7. LIMITS

Let's study.

- Definition of Limit
- Algebra of Limits
- Evaluation of Limits
  - ➢ Direct Method
  - ➢ Factorization Method
  - ➢ Rationalization Method
- Limits of Exponential and Logarithmic Functions

Introduction:

Calculus is one of the important branches of Mathematics. The concept of limit of a function is a fundamental concept in calculus.

Let's understand

Meaning of $x \to a$:

When x takes the values gradually nearer to a, we say that 'x tends to a'. This is symbolically written as '$x \to a$'.

'$x \to a$' implies that $x \neq a$ and hence $(x-a) \neq 0$

Limit of a function :

Let us understand the concept by an example.

Consider the function $f(x) = x + 3$

Take the value of x very close to 3, but not equal to 3; and observe the values of $f(x)$.

	x approaches 3 from left				
x	2.5	2.6	…	2.9	2.99
$f(x)$	5.5	5.6	…	5.9	5.99

	x approaches 3 from right				
x	3.6	3.5	…	3.1	3.01
$f(x)$	6.6	6.5	…	6.1	6.01

From the table we observe that as $x \to 3$ from either side. $f(x) \to 6$.

This idea can be expressed by saying that the limiting value of $f(x)$ is 6 when x approches to 3.

This is symbolically written as,

$$\lim_{x \to 3} f(x) = 6$$

i.e. $$\lim_{x \to 3} (x+3) = 6$$

Thus, limit of the function, $f(x) = x + 3$ as $x \to 3$ is the value of the function at $x = 3$.

Let's learn.

7.1 DEFINITION OF LIMIT OF A FUNCTION:

A function $f(x)$ is said to have the limit l as x tends to a, if for every $\in > 0$ we can find $\delta > 0$ such that, $|f(x) - l| < \in$ whenever $0 < |x-a| < \delta$ and 'l' is a finite real number.

We are going to study the limit of a rational function $\dfrac{P(x)}{Q(x)}$ as $x \to$ a.

Here P(x) and Q(x) are polynomials in x.

We get three different cases.

(1) $Q(a) \neq 0$,

(2) $Q(a) = 0$ and $P(a) = 0$

(3) $Q(a) = 0$ and $P(a) \neq 0$

In case (1) $\lim_{x \to a} \dfrac{P(x)}{Q(x)} = \dfrac{P(a)}{Q(a)}$.

Because as $x \to a$, $P(x) \to P(a)$ and $Q(x) \to Q(a)$

In Case (2) $x - a$ is a factor of $P(x)$ as well as Q(x) so we have express $P(x)$ and $Q(x)$ as $P(x) = (x - a)\, P_1(x)$ and $Q(x) = (x - a)\, Q_1(x)$

Now $\dfrac{P(x)}{Q(x)} = \dfrac{(x-a)P_1(x)}{(x-a)Q_1(x)} = \dfrac{P_1(x)}{Q_1(x)}$.

Note that

$(x-a) \neq 0$ so we can cancel the factor.

In case (3) $Q(a) = 0$ and $P(a) \neq 0$,

$\lim\limits_{x\to a} \dfrac{P(x)}{Q(x)}$ does not exist.

7.1.1 One Sided Limit: You are aware of the fact that when $x \to a$; x approaches a in two directions; which can lead to two limits, known as left hand limit and right hand limit.

7.1.2 Right hand Limit : *If given $\in > 0$ (however small), there exists $\delta > 0$ such that $|f(x) - l_1| < \in$ for all x with $a < x < a + \delta$ then* $\lim\limits_{x\to a^+} f(x) = l_1$

7.1.3 Left hand Limit : *If given $\in > 0$ (however small), there exists $\delta > 0$ such that for $|f(x) - l_2| < \in$ all x with $a - \delta < x < a$ then* $\lim\limits_{x\to a^-} f(x) = l_2$

Example:

Find left hand limit and right hand limit for the following example.

$$f(x) = \begin{cases} 3x+1 & \text{if } x < 1 \\ 7x^2 - 3 & \text{if } x \geq 1 \end{cases}$$

To compute, $\lim\limits_{x\to 1^+} f(x)$, we use the definition for f which applies to $x \geq 1$:

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (7x^2 - 3) = 4$$

Likewise, to compute $\lim\limits_{x\to 1^-} f(x)$, we use the definition for f which applies to x < 1:

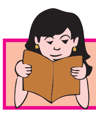
$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (3x + 1) = 4$$

Since left and right-hand limits are equal,

$\lim\limits_{x\to 1} f(x) = 4$

7.1.4 Existence of a limit of a function at a point $x = a$

If $\lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a^-} f(x) = l$, then limit of the function f(x) as $x \to a$ exists and its value is l. If $\lim\limits_{x\to a^+} f(x) \neq \lim\limits_{x\to a^-} f(x)$ then $\lim\limits_{x\to a} f(x)$ does not exist.



