

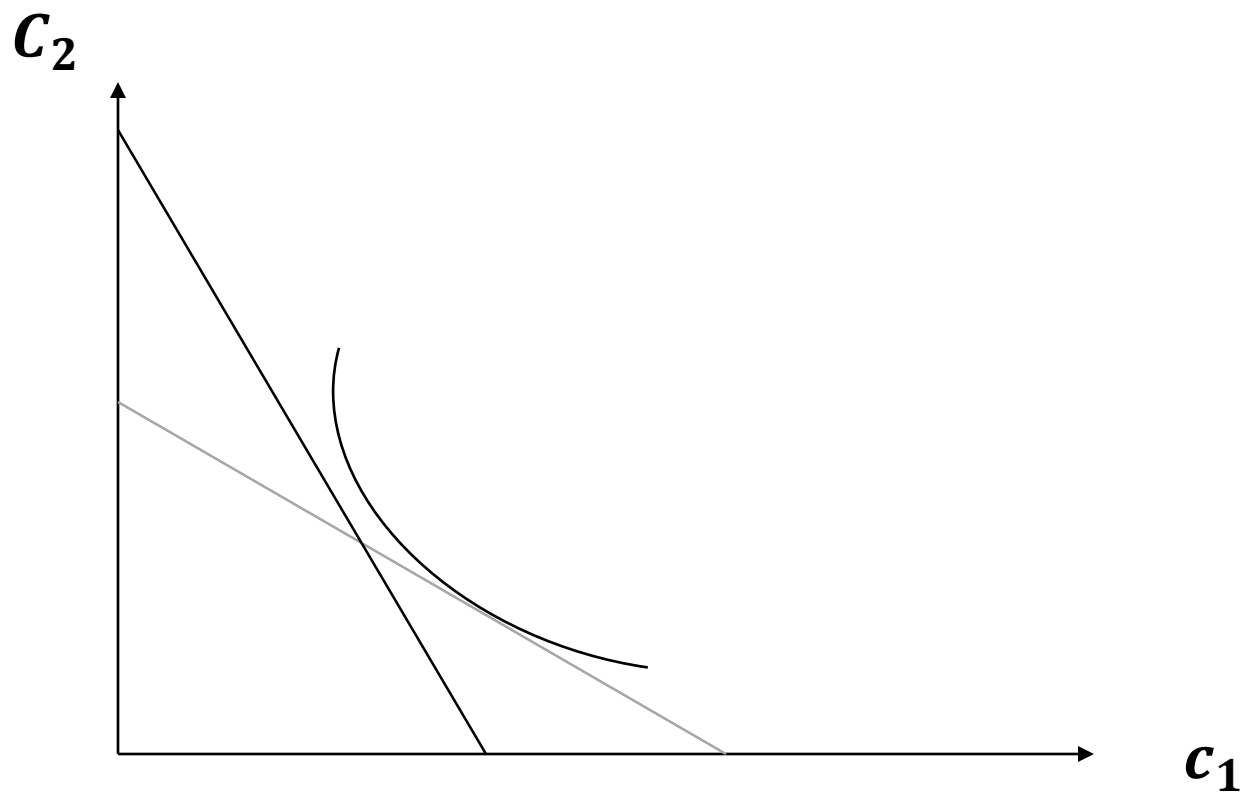
# Reminder

- The 8<sup>th</sup> edition of Intermediate Microeconomics by Hal Varian is compatible with this course.
- This may be the only edition still available at the bookstore.

# Announcements

- Make-up midterm exams?
  - Yes, however, the exam will be slightly harder since students get more time to prepare for this exam.
  - If you need a make-up midterm, the earlier you let me know, the better.
- Mathematical Requirements:
  - Derivatives: Partial Derivatives, Total Derivatives

# Borrower becomes saver



# Valuing Bonds

- A bond is a special type of security that pays a fixed amount  $\$x$  for  $T$  years (its maturity date) and then pays its face value  $\$F$ .
- What is the most that should now be paid for such a bond?

# Valuing Bonds

|               |                 |                     |                     |     |                         |                     |
|---------------|-----------------|---------------------|---------------------|-----|-------------------------|---------------------|
| End of Year   | 1               | 2                   | 3                   | ... | T-1                     | T                   |
| Income paid   | x               | x                   | x                   | x   | x                       | F                   |
| Present value | $\frac{x}{1+r}$ | $\frac{x}{(1+r)^2}$ | $\frac{x}{(1+r)^3}$ | ... | $\frac{x}{(1+r)^{T-1}}$ | $\frac{F}{(1+r)^T}$ |

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \dots + \frac{x}{(1+r)^{T-1}} + \frac{x+F}{(1+r)^T}$$

# Announcements

- Clarification:
- Suppose you win a State lottery. The prize is \$1,000,000 but it is paid over 10 years in equal installments of \$100,000 each. The interest rate,  $r$ , is 0.1. What is the prize actually worth?

$$\begin{aligned}\text{Answer: } PV &= \frac{100,000}{1+r} + \frac{100,000}{(1+r)^2} + \dots + \frac{100,000}{(1+r)^{10}} \\ &= \$614,457\end{aligned}$$

# Announcements

- Corrections on Slide 81 from last week
- Problem set for next week's discussion group is up. Please come prepared.

# Chapter 12: Uncertainty



# Uncertainty

- We continue discussing how the utility framework from EC2101 can be used to address different decisions that households face
  - Last week: Consumption across different periods
  - This week: Consumption across different (uncertain) states of the world

# Uncertainty is Pervasive

- What is uncertain in economic systems?
  - tomorrows prices
  - future wealth
  - future availability of commodities
  - present and future actions of other people.

# Uncertainty is Pervasive

- What are some rational responses to uncertainty?
  - buying insurance (health, life, auto)
  - a portfolio of contingent consumption goods.
- Today, we will use economic models to explain these responses

# Uncertainty

- Budget constraints
- Preferences
  - Expected Utility, Risk Premium, Risk Aversion, Marginal Rate of Substitution
- Insurance
- Diversification and Risk Spreading

# States of Nature

- Definition: different outcomes of some random event
- Example:
  - possible States of Nature:
    - “car damaged in accident” (d) and “no car damage” (nd).
  - Damage occurs with probability  $\pi_d$ , does not with probability  $\pi_{nd}$  ;
$$\pi_a + \pi_{na} = 1.$$
  - Accident causes a loss of \$L.

# Contingencies

- A contingency is something that depends on an event that is not yet certain.
- A contract implemented only when a particular state of Nature occurs is *state-contingent*.
- E.g. the insurer pays only if there is an accident.

# Contingencies

- A *contingent consumption plan* is a specification of what will be consumed in each different state of nature
- E.g. take a vacation only if there is no accident, stay at home if there is an accident.

# State-Contingent Budget Constraints

- How do we model individual choice under uncertainty?
- We can work with the utility framework we are familiar with, defining the consumption in the various states as separate goods.
- Let's look at an example of how insurance works



# Example

- According to Daily Makeover, J-Lo has reportedly insured her booty for a whopping \$300 million dollars. Wow!

## Jennifer Lopez Insures Her Booty



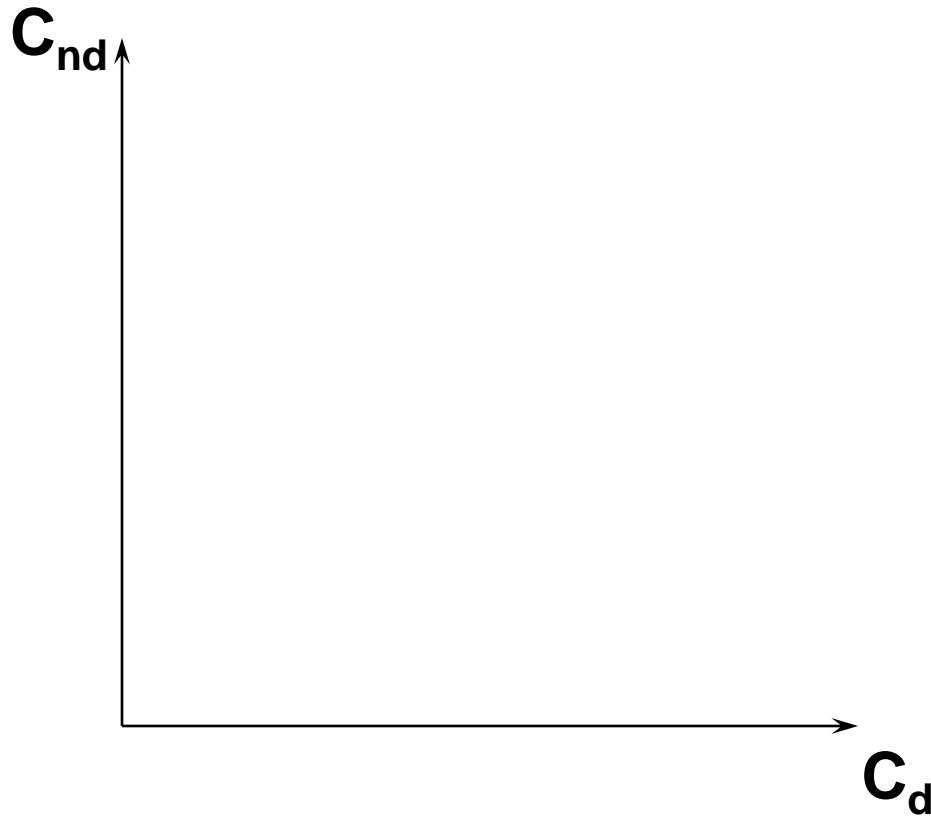
# Example

- Two contingent states:
  - Damage
  - No Damage
- Damage causes a loss of  $\$L$

# State-Contingent Budget Constraints

- Each \$1 of damage insurance costs  $\gamma$ .
- J-Lo has \$m of wealth.
- $C_{nd}$  is consumption value in the no-damage state.
- $C_d$  is consumption value in the damage state.

