5 SETS AND RELATIONS

Let's Study

- Representation of a set
- Types of sets
- Intervals
- Operations on sets
- Ordered pair
- Types of relations

Let's Recall

5.1 Introduction:

The concept of a set was developed by German mathematician George Cantor (1845–1918)

You have already learnt about sets and some basic operations involving them in the earlier standards.

We often talk about group or collection of objects. Surely you must have used the words such as team, bouquet, bunch, flock, family for collection of different objects.

It is very important to determine whether a given object belongs to a given collection or not. Consider the following collections:

i) Successful persons in your city.

ii) Happy people in your town

iii) Clever students in your class.

iv) Days in a week.

v) First five natural numbers.

First three collections are not examples of sets, but last two collections represent sets. This is because in first three collections, we are not sure of the objects. The terms 'successful persons,' 'Happy people', 'Clever student' are all relative terms. Here, the objects are not well–defined. In the last two collections. We can determine the objects clearly. Thus, we can say that objects are well–defined.

5.1.1 Set : Definition:

A collection of well–defined objects is called a set.

The object in a set is called its element or member.

We denote sets by capital letters A,B,C. etc. The elements of a set are represented by small letters a, b, c, x, y, z etc. If x is an element of a set A we write $x \in A$, and read as 'x belongs to A'. If x is not an element of a set A, we write $x \notin A$, and read as 'x does not belong to A.'

e.g. zero is a whole number but not a natural number.

$\therefore$ $0 \in W$ (Where W is the set of whole numbers) and $0 \notin N$ (Where N is the set of natural numbers)

5.1.2 Representation of a set:

There are two methods of representing a set.

1) Roster or Tabular method or List method

2) Set–Builder or Rule Method

3) Venn Diagram

1. Roster Method:

In the Roster method, we list all the elements of the set within braces {,} and separate the elements by commas.

Ex : State the sets using Roster method.

i) B is the set of all days in a week.

B = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

ii) C is the set of all vowels in English alphabets.
C = {a, e, i, o, u}

Let's Note:

1) If the elements are repeated, they are written only once.

2) While listing the elements of a set, the order in which the elements are listed is immaterial.

2. Set-Builder Method:

In the set builder method, we describe the elements of the set by specifying the property which determines the elements of the set uniquely.

Ex : State the sets using set–Builder method.

i) Y = {Jan, Feb, Mar, Apr, ..., Dec}
Y = {x/x is a month of a year}

ii) B = {1, 4, 9, 16, 25, ...}
B = {x/x∈N and x is a square}

3) Venn Diagram:

The pictorial representation of a set is called Venn diagram. English Logician John Venn introduced such diagrams. We can use triangles, circles, rectangles or any closed figure to represent a set.

In a Venn diagram the elements of the sets are shown as points

A = {1,2,3} B = {a,b,c,d,e,f} C = {4,5,6}

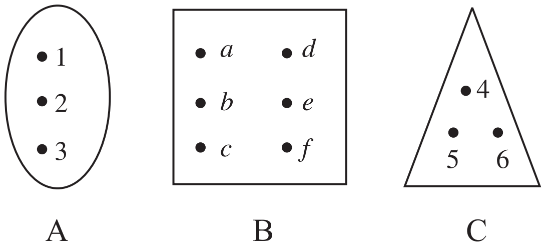

Fig. 5.1

5.1.3 Number of elements of a set:

The number of distinct elements contained in a finite set A is denoted by n(A).

Thus, if A = {5, 2, 3, 4}, then n(A) = 4

n(A) is also called the cardinality of set A.

5.1.4 Types of Sets:

1) Empty Set:

A set containing no element is called an empty or a null set and is denoted by the symbol ϕ or { } or void set.

e.g. A = {x/x∈N, 1<x<2} = { }

Here n(A) = 0

2) Singleton set:

A Set containing only one element is called a singleton set.

e.g. Let A be a set of all integers which are neither positive nor negative.

∴ A = {0} Here n (A) = 1

3) Finite set:

The empty set or set which contains finite number of objects is called a finite set.

e.g set of letters in the word 'BEAUTIFUL'

A = {B, E,A ,U, T, I, F, L}, n(A) = 7

A is a finite set

4) Infinite set:

A set which is not finite, is called an infinite set.

e.g. set of natural numbers, set of rational numbers.

Note :

1) An empty set is a finite set.

2) N, Z, set of all points on a circle, are infinite sets.

5) Subset:

A set A is said to be a subset of set B if every element of A is also an element of B and we write A ⊆ B.

Note: 1) ϕ is subset of every set.

2) A ⊆ A, Every set is subset of itself.

6) Superset: If $A \subseteq B$, then B is called a superset of A and we write, $B \supseteq A$.

7) Proper Subset: A nonempty set A is said to be a proper subset of the set B, if all elements of set A are in set B and at least one element of B is not in A.

i.e. If $A \subseteq B$ and $A \neq B$ then A is called a proper subset of B and we write $A \subset B$.

e.g. Let $A = \{1, 3, 5\}$ and $B = \{1, 3, 5, 7\}$. Then, evey element of A is an element of B but $A \neq B$.

$\therefore$ $A \subset B$, i.e. A is a proper subset of B.

Remark: If there exists even a single element in A which is not in B then A is not a subset of B and we wrtie, $A \not\subset B$.

8) Power Set:

The set of all subsets of a given set A is called the power set of A and is denoted by P(A), Thus, every element of power set A is a set.

e.g. consider the set $A=\{a,b\}$, let us write all subsets of the set A. We know that ϕ is a subset of every set, so ϕ is a subset of A. Also $\{a\}$, $\{b\}$, $\{a,b\}$ are also subsets of A. Thus, the set A has in all four subsets viz. ϕ, $\{a\}$, $\{b\}$, $\{a,b\}$

$\therefore$ $P(A) = \{\phi, \{a\}, \{b\}, \{a,b\}\}$

9) Equal sets:

Two sets are said to be equal if they contain the same elements i.e. if $A \subseteq B$ and $B \subseteq A$.

e.g. Let X be the set of letters in the word 'ABBA' and Y be the set of letters in the word 'BABA'.

$\therefore$ $X = \{A, B\}$, $Y = \{B, A\}$

Thus the sets X and Y are equal sets and we denote it by $X = Y$

10) Equivalent sets:

Two finite sets A and B are said to be equivalent if $n(A) = n(B)$

e.g. $A = \{d, o, m, e\}$

$B = \{r, a, c, k\}$

$n(A) = n(B) = 4$

$\therefore$ A and B are equivalent sets.

11) Universal set: If in a particular discussion all sets under consideration are subsets of a set, say U, then U is called the universal set for that discussion.

e.g. The set of natural numbers N, the set of integers Z are subsets of real numbers R. Thus, for this discussion R is a universal set.

In general universal set is denoted by U or X.

5.1.5 Operations on sets:

1) Complement of a set:

The complement of the set A is denoted by A' or A^c. It is defined as

$A' = \{x / x \in U, x \notin A\}$ = set of all elements in U which are not in A.

Ex. Let $X = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ be the Universal Set and $A = \{2, 4, 6, 8\}$

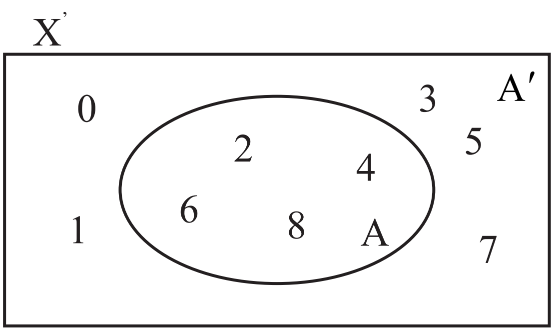

Fig. 5.2

$\therefore$ The complement of the set A is

$A' = \{0, 1, 3, 5, 7\}$

Properties:

i) $(A')' = A$ ii) $\phi' = U$ iii) $U' = \phi$

2) Union of Sets:

The union of two sets A and B is the set of all elements which are in A or in B, (here 'or' is taken in the inclusive sense) and is denoted by $A \cup B$

Thus, $A \cup B = \{x/x \in A \textbf{ or } x \in B\}$

The Union of two sets A and B i.e. $A \cup B$ is represented by a shaded part in Venn–diagram as in fig. 5.3 and fig. 5.4.

