Dynamic Games and Stackelberg Competition

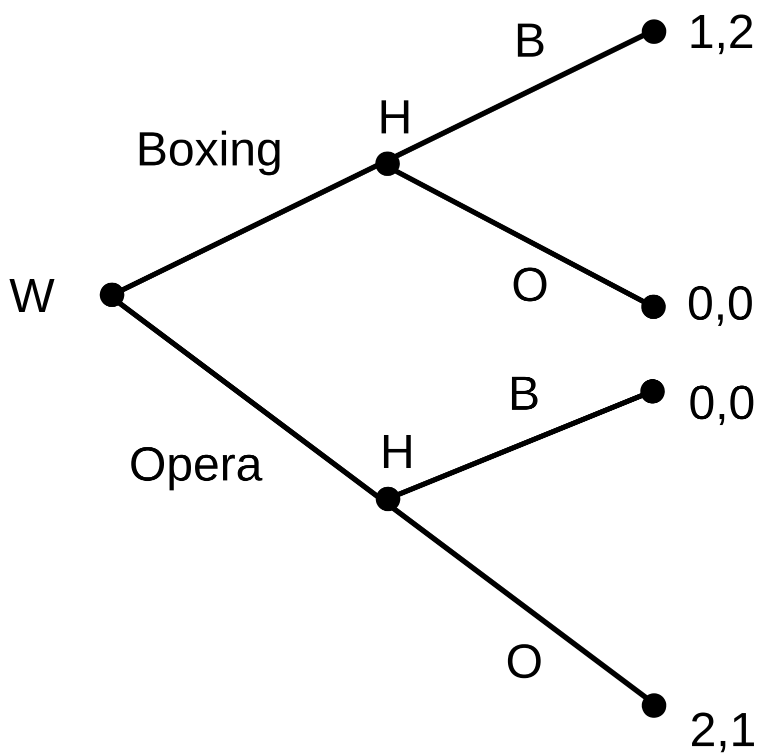
Reference: Pepall, Richards, and Norman, chapter 11
and Church and Ware's Entry Deterrence

Introduction

- In a wide variety of markets firms compete **sequentially**
  - One firm takes an action
  - The second firm observes the action and responds
- Ex. Boeing and Airbus
  - Boeing’s 417-seat 747 dominated the jumbo jet market for years after being introduced in the 1970s
  - Airbus had for some time unsuccessfully tried to challenge Boeing
  - Finally introduced the A380 with between 555 and 800 seats
- This is a **dynamic game**
  - We need to use an equilibrium concept that explicitly accounts for the dynamic nature of the game

Subgame Perfect Nash Equilibrium

- Battle of the Sexes



Husband

Wife		BB	BO	OB	OO
	B	1,2	1,2	0,0	0,0
	O	0,0	2,1	0,0	2,1

Nash-Equilibria: (B,BB), (O,BO), (O,OO)

Subgame Perfect Nash Equilibrium

- In this game there are **multiple NE**. This reduces our ability to generate useful predictions
- We need another **solution concept** that can **narrow down the set of NE** into a smaller set of (often more reasonable) outcomes
- We do so by **eliminating NE that involve non-credible threats/actions**

Subgame Perfect Nash Equilibrium

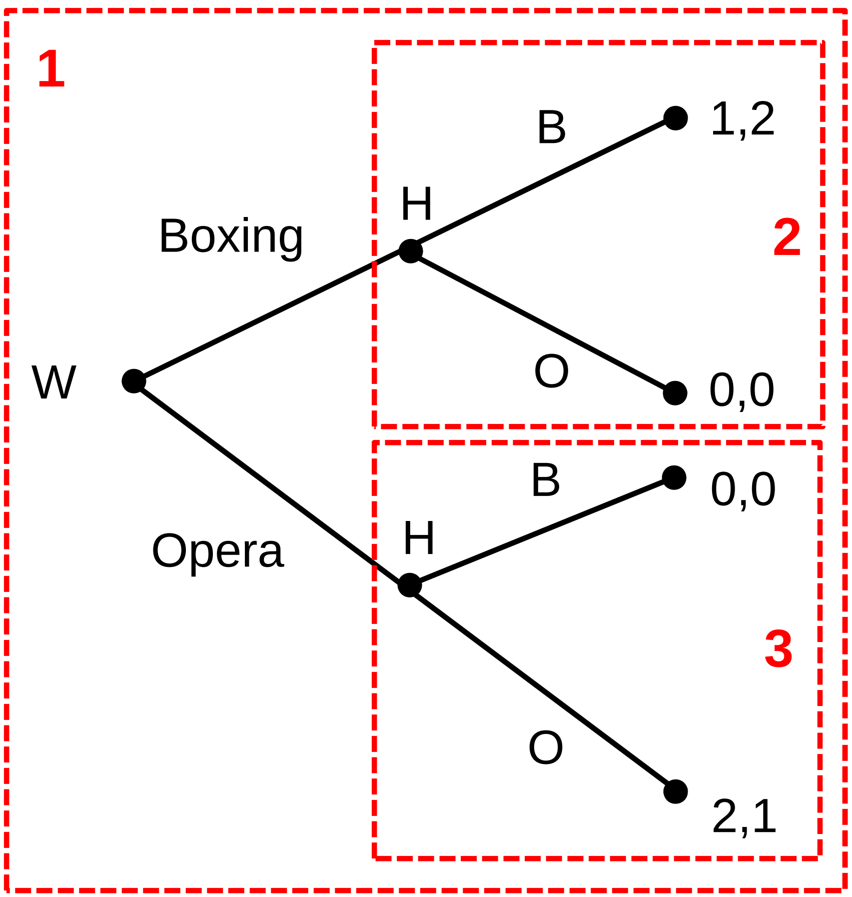
- In the example the husband's strategy (Boxing, Boxing) involves the threat that he will choose Boxing if his wife chooses Opera. But this threat is not credible ➔ If the wife chose Opera, the husband would be best off choosing Opera over Boxing

