Problem Set #2

1. John's utility function is $U(c) = \ln(c)$. He purchases a risky asset that pays \$404 with probability 0.6 and \$1097 with probability 0.4. What is the risk premium associated with this asset?

2. Robinson Crusoe and Chuck Noland consume coconuts and volleyballs. Robinson has an initial endowment of 6 coconuts and 3 volleyballs. Chuck has an endowment of 8 coconuts and 1 volleyball. Robinson's utility function is $U_r(c, v) = \ln(c_r) + \ln(v_r)$. Chuck's utility function is $U_h(c, v) = \ln(c_h) + 2\ln(v_h)$.
    a. Draw an Edgeworth box with Robinson's consumption measured from the lower left and Chuck's consumption measured from the upper right. Place coconuts on the x-axis and volleyballs on the y-axis. Label the endowment point.
    b. Use Robinson and Chuck's utility functions to express the condition that maps out the contract curve in terms of c_r, c_h, v_r and v_h. (Hint: marginal utilities)
    c. Now, use your knowledge about the endowments to express the contract curve only as a function of c_r and v_r .

3. Write down the conditions that define a competitive equilibrium between Robinson and Chuck. There should be five or six conditions depending on how you set them up.
    a. Use the conditions characterizing Robinson's marginal utility and his budget constraint to solve for v_r as a function of p_c, p_v.
    b. Use the conditions characterizing Chuck's marginal utility and his budget constraint to solve for v_h as a function of p_c, p_v.

4. Using the same framework from Question 3,
    a. Let p_v be the numeraire good (set $p_v = 1$).Use your answers from (a) and (b) along with the last two conditions to solve for p_c.
    b. Use p_c to solve for c_r, c_h, v_r and v_h.

5. Linus' utility function is $U_L(a_L, b_L) = a_L + b_L$ while Lucy's utility function is $U_c(a_c, b_c) = a_c b_c$. Linus has an initial endowment of 6 units of a and 1 units of b while Lucy has an initial endowment of 3 units of a and 5 units of b.
    a. Draw the Edgeworth box related to these agents with Linus' consumption plotted from the lower left and good a on the vertical axis. Label the endowment point.
    b. What type of preferences does each agent have? (Hint: how does Linus trade off between a and b? Lucy's preferences are named after two famous economists.)
    c. Derive the marginal rates of substitution for Linus and Lucy. What is peculiar about Linus' MRS? Now, write out the condition describing all Pareto optimal allocations.
    d. If we allow the price of a to be 1, then in a competitive equilibrium, what must the price of b be?
    e. At these equilibrium prices, calculate Linus and Lucy's consumption of a and b.

Answers

1. $EU = 0.6[\ln(404)] + 0.4[\ln(1097)] = 0.6(6) + 0.4(7) = 6.4\,utils$

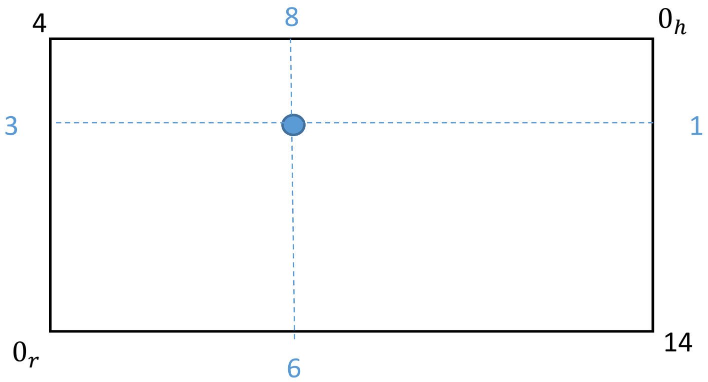
   The $ amount that would make John indifferent to the lottery is a sure payment of: $\exp(6.4) = \$601.84$.

   $EM = 0.6(404) + 0.4(1097) = \$681.2$.

   The risk premium associated with this asset is $681.2 - 601.84 = \$79.36$

2.

   a.



   b. $\frac{MU_{c_r}}{MU_{v_r}} = \frac{MU_{c_h}}{MU_{v_h}}$

   $$\frac{v_r}{c_r} = \frac{v_h}{2c_h}$$

   c. $c_r + c_h = 14$

   $v_r + v_h = 4$

   $c_h = 14 - c_r$

   $v_h = 4 - v_r$

   $$\frac{v_r}{c_r} = \frac{v_h}{2c_h}$$

   $$\frac{v_r}{c_r} = \frac{4 - c_r}{2(14 - c_r)}$$

3.

