EC3322
Industrial Organization I
Semester 1, 2011-2012
Midterm Solutions
September 29, 2011

1. Elasticity of demand is

$$\epsilon = \frac{dQ}{dP}\frac{P}{Q} = -\frac{P}{Q} = -\frac{100 - Q}{Q}.$$

   Set the equation equal to -1 and solve to find $Q = 50$.

2. Optimal pricing is determined by:

$$\frac{P - c}{P} = -\frac{1}{\epsilon}.$$

   Substitute $c = 1$ and the appropriate elasticity of each group to find $p^* = \frac{4}{3}$ for senior citizens and $p^* = 2$ for all other customers.

3. (a) A monopoly chooses Q^* such that $MC = MR$. In the case of first-degree price discrimination, the marginal revenue curve is equal to demand curve. Therefore:

$$20 = 100 - 10Q \Rightarrow Q^* = 8.$$

   (b) Profit per customer is the area under the demand curve and above the marginal cost curve:

$$\pi^* = \frac{1}{2} * 80 * 8 = 320.$$

   (c) Yes, the outcome is efficient since the monopolist supplies the same quantity that the social planner would.

4. (a) In the case of unbundled pricing, the various willingnesses-to-pay for each product are candidates for the profit maximizing prices. The optimal prices are

$P_1^* = 500$ with profit $\pi_1^* = 900$ and $P_2^* = 900$ with profit $\pi_2^* = 600$. Total profit is $\pi^* = 1500$.

(b) The relevant willingnesses-to-pay under pure bundling are the sums of each consumer's willingness-to-pay for each product. The optimal price is $P_B^* = 1000$ with profit $\pi^* = 2000$.

(c) Under mixed bundling, the firm sets individual product prices as well as a bundled price. For each product there is one consumer that values the product for less than MC. The firm will set individual prices so that these consumers won't buy the low-valued product: $P_1^* = 900$, $\pi_1^* = 700$, $P_2^* = 900$, and $\pi_2^* = 600$. The last two consumers have negatively coordinated willingnesses-to-pay. The firm will set a bundled price targeting them: $P_B^* = 1040$ and $\pi_B^* = 1080$. Total profit is $\pi^* = 2380$.

5. (a) Under perfect competition, price is equal to MC: $p^* = 0$, $Q^* = 360$, and $\pi^* = 0$.

(b) The Nash Equilibrium price for Bertrand is $p = MC$: $p^* = 0$, $q^* = 180$, $Q^* = 360$, and $\pi^* = 0$.

(c) Under Cournot, firm 1 maximizes profit:

$$\pi_1 = \left(90 - \frac{1}{4}q_1 - \frac{1}{4}q_2\right) q_1.$$

The best response is: $q_1^R = 180 - \frac{1}{2}q_2$. Imposing symmetry solve to find $q^* = 120$ and $Q^* = 240$. Price is $P^* = 30$ and profit is $\pi^* = 3600$.

(d) The monopoly output can be found by either maximizing the monopolist's profit function or substituting $q_2 = 0$ in firm 1's reaction function in part (c): $q^* = Q^* = 180$, $P^* = 45$, and $\pi^* = 8100$.

(e) There is no DWL under perfect competition or the Bertrand outcome since Q^* is such that total welfare is maximized. Under Cournot, $DWL = 1800$ and under monopoly $DWL = 4050$.