Competition under Product Differentiation

Reference: slides only

Introduction

- Previously we discussed monopoly product differentiation
- We'll now discuss product differentiation under competition
- Firms try to differentiate their products from their competitors' in order to make them less substitutable
- We'll start with horizontal differentiation: **consumers** rank products differently

Introduction

- There are several approaches to horizontal differentiation
- The location-based approach that we have already seen assumes that consumers have preferences over the characteristics of the product as represented by the location of the product on a line or a circle
- The goods approach assumes that consumers have preferences over the goods themselves. We’ll start with the second approach

Bertrand Competition and the Goods Approach

- Suppose that there are two firms each selling one product
- In the goods approach to differentiation we can write a linear demand system as:

$$q_1 = A + B_1 p_1 + B_2 p_2$$

$$q_2 = A + B_3 p_1 + B_4 p_2$$

- This **demand system** allows changes in own price to effect quantity differently than changes in the other product’s price

Bertrand Competition and the Goods Approach

- Gasmi, Vuong, and Laffont (1992) estimated this demand system and marginal costs for Coke and Pepsi. They found:

$$q_c = 64 - 4p_c + 2p_p \text{ and } MC_c = 5$$

$$q_p = 50 - 5p_p + p_c \text{ and } MC_p = 4$$

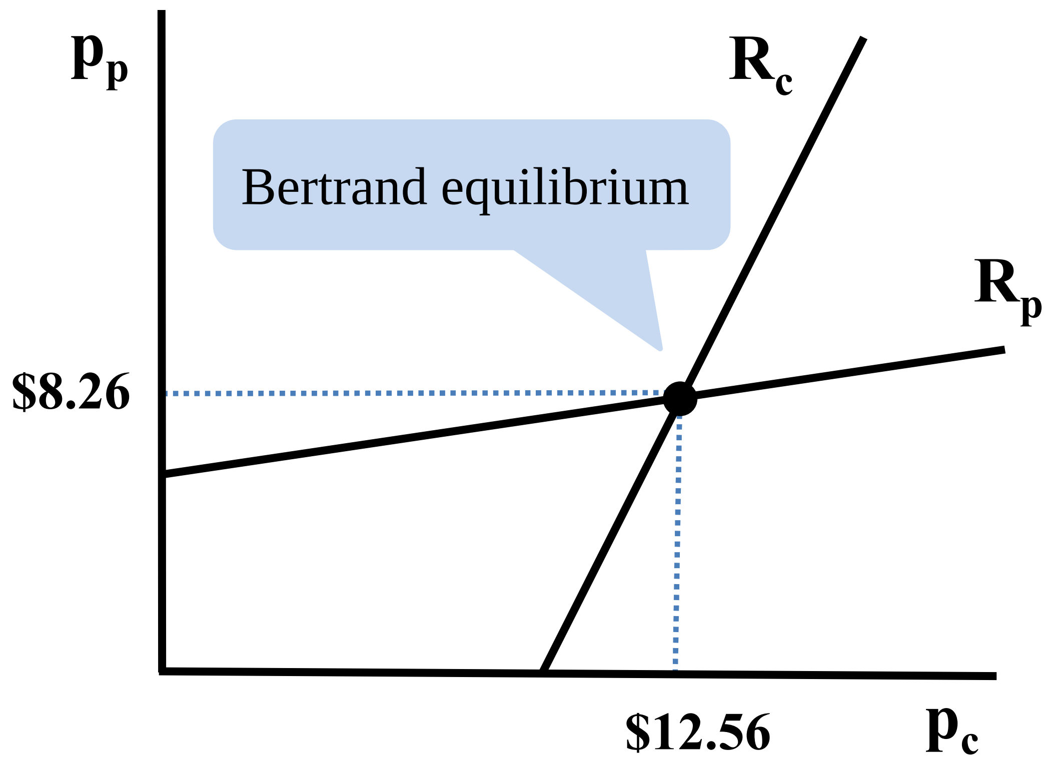
Bertrand Competition and the Goods Approach

- Profits are

$$\pi_c = (p_c - 5)(64 - 4p_c + 2p_p)$$
$$\pi_p = (p_p - 4)(50 - 5p_p + p_c)$$

- Differentiate with respect to price and solve for the best response of each firm:

