Chapter 30: Game Theory Applications

Game Theory Applications

- Games of Coordination
- Games of Competition
- Games of Coexistence
- Games of Commitment
- Bargaining Games

Games of Coordination

- These are games where the payoffs to the players are highest when they can coordinate their strategies.

- Examples you have already seen:
  - Prisoner's Dilemma
  - Battle of the Sexes – Going to the movies
  - Assurance Games – Buying a car
  - Chicken – Cars driving at each other

Prisoner's Dilemma

- In this class of games, each player has a dominant strategy.
- This leads to a dominant strategy equilibrium (DSE)
- However, there is some other outcome that would leave all players better off, that is the DSE is not Pareto optimal.

		Joey	
		Confess	Deny
Chandler	Confess	-6 / -6	-1 / -12
	Deny	-12 / -1	-3 / -3

Prisoner’s Dilemma

- The Cournot game is also a Prisoner’s Dilemma type game
- The dominant strategy equilibrium is

  {SQ: cheat; MH: cheat)

		Malaysia Airlines (MH)	
		Collude	Cheat
Singapore Airlines (SQ)	Collude	1012.5 1012.5	1139.06 759.38
	Cheat	759.38 1139.06	900 900

Prisoner's Dilemma

- In these games, how do we get to the better outcome?
    - {Chandler: Deny; Joey: Deny} and {SQ: Collude; MH: Collude}
- Have punishments for choosing the dominant strategy
    - We discussed the Nash Reversion Strategy and the "tit-for-tat" strategy

Battle of the Sexes

- In the examples of these games that we have seen, there are two players and two PSNE. Each player prefers a different PSNE
  - You prefer the “Star Wars” equilibrium
  - Your bae prefers the “Dumb Chick Flick” equilibrium

		Your bae	
		Star Wars	Dumb Chick Flick
You	Star Wars	10 20	-1 -1
	Dumb Chick Flick	-1 -1	20 10

dunno which of the nash equilibrium that will occur

Battle of the Sexes

Ross

Rachel		**B**	**M**
	B	$\boxed{8},\boxed{4}$	**2,2**
	M	**2,2**	$\boxed{4},\boxed{8}$

Ross wants to go to the museum and Rachel wants to go to Bloomingdales.

The two PSNE are: {Rachel: B, Ross: B} and {Rachel: M, Ross: M}

How about MSNE?

Battle of the Sexes

		Ross	
		B	**M**
Rachel	**B**	**8,4**	**2,2**
	M	**2,2**	**4,8**

π_R is the prob. that Rachel chooses B.
π_S is the prob. that Ross chooses B.

$$EV_R(B) = 8\pi_s + 2(1 - \pi_s) = 2 + 6\pi_s$$
$$EV_R(M) = 2\pi_s + 4(1 - \pi_s) = 4 - 2\pi_s$$

Rachel is indifferent if $EV_R(B) = EV_R(M)$, i.e. $\pi_s = 0.25$

Battle of the Sexes

		Ross	
		B	**M**
Rachel	**B**	**8,4**	**2,2**
	M	**2,2**	**4,8**

π_R is the prob. that Rachel chooses B.
π_S is the prob. that Ross chooses B.

$$EV_R(B) = 8\pi_s + 2(1 - \pi_s) = 2 + 6\pi_s$$
$$EV_R(M) = 2\pi_s + 4(1 - \pi_s) = 4 - 2\pi_s$$

Rachel is indifferent if $EV_R(B) = EV_R(M)$, i.e. $\pi_s = 0.25$

Battle of the Sexes

		Ross	
		B	**M**
Rachel	**B**	**8,4**	**2,2**
	M	**2,2**	**4,8**

π_R is the prob. that Rachel chooses B.
π_S is the prob. that Ross chooses B.

$EV_S(B) = 4\pi_R + 2(1 - \pi_R) = 2 + 2\pi_R$
$EV_S(M) = 2\pi_R + 8(1 - \pi_R) = 8 - 6\pi_s$
Ross is indifferent if $EV_S(B) = EV_S(M)$, i.e. $\pi_R = 0.75$

Battle of the Sexes

		Ross	
		B	**M**
Rachel	**B**	**8,4**	**2,2**
	M	**2,2**	**4,8**

π_R is the prob. that Rachel chooses B.
π_S is the prob. that Ross chooses B.

