Time Series Analysis

~Abhishek Kumar

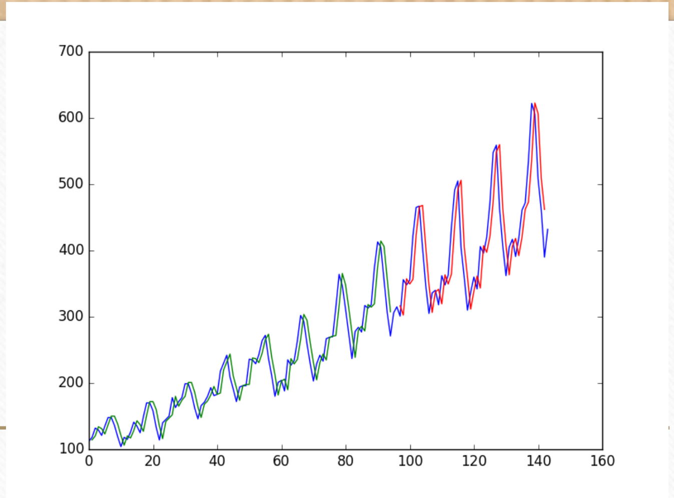
Scope

- Why it is different than regression?
- What is Signal & Noise?
- Autocorrelation, Seasonality, Trending
- Preprocessing & Filtering

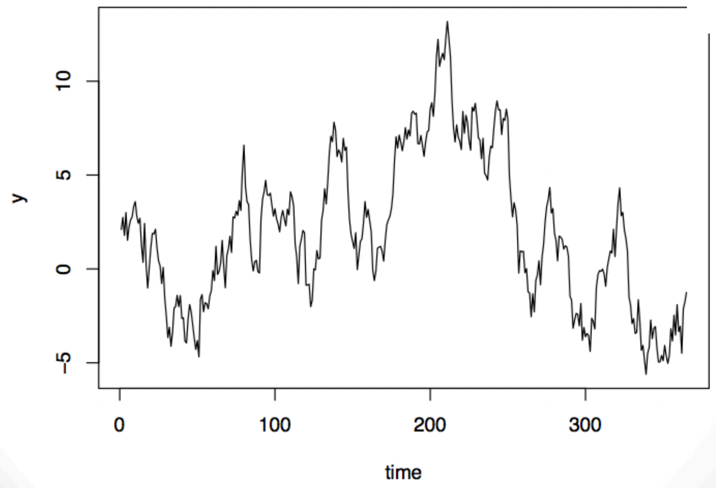
Time Series

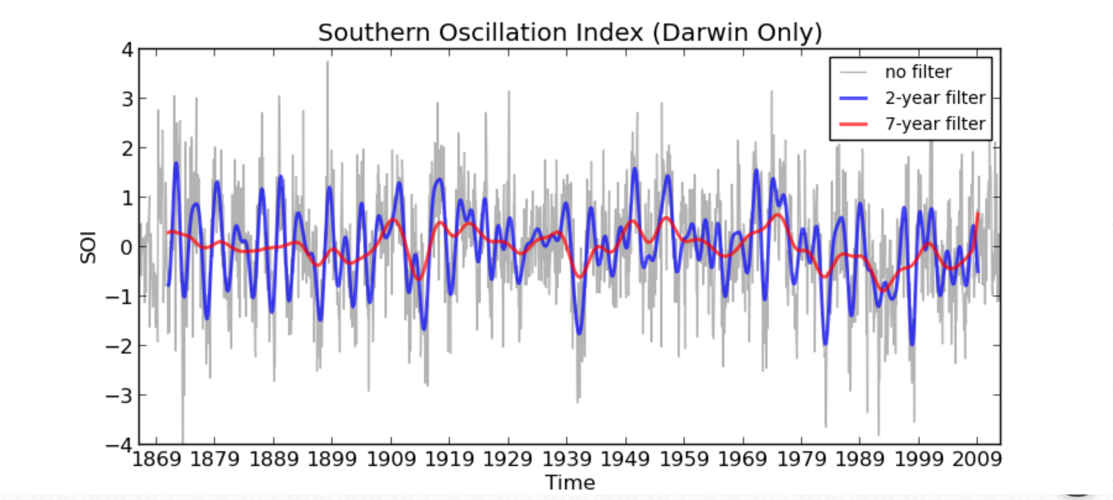
- An order sequence of values of a variable at equally spaced time interval


Sales for all stores
800
700
600
500
400
300
200
100
0
12/21/2017
12/31/2017
1/10/2018
1/20/2018
1/30/2018
2/9/2018
2/19/2018
3/1/2018
3/11/2018


700
600
500
400
300
200
100
0
20
40
60
80
100
120
140
160


25
20
15
10
5
value
1970
1980
1990
2000
2010
date
variable
psavert
uempmed


10
5
0
-5
y
0
100
200
300
time


Southern Oscillation Index (Darwin Only)
no filter
2-year filter
7-year filter
4
3
2
1
0
−1
−2
−3
−4
SOI
1869
1879
1889
1899
1909
1919
1929
1939
1949
1959
1969
1979
1989
1999
2009
Time

Like Regression?

