

Naïve Bayes

~Abhishek Kumar

Bayesian statistic

- Bayesian statistics is a theory in the field of statistics based on the Bayesian interpretation of probability where probability expresses a degree of belief in an event.
- The degree of belief may be based on **prior** knowledge about the event.

Spam Detector

100 e-mails



Spam Detector

25 Spam



75 No spam





“Buy”

25 Spam



75 No spam





“Buy”

25 Spam



75 No spam



Spam Detector



“Buy”

Spam



No spam



Quiz: If an e-mail contains the word “buy”, what is the probability that it is spam?

- 40%
- 60%
- 80%
- 100%

Spam Detector



Quiz: If an e-mail contains the word “buy”, what is the probability that it is spam?

- 40%
- 60%
- 80%
- 100%

Solution:
80%



“Cheap”

Spam



No spam





“Cheap”

Spam



No spam





“Cheap”

Spam



No spam



Quiz: If an e-mail contains the word “cheap”, what is the probability that it is spam?

- 40%
- 60%
- 80%
- 100%



“Cheap”

Spam



60%

No spam



40%

Quiz: If an e-mail contains the word “cheap”, what is the probability that it is spam?

- 40%
- 60%
- 80%
- 100%



“Buy” and “Cheap”

Spam



No spam





“Buy” and “Cheap”

Spam



No spam





“Buy” and “Cheap”

Spam



No spam





“Buy” and “Cheap”

Spam



No spam





“Buy” and “Cheap”

Spam



No spam





“Buy” and “Cheap”

Spam



No spam

Quiz: If an e-mail contains the words “buy” and “cheap”, what is the probability that it is spam?

- 40%
- 60%
- 80%
- 100%



“Buy” and “Cheap” → 100% ?

Spam



100%

No spam

0%

Quiz: If an e-mail contains the words “buy” and “cheap”, what is the probability that it is spam?

- 40%
- 60%
- 80%
- 100%

Solution:
100%



“Buy” and “Cheap”

