Homework #1

1. Emmanuel Saez has the utility function $U(c_1, c_2) = c_1^{\frac{1}{2}} + 2c_2^{\frac{1}{2}}$ where c_1 is his consumption in period 1 and c_1 is his consumption in period 2. He will earn 100 units of the consumption good in period 1 and 100 units of the consumption good in period 2. He can borrow or lend at an interest rate of 10%.
   a. (1 point) Write an equation that describes Emmanuel's budget in terms of c_1 and c_2.
   $$1.1c_1 + c_2 = 110 + 100$$
   $$1.1c_1 + c_2 = 210$$
   b. (1 point) If Emmanuel neither borrows nor lends, what will be his marginal rate of substitution between current and future consumption?
   $$MRS_{c_1,c_2} = \frac{\sqrt{c_2}}{2\sqrt{c_1}} = \frac{10}{20} = \frac{1}{2}$$
   c. (1 point) If Emmanuel does the optimal amount of borrowing or saving, what will be the ratio of his period 2 consumption to his period 1 consumption?
   $$MRS_{c_1,c_2} = (1+r)$$
   $$\frac{\sqrt{c_2}}{2\sqrt{c_1}} = 1.1$$
   $$\sqrt{c_2} = 2.2\sqrt{c_1}$$
   $$c_2 = 2.2^2 c_1$$
   $$c_2 = 4.84c_1$$

2. Stone and Neumark consume two goods, b and f. Stone has an initial endowment of 4 b and 8 f. Neumark has an endowment of 2 b and 6 f. Stone's utility function is $U_s(b,f) = \ln(b_s) + \ln(f_s)$. Neumark's utility function is $U_n(b,f) = b_n f_n$.
   a. (1.5 points) Calculate the marginal rates of substitution for Stone and Neumark. Do you notice anything peculiar about these MRS? Explain.
   $$MRS_s = \frac{f_s}{b_s}, MRS_n = \frac{f_n}{b_n}$$

   The MRS of both agents are the same, even though the utility functions are different. This is because Stone's utility function is the log transformation of Neumark's.

   b. (2.5 points) Assume there are 2000 people in the economy, half have preferences like Stone and the rest have preferences like Neumark. Write out the conditions that satisfy the competitive equilibrium.

   $$\frac{MU_{b_s}}{MU_{f_s}} = \frac{MU_{b_n}}{MU_{f_n}} = \frac{p_b}{p_f} \rightarrow \frac{f_s}{b_s} = \frac{f_n}{b_n} = \frac{p_b}{p_f}$$

   $$p_b b_s + p_f f_s = 4p_b + 8p_f$$

   $$p_b b_n + p_f f_n = 2p_b + 6p_f$$

   $$b_s + b_n = 6$$

   $$f_s + f_n = 14$$

Also acceptable to write the constraints in terms of the two thousand people in the economy rather than for two representative agents.

c. (6 points) Let good b be the numeraire good. Calculate the competitive equilibrium prices and quantities.

$$\frac{f_s}{b_s} = \frac{1}{p_f} \rightarrow f_s = \frac{b_s}{p_f}$$
$$\frac{f_n}{b_n} = \frac{p_b}{p_f} \rightarrow f_n = \frac{b_n}{p_f}$$

Plugging these into the respective budget constraints, we have:

$$2b_s = 4 + 8p_f$$
$$b_s = 2 + 4p_f$$

$$2b_n = 2 + 6p_f$$
$$b_n = 1 + 3p_f$$

Plugging into the feasibility constraint:

$$2 + 4p_f + 1 + 3p_f = 6$$
$$p_f = \frac{3}{7}$$

Using $p_f = \frac{3}{7}$, we have:

$$b_s = 2 + 4\left(\frac{3}{7}\right) = \frac{26}{7}, b_n = 1 + 3\left(\frac{3}{7}\right) = \frac{16}{7}, f_s = \frac{26}{7}\left(\frac{7}{3}\right) = \frac{26}{3}, f_n = \frac{16}{7}\left(\frac{7}{3}\right) = \frac{16}{3}.$$

d. (1 point) Briefly provide an explanation for the difference in the prices of the two goods found in (c).

Since both consumers value goods b and f similarly, the price only reflects the scarcity of the good. b is less abundant, thus more expensive.

3. A monopolist faces a demand curve given by $Q = \frac{100}{P}$ and a cost function of $C = 4Q^2$.

a. (1 point) Calculate price elasticity of demand

$$\varepsilon_d = \frac{\partial Q}{\partial P}\frac{P}{Q} = -\frac{100}{P^2}\frac{P}{Q}$$

$$= -\frac{100}{PQ}$$
$$= -\frac{100}{P}\left(\frac{P}{100}\right) = -1$$

b. (2 points) Calculate the optimal level of output for the monopolist.

Since $\epsilon_d = -1, MR = P\left(1 - \frac{1}{|\epsilon|}\right) = 0.$

Since revenue is constant at 100, the firm will maximize profits at Q as close to zero as possible, since that will minimize costs.

c. (1 point) Now assume the demand curve is given by $Q = 100 - P$. Calculate the price elasticity of demand as a function of P.

$$\frac{\partial Q}{\partial P} = -1$$

$$\varepsilon_d = \frac{\partial Q}{\partial P}\frac{P}{Q} = -1\left(\frac{P}{100 - P}\right)$$

d. (2 points)Calculate the profit maximizing price and level of output.

$$MR = 100 - 2Q$$

$$MC = 8Q$$

$$100 - 2Q = 8Q$$

$$Q = 10$$

$$P = 100 - 10 = 90$$