Let's learn.

7.2 ALGEBRA OF LIMITS:

Let f(x) and g(x) be two functions such that $\lim\limits_{x\to a} f(x) = l$ and $\lim\limits_{x\to a} g(x) = m$, then

1. $\lim\limits_{x\to a} [f(x) \pm g(x)] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x)$
   $= l \pm \mathbf{m}$

2. $\lim\limits_{x\to a} [f(x) \times g(x)] = \lim\limits_{x\to a} f(x) \times \lim\limits_{x\to a} g(x)$
   $= l \times \mathbf{m}$

3. $\lim\limits_{x\to a} [k\,f(x)] = k \lim\limits_{x\to a} f(x) = kl$, where '$k$' is a constant

4. $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} = \dfrac{l}{m}$ provided $m \neq 0$

7.3 EVALUATION OF LIMITS :

Direct Method : In some cases $\lim\limits_{x\to a} f(x)$ can be obtained by just substitution of x by a in $f(x)$

SOLVED EXAMPLES

Ex. 1) $\lim\limits_{r\to 1}\left(\frac{4}{3}\pi r^2\right)=\frac{4}{3}\pi\lim\limits_{r\to 1}\left(r^2\right)=\frac{4}{3}\pi(1)^2=\frac{4}{3}\pi$

Ex. 2) $\lim\limits_{y\to 2}\ [(y^2-3)(y+2)]$

$= \lim\limits_{y\to 2}\ (y^2-3)\ \lim\limits_{y\to 2}\ (y+2)$

$= (2^2-3)(2+2) = (4-3)(4) = 1 \times 4 = 4$

Ex. 3) $\lim\limits_{x\to 3}\left(\frac{\sqrt{6+x}-\sqrt{7-x}}{x}\right)$

$$= \frac{\lim\limits_{x\to 3}\left(\sqrt{6+x}\right)-\lim\limits_{x\to 3}\left(\sqrt{7-x}\right)}{\lim\limits_{x\to 3}(x)}$$

$$= \frac{\sqrt{6+3}-\sqrt{7-3}}{3}$$

$$= \frac{\sqrt{9}-\sqrt{4}}{3}$$

$$= \frac{3-2}{3} = \frac{1}{3}$$

Ex. 4) Discuss the limit of the following function as x tends to 3 if

$$f(x) = \begin{cases} x^2+x+1, & 2\le x\le 3 \\ 2x+1, & 3< x\le 4\end{cases}$$

Solution: we use the concept of left hand limit and right hand limit, to discuss the existence of limit as $x\to 3$

Note : *In both cases **x** takes only positive values.*

For the interval $2 \le x \le 3$; $f(x) = x^2 + x + 1$

$\therefore\ \lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-}\ (x^2 + x + 1)$

$= (3)^2 + 3 + 1$

$= 9 + 3 + 1 = 13$ -----(I)

Similarly for the interval $3 < x \le 4$;
$f(x) = 2x + 1$

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} (2x+1) = (2 \times 3) + 1 = 6 + 1$

$= 7$ -----(II)

From (I) and (II), $\lim\limits_{x\to 3^-} f(x) \neq \lim\limits_{x\to 3^+} f(x)$

$\therefore\ \lim\limits_{x\to 3} f(x)$ does not exist.

Theorem: $\lim\limits_{x\to a}\frac{x^n-a^n}{x-a}=n(a^{n-1})$, for $n\in Q$.

SOLVED EXAMPLES

Ex. 1) $\lim\limits_{x\to 4}\left[\frac{x^n-4^n}{x-4}\right]$ = 48 and $n\in N$, find n.

