Reminder

- The 8th edition of Intermediate Microeconomics by Hal Varian is compatible with this course.
- This may be the only edition still available at the bookstore.

Announcements

- Make-up midterm exams?
  - Yes, however, the exam will be slightly harder since students get more time to prepare for this exam.
  - If you need a make-up midterm, the earlier you let me know, the better.
- Mathematical Requirements:
  - Derivatives: Partial Derivatives, Total Derivatives

Announcements

- Clarification:
- Suppose you win a State lottery. The prize is \$1,000,000 but it is paid over 10 years in equal installments of \$100,000 each. The interest rate, r, is 0.1. What is the prize actually worth?

$$\text{Answer: } PV = \frac{100{,}000}{1+r} + \frac{100{,}000}{(1+r)^2} + \cdots + \frac{100{,}000}{(1+r)^{10}}$$

$$= \$614{,}457$$

Announcements

- Corrections on Slide 81 from last week
- Problem set for next week's discussion group is up. Please come prepared.

Chapter 12: Uncertainty

Uncertainty

- We continue discussing how the utility framework from EC2101 can be used to address different decisions that households face
  - Last week: Consumption across different periods
  - This week: Consumption across different (uncertain) states of the world

Uncertainty is Pervasive

- What is uncertain in economic systems?
  - tomorrows prices
  - future wealth
  - future availability of commodities
  - present and future actions of other people.

Uncertainty is Pervasive

- What are some rational responses to uncertainty?
  - buying insurance (health, life, auto)
  - a portfolio of contingent consumption goods.
- Today, we will use economic models to explain these responses

Uncertainty

- Budget constraints
- Preferences
  - Expected Utility, Risk Premium, Risk Aversion, Marginal Rate of Substitution
- Insurance
- Diversification and Risk Spreading

States of Nature

- Definition: different outcomes of some random event
- Example:
  - possible States of Nature:
    - "car damaged in accident" (d) and "no car damage" (nd).
  - Damage occurs with probability π_d, does not with probability π_{nd} ;

$$\pi_a + \pi_{na} = 1.$$

  - Accident causes a loss of \$L.

Contingencies

- A contingency is something that depends on an event that is not yet certain.
- A contract implemented only when a particular state of Nature occurs is *state-contingent*.
- E.g. the insurer pays only if there is an accident.

Contingencies

- A *contingent consumption plan* is a specification of what will be consumed in each different state of nature
- E.g. take a vacation only if there is no accident, stay at home if there is an accident.

State-Contingent Budget Constraints

- How do we model individual choice under uncertainty?
- We can work with the utility framework we are familiar with, defining the consumption in the various states as separate goods.
- Let’s look at an example of how insurance works

Example

- According to Daily Makeover, J-Lo has reportedly insured her booty for a whopping $300 million dollars. Wow!

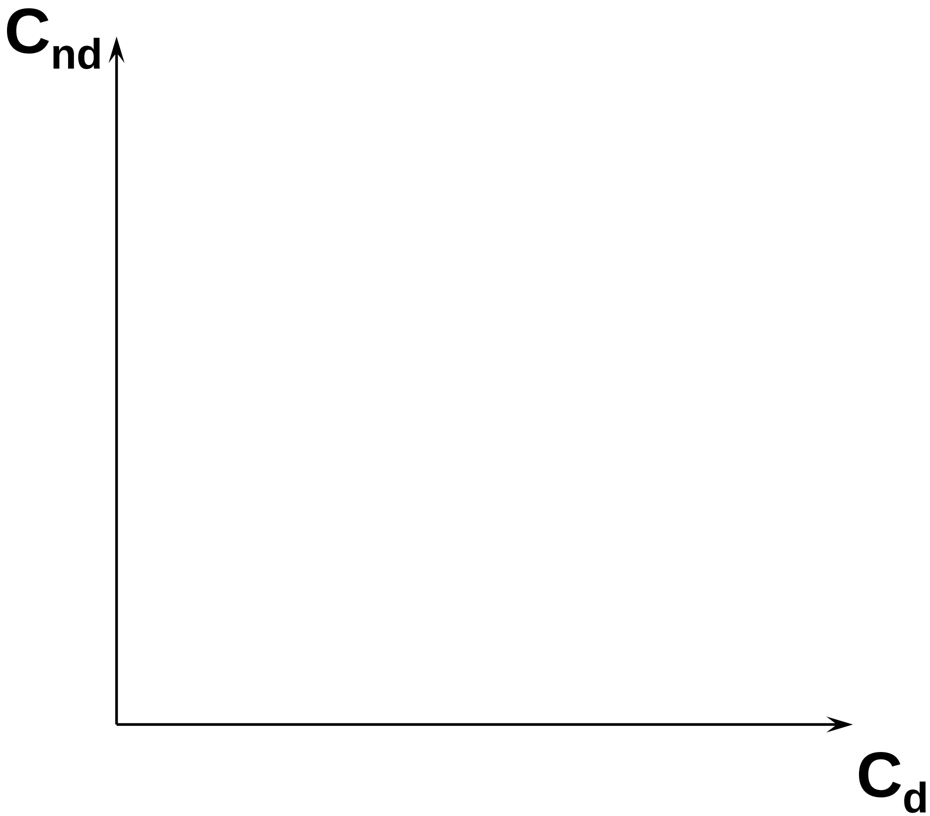
Example

- Two contingent states:
  - Damage
  - No Damage
- Damage causes a loss of $L

State-Contingent Budget Constraints

- Each \$1 of damage insurance costs γ.
- J-Lo has \$m of wealth.
- C_{nd} is consumption value in the no-damage state.
- C_d is consumption value in the damage state.

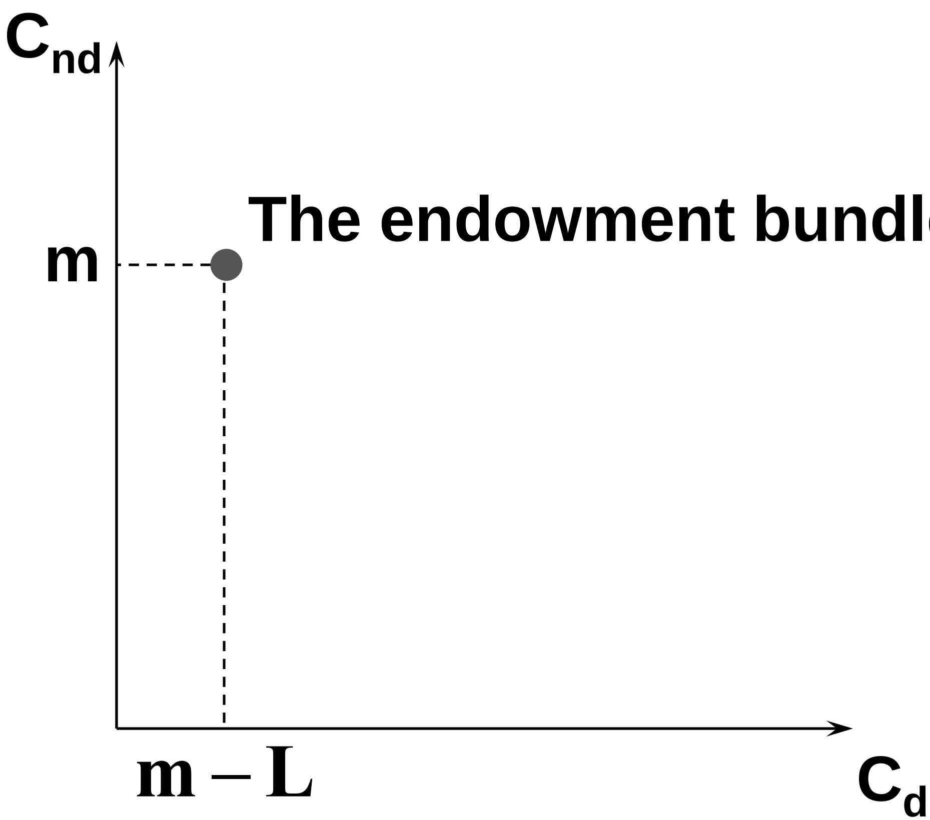
State-Contingent Budget Constraints



State-Contingent Budget Constraints

- Without insurance,
  - $C_d = m - L$
  - $C_{nd} = m.$ unaffected because there is no damage

