Problem Set #9

1. In conjunction with Singapore's National Day celebrations, there will be a fireworks display at Marina Bay. There are 1000 citizens in Singapore and they are interested in only two things, eating laksa and watching fireworks. Fireworks cost 1 bowl of laksa per unit. All Singaporeans have the same utility function $U(l, f) = l + \frac{\sqrt{f}}{20}$, where l is bowls of laksa per year and g is number of units of fireworks exploded at the National Day celebrations. Find the Pareto optimal amount of fireworks for Singapore.

$$MRS = \frac{MU_f}{MU_l} = (1000)\frac{1}{40\sqrt{f}} = 1$$
$$f = 625$$

2. There are two types of laksa sellers. "High quality" sellers make very good laksa which consumers value at $14 per bowl. "Low quality" sellers make less tasty laksa which consumers value at $8 per bowl. At the time of purchase, customers cannot distinguish between a high-quality product and a low-quality product. However, they can determine the quality of the product after purchase. The consumers are risk-neutral; if they have probability q of getting a high-quality product and $1 - q$ of getting a low quality product, then they value this prospect at $14q + 8(1 - q)$. Each type of seller can make a bowl of laksa at a constant unit cost of $11.50. All manufacturers behave competitively.
   a. Suppose that the sale of low quality laksa is illegal so that only high quality laksa is allowed to appear on the market. What is the equilibrium price?
      $11.50
   b. Suppose that there were no high-quality sellers. How many low quality bowls of laksa would be sold in equilibrium?
      Consumers don't value low quality laksa enough that any of it will be sold in equilibrium $(8 < 11.50)$
   c. Could there be an equilibrium in which equal positive quantities of the two types of laksa appear in the market?
      If $q = \frac{1}{2}$, then consumers have an expected value of $14\left(\frac{1}{2}\right) + 8\left(\frac{1}{2}\right) = \$11$ which is less than the cost to manufacture $11.50 so there will be no laksa sold.
   d. What minimum value of q would support an equilibrium where both types of laksa appear in the market?
      $$14q + 8(1 - q) = 11.50$$
      $$6q = 3.50$$
      $$q = 0.583$$
   e. Now suppose that each producer can *choose* to manufacture either a high-quality product or a low-quality product with a unit cost of $11.50 for the former and $11 for the latter. What would we expect to happen in equilibrium?
      In an equilibrium where all producers make high quality products, each seller sells a bowl of laksa for $11.50 and makes zero profit. Every seller has incentive to sell a low quality product for $11 to gain 50 cents in profit. Since all sellers think this way, only low quality products are offered in the market. But since low quality products are valued less than cost ($8<$11) none are purchased, so there is no trade in the market.

In an equilibrium where both laksa sellers exist, $14q + 8(1-q) > 11.50$ so that high quality laksa is still sold. Therefore $q > 0.583$. But every seller can reason that they can make 50 cents more by switching to low quality products. Therefore only low quality products are offered and none are traded.

f. Assuming that each seller is able to make the production choice described in the last question, what good would it do if the government banned production of low-quality laksa?

If there is no ban, there will be no output and no consumers' surplus. If low-quality products are banned, then in equilibrium there is output and positive consumers' surplus.

3. There are 1000 people in an economy who want to sell their used cars. These cars vary in quality. Original owners know exactly what their cars are worth. All used cars look the same to potential buyers until they have bought them; then they find out the truth. For any number X between 0 and 2000, the number of cars of quality lower than X is $\frac{X}{2}$. If a car is of quality X, its original owner will be willing to sell it for any price greater than X. If a buyer knew that a car was of quality X, she would be willing to pay $X + 500$ for it. When buyers are not sure of the quality of a car, they are willing to pay its expected value, given their knowledge of the distribution of qualities on the market, plus 500 dollars. Sellers can extract all the willingness to pay from buyers because buyers are plentiful.

a. Suppose that everybody knows that all the used cars are for sale. Buyers cannot assess the quality of used cars. What price would used cars sell for?

There are 1000 cars each with a value between 0 and 2000. On average they are worth \$1000. This is what they will sell for \$1000+\$500=\$1500.

b. Would every used car owner be willing to sell at this price? If no, which used cars would appear on the market?

Only cars worth \$1500 or less will sell.

c. What is the expected value of a car given the distribution of cars in part (b)? How much are buyers willing to pay for cars given this new distribution?

$EV = \frac{\$1500}{2} = 750$. Buyers are willing to pay $EV + 500 = \$1250$

d. Let X^* be some number between 0 and 2000 and suppose that all cars of quality lower than X^* are sold but original owners keep all cars of quality higher than X^*. What is the expected value of cars as a function of X^*? What would buyers pay for a used car? At this price, which used cars would be for sale?

If X^* is the number of cars sold, then they have an expected value of $\frac{X^*}{2}$. Buyers are willing to pay $\frac{X^*}{2} + 500$ for the cars. Cars worth less than $\frac{X^*}{2} + 500$ are sold

e. Write an equation for the equilibrium value of X^* at which the price that buyers are willing to pay is exactly enough to induce all cars of quality less than X^* into the market, then solve this equation for the equilibrium value of X^*.

$$\frac{X^*}{2} + 500 = X^*$$
$$X^* = \$1000$$

4. Every year, 1000 people in The Big City sell their used cars to get new ones. The original owners of the used cars have no place to keep the old cars and must sell them. Their

used cars vary in their quality. Their owners know exactly what is good and what is bad about their cars, but potential buyers can't tell them apart by looking at them. The owners have no scruples about lying about their old cars.

Each car has a value, V, which a buyer who knew all about its qualities would be willing to pay. There is a very large number of potential buyers, any one of which would be willing to pay $\$V$ for a car of value \$V.

