PRACTICE PROBLEMS 18

Topic: Signaling and adverse selection

VERY IMPORTANT: do **not** look at the answers until you have made a VERY serious effort to solve the problem. If you turn to the answers to get clues or help, you are wasting a chance to test how well you are prepared for the exams. I will **not** give you more practice problems later on.

1. Let the quality of a car be denoted by $\theta \in \{2000, 3000, 4000, 5000\}$. The proportion of cars of quality θ is given as follows:

QUALITY	$\theta = 2{,}000$	$\theta = 3{,}000$	$\theta = 4{,}000$	$\theta = 5{,}000$
PROPORTION	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{2}{8}$	$\frac{2}{8}$

There are 200 cars in total. The utility a seller who sells a car of quality θ at price P is $(P - \theta)$.

Fill in the following table.

If price is	Number of cars offered for sale	Average quality of cars offered for sale
\$2500		
\$3100		
\$4600		
\$5250		
\$6100		

2. Consider the following variation on the lemons model. Let the quality of a car be denoted by θ. θ is uniformly distributed in the interval [2000,6000]. Sellers value their cars at exactly θ, while the buyer has a valuation which is 20% greater: the buyer's utility if she buys a car of quality θ is $U_{buyer} = (1.2)\theta - P$, while the utility of the seller who sells his car of quality θ is $U_{seller} = P - \theta$, where P is the price.

Is there a market for second-hand cars? Is it still true that bad cars drive out good cars?

3. Consider the following modification of Spence's model of signaling in the job market. Education **does** increase productivity. There are two groups in the population.

People in Group I have a productivity of $\left(1 + \frac{y}{4}\right)$ (where y is the amount of education) and the cost of acquiring y units of education is \$y.

People in Group II have a productivity of $\left(2 + \frac{y}{4}\right)$ and the cost of acquiring y units of education is $\$ \frac{y}{2}$.

Find all the signaling equilibria, when the employer's beliefs are as follows:

<< if a person has $y<y^o$, then he/she comes from Group I, while if a person has $y \geq y^o$, then he/she comes from Group II >>

Assume that the employer offers a wage which is equal to his estimate of the productivity of the applicant.

4. An individual owns a house worth \$90,000. This is his only wealth. In the area where she lives there is a 10% chance of a flood that would severely damage the house and lead to a loss equal to \$50,000. The individual's von Neumann-Morgenstern utility of wealth function is $U(w) = \sqrt{w}$.

(a) Measure wealth if there is a flood (W_1) on the horizontal axis and wealth if there is no flood (W_2) on the vertical axis. Describe the following insurance contracts in terms of premium *h* and deductible *D*: $C_1 = (W_1 = 65{,}000,\ W_2 = 80{,}000)$, $C_2 = (W_1 = 70{,}000,\ W_2 = 78{,}000)$.

(b) If the above two contracts were the only contracts offered by insurance companies, which (if any) would the individual choose?

(c) What is the maximum premium that the individual is willing to pay for full insurance?

(d) Draw the fair odds line (give the vertical and horizontal intercepts).

(e) What contract would the individual choose among the ones on the fair odds line? Describe it in terms of premium *h* and deductible *D*.

Now suppose that there are two types of individuals, L and H. They have the same initial wealth, the same utility function and the same potential loss. However, they differ in the probability of loss. One type (H) live in an area where the probability is 10%, while the other type (L) live in an area where the probability is 2.5%.

(f) Suppose that the insurance companies can tell the individuals apart (e.g. by the area in which they live). Define an equilibrium as a set of contracts such that: (1) every contract yields zero expected profits, (2) every individual chooses the best among the available contracts, and (3) there is no contract that if added to the existing contracts would enable the company that offers it to make positive expected profits.

(g) Suppose now that the insurance companies cannot tell individuals apart. Find the *candidate* for a separating equilibrium (that is, a pair of contracts such that profits from each contract are zero, every individual chooses the best among the available contracts, all the H types purchase one contract while all the L types purchase the other contract). Let α be the proportion of type H in the population. If we add the requirement that there is no contract that if added to the existing contracts would enable the company that offers it to make positive expected profits, what other condition is necessary and sufficient for the above candidate for a separating equilibrium to be an actual equilibrium? Express this condition as an inequality.