Supplementary material

IV. Proposed Method

In this part, the problem statement in this study is formally given firstly, and then the details of the proposed method are described.

A. Notation

In this paper, NN_K denotes the selected k-nearest neighbors. $\mathbf{V}_{1e}{}^{L}$, $\mathbf{V}_{1e}{}'^{L}$ denote the Lth-layer deep envelope samples generated by DSEN and DSENLG, respectively. c_L represents the Lth-layer number of clusters and φ denotes an implicit but generic transformation. $\mathbf{\Psi}, \mathbf{\Psi}_{\mathrm{V}}, \mathbf{\Psi}_{\mathrm{e}}$ represents the kernel Gram matrices. $\|\bullet\|_F, \|\bullet\|_2$ are the F-norm and 2-norm. $\oplus$ and $\lfloor \ \rfloor$ denote the concatenation operator and rounding down operator. $\mathbf{I}$ denotes the identity matrix.

B. Problem Formulation

Suppose an imbalanced dataset $\boldsymbol{X} = \{(x_i, y_i)\}_{i=1}^{N}$ where x_i is the ith sample, and y_i is its corresponding label. Through sampling $\boldsymbol{X}$, Q balanced subsets $\boldsymbol{X}_1, \boldsymbol{X}_2, ..., \boldsymbol{X}_Q$ can be obtained, where $\boldsymbol{X}_{i,i=1,2,...,Q} \subset \boldsymbol{X}$ .Traditional imbalanced ensemble (IE) methods trains the base classifier $C_1, C_2, ..., C_Q$ based on the $\boldsymbol{X}_1, \boldsymbol{X}_2, ..., \boldsymbol{X}_Q$, and fuses the base classifiers into a strong classifier C. For a complex distribution of $\boldsymbol{X}$ with high class overlapping and low diversity, the balanced subsets obtained often presents high overlapping and poor diversity, resulting in poor performance of the final classifier C. This paper aims to find a sample transformation mechanism ϕ to improve the quality of subsets, that is $\phi\left(\boldsymbol{X}_{i,i=1,2..,Q}\right) = \boldsymbol{X}'_{i,i=1,2,..,Q}$ .The transformed balanced subsets $\boldsymbol{X}'_{i,i=1,2,..,Q}$ is used to train the base classifiers, thereby maximizing the performance of the final classifier C.

The proposed algorithm DSENLG-IE aims to generate envelope samples to improve the quality of subsets as shown in Fig.3. The method consists of the training and test phases. The training phase includes three components: Subsets Generation (SG), DSENLG network, and ensemble voting mechanism (EVM). The proposed DSENLG is the sample transformation mechanism ϕ discussed above.

C. Subsets Generation (SG)

In this proposed algorithm, subsets are generated through the sample division and fusion method. Firstly, the majority class samples in the imbalanced training set $\boldsymbol{X}_{train}$ are divided into Q subsets, and each subset contains a number of majority class samples equal to minority class samples in the training set. The subset partitioning based on feature weighting is performed [50], rather than random sampling or clustering. Assuming n_1, n_2 denote the number of majority samples $\boldsymbol{X}_{maj} \in \mathrm{R}^{s\times n_1}$ and minority samples $\boldsymbol{X}_{min} \in \mathrm{R}^{s\times n_2}$ in the imbalanced training set respectively, s denotes the number of features. The feature-weighted sum of each sample in the majority class can be calculated as follows.

$$y = \sum_{f=1}^{s} w_{majf} x_{majf}, \quad w_{majf} = x_{majf} / \sum_{f=1}^{s} x_{majf} \tag{1}$$

where x_{majf} denotes the value of the fth feature for x_{maj}, w_{majf} denotes the weight of the fth feature for x_{maj} .Each sample is given an index value y by Eq.(1).The samples in the majority class are sorted according to the corresponding ascending order of y ,and then divided into Q subsets. A balanced training set can be obtained by fusing the majority class samples in each subset with the original minority class samples, resulting $2n_2$ samples in each subset. The relationship between Q and the imbalance ratio (IR) can be obtained as follow.

$$Q = \lfloor n_1/n_2 \rfloor \le IR \tag{2}$$

Eq. (2) can be further expressed as follows

$$Q = \lfloor \rho \cdot IR \rfloor, 0 < \rho \le 1 \tag{3}$$

where, $\lfloor \ \rfloor$ is a rounding down operator, and the number of divided subsets s is correlated with IR.

D.DSENLG Network

1) Deep Sample Envelope Pre-Network (DSEN)

To generate the envelope samples, the deep sample envelope pre-network (DSEN) is proposed. Firstly, the complementary information about samples is enhanced through sample neighborhood concatenation (SNC), and then the deep envelope samples are obtained through multilayer FCM.

a) Sample neighborhood concatenation mechanism (SNC)

Consider a subset $\boldsymbol{X}_1=\left[x_1,x_2,...,x_{2n_2}\right]\in \mathrm{R}^{s\times 2n_2}$ with $2n_2$ samples and s features obtained after SG. For the sample $x, x\in \boldsymbol{X}_1$, the k-nearest neighbors of this sample can be found using the Euclidean distance in the form.

$$d(x,x_i)=\sqrt{(x-x_i)^{\mathrm{T}}(x-x_i)}\ ,1\le i\le 2n_2 \tag{4}$$

Let $NN_K(x)=\{nn_x^i \mid nn_x^i\in X\}_{i=1}^K$ denote the selected k-nearest neighbors of x ,where $K\le 2n_2$,and through Eq.(4),the distances between x and $NN_K(x)$ can be displayed in an ascending order as follows:

$$d(x,nn_x^1)\le d(x,nn_x^2)\le ...\le d(x,nn_x^K) \tag{5}$$

The ranking of $d(x,nn_x^i)$ in the distance sequence $DS=\{d(x,nn_x^i) \mid nn_x^i\in \boldsymbol{X}_1\}_{i=1}^K$ can simply represent the level of similarity between x and nn_x^i . the k-nearest neighbors search can be defined more formally in the following manner.

$$NN_K(x,\boldsymbol{X}_1,K)=\boldsymbol{A} \tag{6}$$

where, $\boldsymbol{A}$ is a set satisfying the following conditions.

$$\boldsymbol{A}\subseteq \boldsymbol{X}_1, \forall x_p\in \boldsymbol{A}, x_q\in \boldsymbol{X}_1-\boldsymbol{A},\ d(x,x_p)\le d(x,x_q) \tag{7}$$

The k-nearest neighbors sample set $\boldsymbol{A}$ is obtained and concatenated with the original sample x to form an envelope sample x_e

$$x_e=x\oplus \boldsymbol{A} \tag{8}$$

where, $\oplus$ denotes the concatenation operator. So, from Eq (8),the envelope dataset $\boldsymbol{X}_{1e}=\left[x_{1e},x_{2e},...,x_{2n_2e}\right]\in \mathrm{R}^{(K+1)s\times 2n_2}$ can be obtained based on the original dataset $\boldsymbol{X}_1\in \mathrm{R}^{s\times 2n_2}$ transformation.

b) Deep envelope sample generation network

After obtaining $\boldsymbol{X}_{1e}$,FCM is used to cluster $\boldsymbol{X}_{1e}$ by minimizing the objective function (9) and $\mathbf{V}_{1e}=\left[v_1,v_2,..,v_c\right]\in \mathrm{R}^{(K+1)s\times c}$ denotes the corresponding prototypes of c clusters.

$$\min J\left(\mathbf{U},\mathbf{V}_{1e}\right)=\sum\nolimits_{i=1}^{c}\sum\nolimits_{p=1}^{2n_2}u_{ip}^m d_{ip}^2,\ \ \text{s.t.}\sum\nolimits_{i=1}^{c}u_{ip}=1 \tag{9}$$

where $d_{ip}=\left\|x_{pe}-v_i\right\|$ stands for the Euclidean distance between the sample and the prototype, u_{ip} is the membership degree of x_{pe} to the ith cluster, $\mathbf{U}=\left(u_{ip}\right)_{c\times 2n_2}$ is the partition matrix, and m denotes a fuzzification coefficient ($m>1$). To minimize (9) by using Lagrange multiplier method, there is

$$\min J\left(\mathbf{U},\mathbf{V}_{1e},\zeta\right)=\sum\nolimits_{i=1}^{c}\sum\nolimits_{p=1}^{2n_2}u_{ip}^m\left\|x_{pe}-v_i\right\|^2+\zeta\left(1-\sum_{i=1}^{c}u_{ip}\right) \tag{10}$$

where ζ is the Lagrange multiplier. To obtain the iterative fashion by setting the partial derivative of Eq. (10) with respect to u_{ip},v_i,ζ to be zero, there is

$$\begin{cases}\partial J\left(\mathbf{U},\mathbf{V}_{1e},\zeta\right)/\partial u_{ip}=mu_{ip}^{m-1}\left\|x_{pe}-v_i\right\|^2-\zeta=0\\ \partial J\left(\mathbf{U},\mathbf{V}_{1e},\zeta\right)/\partial v_i=-2\sum\nolimits_{p=1}^{2n_2}u_{ip}^m(x_{pe}-v_i)=0\\ \partial J\left(\mathbf{U},\mathbf{V}_{1e},\zeta\right)/\partial \zeta=1-\sum\nolimits_{i=1}^{c}u_{ip}=0\end{cases} \tag{11}$$

Through Eq.(11), the partition matrix and prototype formulas are applied in an iterative fashion as follow [51].

$$u_{ip}=\frac{\left\|x_{pe}-v_i\right\|^{-2/(m-1)}}{\sum\nolimits_{j=1}^{c}\left\|x_{pe}-v_j\right\|^{-2/(m-1)}},\ \ v_i=\frac{\sum\nolimits_{p=1}^{2n_2}\left(u_{ip}\right)^m x_{pe}}{\sum\nolimits_{p=1}^{2n_2}\left(u_{ip}\right)^m} \tag{12}$$

Through FCM clustering, the original dataset $\boldsymbol{X}_{1e}$ can be transformed into $\mathbf{V}_{1e}{}^1=\left[v_1,v_2,..,v_{c_1}\right]\in \mathrm{R}^{(K+1)s\times c_1}$ by Eq.(12), and $\mathbf{V}_{1e}{}^1$ as a new dataset can be further transformed into $\mathbf{V}_{1e}{}^2=\left[v_1,v_2,..,v_{c_2}\right]\in \mathrm{R}^{(K+1)s\times c_2}$ by iteratively using Eq.(12). Suppose that L-layer sample transformation based on FCM is performed, the new sample points $\mathbf{V}_{1e}{}^L=\left[v_1,v_2,..,v_{c_L}\right]\in \mathrm{R}^{(K+1)s\times c_L}$ can be obtained accordingly. Hence, through multilayer clustering, the original dataset can be transformed into the new representatives. This method can be referred to as MlFCM and MlFCM is achieved on the basis of single layer FCM(SlFCM). The deep envelope samples $\mathbf{V}_{1e}^{L}$ can be obtained by MlFCM.

To sum up, the proposed DSEN method, which combines the SNC and MlFCM, is shown in Fig.4. Specifically, Fig.4(a) denotes the SNC module, Fig.4(b) denotes the SlFCM module, and Fig.4(c) denotes MlFCM module. The pseudocode description for DSEN is shown in Algorithm 1.

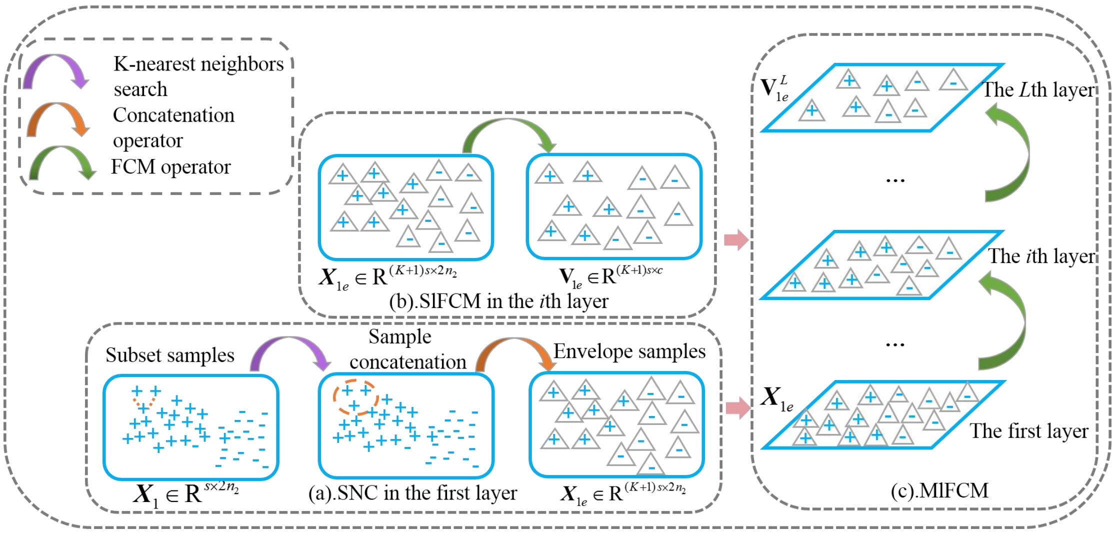

Fig.4. Deep Sample Envelope Pre-Network (DSEN): (a)SNC, (b)SlFCM, (c)MlFCM

Algorithm 1: DSEN

Input: Subset $\boldsymbol{X}_1$, the number of clusters per layer $c_1,..,c_L$, the number of layers of clustering L, the number of nearest neighbor samples K ; fuzzification coefficient m, iteration number w and threshold ε.

Output: Generated deep envelope samples $\mathbf{V}_{1e}{}^{L}$.

Procedure:

1. Find the nearest neighbor samples $\boldsymbol{A}$ for the original sample via (4)(5)(6);
2.Concatenate the original sample with $\boldsymbol{A}$ to form the envelope dataset $\boldsymbol{X}_{1e}$ via (8);
3. For $l = 1 : L$
4. Initialize the partition matrix $\mathbf{U}$ randomly;
5. $w \leftarrow 1$;
6. **Repeat**
7. Update new samples $\mathbf{V}_{1e}{}^{l(w)} = \{v_1, v_2, ..., v_{c_l}\}$ via (12);
8. $w \leftarrow w+1$;
9. **Until** $\left| J\left(\mathbf{U}, \mathbf{V}_{1e}\right)^{(w+1)} - J\left(\mathbf{U}, \mathbf{V}_{1e}\right)^{(w)} \right| < \varepsilon$;
10. **Return** $\mathbf{V}_{1e}{}^{l}$ and $\mathbf{V}_{1e}{}^{l}$ is used as the input of the next layer;
11. End
12. **Return** $\mathbf{V}_{1e}{}^{L}$.

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2) DSEN with Local-global Structure Consistency Mechanism (DSENLG)

The interlayer envelope samples can be obtained based on SlFCM and there might be distribution discrepancy in the interlayer deep envelope samples as shown in Fig. 5(a). In order to enhance consistency of interlayer deep envelope samples, the local-global structure consistency mechanism (LGSCM) is proposed here.

Specifically, $\boldsymbol{X}_{1e} \in \mathrm{R}^{(K+1)s\times 2n_2}, \mathbf{V}_{1e} \in \mathrm{R}^{(K+1)s\times c}$ denote the interlayer envelope sample sets in the SlFCM. Through a projection matrix $\boldsymbol{P} \in \mathrm{R}^{(K+1)s\times d}$,the datasets $\boldsymbol{X}_{1e}, \mathbf{V}_{1e}$ are mapped to a potential common subspace, and a transition data- set $\boldsymbol{X}_M \in \mathrm{R}^{(K+1)s\times 2n_2}$ is generated based on $\mathbf{V}_{1e}$ and a introduced matrix $\boldsymbol{\mathcal{G}} \in \mathrm{R}^{c\times 2n_2}$ in the common subspace. $\boldsymbol{P}$ maps the data space from $\mathrm{R}^{(K+1)s}$ to a latent subspace R^{d} ($(K+1)s \geq d$). By minimizing the local and global distribution between $\boldsymbol{X}_M$ and $\boldsymbol{X}_{1e}$, the interlayer data distribution in the SlFCM is made consistent as shown in Fig.5(b). That is, the distribution relationship of sample sets of the interlayers $\boldsymbol{X}_{1e},\mathbf{V}_{1e}$ is established through the transition dataset $\boldsymbol{X}_M$ .LMSM and GSDM are designed to measure the local and global distribution discrepancy between $\boldsymbol{X}_M$ and $\boldsymbol{X}_{1e}$. Along this direction, the objective function (9) can be further written as

$$
\begin{aligned}
&\min J_1\left(\mathbf{U},\mathbf{V}_{1e}\right)=\underbrace{\sum_{i=1}^{c}\sum_{p=1}^{2n_2}u_{ip}^{m}\left\|x_{pe}-v_i\right\|^2}_{\text{SIFCM}}\\
&\qquad +\sigma\left(\underbrace{\sum_{h,p}^{2n_2}S_{hp}\left\|\varphi\left(x_h\right)-\varphi\left(x_{pe}\right)\right\|_2^2}_{\text{LMSM}}\right.\\
&\left.+\underbrace{\frac{1}{2n_2}\sum_{h=p=1}^{2n_2}\left\|\varphi\left(x_h\right)-\varphi\left(x_{pe}\right)\right\|_2^2}_{\text{GSDM}}\right)\\
&\text{s.t.}\sum_{i=1}^{c}u_{ip}=1
\end{aligned}
\tag{13}
$$

where $\varphi\left(x_h\right)\in\varphi\left(\boldsymbol{X}_M\right)=\varphi\left(\mathbf{V}_{1e}\right)\mathcal{G}$, $\varphi\left(x_{pe}\right)\in\varphi\left(\boldsymbol{X}_{1e}\right)$, σ is a balance factor. φ indicates an implicit but generic transformation. $S_{hp}\in\mathbf{S}$ and $\mathbf{S}$ is the affinity matrix. There are three parts forming the formula (13). The first part denotes the SIFCM which is used to generate the envelope samples, and the second and third parts denote the LMSM and GSDM which are designed to enhance the consistency of the local and global distribution between the interlayer envelope samples. Fig. 5(a) shows the SIFCM; and Fig.5(b) shows the SIFCM with LGSCM(LG).