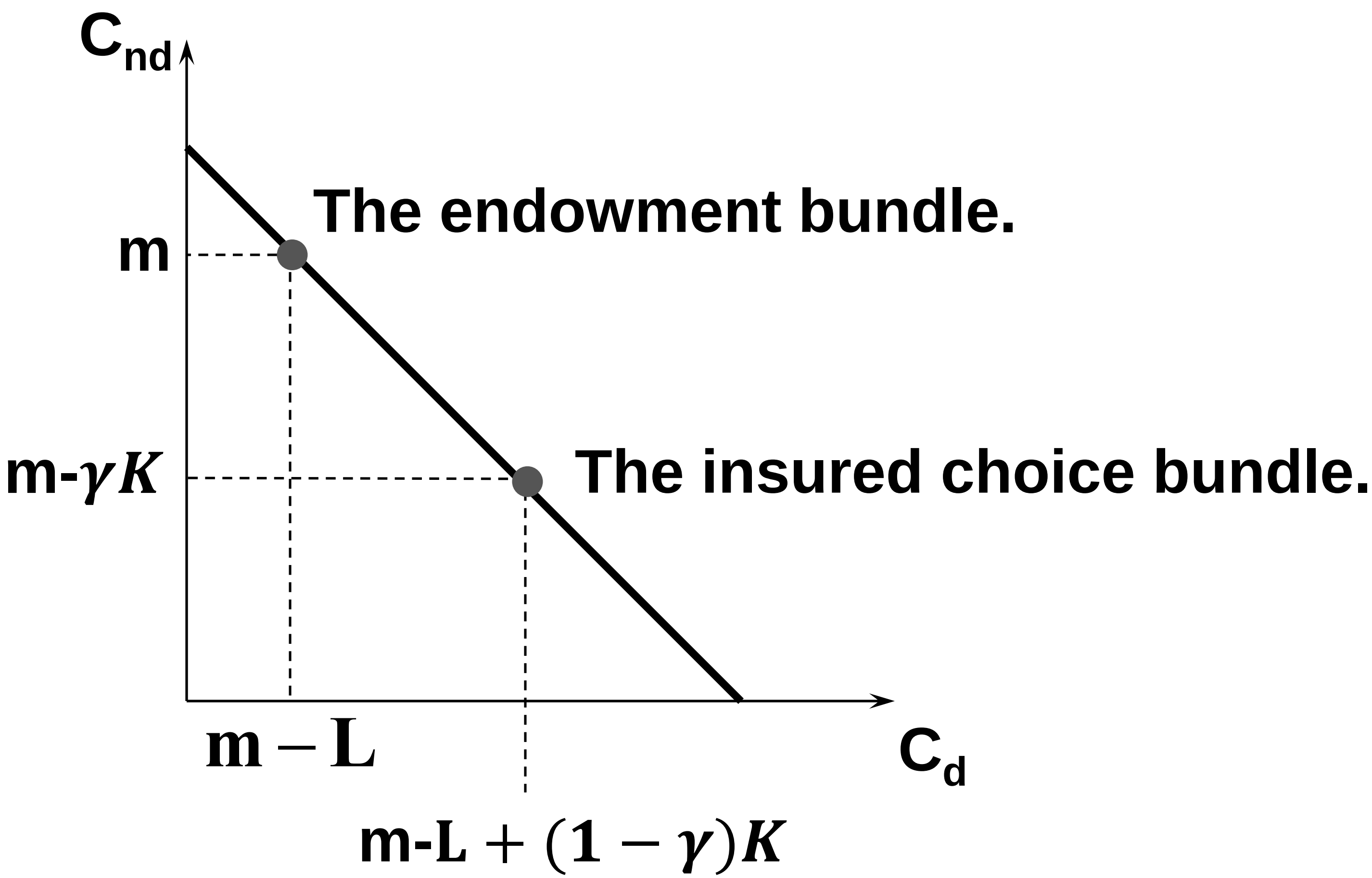
State-Contingent Budget Constraints



State-Contingent Budget Constraints

- Buy \$K of accident insurance at a cost of γK
  - $C_{nd} = m - \gamma K.$
  - $C_d = m - L - \gamma K + K$
    $= m - L + (1 - \gamma)K.$

State-Contingent Budget Constraints



State-Contingent Budget Constraints

- What is the slope of the budget constraint?

Slope = $\frac{rise}{run}$

$$\frac{\Delta C_{nd}}{\Delta C_d} = \frac{\gamma K}{(1-\gamma)K} = \frac{\gamma}{1-\gamma}$$

State-Contingent Budget Constraints

Alternatively,

$$C_d = m - L + (1- \gamma)K.$$

So, $$K = \frac{C_d - m + L}{1-\gamma}$$

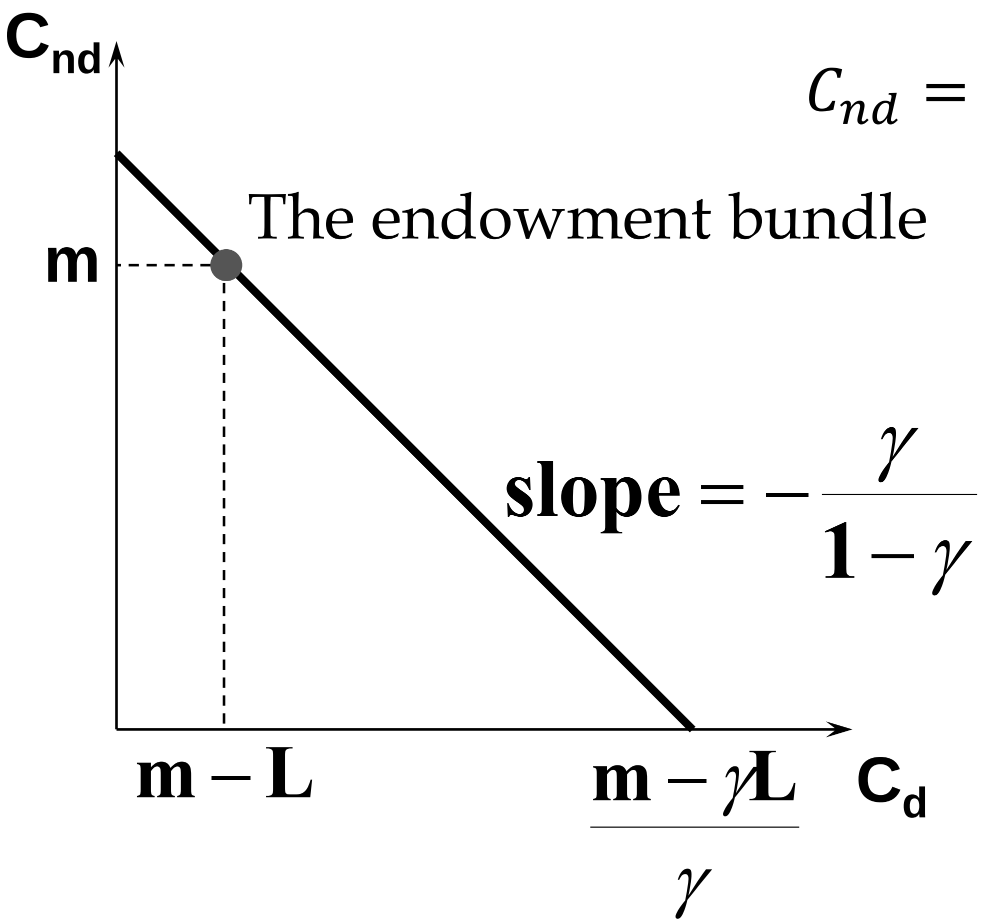
Since, $C_{nd} = m - \gamma K$,

$$C_{nd} = m - \gamma\left(\frac{C_d - m + L}{1-\gamma}\right)$$

Rearranging,

$$C_{nd} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} C_d$$

State-Contingent Budget Constraints



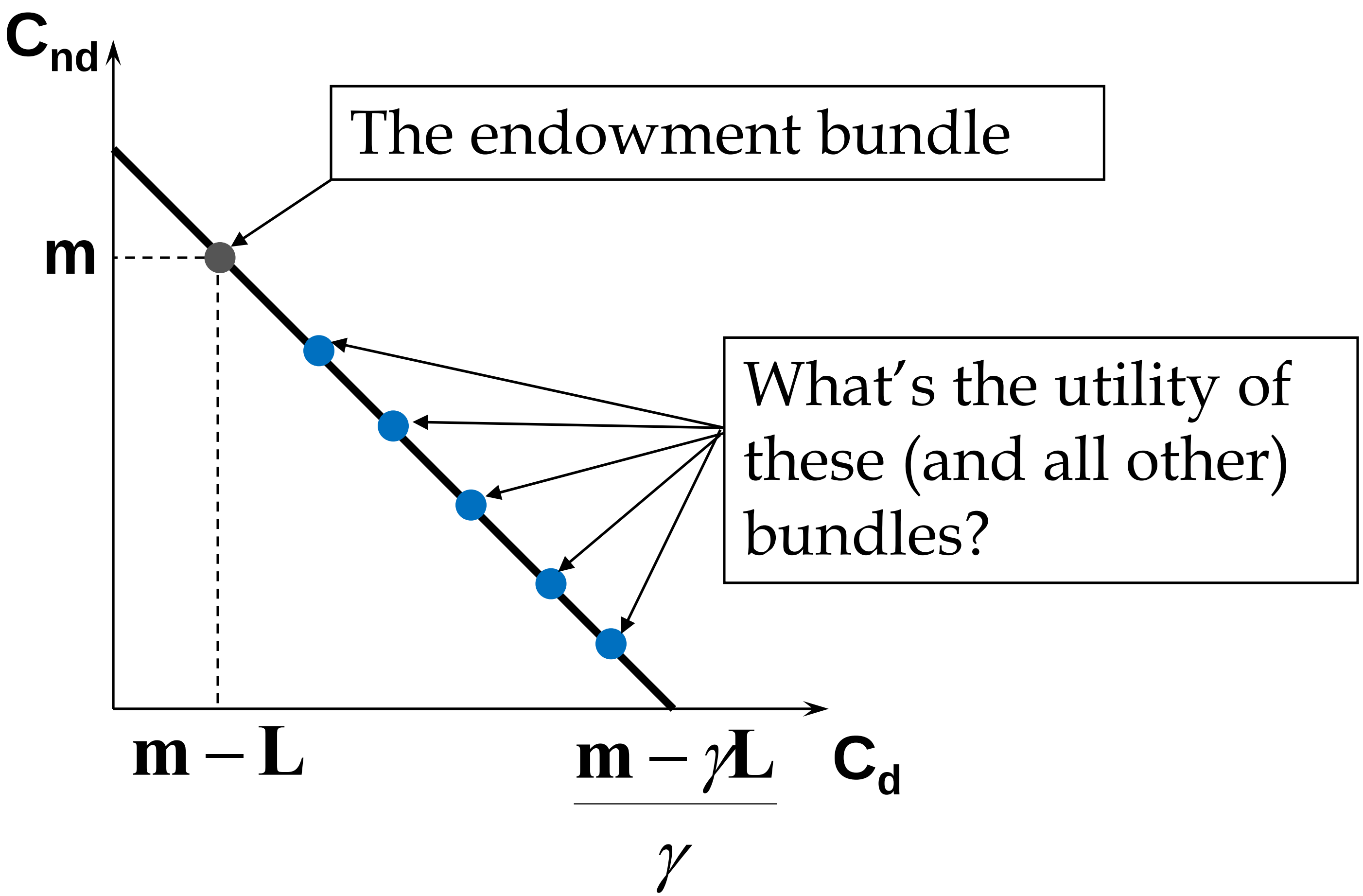
$$C_{nd} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma}\ C_d$$

Where is the most preferred state-contingent consumption plan?

