EC3101: Problem Set #1 for Week 3 Discussion Groups

1. Calculate the following:
    a. The present value of $1,000,000 delivered 20 years in the future if the interest rate is 20%.
    b. The value of $1,000,000 20 years from now if the interest rate is 15%
    c. The present value of a bond which pays a coupon of $100 and has a face value of $1000. The bond matures after 10 years and the interest rate is 7%.
    d. The present value of a consol that pays $4530 annually with an interest rate of 4%.

2. Nickelby has an income of $2,000 this year and expects an income of $1,100 next year. He can borrow and lend money at an interest rate of 10%. Consumption goods cost $1 per unit and there is no inflation.
    a. What is the present value of his endowment?
    b. What is the future value of his endowment?
    c. Write out Nickelby's budget line in "slope and intercept" form (y=mx+c)

3. Suppose Nickelby has the following utility function: $U(c_1, c_2) = C_1 C_2$.
    a. Write an expression for Nickelby's marginal rate of substitution between consumption this year and consumption next year.
    b. Solve the two equations that determine Nickelby's utility maximizing consumption in each period.

4. Seng Kang has a utility function of the form $U(c) = \sqrt{c}$. He is currently a manager at Sheng Siong and earns $50,000 a year. His favorite pastime is playing football and the Tampines Rovers have just made him an offer to play for them professionally. They will pay him $100,000 a year but there is a 20% probability that he will be injured badly enough to render him unable to do any kind of work.
    a. What is Seng Kang's expected utility if he chooses to play for the Rovers?
    b. Will Seng Kang leave his job at Sheng Siong to play for the Rovers?

5. If Seng Kang pays $p for an insurance policy that would give him $100,000 if he suffers a career ending injury while in college, then he would be sure to have an income of $100,000 - p$ no matter what happened to him. Write an equation that can be solved to find the largest price that Seng Kang will be willing to pay for such an insurance policy. Then, solve for p.

Solutions:

1.

    a. $\frac{1,000,000}{1.2^{20}} = 26,084.05$

    b. $1,000,000 \times 1.15^{20} = 16,366,537.4$

    c. $\frac{100}{1.07} + \frac{100}{1.07^2} + \frac{100}{1.07^3} + \cdots + \frac{100}{1.07^9} + \frac{1100}{1.07^{10}} = 1210.71$

    d. $\frac{4530}{.04} = 113,250$

2.

    a. $PV = \$2,000 + \frac{\$1,100}{1.1} = \$3000$

    b. $FV = \$2,000(1.1) + \$1,100 = \$3,300$

    c. $1.1\,c_1 + c_2 = \$3,300$

       $c_2 = -1.1c_1 + 3300$

3.

    a. $MRS = -\frac{MU_{c_1}}{MU_{c_2}} = -\frac{c_2}{c_1}$

    b. $-\frac{c_2}{c_1} = -1.1$

       $c_1 = \dfrac{1}{1.1}c_2$

       Pugging this expressions into 2c, $c_2 = -1.1\left(\frac{1}{1.1}c_2\right) + 3300$

$$c_2 = -c_2 + 3300$$
$$c_2 = 1650$$
$$c_1 = \frac{1650}{1.1} = 1500$$

4.

    a. $0.8\sqrt{100,000} + 0.2\,(0) = 252.98$

    b. Expected utility from working at Sheng Siong is $\sqrt{50000} = 223.6$

       Since his expected utility from playing for the Rovers is higher, he will quit his job at Sheng Siong.

5. Seng Kang will be indifferent between buying the insurance policy and not purchasing the policy when

$$\sqrt{100,000 - p} = 0.8\sqrt{100,000}$$
$$\sqrt{100,000 - p} = 252.98$$

$$100{,}000 - p = 252.98^2$$
$$p = 36{,}000$$