Game Theory and Static Oligopoly

References: Pepall, Richards, and Norman, chapter 9 and 10
Game Theory chapter, Nicholson and Snyder

Introduction

- Oligopoly (small number of firms) differs from the other market structures we've examined so far because oligopolists are concerned with their rivals' actions
- A **competitive** firm potentially faces **many** rivals and, as such, the firm and its rivals are **price takers** ➔ no need to worry about rivals' actions
- A **monopolist** does not have to worry about how rivals will react to its actions simply because there are **no rivals**

Introduction

- An **oligopolist** operates in a market with **few competitors** and needs to anticipate and respond to rivals' actions (e.g., prices, output, advertising) since they affect its own profit
  - ➢ Decisions are strategic
  - When Coke introduced Coke Zero, Pepsi responded with Pepsi Max
  - When Dreamworks learned that Pixar was developing A Bug's Life, they responded with Antz
- To study oligopoly we'll rely extensively on **game theory**, a mathematical approach that formally models strategic behavior.
- We first discuss some game theory concepts

Game Theory

- A **game** is a formal representation of a situation in which individuals or firms interact *strategically*

- A game consists of:
  - Players (e.g., 2 firms)
  - Set of strategies for all players
    - A **strategy** is a *full specification* of a player's behavior at each of his/her decision points
      - E.g., to enter or not to enter
  - Payoffs for each player for all combinations of strategies (e.g., profits)

Game Theory

- The **equilibrium concept** is a **Nash Equilibrium** or a **Subgame Perfect Nash Equilibrium** (to be discussed later)

- A **Nash Equilibrium** is a ***set of strategies*** for which no player wants to change his/her strategy given the strategies played by everyone else

    - each player is playing his/her best response given the actions of the other players

- Another way to think about a Nash Equilibrium:

    - after the game is over, no player regrets his/her strategy after the strategies that the other players played has been revealed

Game Theory

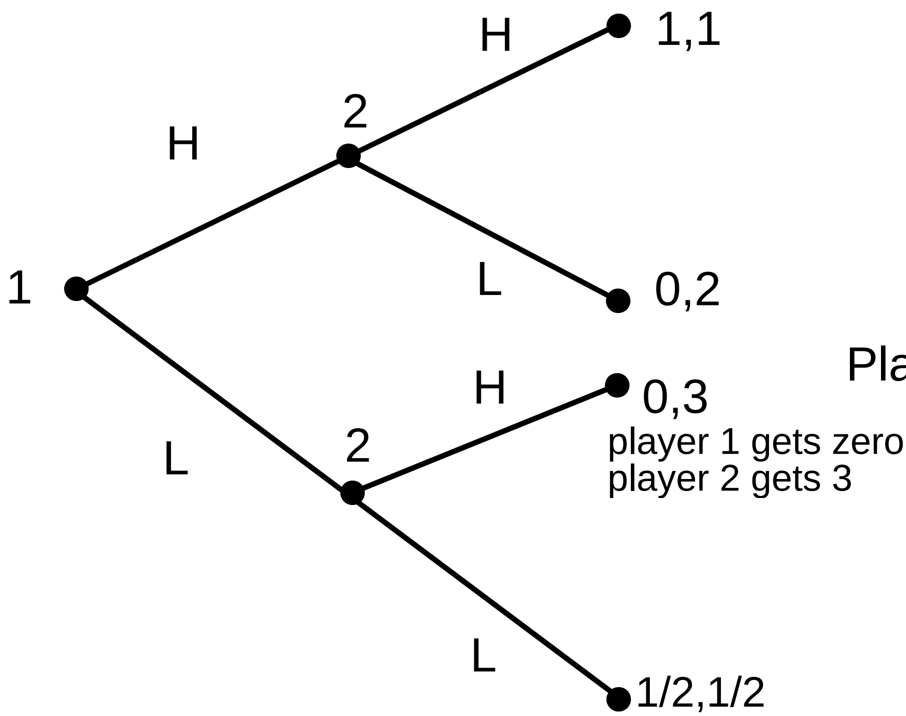
There are two primary ways to represent a game:

- The **extensive form** representation (game tree) shows:
  - who moves when
  - what actions each player can take order of action
  - what players know when they move
  - what the outcomes are
  - what the payoffs are for each outcome
- The **normal form** representation (matrix) is
  - a condensed version of the game that shows the strategies and payoffs but not the timing of the game

Game Theory

- The normal form is usually best suited to describing one-stage games and finding Nash Equilibria
- The extensive form is usually best suited to describing multi-stage games and finding Subgame Perfect Nash Equilibria

Sequential move game (multi-stage)



extensive form

upper node

lower node

Player 2

Player 1

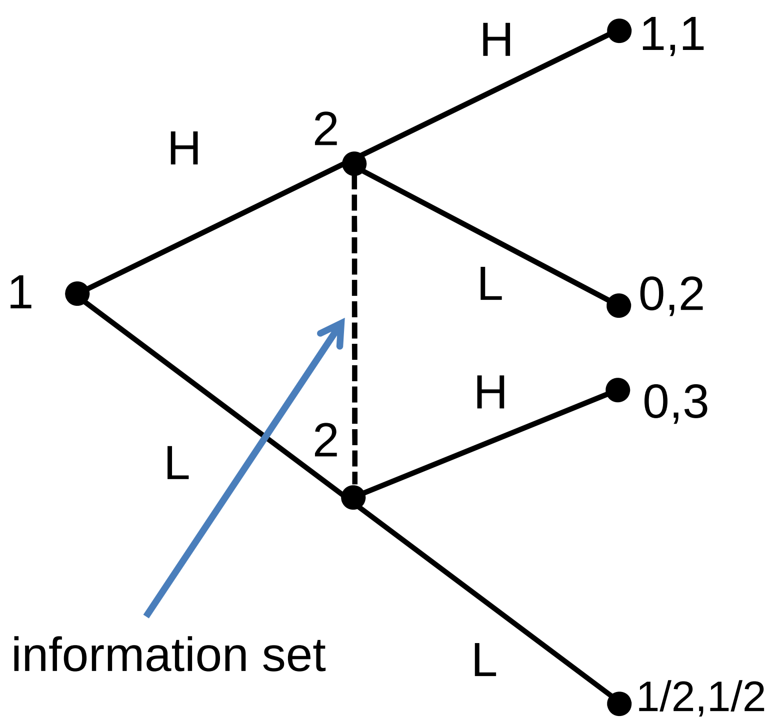
	HH	HL	LH	LL
H	1,1	1,1	0,2	0,2
L	0,3	1/2,1/2	0,3	1/2,1/2

matrix represents all the strategy of player 1 and player 2

normal form

Simultaneous move game (one stage)

- Information set: collection of decision nodes for which the player doesn't know which node she's at



extensive form

		Player 2	
		H	L
Player 1	H	1,1	0,2
	L	0,3	1/2,1/2

normal form

Finding NE, example 1

- Two airlines, SIA and Qantas, offer a daily flight from Singapore to Sydney
- Assume that they have set the same price for the flight but that the departure time is undecided ➔ the **departure time** is the **strategy choice** in this game
- 70% of consumers prefer an evening departure while 30% prefer a morning departure
- If both airlines choose the same departure time, they split the market
- Payoffs (profits) to the airlines are directly proportional to the market share ➔ airlines want to maximize market share
- The airlines choose the departure time simultaneously

Finding NE, example 1

- Payoff matrix:

		Qantas	
		Morning	Evening
SIA	Morning	15,15	30,70
	Evening	70,30	35,35

- In this example, we can find the NE by eliminating dominated strategies
  - A dominated strategy is a strategy for which there is some other strategy that yields a greater payoff regardless of what the other players do
- If we are able to eliminate all but one strategy for each player, these remaining strategies are the NE

Finding NE, example 1

- Elimination of dominated strategies

		Qantas	
		~~Morning~~	Evening
SIA	~~Morning~~	~~15,15~~	~~30,70~~
	Evening	~~70,30~~	35,35

- For Qantas, Morning is dominated by Evening ➔ eliminate Morning
- For SIA, Morning is dominated by Evening ➔ eliminate Morning
- The NE is Evening for SIA and Evening for Qantas, the only remaining strategies for each airline (Evening, Evening)

Finding NE, example 2

- Suppose now that SIA has a frequent flyer program
- When both airlines choose the same departure time, SIA will obtain 60% of the market share

		Qantas	
		Morning	Evening
SIA	Morning	18,12	30,70
	Evening	70,30	42,28

Finding NE, example 2

		Qantas	
		Morning	~~Evening~~
SIA	~~Morning~~	~~18,12~~	~~30,70~~
	Evening	70,30	~~42,28~~

- For SIA Morning is still dominated by Evening
- For Qantas Evening is *not* better than Morning for *all* of SIA's strategies
- But Evening *is* dominated once we eliminate SIA's dominated strategy
- A valid approach to finding a NE is to eliminate dominated strategies by iteration ➔ that is, we can delete strategies that become dominated *after* the deletion of other strategies
- The NE is (Evening, Morning)

Finding NE, example 3

- What if there are **no dominated strategies**?
- Consider a modified version of the airlines game
  - Firms choose prices instead of departure times
  - For simplicity we'll consider only two possible prices
- Setting
  - There are 60 consumers with a reservation price of \$500 for the flight and another 120 consumers with a reservation price of \$220
  - Price discrimination is not possible
    - Airlines choose between a price of \$500 and \$220
  - Airline costs are \$200 per passenger
  - The low price airline gets the whole market. If identical prices are charged, the airlines split the market.