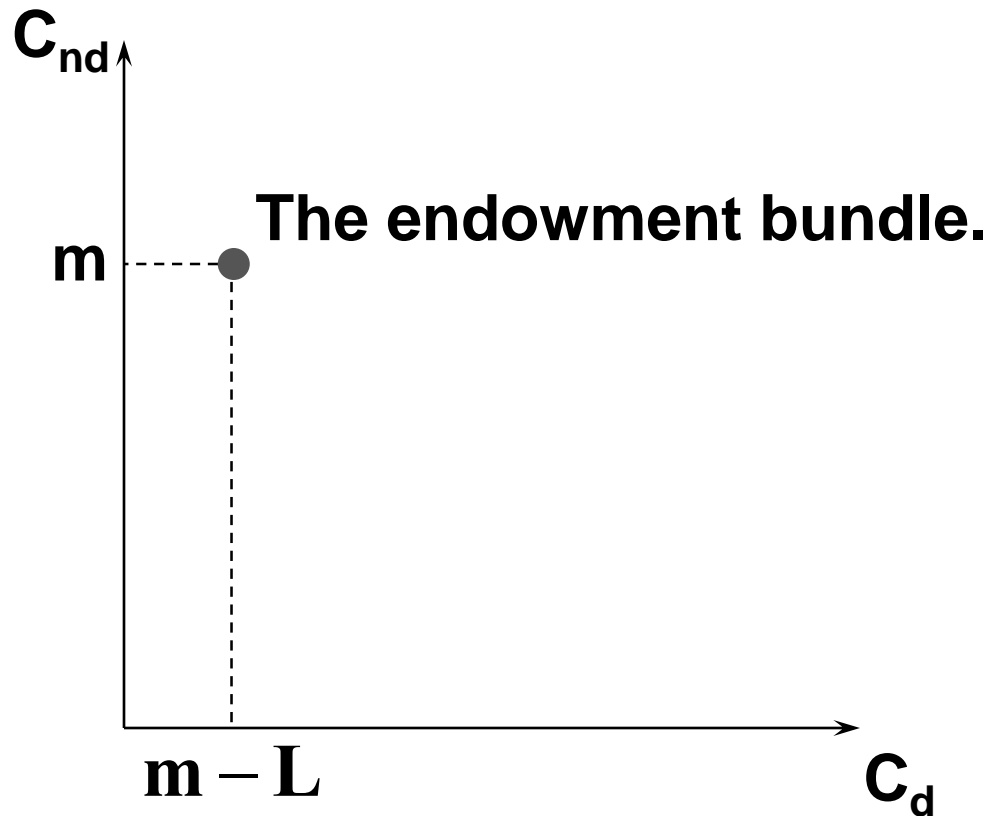
# State-Contingent Budget Constraints



# State-Contingent Budget Constraints

- Without insurance,
  - $C_d = m - L$
  - $C_{nd} = m.$

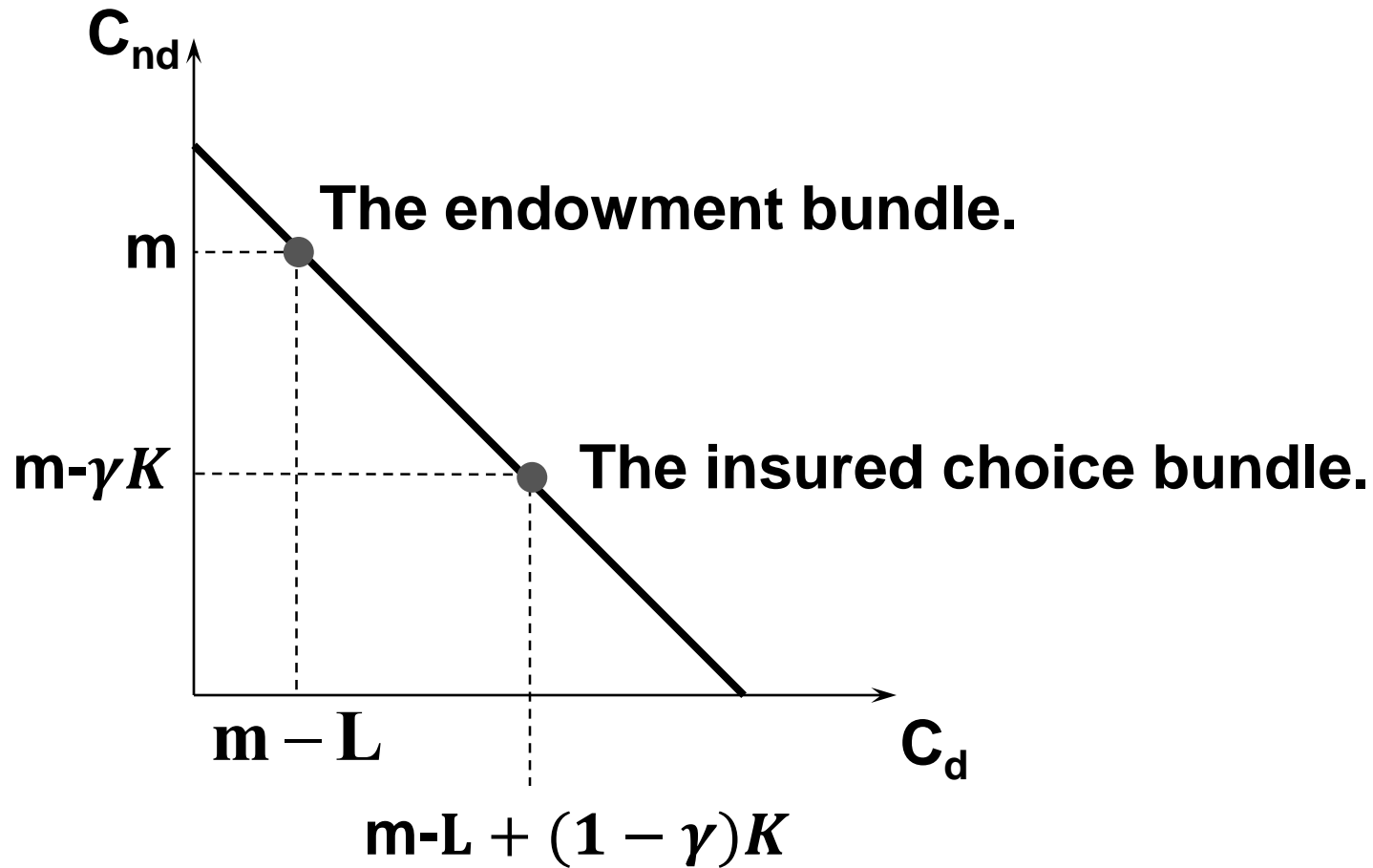
# State-Contingent Budget Constraints



# State-Contingent Budget Constraints

- Buy \$K of accident insurance at a cost of  $\gamma K$ 
  - $C_{nd} = m - \gamma K.$
  - $C_d = m - L - \gamma K + K$   
 $= m - L + (1 - \gamma)K.$

# State-Contingent Budget Constraints





# State-Contingent Budget Constraints

- What is the slope of the budget constraint?

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\frac{\Delta C_{nd}}{\Delta C_d} = \frac{\gamma K}{(1 - \gamma)K} = \frac{\gamma}{1 - \gamma}$$

# State-Contingent Budget Constraints

Alternatively,

$$C_d = m - L + (1 - \gamma)K.$$

So,

$$K = \frac{C_d - m + L}{1 - \gamma}$$

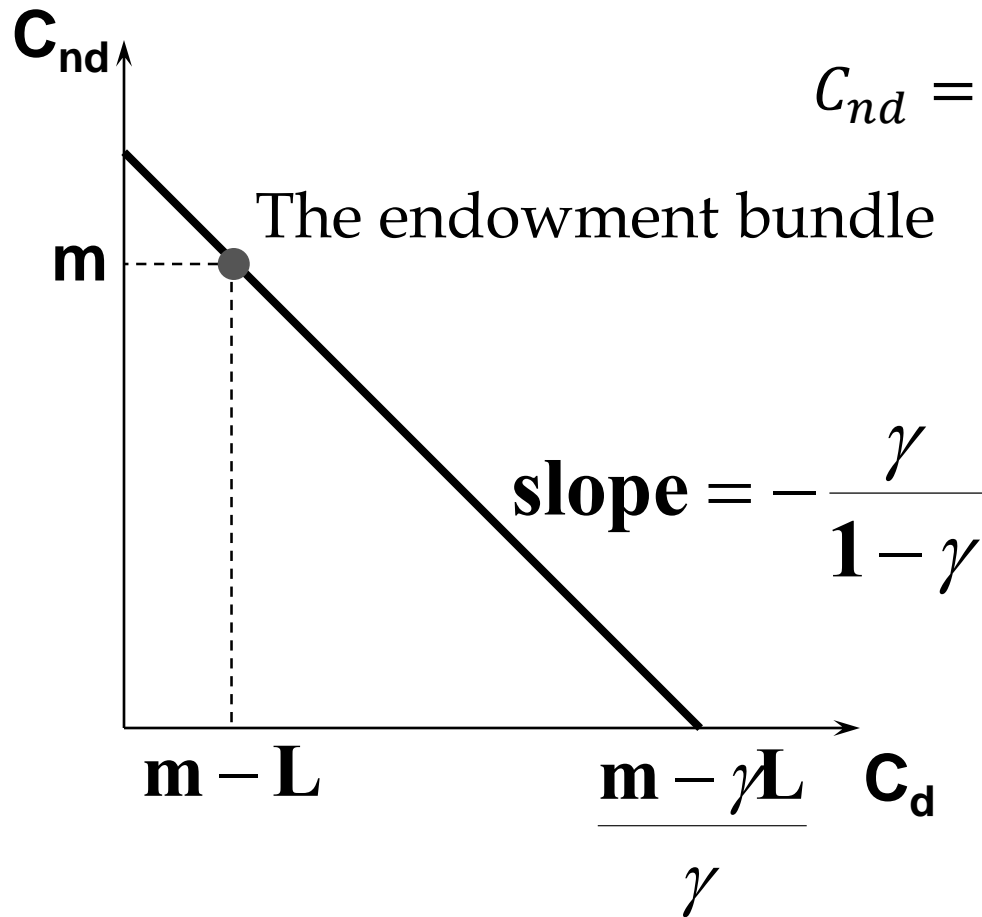
Since,  $C_{nd} = m - \gamma K$ ,

$$C_{nd} = m - \gamma \left( \frac{C_d - m + L}{1 - \gamma} \right)$$

Rearranging,

$$C_{nd} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} C_d$$

# State-Contingent Budget Constraints



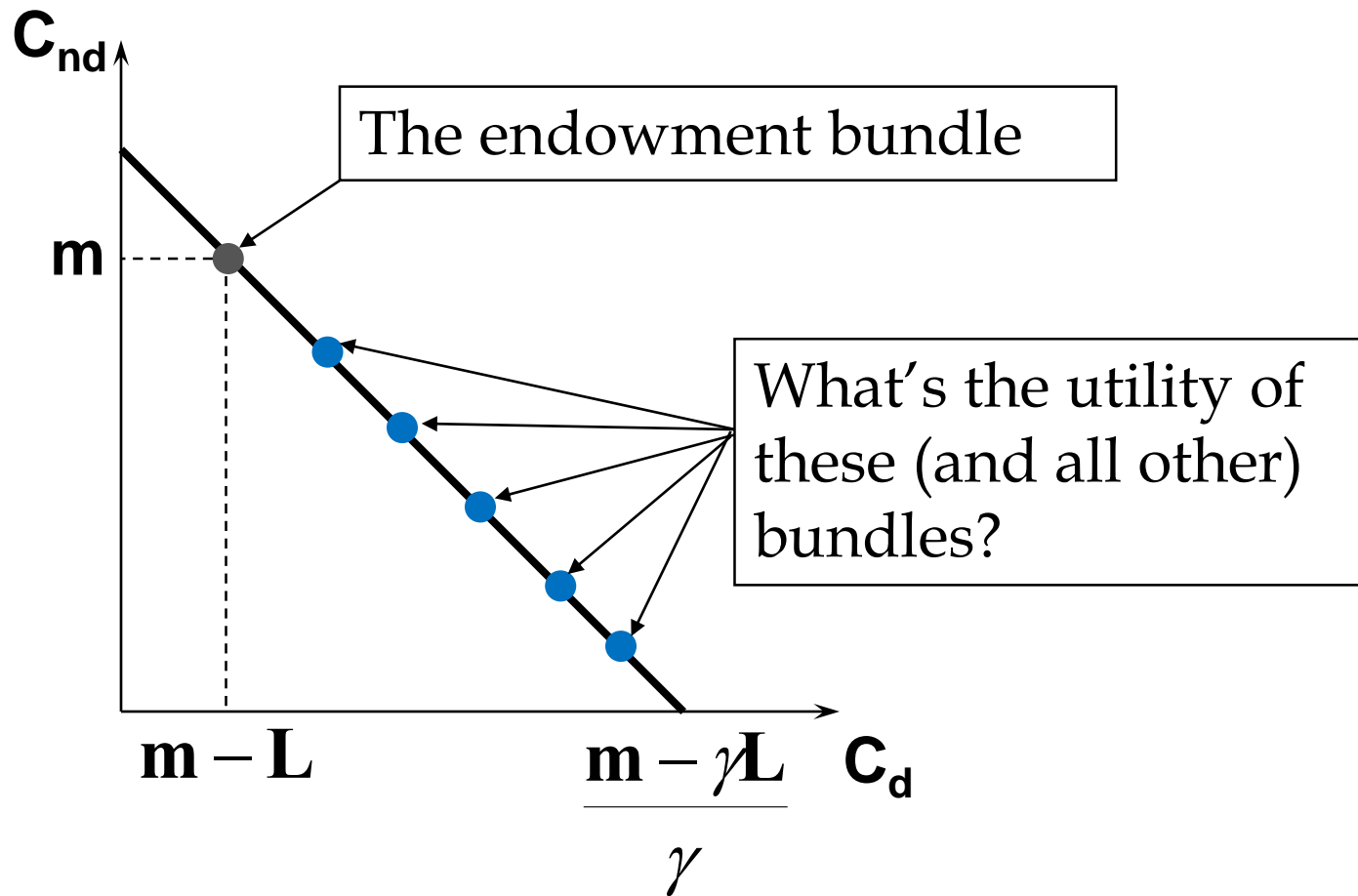
$$C_{nd} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} C_d$$

Where is the most preferred state-contingent consumption plan?

