

# Monopoly Product Variety and Quality

Reference: Pepall, Richards, and Norman, chapter 7

# OJ Insanity



# Introduction

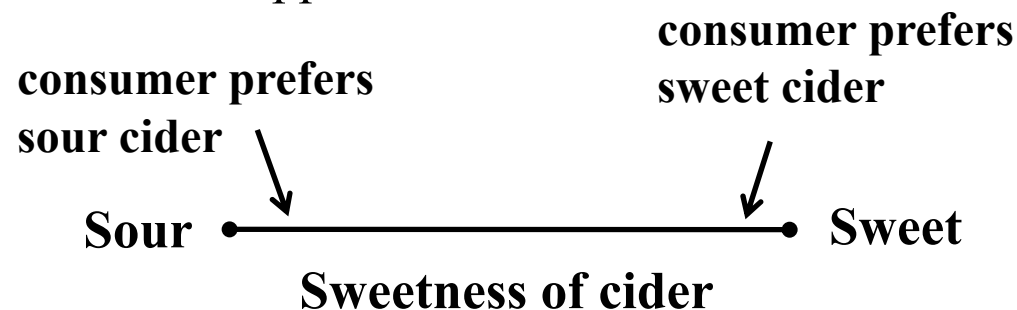
- Firms often produce different varieties of essentially the same product to sell to consumers with different tastes → product differentiation
- One type of product differentiation is called horizontal differentiation: **consumers** rank products differently
  - some people like durians, others can't stand them
  - some people prefer a McDonald's Big Mac to a Burger King Whopper
  - people prefer a petrol station near their home
- An important question in this context: is the level of product variety that is chosen by a monopolist too much or too little?

# Introduction

- For other types of products, consumers agree on how good a product is
- **All consumers** rank the products similarly
  - everyone agrees that Duracell batteries are better than a no-name brand because they last longer
  - a Jaguar is better than a Toyota Vios
- This is called vertical differentiation
- An important question here is: is the quality level chosen by a monopolist too high or too low?
- In this section we address these questions for a monopoly market
- Later in the course we will consider how competition affects both horizontal and vertical product differentiation

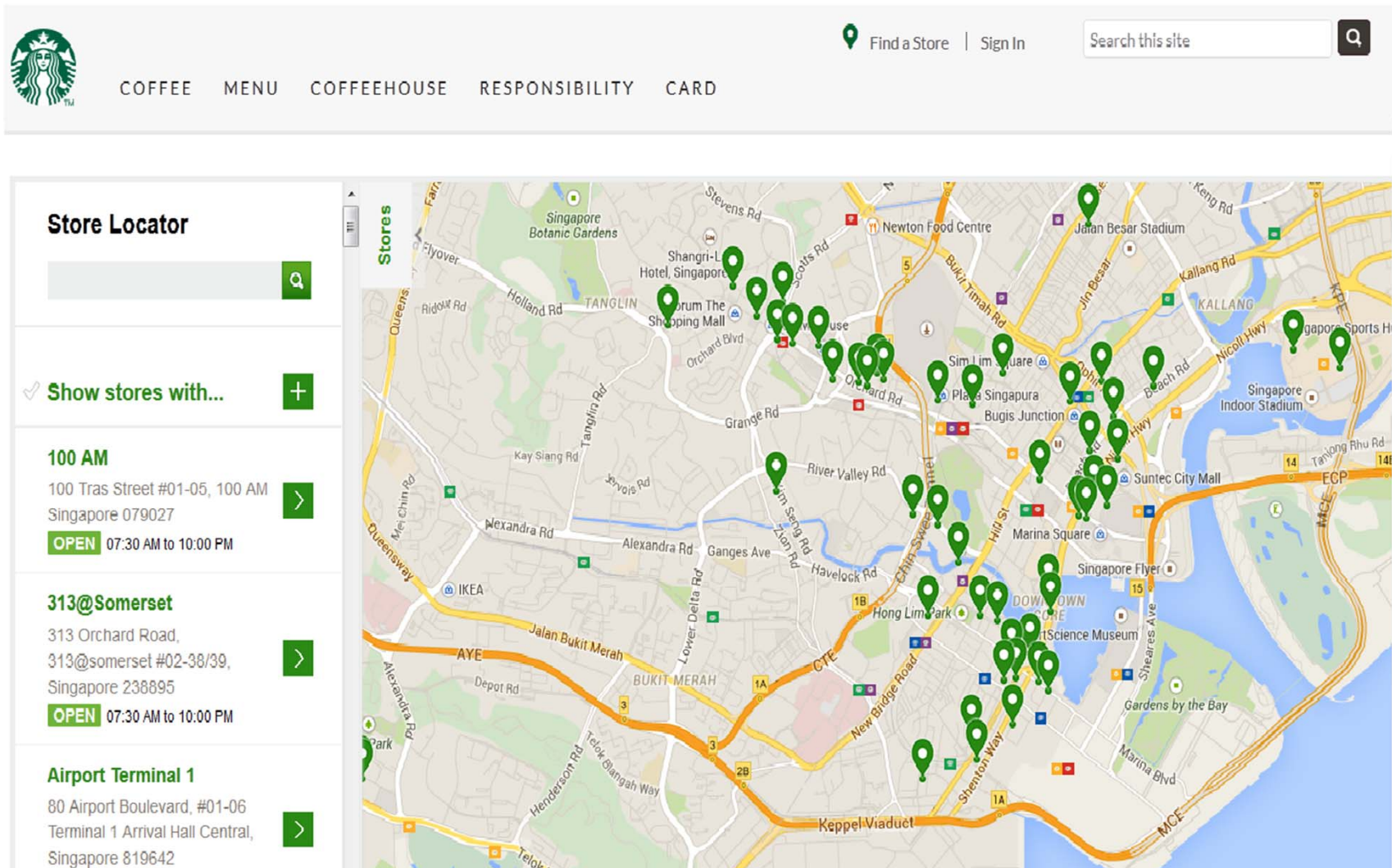
# Location-based models of Horizontal Differentiation

