EC3322
Industrial Organization I
Semester 1, 2015-2016
Tutorial #8

SOLUTIONS

1. The first step is to compute the reaction functions of firm 2 and firm 3 in period 2. Firm 2's profit function is

$$\pi_2 = pq_2 = (60 - 5q_1 - 5q_2 - 5q_3)\, q_2$$

The first-order condition is:

$$60 - 5q_1 - 10q_2 - 5q_3 = 0.$$

Since firm 2 and firm 3 are identical and move at the same time, they will have the same output. Imposing this symmetry in the first-order condition:

$$q_2^R = q_3^R = 4 - \frac{q_1}{3} \;\text{ and }\; Q_{-1}^R = 8 - \frac{2q_1}{3}.$$

Firm 1's profit function when firm 1 anticipates firm 2 and firm 3's reaction to its output is:

$$\begin{aligned} \pi_1 &= \left(60 - 5q_1 - 5Q_{-1}^R\right) q_1 \\ &= \left(60 - 5q_1 - 5\left(8 - \frac{2q_1}{3}\right)\right) q_1 \\ &= \left(20 - \frac{5}{3}q_1\right) q_1. \end{aligned}$$

Firm 1's profit is maximized at $q_1^* = 6$. Plug q_1^* into firm 2 and 3's reaction function to find $q_2^* = 2$ and $q_3^* = 2$. Equilibrium price is $p^* = 10$.

2. (a) The best response of firm 2 is: $q_2^R = \dfrac{300 - q_1}{2}$.

   (b) The SPNE is $(q_1 = 150, q_2^R = \dfrac{300 - q_1}{2})$. The equilibrium output, price, and profits are $q_1^* = 150, q_2^* = 75, p^* = 325, \pi_1^* =$33,750,$\pi_2^* =$16,875.

   (c) To find the limit output substitute q_2^R into firm 2's profit function:

$$\pi_2 = (1000 - 3q_1 - 3q_2 - 100)\, q_2 - F = \left(900 - 3q_1 - 3\left(\frac{300 - q_1}{2}\right)\right)\left(\frac{300 - q_1}{2}\right) - F$$

   This can be simplified to:

$$\pi_2 = 3\left(\frac{300 - q_1}{2}\right)\left(\frac{300 - q_1}{2}\right) - F = 3\left(\frac{300 - q_1}{2}\right)^2 - F$$

   Setting $\pi_2 = 0$, we have:

$$\left(\frac{300 - q_1}{2}\right)^2 = F/3 \quad \Rightarrow \quad \frac{300 - q_1}{2} = \pm\sqrt{F/3}.$$

   From a mathematical perspective, we have two roots, but from an economic perspective only one makes sense. Notice that the left-hand side is the reaction function of firm 2. Since $q_2 \geq 0$, the right-hand side cannot be negative so we can eliminate the negative root. The limit output is:

$$q_1^L = 300 - 2\sqrt{F/3}.$$

   Find firm 1's profit at the limit output by substituting q_1^L into its profit function:

$$\pi_1^L = 1800\sqrt{F/3} - 5F.$$

   (d) If $F = 300$, firm 1 earns \$33,450 when it accommodates entry (Stackelberg profits) and \$16,500 when it deters entry (at the limit output). So firm 1 will accommodate entry. If $F = 2,700$, firm 1 earns \$31,050 when it accommodates entry and \$40,500 when it deters entry. In this case firm 1 finds it profitable to deter entry.

3. (a) (W, W)

   (b) (R, R)

   (c) No, because both Bob and Hank would deviate in the last period, and thus, deviate in the first period as well.

   (d) Play R if the other player played R last period. Play W forever if the other player played W last period.

   (e) The players stick to collusion if

$$V^C = \frac{3}{1-\delta} > V^D = 5 + \frac{\delta}{1-\delta},$$

   Simplify to find that the threshold discount factor is 1/2.

4. (a) $q^N = 24, p^N = 68, \pi^N = 1,152$

   (b) $Q^* = 60, p^* = 140, q^M = 60/4 = 15, \pi^M = 1,800$

   (c) The deviating firm chooses output according to its best-response function:

$$q_i^R = \frac{120 - Q_{-i}}{2} = \frac{120 - 3 * 15}{2} = 37.5.$$

   The equilibrium price is $p^* = 95$, and the profit of the deviating firm is $\pi^D = 2,812.5$. The profit of the firms that don't deviate is $\pi^D = 1,125$

   (d) Collusion is maintained if $\delta > \dfrac{\pi^D - \pi^M}{\pi^D - \pi^N} = \dfrac{2812.5 - 1800}{2812.5 - 1152} = 0.61.$

   (e) The profits when there are two firms are: $\pi^N = 3,200, \pi^C = 3,600, \pi^D = 4,050$. Collusion is maintained if $\delta > \dfrac{\pi^D - \pi^M}{\pi^D - \pi^N} = \dfrac{4,050 - 3,600}{4,050 - 3,200} = 0.53$. Since the threshold discount factor is greater in part (d), collusion is easier to maintain with two firms.