# Preferences Under Uncertainty

- Where is the most preferred state-contingent consumption plan?
- To answer this question, we must know something about the consumer's preferences across various state contingent consumption plans

# Preferences Under Uncertainty

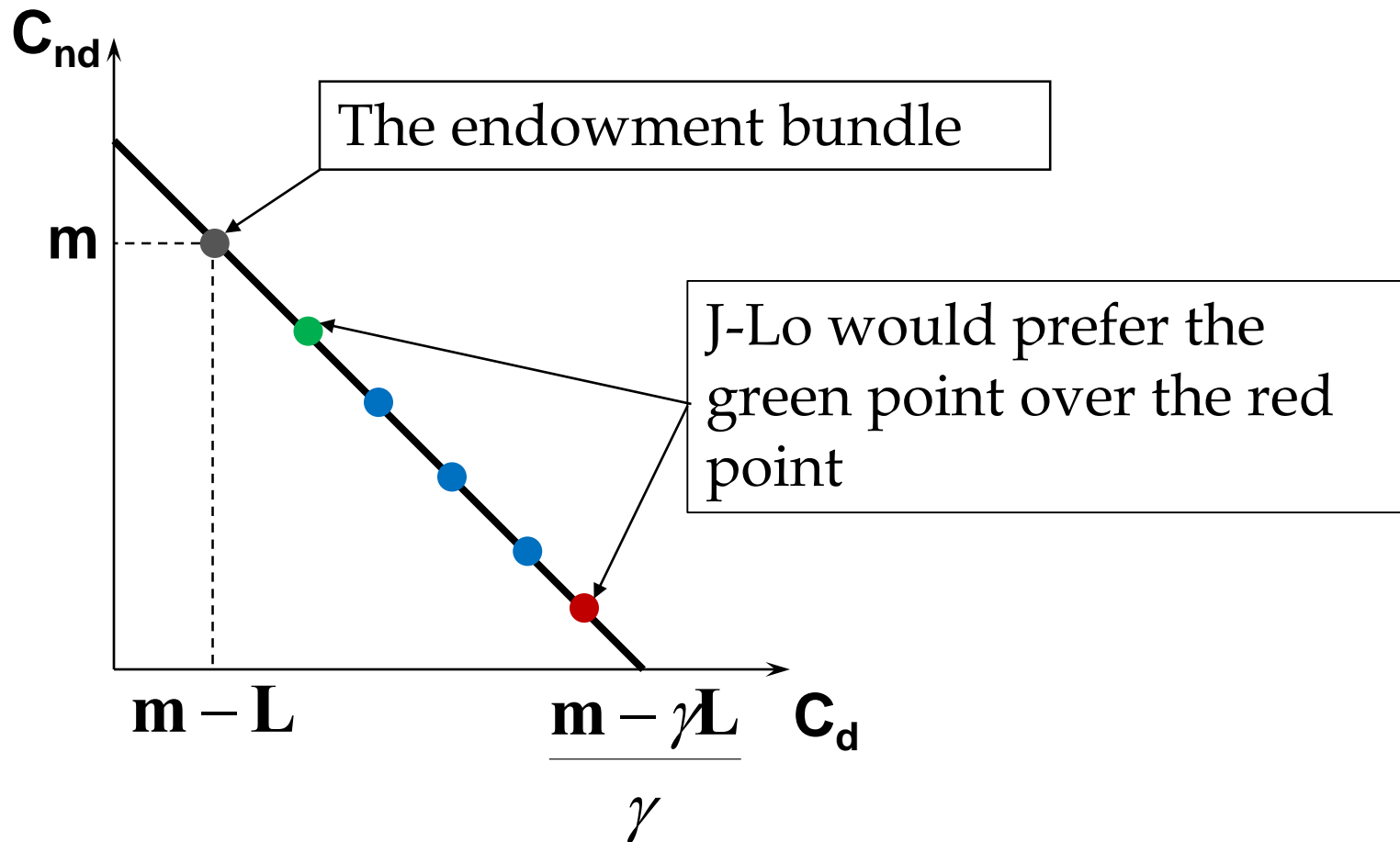


# Preferences Under Uncertainty


- Preferences across various contingent consumption plans depend on how likely the consumer perceives the various states of nature to occur.
- If J-Lo believes that the no damage state is most likely (i.e. 99% no damage vs 1% damage) then she would prefer endowment bundles on the budget constraint that gives her more in state:

$C_{nd}$

# Preferences Under Uncertainty



# Preferences Under Uncertainty

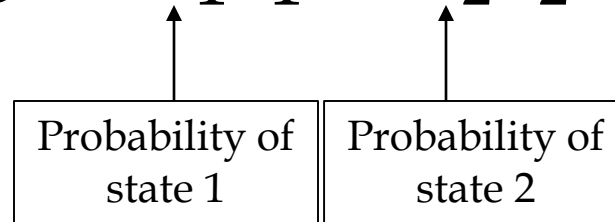
- Expected Utility 
- Risk Aversion
- Risk Premium
- Marginal Rate of Substitution



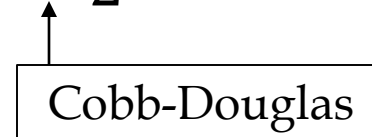
# Preferences Under Uncertainty

- Examples of utility functions in the context of choice under uncertainty:

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1 + \pi_2 c_2$$



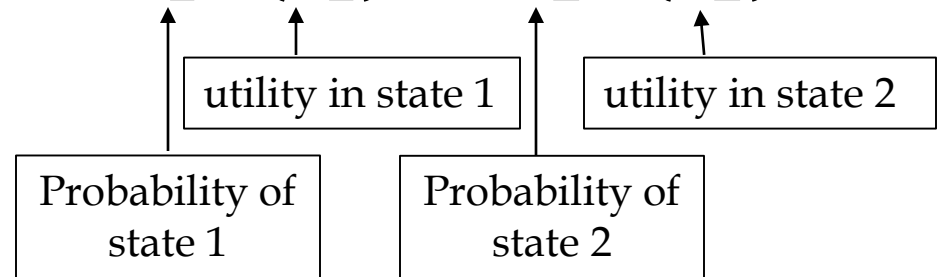
$$u(c_1, c_2, \pi_1, \pi_2) = c_1^{\pi_1} c_2^{1-\pi_1}$$



# Preferences Under Uncertainty

- Expected Utility

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

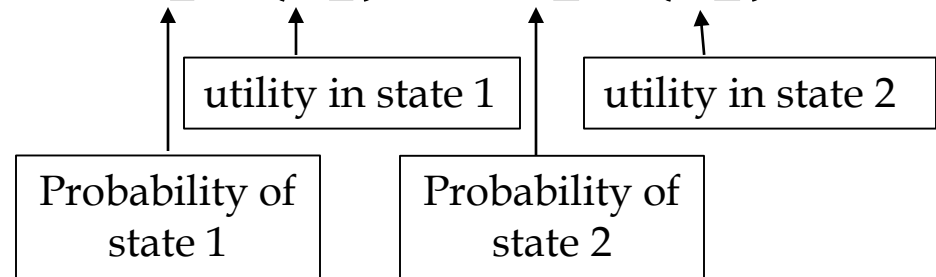


- Utility “a priori” is a weighted sum of the consumptions in each state where the weights are the probabilities of each state occurring

# Preferences Under Uncertainty

- Expected Utility

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$



- Any utility function that follows this form is called an “*expected utility function*” or a “*von-Neumann-Morgenstern utility function*”

# Preferences Under Uncertainty

- If  $v(c_1) = c_1$ , then

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1 + \pi_2 c_2$$

is an expected utility function

- $u(c_1, c_2, \pi_1, \pi_2) = c_1^{\pi_1} c_2^{1-\pi_1}$  is not an expected utility function, however the log transformation is:

$$\ln u(c_1, c_2, \pi_1, \pi_2) = \pi_1 \ln c_1 + (1 - \pi_1) \ln c_2$$

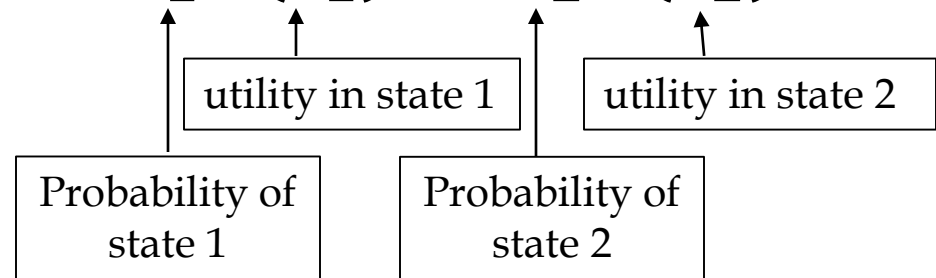
Now we redefine the utility function as:

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 \ln c_1 + (1 - \pi_1) \ln c_2$$

# Preferences Under Uncertainty

- Expected Utility

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$



- The expected utility function maintains the expected utility function under **positive affine transformations**:

$$v(u) = au + b$$

# Preferences Under Utility

- Why do economists like the expected utility function?
  - Read Section 12.4: Why Expected Utility is Reasonable (pg 224-226)
  - You will be responsible to know this material for homework assignments and exams

# Preferences Under Uncertainty

- Consider a lottery.
  - Win \$90 with probability  $1/2$  and win \$1 with probability  $1/2$ .
  - Assume the Cobb-Douglas utility function. With the log transformation:
- $U(\$90) = \ln(90)=4.49$
- $U(\$1) = \ln(1)=0$
- Expected utility is

