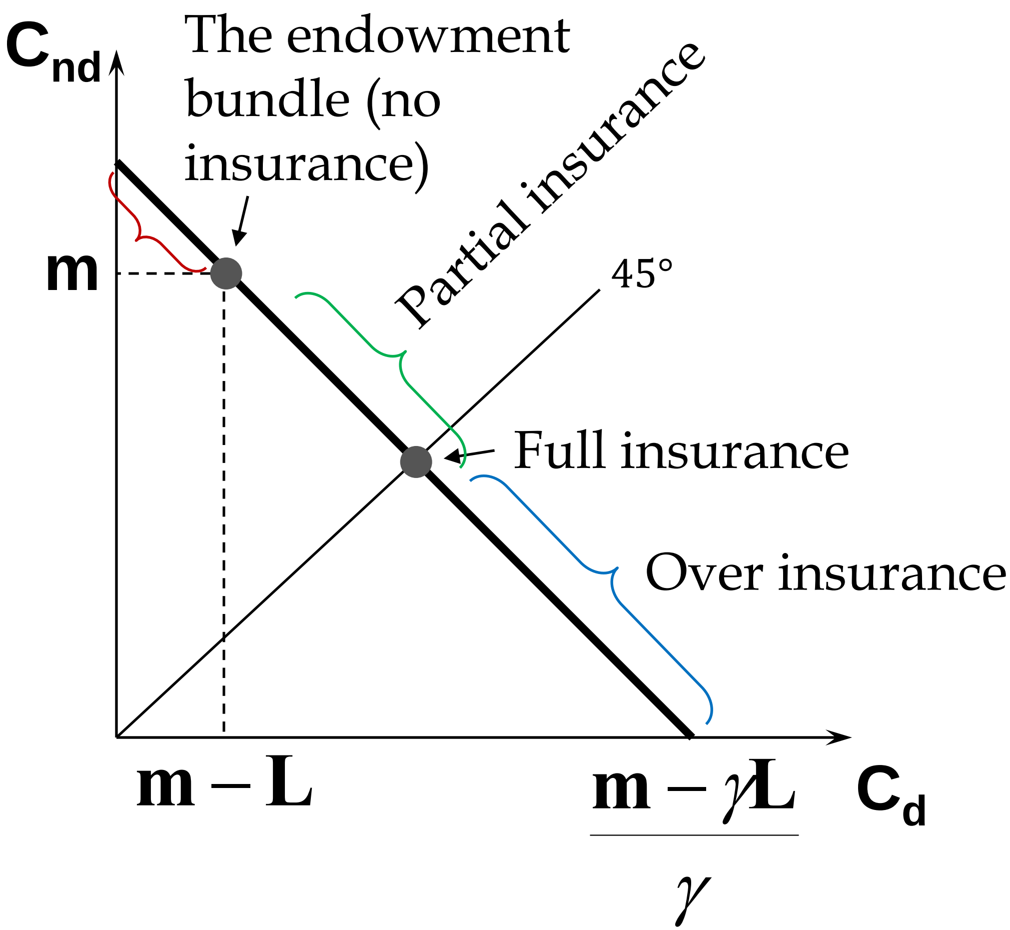
State-Contingent Budget Constraints



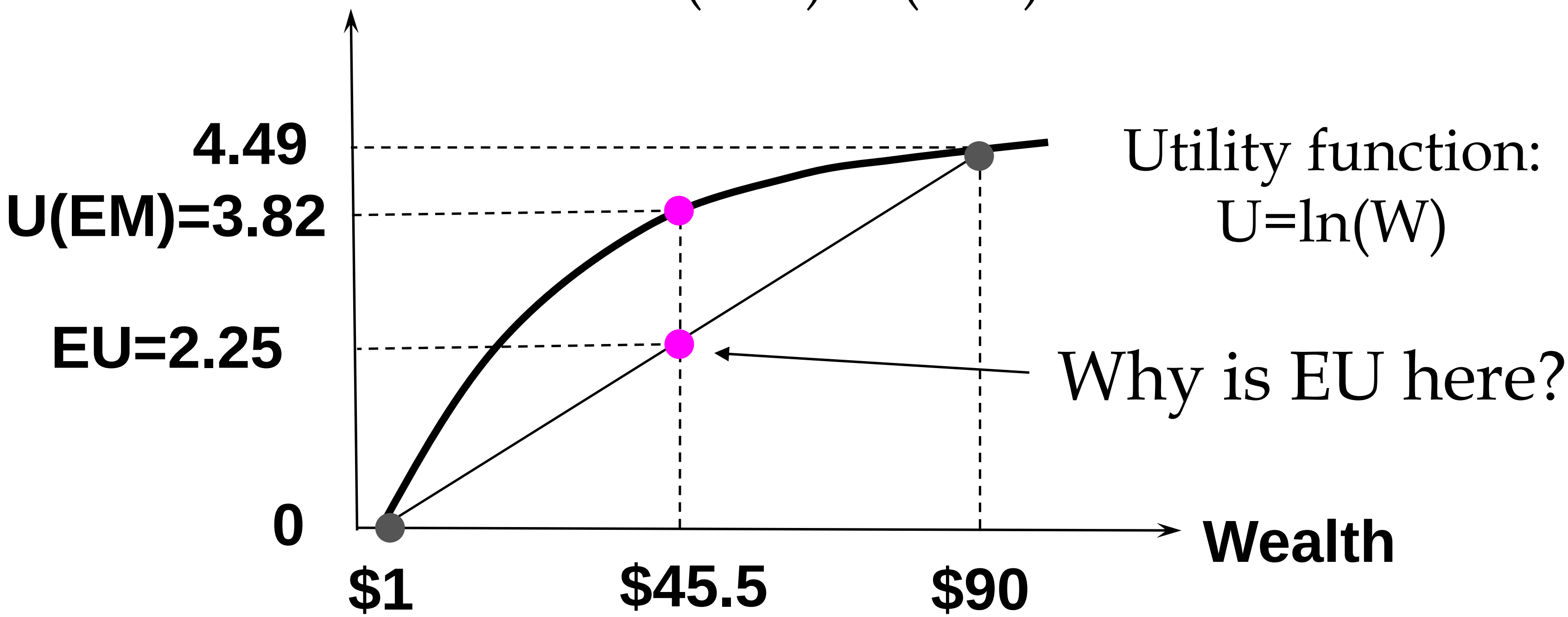
$$\mathbf{slope} = -\frac{\gamma}{1-\gamma}$$

Insurance

- Under-insurance:
  - when insurance firm makes positive
- Full-insurance:
  - when insurance firm makes zero profits
- Over-insurance:
  - When insurance firm makes negative profits

Preferences Under Uncertainty

U(EM) > EU ⇒ risk-aversion.
(3.82)   (2.25)



Because EU is a linear combination of the two grey points, so it must lie on the line that connects the two points

Preferences Under Uncertainty

- U($90) = ln(90) = 4.49
- U($1) = ln(1) = 0
- Expected utility is

$$\frac{1}{2}(4.49) + \frac{1}{2}(0) = 2.25$$

•**Expected Utility** is the probability weighted average of the utilities associated with all the possible outcomes

$$\sum_j prob_j\{U(Outcome_j)\}$$

"Unfair" Insurance

- Suppose insurers make positive expected economic profit.

$$\gamma K - \pi_d K - (1 - \pi_d)0$$
$$= (\gamma - \pi_d)K > 0.$$

Which means $\gamma > \pi_d$

- The insurers charge more than their cost of insuring.

"Unfair" Insurance

- Rational choice requires

$$\frac{\gamma}{1-\gamma} = \frac{\pi_d MU(C_d)}{\pi_{nd} MU(C_{nd})}$$

- Since $\gamma > \pi_d$

$$\frac{\gamma}{1-\gamma} > \frac{\pi_d}{\pi_{nd}}$$

- which means $\frac{MU(C_d)}{MU(C_{nd})}$ >1 so that the first equality can still hold. Therefore:

$$MU(C_d) > MU(C_{nd})$$

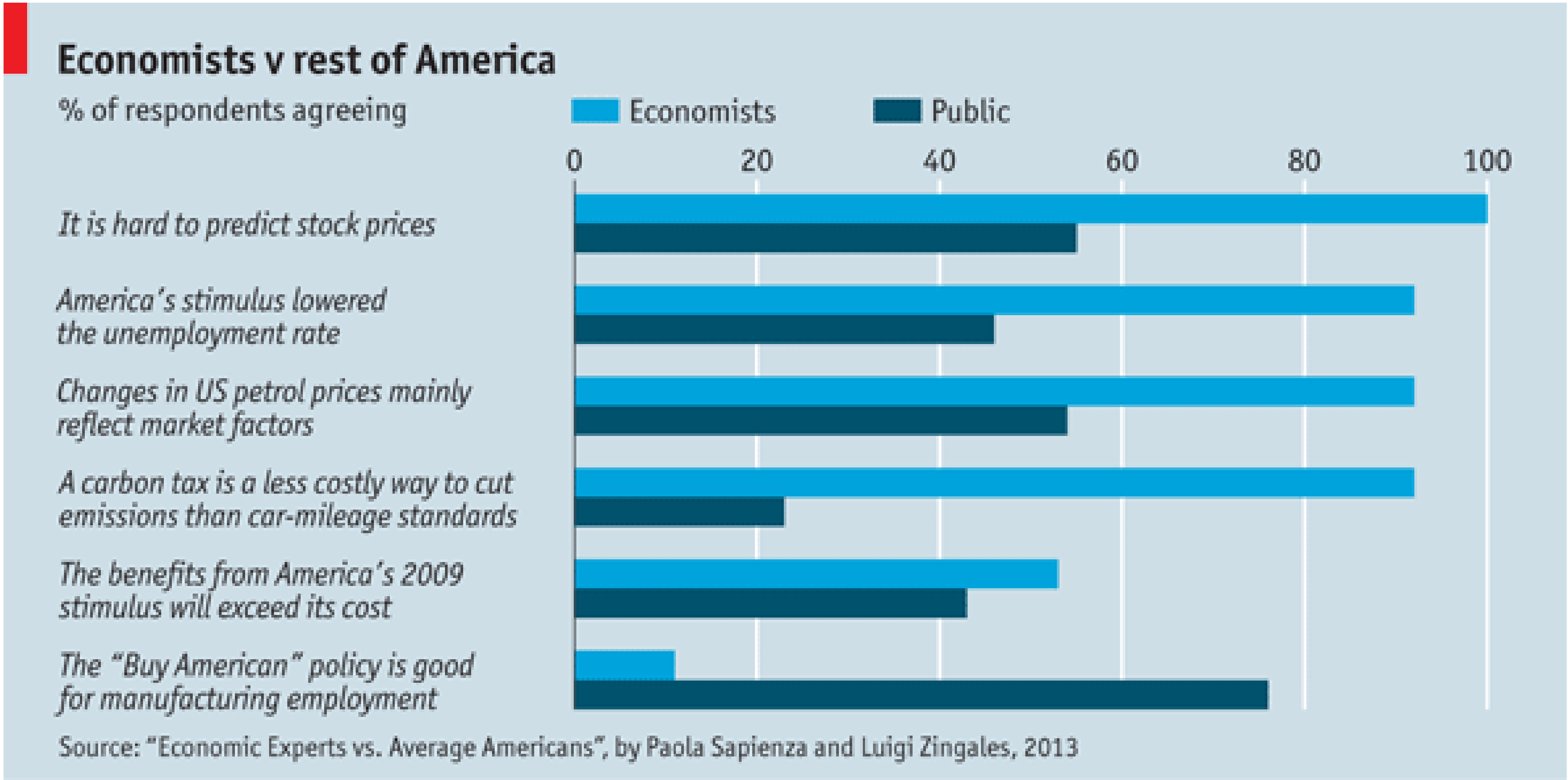
"Unfair Insurance"

- Remember that this consumer is risk adverse. This means MU declines in wealth
- Since the marginal utility when there is damage is higher than the marginal utility when there is no damage:

$$C_d < C_{nd}$$

- In this case, the consumer only partially insures

Chapter 32: Exchange



What's the source of agreement?

Today, we talk about the foundations that lead to such policy agreements

Exchange

- In today’s lecture, we will discuss the Edgeworth Box framework:
  - Pareto Optimality
  - Trade in Competitive Markets
  - First **Fundamental** Welfare Theorem
  - Second **Fundamental** Welfare Theorem
  - Walras’ Law

Pareto Optimality

- An allocation that cannot improve the welfare of a consumer without reducing the welfare of another.
- Someone must be made worse-off to make another better off.

Exchange

- The Edgeworth Box isn't a common tool in economic research.
- However, it can be used to demonstrate two powerful theorems:
  - First Welfare Theorem: "competitive markets are Pareto-optimal"
  - Second Welfare Theorem: "a reallocation of resources prior trade can yield *any* Pareto-optimal outcome"

Exchange

- Why are these theorems fundamental?
  - The first theorem confirms Adam Smith's hypothesis of the "invisible hand". Let the market work and the outcome will be Pareto-efficient.
  - The second theorem shows that redistribution, then letting the market take over allows us to select a desired Pareto-efficient outcome.

Exchange

- These theorems "preach"
  - Non-intervention in competitive markets
  - Lump-sum transfers as a method of redistribution.
- But remember that these are stylized models. There are lots of assumptions that underlie this.

Exchange

- Two consumers, A and B.
- Their endowments of goods 1 and 2 are

$$\omega^A = (\omega_1^A, \omega_2^A) \text{ and } \omega^B = (\omega_1^B, \omega_2^B).$$

- E.g.
- The total quantities available

$$\omega^A = (6,4) \text{ and } \omega^B = (2,2).$$

are $\omega_1^A + \omega_1^B = 6 + 2 = 8$ units of good 1

and $\omega_2^A + \omega_2^B = 4 + 2 = 6$ units of good 2.

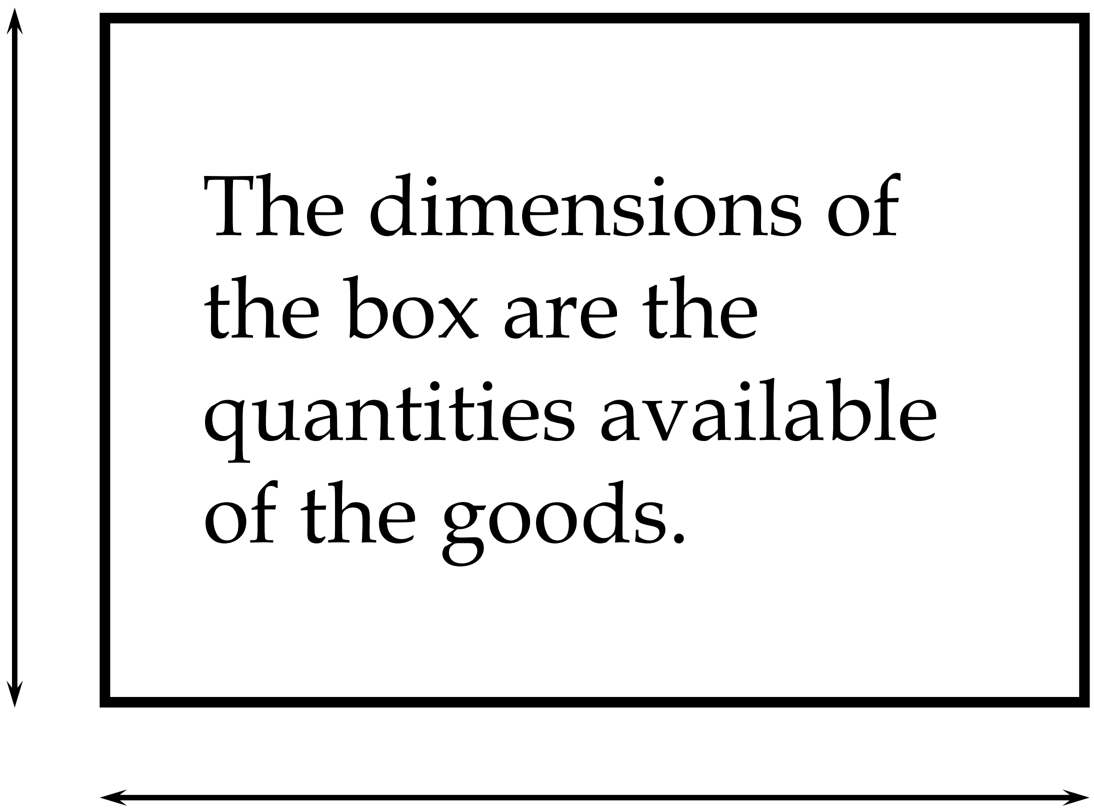
Exchange

- Edgeworth and Bowley devised a diagram, called an Edgeworth box, to show all possible allocations of the available quantities of goods 1 and 2 between the two consumers.

Starting an Edgeworth Box

Starting an Edgeworth Box

Height = $\omega_2^A + \omega_2^B$ $= \mathbf{4 + 2}$ $= \mathbf{6}$



Width = $\omega_1^A + \omega_1^B = \mathbf{6 + 2 = 8}$

Feasible Allocations

- What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?
- How can all of the feasible allocations be depicted by the Edgeworth box diagram?

Feasible Allocations

- What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?
- How can all of the feasible allocations be depicted by the Edgeworth box diagram?
- One feasible allocation is the before-trade allocation; i.e. the endowment allocation.

The Endowment Allocation

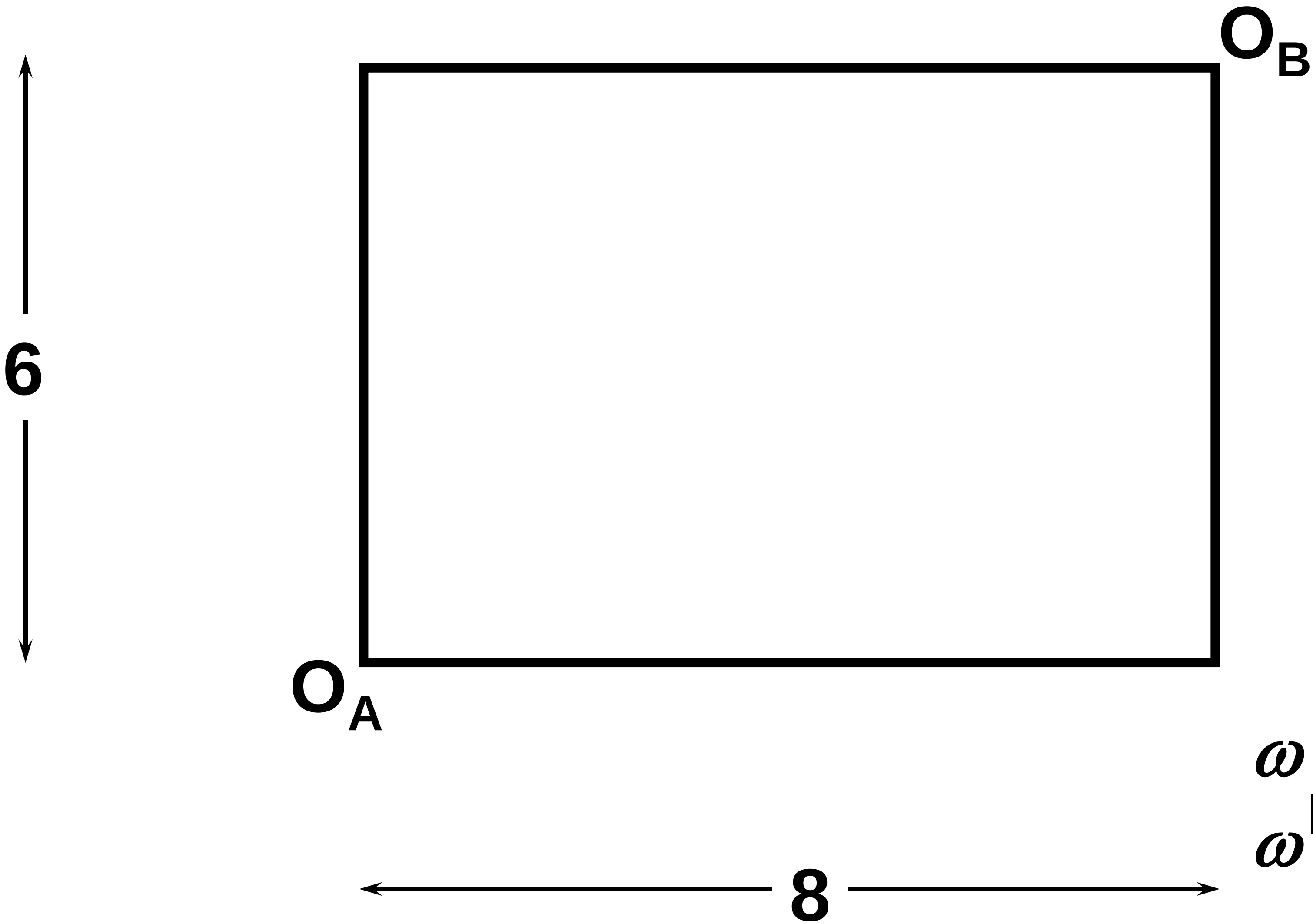
Height = $\omega_2^A + \omega_2^B$ $= 4 + 2$ $= 6$

The endowment allocation is $\omega^A = (6,4)$ and $\omega^B = (2,2)$.