- **Location-based models** are models in which the heterogeneous preferences of consumers are represented by location on a line or circle
- Location can represent:
  - **geography** – physical location
  - **time** – departure time of a flight
  - **product characteristics** – crunchiness of a cereal
- Hotelling's (1929) Linear City Model
  - ex. sweetness of apple cider



- A firm chooses a product type by choosing a location

## Ex. Starbucks



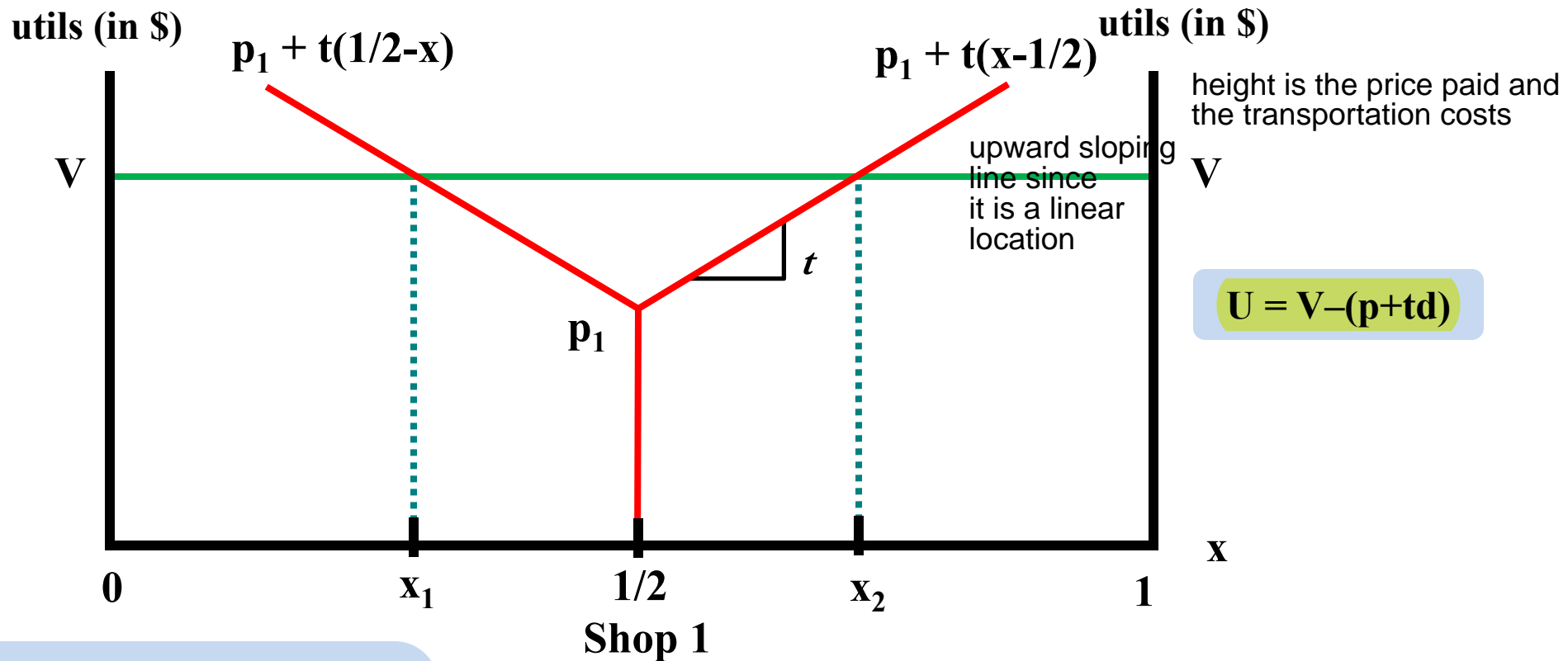
# Hotelling Model

- We'll interpret the line as a street and consider a monopolist who sells a service (e.g., a health club or spa) at its shops along the street
- **Basic Setup**
  - N consumers are **uniformly (evenly) distributed** along a street of length 1
    - $0 \leq x \leq 1$  denotes a location,  $d$  is distance
    - Consumers consider **buying one unit** of the service. The gross value of the service to the consumer is  $V$
  - consumers incur “**transportation**” costs ( $t$ ) per unit of distance traveled to buy the service (utility loss from not consuming their preferred product)
  - utility:  **$U = V - p - td$**   $\rightarrow$  consumer buys if  $U \geq 0$
- The monopolist chooses the price and where to offer the service (where to locate the shops on the street)
- Consider first the case of a **single shop**  $\rightarrow$  it is reasonable to expect the firm to locate the shop in the middle of the line ( $x = 0.5$ )



# Hotelling Model

Suppose that the monopolist sets a price of  $p_1 < V$ .  
Who buys the service?



