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Ecn 200C – Micro Theory – Professor Giacomo Bonanno

ANSWERS TO PRACTICE PROBLEMS $\boxed{18}$

1.

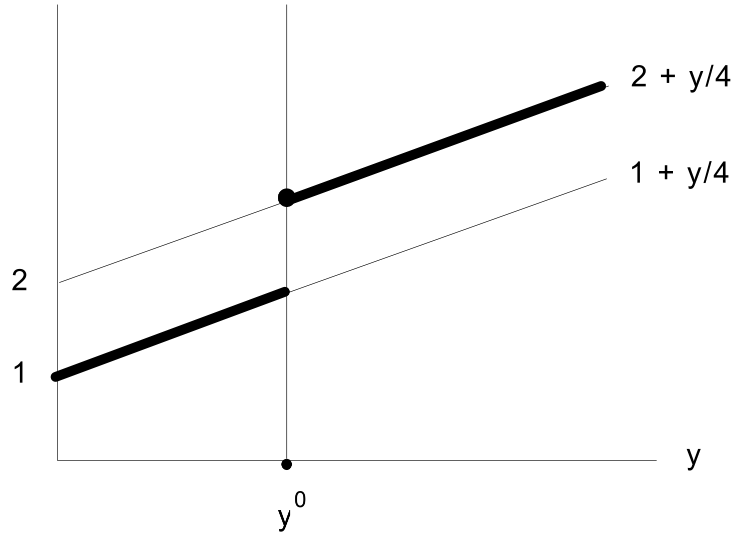
If price is	Number of cars offered for sale	Average quality of cars offered for sale .
\$2500	(1/8)200 = 25	2000
\$3100	[(1/8)+(3/8)]200=100	(1/4)2000+(3/4) 3000 = 2750
\$4600	(6/8)200 = 150	(1/6)2000+(3/6)3000+(2/6)4000 = 3166
\$5250	200 (all the cars)	(1/8)2000+(3/8)(3000)+(2/8)4000+(2/8)5000=3625
\$6100	200 (all the cars)	(1/8)2000+(3/8)(3000)+(2/8)4000+(2/8)5000=3625

2. Given price p, the only car-owners who are willing to sell are those whose θ is less than or equal to p. Thus, given p, the quality of the cars *offered for sale* is uniformly distributed in the interval [2000, P], hence the average quality is $\frac{p+2000}{2}$. Thus the buyer's expected utility if the price is p is:

$$1.2\left[\frac{p+2000}{2}\right] - p = 1200 + \frac{3}{5}p - p = 1200 - \frac{2}{5}p.$$

Hence buyers are willing to buy only if $\left[1200 - \frac{2}{5}p\right] \geq 0$, i.e. only if $p \leq 3000$. When p = 3000 supply equals demand (if there are at least as many buyers as sellers). Thus in this case there is a market for second-hand cars, even though the market is not efficient. Efficiency requires that all cars be sold, because buyers value them more than sellers do. Yet only $\frac{1}{4}$ of the cars are actually sold. It is still the case that bad cars drive good cars out of the market, although not to the point of having only the lowest-quality cars up for sale.

3. The wage offered by the employer is represented by the thick lines in the following diagram:



For example, if $y^0 = 13$, anybody with $y = 12$ will be offered a salary of $1 + 12/4 = 4$ (even if he belongs to Group II, because the employer does not know to which group the applicant belongs) and anybody with $y = 16$ will be offered a salary of $2 + 16/4 = 6$ (even if he belongs to Group I).

First of all, note that the only sensible choices are $y = 0$ and $y = y^0$. In fact, if you increase y by 1 unit, you get an extra \$(1/4) but you pay more than this (you pay \$1 if you belong to Group I and \$ 0.5 if you belong to Group II).

Decision for a Group I person:

	wage	cost	net income
$y = 0$	$1 + 0/4 = 1$	0	1
$y = y^0$	$2 + y^0/4$	y^0	$2 + y^0/4 - y^0$

In equilibrium a Group I person chooses what the employer expects him to choose, namely $y = 0$. However, it must be in his interest to make this choice, i.e. it must be the case that

$$1 > 2 + \frac{y^o}{4} - y^o$$

Decision for a Group II person:

	wage	cost	net income
$y = 0$	$1 + 0/4 = 1$	0	1
$y = y^0$	$2 + y^0/4$	$y^0/2$	$2 + y^0/4 - y^0/2$

In equilibrium a Group II person chooses what the employer expects him to choose, namely $y = y^0$. However, it must be in his interest to make this choice, i.e. it must be the case that

$$2 + \frac{y^o}{4} - \frac{y^o}{2} > 1.$$

These two inequalities are simultaneously satisfied if and only if $\frac{4}{3} < y^o < 4$.

