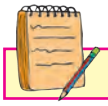
4. BIVARIATE FREQUENCY DISTRIBUTION AND CHI SQUARE STATISTIC



Let's Study

- Meaning of bivariate frequency distribution
- Classification and Tabulation of bivariate data
- Categorical variables
- Contingency table
- Chi square statistic

Let's Recall

- ✓ Concept of univariate data
- ✓ Classification and Tabulation of data
- ✓ Univariate frequency distribution for grouped and ungrouped date
- ✓ Basic Set Theory

Let's Observe…

4.1 Meaning of bivariate frequency distribution:

So far our study was restricted to frequency distributions of single variables. Such frequency distributions are called univariate frequency distributions. We now turn our attention to the study of two variables observed on the same population.

Heights and weights, income and expenditure of a group of individuals, marks obtained by a group of students in two different subjects etc are examples of bivariate data as they involve simultaneous study of two variables.

Frequency distribution of two variables is called a bivariate frequency distribution.

4.2 Classification and Tabulation of Bivariate data:

The given data can be summarized in the form of a two way table called the bivariate frequency table. For this, the values of each variable are grouped into various classes.

Consider the following data which gives a 15 days record of number of ice candies sold and temperature of the day recorded by a shopkeeper.

Daily temperature (Celsius)	33	18	16	34	33	18	16	33
No. of ice candies sold	250	150	250	400	500	250	150	500
Daily temperature (Celsius)	18	33	16	33	18	34	34	
No. of ice candies sold	150	500	250	400	400	500	400	

- Let us denote 'Daily temperature (in Celsius) by X and 'Number of ice candies sold' by Y
- Thus, we have two variables X and Y.
- Observe that variable X takes values from 16 to 34 and variable Y takes values from 150 to 500.
- For a bivariate frequency distribution, we construct a table by taking values of X horizontally at the top and values of Y vertically on the left side as shown below:

X / Y	16	18	33	34
150				
250				
400				
500				

Table 4.1

- Now, on the first day, the temperature was 16 (X) and Number of ice candies sold (Y) was 150. Therefore, we put a tally mark in the cell corresponding to X = 33 and Y=250

X / Y	16	18	33	34
150				
250			|	
400				
500				

Table 4.2

- Proceeding in the same manner, we put tally marks for each pair of observations (x, y) for all 15 days.
- Counting the number of tally marks, total frequency of each cell is written in the brackets ( ), after the tally marks.

X / Y	16	18	33	34	Total (f_y)
150	| (1)	|| (2)			3
250	|| (2)	| (1)	| (1)		4
400		| (1)	| (1)	|| (2)	4
500			||| (3)	| (1)	4
Total (f_x)	3	4	5	3	15

Table 4.3

- Sum of all the frequencies in every row is written in the extreme right column of that row.

  These frequencies together with values of Y give marginal frequency distribution of Y.

Y	150	250	400	500	Total
Frequency	3	4	4	4	15

Table 4.4

- Sum of all the frequencies in every column is written in the bottom row of that column.
- These frequencies together with values of X give marginal frequency distribution of X.

X	16	18	33	34	Total
Frequency	3	4	5	3	15

Table 4.5

- Sum of all frequencies in extreme right columns or bottom row is written in the right bottom cell. This is the total frequency of the data.
- Applying a particular condition on X (or Y), we can obtain conditional frequency distribution of Y (or X)

  Ex. (i) Conditional frequency distribution of X when Y = 250

X	16	18	33	34	Total
Frequency	2	1	1	-	4

Table 4.6

(ii) Conditional frequency distribution of Y when $X \geq 33$

X / Y	33	34	Total (f_y)
150	-	-	-
250	| (1)		1
400	| (1)	|| (2)	3
500	||| (3)	| (1)	4
Total (f_x)	5	3	8

Table 4.7

In general, if X and Y are two variables with m values of X and n values of Y, then the bivariate table will contain cells. By going through the different pairs of observations (x, y) on the two variables and using tally marks, we find frequency for each cell and obtain a bivariate frequency table as shown below.