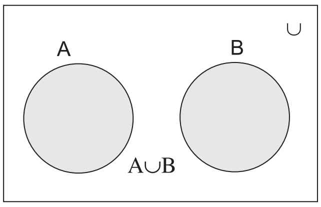

Fig. 5.3

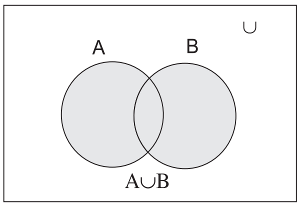

Fig. 5.4

Ex. $A = \{x/x$ is a prime number less than $10\}$

$B = \{x/x \in N$, x is a factor of $8\}$

find $A \cup B$.

Solution : We have $A = \{2,3,5,7\}$

$B = \{1, 2, 4, 8\}$

$\therefore A \cup B = \{1, 2, 3, 4, 5, 7, 8\}$

Properties:

i) $A \cup B = B \cup A$ (Commutativity)

ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associativity)

iii) $A \cup \phi = A$ Identity for union

iv) $A \cup A = A$ Idempotent law

v) If $A \cup A' = U$

vi) If $A \subset B$ then $A \cup B = B$

vii) $U \cup A = U$

viii) $A \subset (A \cup B)$, $B \subset (A \cup B)$

3) Intersection of sets:

The intersection of two sets A and B is the set of all elements which are both in A and B is denoted by $A \cap B$

Thus, $A \cap B = \{x/x \in A \textbf{ and } x \in B\}$

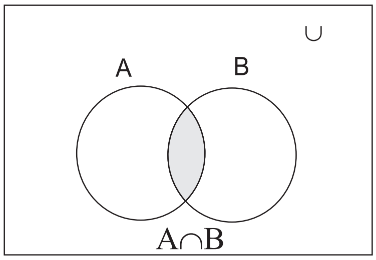

Fig. 5.5

The shaded portion in Fig. 5.5 represents the intersection of A and B i.e. $A \cap B$

Ex. $A = \{1,3,5,7,9\}$

$B = \{1,2,3,4,5,6,7,8\}$

Find $A \cap B$.

Solution: $A \cap B = \{1,3,5,7\}$

Ex. (Activity) : If $A = \{x/x \in N$, x is a factor of $12\}$

$B = \{x/x \in N$, x is a factor of $18\}$

Find $A \cap B$

Solution:

$A = \{\square, \square, \square, \square, \square, \square\}$

$B = \{\square, \square, \square, \square, \square, \square\}$

$\therefore A \cap B = \{\square, \square, \square, \square\}$

= common factors of 12 & 18

Ex. : $A = \{1,3,5,7,9\}$

$B = \{2,4,6,8, 10\}$, $A \cap B = ?$

Solution : $\therefore A \cap B = \{\ \} = \phi$

If $A \cap B = \phi$, A and B are disjoint sets.

Properties:

i) $A\cap B = B \cap A$ (Commutativity)

ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associativity)

iii) $\phi \cap A = \phi$

iv) $A \cap A = A$ Idempotent law

v) $A\cap A = \varnothing$

vi) if $A\subset B$ then $A\cap B = A$

vii) $U\cap A = A$ (Identity for intersection)

viii) $(A\cap B) \subset A$, $(A\cap B) \subset B$

ix) a) $A\cap(B\cup C) = (A\cap B)\cup (A\cap C)$

b) $A\cup(B\cap C) = (A\cup B) \cap (A\cup C)$

..... Distributivity

De Morgan's Laws:

For any two sets A and B

1) $(A\cup B)' = (A'\cap B')$

2) $(A\cap B)' = (A'\cup B')$

Verify the above laws by taking

$U = \{1,2,3,4,5\}$ $A = \{1,3,4\}$ $B = \{4,5\}$

4) Difference of Sets:

Difference of set A and set B is the set of elements which are in A but not in B and is denoted by $A-B$, or $A\cap B'$

The shaded portion in fig. 5.6 represents $A-B$. Thus, $A-B = \{x/x\in A, x\notin B\}$

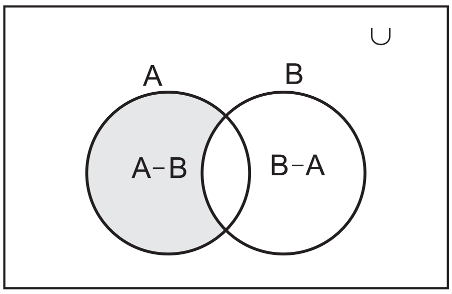

Fig. 5.6

Similary $B-A = \{y/y\in B, y\notin A\}$

Note:

i) $A-B \subseteq A$ and $B-A \subseteq B$.

ii) The sets $A-B$, $A\cap B$ and $B-A$ are mutually disjoint sets, i.e. the intersection of any of these two sets is the null (empty) set.

iii) $A-B = A\cap B'$, $B-A = A'\cap B$

iv) $A\cup B = (A-B)\cup(A\cap B) \cup (B-A)$

Shaded portion in fig. 5.7 represents $A\cup B$

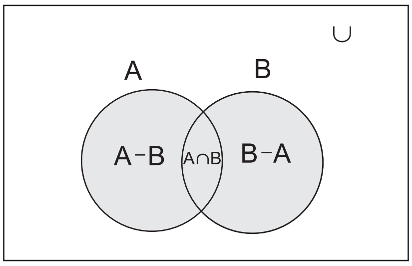

Fig. 5.7

v) $(A-B) \cup (B-A) = A\Delta B$ is called symmetric difference of sets A and B

Illustration :

If $A = \{4,5,6,7,8\}$ and $B = \{3,5,6,8,9\}$ then $A-B = \{4,7\}$, $B-A = \{3,9\}$

and $A\Delta B = (A-B)\cup(B-A) = \{4,7\}\cup\{3,9\} = \{3,4,7,9\}$

Properties:

i) $A\Delta B = (A\cup B)-(A\cap B)$

ii) $A\Delta A = \phi$ iii) $A\Delta\phi = A$

iv) If $A\Delta B = A\Delta C$ then $B=C$

v) $A\Delta B = B\Delta A$

vi) $A\cap(B\Delta C) = (A\cap B)\ \Delta\ (A\cap B)$

Properties of Cardinality of Sets:

For give sets A, B

1) $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

2) When A and B are disjoint sets, then

$n(A\cup B) = n(A) + n(B)$, as $A\cap B = \phi$,

$n(A\cap B) = 0$

3) $n(A \cap B') + n(A \cap B) = n(A)$

4) $n(A' \cap B) + n(A \cap B) = n(B)$

5) $n(A \cap B') + n(A \cap B) + n(A' \cap B) = n(A \cup B)$

6) For any sets A, B, C.

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$

$-n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

7) If $n(A) = m$, $n[P(A)] = 2^m$ Where p(A) is power set of A

8) $n(A \Delta B) = n(A) + n(B) - 2n\,(A \cap B)$

5.1.6 Intervals:

1) Open Interval: Let $a, b \in R$ and $a < b$ then the set $\{x/x \in R,\ a < x < b\}$ is called open interval and is denoted by (a,b). All the numbers between a and b belong to the open interval (a,b) but a, b themselves do not belonging to this interval.

Fig. 5.8

$\therefore\ (a,b) = \{x/x \in R,\ a < x < b\}$

2) Closed Interval: Let $a, b \in R$ and $a < b$ then the set $\{x/x \in R,\ a \le x \le b\}$ is called closed interval and is denoted by [a,b]. All the numbers between a and b belong to the closed interval [a, b]. Also a and b belong to this interval.

