

Chapter 37:  
Public Goods  
(Sections 37.1 - 37.4)

# Public Goods

- Definition and concept
- When to provide a public good
- The free-rider problem
- How much of a public good to provide

# Definition

- A public good is a good that is:
  - Non-excludible: once produced, it is impossible to exclude someone from consuming it
  - Non-rivalrous: one person's consumption of the good doesn't diminish another person's consumption of the good

# Public Goods

	Excludable	Non-excludable
Rivalrous	<b>Private goods</b> (food, clothing, books)	<b>Common goods</b> (fish stocks, public highways when congested)
Non-rivalrous	<b>Club goods</b> (golf courses, cinemas)	<b>Public goods</b> (air, national defense, public highways when not congested)

- Non-rivalrous can mean indivisible or very large/abundant
  - National defense is indivisible
  - A golf course is very large/abundant

# Public Goods

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# When to provide a public good

- Section 37.1 is extensive, and I do not have time to present the whole derivation here. You need to read it, but just grasp the general concept. Do not get bogged down with the details.
  - Second paragraph on pg. 717 until end of section on pg. 718 is less important.
- What these slides will cover is a simpler discussion of the ideas. If we have time, we may come back to the full derivation in Week 13

# When to provide a public good

- Suppose there are two individuals, Chandler and Joey.
  - Joey owns a jewelry store.
  - Chandler owns a television store.
  - Both stores are side-by-side
- They are both interested in getting a security guard which is a public good.
  - The security guard will make the entire area safer. He is non-excludible.
  - The security he provides is non-rivalrous.

# When to provide a public good

- The cost of a security guard is  $c$
- The value of security to Joey is  $r_j$
- The value of security to Chandler is  $r_t$



# When to provide a public good

- Suppose  $r_j > c$  and  $r_t < c$
- Then Joey gets the security
  - Joey has a payoff of  $r_j - c$
  - Chandler free-rides with a payoff of  $r_t$
- Suppose  $r_j < c$  and  $r_t > c$ 
  - Chandler gets a guard and Joey free-rides.

# When to provide a public good

- Suppose  $r_j < c$  and  $r_t < c$ , but  $r_j + r_t > c$ 
  - Then individually, neither will get the security guard, but if they coordinate, they could make themselves better off.
- So the public good should be provided as long as  $r_j + r_t > c$

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# The free-rider problem

- Suppose  $r_j = 8$ ,  $r_t = 8$  and  $c = 10$ .
- If the stores decide independently whether to hire a guard, then:

		Chandler	
		Hire	Do Not Hire
Joey	Hire	-2, -2	-2, 8
	Do Not Hire	8, -2	0, 0

- The Nash equilibrium is {Joey: Do Not Hire; Chandler: Do Not Hire}
- Because each tries to free-ride on the other, they don't do what is best for the collective.

# The free-rider problem

- How do we overcome the free-rider problem?
- If Joey and Chandler can make “side payments” to each other, then we can get to a Pareto improving outcome.
- If Joey hires the guard and Chandler makes him a “side payment” then both agents are better off than in the Nash equilibrium outcome.

# The free-rider problem

- Let's say the "side payment" is \$2:

		Chandler	
		Hire	Do Not Hire
Joey	Hire	-2, -2	0, 6
	Do Not Hire	8, -2	0, 0

# The free-rider problem

- Allowing “side-payments” makes possible supply of a public good when no individual will supply the good unilaterally.
- However, “side-payments” could fail:
  - If voluntary, one has the incentive to under-report the valuation of the good to pay less.
  - This would lead to an under-provision of the good.

# Public Goods

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- How much of a public good to provide



# How much of a public good to provide

- We've discussed when a public good should be produced and the problem of free riding that may cause the good not to be produced.
- Now we look at how much of a public good society should produce.
- Examples:
  - How much public defense should we have?
  - How big should Gardens by the Bay be?
  - How much should society spend on medical research?

