Problem Set #4

1. Consider a Cournot Duopoly. Call them Firm 1 and Firm 2. They face inverse demand P(Q) = 20 - Q, where Q is the total market quantity (Firm 1's quantity plus Firm 2's quantity). Firm 1 can produce at a constant marginal cost of 2 and Firm 2 can produce at a constant marginal cost of 2. There are no fixed costs.
    a. Write out the profit function for each firm.
    b. Find the best response function for each firm.
    c. Find the equilibrium quantity, price, and profits for each firm.
    d. Now instead assume that both firms continue to compete on quantity but Firm 1 chooses its quantities before Firm 2. Calculate the equilibrium quantities, prices and profits associated with this market structure.
2. There are two chicken rice sellers in a food court. The demand for chicken rice from either seller depends on its own price and the price of its rival. The demand function for chicken rice from seller $A$ is $21 - 2P_a + P_b$ while the demand function for chicken rice from seller $B$ is $21 + P_a - 2P_b$. Assume that marginal costs are zero. Each chicken rice seller maximizes its revenue assuming that the other's price is independent of its own price. If the chicken rice sellers are competing against each other, what price and quantities will we see in the market?

3. Consider a duopoly marker where the goods are not identical, that is, firms differentiate their products. For example, airlines try to position their products as unique. Singapore Airlines markets the hospitality of their "Singapore Girls," while Emirates claims to have the "best in-flight entertainment system in the world." Nevertheless, airlines still compete with each other and do not have a monopoly over the market. Suppose Singapore Airlines and Emirates compete on a route. Singapore's demand is given by, $P_A = 100 - Q_A + 2Q_B$ while Emirates' demand is given by $P_B = 120 - Q_B + 3Q_A$. Singapore's cost curve is $C_A(Q_A) = Q_A^2$ and Emirate's cost curve is $C_B(Q_B) = 2Q_B^2$. Find the Cournot equilibrium prices and quantities in this market.
4. Firefly is an airline that has a monopoly on air travel between Singapore and Kuantan, Malaysia. Demand for daily round trips (no one makes one-way trips) is $160 - 2p$, where $q$ is the number of passengers per day. It costs \$2000 to fly an empty plane to Kuantan and an additional \$10 per passenger on board the plane. This means daily costs are $2000 + 10q$.

    a. Calculate the profit maximizing price, quantity and daily profits for Albatross Airlines.

    b. If the annual interest rate is 7%, how much would an investor be willing to spend to obtain the profits from Firefly at the end of the year. (Daily profits are kept under a mattress until the end of the year, then handed to the investor)

    c. A new airline, Malindo Air emerges with identical costs as Firefly and wants to enter the Singapore-Kuantan market making the industry a Cournot duopoly. Would the new entrant make a profit?

Club Med builds a resort on the beaches of Cherating, just north of Kuantan increasing the popularity of Kuantan as a tourist destination. As a result, demand for trips between the cities double to $q = 320 - 4p$.

d. Firefly only has a single plane with 80 seats to ply this route. If Firefly remains the sole airline on this flight, what will $p$ be and what are Firefly's profits?

e. Firefly adds a second plane which makes fixed costs \$4000 while marginal costs are still \$10 per passenger. What is its price, quantity and profits? Does Firefly prefer having one plane or two planes?

f. What if Firefly stuck with a single plane and Malindo Air entered with a single plane with the two firms acting as Cournot oligopolists?

Solutions

1.

a. $\pi_1 = (20 - Q_1 - Q_2)Q_1 - 2Q_1$
$\pi_2 = (20 - Q_1 - Q_2)Q_2 - 2Q_2$

b. $\frac{\partial \pi_1}{\partial Q_1} = 20 - 2Q_1 - Q_2 - 2 = 0$

$$Q_1 = \frac{18 - Q_2}{2}$$

$$\frac{\partial \pi_2}{\partial Q_2} = 20 - 2Q_2 - Q_1 - 2 = 0$$

$$Q_2 = \frac{18 - Q_1}{2}$$

c. $Q_1 = \frac{18 - \left(\frac{18 - Q_1}{2}\right)}{2}$

$$Q_1 = \frac{18 + Q_1}{4}$$

$$\frac{3}{4}Q_1 = \frac{18}{4}$$

$$Q_1 = 6$$

$$Q_2 = \frac{18 - 6}{2} = 6$$

$$Q = 6 + 6 = 12$$

$$P = 20 - 12 = 8$$

$$\pi_1 = (8 - 2) \times 6 = 36$$

$$\pi_2 = (8 - 2) \times 6 = 36$$

d.

$$\pi_{stackelberg\ leader} = \left(20 - Q_1 - (\frac{18 - Q_1}{2})\right) Q_1 - 2Q_1 = \left(11 - \frac{Q_1}{2}\right) Q_1 - 2Q_1$$

$$\frac{\partial \pi}{\partial Q_1} = 11 - Q_1 - 2 = 0$$

$$Q_1 = 9$$

$$Q_2 = \frac{18 - 9}{2} = 4.5$$

$$P = 20 - 9 - 4.5 = 6.5$$

$$\pi_{stackelberg\ leader} = (6.5 - 2) \times 13 = 40.5$$

$$\pi_{stackelberg\ follower} = (6.5 - 2) \times 4.5 = 20.25$$

2. If the firms price independently, then

$$\Pi_a = Q \times P = (21 - 2p_a + p_b)p_a$$

To maximize profits,

$$MR = 21 - 4p_a + p_b = 0$$

$$p_a = \frac{21 + p_b}{4}$$

Similarly

$$\Pi_b = Q \times P = (21 + p_a - 2p_b)p_b$$

To maximize profits,

$$MR = 21 + p_a - 4p_b = 0$$
$$p_b = \frac{21 + p_a}{4}$$

Plugging one best response function into the other:

$$p_a = \frac{21 + \left(\frac{21 + p_a}{4}\right)}{4}$$
$$\frac{15}{16}p_a = \frac{21}{4} + \frac{21}{16}$$
$$p_a = \frac{105}{16} \times \frac{16}{15} = 7$$

$$p_b = \frac{21 + 7}{4} = 7$$
$$Q_a = 21 - 2(7) + (7) = 14$$
$$Q_b = 21 + 7 - 2(7) = 14$$