Finding NE, example 3

		Qantas	
		P_H = \$500	P_L = \$220
SIA	P_H = \$500	9000, 9000	0, 3600
	P_L = \$220	3600, 0	1800, 1800

- If both airlines charge *\$500*, then π = *(500-200)*30 = \$9,000*
- If both airlines charge *\$220*, then π = *(220-200)* 90 = \$1,800*
- If one airlines charges *\$500* while the other airline charges *\$220*, then the high price airlines earns *\$0* and the low price airline earns π = *(220-200)* 180 = \$3,600*

Finding NE, example 3

		Qantas	
		P_H = \$500	P_L = \$220
SIA	P_H = \$500	9000, 9000	0, 3600
	P_L = \$220	3600, 0	1800, 1800

- Notice that there are no dominated strategies to eliminate. Instead we find the best response of each firm.
- SIA reasons:
  - If Qantas chooses the high price, then the best response is to also choose the high price
  - If Qantas chooses the low price, then the best response is the low price
- Qantas has the same reasoning as SIA
- The strategies (low price, low price) is a NE because both strategies are a best response for the strategy of the rival ➔ i.e., there is no incentive for either player to change its strategy
- (high price, high price) is also a NE

Ex. Prisoner's Dilemma

- Common game that shows that a NE is not necessarily Pareto efficient

Criminal 2

Criminal 1		Confess	Don't Confess
	Confess	6,6	1,10
	Don't Confess	10,1	3,3

- The only NE is (C,C) even though (DC,DC) provides shorter jail sentences for both criminals. The NE is inefficient.

Oligopoly models

- In monopoly, we saw that choosing price is the same as choosing quantity
- But in oligopoly the strategic variable matters a great deal
- The nature of competition and the outcome depends on whether firms compete in terms of quantities or in terms of price:
  - Cournot: quantity
  - Bertrand: price
- The **timing** of the decisions is also important
  - A sequential move Cournot game is called Stackelberg
  - Similarly, it's possible that price setters move one after the other
- We'll start with simultaneous move Cournot

The Cournot Model

- Consider the case of **duopoly** (2 competing firms)
- Firms produce a homogenous product with marginal cost **c**
- Inverse market demand is

$$p = A - BQ$$

where Q is total output

- The firms decide~~d~~ on their output simultaneously
  - each firm chooses output based on the expectation of the other firm's output
  - price adjusts to clear market

The Cournot Model

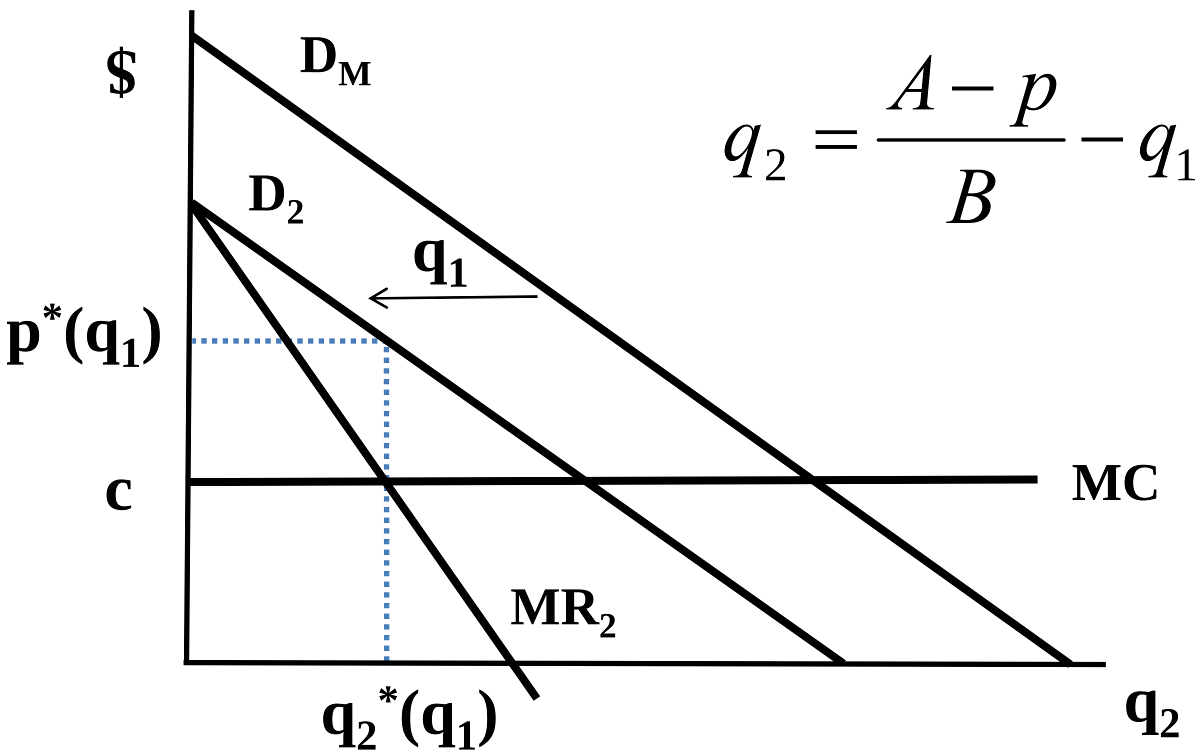
- Suppose firm 2 expected firm 1 to produce q_1 units
- The relationship between the market price and firm 2's output for a given amount of firm 1 output is captured by the residual demand curve of firm 2:

$$q_1 + q_2 = \frac{A - p}{B} \Rightarrow q_2 = \frac{A - p}{B} - q_1$$

The Cournot Model

- Graphically, firm 2's residual demand curve is the market demand curve shifted left by q_1 units
- Firm 2 acts as a monopolist relative to the residual demand

  ➔ $\mathbf{q_2^*(q_1)}$ is firm 2's best response



$$q_2 = \frac{A - p}{B} - q_1$$

The Cournot Model

- By varying the expected output of firm 1 we could find $\mathbf{q_2^*(q_1)}$ for all $\mathbf{q_1}$
  - This is called the **reaction function** (or **best response function**)
- Mathematically, we derive the best response function of firm 2 by setting the marginal revenue of firm 2 equal to marginal cost
- The *inverse* residual demand curve is $p = A - Bq_1 - Bq_2$
  - $MR_2 = A - Bq_1 - 2Bq_2$
- Set $MR_2 = c$ and solve for q_2^*:

