Numerical Modelling of District Heating and Cooling network

Leanne Dong

May 12, 2020

Overview

1. Fundamentals
   - General concepts
   - Graph theory
   - Solution of linear systems
   - Solution of nonlinear systems

2. Hydraulic Model
   - Numerical Solution

3. Thermal model
   - Thermal Calculation

4. Hydraulic-Thermal Model
   - Calculation

Fundamentals
Hydraulic Model
Thermal model
Hydraulic-Thermal Model

General concepts
Graph theory
Solution of nonlinear systems

District Heating and Cooling Networks (DHN)

- The most common method for heating building in cities
- Usually consist of supply and return pipes that deliver heat in form of hot water or stream

Fundamentals
Hydraulic Model
Thermal model
Hydraulic-Thermal Model

General concepts
Graph theory
Solution of nonlinear systems

Common problems

- Shortest Path Problem : Breadth First Search (BFS), Dijkstra's, Bellman-Ford, Floyd-Warshall
- Connectivity: BFS, DFS
- Negative Cycle : Bellman-Ford, Floyd-Warshall
- Traveling Salesman Problem:
- Bridges: These are edges in a graph whose removal could increase the number of connected components in the graph. They are important because they represent the vulnerabilities and bottlenecks with in the graph.
- Minimum Spanning Tree: Kruskal's and Prim's algorithm
- Maximum Network Flow: Ford-Fulkerson and Edmonds Karp& Dinic's algorithms

Fundamentals
Hydraulic Model
Thermal model
Hydraulic-Thermal Model

General concepts
Graph theory
Solution of nonlinear systems

What type of graph problems do we want to study?

Ask yourself:

- Is the graph directed or undirected?
- Are the edges of the graph weighted?
- Is the graph we will counter likely to be sparse or dense with edges?
- Should I use an adjacency matrix (or incidence matrix), adjacency list, an edge list or other structure to represent the graph efficiently?

Fundamentals
Hydraulic Model
Thermal model
Hydraulic-Thermal Model

General concepts
Graph theory
Solution of nonlinear systems

Nonlinear system and root finding

- A system of nonlinear equation is a set of equations

$$f_1(x_1, x_2, \cdot, x_n) = 0,$$
$$f_2(x_1, x_2, \cdot, x_n) = 0,$$
$$\vdots$$
$$f_n(x_1, x_2, \cdot, x_n) = 0.$$

- There are three type of nonlinear system in hydraulic model. The solutions are usually found via **Newton-Raphson** or Hardy Cross Method.
- An example of nonlinear system from the DHN in Scharnhauser Park (Hassine and Eicker 2011)

$$x^2 - 2x - y + 0.5 = 0$$
$$x^2 + 4 * y^2 - 4 = 0$$

- To find the solution (root), we would use the C++ Linear algebra library Eigen.

Fundamentals
Hydraulic Model
Thermal model
Hydraulic-Thermal Model

General concepts
Graph theory
Solution of nonlinear systems

```cpp
#include <iostream>
#include </usr/include/eigen3/Eigen/Dense>
#include <cmath>
#include <vector>

void newton2d()
{
        // F
        auto F = [](const Eigen::Vector2d &x){
                Eigen::Vector2d res;
                res << pow(x(0),2) -2*x(0)-x(1)+0.5, pow(x(0),2) +4*pow(x(1),2)-x(1)-4;
                return res;
        };

        // jacobian of F
        auto DF= [] (const Eigen::Vector2d &x){
                Eigen::Matrix2d J;
                J << 2*x(0)-2    , -1,
                        2*x(0)            , 8*x(1);
                return J;
        };

        Eigen::Vector2d x, x_ast, s;
        x << 2, 0.25; // initial value
        x_ast << 1.9007, 0.3112; // solution
        double tol=1E-10;

        std::vector<double> errors;
        errors.push_back((x-x_ast).norm());

        do
        {
                s = DF(x).lu().solve(F(x));
                x = x-s; // newton iteration
                errors.push_back((x-x_ast).norm());
        }
        while (s.norm() > tol*x.norm());

        // create eigen vector from std::vector
        unsigned int n = errors.size();
        Eigen::Map<Eigen::VectorXd> err(errors.data(), n);

        std::cout << "solution" << std::endl;
        std::cout << x << std::endl;

        std::cout << "Errors:" << std::endl;
        std::cout << err << std::endl;
```

```cpp
#include "newton2d.hpp"

int main()
{
        newton2d();
}
//The solution is found as 1.94 and 0.39.
```

Assumption

- The basic assumption for the calculation is incompressible media.
- Computation of flow distributions in DHN are mainly based on the Kirchhoff law for current and voltage in circuits: The two equations describe flow rate and pressure losses in the network
- Consider the PDE describing an one dimensional flow through a horizontal pipe which can be systematically derived from the Navier-Stokes equations.

$$\frac{l}{A}\frac{d\dot{m}}{dt} + \Delta p + R|\dot{m}|\dot{m} = 0 \tag{1}$$

Note: Δp denoting the difference in pressure head between the two pipes ends and $\dot{m}$ is the mass flow rate. The variable R stands for the hydraulic resistance of the pipe element, which is postulated to be a function of the physical properties such as length, roughness and diameter.

Network Topology

The description of the heating network is based on graph theory.
First we need to form the topological matrice which show the
structure of the mesh in matrix form. (To be added)

Solution of network equation system

Numerical solution via finite elements

By the first law of thermodynamics, the variation of the enthalpy of one pipe element is modelled by the following hyperbolic PDE:

$$\frac{\partial(mf)}{\partial t} = \dot{H} - \dot{H}(x + \Delta x) - d\dot{Q}_l \qquad (2)$$

Model Validation

Network structure

to be added

Calculations

to be added