Problem Set #8

1. Joey has a sandwich shop located next to Chandler's cigarette store in a small shopping mall. The number of customers who frequent the mall to shop at either store depends on the amount of money the store spends on advertising in a day. If Joey spends $\$x_j$ dollars on advertising per day and Chandler spends $\$x_c$ dollars on advertising per day, then the total profits per day Joey's shop is $\Pi_j(x_j, x_c) = (60 + x_c)x_j - 2x_j^2$ and the total profits at Chandler's store is $\Pi_c(x_j, x_c) = (105 + x_j)x_c - 2x_c^2$
   a. Do the firms impose positive or negative externalities on each other? What term in the profit function helps you identify this?

      Positive externalities, the other firm's advertising <u>adds</u> to your profits

   b. If each firm chooses its advertising to maximize its profits independent of the other firm's decision on advertising, how much advertising does each firm choose and how much profits does each firm make?

$$\Pi_j(x_j, x_c) = (60 + x_c)x_j - 2x_j^2$$
$$\frac{\partial \Pi_j}{\partial x_j} = 60 + x_c - 4x_j = 0$$
$$x_j = \frac{60 + x_c}{4}$$

$$\Pi_c(x_j, x_c) = (105 + x_j)x_c - 2x_c^2$$
$$\frac{\partial \Pi_c}{\partial x_c} = 105 + x_j - 4x_c = 0$$
$$x_c = \frac{105 + x_j}{4}$$

Solving simultaneously,

$$x_j = 23$$
$$x_c = 32$$

$$\Pi_j = (60 + 32)(23) - 2(23^2) = \$1058$$
$$\Pi_c = (105 + 23)(32) - 2(32^2) = \$2048$$

   c. At the equilibrium levels of advertising you calculated in (b) what is the effect that Chandler's advertising has on Joey's firm's profits and what is the effect that Joey's advertising has on Chandler's firm's profits? What insight does this finding tell you?

$$\frac{\partial \Pi_j}{\partial x_c} = x_j = 23$$
$$\frac{\partial \Pi_c}{\partial x_j} = x_c = 32$$

   d. If the two firms merged, how much advertising would be spent on sandwiches and cigarettes respectively? Is this more or less than the amount spent by the firms in (b)?

$$\Pi_m = \Pi_j + \Pi_c = (60 + x_c)x_j - 2x_j^2 + (105 + x_j)x_c - 2x_c^2$$

$$\frac{\partial \Pi_m}{\partial x_j} = 60 + x_c - 4x_j + x_c = 0$$
$$x_j = \frac{60 + 2x_c}{4}$$
$$\frac{\partial \Pi_m}{\partial x_j} = 105 + x_j - 4x_c + x_j = 0$$
$$x_c = \frac{105 + 2x_j}{4}$$

Solving simultaneously we have, $x_j = 37.5$ and $x_c = 45$ which is more than the amount spent in (b)

e. Without calculating profits, how do we know if profits in (d) are higher or lower than total profits in (b)

Profits must be higher in (d) because the merger internalizes the externality.

2. There are 1001 residents in the city of Radiator Springs. People in this town enjoy doing two things, driving around and eating In-N-Out burgers. Just like in many other cities around the world, while everyone likes to drive, they also like to complain about the congestion, noise and pollution caused by traffic. A typical resident's utility function is $U(b,d,h) = b + 16d - d^2 - \frac{6h}{1000}$, where b is consumption of burgers, d is number of hours of driving and h is the total amount of driving by other residents in Radiator Springs. The price of an In-N-Out burger is \$1. Each person has an income of \$40 per day. To simplify calculations, we assume, it costs nothing to drive a car.

a. If an individual maximizes his utility by only choosing his amount of driving, how much will he drive in a day?

$$\frac{\partial U(b,d,h)}{\partial d} = 16 - 2d = 0$$
$$d = 8$$

b. If all individuals in the city behave as in (a), what is the amount of h that each individual faces?

$$h = 1000 \times d = 8000$$

c. What is the utility of each resident in Radiator Springs?

$$U(b,d,h) = 40 + 16(8) - 8^2 - 6(8) = 56$$

d. If everyone drives 7 hours a day, what is the utility of each resident?

$$U(b,d,h) = 40 + 16(7) - 7^2 - 6(7) = 61$$

e. The mayor of Radiator Spring, is an economist. He realizes he can make everyone better off by restricting the amount of daily driving of each resident. How much driving should each resident be allowed to maximize the utility of each resident? (Hint: how can you express the residents' utility function such that when they maximize utility, they internalize the externality?)

