200C – Micro Theory – Professor Giacomo Bonanno

# PRACTICE PROBLEMS 9

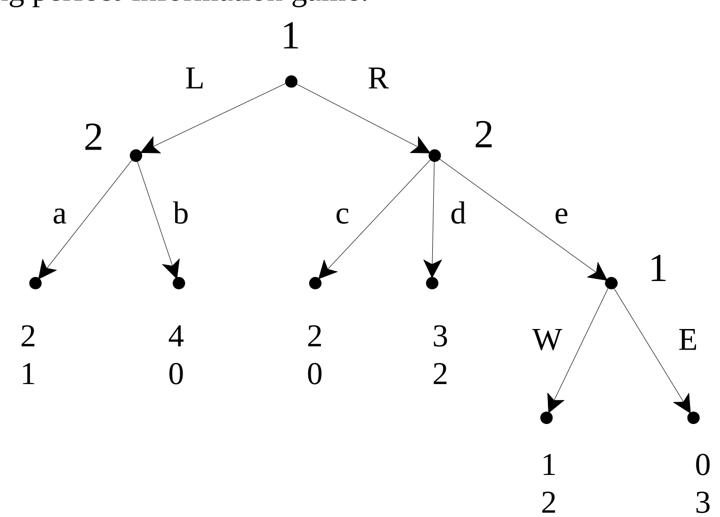
## Topic: Perfect information games and backward induction

**VERY IMPORTANT**: do **not** look at the answers until you have made a VERY serious effort to solve the problem. If you turn to the answers to get clues or help, you are wasting a chance to test how well you are prepared for the exams. I will **not** give you more practice problems later on.

**1.** Consider again the dognapping situation of Problem 1 of Week 1. Suppose that when the dognapper calls you he tells you the following: «First of all, don't call the police, or I'll kill the damn dog! Now listen carefully, I will say this only once. Tonight at 10 pm you leave the money outside the Social Science building near the window of Professor Bonanno's office; I will send a friend to pick the money. If the money is not there or the place is crawling with policemen, I will kill the stupid dog. If I find the money (and no policemen), then tomorrow morning I will set Barky free». Click (he put the phone down).

- **(a)** Represent the situation as an extensive game (with perfect information). You have to decide whether or not to tell the police. If you tell the police then they take over and decide whether or not to pay the ransom. If you don't call the police it is your decision. Write three different games, depending on the type of dognapper you are facing (Nice, Nasty, Professional).
- **(b)** Predict the outcome by solving each game by backward induction.

**2.** Consider the following perfect-information game:




**(a)** Find the backward-induction solutions. (Hint: there is more than one)
**(b)** Write down all the strategies of player 1.
**(c)** Write down all the strategies of player 2.
**(d)** Write the normal form (= matrix) associated with this game.
**(e)** Does player 1 have a dominant strategy?
**(f)** Does player 2 have a dominant strategy?
**(g)** Is there a dominant-strategy equilibrium?
**(h)** Does player 1 have any dominated strategies?
**(i)** Does player 2 have any dominated strategies?
**(j)** What do you get when you apply the iterative elimination of weakly dominated strategies?
**(k)** What are the Nash equilibria?

**3.** Consider an industry where at the moment there is a single firm, from now referred to as the **incumbent** (she). Inverse demand in the industry is given by $p = 13 - x$, where p denotes price and x the total output produced in the industry. Thus if total output is 4, then each unit can be sold at a price of $13 - 4 = 9$. The problem for the incumbent monopolist is that there is an entrepreneur (he) − from now on referred to as the **potential entrant** − who is contemplating entering the industry. The incumbent has the following cost function: $C(q) = q + 6.25$, that is, there is a fixed cost of 6.25 and a constant marginal cost of 1. If the potential entrant enters, then his cost function will be the same as the incumbent's. The incumbent has a first-mover advantage. It can now choose (and commit to) a level of output which can be either 6 or 6.5 or 7 units. The potential entrant then, after having observed the incumbent's output, decides whether or not to enter. If he enters he has to decide what level of output to produce; his only possible choices are 2.5, 2.75 and 3 units. If he stays out his profits are, of course, zero. Assume that the potential entrant decides to enter only if it expects to make positive profits.

**(a)** Represent this situation as an extensive game with perfect information.

**(b)** Solve the game using backward induction.

**(c)** How many strategies does the incumbent have in this game?

**(d)** How many strategies does the potential entrant have in this game? Write down two such strategies.

**(e)** Find a Nash equilibrium of this game which yields a different outcome to the one obtained in part (b). Comment on whether this equilibrium is reasonable or not.

**(f)** Compare the incumbent's output in the solution found under (b) with the output he would produce if there were no threat of entry (e.g. if she were protected from entry by a patent).