Besides the two PSNE, there is an MSNE of $\pi_R = 0.75, \pi_s = 0.25$

8,4 and 4,8

Assurance Games

- In the Assurance Games we've seen, there are two players and two PSNE. One of the PSNE gives a better payoff to BOTH players.
  - Ross and Phoebe have the best payoff when they both buy a Nano

		Phoebe	
		Hummer	Nano
Ross	Hummer	2 / 2	-1 / 3
	Nano	3 / -1	4 / 4

Assurance Games

- The question in this type of game is, "How can each player give the other player ***assurance*** that the better PSNE will prevail?"
- Let's look at a new Assurance game: an arm's race

		Russia	
		Don't stockpile	Stockpile
Ukraine	Don't stockpile	$\boxed{5}$ $\boxed{5}$	4 1
	Stockpile	1 4	$\boxed{3}$ $\boxed{3}$

Assurance Games

- The Nash equilibria are:

		Russia	
		Don’t stockpile	Stockpile
Ukraine	Don’t stockpile	5, 5	1, 4
	Stockpile	4, 1	3, 3

- Communication might help us get to the better equilibrium
- What if Russia went first?

Chicken

- You will see this game in your problem set this week.

Game Theory Applications

- Games of Coordination
- Games of Competition
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Games of Competition

- These are games that are the pole opposite of cooperation. The payoff to one player is equal to the losses of the other.
- Example: Football Penalty Game

		Keeper	
		Defend Left	Defend Right
Kicker	Kick Left	-50 / 50	-80 / 80
	Kick Right	-90 / 90	-20 / 20

Game Theory Applications

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Games of Coexistence

- These are games used to model animal behavior, for example, how members of a species act towards each other.
- **Hawk-Dove Game**
  - A bear can either hawk (be aggressive) when it and another bear encounters food in the forest, or
  - it can dove (be passive and share)

Hawk-Dove Game

		Bear 2	
		Hawk	**Dove**
Bear 1	**Hawk**	**-5,-5**	**8,0**
	Dove	**0,8**	**4,4**

Is there a NE in mixed strategies?

Are there NE in pure strategies?
Yes {1: Hawk, 2: Dove} and {1: Dove, 2: Hawk}.
Notice that purely peaceful coexistence is not a NE.

Coexistence Games; The Hawk-Dove Game

		Bear 2	
		Hawk	**Dove**
Bear 1	**Hawk**	**-5,-5**	**8,0**
	Dove	**0,8**	**4,4**

π_1 is the prob. that 1 chooses Hawk.
π_2 is the prob. that 2 chooses Hawk.

$$EV_1(H) = -5\pi_2 + 8(1-\pi_2) = 8 - 13\pi_2$$
$$EV_1(D) = 0\pi_2 + 4(1-\pi_2) = 4 - 4\pi_2$$

Bear 1 is indifferent when $EV_1(H) = EV_1(D), i.e. \pi_2 = \frac{4}{9}$

Coexistence Games; The Hawk-Dove Game

		Bear 2	
		Hawk	**Dove**
Bear 1	**Hawk**	**-5,-5**	**8,0**
	Dove	**0,8**	**4,4**

π_1 is the prob. that 1 chooses Hawk.
π_2 is the prob. that 2 chooses Hawk.

$$EV_2(H) = -5\pi_1 + 8(1-\pi_1) = 8 - 13\pi_1$$
$$EV_2(D) = 0\pi_1 + 4(1-\pi_1) = 4 - 4\pi_1$$

Bear 1 is indifferent when $EV_2(H) = EV_2(D), i.e. \pi_1 = \frac{4}{9}$

Hawk-Dove Game

		Bear 2	
		Hawk	**Dove**
Bear 1	**Hawk**	**-5,-5**	**8,0**
	Dove	**0,8**	**4,4**

We have a MSNE when both bears play Hawk with probability $\frac{4}{9}$.