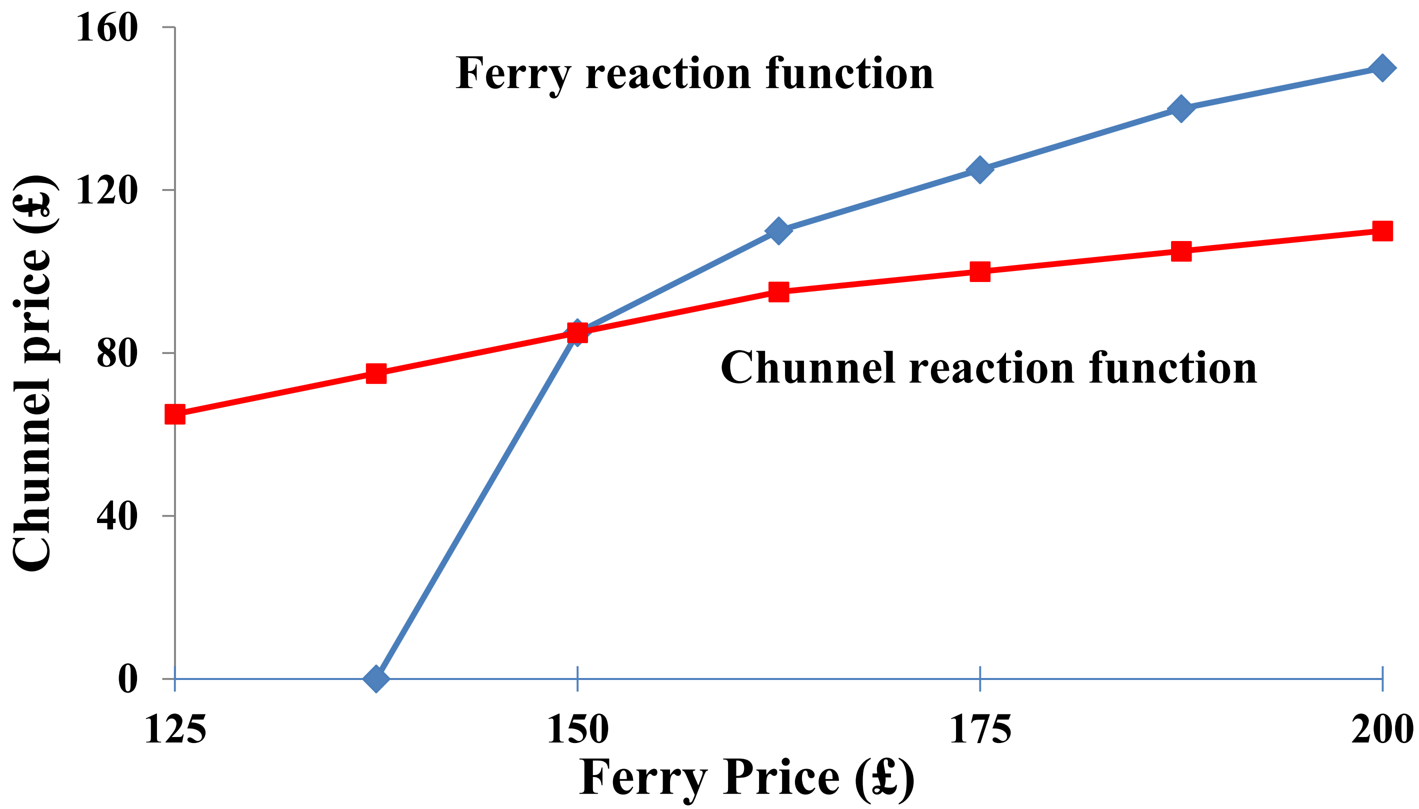
$$p_c = 10.5 + 0.25 p_p \quad \text{and} \quad p_p = 7 + 0.1 p_c$$



- Prices are greater than MC ➔ product differentiation "softens" competition
  - price cutting is less effective when products are differentiated

Empirical Application – The Chunnel

- The Chunnel is the 32-mile long Channel Tunnel owned by Eurotunnel that links France with England
- Eurotunnel competes with ferry services by transporting trucks
- Before the Chunnel opened in 1994 the main ferry company carried about 80% of the cross-channel passenger and freight traffic
- Kay, Manning, and Szymmanski used the differentiated-product Bertrand model to predict the post-Chunnel opening equilibrium



- The model predicted that the price of the chunnel would be £87 and £150 for the ferry

Empirical Application – The Chunnel

- The large difference in price is due to the much lower MC of Eurotunnel relative to the ferry's MC
- The analysis suggested that Eurotunnel would be a formidable competitor of the ferry
  - two years after opening, Eurotunnel had a 44% share of cross-channel truck traffic compared to 40% for the ferry

Location-based models

- Recall: **Location-based models** are models in which consumer preferences are heterogeneous due to differences in location along a line or circle
    - Ex. Hotelling Model uses a line
- Location can represent: **geography, time, product characteristics**
- Firms chooses a product type by choosing a location

Hotelling Model

- **N** consumers are uniformly (evenly) distributed along a street of length 1
- **x** denotes a location, **d** is distance
- Consumers consider **buying one unit**. The gross value of the product is **V**
- Consumers incur "**transportation" costs (t)** (utility loss) per unit of distance traveled
- Utility: **U = V – p - td**

