Problem Set #5

1. Consider the normal-form game below:

			James	
		Left	Center	Right
	Top	0,2	3,1	2,3
Julia	Middle	1,3	4,4	4,1
	Bottom	2,1	2,1	3,2

   a. What are the three "essential elements" of any game?
      Players, Strategies, Payoffs
   b. Who are the players of the above game, and what are their respective actions?
      Julia: Top; Middle; Bottom
      James: Left; Right; Center
   c. Is there a dominant strategy equilibrium to this game? If so, what is it?
      There are no dominant strategy equilibria in this game. Neither player has a dominant strategy in this game
   d. Are there any Nash Equilibria? If so, what are they, and are they Pareto optimal?
      There is one Nash equilibrium: {Julia: Middle; James: Center} and it is Pareto optimal.

2. Consider the following normal representation of a game:

		Player B	
		Left	Right
Player A	Top	a,b	c,d
	Bottom	e,f	g,h

   a. If (top, left) is a dominant strategy equilibrium, then we know that:
      i. a > e
      ii. b > d
      iii. c > g
      iv. f > h
   b. If (top, left) is a Nash equilibrium, then which of the above inequalities must be satisfied?
      i. a > e
      ii. b > d
   c. Based on your answers in (a) and (b), briefly explain why a dominant strategy equilibrium must be a Nash equilibrium.
      Since the two conditions in (b) are satisfied in (a), the dominant strategy equilibrium must be a Nash equilibrium
   d. If (bottom, right) is a Nash equilibrium, then we know that:
      i. g > c
      ii. h > f

3. An investor has approached everyone in this module and made an offer. There are M students in the module. Every student can contribute \$10 to a fund that earns X times the amount contributed by the end of the semester. At the end of the semester, the total value of the fund will be divided equally between all students regardless of how much a student contributed to the fund.

   For example, if there are 2 students in the module and $X = 1.8$, then if both students contribute to the fund, they each receive \$18 at the end of the semester. If only one student contributes to the fund, then the fund has \$18 at the end of semester, and that \$18 is split equally between the students leaving the student who contributed with just \$9, and the student who did not contribute with \$19.

   a. If X is greater than one, what is the Pareto optimal outcome in this game?
   All students contributing their money to the fund (assuming we know nothing about how X and M are related.
   b. If only L students contribute, then what is the payoff to a student who does not contribute?
   $$10 + \frac{10XL}{M} \text{ or } \frac{10XL}{M}$$
   c. L students are currently contributing. If one student who does not contribute changes his mind and decides to contribute, what is his payoff?
   $$\frac{10X(L+1)}{M} \text{ or } \frac{10X(L+1)}{M} - 10$$
   d. If $X = 5$ and $M = 4$, what is the Nash equilibrium outcome of this game?
   If someone does not contribute, their payoff is $10 + \frac{10(5)L}{4} = 10 + 12.5L$
   If someone does contribute, their payoff is $\frac{10(5)(L+1)}{4} = 12.5 + 12.5L$
   Someone who does contribute always gets a payoff that is \$2.5 higher than someone who does not contribute. In equilibrium, everyone contributes.
   e. If $X = 3$ and $M = 4$, what is the Nash equilibrium outcome of this game?
   If someone does not contribute, their payoff is $10 + \frac{10(3)L}{4} = 10 + 7.5L$
   If someone does contribute, their payoff is $\frac{10(3)(L+1)}{4} = 7.5 + 7.5L$
   Someone who does not contribute always gets a payoff that is \$2.5 higher than someone who does contribute. In equilibrium, no one contributes

   f. Use your answer in (b) and (c) to provide an expression (in terms of M and X) that guarantees a dominant strategy equilibrium where no student contributes to the fund
   $$10 + \frac{10XL}{M} > \frac{10X(L+1)}{M}$$
   $$10M + 10XL > 10X(L+1)$$
   $$10M + 10XL > 10XL + 10X$$
   $$\frac{M}{X} > 1$$

4. MacLaren's is a pub in New York City that many people visit to socialize. Individuals are more likely to frequent the bar if they expect other people to be there. In addition, five individuals (Barney, Robin, Ted, Marshall and Lily) are die-hard

supporters of the bar and are there every night. Let E be the number of customers people expected to be at MacLaren's on a given evening. The number of people who actually frequent the establishment on a given night is $A = 5 + 0.5E$. In equilibrium, it must be true that the number of people expected at the pub and the number of people who frequent are the same.

a. What two equations must be solved to obtain the equilibrium number of people who frequent MacLaren's on a given night?

$$A = 5 + 0.5E$$
$$A = E$$

b. What is the equilibrium attendance at MacLaren's?

$$A = 5 + 0.5A$$
$$0.5A = 5$$
$$A = 10$$

c. Suppose one additional person (Tracy) becomes a die-hard supporter of the pub. Write down the new equations determining the equilibrium attendance and solve for that equilibrium.

$$A = 6 + 0.5E$$
$$A = E$$
$$A = 6 + 0.5A$$
$$0.5A = 6$$
$$A = 12$$

d. Now suppose that everyone bases expectations about tonight's attendance on last night's attendance and that last night's attendance is announced to everyone via MetroNews One every morning. Then $E_t = A_{t-1}$, where t is a day, and $t - 1$ is the day preceding it. On day one, 10 people are in attendance at MacLaren's. What is attendance on the second day, third day and fourth day? Assume Tracy is currently a die-hard supporter of the pub.

$$A_2 = 6 + 0.5(10) = 11$$
$$A_3 = 6 + 0.5(11) = 11.5$$
$$A_4 = 6 + 0.5(11.5) = 11.75$$

e. What is the limiting value of the attendance in (d)?

$$\begin{aligned} L &= \lim_{t\to\infty} A_t \\ &= \lim_{t\to\infty} 6 + 0.5A_{t-1} \\ &= 6 + 0.5 \lim_{t\to\infty} A_{t-1} \end{aligned}$$
$$L = 6 + 0.5(L)$$
$$L = \frac{6}{0.5} = 12$$