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

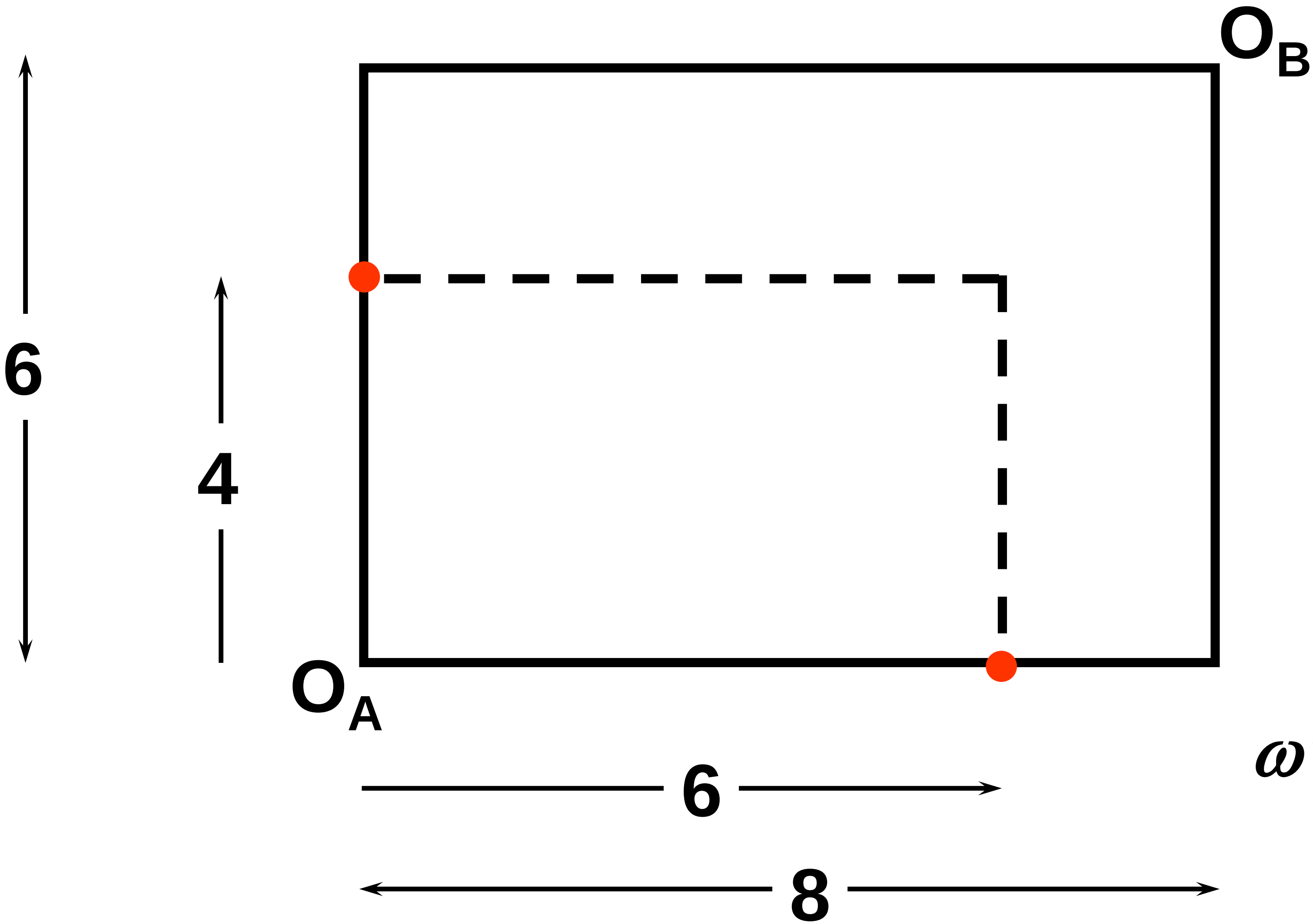
$\omega^A = (6,4)$
$\omega^B = (2,2)$

The Endowment Allocation



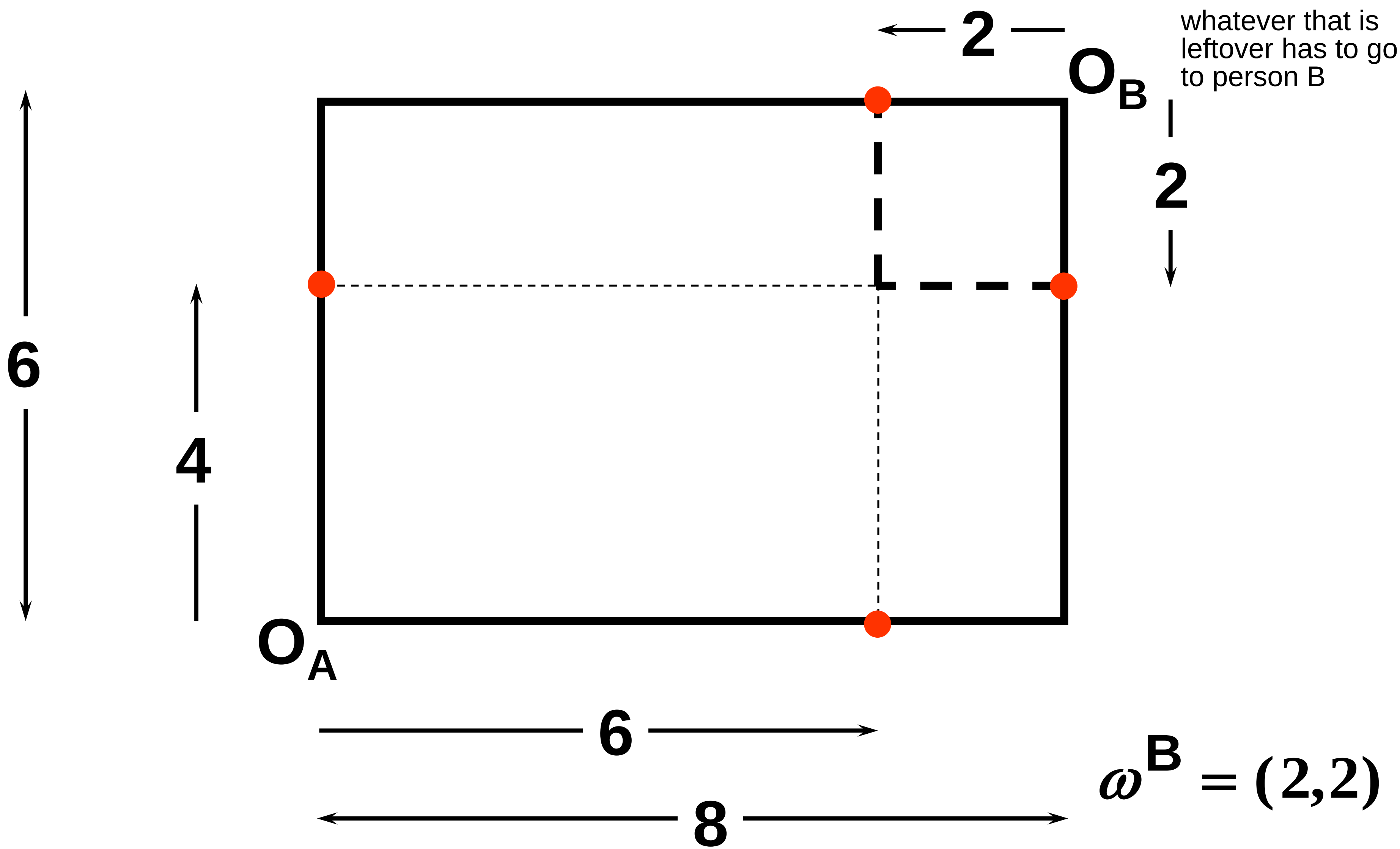
$$\omega^A = (6,4)$$
$$\omega^B = (2,2)$$

The Endowment Allocation



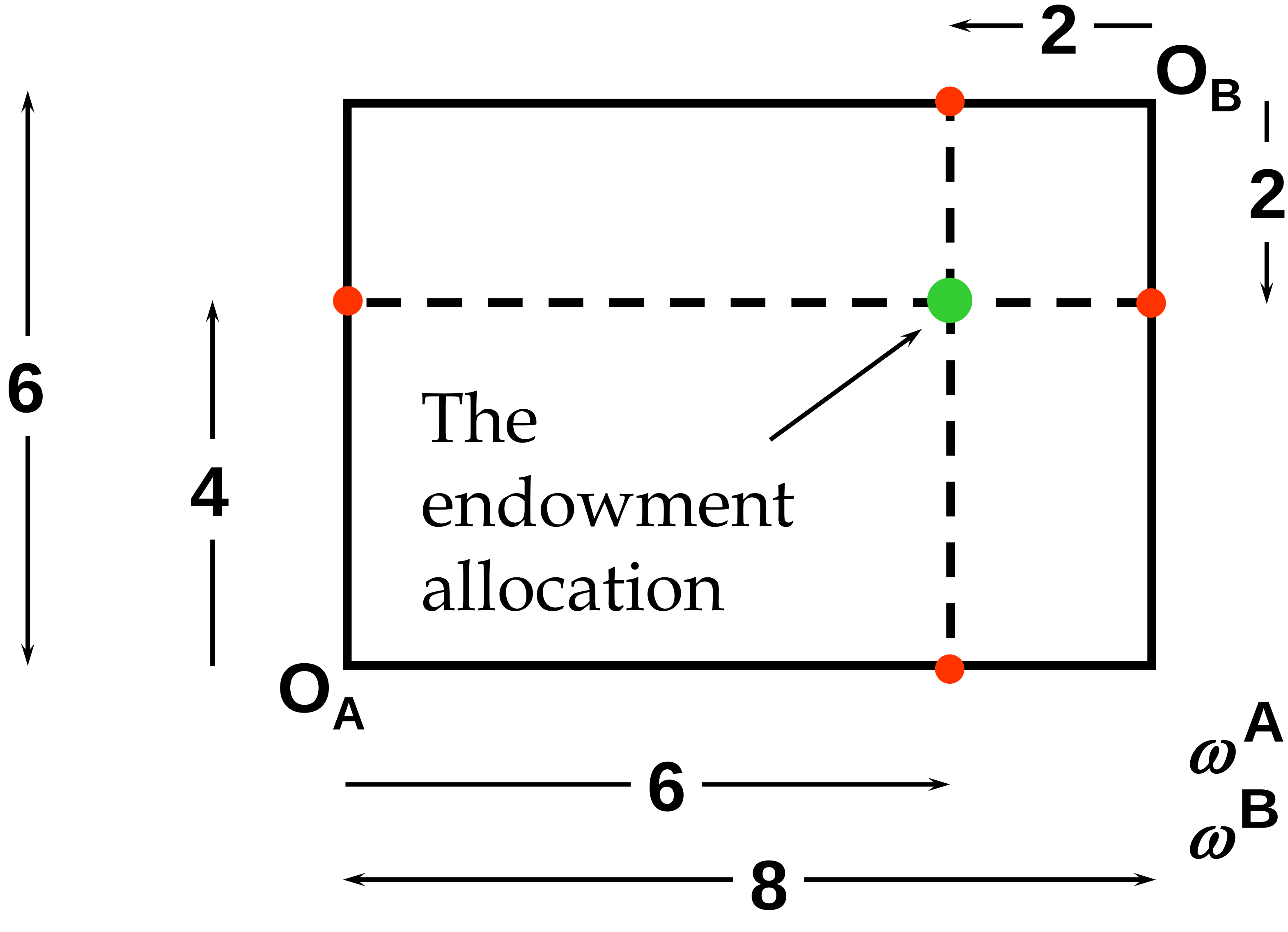
$\omega^A = (6,4)$

The Endowment Allocation



$\omega^B = (2,2)$

The Endowment Allocation



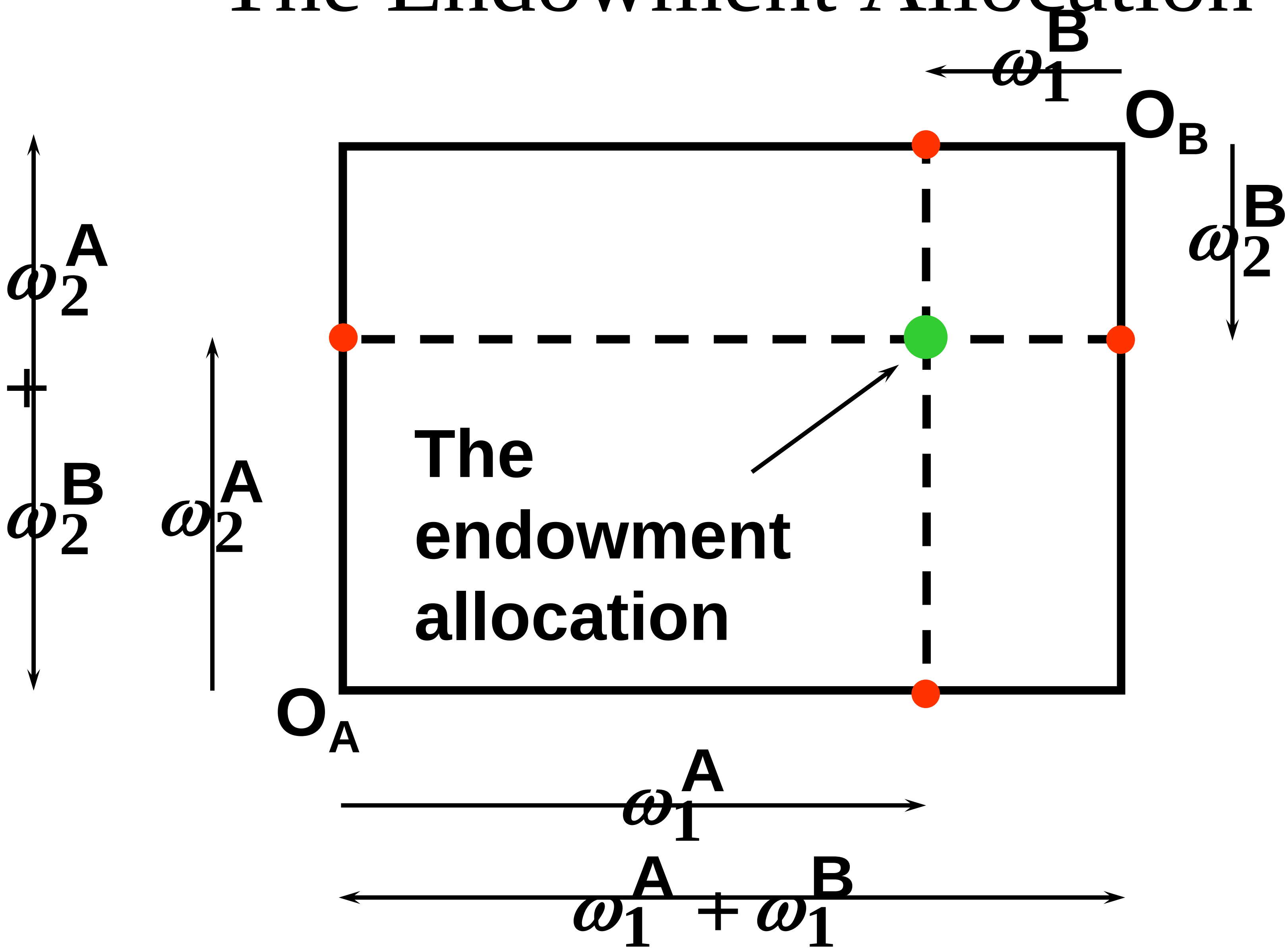
$\omega^A = (6,4)$

$\omega^B = (2,2)$

The Endowment Allocation

More generally, …

The Endowment Allocation



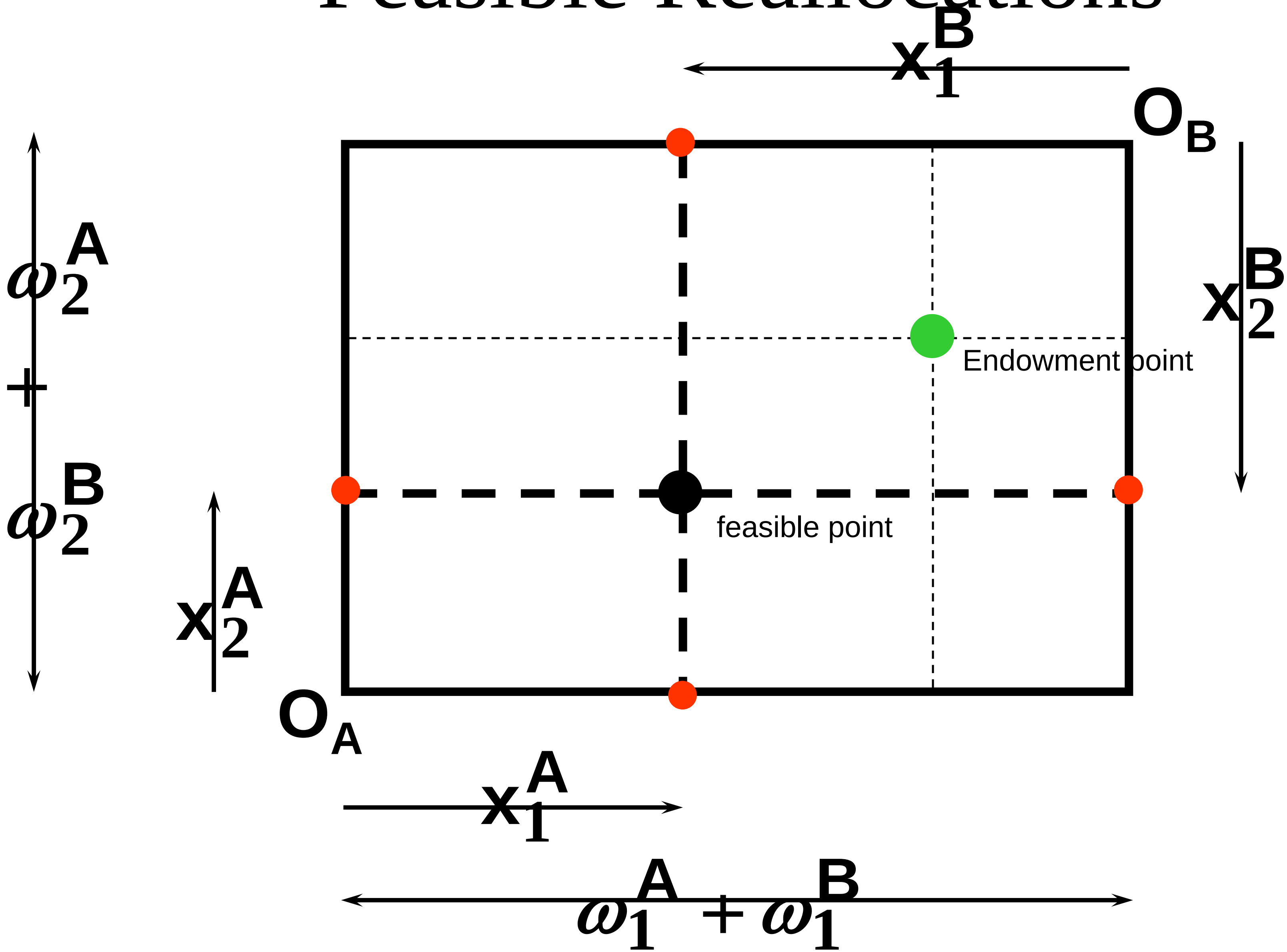
Other Feasible Allocations

- (x_1^A, x_2^A) denotes an allocation to consumer A.
- (x_1^B, x_2^B) denotes an allocation to consumer B.
- An allocation is feasible if and only if

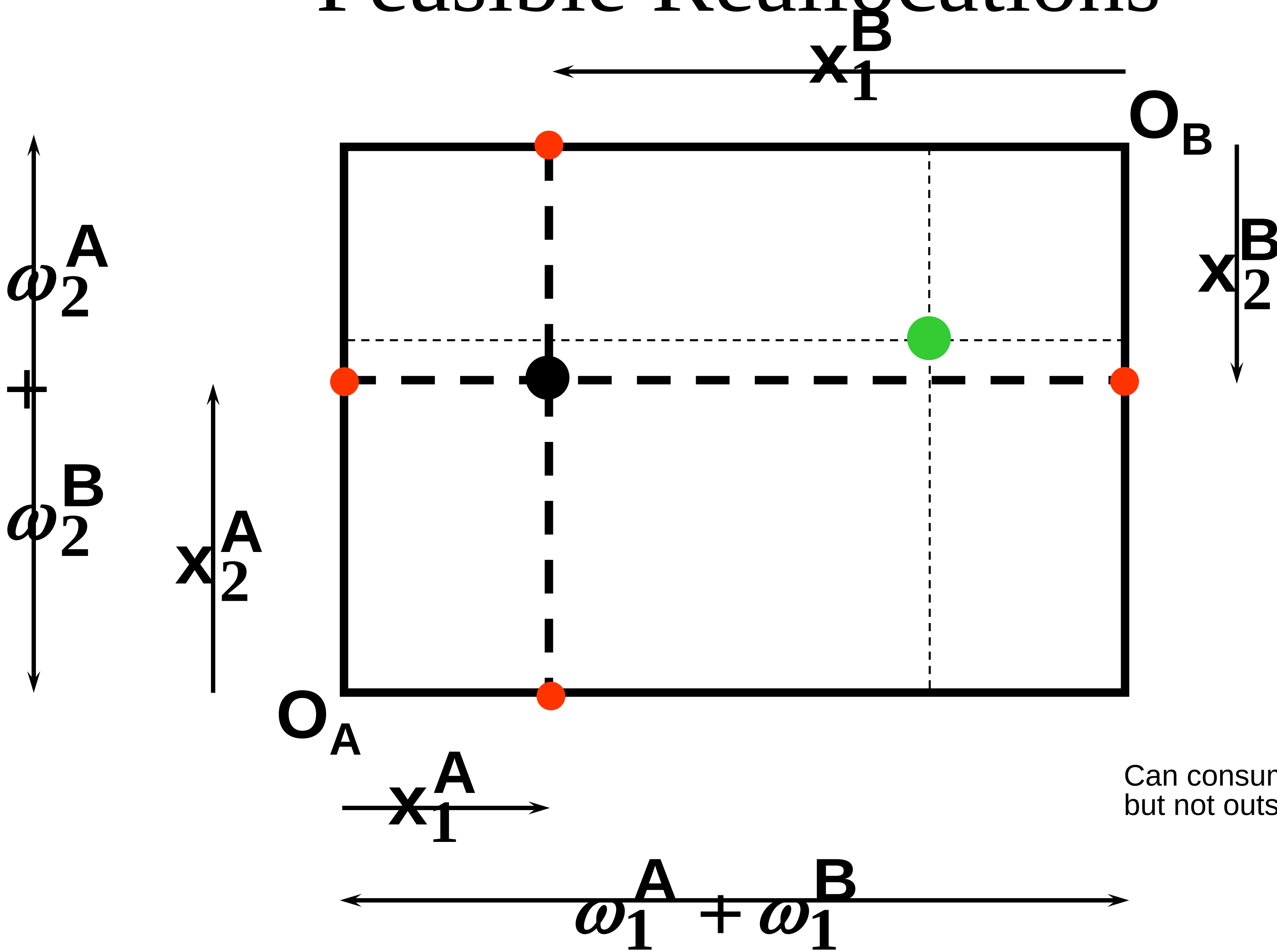
$$x_1^A + x_1^B \le \omega_1^A + \omega_1^B$$