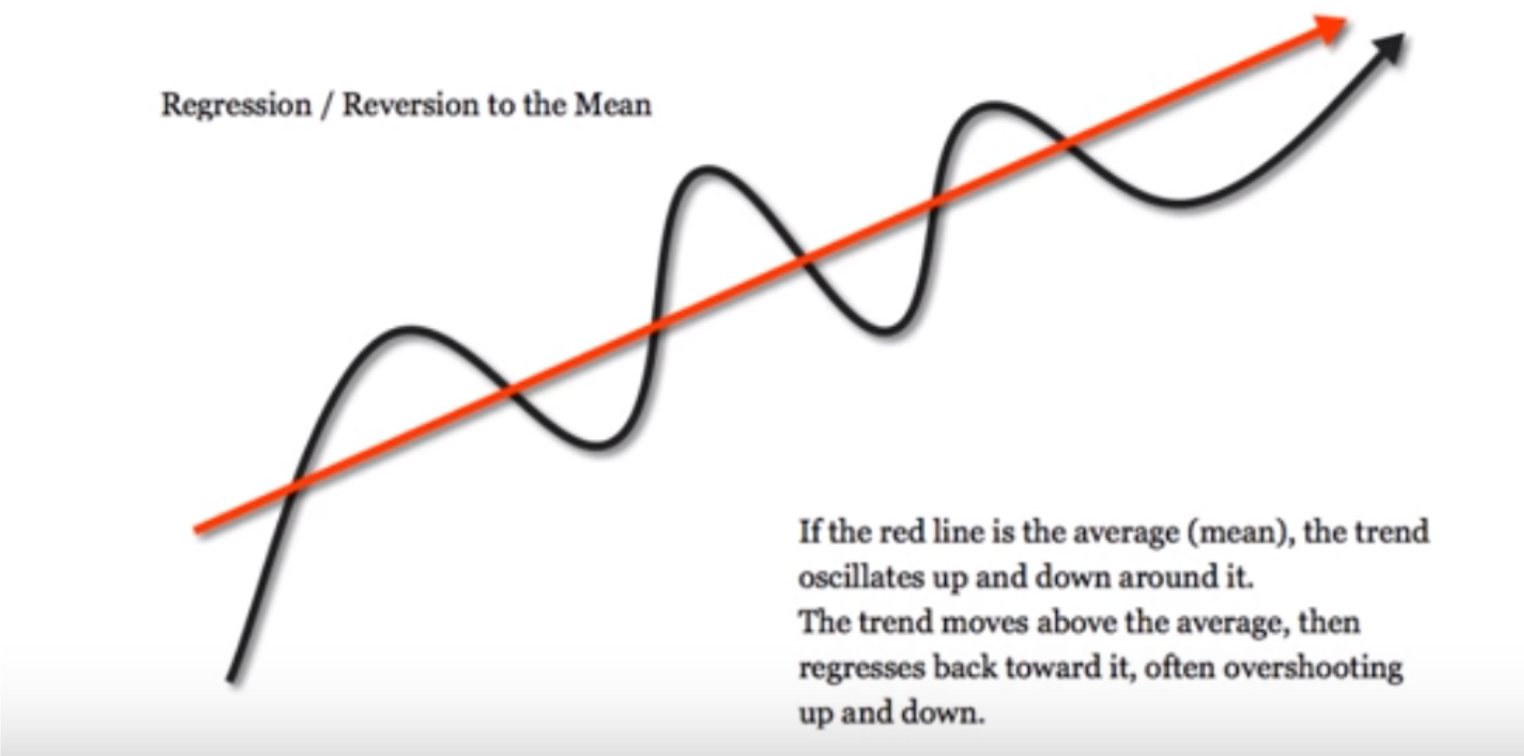
- It is like regression, time series is often focused on identifying underline trends and patterns, describing them mathematically and then make a prediction/forecast about future events.

Whats different?

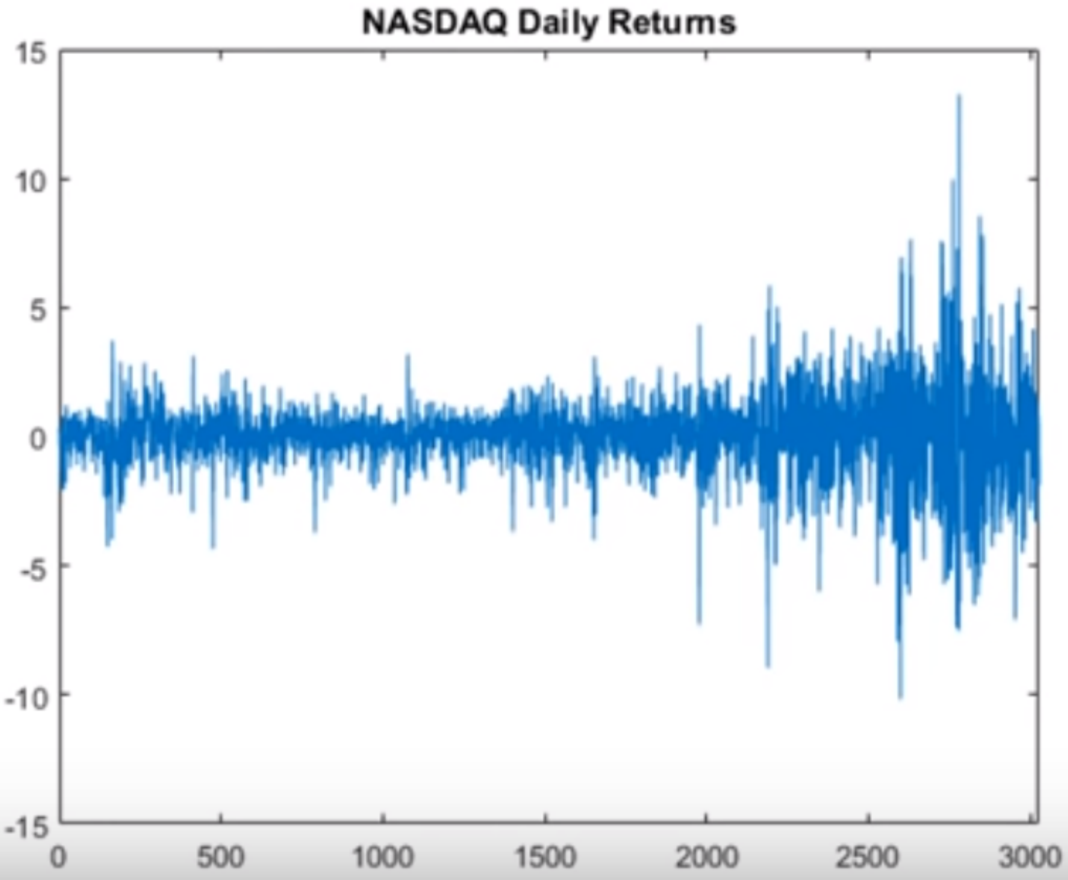
- In many cases, we can describe parts of time series processes in terms of randomized variable with statistical moments.