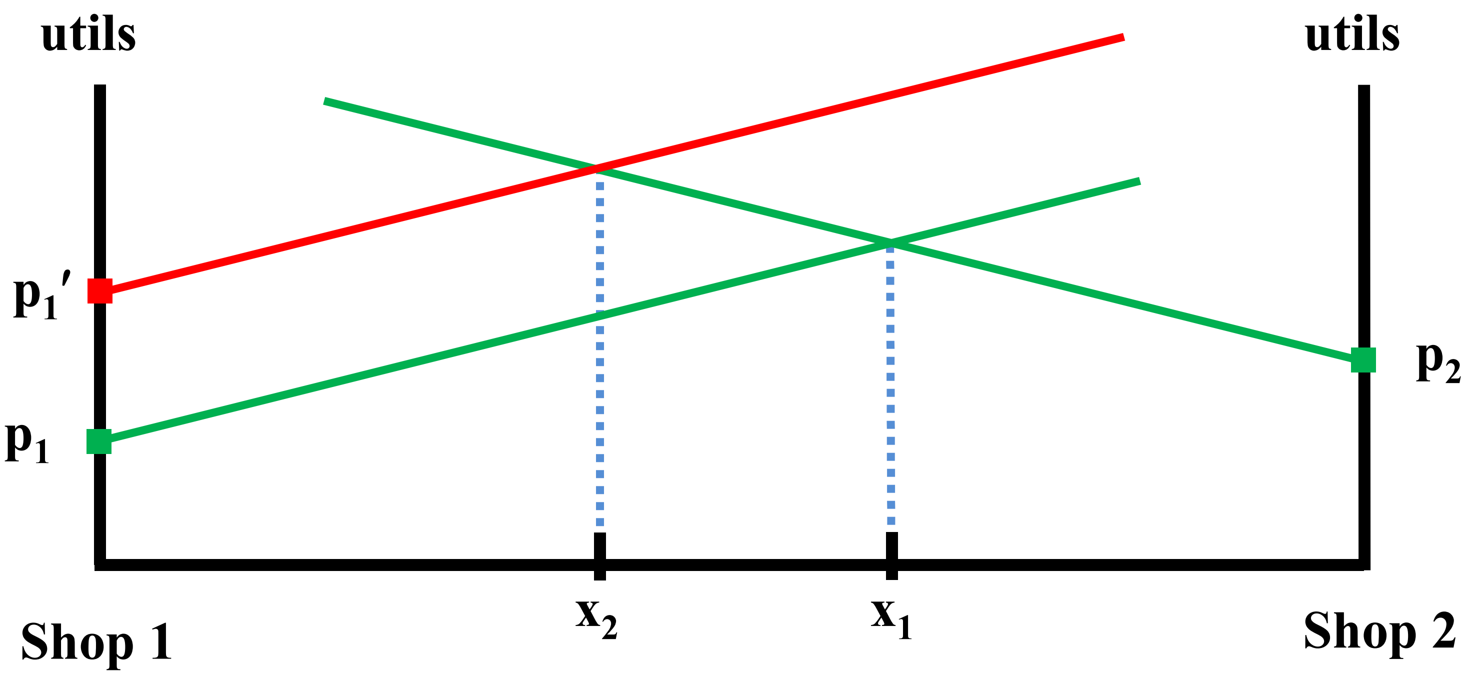
Hotelling Model

- Consider the case of **two competing shops**
- Consumers buy from the shop that has the **lower total price** (product price + transportation costs)
- Suppose that **location of these shops are fixed at each end** of the street and thus **they compete only in price**
- What is **demand for each shop** and **what prices** will they charge?

Hotelling Model

Shop 1 sets price p_1 and shop 2 sets price p_2 simultaneously

What if shop 1 raises its price?



x_2 is to the left: some consumers switch to shop 2

All consumers to the left of x_1 buy from shop 1 and all to the right buy from shop 2

- Find $\mathbf{x_1}$: $p_1 + t x_1 = p_2 + t(1 - x_1) \rightarrow x_1 = \dfrac{p_2 - p_1 + t}{2t}$

- Demand: $D_1 = N\left(\dfrac{p_2 - p_1 + t}{2t}\right)$ *and* $D_2 = N\left(1 - \dfrac{p_2 - p_1 + t}{2t}\right)$

- Firm 1's profit: $\pi_1 = N\,\dfrac{p_2 - p_1 + t}{2t}(p_1 - c)$

- Maximize profit to find shop 1's best response:

$$p_1^R = \frac{p_2 + t + c}{2}$$

when firm 1 increases its price, firm 2 also increases the price.

- Similarly for shop 2 (or by symmetry): $p_2^R = \dfrac{p_1 + t + c}{2}$

Hotelling Model

- Finding the Bertrand-Nash equilibrium
- Best responses

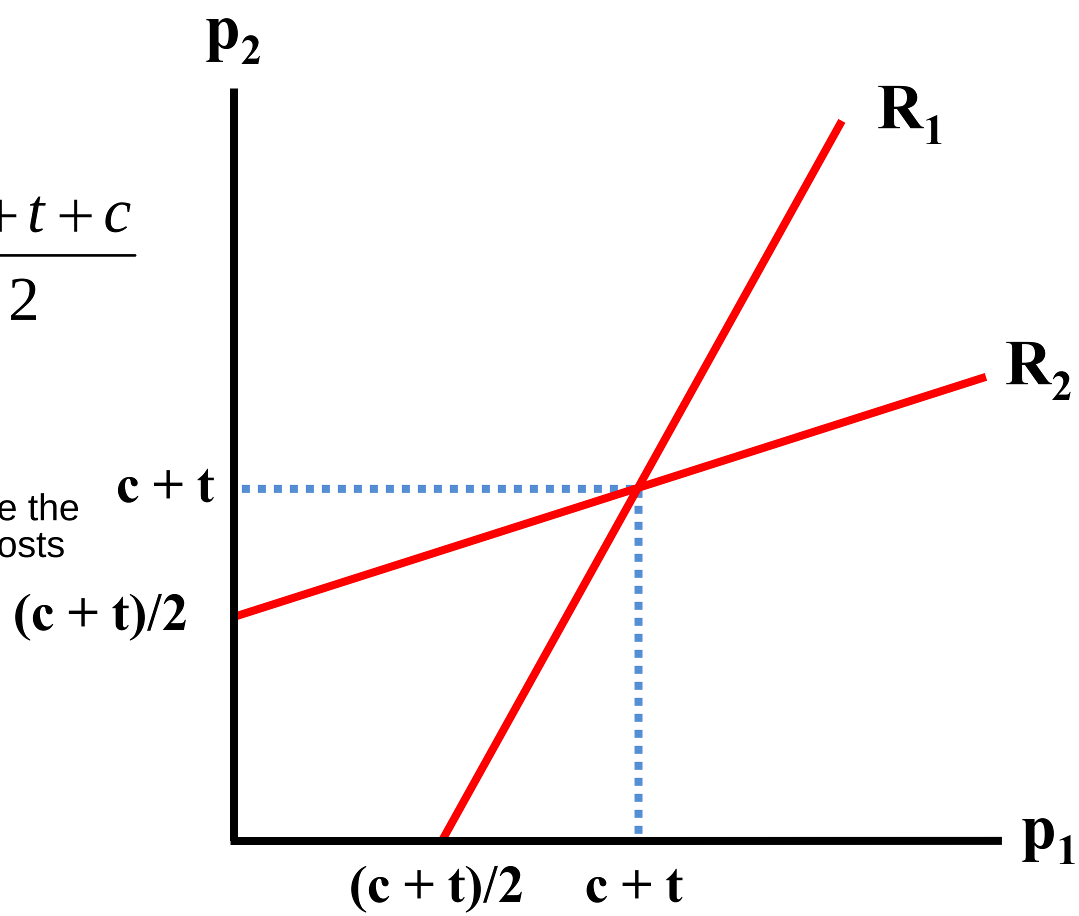
$$p_1^R = \frac{p_2 + t + c}{2},\ p_2^R = \frac{p_1 + t + c}{2}$$

