EC3322
Industrial Organization I
Semester 2, 2009-2010
Midterm Solutions
March 3, 2010

<u>Section 1</u>

1. A perfectly-competitive firm has the long-run cost function $C(q) = 4q^2 + 196$. In the long run, it will supply a positive amount of output so long as price is greater than or equal to: **\$56.** (This is the minimum AC.)

2. A firm sells one million units at a price of \$100 for each unit. The firm's marginal cost is constant at \$40 and its average cost (at the output level of one million units) is \$90. The firm estimates that its elasticity of demand is constant at -2. The firm should: **lower the price.** (Use the Lerner index to show that the optimal price is \$80)

3. If New York City expects than an increase in bus fares will raise mass transit revenue, it must think that the demand for bus travel is: **inelastic.** (When demand is inelastic, an increase in price increase total revenue.)

4. If the sellers in the cigarette industry formed a cartel and decided to set price along a straight-line downward-sloping demand curve, which point would they choose if they wanted to gain the highest total revenue? **The point where demand is unit elastic.** (Suppose the cartel is pricing where the demand curve is unit elastic. Call this price p. If the cartel lowers price, it would be pricing on the inelastic part of the demand curve. But on this part of the demand curve, increasing price increases revenue. Therefore, lowering price from p will not increase revenue. If the cartel raises price above p, it would be pricing on the elastic part of the demand curve. But on this part of the

demand curve, decreasing price increases revenue. Therefore, increasing the price from p will not increase revenue. Therefore, p is the revenue-maximizing price.)

Section 2

Solve the model by backwards induction. That is, start with firm 3's decision, taking q_1 and q_2 as given. Firm 3's profit is:

$$\pi_3 = (120 - q_1 - q_2 - q_3)\, q_3.$$

Find the first-order condition and solve for q_3 to find firm 3's reaction function:

$$q_3^R = \frac{120 - q_1 - q_2}{2}.$$

Now, move back to period 2. Firm 2 takes q_1 as given but anticipates firm 3's reaction to its choice of output, q_2. Therefore, substitute firm 3's reaction function into firm 2's profit function:

$$\pi_2 = \left(120 - q_1 - q_2 - q_3^R\right) q_2 = \left(120 - q_1 - q_2 - \left(\frac{120 - q_1 - q_2}{2}\right)\right) q_2 = \left(\frac{120 - q_1 - q_2}{2}\right) q_2$$

Find the first-order condition and solve for q_2 to find firm 2's reaction function:

$$q_2^R = \frac{120 - q_1}{2}.$$

Now, move back to period 1. Firm 1 anticipates firm 2 and firm 3's reaction to its choice of output, q_1. Therefore, substitute firm 2 and firm 3's reaction function into firm 1's profit function:

$$\begin{aligned} \pi_1 &= \left(120 - q_1 - q_2^R - q_3^R\right) q_1 = \left(120 - q_1 - \frac{120 - q_1}{2} - \frac{120 - q_1 - q_2^R}{2}\right) q_1 \\ &= \left(120 - q_1 - \frac{120 - q_1}{2} - \frac{120 - q_1 - \dfrac{120 - q_1}{2}}{2}\right) q_1 = \frac{120 - q_1}{4} q_1. \end{aligned}$$

Find the first-order condition and solve for q_1 to find $q_1 = 60$. The SPNE is

$$q_1 = 60, q_2^R = \frac{120 - q_1}{2}, q_3^R = \frac{120 - q_1 - q_2}{2}.$$

Firm 2's output is found by plugging firm 1's output into firm 2's reaction function. Firm 3's output is found by plugging firm 1 and firm 2's output into firm 3's reaction function. Price is found by plugging the total quantity into the inverse demand function. The equilibrium outputs and price are: $q_1^* = 60, q_2^* = 30, q_3^* = 15, p^* = 15$.