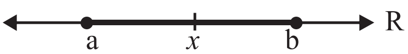

Fig. 5.9

$[a, b] = \{x/x \in R,\ a \le x \le b\}$

3) Semi-closed Interval:

$[a, b) = \{x/x \in R,\ a \le x < b\}$

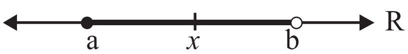

Fig. 5.10

Note that $a \in [a, b)$ but $b \notin [a, b)$

4) Semi–open Interval:

$(a, b] = \{x/x \in R,\ a < x \le b\}$

Fig. 5.11

(a, b] excludes a but includes b.

i.e. $a \notin (a, b]$ but $b \in (a, b]$

5) i) The set of all real numbers greater than a i.e. $(a,\infty) = \{x/x \in R,\ x > a\}$

Fig. 5.12

ii) The set of all real numbers greater than or equal to a

$[a,\infty) = \{x/x \in R,\ x \ge a\}$

Fig. 5.13

6) i) The set of all real numbers less than b. ie. $(-\infty, b)$

$\therefore (-\infty,\ b) = \{x/x \in R,\ x < b\}$

Fig. 5.14

ii) The set of all real numbers less than or equal to b i.e. $(-\infty, b]$

Fig. 5.15

$\therefore (-\infty, b] = \{x/x \in R,\ x \le b\}$

7) The set of all real numbers R is $(-\infty, \infty)$

Fig. 5.16

$R = (-\infty,\infty) = \{x/x \in R,\ -\infty < x < \infty\}$

Rules of inequality of real numbers.

1) If $0<a<b$ and $k>0$ then $a \pm k < b \pm k$, $ka<kb$, $a^k < b^k$

2) If $0<a<b$ and $k<0$ then $ka>kb$, $a^k>b^k$

Ex. : Solve the following inequalities and write the solution set using interval notation.

a) $-7 < 2x + 5 \leq 9$

: Subtracting 5, we get

$-7-5 < 2x \leq 9-5$

$-12 < 2x \leq 4$

Dividing by 2, we get

$-6 < x \leq 2$

So, $x \in (-6,2]$.

b) $x^2 + 2x < 15$

Subtracting 15, we get

$x^2 + 2x - 15 < 0$

Factor $x^2 + 2x - 15$, we get

Say $p(x) = (x - 3)(x + 5) < 0$

As $x-3$ and $x + 5$ is 0, for $x = 3, -5$, consider them as critical points. They divide number line into 3 regions as follow.

Regions	$x<-5$	$(-5<x<3)$	$(x>3)$
Test Points	-6	0	4
Value $(x-3)(x+5)$ of at test points	$(-6-3)(-6+5) = (-9)(-1) = 9$	$(0-3)(0+5) = (-3)(5) = -15$	$(4-3)(4+5) = (1)(9) = 9$
Sign of $p(x)$	$p(x)>0$	$p(x)<0$	$p(x)>0$

Therefore the solution set is $(-5 < x < 3)$

So, x $(-5, 3)$.

c) $\dfrac{x}{x-3} \geq 4$

subtracting, 4, we get

[Note that, as we do not know value of x, we do not multiply by $(x - 3)$]

$$\frac{x}{x-3} - 4 \geq 0$$

$$\frac{x-4(x-3)}{x-3} \geq 0$$

$$\frac{x-4x+12}{x-3} \geq 0$$

$$\frac{-3x+12}{x-3} \geq 0$$

$$\frac{-3(x-4)}{x-3} \geq 0$$

Dividing by -3, we get

Say $p(x) = \dfrac{x-4}{x-3} \leq 0$

Take $x = 3, 4$ as critical points. It divides number line into 3 regions as follows.

Regions	$(x \leq 3)$	$(3 \leq x \leq 4)$	$(x \geq 4)$
Test Points	2	3.5	5
Sign of $x-4$	$-$ve	$-$ve	+ve
Sign of $x-3$	$-$ve	+ve	+ve
Sign of $p(x)$	$p(x)>0$ Except at $x = 3$	$p(x)<0$	$p(x)>0$

The solution set is $(x < 3)$ and $(x \geq 4)$. So $x \in (-\infty, 3) \cup [4, \infty)$.

Here $x = 3$ is not in the solution set, as it makes denominator 0.

Ex. 8 : If A = [-5,3], B = (3,7), and (5,8]. Find (a) $A \cup B$ (b) $B \cup C$ (c) $A \cup C$ (d) $A \cap B$ (e) $B \cap C$ (f) $A \cap C$ (g) $A \cup C'$ (h) $B - C$ (i) $C - B$

Solution :

(a) $A\cup B = [-5, 3] \cup (3, 7) = [-5, 7)$

(b) $B\cup C = (3, 7) \cup (5, 8] = (3, 8]$

(c) $A\cup C = [-5, 3] \cup (5, 8]$

(d) $A\cap B = [-5, 3] \cap (3, 7) = \phi$

(e) $B\cap C = (3, 7) \cap (5, 8] = (5, 7)$

(f) $A\cap C = [-5, 3] \cap (5, 8] = \phi$

(g) $A\cup C' = [-5, 3] \cup (5, 8]' = [-5, 3] \cup \{(-\infty, 5] \cup (8, \infty)\} = (-\infty, 5] \cup (8, \infty)$

(h) $B - C = (3, 7) - (5, 8] = (3, 5]$

(i) $C - B = (5, 8] - (3, 7) = [7, 8]$

SOLVED EXAMPLES

Ex. 1: If $A= \{x/x\in N, x$ is a factor of $6\}$

$B = \{x/x\in N, x$ is a factor of $8\}$

find the A–B and B–A

Solution : $A = \{1, 2, 3, 6\}$

$B = \{1, 2, 4, 8\}$

$\therefore A-B = \{3, 6\}$

$B-A = \{4, 8\}$

Ex.2: $A=\left\{\frac{1}{x} / x\in N, x<8\right\}$

$B=\left\{\frac{1}{2x} / x\in N, x\le 8\right\}$ Find A–B and B–A

Solution: $A=\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6},\frac{1}{7}\right\}$

$B=\left\{\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10},\frac{1}{12},\frac{1}{14},\frac{1}{16}\right\}$

$\therefore A-B=\left\{1,\frac{1}{3},\frac{1}{5},\frac{1}{7}\right\}$

and $B-A=\left\{\frac{1}{8},\frac{1}{10},\frac{1}{12},\frac{1}{14},\frac{1}{16}\right\}$

Ex. 3 : If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$

$C = \{5, 6, 7, 8\}$, $D = \{7, 8, 9, 10\}$

find i) $A\cup B$ ii) $A\cup B\cup C$ iii) $B\cup C\cup D$

Are the sets A, B, C, D equivalent?

Solution: We have

i) $A\cup B = \{1, 2, 3, 4, 5, 6\}$

ii) $A\cup B\cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

iii) $B\cup C\cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

As $n(A) = n(B) = n(C) = n(D) = 4$, the sets A, B, C, D are equivalent.

