5. LOCUS AND STRAIGHT LINE

Let's study.

- Locus
- Slope of a line
- Perpendicular and parallel lines
- Angle between interesting lines
- Equations of lines in different forms :
  - ➢ Slope point form
  - ➢ Slope intercept form
  - ➢ Two points form
  - ➢ Double intercept form
  - ➢ Normal form
  - ➢ General form
- Distance of a point from a line
- Distance between parallel lines

Let's recall.

We are familiar with the perpendicular bisector of a segment, the bisector of an angle, circle and triangle etc.

These geometrical figures are sets of points in plane which satisfy certain conditions.

- The perpendicular bisector of a segment in a plane is the set of points in the plane which are equidistant from the end points of the segment. This set is a line.
- The bisector of an angle is the set of points in the plane of the angle which are equidistant from the arms of the angle. This set is a ray.

Let's learn.

5.1 : DEFINITION : LOCUS : A set of points in a plane which satisfy certain geometrical condition (or conditions) is called a locus.

$L = \{P \mid P$ is a point in a plane and P satisfy given geometrical condition$\}$

Here P is the representative of all points in L. L is called the *locus* of point P. Locus is a set of points. Every locus has corresponding geometrical figure. We may say P is a point in the locus or a point on the locus.

Examples:

- The perpendicular bisector of segment AB is the set $M = \{ P \mid P$ is a point in a plane such that $PA = PB \}$.
- The bisector of angle AOB is the set :

  $D = \{ P \mid P$ is a point in the plane such that P is equidistant from OA and $OB \}$

  $= \{ P \mid \angle \text{POA} = \angle \text{POB} \}$
- The circle with center O and radius 4 is the set L $= \{ P \mid OP = 4$, P is a point in the plane $\}$

The plural of locus is loci.

5.2 : EQUATION OF LOCUS : Every point in XY plane has a pair of co–ordinates. If an equation is satisfied by co–ordinates of all points on the locus and if any point whose co–ordinates satisfies the equation is on the locus then that equation is called the equation of the locus.

Ex.1 We know that the y co–ordinate of every point on the X–axis is zero and this is true for points on the X–axis only. Therefore the equation of the X–axis is $y = 0$. Similarly every point on Y-axis will have x-co-ordinate zero and conversely, so its equation will be $x = 0$.

Note: The x-co-ordinate is also called as abscissa and y-co-ordinate is called as ordinate.

Ex.2 Let $L = \{P \mid OP = 4\}$. Find the equation of L.

Solution : L is the locus of points in the plane which are at 4 unit distance from the origin.

Let $P(x, y)$ be any point on the locus L.

As $OP = 4$, $OP^2 = 16$

$\therefore (x - 0)^2 + (y - 0)^2 = 16$

$\therefore x^2 + y^2 = 16$

This is the equation of the locus L.

The locus is seen to be a circle

Ex.3 Find the equation of the locus of points which are equidistant from A(–3, 0) and B(3, 0). Identify the locus.

Solution : Let $P(x, y)$ be any point on the required locus.

P is equidistant from A and B.

$\therefore$ PA = PB

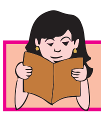
$\therefore PA^2 = PB^2$

$\therefore (x + 3)^2 + (y - 0)^2 = (x - 3)^2 + (y - 0)^2$

$\therefore x^2 + 6x + 9 + y^2 = x^2 - 6x + 9 + y^2$

$\therefore 12x = 0$

$\therefore x = 0$. The locus is the Y–axis.



Let's learn.

Shift of Origin : Let OX, OY be the co-ordinate axes. Let $O'(h, k)$ be a point in the plane. Let the origin be shifted to O′. Let O′X′, O′Y′ be the new co–ordinate axes through O′ and parallel to the axes OX and OY respectively.

Let (x, y) be the co–ordinates of P referred to the co–ordinates axes OX, OY and (X, Y) be the co–ordinates of P referred to the co–ordinate axes O′X′, O′Y′. To find relations between (x, y) and (X, Y).

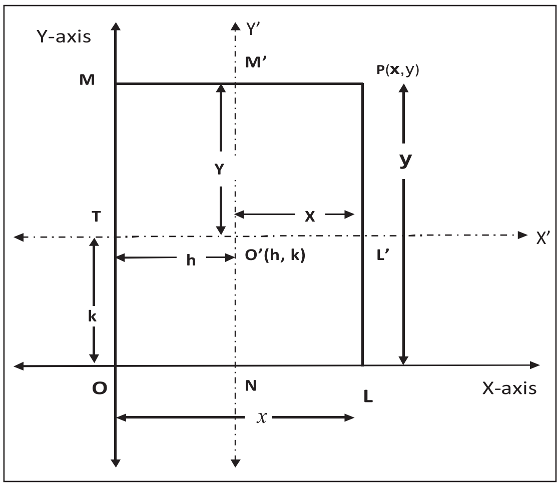


Figure 5.3

Draw $PL \perp OX$ and suppose it intersects $O'X'$ in L'.

Draw $PM \perp OY$ and suppose it intersects $O'Y'$ in M'.

Let $O'Y'$ meet line OX in N and $O'X'$ meet OY in T.

$\therefore$ ON $= h$, $OT = k$, $OL = x$, $OM = y$,

O′L′ = X, O′M′ = Y

Now $x = OL = ON + NL = ON + O'L'$

$= h + X$

and $y = OM = OT + TM = OT + O'M'$

$= k + Y$

$\therefore$ $\boxed{x = X + h,}$ $\boxed{y = Y + k}$

These equations are known as the formulae for shift of origin.

Ex.4 If the origin is shifted to the point O′(3,2) the directions of the axes remaining the same, find the new co–ordinates of the points (a)A(4, 6) (b) B(2,–5).