- Solve to find:

$$p_1^* = p_2^* = c + t$$

price above the marginal costs

- **Profit per unit: *t***
- $\mathbf{x_1}$ = ½ ➔ split market
- Profits per shop: **Nt/2**



Hotelling Model

- Greater distance between **firms** increases product differentiation, which increases the market power of the firms
- Let's look at the polar cases:
  - Shops are located at the endpoints, **0** and **1** (maximum differentiation)
    - Profit: **Nt/2**
  - Both shops are located at $\mathbf{x = 1/2}$ (minimum differentiation)
    - Products are no longer differentiated because it has the same location
    - Bertrand with homogenous goods: $\mathbf{p_1 = p_2 = c, \pi_1 = \pi_1 = 0}$

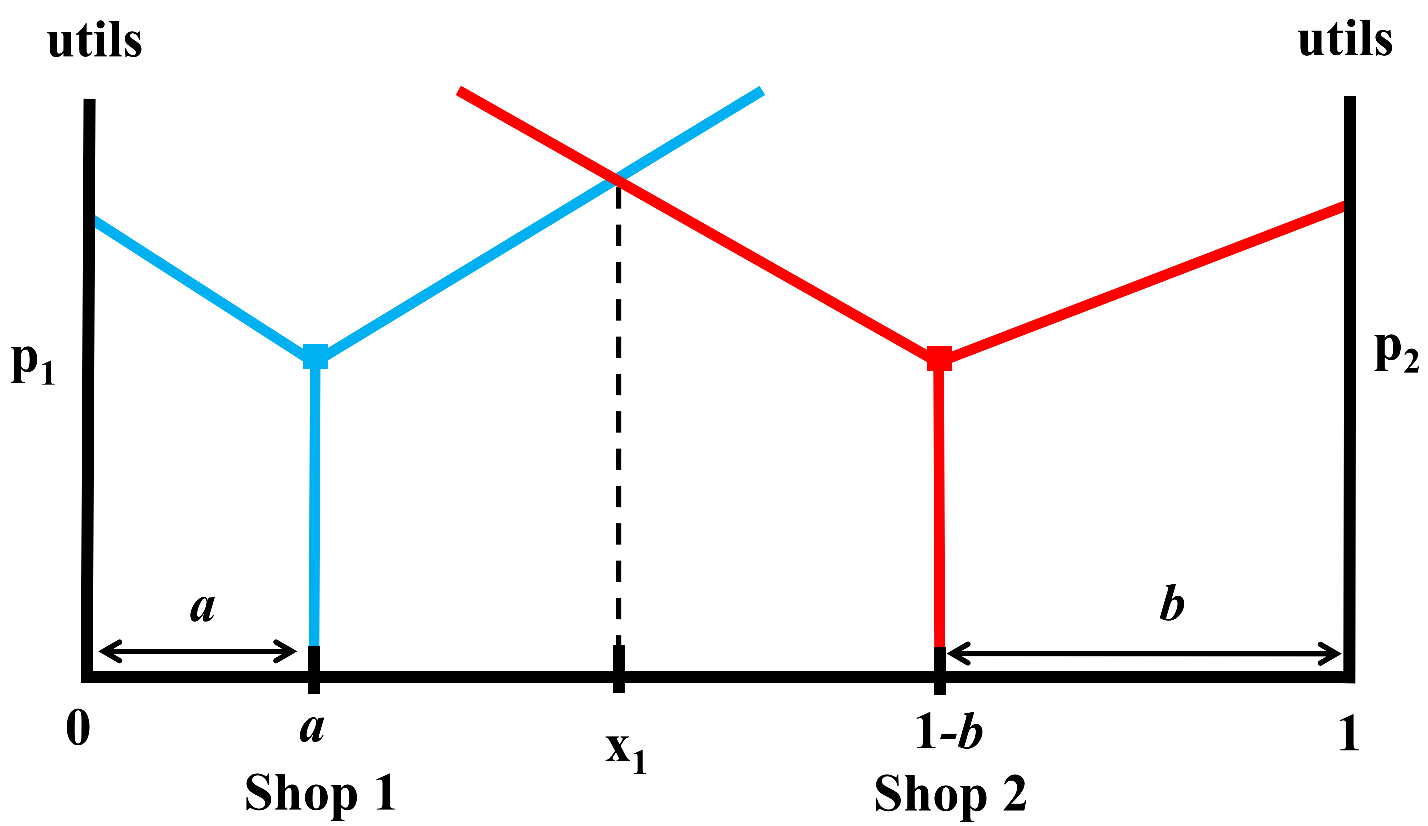
Hotelling Model

- **Transportation costs** also increase product differentiation
  - t is the value that consumers place on getting their most preferred variety
  - firms have more market power as t increases and competition softens ➔ **Prices and profits increase with transportation costs**
  - When $\mathbf{t = 0}$, firms lose all market power ➔ $\boldsymbol{\pi_1 = \pi_1 = 0}$

consumers less likely to switch

Hotelling Model

Now suppose shop 1 is located at ***a*** and shop 2 is located at **1-*b*** (assume that the shops aren't located too near the center)