Ex. 4 : Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be the universal set, $A = \{1, 3, 5, 7, 9\}$

$B = \{2, 3, 4, 6, 8, 10\}$, $C = \{6, 7, 8, 9\}$

find i) A' ii) $(A\cap C)'$ iii) $(A')'$ iv) $(B-C)'$

Solution: We have

i) $A' = \{2, 4, 6, 8, 10\}$

ii) $(A\cap C) = \{7, 9\}$

$\therefore (A\cap C)' = \{1, 2, 3, 4, 5, 6, 8, 10\}$

iii) $(A')' = \{2, 4, 6, 8, 10\}' = \{1, 3, 5, 7, 9\} = A$

iv) $B-C = \{2, 3, 4, 10\}$

$\therefore (B-C)' = \{1, 5, 6, 7, 8, 9\}$

Ex. 5 : Let X be the universal set, for the non–empty sets A and B, verify the De Morgan's laws

i) $(A\cup B)' = A'\cap B'$

ii) $(A\cap B)' = A'\cup B'$

Where $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4, 5\}$ $B = \{1, 2, 5, 6, 7\}$

Solution: $A\cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$(A\cap B) = \{1, 2, 5\}$

$A' = \{6, 7, 8, 9, 10\}$

$B' = \{3, 4, 8, 9, 10\}$

i) $(A\cup B)' = \{8, 9, 10\}$ ...(1)

$A'\cap B' = \{8, 9, 10\}$ ...(2)

from (1) and (2),

$(A\cup B)' = A'\cap B'$

ii) $(A\cap B)' = \{3,4,6,7,8,9,10\}$ ...(3)

$A'\cup B' = \{3,4,6,7,8,9,10\}$ ...(4)

from (3) and (4)

$(A\cap B)' = A'\cup B'$

Ex.6: If $P = \{x/x^2 + 14x + 40 = 0\}$

$Q = \{x/x^2 - 5x + 6 = 0\}$

$R = \{x/x^2 + 17x - 60 = 0\}$ and the universal set $X = \{-20, -10 - 4, 2, 3, 4\}$, find

i) $P\cup Q$ ii) $Q\cap R$ iii) $P\cup(Q\cap R)$

iv) $P\cap(Q\cup R)$ v) $(P\cup Q)'$ vi) $Q'\cap R'$

Solution: $P = \{x/x^2 + 14x + 40 = 0\}$

$\therefore P = \{-10, -4\}$

Similarly $Q = \{3, 2\}$, $R = \{-20, 3\}$ and

$X = \{-20, -10, -4, 2, 3, 4\}$

i) $P\cup Q = \{-10, -4, 3, 2\}$

ii) $Q\cap R = \{3\}$

iii) $P\cup(Q\cap R) = \{-10, -4, 3\}$

iv) $P\cap(Q\cup R) = \phi$

v) $(P\cup Q)' = \{-20, 4\}$

vi) $Q' = \{-20, -10, -4, 4\}$, $R' = \{-10, -4, 2, 4\}$

$Q'\cap R' = \{-10, -4, 4\}$

Ex. 7: If A and B are the subsets of the universal set X and $n(x) = 50$, $n(A) = 35$, $n(B) = 22$ and $n(A'\cap B') = 3$, find

i) $n(A\cup B)$ ii) $n(A\cap B)$

iii) $n(A'\cap B)$ iv) $n(A\cup B')$

Solution:

i) $n(A\cup B) = n(X) - n[(A\cup B)']$

$= n(X) - n(A'\cap B')$ (De Morgan's Law)

$= 50 - 3$

$= 47.$

ii) $n(A\cap B) = n(A) + n(B) - n(A\cup B)$

$n(A\cap B) = 35 + 22 - 47$

$= 10$

iii) $n(A'\cap B) = n(B) - n(A\cap B)$

$= 22 - 10$

$= 12$

iv) $n(A\cup B') = n(X) - n[(A\cup B)']'$

$= n(X) - n(A'\cap B)$

$= 50 - 12$

$= 38$

Ex. 8 : In a board examination, 40 students failed in Physics, 40 in Chemistry and 35. In Maths, 20 failed in Maths and Physics, 17 in Physics and Chemistry, 15 in Maths and Chemistry and 5 in all the three subjects. If 350 students appeared in the examination, how many of them did not fail in any subject?

Solution:

P = set of students failed in Physics

C = set of students failed in Chemistry

M = set of students failed in Maths

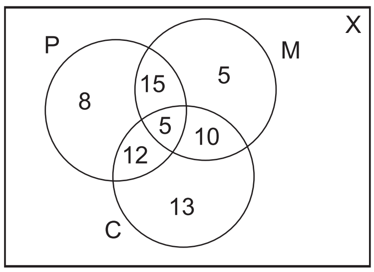

Fig. 5.17

From figure 5.17, we have

$n(X) = 350$, $n(P) = 40$, $n(C) = 40$, $n(M) = 35$

$n(M\cap P) = 20$, $n(P\cap C) = 17$, $n(M\cap C) = 15$

and $n(M\cap P\cap C) = 5$

The number of students who failed in at least one subject = $n(M\cup P\cup C)$

we have,

$n(M\cup P\cup C) = n(M) + n(P) + n(C) - n(M\cap P) - n(P\cap C) - n(M\cap C) + n(M\cap P\cap C)$

$= 35 + 40 + 40 - 20 - 17 - 15 + 5 = 68$

The number of students who did not fail in any subject $= n(X) - n(M\cup P\cup C) = 350 - 68 = 282$

Ex. 9 : A company produces three kinds of products A, B and C. The company studied the perference of 1600 consumers for these 3 products. It was found that the product A was liked by 1250, the product B was liked by 930 and product C was liked by 1000. The proudcts A and B were liked by 650, the products B and C were liked by 610 and the products C and A were liked by 700 consumers. None of the products were liked by 30 consumers.

Find number of consumers who liked.

i) All the three products

ii) Only two of these products.

Solution: Given that totally 1600 consumers were studied. $\therefore n(X) = 1600$, Let A be the set of all consumers who liked product A. Let B the set of all consumers who liked product B and C be the set of all consumers who liked product C.

$n(A) = 1250$,

$n(B) = 930$, $n(C) = 1000$.

$n(A\cap B) = 650$, $n(B\cap C) = 610$

$n(A\cap C) = 700$, $n(A'\cap B'\cap C') = 30$

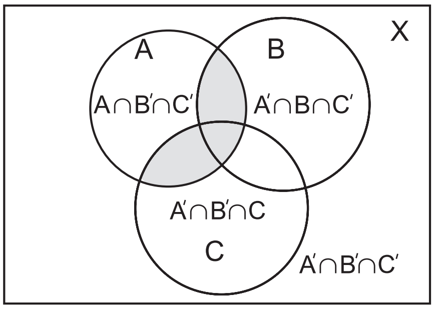

Fig. 5.18

i) $n(A\cup B\cup C) = n(X) - n[(A\cup B\cup C)]'$

$= n(X) - n(A'\cap B'\cap C') = 1600 - 30 = 1570$

Since, $(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)$

$\therefore 1570 = 1250 + 930 + 1000 - 650 - 610 - 700 + n(A\cap B\cap C)$

$\therefore n(A\cap B\cap C) = 1570 + 1960 - 3180$

$= 350$

$\therefore$ The number of consumers who liked all the three products is 350.

ii) $n[(A\cap B)\cap C'] = n(A\cap B) - n[(A\cap B)\cap C]$

$\because n(P\cap Q') = n(P) - n[P\cap Q]$

$= 650 - 350 = 300$

Similarly $n(A'\cap B\cap C) = 610 - 350 = 260$

and $n(A\cap B'\cap C) = 700 - 350 = 350$

$\therefore$ The number of consumers who liked only two of the three products is

$= n(A\cap B\cap C') + n(A\cap B'\cap C) + n(A'\cap B\cap C)$

$= 300 + 350 + 260 = 910$

Maximum & Minimum of Sets:

(a) $\min\{n(A\cup B)\} = \max\{n(A), n(B)\}$

(b) $\max\{n(A\cup B)\} = n(A) + n(B)$

(c) $\min\{n(A\cap B)\} = 0$

(d) $\max\{n(A\cap B)\} = \min\{n(A), n(B)\}$

Illustration:

e.g. If $n(A) = 10$, $n(B) = 20$ then $\min\{n(A\cup B)\} = \max\{10,20\} = 20$ and $\max\{n(A\cup B)\} = 10+20 = 30$, so $20 \le n(A\cup B) \le 30$.