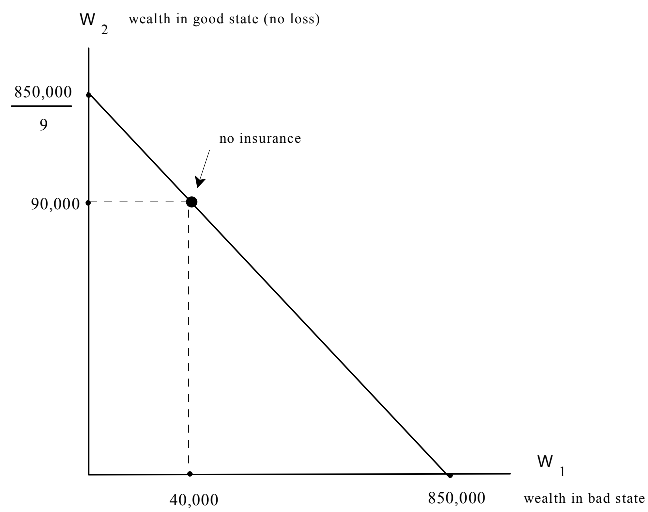
4. **(a)** For contract C_1: h_1 = 10,000, D_1 = 15,000. For contract C_2: h_2 = 12,000, D_2 = 8,000.

(b) The expected utility from no insurance is $EU_0 = \frac{9}{10}\sqrt{90,000} + \frac{1}{10}\sqrt{40,000} = 290$. The expected utility from contract C_1 is $EU_1 = \frac{9}{10}\sqrt{80,000} + \frac{1}{10}\sqrt{65,000} = 280.054$ and the expected utility from contract C_2 is $EU_2 = \frac{9}{10}\sqrt{78,000} + \frac{1}{10}\sqrt{70,000} = 277.814$. Thus the individual would prefer not to insure.

(c) The certain wealth that is on the same indifference curve as the no insurance point is given by the solution to $\sqrt{w} = 290$ which is w = 84,100. Thus the maximum premium that the individual would be willing to pay for full insurance is 90,000 − 84,100 = 5,900.

(d) The fair odds line is a straight line with slope $-\frac{\frac{1}{10}}{\frac{9}{10}} = -\frac{1}{9}$. Thus it has equation

$W_2 = \frac{850,000 - W_1}{9}$. It is shown below.

