Chapter 30:
Game Applications

Big Data and Jobs

- Today we will spend some time talking about Game Theory and Jobs.
- Digression:
  - The age of Big Data and Computer Languages
  - Susan Athey (Professor at Stanford and Researcher at Microsoft) tells her undergraduate economics students "if you don't have programming skills, you're useless to me."

Big Data and Jobs

- Tech firms are the current frontier:
  - Big Data and Economics
    - Organized by Economics Undergraduates at The University of Chicago
    - Panel includes Susan Athey and Hal Varian
    - https://bfi.uchicago.edu/events/how-big-data-changing-economies
  - The Economist Magazine on Big Data Economists
    - http://www.economist.com/news/finance-and-economics/21638152-new-breed-high-tech-economist-helping-firms-crack-new-markets-meet

Good news!

- Ignore Section 30.2: Mixed Strategies
- Ignore Example: Dynamic inefficiency of price discrimination (page 578)

Today's Roadmap

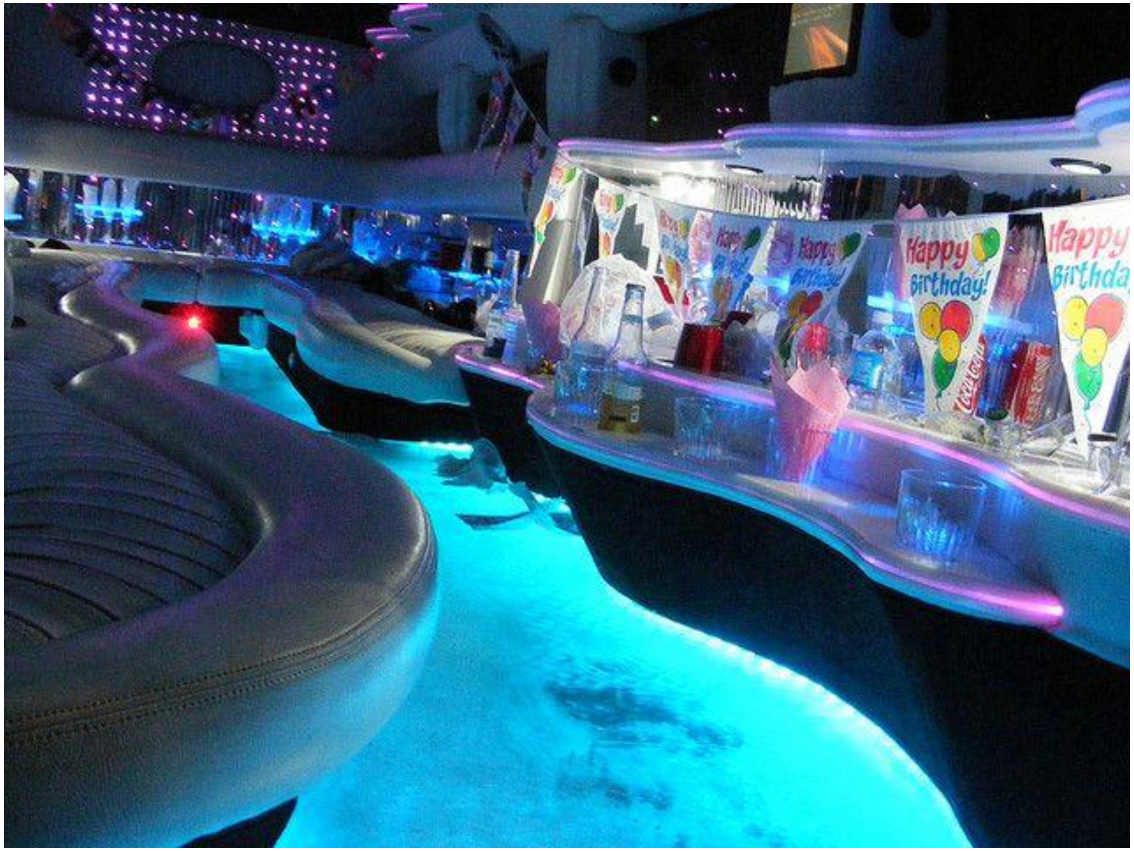
- Reinforcing concepts
  - Simultaneous games:
    - Normal representation
    - Nash equilibrium
  - Sequential games:
    - Extensive representation
    - Nash equilibrium
- Game Theory and Jobs
- Mixed Strategy Equilibria