Everyone between  $x_1$  and  $x_2$  has  $U \geq 0$  and therefore buy

$$V = p_1 + t(x_2 - 1/2) \rightarrow x_2 = \frac{1}{2} + \frac{V - p_1}{t}$$

$$V = p_1 + t(1/2 - x_1) \rightarrow x_1 = \frac{1}{2} - \frac{V - p_1}{t}$$

two marginal consumers



# Hotelling Model

- Total demand is

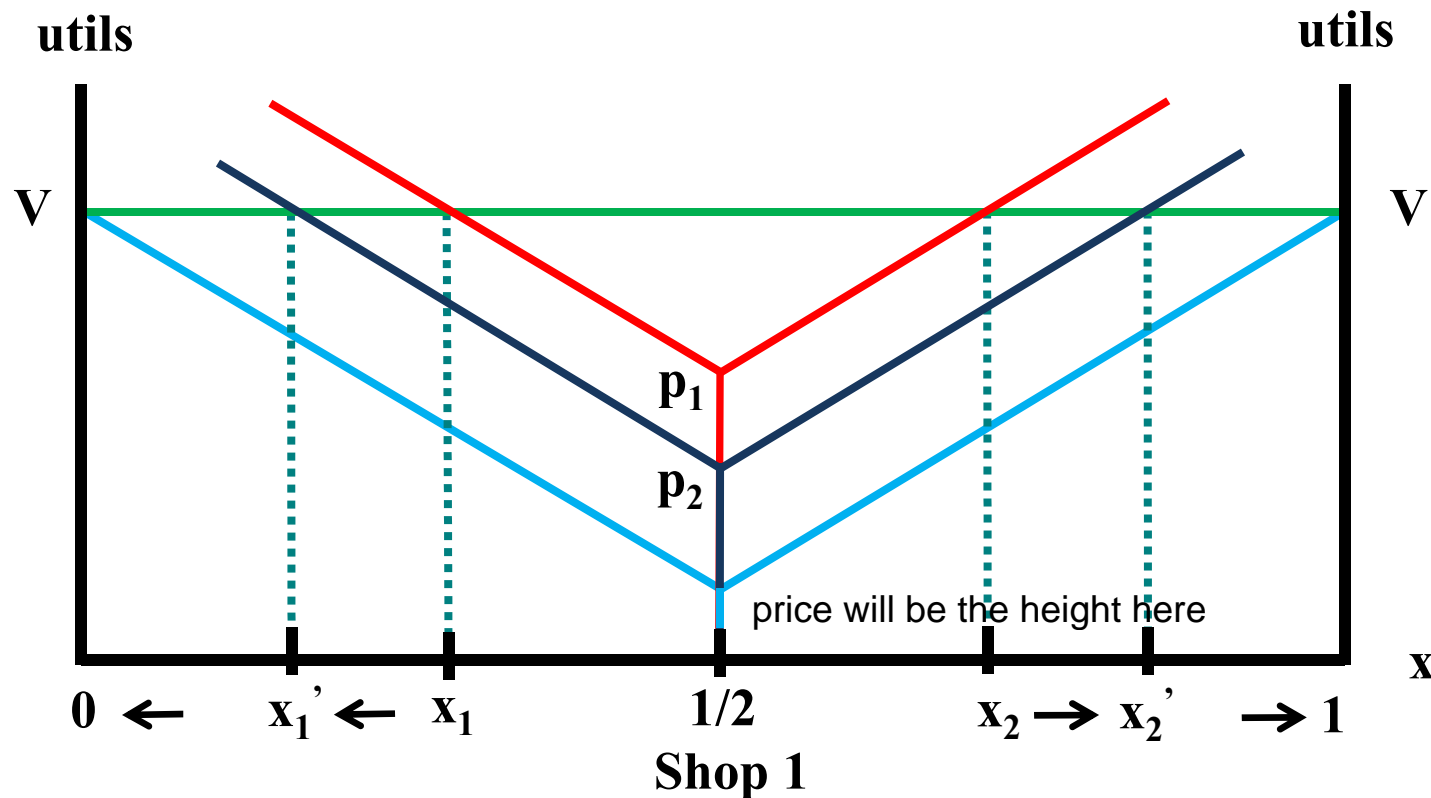
$$Q(p_1, 1) = N(x_2 - x_1) = N\left(\frac{1}{2} + \frac{V - p_1}{t} - \left(\frac{1}{2} - \frac{V - p_1}{t}\right)\right) = 2N\left(\frac{V - p_1}{t}\right)$$

- When  $V - t > c$ , the firm will sell to the whole market
  - In this case we say that the market is covered
  - We will assume throughout that this is the case
- The highest price possible is the price such that the two persons who live furthest away are just willing to buy
  - these persons are  $\frac{1}{2}$  distance away from the shop
  - transport costs are  $t/2$
  - price is then:  $V - p - t/2 = 0 \rightarrow p = V - t/2$

# Hotelling Model

Suppose the firm reduces the price to  $p_2$

If the firm decreases price enough, everyone buys.



a shop wouldn't want to set up shop at 0 or 1 since majority of the consumer will have to pay a lot of transportation cost, can extract higher price if less transportation costs are incurred