# How much of a public good to provide

- Consider the following:
  - $c(G)$  is the cost of providing  $G$  units of a public good.
  - There are two individuals, A and B with endowments  $\omega_A$  and  $\omega_B$ .
  - They can consume  $G$  or a private good,  $X_A$  and  $X_B$ .
  - Set price of  $X$ ,  $p_X = 1$
  - Budget allocations must satisfy  $X_A + X_B + c(G) = \omega_A + \omega_B$

# How much of a public good to provide

- $MRS_A$  and  $MRS_B$  are the marginal rates of substitution between the private good and public good.

$$MRS_A = \frac{MU_G}{MU_{X_A}}, MRS_B = \frac{MU_G}{MU_{X_B}}$$

- The Pareto efficient allocation is given by
$$MRS_A + MRS_B = MC(G)$$

Where  $MC(G)$  is the marginal cost of G

# How much of a public good to provide

- Recall in a competitive equilibrium:

$$MRS_A = MRS_B = MC(G)$$

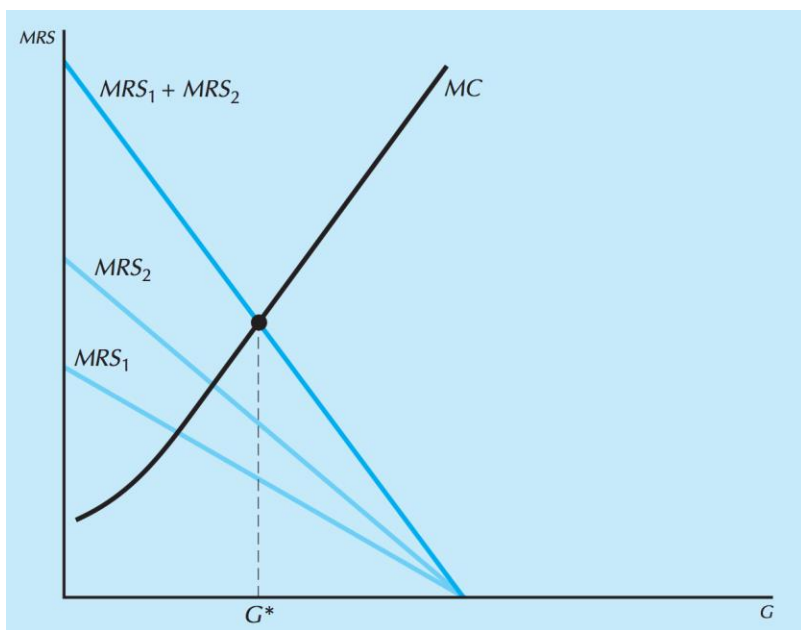
$$\text{since } p_x = 1$$

- Why  $MRS_A + MRS_B = MC(G)$ 
  - Because  $G$  is non-rivalrous, one extra unit of  $G$  is fully consumed by  $A$  and  $B$ .
  - You have to think of  $A$  &  $B$  as if they were one person because they can both simultaneously consume the same good.

# How much of a public good to provide

- MRS is how much  $X$  an agent is willing to give up to gain more  $G$ .
- In other words, it is how much value an agent places on one unit of  $G$  measured in units of  $X$
- The value each agent places on  $G$  must sum up to the value of  $G$
- In terms of the earlier example with Joey and Chandler,  $r_j + r_t = c$

# How much of a public good to provide



- If  $MRS_A + MRS_B > MC(G)$ , increase  $G$
- If  $MRS_A + MRS_B < MC(G)$ , reduce  $G$

# How much of a public good to provide

- How do we figure out what  $MRS_A + MRS_B$  is?
- This is a difficult question.
- If agents think the government wants to elicit their  $MRS$  so they can charge them for public goods accordingly, then agents will under-report their  $MRS$ .

# How much of a public good to provide

- Merging will internalize the externality.
  - If the two stores merge, they no longer have a problem of under-provision of security guards.
- Technology makes it easier to track MRS.
  - GPS tracking means it is easy to figure out agents value of public roads by tracking how much they use it.



