Signaling

- Adverse selection is an outcome of an informational deficiency.
- We saw that high quality sellers (e.g. Peach sellers) lose out when adverse selection is present in the market.
- What if information can be improved by high-quality sellers signaling credibly that they are high-quality?
- E.g.
    - warranties for cars, smartphones, laptops.
    - professional credentials for doctors, architects.

Signaling

- A labor market has two types of workers; high-ability and low-ability.
- A high-ability worker's marginal product is a_H.
- A low-ability worker's marginal product is a_L.

$$a_L < a_H$$

Signaling

- A fraction h of all workers are high-ability.
- $1 - h$ is the fraction of low-ability workers.
- Each worker is paid his expected marginal product (the labor market is competitive).
- If firms knew each worker's type they would
    - pay each high-ability worker $w_H = a_H$
    - pay each low-ability worker $w_L = a_L$.

Signaling

- If firms cannot tell workers types then every worker is paid the same wage rate; i.e. the expected marginal product

$$w_P = (1-h)a_L + ha_H$$

- Since $w_P = (1-h)a_L + ha_H < a_H$, which is the wage rate paid when the firm knows a worker really is high-ability.
- So high-ability workers have an incentive to find a credible signal.

Signaling

- Workers can acquire "education".
- Education costs a high-ability worker c_H per unit
- and costs a low-ability worker c_L per unit.

$$c_L > c_H$$

- Suppose that education has no effect on workers' productivities; i.e., the cost of education is a deadweight loss.

Signaling

- If high-ability workers obtain more “education”, than low ability workers then firms can use “education” as a signal to pay high ability workers a higher salary and low ability workers a low salary.
- That is anyone with a sufficient level of education is paid w_H and everyone with insufficient education is paid w_L.
- We call this outcome a separating equilibrium.
- Firms can separate the high-ability workers from the low-ability workers.

Signaling

- The equilibrium where both types of workers have the same level of education is called a pooling equilibrium.
- Firms cannot distinguish between worker types, thus “pool” all types of workers together and pay them the same wage (w_P).

Signaling

- High-ability workers will acquire e_{H} education units if
  (i) $w_H - w_L = a_H - a_L > c_H e_H$, and
  (ii) $w_H - w_L = a_H - a_L < c_L e_H$
  (iii) $w_H - c_H e_H > (1-h)a_L + h a_H$

- Let's look more carefully into what each equation means…

Signaling

- High-ability workers will acquire e_{H} education units if
  (i) $w_H - w_L = a_H - a_L > c_H e_H$

- If they obtain e_H units of education at cost c_H per unit, in a separating equilibrium, they will enjoy a wage differential over low-ability workers that is greater than the cost of education.
- This says that the high-ability workers see education as worth while.

Signaling

- High-ability workers will acquire e_{H} education units if
  (ii) $w_H - w_L = a_H - a_L < c_L e_H$

- Low-ability workers obtain e_H units of education at cost c_L per unit. The wage differential they will enjoy from getting w_H instead of w_L in a separating equilibrium will be smaller than the cost of education.
- This says acquiring e_{H} education units is not beneficial to low-ability workers.

Signaling

- High-ability workers will acquire e_{H} education units if
(iii)$w_H - c_H e_H > (1-h)a_L + h a_H$

- This equation says that the wage to high-ability workers after obtaining education is better than the average wage they receive if they don't obtain an education.

Signaling

- Suppose (iii) holds, that is education is cheap enough that high-ability workers would prefer having it.
- High-ability workers will acquire e_{H} education units if
  (i) $w_H - w_L = a_H - a_L > c_H e_H$, and
  (ii) $w_H - w_L = a_H - a_L < c_L e_H$.
- This implies:

$$\frac{a_H - a_L}{c_L} < e_H < \frac{a_H - a_L}{c_H}$$

Signaling

- What if these equations don’t hold?

$$\text{(iii)} w_H - c_H e_H > (1-h)a_L + h a_H$$

- If this equation doesn’t hold, there is no incentive for high-ability workers to get an education. so firms cannot tell between high- and low-ability workers and we remain in the pooling equilibrium.

