Demo #1

Solutions

October 4, 2017

1.1 factorielle double

Évaluée la croissance asymptotique de n!!.
Cas simple (2k)!!

$$
\begin{aligned}
(2k)!! &= k! \prod_{i=1}^{k} 2 \\
&= k!\ 2^k \\
&= (k/e)^k \sqrt{2\pi k}\ 2^k \\
&= (k/e)^k \sqrt{2\pi}\ k^{1/2}\ k^{log_k(2^k)} \\
&= (k^k/e^k) \sqrt{2\pi}\ k^{1/2}\ k^{k\ log_k(2)} \\
&= k^k \sqrt{2\pi}\ k^{1/2}\ k^{-k\ log_k(e)} k^{k\ log_k(2)} \\
&= k^{k+1/2-k\ log_k(e)+k\ log_k(2)} \sqrt{2\pi} \\
&< k^{k+1/2} \in O(k^k)
\end{aligned}
$$

1.1 factorielle double

Évaluée la croissance asymptotique de n!!.
Cas (2k+1)!!

$$
\begin{aligned}
(2k+1)!! &= \frac{2k+1!}{k! \prod_{i=1}^{k} 2} = \frac{2k+1!}{k!\ 2^k} \\
&= \frac{(\frac{2k+1}{e})^{2k+1} \sqrt{2\ \pi\ (2k+1)}}{(k/e)^k \sqrt{2\pi k}\ 2^k} \\
&= \frac{(\frac{2k+1}{e})^{2k+1} \sqrt{(2k+1)}}{(k/e)^k \sqrt{k}\ 2^k} \\
&= \frac{(2k+1)^{2k+1} \sqrt{(2+1/k)k}}{e^{2k+1}} * \frac{e^k}{k^k \sqrt{k}\ 2^k}
\end{aligned}
$$

1.1 factorielle double

Évaluée la croissance asymptotique de n!!.
Cas (2k+1)!! suite

$$
\begin{aligned}
(2k+1)!! &= \frac{(2k+1)^{2k+1}\sqrt{(2+1/k)k}}{e^{2k+1}} * \frac{e^k}{k^k\sqrt{k}\ 2^k} \\
&= \frac{(2k+1)^{2k+1}\sqrt{(2+1/k)}}{e^{k+1}} * \frac{1}{k^k\ 2^k} \\
&> \frac{(2k+k)^{2k+1}\sqrt{(2+1/k)}}{e^{k+1}} * \frac{1}{(k)^k\ 2^k} \\
&\geq \frac{3^{2k+1}k^{2k+1}\sqrt{(2+1/k)}}{e^{k+1}} * \frac{1}{(k)^k\ 2^k} \\
&\geq \frac{3^{2k+1}k^{k+1}\sqrt{(2+1/k)}}{e^{k+1}\ 2^k} \\
&\in O(k^k)
\end{aligned}
$$

1.6 Euler en binaire

```
def gcd(x,y):
        if x == 0:
 return y
        if x % 2 == 0 && y % 2 == 0:
                return gcd(x/2,y/2)
        if x % 2 == 0 && y % 2 == 1:
                return gcd(x/2,y)
        if x % 2 == 1 && y % 2 == 0:
                return gcd(y,x)
        if x % 2 == 1 && y % 2 == 1:
                if y > x:
                        return gcd(y,x)
                else :
                        return gcd(x-y,y)
```