Preferences Under Uncertainty

- Where is the most preferred state-contingent consumption plan?
- To answer this question, we must know something about the consumer's preferences across various state contingent consumption plans

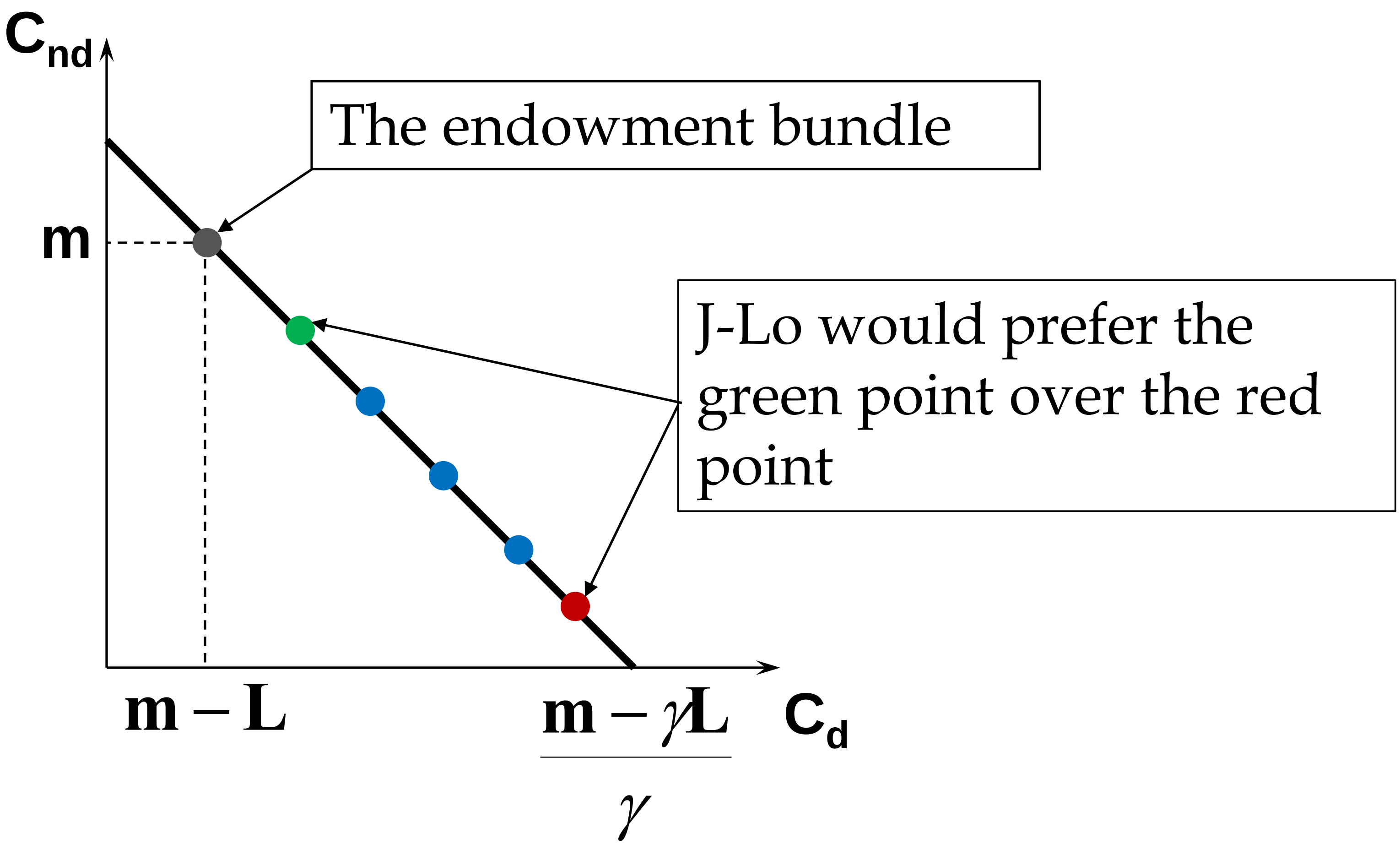
Preferences Under Uncertainty



Preferences Under Uncertainty

- Preferences across various contingent consumption plans depend on how likely the consumer perceives the various states of nature to occur.
- If J-Lo believes that the no damage state is most likely (i.e. 99% no damage vs 1% damage) then she would prefer endowment bundles on the budget constraint that gives her more in state: C_{nd}

Preferences Under Uncertainty



Preferences Under Uncertainty

- Expected Utility
- Risk Aversion
- Risk Premium
- Marginal Rate of Substitution

Preferences Under Uncertainty

- Examples of utility functions in the context of choice under uncertainty:

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1 + \pi_2 c_2$$

Probability of state 1

Probability of state 2

$$u(c_1, c_2, \pi_1, \pi_2) = c_1^{\pi_1} c_2^{1-\pi_1}$$

Cobb-Douglas

Preferences Under Uncertainty

- Expected Utility

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

utility in state 1

utility in state 2

Probability of state 1

Probability of state 2

- Utility “a priori” is a weighted sum of the consumptions in each state where the weights are the probabilities of each state occurring

Preferences Under Uncertainty

- Expected Utility

$$u(c_1, c_2, \pi_1, \pi_2) = \underbrace{\pi_1}_{\text{Probability of state 1}} \underbrace{v(c_1)}_{\text{utility in state 1}} + \underbrace{\pi_2}_{\text{Probability of state 2}} \underbrace{v(c_2)}_{\text{utility in state 2}}$$

- Any utility function that follows this form is called an *"expected utility function"* or a *"von-Neumann-Morgenstern utility function"*

Preferences Under Uncertainty

- If $v(c_1) = c_1$, then
$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1 + \pi_2 c_2$$
is an expected utility function

- $u(c_1, c_2, \pi_1, \pi_2) = c_1^{\pi_1} c_2^{1-\pi_1}$ is not an expected utility function, however the log transformation is:

$$\ln u(c_1, c_2, \pi_1, \pi_2) = \pi_1 \ln c_1 + (1 - \pi_1) \ln c_2$$

Preferences Under Uncertainty

- Expected Utility

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

utility in state 1

utility in state 2

Probability of state 1

Probability of state 2

- The expected utility function maintains the expected utility function under **positive affine transformations:**

$$v(u) = au + b$$

even with the a and b

Preferences Under Utility

- Why do economists like the expected utility function?
  - Read Section 12.4: Why Expected Utility is Reasonable (pg 224-226)
  - You will be responsible to know this material for homework assignments and exams

Preferences Under Uncertainty

- Consider a lottery.
  - Win \$90 with probability 1/2 and win \$1 with probability 1/2.
  - Assume the Cobb-Douglas utility function. With the log transformation:
- $U(\$90) = \ln(90)=4.49$
- $U(\$1) = \ln(1)=0$
- Expected utility is

Preferences Under Uncertainty

- U($90) = ln(90) = 4.49
- U($1) = ln(1) = 0
- Expected utility is

$$\frac{1}{2}(4.49) + \frac{1}{2}(0) = 2.25$$

Preferences Under Uncertainty

- Expected money value of the lottery (or Expected value of the lottery is

$$EM = \frac{1}{2}(\$90) + \frac{1}{2}(\$1) = \$45.50$$

aka known as the average payout of the lottery. different from the concept of utility.

Preferences Under Uncertainty

- EU and EM are not the same thing!
- **Expected (Money) Value of the Lottery** is the probability weighted average of all the outcomes of the lottery

$$\sum_j prob_j(Outcome_j)$$

- **Expected Utility** is the probability weighted average of the utilities associated with all the possible outcomes

$$\sum_j prob_j\{U\left(Outcome_j\right)\}$$

Preferences Under Uncertainty

- Expected Utility
- Risk Aversion
- Risk Premium
- Marginal Rate of Substitution

Preferences Under Uncertainty

- By comparing the expected utility of a bundle to the utility of the expected money value, we can determine the riskiness of a consumer:

$$EU \text{ vs } U(EM)$$

Expected Utility

Utility of the Expected money value

Preferences Under Uncertainty

- EU = 2.245 and EM = \$45.50
- U(EM) = U(\$45.50)