negative number- produce zero

$$q_2^* = \frac{A-c}{2B} - \frac{q_1}{2}$$

The Cournot Model

- Similarly we can find the reaction function of firm 1: $\mathbf{q_1^*(q_2)}$

$$q_1^* = \frac{A - c}{2B} - \frac{q_2}{2}$$

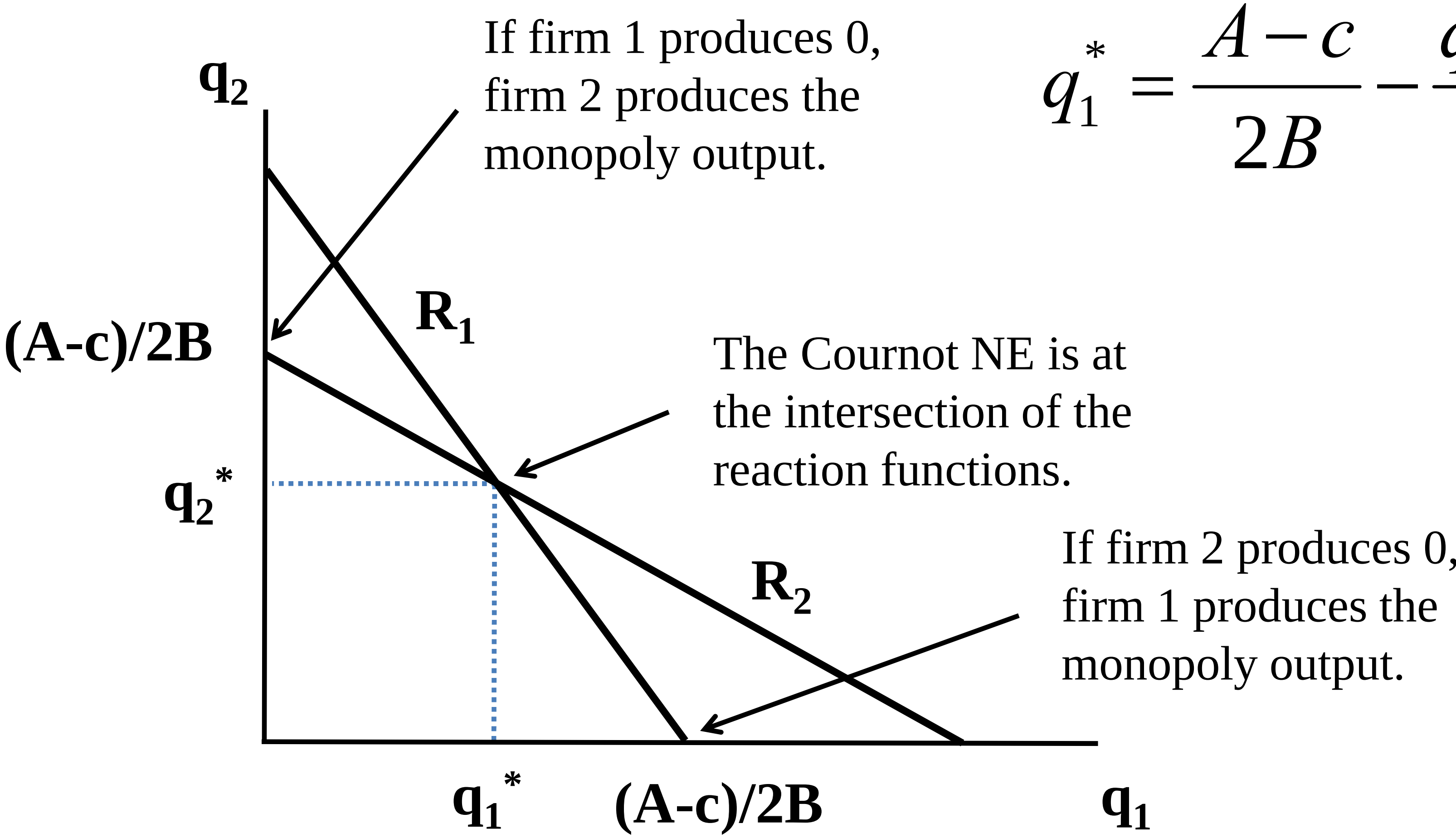
- A **Cournot-Nash Equilibrium** requires that each firm's output satisfies the reaction functions
  - each firm's output must be a best response to its rival's output
  - neither firm has any after-the-fact reason to regret its output choice

The Cournot Model

$$q_2^* = \frac{A-c}{2B} - \frac{q_1}{2}$$

$$q_1^* = \frac{A-c}{2B} - \frac{q_2}{2}$$

- Reaction functions $\mathbf{R_1}$ and $\mathbf{R_2}$



The Cournot Model

- It's easiest to find the Cournot NE mathematically
- Find q_1^* and q_2^* that satisfy both reaction functions:

$$q_2^* = \frac{A-c}{2B} - \frac{q_1^*}{2} \quad \text{and} \quad q_1^* = \frac{A-c}{2B} - \frac{q_2^*}{2}$$

➔ **q_1^* = (A-c)/3B** and **q_2^* = (A-c)/3B**

The Cournot Model

- In equilibrium each firm produces $q_1^* = q_2^* = \dfrac{A-c}{3B}$

- Total output is $Q^* = \dfrac{2(A-c)}{3B}$

- Price is $p^* = \dfrac{A+2c}{3}$

- Profits of each firm is $\pi_1^* = \pi_1^* = \dfrac{(A-c)^2}{9B}$

More generally…

- Let π_1 be firm 1's profit and (s_1, s_2) be firm 1 and 2's (continuous) strategic choice
- We find the best response $s_1^*(s_2)$ by solving the first-order condition:

$$\text{Solve } \frac{\partial \pi_1(s_1, s_2)}{\partial s_1} = 0 \text{ for } s_1^*(s_2)$$

- Similarly we find the best response $s_2^*(s_1)$ of firm 2:

$$\text{Solve } \frac{\partial \pi_2(s_2, s_1)}{\partial s_2} = 0 \text{ for } s_2^*(s_1)$$

- The NE, (s_1^*, s_2^*), is found by solving the two equations simultaneously

The Cournot Model

- Comparison with monopoly and perfect competition:

$$Q^{PC} = \frac{A-c}{B} > Q^{*} = \frac{2(A-c)}{3B} > Q^{M} = \frac{(A-c)}{2B}$$

$$p^{PC} = c < p^{*} = \frac{A+2c}{3} < p^{M} = \frac{A+c}{2}$$

- The Cournot duopoly output is higher than under monopoly but lower than the competitive output
- The price is lower than under monopoly but higher than in perfect competition

➢ With more firms, the outcome is moving from monopoly to perfect competition

The Cournot Model (different costs)

- Marginal cost of firm 1 is $\mathbf{c_1}$ and of firm 2 is $\mathbf{c_2}$
- Demand is still:

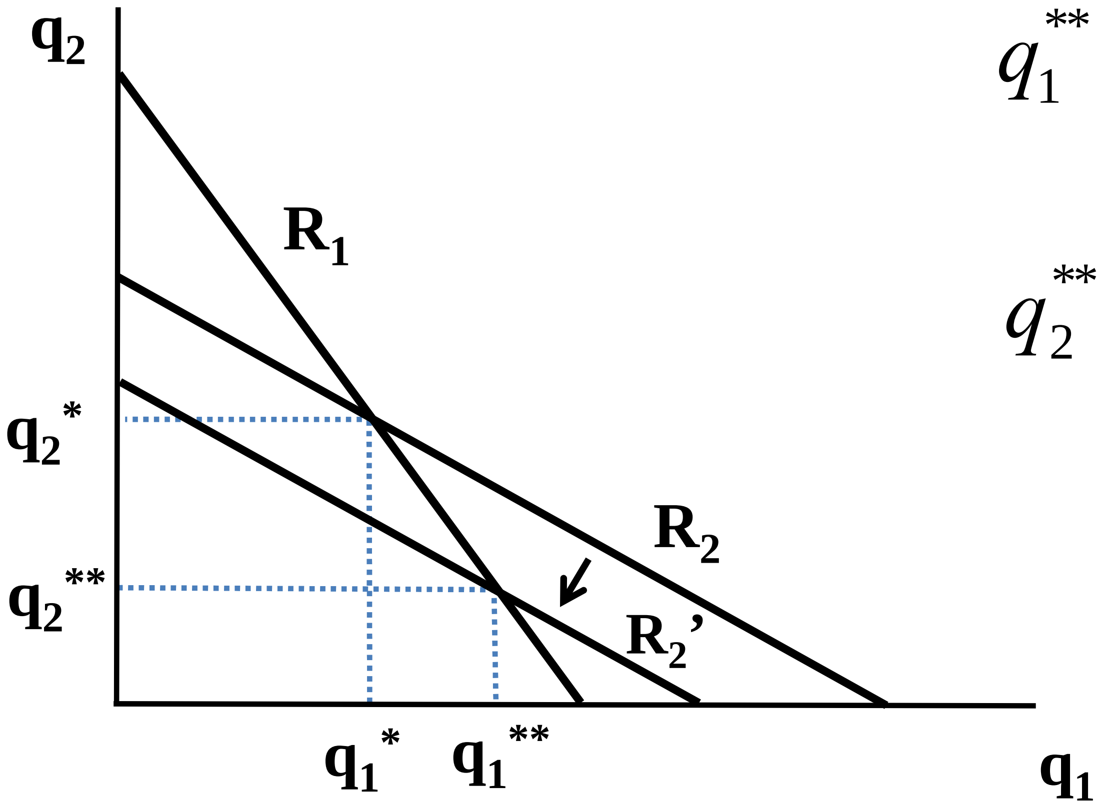
$$p = A - BQ$$

- Like previously, each firm maximizes profit by choosing output so that *MR* is equal to *MC*
- The reactions functions are:

$$q_1^{**} = \frac{A - c_1}{2B} - \frac{q_2}{2} \quad \text{and} \quad q_2^{**} = \frac{A - c_2}{2B} - \frac{q_1}{2}$$