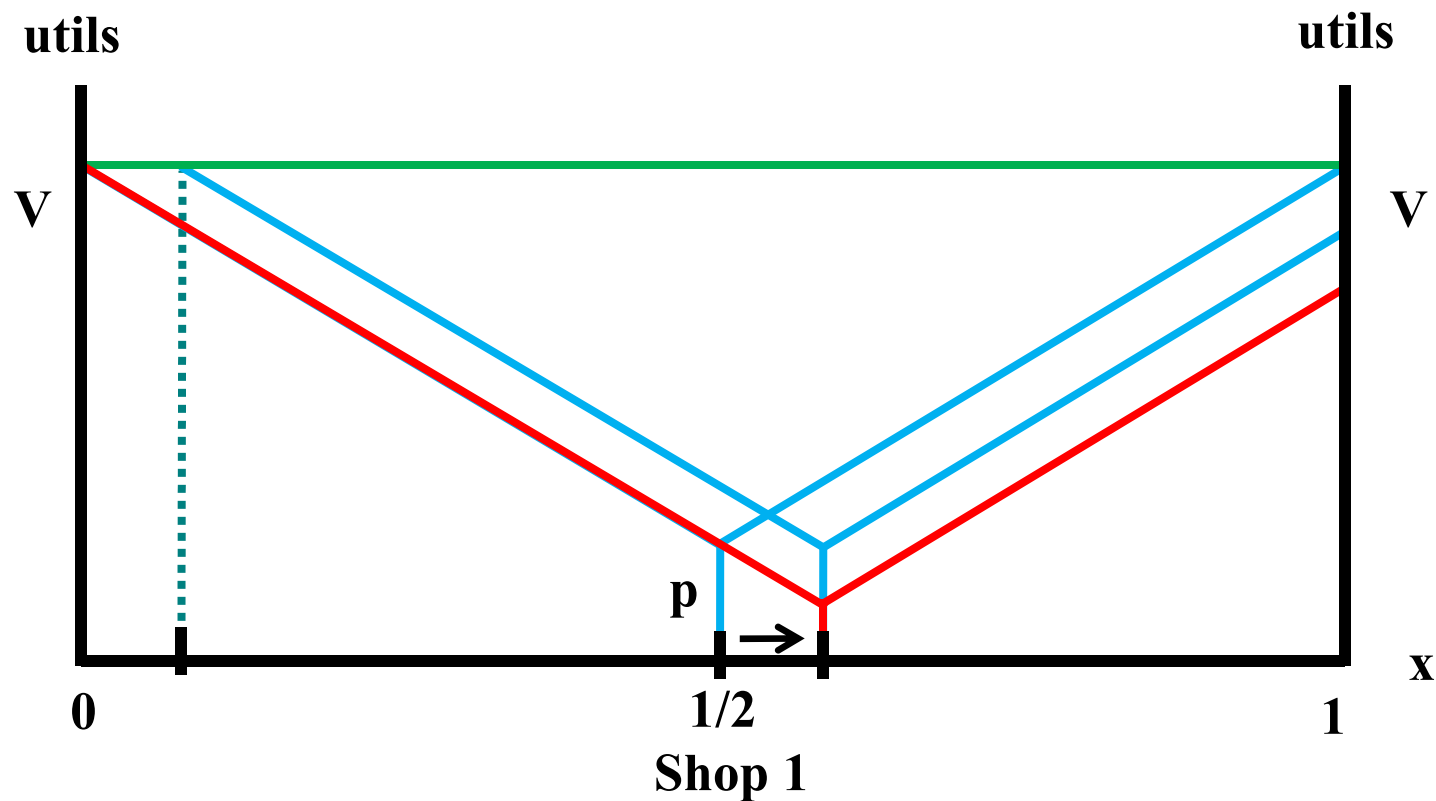
# Hotelling Model

Why does the firm set up in the middle?

Suppose the firm moves from  $x = \frac{1}{2}$

It would have to drop price to cover the market

But sales are the same...profit must be lower than at  $x = \frac{1}{2}$ .



# Hotelling Model

- What is the firm's profit?
- Let marginal cost be  $c$  and set-up costs per shop be  $F$
- Profit is then

$$\pi(1) = N \left( V - \frac{t}{2} - c \right) - F$$

- What if the firm had **two shops**?
- Since each shop has the same costs the monopolist will choose the same price for each shop
- The optimal location is for one shop to be located at  $\frac{1}{4}$  and one at  $\frac{3}{4}$
- What is the maximum price that the monopolist can set and cover the market?

# Hotelling Model

With two shops,  $\frac{1}{4}$  is the maximum distance a person has to travel



# Hotelling Model

- The person  $\frac{1}{4}$  distance away from a shop has transport costs  $t/4$
- Price is then:  $V - p - t/4 = 0 \Rightarrow p = V - t/4$
- Profit is:

$$\pi(2) = N \left( V - \frac{t}{4} - c \right) - 2F$$

price is higher but there is an additional cost of fixed fee  $F$

- What if the firm had **three shops**?
- The optimal locations are  $1/6$ ,  $1/2$ , and  $5/6$  and price is  $p = V - t/6$
- Profit is:

$$\pi(3) = N \left( V - \frac{t}{6} - c \right) - 3F$$

# Hotelling Model

- There is a general pattern emerging
- If the monopolist has  $n$  shops, it locates the  $i^{\text{th}}$  shop at  $\frac{2i-1}{2n}$
- E.g., if  $n = 3$ , then the first shop is located at  $(2*1 - 1)/(2*3) = 1/6$
- Price is  $\mathbf{p = V - t / 2n}$
- Profit is: 
$$\pi(n) = N \left( V - \frac{t}{2n} - c \right) - nF$$

high fixed costs will lead to shops opening less
- The effect of a new shop on profit is: 
$$\frac{d\pi(n)}{dn} = \frac{Nt}{2n^2} - F$$



# Hotelling Model

$$p = V - t / 2n$$

$$\frac{d\pi(n)}{dn} = \frac{Nt}{2n^2} - F$$

- Notice that as the number of shops (products) increases, the higher the price the monopolist charges
  - with more shops the closer a shop will be to each customer's location
- But introducing new shops is costly so there is a tradeoff
- Also notice that that there will be more product variety (more shops):
  - in bigger markets
  - when set up costs are small
  - when transportation cost is high
    - when transportation cost is high, it means that people really value their ideal product so it's better to introduce more products to cater to the different customers

# Product Variety

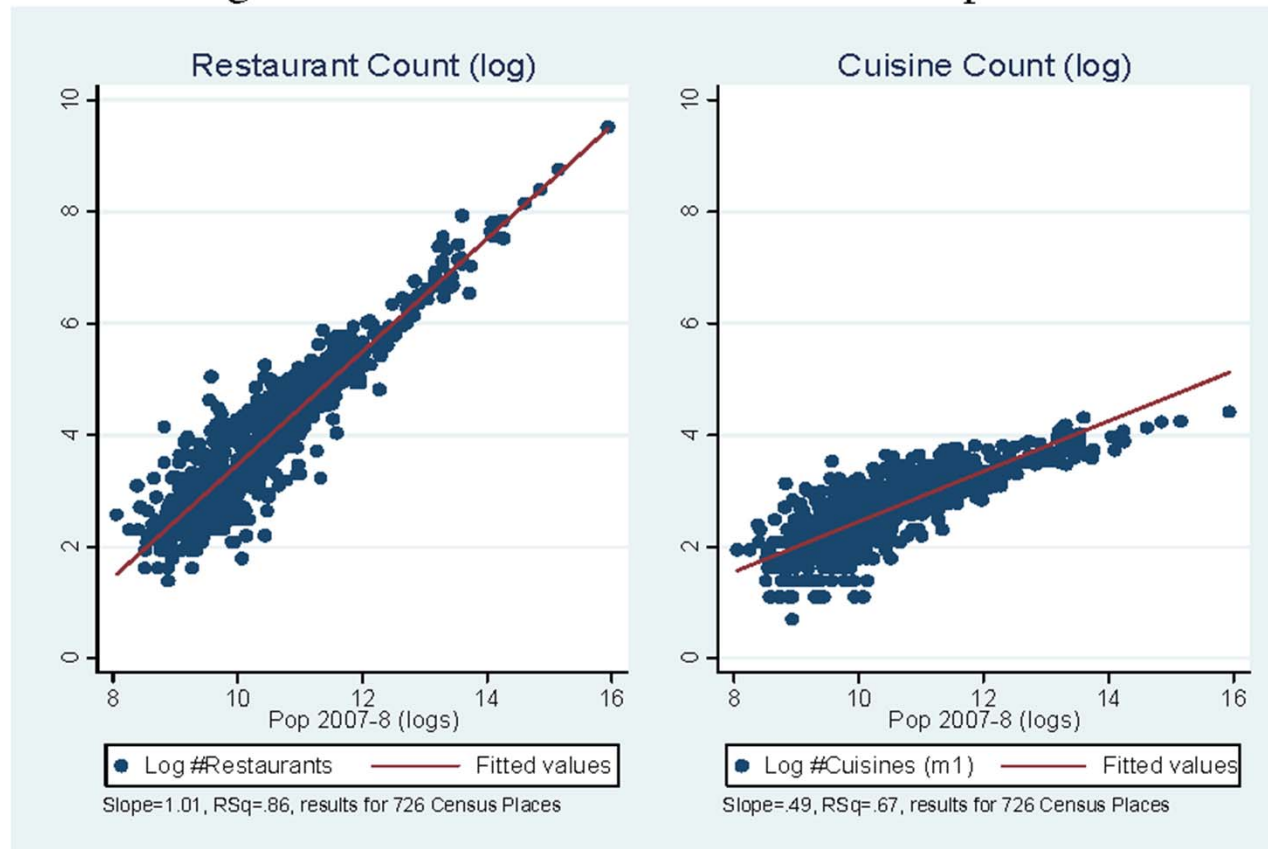
- The monopolist has an incentive to introduce a lot of products in order to extract higher prices