Also $\min\{n(A\cap B)\} = 0$ and $\max\{n(A\cap B)\} = \min\{10,20\} = 10$, so $0 \le n(A\cap B) \le 10$.

Ex. 10) In a survey of 100 consumers 72 like product A and 45 like product B. Find the least and the most number that must have liked both products A and B.

Soln.: Let $n(\mathrm{A})$ and $n(\mathrm{B})$ be number of consumers who like proudct A and B respectively, i.e. $n(\mathrm{A}) = 72$, $n(\mathrm{B}) = 45$.

Since

$n(\mathrm{A}\cup\mathrm{B}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A}\cap\mathrm{B}) = 72 + 45 - n(\mathrm{A}\cap\mathrm{B}) = 117 - n(\mathrm{A}\cap\mathrm{B})$.

But as only 100 consumers are surveyed. Therefore $n(\mathrm{A}\cup\mathrm{B}) \leq 100$ i.e. $117 - n(\mathrm{A}\cap\mathrm{B}) \leq 100$.

$\therefore\ 117 - 100 \leq n(\mathrm{A}\cap\mathrm{B})$, $17 \leq n(\mathrm{A}\cap\mathrm{B})$.

So the least number of consumers who like both the products is 17.

Since $n(\mathrm{A}\cap\mathrm{B}) \leq n(\mathrm{A})$, $n(\mathrm{A}\cap\mathrm{B}) \leq n(\mathrm{B})$, then

$n(\mathrm{A}\cap\mathrm{B}) \leq \min\{n(\mathrm{A}), n(\mathrm{B})\} = \min\{72, 45\} = 45$. So the most number of consumers who like both products is 45.

EXERCISE 5.1

1) Describe the following sets in Roster form

   i) A = $\{x/x$ is a letter of the word 'MOVEMENT'$\}$

   ii) B = $\{x/x$ is an integer, $-\frac{3}{2} < x < \frac{9}{2}\}$

   iii) C = $\{x/x = 2n + 1, n \in \mathrm{N}\}$

2) Describe the following sets in Set–Builder form

   i) $\{0\}$ ii) $(0, \pm1, \pm2, \pm3\}$

   iii) $\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$

   iv) $\{0, -1, 2, -3, 4, -5, 6, ...\}$

3) If A = $\{x/6x^2+x-15 = 0\}$

   B = $\{x/2x^2-5x-3 = 0\}$

   C = $\{x/2x^2-x-3 = 0\}$ then

   find i) $(\mathrm{A}\cup\mathrm{B}\cup\mathrm{C})$ ii) $(\mathrm{A}\cap\mathrm{B}\cap\mathrm{C})$

4) If A, B, C are the sets for the letters in the words 'college', 'marriage' and 'luggage' respective, then verify that $[\mathrm{A}-(\mathrm{B}\cup\mathrm{C})] = [(\mathrm{A}-\mathrm{B})\cap(\mathrm{A}-\mathrm{C})]$

5) If A = $\{1, 2, 3, 4\}$, B = $\{3, 4, 5, 6\}$, C = $\{4, 5, 6, 7, 8\}$ and universal set X = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then verify the following:

   i) $\mathrm{A}\cup(\mathrm{B}\cap\mathrm{C}) = (\mathrm{A}\cup\mathrm{B}) \cap (\mathrm{A}\cup\mathrm{C})$

   ii) $\mathrm{A}\cap(\mathrm{B}\cup\mathrm{C}) = (\mathrm{A}\cap\mathrm{B}) \cup (\mathrm{A}\cup\mathrm{C})$

   iii) $(\mathrm{A}\cup\mathrm{B})' = (\mathrm{A}'\cap\mathrm{B})'$

   iv) $(\mathrm{A}\cap\mathrm{B})' = \mathrm{A}'\cup\mathrm{B}'$

   v) $\mathrm{A} = (\mathrm{A}\cap\mathrm{B})\cup(\mathrm{A}\cap\mathrm{B}')$

   vi) $\mathrm{B} = (\mathrm{A}\cap\mathrm{B})\cup(\mathrm{A}'\cap\mathrm{B})$

   vii) $(\mathrm{A}\cup\mathrm{B}) = (\mathrm{A}-\mathrm{B}) \cup (\mathrm{A}\cap\mathrm{B}) \cup (\mathrm{B}-\mathrm{A})$

   viii) $\mathrm{A} \cap (\mathrm{B}\Delta\mathrm{C}) = (\mathrm{A}\cap\mathrm{B})\ \Delta\ (\mathrm{A}\cap\mathrm{C})$

   ix) $n(\mathrm{A}\cup\mathrm{B}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A}\cap\mathrm{B})$

   x) $n(\mathrm{B}) = (\mathrm{A}'\cap\mathrm{B}) + n(\mathrm{A}\cap\mathrm{B})$

6) If A and B are subsets of the universal set X and $n(\mathrm{X}) = 50$, $n(\mathrm{A}) = 35$, $n(\mathrm{B}) = 20$,

   $n(\mathrm{A}'\cap\mathrm{B}') = 5$, find

   i) $n(\mathrm{A}\cup\mathrm{B})$ ii) $n(\mathrm{A}\cap\mathrm{B})$ iii) $n(\mathrm{A}'\cap\mathrm{B})$ iv) $n(\mathrm{A}\cap\mathrm{B}')$

7) In a class of 200 students who appeared certain examinations, 35 students failed in CET, 40 in NEET and 40 in JEE, 20 failed in CET and NEET, 17 in NEET and JEE, 15 in CET and JEE and 5 failed in all three examinations. Find how many students,

   i) did not fail in any examination.

   ii) failed in NEET or JEE entrance.

8) From amongst 2000 literate individuals of a town, 70% read Marathi newspapers, 50% read English newspapers and 32.5% read both Marathi and English newspapers. Find the number of individuals who read.

   i) at least one of the newspapers.

ii) neither Marathi and English newspaper.

iii) Only one of the newspapers.

9) In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 student take bot tea and coffee, 8 students take both milk and coffee. None of them take tea and milk both and everyone takes atleast one beverage, find the total number of students in the hostel.

10) There are 260 persons with a skin disorder. If 150 had been exposed to the chemical A, 74 to the chemical B, and 36 to both chemicals A and B, find the number of persons exposed to

i) Chemical A but not Chemical B

ii) Chemical B but not Chemical A

iii) Chemical A or Chemical B

11) Write down the power set of A $= \{1,2,3)$

12) Write the following intervals in Set–Builder form

i) $(-3, 0)$, ii) $[6,12]$,

iii) $(6, \infty)$, iv) $(-\infty, 5]$

v) $(2, 5]$ vi) $[-3, 4)$

13) A college awarded 38 medals in volley ball, 15 in football and 20 in basket ball. The medals awarded to a total of 58 players and only 3 players got medals in all three sports. How many received medals in exactly two of the three sports?

14) Solve the following inequalities and write the solution set using interval notation.

i) $-9 < 2x + 7 \leq 19$ ii) $x^2 - x > 20$

iii) $\dfrac{2x}{x-4} \leq 5$ iv) $6x^2 + 1 \leq 5x$

15) If A = $(-7, 3]$, B = $[2, 6]$ and C = $[4, 9]$ then find (i) $A \cup B$ (ii) $B \cup C$ (iii) $A \cup C$ (iv) $A \cap B$ (v) $B \cap C$ (vi) $A \cap C$ (vii) $A' \cap B$ (viii) $B' \cap C'$ (ix) $B - C$ (x) $A - B$

5.2 Relations:

5.2.1 Ordered Pair:

A pair (a, b) of numbers, such that the order, in which the numbers appearis important, is called an ordered pair. Ordered pairs (a,b) and (b, a) are different. In ordered pair (a,b), 'a' is called first component and 'b' is called second component.