- An important features of many time series processes is that their mean and variance change through time

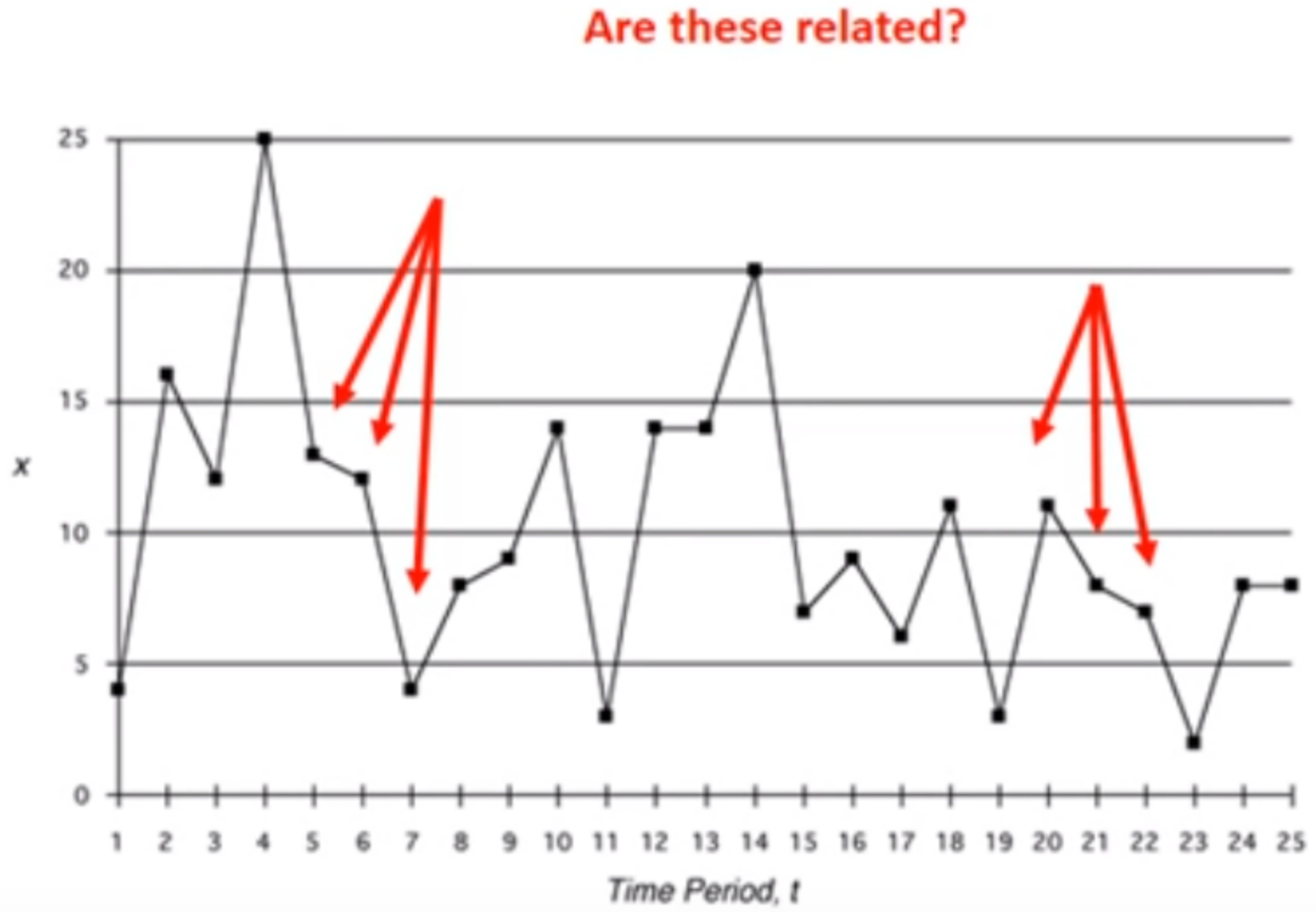
Mean is varying, but oscillation around the mean (the standard deviation) looks constant.



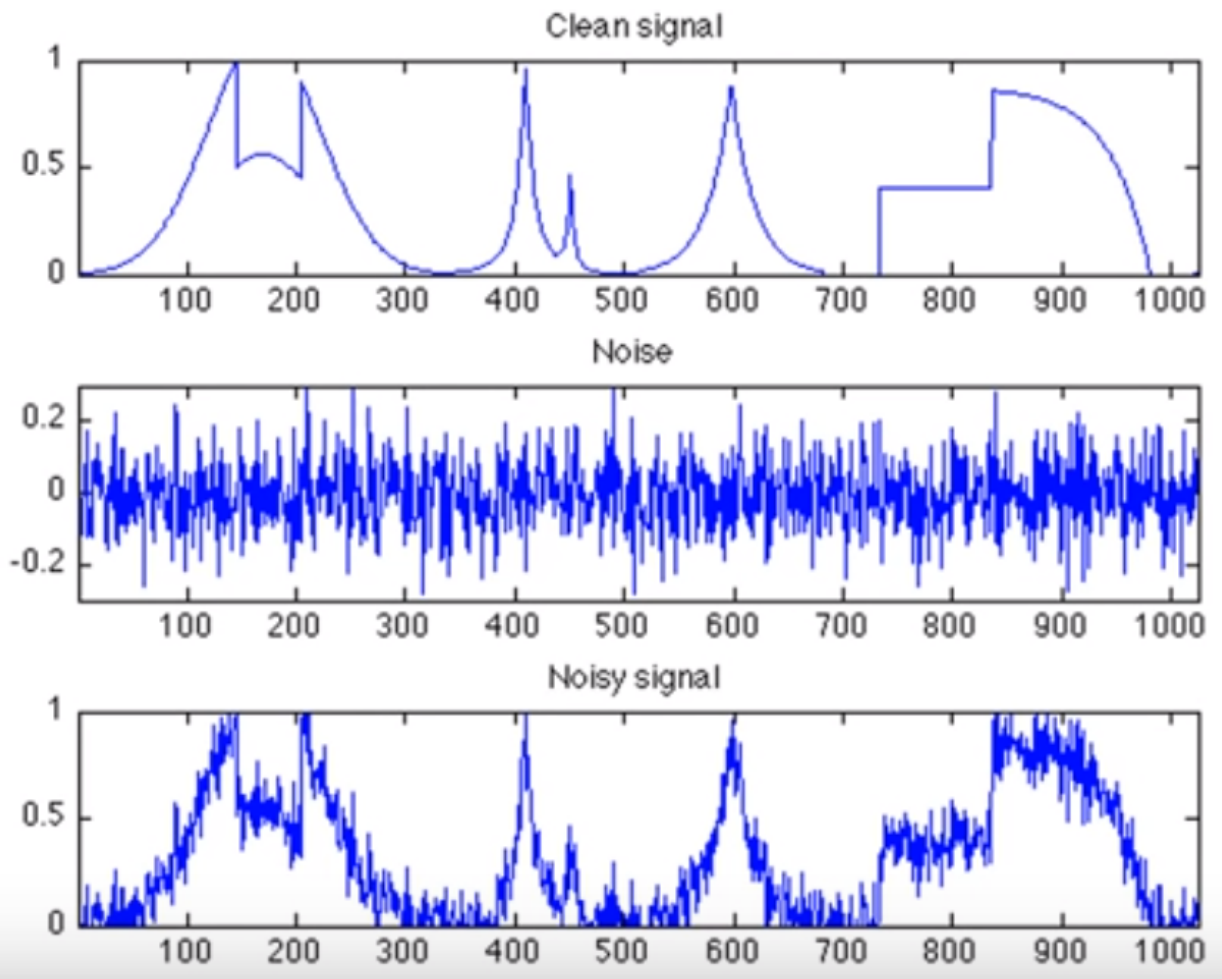
Constant mean, changing standard deviation ("volatility")