We need to internalize the externality. Since $h = 1000 \times d$, let

$$U(b,d,h) = b + 16d - d^2 - \frac{6(1000d)}{1000}$$
$$= b + 10d - d^2$$

$$\frac{\partial U(b,d,h)}{\partial d} = 10 - 2d = 0$$
$$d = 5$$

f. If the mayor wanted to impose the same outcome as (e) by means of a tax on driving, how much would that tax on driving have to be?
The tax each individual faces must equal the cost he bears on everyone else by driving.
The externality, $e = \frac{6h}{1000}$. Each unit of driving, d increases this cost by $\frac{\partial e}{\partial d} = 6$
Therefore, the tax should be $6.
(What if the price of a burger is $2? How does that change the tax necessary to bring about the social optimum? Consider the fact that an individual maximizes utility where his marginal rate of substitution equals the price ratio.)

3. Every morning, 6000 commuters must travel from Tampines to Tuas. Commuters all try to minimize the time it takes to get to work. There are two ways to make the trip. One way is to drive straight across the island, through the heart of Singapore. The other way is to drive north and circle the island. There is no congestion when circling the island but the drive is roundabout and takes 45 minutes. In comparison, driving through the city takes only 20 minutes if uncongested, but this road is prone to congestion. If the number of people who use this road is N, the number of minutes to drive from Tampines to Tuas is $20 + \frac{N}{100}$.

   a. Assuming no tolls are charged for using either road, in equilibrium, how many commuters will drive straight across town?
$$20 + \frac{N}{100} = 45$$
$$N = 2500$$

   b. How much time will be spent commuting from Tampines to Tuas each day?
$$6000 \times 45 = 270{,}000 \; minutes$$

   c. The Land Transport Authority, LTA, decides to limit access across the island so as to minimize the total number of minutes spent per day by everyone commuting from Tampines to Tuas. What is the objective function that LTA is trying to minimize?
$$F = 45(6000 - N) + \left(20 + \frac{N}{100}\right) N$$

   d. Minimize the function you obtained in (c) to determine the number of commuters LTA will permit through the heart of Singapore. How much time is spent commuting by the drivers with LTA's policy?
$$\frac{\partial F}{\partial N} = -45 + 20 + \frac{2N}{100} = 0$$
$$N = \frac{2500}{2} = 1250.$$
$$F = 45(6000 - 1250) + \left(20 + \frac{1250}{100}\right)(1250) = 254{,}375$$

   e. What is the travel time through the city for one commuter based on (d)? Do commuters prefer driving around the island or through the city given LTA's policy? How will drivers change their departure times given the policy?
$$20 + \frac{1250}{100} = 32.5 \; minutes$$

Drivers prefer going through the city. Drivers will have to leave early to be able to drive through the city center before the quota of cars allowed through the city is filled.

f. Commuters value the time saved from commuting at $\$w$ per minute. Instead of imposing a quota, the LTA decides to implement an electronic road pricing system that charges a toll equal to $\$t$ for driving through the city. If LTA charges this toll to minimize the total time spent commuting, how much should t be? How much revenue will this toll raise? (Hint: the toll must make drivers indifferent between driving around the island and through the city given the optimal travel times through the city.)
The toll must make drivers indifferent between driving around the island and driving through the city so

$$45w = 32.5w + t$$
$$t = 12.5w$$

$$Revenue = 12.5w \times 1250 = \$15{,}625w$$

g. If the LTA decides to return these revenues equally to all commuters show that every commuter is better off with this electronic road pricing policy than when no policy was in place (scenario in (a)).

Revenue to each commuter = $\frac{15625w}{6000} = 2.6w$
In (a), all commuters pay a cost of $45w$
If driving around the island, cost is $45w - 2.6w = 42.4\text{w}$
If driving through the island, cost is $32.5w + 12.5w - 2.6w = 42.4w$

4. Suppose that the LTA decides to double the capacity of the road through the island instead of imposing a toll. Now the travel time through the island is given by $20 + \frac{N}{200}$.
   a. In the absence of a toll, how many commuters will drive through the island to get to Tuas?

$$45 = 20 + \frac{N}{200}$$
$$N = 25 \times 200 = 5000$$

   b. How many minutes of commuting time are saved in aggregate by expanding road capacity? Is capacity expansion worth the expenditure of tax dollars?
   No commuting time is saved. This is not a good way to spend tax dollars