# Chapter 38:

# Asymmetric Information

Why you should never trust everything a doctor  
tells you to do

# Today's plan

- Definition and Concepts
- Adverse Selection
  - The Market for Lemons
  - Adverse Selection with Quality Choice
  - Signaling
- Moral Hazard
  - Incentive Contracts

# Information in Competitive Markets

- In purely competitive markets all agents are fully informed about traded commodities and other aspects of the market.
- Both sides of the market (buyers and sellers) know all information there is to know about the product and each other.
- In reality, there are many markets where agents have imperfect information about the product being traded.

# Asymmetric Information in Markets

- Markets where one side and/or the other is imperfectly informed are markets with **imperfect information**.
- Typically, these are markets where the products and or agents have qualities that are difficult to assess
- Imperfectly informed markets with one side better informed than the other are markets with **asymmetric information**.

# Markets with Asymmetric Information

- Seller has more information:
  - A doctor knows more about medical services than the buyer.
  - A used car's owner knows more about it than a potential buyer.
  - A taxi driver knows more about traffic routes and patterns than the customer
  - A job applicant knows his ability better than the firm seeking his employment
- Buyer has more information:
  - An insurance buyer knows more about his riskiness than does the seller.

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  - Contract Theory

# Adverse Selection

- Adverse selection is a scenario where agents with complete information exploit the agents with incomplete information.
- If sellers have complete information, they use it to “trick” buyers into purchasing inferior products.
- If buyers have complete information, they “trick” sellers into selling a product that is not profitable to sell.
- This leads to inefficient market outcomes

# The Market for Lemons

- Consider a used car market with two types of cars:
- Lemons:
  - Seller will accept \$1,000
  - Buyer will pay at most \$1,200.
- Peaches
  - Seller will accept \$2,000
  - Buyer will pay at most \$2,400.
- If buyers can tell the difference between lemons and peaches, then...
- Gains-to-trade are generated when buyers are well informed.

Traded for between  
\$1,000 and \$1,200

Traded for between  
\$2,000 and \$2,400



# The Market for Lemons

- Suppose no buyer can tell a peach from a lemon before buying.
- What is the most a buyer will pay for any car?
- Let  $q$  be the fraction of peaches and  $1 - q$  is the fraction of lemons.
- The buyer and the sellers know  $q$  and buyers are risk neutral
- Expected value to a buyer of any car is at most:  
$$EV = \$1,200(1 - q) + \$2,400q$$

# The Market for Lemons

- Suppose there are lots of peaches such that  $EV > \$2000$ .
- Recall that sellers will accept \$2,000 for a peach and \$1000 for a lemon
- When  $EV > \$2000$ , every seller can negotiate a price between \$2000 and  $EV$ 
  - regardless of whether the car is a lemon or a peach.
- All sellers gain from being in the market.
- Buyers who get lemons lose from being in the market

# The Market for Lemons

- Suppose  $EV < \$2000$ .
- A peach seller cannot negotiate a price above \$2000 and will exit the market.
- So all buyers know that remaining sellers own lemons only.
- Buyers will pay at most \$1200 and only lemons are sold.

# The Market for Lemons

- Hence “too many” lemons “crowd out” the peaches from the market.
- Gains-to-trade are reduced since no peaches are traded.
- The presence of the lemons inflicts an external cost on buyers and peach owners.

# The Market for Lemons

- How many lemons can be in the market without crowding out the peaches?
- Buyers will pay \$2000 for a car only if
$$EV = \$1200(1 - q) + \$2400q \geq \$2000$$
$$q \geq \frac{2}{3}$$

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# Adverse Selection with Quality Choice

- Consider the market for taxi trips. It is a competitive market and taxi drivers can choose the quality of their product.
- Two taxi routes; fast route and slow route.
- Which will the taxi drivers choose?

# Adverse Selection with Quality Choice

- If the buyers knew the two options, he would value the fast route at \$14 and the slow route at \$8.
- Marginal production cost of a fast trips is \$11.
- Marginal production cost of a slow trip is \$10.
- With complete information:
  - fast trips are sold for \$11 and buyers gain \$3.
  - slow trips are never sold since they are valued less than cost.
- Now assume all buyers are unfamiliar with the area thus cannot tell the length of a trip before it occurs.