- Find demand for each shop

if a increases, then cost will increase and marginal consumer moves to the right

$$U=\begin{cases} V-p_1-t(x_1-a) & \text{if buys from shop 1} \\ V-p_2-t((1-b)-x_1) & \text{if buys from shop 2} \end{cases}$$

$$V-p_1-t(x_1-a)=V-p_2-t(1-b-x_1)\rightarrow x_1=\frac{p_2-p_1+t(1+a-b)}{2t}$$

$$\rightarrow \begin{array}{l} D_1=Nx_1=N\left(\dfrac{p_2-p_1+t(1+a-b)}{2t}\right) \\ D_2=N(1-x_1)=N\left(\dfrac{p_1-p_2+t(1+b-a)}{2t}\right) \end{array}$$

- Assume that ***c* = 0**
- Shop 1:

$$\pi_1 = D_1(p_1 - c) = N\left(\frac{p_2 - p_1 + t(1 + a - b)}{2t}\right)p_1$$

$$\frac{\partial \pi_1}{\partial p_1} = 0 \rightarrow \ p_1^R = \frac{p_2 + t(1 - b + a)}{2}$$

- Similarly for shop 2:

$$p_2^R = \frac{p_1 + t(1 + b - a)}{2}$$

- Solve the best response functions to find the Bertrand-Nash equilibrium:

$$p_1^* = \frac{t\left(3-b+a\right)}{3},\ \ D_1 = N\left(\frac{3-b+a}{6}\right),\ \ \pi_1 = \frac{Nt}{18}\left(3-b+a\right)^2$$

$$p_2^* = \frac{t\left(3+b-a\right)}{3},\ \ D_2 = N\left(\frac{3+b-a}{6}\right),\ \ \pi_2 = \frac{Nt}{18}\left(3+b-a\right)^2$$

- Again profit is increasing in ***t***

- Also notice that shop 1 would prefer to increase ***a***, i.e., move closer to shop 2 (and similarly for shop 2):

$$\frac{\partial \pi_1}{\partial a} > 0,\ \ \frac{\partial \pi_2}{\partial b} > 0,$$

- As shop **a** moves farther to the center, there are more "captive" consumers to the left ➔ "**demand effect**"

- But this is true only when the firms are relatively far away from each other. If the **firms are close to each other** they become **less differentiated** and are subject to the Bertrand Paradox ➔ **the "price effect" causes firms to move away from each other**

- We would like to solve a two-stage game in which firms choose locations then price but…

- Unfortunately a Hotelling model with linear transportation costs where firms choose **both prices and locations** isn't well-behaved and an **equilibrium doesn't exist**

Hotelling Model

- With quadratic transportation costs (**$U = V - p - td^2$**), it can be shown that in equilibrium firms will choose the endpoints in order to minimize price competition
  - the price effect dominates the demand effect
  - **maximal differentiation**

Salop's Circle Model

- Salop's circle model avoids the problem of equilibrium non-existence
  - Firms are located around a circle ➔ no endpoints which eliminates the cause of the problems in the Hotelling model
  - We can allow free entry
    - two-stage game: first, entry and location decision, second, price decision

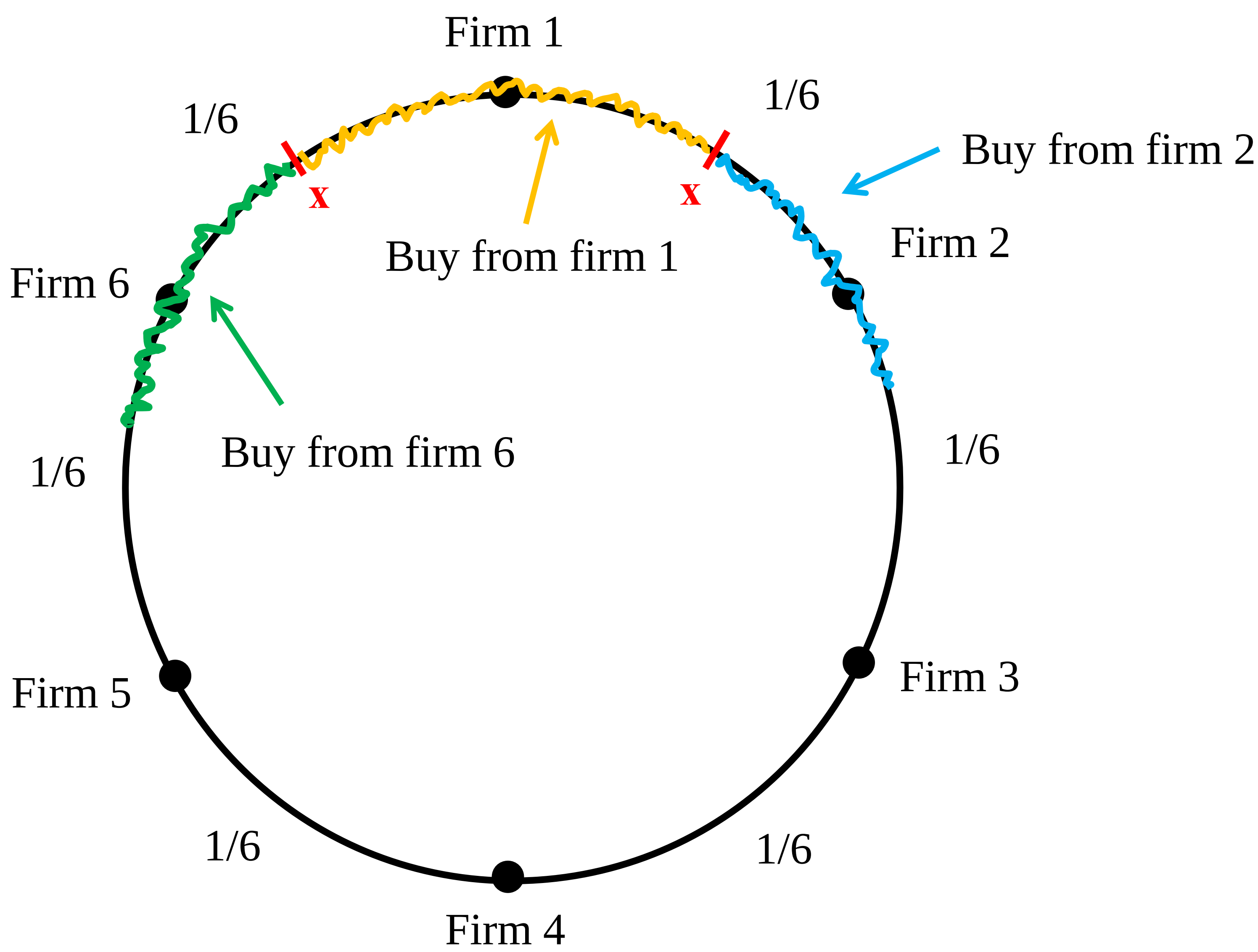
- **Consumers**:
  - ***L*** consumers uniformly located around the circle
  - A consumer's location represents their most preferred type of product
  - Each consumer buys at most one unit
  - Transportation costs per unit of distance: ***t***