The Cournot Model (different costs)

- *MC* affects the location of the reaction functions
- An increase in *MC* of firm 2 will shift its reaction function $\mathbf{R_2}$ inward ➔ firm 2's response to any level of q_1 is **lower when *MC* increases**



$$q_1^{**} = \frac{A - c_1}{2B} - \frac{q_2}{2}$$

$$q_2^{**} = \frac{A - c_2}{2B} - \frac{q_1}{2}$$

The Cournot Model (different costs)

- In equilibrium each firm produces

$$q_1^{**} = \frac{A - 2c_1 + c_2}{3B} \quad \text{and} \quad q_2^{**} = \frac{A - 2c_2 + c_1}{3B}$$

- Total output is $Q^* = \dfrac{2A - c_1 - c_2}{3B}$

- Price is $p^* = \dfrac{A + c_1 + c_2}{3}$

- It's easy to confirm that the firm with the higher *MC* will have lower output and lower profits (try showing it)

- Note that Q^* is produced inefficiently ➔ from a welfare standpoint, the firm with the lower marginal cost should produce all the output

The Cournot Model (N firms)

- Suppose there are N identical firms producing identical products
- Total output is $Q = q_1 + q_2 + \ldots + q_N$
- Demand: $p = A - BQ$
- The sum of output for all firms except firm 1: $Q_{-1} = q_2 + \ldots + q_N$
- So the inverse residual demand function for firm 1 is:

$$p = A - BQ_{-1} - Bq_1$$

The Cournot Model (*N* firms)

- The profit-maximizing output of firm 1 depends on the output of the other firms:

$$MR_1 = MC \quad \rightarrow \quad A - BQ_{-1} - 2Bq_1^* = c$$

$$\rightarrow \quad q_1^* = \frac{A-c}{2B} - \frac{Q_{-1}}{2}$$

- Since all firms are identical, each firm's reaction function will be the same
- Let Q_{-i} be the **total output excluding firm *i***. The reaction function is:

$$MR_i = MC \quad \rightarrow \quad A - BQ_{-i} - 2Bq_i^* = c$$

$$\rightarrow \quad q_i^* = \frac{A-c}{2B} - \frac{Q_{-i}}{2}$$

The Cournot Model (N firms)

rmb to maximise before using the symmetry

- Denote Q_{-i}^* as the total output of all firms except i in the NE

$$q_i^* = \frac{A-c}{2B} - \frac{Q_{-i}^*}{2}$$

- Because the firms are identical in a NE each firm chooses the same output
  - $q^* \equiv q_1^* = q_2^* = \ldots = q_N^*$
- Then we can find the NE from:

$$q^* = \frac{A-c}{2B} - \frac{(N-1)q^*}{2}$$

- The Cournot NE output for each firm is $q^* = \dfrac{A-c}{(N+1)B}$

The Cournot Model (*N* firms)

- In equilibrium each firm produces $q^* = \dfrac{A-c}{(N+1)B}$
- Total output is $Q^* = \dfrac{N(A-c)}{(N+1)B}$
- Price is $p^* = \dfrac{A}{(N+1)} + \dfrac{N}{(N+1)}c$
- Notice that if we set $N = 1$ and $N = 2$, then prices and quantities are identical to the monopoly and duopoly case
- What happens as N gets large? The outcomes converge to the perfectly competitive outcomes:

$$p^* \to c \quad \text{and} \quad Q^* \to \frac{A-c}{B}$$

Empirical Example

- Porter and Spence study the corn wet-milling industry
- Firms in the corn wet-milling industry convert corn into corn syrup
- The industry was a stable oligopoly until the early 1970s. Then high fructose corn syrup (HFCS) became viable as a substitute for sugar to sweeten products
    - Firms had to decide how much to expand capacity in order to meet the expected increase in demand
- Porter and Spencer modeled the capacity expansion decision of the 11 major competitors with a Cournot model
    - In the NE each firm’s choice of capacity was an optimal response to its expectations about its rival’s capacity choices
- They concluded that a moderate amount of additional capacity would be added in response to the new demand

Empirical Example

- Results:

	1973	1974	1975	1976+	Total
Actual capacity expansion	0.6	1.0	1.4	6.2	9.2
Predicted capacity expansion	0.6	1.5	3.5	3.5	9.1

- Though not perfect, their calculated equilibrium was close to the actual capacity expansion in the industry, particularly in 1973 and 1974
- Their research suggests that the Cournot model adapted to specific industry conditions can accurately describe the dynamics of capacity expansion in a homogenous-product oligopoly

Concentration and Profitability

- Consider now the case of N firms with different marginal costs
- We can use the N-firm analysis with some modification
- Recall that demand for firm ***i*** is $p = A - BQ_{-i} - Bq_i$
- Equating MR to MC: $MR_i = MC \;\; \rightarrow \;\; A - BQ_{-i} - 2Bq_i^* = c_i$
- In a NE each FOC must be satisfied so we can replace Q_{-i} with Q_{-i}^*:

$$A - BQ_{-i}^* - 2Bq_i^* = c_i$$

- Since $Q^* = Q_{-i}^* + q_i^*$ we can rewrite the FOC as

$$A - B\left(Q^* - q_i^*\right) - 2Bq_i^* = c_i$$

Concentration and Profitability

- Using $p^* = A - BQ^*$, we have $p^* - c_i = Bq_i^*$
- Divide both sides by p^* and multiply the RHS by Q^*/Q^*:

$$\frac{p^* - c_i}{p^*} = \frac{Bq_i^*}{p^*}\frac{Q^*}{Q^*} \quad \rightarrow \quad \frac{p^* - c_i}{p^*} = \frac{BQ^*}{p^*} s_i^*$$

- Finally, since the elasticity of demand is: $\varepsilon = \dfrac{dQ}{dp}\dfrac{p^*}{Q^*} = -\dfrac{p^*}{BQ^*}$

we can rewrite the FOC as: $\dfrac{p^* - c_i}{p^*} = -\dfrac{s_i^*}{\varepsilon}$

Concentration and Profitability

- The Lerner index, or market power, of each firm is determined by its cost and the elasticity of demand

$$\frac{p^* - c_i}{p^*} = -\frac{s_i^*}{\varepsilon}$$

- What about market power at the industry level?
- Multiply each firm's Lerner index by its market share and then sum them to find the weighted-average Lerner index for the industry
- The LHS is

$$\sum_{i=1}^{N} s_i^* \frac{p^* - c_i}{p^*} = \frac{\sum_{i=1}^{N} s_i^* p^* - \sum_{i=1}^{N} s_i^* c_i}{p^*} = \frac{p^* - \overline{c}}{p^*}$$

where the weighted average of marginal costs is denoted $\overline{c} \equiv \sum_{i=1}^{N} s_i^* c_i$

Concentration and Profitability

- The RHS is

divide back to balance

$$-\sum_{i=1}^{N}\frac{\left(s_i^*\right)^2}{\varepsilon}=-\sum_{i=1}^{N}\frac{\frac{1}{100^2}\left(100s_i^*\right)^2}{\varepsilon}-\frac{1}{\varepsilon}\frac{HHI}{10000}$$

  where HHI is the Herfindahl–Hirschman index

- The industry Lerner index is then

$$\frac{p^*-\bar{c}}{p^*}=-\frac{1}{\varepsilon}\frac{HHI}{10000}$$

- This tells us that as a market becomes more concentrated the average-price margin increases
- Excel collusion example: The price elasticity for lysine, a food additive, is -1.55. Given marginal costs and firm shares find the equilibrium price predicted by a Cournot model. (Ch. 14, problem 12)

Table 12.4 Lerner Estimates for Selected Industries

Author	Year	Industry	Lerner Index
Bresnahan	1981	Automobiles	0.100–0.340
Appelbaum	1982	Rubber	0.049
		Textile	0.072
		Electrical Machinery	0.198
		Tobacco	0.648
Porter	1983	Railroads (in collusive phase)	0.40
Lopez	1984	Food Processing	0.504
Roberts	1984	Coffee Roasting (largest/second largest firms)	0.055/0.025
Spiller-Favaro	1984	Banks (Regulated, large firms/small firms)	0.88/0.21
		Banks (Deregulated, large firms/small firms)	0.40/0.16
Suslow	1986	Aluminum	0.590
Slade	1987	Retail Gasoline	0.100
Buschena and Perloff	1991	Philippines Coconut Oil	0.89
Gasmi, Laffont, & Vuong	1992	Soft Drinks (Coke/Pepsi post 1976)	0.64/0.56
Ellison	1994	Railroads (in collusive phase)	0.472
Taylor & Zona	1997	AT&T (long-distance telephony)	0.88