and $$x_2^A + x_2^B \le \omega_2^A + \omega_2^B.$$

Feasible Reallocations



Feasible Reallocations

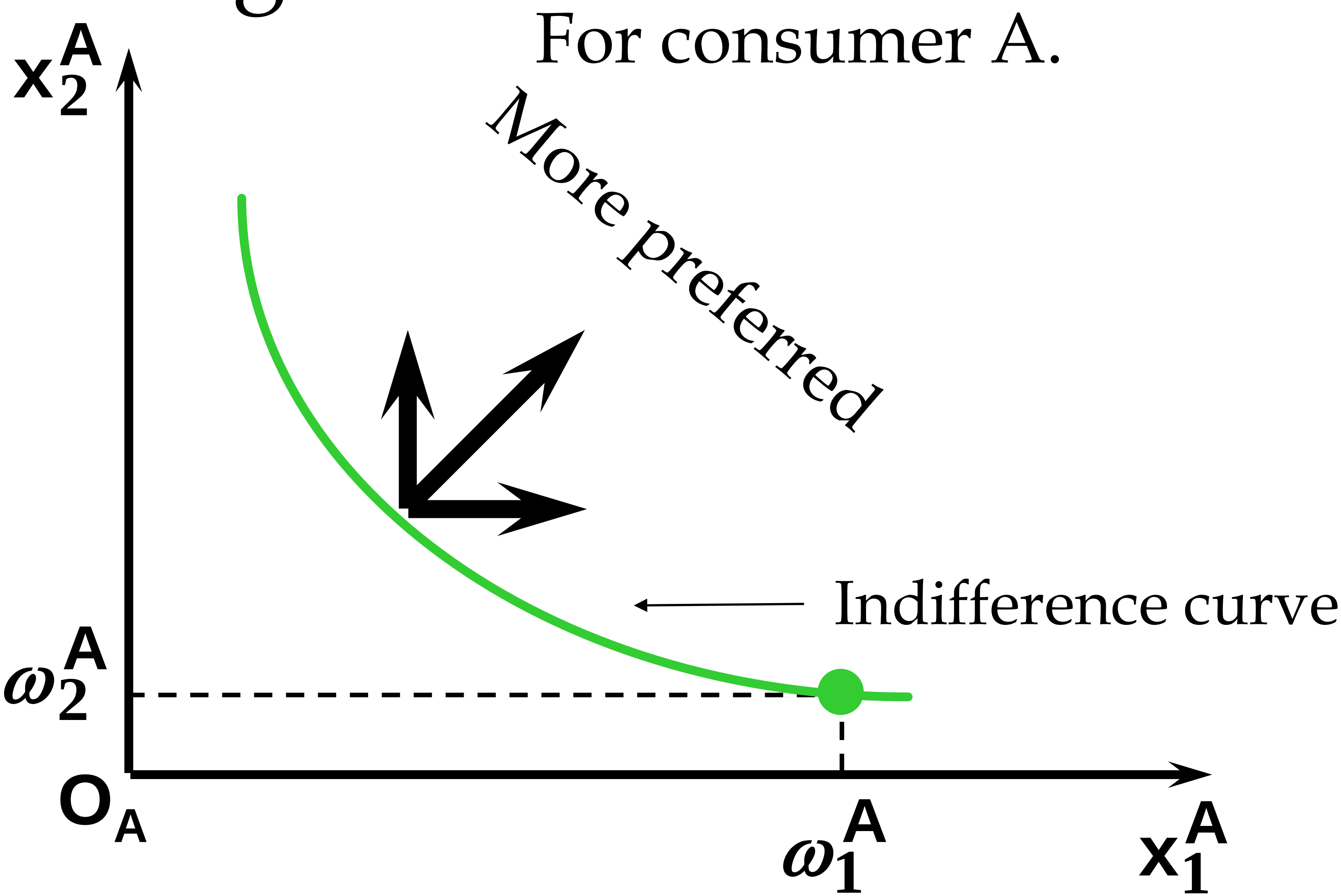


Can consume on the box
but not outside the box

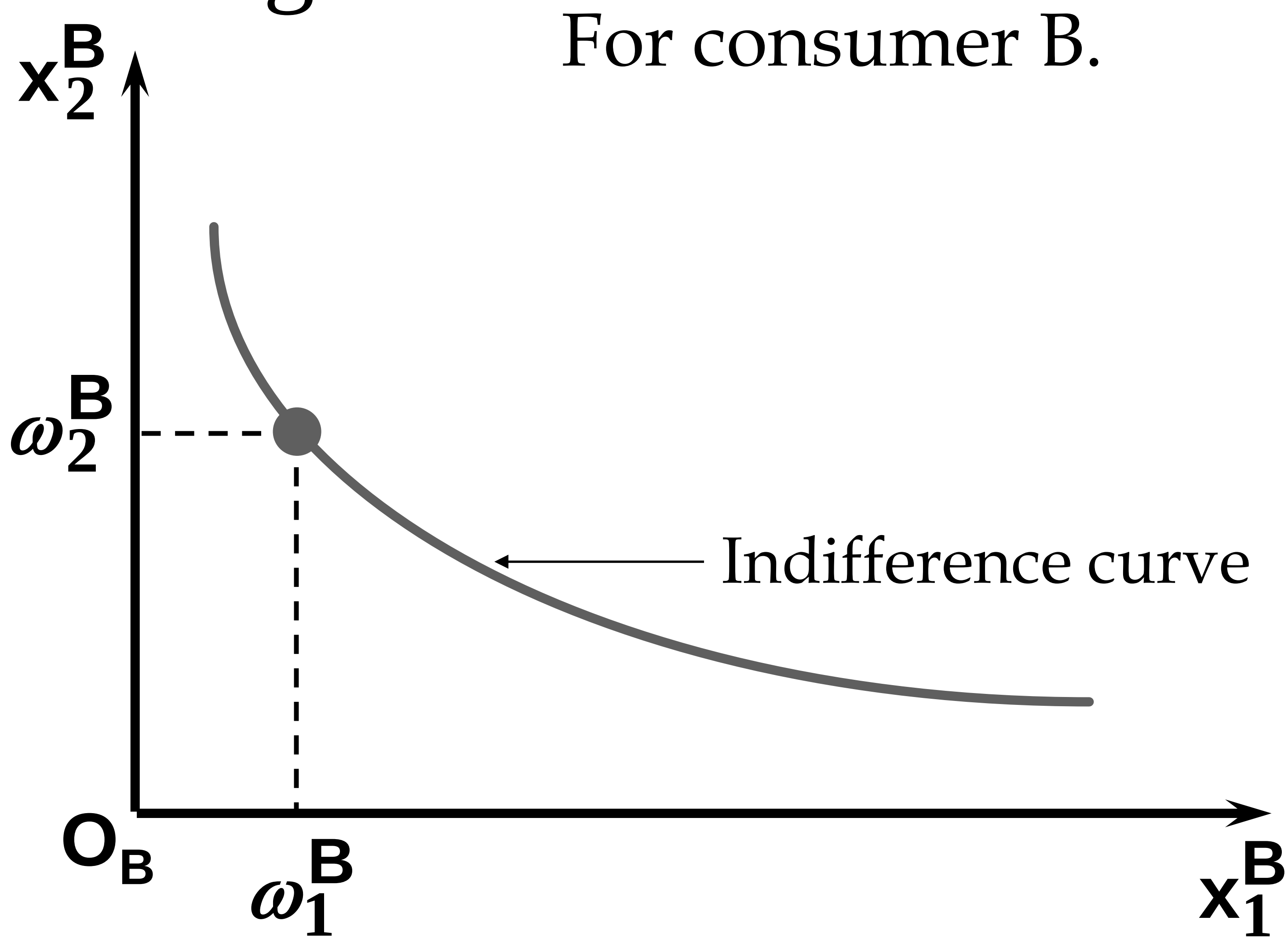
Feasible Reallocations

- All points in the box, including the boundary, represent feasible allocations of the combined endowments.
- Which allocations will be blocked by one or both consumers?
- Which allocations make both consumers better off?

Adding Preferences to the Box

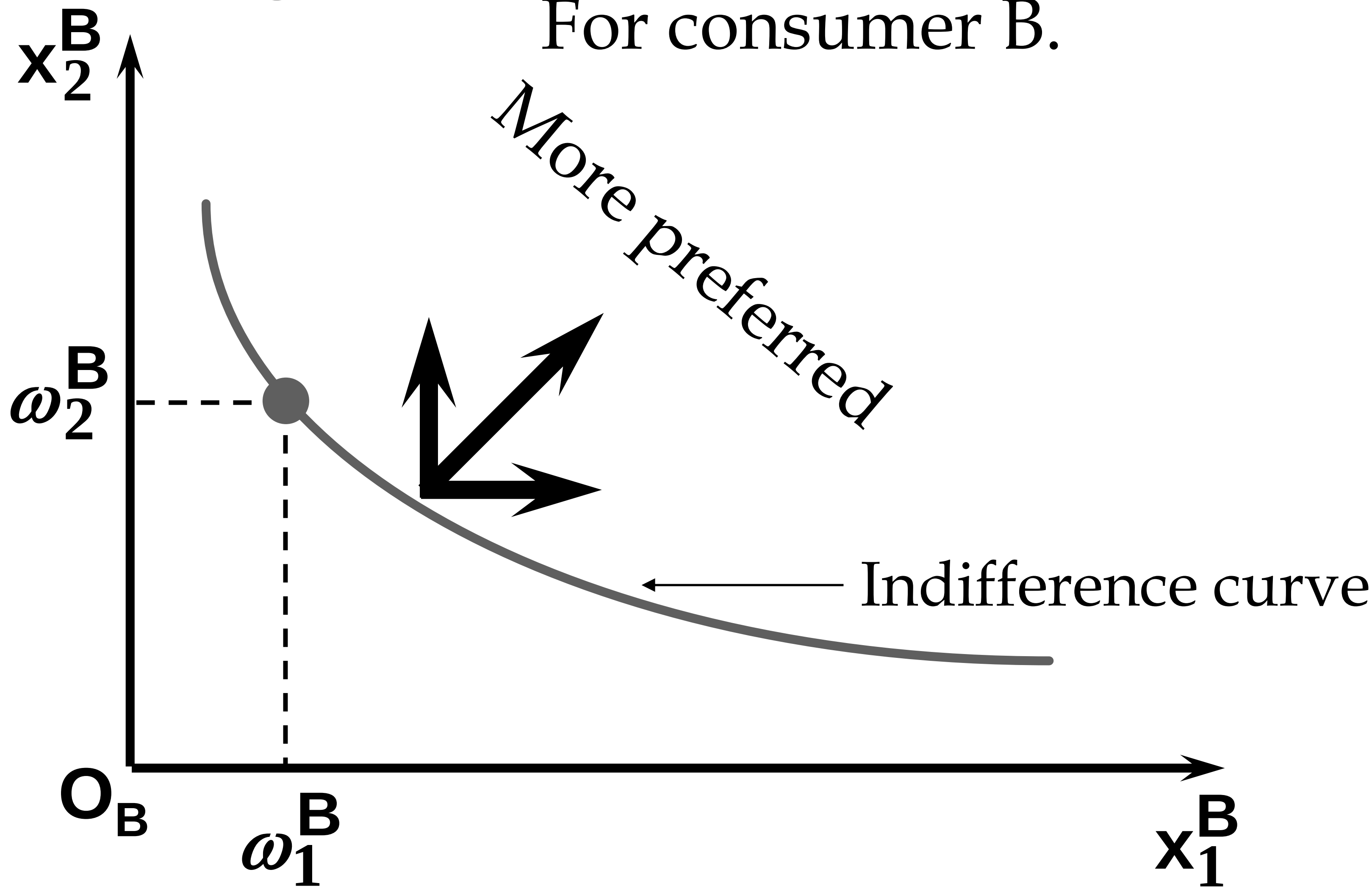


Adding Preferences to the Box



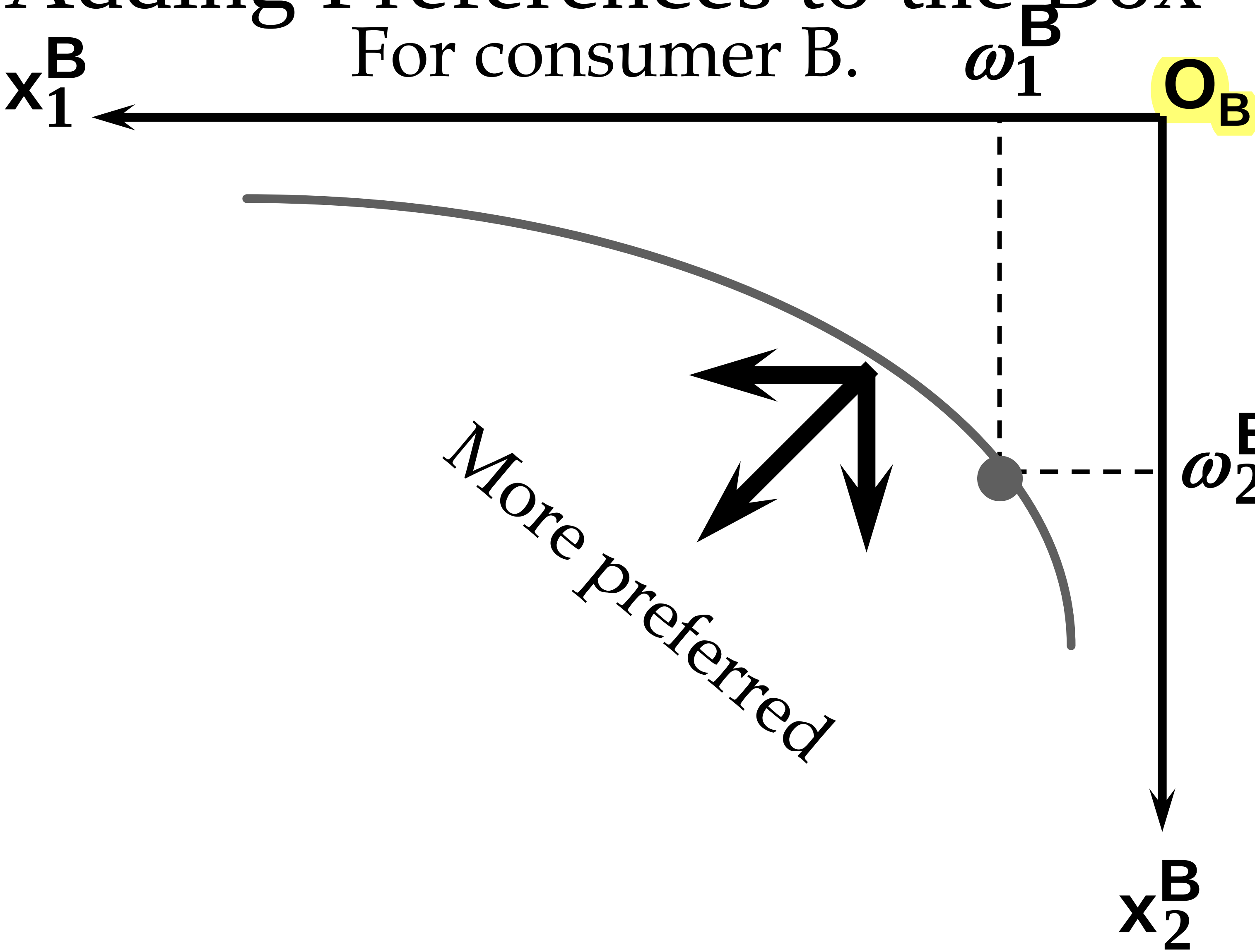
Adding Preferences to the Box

For consumer B.



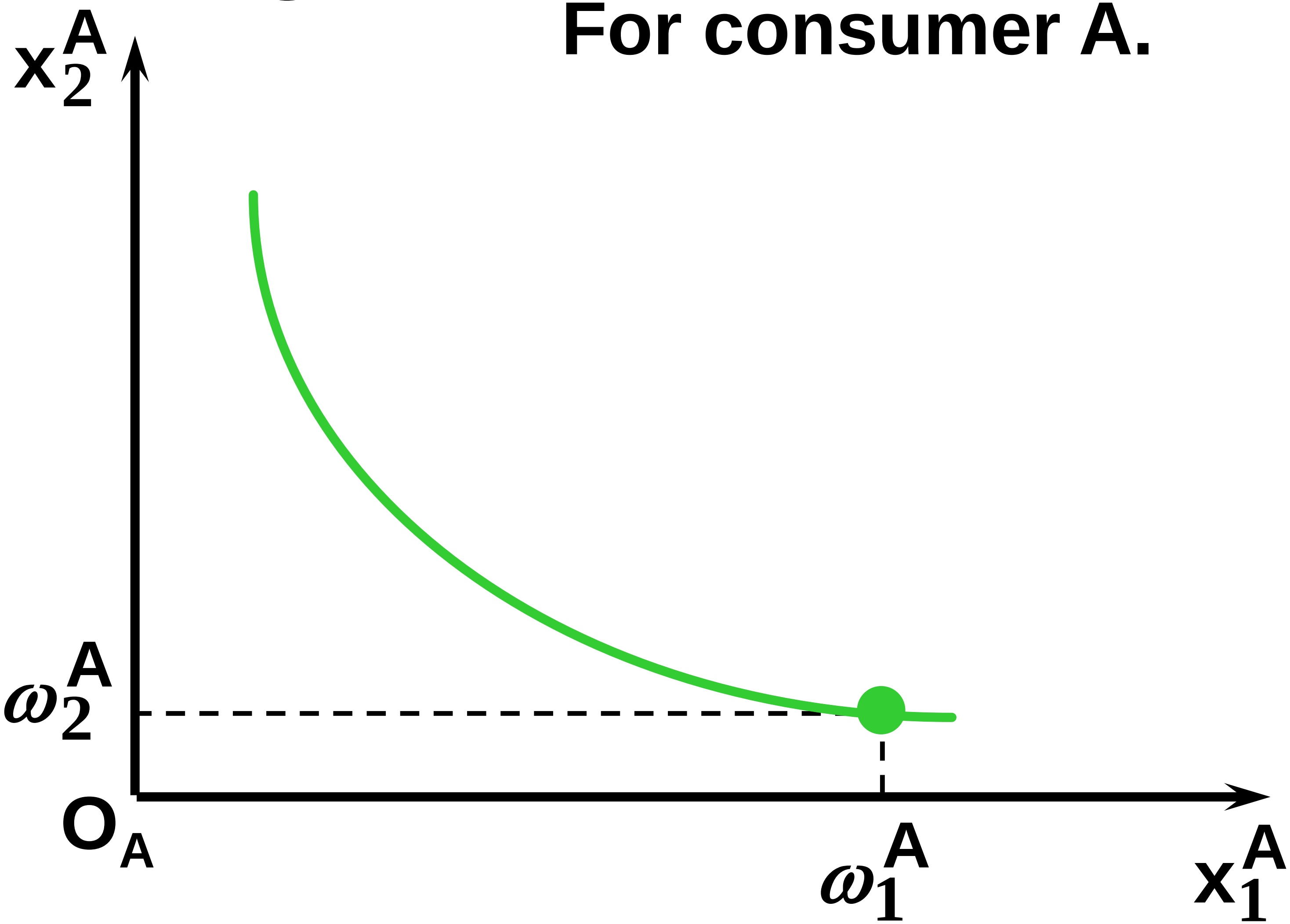
Adding Preferences to the Box

For consumer B.

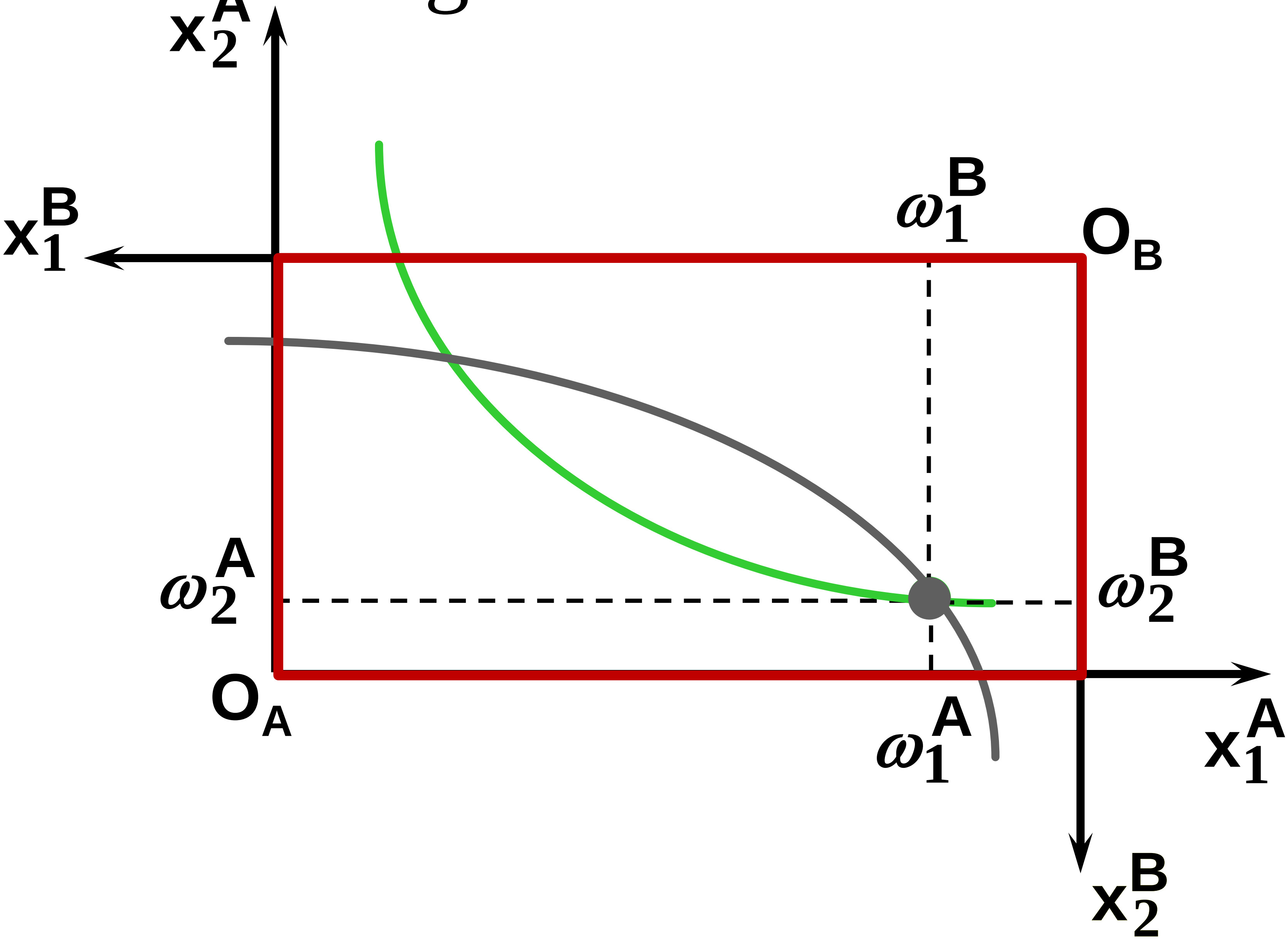


Adding Preferences to the Box

For consumer A.



Edgeworth's Box with their preferences



Pareto-Improvement

- An allocation of the endowment that improves the welfare of a consumer without reducing the welfare of another is a Pareto-improving allocation.