# Preferences Under Uncertainty

- $U(\$90) = \ln(90) = 4.49$

- $U(\$1) = \ln(1) = 0$

- Expected utility is

$$\frac{1}{2}(4.49) + \frac{1}{2}(0) = 2.25$$



# Preferences Under Uncertainty

- Expected money value of the lottery (or Expected value of the lottery is

$$EM = \frac{1}{2}(\$90) + \frac{1}{2}(\$1) = \$45.50$$

# Preferences Under Uncertainty



- EU and EM are not the same thing!
- **Expected (Money) Value of the Lottery** is the probability weighted average of all the outcomes of the lottery

$$\sum_j prob_j(\textit{Outcome}_j)$$

- **Expected Utility** is the probability weighted average of the utilities associated with all the possible outcomes

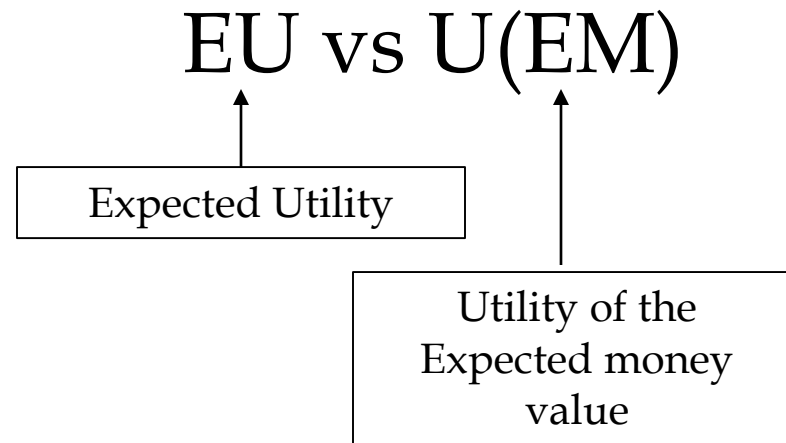
$$\sum_j prob_j\{U(\textit{Outcome}_j)\}$$

# Preferences Under Uncertainty

- Expected Utility 
- Risk Aversion 
- Risk Premium
- Marginal Rate of Substitution

# Preferences Under Uncertainty

- By comparing the expected utility of a bundle to the utility of the expected money value, we can determine the riskiness of a consumer:



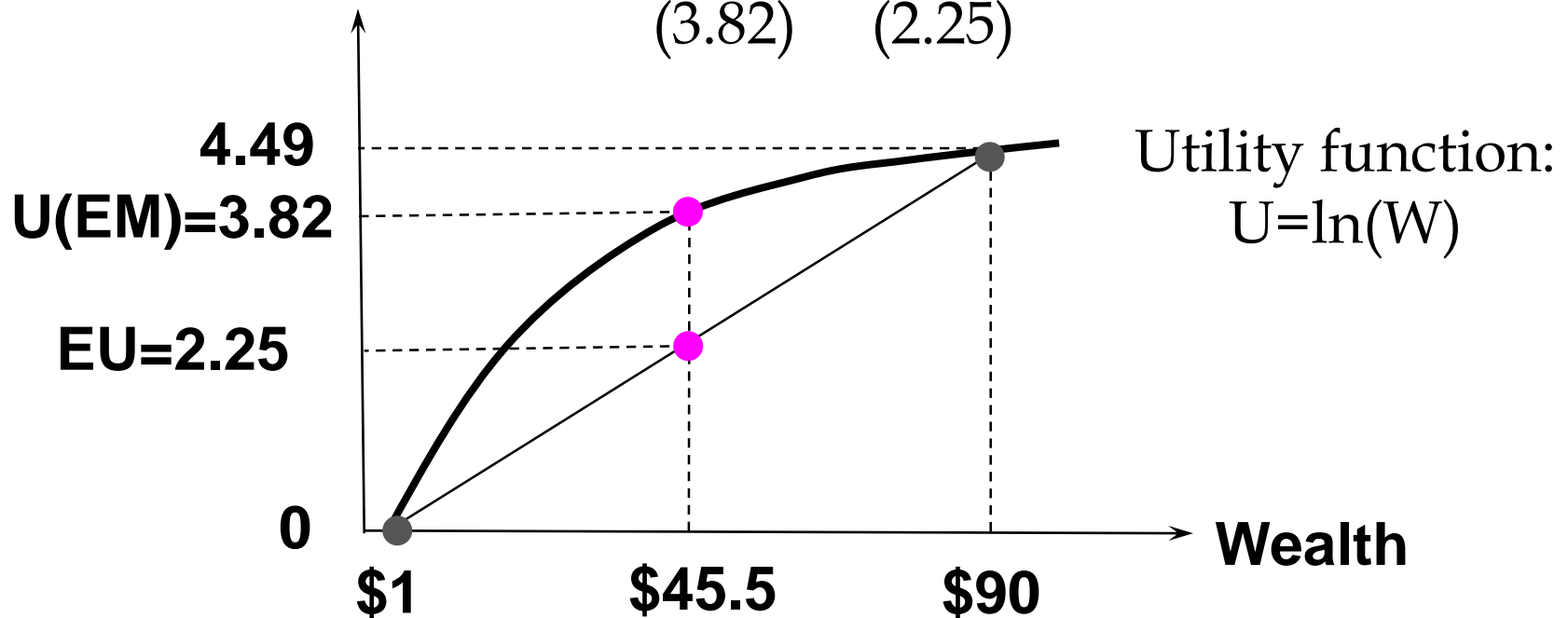
# Preferences Under Uncertainty

- $EU = 2.245$  and  $EM = \$45.50$
- $U(EM) = U(\$45.50)$   
 $= \ln(45.50)$   
 $= 3.82$

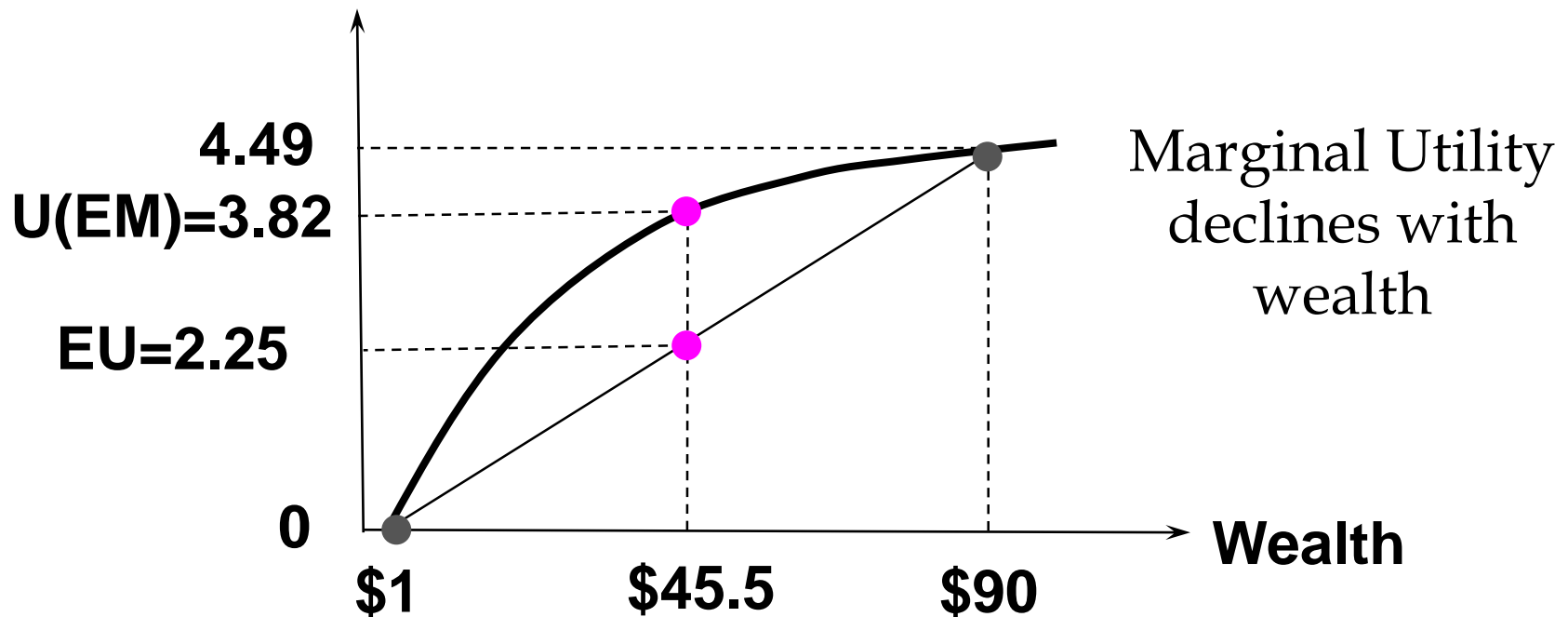
# Preferences Under Uncertainty

$U(EM) > EU \Rightarrow$  risk-aversion.

(3.82)    (2.25)



# Preferences Under Uncertainty



# Preferences Under Uncertainty

$$U(EM) > EU$$

- This implies the consumer would prefer to have the expected money value of the lottery rather than participate in the lottery.
- This is why we call this consumer risk adverse



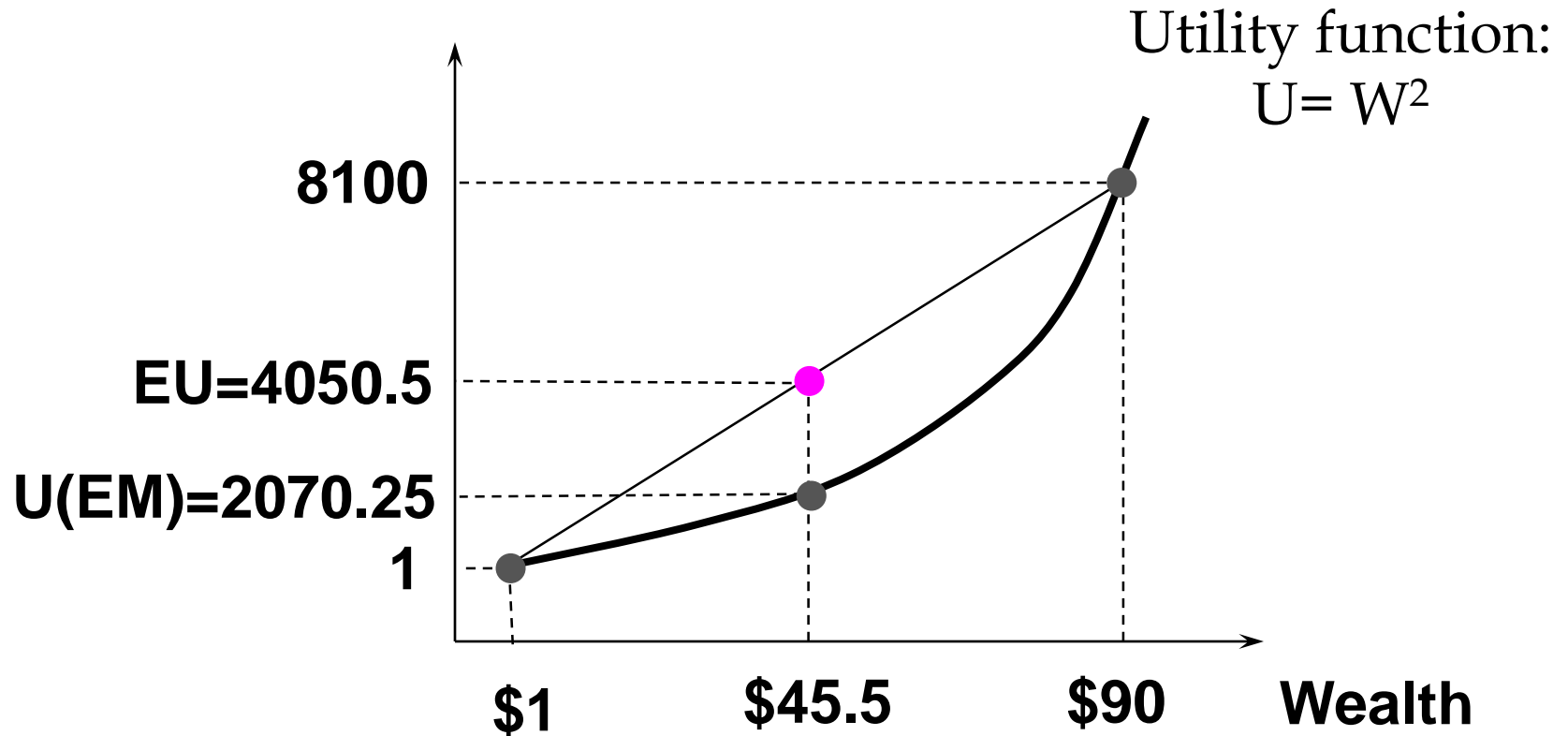
# Preferences Under Uncertainty

- Consider the same lottery
  - Win \$90 with probability  $1/2$  and win \$1 with probability  $1/2$ .
  - Assume the following utility function:  $U(x) = x^2$

