Problem Set #7

1. The Hawk-Dove game was first used by biologist John Maynard Smith to illustrate the uses of game theory in the theory of evolution. Males of certain species frequently come into conflict with other males over the opportunity to mate with females. In an encounter, a male can play "Hawk" and fight the other male until he wins or is badly hurt. Otherwise, he can play "Dove" by making a bold display but retreating if his opponent starts to fight. If two "Hawks" meet, both end up injured. If two "Doves" meet, they both strut their stuff until the female selects one at random or gets bored and wanders off. If one "Hawk" and one "Dove" meet, the Hawk gets the female and the "Dove" slinks off to celibate contemplation.

		Animal B	
		Hawk	Dove
Animal A	Hawk	-5,-5	10,0
	Dove	0,10	4,4

   a. If there is one Hawk and all other males are Doves, then what is the Hawk's payoff in every encounter? 10
   b. If all other males are Doves, what is the payoff of the remaining male if he is also a Dove? 4
   c. Use (a) and (b) to explain why an equilibrium cannot form where all males are Doves. If all males are doves, one can become a Hawk and enjoy a better payoff.
   d. If there is one Dove and all other males are Hawks, then what is the Dove's payoff in every encounter? 0
   e. If all other males are Hawks, what is the payoff of the remaining male if he is also a Hawk? -5
   f. Use (d) and (e) to explain why an equilibrium cannot form where all males are Hawks. If all males are Hawks, one can become a Dove and enjoy a better payoff.
   g. Since there is not an equilibrium where all males adopt the same strategy, find an equilibrium where some fraction of the males are "Doves" and the rest are "Hawks." Use p to denote the proportion who "Dove" and $1-p$ as the proportion who "Hawk."

      If A Hawks, $\pi_{A\,Hawk} = -5(1-p) + 10p = 15p - 5$

      If A Doves, $\pi_{A\,Dove} = 0(1-p) + 4p = 4p$

      To be indifferent $15p - 5 = 4p$

      $$11p = 5$$

      $$p = \frac{5}{11}$$

      In equilibrium, $\frac{5}{11}$ males are Hawks while $\frac{6}{11}$ males are Doves.

2. Monica has a Porsche. She is leaving town for a two-day cooking convention and will not need the car during this time. Joey wants to impress the ladies and is interested in renting Monica's sports car while she is away. The value of Joey having the car is $50 per day. Monica figures the total cost of letting Joey have her car is $35 regardless of how many days Joey uses it. On the evening before the first of these two days, Monica can send a message to Joey, offering to rent the car to him for two days at a specified price. Joey can either accept the offer or reject the offer and make a counteroffer.

   The only problem is that it takes a full day for a counteroffer to be made and accepted. If Joey rejects the original offer, then the car can only be rented for one day and there will be no time for Monica to make a counteroffer.

   Here we will walk through solving the Rubinstein bargaining solution to this problem. Assume that if any player is indifferent between accepting an offer, he or she chooses to do what the other player prefers. Also assume that both players have a discount rate of 0. They value future gains equally as present gains.

   a. If Joey rejects the offer, how much must he offer Monica in order for her to accept?

      Joey offers Monica $35 for the car.

   b. How much does Joey benefit from this offer?

      He obtains $50-$35=$15 surplus from this arrangement.

   c. Since Monica knows that Joey values the car at $50 per day, she knows she must offer him a surplus greater than or equal to the answer in (b), otherwise he will reject the offer. Since Joey values two days of car rental at $100, how much will Monica offer the car to Joey for in the first round.
      She will offer it to him at $85. With this offer, his surplus is $15 and he is indifferent between this offer and what he will offer in the next round.
   d. Your answer in (c) is what Monica will offer Joey in the first round. What are each player's surpluses from this bargaining solution?
      Joey has a surplus of $15 while Monica has a surplus of $50.

3. Suppose the same scenario as Question 2 occurs but now Monica will be out of town for three days and Joey, rather than Monica makes the first offer. Additionally, Monica's total cost is now $20, rather than $35. To find the Rubinstein bargaining solution, start by solving the final round, then working your way up.
   a. Joey makes the offer in the final round. What will he offer Monica and why?
      He will offer her $20 since then she is indifferent between the offer and not renting out her car but Joey prefers renting the car for $20 gaining a surplus of $30.
   b. Knowing that Joey will make the offer in (a), what will Monica offer Joey in the second round and why?
      Monica must make an offer that gives Joey a surplus of at least $30, otherwise he will reject the offer in favour of the offer he will make in the following round.

Knowing that Joey values the car at $100, she will offer him the car at $70, making him indifferent between her offer and his offer in the next round.

c. Knowing that Monica will make Joey the offer in (b), what will Joey offer Monica in the first round and why? What is Joey's and Monica's respective eventual payoffs from this solution?
Joey must make Monica at least as well of as she is with her second round offer. She receives $70 in the second round and after paying the $20 rental cost, has a surplus of $50. So Joey must make an offer to Monica that leaves her a surplus of $50 in the first round. Thus Joey offer's Monica $70 in the first round. He receives a surplus of $150 - $70 = $80.

4. Now suppose that Monica's convention is for four days, not three. Monica's cost for renting out the car is $20. Joey makes the first offer. Solve for the Rubinstein Bargaining solution. Be sure to note what each player's payoff is from this solution.

In the last round (fourth round), Monica make an offer to Joey. Knowing that he values the car at $50, she makes him an offer for $50. He will accept.

In the second to last round (third round), Joey makes an offer. He has to make Monica at least as happy as the last round. In the last round, Monica receives a payoff of $50 - $20 = $30. So Joey must offer Monica $50.

In the third to last round (second round), Monica make an offer that must leave Joey at least as happy as in the next round. Joey receives a payoff of $100 - $50 = $50 in the last round. In this round, Joey values the car at $150, so if Monica makes an offer for $100, Joey is indifferent between this offer and the offer he makes in the next round.

In the first round, Joey makes an offer to Monica. He has to make her at least as happy as in the next round where she has a surplus of $100 - $20 = $80. So Joey will offer $100 for the car.

Monica's payoff will be $100 - $20 = $80.
Joey's payoff will be $200 - $100 = $100.