Signaling

- What if these equations don't hold?

$$\text{(ii) } w_H - w_L = a_H - a_L < c_L e_H$$

- If this equation doesn't hold, then a separating equilibrium cannot be sustained.
- Since $a_H - a_L > c_L e_H$, low-ability workers have incentive to get the same level of education as high ability workers.
- Therefore if the market was in a separating equilibrium, low-ability workers would get the same education as high-ability workers.
- Now we cannot tell between low- and high-ability workers and the separating equilibrium fails (i.e. education is no longer a signal)

Signaling

- What if these equations don't hold?

  (i) $w_H - w_L = a_H - a_L > c_H e_H$

- If this equation doesn't hold, then a separating equilibrium cannot be sustained
- If $a_H - a_L < cHeH$, high-ability workers have no incentive to get an education, because in a separating equilibrium they don't earn enough to justify the cost of education.
- Now we cannot tell between low and high ability workers (i.e. education is not a signal).

Signaling

- Side Note: (i) and (ii) cannot be violated at the same time because $c_H < c_L$.
- That is if $a_H - a_L < c_H e_H$ then $a_H - a_L < c_L e_H$

Signaling

- Q: Given that high-ability workers acquire e_H units of education, how much education should low-ability workers acquire?
- A: Zero. Low-ability workers will be paid $w_L = a_L$ so long as they do not have e_H units of education and they are still worse off if they do.

Signaling

- Signaling can improve information in the market.
- But, total output does not change and education is costly so signaling worsens market's efficiency.
- So improved information need not improve gains-to-trade.
- Nevertheless, high-ability workers are better off and low-ability workers are worse off.
    - High ability workers get paid a_H
    - Low ability workers get paid a_L

Today's plan

- Definition and Concepts
- Adverse Selection
    - The Market for Lemons
    - Adverse Selection with Quality Choice
    - Signaling
- Moral Hazard
  - Incentive Contracts

Moral Hazard

- Adverse Selection arises because agents cannot determine other agents' <u>type</u>.
  - Hidden information
- Moral Hazard arises because agents cannot determine other agents' <u>actions</u>.
  - Hidden action

Moral Hazard

- Consider a health insurance company.
  - Unhealthy people are more willing to buy insurance.
  - This is <u>adverse selection</u>

  - After buying insurance, people have the incentive to live more unhealthily since insurance will cover their medical costs anyway
  - This is <u>moral hazard</u>

Incentive Contracting

- A worker is hired by a principal to do a task.
- Only the worker knows the effort she exerts.
- The effort exerted affects the principal's payoff.

Incentive Contracting

- The principal' s problem: design an incentives contract that induces the worker to exert the amount of effort that maximizes the principal' s payoff.
- First we will assume that there is symmetric information, that is the principal can observe the worker's effort via the output he produces.
- This problem is offered referred to as the "principal-agent problem"

Incentive Contracting

- x is the agent's effort.
- Principal's reward is $y = f(x)$
- An incentive contract is a function $s(y)$ specifying the worker's payment when he produces an amount of output equal to y. The principal's profit is thus

$$\Pi_p = y - s(y) = f(x) - s(f(x))$$

Incentive Contracting

- Let $\tilde{u}$ be the worker's (reservation) utility of not working.
- To get the worker's participation, the contract must offer the worker a utility of at least $\tilde{u}$
- The worker's utility cost of an effort level x is $c(x)$.

Incentive Contracting

- So the principal's problem is

$$\max_x \Pi_p = f(x) - s\big(f(x)\big)$$

$$\text{subject to } s\big(f(x)\big) - c(x) \geq \tilde{u}$$

(participation constraint)

- To maximize his profit the principal designs the contract to provide the worker with only the reservation utility level of utility. That is, ...

Incentive Contracting

- The principal chooses x such that
$$\max_x \Pi_p = f(x) - s\big(f(x)\big)$$
$$\text{subject to } s\big(f(x)\big) - c(x) = \tilde{u}$$
(participation constraint)
- We can substitute the constraint into the optimization function:
$$\max_x \Pi_p = f(x) - c(x) - \tilde{u}$$
- Solving this we have that
$$f'(x) = c'(x)$$

Incentive Contracting

$$f'(x) = c'(x)$$

- The contract that maximizes the principal's profit insists upon the worker effort level x^* that equalizes the worker's marginal effort cost to the principal's marginal payoff from worker effort.
- We will label x such that $f'(x) = c'(x)$ as x^*
- How can the principal induce the worker to choose x = x*?