# Preferences Under Uncertainty

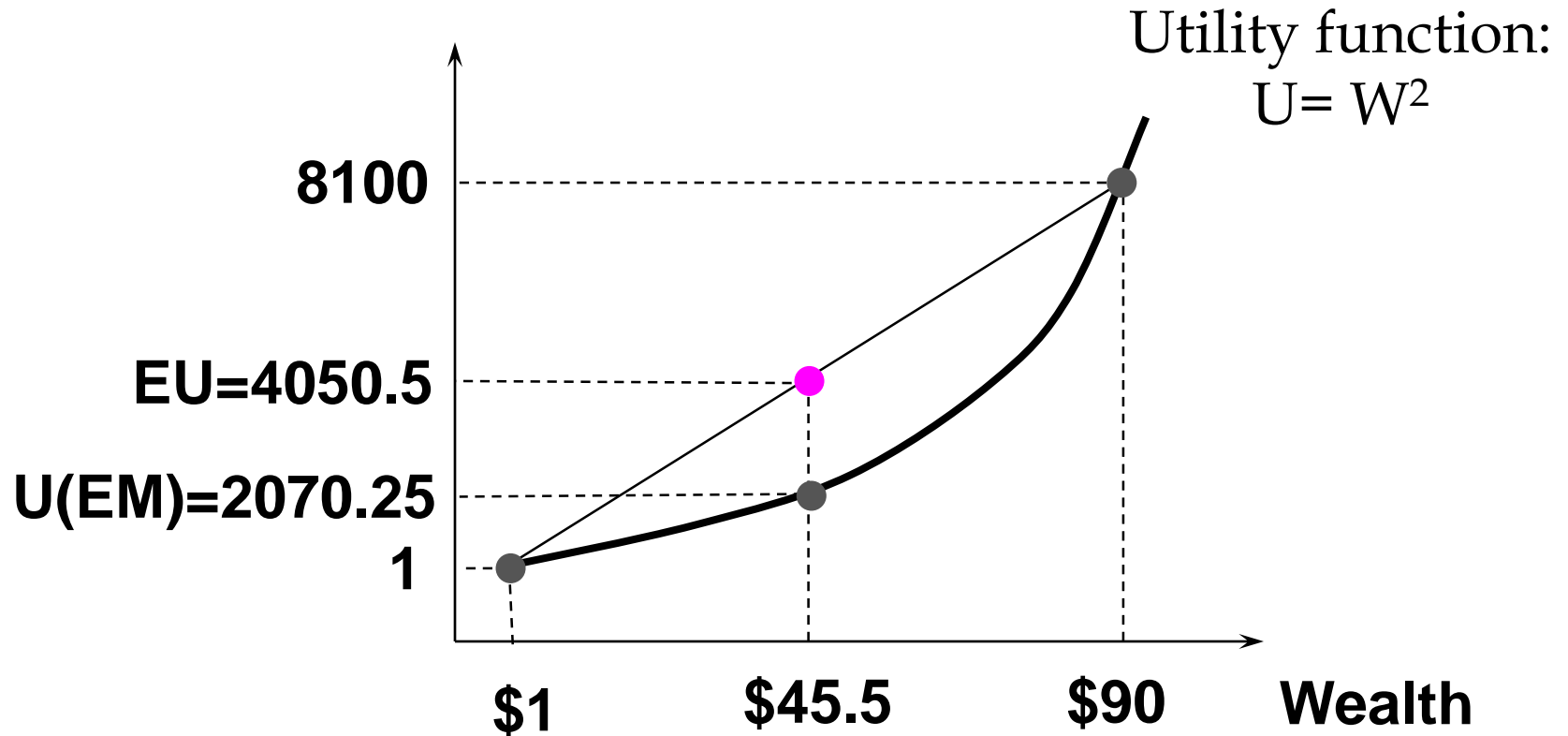
- $EU = \frac{1}{2} (90^2) + \frac{1}{2} (1^2)$   
 $= 4050.5$
- $EM = \frac{1}{2} (\$90) + \frac{1}{2} (\$1) = \$45.50$
- $U(EM) = 45.5^2$   
 $= 2070.25$

# Preferences Under Uncertainty



$U(EM) < EU \Rightarrow$  risk-loving.

# Preferences Under Uncertainty



Marginal Utility rises with wealth

# Preferences Under Uncertainty

- $EU = 7$  and  $EM = \$45$ .
- $U(EM) > EU \Rightarrow$  Guaranteed payout is preferred to the lottery  $\Rightarrow$  risk-aversion.
- $U(EM) < EU \Rightarrow$  the lottery is preferred to guaranteed payout  $\Rightarrow$  risk-loving.
- $U(EM) = EU \Rightarrow$  the lottery is preferred equally to guaranteed payout  $\Rightarrow$  risk-neutral.




# Preferences Under Uncertainty

- In your own time, consider the same lottery with the following utility function:

$$U(x) = 2x + 6$$

- What type of risk profile does this utility function exhibit?
- How does marginal utility change with wealth?

# Preferences Under Uncertainty

- Expected Utility 
- Risk Aversion 
- Risk Premium 
- Marginal Rate of Substitution

# Risk Premium

- The difference between the expected value of a risky income and a certain income to make the consumer indifferent between the two







# Risk Premium

- Suppose the utility function of a consumer is  $U(I) = \sqrt{I}$
- The consumer can buy a risky asset
  - \$900 with probability 60%
  - \$400 with probability 40%
- What is the risk premium associated with this asset?

# Risk Premium

- Expected utility of the asset:  
$$0.6\sqrt{900} + 0.4\sqrt{400} = 26$$
- To get the same utility, the consumer needs to attain income of  $I = 26^2 = 676$
- Expected value of the asset is:  
$$0.6(900) + 0.4(400) = 700$$
- Risk Premium =  $700 - 676 = 24$

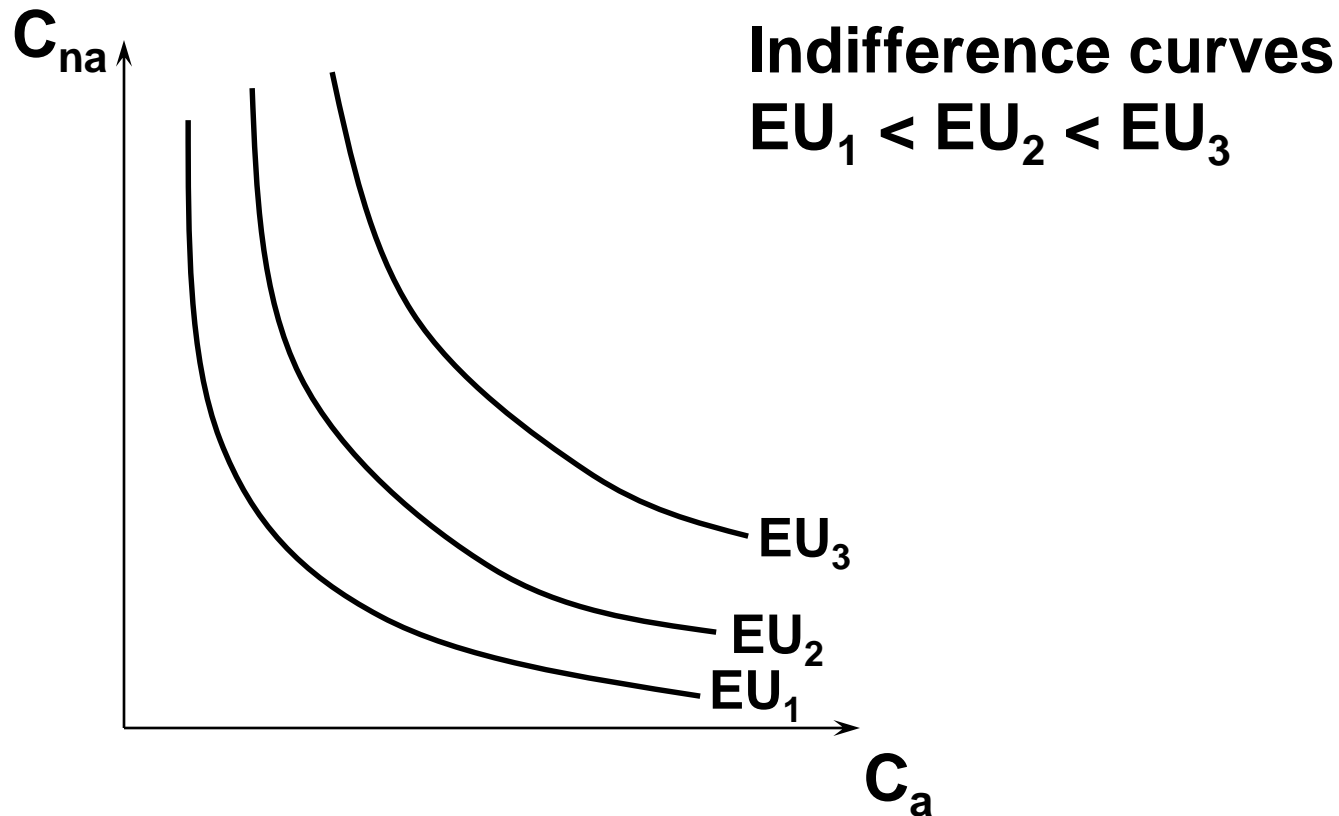
# Preferences Under Uncertainty

- Expected Utility 
- Risk Aversion 
- Risk Premium 
- Marginal Rate of Substitution 

# Preferences Under Uncertainty

- Indifference curves
- State-contingent consumption plans that give equal expected utility are equally preferred.

# Preferences Under Uncertainty



# Preferences Under Uncertainty

- What is the Marginal Rate of Substitution of an indifference curve?
- Get consumption  $c_1$  with prob.  $\pi_1$  and  $c_2$  with prob.  $\pi_2$  ( $\pi_1 + \pi_2 = 1$ ).
- $EU = \pi_1 U(c_1) + \pi_2 U(c_2)$ .
- For constant EU,  $dEU = 0$ .