Games of Coexistence

- What is the interpretation of mixed strategies here?
  - It isn't that the two bears that meet repeatedly in the forest and hawk with probability $\frac{4}{9}$.
  - It is that among all the bears, $\frac{4}{9}$ of them will be hawkish.
- Behavior is genetic. Bears have evolved over time to have "hawk-ish" and "dove-ish" behavior in such proportions.

Games of Coexistence

This is a good question

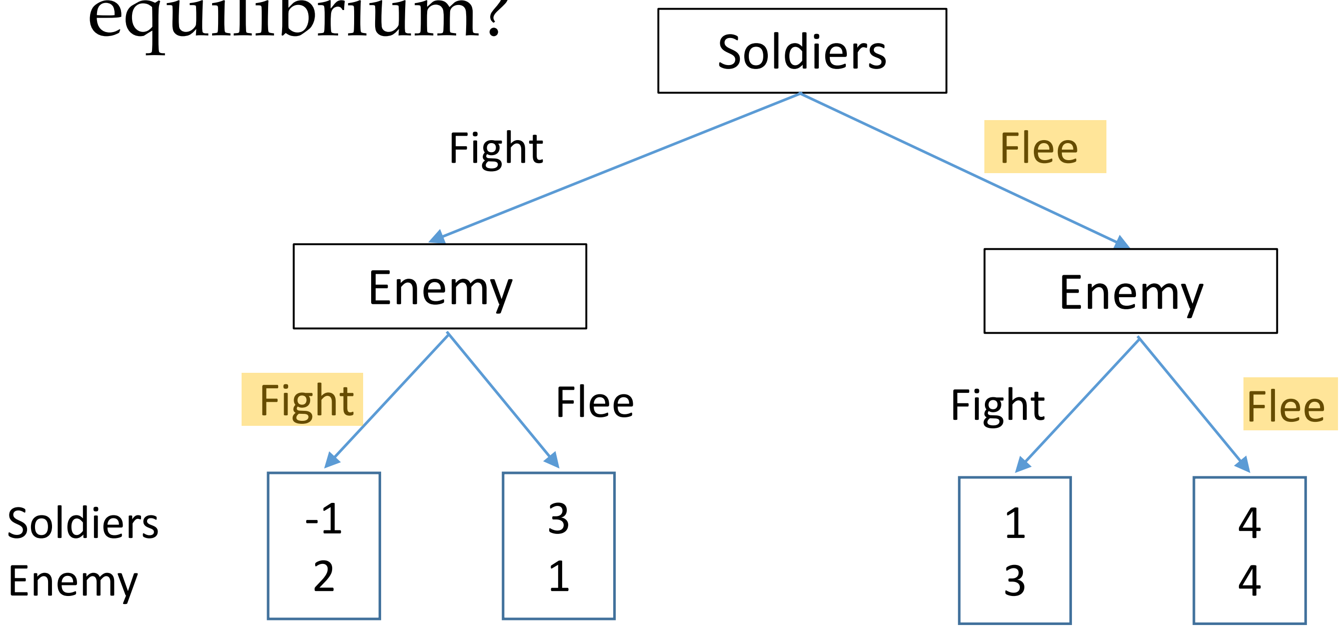
- Let's suppose that there were an equal proportion of hawk-ish and dove-ish bears, i.e. $\pi_1 = \pi_2 = 0.5 > \frac{4}{9}$
- What is the payoff from being hawk-ish and being dove-ish?
  - $EV_1(H) = 8 - 13\pi_2 = 1.5$
  - $EV_1(D) = 4 - 4\pi_2 = 2$
- It's better to be dove-ish, so over time, hawk-ish bears will evolve to become dove-ish until $\pi_1 and\ \pi_2$lowers to $\frac{4}{9}$.

Games of Commitment

- These are sequential games.
- The first player chooses an action which is observed by the second player
- This action is irreversible and observable.
- The first player knows that his action is seen by the second player

General Han Xin goes to war

- Han Xin sent his soldiers to battle. What is the equilibrium?