Incentives Contracting

- $x = x^*$ must be most preferred by the worker.
- So the contract $s(y)$ must satisfy the incentive-compatibility constraint:

$$s(f(x^*)) - c(x^*) \geq s\big(f(x)\big) - c(x), \forall x \geq 0$$

Rental Contracts

- The principal keeps a lump-sum R for himself and the worker gets all profit above R; i.e.

$$s\big(f(x)\big) = f(x) - R$$

- When the worker maximize his payoff:
$\max_{x} s\big(f(x)\big) - c(x) = f(x) - R - c(x)$
- This results in the condition $f'(x) = c'(x)$ which yields the level of effort x^*

Rental Contracts

- How large should be the principal's rental fee R?
- The principal should extract as much rent as possible without causing the worker not to participate, so R should satisfy

$$f(x^*) - c(x^*) - R = \tilde{u}$$
$$R = f(x^*) - c(x^*) - \tilde{u}$$

Wrong in the web lecture

Wage Contracts

- In a wage contract the payment to the worker is
$$s(f(x)) = wx + K$$
where w is the wage per unit of effort. K is a lump-sum payment.
- The worker thus maximizes $wx + K - c(x)$ which yields the condition $w = c'(x)$
- If $w = f'(x)$, then the worker will exert effort x^*
- K is chosen so that the worker is indifferent between participating and not participating:
  - $wx + K - c(x) = \tilde{u}$
  - $K = \tilde{u} - wx + c(x)$

Take it Or Leave it Contracts

- Choose $x = x^*$ and be paid a lump-sum L, or choose $x \neq x^*$and be paid zero.
- The worker's utility from choosing $x \neq x^*$is $-c(x)$, so the worker will choose $x = x^*$.
- L is chosen to make the worker indifferent between participating and not participating.

$$L - c(x) = \tilde{u}$$

Therefore $L = \tilde{u} + c(x)$

Profit Sharing Contracts

- The worker receives a share of the profits:
$$s(x) = \alpha f(x)$$
where $0 < \alpha < 1$
- Maximizing the worker's payoff condition: $\alpha f(x) - c(x)$ yields the condition:
$$\alpha f'(x) = c'(x)$$
- Therefore, $x \neq x^*$

- Principals should never profit-share with their workers.

Incentive Contracting

- In summary,
- A profit maximizing firm must take into consideration these two constraints when hiring a worker:
  - $s\big(f(x)\big) - c(x) = \tilde{u}$ ← Individual Rationality Constraint or Participation Constraint
  - $s(f(x^*) - c(x^*) \geq s\big(f(x)\big) - c(x), \forall x \geq 0$ ← Incentive Compatibility Constraint
- The first ensures that the worker wants to work for you.
- The second ensures that the worker will exert the right amount of effort.

Incentive Contracts with Asymmetric Information

- We've seen that there are various ways for principals to write contracts such that workers exert the level of effort which maximizes the principal's profits under complete information where $y = f(x)$.
- What if the principal cannot perfectly observe the worker's effort level, x?
  - Then we have asymmetric information. The worker knows his effort level but the principal doesn't.
- The principal has to guess the level of effort from the observed output.

Incentive Contracts with Imperfect Information

- Alternatively, there is “noise” or uncertain events that affect the production process that is beyond the control of the worker
  - This is a case of imperfect information. Output and effort are no longer perfectly correlated. i.e neither party knows $f(x)$.

Incentive Contracts with Imperfect Information

- Rent:
  - Since the firm receives a fixed fee, the worker is left to bear all the risk from the noise that comes from production.
  - If the worker is more risk averse than the owner, then the outcome will be inefficient.
- Profit-sharing
  - Since the worker doesn't want to bear all the risk from production noise, he will likely be willing to share some of the profits from production since this offloads some of the risk to the principal.