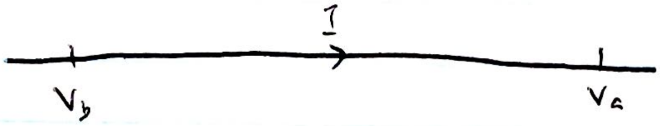
Soln to Diode exercise $\quad \frac{q}{n} = \hat{q}$

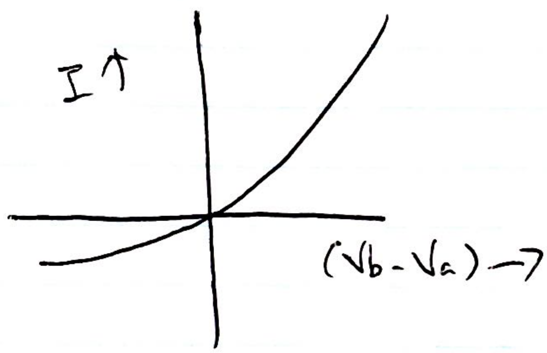
positron: electron with positive charge

(1) $\quad I = I_0 \left(\exp\left(\frac{\hat{q}(V_b - V_a)}{kT} \right) - 1 \right)$



If I is positron current, $V_b > V_a \Rightarrow I > 0$

I_0, $\hat{q}$, kT are some parameter.



$q = 1.602 \times 10^{-19}$ electron charge C

T = temp (kelvin)

k = Bolzman constant 1.380×10^{-23} (J/K)

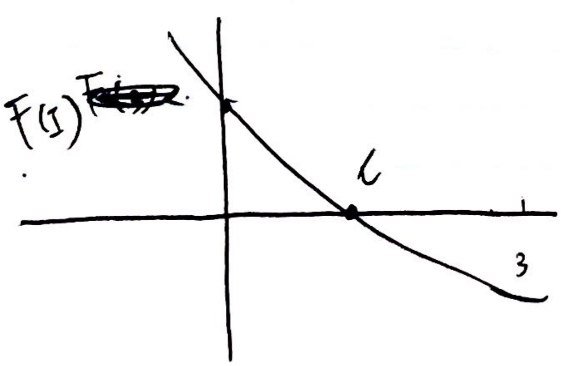
By (1)

(2) $\quad (V_b - V_a) = \frac{kT}{\hat{q}} \ln\left(\frac{I}{I_0} + 1 \right)$

(3) $\quad V_1 - (5 \times 3) - \frac{kT}{\hat{q}} \ln\left(\frac{I_B - 3}{I_0} + 1 \right) = 0 \quad$ for circuit A

(4) $\quad F(I_B) \equiv -5 I_B - 10 I_B - \frac{kT}{\hat{q}} \ln\left(\frac{I_B - 3}{I_0} + 1 \right) = 0 \quad$ for circuit B

Solve (4) for I_B



$F(3) < 0$, $F(0) > 0$, $F'(I_B) < 0$

So $\exists!\ I_B > 0$ lying on $(0, 3)$ with $F(I_B) = 0$

$F'' > 0$ on $(0, \infty)$ means Newton's method solve this quickly

Once I_B is found. we can find V_1, V_2, V_3

V_1 from (3)

$$V_1 - V_2 = 3 \times 5 \implies V_2 = V_1 - 15$$

and

$$V_2 - V_3 = 5 I_B \implies V_3 = V_2 - 5 I_B$$

$$= V_1 - 15 - 15 I_B$$

By KCL. $\sum I_i = 0$. @ each node

(5) node 1 $\quad -3 + \frac{V_1 - V_2}{5} = 0$

(6) node 2 $\quad \frac{V_2 - V_1}{5} + \frac{V_2 - V_3}{5} + I_0\left(\exp\left(\frac{qV_2}{kT}\right) - 1\right) = 0$

(7) node 3 $\quad \frac{V_3 - V_2}{5} + \frac{V_3 - V_0}{10} = 0$

8 node 0 $\quad 3 - I_0\left(\exp\left(\frac{qV_2}{kT}\right) - 1\right) - \frac{V_3}{10} = 0$

but (5), (6) (7) $\implies$ (8) so drop (8)

~~Elim~~ Eliminate V_1, V_3 and get eqn for V_2

$F'(V_2) > 0$

$F''(V_2) > 0$

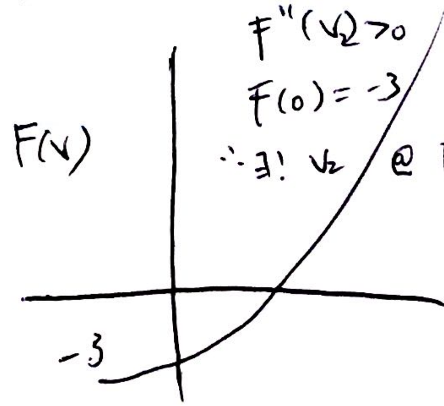
$F(0) = -3$

$\therefore \exists!\ V_2$ @ $F(V_2) = 0$

(7) $\implies V_3 = \frac{2}{3} V_2$ (9)

(1) $\implies V_1 = 15 + V_2$ (10)

(6) $\implies \underbrace{-3 + \frac{V_2}{3} + I_0\left(\exp\left(\frac{qV_2}{kT}\right) - 1\right)}_{F(V_2) \equiv} = 0$



If $\exp(x) \approx 1 + x + \frac{x^2}{2}$ we can get approximation by solving a quadratic

then V_1, V_3 for (9) & (10) and $I_B = \frac{V_2 - V_3}{5}$

Q.E.D.