Salop’s Circle Model

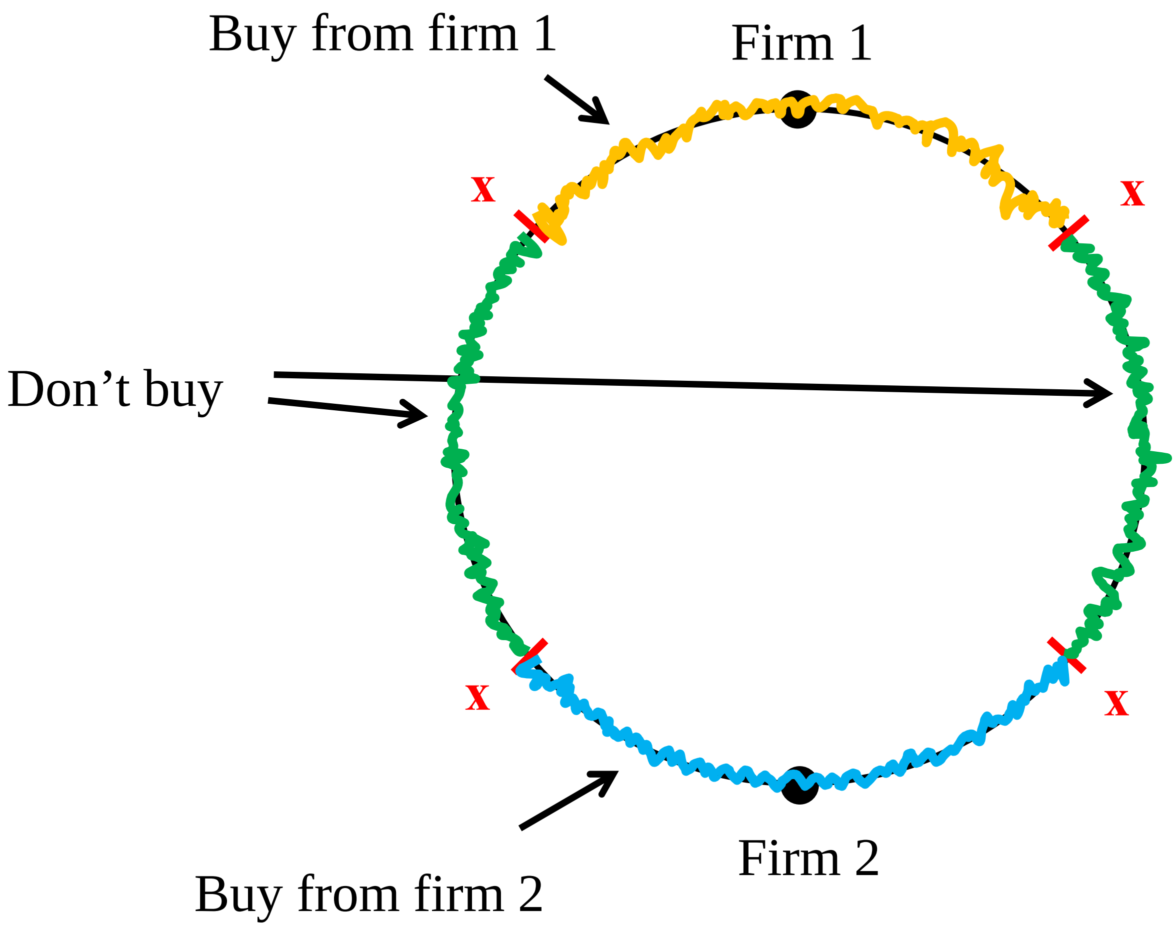
- **Firms**:
  - Firms are located an equal distance from each other on a circle with circumference = 1
    - It can be proven that for quadratic transportation costs firms choose symmetric, equi-distant locations ➔ again, firms want to differentiate their products to soften price competition
  - Fixed costs **F** and marginal cost **c**
    - When **F** is large, few firms enter and the market isn’t covered

      ➔ each firm is a local monopoly
    - When **F** is small, a lot of firms enter and firms are located sufficiently close to each other to cause competition for consumers