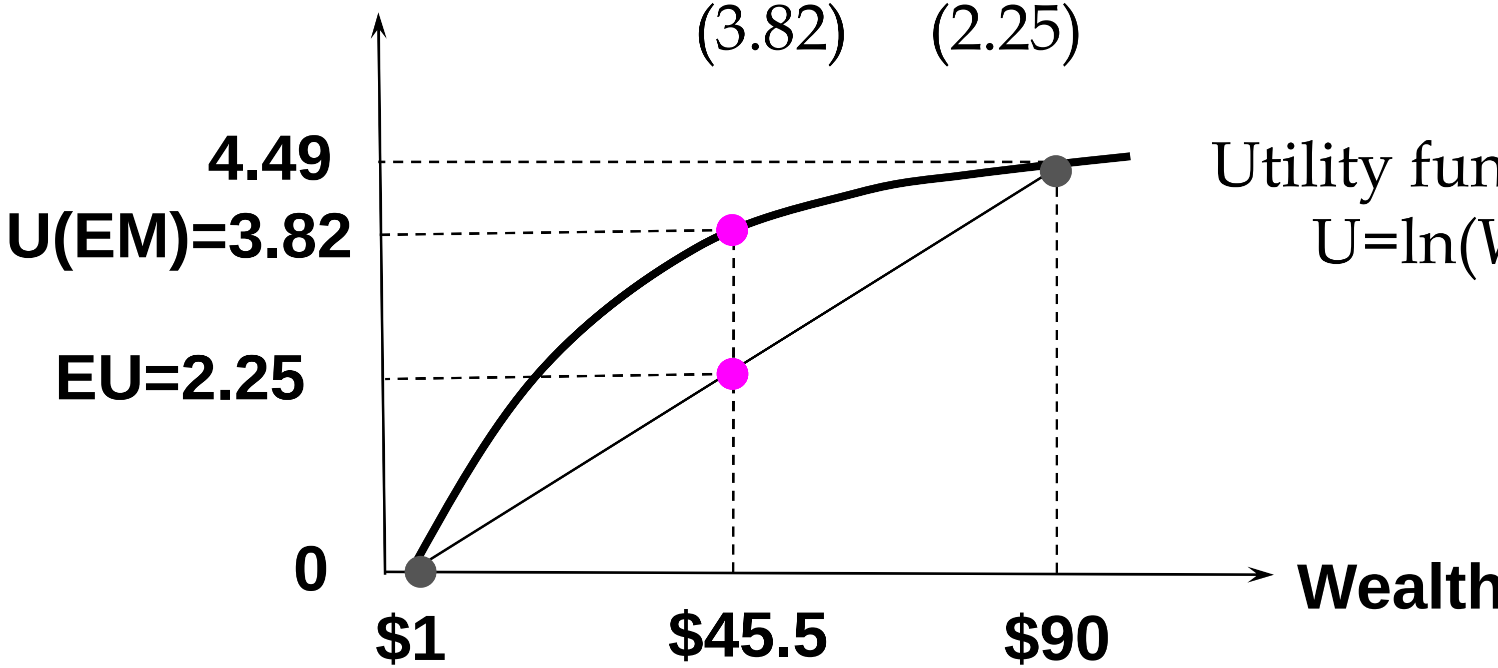
$$= \ln(45.50)$$
$$= 3.82$$

Preferences Under Uncertainty

prefers not playing the game

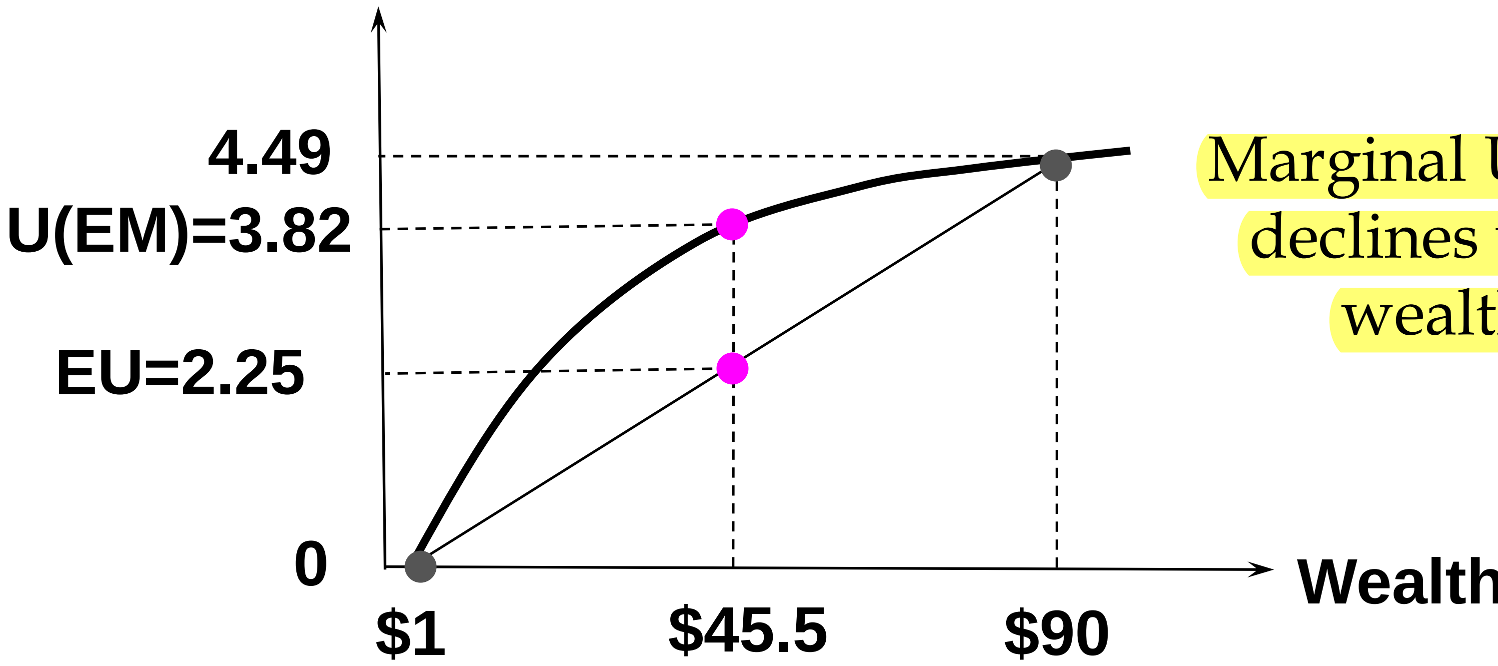
U(EM) > EU $\Rightarrow$ risk-aversion.

(3.82) (2.25)



Utility function:
U=ln(W)

Preferences Under Uncertainty



Marginal Utility declines with wealth

Preferences Under Uncertainty

$$U(EM) > EU$$

- This implies the consumer would prefer to have the expected money value of the lottery rather than participate in the lottery.
- This is why we call this consumer risk adverse

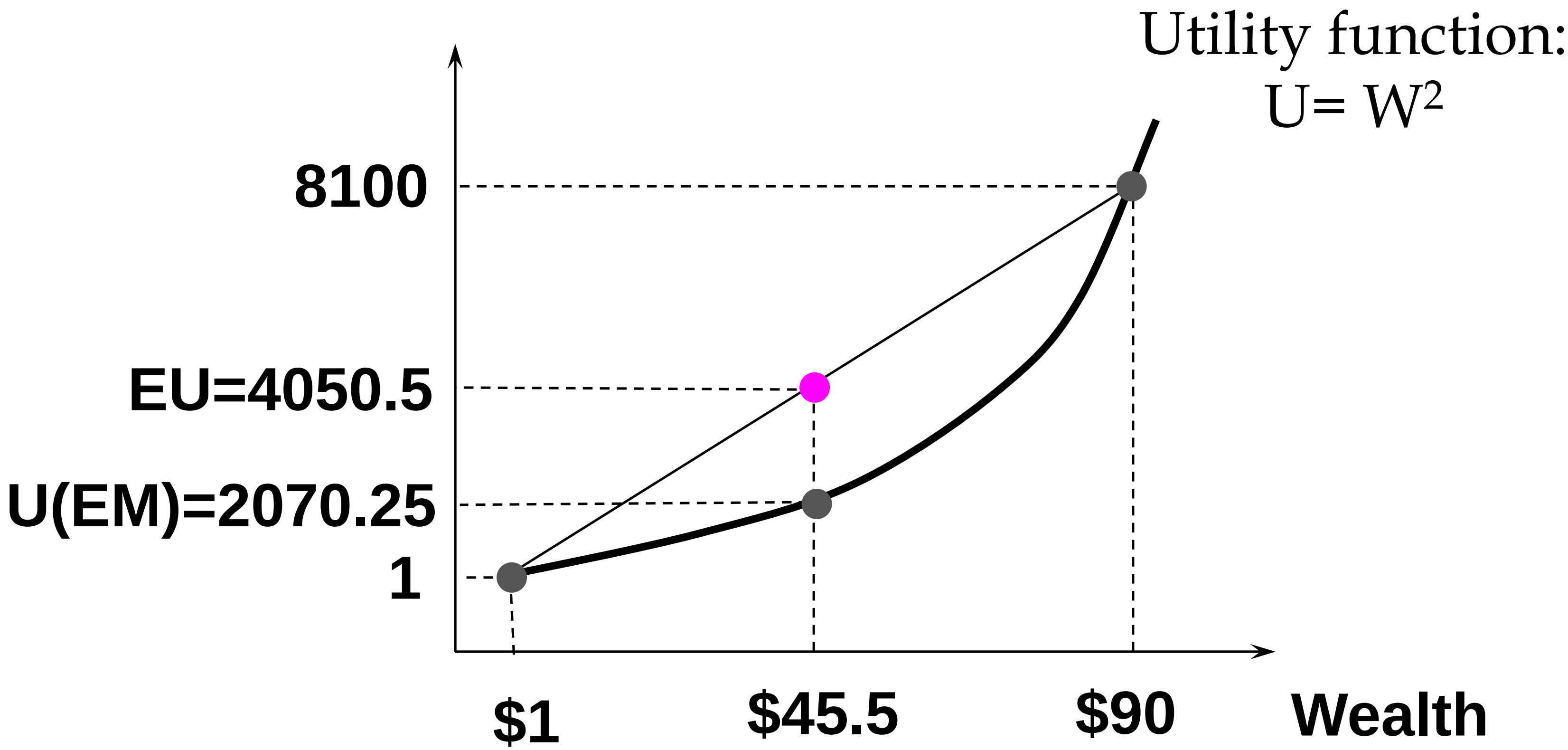
Preferences Under Uncertainty

- Consider the same lottery
  - Win \$90 with probability 1/2 and win \$1 with probability 1/2.
  - Assume the following utility function: $U(x) = x^2$

Preferences Under Uncertainty

- $EU = \frac{1}{2}\,(90^2) + \frac{1}{2}(1^2)$
  $= 4050.5$
- $EM = \frac{1}{2}(\$90) + \frac{1}{2}(\$1) = \$45.50$
- $U(EM) = 45.5^2$
  $= 2070.25$