Solution : We have $(h, k) = (3,2)$

$x = X + h, y = Y + k$

$\therefore$ $x = X + 3.$ and $y = Y + 2$ .... (1)

(a) $(x, y) = (4, 6)$

$\therefore$ From (1), we get $4 = X + 3$, $6 = Y + 2$

$\therefore$ $X = 1$ and $Y = 4$.

New co–ordinates of A are (1, 4)

(ii) $(x, y) = (2,–5)$

from (1), we get $2 = X + 3$, $–5= Y + 2$

$\therefore$ $X = –1$ and $Y = –7$.

New co–ordinates of B are (–1,–7)

Ex.5 The origin is shifted to the point (-2, 1), the axes being parallel to the original axes. If the new co–ordinates of point A are (7, –4), find the old co–ordinates of point A.

Solution : We have $(h, k) = (–2, 1)$

$x = X + h, y = Y + k$

$\therefore$ $x = X – 2, y = Y + 1$

$(X, Y) = (7, -4)$

we get $x = 7 – 2 = 5$,

$y = – 4 + 1 = –3$.

$\therefore$ Old co-ordinates A are (5, -3)

Ex.6 Obtain the new equation of the locus $x^2 – xy – 2y^2 – x + 4y + 2 = 0$ when the origin is shified to (2, 3), the directions of the axes remaining the same.

Solution : Here $(h, k) = (2, 3)$

$\therefore$ $x = X + h$, $y = Y + k$ gives

$\therefore$ $x = X + 2$, $y = Y + 3$

The given equation

$x^2 – xy – 2y^2 – x + 4y + 2 = 0$ becomes

$(X+2)^2 – (X + 2)(Y + 3) – 2(Y + 3)^2$

$– (X + 2) + 4(Y + 3) + 2 = 0$

$\therefore$ $X^2 – XY – 2Y^2 – 10Y – 8 = 0$

This is the new equation of the given locus.

EXERCISE 5.1

1. If A(1,3) and B(2,1) are points, find the equation of the locus of point P such that PA = PB.
2. A(–5, 2) and B(4, 1). Find the equation of the locus of point P, which is equidistant from A and B.
3. If A(2, 0) and B(0, 3) are two points, find the equation of the locus of point P such that AP = 2BP.
4. If A(4, 1) and B(5, 4), find the equation of the locus of point P if $PA^2 = 3PB^2$.
5. A(2, 4) and B(5, 8), find the equation of the locus of point P such that

   $PA^2 - PB^2 = 13$.
6. A(1, 6) and B(3, 5), find the equation of the locus of point P such that segment AB subtends right angle at P. ($\angle$ APB $= 90°$)
7. If the origin is shifted to the point O′(2, 3), the axes remaining parallel to the original axes, find the new co-ordinates of the points

   (a) A(1, 3) (b) B(2, 5)
8. If the origin is shifted to the point O′(1,3) the axes remaining parallel to the original axes, find the old co-ordinates of the points

   (a) C(5, 4) (b) D(3, 3)
9. If the co-ordinates (5,14) change to (8,3) by shift of origin, find the co-ordinates of the point where the origin is shifted.
10. Obtain the new equations of the following loci if the origin is shifted to the point O'(2, 2), the direction of axes remaining the same :

    (a) $3x – y + 2 = 0$

    (b) $x^2 + y^2 – 3x = 7$

    (c) $xy – 2x – 2y + 4 = 0$

5.3 LINE :

The aim of this chapter is to study a line and its equation. The locus of a point in a plane such that the segment joining any two points on the locus lies completely on the locus is called a line.

The simplest locus in a plane is a line. The characteristic property of this locus is that if we find the slope of a segment joining any two points on this locus, then the slope is constant.

Further we know that if a line meets the X-axis in the point A $(a, 0)$, then a is called the x-intercept of the line. If it meets the Y-axis in the point B $(0, b)$ then b is called the y-intercept of the line.

Let's learn.

5.3.1 : Inclination of a line : The smallest angle made by a line with the positive direction of the X–axis measured in anticlockwise sense is called the inclination of the line. We denote inclination by θ. Clearly $0° < \theta < 180°$.

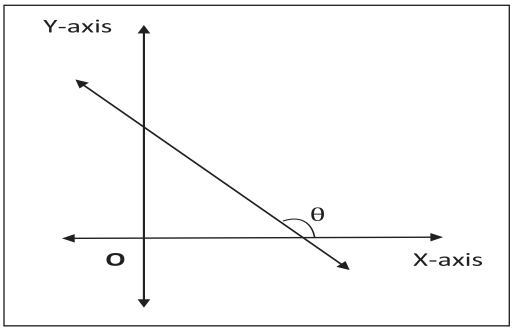


Figure 5.1

Remark : Two lines are parallel if and only if they have the same inclination.

The inclination of the X-axis and a line parallel to the X-axis is Zero. The inclination of the Y-axis and a line parallel to the Y-axis is 90^0.

5.3.2 : Slope of a line : If θ is the inclination of a line then $\tan\theta$ (if it exists) is called the slope of the line.

Let $A(x_1, y_1)$, $B(x_2, y_2)$ be any two points on a non–vertical line whose inclination is θ then verify that

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

The slope of the Y-axis is not defined. Similarly the slope of a line parallel to the Y–axis is not defined. The slope of the X–axis is 0. The slope of a line parallel to the X–axis is also 0.

Remark : Two lines are parallel if and only if they have the same slope.

Ex.1 Find the slope of the line whose inclination is 60^0.

Solution : The tangent ratio of the inclination of a line is called the slope of the line.

Inclination $\theta = 60°$.