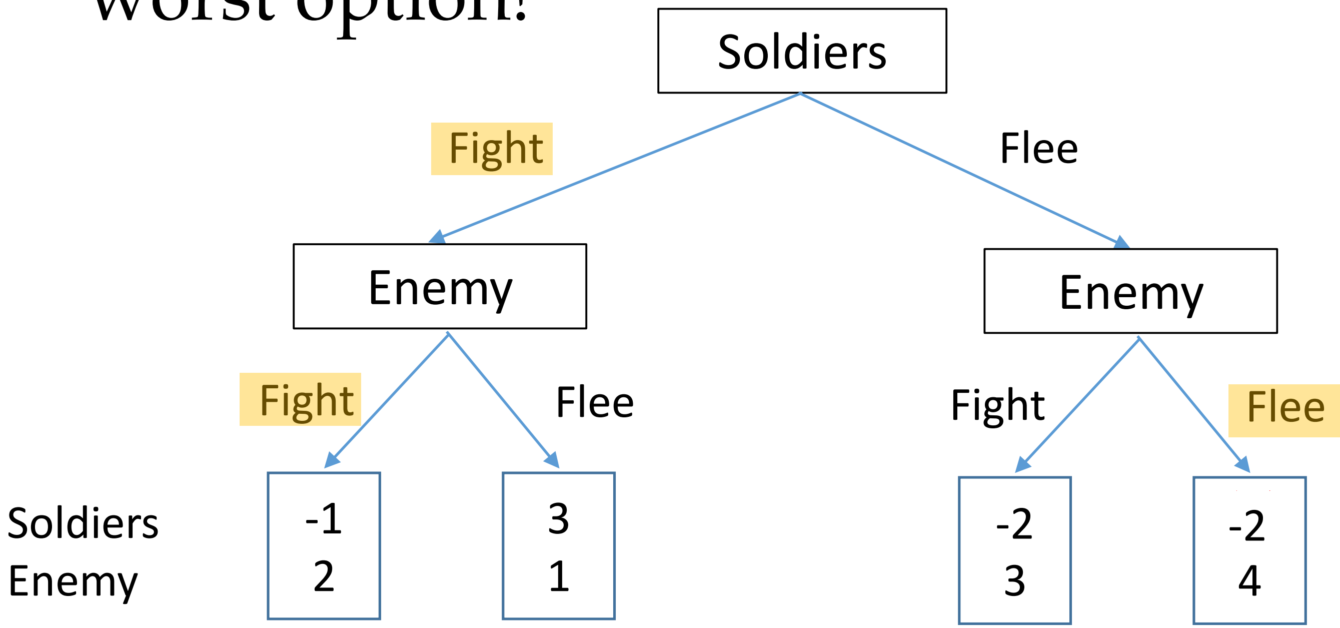


Example: Han Xin goes to war

- General Han Xin knows that he cannot trust his soldiers. They might say they will fight but in reality they will run away.
- This is not the outcome he desires. To get the outcome he wants (his soldiers fighting), he has to create a commitment device.
- Han Xin places his soldiers backed up against a raging river. Now if they flee, they drown!

General Han Xin goes to war

- Drowning kills everyone, so fleeing is now the worst option!



- Han Xin gets the outcome that he wants!

Example: Han Xin goes to war

- The commitment device of placing soldiers by the river changes their incentives, forcing them to obey their General.

- Simple question: In any game, how could you (minimally) change the payoffs to alter the outcome of the game?

Game Theory Applications

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Bargaining Games

- Rubenstein Bargaining Model
  - This model looks at a sequence of choices and then solves for an equilibrium using backward induction.