# Preferences Under Uncertainty

$$EU = \pi_1 U(\mathbf{c}_1) + \pi_2 U(\mathbf{c}_2)$$

# Preferences Under Uncertainty

$$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$$

$$dEU = \pi_1 MU(c_1)dc_1 + \pi_2 MU(c_2)dc_2$$



# Preferences Under Uncertainty

$$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$$

$$dEU = \pi_1 MU(c_1)dc_1 + \pi_2 MU(c_2)dc_2$$

$$dEU = 0 \Rightarrow \pi_1 MU(c_1)dc_1 + \pi_2 MU(c_2)dc_2 = 0$$

# Preferences Under Uncertainty

$$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$$

$$dEU = \pi_1 MU(c_1)dc_1 + \pi_2 MU(c_2)dc_2$$

$$dEU = 0 \Rightarrow \pi_1 MU(c_1)dc_1 + \pi_2 MU(c_2)dc_2 = 0$$

$$\Rightarrow \pi_1 MU(c_1)dc_1 = -\pi_2 MU(c_2)dc_2$$

# Preferences Under Uncertainty

$$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$$

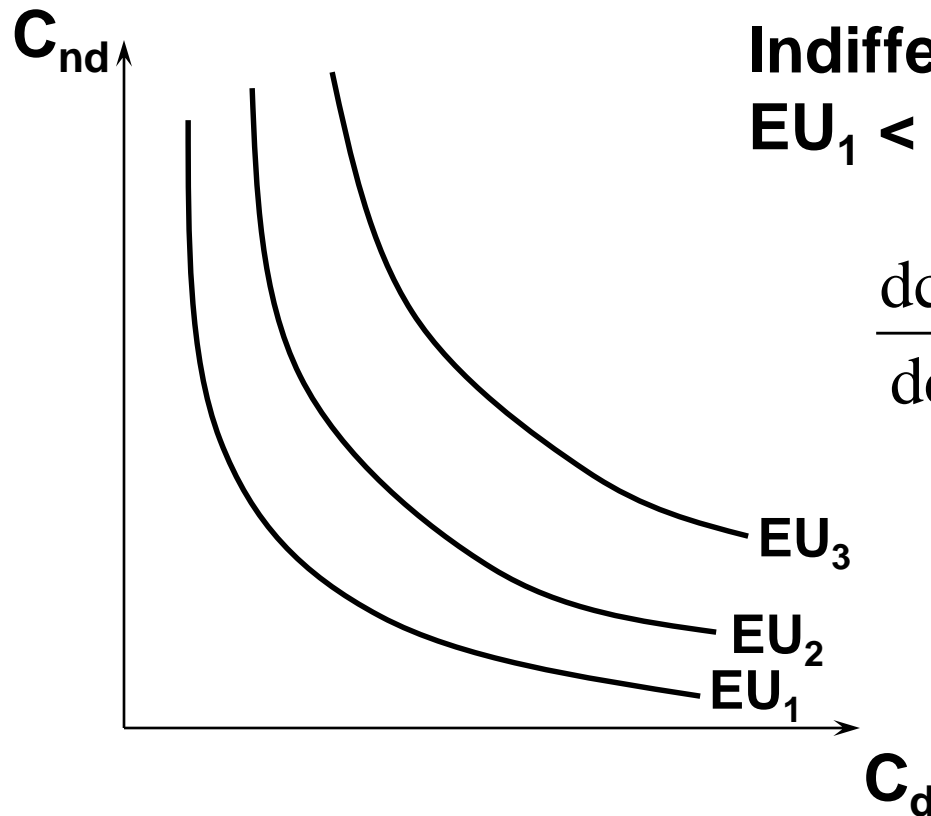
$$dEU = \pi_1 MU(c_1)dc_1 + \pi_2 MU(c_2)dc_2$$

$$dEU = 0 \Rightarrow \pi_1 MU(c_1)dc_1 + \pi_2 MU(c_2)dc_2 = 0$$

$$\Rightarrow \pi_1 MU(c_1)dc_1 = -\pi_2 MU(c_2)dc_2$$

$$\Rightarrow \frac{dc_2}{dc_1} = -\frac{\pi_1 MU(c_1)}{\pi_2 MU(c_2)}.$$





# Preferences Under Uncertainty



**Indifference curves**  
 **$EU_1 < EU_2 < EU_3$**

$$\frac{dc_{nd}}{dc_d} = - \frac{\pi_d MU(c_d)}{\pi_{nd} MU(c_{nd})}$$

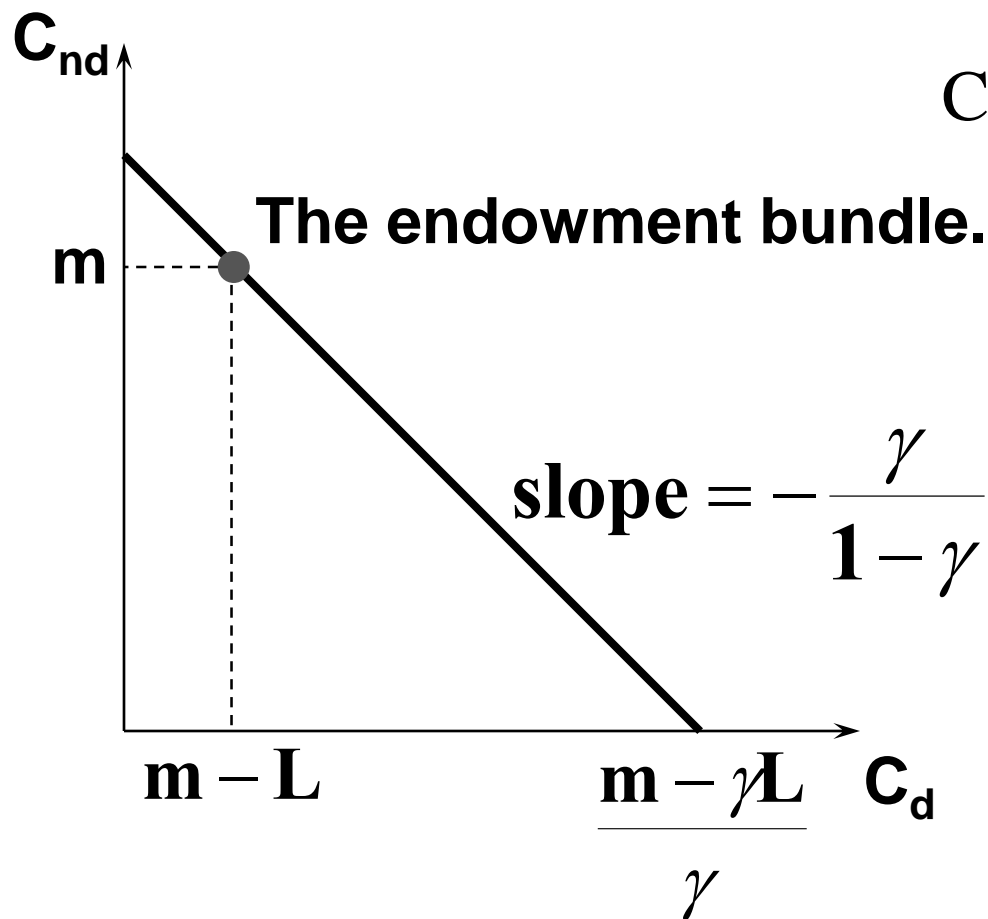
# Preferences Under Uncertainty

- Expected Utility 
- Risk Premium 
- Risk Aversion 
- Marginal Rate of Substitution 

# Choice Under Uncertainty

- How is a rational choice made under uncertainty?
  - Choose the most preferred affordable state-contingent consumption plan.

