Fibonacci rabbits

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This article needs more work.

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There are a number of ways of keeping track of the population of "*[male-female] pairs of Fibonacci rabbits*" pertaining to the Fibonacci numbers.

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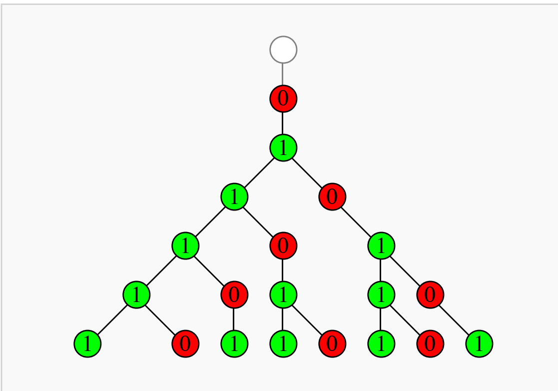
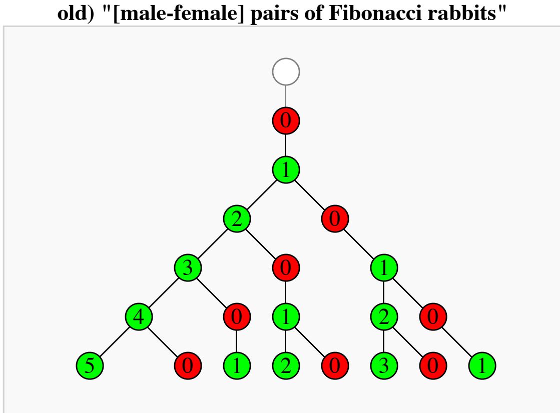
Tree of "[male-female] pairs of Fibonacci rabbits"

Tree of newborn (0) and mature (1) "[male-female] pairs of Fibonacci rabbits"



Tree of newborn (0 months old) and mature (≥ 1 months old) "[male-female] pairs of Fibonacci rabbits"



The **Fibonacci rabbits** tree (more precisely, the tree of *[male-female] pairs of **Fibonacci rabbits***), named after Leonardo of Pisa (known as **Fibonacci**), follows the rules:

- at the beginning of the 0^{th} month, there are no rabbits on the island;
- at the beginning of the 1^{st} month, a *[male-female] pair of newborn rabbits* is dropped on the island;
- thereafter, at the beginning of each month,
  - every *[male-female] pair of newborn rabbits* becomes a *[male-female] pair of mature rabbits*,
  - every *[male-female] pair of mature rabbits* begets a *[male-female] pair of newborn rabbits*, where the offspring stands to the right of the parents;

where

- a *[male-female] pair of newborn rabbits* (red vertex) is represented either by 0 (mature = FALSE, left side tree) or by an age equal to 0 month (right side tree);
- a *[male-female] pair of mature rabbits* (green vertex) is represented either by 1 (mature = TRUE, left side tree) or by an age greater than 0 month (right side tree);
- rabbits never die.

The number of rabbits at the beginning of each month gives the Fibonacci numbers (A000045(n), $n \geq 0$)

{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, ...}

If any *[male-female] pair of rabbits* could beget a new *[male-female] pair of rabbits*, we would have 0 followed by 2^{n-1} pairs at the beginning of the n^{th} month (A131577(n), $n \geq 0$)

{0, 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768, 65536, 131072, 262144, 524288, 1048576, 2097152, 4194304, 8388608, 16777216, 33554432, ...}

A131577(n) @opsp@−@opsp@ A000045(n), $n \geq 0$, i.e. the number of non-existing *[male-female] pair of rabbits* due to the maturation delay (0 followed by A027934$(n-1)$, $n \geq 1$)

{0, 0, 1, 2, 5, 11, 24, 51, 107, 222, 457, 935, 1904, 3863, 7815, 15774, 31781, 63939, 128488, 257963, 517523, 1037630, 2079441, 4165647, 8342240, 16702191, 33433039, ...}

Binary sequence of "[male-female] pairs of Fibonacci rabbits"

The tree of newborn (0) and mature (1) "[male-female] pairs of Fibonacci rabbits" yields the **binary sequence of "[male-female] pairs of Fibonacci rabbits"** $a(n)$, $n \geq 1$, (A036299$(n-2)$, $n \geq 2$)

{0, 1, 10, 101, 10110, 10110101, 1011010110110, 101101011011010110101, 1011010110110101101011011010110110, 1011010110110101101011011010110110101101011011010110101, ...}

A005203 Fibonacci rabbits numbers (or rabbit sequence) converted to decimal, $n \geq 1$.