➢ Refinement: **Subgame Perfect Nash Equilibrium (SPNE)**

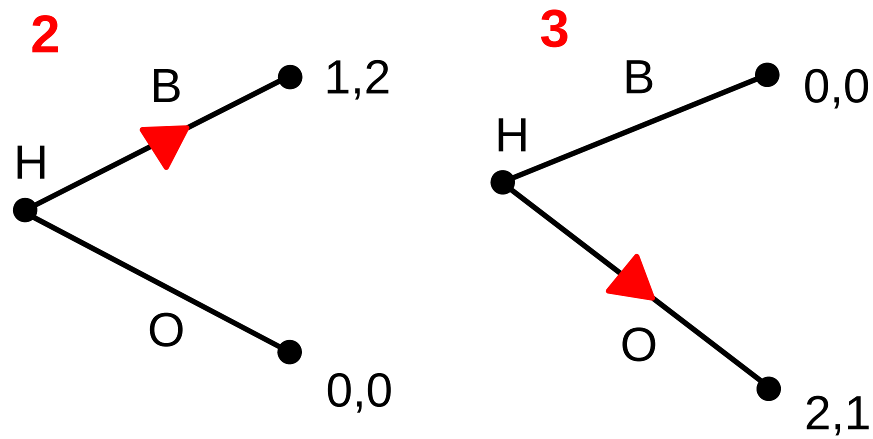
- A **subgame** is part of the game tree including a decision node (not part of an information set) and everything branching below it
  - A subgame can stand alone as a game in itself
- A set of strategies is a SPNE if it induces a NE in each subgame
  - A SPNE is a set of strategies for which each action is a best response in its particular subgame

Subgame Perfect Nash Equilibrium

- 3 subgames



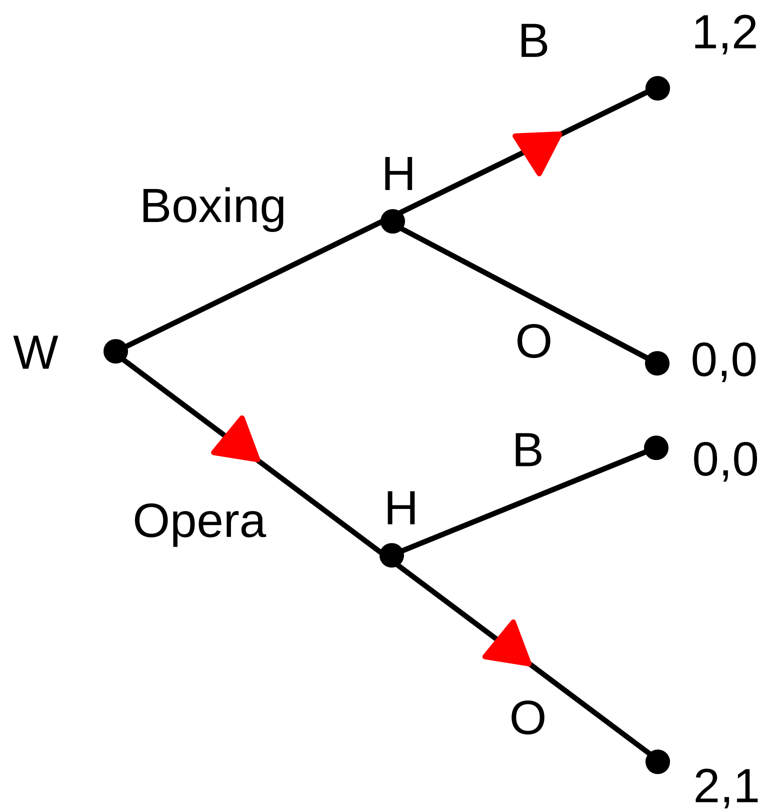
- NE: (B,BB), (O,BO), (O,OO)
- For the 2 proper subgames:



- Any NE that has the husband playing Opera in subgame 2 or playing Boxing in subgame 3 cannot be a SPNE
- Opera is the best choice for the wife when the husband plays (BO)
  ➔ (O, BO) is the SPNE

Subgame Perfect Nash Equilibrium

- This process of finding the SPNE is called backwards induction
- Start by finding the NE (best responses) in the subgames at the bottom of the tree and work backwards



Stackelberg Competition

- Firms choose output sequentially
  - The leader (firm 1) sets output first
  - The follower (firm 2) observes firm 1's output choice and chooses its own output in response

Firm 1 chooses q_1

Firm 2 observes q_1 and then chooses q_2

- We solve by backwards induction to find the SPNE

Stackelberg Competition

- As in our earlier Cournot example let demand be

$$p = A - BQ = A - B(q_1 + q_2)$$

- Marginal cost for each firm is **c**
- **Firm 1** is the **leader** and chooses $\mathbf{q_1}$ ➔ the last stage is **firm 2**'s decision
- Demand for firm 2 for any choice of output $\mathbf{q_1}$ is

$$p = (A - Bq_1) - Bq_2$$

and marginal revenue is

$$MR_2 = (A - Bq_1) - 2Bq_2$$

Stackelberg Competition

- Setting $\mathbf{MR_2 = MC}$ we find firm 2's best response: $$q_2^* = \frac{A-c}{2B} - \frac{q_1}{2}$$

- Firm 1 knows firm 2's best response and can therefore anticipate firm 2's behavior

- Demand for firm 1 is then

$$p = A - Bq_1 - Bq_2^* = \frac{A+c}{2} - \frac{B}{2}q_1$$

Stackelberg SPNE

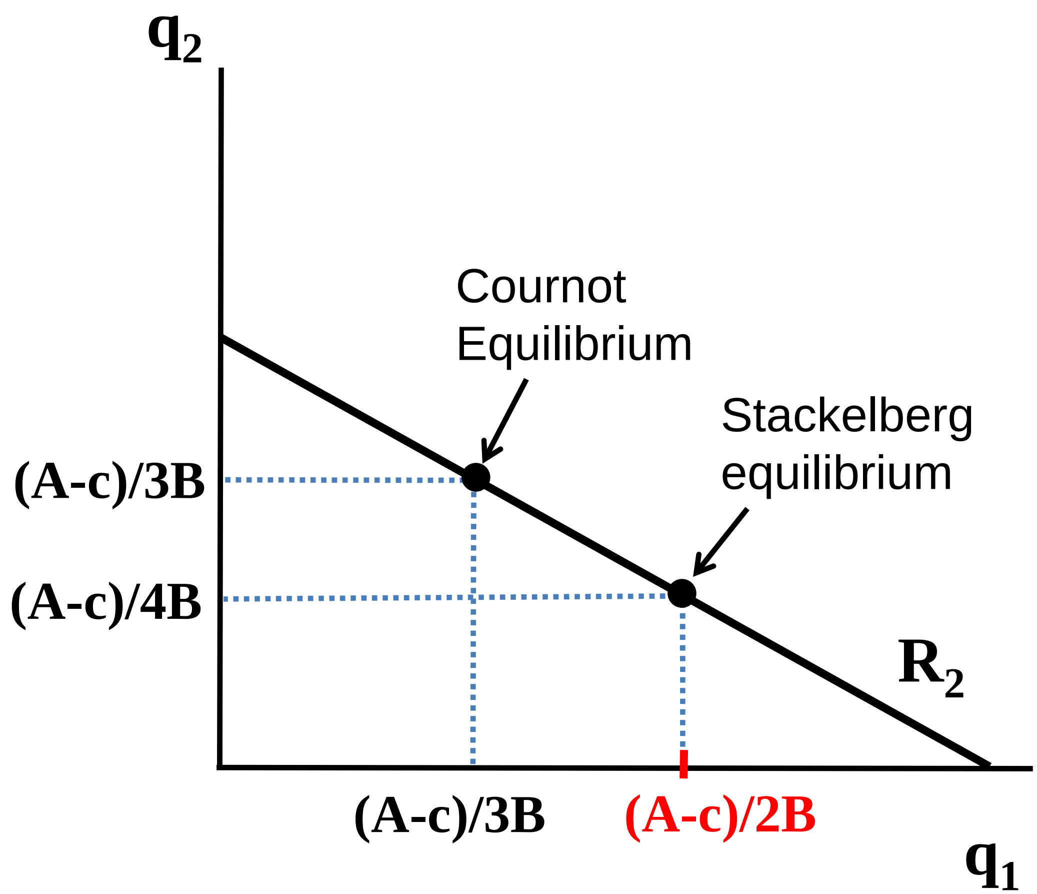
- Solving $\mathbf{MR_1 = MC}$, we find firm 1's optimal choice: $$q_1^* = \frac{A-c}{2B}$$