Note : i) (a,b)=(c, d), if and only if a = c and b = d.

ii) (a, b) = (b, a) if and only if a = b

Ex. : Find x and y when $(x + 3, 2) = (4, y - 3)$

Solution: Using the definition of equality of two ordered pairs, we have

$(x + 3, 2) = (4, y - 3)$

$\Rightarrow x + 3 = 4$ and $2 = y - 3$

$\Rightarrow x = 1$ and $y = 5$

5.2.2 Carstesian Product of two sets:

Let A and B be two non–empty sets then, the cartensian product of A and B is denoted by $A \times B$ and is defined as set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$

Thus, $A \times B = \{(a, b) / a \in A, b \in B\}$

e.g. 1) If A = {1, 2} and B = {a, b, c}

then $A \times B$ = {(1, a) (1, b), (1, c), (2, a), (2, b), (2, c)}

2) If A = ϕ or B = ϕ, we define

$\therefore A \times B = \phi$

Note:

Let A and B be any two finite sets with n(A) = m and n(B) = n then the number of elements in the Cartesian product of two finite sets A and B is given by $n(A \times B) = mn$

Ex. 2 : Let A = {1,3} ; B = {2, 3, 4} Find the number of elements in the Cartesian product of A and B.

Solution: Given A = {1, 3} and B = {2, 3, 4}

$\therefore$ n(A) = 2 and n (B) = 3

$\therefore$ $n(A \times B) = 2 \times 3 = 6$

5.2.3 Cartesian product of a set with itself:

$A \times A = A^2 = \{(a, b) / a, b \in A\}$

Where (a, b) is an ordered pair

$A^3 = A \times A \times A = \{(a, b, c) / a, b, c \in A\}$

Where (a, b, c) is an ordered triplet.

e.g. If $A = \{4, 5\}$ then we have

$A^2 = A \times A = \{(4, 4), (4, 5), (5, 4), (5, 5)\}$

$A^3 = A \times A \times A = \{(4, 4, 4), (4, 4, 5), (4, 5, 4)$ $(5, 5, 5), (5, 4, 4) (5, 4, 5), (5, 5, 4), (4, 5, 5)\}$

Note: If $n(A) = m$ then $n(A \times A \times A) = m^3$. i.e. $n(A^3) = [n(A)]^3$

In general, $n(A \times A \times \ldots\ldots$ r times$) = n(A)^r$

5.2.4 Definitions of relation, Domain, Co- domain and Range of a Relation:

Relation:

The concept of the term Relation is drawn from the meaning of relation in English language, according to which two objects or quantities are related if there is recognizable link between them.

Members in a family are often related to each other. We describe relation as A is friend of B, F is father of S, P is sister of Q etc. In Mathematics we can have different relations. e.g. among integers we can define the relation as m is factor of n. If this relation is denoted by R, then we write 2R4, 3R6, 5R10.

A well defined relation between elements of A and B is given by aRb, $a \in A$ and $b \in B$. Thus the relation gives ordered pair (a,b) and defines a subset of $A \times B$.

e.g. if $A = \{2, 3, 4, 5, 6)$ and $B = \{6, 7, 8, 10\}$ then the relation aRb if a is factor of b gives the subset $\{(2, 6), (2, 8), (2, 10), (3, 6), (4, 8), (5, 10)\}$ of $A \times B$.

On the other hand, if we consider the subset $S' = \{(2, 7), (3, 10), (4, 7)\}$ of $A \times B$, then there is a unique relation R' given by this subset. i.e. 2R'7, 3R'10, 4R'7. Thus S' has defined R'.

For every relation between A and B, there is a unique subset defined in $A \times B$ and for every subset of $A \times B$, there is a unique relation associated with it. Hence we can say that every relation between A and B is a subset of $A \times B$ and all relationships are ordered pairs in $A \times B$.

When A = B, that is when we have relation between the elements of A, that will be subset of $A \times A$. If a is nonempty, any relation in $A \times A$ is called **binary relation** on A.

Ex. : Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 4, 5\}$

Let R be a relation such that $(x, y) \in R$ implies $x < y$. We list the elements of R.

Solution: Here $A = \{1,2, 3, 4, 5\}$ and $B = \{1, 4, 5\}$

$\therefore R = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}$

Arrow diagram for this relation R is given by

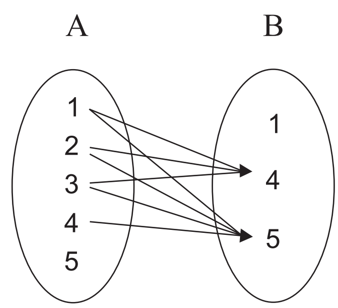

Fig. 5.19

Domain:

The set of all first components of the ordered pairs in a relation R is called the domain of the relation R.

i.e. domain (R) = $\{a/(a, b) \in R\}$

Range:

The set of all second components of all ordered pairs in a relation R is called the range of the relation.

i.e. range (R) = $\{b/(a, b) \in R\}$

Co-domain:

If R is a relation from A to B then set B is called the co-domain of the relation R.

Note : Range is a subset of co-domain (check!).

5.2.5 Binary relation on a set:

Let A be non-empty set then every subset of $A \times A$ is binary relation on A.

Illustrative Examples:

Ex. 1.: Let A = {1, 2, 3} and

R = {(1, 2), (2, 2), (3, 1), (3, 2)}

$A \times A$ = {(1,1), (1,2), (1,3), (2,1), (2,2,), (2,3), (3,1), (3,2), (3,3)}

$R \subset A \times A$ and therefore, R is a binary relation on A.

Ex. 2.: Let N be the set of all natural numbers and R = $\{(a, b) / a, b \in N \text{ and } 2a + b = 10\}$

Since $R \subset N \times N$, R is a binary relation on N.
R = {(1, 8), (2, 6), (3, 4), (4, 2)}

We can state domain, range and co-domain of the relation.

They are as follows:

Domain (R) = {1, 2, 3, 4}

Co-domain (R) = {2, 4, 6, 8}

Co-domain (R) = range (R)

Ex. 3.: If R' is defined in $Z \times Z$ as aR'b if (a+b) is even. Let R be defined $Z \times Z$ as a Rb if (a–b) is even.

Note that R and R' define the same relation and subset related to them is {a, b)/a and b both even or a and b both odd}.

Note : i) Since $\phi \subset A \times A$, ϕ is a relation on A and is called the empty or void relation on A.

ii) Since $A \times A \subset A \times A$, $A \times A$ is a relation on A called the universal relation on A. i.e. $R = A \times A$

Ex. 4 : If A = {2, 4, 6}

then R = $A \times A$ = {(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)} and $R = A \times A$ is the universal relation on A.

Note : The total number of relation that can be defined from a set A to a set B is the number of possible subsets of $A \times B$.

if $n(A) = m_1$ and $n(B) = m_2$

then $n(A \times B) = m_1 m_2$

and the total number of relations is $2^{m_1 m_2}$.

5.2.6 Identity Relation:

If the relation in A is given by aRa for every $a \in A$, then the subset given by this relation is the diagonal subset $\{a,a)/a \in A\}$. Hence, aRb if b = a. This is called identity relation.

5.2.7 Types of relations:

Let A be a non–empty set. Then a binary relation R on A is said to be

(i) Reflexive, if $(a, a) \in R$ for every $a \in A$ i.e. aRa for every $a \in A$

(ii) Symmetric, if $(a, b) \in R$
$\Rightarrow (b, a) \in R$ for all $a, b \in A$
i.e. aRb $\Rightarrow$ bRa for all $a, b \in A$

(iii) Transitive, if $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

Note: Read the symbol '$\Rightarrow$' as 'implies'.

Equivalence relation:

A relation which is reflexive, symmetric and transitive is called an equivalence relation.

Illustrative examples:

Ex. 1: Let R be a relation on Q, defined by

$R = \{(a, b)/a, b\in Q \text{ and } a-b\in Z\}$

Show that R is an equivalence relation.