$\therefore$ slope $= \tan\theta = \tan 60° = \sqrt{3}$.

Ex.2 Find the slope of the line which passes through the points A(2, 4) and B(5, 7).

Solution : The slope of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Slope of the line AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-4}{5-2} = 1$

Note that $x_1 \neq x_2$.

Ex.3 Find the slope of the line which passes through the origin and the point A(-4, 4).

Solution : The slope of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Here A(-4, 4) and O(0, 0).

Slope of the line OA $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-4}{0+4} = -1$.

Note that $x_1 \neq x_2$.

Let's study.

5.3.3 : Perpendicular Lines : Lines having slopes m_1 and m_2 are perpendicular to each other if and only if $m_1 \times m_2 = -1$.

Ex.1 Show that line AB is perpendicular to line BC, where A(1, 2), B(2, 4) and C(0, 5).

Solution : Let slopes of lines AB and BC be m_1 and m_2 respectively.

$\therefore m_1 = \dfrac{4-2}{2-1} = 2$ and

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-4}{0-2} = -\frac{1}{2}$$

Now $m_1 \times m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$

$\therefore$ Line AB is perpendicular to line BC.

Ex.2 A(1,2), B(2,3) and C(–2,5) are the vertices of a Δ ABC. Find the slope of the altitude drawn from A.

Solution : The slope of line BC is

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{-2-2} = -\frac{2}{4} = -\frac{1}{2}$$

Altitude drawn from A is perpendicular to BC.

If m_2 is the slope of the altitude from A then $m_1 \times m_2 = -1$.

$$\therefore m_2 = \frac{-1}{m_1} = 2.$$

The slope of the altitude drawn from A is 2.

5.3.4 : Angle between intersecting lines :

If θ is the acute angle between non–vertical lines having slopes m_1 and m_2 then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|, \text{ where } 1 + m_1 m_2 \neq 0$$

Ex.1 Find the acute angle between lines having slopes 3 and –2.

Solution : Let $m_1 = 3$ and $m_2 = -2$.

Let θ be the acute angle between them.

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{3-(-2)}{1+(3)(-2)}\right| = \left|\frac{5}{-5}\right| = 1$$

$\therefore \theta = 45°$

The acute angle between lines having slopes 3 and –2 is 45°.

Ex.2 If the angle between two lines is 45° and the slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

Solution : If θ is the acute angle between lines having slopes m_1 and m_2 then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Given θ = 45°.

Let $m_1 = \frac{1}{2}$. Let m_2 be slope of the other line.

$$\tan 45° = \left|\frac{\frac{1}{2} - m_2}{1 + \left(\frac{1}{2}\right)m_2}\right| \qquad \therefore 1 = \left|\frac{1 - 2m_2}{2 + m_2}\right|$$

$$\therefore \frac{1-2m_2}{2+m_2} = 1 \text{ or } \frac{1-2m_2}{2+m_2} = -1$$

$\therefore$ $1-2m_2 = 2+m_2$ or $1-2m_2 = -2-m_2$

$\therefore$ $3m_2 = -1$ or $m_2 = 3$

$\therefore$ $m_2 = 3$ or $-\frac{1}{3}$

EXERCISE 5.2

1. Find the slope of each of the following lines which pass through the points :

   (a) (2, –1), (4, 3) (b) (–2, 3), (5, 7)

   (c) (2, 3), (2, –1) (d) (7, 1), (–3, 1)

2. If the X and Y–intercepts of line L are 2 and 3 respectively then find the slope of line L.

3. Find the slope of the line whose inclination is 30^0.

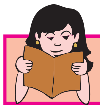
4. Find the slope of the line whose inclination is 45^0.

5. A line makes intercepts 3 and 3 on co–ordiante axes. Find the inclination of the line.

6. Without using Pythagoras theorem show that points A(4,4), B(3, 5) and C(–1, –1) are the vertices of a right angled triangle.

7. Find the slope of the line which makes angle of 45° with the positive direction of the Y–axis measured clockwise.

8. Find the value of k for which points P(k,–1),Q(2,1) and R(4,5) are collinear.



Let's learn.

5.4: EQUATIONS OF LINES IN DIFFERENT FORMS :

We know that line is a locus. Every locus has an equation. An equation in x and y which is satisfied by the co–ordinates of all points on a line and which is not satisfied by the co–ordinates of any point which does not lie on the line is called the equation of the line.

The equation of any line parallel to the Y-axis is of the type $x = k$ (where k is a constant) and the equation of any line parallel to the X-axis is of the type $y = k$. This is all about vertical and horizontal lines.

Let us obtain equations of non–vertical and non –horizontal lines in different forms:

5.4.1 : Slope-Point Form : To find the equation of a line having slope m and which passes through the point A(x_1, y_1).

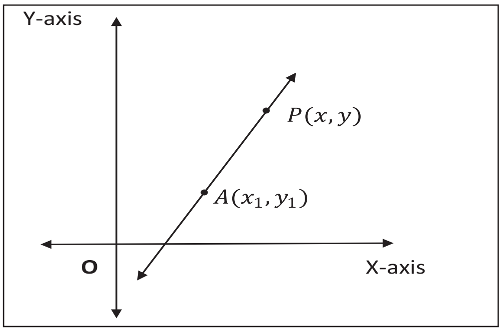


Figure 5.9

The equation of the line having slope m and passing through A(x_1, y_1) is $(y-y_1) = m\ (x-x_1)$.

Remark : In particular if the line passes through the origin O(0,0) and has slope m, then its equation is $y - 0 = m\ (x - 0)$

i.e. $y = mx$

Ex.1) Find the equation of the line passing through the point A(2, 1) and having slope –3.