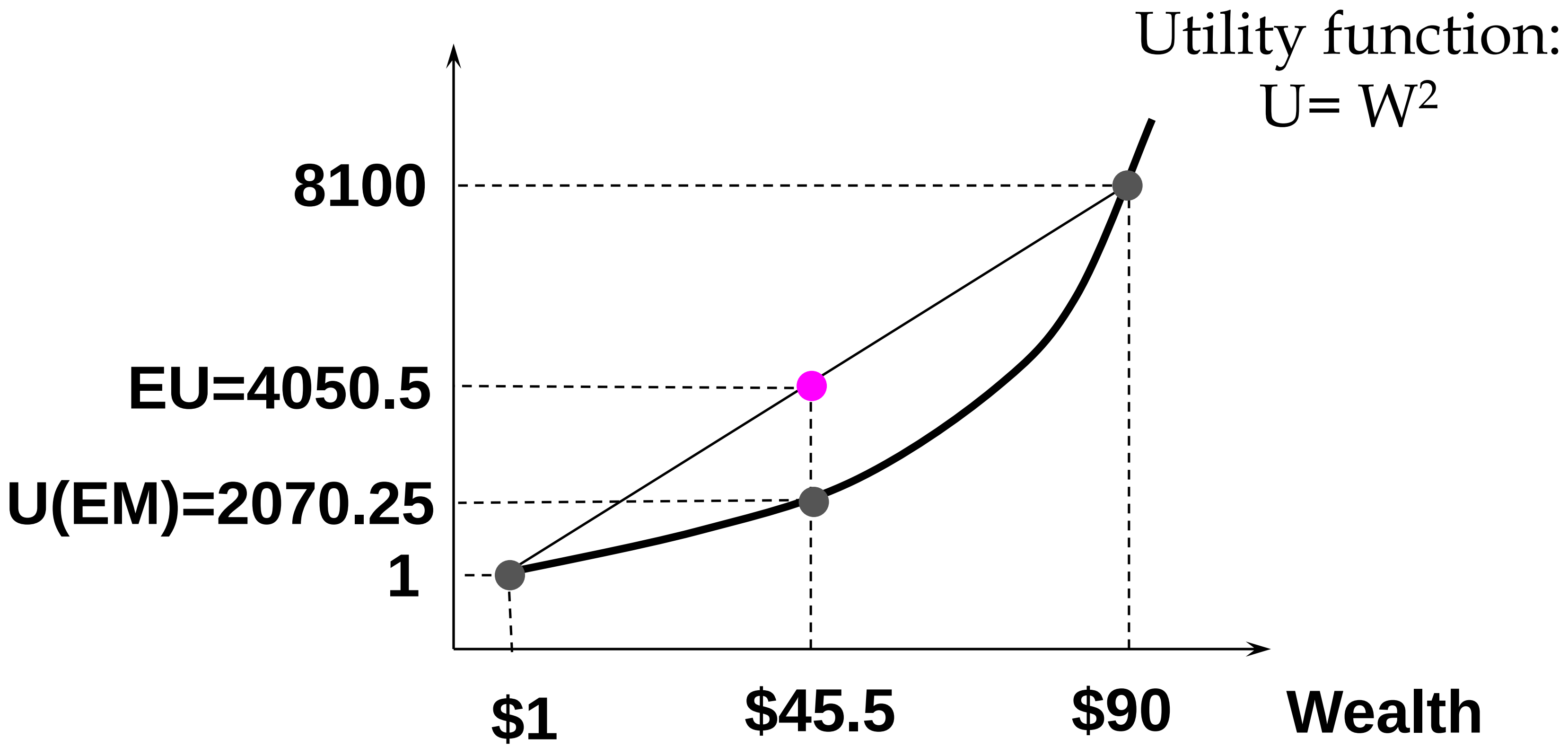
Preferences Under Uncertainty



U(EM) < EU $\Rightarrow$ risk-loving.

higher utility from playing that game

Preferences Under Uncertainty



Marginal Utility rises with wealth

Preferences Under Uncertainty

- EU = 7 and EM = \$45.
- U(EM) > EU $\Rightarrow$ Guaranteed payout is preferred to the lottery $\Rightarrow$ risk-aversion.
- U(EM) < EU $\Rightarrow$ the lottery is preferred to guaranteed payout$\Rightarrow$ risk-loving.
- U(EM) = EU $\Rightarrow$ the lottery is preferred equally to guaranteed payout$\Rightarrow$ risk-neutral.

Preferences Under Uncertainty

- In your own time, consider the same lottery with the following utility function:

$$U(x) = 2x + 6$$

  - What type of risk profile does this utility function exhibit?
  - How does marginal utility change with wealth?

Preferences Under Uncertainty

- Expected Utility
- Risk Aversion
- Risk Premium
- Marginal Rate of Substitution

Risk Premium

- The difference between the expected value of a risky income and a certain income to make the consumer indifferent between the two

Risk Premium

- Suppose the utility function of a consumer is $U(I) = \sqrt{I}$
- The consumer can buy a risky asset
    - \$900 with probability 60%
    - \$400 with probability 40%
- What is the risk premium associated with this asset?

Risk Premium

- Expected utility of the asset:
$$0.6\sqrt{900} + 0.4\sqrt{400} = 26$$
- To get the same utility, the consumer needs to attain income of $I = 26^2 = 676$
- Expected value of the asset is:
$$0.6(900) + 0.4(400) = 700$$
- Risk Premium = $700 - 676 = 24$

Give 24 extra dollars for playing this game

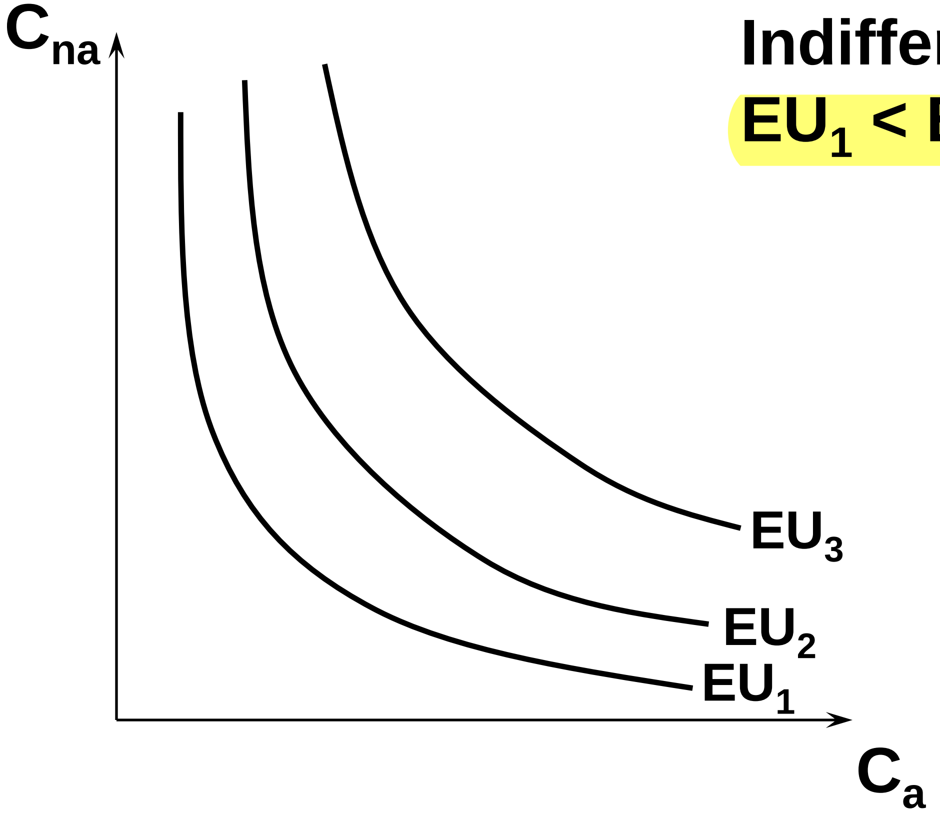
Preferences Under Uncertainty

- Expected Utility
- Risk Aversion
- Risk Premium
- Marginal Rate of Substitution

Preferences Under Uncertainty

- Indifference curves
- State-contingent consumption plans that give equal expected utility are equally preferred.

Preferences Under Uncertainty



Indifference curves

$EU_1 < EU_2 < EU_3$

Preferences Under Uncertainty

- What is the Marginal Rate of Substitution of an indifference curve?
- Get consumption c_1 with prob. π_1 and c_2 with prob. π_2 $(\pi_1 + \pi_2 = 1)$.
- $EU = \pi_1 U(c_1) + \pi_2 U(c_2)$.
- For constant EU, $dEU = 0$.

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_{\mathbf{1}}\mathbf{U}(\mathbf{c}_{\mathbf{1}}) + \pi_{\mathbf{2}}\mathbf{U}(\mathbf{c}_{\mathbf{2}})$$

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U(c_1)} + \pi_2 \mathbf{U(c_2)}$$

$$\mathbf{dEU} = \pi_1 \mathbf{MU(c_1)dc_1} + \pi_2 \mathbf{MU(c_2)dc_2}$$

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U}(\mathbf{c}_1) + \pi_2 \mathbf{U}(\mathbf{c}_2)$$

$$\mathbf{dEU} = \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 + \pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2$$

$$\mathbf{dEU} = \mathbf{0} \Rightarrow \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 + \pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2 = \mathbf{0}$$

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U(c_1)} + \pi_2 \mathbf{U(c_2)}$$

$$\mathbf{dEU} = \pi_1 \mathbf{MU(c_1)dc_1} + \pi_2 \mathbf{MU(c_2)dc_2}$$

$$\mathbf{dEU} = \mathbf{0} \Rightarrow \pi_1 \mathbf{MU(c_1)dc_1} + \pi_2 \mathbf{MU(c_2)dc_2} = \mathbf{0}$$

$$\Rightarrow \pi_1 \mathbf{MU(c_1)dc_1} = -\pi_2 \mathbf{MU(c_2)dc_2}$$

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U}(\mathbf{c}_1) + \pi_2 \mathbf{U}(\mathbf{c}_2)$$

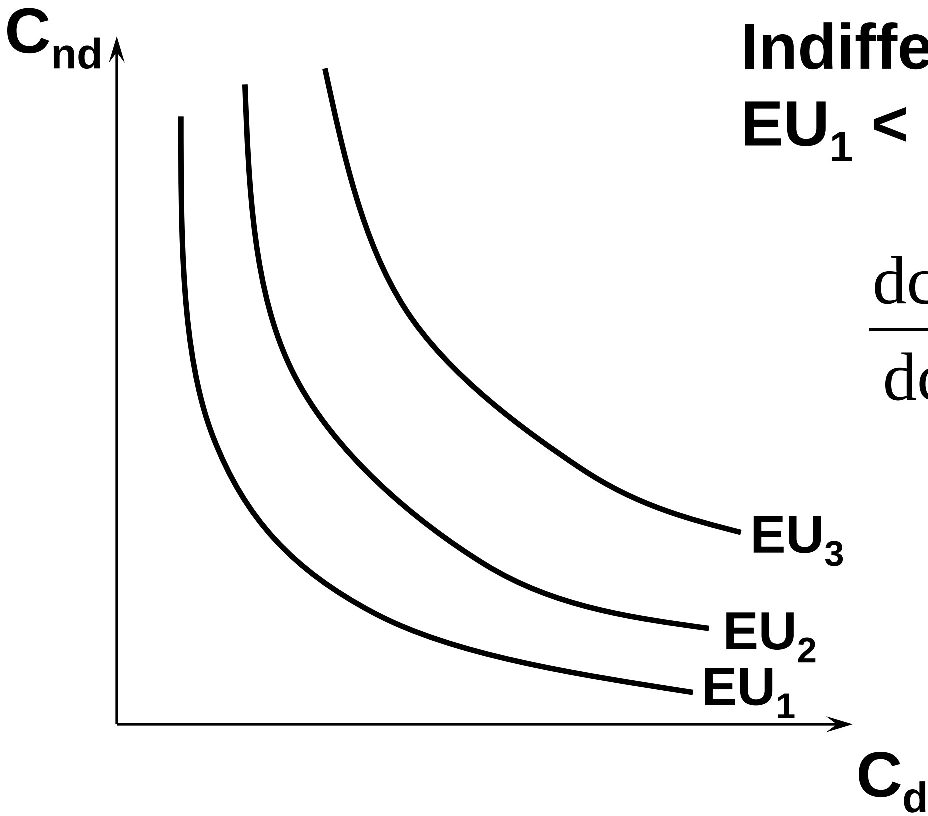
$$\mathbf{dEU} = \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 + \pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2$$

$$\mathbf{dEU} = \mathbf{0} \Rightarrow \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 + \pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2 = \mathbf{0}$$

$$\Rightarrow \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 = -\pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2$$

$$\Rightarrow \frac{\mathbf{dc}_2}{\mathbf{dc}_1} = -\frac{\pi_1 \mathbf{MU}(\mathbf{c}_1)}{\pi_2 \mathbf{MU}(\mathbf{c}_2)}.$$