Not if the value of x(t) depends in any significant way on the value of x(t-1).



Time series allow us to replicate every element of the process by decomposing the mathematical process into combination of signals and noise without necessarily knowing the underline causes.


Clean signal
1
0.5
0
100
200
300
400
500
600
700
800
900
1000
Noise
0.2
0
-0.2
100
200
300
400
500
600
700
800
900
1000
Noisy signal
1
0.5
0
100
200
300
400
500
600
700
800
900
1000

Signal & Noise

- Signal: Data pattern we are interested in.
- Noise: random process, with mean 0 and constant variance
- A person should be able to tell the difference between signal and noise.
- Model can never be perfect, because there is always something left and that we call random process or noise/ white noise

Non-stationary

- Change over time, may be your signal remain constant but your noise can vary with time (NASDAQ)

Seasonality

- Periodic patterns

Autocorrelation

- If value of x is dependent of any given point of time depends in anyway to the value of x to the time period before.
- If trends tend to persist within time series but overall there is no trend this is called memory in time series or autocorrelation.
- To detect randomness in the data
- Both Noise and signal can have autocorrelation.

White Noise

- If we eliminate the signal(trends, periodicity, autocorrelation) then we are left with white noise.
- White Noise is a completely random process, whose sample are regarded as a sequence of uncorrelated random variables with zero mean and constant variance.

Preprocessing & Filtering

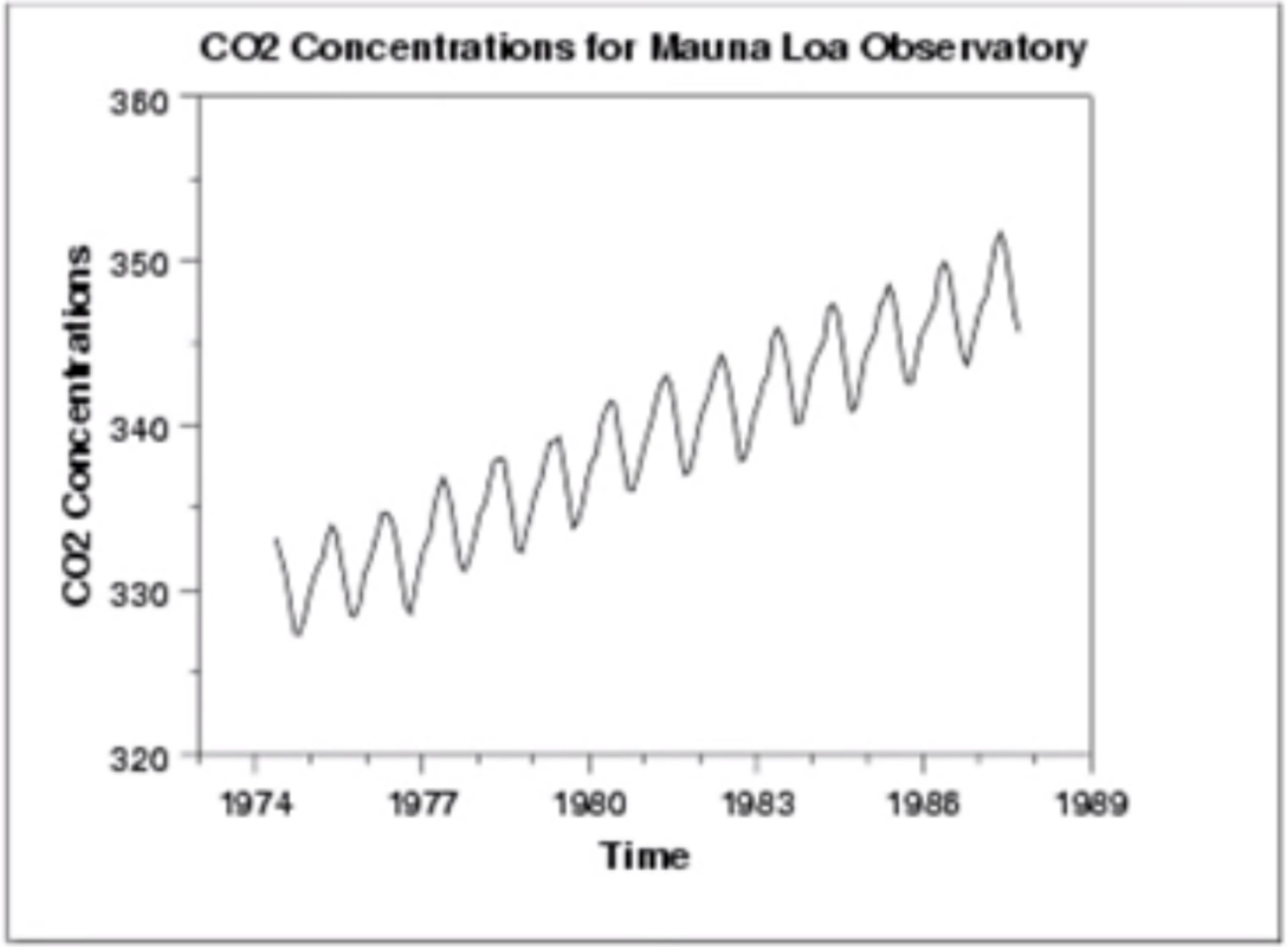
- Take most of the time.
- Filtering: Changing the attribute of a time series or deconstructing it into its component part