Bargaining Games

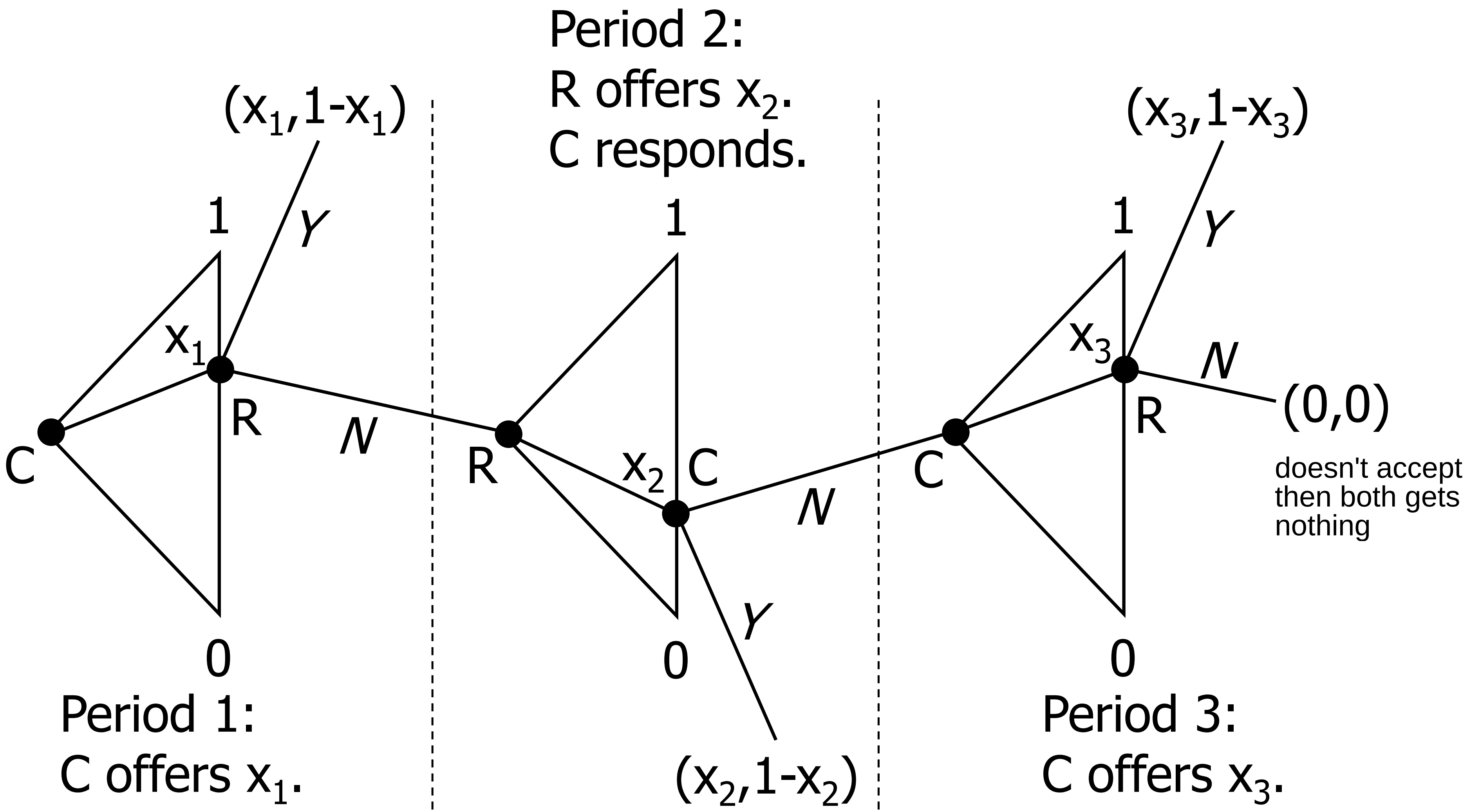
- Example: Cake division
  - Chandler and Rachel have to divide a cake between them.
  - They each alternate making offers, with Chandler making the first offer.
  - If Rachel accepts the offer, the game ends immediately.
  - If Rachel rejects the offer, then it is her turn to make an offer
  - Chandler and Rachel agree that there will be a maximum of k offers.
  - If no agreement is made by round k, neither player gets any cake

Bargaining Games

Bargaining Games

- Let's solve for this game when k=3.
- Chandler's discount factor is $0 < \alpha < 1$
- Rachel's discount factor is $0 < \beta < 1$
  - Discount factor: \$1 in the next period is worth \$$\alpha$ today.
- If a player is indifferent between accepting the offer and not, he/she will choose the option preferred by the opponent.
- The idea is the opponent can give an "arbitrarily small amount" to the other player to make him/her accept the option preferred by the opponent. This arbitrarily small amount is rounded to zero

Strategic Bargaining



Period 1:
C offers x_1.
R responds.