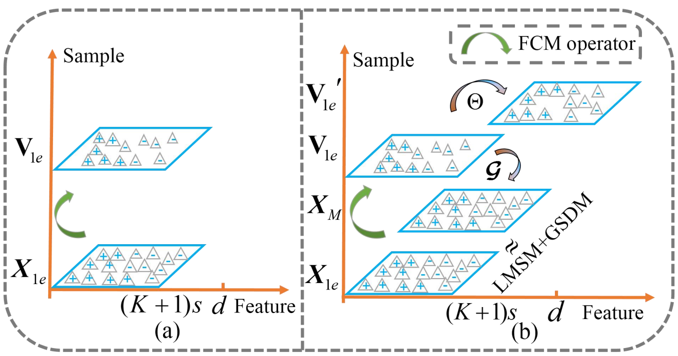


Fig.5. Local-global structure consistency mechanism (LGSCM)-LG: (a) SIFCM;(b) SIFCM with LG

a) Local manifold structure metric (LMSM)

LMSM is proposed to enhance the consistency of local structure distribution between $\boldsymbol{X}_{1e}$ and $\mathbf{V}_{1e}$ indirectly, by constraining the generative transition dataset $\boldsymbol{X}_M$ .So local structure preservation can be defined as follow.

$$
\begin{aligned}
\mathcal{L}_{\text{LMSM}(\mathrm{F}_{X_M},\mathrm{F}_{X_{1e}})}&=\sum_{h,p}^{2n_2}S_{hp}\left\|\varphi\left(x_h\right)-\varphi\left(x_{pe}\right)\right\|_2^2\\
&=Tr\left(\varphi\left(\boldsymbol{X}_M\right)\mathbf{D}\left(\varphi\left(\boldsymbol{X}_M\right)^{\mathrm{T}}\right)\right)\\
&\quad+Tr\left(\varphi\left(\boldsymbol{X}_{1e}\right)\mathbf{D}\left(\varphi\left(\boldsymbol{X}_{1e}\right)^{\mathrm{T}}\right)\right)\\
&\quad-2Tr\left(\varphi\left(\boldsymbol{X}_M\right)\mathbf{S}\left(\varphi\left(\boldsymbol{X}_{1e}\right)^{\mathrm{T}}\right)\right)
\end{aligned}
\tag{14}
$$

where F_{X_M} and $\mathrm{F}_{X_{1e}}$ denote the distribution of $\boldsymbol{X}_M$ and $\boldsymbol{X}_{1e}$, respectively. $Tr(\bullet)$ is the matrix trace. The diagonal matrix $\mathbf{D}$ is a matrix with entries $\mathrm{D}_{hh}=\sum_{p=1}^{2n_2}\mathrm{S}_{hp}$ and $\mathbf{S}$ is the affinity matrix, calculated as

$$
\mathrm{S}_{hp}=\begin{cases}1, & \text{if } x_h\in NN_K\left(x_{pe}\right)\|x_{pe}\in NN_K\left(x_h\right)\\ 0, & \text{otherwise}\end{cases}
\tag{15}
$$

Let $\boldsymbol{P}^{\mathrm{T}}=\Theta^{\mathrm{T}}\varphi\left(\boldsymbol{X}_r\right)^{\mathrm{T}}$, $\boldsymbol{X}_r=\left[\mathbf{V}_{1e},\boldsymbol{X}_{1e}\right]$, the projected $\boldsymbol{X}_{1e},\mathbf{V}_{1e}$ can be denoted as $\Theta^{\mathrm{T}}\varphi\left(\boldsymbol{X}_r\right)^{\mathrm{T}}\varphi\left(\boldsymbol{X}_{1e}\right)$ and $\Theta^{\mathrm{T}}\varphi\left(\boldsymbol{X}_r\right)^{\mathrm{T}}\varphi\left(\mathbf{V}_{1e}\right)$. After projection, Eq.(14) can be further expressed as

$$\begin{aligned}&\min_{\Theta,\mathcal{G}}\frac{1}{4n_2{}^2}Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{D}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\right)^{\mathrm{T}}\right)\\&+\frac{1}{4n_2{}^2}Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\mathbf{D}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\right)\\&-\frac{2}{4n_2{}^2}Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{S}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\right)\end{aligned}\tag{16}$$

where $\boldsymbol{\Psi}_{\mathrm{V}}=\varphi\left(\boldsymbol{X}_r\right)^{\mathrm{T}}\varphi\left(\mathbf{V}_{1e}\right)$, $\boldsymbol{\Psi}_{\mathrm{e}}=\varphi\left(\boldsymbol{X}_r\right)^{\mathrm{T}}\varphi\left(\boldsymbol{X}_{1e}\right)$ are kernel matrices.

b) Global structure distribution metric (GSDM)

GSDM is proposed to reduce the discrepancy of global distribution between the $\boldsymbol{X}_{1e}$ and $\mathbf{V}_{1e}$ indirectly, by constraining the generative transition dataset $\boldsymbol{X}_M$ .So the GSDM can be expressed as follows.

$$\mathcal{L}_{\mathrm{GSDM}(\mathrm{F}_{X_M},\mathrm{F}_{X_{1e}})}=\frac{1}{2n_2}\sum_{h=p=1}^{2n_2}\left\|\varphi\left(x_h\right)-\varphi\left(x_{pe}\right)\right\|_2^2\tag{17}$$

Similarly, after projection, Eq.(17) can be further expressed as

$$\min_{\Theta,\mathcal{G}}\frac{1}{2n_2}\left\|\Theta^{\mathrm{T}}\left(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}-\boldsymbol{\Psi}_{\mathrm{e}}\right)\mathbf{1}\right\|_2^2\tag{18}$$

where **1** represents a column vector with all elements set to 1.

c) Joint optimization

The proposed LGSCM aims to align the distribution of interlayer deep envelope samples and preserve local manifold structure. Therefore, through the projection matrix $\boldsymbol{P}$, in the common subspace, the local and global distribution discrepancy between $\boldsymbol{X}_{1e}$ and $\mathbf{V}_{1e}$ can be minimized by combining LMSM(16),GSDM (18) and low rank constraint (LRC) regularization [52]. So Eq.(13) can be written as follows

$$\begin{aligned}&\min J_2\left(\mathbf{U},\mathbf{V}_{1e},\Theta,\mathcal{G}\right)=\sum_{i=1}^{c}\sum_{p=1}^{2n_2}u_{ip}^m\left\|x_{pe}-v_i\right\|^2\\&+\sigma\begin{pmatrix}\frac{1}{4n_2{}^2}Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{D}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\right)^{\mathrm{T}}\right)\\+\frac{1}{4n_2{}^2}Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\mathbf{D}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\right)-\\\frac{2}{4n_2{}^2}Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{S}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\right)\\+\frac{\lambda}{2n_2}\left\|\Theta^{\mathrm{T}}\left(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}-\boldsymbol{\Psi}_{\mathrm{e}}\right)\mathbf{1}\right\|_2^2+\lambda_1\left\|\mathcal{G}\right\|_*\end{pmatrix}\\&\text{s.t.}\sum_{i=1}^{c}u_{ip}=1\quad\text{and }\Theta^{\mathrm{T}}\boldsymbol{\Psi}\Theta=\mathbf{I}\end{aligned}\tag{19}$$

where, $\boldsymbol{\Psi}=\varphi\left(\boldsymbol{X}_r\right)^{\mathrm{T}}\varphi\left(\boldsymbol{X}_r\right)$ is kernel matrix and the nuclear norm $\left\|\mathcal{G}\right\|_*$ is the low rank constraint. The low-rank structure of $\mathcal{G}$ is considered for enhancing the correlation of the interlayer envelope sample sets, and λ,λ_1 are tradeoff parameters.

The optimization problem in (19) can be divide into two subproblems, and can be optimized iteratively. The first subproblem mainly focuses on variables $\mathbf{U},\mathbf{V}_{1e}$, and the optimization process of $\mathbf{U},\mathbf{V}_{1e}$ can be followed as Eqs.(10)(11)(12). The second subproblem mainly focuses on variables $\Theta,\mathcal{G}$, and the optimization process of $\Theta,\mathcal{G}$ can be solved with the augmented Lagrange function by introducing an auxiliary variable $\mathcal{H}$ as follow,

$$\begin{aligned}&\min_{\Theta,\mathcal{G},\mathcal{H}} \frac{1}{4{n_2}^2}(Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{D}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\right)^{\mathrm{T}}\right)\\&+Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\mathbf{D}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\right)-2Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{S}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\right))\\&+\frac{\lambda}{2n_2}\Theta^{\mathrm{T}}(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{1}(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G})^{\mathrm{T}}-\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{1}\left(\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\\&-\boldsymbol{\Psi}_{\mathrm{e}}\mathbf{1}(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G})^{\mathrm{T}}+\boldsymbol{\Psi}_{\mathrm{e}}\mathbf{1}(\boldsymbol{\Psi}_{\mathrm{e}})^{\mathrm{T}})\Theta+\lambda_1\left\|\mathcal{H}\right\|_*\\&+Tr\left(\mathbf{W}_1^{\mathrm{T}}\left(\mathcal{G}-\mathcal{H}\right)\right)+\frac{\delta}{2}\left(\left\|\mathcal{G}-\mathcal{H}\right\|_F^2\right)\end{aligned}\tag{20}$$

where $\mathbf{W}_1$ is the Lagrange multiplier, and δ is a penalty parameter. The optimization process of the three variables $\Theta,\mathcal{G},\mathcal{H}$ can be carried out, where one of the variables is solved while the remaining two are fixed. Fixing $\mathcal{H},\mathcal{G}$,to solve Θ, the problem (20) transforms to the following optimization problem (21),

$$\begin{aligned}&\min_{\Theta} \frac{1}{4{n_2}^2}(Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{D}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\right)^{\mathrm{T}}\right)\\&+Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\mathbf{D}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\right)-2Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{S}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\right))\\&+\frac{\lambda}{2n_2}\Theta^{\mathrm{T}}(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{1}(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G})^{\mathrm{T}}-\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{1}\left(\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\\&-\boldsymbol{\Psi}_{\mathrm{e}}\mathbf{1}(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G})^{\mathrm{T}}+\boldsymbol{\Psi}_{\mathrm{e}}\mathbf{1}(\boldsymbol{\Psi}_{\mathrm{e}})^{\mathrm{T}})\Theta\\&\text{s.t.}\quad \Theta^{\mathrm{T}}\boldsymbol{\Psi}\Theta=\mathbf{I}\end{aligned}\tag{21}$$

Fixing $\Theta,\mathcal{H}$,to solve $\mathcal{G}$, the problem (20) transforms to the following optimization problem (22),

$$\begin{aligned}&\min_{\mathcal{G}} \frac{1}{4{n_2}^2}(Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{D}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\right)^{\mathrm{T}}\right)-2Tr\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{S}\left(\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}\right))\\&+\frac{\lambda}{2n_2}\Theta^{\mathrm{T}}(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{1}(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G})^{\mathrm{T}}-\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G}\mathbf{1}\left(\boldsymbol{\Psi}_{\mathrm{e}}\right)^{\mathrm{T}}-\boldsymbol{\Psi}_{\mathrm{e}}\mathbf{1}(\boldsymbol{\Psi}_{\mathrm{v}}\mathcal{G})^{\mathrm{T}})\Theta\\&+Tr\left(\mathbf{W}_1^{\mathrm{T}}\left(\mathcal{G}-\mathcal{H}\right)\right)+\frac{\delta}{2}\left(\left\|\mathcal{G}-\mathcal{H}\right\|_F^2\right)\end{aligned}\tag{22}$$

Fixing $\Theta,\mathcal{G}$,to solve $\mathcal{H}$, the problem (20) transforms to the following optimization problem (23)

$$\min_{\mathcal{H}}\lambda_1\left\|\mathcal{H}\right\|_*+Tr\left(\mathbf{W}_1^{\mathrm{T}}\left(\mathcal{G}-\mathcal{H}\right)\right)+\frac{\delta}{2}\left(\left\|\mathcal{G}-\mathcal{H}\right\|_F^2\right)\tag{23}$$

The specific optimization process of $\Theta,\mathcal{G},\mathcal{H}$ can be found in [33], which involves eigenvalue decomposition, gradient descent and singular value thresholding (SVT). When optimal Θ is obtained, the new envelope sample set is obtained by $\mathbf{V}_{1e}{'}=\Theta^{\mathrm{T}}\boldsymbol{\Psi}_{\mathrm{V}}$ and then input the next layer of DSEN. After L-layer DSEN, the final deep envelope sample can be obtained $\mathbf{V}_{1e}{'}^{L}$. So the DSEN and LGSCM are combined to form a new network called DSENLG and the deep envelope sample $\mathbf{V}_{1e}{'}^{L}$ generated via DSENLG for tackling the class imbalance problem. The pseudocode description for DSENLG is shown in Algorithm 2.

Algorithm 2: DSENLG

Input: Envelope sample sets $\boldsymbol{X}_{1e}$, the number of clusters per layer $c_1,..,c_L$, the number of layers of clustering L ,parameters σ,λ,λ_1.

Output: Generated deep envelope sample set $\mathbf{V}_{1e}{'}^{L}$

Procedure:

1. For $l=1:L$
2. Initialize: $\mathbf{W}_1=\mathcal{H}=\mathcal{G}=0$, $\delta=0.1$, $\mathbf{U}$;
3. Compute $\mathbf{V}_{1e}$ based on $\boldsymbol{X}_{1e}$ via (10)(11)(12);
4. Compute $\boldsymbol{\Psi}=\varphi\left(\boldsymbol{X}_r\right)^{\mathrm{T}}\varphi\left(\boldsymbol{X}_r\right)$, $\boldsymbol{\Psi}_{\mathrm{V}}=\varphi\left(\boldsymbol{X}_r\right)^{\mathrm{T}}\varphi\left(\mathbf{V}_{1e}\right)$, $\boldsymbol{\Psi}_{\mathrm{e}}=\varphi\left(\boldsymbol{X}_r\right)^{\mathrm{T}}\varphi\left(\boldsymbol{X}_{1e}\right)$, $\boldsymbol{X}_r=\left[\mathbf{V}_{1e},\boldsymbol{X}_{1e}\right]$;
5. **While** not converge do
6. Optimize Θ based on (21);
7. Optimize $\mathcal{G}$ based on (22);

8. Optimize $\mathcal{H}$ based on (23);
9. **End While**
10. Compute $\mathbf{V}_{1e}{}'^{l} = \Theta^{\mathrm{T}}\mathbf{\Psi}_{\mathrm{V}}$;
11. $\boldsymbol{X}_{1e} \leftarrow \mathbf{V}_{1e}{}'^{l}$;
12. End
13.**Return** $\mathbf{V}_{1e}{}'^{L}$.

E. Ensemble Voting Mechanism (EVM)

As existing imbalanced ensemble algorithms, multiple balanced subsets are obtained. They are Q balanced training sets $\boldsymbol{X}_1, \boldsymbol{X}_2, ..., \boldsymbol{X}_Q$ .Specifically, Q balanced training sets $\boldsymbol{X}_1, \boldsymbol{X}_2, ..., \boldsymbol{X}_Q$ are input to the DSENLG to train the network respectively and obtain the corresponding deep envelope training sets $\mathbf{V}_{1e}{}'^{L}, \mathbf{V}_{2e}{}'^{L}, ...\mathbf{V}_{Qe}{}'^{L}$ .Then the deep envelope training sets are used for classification model training and $C_1, C_2, ..., C_Q$ classifiers are obtained. The Q base classifiers can be ensembled as a strong classifier C for the final prediction. Please see Fig. 6.

For a sample x_{test} in the test set $\boldsymbol{X}_{test}$, the deep envelope samples $x_{test\ 1e}{}^{L}, x_{test\ 2e}{}^{L}, ..., x_{test\ Qe}{}^{L}$ can be obtained via trained DSENLG, and input to the base classifiers respectively. Since the output of base classifiers $C_1, C_2, ..., C_Q$ is the predicted label, the prediction result can be written by:

$$C(x_{test\ 1e}{}^{L}, x_{test\ 2e}{}^{L}, ..., x_{test\ Qe}{}^{L}) = [C_1(x_{test\ 1e}{}^{L}), C_2(x_{test\ 2e}{}^{L}) \\ , ..., C_Q(x_{test\ Qe}{}^{L})] \tag{24}$$

where $C_1(x_{test\ 1e}{}^{L}), C_2(x_{test\ 2e}{}^{L}), ..., C_Q(x_{test\ Qe}{}^{L})$ denote the output of base classifiers. After that, a voting mechanism is adopted to determine the final prediction results of the test sample. The IE method based on DSENLG (DSENLG-IE) is shown in Algorithm 3.

Algorithm 3: DSENLG-IE

Input: Training set $\boldsymbol{X}_{train}$, the number of layers of the DSENLG L and test set $\boldsymbol{X}_{test}$
Output: Test set prediction results r
Procedure:
1. Divide the training set $\boldsymbol{X}_{train}$ into Q subsets and obtain Q balanced training sets $\boldsymbol{X}_1, \boldsymbol{X}_2, ..., \boldsymbol{X}_Q$ through SG;
2. For $q = 1, .., Q$
3. The deep envelope training set $\mathbf{V}_{qe}{}'^{L}$ and test set $\boldsymbol{X}_{test\ qe}{}^{L}$ are obtained by DSENLG;
4. The base classifier C_q is trained by using $\mathbf{V}_{qe}{}'^{L}$;
5. The predict result of the deep envelope test set are obtained based on the trained classifier C_q ;
6. End
7. The voting mechanism is adopted to get the final prediction r of the test set;
8. **Return** r

F. Example for Implementation

For an illustration, an imbalanced dataset Ecoli1 from the KEEL database is selected to implement the proposed algorithm. Ecoli1 has 336 samples and 7 features, with an imbalance ratio of 3.36. At first, the dataset is divided into training set $\boldsymbol{X}_{trian}$ and test set $\boldsymbol{X}_{test}$ by 5-fold cross validation, and then the feature-weighted sum y of each majority class sample in the $\boldsymbol{X}_{trian}$ is calculated by Eq.(1). According to the ascending order of y , three balanced subsets $\boldsymbol{X}_1, \boldsymbol{X}_2, \boldsymbol{X}_3$ can be obtained ($Q = 3$). For subset $\boldsymbol{X}_1$, the samples in the $\boldsymbol{X}_1$ first find their three nearest neighbor samples ($K = 3$) and then concatenate with them to form the envelope dataset $\boldsymbol{X}_{1e}$ via Eq.(8). The FCM is used to cluster $\boldsymbol{X}_{1e}$ to obtain $\mathbf{V}_{1e}$ and LGSCM is used to enhance consistency of $\boldsymbol{X}_{1e}, \mathbf{V}_{1e}$ and $\mathbf{V}_{1e}{}'$ can be obtained. The joint optimization of FCM and LGSCM as shown in Eq.(19) means a layer of DSENLG, after three layers of DSENLG, the $\mathbf{V}_{1e}{}'^{3}$ ($L = 3$) can be obtained and used to train the base classifier C_1 .The other two subsets $\boldsymbol{X}_2, \boldsymbol{X}_3$ can be transformed in the same way to obtain the envelope sample set $\mathbf{V}_{2e}{}'^{3}, \mathbf{V}_{3e}{}'^{3}$, and $\mathbf{V}_{2e}{}'^{3}, \mathbf{V}_{3e}{}'^{3}$ are used to train the base classifier C_2, C_3 .

For a sample x_{test} in the test set $\boldsymbol{X}_{test}$, the deep envelope samples $x_{test\ 1e}{}^{3}, x_{test\ 2e}{}^{3}, x_{test\ 3e}{}^{3}$ can be obtained via the trained DSENLG, and then input to the C_1, C_2, C_3 to obtain the predicted labels, respectively. After that, the final prediction result of the test sample is determined by the voting mechanism.

V. Time Complexity Analysis

A theoretical analysis for DSENLG-IE with respect to the computational complexity was conducted. The time complexity $T_{\text{DSENLG-IE}}$ is computed by

$$T_{\text{DSENLG-IE}} = T_{\text{SG}} + Q \cdot T_{\text{DSENLG}} + T_{\text{EVM}} \tag{25}$$

where $T_{\text{SG}}, T_{\text{DSENLG}}, T_{\text{EVM}}$ denote computational costs for the SG, DSENLG network and EVM, respectively.

T_{SG} is affected by the number of majority samples n_1 and features s, so T_{SG} can be given by

$$T_{\text{SG}} = O(n_1 \cdot s) \tag{26}$$

$T_{\text{DSEN-LG}}$ contains the computational costs of SNC, MIFCM and LGSCM as follows

$$T_{\text{DSENLG}} = T_{\text{DSEN}} + T_{\text{GS\ M}} = T_{\text{SNC}} + T_{\text{MIFCM}} + T_{\text{LGSCM}} \tag{27}$$

T_{SNC} is related to the number of data $2n_2$ and features s, which can be given by

$$T_{\text{SNC}} = O(2n_2 s) \tag{28}$$

T_{MIFCM} is related to the number of iterations w, the number of features s, the number of clusters c, the number of data $2n_2$ and the number of layers of clustering L. As the time complexity of the FCM algorithm is $O\left(wsc^2 2n_2\right)$ and $c = 2(n_2 - L)$, the T_{MIFCM} can be given by

$$T_{\text{MIFCM}} = O\left(wsLn_2{}^{3}\right) \tag{29}$$

T_{LGSCM} includes computational costs of updating $\Theta, \mathcal{H}, \mathcal{G}$. Suppose the number of iterations is w_1. Then the computational costs of LGSCM can be given by

$$T_{\text{LGSCM}} = O\left(w_1 L n_2{}^{3}\right) + O\left(w_1 L n_2{}^{2}\right) \tag{30}$$

T_{EVM} is related to the number of base classifiers Q. So T_{EVM} can be given by

$$T_{\text{EVM}} = Q \tag{31}$$

Therefore, the total time complexity of DSENLG-IE is approximately equal to:

$$T_{\text{DSENLG-IE}} = O(n_1 \cdot s) + Q \cdot \begin{pmatrix} O(n_2 s) + O\left(wsLn_2{}^{3}\right) \\ + O\left(w_1 L n_2{}^{3}\right) + O\left(w_1 L n_2{}^{2}\right) \end{pmatrix} \tag{32}$$

VI. Experimental Results and Their Analysis

A. Experiment Conditions

Forty-six popular publicly available datasets in Table I are employed to evaluate the proposed algorithm. The MIFCM and IE are compared to the proposed method. MIFCM means the original dataset is clustered by MIFCM. IE denotes the traditional imbalanced ensemble method with bagging. Additionally, another three groups of ensemble algorithms have also been adopted as the comparison algorithms. The first group comprises eight classical IE algorithms: RUSBoost(RBO) [20], SMOTEBoost(SBO) [21], UnderBagging(UBAG) [22], SMOTEBagging(SBAG) [10], BalancedBagging(BBAG) [35], EasyEnsemble(EYEE) [36], BalanceCascade(BACE) [36], GBDT [8]. Each one of these methods represents a distinct combination of an ensemble method (e.g., bagging, boosting and hybrid method). For comparison with more sophisticated algorithms, the second group uses six state-of-the-art (SOTA) IE algorithms: CBIS [24], SPE [38], EASE [39], HOEC [40], HD-Ensemble [41], Imbalance-XGBoost [9]. The third group includes six deep learning methods (DL) based imbalanced classification methods: CNN+SMOTE [14], CNN+AE+GAN [15], BED [16], RVGAN-TL [18], EAL-GAN [47], DLE-ISMOTE [49].