Pareto-Improvement

- A Pareto-efficient allocation can be described as an allocation where:
  1. ~~there is no way to make all the people involved better off; or~~

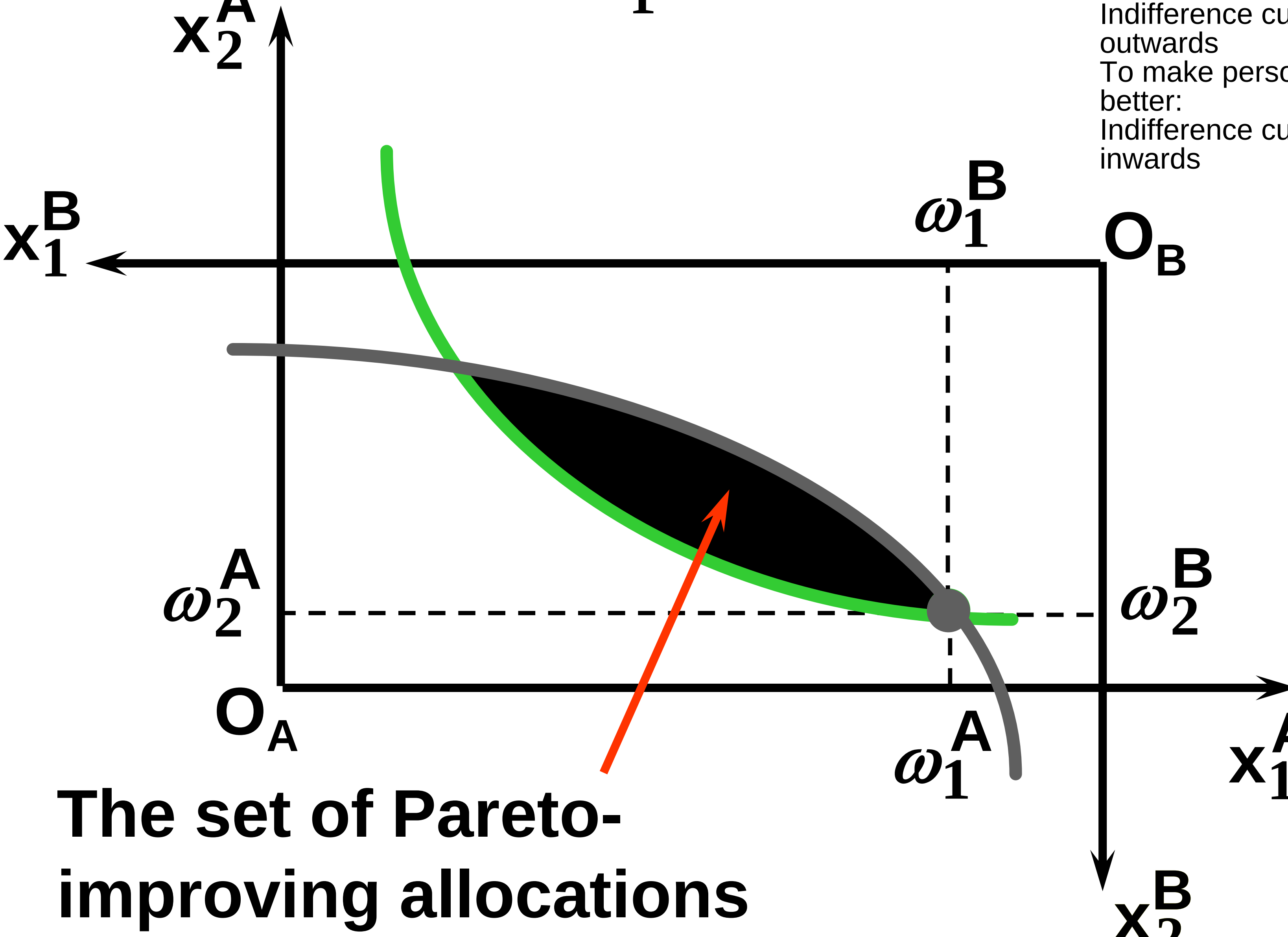
     only this definition is important
  2. there is no way to make some individual better off without making someone else worse off; or
  3. ~~all of the gains from trade have been exhausted; or~~
  4. ~~there are no mutually advantageous trades to be made~~

Pareto-Improvement

- Where are the Pareto-improving allocations?

Pareto-Improvements

To make person A better:
Indifference curve outwards
To make person B better:
Indifference curve goes inwards



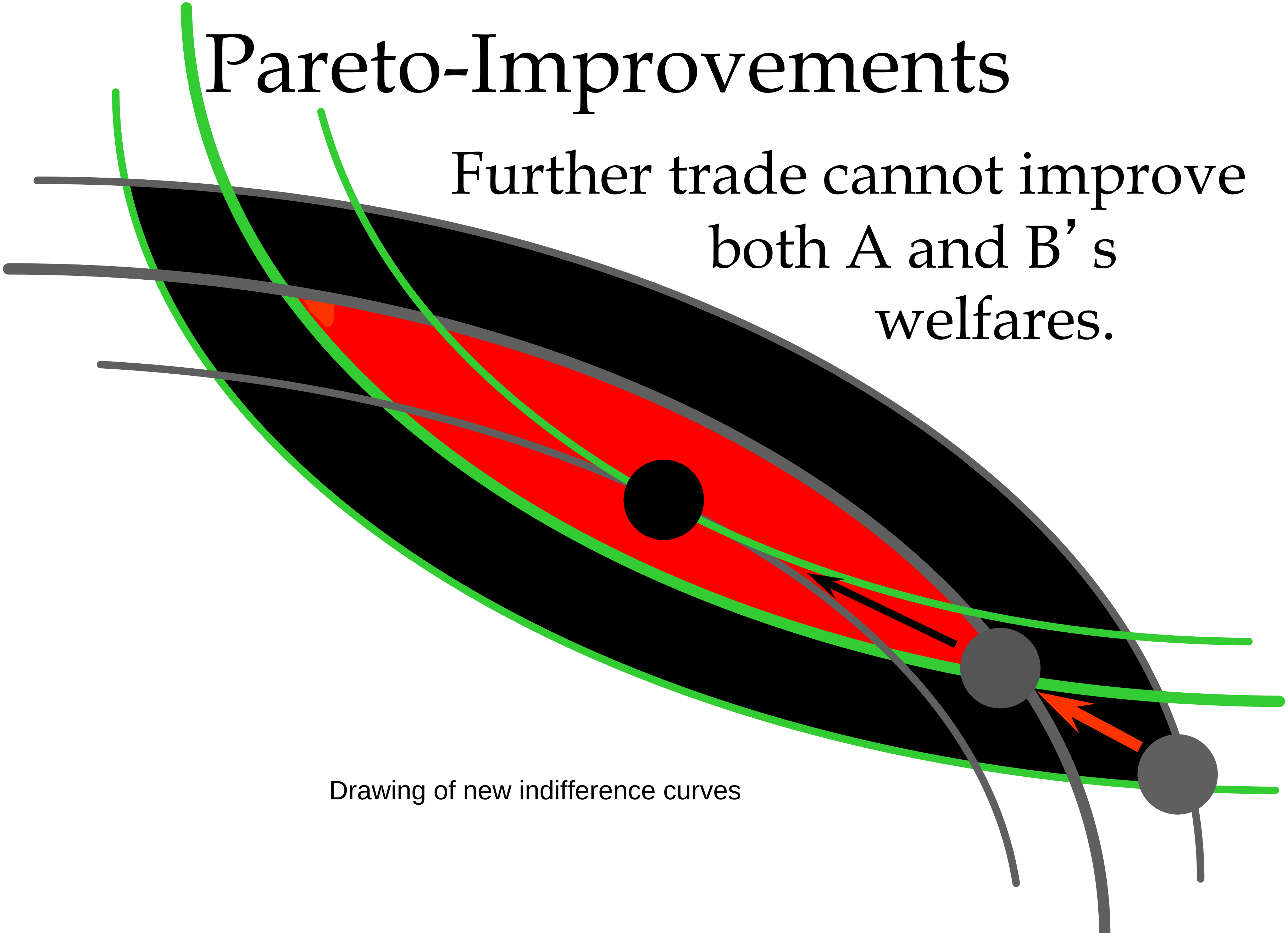
The set of Pareto-improving allocations

Pareto-Improvements

- Since each consumer can refuse to trade, the only possible outcomes from exchange are Pareto-improving allocations.
- But which particular Pareto-improving allocation will be the outcome of trade?

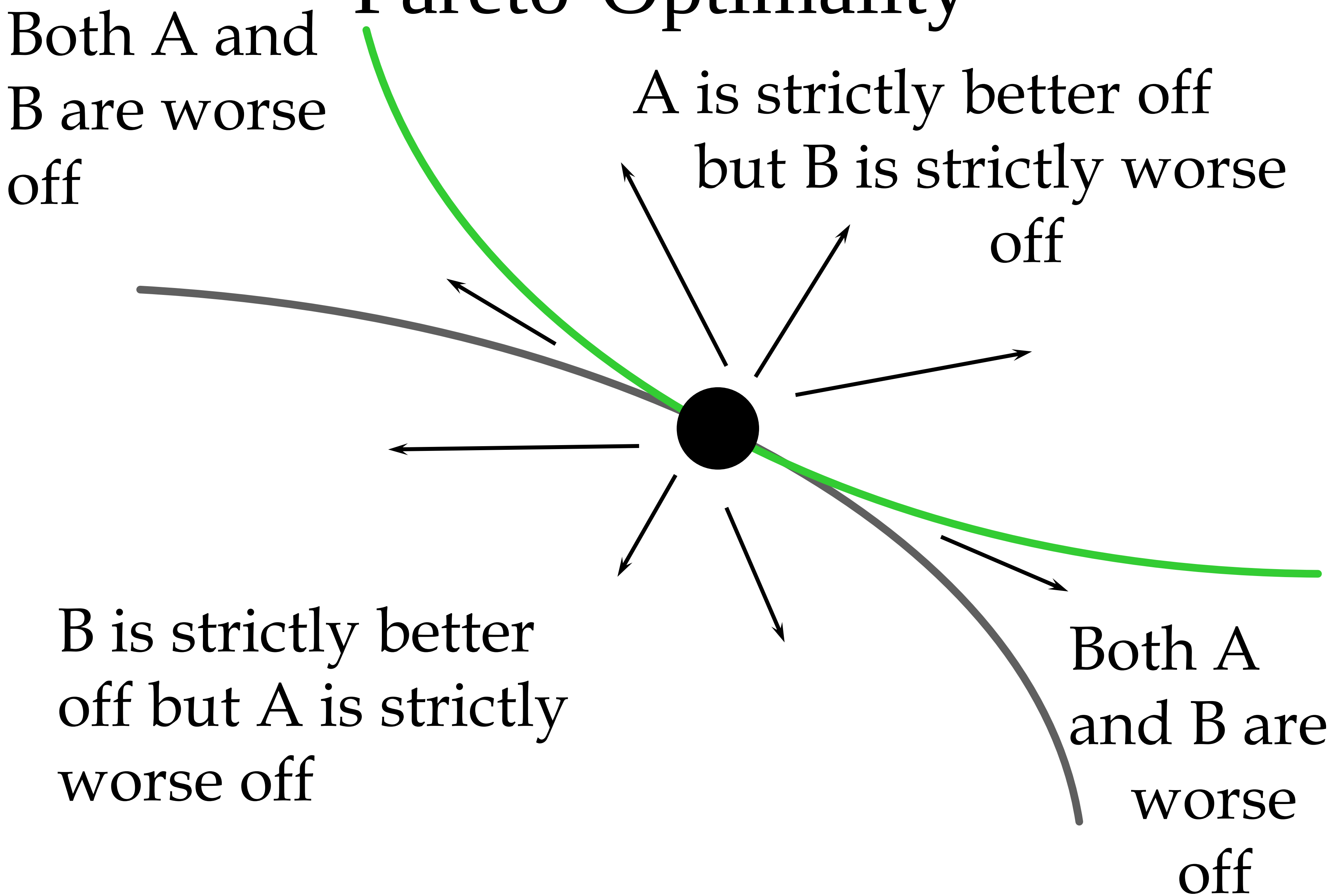
Pareto-Improvements

Further trade cannot improve both A and B’s welfares.

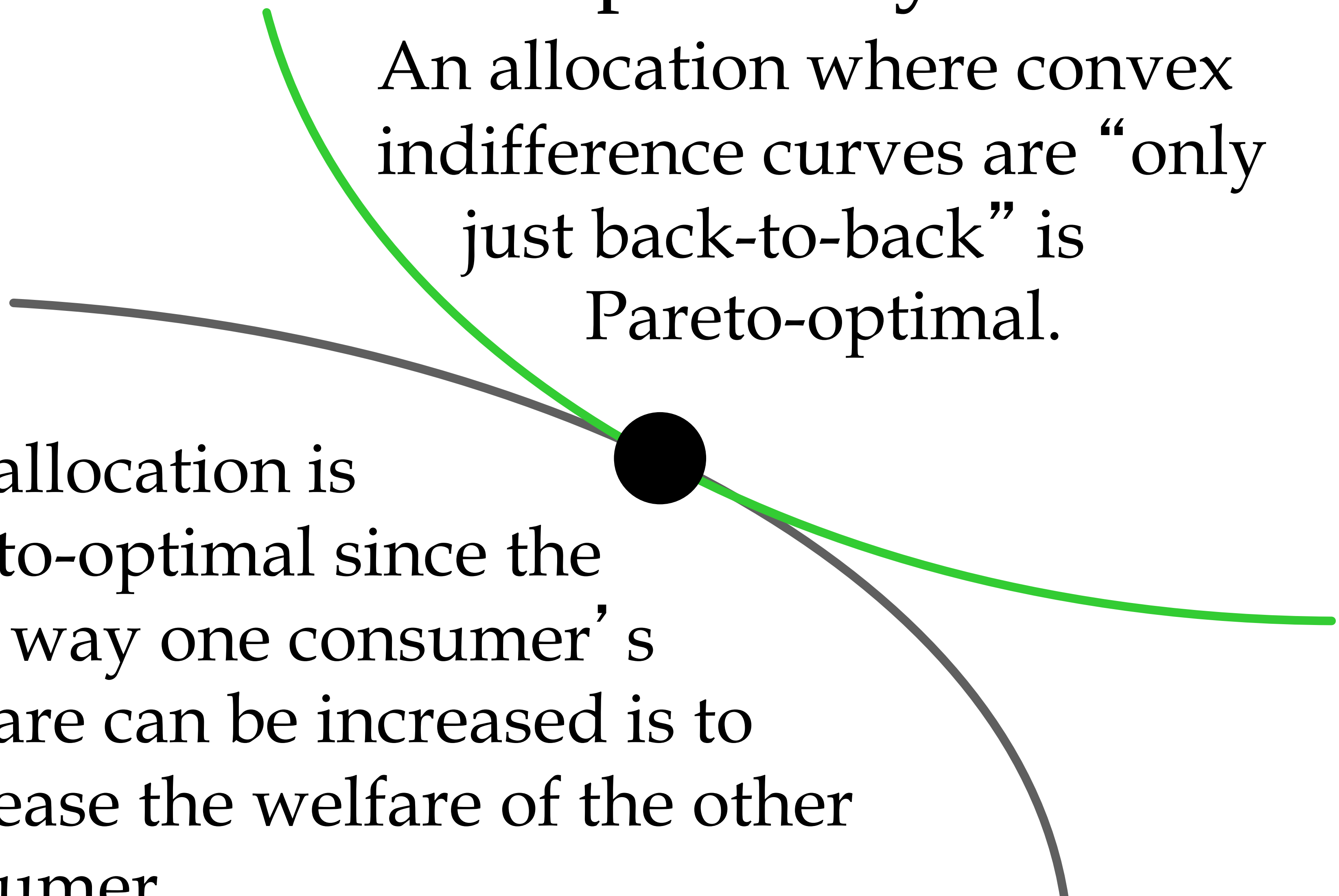


Drawing of new indifference curves

Pareto-Optimality



Pareto-Optimality



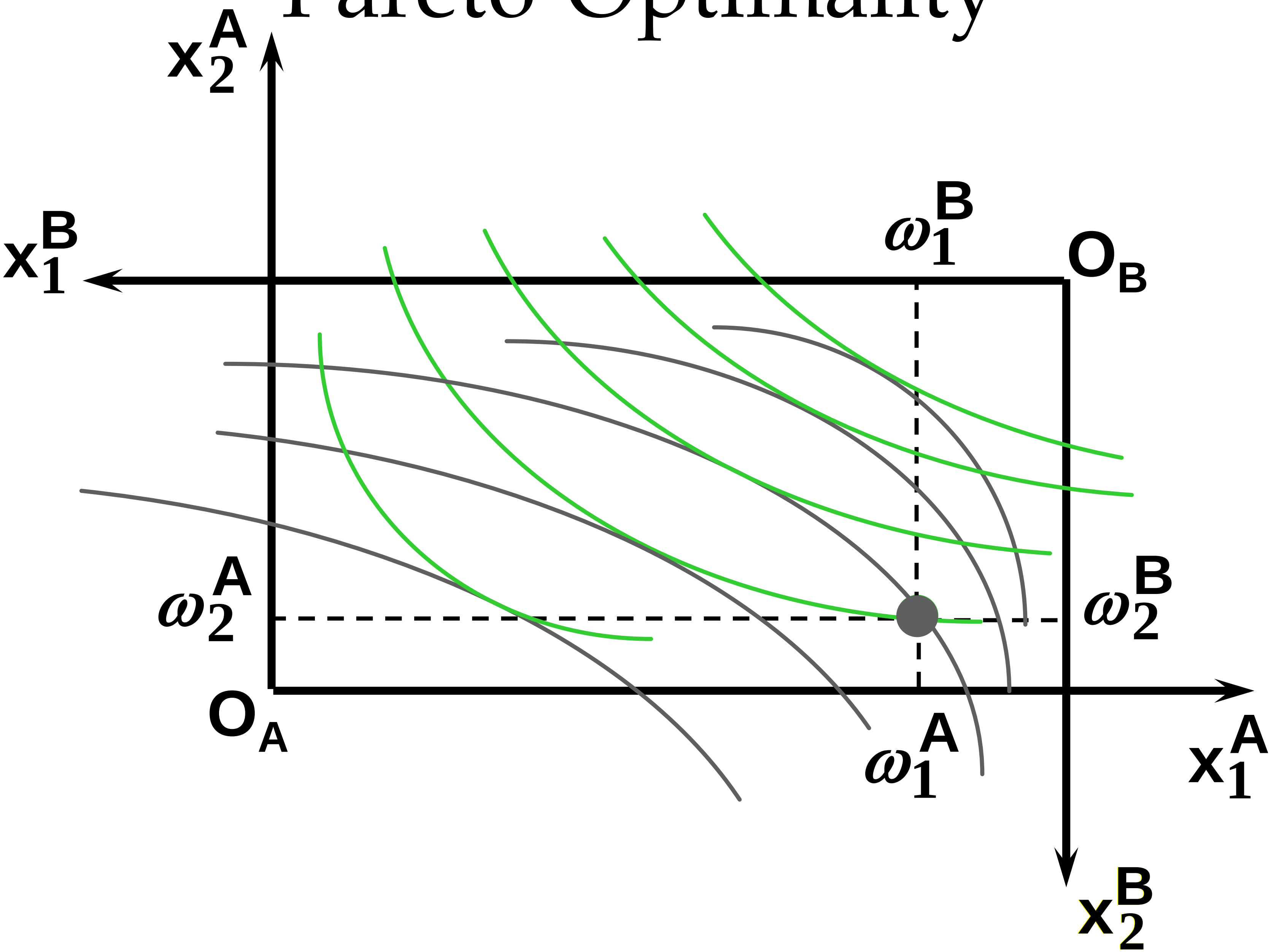
An allocation where convex indifference curves are “only just back-to-back” is Pareto-optimal.

The allocation is Pareto-optimal since the only way one consumer’ s welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality

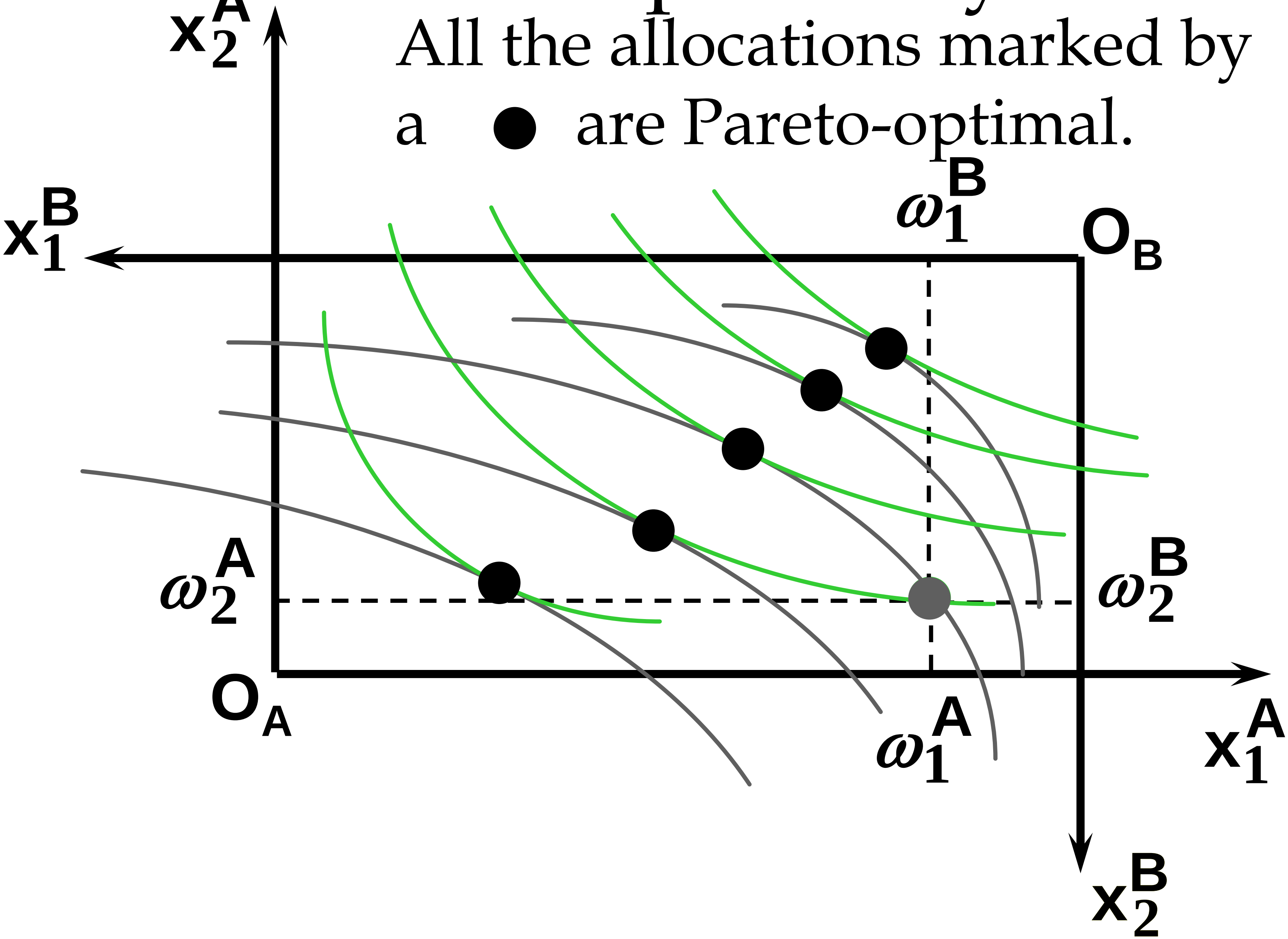
- Where are all of the Pareto-optimal allocations of the endowment?