Bertrand Competition

- In the section we discuss oligopoly price setting
- We'll rework the basic Cournot model into a Bertrand model and see how dramatically the results change. Besides the strategic variable changing from quantity to price, all other assumptions are the same:
    - two firms sell an identical product
    - each firm has constant marginal cost **c**
    - inverse demand: **P = A – BQ**
    - direct demand: **Q = a – bP**, where **a = A/B** and **b = 1/B**

Bertrand Competition

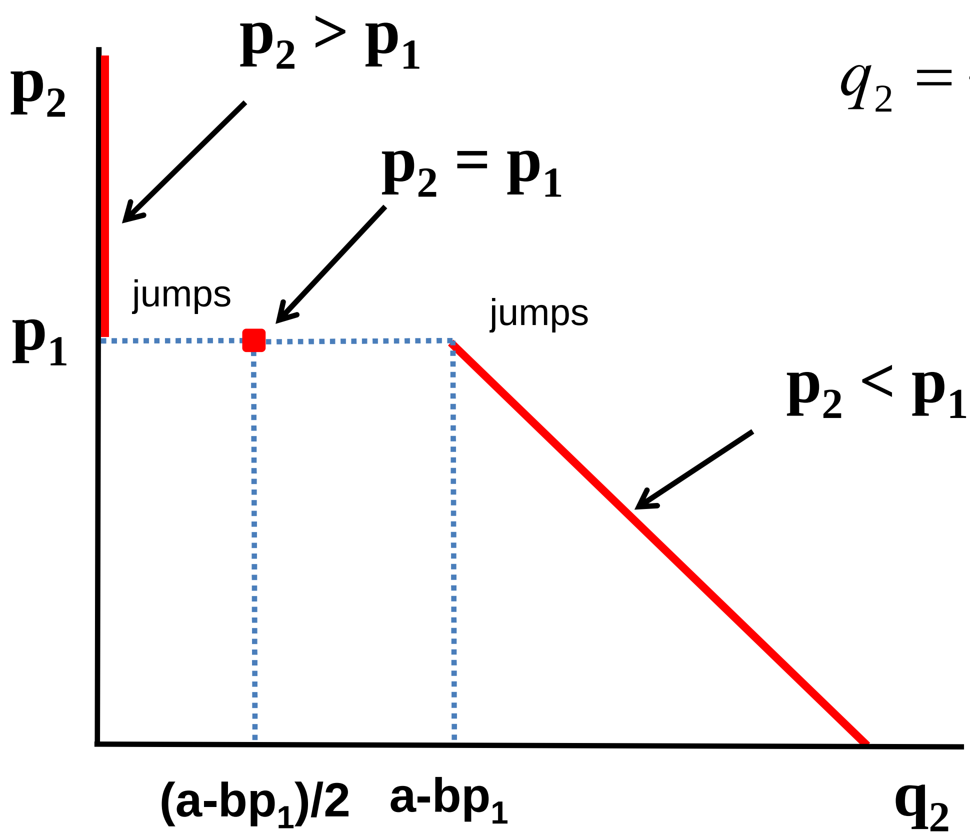
- First, we need to find each firm's residual demand (demand conditional upon the price charged by the other firm):
- Take firm 2. Assume that firm 1 has set a price of $\boldsymbol{p_1}$
  - If firm 2 sets a price greater than $\boldsymbol{p_1}$, then it sells nothing
  - If firm 2 sets a price less than $\boldsymbol{p_1}$, then it supplies the whole market
  - If firm 2 sets a price equal to $\boldsymbol{p_1}$, then consumers are indifferent between the two firms. We'll assume in this case that the firms split the market
- So demand for firm 2 is

$$q_2 = \begin{cases} 0 & \text{if } p_2 > p_1 \\ (a - bp_1)/2 & \text{if } p_2 = p_1 \\ a - bp_2 & \text{if } p_2 < p_1 \end{cases}$$

Bertrand Competition

- Demand is discontinuous (unlike in the Cournot model).

$$q_2 = \begin{cases} 0 & \text{if } p_2 > p_1 \\ (a - bp_1)/2 & \text{if } p_2 = p_1 \\ a - bp_2 & \text{if } p_2 < p_1 \end{cases}$$



Bertrand Competition

- The discontinuity in demand carries over into a discontinuity in profits:

$$\pi_2 = \begin{cases} 0 & \text{if } p_2 > p_1 \\ (p_1 - c)(a - bp_1)/2 & \text{if } p_2 = p_1 \\ (p_2 - c)(a - bp_2) & \text{if } p_2 < p_1 \end{cases}$$

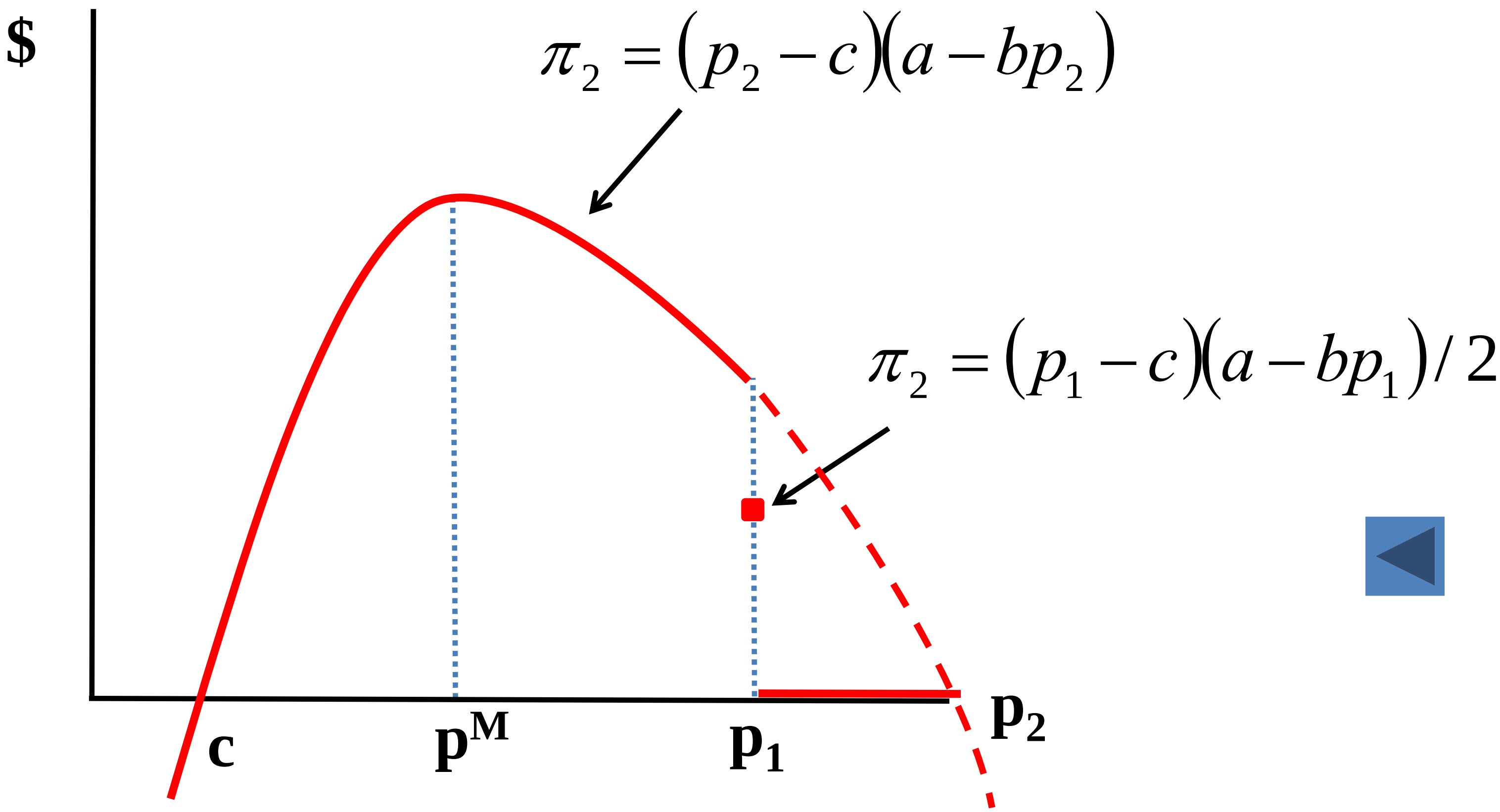
- Now we turn to finding the firm's ***best response function***
- The discontinuities will make the derivation more complex than in the case of Cournot