Bivariate frequency distribution table

X / Y	Values						Total frequency of Y ($f_{\cdot j}$)
Values		x_1	x_2	…	x_i	…	x_m	
	y_1	f_{11}	f_{21}	…	f_{i1}	…	f_{m1}	$f.1$
	y_2	f_{12}	f_{22}		f_{i2}		f_{m2}	$f.2$
		…			…		…	…
	y_j	f_{1j}	f_{2j}	…	f_{ij}	…	f_{mj}	$f.j$
		...			…		…	…
	y_n	f_{1n}	f_{2n}	…	f_{in}	…	f_{mn}	$f.n$
Total frequency of X ($f_{i\cdot}$)		f_1	f_2	…		…	f_m	N = $\Sigma_{i=1}^{m} f_{ij} = f_{\cdot j}$, $\sum_{j=1}^{m} f_{ij} = f_{i}$.

Table no.4.8

The i^{th} value of X is denoted by x_i and the j^{th} value of Y is denoted by y_j. Frequency associated with the pair (x_i, y_j) is denoted by fij and

Total frequency is also obtained as

$$N = \sum_{i=1}^{m} f_{ij} = f_{\cdot j}, \quad \sum_{i=1}^{m} f_{ij} = f_{\cdot i}.$$

4.2.1 Marginal frequency distributions:

The frequency distribution of the values of variable X together with their frequency totals (f_x) is called marginal frequency distribution of X.

Similarly, the frequency distribution of the values of variable Y together with their frequency totals (f_y) is called marginal frequency distribution of Y.

Marginal frequency distribution of X

X	x_1	x_2	…	…	x_m	Total
Freq.	f_1	f_2	…	…	fm	N

Table no 4.9

Marginal frequency distribution of Y

Y	y_1	y_2	…	…	y_n	Total
Freq.	f_1	f_2	…	…	f_n	N

Table no 4.10

4.2.2 Conditional frequency distributions:

Frequency distribution of one variable, when the value of the other variable is given, is called conditional frequency distribution.

Conditional frequency distribution of X when $Y=y_j$

X	x_1	x_2	…	x_i	…	x_m	Total
Freq	f_{1j}	f_{2j}	…	f_{ij}	…	f_{mj}	$f_{\cdot j}$

Table no 4.11

Conditional frequency distribution of Y when $X=x_i$

Y	y_1	y_2	…	y_j	…	Y_n	Total
Freq	f_{i1}	f_{i2}	…	f_{ij}	⋯	f_{in}	$f_{\cdot j}$

Table no 4.12

SOLVED EXAMPLES

Ex.1. Following data gives age (X in years) of students in a particular school and their marks in G.K. test. Prepare a bivariate frequency distribution.

X : 11 11 10 13 12 10 12 12 13 10
11 13 12 12 12 11 13 10 10 11

Y : 22 24 23 21 24 21 22 23 24 24
22 23 21 24 21 22 24 22 23 24

Also obtain

i) Marginal frequency distributions of age and marks in G.K.

ii) Conditional frequency distribution of age when marks in G.K. are 23

iii) Conditional frequency distribution of marks in G.K. when the age is 11 years.

Solution:

Write values of X horizontally and Y vertically.

X takes values from 10 to 13 and Y takes values from 21 to 24. Therefore the two way table will contain 16 cells.

The first student is 11 years old and has 22 marks in G.K.

Therefore, we put tally mark in the cell corresponding to X=11 and Y=22 etc.

Bivariate frequency distribution table of X and Y

X / Y	10	11	12	13	f_y
21	| (1)	-	|| (2)	| (1)	4
22	| (1)	||| (3)	| (1)	-	5
23	|| (2)	-	| (1)	| (1)	4
24	| (1)	|| (2)	|| (2)	|| (2)	7
f_x	5	5	6	4	20

Table 4.13

(i) Marginal frequency distribution of X :

X	10	11	12	13	Total
F	5	5	6	4	20

Table 4.14

(ii) Marginal frequency distribution of Y :

Y	21	22	23	24	Total
F	4	5	4	7	20

Table 4.15

(iii) Conditional frequency distribution of X when Y=23

X	10	11	12	13	Total
F	2	0	1	1	4

Table 4.16

(iv) Conditional frequency distribution of Y when X=11

Y	21	22	23	24	Total
F	0	3	0	2	5

Table 4.17

Ex.2. Construct the frequency distribution table for the data on heights (X in cm) and weights (Y in kg) of 20 students are given below:

Height (cm)	138	139	147	144	163	142	149	140	138	139
Weight (kg)	62	56	63	54	45	46	51	43	36	58
Height (cm)	148	155	141	160	153	150	152	162	146	160
Weight (kg)	61	60	55	59	58	43	38	51	57	60

Also obtain

(i) Marginal frequency distributions of height (X) and weight (Y) of the students

(ii) Conditional frequency distribution of X when Y is more than or equal to 55 kgs

(iii) Conditional frequency distribution of Y when X lies between 135 to 145 cms

Solution: Bivariate frequency distribution table for height and weight

X / Y	135-145	145-155	155-165	f_y
35-45	|| (2)	|| (2)	-	4
45-55	|| (2)	| (1)	| (1)	4
55-65	|||| (4)	|||| (4)	|||| (4)	12
f_x	8	7	5	20

Table 4.18

(i) Marginal frequency distribution of X :

X	135-145	145-155	155-165	Total
F	8	7	5	20

Table 4.19

Marginal frequency distribution of Y:

Y	35-45	45-55	55-65	Total
F	4	4	12	20

Table 4.20

(i) Conditional frequency distribution of X when Y is more than or equal to 55 kg

X	135-145	145-155	155-165	Total
F	4	4	4	12

Table 4.21

(ii) Conditional frequency distribution of Y when X lies in 135-145

X	35-45	45-55	55-65	Total
F	2	2	4	8

Table 4.22

EXERCISE 4.1

1) Following table gives income (X) and expenditure (Y) of 25 families :

X / Y	200-300	300-400	400-500
200-300	6	6	1
300-400	-	4	6
400-500	-	-	2

Find

(i) Marginal frequency distributions of income and expenditure.

(ii) Conditional frequency distribution of X when Y is between 300-400.

(iii) Conditional frequency distribution of Y when X is between 200-300.

(iv) How many families have their income Rs. 300 and more and expenses Rs. 400 and less?

2) Two dice are thrown simultaneously 25 times. The following pairs of observations are obtained.

(2, 3) (2, 5) (5, 5) (4, 5) (6, 4) (3, 2)
(5, 2) (4, 1) (2, 5) (6, 1) (3, 1) (3, 3)
(4, 3) (4, 5) (2, 5) (3, 4) (2, 5) (3, 4)
(2, 5) (4, 3) (5, 2) (4, 5) (4, 3) (2, 3)
(4, 1)

Prepare a bivariate frequency distribution table for the above data. Also obtain the marginal distributions.

3) Following data gives the age of husbands (X) and age of wives (Y) in years. Construct a bivariate frequency distribution table and find the marginal distributions.

X	27	25	28	26	29	27	28	26	25	25	27
Y	21	20	20	21	23	22	20	20	19	19	23
X	26	29	25	27	26	25	28	25	27		
Y	19	23	23	22	21	20	22	23	22		

Find conditional frequency distribution of age of husbands when the age of wife is 23 years.

4) Construct a bivariate frequency distribution table of the marks obtained by students in English(X) and Statistics(Y).

Marks in Statistics (X)	37	20	46	28	35	26	41	48	32	23	20	39	47	33	27	26
Marks in English (Y)	30	32	41	33	29	43	30	21	44	38	47	24	32	31	20	21

Construct a bivariate frequency distribution table for the above data by taking class intervals 20-30, 30-40, ... etc. for both X and Y. Also find the marginal distributions and conditional frequency distribution of Y when X lies between 30−40.

5) Following data gives height in cm (X) and weight in kgs (Y) of 20 boys. Prepare a bivariate frequency table taking class intervals 150-154, 155-159…etc. for X and 35-39, 40-44…etc for Y. Also find

(i) Marginal frequency distributions.

(ii) Conditional frequency distribution of Y when $155 \leq X \leq 159$

(152, 40) (160, 54) (163, 52) (150, 35)
(154, 36) (160, 49) (166, 54) (157, 38)
(159, 43) (153, 48) (152, 41) (158, 51)
(155, 44) (156, 47) (156, 43) (166, 53)
(160, 50) (151, 39) (153, 50) (158, 46)

CHI SQUARE STATISTIC

Let's Learn

It is interesting to find that there is a relationship between age and type of songs people prefer to listen, gender and driving skills, day of a week and tendency of students to remain absent etc. Various types of statistical relationships

may exist among variables of interest. Each of these has some form of association between the variables.