Preprocessing & Filtering

- Detrending
- Autocorrelation
- Outliers
- “Low pass” filters/ Smoothing

Detrending

- It is used to make series stationary

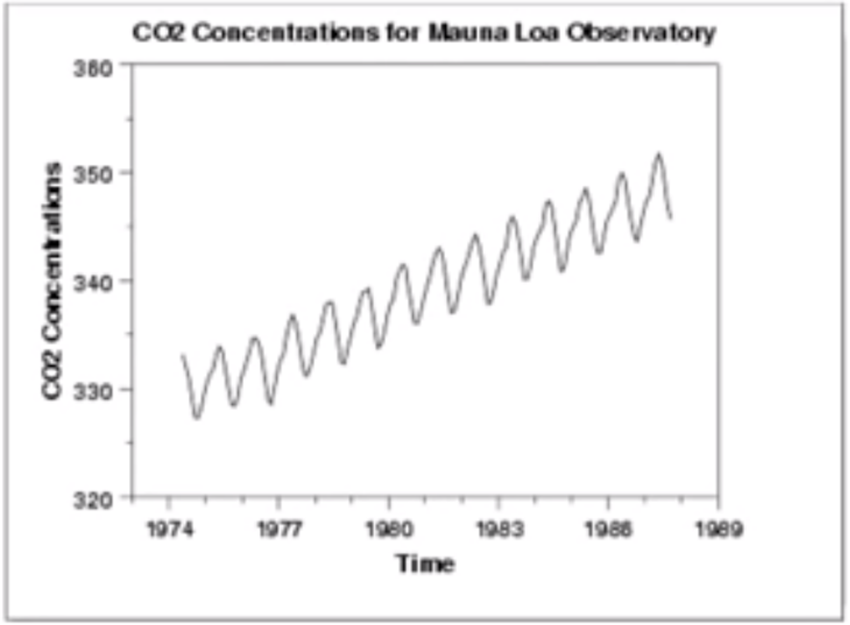

CO2 Concentrations for Mauna Loa Observatory
360
350
340
330
320
CO2 Concentrations
1974
1977
1980
1983
1986
1989
Time

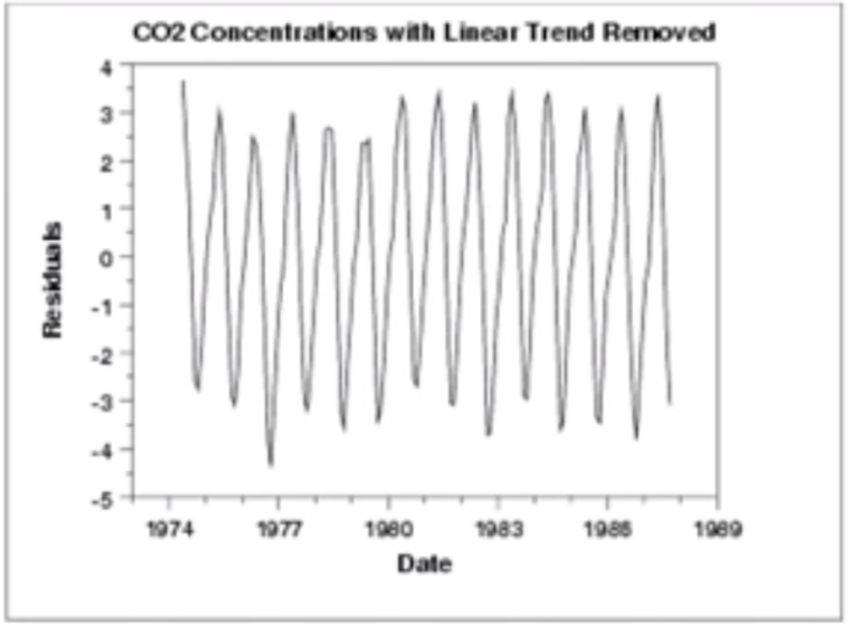
We can fit a linear model to the data using least squares regression:

$$CO_2 = b(Time) + C$$

We can subtract this linear trend from our original data to get "de-trended" CO_2 data.

