$\begin{array}{c} {\rm EC3322} \\ {\rm Industrial~Organization~I} \\ {\rm Semester~1,~2015\text{--}2016} \\ {\rm Tutorial~\#8} \end{array}$

You will receive full credit if you present your attempt at the solution during tutorial, whether or not you have the correct answer. Also, feel free to discuss the questions and answers with other students who have not yet attended tutorial. However, I request that you do not ask former students of this module or current students who attend an earlier tutorial than you for the answers before your own tutorial has taken place.

- 1. (Final Exam 2011 Semester 2) Consider a two-period Stackelberg model with three firms. Firm 1 chooses q_1 in period 1. Firm 2 chooses q_2 and firm 3 chooses q_3 simultaneously in period 2. The market inverse demand function is p = 60 5Q and marginal cost and fixed cost is zero. Solve for the equilibrium output level of each firm and the market price in the Stackelberg SPNE.
- 2. Consider a market with two firms that choose quantities. Firm 1 is already in the market, and firm 2 is a potential entrant. Market demand is p = 1000 3Q. The sequence of actions are as following. First, firm 1 chooses its production level q_1 . Second, firm 2, having observed q_1 , decides whether to enter and, if it enters, its production level, q_2 . Each firm has cost function $C(q_i) = 100q_i + F$, where F is a fixed cost. If firm 2 decides not to enter the market, it produces zero quantity and earns zero profit.
 - (a) Derive firm 2's optimal quantity as a function of q_1 assuming that firm 2 enters.
 - (b) Suppose F = 0. Derive the subgame perfect Nash equilibrium. Also derive equilibrium price and profits.
 - (c) Now suppose that F > 0. Compute the limit output, q_1^L , as a function of F. Also find firm 1's profit if it produces the limit output.
 - (d) Would firm 1 deter entry if F = 300? If F = 2700?

3. (Final, Sem 1, 2013-14) Consider the following payoff matrix:

$\begin{array}{c|cccc} & & & & & \\ & & W & R \\ \\ Bobby & W & 1,1 & 5,0 \\ R & 0,5 & 3,3 \end{array}$

- (a) If the game is played once, what is the Nash equilibrium (or equilibria)?
- (b) If the players were able to write and enforce a collusive contract before playing the game, what strategies would they choose to maximize their joint payoffs?
- (c) Assume now that the game is repeated twice. Are there any strategies that would allow both players to reach the collusive outcome described in (b) as a sub-game perfect Nash equilibrium (SPNE)? Explain.

Assume now that the game is repeated infinitely many times and that both players discount future payoffs according to the discount factor $0 < \delta < 1$.

- (d) Describe the trigger strategy that leads to sustainable collusion when δ is large enough.
- (e) What is the smallest discount factor (δ) that sustains collusion?
- 4. The demand faced by four firms is given by p = 260 2Q. Each firm has a marginal cost of MC = 20.
 - (a) Derive the one period Cournot Nash equilibrium, and the price and profits of all firms in equilibrium.
 - (b) What is output, price, and profit of each firm if they collude to produce the monopoly output? Assume that there is an equal division of monopoly output among firms.
 - (c) Suppose that one firm decides to cheat on the agreement, and each of the three other firms continue to produce one fourth of the monopoly output. How much will

- the deviating firm produce? Derive the price, the deviating firm's profit, and the other firms' profit.
- (d) Suppose now that the game is infinitely repeated. Derive the minimum value of the discount factor necessary to sustain the collusive agreement under the trigger strategy discussed in lecture.
- (e) Suppose now that there are only 2 firms. Re-derive your answer in part (d). Is it more difficult to sustain collusion with fewer firms?