Preferences Under Uncertainty



Indifference curves

$EU_1 < EU_2 < EU_3$

$$\frac{dc_{nd}}{dc_d} = -\frac{\pi_d MU(c_d)}{\pi_{nd} MU(c_{nd})}$$

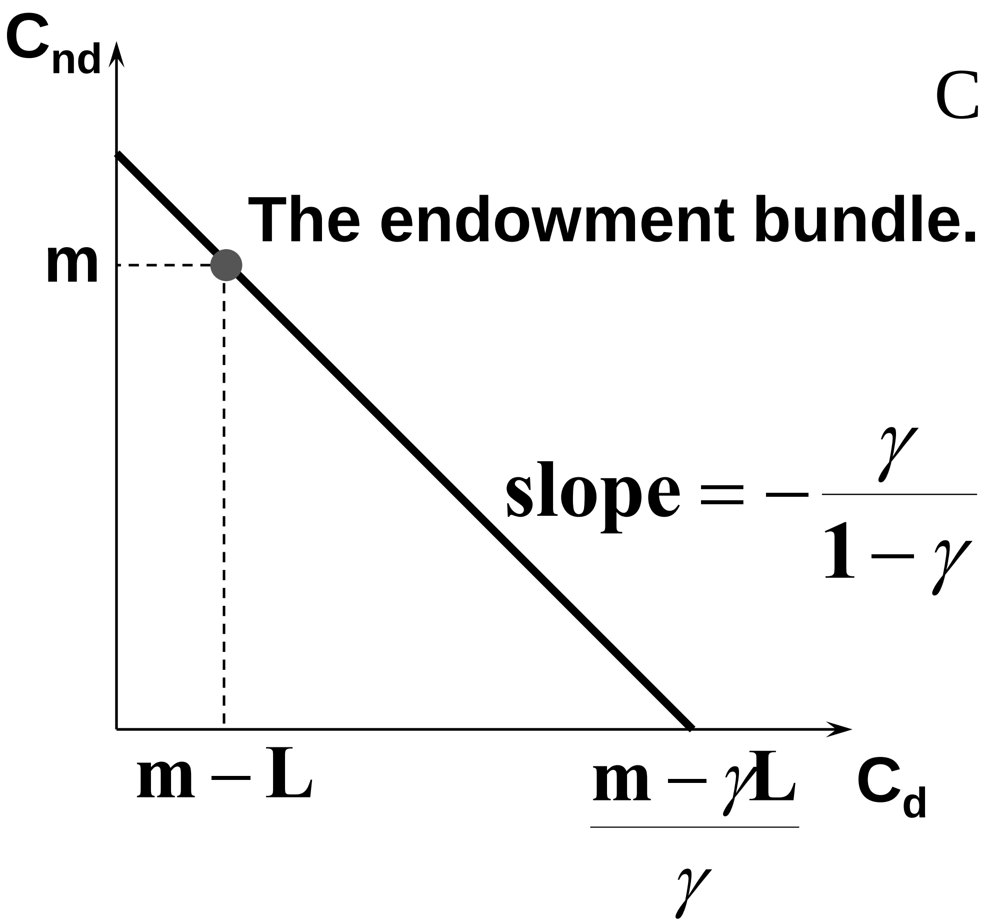
Preferences Under Uncertainty

- Expected Utility ★
- Risk Premium ★
- Risk Aversion ★
- Marginal Rate of Substitution ★

Choice Under Uncertainty

- How is a rational choice made under uncertainty?
  - Choose the most preferred affordable state-contingent consumption plan.

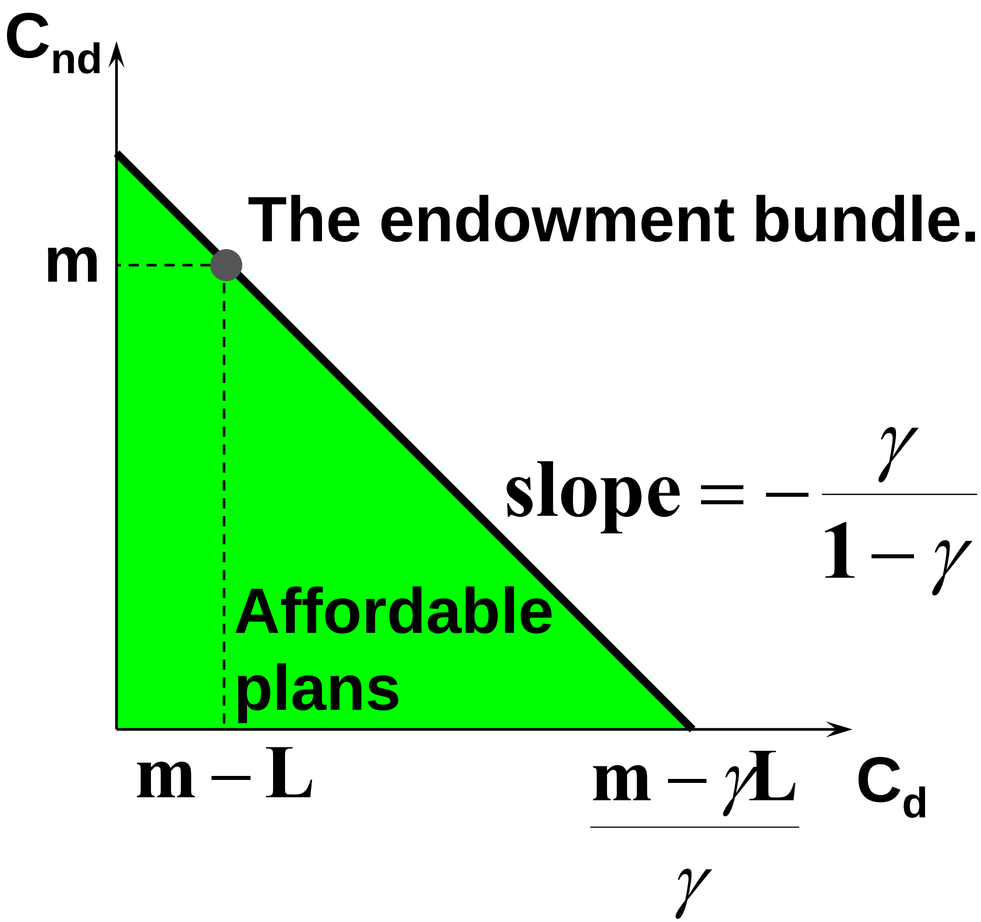
State-Contingent Budget Constraints



$$C_{nd} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_d$$

Where is the most preferred state-contingent consumption plan?