Section 3

1. $Q^* = 40, p^* = 20, \pi^* = 400$
2. Firm 1 can undercut the other firms by setting a price less than the mc of firm 2. Since firm 1's profit is increasing in price up to \$20, firm 1 sets price $p_1^* = 15 - \epsilon$. Firm 2 and firm 3 would make losses if they charged at p_1^* or lower. The Nash equilibrium is thus: $p_1^* = 15 - \epsilon, p_2^* > p_1^*, p_3^* > p_1^*$. Outputs in the equilibrium are: $q_1^* = 60, q_2^* = 0, q_3^* = 0$. Profits are: $\pi_1^* = 300, \pi_2^* = 0, \pi_3^* = 0$.
3. Since firm 1's profit-maximizing (monopoly) price is less than the mc of firm 3, firm 1 sets price $p_1^* = 20$. Firm 3 would make losses if it charged at p_1^* or lower. The Nash equilibrium is thus: $p_1^* = 20, p_3^* > p_1^*$. Outputs in the equilibrium are: $q_1^* = 40, q_3^* = 0$. Profits are: $\pi_1^* = 400, \pi_3^* = 0$.

Section 4

1. $Q^* = 40, p^* = 80, \pi^* = 800$
2. The fringe firms are price takers. They choose output such that $p = mc$:

$$p = mc \Leftrightarrow p = 6q + 60 \Leftrightarrow q = \frac{p - 60}{6}.$$

The shutdown price of a fringe firm is \$60. Thus, the supply function is:

$$q_f = \begin{cases} \dfrac{p-60}{6} & , \ p \geq 60 \\ 0 & , \ p < 60 \end{cases}$$

Find the supply function of the competitive fringe as a whole by aggregating the individual firm supply functions: $Q_f = 12q_f$:

$$Q_f = \begin{cases} 2p - 120 & , \ p \geq 60 \\ 0 & , \ p < 60 \end{cases}$$

3. Find Apple's residual demand curve by subtracting market demand by the supply of the competitive fringe, $D_A(p) = D(p) - Q_f(p)$:

$$D_A(p) = \begin{cases} 320 - 4p & , \quad 60 \le p \le 80 \\ 200 - 2p & , \quad p < 60 \\ 0 & , \quad p > 80 \end{cases}$$

Rewrite the residual demand curve as the inverse demand curve:

$$p_A(q) = \begin{cases} 80 - 0.25q & , \quad q \le 80 \\ 100 - 0.5q & , \quad 80 < q \le 200 \end{cases}$$

Find MR by doubling the magnitude of the slope of the inverse demand curve:

$$MR_A(q) = \begin{cases} 80 - 0.5q & , \quad q \le 80 \\ 100 - q & , \quad 80 < q \le 200 \end{cases}$$

Now, we're ready to solve for the equilibrium. With $mc = 60$, Apple does not find it profitable to set a price that keeps the fringe from producing (see graph). That is, Apple sets q_A by setting the portion of the MR curve below 80 units equal to mc:

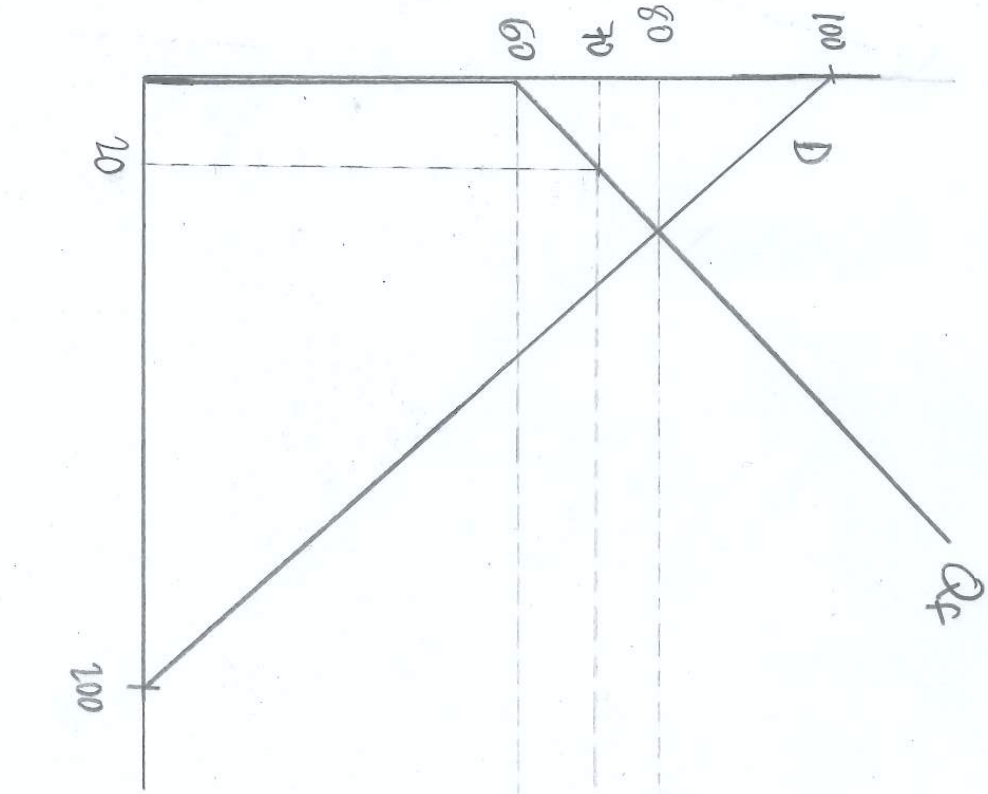
$$MR = mc \Leftrightarrow 80 - 0.5q_A = 60 \Leftrightarrow q_A^* = 40$$