Bertrand Competition

- Firm 2's best response is the p_2 that maximizes firm 2's profit given p_1
  - o $p_1 > p^M$ ➔ firm 2 could capture the entire market by choosing a price lower than p_1 ➔ firm 2's best response *is* the monopoly price p^M ➔ $p_2^* = p^M$ *if* $p_1 > p^M$
  - o $c < p_1 \leq p^M$ ➔ Firm 2 can gain the whole market by undercutting firm 1 and firm 2's profit is increasing in price (up to p^M) ➔ firm 2's best response is a price just below p_1 ➔ $p_2^* = p_1 - \varepsilon$ *if* $c < p_1 \leq p^M$
  - o $p_1 < c$ ➔ If firm 2 matches or beats firm 1's price, it would make a loss ➔ firm 2 should set a high price ➔ $p_2^* > p_1$ *if* $p_1 < c$
  - o $p_1 = c$ ➔ firm 2 again doesn't have any incentive to undercut firm 1 ➔ same as previous case except firm 2 can also earn zero profits at $p_2^* = c$ ➔ $p_2 \geq p_1$ *if* $p_1 = c$

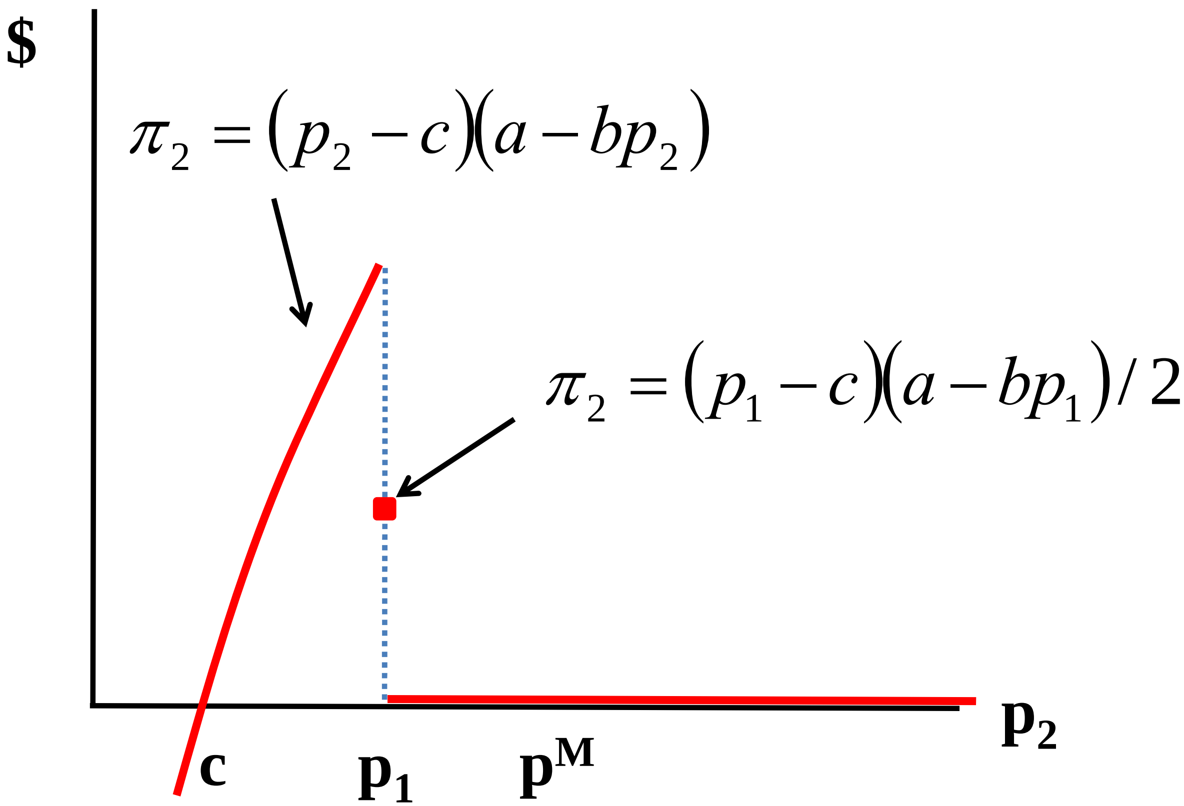
Bertrand Competition

- Suppose that firm 1 sets a price greater than the monopoly price ($p_1 > p^M$)
- Firm 2's profits look like this:



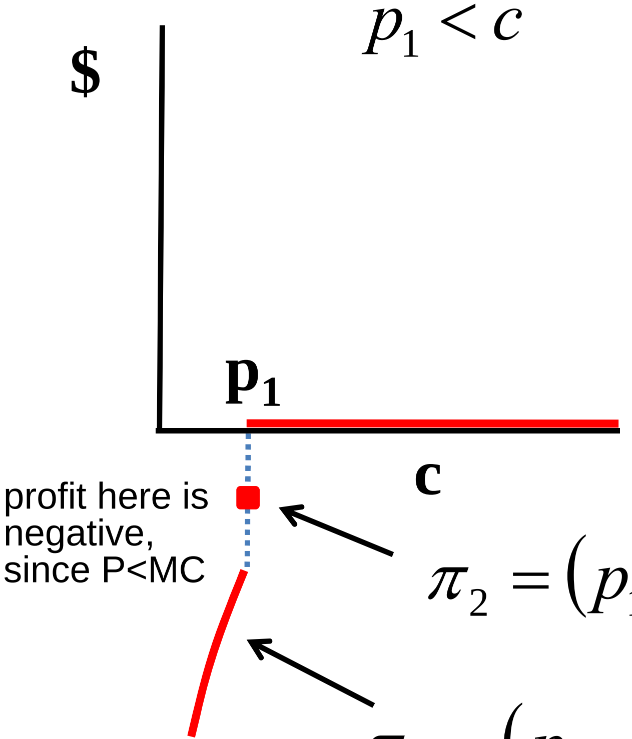
Bertrand Competition

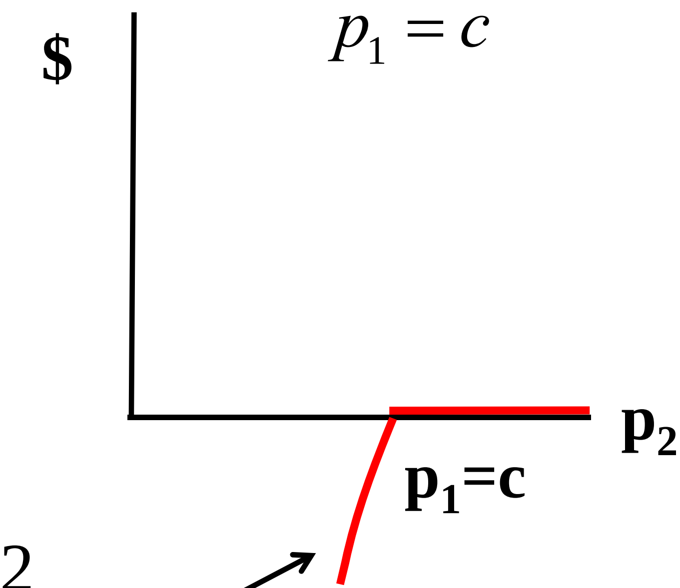
- Suppose that firm 1 sets a price less than the monopoly price (**c** < $\boldsymbol{p_1 \leq p^M}$)
- Firm 2's profits look like this:



Bertrand Competition

- Suppose that firm 1 sets a price less than or equal to **mc** (***p*₁ ≤ *c***)
- Firm 2's profits look like this:





$$\pi_2 = (p_1 - c)(a - bp_1)/2$$

$$\pi_2 = (p_2 - c)(a - bp_2)$$

Bertrand Competition

- Firm 2's best response function is:

$$
\begin{aligned}
p_2^* &= \frac{a+bc}{2b} && \text{if } p_1 > p^M = (a+bc)/2b \\
p_2^* &= p_1 - \varepsilon && \text{if c} < p_1 \leq p^M = (a+bc)/2b \\
p_2^* &\geq p_1 && \text{if } p_1 = c \\
p_2^* &> p_1 && \text{if } p_1 < c
\end{aligned}
$$