3.

$$\pi_A = (100 - Q_A + 2Q_B)Q_A - Q_A^2$$
$$\frac{\partial \pi}{\partial Q_A} = 100 - 2Q_A + 2Q_B - 2Q_A = 0$$
$$4Q_A = 100 + 2Q_B$$
$$Q_A = 25 + 0.5Q_B$$

$$\pi_B = (120 - Q_B + 3Q_A)Q_B - 2Q_B^2$$
$$\frac{\partial \pi}{\partial Q_A} = 120 - 2Q_B + 3Q_A - 4Q_B = 0$$
$$6Q_B = 120 + 3Q_A$$
$$Q_B = 20 + 0.5Q_A$$

Plugging one into the other, you have:

$$Q_A = 25 + 0.5(20 + 0.5Q_A)$$
$$= 25 + 10 + 0.25Q_A$$

$$0.75Q_A = 35$$

$$Q_A = \frac{35}{0.75} = 46.67$$

$$Q_B = 20 + 0.5(46.67) = 43.33$$

4.

a.

$$q = 160 - 2p$$
$$p = 80 - \frac{1}{2}q$$
$$\pi = \left(80 - \frac{1}{2}q\right)q - 2000 - 10q$$
$$\frac{\partial \pi}{\partial q} = 80 - q - 10 = 0$$
$$q = 70$$
$$p = 80 - \frac{1}{2}(70) = 45$$
$$\pi = (45 - 10)70 - 2000 = 450$$

b. If the investor receives $x + (450 \times 365)$ at the end of the year, then

$$0.07x = 450 \times 365$$
$$x = \$2.346\ million$$

If the investor receives $450 \times 365$ at the end of the year, then $x = \frac{450\times365}{1.07} = \$153{,}504$

c.

$$\pi_{Malindo} = \left(80 - \frac{1}{2}(q_M + q_F)\right)q_M - 2000 - 10q_M$$

$$\frac{\partial \pi}{\partial q} = 80 - q_M - \frac{1}{2}q_F - 10 = 0$$
$$q_M = 70 - 0.5q_F$$
$$q_F = 70 - 0.5q_M$$

$$q_M = 70 - 0.5(70 - 0.5q_M)$$
$$q_M = 70 - 35 + 0.25q_M$$
$$.75q_M = 35$$
$$q_M = 46.67$$
$$q_F = 46.67$$
$$p = 80 - 46.67 = 33.33$$

$$\pi_M = (33.33 - 10)46.67 - 2000 = -911$$

No the firm will not enter because they would make a loss

d. If Firefly's quantity was unconstrained:

$$p = 80 - \frac{1}{4}q$$
$$\pi = \left(80 - \frac{1}{4}q\right)q - 2000 - 10q$$
$$\frac{\partial \pi}{\partial q} = 80 - 0.5q - 10 = 0$$
$$q = 70 \times 2 = 140$$

Since it cannot produce 140, it will produce the closest to that, which is 80.

$$p = 80 - \frac{1}{4}(80) = 60$$
$$\pi = (60 - 10)80 - 2000 = 2000$$

e.

$$p = 80 - \frac{1}{4}q$$

$$\pi = \left(80 - \frac{1}{4}q\right)q - 4000 - 10q$$

$$\frac{\partial\pi}{\partial q} = 80 - 0.5q - 10 = 0$$

$$q = 70 \times 2 = 140$$

$q = 140$, so $p = 80 - \frac{1}{4}(140) = 45$

$$\pi = (45 - 10)140 - 4000 = 900$$

No, they should not use a second plane because profits will shrink from 2000 to 900.

f.

$$\pi_{Malindo} = \left(80 - \frac{1}{4}(q_M + q_F)\right)q_M - 2000 - 10q_M$$

$$\frac{\partial\pi}{\partial q} = 80 - \frac{1}{2}q_M - \frac{1}{4}q_F - 10 = 0$$

$$q_M = 140 - 0.5q_F$$

$$q_F = 140 - 0.5q_M$$

$$q_M = 140 - 0.5(140 - 0.5q_M)$$

$$q_M = 140 - 70 + 0.25q_M$$

$$.75q_M = 70$$

$$q_M = 93.33$$

$$q_F = 93.33$$

$$p = 80 - 46.67 = 33.33$$

$$\pi_{Malindo} = (33.33 - 10) * 93.33 - 2000 = 177.3889$$

$$\pi_{Firefly} = (33.33 - 10) * 93.33 - 2000 = 177.3889$$

But since each plane only carries 80 passengers, $q_m = q_F = 80$.

$$p = 80 - 40 = 40$$

$$\pi_M = (40 - 10) \times 80 - 2000 = 400$$

$$\pi_F = (40 - 10) \times 80 - 2000 = 400$$

Note that profits are higher with the capacity constraint than if they had no constraint!