- Solve  $\frac{d\pi(n)}{dn} = \frac{Nt}{2n^2} - F = 0$

- The monopolist will introduce  $n^* = \sqrt{\frac{tN}{2F}}$  products

# Ex. City Size and Product Variety (Schiff)

Figure 1: Restaurants and Cuisines vs Population



# Product Variety

- Are too many products introduced? That is, is the profit-maximizing number of products the same as the socially optimal number?
- The socially optimal number of products maximizes total welfare,  $W$ :
  - $W = NV - cN - T(n) - nF$
- The total gross benefit that the customers receive is  $NV$  and the total variable cost of producing the product is  $cN$ 
  - these do not depend on  $n$  and thus can be ignored

marginal benefit of the additional food is decreasing

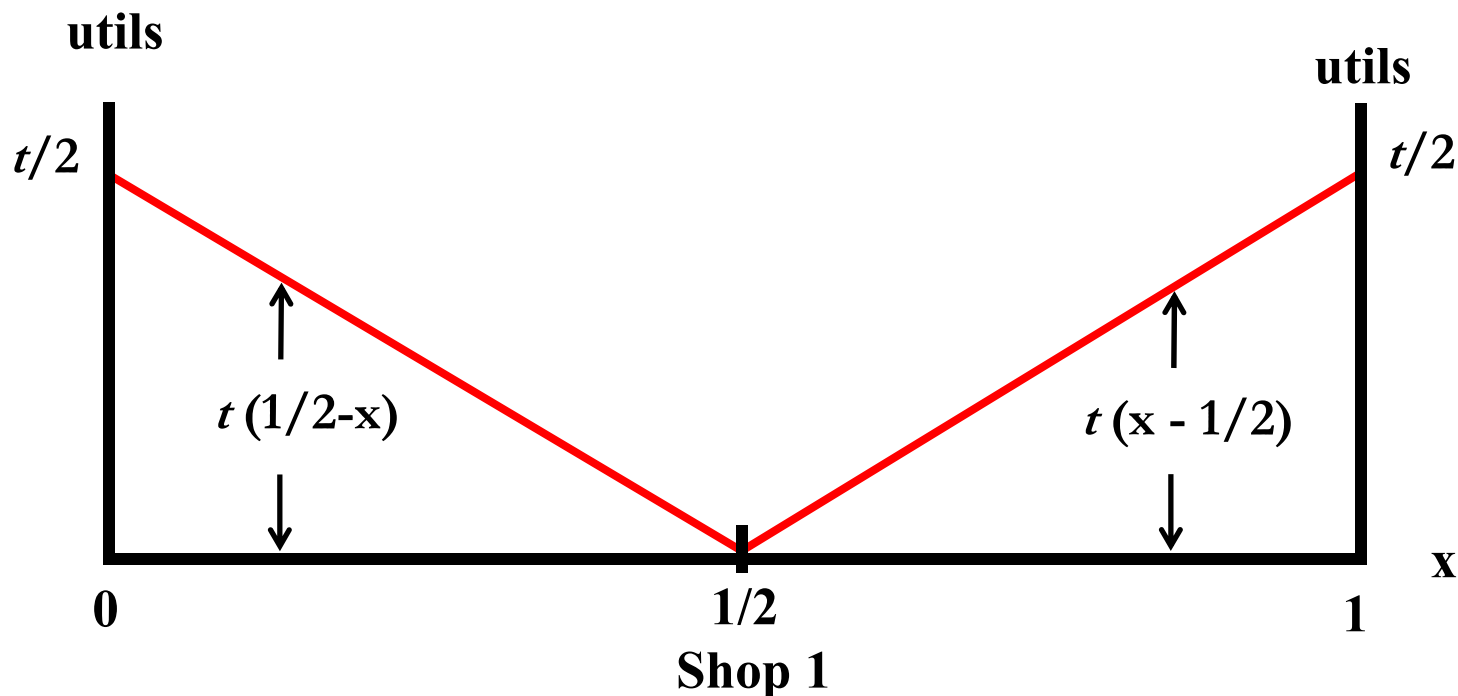
# Product Variety

- The factors that affect total welfare and depend on the number of products are total transportation costs,  $T(n)$ , and the setup costs,  $nF$ 
  - Maximizing  $W$  is equivalent to minimizing the costs  $T(n) + nF$
- The social planner, just like the monopolist, will space the shops evenly
- This makes computing  $T(n)$  relatively straightforward
- We start again with one shop

