

## Logistic Regression

Logistic regression is a learning algorithm used in a supervised learning problem when the output  $y$  are all either zero or one. The goal of logistic regression is to minimize the error between its predictions and training data.

Example: Cat vs No - cat

Given an image represented by a feature vector  $x$ , the algorithm will evaluate the probability of a cat being in that image.

$$\text{Given } x, \hat{y} = P(y = 1|x), \text{ where } 0 \leq \hat{y} \leq 1$$

The parameters used in Logistic regression are:

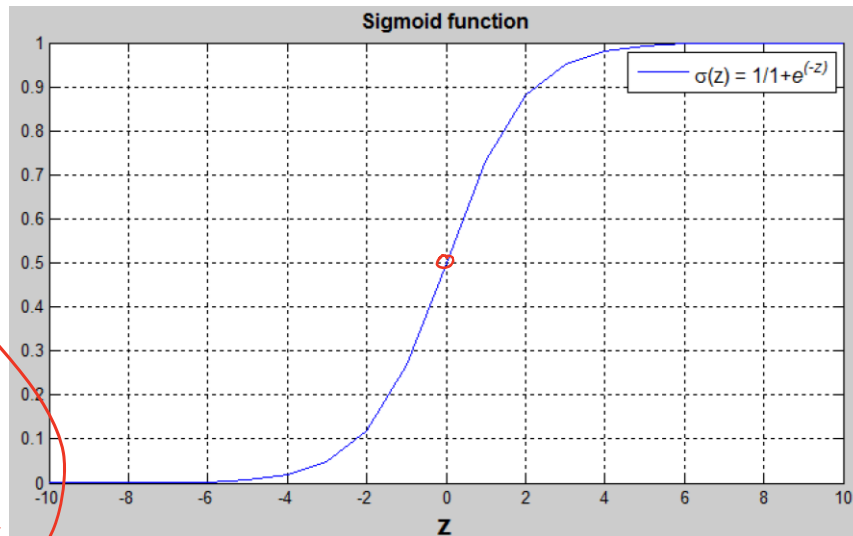
- The input features vector:  $x \in \mathbb{R}^{n_x}$ , where  $n_x$  is the number of features
- The training label:  $y \in \{0,1\}$
- The weights:  $w \in \mathbb{R}^{n_x}$ , where  $n_x$  is the number of features
- The threshold:  $b \in \mathbb{R}$
- The output:  $\hat{y} = \sigma(w^T x + b)$
- Sigmoid function:  $s = \sigma(w^T x + b) = \sigma(z) = \frac{1}{1 + e^{-z}}$

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$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

在神经网络中分开  
表示  $w$  和  $b$  更方便

$$\begin{aligned} x_0 &= 1, x \in \mathbb{R}^{n_x+1} \\ \hat{y} &= \sigma(Q^T x) \\ Q &= \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_{n_x} \end{bmatrix} \begin{matrix} \} b \leftarrow \\ \} w \leftarrow \end{matrix} \end{aligned}$$



$(w^T x + b)$  is a linear function ( $ax + b$ ), but since we are looking for a probability constraint between  $[0,1]$ , the sigmoid function is used. The function is bounded between  $[0,1]$  as shown in the graph above.

Some observations from the graph:

sigmoid 将  $(-\infty, +\infty)$  映射到  $(0, 1)$

- If  $z$  is a large positive number, then  $\sigma(z) = 1$
- If  $z$  is small or large negative number, then  $\sigma(z) = 0$
- If  $z = 0$ , then  $\sigma(z) = 0.5$