Trend Stationary



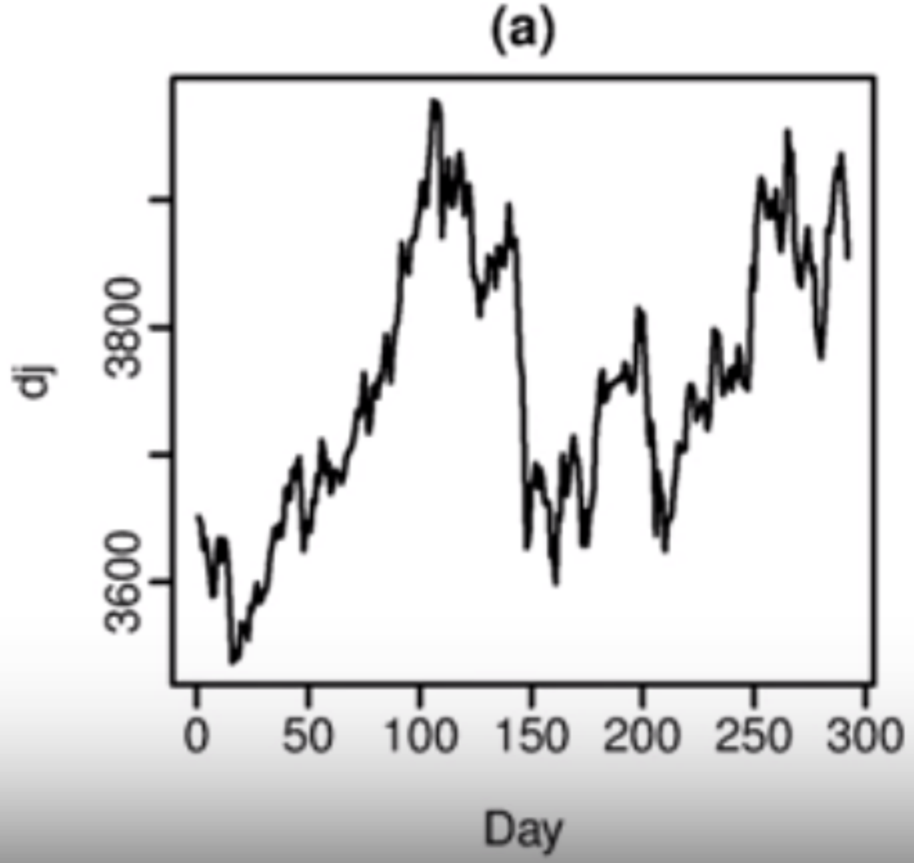


Now we're left with seasonality

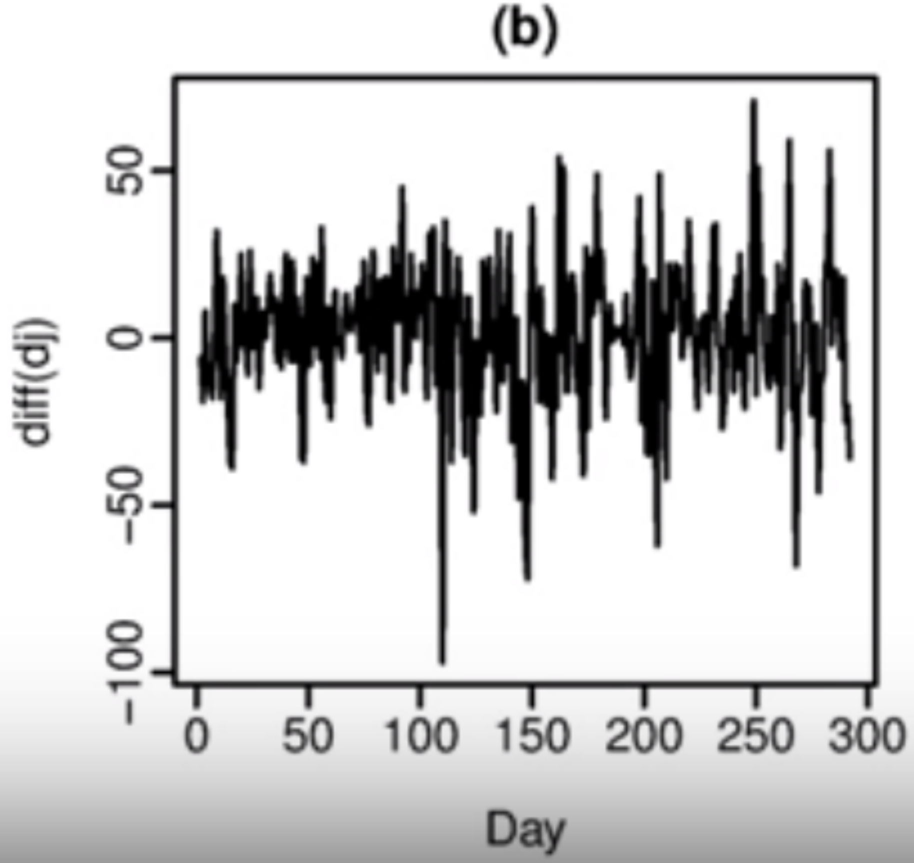
Differencing

- If the mean, variance, autocorrelation of the original time series is not constant in time, even after detrending, in that case we have to transform it into a series of period to period and season to season differences.
- Such a series is called difference stationary.