Equilibrium outcomes

- Substituting $\mathbf{q_1^*}$ into firm 2's reaction function we have: $$q_2^* = \frac{A-c}{4B}$$

Stackelberg Competition

- The reaction function of firm 2 is the same as in Cournot
- The leader chooses the location on $\mathbf{R_2}$ by its choice of output
- The leader chooses a higher output than in Cournot, the follower reacts by producing less
- The leader gets a greater market share and a larger profit than the follower ➔ this is the first-mover advantage



Stackelberg Competition

- Aggregate output and price:

$$Q^* = q_1^* + q_2^* = \frac{3(A-c)}{4B} \quad \text{and} \quad p^* = \frac{A+3c}{4}$$

- Profit:

$$\pi_1 = \frac{(A-c)^2}{8B} \quad \text{and} \quad \pi_2 = \frac{(A-c)^2}{16B}$$

- Remember in the Cournot equilibrium:

$$q_1^c = q_2^c = \frac{(A-c)}{3B}, \quad Q = \frac{2(A-c)}{3B}, \quad p^* = \frac{A+2c}{3}$$

- Profit:

$$\pi_1 = \pi_2 = \frac{(A-c)^2}{9B}$$

Sequential Price Competition

- We’ve seen that in a Stackelberg model with quantity choice there is a first-mover advantage. But is moving first always better than moving second?
- Consider price competition with a homogenous product and identical marginal costs
- Is **p = mc** still the outcome?
  - Would the leader raise the price above **mc**? The follower would undercut to earn all profits
  - Would the leader lower the price below **mc**? The follower wouldn’t match or undercut price in order to avoid losses
  - ➢ There is no incentive for the leader to deviate from **p = mc** and therefore the Stackelberg outcome is the same as in the simultaneous-move model

Stackelberg – Entry deterrence

- In the previous discussion of Stackelberg we implicitly assumed that the leader would accommodate entry by the follower (i.e., the follower has positive output)
- However, the leader may be able to **deter entry**
- The profit of the follower is a function of both firm 1 and firm 2's output:

$$\pi_2(q_2, q_1)$$

- We can replace $\boldsymbol{q_2}$ with firm 2's reaction function, $\boldsymbol{R_2(q_1)}$