# Total transportation costs with 1 shop

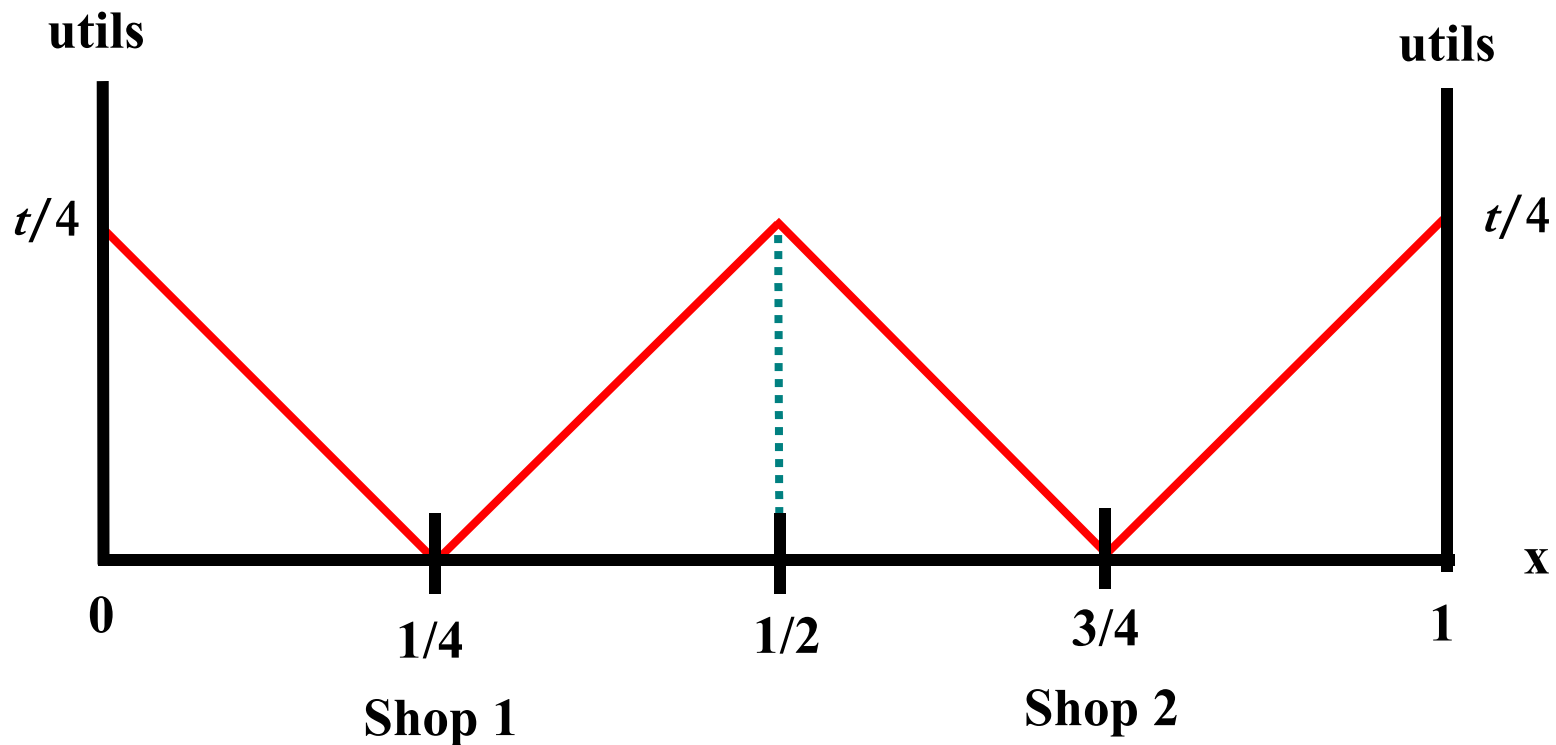
- The figure shows the transportation cost of a consumer at  $x$
- $T(1)$  is the area under the lines times the number of consumers:

$$T(1) = N \cdot 2 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{t}{2} = \frac{tN}{4}$$



# Total transportation costs with 2 shops

- With two shops:  $T(2) = N \cdot 4 \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{t}{4} = \frac{tN}{8}$   
 area under the lines





# Transportation costs with $n$ shops

- With  $n$  shops:  $T(n) = \frac{tN}{4n}$
- Thus, the total cost of  $n$  products is:  $\frac{tN}{4n} + nF$
- The welfare maximizing number of products:  $n^w = \sqrt{\frac{tN}{4F}}$
- We can compare this to the monopoly's choice:  $n^* = \sqrt{\frac{tN}{2F}}$ 
  - **The market produces too much variety**
- The firm values the higher price/extra revenue from an additional shop/product but from a social standpoint this is just a transfer from the consumer to the firm and does not affect total welfare
  - **The firm overvalues new products**

# Vertical differentiation

- Vertical differentiation means that the ranking of products is the same for all consumers
  - if two different products were sold at the same price, everyone would buy the same product
- Even though consumers rank the products similarly, their willingness to pay for the products differs
  - some consumers will pay more for a high quality product than other consumers due to, for example, differing incomes
- We will use this framework to understand what product quality a monopolist chooses and whether it is socially optimal

# Vertical differentiation

- To keep the analysis simple we will assume that the firm offers only one product
- The choice variables for the firm are quantity and quality
- The demand curve depends on the quality,  $z$ :  $P = P(Q, z)$ 
  - Each consumer buys at most one unit of the good
    - The demand curve is a ranking of the consumers' WTP for a product of quality  $z$
  - an increase in quality increases WTP, rotating the demand curve out
  - but increases in quality are costly so the firm faces a tradeoff

# Vertical differentiation

- Suppose demand and costs are given by:

$$P = z(50 - Q) \quad \text{and} \quad C(Q, z) = 5z^2$$

- Profit is then:

$$\pi(Q, z) = z(50 - Q)Q - 5z^2$$

- First-order conditions:

$$\frac{\partial \pi(Q, z)}{\partial Q} = 0 \Rightarrow z(50 - 2Q) = 0 \Rightarrow Q^* = 25$$

