Homework #2

1. Consider the video clip from A Beautiful Mind (the link is provided here and is also available on IVLE). Explain why the description of the Nash equilibrium in the video clip is wrong by listing the following:
   a. (2 points) An explanation of why the outcome described in the scene is not a Nash equilibrium.
      If everyone talks to the brunettes, there is a profitable deviation to talk to the blonde.
   b. (2 points) What the correct Nash equilibrium is (HINT: think about unilateral incentives to deviate).
      One person talks to the blonde and everyone else talks to the brunettes
   c. (1 point) Whether or not the outcome is efficient, and if not, what is the Pareto efficient outcome?
      The outcome Nash (Russell Crowe) describes is not efficient, the real Nash equilibrium is efficient

   Your explanation <u>must not</u> exceed half a page of an A4 sized paper in length and will be penalized otherwise. Please type in Times New Roman 12-point font, 1.5 line spacing.

2. Consider the game "matching pennies." In this game, two players toss coins simultaneously and if the face of the two coins (heads or tails) match, Player 1 gets to keep the coins. If the coins do not match, then Player 2 gets to keep coins.

		Player 2: Head	Player 2: Tail
Player 1	Head	<u>+1</u>, -1	-1, +1
	Tail	-1, +1	<u>+1</u> , -1

   a. (1 point) In the payoff matrix, underline the payoffs to Player 1 corresponding to the best responses of Player 1 and circle the payoffs to Player 2 corresponding to the best responses for Player 2.
   b. (3 points) Solve for all the Nash equilibria in this game:
      There are no pure strategy Nash equilibria.
      Mixed strategy Nash equilibria:
      Let Player 1 play heads with probability p and Player 2 play heads with probability q
      Player 1 is indifferent between heads and tails when
      $$q-(1-q)=-q+(1-q)$$
      $$q=\frac{1}{2}$$
      Player 2 is indifferent between heads and tails when
      $$p-(1-p)=-p+(1-p)$$
      $$p=\frac{1}{2}$$

   How do risk attitudes affect the outcome in the matching pennies game? Suppose Player 1 is risk neutral, that is, he is indifferent between receiving nothing with certainty and the lottery that pays one penny with probability one half and costs one penny with probability one half. Player 2 is indifferent between receiving K with certainty and the lottery that pays one penny with probability one half and costs one penny with probability one half (K may be negative.) Normalize both players' von Neumann-Morgenstern utility function so that the payoff for losing is -1

c. (2 points) Compute the payoff for winning as a function of K. Use "$U(x)$" to denote the utility of Player 2.

$$0.5U(1) + 0.5U(-1) = U(K)$$

Since $U(-1) = -1$,

$$0.5U(1) - 0.5 = U(K)$$
$$U(1) = 2U(K) + 1$$

d. (3 points) Compute the equilibrium of the game for each possible value of K

There are still no pure strategy Nash equilibria in this game.

Player 1 is still indifferent between heads and tails when $q = \frac{1}{2}$

Player 2 is indifferent between heads and tails when:

$$(2U(K) + 1)p - (1 - p) = -p + (2U(K) + 1)(1 - p)$$
$$(2U(K) + 2)p - 1 = -(2U(K) + 2)p + 2U(K) + 1$$
$$2(2U(K) + 2)p = 2U(K) + 2$$
$$p = \frac{1}{2}$$

The result is independent of K. The same equilibrium occurs as when Player 2 is risk neutral.

3. Two criminals have just made away with \$10. Both are profit maximizers. Criminal 1 proposes how to split the winnings and Criminal 2 can accept or reject the offer. If the offer is accepted, both criminals receive the proposed share, otherwise, they get into a huge argument during which a dog runs by and snags all the money leaving both criminals with nothing. Assume that player 2 accepts the offer if he is indifferent.

a. (1 point) Let x_1 denote Criminal 1's takings and x_2 denote Criminal 2's takings. What is the equilibrium allocation of this game?

$$x_1 = 10, x_2 = 0$$

b. (2 points) Now suppose Criminal 2 is spiteful. He takes displeasure in Criminal 1's takings and his payoff is defined by $x_2 - \frac{1}{2}x_1$. What is the equilibrium in this game?

If Criminal 2 rejects the offer, his payoff is 0. Criminal 1 must offer a split that equals that so $x_2 - \frac{1}{2}x_1 = 0$

Since $x_1 + x_2 = 10,\ x_2 - \frac{1}{2}(10 - x_2) = 0.$

$$\frac{3}{2}x_2 = 5$$
$$x_2 = \frac{10}{3}$$
$$x_1 = \frac{20}{3}$$

c. (1 point) Now suppose that Criminal 2 is not spiteful but Criminal 1 is altruistic, that is, suppose Criminal 1's payoff is $ln(x_1) + 0.5ln(x_2)$. What is the equilibrium?

Criminal 2 will accept any positive value of x_2. Criminal 1 will maximize his payoff subject to the constraint $x_1 + x_2 = 10$

$$\frac{\partial U}{\partial x_1} = \frac{1}{x_1} - \frac{0.5}{10 - x_1} = 0$$
$$10 - x_1 = 0.5x_1$$

$$x_1 = \frac{20}{3}$$
$$x_2 = \frac{10}{3}$$