

# Numerical Modelling of District Heating and Cooling network

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# District Heating and Cooling Networks (DHN)

- The most common method for heating building in cities
- Usually consist of supply and return pipes that deliver heat in form of hot water or steam

# Common graph problems

- Shortest Path Problem : Breadth First Search (BFS), Dijkstra's, Bellman-Ford, Floyd-Warshall
- **Connectivity/Path finding: A\* algorithm**, BFS, DFS
- Negative Cycle : Bellman-Ford, Floyd-Warshall
- Traveling Salesman Problem:
- Bridges: These are edges in a graph whose removal could increase the number of connected components in the graph. They are important because they represent the vulnerabilities and bottlenecks within the graph.
- Minimum Spanning Tree: Kruskal's and Prim's algorithm
- Maximum Network Flow: Ford-Fulkerson and Edmonds Karp & Dinic's algorithms

# Our graph problems

**Problem 1:** : Suppose there are 1000 building in a town, can we implement an algorithm that connects all the thousand building?  
Implement a A\* algorithm.

# What type of graph problems do we want to study?

Ask yourself:

- Is the graph directed or undirected?
- Are the edges of the graph weighted?
- Is the graph we will counter likely to be sparse or dense with edges?
- Should I use an adjacency matrix (or incidence matrix), adjacency list, an edge list or other structure to represent the graph efficiently?

# Nonlinear system and root finding

- A system of nonlinear equation is a set of equations

$$f_1(x_1, x_2, \cdot, x_n) = 0,$$

$$f_2(x_1, x_2, \cdot, x_n) = 0,$$

$$\vdots$$

$$f_n(x_1, x_2, \cdot, x_n) = 0.$$

- There are three type of nonlinear system in hydraulic model. The solutions are usually found via **Newton-Raphson** or Hardy Cross Method.
- An example of nonlinear system from the DHN in Scharnhauser Park (Hassine and Eicker 2011)

$$x^2 - 2x - y + 0.5 = 0$$

$$x^2 + 4 * y^2 - 4 = 0$$

- To find the solution (root), we would use the C++ Linear algebra library **Eigen**.

```
#include <iostream>
#include </usr/include/eigen3/Eigen/Dense>
#include <cmath>
#include <vector>

void newton2d()
{
    // F
    auto F = [] (const Eigen::Vector2d &x){
        Eigen::Vector2d res;
        res << pow(x(0),2) -2*x(0)-x(1)+0.5, pow(x(0),2) +4*pow(x(1),2)-x(1)-4;
        return res;
    };

    // jacobian of F
    auto DF= [] (const Eigen::Vector2d &x){
        Eigen::Matrix2d J;
        J << 2*x(0)-2, -1,
            2*x(0), 8*x(1);
        return J;
    };

    Eigen::Vector2d x, x_ast, s;
    x << 2, 0.25; // initial value
    x_ast << 1.9007, 0.3112; // solution
    double tol=1E-10;

    std::vector<double> errors;
    errors.push_back((x-x_ast).norm());

    do
    {
        s = DF(x).lu().solve(F(x));
        x = x-s; // newton iteration
        errors.push_back((x-x_ast).norm());
    }
    while (s.norm() > tol*x.norm());

    // create eigen vector from std::vector
    unsigned int n = errors.size();
    Eigen::Map<Eigen::VectorId> err(errors.data(), n);

    std::cout << "solution" << std::endl;
    std::cout << x << std::endl;

    std::cout << "Errors:" << std::endl;
    std::cout << err << std::endl;

#include "newton2d.hpp"

int main()
{
    newton2d();
}

//The solution is found as 1.94 and 0.39.
```



# Background

Please refer to LC Evans control theory course

<https://math.berkeley.edu/~evans/control.course.pdf>

# Problem

Control problem: (1) how do you adjust the flow rate in individual substations to that the return temperature has a given value (for example always  $40^{\circ}\text{C}$ )?

# Assumption

- The basic assumption for the calculation is incompressible media.
- Computation of flow distributions in DHN are mainly based on the Kirchhoff law for **current** and **voltage** in circuits: The two equations describe **flow rate** and **pressure losses** in the network
- Consider the PDE describing an one dimensional flow through a horizontal pipe which can be systematically derived from the Navier-Stokes equations.

$$\frac{l}{A} \frac{d\dot{m}}{dt} + \Delta p + R|\dot{m}|\dot{m} = 0 \quad (1)$$

Note:  $\Delta p$  denoting the difference in pressure head between the two pipes ends and  $\dot{m}$  is the mass flow rate. The variable  $R$  stands for the hydraulic resistance of the pipe element, which is postulated to be a function of the physical properties such as length, roughness and diameter.

# Network Topology

The description of the heating network is based on graph theory. First we need to form the topological matrix which show the structure of the mesh in matrix form. (To be added)

# Solution of network equation system

# Numerical solution via finite elements

By the first law of thermodynamics, the variation of the enthalpy of one pipe element is modelled by the following hyperbolic PDE:

$$\frac{\partial(mf)}{\partial t} = \dot{H} - \dot{H}(x + \Delta x) - d\dot{Q}_l \quad (2)$$

# Model Validation

# Network structure

to be added



# Calculations

to be added