Decision tree C4.5 was adopted as the base classifier in the experiment. 5-fold cross validation procedure (5-CV) was adopted and the 5-CV procedure was repeated 10 times on every experimental dataset to eliminate the effect of randomness.

TABLE I
CHARACTERISTICS OF 46 IMBALANCED DATASETS

Dataset	f	s	IR	Dataset	f	s	IR	Dataset	f	s	IR
Iris0	4	150	2	Glass016vs2	9	192	10.29	Yeast5	8	1484	32.73
Glass0	9	214	2.06	Ecoli0147vs2356	7	336	10.59	Ozone-onehr	72	2536	33.74
Vertebral	6	310	2.1	climate	18	540	10.7	krvsk3vs11	6	2935	35.23
Haberman	3	306	2.78	Glass2	9	214	11.59	Abalone21vs8	8	581	40.5
Vehicle1	18	846	2.9	german	24	324	12.5	Yeast6	8	1484	41.4
Ecoli1	7	336	3.36	Shuttle-c0-vs-c4	9	1829	13.87	Winequality-white3vs7	11	900	44
New-thyroid1	5	215	5.14	Yeast1vs7	8	459	14.3	Winequality-red8vs67	11	855	46.5
Ecoli2	7	336	5.46	Ecoli4	7	336	15.8	krvsk0vs8	6	1460	53.07
Musk	166	6598	5.48	Page-blocks13vs4	10	472	15.86	Shuttle-2vs5	9	3316	66.67
Glass6	9	214	6.38	Dermatology-6	34	358	16.9	kddbufferoverflowvsback	41	2233	73.43
Yeast3	8	1484	8.10	svmguide3	22	312	18.5	krvsk0vs15	6	2193	80.22
Ecoli3	7	336	8.6	Yeast1458vs7	8	693	22.1	kddrootkitback	41	2225	100.14
Page-blocks0	10	5472	8.79	Yeast4	8	1484	28.10	skinnonskin	3	20034	588.24
Yeast2vs4	8	514	9.08	Winequality-red-4	11	1599	29.17	cod	8	19871	763.27
Yeast05679vs4	8	528	9.35	Yeast1289vs7	8	947	30.57				
Vowel0	10	988	9.98	Abalone3vs11	8	502	32.47				

1) Evaluation Metrics and Parameter Setting

To assess the performance of the methods, this paper used AUC, F-measure (F-M), G-mean (G-M), Matthews correlation coefficient (Mcc) criteria. Moreover, the nonparametric statistical test methods were adopted to detect statistical differences between all the methods.

For the proposed method, three parameters need to be determined before running the learning procedure: (1) ρ, as defined in Eq.(3),which determines the number of subsets, (2) K ,which means the number of nearest neighbor samples for SNC, (3) L,which is used to determine the number of layers for DSEN-LG. $\rho=1, K=3, L=3$ in this paper. In SIFCM, the difference in the number of samples before and after clustering is set to 1, the fuzzification coefficient $m=2$ and $\varepsilon=10^{-5}$ .In LGSCM, $\sigma=0.01, \lambda=\lambda_1=1$ and the Gaussian kernel function $k\left(x_i, x_j\right)=\exp(-\left\|x_i-x_j\right\|^2/2\gamma^2)$ was used in the study, where $\gamma=1.2$. All the results are obtained under this setting.

For all compared methods except CBIS, HOEC, HD-Ensemble, CNN+SMOTE, CNN+AE+GAN and EAL-GAN, the number of the base classifiers are set to. For SMOTEBoost, SMOTEBagging, CNN+SMOTE and DLE-ISMOTE, the number of neighbors is set to 3. For SPE, the number of bins is assigned 20 as to the original paper[38]. For Imbalance-XGBoost, the focal loss is used and focal_gamma = [1.0,1.5,2.0,2.5,3.0] following the original paper[9]. One layer with ReLU activation function as feature learning, and the other layer with Sigmoid activation function for classification. In general, some parameters' setup follow the description of the papers; other parameters which the papers didnot said are setup with default values.

2) Verification of DSENLG by Ablation Method

To demonstrate the effectiveness of deep envelope samples obtained by DSEN-LG, ablation method was adopted to compare the proposed algorithm with the MIFCM and IE. Table II is the comparison results between the IE, MIFCM and proposed DSENLG-IE. From Table II, the proposed algorithm shows a large improvement in performance on all four metrics compared to MIFCM and IE method for most datasets. This indicates envelope samples generated through DSEN-LG network are of high quality and very effective. The DSENLG-IE is better than the IE. It means that the multilayer clustering can obtain envelope samples with high-quality, which are more helpful for imbalanced learning. The DSENLG-IE is better than the MIFCM. It means that the LGSCM can well enhance the consistency of the interlayer samples of MIFCM, thereby contributing to improving the quality of the envelope samples.

TABLE II
ABLATION METHOD FOR THE PROPOSED METHOD

Dataset	Measure	IE	MIFCM	DSENLG-IE	Dataset	Measure	IE	MIFCM	DSENLG-IE
Iris0	AUC	98.80±2.78	78.15±4.03	**100.0±0.00**	Glass0	AUC	74.62±6.70	67.11±3.92	**76.35±6.29**
	F-M	98.70±3.12	35.69±3.64	**100.0±0.00**		F-M	65.52±10.0	59.79±2.90	**67.19±9.05**
	G-M	98.75±2.95	75.69±4.34	**100.0±0.00**		G-M	73.02±7.98	58.12±6.76	**74.24±7.32**
	Mcc	98.23±4.09	34.68±5.10	**100.0±0.00**		Mcc	51.64±13.5	38.10±5.74	**57.95±12.6**
Vertebral	AUC	76.96±5.15	68.33±10.6	**83.98±7.29**	Haberman	AUC	55.41±7.27	53.31±6.87	**61.81±9.38**
	F-M	68.64±7.27	58.33±14.7	**78.41±7.08**		F-M	37.86±9.36	38.89±7.01	**43.65±8.57**
	G-M	76.10±5.89	68.31±10.1	**82.98±8.10**		G-M	54.11±8.89	52.29±6.58	**60.19±7.30**
	Mcc	54.69±10.0	34.44±14.4	**71.45±8.29**		Mcc	9.76±13.32	5.98±12.31	**21.59±8.67**
Vehicle1	AUC	66.52±3.54	63.18±4.31	**82.70±6.54**	Ecoli1	AUC	85.89±4.51	81.85±5.21	**92.47±4.39**
	F-M	50.54±4.09	46.30±5.32	**67.23±6.25**		F-M	69.34±6.91	64.82±6.97	**80.42±7.52**
	G-M	66.40±3.56	62.65±4.62	**81.74±7.15**		G-M	84.94±5.21	81.19±5.29	**92.09±4.84**
	Mcc	29.33±6.45	23.97±7.87	**57.01±9.87**		Mcc	61.84±8.75	54.87±9.68	**80.48±9.12**
New-thyr	AUC	95.06±4.83	70.00±11.7	**99.80±1.41**	Ecoli2	AUC	71.31±5.20	74.33±4.32	**93.62±7.25**
	F-M	85.92±10.2	53.99±23.6	**99.78±1.57**		F-M	39.84±5.18	42.42±3.79	**82.79±7.79**

oid1	G-M	94.85±5.12	60.96±18.8	**99.79±1.49**		G-M	66.16±7.71	70.86±5.04	**92.76±7.51**
	Mcc	84.10±11.4	58.01±19.1	**100.0±0.00**		Mcc	31.95±6.89	35.53±5.89	**82.01±7.24**
Musk	AUC	86.32±2.74	55.52±5.58	**98.56±0.76**	Glass6	AUC	92.02±5.97	72.97±8.45	**98.13±5.07**
	F-M	58.88±4.88	16.46±1.63	**92.59±4.20**		F-M	77.01±12.4	37.50±15.0	**95.91±7.79**
	G-M	85.61±3.01	31.90±2.62	**98.55±0.77**		G-M	91.83±6.13	67.78±9.49	**97.96±5.68**
	Mcc	55.00±5.43	11.35±5.22	**91.50±4.82**		Mcc	74.46±13.7	32.56±17.9	**95.77±7.92**
Yeast3	AUC	91.46±2.54	67.95±3.16	**97.71±1.99**	Ecoli3	AUC	86.28±3.68	78.15±4.03	**95.70±4.69**
	F-M	69.97±4.52	28.62±1.78	**83.33±1.02**		F-M	50.22±5.68	35.69±3.64	**73.37±6.77**
	G-M	91.40±2.57	64.06±2.77	**97.68±2.09**		G-M	85.79±3.59	75.69±4.34	**95.50±4.98**
	Mcc	67.93±4.81	22.81±3.97	**82.56±1.12**		Mcc	49.37±6.15	34.68±5.10	**74.30±6.93**
Page-blocks0	AUC	92.68±1.05	69.23±2.92	**98.14±0.39**	Yeast 2vs4	AUC	92.40±4.01	84.37±4.90	**99.44±1.37**
	F-M	64.61±3.00	35.59±3.62	**90.43±3.06**		F-M	65.58±8.91	51.82±7.79	**76.08±11.6**
	G-M	92.56±1.05	67.98±4.15	**98.12±0.40**		G-M	92.23±4.06	84.26±4.93	**99.44±1.39**
	Mcc	64.06±2.90	27.96±4.28	**89.80±3.21**		Mcc	64.96±9.15	49.43±8.70	**76.48±11.7**
Yeast 05679vs4	AUC	75.05±5.13	67.02±6.83	**96.77±1.11**	Vowel0	AUC	95.34±1.66	83.94±5.31	**100.0±0.00**
	F-M	31.67±3.73	26.19±4.51	**72.73±2.89**		F-M	72.67±5.09	42.88±5.40	**100.0±0.00**
	G-M	72.78±4.93	65.00±6.46	**96.72±1.15**		G-M	95.28±1.66	83.05±5.95	**100.0±0.00**
	Mcc	29.87±6.11	20.26±8.16	**73.11±2.92**		Mcc	72.34±4.86	42.98±6.50	**100.0±0.00**
Glass 016vs2	AUC	70.45±12.5	58.71±2.73	**89.39±11.6**	Ecoli 0147vs2356	AUC	75.39±5.08	74.25±7.69	**97.81±3.15**
	F-M	**29.99±11.1**	19.07±2.87	22.22±10.9		F-M	29.94±3.56	31.70±7.55	**81.97±10.5**
	G-M	68.78±13.3	41.15±7.41	**88.76±12.5**		G-M	73.50±4.68	70.62±10.6	**97.73±3.37**
	Mcc	25.27±15.8	13.45±3.14	**31.38±12.9**		Mcc	29.04±5.71	31.85±9.79	**77.34±10.8**
climate	AUC	85.60±4.43	50.00±0.00	**79.93±4.80**	Glass2	AUC	71.87±10.4	61.98±2.76	**87.69±4.45**
	F-M	47.50±6.38	0.000±0.00	**70.60±4.56**		F-M	**26.12±7.01**	18.52±2.56	24.72±9.54
	G-M	85.35±4.44	0.000±0.00	**74.74±4.30**		G-M	70.07±10.1	48.62±5.79	**86.70±5.05**
	Mcc	47.25±6.91	0.000±0.00	**73.87±4.09**		Mcc	24.09±11.5	15.57±2.65	**32.47±9.00**
german	AUC	54.17±8.29	54.00±14.6	**84.48±9.24**	Shuttle-c0-vs-c4	AUC	99.07±0.33	90.57±9.19	**100.0±0.00**
	F-M	14.75±4.44	14.16±5.96	**23.08±5.05**		F-M	88.76±3.57	84.48±13.0	**100.0±0.00**
	G-M	52.72±7.71	52.06±14.5	**83.05±10.6**		G-M	99.07±0.33	89.71±10.2	**100.0±0.00**
	Mcc	4.500±8.90	3.760±14.7	**29.23±10.4**		Mcc	88.55±3.51	83.83±13.7	**100.0±0.00**
Yeast 1vs7	AUC	71.74±6.88	60.30±7.95	**83.72±6.06**	Ecoli4	AUC	80.30±5.24	74.92±2.66	**98.54±4.88**
	F-M	22.83±4.01	16.92±4.73	**30.00±8.23**		F-M	26.51±5.03	20.29±1.70	**87.87±8.94**
	G-M	70.50±7.30	58.68±9.12	**82.12±6.46**		G-M	78.51±6.00	70.50±3.81	**98.37±5.72**
	Mcc	22.17±7.03	10.74±8.29	**34.50±8.12**		Mcc	30.37±5.55	23.73±2.37	**88.61±8.51**
Page-blocks 13vs4	AUC	94.47±2.45	72.19±15.8	**98.50±1.38**	Dermatology-6	AUC	91.24±5.72	97.78±1.30	**100.0±0.00**
	F-M	55.13±11.1	45.72±22.2	**77.11±9.46**		F-M	58.79±11.0	74.47±12.3	**100.0±0.00**
	G-M	94.27±2.64	65.29±21.3	**98.49±1.41**		G-M	90.94±5.93	97.75±1.33	**100.0±0.00**
	Mcc	58.41±9.70	44.34±25.5	**78.88±7.20**		Mcc	60.14±10.4	75.74±11.3	**100.0±0.00**
svmguide 3	AUC	78.72±10.0	51.98±7.20	**80.70±8.98**	Yeast 1458vs7	AUC	58.38±8.88	51.85±7.52	**72.14±9.29**
	F-M	**24.49±6.03**	9.520±2.70	15.38±3.27		F-M	10.20±2.41	8.580±1.60	**15.08±3.70**
	G-M	77.36±10.8	49.86±10.3	**78.36±7.87**		G-M	55.14±7.50	43.88±6.27	**68.13±10.8**
	Mcc	**27.62±9.51**	1.760±10.4	17.37±5.21		Mcc	7.020±7.41	1.700±7.15	**18.43±7.42**
Yeast4	AUC	84.70±3.80	74.62±5.99	**87.71±4.70**	Winequality-red-4	AUC	62.93±4.27	41.53±9.49	**71.33±9.39**
	F-M	20.68±2.11	14.56±2.38	**43.34±6.42**		F-M	8.790±0.83	4.750±3.94	**17.53±5.74**
	G-M	83.90±3.54	73.78±5.68	**85.19±4.89**		G-M	57.22±3.80	36.21±8.60	**69.22±9.92**
	Mcc	28.11±3.25	18.71±4.68	**47.68±6.85**		Mcc	9.680±3.17	5.650±8.11	**19.25±8.39**
Yeast 1289vs7	AUC	64.99±5.72	61.78±8.39	**81.23±8.50**	Abalone 3vs11	AUC	96.67±6.73	99.99±0.07	**100.0±0.00**
	F-M	9.510±1.31	9.060±2.42	**29.30±1.67**		F-M	96.00±8.08	99.71±2.02	**100.0±0.00**
	G-M	63.11±4.76	60.78±8.14	**71.24±8.26**		G-M	96.33±7.41	99.99±0.07	**100.0±0.00**
	Mcc	10.60±4.03	8.410±6.05	**34.32±2.50**		Mcc	96.25±7.58	99.72±1.96	**100.0±0.00**
Yeast5	AUC	94.90±1.31	86.39±1.55	**97.55±5.15**	Ozone-onehr	AUC	68.32±2.51	60.79±2.25	**84.95±1.95**
	F-M	38.17±6.16	18.49±1.85	**63.43±9.52**		F-M	8.760±0.62	7.080±0.41	**58.20±4.42**
	G-M	94.75±1.39	85.29±1.82	**97.52±5.95**		G-M	62.31±3.03	48.59±2.24	**77.24±3.23**
	Mcc	46.02±5.24	27.22±2.04	**66.92±8.83**		Mcc	12.57±1.60	8.480±1.68	**44.99±4.56**
krvsk 3vs11	AUC	98.54±0.32	64.21±1.65	**100.0±0.00**	Abalone21 vs8	AUC	54.44±3.64	74.04±11.8	**91.95±1.57**
	F-M	66.39±5.17	7.360±0.36	**100.0±0.00**		F-M	5.140±0.89	10.47±3.70	**51.85±3.22**
	G-M	98.53±0.32	53.23±3.08	**100.0±0.00**		G-M	29.77±8.94	72.44±11.5	**91.17±1.68**
	Mcc	69.51±4.30	10.42±0.85	**100.0±0.00**		Mcc	4.620±2.49	15.47±7.85	**56.87±2.91**
Yeast6	AUC	82.39±4.41	79.01±5.10	**96.01±2.03**	Winequality-white3vs7	AUC	77.32±5.75	67.99±12.4	**92.63±6.79**
	F-M	14.99±1.65	11.70±1.55	**30.30±4.05**		F-M	10.48±1.68	10.88±4.87	**35.39±22.7**
	G-M	81.89±4.09	77.89±4.67	**95.92±3.47**		G-M	75.89±4.90	64.82±15.5	**91.99±8.05**
	Mcc	22.54±2.94	18.57±3.38	**40.54±3.30**		Mcc	16.91±3.49	13.14±8.99	**46.85±20.3**
Winequality-red 8vs67	AUC	71.33±8.76	64.28±6.25	**76.50±8.75**	krvsk0vs8	AUC	97.16±0.73	60.36±13.3	**98.17±2.02**
	F-M	7.740±1.68	6.170±1.17	**10.18±2.75**		F-M	40.80±6.94	4.750±1.43	**58.30±3.89**
	G-M	68.96±7.87	61.80±4.64	**75.24±8.72**		G-M	97.12±0.75	55.30±11.0	**98.14±2.06**
	Mcc	12.30±5.00	8.200±3.48	**16.19±5.53**		Mcc	49.16±5.63	57.20±7.22	**64.55±3.31**
Shuttle-2vs5	AUC	99.30±0.23	57.96±11.5	**100.0±0.00**	kddbufferoverflowvsback	AUC	98.17±3.87	75.00±5.34	**100.0±0.00**
	F-M	68.77±7.45	11.76±13.0	**100.0±0.00**		F-M	97.96±4.37	66.67±8.73	**100.0±0.00**
	G-M	99.29±0.24	54.37±10.7	**100.0±0.00**		G-M	98.06±4.12	70.71±3.69	**100.0±0.00**
	Mcc	71.99±6.18	15.63±14.7	**100.0±0.00**		Mcc	98.04±4.17	70.47±10.3	**100.0±0.00**
krvsk 0vs15	AUC	98.25±0.51	78.07±13.3	**100.0±0.00**	kdd root kitback	AUC	96.70±7.17	70.00±8.12	**98.76±5.04**
	F-M	42.81±7.48	9.900±6.29	**100.0±0.00**		F-M	**95.93±9.09**	57.14±10.3	87.19±2.95
	G-M	98.24±0.52	70.06±18.2	**100.0±0.00**		G-M	96.31±8.15	63.25±8.36	**98.58±5.93**
	Mcc	51.26±5.98	18.52±4.89	**100.0±0.00**		Mcc	**96.28±8.20**	63.03±10.5	84.53±2.64

skinnonsk in	AUC	89.55±0.29	55.82±0.17	**99.29±0.86**	cod	AUC	96.70±5.64	53.34±0.43	**98.62±0.19**
	F-M	1.600±0.12	0.380±0.02	**78.32±3.93**		F-M	23.02±15.2	0.280±0.04	**54.67±1.64**
	G-M	88.93±0.33	34.13±0.49	**99.11±0.86**		G-M	96.62±5.84	25.78±1.66	**98.61±0.19**
	Mcc	7.990±0.33	1.490±0.04	**82.18±3.83**		Mcc	33.40±15.9	0.960±0.02	**61.48±1.97**

B. Ablation Study for Verification of DSENLG

Fig.7 compared intuitively the envelope samples' distribution generated by the proposed algorithm and the original samples' distribution of the compared algorithms SPE, EASE and Imbalance-XGBoost on Ecoli1. It can be seen the envelope samples are more separable than the original samples.