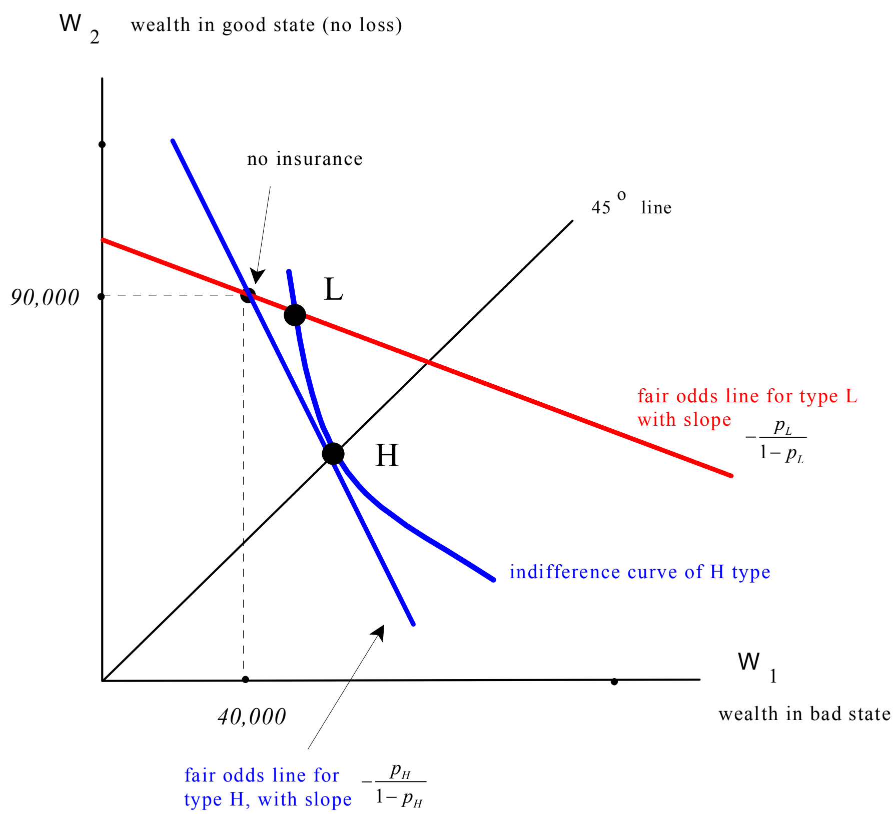


(e) Since she is risk averse she would choose the full insurance contract, given by the solution to $W = \frac{850,000 - W}{9}$ which is W = 85,000. Thus the contract would have a premium h = 5,000 and zero deductible.

(f) The equilibrium would involve full insurance for both groups. Thus the H type would get the same contract as under (e): h = 5,000, D = 0. For the L type we need to first determine their fair odds line, which has slope $-\frac{\frac{1}{40}}{\frac{39}{40}} = -\frac{1}{39}$ and is thus given by the equation

$W_2 = \frac{3,550,000 - W_1}{39}$. The solution to $W = \frac{3,550,000 - W}{39}$ is W = 88,750. Thus their contract would be h = 1,250, D = 0.

(g) A candidate for a separating equilibrium is the pair of contracts represented by points H and L in the diagram below.