It is common to use the 'chi-square' statistic for measuring relationship between categorical variables.

Let's Note

In general, any quantity calculated from data is called a statistic. For example, mean is a statistic.

4.3 Categorical Variables:-

A variable that takes non numerical values is called a categorical variable.

Ex- 1) Gender - 'Male', 'Female' or 'Other'

2) Blood group- A, B, AB, O

3) Mother tongue of a student.

4.4 Contingency Table-

To study two categorical variables, data can be summarized in the form of a two-way frequency table. Such a bivariate frequency distribution for categorical variables is called a contingency table.

For Example – We may construct a contingency table to study the relationship between Gender and Food preference.

	Sweet	Spicy
Female	a	b
Male	c	d

Table 4.23

Where, a, b, c, d denote the frequencies.

SOLVED EXAMPLES

Ex.1 A survey was carried out to determine whether there is an association between age group and preference for ice cream flavor (Chocolate, Vanilla). Out of a sample of 180 persons, 72 were teenagers of which 40 liked chocolate flavor. 58 adults liked vanilla. Construct a two way contingency table.

Solution: The above data can be arranged in the following contingency table.

Age group	Ice-cream flavor		Total
	Chocolate	Vanilla	
Teenagers	40		72
Adults		58	
Total			180

Table 4.24

Let us complete the above table by finding missing frequencies.

Age group	Ice-cream flavor		Total
	Chocolate	Vanilla	
Teenagers	40	32	72
Adults	50	58	108
Total	90	90	180

Table 4.25

Let's Construct

A sample of 400 students was taken for survey. 140 of them were Music students. 190 of them were good in Mathematics. Prepare a 2x2 contingency table from this information.

Let's Learn

4.5 Chi-square Statistic (χ^2) :-

'Chi-square' statistic is denoted by χ^2 where, χ is a Greek letter, written as 'Chi' read like 'Kai' or 'Ki' in 'Kite'.

The χ^2-statistic is used to measure the relationship between two categorical variables.

χ^2 is always non-negative.

Formula for computing χ^2 statistic -

$$\chi^2 = \Sigma\left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right]$$

Where,

χ^2 = Chi-square value

O_{ij} = Observed frequencies

E_{ij} = Expected frequencies

It is common to write

χ^2 as $\chi^2 = \Sigma\left[\frac{(O-E)^2}{E}\right]$

The **Expected frequencies** can be obtained by using the formula:

$$E_{ij} = \frac{R_i \times C_j}{N}$$

Where,

R_i = Row Total

C_j = Column Total

N = Grand Total that is Total frequency

SOLVED EXAMPLES

Ex.1: Following data is obtained from a sample of 200 people of which 80 were men and 120 were women.

	Right-handed	Left- handed
Men	40	40
Women	35	85

Table 4.26

Find the value of χ^2 statistic.

Solution:

Let us find the row and column totals and total frequency from the given 2 × 2 contingency table.

	Right Handed	Left Handed	Total
Men	40	40	80
Women	35	85	120
Total	75	125	200

Table 4.27

Expected frequencies are given by

$$E_{ij} = \frac{R_i \times C_j}{N}$$

Therefore,

$$E_{11} = \frac{R_1 \times C_1}{N} = \frac{80 \times 75}{200} = 30,$$

$$E_{12} = \frac{R_1 \times C_2}{N} = \frac{80 \times 125}{200} = 50$$

$$E_{21} = \frac{R_2 \times C_1}{N} = \frac{120 \times 75}{200} = 45,$$

$$E_{22} = \frac{R_2 \times C_2}{N} = \frac{120 \times 125}{200} = 75$$

Write the table of expected frequencies

We have,

$$\chi^2 = \Sigma\left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right]$$

$$= \frac{(40-30)^2}{30} + \frac{(40-50)^2}{50} + \frac{(35-45)^2}{45} + \frac{(85-75)^2}{75}$$

$$= \frac{100}{30} + \frac{100}{50} + \frac{100}{45} + \frac{100}{75}$$

$= 3.33 + 2.22 + 2.00 + 1.33$

$\chi^2 = 8.88$

Ex.2: A sample of boys and girls were asked to choose one colour from three options – pink, blue and orange to paint their room with the following results.

	Pink	Blue	Orange
Boys	27	63	10
Girls	41	45	14

Calculate χ^2 - statistic.