# Adverse Selection with Quality Choice

- Suppose every taxi driver takes only fast trips
- Every passenger pays \$11 and sellers' profit per trip is zero.
- But then any single driver can switch to a slow trip for which passengers will still pay \$11, increasing his profit to  $\$11 - \$10 = \$1$ .
- Since all drivers have this incentive, an equilibrium where only fast trips take place is not sustainable.

# Adverse Selection with Quality Choice

- Now we know there is no market equilibrium in which only fast trips are provided.
- Is there an equilibrium where both types of trips are supplied?

# Adverse Selection with Quality Choice

- A fraction  $q$  of drivers make fast trips;  $0 < q < 1$ .
- Buyers' expected value of a trip is  
 $EV = 14q + 8(1 - q) = 8 + 6q$ .
- Fast trip drivers must recover costs so

$$EV = 8 + 6q \geq 11$$
$$q \geq \frac{1}{2}$$

# Adverse Selection with Quality Choice

- So at least half of the drivers must provide fast trips for there to be a market where both types of trips exist.
- But every driver can reason that if he switches to making slow trips he can increase his profit by \$1.

# Adverse Selection with Quality Choice

- Since all drivers can reason this way, this will cause all drivers to provide slow trips only.
- So there is no equilibrium in which both types of trips are supplied.
- Drivers ignore the externality they incur on other drivers by switching to a slow trip.

# Adverse Selection with Quality Choice

- Is there a market equilibrium in which only slow trips are supplied?
- All drivers make only slow trips.
- Buyers pay at most \$8 for a trip, while marginal production cost is \$10.
- There is no market equilibrium in which only slow trips are made.

# Adverse Selection with Quality Choice

- The market has no equilibrium
  - with just one-type of trip traded
  - with both types of trips traded
- so the market has no equilibrium at all.
- Adverse selection has destroyed the entire market!

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# Signaling

- Adverse selection is an outcome of an informational deficiency.
- What if information can be improved by high-quality sellers signaling credibly that they are high-quality?
- E.g.
  - warranties for cars, smartphones, laptops.
  - professional credentials for doctors, architects.

# Signaling

- A labor market has two types of workers; high-ability and low-ability.
- A high-ability worker's marginal product is  $a_H$ .
- A low-ability worker's marginal product is  $a_L$ .

$$a_L < a_H$$

# Signaling

- A fraction  $h$  of all workers are high-ability.
- $1 - h$  is the fraction of low-ability workers.
- Each worker is paid his expected marginal product.
- If firms knew each worker's type they would
  - pay each high-ability worker  $w_H = a_H$
  - pay each low-ability worker  $w_L = a_L$ .

# Signaling

- If firms cannot tell workers' types then every worker is paid the same wage rate; i.e. the expected marginal product

$$w_P = (1 - h)a_L + ha_H$$

- Since  $w_P = (1 - h)a_L + ha_H < a_H$ , the wage rate paid when the firm knows a worker really is high-ability.
- So high-ability workers have an incentive to find a credible signal.

# Signaling

- Workers can acquire “education”.
- Education costs a high-ability worker  $c_H$  per unit
- and costs a low-ability worker  $c_L$  per unit.

$$c_L > c_H$$

- Suppose that education has no effect on workers’ productivities; i.e., the cost of education is a deadweight loss.

# Signaling

- High-ability workers will acquire  $e_H$  education units if
  - (i)  $w_H - w_L = a_H - a_L > c_H e_H$ , and
  - (ii)  $w_H - w_L = a_H - a_L < c_L e_H$
  - (iii)  $w_H - c_H e_H > (1 - h)a_L + ha_H$
- (i) says acquiring  $e_H$  units of education benefits high-ability workers.
- (ii) says acquiring  $e_H$  education units is not beneficial to low-ability workers.
- This means that acquiring education allows high ability workers to distinguish themselves from low ability workers
- (iii) says high ability wage after acquiring education is greater than average wage received without education

# Signaling

- Suppose (iii) holds, that is education is cheap enough that high ability workers would prefer having it.
- High-ability workers will acquire  $e_H$  education units if
  - (i)  $w_H - w_L = a_H - a_L > c_H e_H$ , and
  - (ii)  $w_H - w_L = a_H - a_L < c_L e_H$ .