Small F: market is covered as every consumer buys a unit



Large F: market is partially uncovered ➔ some consumers don't make a purchase



- We assume F is small so that the market is covered and firms compete
- For price p_1 of firm 1 and price p of the adjacent firms, we derive the location of the indifferent consumer, $x \in (0, 1/N)$, and demand for firm 1:

p assumes that the firm on the left and right have the same price

$$V - p_1 - tx = V - p - t\left(\frac{1}{N} - x\right)$$

$$\rightarrow D_1(p_1, p) = 2xL = L\left(\frac{p - p_1}{t} + \frac{1}{N}\right)$$

- Therefore:

$$\pi_1 = L(p_1 - c)\left(\frac{p - p_1}{t} + \frac{1}{N}\right) - F$$

$$\frac{\partial \pi_1}{\partial p_1} = 0 \rightarrow p_1^* = \frac{t}{2N} + \frac{p + c}{2}$$

- By symmetry, in equilibrium we have $\boldsymbol{p_1^* = p^*}$ and thus

$$p^* = c + \frac{t}{N} \quad \text{and} \quad \pi^* = \left(p^* - c\right)\frac{L}{N} - F = \frac{tL}{N^2} - F$$

- Price and profit increase if transportation costs go up, decrease if there are more firms price effect
- Profit is larger when the market is larger
- Suppose that there is free entry of firms ➔ entry will take place until $\boldsymbol{\pi = 0}$

$$\pi^* = 0 \rightarrow N^* = \sqrt{\frac{tL}{F}} \quad and \quad p^* = c + \sqrt{\frac{tF}{L}}$$

- With free entry, an increase in fixed costs causes a decrease in the number of firms
- An increase in the transportation costs or the market size increases the number of firms
- When fixed costs fall to zero, the numbers of firms becomes very large:

$$N^* = \lim_{F \to 0} \sqrt{\frac{tL}{F}} = \infty$$

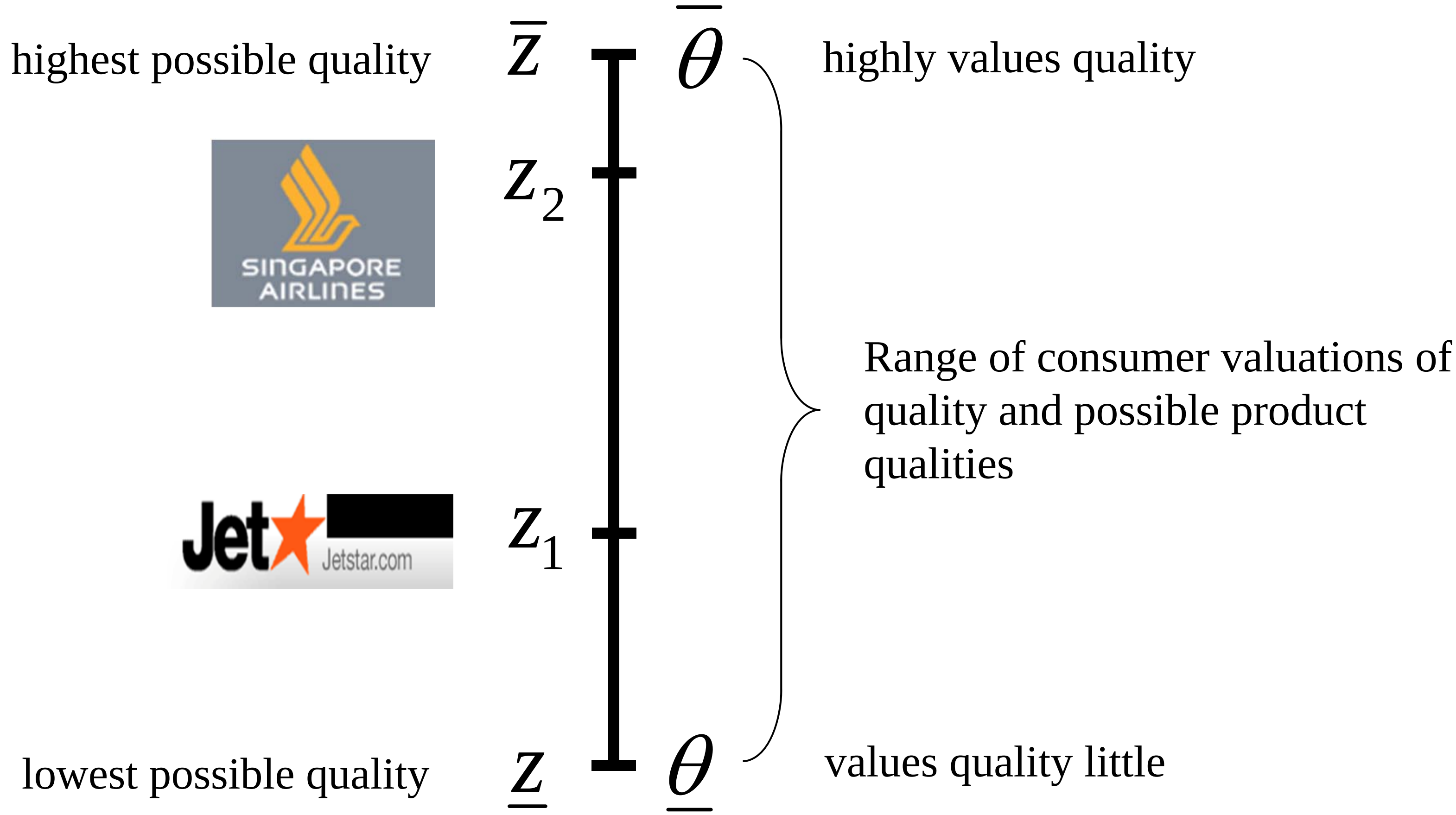
  - If **F** was very small, firms would flood the market with “niche” products. (e.g., orange juice)
  - If **F** is not too small, entry is limited because, at some point, demand isn’t sufficient to cover the fixed costs of new products (e.g., cars)