Solution: Given $\lim\limits_{x\to 4}\left[\frac{x^n-4^n}{x-4}\right] = 48$

$\therefore\ n(4)^{n-1} = 48 = 3 \times 16 = 3(4)^2$

$\therefore\ n(4)^{n-1} = 3(4)^{3-1}$... by observation

$\therefore\ n = 3.$

Ex. 2) Evaluate $\lim\limits_{x\to 1}\left[\frac{2x-2}{\sqrt[3]{26+x}-3}\right]$

Solution: Put $26 + x = t^3$ $\therefore\ x = t^3 - 26$
As $x \to 1$, $t \to 3$

$$\therefore\ \lim\limits_{x\to 1}\left[\frac{2x-2}{\sqrt[3]{26+x}-3}\right]$$

$$= \lim\limits_{t\to 3}\left[\frac{2\left(t^3-26\right)-2}{\sqrt[3]{t^3}-3}\right]$$

$$= 2\lim\limits_{t\to 3}\left[\frac{t^3-3^3}{t-3}\right]$$

As $\lim\limits_{x\to a}\left[\frac{x^n-a^n}{x-a}\right] = na^{n-1}$

$= 2 \times 3(3)^{3-1}$

$= 2 \times 3^3 = 2 \times 27$

$= 54$

EXERCISE 7.1

Q.I Evaluate the Following limits :

1. $\lim_{x \to 3} \left[\frac{\sqrt{x+6}}{x} \right]$

2. $\lim_{x \to 2} \left[\frac{x^{-3} - 2^{-3}}{x-2} \right]$

3. $\lim_{x \to 5} \left[\frac{x^3 - 125}{x^5 - 3125} \right]$

4. If $\lim_{x \to 1} \left[\frac{x^4 - 1}{x-1} \right] = \lim_{x \to a} \left[\frac{x^3 - a^3}{x-a} \right]$ find all possible values of a.

Q.II Evaluate the Following limits :

1. $\lim_{x \to 7} \left[\frac{\left(\sqrt[3]{x} - \sqrt[3]{7}\right)\left(\sqrt[3]{x} + \sqrt[3]{7}\right)}{x-7} \right]$

2. If $\lim_{x \to 5} \left[\frac{x^k - 5^k}{x-5} \right] = 500$ find all possible values of k.

3. $\lim_{x \to 0} \left[\frac{(1-x)^8 - 1}{(1-x)^2 - 1} \right]$

Q.III Evaluate the Following limits :

1. $\lim_{x \to 0} \left[\frac{\sqrt[3]{1+x} - \sqrt{1+x}}{x} \right]$

2. $\lim_{y \to 1} \left[\frac{2y-2}{\sqrt[3]{7+y} - 2} \right]$

3. $\lim_{z \to a} \left[\frac{(z+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{z-a} \right]$

4. $\lim_{x \to 5} \left[\frac{x^3 - 125}{x^2 - 25} \right]$

Let's learn.

7.4 FACTORIZATION METHOD :

Consider the problem of evaluating,

$\lim_{x \to a} \frac{f(x)}{g(x)}$ where $g(a) \neq 0$.

SOLVED EXAMPLES

Ex. 1) Evaluate $\lim_{z \to 3} \left[\frac{z(2z-3)-9}{z^2 - 4z + 3} \right]$

Solution: If we substitute z = 3 in numerator and denominator,

we get $z(2z - 3) - 9 = 0$ and $z^2 - 4z + 3 = 0$.

So $(z - 3)$ is a factor in the numerator and denominator.

$$\therefore \lim_{z \to 3} \left[\frac{z(2z-3)-9}{z^2 - 4z + 3} \right]$$

$$= \lim_{z \to 3} \left[\frac{2z^2 - 3z - 9}{z^2 - 4z + 3} \right]$$

$$= \lim_{z \to 3} \left[\frac{(z-3)(2z+3)}{(z-3)(z-1)} \right]$$

$$= \lim_{z \to 3} \left[\frac{(2z+3)}{(z-1)} \right] \quad \because (z-3 \neq 0)$$

$$= \frac{2(3)+3}{3-1}$$

$$= \frac{9}{2}.$$

Ex. 2) Evaluate $\lim_{x \to 4}\left[\frac{(x^3-8x^2+16x)^9}{\left(x^2-x-12\right)^{18}}\right]$

Solution : $\lim_{x \to 4}\left[\frac{\left[x(x-4)^2\right]^9}{(x-4)^{18}(x+3)^{18}}\right]$