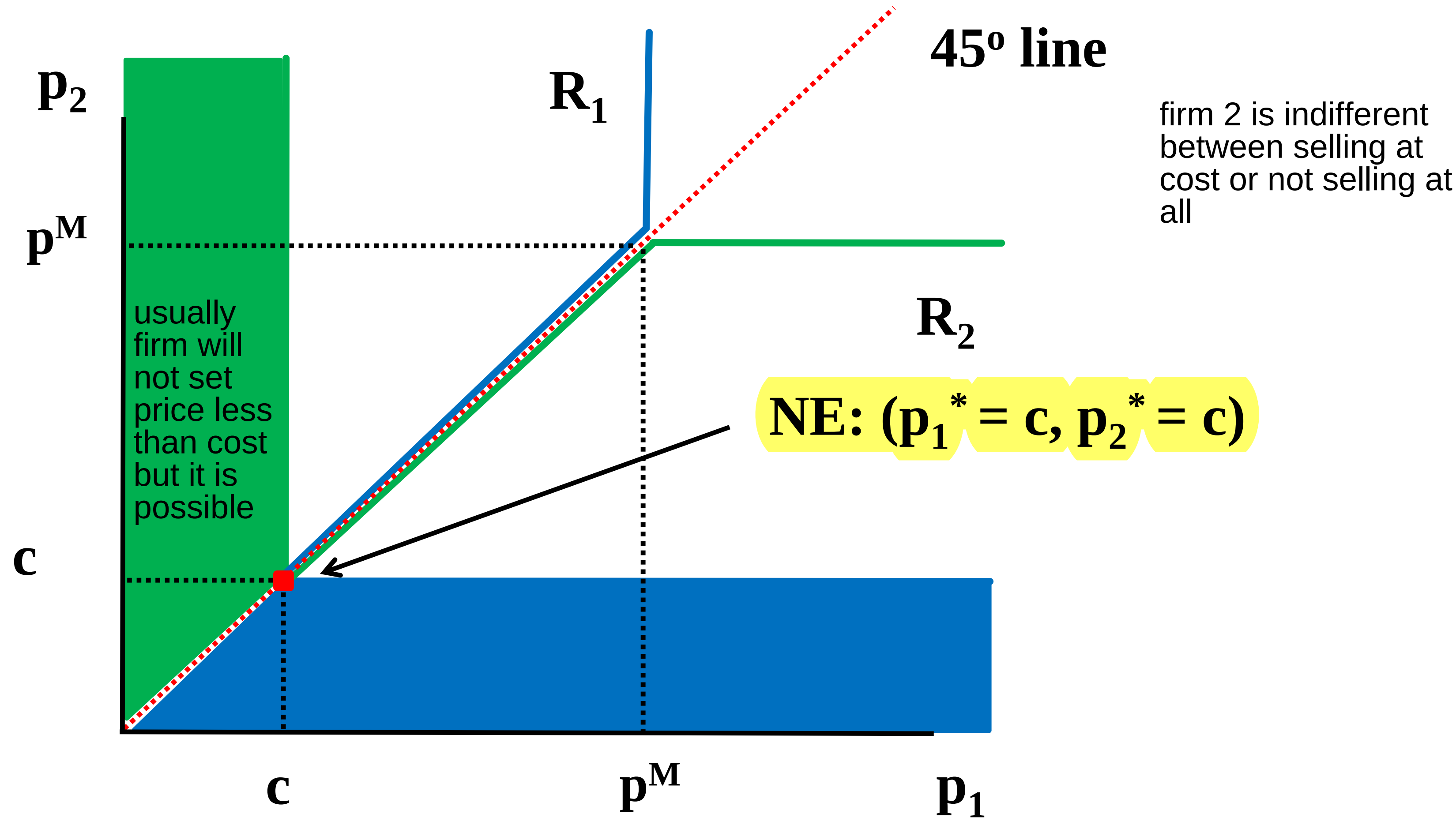
- By symmetry firm 1's best response function is:

$$
\begin{aligned}
p_1^* &= \frac{a+bc}{2b} && \text{if } p_2 > p^M = (a+bc)/2b \\
p_1^* &= p_2 - \varepsilon && \text{if c} < p_2 \leq p^M = (a+bc)/2b \\
p_1^* &\geq p_2 && \text{if } p_2 = c \\
p_1^* &> p_2 && \text{if } p_2 < c
\end{aligned}
$$

Bertrand Competition

base on price

- We can now find the NE by graphing the best response functions



Bertrand Competition

- The previous slides show how to systematically find the NE
- If we already have a hunch that $\mathbf{p_1 = p_2 = c}$ is a NE, it is easy to prove
- **Proof that $\mathbf{p_1 = p_2 = c}$ is a NE**
  - At $p_1 = p_2 = c$ both firms earn zero profits
  - If either firm raises its price, it will make no sales and earn zero profits
  - If either firm lowers its price, it will increases sales but make losses
  - ➢ Neither firm benefits by deviating ➔ $\mathbf{p_1 = p_2 = c}$ is a NE

Empirical evidence: Airline industry

- Consistent with Bertrand pricing behavior, many airlines follow a policy of reduced pricing on routes on which they face competition, especially from low-price airlines
- The carriers' rationale for this behavior is consistent with the Bertrand model – each carrier fears that if its fares are even slightly higher than the competition it will lose a large part of the market
- Consider the following fares offered on the internet for two pairs of routes of almost identical distances (so costs are similar)

Empirical evidence: Airline industry

Route	US Airways	Southwest Airlines
Syracuse, NY to Baltimore, MD	$173	No flights
Albany, NY to Baltimore, MD	$115	$120
Syracuse, NY to Raleigh-Durham, NC	$216	No flights
Albany, NY to Raleigh-Durham, NC	$167	$172

- US Airways competed with Southwest in the Albany market but not in the Syracuse market where it was a monopolist for flights to Baltimore and Raleigh-Durham
- In the Albany market US Airways matched Southwest's price
- US Airways charged 50.4% higher for Syracuse-Baltimore than Albany-Baltimore
- US Airways charged 22.7% higher for Syracuse-Raleigh-Durham than Albany-Raleigh-Durham

Empirical evidence: Airline industry

- If Southwest charged prices closed to marginal cost, then the Albany fares can be interpreted as Bertrand equilibrium prices
- Alternatively, we can take a less strict interpretation of the model
  - Competition causes a large decrease in price
- Why? US Airways reasoned that if its Albany prices were higher than Southwest's, it would fly mostly empty planes from Albany to Baltimore or Raleigh-Durham
  - In the short-run, quantity is fixed by the choice of capacity

Bertrand Competition

- The Bertrand model shows that competition in prices gives a result **very different** from the Cournot model
- Many firms seem to set prices (and not quantities) so logically we might prefer the Bertrand model to the Cournot approach in these cases (though later we will think about this a bit more carefully)
- But the result is “not nice” ➔ there are only 2 firms and yet **p = mc,** just like under perfect competition (although we just saw some evidence of Bertrand pricing in the airline industry so for some markets it may be a suitable model of behavior) ➔ **Bertrand Paradox**
- Two possible extensions that soften this outcome:
    - So far firms set prices and quantities adjust ➔ what if firms had **capacity constraints?**
    - What happens if **products are differentiated?**

      not much difference in customers

Capacity Constraints

- For the $\mathbf{p_1 = p_2 = c}$ to be an equilibrium, **both firms need enough capacity to satisfy <u>all demand</u>** at $\mathbf{p_1 = p_2 = c}$
  - o With enough capacity each firm has a big incentive to undercut each other until price is equal to marginal cost
  - o With limited capacity each firm can raise prices without losing all its sales ➔ $\mathbf{p_1 = p_2 = c}$ is no longer a NE
  - ➢ *capacity constraints* can affect the equilibrium
- Consider an example
  - o Daily demand for a product: **Q = 6000 – 60P**
  - o Suppose there are two firms. **Firm 1** has daily capacity of **1,000** and **firm 2** has daily capacity of **1,400**
  - o Marginal cost for both firms is **c = $10**
  - o Is $\mathbf{p_1 = p_2 = c}$ still an equilibrium? ➔ quantity demanded at **$10** is **5,400**, far exceeding the total capacity of the two firms