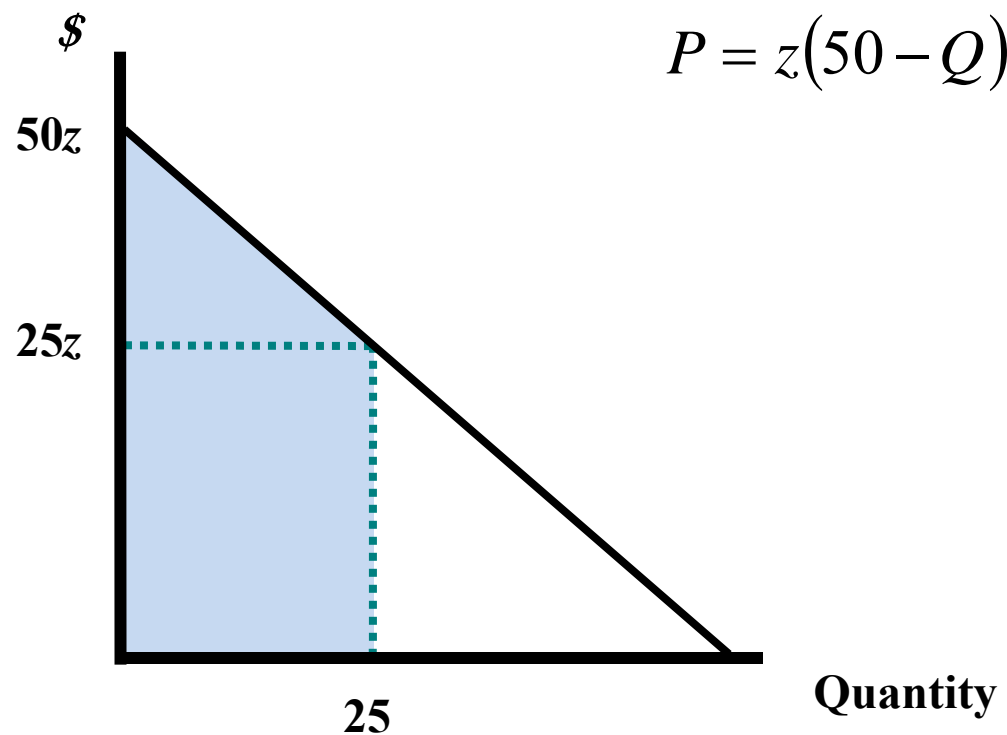
$$\frac{\partial \pi(Q, z)}{\partial z} = 0 \Rightarrow (50 - Q)Q - 10z = 0 \Rightarrow z^* = 62.5$$

# Vertical differentiation

- How does the monopolist's choice compare to the socially efficient outcome?
- The social planner would choose a higher level of quantity than the monopolist
  - *mc* of producing an additional unit for a given quality is zero
- This will affect the optimal choice of quality
- We wish to abstract from this effect so we assume that the social planner chooses the same quantity as the monopolist
- The social planner chooses  $z$  to maximize total welfare, which is the total benefit of 25 units of a product of quality  $z$  minus the cost of developing the product

# Vertical differentiation

- Total benefit is the area under the demand curve up to  $Q = 25$ :  $1875z/2$



# Vertical differentiation

- Total welfare is then: 
$$W = \frac{1875}{2} z - 5z^2$$
- Maximize  $W$  to find the socially optimal choice of quality:  $z^w = 93.75$ 
  - the monopolist chooses too little quality
- Underprovision of quality, however, is not a general result
- In some cases, the monopolist chooses too little quality, in other cases it chooses too much quality