$= \lim_{x \to 4}\left[\frac{(x-4)^{18}x^9}{(x-4)^{18}(x+3)^{18}}\right]$

$= \lim_{x \to 4}\left[\frac{x^9}{(x+3)^{18}}\right] \because (x-4) \neq 0$

$= \frac{4^9}{7^{18}}$

Ex. 3) Evaluate $\lim_{x \to 1}\left[\frac{1}{x-1}+\frac{2}{1-x^2}\right]$

Solution : $\lim_{x \to 1}\left[\frac{1}{x-1}+\frac{2}{(1-x)(x+1)}\right]$

$= \lim_{x \to 1}\left[\frac{1}{x-1}-\frac{2}{(x-1)(x+1)}\right]$

$= \lim_{x \to 1}\left[\frac{1+x-2}{(x-1)(x+1)}\right]$

$= \lim_{x \to 1}\left[\frac{x-1}{(x-1)(x+1)}\right]$

Since $x \to 1, (x-1 \neq 0)$

$= \lim_{x \to 1}\left[\frac{1}{(x+1)}\right]$

$= \frac{1}{2}$

Ex. 4) Evaluate $\lim_{x \to 1}\left[\frac{x^3+x^2-5x+3}{x^2-1}\right]$

Solution : In this case $(x-1)$ is a factor of the numerator and denominator.

To find another factor we use synthetic division, Numerator: $x^3 + x^2 - 5x + 3$

1	1	1	–5	3
		1	2	–3
	1	2 (=1+1)	–3(= –5+2)	0(= –3+3)

$\therefore x^3 + x^2 - 5x + 3 = (x-1)(x^2 + 2x - 3)$

Denominator: $x^2 - 1 = (x+1)(x-1)$

$= \lim_{x \to 1}\left[\frac{x^3+x^2-5x+3}{x^2-1}\right]$

$= \lim_{x \to 1}\left[\frac{(x-1)(x^2+2x-3)}{(x+1)(x-1)}\right]$

$= \lim_{x \to 1}\left[\frac{(x^2+2x-3)}{(x+1)}\right] \quad \begin{pmatrix} x \to 1 \\ x \neq 1 \\ x-1 \neq 0 \end{pmatrix}$

$= \frac{1+2-3}{1+1}$

$= 0$

Ex. 5) $\lim_{x \to 1}\left[\frac{\frac{1}{x}-1}{x-1}\right]$

$= \lim_{x \to 1}\left[\frac{(1-x)}{(x-1)\times x}\right]$

$= \lim_{x \to 1}\left[\frac{-(x-1)}{(x-1)\times x}\right]$

since $x-1 \neq 0$

$= \lim_{x \to 1}\left[\frac{-1}{x}\right] = -\lim_{x \to 1}\left[\frac{1}{x}\right] = -\frac{1}{1}$

$= -1$

EXERCISE 7.2

Q.I Evaluate the following limits :

1. $\lim_{z \to 2}\left[\dfrac{z^2-5z+6}{z^2-4}\right]$

2. $\lim_{x \to -3}\left[\dfrac{x+3}{x^2+4x+3}\right]$

3. $\lim_{y \to 0}\left[\dfrac{5y^3+8y^2}{3y^4-16y^2}\right]$

4. $\lim_{x \to -2}\left[\dfrac{-2x-4}{x^3+2x^2}\right]$

Q.II Evaluate the following limits :

1. $\lim_{u \to 1}\left[\dfrac{u^4-1}{u^3-1}\right]$

2. $\lim_{x \to 3}\left[\dfrac{1}{x-3}-\dfrac{9x}{x^3-27}\right]$

3. $\lim_{x \to 2}\left[\dfrac{x^3-4x^2+4x}{x^2-1}\right]$

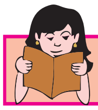
Q.III Evaluate the following limits :

1. $\lim_{x \to -2}\left[\dfrac{x^7+x^5+160}{x^3+8}\right]$

2. $\lim_{y \to \frac{1}{2}}\left[\dfrac{1-8y^3}{y-4y^3}\right]$

3. $\lim_{v \to \sqrt{2}}\left[\dfrac{v^2+v\sqrt{2}-4}{v^2-3v\sqrt{2}+4}\right]$

4. $\lim_{x \to 3}\left[\dfrac{x^2+2x-15}{x^2-5x+6}\right]$



Let's learn.

7.5 RATIONALIZATION METHOD :

If the function in the limit involves a square root, it may be possible to simplify the expression by multiplying and dividing by the conjugate. This method uses the algebraic identity.