Pareto-Optimality



Pareto-Optimality

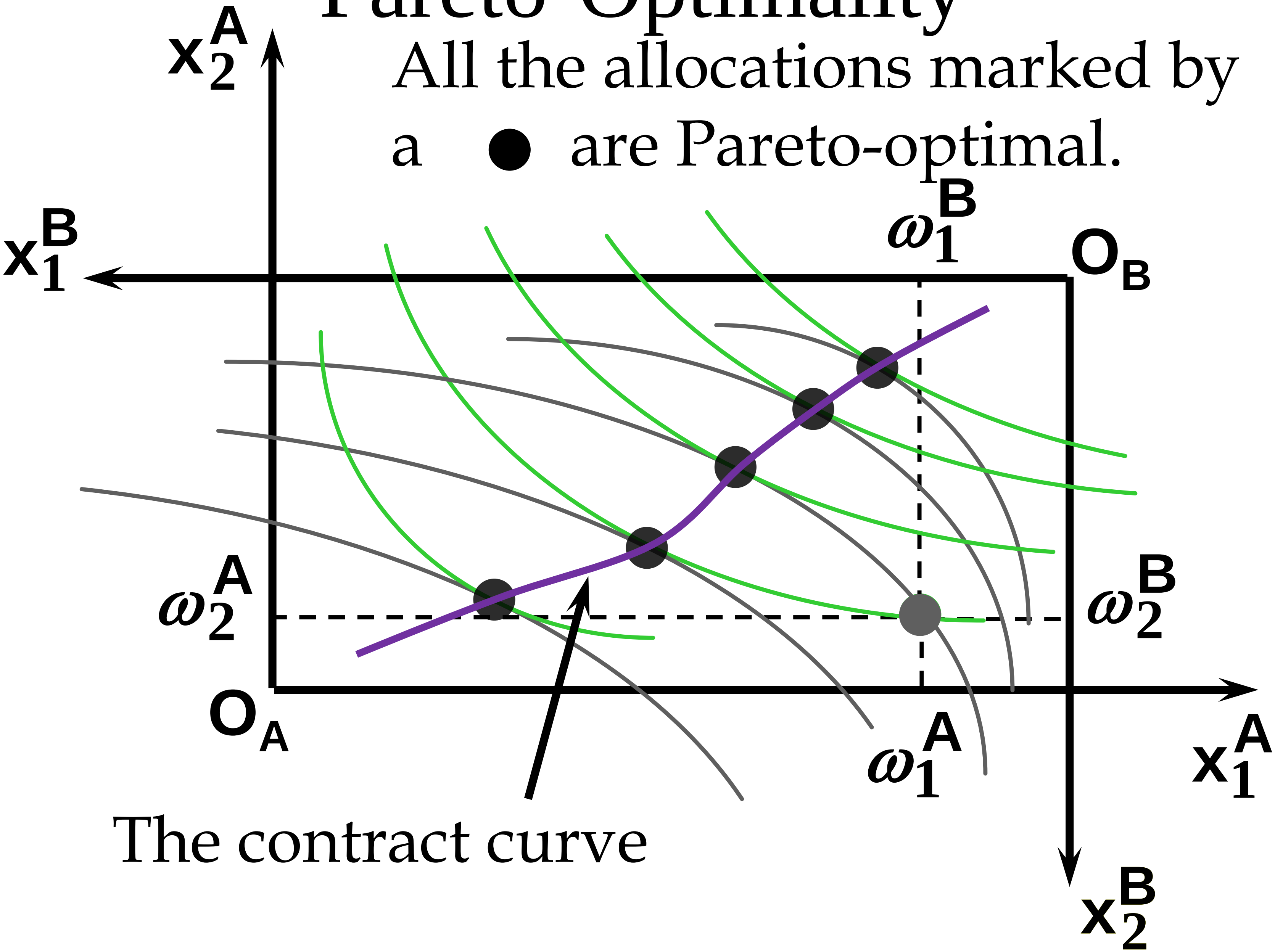
All the allocations marked by a ● are Pareto-optimal.



Pareto-Optimality

- The contract curve is the set of all Pareto-optimal allocations.

Pareto-Optimality



Pareto-Optimality

- What defining feature characterizes the contract curve?
  - The indifference curves of both consumers are always tangent

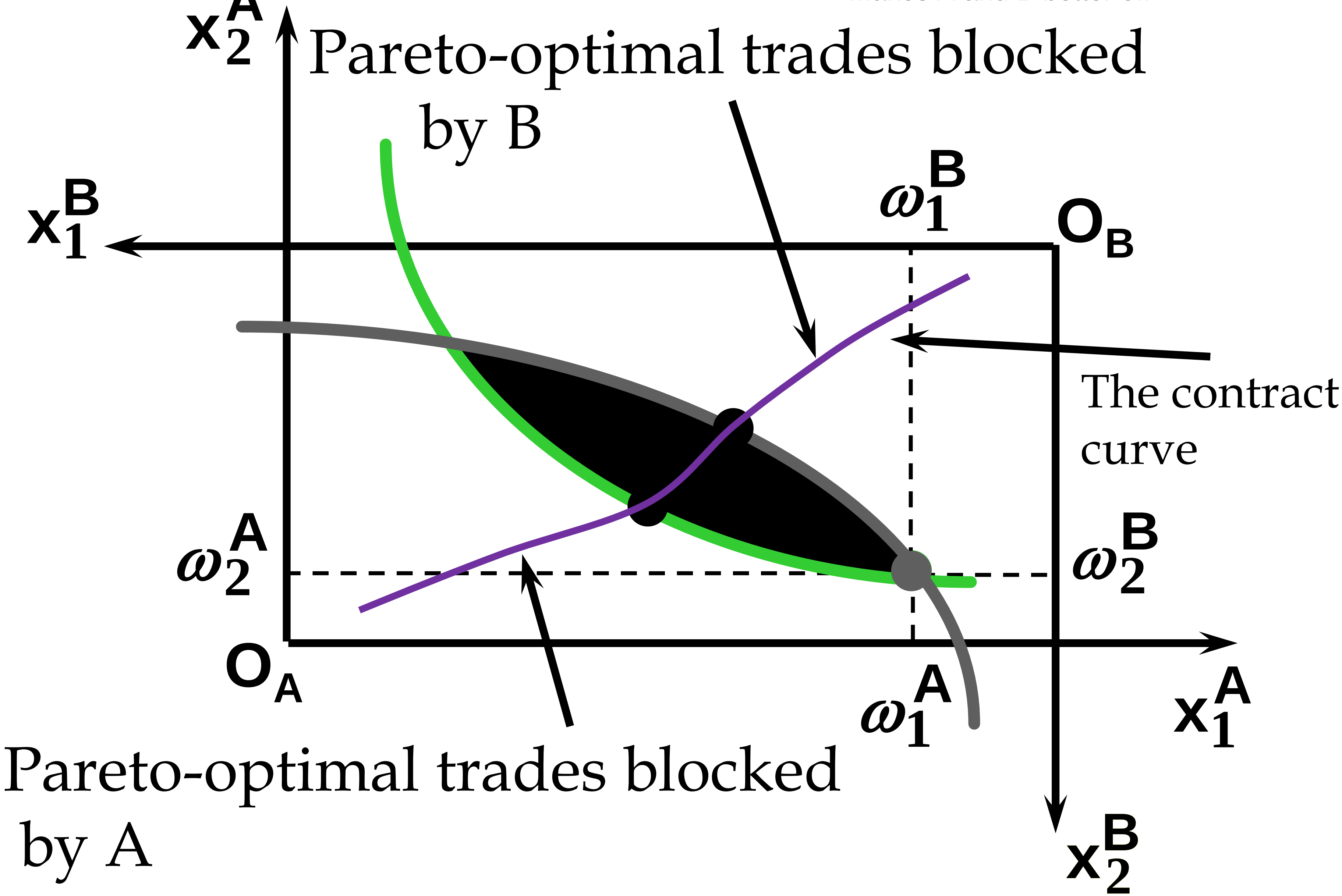
$$\frac{MU_{x^A{}_1}}{MU_{x^A{}_2}} = \frac{MU_{x^B{}_1}}{MU_{x^B{}_2}}$$

Pareto-Optimality

- But to which of the many allocations on the contract curve will consumers trade?
- That depends upon how trade is conducted.
- In perfectly competitive markets? By one-on-one bargaining?

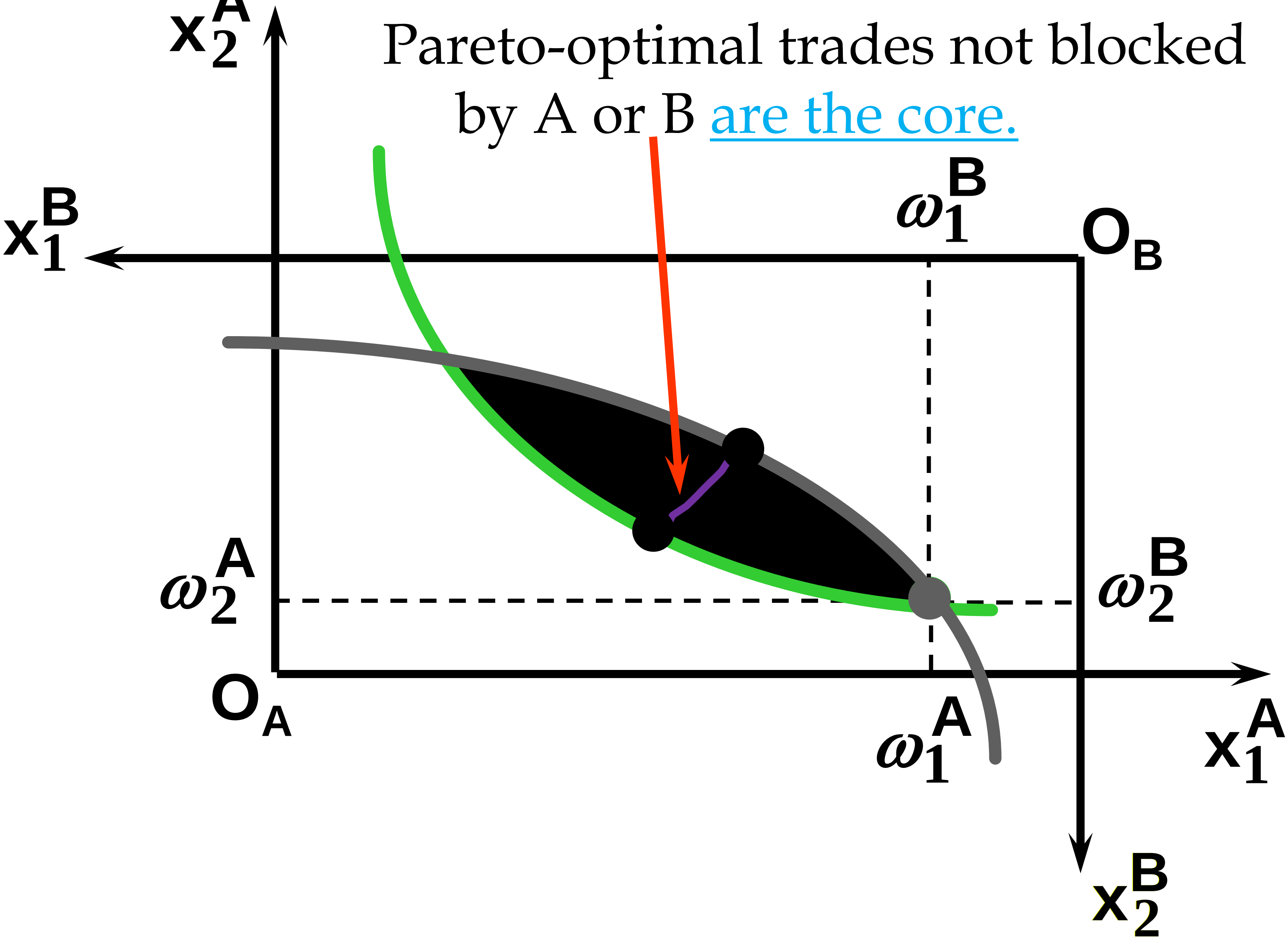
The Core

Any point on the contract curve that makes A and B better off



The Core

Pareto-optimal trades not blocked by A or B are the core.



The Core

- The core is the set of all Pareto-optimal allocations that are welfare-improving for both consumers relative to their own endowments.
- Rational trade should achieve a core allocation.

The Core

- But which core allocation?
- Again, that depends upon the manner in which trade is conducted.
- If there are just two agents, A and B, then the outcome depends on their negotiation skills. If A is a more astute trade, he may persuade B to agree to an allocation more in A's favor than B's.

Trade in Competitive Markets

- Consider trade in perfectly competitive markets.
  - A competitive market cannot have just two consumers.
  - Here we assume A and B are two types of consumers. There are many consumers of type A and B.

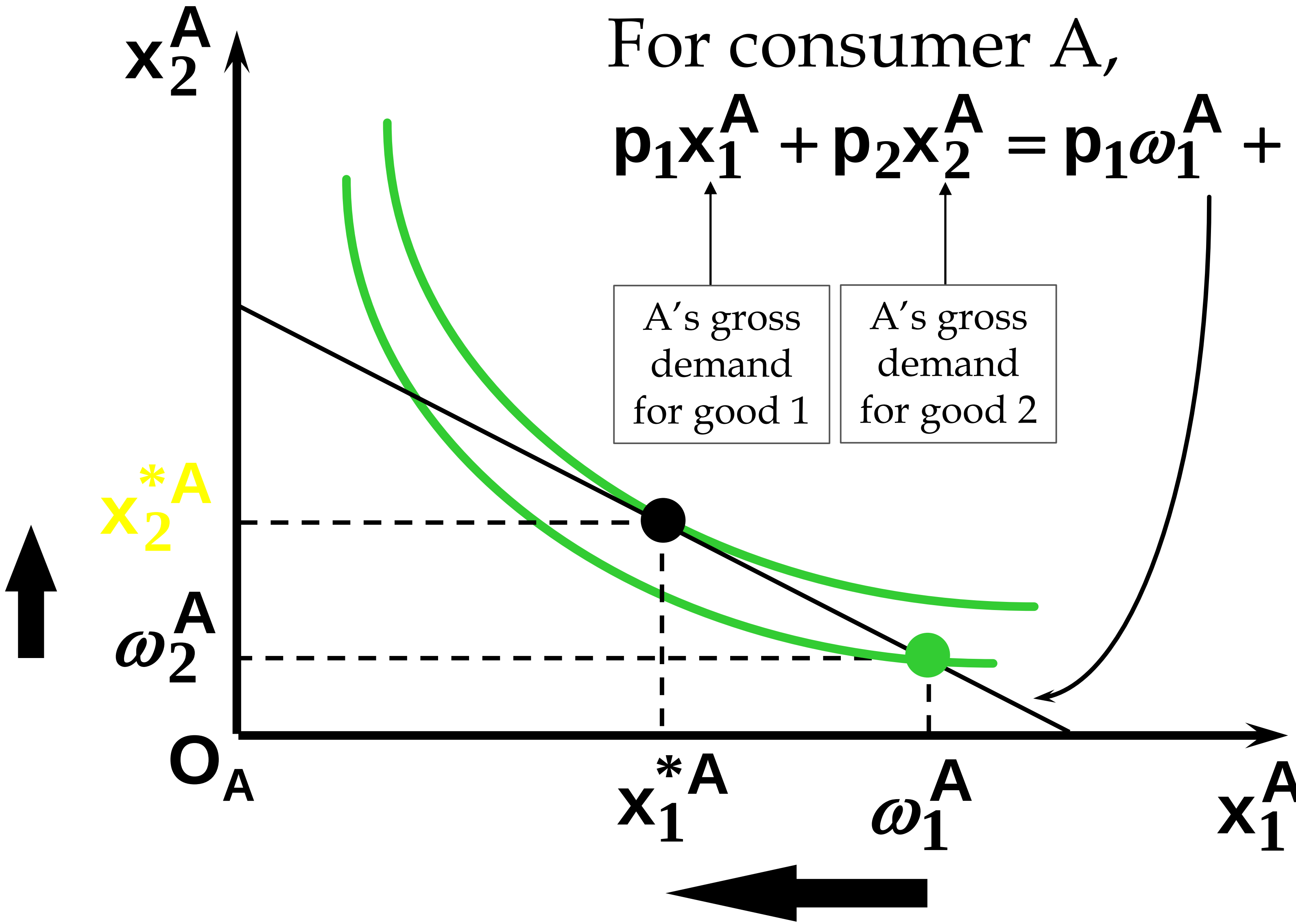
Trade in Competitive Markets

- Take as given arbitrary prices, p_1 and p_2
- Each consumer is a price-taker trying to maximize her own utility given p_1, p_2 and her own endowment. That is,

Trade in Competitive Markets

For consumer A,

$$p_1x_1^A + p_2x_2^A = p_1\omega_1^A + p_2\omega_2^A$$



Trade in Competitive Markets

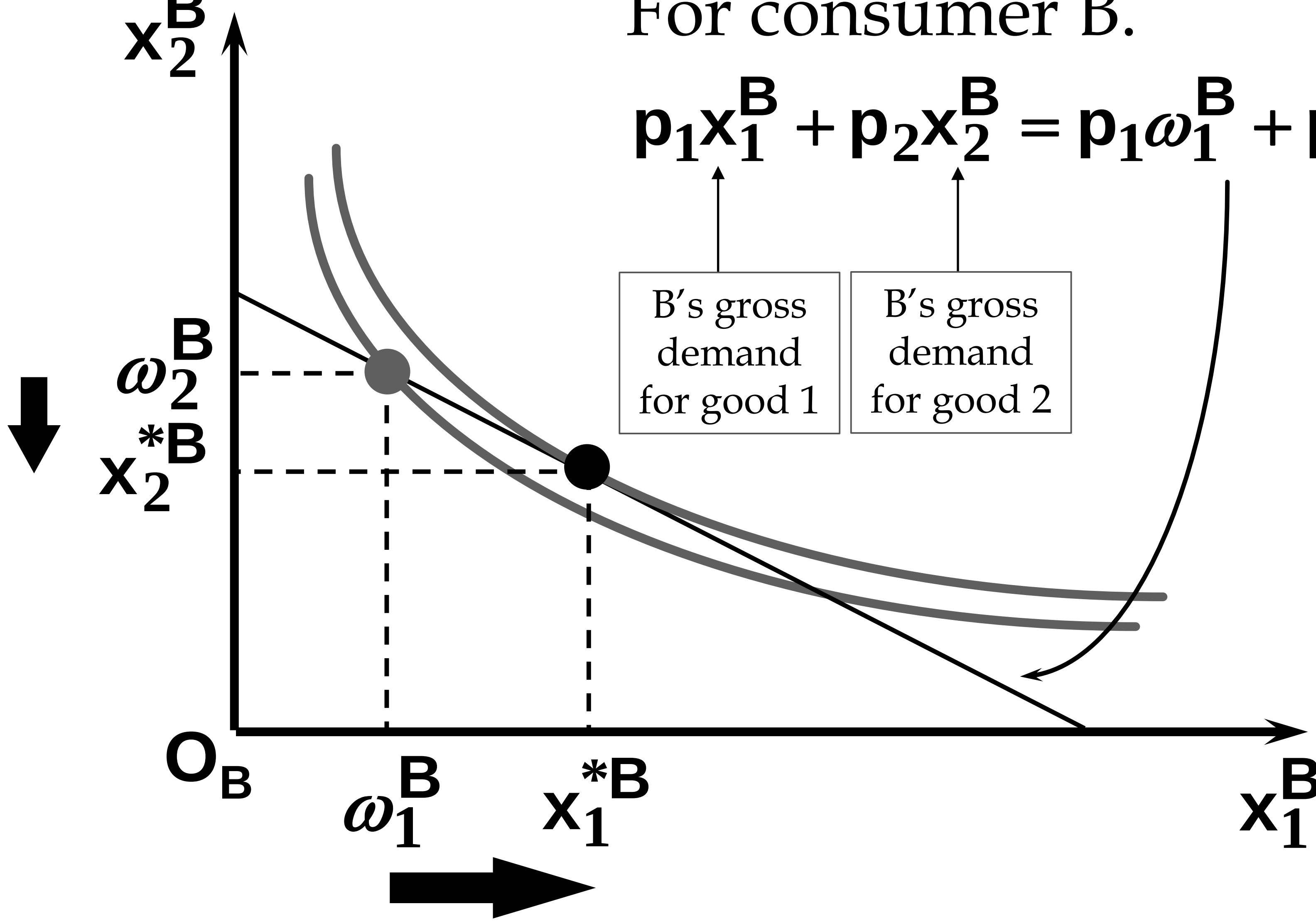
- So given p_1 and p_2, consumer A's net demands for commodities 1 and 2 are

$$\underbrace{x_1^{*A} - \omega_1^A}_{\text{A's excess (or net) demand for good 1}} \quad \text{and} \quad \underbrace{x_2^{*A} - \omega_2^A}_{\text{A's excess (or net) demand for good 2}}.$$

Trade in Competitive Markets

For consumer B.

$$p_1x_1^B + p_2x_2^B = p_1\omega_1^B + p_2\omega_2^B$$



Trade in Competitive Markets

- So given p_1 and p_2, consumer B' s net demands for commodities 1 and 2 are

$$\underbrace{x_1^{*B} - \omega_1^B}_{\text{B's excess (or net) demand for good 1}} \quad \text{and} \quad \underbrace{x_2^{*B} - \omega_2^B}_{\text{B's excess (or net) demand for good 2}}.$$

Trade in Competitive Markets

- A general equilibrium occurs when prices p_1 and p_2 cause both the markets for commodities 1 and 2 to clear; i.e.

everything that is endowed is consumed.

