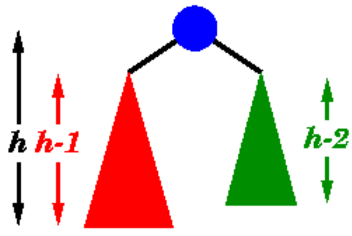
8.3 AVL Trees

An **AVL tree** is another balanced binary search tree. Named after their inventors, **A**delson-**V**elskii and **L**andis, they were the first dynamically balanced trees to be proposed. Like red-black trees, they are not perfectly balanced, but pairs of sub-trees differ in height by at most 1, maintaining an ***O*(log*n*)** search time. Addition and deletion operations also take ***O*(log*n*)** time.

Definition of an AVL tree

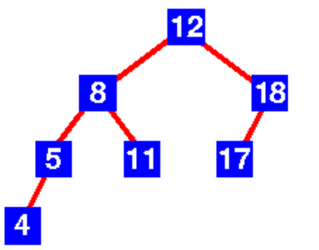
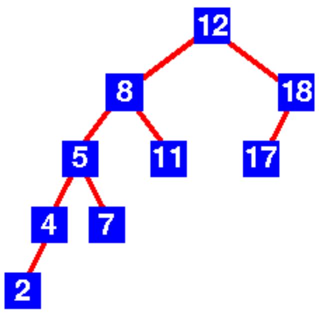
An AVL tree is a binary search tree which has the following properties:

1. The sub-trees of every node differ in height by at most one.
2. Every sub-tree is an AVL tree.



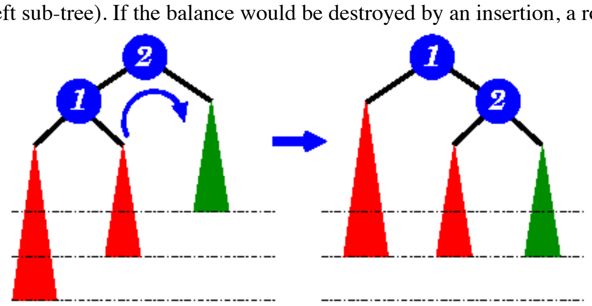
Balance requirement for an AVL tree: the left and right sub-trees differ by at most 1 in height.

You need to be careful with this definition: it permits some apparently unbalanced trees! For example, here are some trees:

Tree	AVL tree?
	Yes Examination shows that *each* left sub-tree has a height 1 greater than each right sub-tree.
	No Sub-tree with root 8 has height 4 and sub-tree with root 18 has height 2

Insertion

As with the red-black tree, insertion is somewhat complex and involves a number of cases. Implementations of AVL tree insertion may be found in many textbooks: they rely on adding an extra attribute, the **balance factor** to each node. This factor indicates whether the tree is *left-heavy* (the height of the left sub-tree is 1 greater than the right sub-tree), *balanced* (both sub-trees are the same height) or *right-heavy* (the height of the right sub-tree is 1 greater than the left sub-tree). If the balance would be destroyed by an insertion, a rotation is performed to correct the balance.



A new item has been added to the left subtree of node 1, causing its height to become 2 greater than 2's right sub-tree (shown in green). A right-rotation is performed to correct the imbalance.

AVL Tree Animation
This animation was written by Seo MinGoo, Min ChangGi and John Morris.

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Key terms

AVL trees
: Trees which remain **balanced** - and thus guarantee **O(logn)** search times - in a dynamic environment. Or more importantly, since any tree can be re-balanced - but at considerable cost - can be re-balanced in **O(logn)** time.

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