The distribution of values of used cars on the market is as follows: for any value V between 0 and \$2000, the number of used cars available for sale that are worth less than $\$V$ is $\frac{V}{2}$. Potential used-car buyers are all risk-neutral. That is, if they don't know the value of a car for certain, they value it at its expected value, given the information they have.

A garage in The Big City will test out any used car and find its true value, V. The garage is perfectly accurate in its appraisals. The cost of an appraisal is \$200.

a. If nobody had their car appraised, what would the market price for used cars in The Big City be and what would be the total revenue received by used-car owners for their cars?

   \$1000 per car. Total revenue is \$1 000 000.

b. If all the cars that are worth more than $\$X$ are appraised and all the cars that are worth less than $\$X$ are sold without appraisal, what will the market price of un-appraised used cars be (as a function of X)?

   $\frac{\$X}{2}$ is the expected value of a car worth less than \$X

c. If all the cars worth more than $\$X$ are appraised and all the cars that are worth less than $\$X$ are sold without appraisal, then if your car is worth $\$X$, how much money would you have left if you had it appraised and then sold if for its true value? How much money would you get if you sold it without having it appraised?

   $\$X - 200$ if appraised

   $\frac{\$X}{2}$ if not appraised

d. In equilibrium, there will be a car of marginal quality such that all cars better than this car will be appraised and all cars worse than it will be sold without an appraisal. The owner of this car will be just indifferent between selling his car un-appraised and having it appraised. What will be the value of this marginal car?

$$X - 200 = \frac{X}{2}$$
$$X = \$400$$

e. In equilibrium, how many cars will be sold un-appraised and what will they sell for?

   Since $X = 400$, there will be $\frac{X}{2} = 200$ cars sold un-appraised. They will sell for $\frac{X}{2} = \$200$.

f. In equilibrium what will be the total net revenue of all owners of used cars, after the garage has been paid for its appraisals?

$$Revenue = 1{,}000{,}000 - 800 \times 200 = 840{,}000$$

5. Old Macdonald has a farm (E.I.E.I.O). The farm produces hay. He has a single employee, Jack. If Jack works for x hours he can produce x bales of hay. Each bale of hay sells for \$1. The cost to Jack of working x hours is $c(x) = \frac{x^2}{10}$.
   a. What is the efficient number of bales of hay for Jack to cut?
$$MR = MC$$
$$1 = \frac{1}{5}x$$
$$x^* = 5$$
   b. If the most that Jack could earn elsewhere is zero, how much would MacDonald have to pay him to get him to work the efficient amount?
   Since his opportunity cost is zero, he must have earnings of zero after exerting his effort so pay him $c(x) = \frac{5^2}{10} = \$2.50$
   c. What is MacDonald's net profit?
$$\$5 - \$2.50 = \$2.50$$
   d. Suppose that Jack would receive \$1 for passing out leaflets for a grocery store chain, an activity that involves no effort whatsoever How much would he have to receive from MacDonald for producing the efficient number of bales of hay instead of passing out leaflets?
   His reservation utility is now \$1, so he must be compensated to obtain a surplus of \$1, so pay him $c(x) + 1 = \$3.50$.
   e. Suppose that the opportunity to pass out leaflets is no longer available, but that MacDonald decides to rent his farm out to Jack for a flat fee. How much would he rent it for?
$$R = Revenue - Cost - Reservation\ Wage = \$2.50$$
6. A firm has hired James to work for them. James can choose high (H) or low (L) effort. If he chooses high effort, he incurs a personal cost of 4. This cost is always known to the firm. In the case of high effort, output is high with probability one. If James chooses low effort, he incurs a personal cost of 0. In this case, output is high with probability $\frac{1}{3}$ and low with probability $\frac{2}{3}$. This is also always known to the firm. When output is high, the firm receives revenue of θ and zero otherwise. James always exerts high effort when he is indifferent between high and low effort.
   a. Under complete information (i.e the firm can observe James' effort), what is the lowest bonus for high effort (denoted b) that the firm can pay to induce James to exert high effort? Assume the firm pays James nothing if he exerts low effort.
   James must prefer exerting high effort over low effort:
$$b - 4 \geq 0$$
$$b \geq 4$$
   The lowest bonus the firm can pay James is 4.
   b. When there is complete information, under what condition on θ would the firm find it worthwhile to pay this bonus?
   The firm finds it worthwhile to pay the bonus when
$$\theta - b \geq \frac{1}{3}\theta$$
$$\theta - 4 \geq \frac{1}{3}\theta$$
$$\theta \geq 6$$

c. When output is observable, but effort is NOT, what is the lowest bonus, b, for high output that the firm can pay to induce the worker to exert high effort? (The bonus is paid as long as the firm observes high output, θ)
James payoff for exerting high effort is $b - 4$. His payoff for exerting low effort is $\frac{1}{3}b$.
To exert high effort, $b - 4 \geq \frac{1}{3}b$

$$\frac{2}{3}b \geq 4$$
$$b \geq 6$$

The lowest bonus he can pay is $b = 6$

d. Under what condition on θ would the firm find it worthwhile to pay this bonus under this case of asymmetric information?

$$\theta - b \geq \frac{1}{3}\theta$$
$$\theta - 6 \geq \frac{1}{3}\theta$$
$$\theta \geq 9$$

e. Note your answers to (a) and (c). Briefly explain why a higher bonus has to be paid to induce high effort under asymmetric information
When the firm cannot observe effort, it is harder to motivate the worker to induce high effort because there is still a $\frac{1}{3}$ chance he will obtain high output with low effort and gets the bonus. Therefore, to induce high effort, the firm has to pay a larger bonus.