Solution:

	Pink	Blue	Orange	Total
Boys	27	63	10	100
Girls	41	45	14	100
Total	68	108	24	200

Expected frequencies are given by

$$E_{11} = \frac{100 \times 68}{200} = 34$$

$$E_{12} = \frac{100 \times 108}{200} = 54$$

$$E_{13} = \frac{100 \times 24}{200} = 12$$

$$E_{21} = \frac{100 \times 68}{200} = 34$$

$$E_{22} = \frac{100 \times 108}{200} = 54$$

$$E_{23} = \frac{100 \times 24}{200} = 12$$

Write the table of expected frequencies

Now,

$$\chi^2 = \Sigma\left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right]$$

$$= \frac{(27-34)^2}{34} + \frac{(63-54)^2}{54} + \frac{(10-12)^2}{12}$$

$$+ \frac{(41-34)^2}{34} + \frac{(45-54)^2}{54} + \frac{(14-12)^2}{12}$$

$$= 2.88+3+0.67$$

$$= 6.55$$

Activity 4.1

1) A group of 200 teenagers was asked to choose the type of movie they like to watch. Following results were obtained, where the numbers in each cell give the observed frequencies while those inside the bracket ( ) give the expected frequencies.

	Melodra-matic	Action	Total
Boys	25 (45)	75 (55)	100
Girls	65 (45)	35 (55)	100
Total	90	110	200

Calculate the χ^2 – statistic.

2) In a Commerce college, students can choose either Mathematics or Secretarial Practice (S.P.) as an optional subject. Following information is collected from a sample of 150 students of the college.

	Mathematics	S. P.	Row Total
Boys	50	25	75
Girls	39	36	75
Column Total	89	61	N = 150

Complete the table given below:

	Mathematics		Secretarial Practice	
	Observed freq.	Expected freq.	Observed freq.	Expected freq.
Boys	50			
Girls			36	

EXERCISE 4.2

1) Following table shows the classification of applications for secretarial and for sales positions according to gender. Calculate the value of χ^2 statistic.

	Offered	Denied
Male	75	150
Female	25	50

2) 200 teenagers were asked which take out food do they prefer – French fries, burger or pizza. The results were –

	French Fries	Burger	Pizza
Boys	6	20	24
Girls	18	40	92

Compute χ^2 statistic.

3) A sample of men and women who had passed their driving test either in 1st attempt or in 2nd attempt were surveyed. Compute χ^2 statistic.

Passed in →	First attempt	Second attempt
Men	32	28
Women	8	12

4) 800 people were asked whether they wear glasses for reading with following results.

Age	Wear glasses	Do not wear glasses
≤ 30	310	90
> 30	290	110

Compute the χ^2 square statistic.

5. Out of a sample of 120 persons in a village, 80 were administered a new drug for preventing influenza and out of them 18 were attacked by influenza. Out of those who were not administered the new drug, 10 persons were not attacked by influenza:

Prepare: (a) a two-way table showing frequencies.

(b) Compute the χ^2 square statistic.

Let's Remember

- Chi square statistic -

  Formula for computing χ^2 statistic –

$$\chi^2 = \Sigma\left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right] \quad \text{or}$$

$$\chi^2 = \Sigma\left[\frac{(O-E)^2}{E}\right]$$

- Formula for Expected Frequencies -

$$E_{ij} = \frac{R_i \times C_j}{N}$$

MISCELLANEOUS EXERCISE - 4

1) Following data gives the coded price (x) and demand (y) of a commodity.

Price	5	7	9	8	10	7	9	8	5	11	11	10	2	3	9
De-mand	9	15	13	15	14	10	11	14	10	14	6	14	15	11	12
Price	2	4	3	14	6	10	7	15	8	6	5	6	11	14	15
De-mand	6	11	8	11	10	15	9	15	13	9	14	10	7	5	6

Classify the data by taking classes 0-4, 5-9 etc. for x and 5-8, 9-12 etc. for y

Also find

(i) marginal frequency distribution of x and y.

(ii) Conditional frequency distribution of y when x is less than 10

2) Following data gives the age in years and marks obtained by 30 students in an intelligence test.