- This implies:

$$\frac{a_H - a_L}{c_L} < e_H < \frac{a_H - a_L}{c_H}$$

# Signaling

- Q: Given that high-ability workers acquire  $e_H$  units of education, how much education should low-ability workers acquire?
- A: Zero. Low-ability workers will be paid  $w_L = a_L$  so long as they do not have  $e_H$  units of education and they are still worse off if they do.



# Signaling

- Signaling can improve information in the market.
- But, total output does not change and education is costly so signaling worsens market's efficiency.
- So improved information need not improve gains-to-trade.
- Nevertheless, high-ability workers are better off and low-ability workers are worse off.
  - High ability workers get paid  $a_H$
  - Low ability workers get paid  $a_L$

# Signaling

- The equilibrium where high-ability workers attain education is called a **separating equilibrium**.
- Firms can tell apart or “separate” the two types of workers
  - It occurs when (i), (ii) and (iii) are met.
- The equilibrium where high-ability workers do not attain education is called a **pooling equilibrium**. Firms “pool” all types of workers together and pay them the same wage.

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# Moral Hazard

- Adverse Selection arises because agents cannot determine other agents' type.
  - Hidden information
- Moral Hazard arises because agents cannot determine other agents' actions.
  - Hidden action

# Moral Hazard

- Consider a health insurance company.
  - Unhealthy people are more willing to buy insurance.
  - This is adverse selection
- After buying insurance, people have the incentive to live more unhealthily since insurance will cover their medical costs anyway
- This is moral hazard

# Incentive Contracting

- A worker is hired by a principal to do a task.
- Only the worker knows the effort she exerts.
- The effort exerted affects the principal's payoff.
- First we assume that the principal can observe the worker's effort (i.e. no asymmetric information)

# Incentive Contracting

- The principal's problem: design an incentives contract that induces the worker to exert the amount of effort that maximizes the principal's payoff.
- First we will assume that there is symmetric information, that is the principal can observe the worker's effort via the output he produces.
- This problem is often referred to as the "principal-agent problem"

# Incentive Contracting

- $x$  is the agent's effort.
- Principal's reward is  $y = f(x)$
- An incentive contract is a function  $s(y)$  specifying the worker's payment when he produces an amount of output equal to  $y$ . The principal's profit is thus

$$\Pi_p = y - s(y) = f(x) - s(f(x))$$



# Incentive Contracting

- Let  $\tilde{u}$  be the worker's (reservation) utility of not working.
- To get the worker's participation, the contract must offer the worker a utility of at least  $\tilde{u}$
- The worker's utility cost of an effort level  $x$  is  $c(x)$ .

# Incentive Contracting

- So the principal's problem is

$$\begin{aligned} \max_x \Pi_p &= f(x) - s(f(x)) \\ \text{subject to } s(f(x)) - c(x) &\geq \tilde{u} \end{aligned}$$

(participation constraint)

- To maximize his profit the principal designs the contract to provide the worker with only the reservation utility level of utility. That is, ...

# Incentive Contracting

- The principal chooses  $x$  such that

$$\max_x \Pi_p = f(x) - s(f(x))$$

$$\text{subject to } s(f(x)) - c(x) \textcircled{=} \tilde{u}$$

(participation constraint)

- We can substitute the constraint into the optimization function:

$$\max_x \Pi_p = f(x) - c(x) - \tilde{u}$$

- Solving this we have that

$$f'(x) = c'(x)$$

# Incentive Contracting

$$f'(x) = c'(x)$$

- The contract that maximizes the principal's profit insists upon the worker effort level  $x^*$  that equalizes the worker's marginal effort cost to the principal's marginal payoff from worker effort.
- We will label  $x$  such that  $f'(x) = c'(x)$  as  $x^*$
- How can the principal induce the worker to choose  $x = x^*$ ?