Assurance Games: Vehicle Size

- Ross and Phoebe are both buying new cars. They have two options:

Hummer Nano

Vehicle Size



Vehicle Size

- What are Ross' best responses?
  - If Phoebe gets a Hummer, he should get a Hummer
  - If Phoebe gets a Nano, he should get a Nano

column player

		Phoebe	
		Hummer	Nano
row player: Ross	Hummer	2 / $\boxed{2}$	-1 / 3
	Nano	3 / -1	4 / $\boxed{4}$

Vehicle Size

- What are Phoebe's best responses?
  - If Ross gets a Hummer, she should get a Hummer
  - If Ross gets a Nano, she should get a Nano

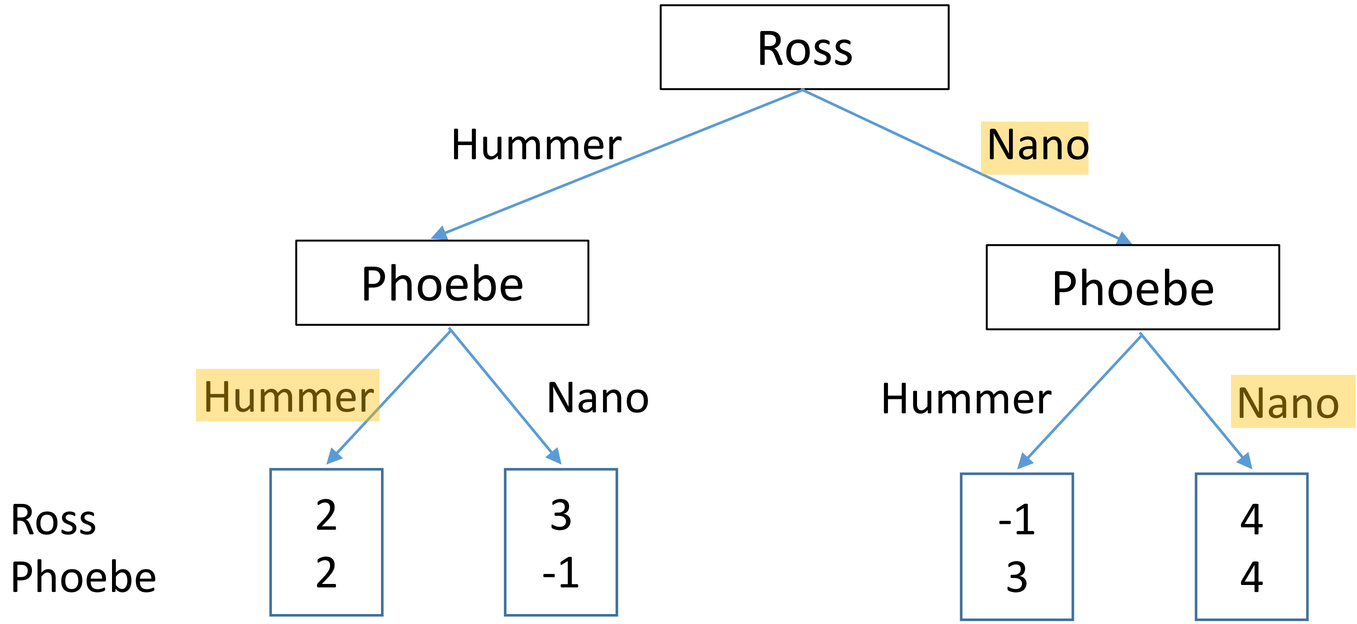
		Phoebe	
		Hummer	Nano
Ross	Hummer	$\boxed{2}$ $\boxed{2}$	-1 3
	Nano	3 -1	$\boxed{4}$ $\boxed{4}$

Vehicle Size

- How many dominant strategy equilibria are there?
  - None
- How many Nash equilibria are there?
  - Two