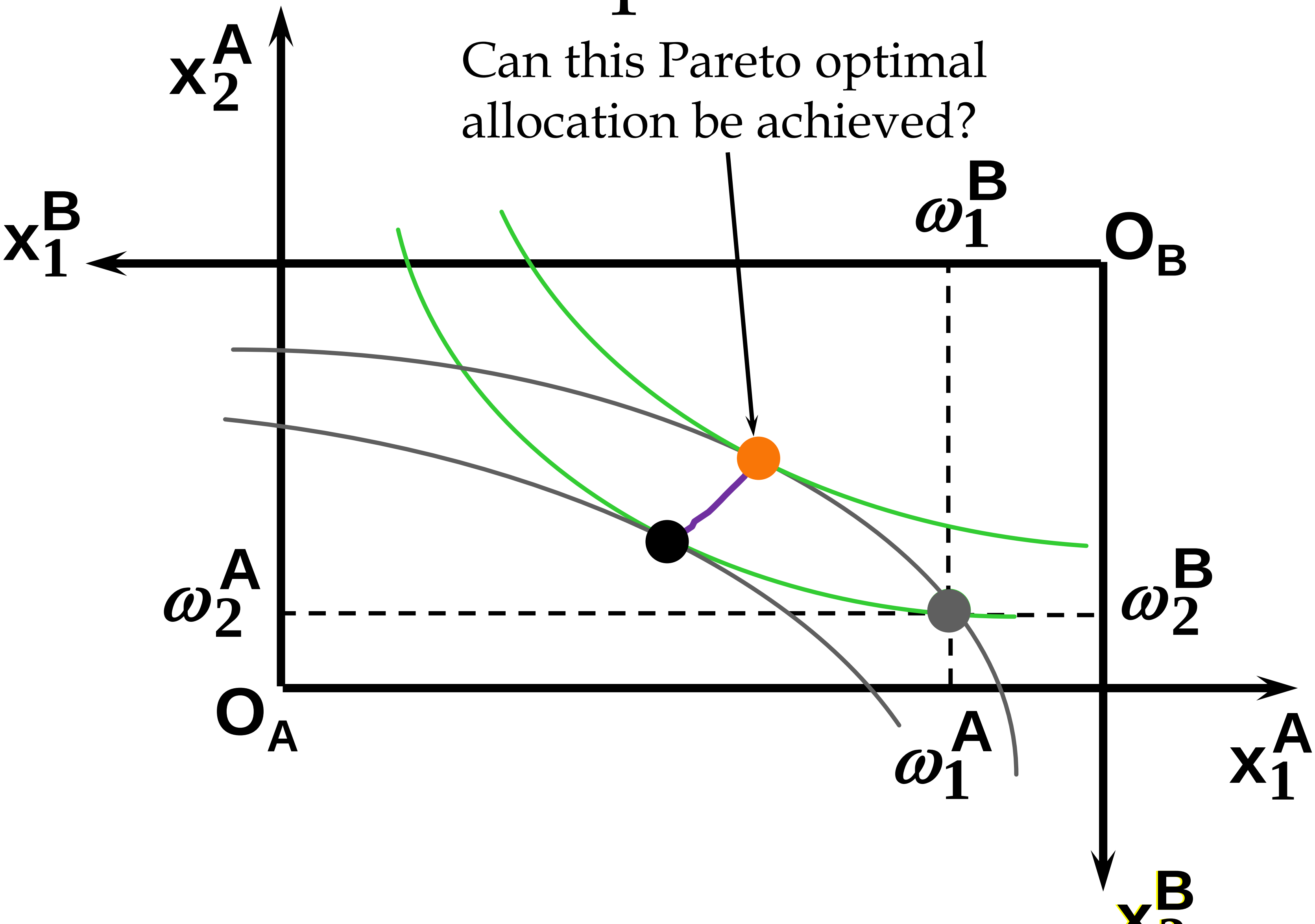
$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

and

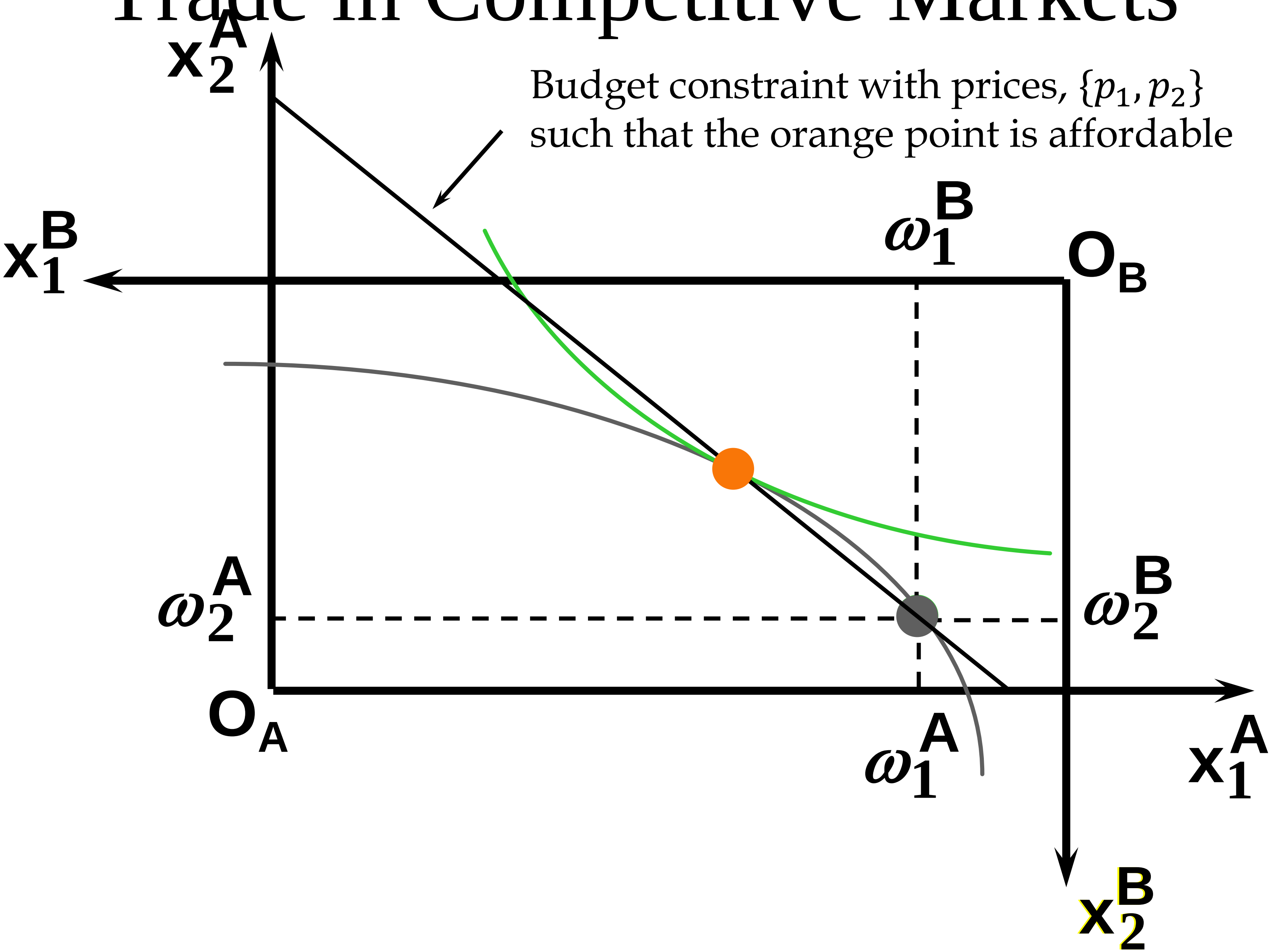
$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B.$$

- This means that the allocation must be feasible (on or inside the Edgeworth box)

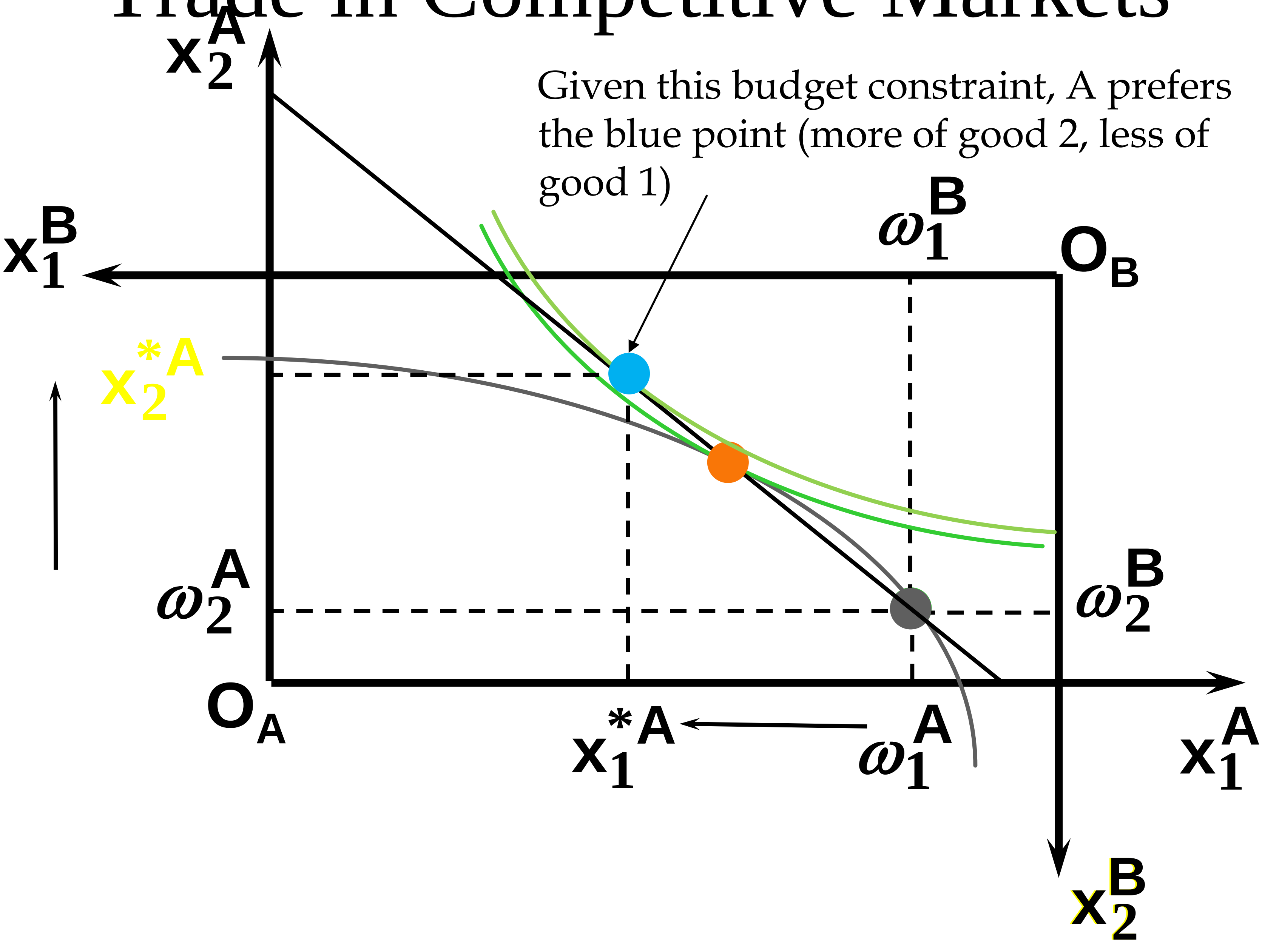
Trade in Competitive Markets



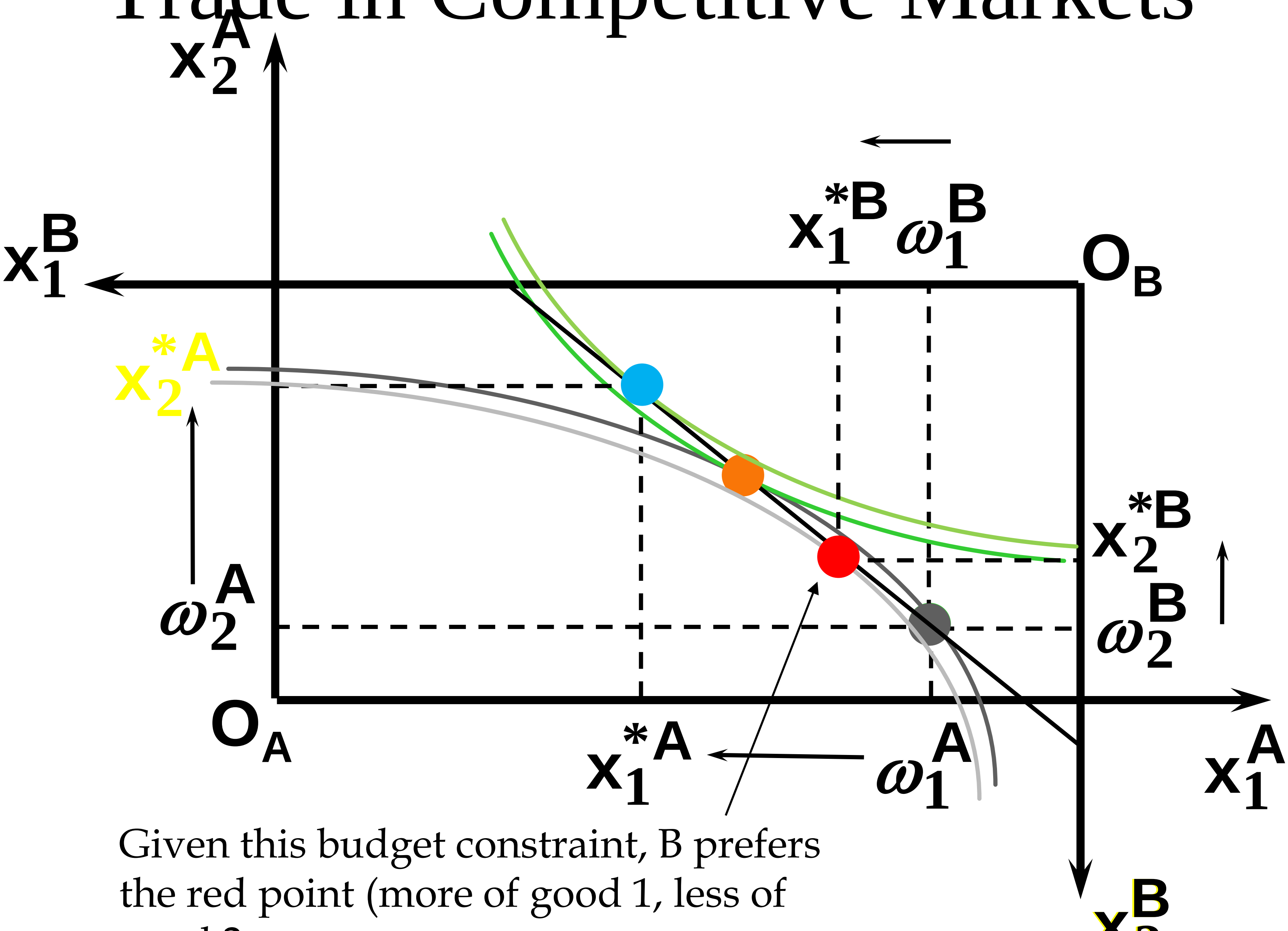
Trade in Competitive Markets



Trade in Competitive Markets



Trade in Competitive Markets

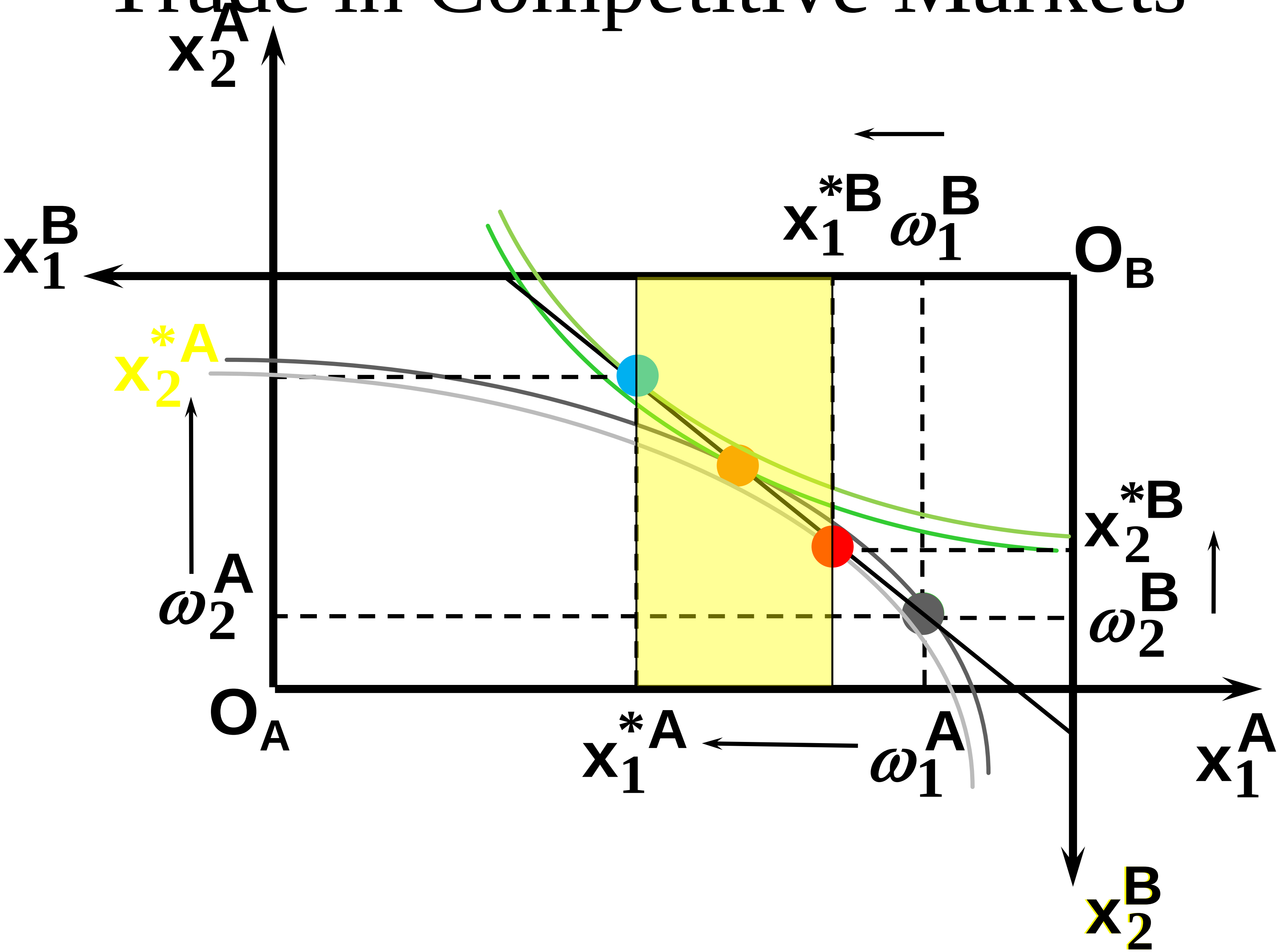


Given this budget constraint, B prefers the red point (more of good 1, less of good 2

Trade in Competitive Markets

- However, the red and blue points do not overlap.
- This means sum of good 1 demanded by A & B is less than the total endowment of good 1.

Trade in Competitive Markets

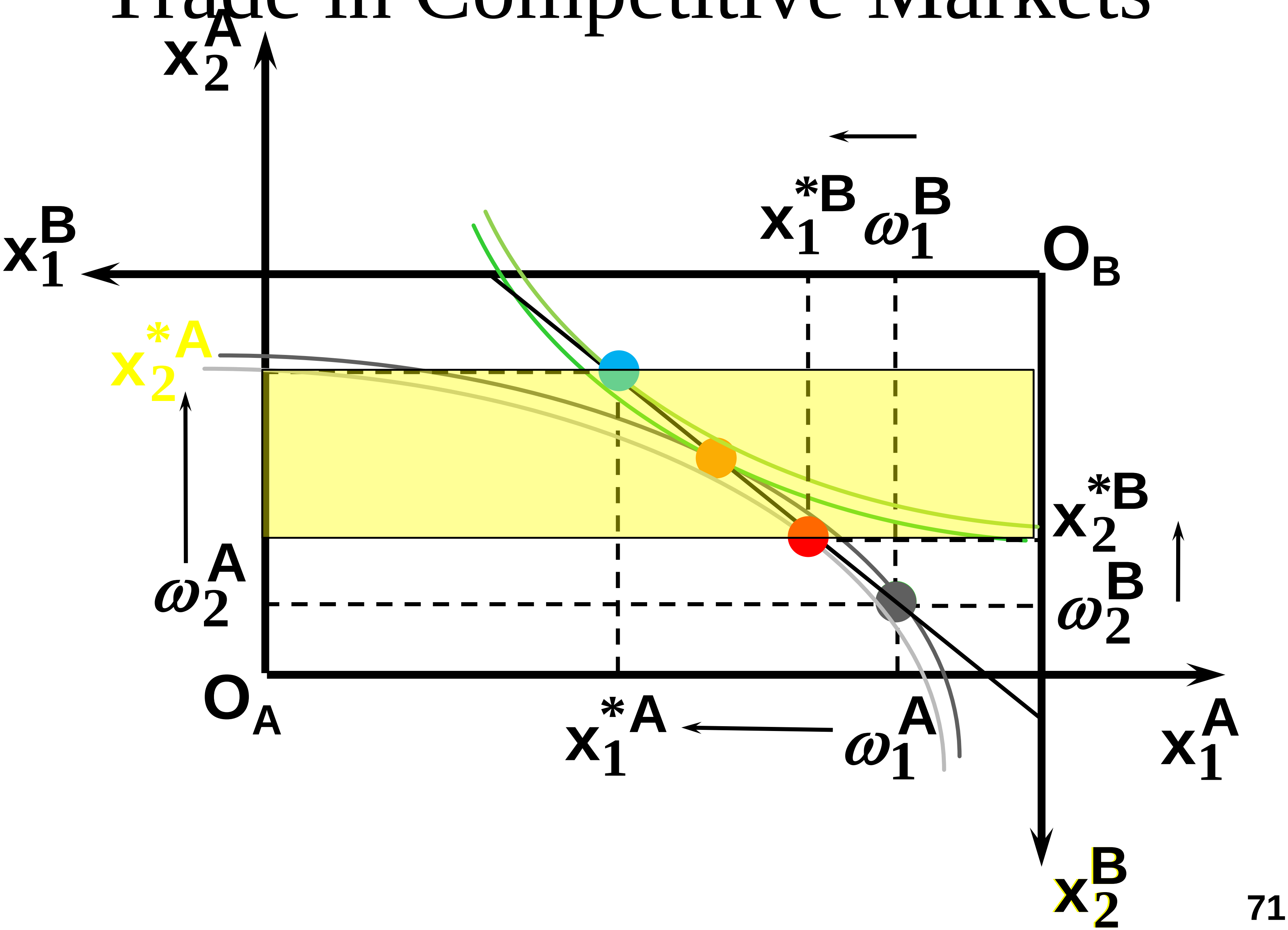


Trade in Competitive Markets

- The sum of good 1 demanded by A & B is less than the total endowment of good 1.
- There is excess supply of good 1 by the amount in yellow

$$x_1^{*A} + x_1^{*B} < \omega_1^A + \omega_1^B$$

Trade in Competitive Markets



Trade in Competitive Markets

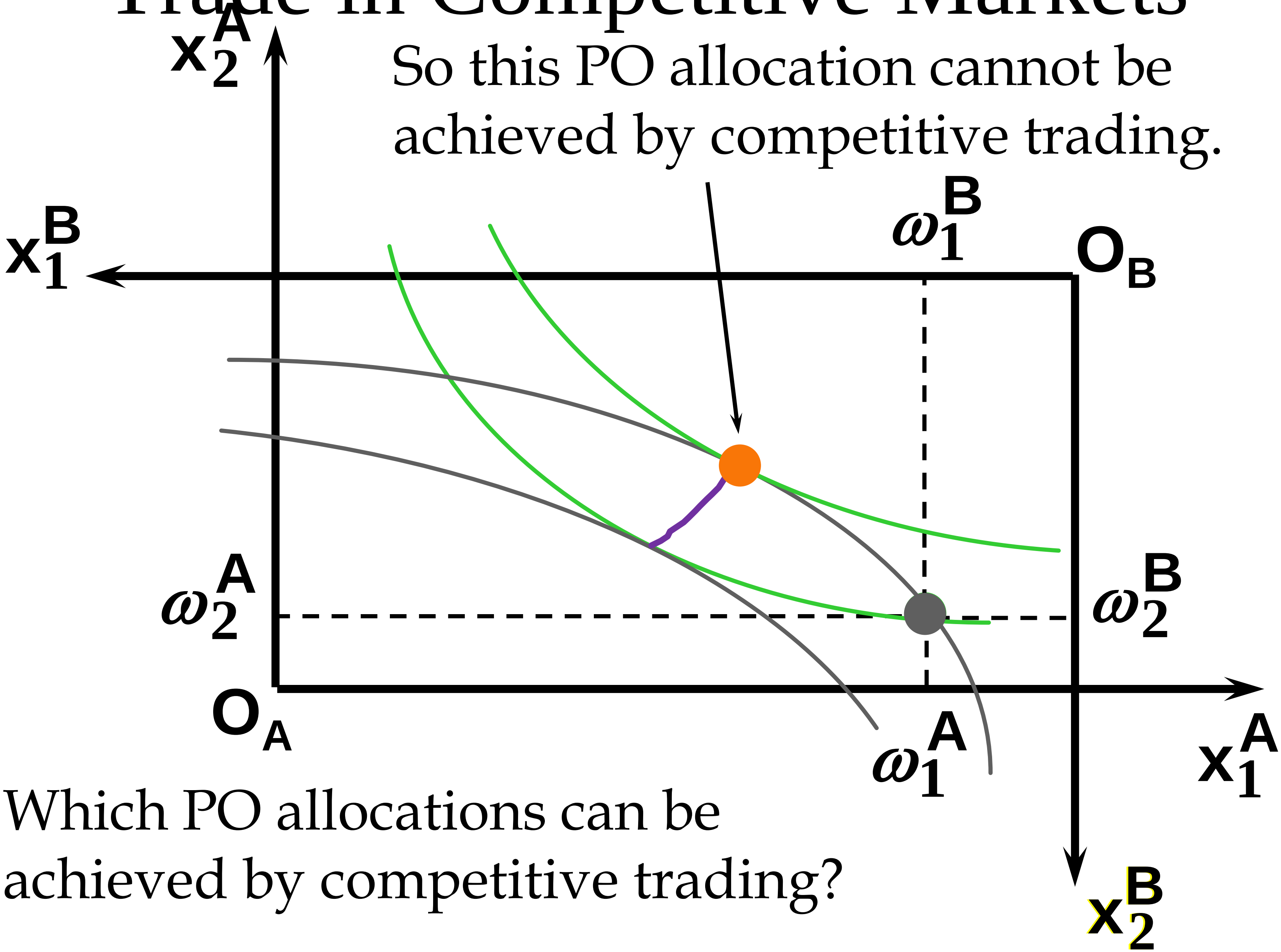
- Likewise, the sum of good 2 demanded by A and B is more than the total endowment of good 2.
- There is excess demand for good 2:

$$x_2^{*A} + x_2^{*B} > \omega_2^A + \omega_2^B$$

Trade in Competitive Markets

- Given prices p_1 and p_2 there is an
  - excess supply of good 1
  - excess demand for good 2.
- Neither market clears so the prices p_1 and p_2 do not cause a general equilibrium.