Solution : Given line passes through the point A(2, 1) and slope of the line is –3.

The equation of the line having slope m and passing through A(x_1, y_1) is $(y-y_1) = m(x-x_1)$.

$\therefore$ the equation of the required line is

$y - 1 = -3(x - 2)$

$\therefore$ $y - 1 = -3x + 6$

$\therefore$ $3x + y - 7 = 0$

5.4.2 : Slope-Intercept form : The equation of a line having slope m and which makes intercept c on the Y–axis is y = mx + c.

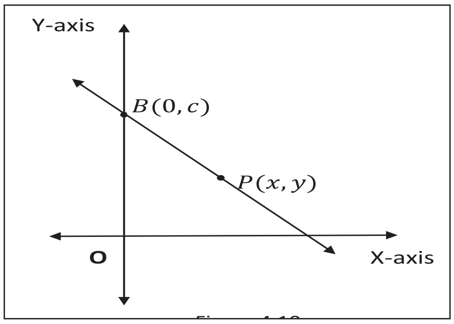


Figure 5.10

Ex.2 Obtain the equation of the line having slope 3 and which makes intercept 4 on the Y–axis.

Solution: The equation of line having slope m and which makes intercept c on the Y–axis is

$y = mx + c.$

$\therefore$ the equation of the line giving slope 3 and making y–intercept 4 is $y = 3x + 4$.

5.4.3 : Two-points Form : The equation of a line which passes through points A(x_1, y_1) and B(x_2, y_2) is $\dfrac{y-y_1}{x-x_1} = \dfrac{y_1-y_2}{x_1-x_2}$

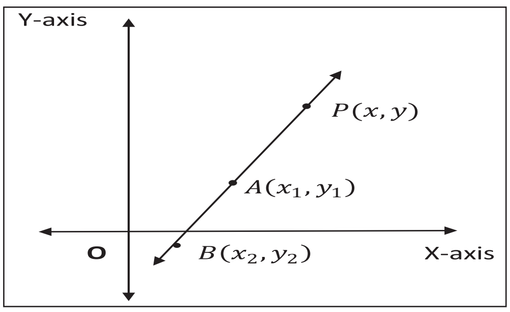


Figure 5.11

Ex.3 Obtain the equation of the line passing through points A(2, 1) and B(1, 2).

Solution : The equation of the line which passes through points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}.$$

$\therefore$ The equation of the line passing through points A(2, 1) and B(1, 2) is $\frac{x-2}{1-2} = \frac{y-1}{2-1}$

$$\therefore \frac{x-2}{-1} = \frac{y-1}{1}$$

$$\therefore x - 2 = -y + 1$$

$$\therefore x + y - 3 = 0$$

5.4.4 : Double-Intercept form : The equation of the line which makes non–zero intercepts a and b on the X and Y axes respectively is

$$\frac{x}{a} + \frac{y}{b} = 1$$

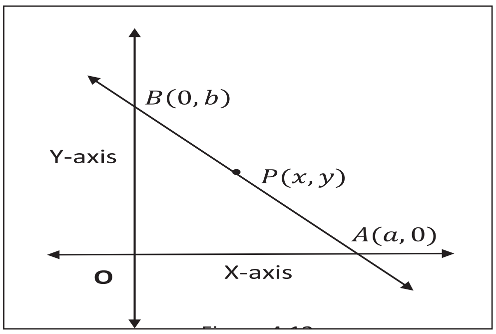


Figure 5.12

Ex.1 Obtain the equation of the line which makes intercepts 3 and 4 on the X and Y axes respectively.

Solution : The equation of the line which makes intercepts a and b on the X and Y co–ordiante axes $\frac{x}{a} + \frac{y}{b} = 1$

The equation of the line which makes intercepts 3 and 4 on the co–ordiante axes is $\frac{x}{3} + \frac{y}{4} = 1$

$$\therefore 4x + 3y - 12 = 0.$$

5.4.5 : Normal Form : Let L be a line and segment ON be the perpendicular (normal) drawn from the origin to line L.

If ON = p. Let the ray ON make angle α with the positive X–axis.

$\therefore$ N = (pcosα, psinα)

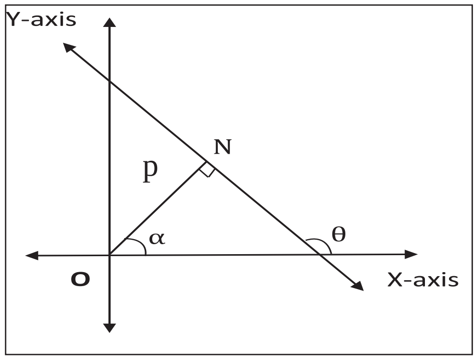


Figure 5.13

Let θ be the inclination of the line L. The equation of the line, the normal to which from the origin has length p and the normal makes angle α with the positive directions of the X–axis, is $x\cos\alpha + y\sin\alpha = p$.

Ex.1 The perpendicular drawn from the origin to a line has length 5 and the perpendicular makes angle 30^0 with the positive direction of the X–axis. Find the equation of the line.

Solution : The perpendicular (normal) drawn from the origin to the line has length 5.

$$\therefore p = 5$$

The perpendicular (normal) makes angle 30^0 with the positive direction of the X–axis.

$$\therefore \theta = 30^0$$

The equation of the required line is

$$x \cos\alpha + y \sin\alpha = p$$

$$\therefore x \cos 30^0 + y \sin 30^0 = p$$

$$\therefore \quad \frac{\sqrt{3}}{2}x+\frac{1}{2}y=5$$

$$\therefore \sqrt{3}x+y-10=0$$

Ex.2 Reduce the equation $\sqrt{3}x-y-2=0$ into normal form. Find the values of p and α.