# Incentives Contracting

- $x = x^*$  must be most preferred by the worker.
- So the contract  $s(y)$  must satisfy the **incentive-compatibility** constraint:

$$s(f(x^*)) - c(x^*) \geq s(f(x)) - c(x), \forall x \geq 0$$

# Rental Contracts

- The principal keeps a lump-sum  $R$  for himself and the worker gets all profit above  $R$ ; i.e.

$$s(f(x)) = f(x) - R$$

- When the worker maximize his payoff:

$$\max_x s(f(x)) - c(x) = f(x) - R - c(x)$$

- This results in the condition  $f'(x) = c'(x)$  which yields the level of effort  $x^*$

# Rental Contracts

- How large should be the principal's rental fee  $R$ ?
- The principal should extract as much rent as possible without causing the worker not to participate, so  $R$  should satisfy

$$s(f(x^*)) - c(x^*) - R = \tilde{u}$$
$$R = s(f(x^*)) - c(x^*) - \tilde{u}$$

# Wage Contracts

- In a wages contract the payment to the worker is
$$s(f(x)) = wx + K$$

where  $w$  is the wage per unit of effort.  $K$  is a lump-sum payment.

- The worker thus maximizes  $wx + K - c(x)$  which yields the condition  $w = c'(x)$
- If  $w = f'(x)$ , then the worker will exert effort  $x^*$
- $K$  is chosen so that the worker is indifferent between participating and not participating:
  - $wx + K - c(x) = \tilde{u}$
  - $K = \tilde{u} - wx + c(x)$



# Take it Or Leave it Contracts

- Choose  $x = x^*$  and be paid a lump-sum  $L$ , or choose  $x \neq x^*$  and be paid zero.
- The worker's utility from choosing  $x \neq x^*$  is  $-c(x)$ , so the worker will choose  $x = x^*$ .
- $L$  is chosen to make the worker indifferent between participating and not participating.

# Profit Sharing Contracts

- The worker receives a share of the profits:

$$s(x) = \alpha f(x)$$

where  $0 < \alpha < 1$

- Maximizing the worker's payoff condition:  
 $\alpha f(x) - c(x)$  yields the condition:

$$\alpha f'(x) = c'(x)$$

- Therefore,  $x \neq x^*$
- Principals should never profit-share with their workers.

# Incentive Contracting

- In summary,
- A profit maximizing firm must take into consideration these two constraints when hiring a worker:
  - $s(f(x)) - c(x) = \tilde{u}$  ← Individual Rationality Constraint or Participation Constraint
  - $s(f(x^*)) - c(x^*) \geq s(f(x)) - c(x), \forall x \geq 0$  ← Incentive Compatibility Constraint
- The first ensures that the worker wants to work for you.
- The second ensures that the worker will exert the right amount of effort.

# Incentive Contracts with Asymmetric Information

- We've seen that there are various ways for principals to write contracts such that workers exert the level of effort which maximizes the principal's profits under complete information where  $y = f(x)$ .
- What if the principal cannot perfectly observe the worker's effort level,  $x$ ?

# Incentive Contracts with Asymmetric Information

- Output and effort are no longer perfectly correlated. i.e we don't know  $f(x)$ .
- There is “noise” or uncertain events that affect the production process that is beyond the control of the worker
- The principal has to guess the level of effort from the observed output.

# Incentive Contracts with Asymmetric Information

- Rent:
  - Since the firm receives a fixed fee, the worker is left to bear all the risk from the noise that comes from production.
  - If the worker is more risk averse than the owner, then the outcome will be inefficient.
- Profit-sharing
  - Since the worker doesn't want to bear all the risk from production noise, he will likely be willing to share some of the profits from production since this offloads some of the risk to the principal.

# Restoring Symmetric Information

- Technology has done a lot in recent years to create symmetry of information in markets
  - Websites provide buyers with reviews about sellers giving buyers insight into sellers' reputation before a transaction takes place (Yelp, TripAdvisor, Amazon, Uber, Airbnb)
  - GPS and online maps give passengers the same information that taxi drivers know.
  - Employers can track employees more closely so they know how effort correlates with output.
- But there are many markets where technology hasn't been harnessed to create symmetry yet.
  - You could solve these problems!