Here, we do the following steps:

Step 1. **Rationalize the factor containing square root.**

Step 2. **Simplify.**

Step 3. **Put the value of *x* and get the required result.**

SOLVED EXAMPLES

Ex. 1) Evaluate $\lim_{z \to 0}\left[\dfrac{(b+z)^{\frac{1}{2}}-(b-z)^{\frac{1}{2}}}{z}\right]$

Solution: $\lim_{z \to 0}\left[\dfrac{(b+z)^{\frac{1}{2}}-(b-z)^{\frac{1}{2}}}{z}\right]$

$$= \lim_{z \to 0}\left[\frac{\sqrt{b+z}-\sqrt{b-z}}{z}\right]$$

$$= \lim_{z \to 0}\left[\frac{\sqrt{b+z}-\sqrt{b-z}}{z}\times\frac{\sqrt{b+z}+\sqrt{b-z}}{\sqrt{b+z}+\sqrt{b-z}}\right]$$

$$= \lim_{z \to 0}\left[\frac{\left(\sqrt{b+z}\right)^2-\left(\sqrt{b-z}\right)^2}{z}\times\frac{1}{\sqrt{b+z}+\sqrt{b-z}}\right]$$

$$= \lim_{z \to 0}\left[\frac{(b+z)-(b-z)}{z}\times\frac{1}{\sqrt{b+z}+\sqrt{b-z}}\right]$$

$$= \lim_{z \to 0}\left[\frac{2z}{z}\times\frac{1}{\sqrt{b+z}+\sqrt{b-z}}\right]$$

$$= \lim_{z\to 0}\left[\frac{2}{\sqrt{b+z}+\sqrt{b-z}}\right]$$

$$= \frac{2}{\sqrt{b+0}+\sqrt{b-0}}$$

$$= \frac{2}{2\sqrt{b}}$$

$$= \frac{1}{\sqrt{b}}$$

Ex. 2) Evaluate $\lim_{x\to 0}\left[\frac{1-\sqrt{x^2+1}}{x^2}\right]$

Solution : $\lim_{x\to 0}\left[\frac{1-\sqrt{x^2+1}}{x^2}\right]$

$$= \lim_{x\to 0}\left[\frac{1-\sqrt{x^2+1}}{x^2}\frac{\left(1+\sqrt{x^2+1}\right)}{\left(1+\sqrt{x^2+1}\right)}\right]$$

$$= \lim_{x\to 0}\left[\frac{1-\left(x^2+1\right)}{x^2}\frac{1}{\left[1+\sqrt{x^2+1}\right]}\right]$$

$$= \lim_{x\to 0}\left[\frac{\left(-x^2\right)}{x^2}\frac{1}{\left[1+\sqrt{x^2+1}\right]}\right]$$

$$= \lim_{x\to 0}\left[\frac{-1}{\left[1+\sqrt{x^2+1}\right]}\right]$$

$$= \frac{-1}{1+1}$$

$$= \frac{-1}{2}$$

EXERCISE 7.3

Q.I Evaluate the following limits :

1. $\lim_{x\to 0}\left[\frac{\sqrt{6+x+x^2}-\sqrt{6}}{x}\right]$

2. $\lim_{y\to 0}\left[\frac{\sqrt{1-y^2}-\sqrt{1+y^2}}{y^2}\right]$

3. $\lim_{x\to 2}\left[\frac{\sqrt{2+x}-\sqrt{6-x}}{\sqrt{x}-\sqrt{2}}\right]$

Q.II Evaluate the following limits :

1. $\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

2. $\lim_{x\to 2}\left[\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right]$

Q.III Evaluate the Following limits :

1. $\lim_{x\to 1}\left[\frac{x^2+x\sqrt{x}-2}{x-1}\right]$

2. $\lim_{x\to 0}\left[\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}\right]$

3. $\lim_{x\to 4}\left[\frac{x^2+x-20}{\sqrt{3x+4}-4}\right]$

4. $\lim_{x\to 2}\left[\frac{x^3-8}{\sqrt{x+2}-\sqrt{3x-2}}\right]$

Q.IV Evaluate the Following limits :

1. $\lim_{y\to 2}\left[\frac{2-y}{\sqrt{3-y}-1}\right]$

2. $\lim_{z\to 4}\left[\frac{3-\sqrt{5+z}}{1-\sqrt{5-z}}\right]$

7.6 LIMITS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS :

We use the following results without proof.

