# Lecture 11 Quiz

Back to Week weekNumber



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Quiz passed!



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1/1 points

1. If  $\Delta E = -3$ , then:



P(s = 1) increases when T increases.

## Correct

At T=1, P(s=1) is 0.05, i.e. fairly close to 0. Increasing the temperature brings that probability closer to 0.5, i.e. it increases it.



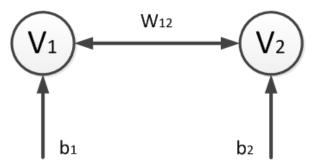
P(s = 1) decreases when T increases.



1/1 points

2

The Hopfield network shown below has two visible units:  $V_1$  and  $V_2$ . It has a connection between the two units, and each unit has a bias.



Let  $W_{12}=-10$ ,  $b_1=1$ , and  $b_2=1$  and the initial states of  $V_1=0$  and  $V_2=0$ .

If the network always updates both units simultaneously, then what is the lowest energy value that it will encounter (given those initial states)?

If the network always updates the units one at a time, i.e. it alternates between updating  $V_1$  and updating  $V_2$ , then what is the lowest energy value that it will encounter (given those initial states)?

Write those two numbers with a comma between them. For example, if you think that the answer to that first question is 4, and that the answer to the second question is -7, then write this: **4, -7** 

0, -1

Correct Response

From the initial state, both units will want to turn on.

If we update both of them at the same time, then both will turn on, leading to a configuration with energy 8. Next, both units will want to turn off,

bringing us back to the initial state, which has energy 0. We'll only ever alternate between those two states, so the lowest energy we'll see is

0.

If we update one unit, say  $V_1$ , first, then it will turn on. Now we're in a state with energy -1. From that state, neither unit will want to

change its state, so we'll stay in that state forever.

**V** 

1/1 points

3.

This question is about Boltzmann Machines, a.k.a. a stochastic Hopfield networks. Recall from the lecture that when we pick a new state  $s_i$  for unit i, we do so in a stochastic way:  $p(s_i=1) = \frac{1}{1+exp(-\Delta E/T)}, \text{ and } p(s_i=0) = 1-p(s_i=1). \text{ Here, } \Delta E \text{ is the } energy \text{ } gap\text{, i.e. the } energy \text{ } when \text{ the unit is off, minus the energy } when \text{ the unit is on. } T \text{ is the } temperature. We can run our system with any temperature that we like, but the most commonly used temperatures are <math>0$  and 1.

When we want to explore the configurations of a Boltzmann Machine, we initialize it in some starting configuration, and then repeatedly choose a unit at random, and pick a new state for it, using the probability formula described above.

Consider two small Boltzmann Machines with 10 units, with the same weights, but with different temperatures. One, the "cold" one, has temperature 0. The other, the "warm" one, has temperature 1. We run both of them for 1000 iterations (updates), as described above, and then we look at the configuration that we end up with after those 1000 updates.

Which of the following statements are true? (Note that an "energy minimum" is what could also reasonably be called a "local energy minimum")

If the cold one is exponentially unfortunate, it could end up in a configuration that's not an energy minimum.

Correct

If the random choice of which unit to update is always the same, then there won't be much progress towards an energy minimum. However, that's very unlikely.

The warm one could end up in a configuration that's not an energy minimum.

Correct

The warm one could end up anywhere, because it's truly stochastic.

For the cold one,  $P(s_i = 1)$  can be any value between 0 and 1, depending on the weights.

Un-selected is correct

#### Correct

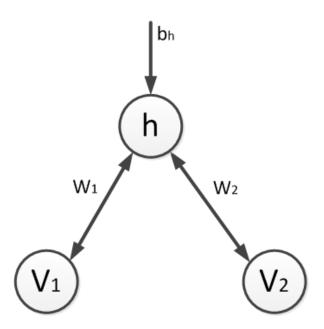
The warm one will jump around between configurations, a lot. The cold one won't: it always reduces energy.

likely to have encountered more different configurations than the cold one.



points

The Boltzmann Machine shown below has two visible units  $V_1$  and  $V_2$ , and one hidden unit h.



Let 
$$W_1 = 3$$
,  $W_2 = -1$ , and  $b_h = -2$ .

What is  $P(V_1=1,V_2=0)$ ? Write your answer with at least three digits after the decimal point.

4. Hint: if you feel confused about the purpose of a hidden unit, revisit the lecture videos.

### 0.468649



This Boltzmann Machine has eight configurations. Each has an energy, and from the energies you can calculate the probabilities. To get the requested

probability, we add up two probabilities:

$$P(V_1 = 1, V_2 = 0) = P(h = 0, V_1 = 1, V_2 = 0) + P(h = 1, V_1 = 1, V_2 = 0).$$

In detail, it's as follows.

$$E(h = 0, V_1 = 0, V_2 = 0) = 0$$

$$E(h = 0, V_1 = 0, V_2 = 1) = 0$$

$$E(h = 0, V_1 = 1, V_2 = 0) = 0$$

$$E(h = 0, V_1 = 1, V_2 = 1) = 0$$

$$E(h = 1, V_1 = 0, V_2 = 0) = 2$$

$$E(h = 1, V_1 = 0, V_2 = 1) = 2 + 1$$

$$E(h = 1, V_1 = 1, V_2 = 0) = 2 + -3$$

$$E(h = 1, V_1 = 1, V_2 = 1) = 2 + -3 + 1$$

Thus,

$$\sum_{s} exp(-E(s)) = exp(0) + exp(0) + exp(0) + exp(0) + exp(-2) + exp(-3) + exp(1) + exp(0) \approx 7.903404$$
 . Now we can convert the

energies into probabilities. We're only interested in two probabilities, as mentioned above.

$$P(h = 0, V_1 = 1, V_2 = 0) \approx \frac{exp(0)}{7.903404} \approx 0.1265$$

$$P(h = 1, V_1 = 1, V_2 = 0) \approx \frac{exp(1)}{7.903404} \approx 0.3440$$

That makes for a total of about 0.4705.



0/1 points

5.

What is the energy of the configuration with the lowest energy? What is the energy of the configuration with the second lowest energy (considering all configurations, not just those that are energy minima)?

Write your answer with a comma between the two numbers. For example, if you think that the energy of the lowest energy configuration is -17, and that

the energy of the second lowest energy configuration is -13, then you should write this: -17, -13

10111, 00111



Incorrect Response

## **Acceptable response**

-7, -6

You can simply try all 32 configurations to find the answer, or you can do some clever elimination, such as "if we want low energy, then unit c will definitely be on, because it only has positive connections."