Trade in Competitive Markets



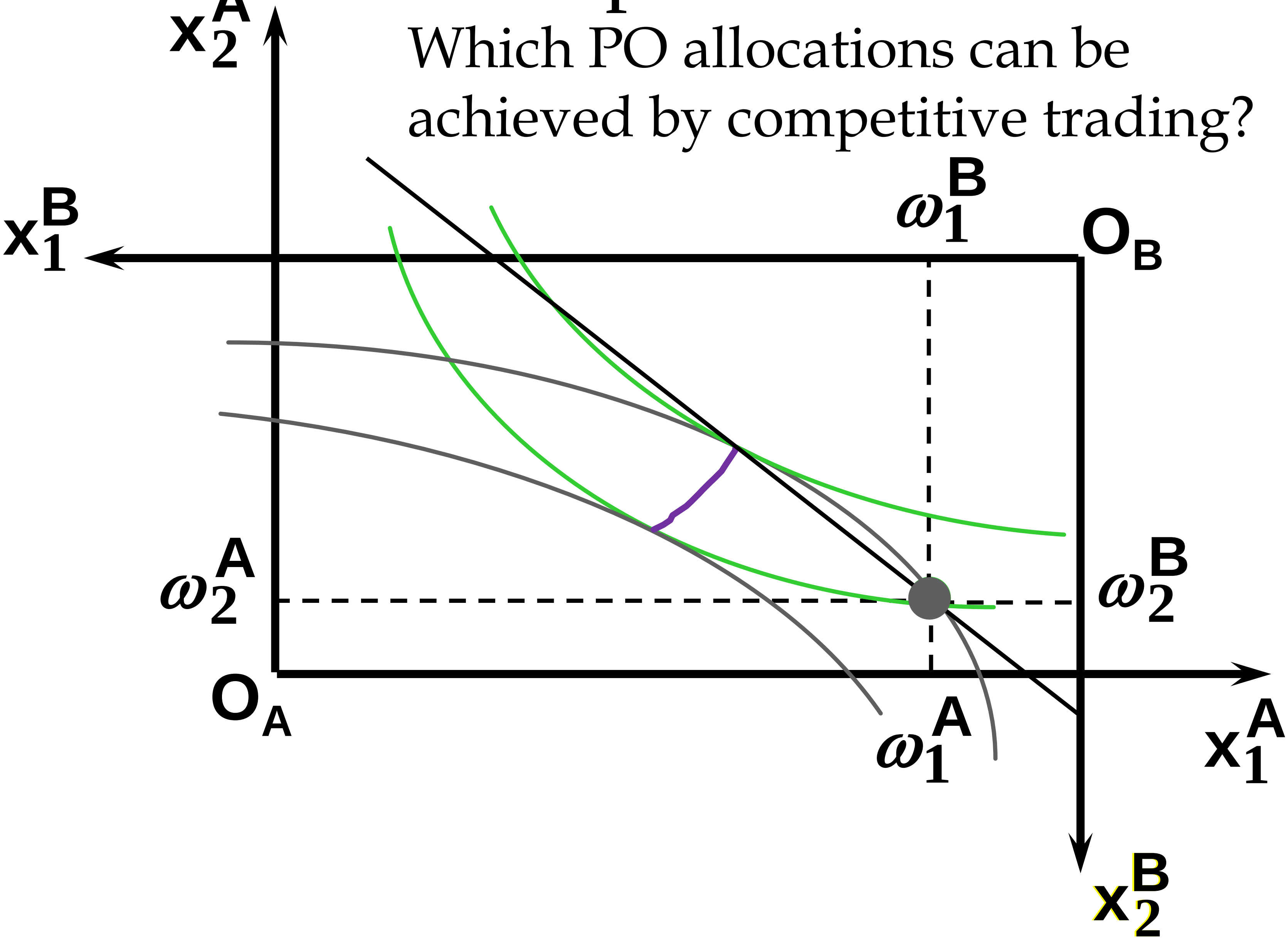
Which PO allocations can be achieved by competitive trading?

Trade in Competitive Markets

- Since there is an excess demand for commodity 2, p_2 will rise.
- Since there is an excess supply of commodity 1, p_1 will fall.
- The slope of the budget constraints is $-\frac{p_1}{p_2}$ so the budget constraints will pivot about the endowment point and become less steep.

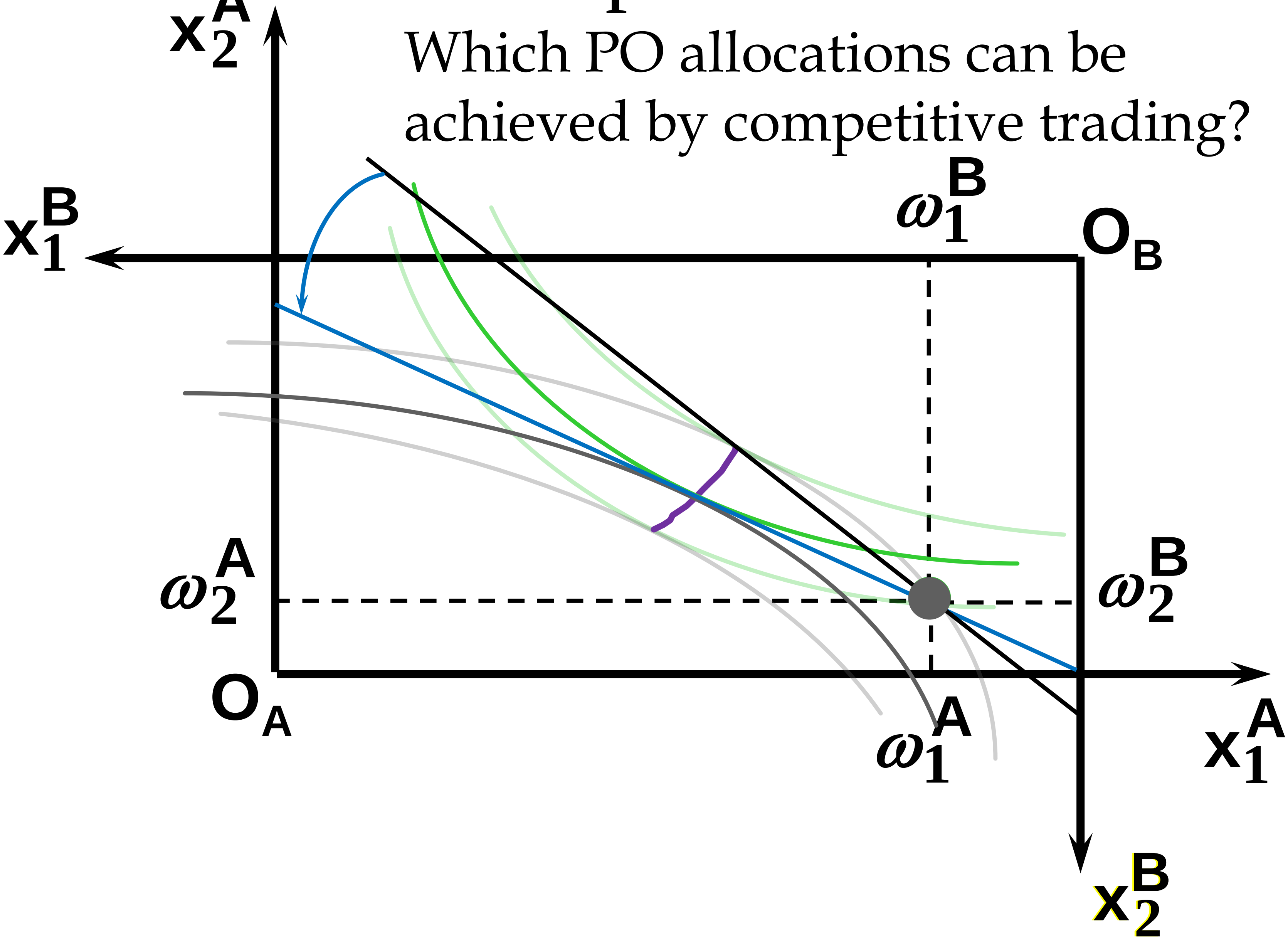
Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?



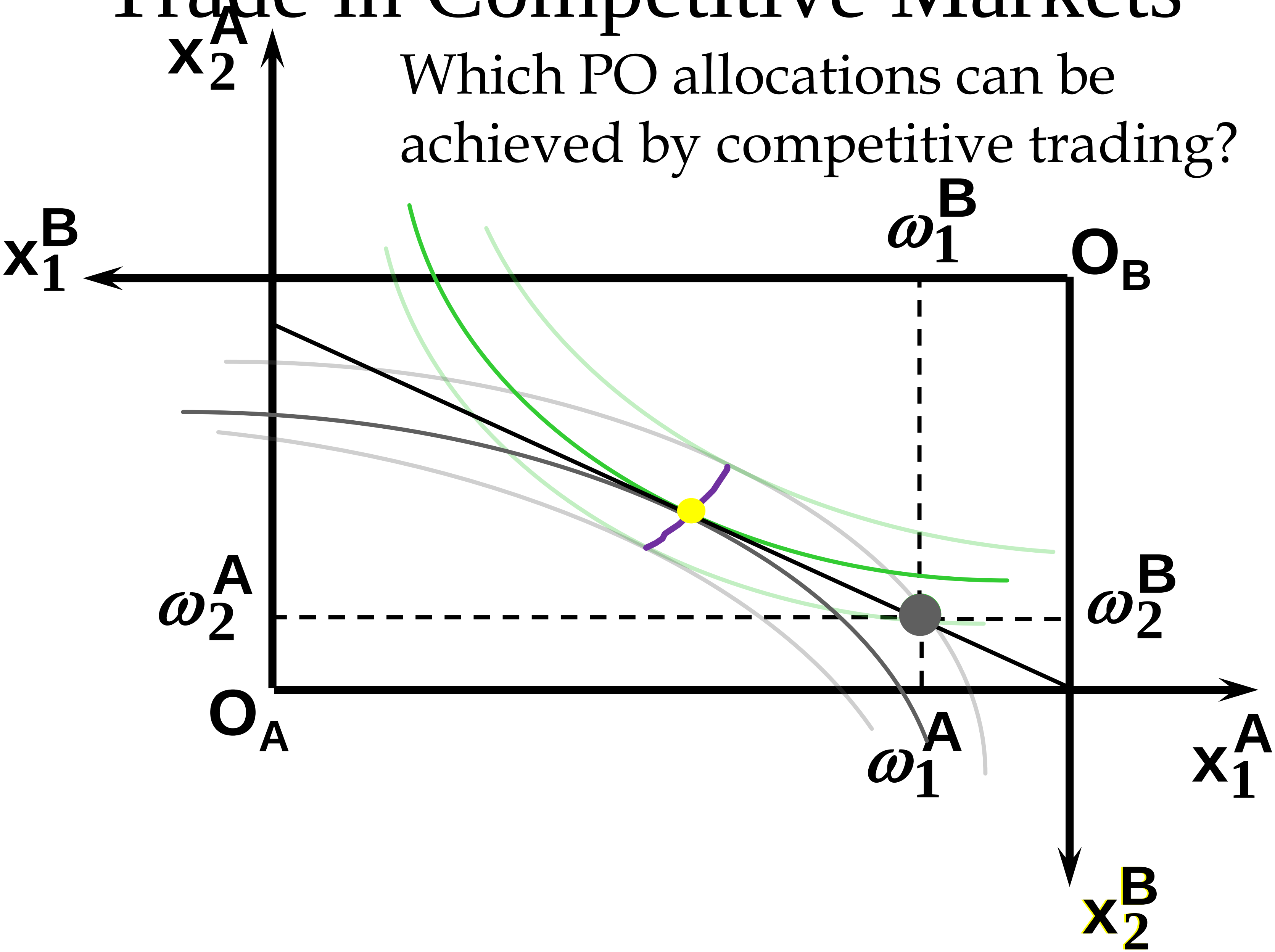
Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?

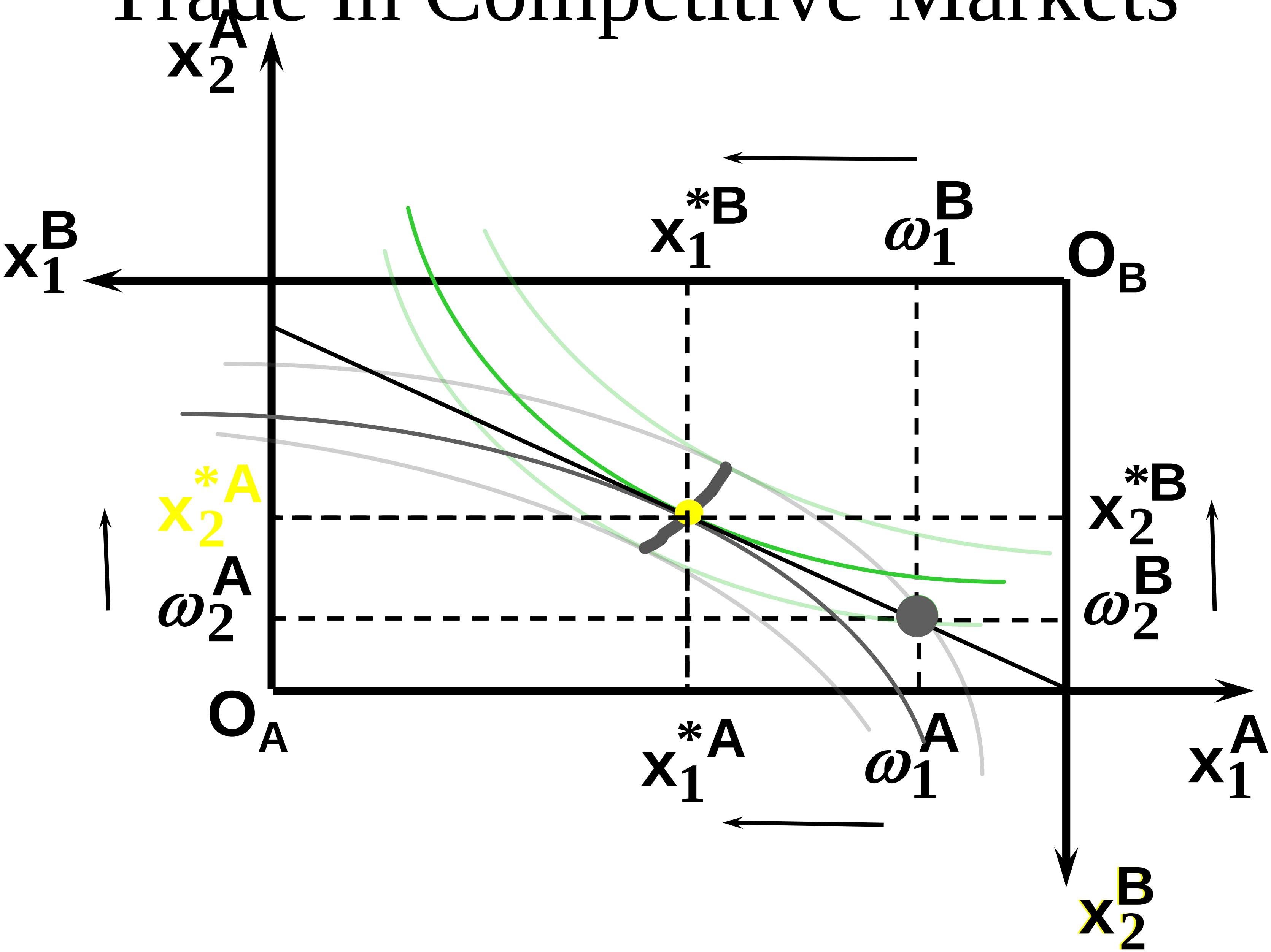


Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?



Trade in Competitive Markets



Trade in Competitive Markets

- In a competitive equilibrium,

$$MRS_A = MRS_B = -\frac{p_1}{p_2}$$

$$-\frac{MU_{x^A{}_1}}{MU_{x^A{}_2}} = -\frac{MU_{x^B{}_1}}{MU_{x^B{}_2}} = -\frac{p_1}{p_2}$$

$$\frac{MU_{x^A{}_1}}{MU_{x^A{}_2}} = \frac{MU_{x^B{}_1}}{MU_{x^B{}_2}} = \frac{p_1}{p_2}$$

Trade in Competitive Markets

- At the new prices p_1 and p_2 both markets clear; there is a general equilibrium.

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

Trade in Competitive Markets

- Trading in competitive markets achieves a particular Pareto-optimal allocation of the endowments.
- This is an example of the First Fundamental Theorem of Welfare Economics.

First Fundamental Theorem of Welfare Economics

- Given that consumers' preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy's endowment.

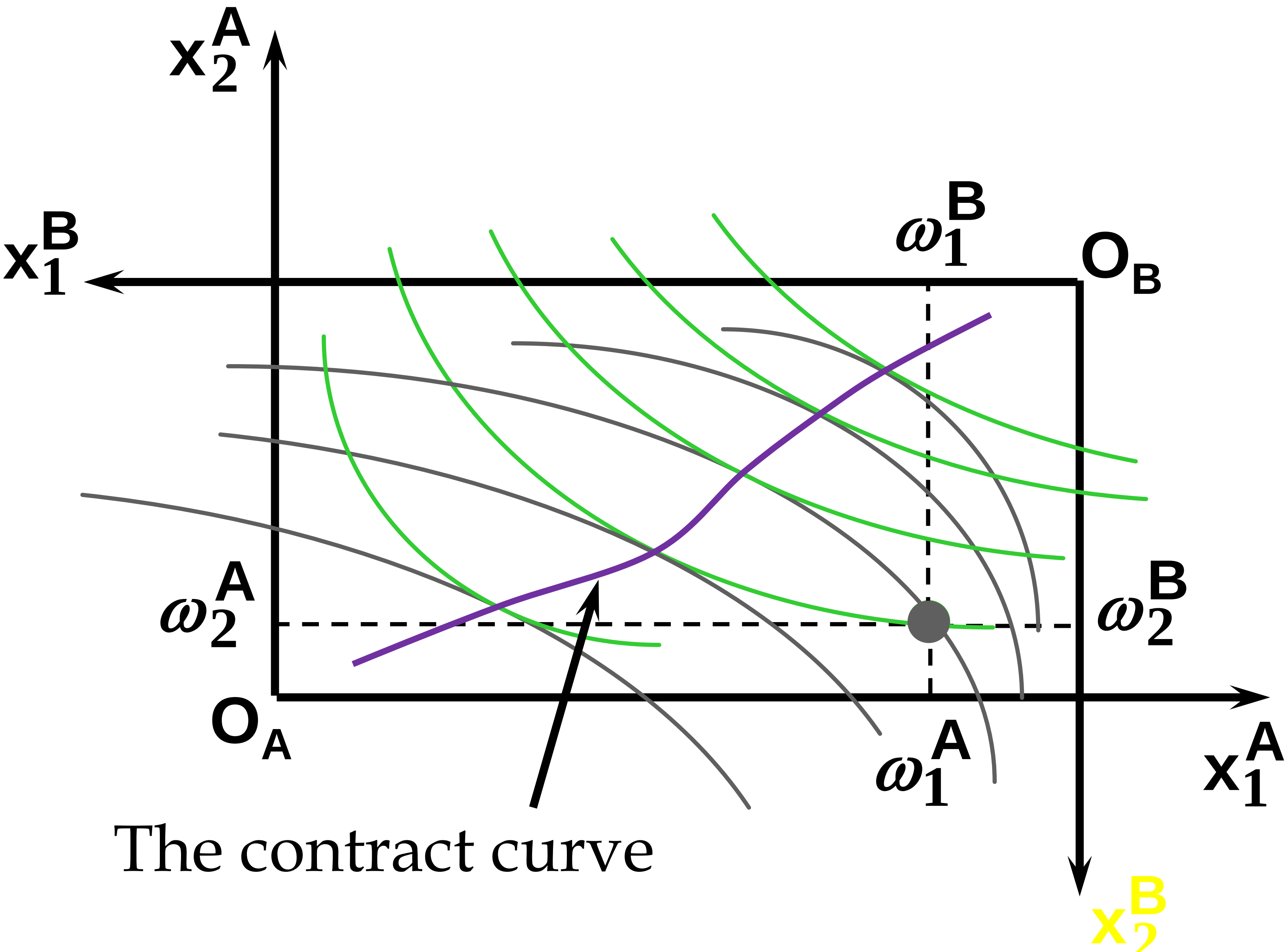
Second Fundamental Theorem of Welfare Economics

- The First Theorem is followed by a second that states that:
  - any Pareto-optimal allocation (i.e. any point on the contract curve) can be achieved by trading in competitive markets provided that endowments are first appropriately rearranged amongst the consumers.