1. $\lim_{x\to 0}\frac{a^x-1}{x}=\log a$, a > 0,

2. $\lim_{x\to 0}\frac{e^x-1}{x}=\log e=1$

3. $\lim_{x\to 0}[1+x]^{\frac{1}{x}}=e$

4. $\lim_{x\to 0}\frac{\log(1+x)}{x}=1$

SOLVED EXAMPLES

Ex.1) Evalute : $\lim_{x\to 0}\left[\frac{7^x-1}{x}\right]$

Solution : $\lim_{x\to 0}\left[\frac{7^x-1}{x}\right]$

$= \log 7$

Ex.2) Evaluate : $\lim_{x\to 0}\left[\frac{5^x-3^x}{x}\right]$

Solutions : $\lim_{x\to 0}\left[\frac{5^x-3^x}{x}\right]$

$=\lim_{x\to 0}\left[\frac{5^x-1-3^x+1}{x}\right]$

$=\lim_{x\to 0}\left[\frac{(5^x-1)}{x}-\frac{(3^x-1)}{x}\right]$

$=\lim_{x\to 0}\left[\frac{(5^x-1)}{x}\right]-\lim_{x\to 0}\left[\frac{(3^x-1)}{x}\right]$

$= \log 5 - \log 3$

$= \log\left(\frac{5}{3}\right)$

Ex.3) Evaluate : $\lim_{x\to 0}\left[1+\frac{5}{6}x\right]^{\frac{1}{x}}$

Solutions : $\lim_{x\to 0}\left[1+\frac{5}{6}x\right]^{\frac{1}{x}}$

$=\lim_{x\to 0}\left[\left(1+\frac{5}{6}x\right)^{\frac{1}{\left(\frac{5}{6}\right)x}}\right]^{\frac{5}{6}}$

$= e^{\frac{5}{6}}$

Ex.4) Evaluate : $\lim_{x\to 0}\left[\frac{\log(1+4x)}{x}\right]$

Solutions : $\lim_{x\to 0}\left[\frac{\log(1+4x)}{x}\right]$

$=\lim_{x\to 0}\left[\frac{\log(1+4x)}{4x}\times 4\right]$

$= 4 \times 1$

$= 4$

Ex.5) Evaluate :

$\lim_{x\to 0}\left[\frac{8^x-4^x-2^x+1}{x^2}\right]$

Solutions : Given $\lim_{x\to 0}\left[\frac{8^x-4^x-2^x+1}{x^2}\right]$

$=\lim_{x\to 0}\left[\frac{(4\times 2)^x-4^x-2^x+1}{x^2}\right]$

$=\lim_{x\to 0}\left[\frac{4^x.2^x-4^x-2^x+1}{x^2}\right]$

$=\lim_{x\to 0}\left[\frac{4^x\left(2^x-1\right)-(2^x-1)}{x^2}\right]$

$$= \lim_{x\to 0}\left[\frac{\left(2^x-1\right).\left(4^x-1\right)}{x^2}\right]$$

$$= \lim_{x\to 0}\left[\frac{\left(2^x-1\right)}{x}\right] \times \lim_{x\to 0}\left[\frac{\left(4^x-1\right)}{x}\right]$$

$= (log2)\ (log4)$

EXERCISE 7.4

I] Evaluate the following :

1) $\lim_{x\to 0}\left[\frac{9^x-5^x}{4^x-1}\right]$

2) $\lim_{x\to 0}\left[\frac{5^x+3^x-2^x-1}{x}\right]$

3) $\lim_{x\to 0}\left[\frac{log\left(2+x\right)-log\left(2-x\right)}{x}\right]$

II] Evaluate the following :

1) $\lim_{x\to 0}\left[\frac{3^x+3^{-x}-2}{x^2}\right]$

2) $\lim_{x\to 0}\left[\frac{3+x}{3-x}\right]^{\frac{1}{x}}$

3) $\lim_{x\to 0}\left[\frac{log\left(3-x\right)-log\left(3+x\right)}{x}\right]$

III] Evaluate the following :

1) $\lim_{x\to 0}\left[\frac{a^{3x}-b^{2x}}{\log(1+4x)}\right]$

2) $\lim_{x\to 0}\left[\frac{\left(2^x-1\right)^2}{(3^x-1).\log(1+x)}\right]$

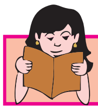
3) $\lim_{x\to 0}\left[\frac{15^x-5^x-3^x+1}{x^2}\right]$

4) $\lim_{x\to 2}\left[\frac{3^{\frac{x}{2}}-3}{3^x-9}\right]$

IV] Evaluate the following :

1) $\lim_{x\to 0}\left[\frac{(25)^x-2\left(5\right)^x+1}{x^2}\right]$

2) $\lim_{x\to 0}\left[\frac{(49)^x-2\left(35\right)^x+\left(25\right)^x}{x^2}\right]$



Let's learn.