Solution: Given $R = \{(a, b)/a, b\in Q \text{ and } a-b\in Z\}$

i) Let $a \in Q$ then $a - a = 0\in Z$

$\therefore (a, a)\in R$

So, R is reflexive.

ii) $(a, b)\in R \Rightarrow (a-b)\in Z$

i.e. $(a-b)$ is an integer

$\Rightarrow -(a-b)$ is an integer

$\Rightarrow (b-a)$ is an integer

$\Rightarrow (b, a) \in R$

Thus $(a, b) \in R \Rightarrow (b, a) \in R$

$\therefore$ R is symmetric.

iii) $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a-b)$ is an integer and $(b-c)$ is an integer

$\Rightarrow \{(a-b) + (b-c)\}$ is an integer

$\Rightarrow (a-c)$ is an integer

$\Rightarrow (a, c) \in R$

Thus $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

$\therefore$ R is transitive.

Thus, R is reflexive, symmetric and transitive.

$\therefore$ R is an equivalence relation.

Ex. 2 : Show that the relation "is congruent to" on the set of all triangles in a plane is an equivalence relation.

Solution: Let S be the set of all triangles in a plane. Then, the congruence relation on S is

i) Reflexive, since $\Delta \cong \Delta$ for every $\Delta \in S$

ii) Symmetric, since $\Delta_1 \cong \Delta_2 \Rightarrow \Delta_2 \cong \Delta_1$ for all $\Delta_1, \Delta_2 \in S$.

iii) Transitive, since $\Delta_1 \cong \Delta_2$ and $\Delta_2 \cong \Delta_3$, then $\Delta_1 \cong \Delta_3$ for all $\Delta_1, \Delta_2, \Delta_3 \in S$. Hence, the given relation is an equivalence relation.

Congruence Modulo:

Let m be any positive integer. $a \equiv b (\mathrm{mod}\, m) \Leftrightarrow (a - b)$ is divisible by m. read as 'a is congruent to b modulo m' e.g: $14 \equiv 4 \pmod 5$ since $14 - 4 = 10$ is divisible by 5.

Ex. $R = \{(a,b) : a,b \in Z, a \equiv b \pmod m\}$ is an equivalence relation.

Solution: Since $a - a = 0$ is divisible by m $\therefore a \equiv a \pmod m$ $\therefore R$ is reflexive.

If $a \equiv b \pmod m$ i.e. $a - b$ is divisible by m then $b - a$ is also divisible by m

i.e. $b \equiv a \pmod m$. $\therefore$ R is symmetric.

If $a \equiv b \pmod m$ and $b \equiv c \pmod m$

i.e. $a - b$ is divisible by m and $b - c$ is divisible by m.

Then $a - c = a - b + b - c = (a - b) + (b - c)$ is also divisible by m. i.e. $a \equiv c \pmod m$.

$\therefore R$ is transitivie.

So, R is an equivalence relation.

Note:

Since each relation defines a unique subset of $A\times B$.

1) If $n(A) = m$ and $n(B) = n$ then number of relations S from A to B is 2^{mn}.

2) If R is relation on A and $n(A) = m$ then number of relations on A is 2^{m^2}

SOLVED EXAMPLES

Ex. 1: If $(x+1, y-2) = (3, 1)$ find the value of x and y.

Solution: Since the order pairs are equal, the coresponding elements are equal.

$\therefore\ x + 1 = 3$ and $y - 2 = 1$

$\therefore\ x = 2$ and $y = 3$

Ex. 2: If A = {1, 2}, find A×A

Solution: We have A = {1, 2}

$\therefore$ A×A = {(1, 1), (1, 2), (2, 1), (2, 2)}

Ex. 3: If A = {1, 3, 5} and B = {2, 3} find A×B and B×A show that A×B ≠ B×A

Solution: We have A×B = {(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)} and B×A = {(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)}

All elements in A×B, B×A except (3, 3) are different.

$\therefore$ A×B ≠ B×A

Ex. 4 : (Activity) If A×B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B

Solution: Clearly, we have

A = Set of all first components of A×B

$\therefore$ A = {$\boxed{}$, $\boxed{}$}

B = Set of all second components of A×B

$\therefore$ B = {$\boxed{}$, $\boxed{}$}

Ex. 5: A and B are two sets given in such a way that A×B contains 6 elements. If three elements of A×B are (1, 3), (2, 5) and (3, 3), find its remaining elements.

Solution: Since (1, 3), (2, 5) and (3, 3) are in A×B it follows that 1, 2, 3 are elements of A and 3, 5 are elements of B. As A×B contains 6 elements and if we let.

A = {1, 2, 3} and B = {3, 5}, then

A×B = {(1, 3), (1, 5), (2, 3), (2, 5), (3, 3) (3, 5)} giving n (A×B) = 6.

Hence, the remaining elements of A×B are (1, 5), (2, 3) and (3, 5).

Ex. 6: Express $\{(x, y)/x^2 + y^2 = 25$ where $x, y \in$ W} as a set of ordered pairs.

Solution: We have $x^2 + y^2 = 25$

$\therefore\ x = 0, y = 5 \Rightarrow x^2 + y^2 = (0)^2 + (5)^2 = 25$

$x = 4, y = 3 \Rightarrow x^2 + y^2 = (4)^2 + (3)^2 = 25$

$x = 5, y = 0 \Rightarrow x^2 + y^2 = (5)^2 + (0)^2 = 25$

$x = 3, y = 4 \Rightarrow x^2 + y^2 = (3)^2 + (4)^2 = 25$

$\therefore$ The set = {(0, 5), (3, 4), (4, 3), (5, 0)}

Ex. 7: Let A = {1, 2, 3} and B = {2, 4, 6} Show that R = {(1, 2), (1, 4), (3, 2), (3, 4)} is a relation from A to B. Find i) domain (R) ii) Co-domain (R) iii) Range (R).

Solution : Here A = {1, 2, 3} , B = {2, 4, 6}

and R = {(1, 2), (1, 4), (3, 2), (3, 4)}

Since R⊂A×B, R is a relation from A to B

i) Domain (R) = Set of first components of R = {1, 3}

ii) Co-domain (R) = B = {2, 4, 6}

iii) Range (R) = Set of second components of R = {2, 4}

Ex. 8: Let A = {1, 2, 3, 4, 5} and B = {1, 4, 5}. Let R be a relation from A to B. Such that $(x, y) \in$ R if $2x < y$.

i) List the elements of R.

ii) Find the domain, co-domain and range of R.

Solution : A = {1, 2, 3, 4, 5} and B = {1, 4, 5}

i) The elements of R are as follows:

R = {(1, 4), (1, 5), (2, 5),}

ii) Domian (R) = {1, 2}

Range (R) = {4, 5}

Co-domain (R) = {1, 4, 5} = B

Ex. 9: Let $A = \{1, 2, 3, 4, 5, 6\}$ Define a relation R from A to A by $R = \{(x, y) / y = x + 1\}$.
Write down the domain, co-domain and range of R.

Solution : A relation R from A to A is given by

$R = (x, y) / y = x + 1\}$ is given by

$R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)$

Domain (R) = $\{1, 2, 3, 4, 5\}$

Range (R) = $\{2, 3, 4, 5, 6\}$

Co-domain (R) = $\{1, 2, 3, 4, 5, 6\}$

Note:

1) R represents set of real numbers on number line.
2) $R \times R$ represents set of all ordered pairs on cartesian plane.
3) $R \times R \times R$ represents set of all ordered triplets in three dimensional space.

EXERCISE 5.2

1) If $(x-1, y+4) = (1, 2)$ find the values of x and y.

2) If $(x + \frac{1}{3}, \frac{y}{3} - 1) = (\frac{1}{2}, \frac{3}{2})$, find x and y,

3) If $A = \{a, b, c\}$, $B = (x, y\}$ find $A\times B$, $B\times A$, $A\times A$, $B\times B$.