Spam

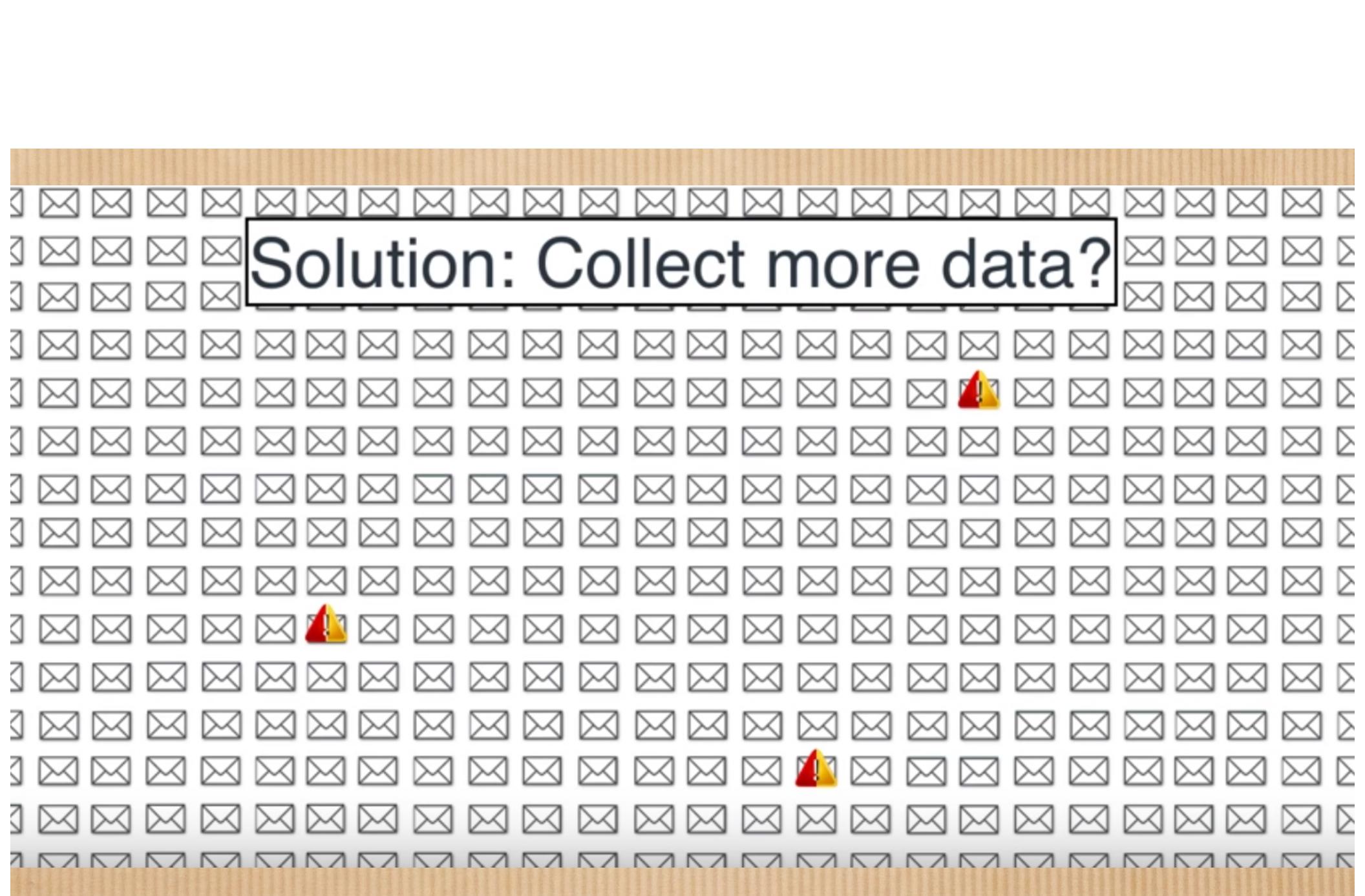


12 e-mails

No spam



0 e-mails?



Solution: Collect more data?

Spam Detector



“Buy” and “Cheap”

Spam



12 e-mails

No spam



0 e-mails?



100 e-mails



100 e-mails

5 “Buy”

5% “Buy”

10 “Cheap”

10% “Cheap”



100 e-mails

5 “Buy”

10 “Cheap”

5% “Buy”

10% “Cheap”

0.5% “Buy” and “Cheap”



100 e-mails

5 "Buy"

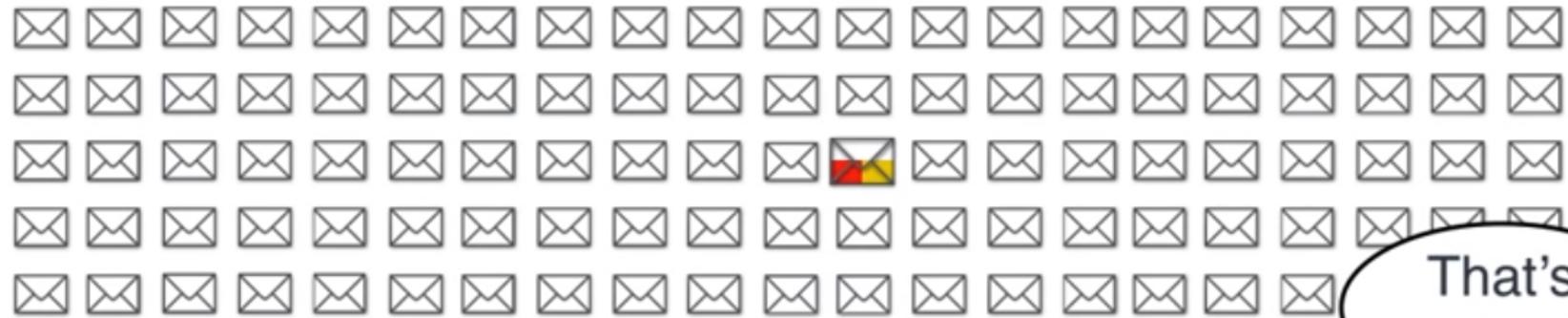
10 "Cheap"

5% "Buy"

10% "Cheap"

Independent

0.5% "Buy" and "Cheap"



100 e-mails

5 "Buy"

10 "Cheap"

5% "Buy"

10% "Cheap"

Independent

0.5% "Buy" and "Cheap"

That's
naive!

Spam Detector

Spam



25 e-mails

20 "Buy"

15 Cheap

$$\frac{4}{5} \rightarrow 12/25 \times 25 = 12 \text{ "Buy" and "Cheap"}$$

$\frac{3}{5}$

Spam Detector

No spam



75 e-mails

5 "Buy"

10 "Cheap"

$$\frac{1}{15} \rightarrow \frac{2}{225}$$

$$\frac{2}{15} \times 75 = \frac{2}{3} \text{ "Buy" and "Cheap"}$$

Spam Detector



“Buy” and “Cheap”

Spam



⚠ 12

No spam



⚠ 2/3

Spam Detector



“Buy” and “Cheap”

Spam

No spam

⚠ 12

⚠ 2/3

Quiz: If an e-mail contains the words “buy” and “cheap”, what is the probability that it is spam?

Spam Detector



“Buy” and “Cheap”

Spam

No spam

12
94.737%

2/3
5.263%

Quiz: If an e-mail contains the words “buy” and “cheap”, what is the probability that it is spam?

$$\frac{12}{12 + 2/3} = \frac{36}{38} = 94.737\%$$

Goal

- Learning function $f(x) \rightarrow y$
- y = one of the classes (eg: Spam/ham , digits 0-9)
- $x = x_1, \dots, x_d$ – value of attributes (numerical or categorical)

Probabilistic Classifier

- Most probable class given observation
- Bayesian Probability of class:

$$\hat{y} = \arg \max_y P(y|x)$$

$$P(y|x) = \frac{P(x|y)P(y)}{\sum P(x|y')P(y')}$$

class model prior

Bayesian Classification: Components

$$P(y|x) = \frac{P(x|y)P(y)}{\sum_{y'} P(x|y')P(y')}$$

Example:
y ... UK patient has Ebola
x ... observed symptoms

P(y): Prior probability of each class

- Encodes which classes are rare or common
- Apriori much more likely to have something other than Ebola

Bayesian Classification: Components

$$P(y|x) = \frac{P(x|y)P(y)}{\sum_{y'} P(x|y')P(y')}$$

Example:
y ... UK patient has Ebola
x ... observed symptoms

P(x/y): class conditional model

- Describe how likely to see an observation x for class y
- Assuming that its Ebola, do symptoms look plausible?