{0, 1, 2, 5, 22, 181, 5814, 1488565, 12194330294, 25573364166211253, 439347050970302571643057846, 1582914572028944779780087453732128257990418 1, ...}

Sequence of "[male-female] pairs of Fibonacci rabbits"

The tree of newborn (0 month old) and mature (≥ 1 months old) "[male-female] pairs of Fibonacci rabbits" yields the infinite sequence of finite sequences

{{0}, {1}, {2, 0}, {3, 0, 1}, {4, 0, 1, 2, 0}, {5, 0, 1, 2, 0, 3, 0, 1}, {6, 0, 1, 2, 0, 3, 0, 1, 4, 0, 1, 2, 0}, {7, 0, 1, 2, 0, 3, 0, 1, 4, 0, 1, 2, 0, 5, 0, 1, 2, 0, 3, 0, 1}, ...}

whose concatenation yields the **sequence of "[male-female] pairs of Fibonacci rabbits"** $a(n)$, $n \geq 1$, (A035614$(n-1)$, $n \geq 1$)

{0, 1, 2, 0, 3, 0, 1, 4, 0, 1, 2, 0, 5, 0, 1, 2, 0, 3, 0, 1, 6, 0, 1, 2, 0, 3, 0, 1, 4, 0, 1, 2, 0, 7, 0, 1, 2, 0, 3, 0, 1, 4, 0, 1, 2, 0, 5, 0, 1, 2, 0, 3, 0, 1, ...}

Substitution rules

Substitution rule for the binary sequence of "[male-female] pairs of Fibonacci rabbits"

The **Fibonacci rabbits** binary sequence (more precisely, the binary sequence of *[male-female] pairs of **Fibonacci rabbits***) is defined by the initial conditions:

- $a(0) =$; (the empty string: there are no rabbits on the island)
- $a(1) = 0$; (a *[male-female] pair of newborn rabbits* is dropped on the island)

and the substitution rules, for $n > 1$:

- $0 \to 1$

  (a *[male-female] pair of newborn rabbits* becomes a *[male-female] pair of mature rabbits*);

- $1 \to 10$

  (a *[male-female] pair of mature rabbits* begets a *[male-female] pair of newborn rabbits*, where the offspring stands to the right of the parents);

where

- a *[male-female] pair of newborn rabbits* is represented by 0 (i.e. mature = FALSE);
- a *[male-female] pair of mature rabbits* is represented by 1 (i.e. mature = TRUE).

Substitution rule for the sequence of "[male-female] pairs of Fibonacci rabbits"

The **Fibonacci rabbits** sequence (more precisely, the sequence of *[male-female] pairs of **Fibonacci rabbits***) is defined by the initial conditions:

- $a(0) =$; (the empty string: there are no rabbits on the island)
- $a(1) = 0$; (a *[male-female] pair of newborn rabbits*, i.e. 0 month old, is dropped on the island)

and the substitution rules, for $n > 1$:

- $0 \to \text{succ}(0)$

  where $\text{succ}(0) = 1$ is the successor of 0 (a *[male-female] pair of newborn rabbits* becomes a *[male-female] pair of mature rabbits*, i.e. more than 0 month old);

- $k \to \mathrm{succ}(k), 0$

  for $k \geq 1$, where $\mathrm{succ}(k)$ is the successor of k (a *[male-female] pair of mature rabbits* begets a *[male-female] pair of newborn rabbits*, where the parent's age increases by 1 month, and where the offspring stands to the right of the parents).

"[male-female] pairs of Fibonacci rabbits" per generation

The number of *[male-female] pairs of Fibonacci rabbits* per generation are expressed by the Fibonacci polynomials.