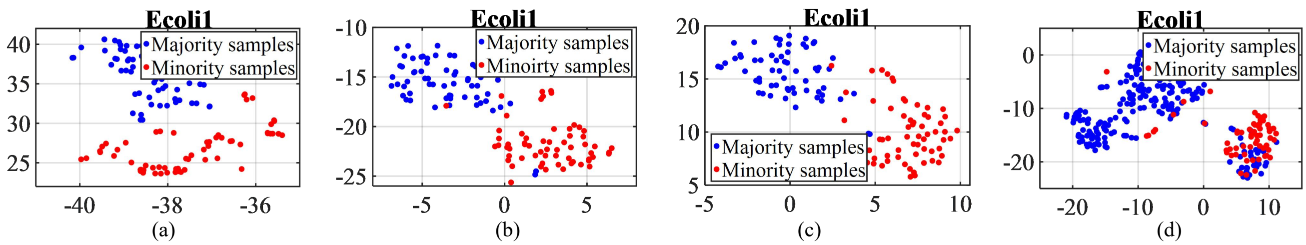


Fig.7. Sample distribution with different algorithms for Ecoli1: (a) is the envelope samples distribution with DSENLG-IE; (b) is the original samples distribution with SPE; (c) is the original samples distribution with EASE; (d) is the original samples distribution with Imbalance-XGBoost. The envelope samples are more separable than original samples

Besides, four diversity indicators such as Disagreement(dis), Correlation coefficient(ς), Q-statistic, and Kappa (κ) are applied to measure the diversity of base classifiers. Higher values of dis are associated with higher diversity, and, conversely, smaller values of ς , Q-statistic, κ are associated with higher diversity. Table III records the results of four diversity indicators on Ecoli3 and Yeast1458vs7 obtained using DSENLG-IE, BBAG, SBAG and UBAG, and Kappa-AUC, F-M, G-M and Mcc diagrams are designed in Fig.8. From Table III, it can be observed that DSENLG-IE got better scores on each indicator. That is, DSENLG-IE has higher diversity. In the Fig.8, it can also be seen the points obtained by DSENLG-IE are located in the upper left corner of the figure. It means the kappa values are smaller and AUC, F-M, G-M and Mcc values are higher with the proposed algorithm, indicating the base classifiers of the proposed algorithm have higher diversity and higher performance than other imbalanced ensemble methods.

TABLE III
DIVERSITY ANALYSIS OF BASE CLASSIFIER

Dataset	Indicators	DSENLG-IE	BBAG [35]	SBAG [10]	UBAG [22]
Ecoli3	dis	**0.1190**	0.1134	0.0452	0.1029
	ς	**0.0039**	0.5024	0.7619	0.5176
	Q-statistic	**0.2252**	0.8128	0.9753	0.8091
	κ	**0.0026**	0.4882	0.7590	0.5032
Yeast14 58vs7	dis	**0.5002**	0.3814	0.1326	0.4231
	ς	**0.0053**	0.3041	0.5790	0.2108
	Q-statistic	**0.0185**	0.4746	0.7727	0.3458
	κ	**0.0055**	0.3072	0.5597	0.2484

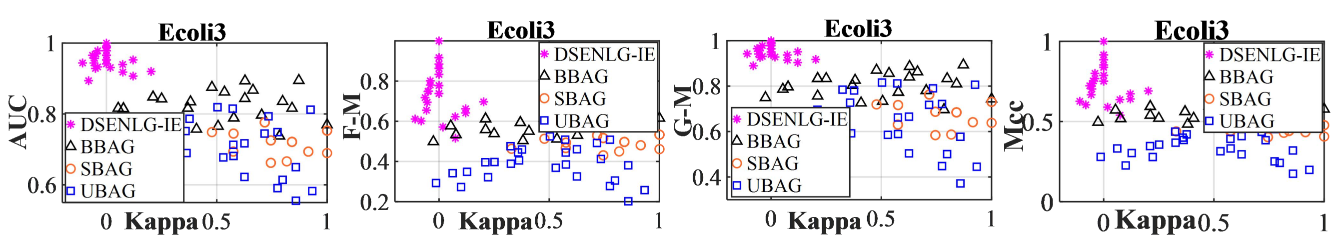


Fig.8. Diversity and performance analysis of base classifiers performance for Ecoli3.

Table III shows the high diversity of base classifier in DSENLG-IE. The base classifiers are constructed based on the envelope samples and the final prediction results through the voting mechanism. Except for the diversity of base classifiers, Table IV discusses the predictive accuracy of the base classifiers for minority samples in DSENLG-IE and IE for dataset 'Ecoli1'. The sample number (SN) for the minority samples is '1-16', and the actual labels (AL) for the minority samples are '0'. The number of basic classifier (BC) is 3 and the prediction fusion (PF) of three basic classifiers can be obtained by voting mechanism. As shown in Table IV, for the sample 12, the first base classifier (BC1) prediction is inconsistent with the actual label (AL), however, the labels of base classifier 2 (BC2) and base classifier 3 (BC3) are predicted correctly to realize error correction. Similarly, for the samples

14, 15 and 16, the second base classifier prediction is inconsistent with the actual label, but the base classifier 1 and 3 predicted correctly to realize error correction. The proposed algorithm guarantees both high prediction accuracy and diversity of base classifiers. However, for traditional method, the diversity is limited.

TABLE IV
ERROR CORRECTION OF BASE CLASSIFIER FOR ECOLI1

		DSENLG-IE					IE			
SN	AL	PF	BC1	BC2	BC3		PF	BC1	BC2	BC3
1	0	0	0	0	0		0	0	0	0
…	…	…	…	…	…	…	…	…	…	…
11	0	0	0	0	0		0	0	0	0
12	**0**	**0**	**1**	**0**	**0**		**0**	**0**	**0**	**0**
13	0	0	0	0	0		0	0	0	0
14	**0**	**0**	**0**	**1**	**0**		**0**	**0**	**0**	**0**
15	**0**	**0**	**0**	**1**	**0**		**0**	**0**	**0**	**0**
16	**0**	**0**	**0**	**1**	**0**		**0**	**0**	**0**	**0**

C. Algorithm Comparison

1) Comparison with Classical IE Methods

Tables V lists the average AUC, F-M, G-M and Mcc values obtained by classical imbalanced ensemble methods and proposed DSENLG-IE method. It can be seen an overwhelming improvement of DSENLG-IE over the other imbalanced ensemble methods on all four criteria. In particular, when considering AUC and G-M, it is observable the method proposed in this paper provided the best performance on 40 and 39 datasets respectively, and never showed the worst performance on any dataset. For F-M and Mcc, the proposed method provided the best performance on 29 and 31 datasets respectively. Thus, DSENLG-IE perform best in most imbalanced datasets.

TABLE V
COMPARISON RESULTS OF THE ENSEMBLE METHODS ON 46 EXPERIMENTAL DATASETS

Data set	Measure	RBO[20]	SBO[21]	UBAG[22]	SBAG[10]	BBAG[35]	EYEE[36]	BACE[36]	GBDT[8]	DSENLG-IE
Iris0	AUC	99.90±0.70	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	99.00±2.00	**100.0±0.00**
	F-M	99.89±0.74	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	98.95±2.11	**100.0±0.00**
	G-M	99.90±0.72	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	98.97±2.05	**100.0±0.00**
	Mcc	99.85±1.04	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	98.52±2.97	**100.0±0.00**
Glass0	AUC	78.40±6.84	72.03±2.65	79.19±2.50	77.84±6.31	**80.97±5.19**	79.54±7.01	77.14±7.00	76.60±6.44	76.35±6.29
	F-M	70.27±8.72	62.09±3.89	71.81±4.20	69.88±8.30	**73.46±6.27**	71.62±8.73	68.37±8.54	68.42±8.84	67.19±9.05
	G-M	77.56±7.42	70.92±3.41	78.93±2.16	77.25±6.68	**80.87±5.20**	79.33±7.27	76.50±7.67	75.63±7.10	74.24±7.32
	Mcc	56.87±11.7	44.51±4.29	57.17±8.05	55.41±12.2	**59.49±9.73**	56.90±12.9	52.34±12.2	54.14±12.9	57.95±12.6
Vertebral	AUC	74.00±4.48	74.57±5.36	82.40±3.88	80.36±3.97	82.64±2.36	79.62±4.63	78.43±6.03	78.29±6.00	**83.98±7.29**
	F-M	64.32±6.09	65.33±7.93	75.35±4.93	73.18±5.46	75.78±3.07	71.21±5.41	70.15±7.39	70.39±8.03	**78.41±7.08**
	G-M	73.04±5.27	73.19±6.35	82.19±3.96	80.04±4.12	82.45±2.54	79.39±4.53	78.06±6.28	77.42±6.63	**82.98±8.10**
	Mcc	47.69±7.66	50.96±10.3	63.11±7.42	60.26±8.11	63.76±4.74	55.99±8.84	55.59±10.7	57.51±10.9	**71.45±8.29**
Haberman	AUC	53.29±4.33	57.41±6.98	59.47±6.51	52.00±8.36	58.89±1.56	56.06±2.80	51.95±4.99	54.43±4.35	**61.81±9.38**
	F-M	30.50±7.85	40.62±8.01	43.01±7.45	30.40±9.85	42.16±2.11	40.10±2.00	34.04±6.35	23.07±10.6	**43.65±8.57**
	G-M	47.55±7.64	56.81±7.09	58.82±6.65	46.75±8.88	57.88±1.70	55.52±2.38	50.65±5.46	36.59±12.5	**60.19±7.30**
	Mcc	6.240±8.40	13.67±13.1	17.31±11.9	4.970±16.9	16.99±2.99	11.04±5.38	3.620±9.07	13.60±13.2	**21.59±8.67**
Vehicle1	AUC	66.51±5.23	70.29±4.56	78.03±3.79	72.62±3.32	75.10±4.23	79.12±3.30	76.12±3.77	53.78±2.83	**82.70±6.54**
	F-M	49.55±8.93	55.56±6.30	64.67±4.67	59.01±4.72	61.53±5.43	65.87±4.05	62.55±4.55	14.91±9.90	**67.23±6.25**
	G-M	63.22±7.82	69.06±5.12	77.91±3.81	71.53±3.81	74.72±4.60	79.03±3.28	75.88±3.96	26.58±12.7	**81.74±7.15**
	Mcc	34.03±9.72	39.60±8.56	51.20±6.83	44.54±6.31	47.00±7.45	52.93±5.97	48.29±6.45	17.91±9.50	**57.01±9.87**
Ecoli1	AUC	84.31±5.21	86.15±5.44	87.70±4.01	88.00±4.28	87.17±4.89	88.39±5.33	88.17±3.95	82.52±7.21	**92.47±4.39**
	F-M	75.80±7.99	77.33±7.31	77.14±5.42	80.38±6.32	77.44±6.94	78.07±7.29	77.96±5.23	75.53±11.4	**80.42±7.52**
	G-M	83.68±5.71	85.71±5.90	87.52±4.19	87.70±4.53	86.93±5.13	88.18±5.60	88.03±4.06	80.73±9.29	**92.09±4.84**
	Mcc	69.04±10.3	70.84±9.35	70.45±7.01	74.78±8.27	70.82±9.06	71.79±9.59	71.48±6.94	70.97±12.3	**80.48±9.12**
New-thyroid1	AUC	98.84±2.27	98.10±2.93	98.47±1.74	97.96±2.98	98.23±2.33	98.42±1.62	98.22±2.67	96.49±5.29	**99.80±1.41**
	F-M	97.10±3.97	96.23±4.63	93.30±7.09	95.61±5.14	94.06±6.43	92.99±6.77	95.81±4.94	95.22±6.30	**99.78±1.57**
	G-M	98.81±2.34	98.05±3.02	98.44±1.79	97.91±3.07	98.20±2.39	98.39±1.66	98.17±2.76	96.29±5.71	**99.79±1.49**
	Mcc	96.62±4.65	95.64±5.40	92.36±7.96	94.92±5.99	93.19±7.32	92.00±7.62	95.16±5.71	94.75±6.81	**100.0±0.00**
Ecoli2	AUC	90.14±5.05	84.02±6.30	89.29±7.92	87.75±7.21	89.01±7.40	87.02±6.84	87.16±7.98	87.12±4.49	**93.62±7.25**
	F-M	**83.43±3.96**	70.21±8.79	77.19±12.8	82.55±7.52	77.28±11.3	74.32±10.0	77.22±9.39	80.65±7.76	82.79±7.79
	G-M	89.55±5.73	83.25±7.12	88.85±8.37	86.57±8.19	88.62±7.72	86.50±7.31	86.07±9.36	86.38±6.16	**92.76±7.51**
	Mcc	81.38±3.88	64.79±10.7	73.38±15.3	81.37±7.62	73.33±13.6	70.00±12.2	74.26±10.4	78.60±7.58	**82.01±7.24**
Musk	AUC	87.03±1.32	92.67±0.51	95.21±0.61	91.38±1.40	93.22±1.05	95.00±0.77	96.55±1.29	67.55±2.05	**98.56±0.76**
	F-M	78.83±1.23	86.87±1.47	88.91±1.86	87.23±1.76	85.82±2.71	88.63±1.97	**93.87±1.39**	51.84±4.40	92.59±4.20
	G-M	86.49±1.60	92.55±0.52	95.20±0.62	91.10±1.51	93.16±1.06	94.98±0.78	96.51±1.33	59.16±3.40	**98.55±0.77**
	Mcc	75.18±1.51	84.46±1.74	86.95±2.15	85.08±2.01	83.24±3.18	86.62±2.25	**92.77±1.63**	55.94±3.42	91.50±4.82
Glass6	AUC	91.98±6.05	89.92±7.78	92.84±3.64	89.98±7.16	91.17±6.87	93.05±4.18	92.57±2.30	86.74±8.27	**98.13±5.07**
	F-M	85.39±9.35	83.73±10.2	84.02±12.3	82.47±8.23	81.87±12.1	85.17±5.98	81.80±6.30	80.23±11.9	**95.91±7.79**
	G-M	91.55±6.54	89.03±8.70	92.58±3.76	89.28±7.80	90.80±7.26	92.84±4.32	92.34±2.39	85.22±10.6	**97.96±5.68**