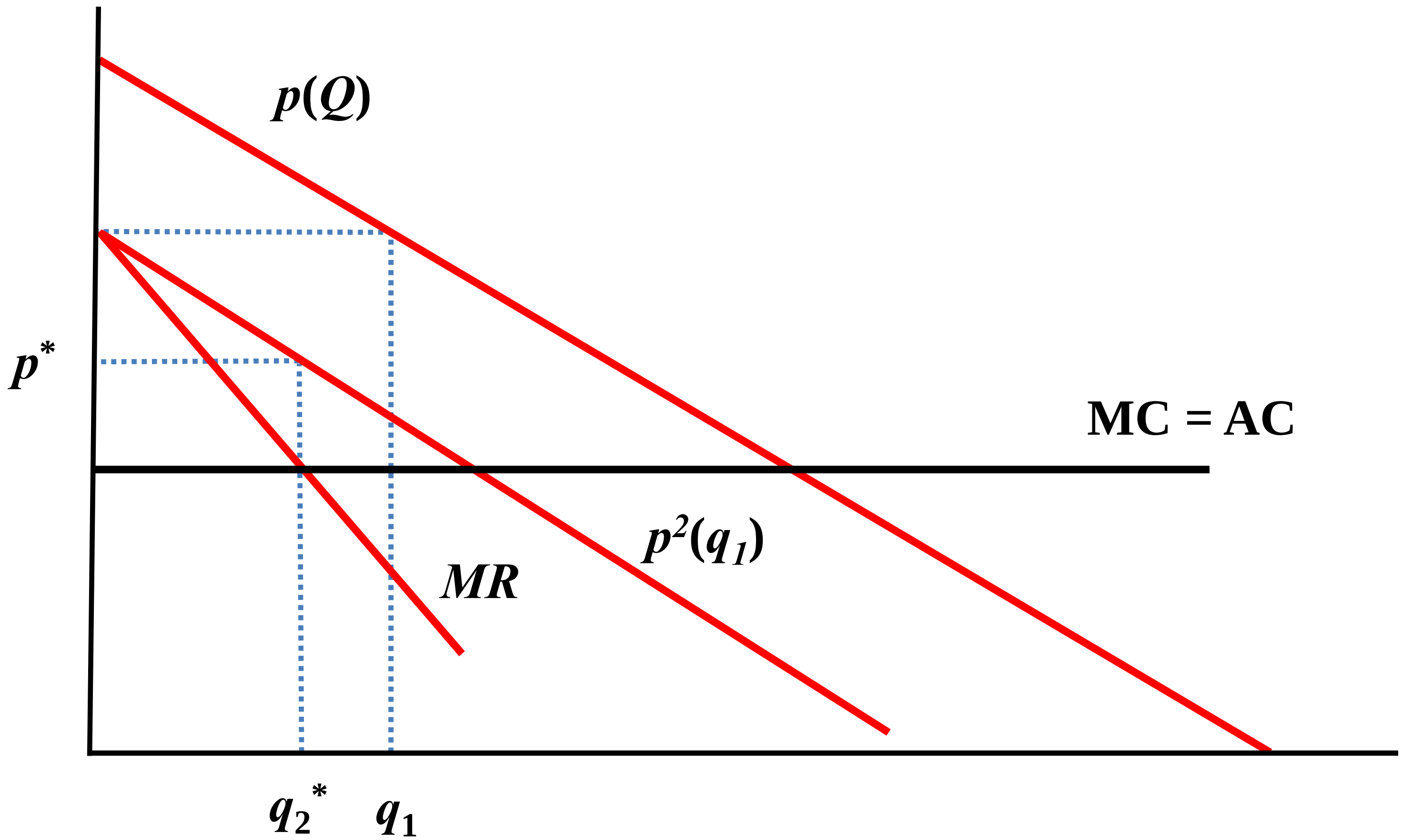
$$\pi_2(R_2(q_1), q_1)$$

- The minimum level of firm 1's output that deters entry by the follower is called the limit output, $\boldsymbol{q_1^L}$. The limit output is defined by:

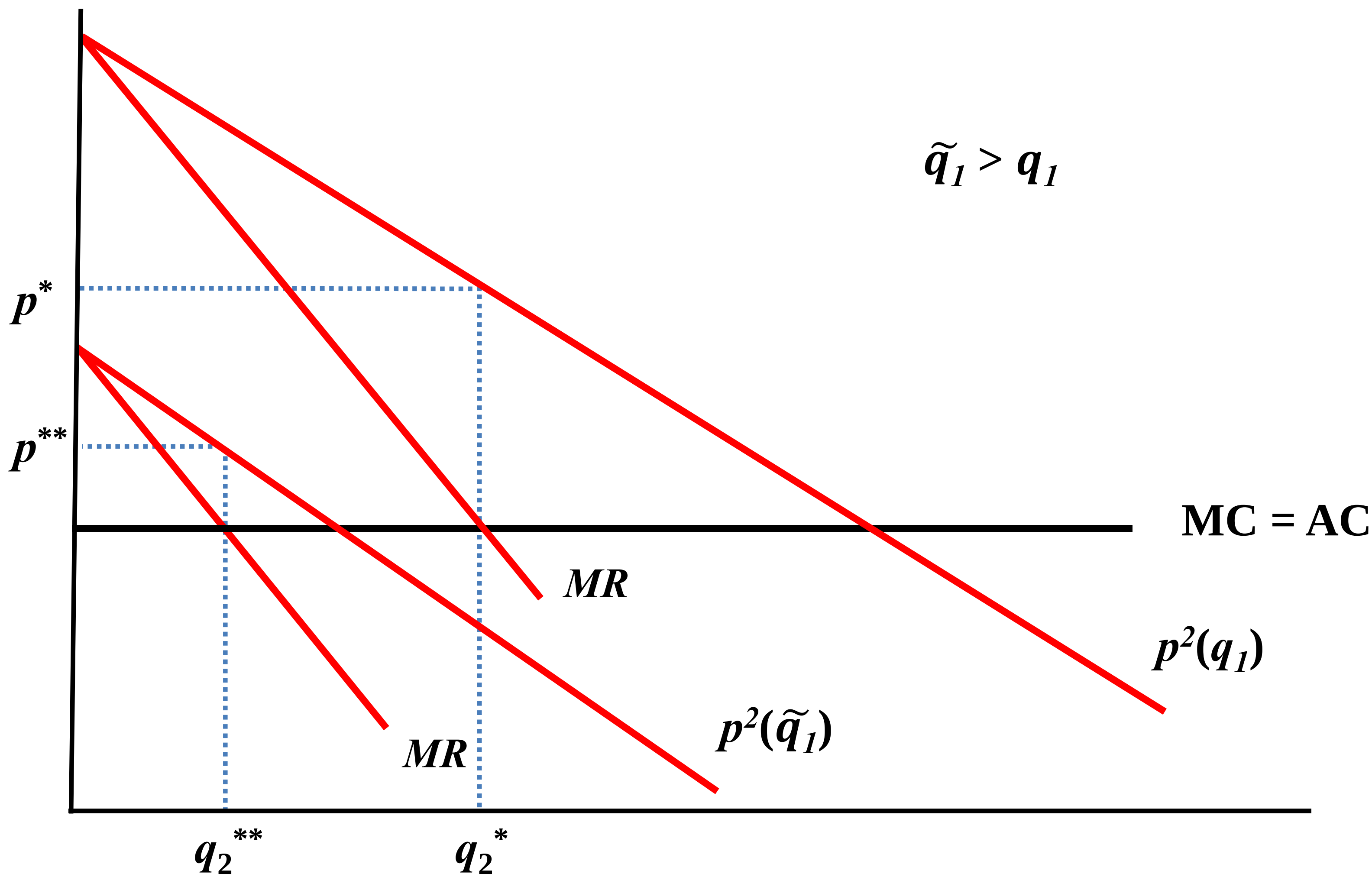
$$\pi_2(R_2(q_1^L), q_1^L) = 0$$

Stackelberg – Entry deterrence

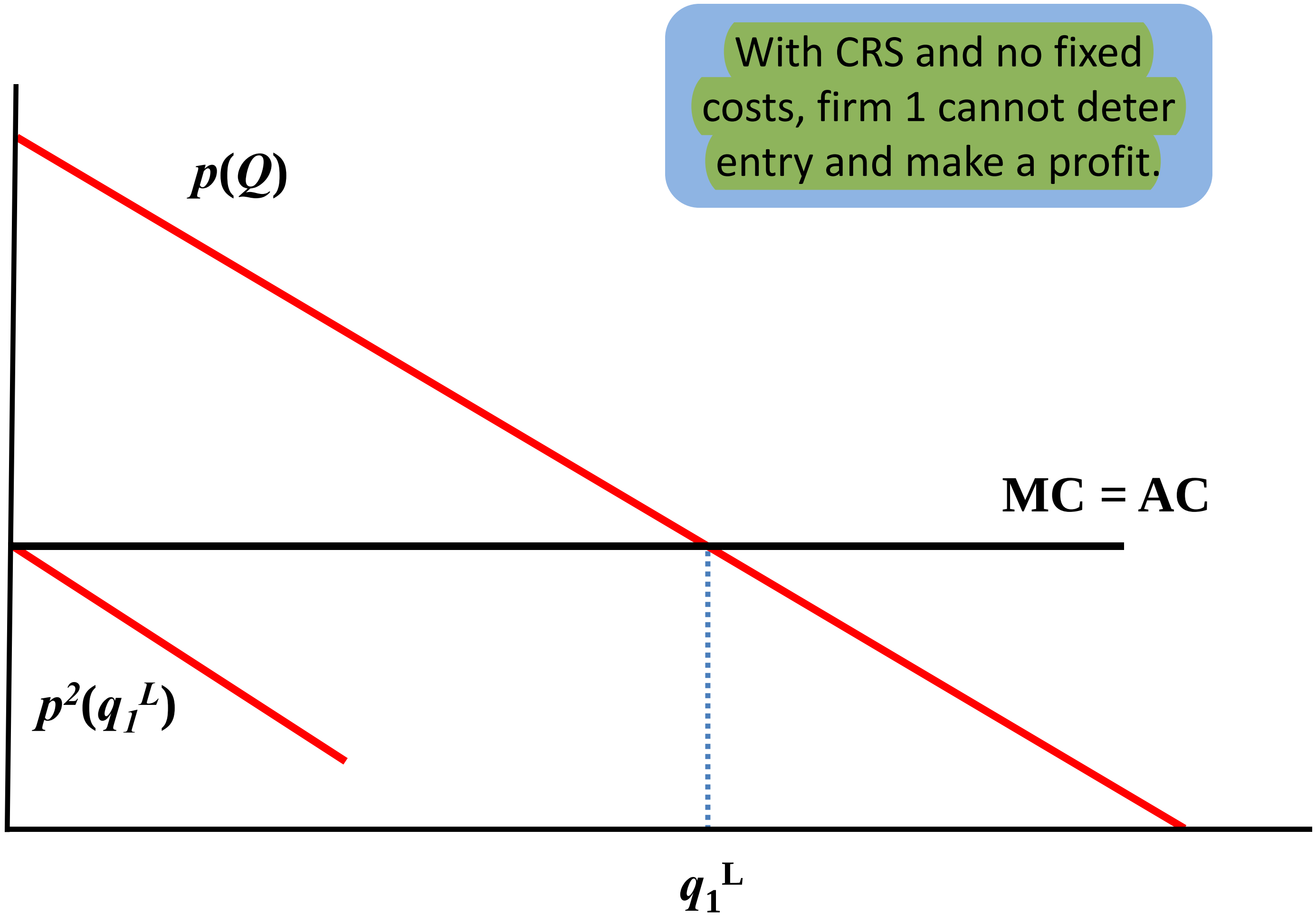
- Suppose that **MC = c** and that there are no fixed costs
- Consider the decision of firm 2 given the output if firm 1