Age	16	17	22	19	21	16
Marks	16	19	39	50	48	41
Age	21	20	20	23	22	19
Marks	59	44	42	62	37	67

Age	23	20	22	22	23	22
Marks	45	57	35	37	38	56
Age	17	18	16	21	19	20
Marks	54	61	47	67	49	56
Age	17	18	23	21	20	16
Marks	51	42	65	56	52	48

Prepare a bivariate frequency distribution by taking class intervals 16-18, 18-20,…etc. for age and 10-20, 20-30… etc. for marks.

Find

(i) Marginal frequency distributions.

(ii) Conditional frequency distribution of marks obtained when age of students is between 20-22.

3) Following data gives Sales (in Lakh Rs.) and Advertisement Expenditure (in Thousand Rs.) of 20 firms

(115, 61) (120, 60) (128, 61) (121, 63)

(137, 62) (139, 62) (143, 63) (117, 65)

(126, 64) (141, 65) (140, 65) (153, 64)

(129, 67) (130, 66) (150, 67) (148, 66)

(130, 69) (138, 68) (155, 69) (172, 68)

i) Construct a bivariate frequency distribution table for the above data by taking classes 115-125, 125-135, ...etc. for sales and 60-62, 62-64, ...etc. for advertisement expenditure.

ii) Find marginal frequency distributions

iii) Conditional frequency distribution of Sales when the advertisement expenditure is between 64-66 (Thousand Rs.)

iv) Conditional frequency distribution of advertisement expenditure when the sales are between 125-135 (lakh Rs.)

4) Prepare a bivariate frequency distribution for the following data, taking class intervals for X as 35-45, 45-55…etc and for Y as 115-130, 130-145…etc. where, X denotes the age in years and Y denotes blood pressure for a group of 24 persons.

(55, 151) (36, 140) (72, 160) (38, 124)

(65, 148) (46, 130) (58, 152) (50, 149)

(38, 115) (42, 145) (41, 163) (47, 161)

(69, 159) (60, 161) (58, 131) (57, 136)

(43, 141) (52, 164) (59, 161) (44, 128)

(35, 118) (62, 142) (67, 157) (70, 162)

Also find

(i) Marginal frequency distribution of X

(ii) Conditional frequency distribution of Y when $X \leq 45$

5) Thirty pairs of values of two variables X and Y are given below. Form a bivariate frequency table. Also find marginal frequency distributions of X and Y.

X	110	88	91	115	97	85	85	91	120	95
Y	500	800	870	599	625	650	905	700	850	824
X	82	105	99	90	108	124	90	90	111	89
Y	970	609	990	735	600	735	729	840	999	780
X	112	100	87	92	91	82	96	120	121	122
Y	638	850	630	720	695	923	555	810	805	526

6) Following table shows how the samples of Mathematics and Economics scores of 25 students are distributed :

Marks in Economics	Marks in Mathematics	
	40-70	70-100
40-70	20	15
70-100	5	10

Find the value of χ^2 statistic.

7) Compute χ^2 statistic from following data:

	Graduates	Post-Graduates
Male	28	22
Female	32	18

8) Attitude of 250 employees towards a proposed policy of the company is

observed in the following table. Calculate χ^2 statistics.

	Favor	Indifferent	Oppose
Male	68	46	36
Female	27	49	24

9) In a certain sample of 1000 families, 450 families are consumers of tea. Out of 600 Hindu families, 286 families consume tea. Calculate χ^2 statistic.

10) A sample of boys and girls were asked to choose their favourite sport, with the following results. Find the value of χ^2 statistics.

	Football	Cricket	Hockey	Basketball
Boys	86	60	44	10
Girls	40	30	25	5

Activity 4.2

Measure weight and height of at least 20 students in your class/college and prepare a bivariate frequency table.

Activity 4.3

Complete the bivariate frequency distribution in the following table for the given data on heights (X cm) and weights (Y kg) of 20 students.

(150, 65) (169, 71) (136, 68) (130, 66)
(137, 65) (147, 70) (125, 64) (118, 62)
(129, 66) (142, 70) (128, 63) (160, 70)
(140, 65) (123, 64) (138, 67) (171, 67)
(131, 67) (121, 65) (150, 68) (130, 66)

Weight in kg (Y)	Height in cm (X)						f_y
	115-125	125-135	135-145	145-155	155-165	165-175	
62-64	|	|					
64-66			||				
66-68							7
68-70			|				
70-72					|		
f_x				6			20