	Mcc	83.62±10.5	82.59±11.0	82.61±12.6	80.42±9.48	79.57±13.8	83.21±7.08	80.00±6.29	79.16±11.1	**95.77±7.92**
Yeast3	AUC	84.85±4.52	87.88±3.09	93.94±2.08	88.45±2.14	91.10±2.41	89.39±2.54	86.74±2.54	85.75±4.49	**97.71±1.99**
	F-M	62.65±4.75	65.88±5.13	77.50±4.20	81.25±5.94	69.77±3.41	72.73±3.61	71.23±2.32	68.76±7.76	**83.33±1.02**
	G-M	84.63±5.65	87.83±3.41	93.94±2.12	87.92±2.28	91.10±2.51	89.28±2.57	86.38±2.76	84.62±6.16	**97.68±2.09**
	Mcc	58.54±4.78	62.65±3.41	75.67±4.59	79.04±6.87	67.47±4.00	69.70±4.14	67.65±2.72	81.36±7.58	**82.56±1.12**
Ecoli3	AUC	78.70±8.84	77.00±8.62	86.93±7.10	76.72±8.26	85.94±7.35	87.04±5.71	85.67±6.93	66.33±9.03	**95.70±4.69**
	F-M	59.14±12.8	55.54±13.0	62.06±10.0	58.74±13.6	62.22±10.6	62.46±8.16	64.54±9.51	44.79±11.2	**73.37±6.77**
	G-M	75.91±12.1	74.05±12.0	86.48±7.68	73.34±11.6	85.35±8.12	86.69±6.07	84.97±7.78	53.63±11.4	**95.50±4.98**
	Mcc	55.88±13.1	51.21±14.0	59.41±11.5	55.02±14.6	59.28±12.0	59.80±9.09	61.51±10.8	46.14±11.6	**74.30±6.93**
Page-blocks0	AUC	87.48±3.40	93.79±1.72	95.67±1.09	93.89±1.39	95.15±1.12	95.70±1.07	95.32±0.88	80.54±2.58	**98.14±0.39**
	F-M	78.10±4.43	84.13±3.75	81.20±2.29	86.29±2.07	81.49±2.55	81.25±2.20	85.73±2.09	73.79±3.91	**90.43±3.06**
	G-M	86.78±3.98	93.71±1.78	95.66±1.09	93.79±1.46	95.14±1.13	95.69±1.07	95.29±0.89	78.23±3.27	**98.12±0.40**
	Mcc	75.84±4.67	82.51±3.99	79.93±2.43	84.77±2.30	80.07±2.66	79.98±2.35	84.34±2.24	73.28±3.68	**89.80±3.21**
Yeast2vs4	AUC	93.92±6.61	98.92±9.14	95.70±3.44	94.37±5.44	98.39±5.01	95.16±2.31	98.39±7.62	90.00±5.50	**99.44±1.37**
	F-M	67.14±12.8	69.74±11.8	65.61±7.49	73.47±10.1	66.13±8.34	66.13±9.15	71.55±7.84	69.82±8.79	**76.08±11.6**
	G-M	93.84±7.99	98.92±10.4	95.60±3.48	94.30±6.16	98.37±5.23	95.04±2.28	98.37±8.24	89.44±7.21	**99.44±1.39**
	Mcc	64.20±14.1	67.20±10.9	63.52±8.13	71.22±10.7	64.11±9.23	64.08±9.92	69.60±8.43	69.42±8.80	**76.48±11.7**
Yeast05679vs4	AUC	89.21±5.02	89.74±3.98	93.42±4.99	88.44±4.72	93.16±3.99	91.88±3.43	89.21±6.44	84.47±7.79	**96.77±1.11**
	F-M	60.00±8.18	62.07±5.55	**81.82±2.86**	76.19±7.59	62.86±5.80	64.00±5.51	60.00±8.33	77.78±6.97	72.73±2.89
	G-M	89.21±7.49	89.74±5.85	93.34±5.29	88.03±5.79	92.91±4.71	86.12±3.84	89.21±8.15	83.22±6.30	**96.72±1.15**
	Mcc	58.62±8.79	60.60±5.94	**80.12±3.73**	73.68±9.20	62.90±6.71	60.98±6.25	58.62±9.68	76.29±7.08	73.11±2.92
Vowel0	AUC	94.55±4.45	95.20±3.48	96.85±1.94	96.27±2.71	96.41±2.33	97.19±1.72	97.15±2.02	90.57±5.83	**100.0±0.00**
	F-M	88.72±6.38	89.43±5.55	83.01±5.73	92.31±4.34	82.90±6.01	83.98±5.09	90.37±4.72	86.56±7.07	**100.0±0.00**
	G-M	94.35±4.74	95.07±3.66	96.82±1.96	96.18±2.85	96.38±2.36	97.17±1.73	97.12±2.06	89.88±6.69	**100.0±0.00**
	Mcc	87.76±6.99	88.55±6.00	82.30±5.74	91.66±4.71	82.05±6.21	83.25±5.20	89.64±5.03	86.01±6.89	**100.0±0.00**
Glass016vs2	AUC	80.48±11.9	84.64±11.8	81.79±14.4	72.14±7.86	79.05±12.1	88.57±12.4	80.00±13.8	63.81±2.70	**89.39±11.6**
	F-M	57.14±19.3	**66.67±18.8**	54.55±14.0	50.00±18.2	50.00±12.6	42.86±12.1	30.00±9.44	33.33±7.22	22.22±10.9
	G-M	79.28±29.2	84.09±30.0	81.50±23.6	68.66±25.9	78.07±21.5	87.83±22.7	77.46±21.4	56.06±10.4	**88.76±12.5**
	Mcc	53.56±22.5	**62.88±20.4**	50.26±19.0	44.29±20.3	46.34±16.5	45.87±15.9	32.54±16.3	27.62±8.11	31.38±12.9
Ecoli0147vs2356	AUC	81.78±8.88	82.11±9.93	86.16±7.91	83.53±11.3	84.90±8.10	85.40±8.96	89.01±8.29	70.72±12.9	**97.81±3.15**
	F-M	67.54±14.2	61.42±14.4	63.72±11.6	73.42±17.9	63.96±12.2	61.06±12.4	73.94±13.0	52.88±13.7	**81.97±10.5**
	G-M	79.42±11.4	80.08±12.5	85.17±9.74	80.76±14.9	83.86±9.19	84.38±10.5	88.17±9.49	60.73±12.3	**97.73±3.37**
	Mcc	66.05±15.2	59.07±15.7	61.81±12.4	72.96±17.9	61.43±13.3	58.96±13.7	72.57±14.0	54.93±13.3	**77.34±10.8**
climate	AUC	70.39±8.93	72.73±6.92	**85.62±6.38**	80.81±8.66	81.82±7.45	85.35±5.39	81.31±6.74	75.00±6.45	79.93±4.80
	F-M	45.73±15.4	45.83±10.9	54.29±8.92	60.00±14.0	51.49±8.40	60.87±6.79	63.16±8.85	66.67±17.1	**70.60±4.56**
	G-M	64.69±13.2	69.29±9.46	**85.25±7.24**	79.56±12.2	80.95±8.31	85.02±5.76	79.98±7.87	70.71±17.3	74.74±4.30
	Mcc	41.41±16.9	40.88±12.2	52.64±10.0	56.31±14.5	48.64±10.0	58.18±7.68	59.71±10.0	68.97±17.7	**73.87±4.09**
Glass2	AUC	69.83±4.81	60.10±8.63	70.07±13.8	57.13±7.50	66.55±9.52	69.46±6.24	67.10±11.1	59.94±5.29	**87.69±4.45**
	F-M	**33.59±2.85**	21.46±11.9	28.50±11.9	20.38±17.0	27.68±10.8	28.83±6.06	23.91±6.97	28.57±15.5	24.72±9.54
	G-M	67.20±7.17	44.22±22.8	66.30±21.0	32.37±26.5	63.51±12.6	68.11±8.18	65.10±13.0	48.70±18.8	**86.70±5.05**
	Mcc	28.44±3.93	13.20±14.0	24.26±16.7	15.95±18.1	21.78±13.2	24.12±7.88	19.05±12.1	22.60±18.1	**32.47±9.00**
german	AUC	67.35±11.2	82.52±10.9	82.73±9.16	81.15±10.8	83.68±8.56	82.85±10.6	**84.67±8.61**	82.67±13.2	84.48±9.24
	F-M	39.00±21.0	64.44±17.1	59.41±13.7	**73.43±9.06**	62.06±14.3	59.68±16.7	69.25±13.1	72.18±12.3	23.08±5.05
	G-M	55.82±24.5	79.95±14.4	80.88±11.7	77.11±17.0	82.04±11.1	80.68±13.8	82.98±10.7	78.44±10.0	**83.05±10.6**
	Mcc	36.42±22.6	63.10±17.3	58.36±14.4	**74.71±7.98**	60.65±14.6	58.58±17.5	68.14±13.7	73.03±10.9	29.23±10.4
Shuttle-c0-vs-c4	AUC	98.90±6.99	99.95±0.08	99.91±0.08	99.95±0.09	99.91±0.11	99.90±0.12	99.91±0.04	99.43±1.12	**100.0±0.00**
	F-M	96.70±13.8	99.37±1.14	98.73±1.17	99.29±1.23	98.74±1.46	98.71±1.52	98.82±1.18	98.83±1.51	**100.0±0.00**
	G-M	97.90±13.9	99.95±0.08	99.91±0.08	99.95±0.09	99.91±0.11	99.90±0.12	99.91±0.08	99.42±1.14	**100.0±0.00**
	Mcc	96.61±13.8	99.33±1.21	98.65±1.23	99.25±1.30	98.66±1.54	98.63±1.60	98.74±1.25	98.76±1.60	**100.0±0.00**
Yeast1vs7	AUC	61.35±3.44	63.74±8.35	71.96±8.88	60.15±9.02	70.55±11.5	75.41±11.6	73.26±7.97	59.31±7.53	**83.72±6.06**
	F-M	28.49±5.22	25.59±10.3	31.13±9.80	28.31±8.71	28.48±10.8	**33.51±9.57**	27.18±5.93	27.29±8.31	30.00±8.23
	G-M	49.89±7.80	56.75±14.6	70.16±10.8	41.49±10.8	67.17±16.7	73.62±13.5	72.42±7.94	38.39±11.7	**82.12±6.46**
	Mcc	25.03±4.71	20.15±12.0	28.04±12.0	25.61±8.00	25.36±14.1	31.72±13.4	25.91±8.63	29.97±10.2	**34.50±8.12**
Ecoli4	AUC	89.67±9.28	86.71±7.42	90.61±5.46	89.84±9.40	91.99±5.99	92.15±5.92	91.71±6.46	82.18±12.6	**98.54±4.88**
	F-M	76.06±15.3	74.81±7.46	69.88±5.80	85.48±13.3	63.43±7.79	68.55±16.5	81.11±11.8	71.84±13.9	**87.87±8.94**
	G-M	88.61±10.9	85.37±8.79	90.15±5.79	88.65±10.8	91.73±6.23	91.82±6.18	91.25±6.83	76.01±11.3	**98.37±5.72**
	Mcc	75.61±15.9	74.82±7.53	69.47±5.93	85.79±12.8	63.84±8.42	69.40±15.8	80.07±12.6	73.09±13.6	**88.61±8.51**
Page-blocks13vs4	AUC	98.05±4.29	96.21±6.54	98.74±0.97	97.34±3.94	98.90±1.00	98.80±0.70	**99.76±0.37**	96.25±5.54	98.50±1.38
	F-M	89.57±10.5	92.61±9.92	84.63±9.25	94.28±6.33	86.62±10.6	84.71±8.36	**96.77±4.97**	92.92±7.63	77.11±9.46
	G-M	97.95±4.64	95.86±7.52	98.73±0.99	97.23±4.12	98.89±1.02	98.78±0.78	**99.76±0.38**	96.00±6.03	98.49±1.41
	Mcc	89.52±10.6	92.52±9.94	84.88±8.68	94.11±6.60	86.89±10.2	84.94±7.88	**96.71±5.01**	92.89±7.62	78.88±7.20
Dermatology-6	AUC	99.99±0.10	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	97.72±4.79	**100.0±0.00**	99.93±0.22	97.37±5.54	**100.0±0.00**
	F-M	99.78±1.56	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	96.98±5.71	**100.0±0.00**	98.89±3.33	95.05±7.25	**100.0±0.00**
	G-M	99.99±0.10	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	97.56±5.14	**100.0±0.00**	99.93±0.22	97.14±6.15	**100.0±0.00**
	Mcc	99.78±1.57	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	97.03±5.62	**100.0±0.00**	98.88±3.37	95.13±6.99	**100.0±0.00**
svmguide3	AUC	57.08±11.8	62.67±11.8	67.89±12.8	55.48±7.56	67.10±16.1	66.56±16.0	72.36±8.46	57.03±9.75	**80.70±8.98**
	F-M	16.26±12.5	**25.01±18.3**	21.24±10.3	15.37±10.1	20.78±12.5	20.12±12.1	20.52±5.21	18.94±14.2	15.38±3.27
	G-M	30.34±21.3	48.28±26.6	62.41±22.3	21.16±27.0	59.33±27.7	57.87±28.7	71.32±8.23	23.80±19.8	**78.36±7.87**
	Mcc	12.13±10.1	21.18±20.0	19.28±14.1	15.23±12.2	18.50±17.6	17.82±17.2	**21.30±8.00**	19.71±16.1	17.37±5.21
Yeast1458vs7	AUC	56.01±10.1	57.18±10.0	62.65±8.40	51.06±3.30	63.31±13.0	60.66±12.9	63.30±7.91	50.42±2.27	**72.14±9.29**
	F-M	13.93±10.4	13.86±10.6	14.49±4.43	4.440±3.89	14.54±6.69	13.74±7.82	11.78±2.29	2.090±7.12	**15.08±3.70**
	G-M	40.01±22.8	43.72±25.1	58.89±10.2	8.100±16.2	59.06±16.8	55.62±17.5	60.54±6.59	3.260±11.0	**68.13±10.8**
	Mcc	8.670±13.0	9.23±12.83	11.94±7.61	2.970±9.22	12.35±11.7	10.31±12.5	10.95±6.52	1.700±9.33	**18.43±7.42**
Yeas	AUC	74.42±8.23	75.98±1.78	82.24±6.18	68.48±4.35	81.61±9.49	79.25±4.25	81.04±3.07	60.13±1.41	**87.71±4.70**

t4	F-M	38.82±9.91	38.81±3.85	29.76±4.97	40.14±7.27	30.87±7.82	28.43±4.16	28.98±4.05	27.75±3.23	**43.34±6.42**
	G-M	70.40±11.1	73.49±2.45	81.81±6.61	61.36±6.88	80.63±10.4	78.53±4.84	80.72±3.24	45.94±2.96	**85.19±4.89**
	Mcc	37.77±10.9	38.14±3.40	33.62±6.60	38.73±7.26	34.19±10.2	31.23±4.68	32.47±4.44	27.45±3.60	**47.68±6.85**
Winequality-red-4	AUC	54.75±5.08	55.89±5.23	67.73±6.74	51.61±2.87	62.18±6.80	67.51±6.66	55.46±7.47	51.52±3.46	**71.33±9.39**
	F-M	9.950±5.88	11.29±6.04	16.59±4.02	6.010±7.74	13.29±4.48	16.66±4.36	7.580±1.82	5.330±9.51	**17.53±5.74**
	G-M	36.12±15.6	39.70±13.9	65.02±9.21	13.04±16.2	56.95±11.1	64.74±9.08	54.65±7.23	10.27±16.9	**69.22±9.92**
	Mcc	6.370±6.67	7.800±6.88	16.80±6.17	4.860±8.84	11.70±6.48	16.75±6.31	3.910±5.36	4.490±10.9	**19.25±8.39**
Yeast1289vs7	AUC	54.05±8.28	65.20±3.98	74.33±8.92	54.29±4.40	66.69±11.6	74.17±11.5	61.98±8.57	57.86±6.35	**81.23±8.50**
	F-M	11.70±7.50	19.22±5.55	17.85±4.74	13.89±11.3	14.72±6.27	18.98±6.23	8.970±2.33	22.87±6.86	**29.30±1.67**
	G-M	27.37±21.2	59.79±5.85	71.66±11.1	24.38±19.9	62.16±17.1	**72.45±13.0**	59.67±9.25	33.61±12.2	71.24±8.26
	Mcc	8.060±9.27	17.95±5.94	20.99±7.69	13.21±13.1	14.99±10.1	21.74±9.80	8.590±6.12	25.22±9.04	**34.32±2.50**
Abalone3vs11	AUC	99.51±0.48	99.98±0.09	99.63±0.45	99.98±0.10	99.63±0.45	99.60±0.43	99.90±0.21	99.93±0.18	**100.0±0.00**
	F-M	87.65±10.8	99.43±2.80	90.64±10.5	99.43±2.80	90.64±10.5	89.71±10.0	97.14±5.71	98.00±4.96	**100.0±0.00**
	G-M	99.50±0.49	99.98±0.10	99.63±0.45	99.98±0.10	99.63±0.45	99.60±0.43	99.90±0.21	99.93±0.18	**100.0±0.00**
	Mcc	88.33±10.1	99.45±2.71	91.14±9.82	99.45±2.71	91.14±9.82	90.22±9.36	97.23±5.54	98.06±4.80	**100.0±0.00**
Yeast5	AUC	89.95±7.84	87.01±8.12	95.40±3.60	86.57±6.90	95.36±3.54	95.07±3.92	93.41±6.76	74.50±8.43	**97.55±5.15**
	F-M	61.05±10.3	67.05±12.2	57.02±7.87	**69.70±9.17**	57.63±7.60	55.29±6.57	67.23±9.45	55.94±14.5	63.43±9.52
	G-M	89.10±9.51	85.67±9.92	95.32±3.70	85.26±8.38	95.28±3.64	94.97±4.05	92.97±7.85	69.12±12.7	**97.52±5.95**
	Mcc	62.30±10.5	66.80±12.6	60.77±7.09	**69.50±9.31**	61.26±6.71	59.25±6.21	68.47±9.54	56.55±14.6	66.92±8.83
Ozone-onehr	AUC	64.75±7.27	64.27±6.17	80.79±5.34	67.65±4.51	78.30±5.52	80.40±5.03	80.15±5.54	59.59±3.13	**84.95±1.95**
	F-M	36.36±7.63	33.33±7.23	30.11±3.58	47.62±5.14	30.14±3.69	24.81±3.63	20.58±3.13	27.27±8.65	**58.20±4.42**
	G-M	55.95±13.4	55.25±11.1	**80.26±5.94**	59.64±4.41	77.36±6.42	79.89±5.57	79.84±5.82	44.54±4.53	77.24±3.23
	Mcc	37.52±8.68	34.28±8.13	37.72±4.99	**49.58±5.11**	33.95±5.03	29.22±4.90	25.83±4.70	27.86±8.26	44.99±4.56
Krvsk3vs11	AUC	98.71±1.84	96.49±3.57	97.82±2.42	95.12±4.59	97.41±2.63	97.35±2.52	98.18±2.52	93.62±4.10	**100.0±0.00**
	F-M	93.03±5.14	94.65±4.67	83.64±5.94	93.56±5.58	83.94±7.24	83.39±6.66	95.28±5.05	92.18±4.68	**100.0±0.00**
	G-M	98.68±1.88	96.35±3.83	97.78±2.50	94.87±4.92	97.36±2.71	97.30±2.59	98.13±2.60	93.30±4.43	**100.0±0.00**
	Mcc	93.04±5.06	94.66±4.57	84.12±5.51	93.64±5.45	84.27±6.97	83.78±6.33	95.22±5.09	92.28±4.51	**100.0±0.00**
Abalone21vs8	AUC	84.83±2.86	83.56±4.31	85.87±9.92	79.03±5.03	87.35±2.63	86.51±1.43	88.42±1.22	69.88±1.36	**91.95±1.57**
	F-M	50.08±3.52	56.34±3.61	35.27±3.23	**59.40±3.03**	35.88±2.99	35.60±1.33	45.06±3.08	44.24±2.63	51.85±3.22
	G-M	81.93±8.40	78.58±4.23	84.75±9.25	71.06±7.78	85.17±8.05	84.38±6.85	86.97±3.50	55.80±3.03	**91.17±1.68**
	Mcc	51.94±3.81	57.56±3.16	39.96±3.97	**61.02±3.14**	41.06±4.53	40.52±1.79	49.28±3.43	45.03±2.75	56.87±2.91
Yeast6	AUC	94.31±9.34	90.79±1.35	95.34±6.56	92.51±8.61	96.37±6.84	**98.10±7.43**	93.60±6.32	91.65±8.30	96.01±2.03
	F-M	29.79±11.1	48.00±9.51	34.15±5.43	**80.00±1.49**	40.00±6.04	56.00±6.66	27.45±6.00	60.00±7.74	30.30±4.05
	G-M	94.14±12.2	90.65±11.6	95.23±7.13	92.26±1.25	96.30±7.64	**98.09±8.29**	93.38±7.34	91.46±8.16	95.92±3.47
	Mcc	39.38±11.6	51.86±8.15	43.21±6.52	**79.67±1.52**	48.15±6.96	61.17±7.96	37.25±6.44	61.77±7.16	40.54±3.30
Winequality-white3vs7	AUC	71.31±11.1	72.73±10.6	81.53±12.6	84.09±10.4	84.38±12.7	82.10±12.6	81.25±11.5	87.50±9.37	**92.63±6.79**
	F-M	21.05±15.8	28.57±11.0	21.43±8.81	31.58±25.0	33.33±9.06	23.08±7.87	20.69±8.16	28.57±9.14	**35.39±22.7**
	G-M	68.05±29.6	69.08±23.5	81.27±20.1	83.60±29.2	83.85±18.4	81.79±20.1	81.01±19.4	86.60±8.17	**91.99±8.05**
	Mcc	18.76±17.0	29.25±12.8	27.35±12.0	36.36±27.1	37.84±12.1	28.89±11.3	26.64±10.2	27.48±10.3	**46.85±20.3**
Winequality-red8vs67	AUC	57.37±1.13	57.09±7.51	61.09±2.44	54.45±6.64	60.90±2.41	60.57±2.37	58.00±8.88	54.22±6.40	**76.50±8.75**
	F-M	10.23±1.12	9.840±8.98	7.190±3.81	**13.27±3.80**	7.380±4.40	7.050±4.08	5.120±1.44	11.16±1.52	10.18±2.75
	G-M	30.68±3.08	33.70±7.75	55.00±1.87	17.75±4.81	52.70±4.18	53.35±3.06	52.37±7.59	18.36±2.45	**75.24±8.72**
	Mcc	9.070±1.38	8.300±8.68	7.270±3.16	14.33±2.16	7.430±8.49	7.010±8.23	5.010±7.21	10.71±1.68	**16.19±5.53**
krvsk0vs8	AUC	86.32±8.68	89.42±7.19	91.72±8.44	85.67±4.22	90.72±7.28	94.10±5.58	94.60±7.21	70.19±10.7	**98.17±2.02**
	F-M	53.45±8.77	74.17±8.89	31.87±7.68	**77.07±6.94**	32.43±8.99	33.68±6.15	59.07±8.41	46.24±19.4	58.30±3.89
	G-M	84.72±8.12	88.48±8.26	91.20±9.49	83.25±4.45	90.37±7.80	93.89±5.99	94.20±7.15	61.25±17.8	**98.14±2.06**
	Mcc	55.36±8.47	74.58±8.89	39.95±8.90	**78.68±5.07**	39.93±9.18	42.35±6.50	62.46±8.48	47.53±19.0	64.55±3.31
Shuttle-2vs5	AUC	99.97±0.09	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
	F-M	98.24±5.38	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
	G-M	99.97±0.09	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
	Mcc	98.35±5.01	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
kddbufferoverflowvsback	AUC	99.49±2.59	99.33±3.27	99.16±2.51	99.00±3.96	99.82±1.17	98.33±4.41	99.67±2.33	**100.0±0.00**	**100.0±0.00**
	F-M	98.81±3.96	99.20±3.92	98.63±3.92	98.80±4.75	98.76±3.18	97.77±5.54	99.60±2.80	**100.0±0.00**	**100.0±0.00**
	G-M	99.45±2.83	99.27±3.60	99.12±2.63	98.90±4.36	99.81±1.22	98.20±4.80	99.63±2.57	**100.0±0.00**	**100.0±0.00**
	Mcc	98.84±3.82	99.26±3.63	98.64±3.92	98.89±4.40	98.80±3.08	97.87±5.25	99.63±2.59	**100.0±0.00**	**100.0±0.00**
krvsk0vs15	AUC	98.90±3.37	94.25±6.90	96.99±6.26	91.63±6.54	95.51±8.29	99.51±0.50	99.91±0.15	89.15±9.00	**100.0±0.00**
	F-M	96.41±8.16	91.94±8.81	84.72±6.46	89.36±7.96	79.38±8.41	84.56±3.36	94.16±8.39	83.60±13.2	**100.0±0.00**
	G-M	98.83±3.61	93.77±7.62	96.72±6.95	90.46±7.25	94.88±4.31	98.27±5.74	99.91±0.15	87.84±10.8	**100.0±0.00**
	Mcc	96.57±7.83	92.29±8.44	85.45±5.77	90.38±7.32	80.44±7.84	85.41±2.67	94.50±7.65	84.45±12.4	**100.0±0.00**
kddrootkitback	AUC	96.75±5.50	95.45±5.63	93.99±7.07	94.35±7.71	93.79±7.36	93.34±7.69	95.69±5.54	94.04±5.78	**98.76±5.04**
	F-M	89.51±19.1	**94.68±6.66**	92.41±9.18	93.22±9.70	91.70±10.1	91.03±10.3	94.30±6.59	92.51±6.89	87.19±2.95
	G-M	96.53±5.92	95.15±6.01	93.46±7.95	93.77±8.72	93.22±9.70	92.69±8.73	95.41±5.91	93.65±6.17	**98.58±5.93**
	Mcc	90.51±17.0	**94.92±6.37**	92.88±8.45	93.73±8.78	92.10±9.63	91.56±9.54	94.55±6.32	92.83±6.63	84.53±2.64
skinnonskin	AUC	99.72±0.11	98.68±2.80	99.38±0.08	97.09±3.56	99.38±0.10	99.49±0.09	**99.98±0.00**	97.09±4.34	99.29±0.86
	F-M	39.67±9.84	95.40±4.47	21.76±2.64	**95.52±4.74**	21.97±2.75	25.33±3.52	88.62±7.19	92.61±7.70	78.32±3.93
	G-M	99.72±0.11	98.63±2.91	99.38±0.08	96.98±3.71	99.38±0.10	99.49±0.09	**99.98±0.00**	96.93±4.61	99.11±0.86
	Mcc	49.49±7.79	95.54±4.34	34.68±2.39	**95.63±4.67**	34.87±2.49	37.83±3.03	89.36±6.49	92.80±7.59	82.18±3.83
cod	AUC	96.92±4.43	92.55±7.87	96.90±4.11	92.96±7.45	97.08±3.71	96.88±4.35	96.03±5.38	82.91±11.5	**98.62±0.19**
	F-M	16.42±8.10	82.76±6.13	10.99±1.63	**87.75±9.06**	11.16±1.34	11.11±1.64	47.01±8.28	64.39±20.9	54.67±1.64

G-M	96.78±4.77	91.81±9.01	96.78±4.36	92.31±8.49	96.99±3.88	96.75±4.67	95.78±5.79	79.19±17.7	**98.61±0.19**
Mcc	28.48±6.84	83.48±6.09	23.28±1.75	**88.24±9.63**	23.55±1.64	23.45±2.15	53.89±7.37	65.14±20.7	61.48±1.97

Assuming the first rank for the method with the best performance and the ninth rank for the method with the worst performance, so for AUC, F-M, G-M and Mcc, the average ranks of each method on the experimental datasets can be calculated and analyzed. Table VI gives the average ranks of AUC, F-M, G-M and Mcc of each method on the 46 datasets. Table VI shows the proposed DSENLG-IE method achieves the lowest average ranks. So the performance of proposed method is the best.