- What if this game was sequential? Ross gets to pick before Phoebe

Vehicle Size (Sequential)

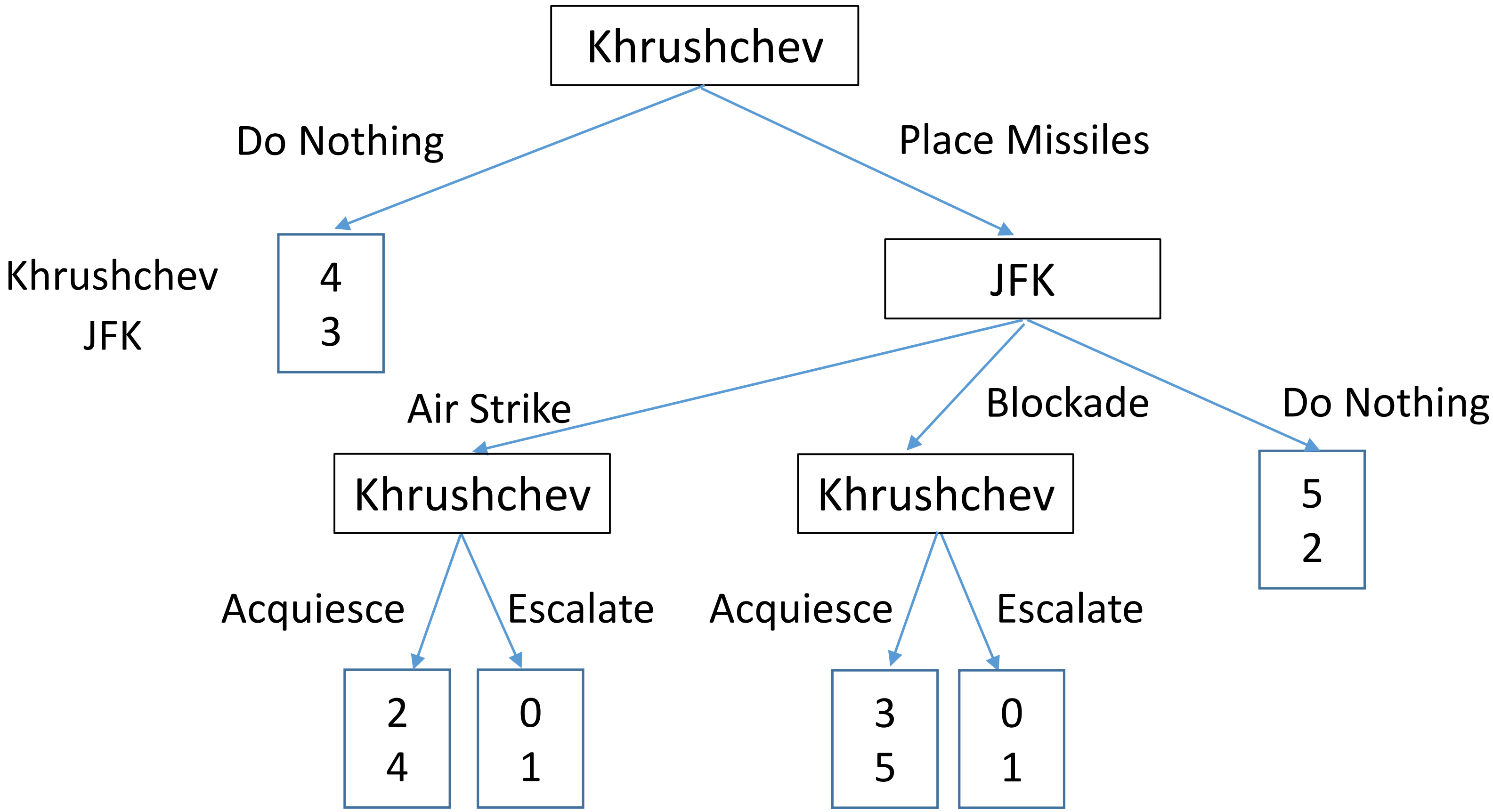


- If Ross chooses the Hummer, Phoebe will choose the Hummer.
- If Ross chooses the Nano, Phoebe will choose the Nano.
- Ross will choose the Nano.