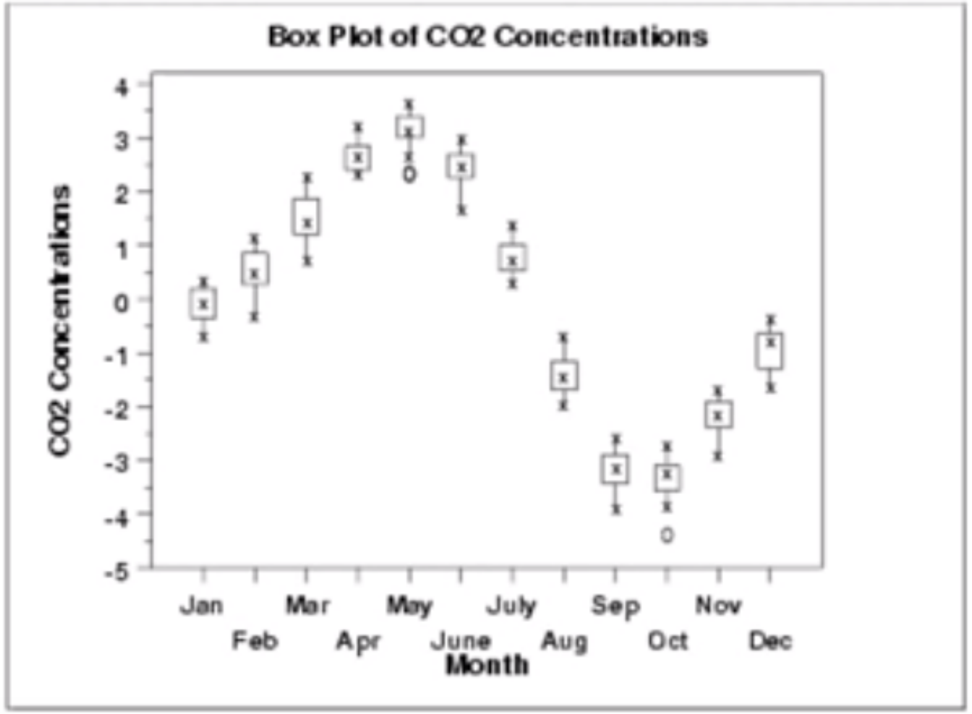
Dow Jones Index

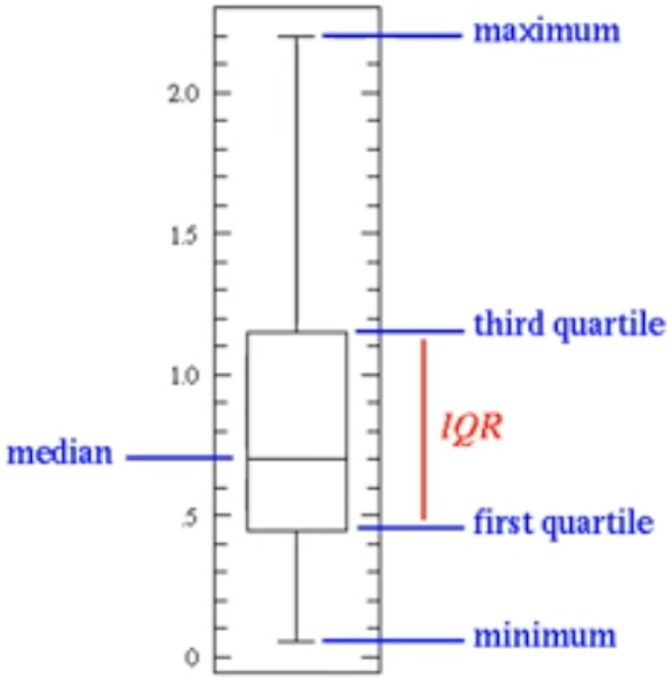


Differenced Dow Jones Index

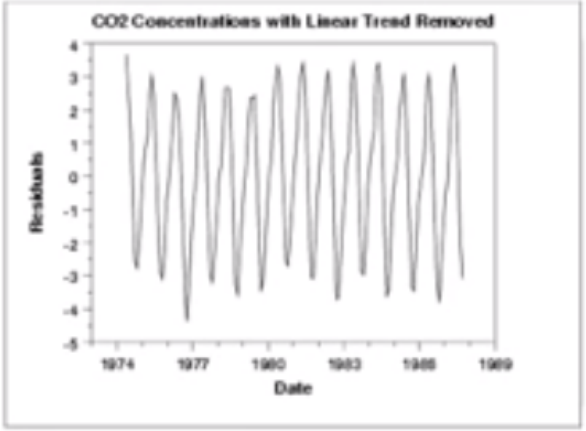


Many time series display **seasonality** (periodic fluctuations). If seasonality is present, it must be incorporated into a time series model. **How do we detect it?**





We can likewise subtract the seasonal or periodic trend from the data, leaving a de-trended process.



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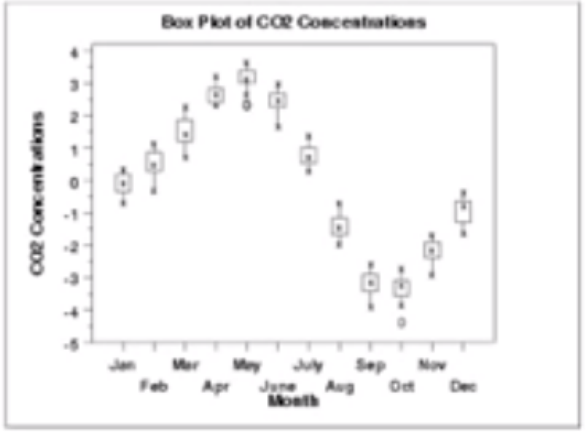

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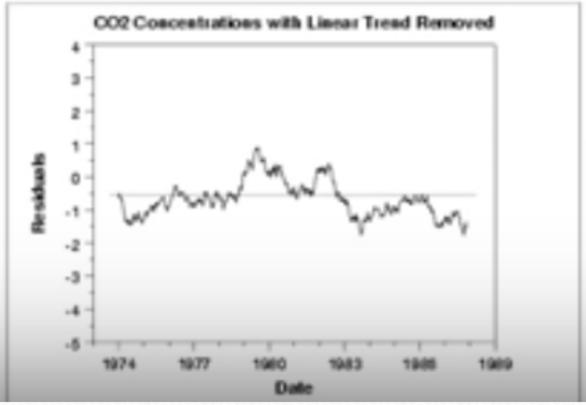
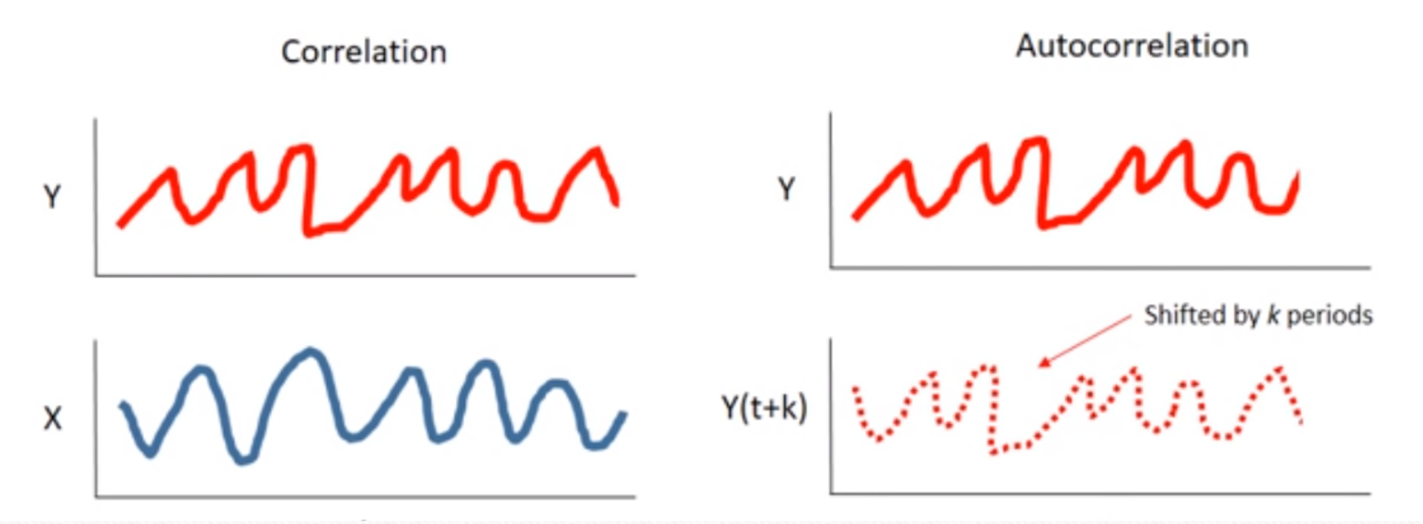
We might get something that looks like this:



Is this a random process? (can we replicate it by just sampling from a known distribution?)

Correlation vs Autocorrelation

- We are just measuring the correlation between time series and itself, at different time lags.



We can test the autocorrelation at as many lags as we want, depending on the length of the time series. Then we can plot autocorrelation as a function of lag.