Period 3:
C offers x_3.
R responds.

Bargaining Games

- Backwards induction:
- Start with period 3.
- Chandler makes an offer.
- If Rachel rejects the offer, she gets 0.
- Chandler knows this, so he what will he offer Rachel?
- He can offer her 0 and take 1, and she will accept the offer.

Bargaining Games

- Backwards induction:
- Now in period 2.
- Rachel makes an offer.
- If Chandler rejects the offer, he knows he gets 1 in the next period.
- Rachel knows this, so what will she offer Chandler?
- She must make him indifferent between 1 in the next period and the amount he receives now. The PV of 1 in period 3 to Chandler is α.
- Rachel offers α to Chandler and he will accept.

Bargaining Games

- Backwards induction:
- Now in period 1.
- Chandler makes an offer.
- If Rachel rejects the offer, she knows she gets $1 - \alpha$ in the next period.
- Chandler knows this, so what will he offer Rachel?
- He must make her indifferent between $1 - \alpha$ in the next period and the amount she receives now. The PV of $1 - \alpha$ in period 2 is $\beta(1 - \alpha)$.
- Chandler offers $\beta(1 - \alpha)$ and Rachel accepts.

Bargaining Games

- The game ends in the first period!
- Rachel gets $\beta(1-\alpha)$ of the cake
- Chandler gets $1-\beta(1-\alpha)$ of the cake

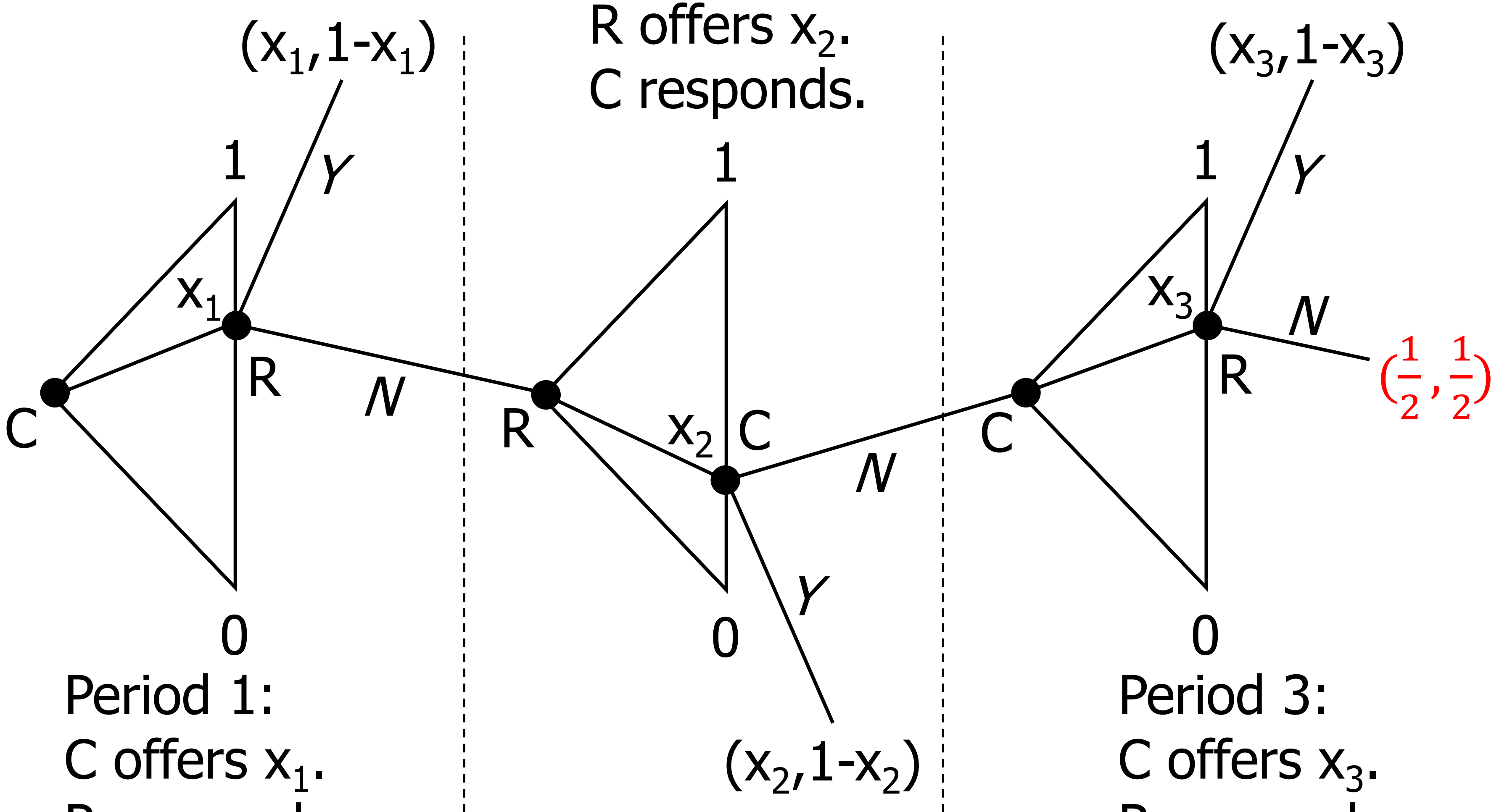
- The more patient you are, the more cake you get.