$$\frac{MU_{c_r}}{MU_{v_r}} = \frac{p_c}{p_v}$$

$$p_c c_r + p_v v_r = 6p_c + 3p_v$$

$$\frac{MU_{c_h}}{MU_{v_h}} = \frac{p_c}{p_v}$$

$$p_c c_h + p_v v_h = 8p_c + p_v$$

$$c_r + c_h = 14$$

$$v_r + v_h = 4$$

a. $\frac{v_r}{c_r} = \frac{p_c}{p_v}$

$c_r = \frac{p_v}{p_c} v_r$ (eq. 1)

$$p_c c_r + p_v v_r = 6p_c + 3p_v$$

Substituting (eq. 1) for c_r

$$p_c \frac{p_v}{p_c} v_r + p_v v_r = 6p_c + 3p_v$$

$$2p_v v_r = 6p_c + 3p_v$$

$$v_r = \frac{6p_c + 3p_v}{2p_v}$$

b. $\frac{v_h}{2c_h} = \frac{p_c}{p_v}$

$c_h = \frac{p_v}{2p_c} v_h$ (eq. 2)

$p_c c_h + p_v v_h = 8p_c + p_v$

Substituting (eq. 2) for c_h

$$p_c \frac{p_v}{2p_c} v_h + p_v v_h = 8p_c + p_v$$

$$\frac{p_v}{2} v_h + p_v v_h = 8p_c + p_v$$

$$\frac{3}{2} p_v v_h = 8p_c + p_v$$

$$v_h = \frac{2(8p_c + p_v)}{3p_v}$$

4.

a.

$$v_r = \frac{6p_c + 3p_v}{2p_v} = \frac{6p_c + 3}{2}$$

$$v_h = \frac{2(8p_c + p_v)}{3p_v} = \frac{16p_c + 2}{3}$$

We know that, $v_r + v_h = 4$.

Therefore,

$\frac{6p_c+3}{2} + \frac{16p_c+2}{3} = 4$

$18p_c + 9 + 32p_c + 4 = 24$

$50p_c = 11$

$p_c = \frac{11}{50}$

b.

$$v_r = \frac{6p_c + 3}{2} = \frac{6\left(\frac{11}{50}\right) + 3}{2} = \frac{\frac{66}{50} + \frac{150}{50}}{2} = \frac{216}{100} = 2.16$$

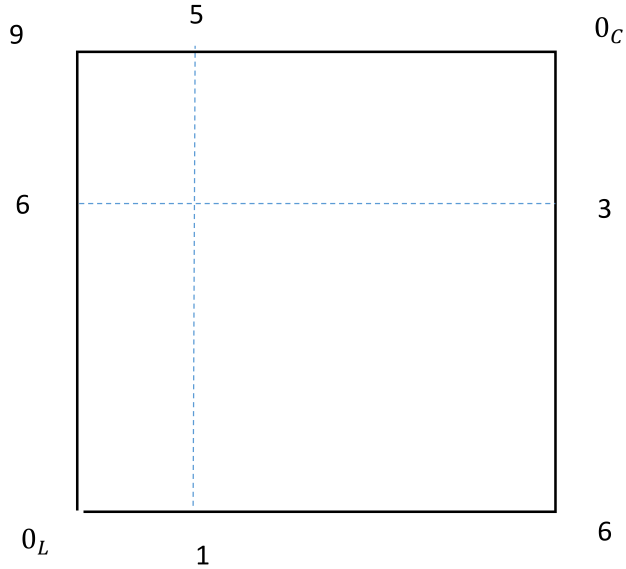
$$v_h = \frac{16p_c + 2}{3} = \frac{16\left(\frac{11}{50}\right) + 2}{3} = \frac{\frac{176}{50} + \frac{100}{50}}{3} = \frac{276}{150} = 1.84$$

$$c_r = \frac{p_v}{p_c} v_r = \frac{50}{11} \times \frac{216}{100} = 9.818$$

$$c_h = \frac{p_v}{2p_c} v_h = \frac{50}{22} \times \frac{276}{150} = 4.18$$

5.

a.



b. Linus: perfect substitutes
Lucy: Cobb-Douglas

c. Linus: 1
Lucy: $\frac{a_c}{b_c}$
Linus' MRS is constant at 1 and independent of the amount of a and b consumed.

$$\frac{a_c}{b_c} = 1$$

d. $\frac{a_c}{b_c} = 1$

$p_a a_L + p_b b_L = 6p_a + p_b$

$p_a a_c + p_b b_c = 3p_a + 5p_b$
$a_L + a_c = 9$

$b_L + b_c = 6$

e. $p_b = 1$

f. $\frac{a_c}{b_c} = 1$. Therefore, $a_c = b_c$. Lucy's budget constraint is $a_c + b_c = 8$. So $a_c = b_c = 4$.

This means $a_L = 9 - 4 = 5$ and $b_L = 6 - 4 = 2$.