Capacity Constraints

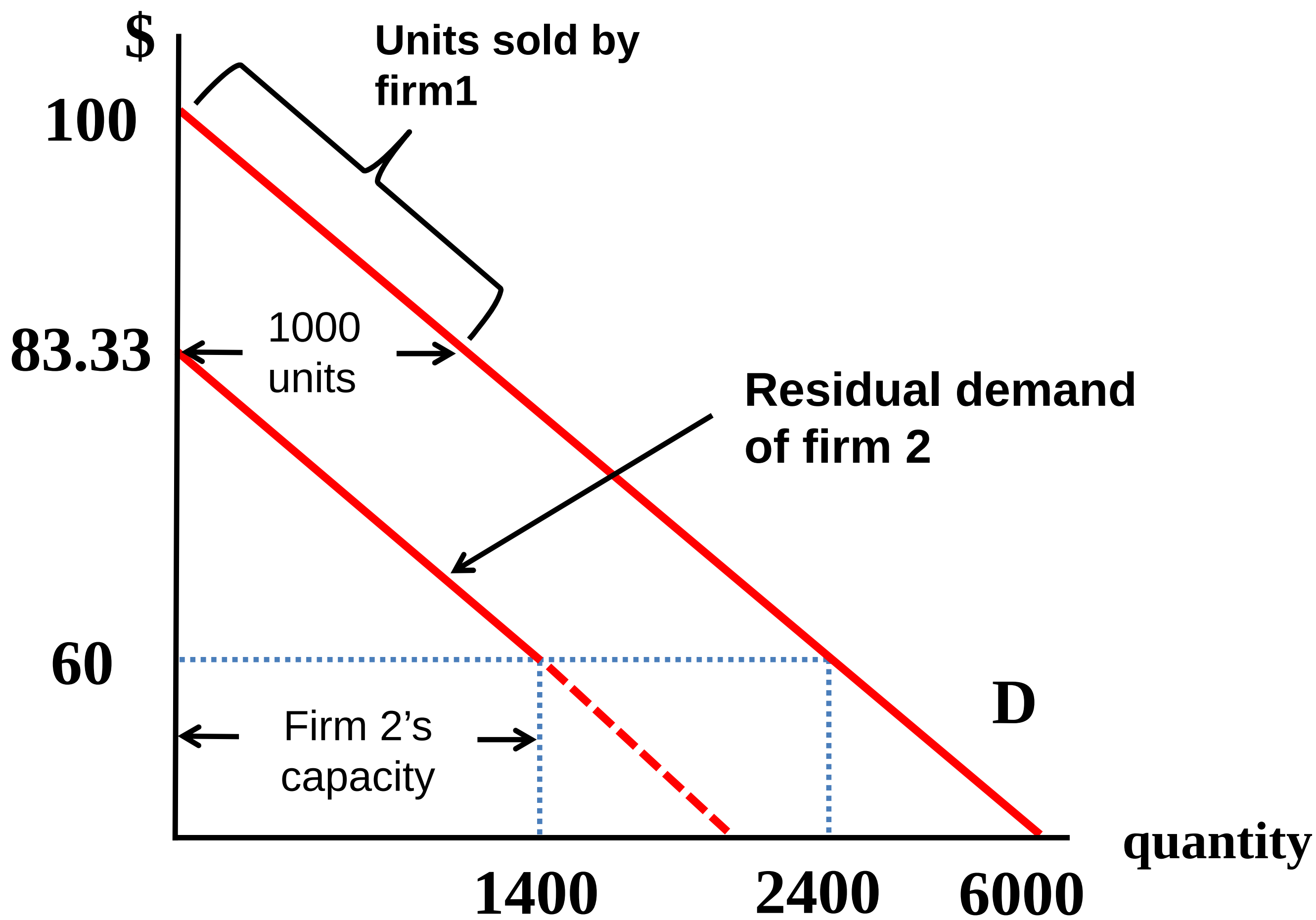
- Consider **firm 2**'s reasoning:
  - Normally raising price decreases quantity sold
  - But where can consumers go? Firm 1 is already at capacity
  - **Some buyers will still buy** from firm 2 even if $p_2 > p_1$
  - So firm 2 can price above **mc** and make profit on the buyers who remain
- We will show that in the NE both firms use all their capacity and the price is the market-clearing price
  - **2400 = 6000 – 60p ➔ both firms set $p_1 = p_2 = 60$ in the NE**

Capacity Constraints

- First, we derive the **residual demand**
- Suppose **p = $60**.
  - Firm 1 sells **1,000 units**
  - Residual demand of firm 2: **Q = 5000 – 60p** or **p = 83.33 – Q / 60**
  - Marginal revenue is then **MR = 83.33 – Q / 30**

Capacity Constraints

- Firm 2's residual demand

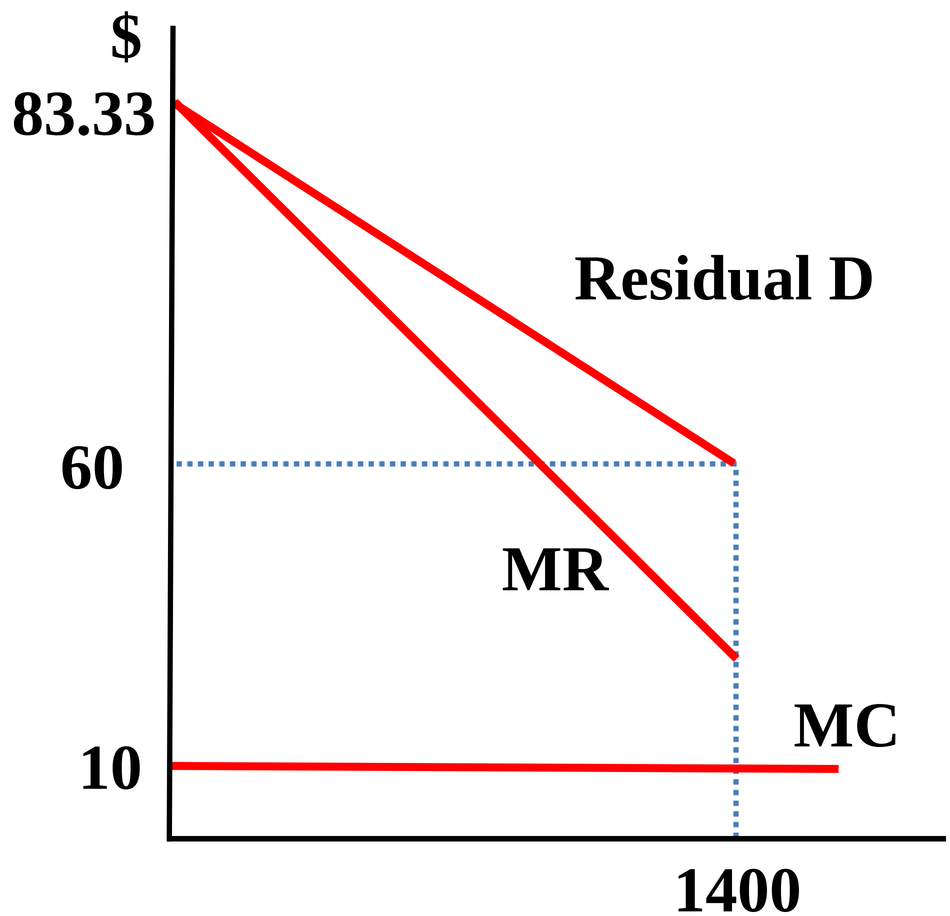


Capacity Constraints

- Does firm 2 want to deviate from **p = 60**?
- Lowering its price does not lead to any more customers since it is at capacity
- Raising price and losing customers will decrease profits because **MR > MC**

➢ **It is not profitable for firm 2 to deviate**

- Same logic applies to firm 1 so $\mathbf{p_1 = p_2 = 60}$ is a NE



When you decrease quantity when MR>MC, then profit is decreased

Capacity Constraints

- Logic is quite general
  - Firms are **unlikely to choose sufficient capacity** to serve the whole market when price equals marginal cost since they get only a fraction of the market in equilibrium
  - So the capacity of each firm is less than needed to serve the whole market
  - But then there is no incentive to cut the price to marginal cost
- So **we avoid the Bertrand Paradox when firms are capacity constrained** (or in other words, when firms don't have excessive capacity)

Product Differentiation

- The analysis so far assumes that firms sells homogenous products
- Another extension that removes the Bertrand paradox is **production differentiation**
- When firms **differentiate** their products, a firm doesn't lose all demand when it raises price above its rival's price
- We will discuss this in detail when we talk more about **product differentiation under competition**

Cournot vs. Bertrand

- Which of the two modelling assumptions is more realistic? Do firms set prices or quantities?
- The answer depends, not surprisingly, on what industry we are studying
- Most industries involve firms directly setting prices, so perhaps Bertrand (price-setting) competition is the more realistic approach
- However, if the firms' capacities are fixed, then a firm's price really may be determined by its available capacity
- In such situations it is typical to model firms as competing in quantities (Cournot) since choosing capacity determines how much is produced which then in turn determines the price firms have to set to clear the market
- Examples might include industries such as airlines, hotels, cars, computers

Cournot vs. Bertrand

- There are other situations in which output is not capacity constrained, or is easily adjusted to meet the quantity demanded at whatever price is set
- For instance, a software provider, a publisher, an insurance company, or a bank can easily handle any increase in quantity demanded when it lowers its price

➢ The standard approach is to adopt the Cournot modelling assumption if quantity is hard to adjust quickly, and the Bertrand modelling assumption if quantity is easy to adjust quickly