Problem Set #6

1. Consider again the, "Battle of the Sexes," game below, where both players make simultaneous action choices.

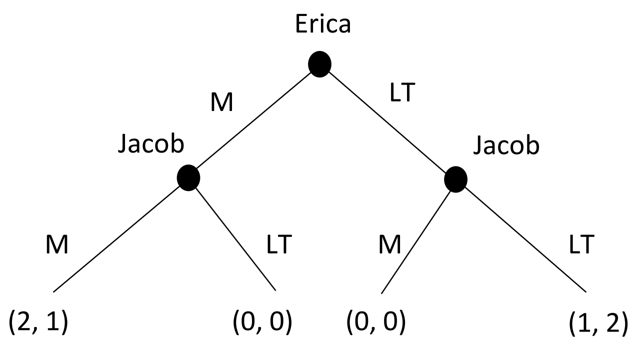
		Jacob	
		Movies	Laser Tag
Erica	Movies	2,1	0,0
	Laser Tag	0,0	1,2

a. What are the pure strategy Nash equilibria of this game?

NE: {(Movies, Movies); (Laser Tag, Laser Tag)}

Now consider the case where Erica gets to move first, e.g. she tells Jacob which entertainment option she is going to attend, and then Jacob responds with his action.

b. In extensive form, draw out the sequential version of this game?



c. Using backward induction, what Nash equilibrium will occur in this version of the game?

$$\{(Movies, Movies)\}$$

d. Assume this is a one-off game. Jacob and Erica will never encounter each other ever again after this. Jacob *threatens* Erica by saying that he's going to play laser tag no matter what Erica does. Will this convince Erica to go play laser tag? Explain why or why not.

No. Since Erica knows Jacob is a rational player, she knows that Jacob's threat is a non-credible threat. It is non-credible, because once Erica decides to go to the movies, Jacob will maximize his payoff by also going to the movies, instead of going to play laser tag.

2. Two psychologists put two goats, one big and one little, into a pen. There was a lever in the pen that, if pressed, would channel goat feed into a trough on the other side of the pen. If the little goat pressed the lever, the big goat would wait at the trough and get all the goat feed and the little goat would get none. If the big goat pressed the lever, the little goat, already waiting at the trough, would get some of the goat feed before the big

goat got to the trough to push him away. Here is the payoffs for these goats in normal form:

		Big Goat	
		Press	Wait
Little Goat	Press	-1,9	-1,10
	Wait	6,4	0,0

a. Are there dominant strategies for the Little Goat and Big Goat in this game?
   Little Goat's dominant strategy is "Wait." Big Goat has no dominant strategy
b. Find all Nash equilibria for this game.
   {Little Goat: Wait; Big Goat: Press}
c. Explain why the phrase "the meek shall inherit the earth" is relevant to this game?
   The Nash equilibrium ends up being the most desirable outcome for Little Goat and the least desirable outcome for Big Goat.

3. Dominic Toretto and Lance Nguyen are engaged in a face-off. They are going to drive in souped-up cars towards each other at great speed. The best thing that can happen to a player is to not swerve and have the other guy swerve. Then, the one who swerves is a "chicken," getting a payoff of one, and the one who doesn't is a "hero," getting a payoff of two. If both players swerve, they are both "chickens," so both get a payoff of 1. If neither "swerves, they both end up in a hospital with a payoff of zero.
   a. Depict the game in normal representation form.

		Lance	
		Swerve	Don't Swerve
Dominic	Swerve	1,1	1,2
	Don't swerve	2,1	0,0

   b. Does this game have any dominant strategies and for whom?
      No dominant strategies
   c. What are the pure strategy Nash equilibria in this game?
      {Dominic: Don't swerve; Lance: Swerve}
      {Dominic: Swerve, Lance: Don't swerve}
   d. Find a Nash equilibrium in mixed strategies for this game. Clearly state how you have defined the probabilities across strategies for each player.
      Dominic: Swerve with probability p and don't swerve with probability $(1-p)$
      Lance: Swerve with probability q and don't swerve with probability $(1-q)$

      For Dominic to be indifferent between swerving and not swerving:

Payoff from swerving = Payoff from not swerving

$$q+(1-q)=2q+0(1-q)$$

$$q=\frac{1}{2}$$

For Lance to be indifferent between swerving and not swerving:

Payoff from swerving = Payoff from not swerving

$$p + (1 - p) = 2p + 0(1 - p)$$

$$p = \frac{1}{2}$$

4. Consider again the penalty kick game. This time, the payoff matrix has been generalized to the following:

		Kicker	
		Kick Left	Kick Right
Goalie	Jump Left	1,0	0,1
	Jump Right	1-p,p	1,0

The probability that the kicker will score if he kicks to the left and the goalie jumps to the right is p. We want to see how the equilibrium probabilities change as p changes.

a. The goalie jumps left with probability π_G. If the kicker kick's right, what is his probability of scoring?

$$\pi_G(1) + (1 - \pi_G)(0) = \pi_G$$

b. Then, what is the kicker's probability of scoring when he kicks left?

$$\pi_G(0) + (1 - \pi_G)(p) = (1 - \pi_G)(p)$$

c. Find the probability π_G that makes kicking left and kicking right lead to the same probability of scoring for the kicker. (Your answer should be a function of p)

$$\pi_G = (1 - \pi_G)p$$

$$\pi_G(1 + p) = p$$

$$\pi_G = \frac{p}{1 + p}$$

d. If the kicker kicks left with probability π_K, then if the goalie jumps left, what is the probability that the kicker will not score?

$$\pi_K(1) + (1 - \pi_K)(0) = \pi_K$$

e. If the kicker kicks left with probability π_K, then if the goalie jumps right, what is the probability that the kicker will not score?

$$\pi_K(1 - p) + (1 - \pi_K)$$

f. Find the probability, π_K that makes the goalie indifferent between jumping left or jumping right.

$$\pi_K = \pi_K(1 - p) + (1 - \pi_K)$$

$$\pi_K(p + 1) = 1$$

$$\pi_K = \frac{1}{p + 1}$$

g. The variable p tells us how good the kicker is at kicking the ball into the left side of the goal when it is undefended. As p increases, how does that change the probability that the kicker kicks left?

It decreases the probability that the kicker kicks left.

h. Why would improving the kicker's payoff from kicking left result in the kicker kicking left less often in equilibrium? (Hint: notice what happens to π_G as p changes)

Notice that the kicker is better at kicking right than left as long as $p < 1$. In other words, left is his weak side. As the kicker gets better at kicking left, it increases the probability that the goalie will jump left, thus decreasing the probability that the goalie jumps right. This gives the kicker an incentive to use his strong side (right) more often.