Stackelberg – Entry deterrence



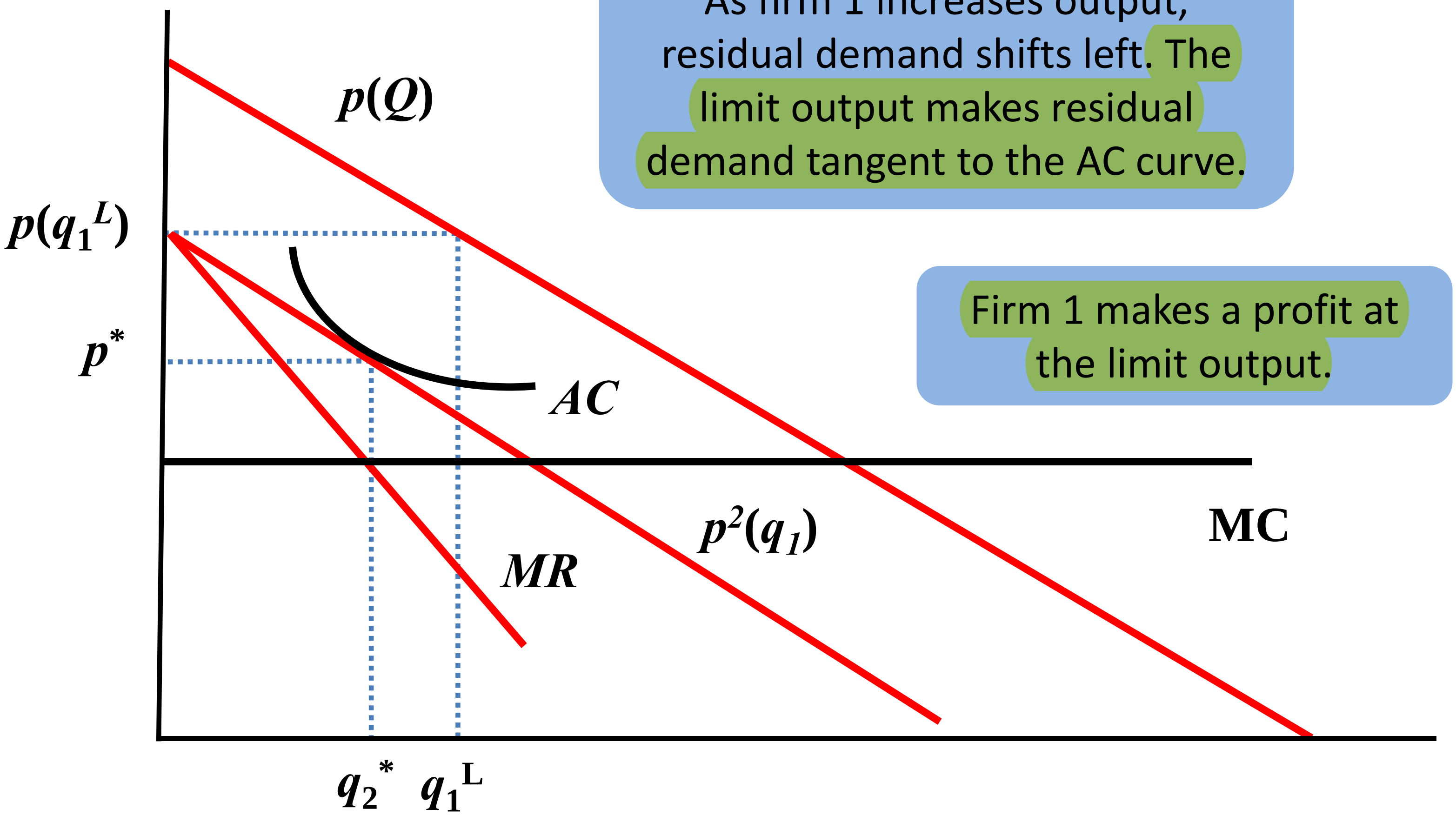
Stackelberg – Entry deterrence



Stackelberg – Entry deterrence

fixed costs has made firm 1's job of keeping firm 2 out of the market easier.

- Suppose that **MC = c** and fixed costs are **F** (economies of scale)



As firm 1 increases output, residual demand shifts left. The limit output makes residual demand tangent to the AC curve.

Firm 1 makes a profit at the limit output.