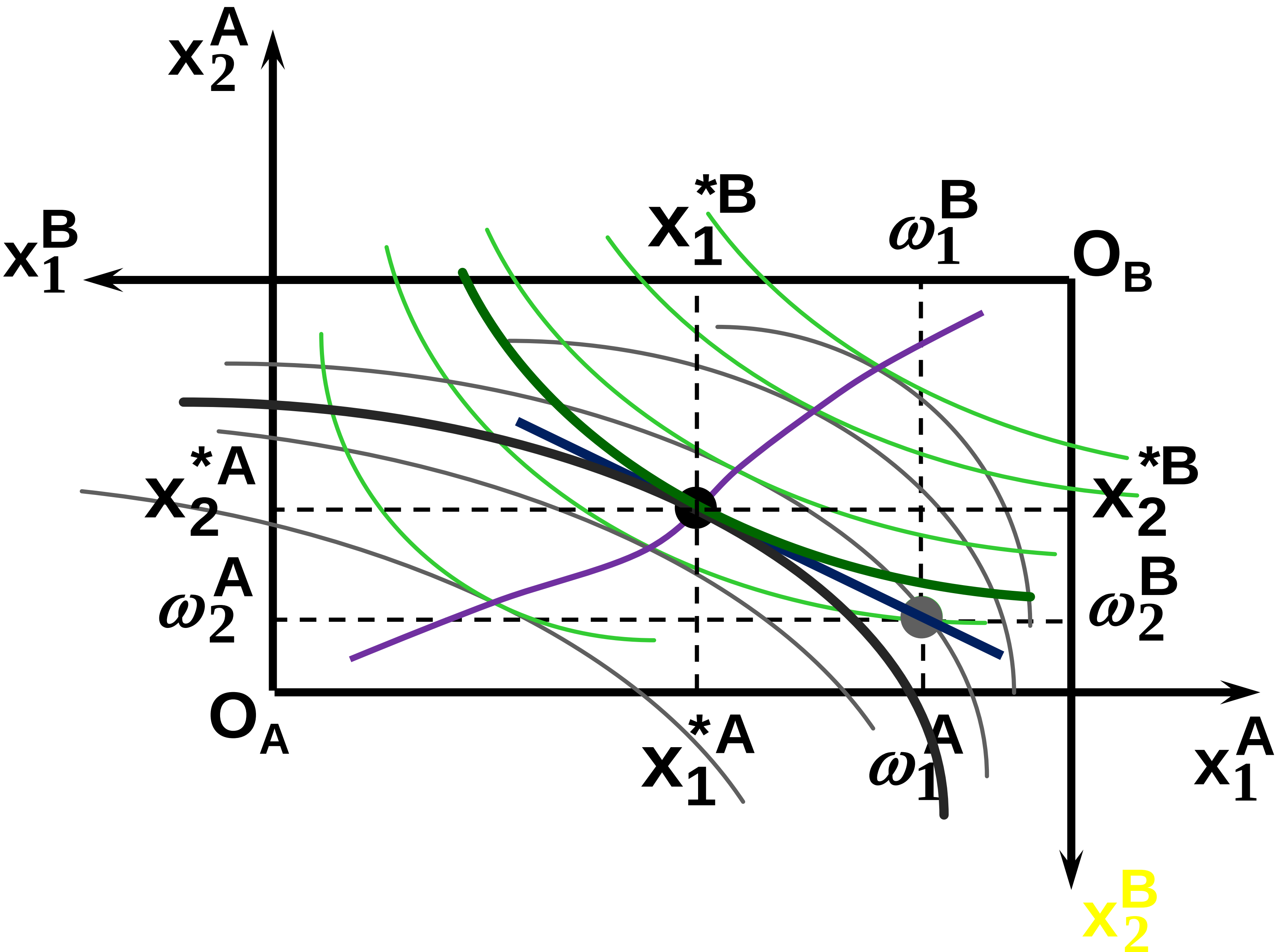
Second Fundamental Theorem of Welfare Economics

- Given that consumers' preferences are well-behaved, for any Pareto-optimal allocation there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation implementable by trading in competitive markets.

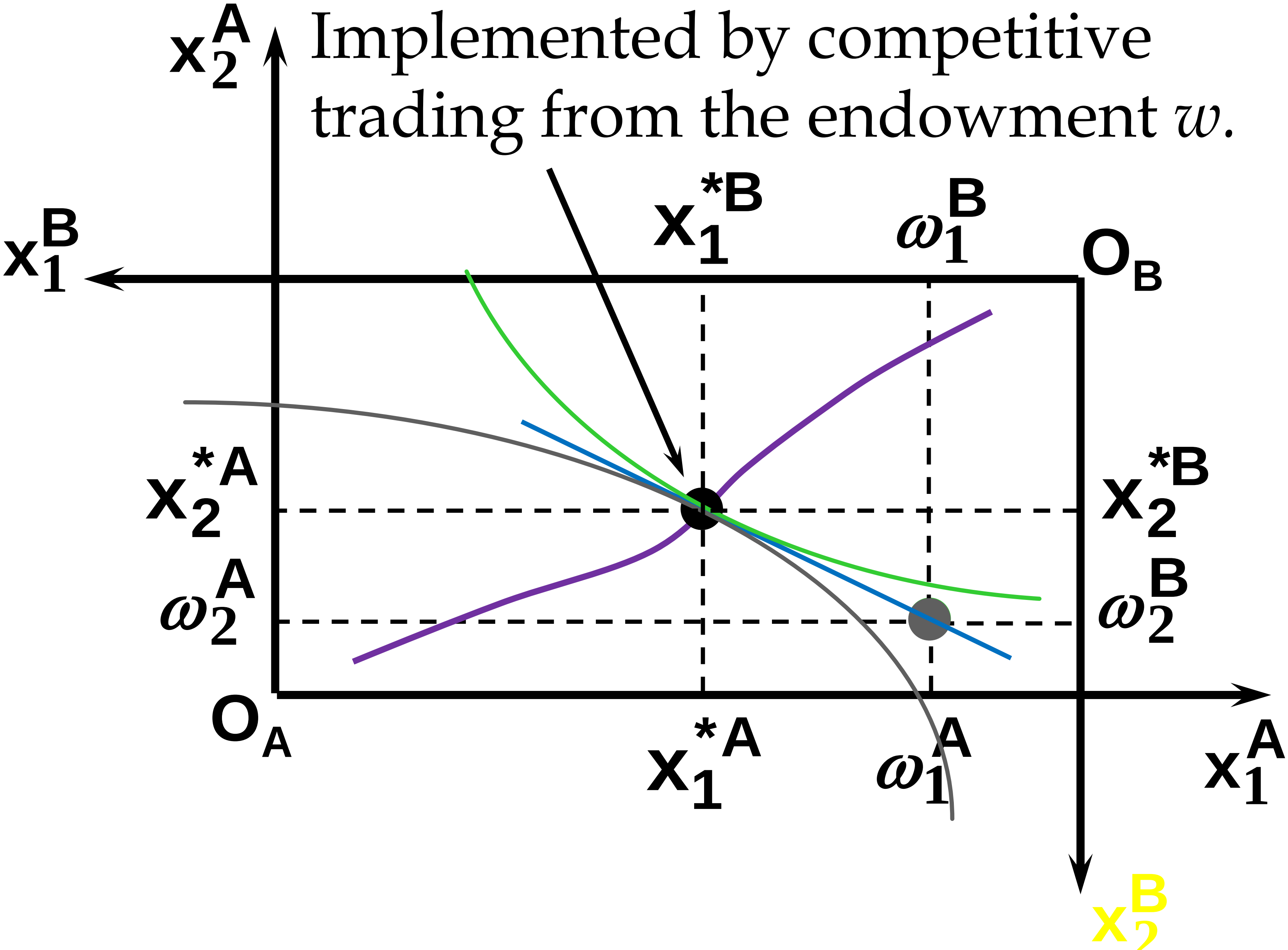
Second Fundamental Theorem



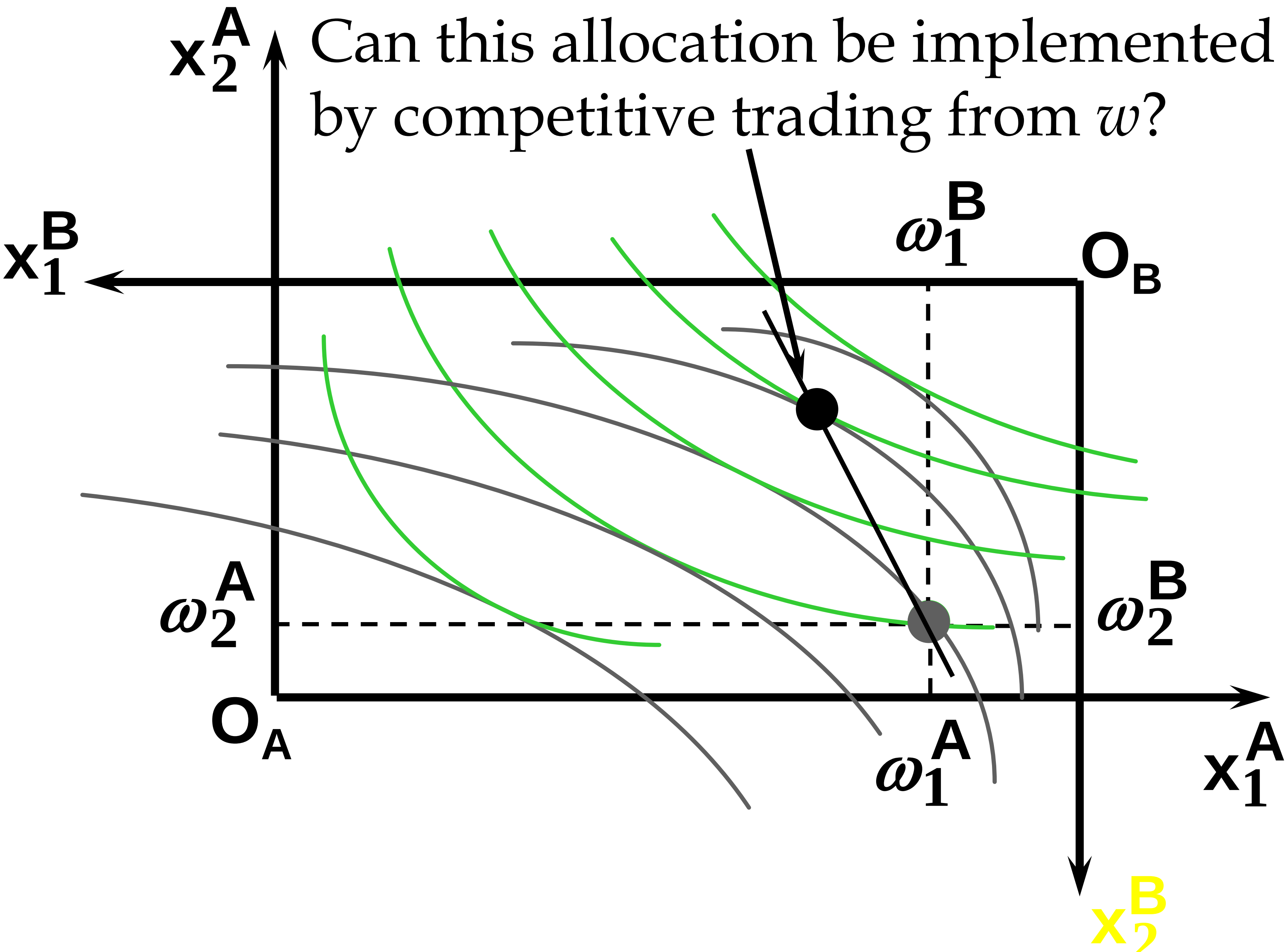
Second Fundamental Theorem



Second Fundamental Theorem

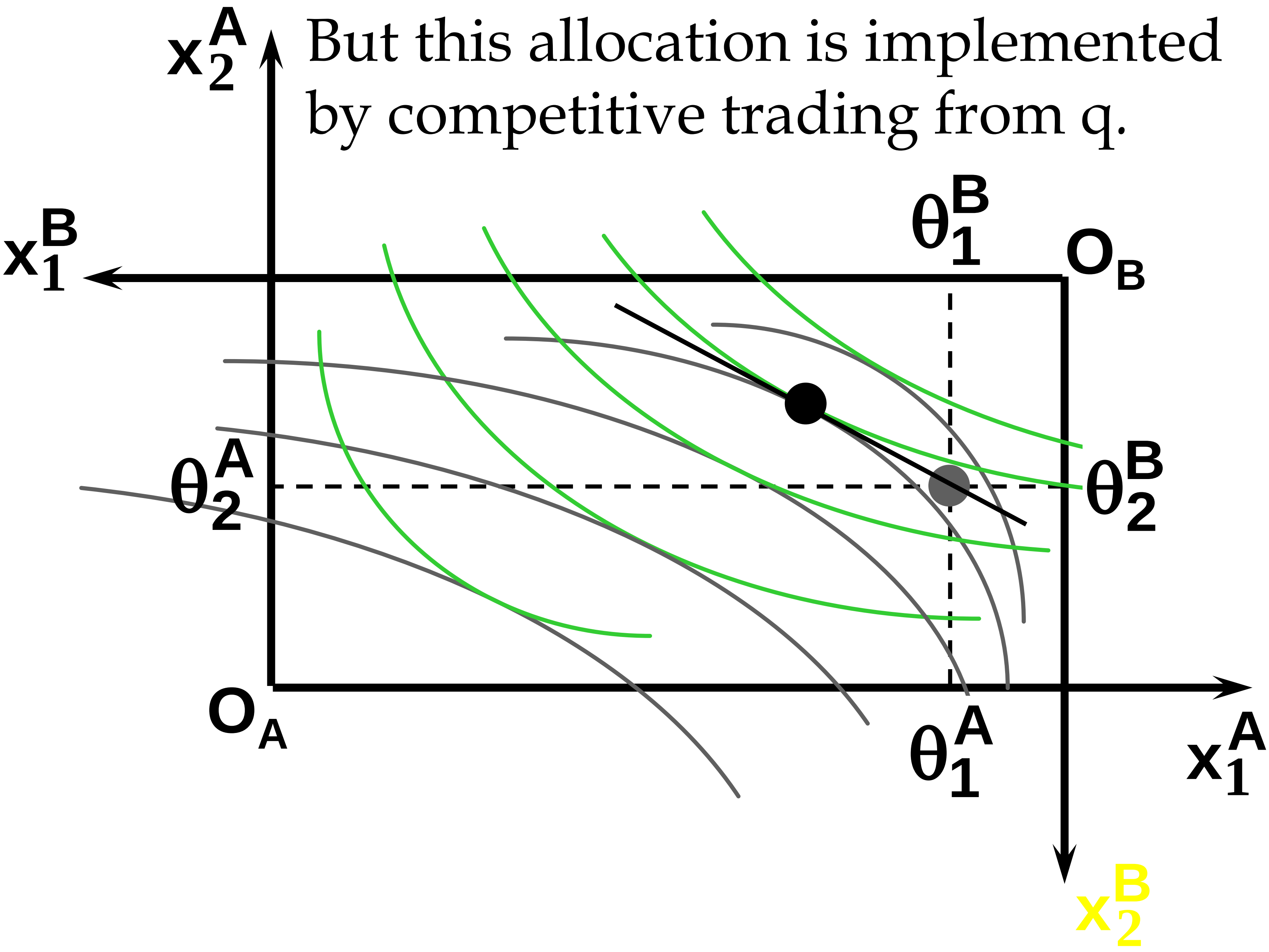


Second Fundamental Theorem



Second Fundamental Theorem

But this allocation is implemented by competitive trading from q.



Second Fundamental Theorem of Welfare Economics

- So if we want to achieve a particular Pareto-optimal outcome, redistribute prior to trading then the market will achieve that outcome for us.

Why are these theorems so fundamental?

- What does the First Welfare Theorem imply?

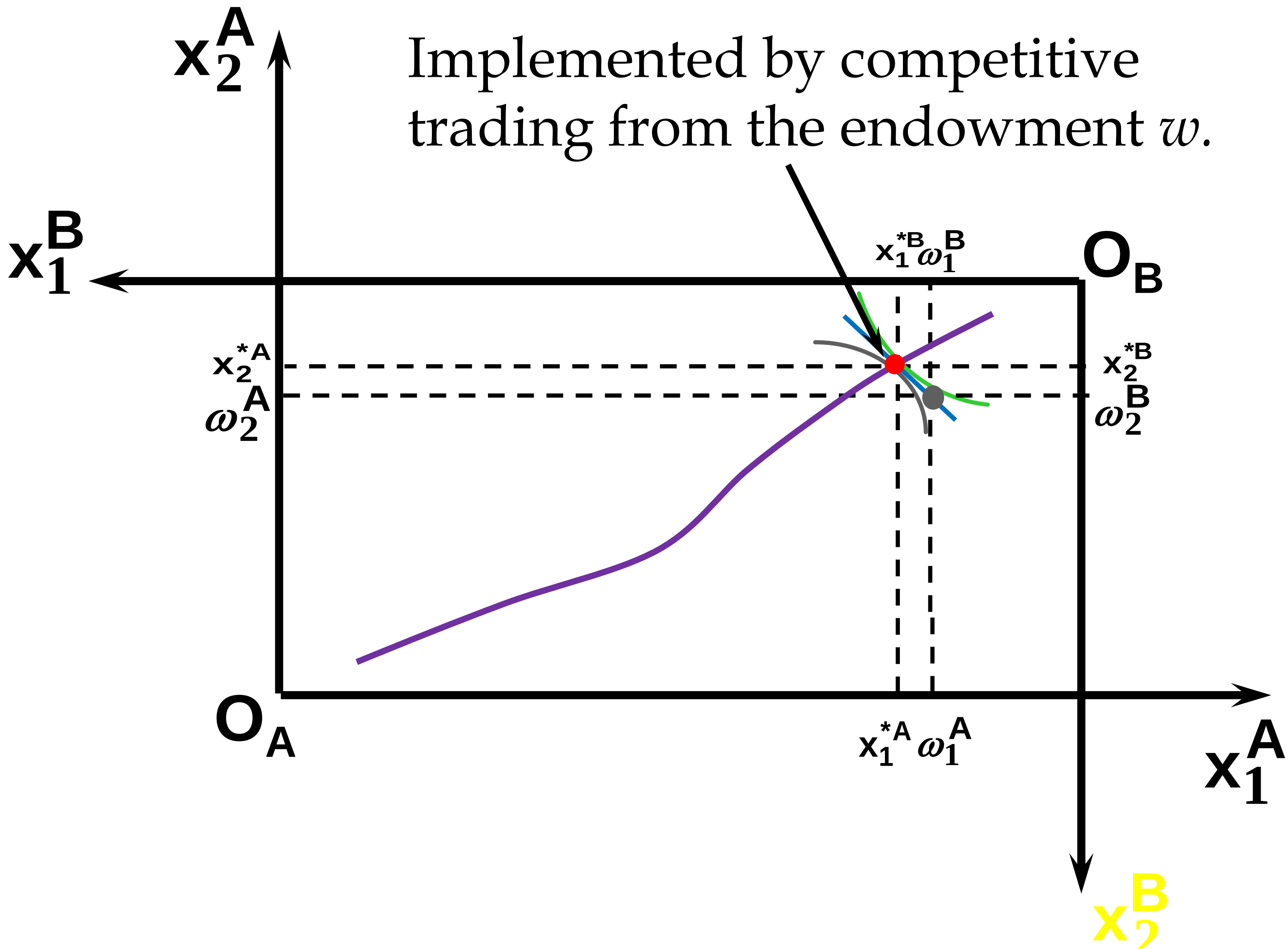
1. The competitive equilibrium outcome is hard to beat. The invisible hand does amazing things.
2. Governments shouldn't intervene in competitive markets because how likely is it that they could they do better than that?
   - The government doesn't knows every person's utility function and MRS between goods!

Why are these theorems so fundamental?

- But is the competitive outcome fair?

1. Initial endowments are very uneven.
   – Suppose agent A has a lot of both goods and agent B has a little of both goods.

Second Fundamental Theorem



Why are these theorems so fundamental?

- But is the competitive outcome fair?

1. Initial endowments are very uneven.
   - Suppose agent A has a lot of both goods and agent B has a little of both goods.
   - The competitive outcome, though Pareto-optimal, is skewed in favor of agent A

- That's why we have the Second Welfare Theorem

Why are these theorems so fundamental?

- What does the Second Welfare Theorem imply?

1. If we are unhappy with the competitive outcome, redistribution then letting the market work will allow us to achieve a more desirable (equitable?) outcome
2. If a government is unhappy with the competitive outcome, it shouldn't intervene in the market, but reallocate endowments prior to market activity.

Why are these theorems so fundamental?

- Policies like price restrictions and trade quotas distort the market and could force agents to a non-Pareto optimal outcome.
- The Second Welfare Theorem says we should avoid these policies in favor of redistribution.

- But what's the catch?

Why are these theorems so fundamental?

- The only way to enforce this type of redistribution is through a lump-sum (i.e. fixed) tax. These are extremely uncommon and widely unpopular.
- Other taxes won't work. Example:
  - An income tax may cause some people to work less, thus changing the size of the Edgeworth box and violating the assumptions of the framework.

People don't want to work more as they are taxed more

Why are these theorems so fundamental?

- Other limitations:
  - Only applies to perfectly competitive markets. Can you think of markets that are not perfectly competitive?
  - Preferences must be convex (diminishing marginal returns)
  - There can be no externalities.

Walras' Law

- Walras' Law is an identity; i.e. a statement that is true for any positive prices (p_1,p_2), whether these are equilibrium prices or not.

Walras' Law

- Every consumer's preferences are well-behaved so, for any positive prices (p_1,p_2), each consumer spends all of his budget.
- For consumer A:

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

For consumer B:

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Walras' Law

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Summing gives

$$p_1(x_1^{*A} + x_1^{*B}) + p_2(x_2^{*A} + x_2^{*B}) = p_1(\omega_1^A + \omega_1^B) + p_2(\omega_2^B + \omega_2^B).$$

Rearranging,

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) + p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0.$$

Implications of Walras' Law

Suppose the market for commodity A is in equilibrium; that is,

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B = 0.$$

Then

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) + p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B = 0.$$

Implications of Walras' Law

So one implication of Walras' Law for a two-commodity exchange economy is that if one market is in equilibrium then the other market must also be in equilibrium.

Implications of Walras' Law

What if, for some positive prices p_1 and p_2, there is an excess quantity supplied of commodity 1? That is,

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B < 0.$$

Then

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) + p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B > 0.$$

Implications of Walras' Law

So a second implication of Walras' Law for a two-commodity exchange economy is that an excess supply in one market implies an excess demand in the other market.