TABLE VI
AVERAGE RANKS OF ALL COMPARED ENSEMBLE METHODS

Algorithm	AUC	F-M	G-M	Mcc
DSENLG-IE	**1.500**	**2.717**	**1.630**	**2.326**
RBO[20]	6.413	5.782	6.326	6.152
SBO[21]	6.108	4.369	6.087	4.587
UBAG[22]	3.478	5.304	3.413	5.261
SBAG[10]	6.369	3.761	6.391	3.652
BBAG[35]	4.021	5.413	4.043	5.695
EYEE[36]	3.804	5.608	3.782	5.674
BACE[36]	4.109	4.804	4.022	4.826
GBDT[8]	7.848	5.869	7.935	5.522

Whether there were the statistically significant differences with other imbalanced ensemble methods in terms of average ranks was analyzed. the results of Holm's test are recorded in Table VII. In the Holm's test, the proposed method was taken as the control method, and the level of significance is α / n_c ,where $\alpha = 0.05$ and n_c is the number of comparisons between algorithms. The results of Holm's test are recorded in Table VII. From Table VII, It's obvious that all the hypothesis of equivalence have been rejected, indicating the proposed method DSENLG-IE performs better than the other imbalanced ensemble methods significantly. The results indicate that the deep envelope samples generated by DSEN-LG are more competitive.

TABLE VII
P-VALUES FROM HOLM'S TEST FOR ALL COMPARED METHODS

Method	AUC		F-M		G-M		Mcc	
	$\alpha_{0.05}$ (α / n_c)	P-value	$\alpha_{0.05}$ (α / n_c)	P-value	$\alpha_{0.05}$ (α / n_c)	P-value	$\alpha_{0.05}$ (α / n_c)	P-value
RBO[20]	0.0071	**1.05E-28**	0.0071	**9.59E-09**	0.0083	**4.51E-27**	0.0063	**4.62E-13**
SBO[21]	0.01	**7.31E-26**	0.025	**1.70E-03**	0.01	**2.72E-24**	0.025	**1.27E-05**
UBAG [22]	0.05	**1.82E-06**	0.0125	**1.11E-06**	0.05	**1.77E-05**	0.0125	**1.87E-08**
SBAG[10]	0.0083	**2.70E-28**	0.05	**4.67E-02**	0.0071	**4.51E-27**	0.05	**9.86E-03**
BBAG[35]	0.0167	**1.62E-09**	0.01	**4.00E-07**	0.0125	**8.53E-09**	0.0071	**1.38E-10**
EYEE[36]	0.025	**3.16E-08**	0.0083	**5.82E-08**	0.025	**2.52E-07**	0.0083	**1.79E-10**
BACE[36]	0.0125	**4.65E-10**	0.0167	**7.85E-05**	0.0167	**1.14E-08**	0.0167	**1.47E-06**
GBDT[8]	0.0063	**4.71E-43**	0.0063	**3.77E-09**	0.0063	**2.64E-42**	0.01	**1.05E-09**

2) Comparison with State-of-the-art IE Methods

Table VIII records comparison results between the proposed DSENLG-IE method and six SOTA IE methods. The comparisons in Table VIII clearly demonstrated that the proposed DSENLG-IE provide better performance in terms of the four metrics than compared methods, suggesting that DSENLG-IE generates high-quality and high- separability envelope samples.

TABLE VIII
COMPARISON RESULTS WITH STATE-OF-THE-ART IMBALANCED ENSEMBLE METHODS

Dataset	Iris0				Glass0			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	99.00	--	--	--	88.50	--	--	--
HD-Ensemble[41]	--	--	--	--	--	--	--	--
EASE[39]	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	74.73±6.66	65.85±8.34	74.45±6.85	47.40±12.8
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**78.95±6.88**	**71.31±8.55**	**78.67±6.97**	56.55±13.5
Imbalance-XGBoost[9]	98.90±2.07	98.77±2.38	98.88±2.13	98.25±3.40	76.44±5.55	67.95±7.63	75.49±6.34	53.46±10.5

Dataset	Vertebral				Haberman			
DSENLG-IE	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	76.35±6.29	67.19±9.05	74.24±7.32	**57.95±12.6**
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	**64.80**	--	--	--
HD-Ensemble[41]	--	--	--	--	--	--	--	--
EASE[39]	77.55±5.01	68.94±6.12	77.25±5.12	52.69±9.54	57.73±8.78	41.78±9.68	56.24±8.28	13.95±15.7
HOEC[40]	--	--	--	--	62.42±1.93	--	--	--
SPE[38]	78.93±6.12	70.89±7.88	78.53±6.54	56.70±11.4	60.02±6.32	**43.82±7.77**	59.31±7.02	17.92±11.3
Imbalance-XGBoost[9]	79.28±6.07	71.49±8.06	78.75±6.65	58.11±11.3	56.06±6.19	32.83±11.5	48.27±11.6	12.94±13.5
DSENLG-IE	**83.98±7.29**	**78.41±7.08**	**82.98±8.10**	**71.45±8.29**	61.81±9.38	43.65±8.57	**60.19±7.30**	**21.59±8.67**
Dataset	Vehicle1				Ecoli1			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	82.50	--	--	--	**95.70**	--	--	--
HD-Ensemble[41]	--	--	--	--	--	--	--	--
EASE[39]	72.21±3.97	57.13±4.82	72.05±4.04	39.96±7.13	86.43±2.94	76.61±4.16	86.17±3.11	69.79±5.47
HOEC[40]	75.96±1.35	--	--	--	88.16±0.87	--	--	--
SPE[38]	77.44±3.50	63.92±4.39	77.31±3.68	50.11±6.22	86.33±4.22	78.46±5.92	85.88±4.66	72.40±7.57
Imbalance-XGBoost[9]	69.99±3.92	55.32±5.73	67.96±4.81	40.75±7.38	84.71±6.62	76.63±9.26	83.80±7.64	70.74±11.0
DSENLG-IE	**82.70±6.54**	**67.23±6.25**	**81.74±7.15**	**57.01±9.87**	92.47±4.39	**80.42±7.52**	**92.09±4.84**	**80.48±9.12**
Dataset	New-thyroid1				Ecoli2			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	99.70	--	--	--	93.40	--	--	--
HD-Ensemble[41]	--	--	--	--	--	--	--	--
EASE[39]	98.84±2.22	97.13±4.08	98.81±2.29	96.68±4.72	86.45±5.62	72.88±10.4	86.14±5.79	68.01±12.4
HOEC[40]	--	--	--	--	91.28±1.53	--	--	--
SPE[38]	98.21±2.89	96.82±4.74	98.15±2.99	96.37±5.41	89.92±6.36	80.67±7.85	89.38±7.10	77.87±8.93
Imbalance-XGBoost[9]	96.36±4.44	93.68±6.26	96.22±4.67	92.68±7.30	84.47±6.84	75.63±10.6	83.23±7.92	72.18±12.2
DSENLG-IE	**99.80±1.41**	**99.78±1.57**	**99.79±1.49**	**1±0**	**93.62±7.25**	**82.79±7.79**	**92.76±7.51**	**82.01±7.24**
Dataset	Musk				Glass6			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	93.40	--	--	--
HD-Ensemble[41]	--	--	--	--	--	--	--	--
EASE[39]	95.18±0.79	88.56±0.71	95.16±0.81	86.54±0.82	91.51±6.12	81.26±8.85	91.14±6.55	78.99±10.1
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	97.17±0.98	**95.97±1.21**	97.14±1.00	**95.27±1.42**	91.64±5.68	83.00±8.67	91.30±6.07	80.74±9.97
Imbalance-XGBoost[9]	92.25±1.43	89.63±2.12	91.99±1.53	88.01±2.45	89.68±8.85	82.29±13.6	88.56±11.1	80.91±13.6
DSENLG-IE	**98.56±0.76**	92.59±4.20	**98.55±0.77**	91.50±4.82	**98.13±5.07**	**95.91±7.79**	**97.96±5.68**	**95.77±7.92**
Dataset	Yeast3				Ecoli3			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	96.90	--	--	--	93.30	--	--	--
HD-Ensemble[41]	--	--	--	--	--	--	--	--
EASE[39]	88.49±4.32	73.00±6.18	88.14±4.71	69.99±6.96	81.43±6.36	58.59±8.97	80.38±7.35	54.40±10.3
HOEC[40]	--	--	--	--	87.34±1.96	--	--	--
SPE[38]	88.77±3.79	75.68±5.65	88.39±4.16	72.83±6.34	83.68±7.73	61.37±8.98	82.59±9.19	57.91±10.5
Imbalance-XGBoost[9]	84.42±3.93	73.96±5.68	83.28±4.76	71.16±6.04	75.70±9.93	56.94±16.7	71.57±14.1	53.86±17.1
DSENLG-IE	**97.71±1.99**	**83.33±1.02**	**97.68±2.09**	**82.56±1.12**	**95.70±4.69**	**73.37±6.77**	**95.50±4.98**	**74.30±6.93**
Dataset	Page-blocks0				Yeast2vs4			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	**98.70**	--	--	--	98.00	--	--	--
HD-Ensemble[41]	--	--	--	--	98.33±1.10	--	94.20±3.70	--
EASE[39]	93.24±1.36	83.68±1.74	93.14±1.42	81.93±1.94	98.91±5.64	75.43±8.74	98.91±6.18	73.09±9.85
HOEC[40]	92.94±0.30	--	--	--	--	--	--	--
SPE[38]	93.24±1.73	86.24±2.03	93.09±1.83	84.72±2.26	**99.46±1.53**	75.31±9.87	**99.46±1.40**	73.04±10.7
Imbalance-XGBoost[9]	92.11±2.08	85.80±2.76	91.87±2.24	84.23±3.05	95.00±6.19	76.58±10.8	**94.87±7.37**	74.53±11.8
DSENLG-IE	98.14±0.39	**90.43±3.06**	**98.12±0.40**	**89.80±3.21**	99.44±1.37	**76.08±11.6**	99.44±1.39	**76.48±11.7**
Dataset	Yeast05679vs4				Vowel0			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	98.10	--	--	--
HD-Ensemble[41]	90.84±4.10	--	82.27±7.40	--	99.99±0.20	--	97.53±1.40	--
EASE[39]	89.27±8.38	60.00±13.6	89.27±10.4	58.68±15.4	97.48±2.14	93.29±4.99	97.44±2.19	92.82±5.22
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	90.83±5.67	66.67±7.16	90.83±6.32	65.05±8.48	96.39±3.45	93.80±4.52	96.27±3.68	93.33±4.78
Imbalance-XGBoost[9]	92.92±6.92	**78.26±11.8**	92.87±7.04	**76.49±12.1**	95.24±3.60	90.52±4.90	95.09±3.81	89.71±5.32
DSENLG-IE	**96.77±1.11**	72.73±2.89	**96.72±1.15**	73.11±2.92	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
Dataset	Glass016vs2				Ecoli0147vs2356			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	71.30	--	--	--	--	--	--	--
HD-Ensemble[41]	86.06±8.70	--	77.11±13.3	--	--	--	--	--
EASE[39]	60.14±13.0	22.84±16.2	48.03±27.9	14.40±19.3	87.17±7.64	73.12±12.7	86.16±8.75	71.93±13.4
HOEC[40]	--	--	--	--	84.71±1.33	--	--	--
SPE[38]	64.18±14.5	24.65±10.6	60.18±20.4	17.15±17.1	84.76±8.88	63.31±11.9	83.43±10.6	61.22±13.2

Imbalance-XGBoost[9]	51.85±6.62	**28.57±16.4**	49.28±22.6	21.76±17.1	79.41±10.9	66.98±18.3	75.40±15.6	67.33±16.7
DSENLG-IE	**89.39±11.6**	22.22±10.9	**88.76±12.5**	**31.38±12.9**	**97.81±3.15**	**81.97±10.5**	**97.73±3.37**	**77.34±10.8**
Dataset	climate				Glass2			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	76.60	--	--	--
HD-Ensemble[41]	--	--	--	--	86.65±7.41	--	76.44±14.1	--
EASE[39]	77.86±5.01	49.80±5.75	76.38±6.07	45.74±6.72	63.35±12.3	**25.13±13.6**	54.23±13.7	18.53±16.7
HOEC[40]	**85.61±1.65**	--	--	--	77.96±2.12	--	--	--
SPE[38]	80.86±6.40	45.64±8.02	**80.34±7.62**	43.23±9.28	72.52±12.2	24.07±9.93	71.19±13.0	18.21±14.3
Imbalance-XGBoost[9]	70.11±8.66	51.10±17.1	62.05±15.8	51.54±16.8	53.43±7.19	25.00±18.2	49.35±15.2	22.66±19.8
DSENLG-IE	79.93±4.80	**70.60±4.56**	74.74±4.30	**73.87±4.09**	**87.69±4.45**	24.72±9.54	**86.70±5.05**	32.47±9.00
Dataset	german				Shuttle-c0-vs-c4			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	**100.0**	--	--	--
HD-Ensemble[41]	80.01±9.90	--	69.61±16.0	--	**100.0±0.00**	--	**100.0±0.00**	--
EASE[39]	**85.67±10.2**	**74.84±16.8**	83.64±13.5	73.76±17.4	99.53±1.21	99.15±1.37	99.52±1.24	99.10±1.44
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	85.30±8.69	66.28±11.6	**83.85±10.4**	65.03±12.1	99.50±1.01	98.91±1.35	99.50±1.03	98.84±1.43
Imbalance-XGBoost[9]	82.27±9.96	73.86±15.1	79.45±12.7	**74.47±14.3**	99.94±0.09	99.17±1.26	99.94±0.09	99.12±1.33
DSENLG-IE	84.48±9.24	23.08±5.05	83.05±10.6	29.23±10.4	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
Dataset	Yeast1vs7				Ecoli4			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	77.50	--	--	--	96.40	--	--	--
HD-Ensemble[41]	**84.41±8.70**	--	77.67±7.70	--	**98.83±1.90**	--	94.05±4.80	--
EASE[39]	74.22±8.44	**36.93±10.7**	72.04±11.1	34.10±12.3	89.80±8.76	79.61±12.4	88.78±10.1	79.40±12.7
HOEC[40]	77.07±1.94	--	--	--	--	--	--	--
SPE[38]	72.46±7.07	26.67±5.30	71.78±7.72	24.96±7.80	90.88±8.87	76.76±12.1	89.90±10.6	77.03±11.6
Imbalance-XGBoost[9]	61.99±8.12	33.11±9.73	44.70±12.3	34.51±11.2	81.72±12.6	71.95±20.9	78.04±16.7	73.19±20.1
DSENLG-IE	83.72±6.06	30.00±8.23	**82.12±6.46**	**34.50±8.12**	98.54±4.88	**87.87±8.94**	**98.37±5.72**	**88.61±8.51**
Dataset	Page-blocks13vs4				Dermatology-6			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	--	--	--	--
HD-Ensemble[41]	--	--	--	--	--	--	--	--
EASE[39]	99.57±1.40	96.44±4.46	99.56±1.48	96.37±4.53	99.87±0.28	98.00±4.27	99.87±0.28	97.98±4.31
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	**99.78±0.34**	**96.83±4.70**	**99.77±0.34**	**96.77±4.76**	99.94±0.20	99.11±3.01	99.94±0.20	99.10±3.05
Imbalance-XGBoost[9]	97.03±5.90	92.58±9.80	96.77±6.77	92.47±9.91	97.87±5.18	95.62±7.05	97.67±5.78	95.69±6.79
DSENLG-IE	98.50±1.38	77.11±9.46	98.49±1.41	78.88±7.20	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
Dataset	svmguide3				Yeast1458vs7			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	63.80	--	--	--
HD-Ensemble[41]	79.43±10.8	--	67.37±18.7	--	69.16±10.9	--	63.08±10.7	--
EASE[39]	71.22±12.7	**29.80±14.6**	65.37±21.3	**27.97±17.1**	62.92±9.26	**17.33±7.88**	55.63±18.3	14.44±10.3
HOEC[40]	--	--	--	--	66.08±3.44	--	--	--
SPE[38]	63.50±10.6	14.42±5.91	60.22±10.6	12.17±10.0	58.98±7.58	10.71±2.34	57.55±7.01	7.380±6.24
Imbalance-XGBoost[9]	53.58±7.79	10.73±10.4	12.99±14.7	10.83±12.9	51.09±3.08	4.210±9.68	6.520±14.9	4.460±12.3
DSENLG-IE	**80.70±8.98**	15.38±3.27	**78.36±7.87**	17.37±5.21	**72.14±9.29**	15.08±3.70	**68.13±10.8**	**18.43±7.42**
Dataset	Yeast4				Winequality-red-4			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	**91.40**	--	--	--	--	--	--	--
HD-Ensemble[41]	--	--	--	--	--	--	--	--
EASE[39]	75.84±6.81	39.11±7.71	72.67±9.10	38.48±8.61	62.14±6.95	15.80±6.17	53.65±14.1	13.81±7.81
HOEC[40]	79.29±1.23	--	--	--	61.84±2.14	--	--	--
SPE[38]	81.80±7.09	30.25±5.45	80.96±8.71	33.82±7.09	66.32±6.91	11.60±2.22	65.49±7.80	12.34±5.06
Imbalance-XGBoost[9]	65.18±6.67	37.70±4.43	54.15±3.71	37.38±5.05	51.31±2.67	4.700±7.94	9.260±15.1	4.55±9.43
DSENLG-IE	87.71±4.70	**43.34±6.42**	**85.19±4.89**	**47.68±6.85**	**71.33±9.39**	**17.53±5.74**	**69.22±9.92**	**19.25±8.39**
Dataset	Yeast1289vs7				Abalone3vs11			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	60.50	--	--	--	--	--	--	--
HD-Ensemble[41]	78.14±8.20	--	68.73±13.1	--	--	--	--	--
EASE[39]	70.70±8.00	23.96±6.91	66.80±10.5	24.11±8.67	99.93±0.18	98.00±4.96	99.93±0.18	98.06±4.80
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	65.18±8.30	10.64±2.66	64.10±8.79	11.18±6.03	99.97±0.12	99.14±3.39	99.97±0.12	99.17±3.29
Imbalance-XGBoost[9]	55.95±6.68	16.42±6.96	25.16±14.9	17.11±8.71	99.56±2.33	96.74±6.18	99.53±2.56	96.86±5.96
DSENLG-IE	**81.23±8.39**	**29.30±1.67**	**71.24±8.26**	**34.32±2.50**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
Dataset	Yeast5				Ozone-onehr			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	97.00	--	--	--	--	--	--	--
HD-Ensemble[41]	**99.12±0.50**	--	95.89±0.90	--	--	--	--	--
EASE[39]	86.04±7.59	**68.27±11.1**	84.57±9.31	**68.05±11.4**	72.22±5.23	32.02±6.19	68.03±7.31	31.44±6.93
HOEC[40]	--	--	--	--	73.97±1.88	--	--	--

