Homework #1

1. Emmanuel Saez has the utility function $U(c_1, c_2) = c_1^{\frac{1}{2}} + 2c_2^{\frac{1}{2}}$ where $c_1$ is his consumption in period 1 and $c_1$ is his consumption in period 2. He will earn 100 units of the consumption good in period 1 and 100 units of the consumption good in period 2. He can borrow or lend at an interest rate of 10%.
    a. (1 point) Write an equation that describes Emmanuel's budget in terms of $c_1$ and $c_2$ and the future value of his earnings.
    b. (1 point) If Emmanuel neither borrows nor lends, what will be his marginal rate of substitution between current and future consumption?
    c. (1 point) If Emmanuel does the optimal amount of borrowing or saving, what will be the ratio of his period 2 consumption to his period 1 consumption?

2. Stone and Neumark consume two goods, $b$ and $f$. Stone has an initial endowment of 4 $b$ and 8 $f$. Neumark has an endowment of 2 $b$ and 6 $f$. Stone's utility function is $U_s(b, f) = \ln(b_s) + \ln(f_s)$. Neumark's utility function is $U_n(b, f) = b_n f_n$.
    a. (1.5 points) Calculate the marginal rates of substitution for Stone and Neumark. Do you notice anything peculiar about their MRS's?
    b. (2.5 points) Assume there are 2000 people in the economy, half have preferences like Stone and the rest have preferences like Neumark. Write out the conditions that satisfy the competitive equilibrium.
    c. (6 points) Let good $b$ be the numeraire good. Calculate the competitive equilibrium prices and quantities.
    d. (1 point) Briefly provide an explanation for the difference in the prices of the two goods found in (c).

3. A monopolist faces a demand curve given by $Q = \frac{100}{P}$ and a cost function of $C = 4Q^2$.
    a. (1 point) Calculate price elasticity of demand
    b. (2 points) Calculate the optimal level of output for the monopolist.
    c. (1 point) Now assume the demand curve is given by $Q = 100 - P$. Calculate the price elasticity of demand as a function of $P$.
    d. (2 points) Calculate the profit maximizing price and level of output.