Some Standard Results

1. $\lim_{x\to a} k = k$, where k is a constant

2. $\lim_{x\to a} x = a$

3. $\lim_{x\to a} x^n = a^n$

4. $\lim_{x\to a} \sqrt[r]{x} = \sqrt[r]{a}$

5. If p(x) is a polynomial, then $\lim_{x\to a} p(x) = p(a)$

6. $\lim_{x\to a} \frac{x^n-a^n}{x-a} = n(a^{n-1})$, for $n \in Q$

7. $\lim_{x\to 0}\left(\frac{e^x-1}{x}\right) = \log e = 1$

8. $\lim_{x\to 0}\frac{\log(1+x)}{x} = 1$

MISCELLANEOUS EXERCISE - 7

I. If $\lim_{x\to 2}\frac{x^n-2^n}{x-2} = 80$ then find the value of n.

II. Evaluate the following Limits.

1) $\lim_{x\to a}\frac{(x+2)^{\frac{5}{3}}-(a+2)^{\frac{5}{3}}}{x-a}$

2) $\lim_{x\to0}\frac{(1+x)^n-1}{x}$

3) $\lim_{x\to2}\left[\frac{(x-2)}{2x^2-7x+6}\right]$

4) $\lim_{x\to1}\left[\frac{x^3-1}{x^2+5x-6}\right]$

5) $\lim_{x\to3}\left[\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}\right]$

6) $\lim_{x\to4}\left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right]$

7) $\lim_{x\to0}\left[\frac{5^x-1}{x}\right]$

8) $\lim_{x\to0}\left(1+\frac{x}{5}\right)^{\frac{1}{x}}$

9) $\lim_{x\to0}\left[\frac{\log(1+9x)}{x}\right]$

10) $\lim_{x\to0}\frac{(1-x)^5-1}{(1-x)^3-1}$

11) $\lim_{x\to0}\left[\frac{a^x+b^x+c^x-3}{x}\right]$

12) $\lim_{x\to0}\frac{e^x+e^{-x}-2}{x^2}$

13) $\lim_{x\to0}\left[\frac{x(6^x-3^x)}{(2^x-1).\log(1+x)}\right]$

14) $\lim_{x\to0}\left[\frac{a^{3x}-a^{2x}-a^x+1}{x^2}\right]$

15) $\lim_{x\to0}\left[\frac{(5^x-1)^2}{x.\log(1+x)}\right]$

16) $\lim_{x\to0}\left[\frac{a^{4x}-1}{b^{2x}-1}\right]$

17) $\lim_{x\to0}\left[\frac{log100+\log(0.01+x)}{x}\right]$

18) $\lim_{x\to0}\left[\frac{\log(4-x)-\log(4+x)}{x}\right]$

19) Evaluate the limit of the function if exist at $x = 1$ where $f(x) = \begin{cases} 7-4x & x<1 \\ x^2+2 & x\geq1 \end{cases}$

Activity 7.1

Evaluate : $\lim_{x\to0}\left[\frac{e^x-x-1}{x}\right]$

Solution : $= \lim_{x\to0}\left[\frac{(e^x-1)-\boxed{}}{x}\right]$

$= \lim_{x\to0}\frac{\boxed{}}{x}-\frac{x}{x}$

$= \lim_{x\to0}\left[\frac{e^x-1}{x}\right]-\boxed{}$

$= \boxed{}-1$

$= \boxed{}$

Activity 7.2

Carry out the following activity.

Evaluate: $\lim_{x\to1}\left[\frac{1}{x-1}+\frac{2}{1-x^2}\right]$

Solution : $\lim_{x\to1}\left[\frac{1}{x-1}-\frac{2}{\boxed{}}\right]$

$= \lim_{x\to1}\left[\frac{\boxed{}}{(x-1)(x+1)}\right]$

$= \lim_{x\to1}\frac{1}{x+1}$

$= \boxed{}$