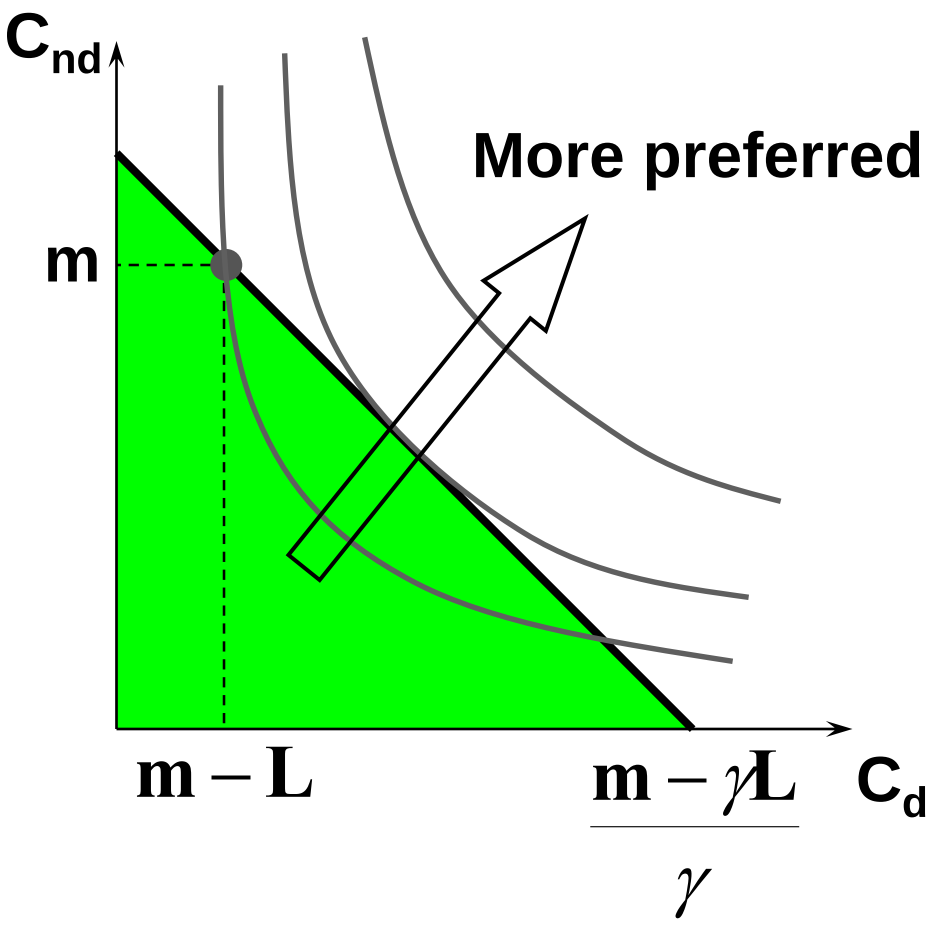
State-Contingent Budget Constraints



$$C_{nd} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_d$$

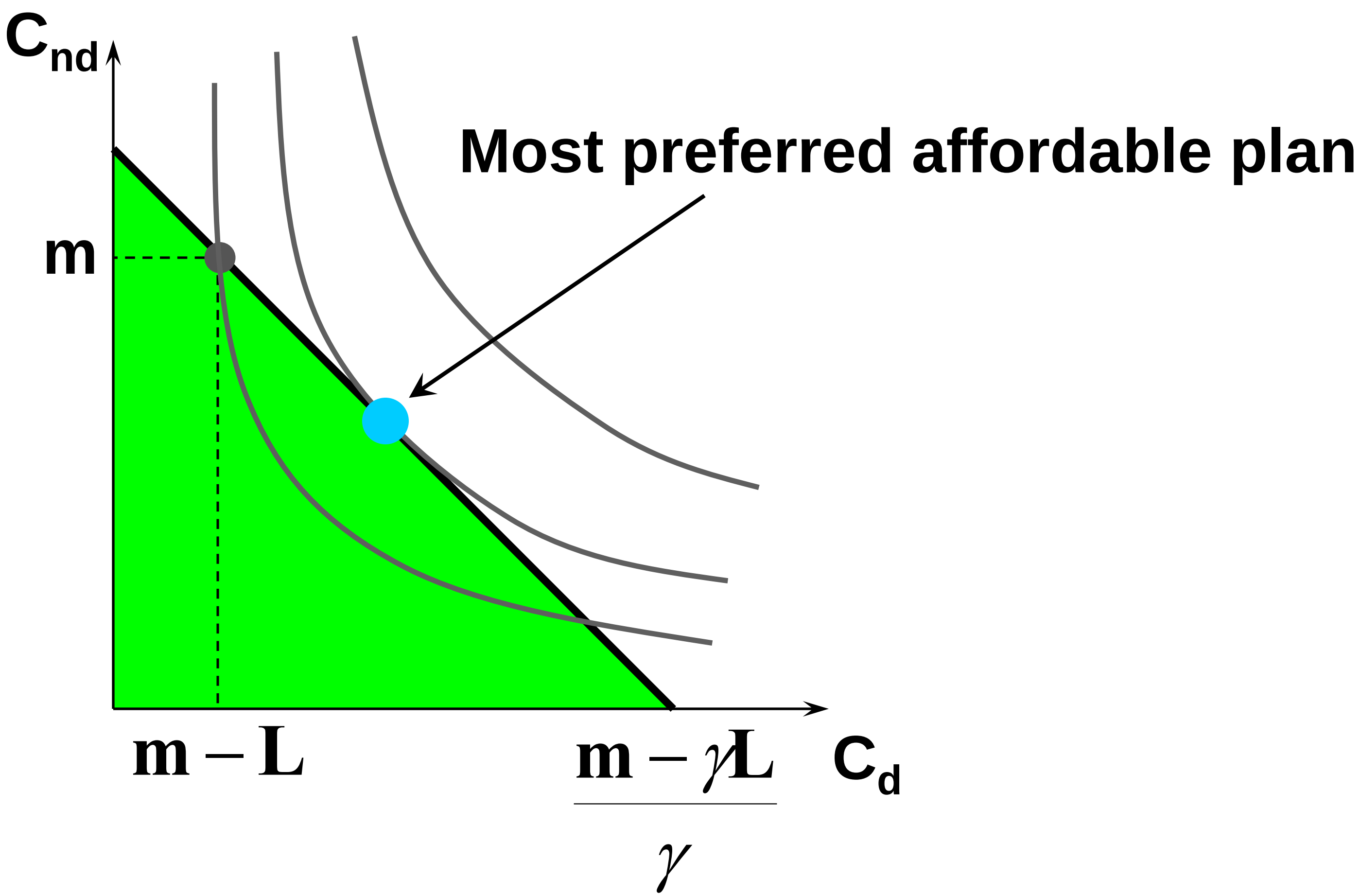
Where is the most preferred state-contingent consumption plan?

State-Contingent Budget Constraints

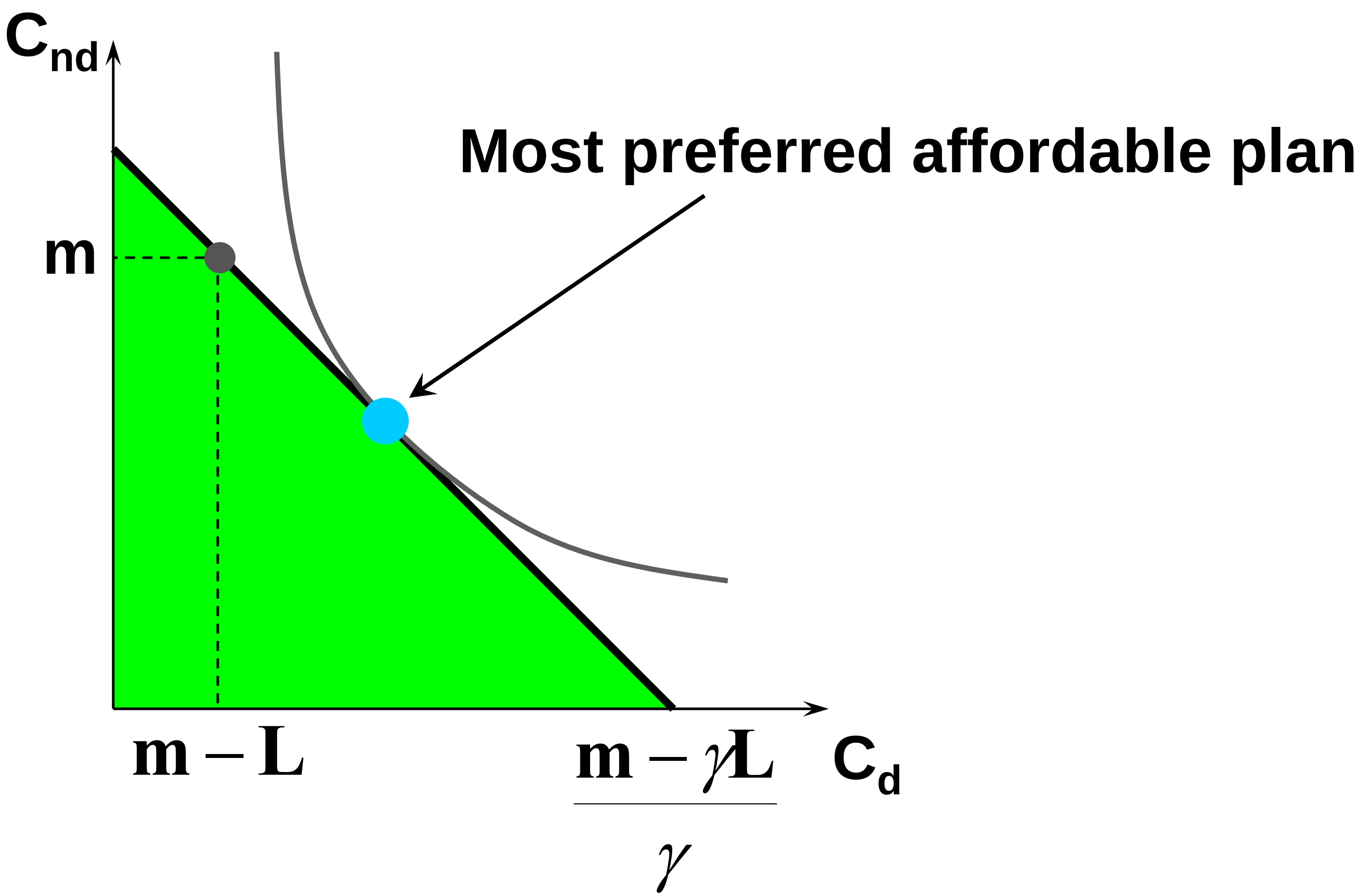


Where is the most preferred state-contingent consumption plan?