Stackelberg – Entry deterrence

- Suppose that demand is $\boldsymbol{p = A - BQ}$ and the cost function is $\boldsymbol{C = cq_i + F}$
- To find the limit output we first find firm 2's reaction function.

$$\pi_2(q_2, q_1) = (A - Bq_1 - Bq_2)q_2 - cq_2 - F$$

$$\rightarrow q_2^* = \frac{A - Bq_1 - c}{2B}$$

- Plug the reaction function back into the profit function:

$$\pi_2^*(q_1) = \frac{(A - Bq_1 - c)^2}{4B} - F$$

- The limit output is the $\boldsymbol{q_1}$ that sets profit equal to zero:

$$q_1^L = \frac{A - c - \sqrt{4BF}}{B}$$

Stackelberg – Entry deterrence

- The profit-maximizing choice for firm 1 depends on whether profits are higher when accommodating entry or deterring entry
- This will depend on how much firm 1 must expand output to deter entry
  - when fixed costs are small firm 1 has to expand output a lot to deter entry ➔ accommodation is likely better
  - when fixed costs are large firm 1 does not have to expand output a lot ➔ deterrence is likely better
- If firm 1 accommodates, it receives profits: $\pi_1^S = \frac{(A-c)^2}{8B} - F$
- If firm 1 deters entry, it earns profits at the limit output:

$$\pi_1^L = \frac{A - c - \sqrt{4BF}}{B}\sqrt{4BF} - F$$

Stackelberg – Entry deterrence

- Whether entry deterrence or accommodation is optimal depends on the magnitude of fixed costs
- Let $A = 28$, $c = 4$, and $B = 1$.
- *Post-entry* profits for various fixed costs are:

Fixed Costs	Stackelberg Profits	Entry Deterrence Profits
1	72	44
4	72	80
9	72	108

- For values of F less than approximately 3, accommodation is more profitable. For values greater than 3, deterrence is more profitable

Empirical Application: Entry Deterrence in the Pharmaceutical Market

- Glenn Ellison and Sara Ellison look at the use of advertising by pharmaceutical companies about to lose their patents
- Advertising by the incumbent has considerable spillover to the generic product because it calls attention to the function of the drug, its potential benefit, its proper use, etc.
- So advertising by the incumbent today will help tomorrow's entrant. Hence if incumbents want to deter entry they should reduce advertising prior to the expected emergence of a rival

Empirical Application: Entry Deterrence in the Pharmaceutical Market

- They test their prediction with so-called detail advertising
- Detail advertising is promotional efforts by the drug companies to influence physicians' prescribing practices
- Their sample consists of 64 drugs whose patents expired between 1986 and 1992

Empirical Application: Entry Deterrence in the Pharmaceutical Market

- The relationship between entry and strategic deterrence is likely non-monotonic
  - Deterring entry is probably not worth it in markets where entry is very likely or very unlikely
- Thus, they predicted that deterrence efforts will first rise as the probability of entry rises from a low value to an intermediate value and then fall as the probability of entry rises from an intermediate value to a high value

Empirical Application: Entry Deterrence in the Pharmaceutical Market

- First, they grouped the markets into three types based on the characteristics of the product:
  - low probability of entry
  - intermediate probability of entry
  - high probability of entry

Empirical Application: Entry Deterrence in the Pharmaceutical Market

- Next, they looked at the time trend in the value of detail advertising relative to its average for each month starting 36 months prior to patent expiration and the 12 months after

$$\frac{Advertising_{it}}{AveAdvertising_{it}} = 1 + \left(\beta_1 LowEntry_i + \beta_2 IntEntry_i + \beta_3 HighEntry_i\right) Time + \varepsilon_{it}$$

- *Time* is a time trend that increases by one as months move closer to the expiration date
- $LowEntry_i$, $IntEntry_i$ and $HighEntry_i$ are dummy variables that indicate the category that market i is in

Empirical Application: Entry Deterrence in the Pharmaceutical Market

$$\frac{\partial\left(\dfrac{\text{Advertising}_{\text{it}}}{\text{AveAdvertising}_{\text{it}}}\right)}{\partial Time} = \beta_1 \text{Low}Entry_i + \beta_2 IntEntry_i + \beta_3 HighEntry_i$$

- The hypothesis is that $\boldsymbol{\beta_2}$ will be significantly less than $\boldsymbol{\beta_1}$ or $\boldsymbol{\beta_3}$,

Coefficient	Estimated Value	Standard Error
$\boldsymbol{\beta_1}$	-0.007	0.013
$\boldsymbol{\beta_2}$	-0.032	0.009
$\boldsymbol{\beta_3}$	0.009	0.007

- As predicted, $\boldsymbol{\beta_2}$ is noticeably smaller than the other two coefficients
- This result implies that the incumbent's detailed advertising falls by 3% per month in markets with an intermediate chance of entry, providing some evidence of strategic deterrence in pharmaceutical markets