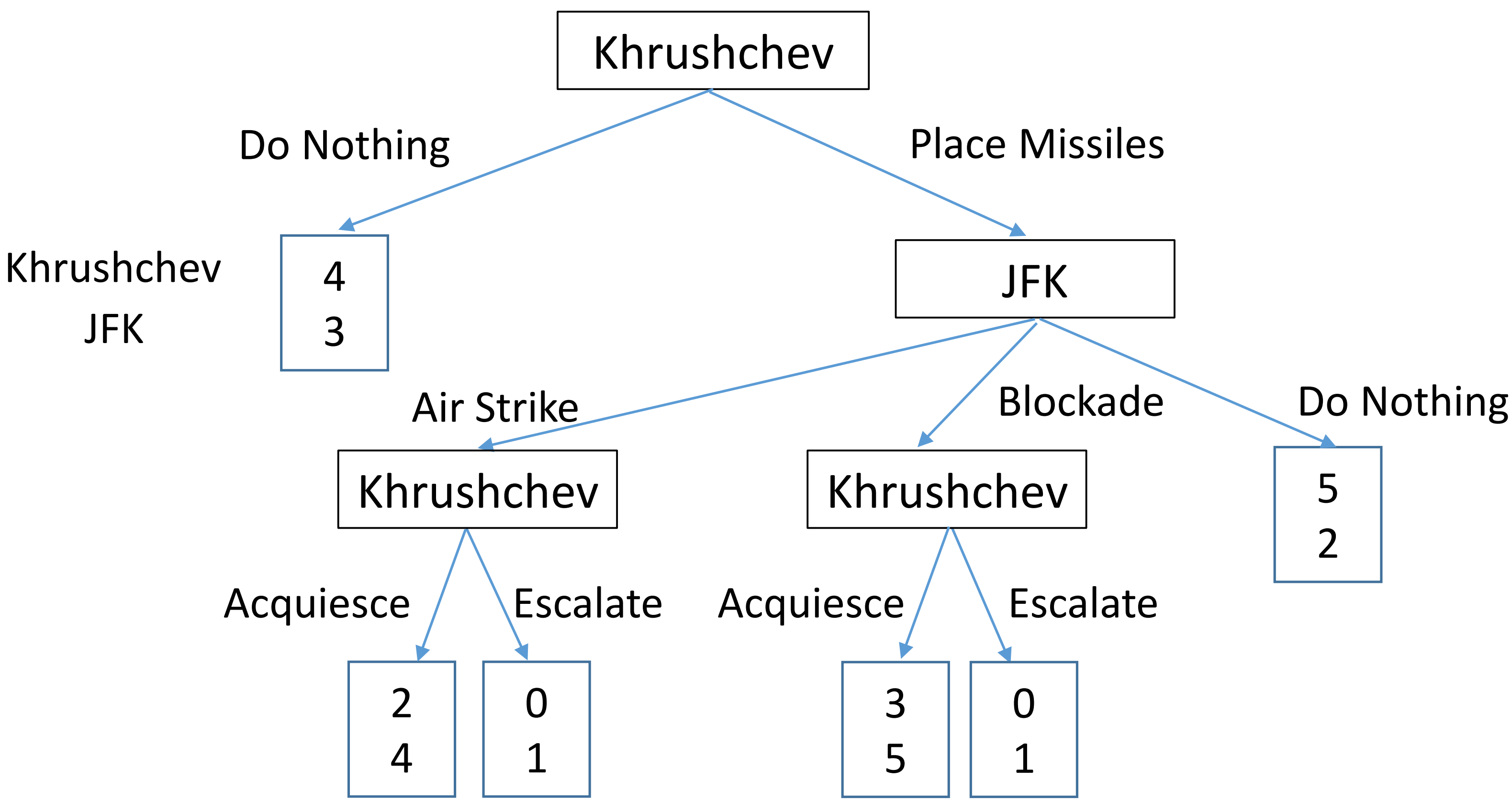
Cuban Missile Crisis

- In 1962, the Soviet Union installed nuclear missiles in Cuba
- U.S. President John F. Kennedy discussed the options:
  - Do nothing
  - Air strike on the missiles
  - Naval blockade
- Let's look at the game in extensive form:

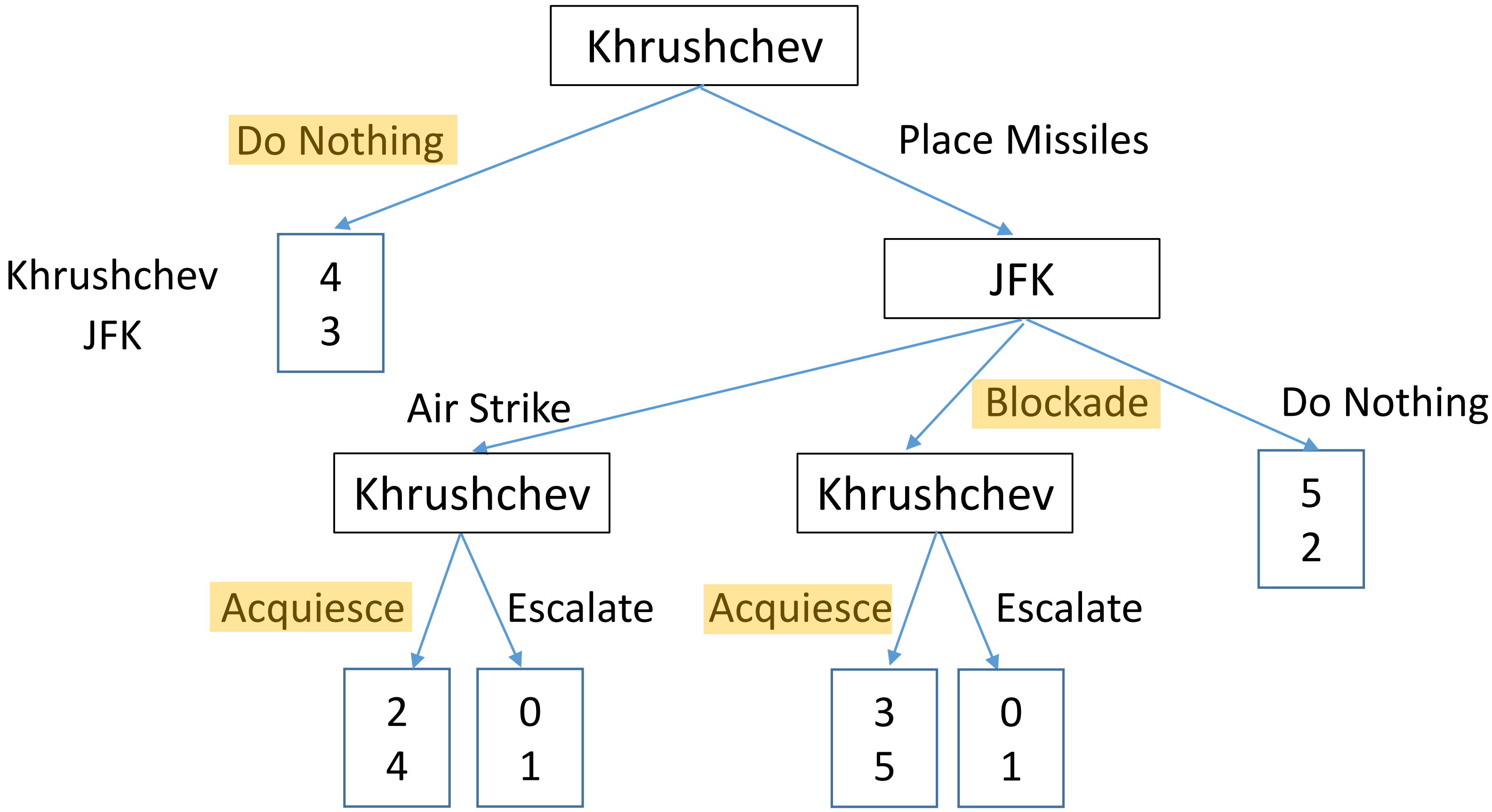
Cuban Missile Crisis



- Khrushchev doesn't want to escalate because it will be costly.
- He also does not want an air strike because that will cause him to lose his missiles.
- He would most like that JFK do nothing



Cuban Missile Crisis



Cuban Missile Crisis

- JFK decided on a naval blockade
- Negotiations ensued, and Khrushchev threatened to escalate the situation.
- Both sides believed that nuclear war was a possibility. This was the closest the cold war got to full escalation.
- Finally, the Soviet Union agreed to remove the missiles if the United States agreed not to invade Cuba.