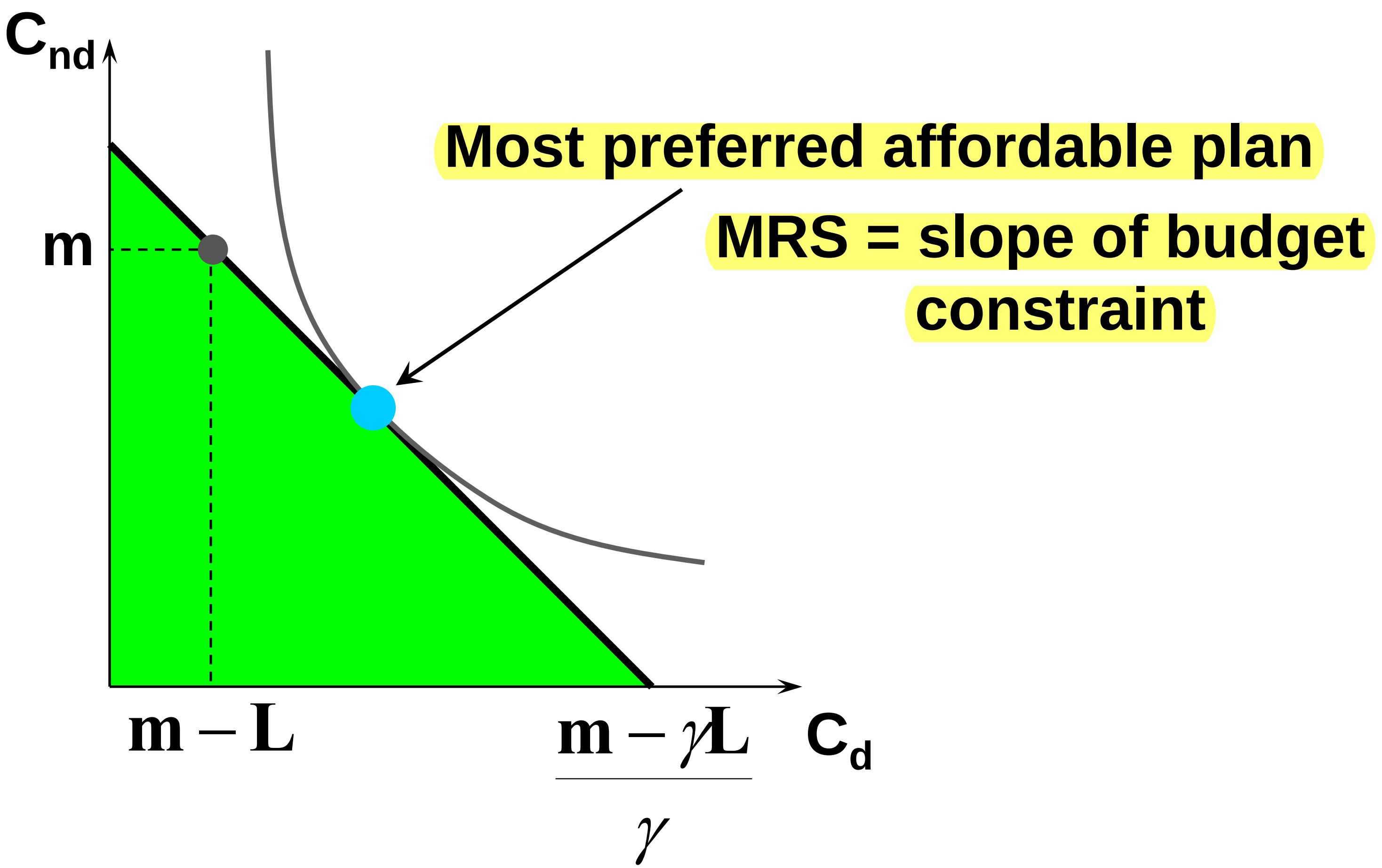
State-Contingent Budget Constraints



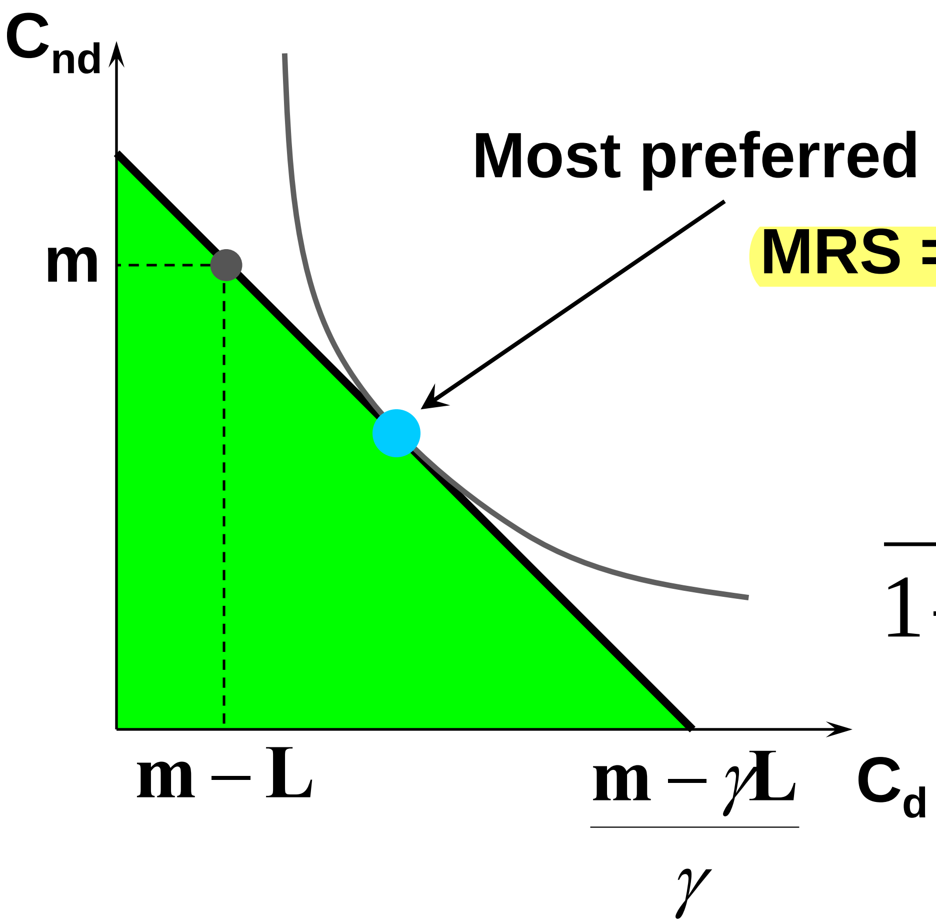
State-Contingent Budget Constraints



State-Contingent Budget Constraints



State-Contingent Budget Constraints



MRS = slope of budget constraint; i.e.

$$\frac{\gamma}{1-\gamma} = \frac{\pi_d \mathrm{MU}(\mathrm{c_d})}{\pi_{nd} \mathrm{MU}(\mathrm{c_{nd}})}$$

Competitive Insurance

- Suppose entry to the insurance industry is free.
- The insurance firm's profit function is given by:

$$\overbrace{\gamma K}^{\text{Revenue}} - \overbrace{\pi_d K}^{\text{Insurance payout if damage}} - \overbrace{(1 - \pi_d)0}^{\text{Insurance payout if no damage}}$$

$$= (\gamma - \pi_d)K$$

Competitive Insurance

- Since entry is free, firms freely enter until there are zero profits

$$(\gamma - \pi_d)K = 0.$$

- This means with free entry, $\gamma = \pi_d$.
- If price of $1 insurance equals the probability of damage, therefore insurance is fair.

Competitive Insurance

- The utility maximizing condition is that:

$$\frac{\gamma}{1-\gamma} = \frac{\pi_d MU(c_d)}{\pi_{nd} MU(c_{nd})}$$

Since $\gamma = \pi_d$,

$$\frac{\pi_d}{1-\pi_d} = \frac{\pi_d MU(c_d)}{\pi_{nd} MU(c_{nd})}$$

$$\frac{\pi_d}{\pi_{nd}} = \frac{\pi_d MU(c_d)}{\pi_{nd} MU(c_{nd})}$$

Competitive Insurance

$$\frac{\pi_d}{\pi_{nd}} = \frac{\pi_d MU(c_d)}{\pi_{nd} MU(c_{nd})}$$

- Therefore, $\frac{MU(c_d)}{MU(C_{nd})} = 1$ or

$$MU(c_d) = MU(C_{nd})$$

Marginal utility of wealth must be the same in both states.

Competitive Insurance

- How much fair insurance does a risk-averse consumer buy?
- For marginal utilities to be equal in both states, consumption must be equal in both states: $c_1 = c_2$
- This means the consumer fully insures!

Competitive Insurance

- The consumer will choose to fully insure if he or she is
  - risk adverse
  - maximizes Expected Utility
  - is offered fair insurance against loss

“Unfair” Insurance

- Suppose insurers make positive expected economic profit.

$$\gamma K - \pi_d K - (1 - \pi_d)0$$
$$= (\gamma - \pi_d)K > 0.$$

Which means $\gamma > \pi_d$

- The insurers charge more than their cost of insuring.

“Unfair” Insurance

- Rational choice requires

$$\frac{\gamma}{1-\gamma} = \frac{\pi_d MU(C_d)}{\pi_{nd} MU(C_{nd})}$$

- Since $\gamma > \pi_d$

$$\frac{\gamma}{1-\gamma} = \frac{\pi_d}{\pi_{nd}}$$

- which means $\frac{MU(C_d)}{MU(C_{nd})}$ >1

$$MU(C_d) > MU(C_{nd})$$

“Unfair Insurance”

- Remember that this consumer is risk adverse. This means MU declines in wealth
- Since the marginal utility when there is damage is higher than the marginal utility when there is no damage:

$$C_d < C_{nd}$$

- In this case, the consumer only partially insures

Uncertainty is Pervasive

- What are rational responses to uncertainty?
  - buying insurance (health, life, auto)
  - a portfolio of contingent consumption goods.

Uncertainty is Pervasive

- What are rational responses to uncertainty?
  - ✓ buying insurance (health, life, auto)
  - a portfolio of contingent consumption goods.

Uncertainty is Pervasive

- What are rational responses to uncertainty?
  - ✓ • buying insurance (health, life, auto)
  - ? • a portfolio of contingent consumption goods.

Diversification

- Firm A sells Chelsea FC jerseys and Firm B sell Manchester City FC jerseys
- Shares cost $10.
- Either Chelsea or Manchester City will win the Premier League. Winning boosts sales of jerseys.

Diversification

- With prob. ½, Chelsea wins.
  - A's profit is \$100 per share and B's profit is \$20 per share.
- With prob. ½, Man City wins.
  - A's profit is \$20 per share and B's profit is \$100 per share.
- You have \$100 to invest. How?

Diversification

- Buy only firm A's stock?
- $100/10 = 10 shares.
- You earn $1000 with prob. 1/2 and $200 with prob. 1/2.
- Expected earning: $500 + $100 = $600

Diversification

- Buy only firm B's stock?
- \$100/10 = 10 shares.
- You earn \$1000 with prob. 1/2 and \$200 with prob. 1/2.
- Expected earning: \$500 + \$100 = \$600

Diversification

- Buy 5 shares in each firm?
- You earn $500 from one firm and $100 from the other regardless of who wins the premier league title.
  - Either $500 from A and $100 from B
  - Or $500 from B and $100 from A
- Diversification has maintained expected earning and lowered risk.

No opportunity cost in diversification

Diversification

- Diversification can lower expected earnings in exchange for lowered risk.
- Of course, in real life scenarios, outcomes are never perfectly correlated like in this example, making diversification less powerful of a tool for minimizing risk

Risk Spreading

- 100 risk-neutral persons each independently risk a \$10,000 loss.
- Loss probability = 0.01.
- Initial wealth is \$40,000.
- No insurance: expected wealth is

$$\mathbf{0 \cdot 99} \times \$\mathbf{40{,}000} + \mathbf{0 \cdot 01}(\$\mathbf{40{,}000} - \$\mathbf{10{,}000})$$

$$= \$\mathbf{39{,}900}.$$

Risk Spreading

- On average one person will suffer a loss each year.
- If all 100 people band together, they can each agree to pay $100 to the person who suffers a loss.

Risk Spreading

- Expected wealth is now:
  $$0.99 \times (\$39{,}900) + 0.01 \times (\$39{,}900 - 10{,}000 + 10{,}000)$$
- Every individual makes $39,900 with certainty now. No risk! (assuming the probabilities of loss are true)