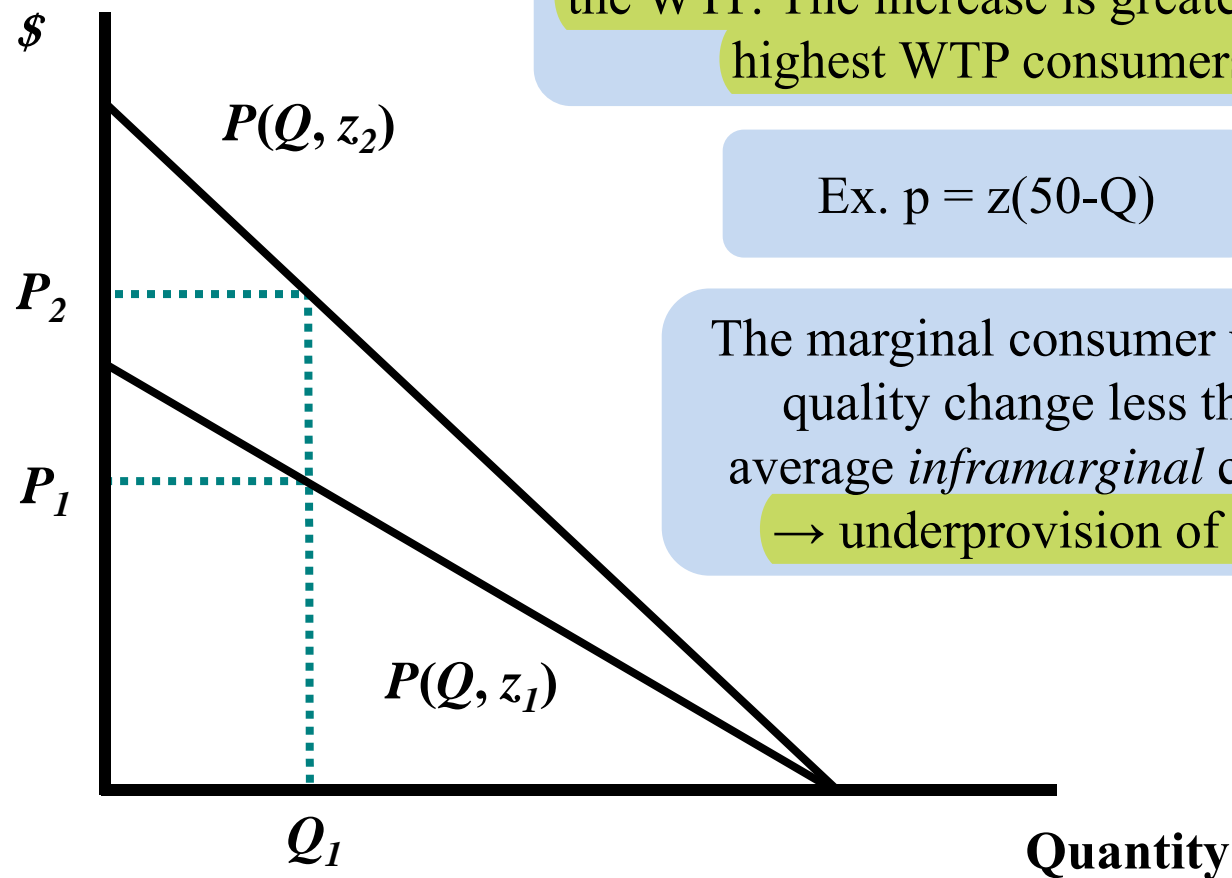
# Vertical differentiation

- A social planner cares about the increase in consumer surplus resulting from an increase in quality
  - An increase in quality affects all consumers of the good, providing them with greater surplus
- In contrast, the monopolist is only interested in the higher price it can get from the marginal consumer being willing to pay more for greater quality
- In other words, the social planner cares about the effect of quality across all consumers who buy the product (these are called *inframarginal* consumers) while the monopolist cares about the effect of quality on the *marginal* consumer

# Vertical differentiation

- For a given quantity, if the impact of quality on the willingness to pay of the average *inframarginal* consumer is greater than the impact on the *marginal* consumer, then a monopolist will undersupply quality
- This is likely to hold if consumers who value the product highly also tend to have a higher valuation for quality improvements
- For example, consumers who value the speed of computers are also likely to put a higher valuation on any improvement in the speed than a consumer who is only just willing to buy a computer at the current price

# Case 1: underprovision of quality

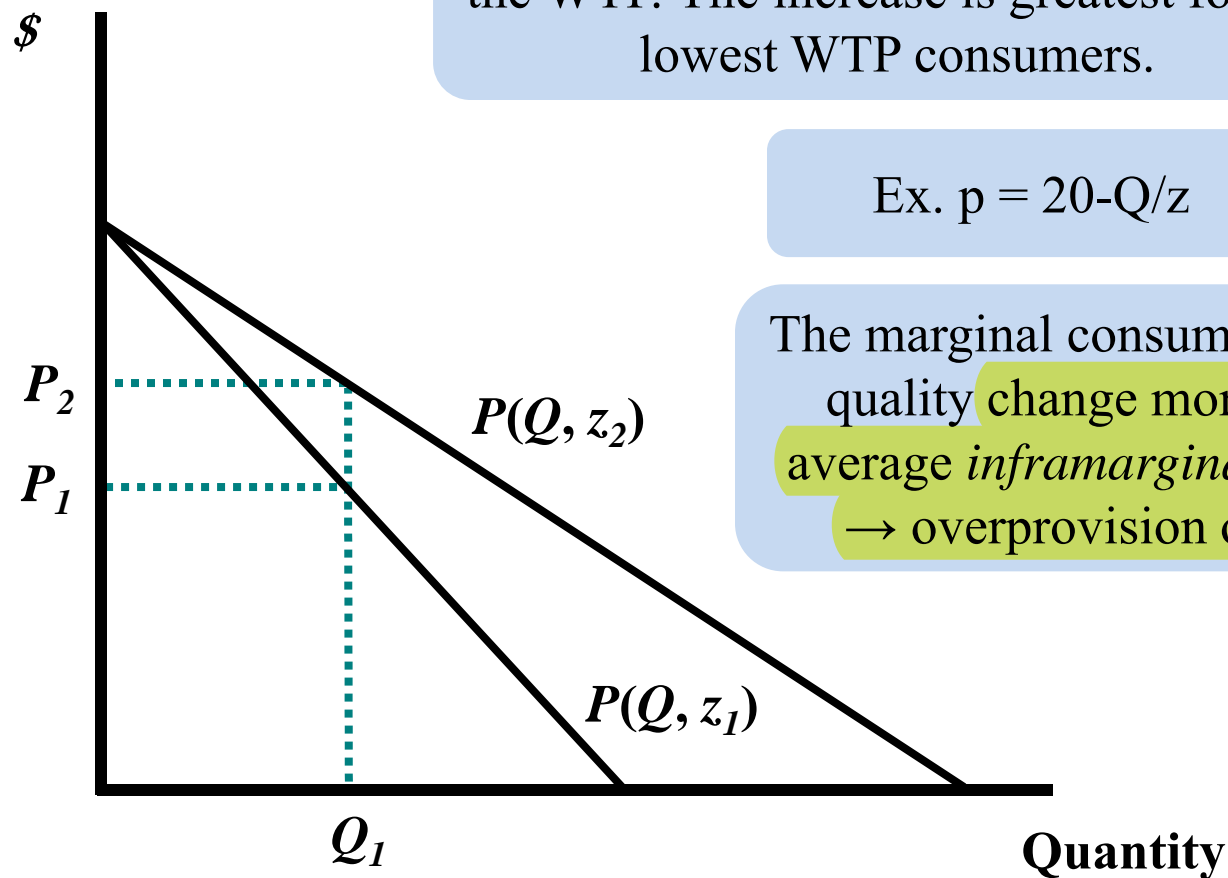


Case 1. An increase in quality increases the WTP. The increase is greatest for the highest WTP consumers.

$$\text{Ex. } p = z(50 - Q)$$

The marginal consumer values the quality change less than the average *inframarginal* consumer  
→ underprovision of quality

## Case 2: overprovision of quality



Case 2. An increase in quality increases the WTP. The increase is greatest for the lowest WTP consumers.

$$\text{Ex. } p = 20 - Q/z$$

The marginal consumer values the quality change more than the average *inframarginal* consumer  
→ overprovision of quality