SPE[38]	93.64±6.24	60.38±9.32	93.31±6.98	62.81±9.23	81.96±4.76	24.74±3.09	**81.62±5.19**	29.87±4.25
Imbalance-XGBoost[9]	79.10±8.98	62.77±13.6	75.55±11.9	62.91±13.5	55.02±4.91	15.68±3.81	53.23±8.41	17.66±5.49
DSENLG-IE	97.55±5.15	63.43±9.52	**97.52±5.95**	66.92±8.83	**84.95±1.95**	**58.20±4.42**	77.24±3.23	**44.99±4.56**
Dataset	krvsk3vs11				Abalone21vs8			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	--	--	--	--
HD-Ensemble[41]	**100.0±0.00**	--	99.87±0.10	--	--	--	--	--
EASE[39]	96.59±2.91	93.75±3.94	96.48±3.06	93.77±3.96	80.41±1.50	**57.06±2.63**	73.1±2.79	**57.49±2.70**
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	98.01±3.00	97.07±3.69	97.94±3.15	97.09±3.63	88.42±2.75	46.72±5.15	85.25±2.12	50.88±4.97
Imbalance-XGBoost[9]	94.50±3.94	92.81±4.90	94.24±4.26	92.86±4.78	71.90±1.69	47.27±3.08	56.40±3.49	49.19±3.18
DSENLG-IE	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**91.95±1.57**	51.85±3.22	**91.17±1.68**	56.87±2.91
Dataset	Yeast6				Winequality-white3vs7			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	88.40	--	--	--	--	--	--	--
HD-Ensemble[41]	94.19±3.80	--	86.59±6.10	--	--	--	--	--
EASE[39]	91.65±7.97	42.11±9.21	91.46±8.29	41.90±9.53	86.08±2.12	**50.00±20.2**	85.36±3.00	**51.61±21.0**
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	**96.21±5.68**	38.89±8.67	**96.13±6.07**	47.23±9.97	81.82±12.9	22.22±5.92	81.53±20.4	28.10±9.84
Imbalance-XGBoost[9]	84.51±9.89	**52.63±8.82**	83.49±10.3	**53.17±8.34**	74.43±9.61	23.79±22.9	70.31±9.07	25.32±24.8
DSENLG-IE	96.01±2.03	30.30±4.05	95.92±3.47	40.54±3.30	**92.63±6.79**	35.39±22.7	**91.99±8.05**	46.85±20.3
Dataset	Winequality-red8vs67				krvsk0vs8			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	--	--	--	--
HD-Ensemble[41]	--	--	--	--	**100.0±0.00**	--	**99.57±0.20**	--
EASE[39]	61.96±1.11	9.560±5.26	49.52±2.58	9.770±8.48	86.35±1.09	**70.42±1.80**	84.24±1.39	**70.75±1.81**
HOEC[40]	68.09±3.80	--	--	--	--	--	--	--
SPE[38]	58.27±1.09	5.330±1.87	56.57±1.02	4.740±6.23	93.97±6.99	53.39±9.04	93.57±8.00	57.71±8.77
Imbalance-XGBoost[9]	52.86±6.10	8.730±1.78	10.92±2.19	9.610±2.07	71.73±1.71	51.59±1.06	63.21±1.93	54.08±1.01
DSENLG-IE	**76.50±8.75**	**10.18±2.75**	**75.24±8.72**	**16.19±5.53**	98.17±2.02	58.30±3.89	98.14±2.06	64.55±3.31
Dataset	Shuttle-2vs5				kddbufferoverflowvsback			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	--	--	--	--
HD-Ensemble[41]	**100.0±0.00**	--	99.86±0.10	--	**100.0±0.00**	--	**100.0±0.00**	--
EASE[39]	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	99.69±1.51	**100.0±0.00**	99.70±1.47
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	99.85±1.08	**100.0±0.00**	99.85±1.05
Imbalance-XGBoost[9]	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
DSENLG-IE	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
Dataset	krvsk0vs15				kddrootkitback			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	--	--	--	--
HD-Ensemble[41]	**100.0±0.00**	--	**100.0±0.00**	--	**100.0±0.00**	--	**100.0±0.00**	--
EASE[39]	98.42±3.75	97.07±5.33	98.32±4.01	97.14±5.25	97.75±4.25	85.71±4.74	97.62±4.50	86.50±4.54
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	99.98±0.04	98.63±3.15	99.98±0.04	98.67±3.07	95.40±7.49	**94.64±10.2**	95.02±8.59	**94.98±9.48**
Imbalance-XGBoost[9]	91.89±8.95	87.38±12.8	90.92±10.6	88.17±11.6	94.25±6.64	93.34±7.95	93.79±7.30	93.74±7.35
DSENLG-IE	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	98.76±5.04	87.19±2.95	98.58±5.93	84.53±2.64
Dataset	skinnonskin				cod			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
CBIS[24]	--	--	--	--	--	--	--	--
HD-Ensemble[41]	**100.0±0.00**	--	99.93±0.00	--	96.23±5.60	--	83.06±16.7	--
EASE[39]	**100.0±0.00**	**98.67±2.67**	**100.0±0.00**	**98.71±2.59**	90.29±7.36	63.47±9.91	89.38±8.32	64.22±9.50
HOEC[40]	--	--	--	--	--	--	--	--
SPE[38]	98.52±2.96	96.08±5.05	98.46±3.08	96.20±4.89	92.74±7.83	75.15±11.3	92.02±8.92	76.69±10.6
Imbalance-XGBoost[9]	96.31±6.24	93.76±9.07	95.97±7.10	94.10±8.33	86.26±11.1	**76.57±18.1**	83.93±14.4	**77.76±18.1**
DSENLG-IE	99.29±0.86	78.32±3.93	99.11±0.86	82.18±3.83	**98.62±0.19**	54.67±1.64	**98.61±0.19**	61.48±1.97

To verify the statistically significant difference between the methods, Wilcoxon paired signed-rank test was adopted, and six comparisons DSENLG-IE vs CBIS, DSENLG-IE vs HD-Ensemble, DSENLG-IE vs EASE, DSENLG-IE vs HOEC, DSENLG-IE vs SPE and DSENLG-IE vs Imbalance-XGBoost were tested. Table IX records the results. In Table IX, R+ is the sum of ranks for the datasets in which the first algorithm outperforms the second algorithm and R- is the sum of ranks for the second algorithm outperforms the first algorithm. It can be found R+ is always larger than R-, and all P-values are smaller than 0.05. P-value <0.05 means the hypothesis of equivalence in six comparisons were rejected. Thus, it can be stated that DSENLG-IE is clearly better than the six state-of-the-art imbalanced ensemble methods.

TABLE IX
RESULT OF WILCOXON PAIRWISE TEST

Comparison	Measure	R+	R-	P-value	Hypothesis (0.05)
DSENLG-IE vs CBIS	AUC	189	64	4.24E-02	**Rejected**
	F-M	--	--	--	--
	G-M	--	--	--	--
	Mcc	--	--	--	--
DSENLG-IE vs HD-Ensemble	AUC	118	35	4.95E-02	**Rejected**
	F-M	--	--	--	--
	G-M	178	12	8.37E-04	**Rejected**
	Mcc	--	--	--	--
DSENLG-IE vs EASE	AUC	924	22	5.16E-08	**Rejected**
	F-M	666.5	323.5	4.53E-02	**Rejected**
	G-M	909	37	1.40E-07	**Rejected**
	Mcc	797	193	4.24E-04	**Rejected**
DSENLG-IE vs HOEC	AUC	114	6	8.54E-04	**Rejected**
	F-M	--	--	--	--
	G-M	--	--	--	--
	Mcc	--	--	--	--
DSENLG-IE vs SPE	AUC	897.5	48.5	2.96E-07	**Rejected**
	F-M	722	268	8.07E-03	**Rejected**
	G-M	856	90	3.75E-06	**Rejected**
	Mcc	786	204	6.84E-04	**Rejected**
DSENLG-IE vs Imbalance-XGBoost	AUC	988	2	8.75E-09	**Rejected**
	F-M	719	271	8.95E-03	**Rejected**
	G-M	986	4	1.00E-08	**Rejected**
	Mcc	792	198	5.28E-04	**Rejected**

3) Comparison with Deep Learning based Imbalanced Methods

Six DL based imbalanced classification methods are chosen for comparison. Since the DL based methods are time consuming, 15 representative datasets are chosen for comparison. Table X records comparison results in this section, and the supplementary material gives the complete results. The best results are shown in boldface. It can be seen the performance of DSENLG-IE is superior to the other deep learning based imbalanced methods on all four criteria. In particular, when considering AUC and G-M, it is observable DSENLG-IE provided the best performance on 12 datasets respectively. For F-M and Mcc, the proposed method provided the best performance on 10 and 11 datasets respectively. The proposed method never showed the worst performance on any datasets.

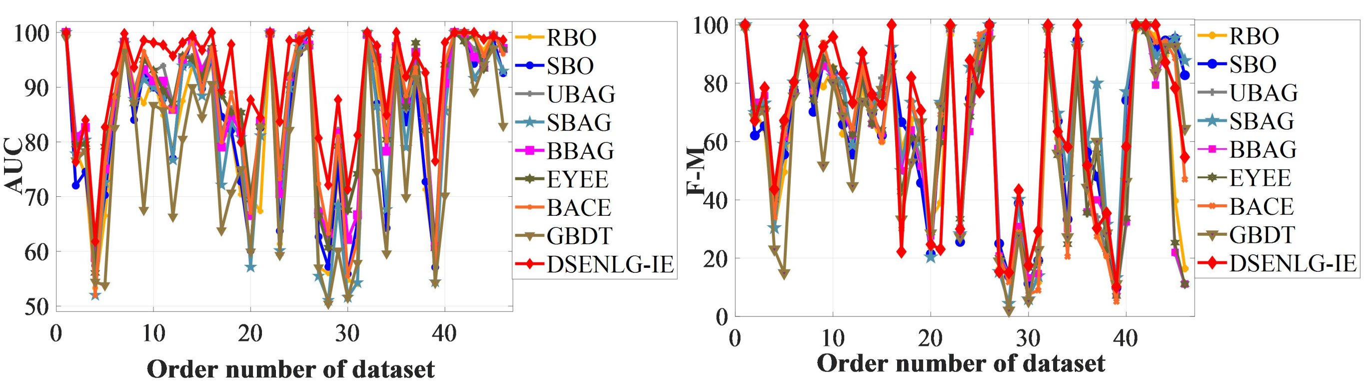
TABLE X
COMPARISON WITH DEEP LEARNING BASED IMBALANCED CLASSIFICATION METHODS

Datasets	Measure	CNN+SMOTE [14]	CNN+AE+GAN [15]	BED[16]	RVGAN-TL [18]	EAL-GAN [47]	DLE-ISMOTE [49]	DSENLG-IE
Ecoli1	AUC	65.22±3.81	83.76±5.41	84.95±2.51	81.23±5.10	92.26±4.41	76.63±5.99	**92.47±4.39**
	F-M	46.68±5.09	43.03±6.90	75.84±2.68	72.52±6.54	69.68±9.60	58.26±7.10	**80.42±7.52**
	G-M	64.05±3.99	82.95±5.56	84.67±2.69	79.91±5.83	86.24±5.18	74.63±9.10	**92.09±4.84**
	Mcc	28.24±7.33	43.10±6.71	68.23±3.49	65.86±7.84	60.68±12.4	45.96±9.19	**80.48±9.12**
Musk	AUC	87.58±3.12	91.59±0.86	93.66±0.53	87.97±3.10	89.49±0.84	95.39±0.43	**98.56±0.76**
	F-M	77.16±6.51	83.41±0.33	85.47±1.20	84.03±4.04	67.98±1.17	86.43±1.25	**92.59±4.20**
	G-M	87.21±3.23	91.48±0.92	93.63±0.54	87.21±3.56	80.36±1.22	95.39±0.43	**98.55±0.77**
	Mcc	73.04±7.89	80.37±0.42	82.89±1.37	82.04±4.18	62.16±1.38	84.22±1.43	**91.50±4.82**
Ecoli3	AUC	82.88±8.31	83.25±1.29	91.97±2.12	74.43±8.83	92.00±5.74	84.52±1.83	**95.70±4.69**
	F-M	48.10±6.36	49.44±3.05	59.41±6.79	56.21±15.6	60.86±15.3	52.27±2.27	**73.37±6.77**
	G-M	82.09±8.51	83.21±1.34	91.59±2.31	70.31±11.8	86.50±6.64	84.51±1.85	**95.50±4.98**
	Mcc	45.76±9.45	56.65±3.09	59.67±6.42	52.39±17.3	56.30±17.1	49.63±2.26	**74.30±6.93**
Glass016 vs2	AUC	60.25±7.65	60.65±9.32	71.33±7.75	59.35±16.0	63.89±12.9	66.98±12.1	**89.39±11.6**
	F-M	21.51±6.01	23.14±8.72	24.88±4.16	19.67±24.1	**37.04±12.5**	22.69±7.69	22.22±10.9
	G-M	56.29±8.42	59.52±10.7	68.87±6.82	32.69±15.6	59.92±9.97	61.43±14.0	**88.76±12.5**
	Mcc	12.93±9.58	15.23±8.73	23.36±8.30	14.06±25.6	31.01±13.9	19.47±12.6	**31.38±12.9**
Shuttle-c 0-vs-c4	AUC	99.57±0.88	99.97±0.07	**100.0±0.00**	**100.0±0.00**	99.60±1.20	97.91±0.22	**100.0±0.00**
	F-M	99.18±1.12	99.59±0.91	**100.0±0.00**	**100.0±0.00**	99.40±1.43	97.87±0.54	**100.0±0.00**
	G-M	99.57±0.89	99.97±0.07	**100.0±0.00**	**100.0±0.00**	99.05±2.34	97.89±0.28	**100.0±0.00**
	Mcc	99.14±1.18	99.56±0.96	**100.0±0.00**	**100.0±0.00**	99.35±4.53	97.75±0.61	**100.0±0.00**
Dermatol ogy-6	AUC	**100.0±0.00**	**100.0±0.00**	92.35±6.65	97.35±7.46	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
	F-M	**100.0±0.00**	**100.0±0.00**	89.21±6.19	94.44±9.24	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
	G-M	**100.0±0.00**	**100.0±0.00**	91.81±7.14	96.92±8.74	99.77±0.82	**100.0±0.00**	**100.0±0.00**
	Mcc	**100.0±0.00**	**100.0±0.00**	89.34±6.08	94.72±9.45	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
Yeast4	AUC	82.79±3.25	79.70±3.35	85.73±0.80	63.52±7.55	87.64±6.04	85.52±1.79	**87.71±4.70**
	F-M	29.63±2.04	29.83±5.38	25.08±1.55	33.28±7.16	30.45±11.3	29.10±2.52	**43.34±6.42**
	G-M	82.50±3.49	79.07±3.80	**85.62±0.84**	49.48±9.37	45.33±11.3	85.45±1.81	85.19±4.89
	Mcc	33.81±2.29	32.50±5.17	31.79±1.54	32.56±8.99	27.98±11.7	34.71±2.55	**47.68±6.85**

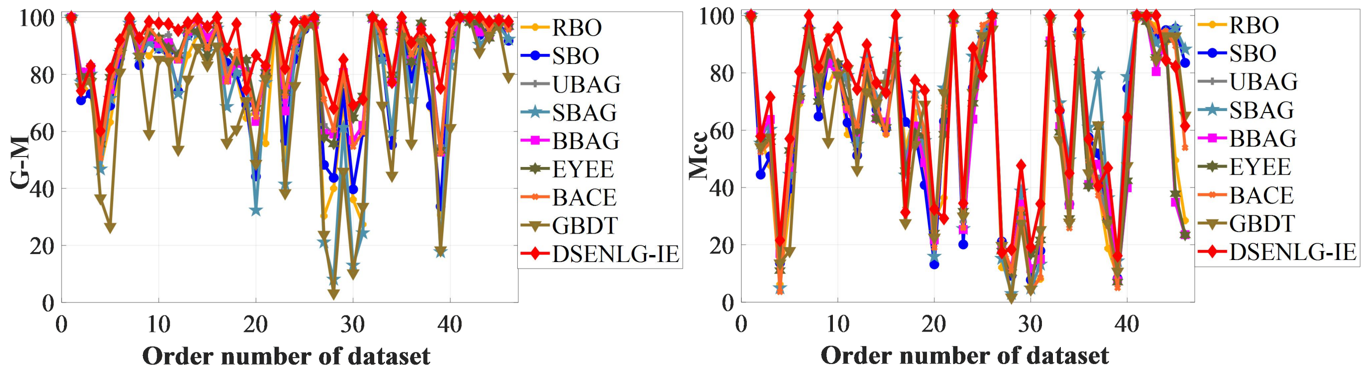
Abalone3 vs11	AUC	99.38±0.45	99.90±0.23	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
	F-M	77.33±5.96	96.00±8.94	**100.0±0.00**	**100.0±0.00**	98.77±1.67	**100.0±0.00**	**100.0±0.00**
	G-M	99.38±0.23	99.90±0.23	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
	Mcc	78.98±5.03	96.25±8.39	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
Yeast5	AUC	92.42±5.13	91.83±3.98	**97.85±0.16**	79.91±7.21	97.15±3.91	96.87±0.86	97.55±5.15
	F-M	49.22±11.2	43.35±5.60	59.25±1.68	**66.38±8.78**	55.56±2.42	48.10±7.07	63.43±9.52
	G-M	92.28±5.60	91.68±4.08	**97.82±0.16**	76.85±9.77	80.94±2.64	96.82±0.89	97.52±5.95
	Mcc	55.20±11.3	48.60±4.56	63.47±1.37	**67.46±7.17**	54.17±2.47	54.52±5.79	66.92±8.83
krvsk3vs 11	AUC	92.06±2.42	87.60±2.39	98.47±1.78	99.39±1.21	99.53±0.09	98.07±1.41	**100.0±0.00**
	F-M	32.74±2.48	26.54±1.14	89.82±5.37	99.37±1.25	93.75±2.40	59.26±2.63	**100.0±0.00**
	G-M	91.98±2.36	87.56±2.37	98.45±1.80	99.38±1.23	95.14±1.60	98.05±1.42	**100.0±0.00**
	Mcc	40.62±2.57	33.86±1.79	89.86±5.27	99.37±1.27	93.57±2.48	63.63±2.60	**100.0±0.00**
Yeast6	AUC	87.39±5.17	86.63±7.44	88.29±2.91	70.98±6.78	93.36±3.55	93.45±1.86	**96.01±2.03**
	F-M	27.53±5.67	24.85±5.50	25.86±1.47	48.21±15.4	**54.76±9.82**	27.17±2.61	30.30±4.05
	G-M	87.24±5.32	86.39±7.54	88.23±2.82	64.15±11.6	70.20±4.96	93.21±1.92	**95.92±3.47**
	Mcc	34.59±6.23	32.17±7.29	33.70±2.11	48.53±16.3	**53.76±10.1**	36.95±2.42	40.54±3.30
Winequal ity-red 8vs67	AUC	64.34±7.62	60.26±3.15	66.13±6.21	64.70±1.93	52.94±16.9	57.22±8.10	**76.50±8.75**
	F-M	8.96±2.88	5.500±0.41	8.010±1.42	**38.00±6.53**	4.000±10.0	9.57±7.91	10.18±2.75
	G-M	62.50±9.34	60.02±3.27	65.87±6.19	54.47±3.72	0.000±0.00	57.14±8.65	**75.24±8.72**
	Mcc	10.27±5.50	5.67±1.30	10.01±3.87	**41.11±10.4**	1.938±10.3	5.810±7.54	16.19±5.53
Shuttle-2 vs5	AUC	99.94±0.08	99.89±0.04	99.91±0.08	**100.0±0.00**	**100.0±0.00**	99.92±0.08	**100.0±0.00**
	F-M	96.36±4.98	93.51±2.37	94.53±4.71	**100.0±0.00**	**100.0±0.00**	95.45±4.55	**100.0±0.00**
	G-M	99.94±0.08	99.89±0.04	99.91±0.08	**100.0±0.00**	**100.0±0.00**	99.92±0.08	**100.0±0.00**
	Mcc	96.46±4.85	93.62±2.26	94.66±4.50	**100.0±0.00**	**100.0±0.00**	95.57±4.43	**100.0±0.00**
kddrootki tback	AUC	99.93±0.06	90.50±10.2	**100.0±0.00**	**100.0±0.00**	95.38±0.92	93.75±6.25	98.76±5.04
	F-M	94.14±5.41	85.32±13.8	**100.0±0.00**	**100.0±0.00**	91.90±1.74	92.86±7.14	87.19±2.95
	G-M	99.93±0.06	89.35±12.1	**100.0±0.00**	**100.0±0.00**	92.98±1.61	93.30±6.70	98.58±5.93
	Mcc	94.34±5.22	89.28±12.1	**100.0±0.00**	**100.0±0.00**	91.82±1.76	93.25±6.75	84.53±2.64
cod	AUC	94.63±5.37	93.24±4.84	89.62±0.01	87.49±7.64	**99.26±2.15**	97.54±4.28	98.62±0.19
	F-M	8.04±0.85	37.68±2.35	20.76±1.92	**78.27±9.22**	46.33±3.09	27.74±1.42	54.67±1.64
	G-M	94.44±5.59	93.11±4.95	89.11±0.01	86.16±8.68	95.92±9.24	97.43±4.52	**98.61±0.19**
	Mcc	19.33±2.02	24.57±10.7	30.69±1.64	**79.14±9.08**	46.26±3.09	38.56±1.09	61.48±1.97