Solution : Comparing $\sqrt{3}x-y-2=0$ with $ax+by+c=0$ we get $a=\sqrt{3}$, $b=-1$ and $c=-2$.

$$\sqrt{a^2+b^2}=\sqrt{3+1}=2$$

Divide the given equation by 2.

$$\frac{\sqrt{3}}{2}x-\frac{1}{2}y=1$$

$\therefore$ cos 30^0 x – sin 30^0 y = 1 is the required normal form of the given equation.

p = 1 and θ = 30^0

SOLVED EXAMPLES

Find the equation of the line :

(i) parallel to the X–axis and 3 units below it

(ii) passing through the origin and having inclination 30^0

(iii) passing through the point $A(5,2)$ and having slope 6.

(iv) passing through the points $A(2–1)$ and $B(5,1)$

(v) having slope $-\frac{3}{4}$ and y–intercept 5,

(vi) making intercepts 3 and 6 on the X and Y axes respectively.

Solution :

(i) Equation of a line parallel to the X–axis is of the form $y=k$,

$\therefore$ the equation of the required line is $y = -3$

(ii) Equation of a line through the origin and having slope m is of the form $y = mx$.

Here, m = tan 30^0 = $\frac{1}{\sqrt{3}}$.

$\therefore$ the equation of the required line is

$$y = \frac{1}{\sqrt{3}}x$$

(iii) By using the point–slope form is

$y-y_1$ = m $(x-x_1)$

equation of the required line is

$(y-2)$ = 6 $(x-5)$

i.e. $6x - y - 28 = 0$

(iv) Here (x_1, y_1) = (2–1); (x_2, y_2) = (5,1)

By using the two points form

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$$

the equation of the required line is

$$\frac{x-2}{5-2}=\frac{y+1}{1+1}$$

$$\therefore \ 2(x-2)=3(y+1)$$

$\therefore$ 2x – 3y – 7 = 0

(v) Given $m=-\frac{3}{4}$, $c=5$

By using the slope intercept form $y=mx+c$

the equation of the required line is

$$y=-\frac{3}{4}x+5 \quad \therefore \ 3x+4y-20=0$$

(vi) x –intercept = a = 3;

y–intercept = b = 6.

By using the double intercept form

$$\frac{x}{a}+\frac{y}{b}=1$$

the equation of the required line is

$$\frac{x}{3}+\frac{y}{6}=1$$

$$2x+y-6=0$$

EXERCISE 5.3

1. Write the equation of the line :
   a) parallel to the X–axis and at a distance of 5 unit form it and above it.
   b) parallel to the Y– axis and at a distance of 5 unit form it and to the left of it.
   c) parallel to the X– axis and at a distance of 4 unit form the point (–2, 3).
2. Obtain the equation of the line :
   a) parallel to the X–axis and making an intercept of 3 unit on the Y–axis.
   b) parallel to the Y–axis and making an intercept of 4 unit on the X–axis.
3. Obtain the equation of the line containing the point :
   a) $A(2,-3)$ and parallel to the Y–axis.
   b) $B(4,-3)$ and parallel to the X–axis.
4. Find the equation of the line passing through the points $A(2,0)$ and $B(3,4)$.
5. Line $y=mx+c$ passes through points A(2,1) and B(3, 2). Determine m and C.
6. The vertices of a triangle are $A(3,4)$, $B(2,0)$ and $C(1,6)$ Find the equations of
   (a) side BC (b) the median AD
   (c) the line passing through the mid points of sides AB and BC.
7. Find the X and Y intercepts of the following lines :
   (a) $\frac{x}{3}+\frac{y}{2}=1$ (b) $\frac{3x}{2}+\frac{2y}{3}=1$
   (c) $2x-3y+12=0$
8. Find the equations of the lines containing the point $A(3,4)$ and making equal intercepts on the co–ordinates axes.
9. Find the equations of the altitudes of the triangle whose vertices are $A(2, 5)$, $B(6,-1)$ and $C(-4, -3)$.

5.5 : General form of equation of line: We can write equation of every line in the form $ax+by+c=0$. where a, b, c$\in$R and all are not simultaneously zero.

This form of equation of a line is called the general form.

The general form of $y=3x+2$ is $3x-y+2=0$

The general form of $\frac{x}{2}+\frac{y}{3}=1$ is $3x+2y-6=0$

The slope of the line $ax+by+c=0$ is $-\frac{a}{b}$.

the x–intercept is $-\frac{c}{a}$ and

the y–intercept is $-\frac{c}{b}$.

Ex.1 Find the slope and intercepts made by the following lines :

(a) $x+y+10=0$ (b) $2x+y+30=0$

(c) $x + 3y - 15 = 0$

Solution:

(a) Comparing equation $x+y+10=0$

with $ax+by+c=0$,

we get $a=1$, $b=1$, $c=10$

$\therefore$ Slope of this line $= -\frac{a}{b} = -1$

The X–intercept is $-\frac{c}{a} = -\frac{10}{1} = -10$

The Y–intercept is $= -\frac{c}{b} = -\frac{10}{1} = -10$

(b) Comparing the equation $2x+y+30=0$

with $ax+by+c=0$.

we get $a=2$, $b=1$, $c=30$

$\therefore$ Slope of this line $= -\frac{a}{b} = -2$

The X–intercept is $-\frac{c}{a} = -\frac{30}{2} = -15$

The Y–intercept is $-\frac{c}{b} = -\frac{30}{1} = -30$

(c) Comparing equation $x+3y-15=0$ with

$ax+by+c=0$.

we get $a=1$, $b=3$, $c=-15$

$\therefore$ Slope of this line $= -\frac{a}{b} = -\frac{1}{3}$

The x– intercept is $-\frac{c}{a} = -\frac{-15}{1} = 15$

The y– intercept is $-\frac{c}{b} = -\frac{-15}{3} = 5$

Ex.2 Find the acute angle between the following pairs of lines :

a) $12x-4y=5$ and $4x+2y=7$

b) $y=2x+3$ and $y=3x+7$

Solution :

(a) Slopes of lines $12x-4y=5$ and $4x+2y=7$ are $m_1=3$ and $m_2=-2$.