Cuban Missile Crisis

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- Negotiations ensued, and Khrushchev threatened to escalate the situation.
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Cuban Missile Crisis

- If the Soviet Union had understood the game from the beginning, they would never have placed missiles in the first place!
- But it is not easy to determine what the game is (i.e. what are the strategies, what are the payoffs)
- The ability to formulate games to predict outcomes has great value.

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- Game Theory and Jobs
- Mixed Strategy Equilibria

Game Theory and Jobs

- Game theorists are demanded by governments' foreign service, military and private defense contractors to work out the strategic interactions between agents in situations like these in the international arena.
- Game theorists also work in Industrial Organization (e.g. Cournot, Stackelberg) informing firms and the government on firm strategy and its impacts on profits/welfare

Game Theory and Jobs

- Game theorists are also demanded by political parties looking for strategies to win elections.
  - Recall last week's example of the sequential game of political leanings (field of political economy/public choice)
- Tech firms like Google, Ebay, Yahoo, Facebook and Amazon hire game theorists to develop auctions for their products (like ads on their websites)
  - Take Game Theory module to learn about auctions.

Game Theory and Jobs

- Environmental economics
  - We will see more examples of this in Week 11
- Engineering and Computer Science:
  - http://www.technologyreview.com/article/515481/gaming-the-system/
- Game theory is also used in Evolutionary Biology, Philosophy, Transportation…

Game Theory and Jobs

- The <u>real skill</u> is not the ability to solve these models but to formulate them. Can you observe real-life situations and determine what the structure of the game is:
  - Players
  - Strategies
  - Information Set (full information, sequential or simultaneous)
  - Payoffs
- Unfortunately, this skill is harder to test on exams, but may be the more important that your ability to find Dominant and Nash equilibria.

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Football Penalty Game

Football Penalty Game

- The goal of the kicker is to score a goal.
  - If he kicks left, he has an 80% probability of scoring assuming the keeper defends right. If the keeper defends left, the probability of scoring falls to 50%
  - If he kicks right, he has a 90% probability of scoring assuming the keeper defends left. If the keeper defends right, the probability of scoring falls to 20%

Football Penalty Game

- This is the normal representation of the game.
  - Kicker's best response is "kick right" if keeper defends left, and "kick left" keeper defends right
  - Keeper's best response is "defend left" if kicker kicks left, and "defend right" if kicker kicks right
- There are no Nash equilibria in this game….

		Keeper	
		Defend Left	Defend Right
Kicker	Kick Left	50, $\boxed{-50}$	$\boxed{80}$, -80
	Kick Right	$\boxed{90}$, -90	20, $\boxed{-20}$

direct opposite payouts

Football Penalty Game

- To be precise, there are no pure strategy Nash equilibria (PSNE) in this game.
- A pure strategy Nash equilibrium is one where a player selects a course of action with probability 1.
- This is in contrast to mixed strategy Nash equilibria (MSNE) where players select courses of action with probabilities less than 1.

Football Penalty Game

- This means that the players randomize over their strategies. They play each strategy with probabilities less than one.
- Is there an MSNE in this game?

Football Penalty Game

- How do we look for mixed strategy equilibria?
  - Assume that Kicker can kick left with probability p and kick right with probability $(1-p)$.
  - Keeper can defend left with probability q and defend right with probability $(1-q)$.

		Keeper	
		Defend Left	Defend Right
Kicker	Kick Left	-50 50	-80 80
	Kick Right	-90 90	-20 20

Football Penalty Game

- Let’s say Kicker chooses $p = 0.5$.
  - Then if Keeper defends left, his expected payoff is $(-50 \times 0.5) + (-90 \times 0.5) = -70$
  - And if Keeper defends right, his expected payoff is $(-80 \times 0.5) + (-20 \times 0.5) = -50$
  - So Keeper will defend right.