4) If $P = \{1, 2, 3)$ and $Q = \{1,4\}$, find sets $P \times Q$ and $Q \times P$

5) Let $A = \{1, 2, 3, 4)$, $B = \{4, 5, 6\}$, $C = \{5, 6\}$.

Verify, i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

6) Express $\{(x, y) / x^2 + y^2 = 100$, where $x, y \in W\}$ as a set of ordered pairs.

7) Let $A = \{6, 8\}$ and $B = \{1, 3, 5\}$
Show that $R_1 = \{(a, b)/a\in A, b\in B, a-b$ is an even number$\}$ is a null relation. $R_2 = \{(a, b)/a\in A, b\in B, a+b$ is odd number$\}$ is an universal relation.

8) Write the relation in the Roster form. State its domain and range.

i) $R_1 = \{(a, a^2) / a$ is prime number less than $15\}$

ii) $R_2 = \{(a, \frac{1}{a}) / 0<a \leq 5, a\in N\}$

iii) $R_3 = \{(x, y) / y = 3x, y\in \{3, 6, 9, 12\}, x\in \{1, 2, 3\}$

iv) $R_4 = \{(x, y) / y > x+1, x = 1, 2$ and $y = 2, 4, 6\}$

v) $R_5 = \{(x, y) / x+y = 3, x, y\in \{0, 1, 2, 3\}$

vi) $R_6 = \{(a, b) / a \in N, a < 6$ and $b = 4\}$

vii) $R_7 = \{(a, b) / a, b \in N, a + b = 6\}$

viii) $R_8 = \{(a, b) / b = a + 2, a \in z, 0 < a < 5\}$

9) Identify which of if the following relations are reflexive, symmetric, and transitive. [Activity]

Relation	Reflexive	Symmetrice	Transitive
$R = \{(a,b) : a,b \in Z, a-b$ is an integer$\}$			
$R = \{(a,b) : a,b \in N, a+b$ is even$\}$	✓	✓	×
$R = \{(a,b) : a,b \in N, a$ divides $b\}$			
$R = \{(a,b) : a,b \in N, a^2 - 4ab + 3b^2 = 0\}$			
$R = \{(a,b) : a$ is sister of b and $a,b \in G =$ Set of girls$\}$			
$R = \{(a,b) :$ Line a is perpendicular to line b in a place$\}$			
$R = \{(a,b) : a,b \in R, a < b\}$			
$R = \{(a,b) : a,b \in R, a \leq b^3\}$			

Let's Remember

- $A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$
- $A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$
- $A' = \{x \mid x \in U, x \notin A\}$ where U is universal set.
- $A - B = \{x \mid x \in A, x \notin B\}$
- $A \Delta B = (A - B) \cup (B - A)$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $A \times B = \{(x, y) \mid x \in A, y \in B\}$
- Relation R from set A to set B is a subset of $A \times B$.
- Domain of R = $\{x \mid (x, y) \in R, x \in A, y \in B\} \subseteq A$.
- Range of R = $\{y \mid (x, y) \in R, x \in A, y \in B\} \subseteq$ B (Co-domain of R).
- R is a relation on set A and if $(x, x) \in R$ for all $x \in A$ then R is reflexive.
- R is a relation on set A and if $(x, y) \in R$ then $(y, x) \in R$ for all $x, y \in A$ then R is symmetric.
- R is a relation on set A and if $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$ for all $x, y, z \in A$ then R is transitive.
- If R is reflexive, symmetric and transitive then R is an equivalence relation.

MISCELLANEOUS EXERCISE 5

(I) Select the correct answer from given alternative (Q:1 to Q:10).

1) For the set A = {a, b, c, d, e} the correct statement is

A) $\{a, b\} \in A$ B) $\{a\} \in A$

C) $a \in A$ D) $a \notin A$

2) If $aN = \{ax : x \in N\}$, then set $6N \cap 8N$ =

A) 8N B) 48N C) 12N D) 24N

3) If set A is empty set then n[P [P [P (A)]]] is

A) 6 B) 16 C) 2 D) 4

4) In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus. Then, persons travelling by car or bus are

A) 80% B) 40% C) 60% D) 70%

5) If the two sets A and B are having 43 elements in common, then the number of elements common to each of the sets $A \times B$ and $B \times A$ is

A) 43^2 B) 2^{43} C) 43^{43} D) 2^{86}

6) Let R be a relation on the set N be defined by $\{(x, y) / x, y \in N, 2x + y = 41\}$ Then R is

A) Reflexive B) Symmetric

C) Transitive D) None of these

7) The relation ">" in the set of N (Natural number) is

A) Symmetric B) Reflexive

C) Transitive D) Equivalent relation

8) A relation between A and B is

A) only $A \times B$

B) An Universal set of $A \times B$

C) An equivalent set of $A \times B$

D) A subset of $A \times B$

9) If $(x,y) \in N \times N$, then $xy = x^2$ is a relation which is

A) Symmetric B) Reflexive

C) Transitive D) Equivalence

10) If A = {a, b, c} The total no. of distinct relations in $A \times A$ is

A) 3 B) 9 C) 8 D) 2^9

(II) Answer the following.

1) Write down the following sets in set builder form

i) {10, 20, 30, 40, 50},

ii) {a, e, i, o, u)

ii) {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

2) If U = $\{x/x \in N, 1 \leq x \leq 12\}$

A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11}

C = {3, 5, 8 9, 12}

Write down the sets

i) $A \cup B$ ii) $B \cap C$ iii) A-B

iv) $B \cap C'$ v) $A \cup B \cup C$ vi) $A \cap (B \cup C)$

3) In a survey of 425 students in a school, it was found that 115 drink apple juice, 160 drink orange juice and 80 drink both apple as well as orange juice. How many drink neither apple juice nor orange juice?

4) In a school there are 20 teachers who teach Mathematics or Physics. Of these, 12 teach Mathematics and 4 teach both Physics and Mathematics. How many teachers teach Physics?

5) i) If A = {1, 2, 3} and B = {2, 4}, state the elements of $A \times A$, $A \times B$, $B \times A$, $B \times B$, $(A \times B) \cap (B \times A)$

ii) If A = {–1, 1} , find $A \times A \times A$

6) If A = {1, 2, 3}, B = {4, 5, 6} check if the following are relations from A to B. Also write its domain and range.

i) R_1 = {(1, 4), (1, 5), (1, 6)}

ii) R_2 = {(1, 5), (2, 4), (3, 6)}

iii) R_3 = {(1, 4), (1, 5), (3, 6), (2, 6), (3, 4)}

ii) R_4 = {(4, 2), (2, 6), (5, 1), (2, 4)}

7) Determine the domain and range of the following relations.

i) $R = \{(a, b) / a \in N, a < 5, b = 4\}$

ii) $R = \{(a, b) / b = |a-1| a \in z, |a| < 3\}$

8) Find $R : A \rightarrow A$ when A = {1,2,3,4} such that

i) $R = (a, b) / a-b = 10\}$

ii) $R = \{(a, b) / |a-b|) \geq 0\}$

9) $R = \{1,2,3\} \rightarrow \{1,2,3\}$ given by R = {(1,1) (2,2), (3,3), (1,2) (2,3)} Check if R is

a) relflexive b) symmentric

c) transitive

10) Check if $R : Z \rightarrow Z$, $R = \{(a, b) | 2$ divides a-b} is equivalence relation.

11) Show that the relation R in the set A = {1, 2, 3, 4, 5} Given by $R = \{(a, b) / |a-b|$ is even} is an equivalence relation

12) Show that following are equivalence relation

a) R in A is set of all books. given by R = { (x, y) / x and y have same number of pages}

b) R in $A = \{x \in Z | 0 \leq x \leq 12\}$ given by $R = \{(a, b) / |a-b|$ is a multiple of 4}

c) R in $A = \{x \in N/x \leq 10\}$ given by $R = \{(a, b) | a=b\}$