If θ is the acute angle between lines having slope m_1 and m_2 then

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\therefore \tan\theta = \left|\frac{3-(-2)}{1+(3)(-2)}\right| = \left|\frac{5}{-5}\right| = 1$$

$\therefore \tan\theta = 1 \qquad \therefore \theta = 45^0$

(b) Slopes of lines $y=2x+3$ and $y=3x+7$ are $m_1=2$ and $m_2=3$

The acute angle θ between lines having slopes m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\therefore \tan\theta = \left|\frac{2-3}{1+(2)(3)}\right| = \left|\frac{-1}{7}\right| = \frac{1}{7}$$

$$\therefore \tan\theta = \frac{1}{7}.$$

Ex.3 Find the acute angle between the lines $y-\sqrt{3}x+1=0$ and $\sqrt{3}y-x+7=0$.

Solution : Slopes of the given lines are $m_1=\sqrt{3}$ and $m_2=\frac{1}{\sqrt{3}}$.

The acute angle θ between lines having slopes m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\therefore \tan\theta = \left|\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\times\frac{1}{\sqrt{3}}}\right| = \left|\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+1}\right| = \left|\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{2}\right|$$

$$=\left|\frac{3-1}{2\sqrt{3}}\right|=\frac{1}{\sqrt{3}}$$

$$=\left|\frac{2}{2\sqrt{3}}\right|=\frac{1}{\sqrt{3}}$$

$\therefore\ \theta = 30^0$

Ex.4 Show that following pairs of lines are perpendicular to each other.

a) $2x-4y=5$ and $2x+y=17$.

b) $y=2x+23$ and $2x+4y=27$

Solution :

(i) Slopes of lines $2x-4y=5$ and

$2x+y=17$ are $m_1=\frac{1}{2}$ and $m_2=-2$

Since $m_1.m_2 = \frac{1}{2}\times(-2) = -1$,

given lines are perpendicular to each other.

(ii) Slopes of lines $y = 2x + 23$ and

$2x+4y = 27$ are $m_1=-\frac{1}{2}$ and $m_2 = 2$.

Since $m_1.m_2 = -\frac{1}{2}\times(2) = -1$, given lines are perpendicular to each other.

Ex.5 Find equations of lines which pass through the origin and make an angle of 45^0 with the line $3x - y = 6$.

Solution : Slope of the line $3x - y = 6$. is 3. Let m be the slope of one of the required line. The angle between these lines is 45^0.

$$\therefore\ \tan 45^0 = \left|\frac{m-3}{1+(m)(3)}\right|$$

$$\therefore\ 1 = \left|\frac{m-3}{1+3m}\right| \qquad \therefore\ |1+3m| = |m-3|$$

$$\therefore\ 1+3m=m-3 \quad \text{or} \quad 1+3m=-(m-3)$$

$$\therefore\ m=-2 \ \text{ or } \ \frac{1}{2}$$

Slopes of required lines are $m_1 = -2$ and $m_2 = \frac{1}{2}$

Required lines pass through the origin.

$\therefore$ Their equations are $y=-2x$ and $y=\frac{1}{2}x$

$\therefore\ 2x+y=0$ and $x-2y=0$

Ex.6 A line is parallel to the line $2x+y=7$ and passes through the origin. Find its equation.

Solution : Slope of the line $2x+y=7$ is -2.

$\therefore$ slope of the required line is also -2

Required line passes through the origin.

$\therefore$ It's equation is $y=-2x$

$\therefore\ 2x+y=0$.

Ex.7 A line is parallel to the line $x+3y=9$ and passes through the point $A(2,7)$. Find its equation.

Solution : Slope of the line $x+3y=9$ is $-\frac{1}{3}$

$\therefore$ slope of the required line is $-\frac{1}{3}$

Required line passes through the point A(2,7).

$\therefore$ It's equation is given by the formula

$(y - y_1) = m\ (x - x_1)$

$\therefore\ (y-7)=-\frac{1}{3}(x-2)$

$\therefore\ 3y - 21 = - x + 2$

$\therefore\ x+3y=23$.

Ex.8 A line is perpendicular to the line $3x+2y-1=0$ and passes through the point $A(1,1)$. Find its equation.

Solution : Slope of the line $3x+2y-1=0$ is $-\frac{3}{2}$

Required line is perpendicular it.

The slope of the required line is $\frac{2}{3}$.

Required line passes through the point $A(1,1)$.

$\therefore$ It's equation is given by the formula

$(y-y_1)=m(x-x_1)$

$\therefore \quad (y-1)=\frac{2}{3}(x-1)$

$\therefore \quad 3y-3=2x-2$

$\therefore \quad 2x-3y+1=0$.

5.5.1 : Point of intersection of lines : The co–ordinates of the point of intersection of two intersecting lines can be obtained by solving their equations simultaneously.

SOLVED EXAMPLES

Ex.1 Find the co–ordinates of the point of intersection of lines $x+2y=3$ and $2x-y=1$.

Solution : Solving equations $x+2y=3$ and $2x - y = 1$simultaneously, we get $x=1$ and $y=1$.

