Problem Set #3

1.
    a. The monopolist faces a demand curve given by $Q = 120 - 2P$ and a cost function of $C = 2Q$. What is the optimal level of output and price?
    b. The monopolist faces a demand curve given by $Q = 7p^{-3}$ and a cost function of $C = 2Q$. What is the optimal level of output and price?
    c. The monopolist faces a demand curve given by $Q = \frac{100}{P}$ and a cost function of $C = 4Q^2$. Calculate the price elasticity of demand. Next, calculate the optimal level of output for the monopolist.

2. A monopolist has an inverse demand curve given by $P = 24 - Q$ and a cost curve given by $C = 2Q^2$.
    a. Write down the monopolist's profit function.
    b. Calculate the firm's profit maximizing level of output.
    c. Suppose the government imposes a tax of $9 per unit of output. Write the monopolist's profit function given this tax.
    d. Calculate the monopolist's profit maximizing level of output and profits given this tax. Also calculate government revenue.
    e. Now suppose the government imposes a lump sum tax of $22.5 on the firm. Write out the firm's profit function given this tax.
    f. Calculate the firm's profit maximizing level of output as well as the firm's profits. Also calculate government revenue.
    g. Draw some implications from the two different tax schemes?

3. In North Pole, Alaska, there is only one gossip magazine, *The Reindeer Scoop*. The demand for this magazine depends on the price and the amount of scandal reported. The demand function is $Q = 15S^{\frac{1}{2}}P^{-3}$, where Q is the number of issues sold per day, S is the number of column inches of scandal reported in the paper, and P is the price. Scandals are not a scarce commodity in North Pole, Alaska (especially because of the elves). However, it takes resources to write, edit, and print stories of scandal. The cost of reporting S units of scandal is $\$10S$. These costs are independent of the number of magazines sold. In addition, it costs $0.10 to print and distribute each magazine. Therefore total costs are $10S + 0.1Q$.
    a. Calculate the price elasticity of demand for $The\ Reindeer\ Scoop$.
    b. Calculate the profit maximizing price, making use of the identity $MR = P(1 + \frac{1}{\epsilon})$.
    c. How many copies of the magazine will sell if there are 100 column inches of scandal in it? (Round to the nearest integer) Write the general expression for the number of copies sold as a function of S (i.e. $Q(S) = \cdots$)
    d. Write out the magazine company's profit function as a function of S assuming that the price it charges is the one calculated in (b)
    e. Make use of your answer in (d) to calculate the profit maximizing amount of scandal.

Solutions

1.

a. $P = 60 - \frac{Q}{2}$

$$\pi = \left(60 - \frac{Q}{2}\right)Q - 2Q$$

$$\frac{\partial \pi}{\partial Q} = 60 - Q - 2 = 0$$

$$Q = 58$$

$$P = 60 - \frac{58}{2} = 31$$

b. $P = \frac{7^{\frac{1}{3}}}{Q^{\frac{1}{3}}}$

$$\pi = \frac{7^{\frac{1}{3}}}{Q^{\frac{1}{3}}}Q - 2Q$$

$$\frac{\partial \pi}{\partial Q} = 2\left(\frac{2}{3}\right)Q^{\frac{-1}{3}} - 2 = 0$$

$$\frac{7^{\frac{1}{3}}\left(\frac{2}{3}\right)}{Q^{\frac{1}{3}}} = 2$$

$$Q = 7\left(\frac{1}{3}\right)^{3} = \frac{7}{27}$$

$$P = \frac{7^{\frac{1}{3}}}{\left(\frac{7}{27}\right)} = 3$$

c. $\epsilon_d = \frac{\partial Q}{\partial P}\frac{P}{Q} = -\frac{100}{P^2}\frac{P}{Q}$

$$= -\frac{100}{PQ}$$

$$= \frac{100}{P}\frac{P}{100} = -1$$

Since $\epsilon_d = -1$, $MR = P\left(1 - \frac{1}{|\epsilon|}\right) = 0$

Marginal revenue is zero everywhere and marginal cost will only equal zero where $Q = 0$.

Revenue is constant at 100 as long as the firm produces a positive quantity, so the firm will want to minimize cost but avoid producing zero. This occurs where Q is the smallest possible number closest to zero.

2.

a. $\pi = (24 - Q)Q - 2Q^2$

b. $\frac{\partial \pi}{\partial Q} = 24 - 2Q - 4Q = 0$

$$24 = 6Q$$

$$Q = 4$$

c. $\pi = (24 - Q - 9)Q - 2Q^2$

$\pi = (15 - Q)Q - 2Q^2$

d. $\frac{\partial\pi}{\partial Q} = 15 - 2Q - 4Q = 0$

$$15 = 6Q$$
$$Q = 2.5$$

$\pi = (15 - 2.5)2.5 - 2(2.5)^2 = 18.75$

$Government\ Revenue = 9 \times 2.5 = 22.5$

e. $\pi = (24 - Q)Q - 2Q^2 - 22.5$

f. $\frac{\partial\pi}{\partial Q} = 24 - 2Q - 4Q = 0$

$$24 = 6Q$$
$$Q = 4$$

$\pi = (24 - 4)4 - 2(4)^2 - 22.5$
$= 80 - 32 - 22.5 = 25.5$
$Government\ Revenue = 22.5$

g. The per-unit tax reduces the quantity produced by the monopolist while the lump-sum tax is non-distortionary (it does not affect the quantity decision of the firm, as long as the lump-sum tax does not exceed the firm's profit, i.e. the firm doesn't want to shut down)
The lump-sum tax raises the same amount of revenue while allowing for a larger quantity to be produced and higher profits for the firm.

3.

a. $\varepsilon_d = \frac{\partial Q}{\partial P}\frac{P}{Q} = 15S^{\frac{1}{2}}(-3)(P^{-4})\left(\frac{P}{Q}\right)$
$= -\dfrac{45S^{\frac{1}{2}}P^{-3}}{15S^{\frac{1}{2}}P^{-3}} = -3$

b. $MR = P\left(1 - \frac{1}{3}\right) = \frac{2}{3}P$
$MC = \dfrac{1}{10}$
$MR = MC$
$\dfrac{2}{3}P = \dfrac{1}{10}$
$P = \dfrac{3}{20} = 0.15$

c. $Q = 15\sqrt{100}(0.15^3)$
$= 44{,}444$

$Q = 4444.44\sqrt{S}$

d. $\pi = 0.15Q - 10S - 0.1Q$
$= 0.05(4444.44\sqrt{S} - 10S$
$= 222.22\sqrt{S} - 10S$

e. $\frac{\partial\pi}{\partial S} = \frac{1}{2}(222.22)S^{\frac{1}{2}} - 10 = 0$

$$S = \frac{111.11}{10} = 123.457$$