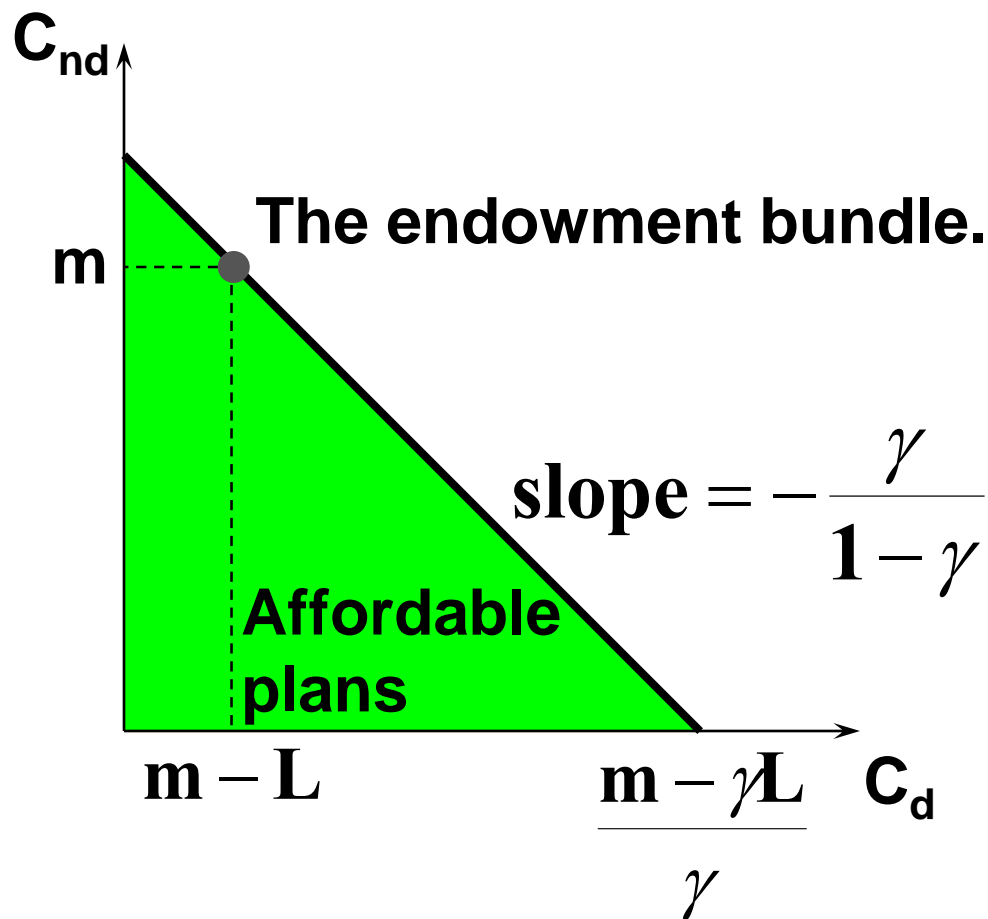
# State-Contingent Budget Constraints



$$C_{nd} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} C_d$$

**Where is the  
most preferred  
state-contingent  
consumption plan?**

# State-Contingent Budget Constraints

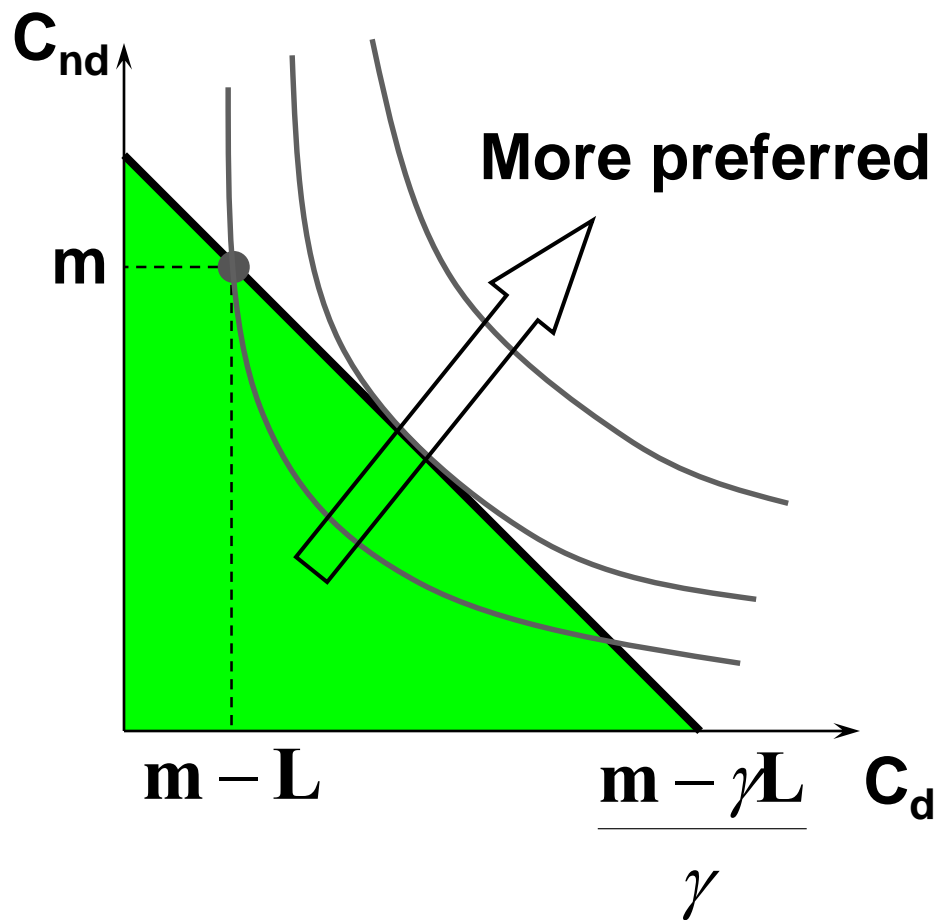


$$C_{nd} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} C_d$$

**Where is the most preferred state-contingent consumption plan?**

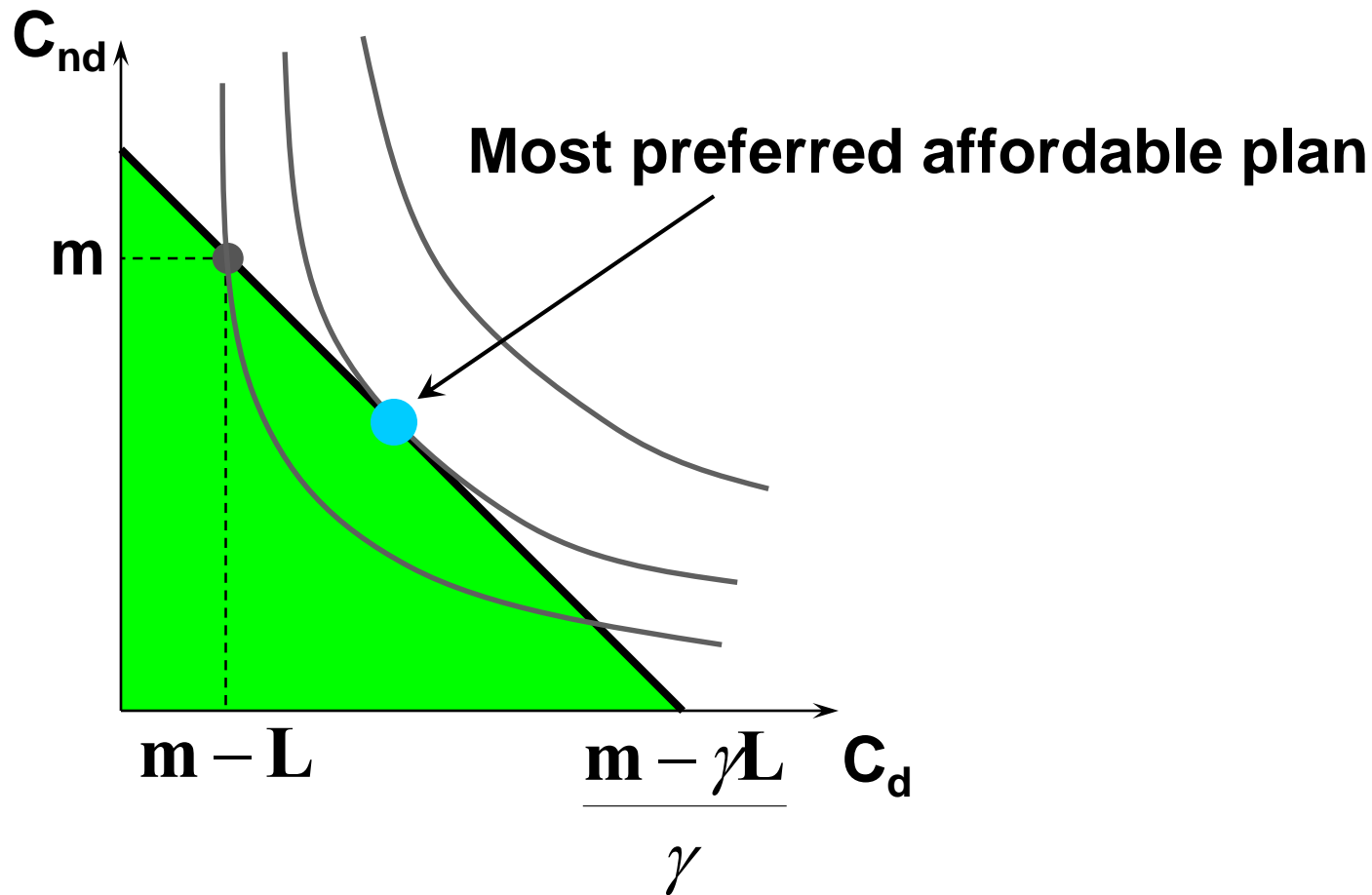


# State-Contingent Budget Constraints

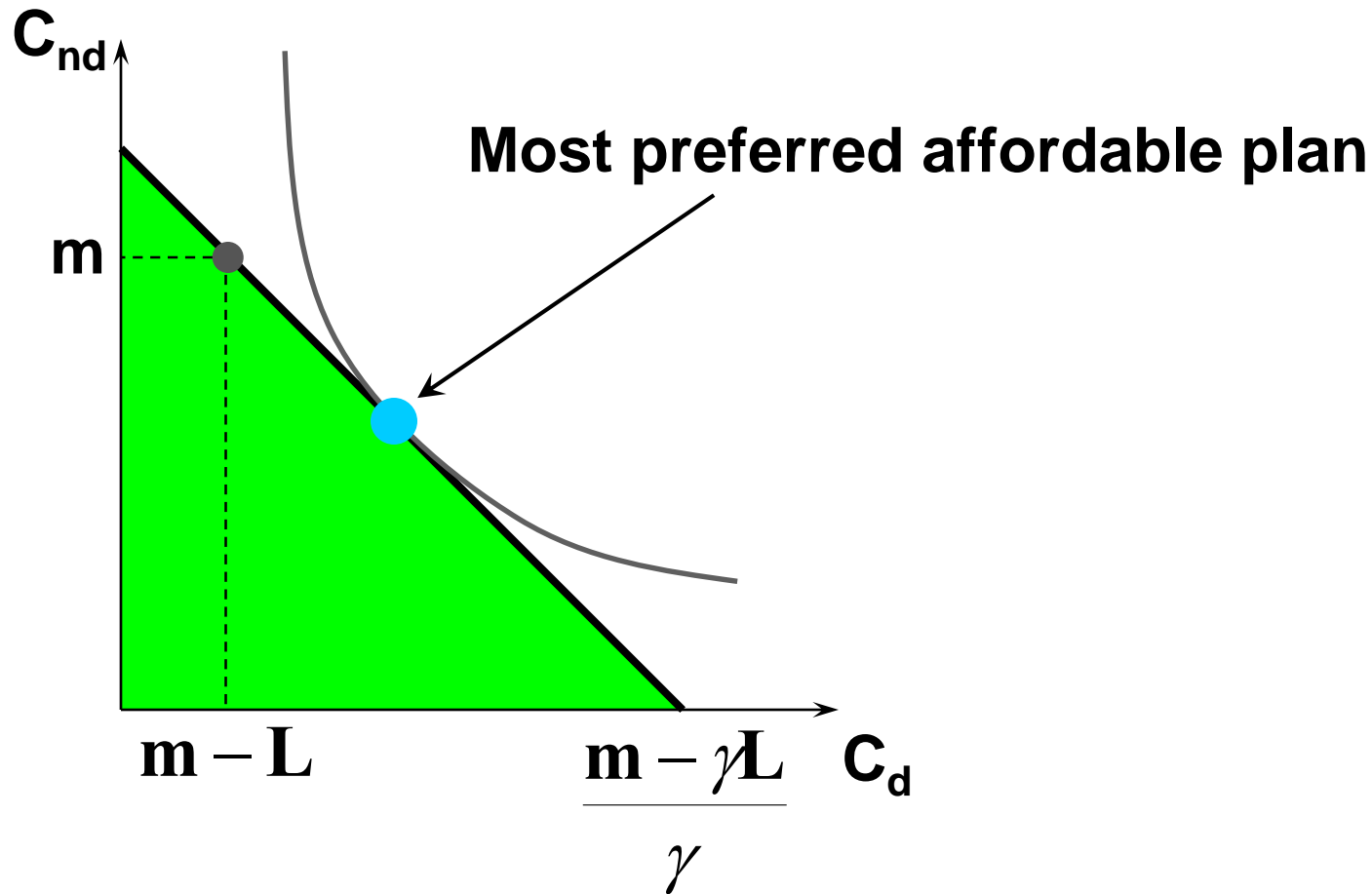


**Where is the most preferred state-contingent consumption plan?**

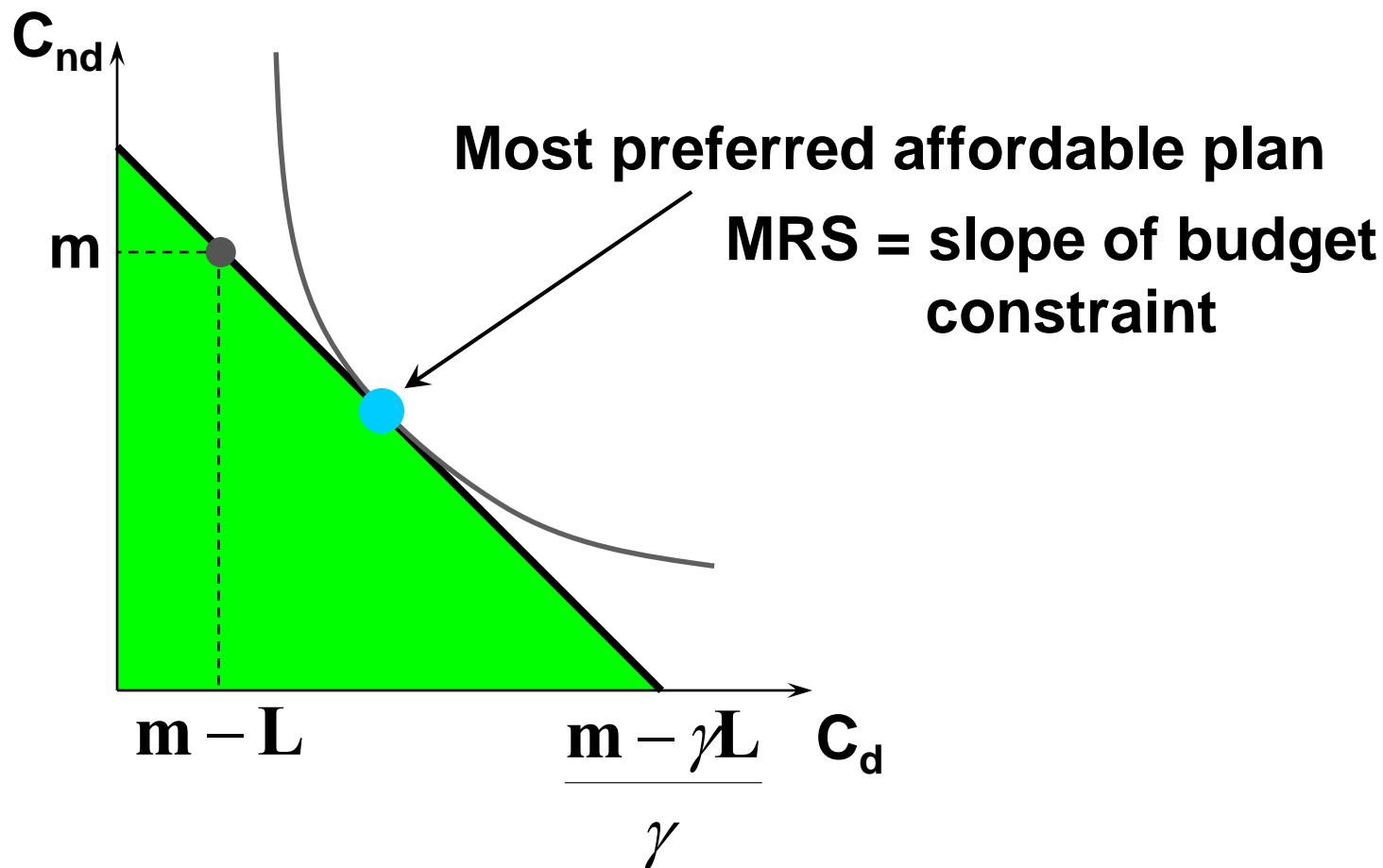
# State-Contingent Budget Constraints



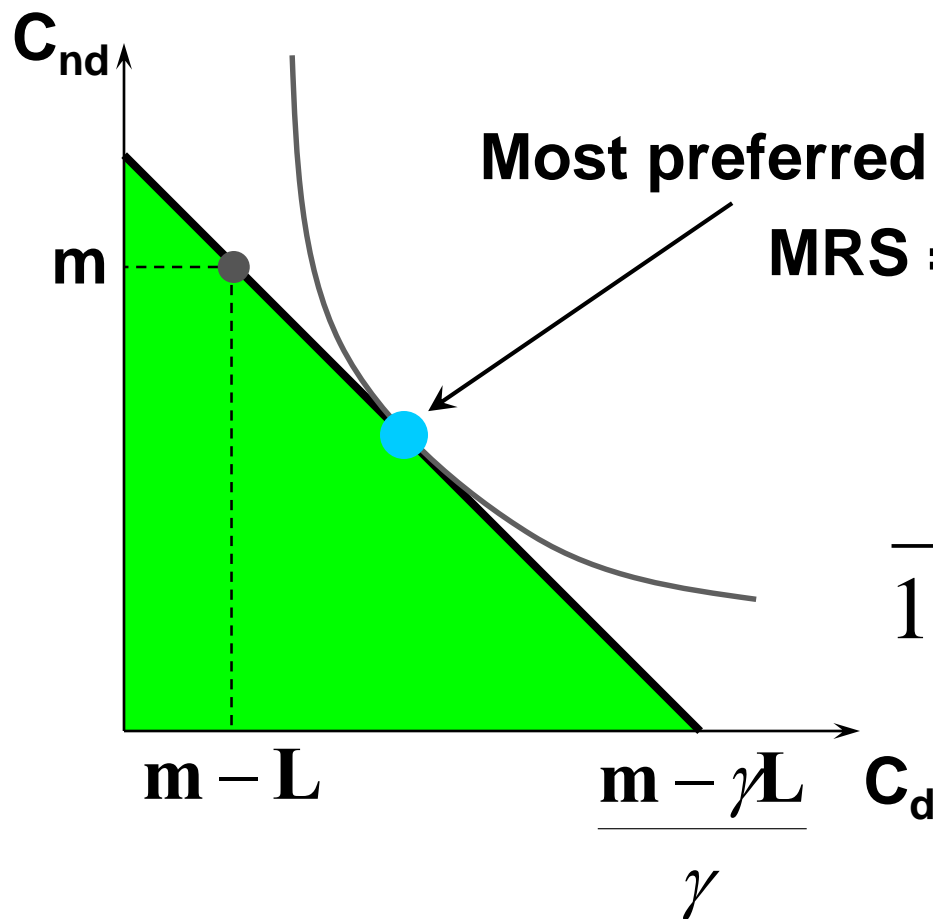
# State-Contingent Budget Constraints



# State-Contingent Budget Constraints



# State-Contingent Budget Constraints



**MRS = slope of budget constraint; i.e.**

$$\frac{\gamma}{1 - \gamma} = \frac{\pi_d \text{MU}(c_d)}{\pi_{nd} \text{MU}(c_{nd})}$$

# Competitive Insurance

- Suppose entry to the insurance industry is free.
- The insurance firm's profit function is given by:

$$\begin{aligned} & \gamma K - \pi_d K - (1 - \pi_d)0 \\ & = (\gamma - \pi_d)K \end{aligned}$$

# Competitive Insurance

- Since entry is free, firms freely enter until there are zero profits

$$(\gamma - \pi_d)K = 0.$$

- This means with free entry,  $\gamma = \pi_d$ .
- If price of \$1 insurance equals the probability of damage, therefore insurance is fair.

# Competitive Insurance

- The utility maximizing condition is that:

$$\frac{\gamma}{1 - \gamma} = \frac{\pi_d MU(c_d)}{\pi_{nd} MU(c_{nd})}$$

Since  $\gamma = \pi_d$ ,

$$\frac{\pi_d}{1 - \pi_d} = \frac{\pi_d MU(c_d)}{\pi_{nd} MU(c_{nd})}$$

$$\frac{\pi_d}{\pi_{nd}} = \frac{\pi_d MU(c_d)}{\pi_{nd} MU(c_{nd})}$$



# Competitive Insurance

$$\frac{\pi_d}{\pi_{nd}} = \frac{\pi_d MU(c_d)}{\pi_{nd} MU(c_{nd})}$$

- Therefore,  $\frac{MU(c_d)}{MU(c_{nd})} = 1$  or

$$MU(c_d) = MU(c_{nd})$$

Marginal utility of wealth must be the same in both states.

# Competitive Insurance

- How much fair insurance does a risk-averse consumer buy?
- For marginal utilities to be equal in both states, consumption must be equal in both states:  $c_1 = c_2$
- This means the consumer fully insures!

# Competitive Insurance

- The consumer will choose to fully insure if he or she is
  - risk adverse
  - maximizes Expected Utility
  - is offered fair insurance against loss

# “Unfair” Insurance

- Suppose insurers make positive expected economic profit.

$$\begin{aligned}\gamma K - \pi_d K - (1 - \pi_d)0 \\ = (\gamma - \pi_d)K > 0.\end{aligned}$$

Which means  $\gamma > \pi_d$

- The insurers charge more than their cost of insuring.

# “Unfair” Insurance

- Rational choice requires

$$\frac{\gamma}{1 - \gamma} = \frac{\pi_d MU(C_d)}{\pi_{nd} MU(C_{nd})}$$

- Since  $\gamma > \pi_d$

$$\frac{\gamma}{1 - \gamma} > \frac{\pi_d}{\pi_{nd}}$$