$\therefore$ Given lines intersect in point (1, 1)

Ex.2 Find the equation of line which is parallel to the X–axis and which passes through the point of intersection of lines $x+2y=6$ and $2x-y=2$

Solution : Solving equations $x+2y=6$ and $2x-y=2$ simultaneously, we get $x = 2$ and $y=2$.

$\therefore$ The required line passes through the point (2, 2) and has slope 0.

(As it is parallel to the X–axis)

$\therefore$ Its equation is given by $(y-y_1) = m(x - x_1)$

$(y-2) = 0(x-2)$

$\therefore \quad y = 2$.

Let's learn.

5.5.2 : The distance of the Origin from a Line :

The distance of the origin from the line $ax+by+c=0$ is given by $p=\left|\frac{c}{\sqrt{a^2+b^2}}\right|$

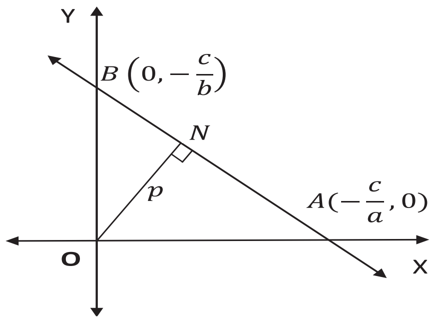


Figure 5.14

5.5.3 : The distance of the point (x_1,y_1) from a line: The distance of the point P (x_1,y_1) from line $ax+by+c=0$is given by

$$p=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

5.5.4 : The distance between two parallel lines :

The distance between two parallel lines $ax+by+c_1=0$ and $ax+by+c_2=0$ is given

by $p=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

SOLVED EXAMPLES

Ex.1 Find the distance of the origin from the line $3x+4y+15=0$

Solution : The distance of the origin from the line $ax + by + c = 0$ is given by

$$p=\left|\frac{c}{\sqrt{a^2+b^2}}\right|$$

$\therefore$ The distance of the origin from the line $3x+4y+15=0$ is given by

$$p=\left|\frac{15}{\sqrt{3^2+4^2}}\right| = \frac{15}{5} = 3$$

Ex.2 Find the distance of the point $P(2,5)$ from the line $3x+4y+14=0$

Solution : The distance of the point $P(x_1, y_1)$ from the line $ax+by+c=0$ is given by

$$p=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$\therefore$ The distance of the point $P(2,5)$ from the line $3x+4y+14=0$ is given by

$$p=\left|\frac{3(2)+4(5)+14}{\sqrt{3^2+4^2}}\right|=\frac{40}{5}=8$$

Ex.3 Find the distance between the parallel lines $6x+8y+21=0$ and $3x+4y+7=0$.

Solution : We write equation $3x+4y+7=0$ as $6x+8y+14=0$ in order to make the coefficients of x and coefficients of y in both equations to be same.

Now by using formula $p=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

We get the distance between the given parallel lines as

$$p=\left|\frac{21-14}{\sqrt{6^2+8^2}}\right|=\frac{7}{10}$$

Let's remember!

- **Locus :** A set of points in a plane which satisfy certain geometrical condition (or conditions) is called a locus.
- **Equation of Locus :** Every point in XY plane has Cartesian co–ordinates. An equation which is satisfied by co–ordinates of all points on the locus and which is not satisfied by the co–ordinates of any point which does not lie on the locus is called the equation of the locus.
- **Inclination of a line :** The smallest angle θ made by a line with the positive direction of the X–axis, measured in anticlockwise sense, is called the inclination of the line. Clearly $0^0 < \theta < 180^0$.
- **Slope of a line :** If θ is the inclination of a line then $\tan\theta$ (if it exists) is called the slope of the line.

  If $A(x_1, y_1)$, $B(x_2, y_2)$ are any two points on the line whose inclination is θ then

  $$\tan\theta = \frac{y_2-y_1}{x_2-x_1} \quad (\text{if } x_1 \neq x_2)$$

- **Perpendicular and parallel lines :** Non-vertical lines having slopes m_1 and m_2 are **perpendicular** to each other if and only if $m_1m_2=-1$.

Two lines are **parallel** if and only if they have the same slope, that is $m_1 = m_2$.

- **Angle between intersecting lines :** If θ is the acute angle between lines having slopes m_1 and m_2 then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ provided $m_1 m_2 + 1 \neq 0$.
- **Equations of line in different forms :**
- Slope point form : $(y - y_1) = m(x - x_1)$
- Slope intercept form : $y = mx + c$
- Two points form : $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$
- Double intercept form : $\frac{x}{a} + \frac{y}{b} = 1$
- Normal form : $x\cos\alpha + y\sin\alpha = p$
- General form : $ax + by + c = 0$
- **Distance of a point from a line :**
- The distance of the origin from the line $ax + by + c = 0$ is given by $p = \left|\frac{c}{\sqrt{a^2 + b^2}}\right|$
- The distance of the point $P(x_1, y_1)$ from line $ax + by + c = 0$ is given by

$$p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

- **The distance between the Parallel lines:** The distance between the parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is give by $p = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

EXERCISE 5.4

1) Find the slope, *x*–intercept, *y*–intercept of each of the following lines.

   a) $2x + 3y - 6 = 0$

   b) $x + 2y = 0$

2) Write each of the following equations in $ax + by + c = 0$ form.

   a) $y = 2x - 4$ b) $y = 4$

   c) $\frac{x}{2} + \frac{y}{4} = 1$ d) $\frac{x}{3} = \frac{y}{2}$

3) Show that lines $x - 2y - 7 = 0$ and $2x - 4y + 5 = 0$ are parallel to each other.

4) If the line $3x + 4y = p$ makes a triangle of area 24 square unit with the co–ordinate axes then find the value of p.