Vertical Product Differentiation

- Vertical differentiation means that **consumers** rank the products similarly
- Consumers have a different **willingness to pay for quality**
- **Setup:**
  - There are two firms that compete in prices. Firm 1 produces a product with quality $\mathbf{z_1}$. Firm 2 produces a product with quality $\mathbf{z_2}$ where $\mathbf{z_2 > z_1}$
  - **N** consumers with location θ are uniformly distributed on a quality interval of length 1
    - θ is a measure of the value consumers place on quality
  - Utility function:

$$U = \begin{cases} \theta z_i - p & \text{if customer buys a unit with quality } z \\ 0 & \text{if does not buy} \end{cases}$$

Vertical Product Differentiation



Vertical Product Differentiation

- **More setup:**
  - Denote the quality differential between the two products as

$$\Delta_z \equiv z_2 - z_1$$

  - We'll assume that the whole market is **covered** (everyone buys a unit):

$$U = \theta z_i - p \geq 0 \text{ or } \theta z_i \geq p \ \ \forall \theta$$

  - We'll also assume that: $\overline{\theta} \geq 2\underline{\theta}$

Vertical Product Differentiation

- The marginal consumer is indifferent between buying the high quality good and the low quality if:

$$\theta z_2 - p_2 = \theta z_1 - p_1 \rightarrow \tilde{\theta} = \frac{p_2 - p_1}{z_2 - z_1} = \frac{p_2 - p_1}{\Delta_z}$$

- This implies that all consumers with preferences in the interval

$$\theta \in \left(\underline{\theta}, \tilde{\theta}\right)$$

buys the **low quality** good. Consumers in the interval

$$\theta \in \left(\tilde{\theta}, \overline{\theta}\right)$$

buys the **high quality** good

The demand functions are then:

$$D_1(p_1, p_2) = N(\tilde{\theta} - \underline{\theta}) = N\left(\frac{p_2 - p_1}{\Delta_z} - \underline{\theta}\right)$$

$$D_2(p_1, p_2) = N(\overline{\theta} - \tilde{\theta}) = N\left(\overline{\theta} - \frac{p_2 - p_1}{\Delta_z}\right)$$

and profits are

$$\pi_1 = (p_1 - c)D_1(p_1, p_2) = (p_1 - c)N\left(\frac{p_2 - p_1}{\Delta_z} - \underline{\theta}\right)$$

$$\pi_2 = (p_2 - c)D_2(p_1, p_2) = (p_2 - c)N\left(\overline{\theta} - \frac{p_2 - p_1}{\Delta_z}\right)$$

Vertical Product Differentiation

- FOCs:

$$\frac{\partial \pi_1}{\partial p_1} = \frac{N\left(c + p_2 - 2p_1 - \underline{\theta}\Delta_z\right)}{\Delta_z} = 0$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{N\left(c + p_1 - 2p_2 + \overline{\theta}\Delta_z\right)}{\Delta_z} = 0$$

- Reaction functions

$$\begin{aligned} p_1 &= \frac{c + p_2 - \underline{\theta}\Delta_z}{2} \\ p_2 &= \frac{c + p_1 + \overline{\theta}\Delta_z}{2} \end{aligned} \quad \rightarrow \quad \begin{aligned} p_1^* &= c + \frac{\left(\overline{\theta} - 2\underline{\theta}\right)}{3}\left(z_2 - z_1\right) \\ p_2^* &= c + \frac{\left(2\overline{\theta} - \underline{\theta}\right)}{3}\left(z_2 - z_1\right) \end{aligned}$$

Vertical Product Differentiation

$$D_1 = N\frac{\left(\overline{\theta}-2\underline{\theta}\right)}{3} \qquad \pi_1 = N\frac{\left(\overline{\theta}-2\underline{\theta}\right)^2}{9}\left(z_2-z_1\right)$$

$$D_2 = N\frac{\left(2\overline{\theta}-\underline{\theta}\right)}{3} \qquad \pi_2 = N\frac{\left(2\overline{\theta}-\underline{\theta}\right)^2}{9}\left(z_2-z_1\right)$$

- Summary:
  - Undifferentiated firms ($\mathbf{z_1 = z_2}$) will charge $\mathbf{p_1 = p_2 = c}$ and make zero profits
  - Profits are increasing in the quality differential ($\mathbf{z_2 - z_1}$)

Vertical Product Differentiation

- Now consider a game with **quality choice**
- The firms play a **two-stage game.** Firms choose quality first and then compete in price
- Assume for simplicity that quality choice is costless
- Since profit depends on $\mathbf{z_2 - z_1}$ firm 1 wants to choose $\mathbf{z_1}$ as low as possible and firm 2 want to choose $\mathbf{z_2}$ as high as possible
- There are two pure strategy Nash equilibria:
  - $z_1^* = \bar{z}, z_2^* = \underline{z}, p_1^*, p_2^*$
  - $z_1^* = \underline{z}, z_2^* = \bar{z}, p_1^*, p_2^*$

Vertical Product Differentiation

- In equilibrium, we have **maximal differentiation** ➔ firms differentiate in order to relax price competition
- In a Stackelberg setting, the first mover will choose the high quality while the second mover chooses the low quality
- We have a setting in which choice of quality is costless and yet the low quality firm gains from reducing the quality level to the minimum

Final Exam comments

- 2 December 2015 (Wednesday), 1pm
- I will hold my usual offices hours (Tues 4pm – 5:30pm)
- For other times, please make appointments via Google docs
- Final exam is cumulative with emphasis on the material after the midterm
- Please, no calculators
- Please complete the feedback exercise!!!

Thank you for a very enjoyable semester!