- which means  $\frac{MU(C_d)}{MU(C_{nd})} > 1$  so that the first equality can still hold. Therefore:

$$MU(C_d) > MU(C_{nd})$$

# “Unfair Insurance”

- Remember that this consumer is risk adverse. This means MU declines in wealth
- Since the marginal utility when there is damage is higher than the marginal utility when there is no damage:
$$C_d < C_{nd}$$
- In this case, the consumer only partially insures

# Uncertainty is Pervasive

- What are rational responses to uncertainty?
  - buying insurance (health, life, auto)
  - a portfolio of contingent consumption goods.

# Uncertainty is Pervasive

- What are rational responses to uncertainty?
- ✓
  - buying insurance (health, life, auto)
  - a portfolio of contingent consumption goods.



# Uncertainty is Pervasive

- What are rational responses to uncertainty?
- ✓ • buying insurance (health, life, auto)
- ? • a portfolio of contingent consumption goods.

# Diversification

- Firm A sells Chelsea FC jerseys and Firm B sell Manchester City FC jerseys
- Shares cost \$10.
- Either Chelsea or Manchester City will win the Premier League. Winning boosts sales of jerseys.

# Diversification

- With prob.  $\frac{1}{2}$ , Chelsea wins.
  - A' s profit is \$100 per share and B' s profit is \$20 per share.
- With prob.  $\frac{1}{2}$ , Man City wins.
  - A' s profit is \$20 per share and B' s profit is \$100 per share.
- You have \$100 to invest. How?

# Diversification

- Buy only firm A's stock?
- $\$100/10 = 10$  shares.
- You earn \$1000 with prob.  $1/2$  and \$200 with prob.  $1/2$ .
- Expected earning:  $\$500 + \$100 = \$600$

# Diversification

- Buy only firm B' s stock?
- $\$100/10 = 10$  shares.
- You earn \$1000 with prob.  $1/2$  and \$200 with prob.  $1/2$ .
- Expected earning:  $\$500 + \$100 = \$600$

# Diversification

- Buy 5 shares in each firm?
- You earn \$500 from one firm and \$100 from the other regardless of who wins the premier league title.
  - Either \$500 from A and \$100 from B
  - Or \$500 from B and \$100 from A
- Diversification has maintained expected earning and lowered risk.

# Diversification

- Diversification can lower expected earnings in exchange for lowered risk.
- Of course, in real life scenarios, outcomes are never perfectly correlated like in this example, making diversification less powerful of a tool for minimizing risk

# Risk Spreading

- 100 risk-neutral persons each independently risk a \$10,000 loss.
- Loss probability = 0.01.
- Initial wealth is \$40,000.
- No insurance: expected wealth is  
$$0.99 \times \$40,000 + 0.01(\$40,000 - \$10,000)$$
$$= \$39,900.$$



# Risk Spreading

- On average one person will suffer a loss each year.
- If all 100 people band together, they can each agree to pay \$100 to the person who suffers a loss.

# Risk Spreading

- Expected wealth is now:  
$$0.99 \times (\$39,900) + 0.01$$
$$\times (\$39,900 - 10,000 + 10,000)$$
- Every individual makes \$39,900 with certainty now. No risk! (assuming the probabilities of loss are true)