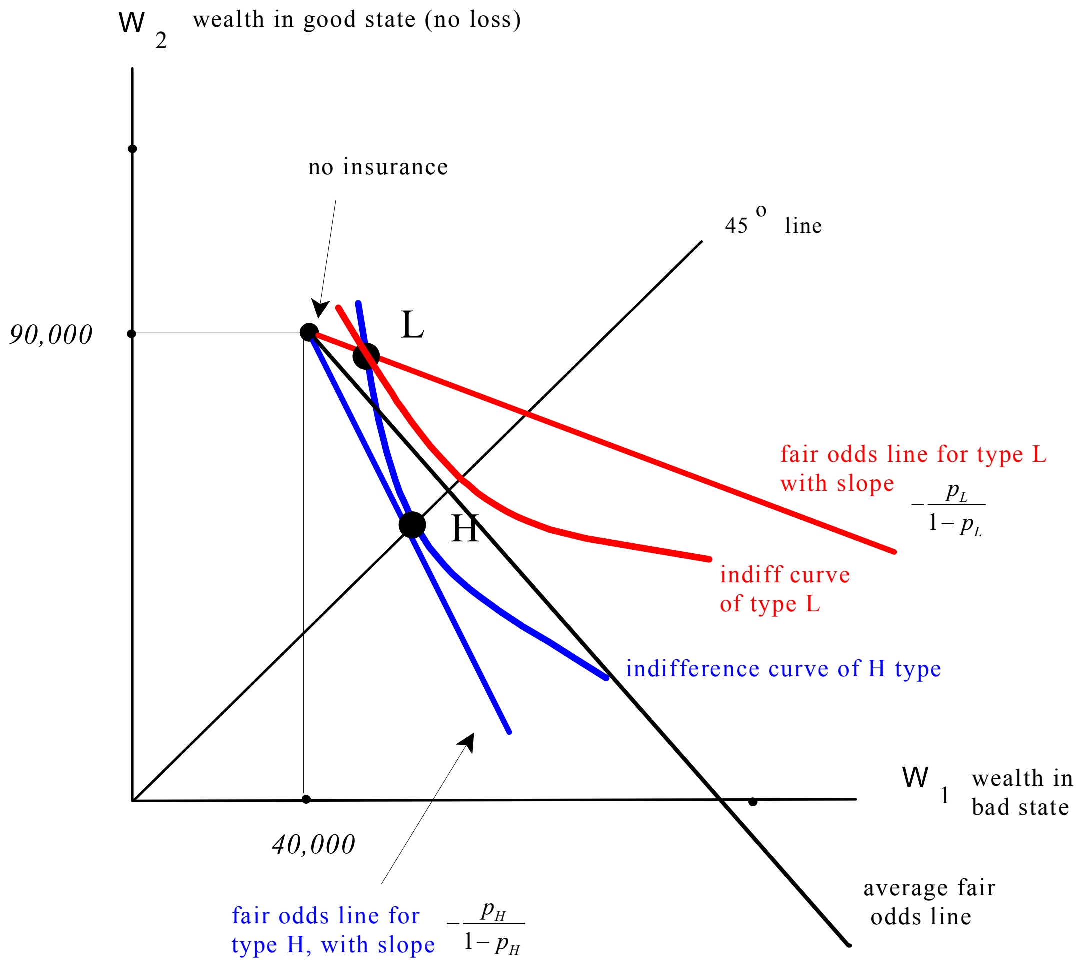


Point H is the full insurance contract for the H type, which we found under (f): h = 5,000, D = 0. To find point L we need to calculate the intersection between the fair odds line for the L type, which has equation $W_2 = \dfrac{3{,}550{,}000 - W_1}{39}$ and the indifference curve of the H type that goes through point H. Expected utility for the H type at point H is $\sqrt{85{,}000} = 291.548$. Thus the indifference curve through point H is given by the equation $\dfrac{1}{10}\sqrt{W_1} + \dfrac{9}{10}\sqrt{W_2} = \sqrt{85{,}000}$. Solving for W_2 we get the equation for the indifference curve:

$W_2 = \dfrac{8{,}500{,}000 + W_1 - \sqrt{34{,}000{,}000\,W_1}}{81}$. Thus contract L is given by the solution to

$\dfrac{3{,}550{,}000 - W_1}{39} = \dfrac{8{,}500{,}000 + W_1 - \sqrt{34{,}000{,}000\,W_1}}{81}$, which is $W_1 = 47{,}718.81$ and the corresponding W_2 is $\dfrac{3{,}550{,}000 - 47{,}718.81}{39} = 89{,}802.08$. Thus the contract for the L type has $h = 90{,}000 - 89{,}802.08 = 197.92$ and $D = 42{,}083.27$.

In order for this to be an equilibrium, there cannot be any points that lie strictly between the indifference curve for the L-type that goes through contract L and the average fair odds line. This we need the average fair-odds line to be below the L-indifference curve that goes though contract L, as shown in the following figure.



The average fair odds line has slope $-\dfrac{\bar{p}}{1-\bar{p}}$ where $\bar{p} = \alpha\dfrac{1}{10} + (1-\alpha)\dfrac{1}{40} = \dfrac{1+3\alpha}{40}$. Thus its slope is equal $-\dfrac{1+3\alpha}{39-3\alpha}$ and the line is defined by the equation $W_2 = \dfrac{50,000}{3}\dfrac{71-3\alpha}{13-\alpha} - \dfrac{1+3\alpha}{39-3\alpha}W_1$.
Next we calculate the equation of the L-indifference curve that goes through point L. L-type expected utility from contract L is $\mathrm{EU_L} = \dfrac{39}{40}\sqrt{89,802.08} + \dfrac{1}{40}\sqrt{47,718.81} = 297.64$. Thus the indifference curve is given by the equation $\dfrac{1}{40}\sqrt{W_1} + \dfrac{39}{40}\sqrt{W_2} = 297.64$, which can be written as $W_2 = 93,190.87 - 15.66\sqrt{W_1} + 0.001W_1$. Thus the necessary and sufficient condition for the pair of contracts found above to be an equilibrium is the following inequality, which says that the average fair odds line is weakly below the L-indifference curve:

$$\frac{50,000}{3}\frac{71-3\alpha}{13-\alpha} - \frac{1+3\alpha}{39-3\alpha}W_1 \;\leq\; 93,190.87 - 15.66\sqrt{W_1} + 0.001W_1, \qquad \text{for every } W_1 \geq 40,000$$

When $\alpha = 1$ this inequality is satisfied and when $\alpha = 0$ it is not. Thus the inequality can be interpreted and a condition on α namely that α be “sufficiently large”.