Bargaining Games II

- Let’s change the game.
- If no agreement is reached by the third round, they each split the pie equally.

Strategic Bargaining

Period 2:
R offers x_2.
C responds.



Period 1:
C offers x_1.
R responds.

Period 3:
C offers x_3.
R responds.

Bargaining Games II

- In period 3:
  - Chandler makes an offer
  - If Rachel rejects the offer, she gets $\frac{1}{2}$.
  - Chandler knows this so he will offer Rachel $\frac{1}{2}$ and he receives $\frac{1}{2}$.

Bargaining Games II

- Now in period 2:
  - Rachel makes an offer
  - If Chandler rejects the offer, he receives $\frac{1}{2}$ in period 3 which he values at $\frac{1}{2}\alpha$ in period 2.
  - Rachel knows this so she will offer Chandler $\frac{1}{2}\alpha$ and Chandler will accept.

Bargaining Games II

- Now in period 1
  - Chandler makes an offer.
  - If Rachel rejects the offer she will receive $1 - \frac{1}{2}\alpha$ which she values at $\beta(1 - \frac{1}{2}\alpha)$ in period 1.
  - Chandler knows this so he will offer Rachel $(1 - \frac{1}{2}\alpha)\beta$ and she will accept
- The game ends in the first period:
  - Chandler getting $1 - \beta\left(1 - \frac{1}{2}\alpha\right)$
  - Rachel getting $\beta\left(1 - \frac{1}{2}\alpha\right)$

Bargaining Games II

- We see that when the failure to reach an agreement leads to a (0,0) allocation:
  - Rachel receives $\beta(1-\alpha)$

- When the failure to reach an agreement leads to a $\left(\frac{1}{2},\frac{1}{2}\right)$ allocation:
  - Rachel receives $\beta(1-\frac{1}{2}\alpha)$

Bargaining Games II

$$\beta\left(1-\frac{1}{2}\alpha\right) > \beta(1-\alpha)$$

- By giving Rachel a better allocation in the event of a failure to reach an agreement, (she receives $\frac{1}{2}$ instead of 0) we increase her bargaining power, giving her a better allocation in the end.
- Even though Chandler also has a better allocation in the event of a failure to reach agreement, he is worse off.
- The allocation in the event of a failure to reach an agreement is only relevant to the person accepting/rejecting the offer in the final period.

Bargaining Games

- What if Rachel and Chandler were allowed to bargain for 4 rounds instead of 3?

Each round the value may change, aka car diminishes over time

What if each round the person makes 2 offers, and then the next round the other person makes 2 offers?