D.Robust Analysis

Robustness checking of the proposed model is conducted with three groups of experiments. In 1st group of experiment, the stability of the accuracies on all datasets and four evaluation criteria are shown in Fig.9. More stable the accuracy is, more robust the method is. In 2nd group of experiment, different methods are compared when the data has noise (noise rate 10%) as shown in Table XI. Higher the accuracy is, more robust the method is. In 3rd group of experiment, different methods are compared when the class label has noise (noise rate 10%) as shown in Table XII. Higher the accuracy is, more robust the method is. From the Fig.9, it can be seen that the performance of the proposed algorithm is most stable, i.e., the overall performance of the proposed algorithm tends to be better as the IR increases, indicating its good robustness. In the Tables XI- XII, it can be observed that the performance of the proposed DSENLG-IE algorithm is optimal on four evaluation criteria in most cases. For data with noise, the proposed method provided the best performance on 15,7,15 and 9 datasets for AUC, F-M, G-M and Mcc respectively. For class label with noise, the proposed method provided the best performance on 13,11,10 and 9 datasets for AUC, F-M, G-M and Mcc respectively. This indicate the proposed DSENLG-IE algorithm has a strong robustness.



(a) (b)



(c) (d)

Fig.9. Stability of classification accuracy on different datasets: (a) AUC, (b) F-M, (c) G-M, (d) Mcc

TABLE XI

COMPARISON RESULTS WHEN DATA WITH NOISE (NOISE RATE 10%)

Dataset	Ecoli1				Musk			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	80.23±3.55	67.02±4.54	79.93±3.77	56.60±6.00	91.99±1.18	**81.53±1.94**	91.95±1.21	78.27±2.30
SPE[38]	82.00±5.36	69.48±7.69	81.66±5.65	60.12±10.3	90.06±1.48	79.21±2.28	89.94±1.55	75.41±2.72
Imbalance-XGBoost[9]	80.87±5.91	70.36±8.93	79.89±6.82	62.19±11.4	88.97±1.35	85.25±1.85	88.41±1.50	**83.18±2.08**
DSENLG-IE	**87.58±6.23**	**74.01±10.4**	**86.93±6.88**	**67.21±13.4**	**92.33±5.78**	77.42±1.47	**92.15±5.88**	74.68±1.65
Dataset	Ecoli3				Glass016vs2			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	81.57±7.52	57.20±10.3	80.48±8.76	53.21±11.6	60.36±12.9	**23.02±14.5**	50.63±25.1	**14.71±18.5**
SPE[38]	77.54±8.46	56.38±13.1	75.03±10.9	51.95±14.8	57.81±12.7	19.73±12.5	47.54±25.9	10.05±16.6
Imbalance-XGBoost[9]	73.21±9.90	53.54±17.7	68.11±14.6	49.82±19.5	50.57±4.99	4.970±12.6	7.190±17.9	1.580±13.7
DSENLG-IE	**95.51±3.06**	**75.34±14.3**	**95.35±3.22**	**75.04±14.0**	**61.42±15.4**	22.62±9.64	**58.55±15.7**	13.46±19.1
Dataset	Shuttle-c0-vs-c4				Dermatology-6			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	98.90±1.71	98.23±2.00	98.88±1.76	98.14±2.10	97.09±5.78	94.34±8.48	96.85±6.38	94.33±8.49
SPE[38]	98.83±1.57	98.35±1.81	98.81±1.61	98.26±1.90	96.38±5.58	94.23±7.04	96.13±5.98	94.25±7.06
Imbalance-XGBoost[9]	99.31±1.18	98.32±2.14	99.30±1.20	98.22±2.28	93.88±8.35	90.85±11.6	93.16±9.97	91.15±10.6
DSENLG-IE	**99.99±0.03**	**99.92±0.40**	**99.99±0.03**	**99.92±0.42**	**99.90±0.39**	**98.61±5.14**	**99.90±0.40**	**98.64±4.99**
Dataset	Yeast4				Abalone3vs11			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	76.12±6.51	37.03±8.74	73.34±8.94	36.74±9.41	92.08±10.1	83.66±15.5	90.99±12.0	84.30±14.8
SPE[38]	77.78±6.12	36.07±5.89	75.63±8.26	36.66±6.68	95.03±8.82	86.00±14.1	94.34±10.5	86.54±13.5
Imbalance-XGBoost[9]	61.81±5.69	30.75±12.7	47.67±13.2	31.26±13.4	91.73±11.1	82.36±18.3	89.82±16.9	82.99±17.9
DSENLG-IE	**80.45±6.53**	**39.61±4.87**	**78.33±6.78**	**37.95±4.83**	**99.24±5.30**	**98.51±8.68**	**98.99±7.07**	**98.71±7.40**
Dataset	Yeast5				krvsk3vs11			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	84.25±4.79	65.03±5.99	82.77±6.14	47.14±12.4	93.92±3.78	87.10±5.59	93.66±4.04	86.89±5.72
SPE[38]	87.22±7.63	**66.71±11.3**	85.99±9.27	47.73±9.83	94.17±4.05	88.41±6.06	93.91±4.36	88.23±6.17
Imbalance-XGBoost[9]	75.99±8.33	56.64±14.4	71.25±12.8	48.61±18.3	88.88±5.82	81.66±8.18	87.97±6.79	81.61±8.09
DSENLG-IE	**92.64±9.30**	63.43±11.4	**91.82±10.8**	**65.95±9.95**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**	**100.0±0.00**
Dataset	Yeast6				Winequality-red8vs67			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	79.25±8.87	46.98±11.8	76.07±12.2	47.14±12.4	65.27±11.9	**11.61±6.52**	55.83±25.2	12.79±9.90
SPE[38]	84.01±8.16	46.00±9.10	82.43±9.94	47.73±9.83	62.28±12.5	7.190±3.86	55.81±23.3	7.800±8.14
Imbalance-XGBoost[9]	69.84±9.39	**47.21±18.2**	60.96±16.9	**48.61±18.3**	52.16±5.24	6.800±15.7	8.460±19.4	7.780±18.7
DSENLG-IE	**90.05±7.70**	32.39±8.86	**89.15±8.75**	39.52±7.60	**79.64±8.86**	10.53±2.05	**76.99±8.21**	**18.15±5.02**
Dataset	Shuttle-2vs5				kddrootkitback			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	97.92±3.12	97.77±3.40	97.85±3.26	97.82±3.30	95.38±6.35	82.80±8.74	95.02±6.91	83.07±8.47
SPE[38]	97.94±2.93	**97.81±3.17**	97.88±3.05	**97.85±3.09**	95.53±5.86	93.04±8.08	95.22±6.31	93.27±7.88
Imbalance-XGBoost[9]	97.62±2.98	97.47±3.22	97.54±3.10	97.51±3.14	95.09±5.94	**93.41±7.56**	94.75±6.39	**93.68±7.29**
DSENLG-IE	**99.94±0.23**	97.31±9.23	**99.94±0.23**	97.59±8.20	**99.68±0.10**	88.96±6.17	**99.68±0.10**	85.08±5.38
Dataset	cod							
Measure	AUC	F-M	G-M	Mcc				
EASE[39]	86.02±7.25	**72.36±11.4**	84.38±9.22	**73.24±10.8**				
SPE[38]	88.40±7.90	68.21±9.94	87.17±9.11	69.34±10.1				
Imbalance-XGBoost[9]	82.25±0.04	68.51±16.5	79.20±13.4	69.68±15.8				

DSENLG-IE	**92.33±1.16**	51.71±0.29	**92.00±1.26**	58.54±0.84

TABLE XII

COMPARISON RESULTS WHEN CLASS LABEL WITH NOISE (NOISE RATE 10%)

Dataset	Ecoli1				Musk			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	85.24±6.59	74.28±8.85	84.98±6.89	66.44±12.0	68.59±1.26	49.39±1.51	68.55±1.25	31.84±2.32
SPE[38]	85.82±4.38	76.44±6.29	85.45±4.71	69.61±8.20	69.76±1.82	51.29±2.26	69.48±1.91	34.93±3.25
Imbalance-XGBoost[9]	85.53±6.08	77.42±8.29	84.85±6.78	71.42±10.3	72.68±1.67	**60.31±2.94**	68.85±2.34	**53.94±3.08**
DSENLG-IE	**88.16±8.19**	**79.09±13.9**	**87.37±9.35**	**73.44±17.7**	**74.41±8.60**	57.51±11.1	**72.10±9.49**	47.42±17.0
Dataset	Ecoli3				Glass016vs2			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	59.98±6.15	34.83±6.54	59.08±6.81	15.84±9.62	55.82±11.2	23.87±12.9	49.04±20.1	8.710±17.2
SPE[38]	56.84±6.80	29.61±10.6	49.94±10.3	13.26±13.1	56.79±9.64	24.39±12.7	47.13±19.2	11.29±15.6
Imbalance-XGBoost[9]	57.99±5.13	29.53±10.4	45.83±11.1	19.76±12.1	52.59±5.93	13.61±14.2	22.40±22.6	7.370±16.7
DSENLG-IE	**64.29±2.97**	**39.90±2.08**	**61.76±3.48**	**23.04±0.22**	**57.32±12.1**	**31.80±13.7**	**54.13±14.4**	**12.69±20.1**
Dataset	Shuttle-c0-vs-c4				Dermatology-6			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	64.43±2.96	37.62±3.54	63.15±3.53	17.05±9.80	61.12±6.31	31.58±7.95	58.94±9.07	17.05±9.80
SPE[38]	64.94±2.97	38.46±3.66	63.59±3.42	16.58±13.1	60.51±8.05	31.54±9.72	58.28±10.1	16.58±13.1
Imbalance-XGBoost[9]	69.07±2.07	**53.47±3.60**	62.40±3.62	35.87±16.0	63.92±7.47	39.38±16.1	53.54±14.4	35.87±16.0
DSENLG-IE	**69.66±6.37**	45.48±8.73	**67.63±7.92**	**36.24±21.6**	**73.63±13.04**	**45.19±15.6**	**70.59±15.9**	**36.24±21.6**
Dataset	Yeast4				Abalone3vs11			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	55.11±4.23	24.09±3.71	54.47±4.92	7.010±5.81	58.21±6.15	26.49±6.53	**56.06±8.33**	11.86±8.96
SPE[38]	54.82±3.48	22.82±4.47	49.62±5.61	7.630±5.56	59.29±5.72	28.12±7.27	54.03±8.55	15.47±9.04
Imbalance-XGBoost[9]	53.54±2.08	14.64±6.04	29.02±8.20	12.93±7.82	**60.85±5.47**	**33.36±12.6**	47.26±11.3	**32.64±13.2**
DSENLG-IE	**66.11±2.26**	**39.10±2.38**	**57.84±2.57**	**19.47±4.21**	53.06±3.23	24.02±1.42	29.17±6.13	7.500±7.17
Dataset	Yeast5				krvsk3vs11			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	52.65±3.98	21.54±2.79	52.42±3.98	3.490±5.24	56.24±2.09	23.70±2.16	53.94±2.89	8.950±2.99
SPE[38]	56.39±4.63	24.07±4.69	54.25±6.07	9.100±6.60	56.92±3.23	24.41±3.36	54.55±4.09	10.00±4.66
Imbalance-XGBoost[9]	55.52±2.68	20.34±7.16	36.19±7.71	17.11±6.73	58.79±2.17	29.25±5.81	42.82±5.23	**31.62±5.81**
DSENLG-IE	**60.89±3.56**	**31.85±9.14**	**55.43±7.16**	**18.43±5.36**	**62.21±4.93**	**30.47±5.53**	**60.44±5.86**	18.27±7.64
Dataset	Yeast6				Winequality-red8vs67			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	55.22±4.03	22.37±3.81	53.95±5.49	7.000±5.45	51.05±5.43	19.03±4.84	48.45±6.87	1.480±7.48
SPE[38]	54.79±4.36	21.95±4.00	52.77±5.59	6.610±5.99	51.16±5.64	19.27±5.01	**49.17±7.22**	1.550±7.67
Imbalance-XGBoost[9]	53.59±2.53	14.96±7.06	29.20±10.0	13.03±8.73	50.27±2.18	6.840±5.90	17.02±12.9	0.780±6.80
DSENLG-IE	**63.45±6.66**	**28.57±4.47**	**54.08±8.95**	**15.80±9.95**	**51.74±1.35**	**22.28±0.62**	20.86±3.83	**5.620±4.04**
Dataset	Shuttle-2vs5				kddrootkitback			
Measure	AUC	F-M	G-M	Mcc	AUC	F-M	G-M	Mcc
EASE[39]	54.32±1.74	20.64±1.58	**52.27±2.16**	5.860±2.37	**54.36±3.48**	**20.59±2.67**	**53.82±3.99**	5.530±4.42
SPE[38]	53.95±2.55	20.26±2.40	51.54±3.36	5.390±3.48	52.39±2.99	19.18±2.26	51.78±3.43	3.030±3.79
Imbalance-XGBoost[9]	55.73±1.68	20.54±5.15	34.46±5.06	**25.56±6.21**	54.11±1.90	15.64±5.89	29.69±6.52	**18.28±7.69**
DSENLG-IE	**56.30±4.13**	**22.44±1.95**	46.95±5.94	9.220±5.61	51.92±4.06	20.54±1.64	16.48±14.1	4.680±6.85
Dataset	cod							
Measure	AUC	F-M	G-M	Mcc				
EASE[39]	50.43±1.28	16.98±0.81	**50.35±1.31**	0.520±1.54				
SPE[38]	50.34±1.10	17.03±0.68	50.32±1.10	0.400±1.33				
Imbalance-XGBoost[9]	50.48±0.26	2.080±1.02	9.840±3.08	**6.510±2.90**				
DSENLG-IE	**50.49±0.55**	**18.52±0.18**	12.74±2.99	2.050±2.28				

E. Parameter Analysis

Fig. 10 visualize the effects of three parameters ρ, K, L on the performance of DSENLG-IE. Let $\rho = 0.5, 0.6, 0.7, 0.8, 0.9, 1$, $K = 0, 1, 2, 3, 4, 5$. Four datasets which represented two types of datasets are chosen (e.g., high- and low-IR), including Ecoli2, Ecoli3, Yeast1458vs7 and Yeast5. As depicted in Fig.10, the performance of DSENLG-IE on the four datasets increases when ρ increases. The possible reason is that SG is performed based on feature weighting, rather than random sampling or clustering. It ensures that the majority class samples in each subset are different, so an increase in ρ ensures that more majority class samples are used for training. For *K*, *L*, the performance of DSENLG-IE on the four datasets increases in the preliminary stage. $K = 0$, $L = 0$ mean there are no SNC and multilayer sample transformation, and the increase of algorithm performance in the preliminary stage further illustrates the effectiveness of proposed DSEN-LG network. After *K*, *L* reach a certain value, the performance of DSENLG-IE decreases. The possible reason is that a large K can lead to an increase in the dimensionality of the envelope sample and thus cause dimensional redundancy, which causes poor performance and if L is too large, poor quality deep envelope samples will be generated due to the fact that the high-layer samples contain less information. In summary, ρ could be selected between 0.7 and 1, K could be selected between 1 and 4, L could be selected between 3 and 6 considering four criteria.

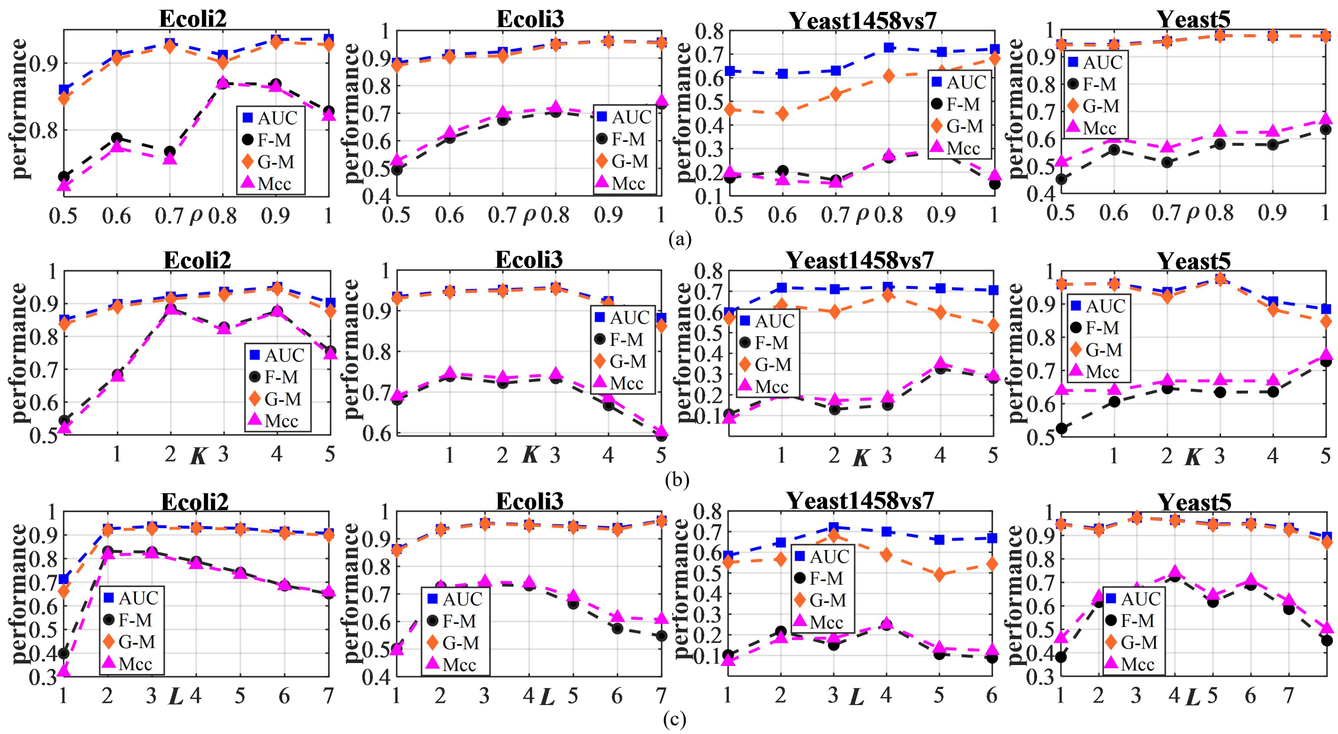


Fig.10. Performance with different parameters on the DSENLG-IE: (a) is for different ρ , (b) is for different K ,(c) is for different L