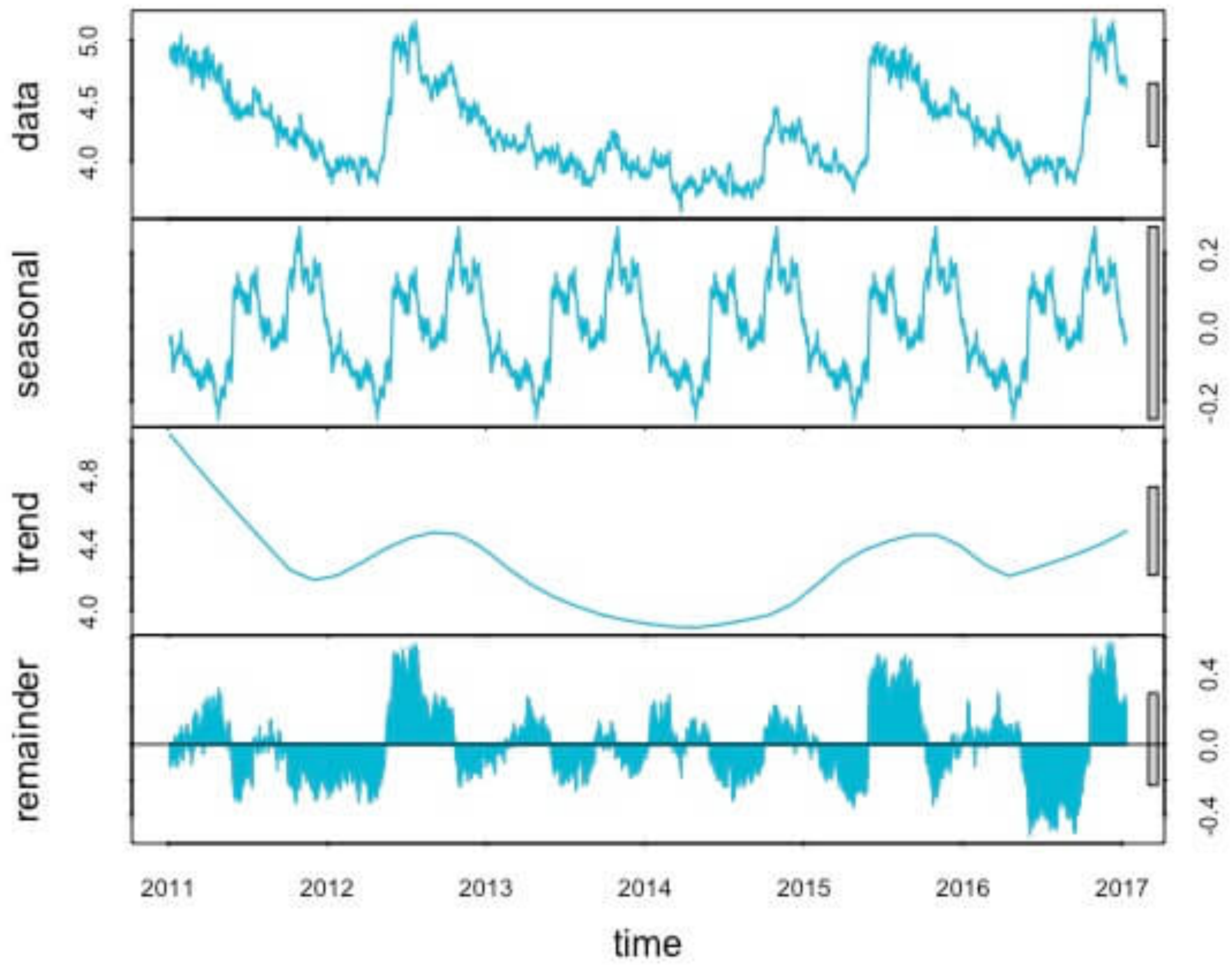
What does this tell us?

We'll learn how to construct models that incorporate autocorrelation in a later lecture.

Correlation

$$r_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$

$$r_{X,Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \ \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

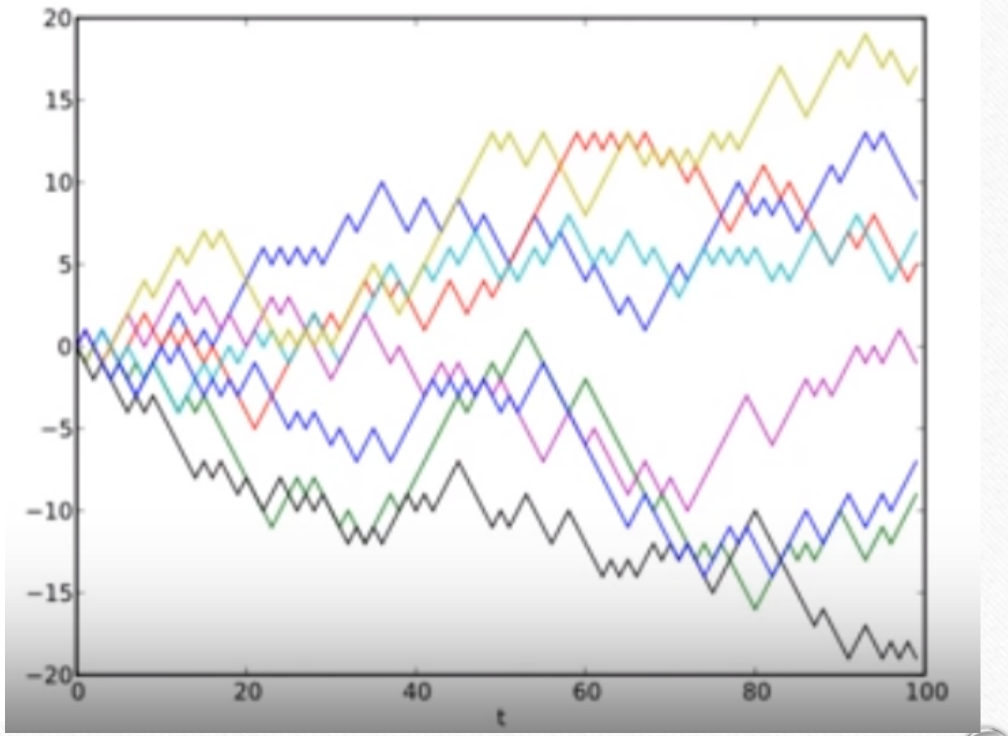


data
seasonal
trend
remainder
5.0
4.5
4.0
0.2
0.0
-0.2
4.8
4.4
4.0
0.4
0.0
-0.4
2011
2012
2013
2014
2015
2016
2017
time

Random Walk Model

- If the differences of Y is stationary and also completely random(not autocorrelated) then Y is described as Random walk model, which means each value is a random step away from previous value.

$$y_t = y_{t-1} + w_t$$



Discussion

Thank you!!!!!!