The infinite Fibonacci word

A005614 The infinite Fibonacci word (start with 1, apply 0 -> 1, 1 -> 10, iterate, take limit).

{1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, ...}

Properties

Theorem.

The n^{th} term, $n \geq 3$, of the binary Fibonacci rabbits sequence may be obtained by the concatenation of the $(n-1)^{\text{th}}$ term and [appended by] the $(n-2)^{\text{th}}$ term.

Proof. (by induction)

(Using . (dot) as the concatenation operator.)
Base case:

$a(1) = 0, a(2) = 1$ and $a(3) = 10 = 1\ .\ 0 = a(2)\ .\ a(1)$;

Inductive step:

Assume the truth of $a(k) = a(k-1)\ .\ a(k-2)$ for $3 \leq k \leq n-1$;
then we have to show that this implies $a(n) = a(n-1)\ .\ a(n-2)$... □

Thus, using . (dot) as the concatenation operator

$$a(n) = a(n-0)$$
$$a(n) = a(n-1)\ .\ a(n-2)$$
$$a(n) = a(n-2)\ .\ a(n-3)\ .\ a(n-3)\ .\ a(n-4)$$
$$a(n) = a(n-3)\ .\ a(n-4)\ .\ a(n-4)\ .\ a(n-5)\ .\ a(n-4)\ .\ a(n-5)\ .\ a(n-5)\ .\ a(n-6)$$
...

which suggests the infinite sequence of finite sequences

{{0}, {1, 2}, {2, 3, 3, 4}, {3, 4, 4, 5, 4, 5, 5, 6}, {4, 5, 5, 6, 5, 6, 6, 7, 5, 6, 6, 7, 6, 7, 7, 8}, {5, 6, 6, 7, 6, 7, 7, 8, 6, 7, 7, 8, 7, 8, 8, 9, 6, 7, 7, 8, 7, 8, 8, 9, 7, 8, 8, 9, 8, 9, 9, 10}, ...}

whose concatenation gives (matches (proof?) A014701 Number of multiplications to compute n-th power by method of Chandra-sutra.)

{0, 1, 2, 2, 3, 3, 4, 3, 4, 4, 5, 4, 5, 5, 6, 4, 5, 5, 6, 5, 6, 6, 7, 5, 6, 6, 7, 6, 7, 7, 8, 5, 6, 6, 7, 6, 7, 7, 8, 6, 7, 7, 8, 7, 8, 8, 9, 6, 7, 7, 8, 7, 8, 8, 9, 7, 8, 8, 9, 8, 9, 9, 10, ...}

Equivalently

$$a(n) = a(n-1)\, 2^{1+\lfloor \log_2(a(n-2)) \rfloor} + a(n-2) = a(n-1)\, 2^{F(n-2)} + a(n-2), \quad n \geq 2.$$

Asymptotic behavior

$$\frac{a(n)}{a(n-1)} \sim 2^{F(n-2)} \sim 2^{\{\frac{\phi^{n-2}}{\sqrt{5}}\}}$$

$$a(n) \sim a(n-1)\, 2^{F(n)-F(n-1)}$$

$$a(n)\, 2^{F(n-1)} \sim a(n-1)\, 2^{F(n)}$$

$$a(n)\, 2^{\{\frac{\phi^{n-1}}{\sqrt{5}}\}} \sim a(n-1)\, 2^{\{\frac{\phi^{n}}{\sqrt{5}}\}}$$

$$\{a(n)\}^{\sqrt{5}}\, 2^{\{\phi^{n-1}\}} \sim \{a(n-1)\}^{\sqrt{5}}\, 2^{\{\phi^{n}\}}$$

External links

- The Golden String of 0s and 1s (http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibrab.html) , © 1996-2011 Dr Ron Knott.
- Weisstein, Eric W. (http://mathworld.wolfram.com/about/author.html) , Rabbit Sequence (http://mathworld.wolfram.com/RabbitSequence.html) , from MathWorld—A Wolfram Web Resource. [http://mathworld.wolfram.com/RabbitSequence.html]