		Keeper	
		Defend Left	Defend Right
Kicker	Kick Left	-50 50	-80 80
	Kick Right	-90 90	-20 20

Football Penalty Game

- But if Keeper defends right, then Kicker will kick left.
- If Kicker kicks left, keeper will now defend left.
- We are caught in an infinite sequence of changes in strategy.
- So $p = 0.5$ is not a MSNE

		Keeper	
		Defend Left	Defend Right
Kicker	Kick Left	50, -50	80, -80
	Kick Right	90, -90	20, -20

Football Penalty Game

- Therefore, we need to find a value p that will not cause keeper to want to deviate to left with probability 1 or deviate to right with probability 1.
- This means we want to find a value p such that keeper is indifferent between left and right.

Football Penalty Game

- If keeper defends left, his payoff, $E_{keep}(left)$ is $$-50p - 90(1-p)$$
- If keeper defends right, his payoff, $E_{keep}(right)$ is $$-80p - 20(1-p)$$
- Kicker is indifferent when

$$E_{keep}(left) = E_{keep}(right)$$

		Keeper	
		Defend Left	Defend Right
Kicker	Kick Left	50, -50	80, -80
	Kick Right	90, -90	20, -20

Football Penalty Game

$$E_{keep}(left) = E_{keep}(right)$$
$$-50p - 90(1-p) = -80p - 20(1-p)$$
$$30p = 70(1-p)$$
$$100p = 70$$
$$\underline{p = 0.7}$$

- Keeper is indifferent between defending left and right when keeper kicks left with probability $p = 0.7$.

Football Penalty Game

- If kicker kicks left, his payoff, $E_{kick}(left)$ is
$$50q + 80(1 - q)$$
- If kicker kicks right, his payoff, $E_{kick}(right)$ is
$$90q + 20(1 - q)$$
- Kicker is indifferent when

$$E_{kick}(left) = E_{kick}(right)$$

		Keeper	
		Defend Left	Defend Right
Kicker	Kick Left	50, -50	80, -80
	Kick Right	90, -90	20, -20

Football Penalty Game

$$E_{kick}(left) = E_{kick}(right)$$
$$50q + 80(1 - q) = 90q + 20(1 - q)$$
$$60(1 - q) = 40q$$
$$100q = 60$$
$$\underline{q = 0.6}$$

- Kicker is indifferent between kicking left and right when keeper defends left with probability $q = 0.6$.

Football Penalty Game

- What does this mean?
- When $q = 0.6$, kicker is equally happy kicking left or right.
  - His best response is left, right, or any mixture of left and right.
- When $p = 0.7$, keeper is equally happy defending left or right.
  - His best response is left, right or any mixture of left and right.

Football Penalty Game

- Now, it is easy for us to find a Nash equilibrium.
- When $q = 0.6$, any value of p is a best response for kicker.
- When $p = 0.7$, any value of q is a best response for keeper.
- Therefore, at $\{q = 0.6, p = 0.7\}$, both players are playing a best response to the other.
- That is a Nash equilibrium!

Rachel and Monica go shopping

- Rachel and Monica are going shopping. They can either go to Sachs 5th Avenue or Bloomingdales.
- Rachel really wants to go shopping at Bloomingdales. Monica prefers to just go where Rachel is going.
- Here is their game in normal form:

		Rachel	
		Sachs 5th Ave	Bloomingdales
Monica	Sachs 5th Ave	4 6	5 3
	Bloomingdales	3 4	7 5

Rachel and Monica go shopping

- What are the Nash equilibria in this game?
  - PSNE?
    - Monica's best responses are:
    - Rachel's best responses are:

		Rachel	
		Sachs 5^{th} Ave	Bloomingdales
Monica	Sachs 5^{th} Ave	4 $\boxed{6}$	$\boxed{5}$ 3
	Bloomingdales	3 4	$\boxed{7}$ $\boxed{5}$

Rachel and Monica go shopping

- There is one PSNE: {Monica: Bloomingdales, Rachel: Bloomingdales} put the agent before the strategy so that can be clear

- How about MSNE?