Bayesian Classification: Components

$$P(y|x) = \frac{P(x|y)P(y)}{\sum_{y'} P(x|y')P(y')}$$

Example:
y ... UK patient has Ebola
x ... observed symptoms

P(x): normalize probabilities across observations

- Doesn't affect which class is most likely (argmax)

Types of Naïve Bayes Models

- **Multinomial:** good when your features describe discrete frequency counts(e.g: word counts)
- **Bernoulli:** good for making prediction from binary features
- **Gaussian:** good for making prediction from normally distributed features

Discrete Example: Spam

- Separate words from valid email, attributes are words
- D1: “send us your password” spam
- D2: “send us your reviews” ham
- D3: “review your password” ham
- D4: “review us” spam
- D5: “send your password” spam
- D6: “send us your account” spam

Discrete Example: Spam

- Separate words from valid email, attributes are words
- D1: “send us your password” spam $P(\text{spam}) = 4/6$ $P(\text{ham}) = 2/6$
- D2: “send us your reviews” ham
- D3: “review your password” ham
- D4: “review us” spam
- D5: “send your password” spam
- D6: “send us your account” spam

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- D2: “send us your reviews” ham
- D3: “review your password” ham
- D4: “review us” spam
- D5: “send your password” spam
- D6: “send us your account” spam

spam	ham	
2/4	1/2	password
1/4	2/2	review
3/4	1/2	send
3/4	1/2	us
3/4	1/2	your
1/4	0/2	account

Discrete Example: Spam

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- D2: “send us your reviews” ham
- D3: “review your password” ham
- D4: “review us” spam
- D5: “send your password” spam
- D6: “send us your account” spam
- New email: “review us now”

spam	ham	
2/4	1/2	password
1/4	2/2	review
3/4	1/2	send
3/4	1/2	us
3/4	1/2	your
1/4	0/2	account

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- D3: “review your password” ham
- D4: “review us” spam
- D5: “send your password” spam

spam	ham	
2/4	1/2	password
1/4	2/2	review
3/4	1/2	send
3/4	1/2	us
3/4	1/2	your
1/4	0/2	account

$$P(\text{review us} | \text{spam}) = P(0,1,0,1,0,0 | \text{spam}) = (1 - \frac{2}{4})(\frac{1}{4})(1 - \frac{3}{4})(\frac{3}{4})(1 - \frac{3}{4})(1 - \frac{1}{4})$$

$$P(\text{review us} | \text{ham}) = P(0,1,0,1,0,0 | \text{ham}) = (1 - \frac{1}{2})(\frac{2}{2})(1 - \frac{1}{2})(\frac{1}{2})(1 - \frac{1}{2})(1 - \frac{0}{2})$$

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- D3: “review your password” ham
- D4: “review us” spam
- D5: “send your password” spam
- D6: “send us your account” spam

spam	ham	
2/4	1/2	password
1/4	2/2	review
3/4	1/2	send
3/4	1/2	us
3/4	1/2	your
1/4	0/2	account

New email: “review us now”

$$P(\text{ham} \mid \text{review us}) = 0.87$$

Problem With Naïve Bayes

- Zero frequency problem:

any email containing ‘account’ is spam $P(\text{account} \mid \text{ham}) = 0$

Solution: never allows zero probabilities

Laplace smoothing: Add a small positive number to all counts

It’s a very common problem (Zipf’s law: 50% words occur once)

Problem With Naïve Bayes

- Assume word independence:

Every word contributes independently to $P(\text{spam} \mid \text{email})$

Fooling Naïve Bayes by adding lots of hammy words into spam email.

Advantages

1. Very simple and easy to implement
2. Need less training data
3. Handle both continuous and discrete data
4. As it is fast, it can be used in real time prediction
5. Not sensitive to irrelevant features
6. No requirement of fill in or explicitly model missing value

Applications in real world

Facial recognition



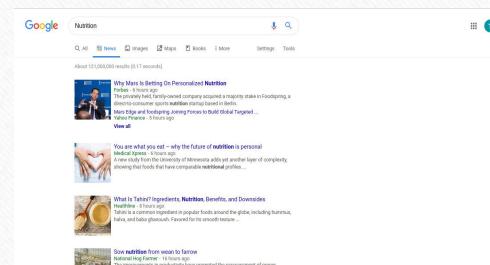
Medical Diagnosis



Weather Prediction



News Classification



Python code implementation

- Jupyter Notebook



Discussion



Thank you!!!!!!