Find price by plugging $q_A^* = 40$ into Apple's inverse demand function: $p^* = 80 - 0.25 * 40 = 70$. Apple's profit is: $\pi_A^* = (70 - 60)40 = 400$. The quantity supplied by the fringe is $Q_f^* = 2 * 70 - 120 = 20$.

4. When Apple's mc is low, it operates on the part of the MR curve above 80 units, keeping the fringe from producing (see graph).

$$MR = mc \Leftrightarrow 100 - q_A = 10 \Leftrightarrow q_A^* = 90$$

Find price by plugging $q_A^* = 90$ into Apple's inverse demand function: $p^* = 100 - 0.5 * 90 = 55$. Apple's profit is: $\pi_A^* = (55 - 10)90 = 4050$. The fringe does not produce since $p^* = 55$ is below the fringe's shutdown price.



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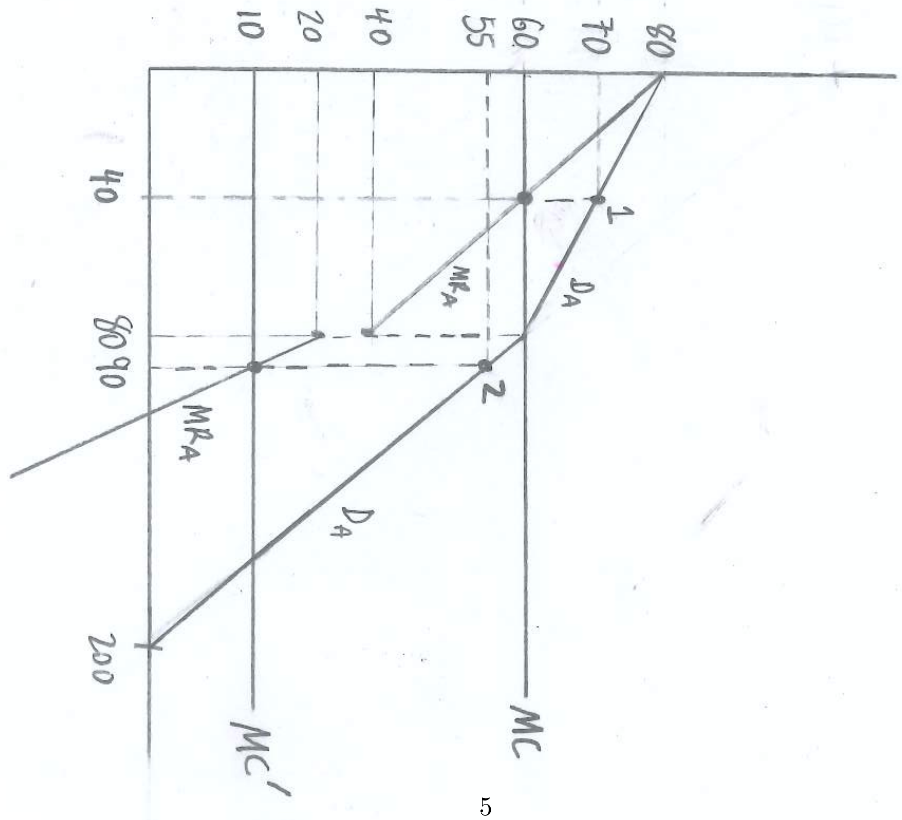

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