		Rachel	
		Sachs 5^{th} Ave	Bloomingdales
Monica	Sachs 5^{th} Ave	4 6	5 3
	Bloomingdales	3 4	7 5

Rachel and Monica go shopping

- Step 1: Define the probabilities:
  - Probability Monica goes to Sachs is p
  - Probability Monica goes to Bloomingdales is $1 - p$
  - Probability Rachel goes to Sachs is q
  - Probability Rachel goes to Bloomingdales is $1 - q$

		Rachel	
		Sachs 5^{th} Ave	Bloomingdales
Monica	Sachs 5^{th} Ave	4 6	5 3
	Bloomingdales	3 4	7 5

Rachel and Monica go shopping

- Step 2: Express Monica's expected payoffs in terms of Rachel's probabilities.
- $E_{Monica}(Sachs) = 6q + 3(1 - q) = 3 + 3q$
- $E_{Monica}(Bloomies) = 4q + 5(1 - q) = 5 - q$

		Rachel	
		Sachs 5th Ave	Bloomingdales
Monica	Sachs 5th Ave	4 6	5 3
	Bloomingdales	3 4	7 5

Rachel and Monica go shopping

- Step 3: Set Monica's expected payoffs equal to each other, $E_{Monica}(Sachs) = E_{Monica}(Bloomies)$:

$$3 + 3q = 5 - q$$
$$4q = 2$$
$$q = 0.5$$ she is indifferent

		Rachel	
		Sachs 5th Ave	Bloomingdales
Monica	Sachs 5th Ave	6, 4	3, 5
	Bloomingdales	4, 3	5, 7

Rachel and Monica go shopping

- Step 2: Express Rachel's expected payoffs in terms of Monica's probabilities.
- $E_{Rachel}(Sachs) = 4p + 3(1-p) = 3 + p$
- $E_{Rachel}(Bloomies) = 5p + 7(1-p) = 7 - 2p$

		Rachel	
		Sachs 5th Ave	Bloomingdales
Monica	Sachs 5th Ave	6 \ 4	3 \ 5
	Bloomingdales	4 \ 3	5 \ 7

Rachel and Monica go shopping

- Step 3: Set Rachel's expected payoffs equal to each other, $E_{Rachel}(Sachs) = E_{Rachel}(Bloomies)$

$$3 + p = 7 - 2p$$

$$3p = 4$$

$$p = 1.33$$

Rachel is only indifferent at p=1.33

A probability that is greater than 1!

		Rachel	
		Sachs 5^{th} Ave	Bloomingdales
Monica	Sachs 5^{th} Ave	6 \ 4	3 \ 5
	Bloomingdales	4 \ 3	5 \ 7

Rachel and Monica go shopping

- Rachel is indifferent between Sachs and Bloomingdales when Monica plays Sachs with probability 1.3
- For all values of p that make sense $(0 \leq p \leq 1)$, what should Rachel do?

		Rachel	
		Sachs 5th Ave	Bloomingdales
Monica	Sachs 5th Ave	4 6	5 3
	Bloomingdales	3 4	7 5

Rachel and Monica go shopping

- Notice that Rachel has a dominant strategy: Bloomingdales.
- Therefore, for all sensible values of p, Rachel will choose Bloomingdales (i.e. $q = 0$).
- Since Monica is only indifferent between Sachs and Bloomingdales when $q = 0.5$, an MSNE does not exist.

		Rachel	
		Sachs 5th Ave	Bloomingdales
Monica	Sachs 5th Ave	4 6	$\boxed{5}$ 3
	Bloomingdales	3 4	$\boxed{7}$ 5

Nash Equilibria

- A game can have no MSNE, 1 MSNE or more than 1 MSNE.
- A game can have no PSNE, 1 PSNE or more than 1 PSNE.
- A game can have PSNE and no MSNE, or MSNE and no PSNE, or both PSNE and MSNE.
- If a game has a DSE (dominant strategy) then that is the only Nash equilibrium for that game.

A pure strategy nash eqn can be considered a mixed strategy nash eqn (with probabiliy 1 and 0)