5) Find the co–ordinates of the circumcenter of the triangle whose vertices are $A(-2,3), B(6,-1), C(4,3)$.

6) Find the equation of the line whose X–intercept is 3 and which is perpendicular to the line $3x - y + 23 = 0$.

7) Find the distance of the point $A(-2,3)$ from the line $12x - 5y - 13 = 0$.

8) Find the distance between parallel lines $9x + 6y - 7 = 0$. and $9x + 6y - 32 = 0$.

9) Find the equation of the line passing through the point of intersection of lines $x + y - 2 = 0$ and $2x - 3y + 4 = 0$ and making intercept 3 on the X–axis.

10) D(–1,8), E(4,–2), F(–5,–3) are midpoints of sides BC, CA and AB of ΔABC. Find

    (i) equations of sides of ΔABC.

    (ii) co–ordinates of the circumcenter of ΔABC.

MISCELLANEOUS EXERCISE - 5

1. Find the slopes of the lines passing through the following of points :

   (a) (1, 2), (3, –5) (b) (1, 3), (5, 2)

   (c) (–1, 3), (3, –1) (d) (2, –5), (3, –1)

2. Find the slope of the line which

   a) makes an angle of 30° with the positive X–axis.

   b) makes intercepts 3 and –4 on the axes.

   c) passes through the points $A(-2,1)$ and the origin .

3. Find the value of k

   a) if the slope of the line passing through the points (3, 4), (5, k) is 9.

   b) the points (1, 3), (4, 1), (3, k) are collinear

   c) the point P(1,k) lies on the line passing through the points A(2, 2) and B(3, 3).

4. Reduce the equation $6x+3y+8=0$ into slope–intercept form. Hence find its slope.

5. Verify that A(2, 7) is not a point on the line $x+2y+2=0$.

6. Find the X–intercept of the line $x+2y-1=0$.

7. Find the slope of the line $y-x+3=0$.

8. Does point $A(2,3)$ lie on the line $3x+2y-6=0$? Give reason.

9. Which of the following lines pass through the origin ?

   (a) $x=2$ (b) $y=3$

   (c) $y=x+2$ (d) $2x-y=0$

10. Obtain the equation of the line which is :

    a) parallel to the X–axis and 3 unit below it.

    b) parallel to the Y–axis and 2 unit to the left of it.

    c) parallel to the X–axis and making an intercept of 5 on the Y–axis.

    d) parallel to the Y–axis and making an intercept of 3 on the X–axis.

11. Obtain the equation of the line containing the point

    a) (2,3) and parallel to the X–axis.

    b) (2,4) and perpendicular to the Y–axis.

    c) (2,5) and perpendicular to the X–axis.

12. Find the equation of the line :

    a) having slope 5 and containing point $A(-1,2)$.

    b) containing the point (2, 1) and having slope 13.

    c) containing the point T(7,3) and having inclination 90^0.

    d) containing the origin and having inclination 90^0.

    e) through the origin which bisects the portion of the line $3x+2y=2$ intercepted between the co–ordinate axes.

13. Find the equation of the line passing through the points A(–3,0) and B(0,4).

14. Find the equation of the line :

a) having slope 5 and making intercept 5 on the X–axis.

b) having an inclination 60^0 and making intercept 4 on the Y–axis.

15. The vertices of a triangle are A(1,4), B(2,3) and C(1,6). Find equations of

(a) the sides (b) the medians

(c) Perpendicular bisectors of sides

(d) altitudes of ΔABC.

ACTIVITIES

Activity 5.1 :

Complete the activity and decide whether the line $2x - 4y = 5$ and $2x + y = 17$ are perpendicular to each other.

Slope of line $2x - 4y = 5$ is $\boxed{}$.

Slope of line $2x + y = 17$ is $\boxed{}$.

Product of slopes of these line = $\boxed{}$.

$\therefore$ The given lines are $\boxed{}$.

Activity 5.2 :

Complete the activity and obtain the equation of the line having slope 2 and which makes intercept 3 on the Y-axis.

Here c = $\boxed{}$, m = $\boxed{}$

The equation of line y = $\boxed{}$ + c

$\therefore$ The equation of line is $\boxed{}$

Activity 5.3 :

Complete the activity and find the equation of the line which is parallel to the line $3x + y = 5$ and passes through the origin.

Slope of line $3x + y = 5$ is $\boxed{}$

The required line passes through the origin

It's equation is y = $\boxed{}$

The equation of required line is $\boxed{}$.

Activity 5.4 :

Complete the following activity and find the value of p, if the area of the triangle formed by the axes and the line $3x + 4y = P$, is 24.

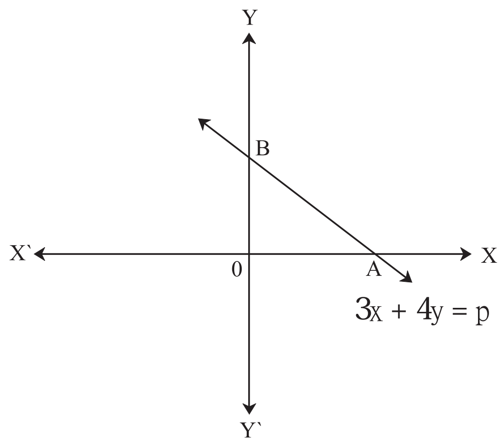


Fig. 5.15

Here, A = ($\boxed{}$, 0) B = (0, $\boxed{}$)

$$A(\Delta AOB) = \frac{1}{2} (OA)(OB)$$

$$\therefore \quad 24 = \frac{1}{2} \boxed{}\,\boxed{}$$

$$\therefore \quad P^2 = \boxed{}$$

$$\therefore \quad P = \boxed{}$$