Problem Set #7

1. The Hawk-Dove game was first used by biologist John Maynard Smith to illustrate the uses of game theory in the theory of evolution. Males of certain species frequently come into conflict with other males over the opportunity to mate with females. In an encounter, a male can play "Hawk" and fight the other male until he wins or is badly hurt. Otherwise, he can play "Dove" by making a bold display but retreating if his opponent starts to fight. If two "Hawks" meet, both end up injured. If two "Doves" meet, they both strut their stuff until the female selects one at random or gets bored and wanders off. If one "Hawk" and one "Dove" meet, the Hawk gets the female and the "Dove" slinks off to celibate contemplation.

		Animal B	
		Hawk	Dove
Animal A	Hawk	-5,-5	10,0
	Dove	0,10	4,4

   a. If there is one Hawk and all other males are Doves, then what is the Hawk's payoff in every encounter?
   b. If all other males are Doves, what is the payoff of the remaining male if he is also a Dove?
   c. Use (a) and (b) to explain why an equilibrium cannot form where all males are Doves.
   d. If there is one Dove and all other males are Hawks, then what is the Dove's payoff in every encounter?
   e. If all other males are Hawks, what is the payoff of the remaining male if he is also a Hawk?
   f. Use (d) and (e) to explain why an equilibrium cannot form where all males are Hawks.
   g. Since there is not an equilibrium where all males adopt the same strategy, find an equilibrium where some fraction of the males are "Doves" and the rest are "Hawks." Use p to denote the proportion who "Dove" and $1 - p$ as the proportion who "Hawk."

2. Monica has a Porsche. She is leaving town for a two-day cooking convention and will not need the car during this time. Joey wants to impress the ladies and is interested in renting Monica's sports car while she is away. The value of Joey having the car is $50 per day. Monica figures the total cost of letting Joey have her car is $35 regardless of how many days Joey uses it. On the evening before the first of these two days, Monica can send a message to Joey, offering to rent the car to him for two days at a specified price. Joey can either accept the offer or reject the offer and make a counteroffer.

   The only problem is that it takes a full day for a counteroffer to be made and accepted. If Joey rejects the original offer, then the car can only be rented for one day and there will be no time for Monica to make a counteroffer.

   Here we will walk through solving the Rubinstein bargaining solution to this problem. Assume that if any player is indifferent between accepting an offer, he or she chooses to do what the other player prefers. Also assume that both players have a discount rate of 0. They value future gains equally as present gains.

   a. If Joey rejects the offer, how much must he offer Monica in order for her to accept?
   b. How much does Joey benefit from this offer?
   c. Since Monica knows that Joey values the car at $50 per day, she knows she must offer him a surplus greater than the answer in (b), otherwise he will reject the offer. Since Joey values two days of car rental at $100, how much will Monica offer the car to Joey for in the first round.
   d. Your answer in (c) is what Monica will offer Joey in the first round. What are each player's surpluses from this bargaining solution?

3. Suppose the same scenario as Question 2 occurs but now Monica will be out of town for three days and Joey, rather than Monica makes the first offer. Additionally, Monica's total cost is now $20, rather than $35. To find the Rubinstein bargaining solution, start by solving the final round, then working your way up.
   a. Joey makes the offer in the final round. What will he offer Monica and why?
   b. Knowing that Joey will make the offer in (a), what will Monica offer Joey in the second round and why?
   c. Knowing that Monica will make Joey the offer in (b), what will Joey offer Monica in the first round and why? What is Joey's and Monica's respective eventual payoffs from this solution?

4. Now suppose that Monica's convention is for four days, not three. Monica's cost for renting out the car is $20. Joey makes the first offer. Solve for the Rubinstein Bargaining solution. Be sure to note what each player's payoff is from this solution.