6. PERMUTATIONS AND COMBINATIONS

Let's Study

- Fundamental principles of counting
- Factorial function
- Permutations
- Combinations

Let's Recall

1. The number system.
2. The four basic mathematical operations: addition, subtraction, multiplication, division.

INTRODUCTION

Counting is a fundamental activity in mathematics. Learning to count was our first step in learning mathematics. After learning to count objects one by one, we used operations of addition and multiplication to make counting easy. We shall now learn two more methods of counting to make counting easier. These two methods are known as permutations and combinations. Permutations refer to the number of different rearrangements of given objects, when the order of objects in the arrangements is important. Combinations are related to the number of different selections from a given set of objects, when the order of objects in the selections is not important.

The theory of permutations and combinations forms the basis of many topics of mathematics, including number theory, graph theory, discrete mathematics, algebra, geometry, probability, and other disciplines like statistics, cryptology, security systems, and so on. The theory of permutations and combinations is central in problems of counting such a large number of objects that it is impossible to count them manually. The theory of permutations and combinations enables us to count objects without listing or enumerating them.

Let us begin with the following simple example. Every smartphone requires a password to unlock it. A password is formed by four of the ten digits on the screen. The order of these four digits cannot be changed for password to work. How many distinct passwords are possible? In other words, how many users can have distinct passwords? Note that a password consists of four digits. The first digit of a password can be any of the ten digits, the second digit can be any of the ten digits, and similarly for the third and fourth digits. This gives a total of $10 \times 10 \times 10 \times 10 = 10{,}000$ as the number of distinct possible passwords.

Consider one more example which is not as easy as the last example. The school cricket team has eleven players. The school wants a photograph of these players, along with the principal and the two vice principals of the school, for school magazine. Seven chairs are arranged in a row for the photograph. Three chairs in the middle are reserved for the principal and the two vice principals. Four players will occupy the remaining four chairs and seven players will stand behind the chairs. The question then is: "In how many different ways can the eleven players take positions for the photograph?" This example will be considered later in the chapter.. Till then, we can try, on our own, to find the number of different ways in which the eleven players can sit or stand for the photo.

Let's Learn

We shall begin by learning some techniques that are useful in counting such arrangements without actually writing them down. Since this chapter is related to counting, we must understand three principles of counting that are fundamental to all methods of counting, including permutations and combinations.

6.1 Fundamental principles of counting

We have learnt in set theory that subsets of a set can be represented diagrammatically in the form of a Venn diagram. An alternative method is to draw a tree diagram if the subsets are disjoint. For example, the English alphabet has 26 letters from A to Z. These 26 letters can be divided in two groups: 5 vowels and 21 consonants. This division can be presented diagrammatically as follows.

Letters

Vowels Consonants

Fig. 6.1

Diagrams of this nature are called tree diagrams. A tree diagram shows the division of a set into disjoint subsets. A complete tree diagram shows different possible divisions of a given set. If the given set represents an action, then the branches of the tree represent different possible outcomes of the action. If the given set consists of objects, then the branches of the tree represent a division of the set according to some characteristic of the objects. If we are studying an experiment, then the branches represent different possible outcomes of the experiment. The collection of all possible outcomes of an experiment is called its Sample space. Can we think of some examples where tree diagrams cans be used?

Tree diagrams, along with the fundamental principles of counting, are powerful tools in counting the total number of possible outcomes of an experiment. The fundamental principles of counting use additions or multiplications to determine the total number of outcomes. The fundamental principles of counting provide an efficient way of finding the number of different ways to carry out two or more activities, either simultaneously or successively (that is, one after another).

The fundamental principles of counting can be effectively used for determining the number of possible outcomes when two or more characteristics are involved. Consider the situation where a boy wants to go for shopping on Sunday. He wants to wear a good shirt when he goes for shopping. He has three T–shirts of three different colours: white, gray, and blue. He also has four shirts of four different colours: red, green, yellow, and orange. How many choices does he have to wear? For this, the boy must first decide whether he wants to wear a shirt or a T–shirt. If he decides to wear a shirt, then he has four choices. If he decides to wear a T–shirt, then he has three choices. This situation can be represented using a tree diagram as follows.

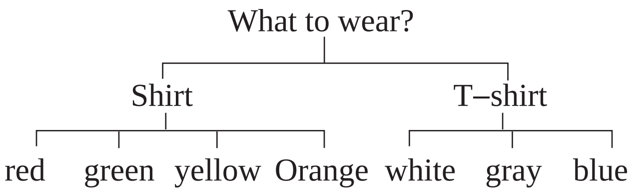

Fig. 6.2

The tree diagram shows that the boy has seven choices in all. These seven choices can be presented in the form of a list as follows.

1. Red shirt
2. Green shirt
3. Yellow shirt
4. Orange shirt
5. White T–shirt
6. Gray T–shirt
7. Blue T–shirt

This example shows that the total number of outcomes is obtained by adding the number of outcomes of each characteristic when only one characteristic can be chosen. This is called the addition principle of counting, which is one of the fundamental principles of counting.

6.1.1 ADDITION PRINCIPLE

Consider a situation where Raju has 3 red shirts (R1, R2, R3) and 2 green shirts (G1, G2). When Raju wants to wear a shirt, he has to first select the colour (red or green) and then a shirt of the chosen colour. Can we find the number of ways Raju can select a shirt to wear?

Let us try to imagine what Raju must be thinking. Do we also think in the following way?

Raju has to perform the following tasks: (i) select a red shirt, or (ii) select a green shirt. He, however, cannot select two shirts. Let us answer one question at a time to understand the situation.

Q.1 In how many ways can the first task be accomplished?

Ans. The first task (of selecting the colour) can be accomplished in two ways.

Q.2 In how many ways can the second task be accomplished?

Ans. The answer depends on the outcome of the first task in the following way.

If the red colour is selected in the first task, then the second task can be accomplished in three ways.

If the green colour is selected in the first task, then the second task can be accomplished in two ways.

Q.3 As the result of accomplishing the two tasks, how many ways are available for selecting a shirt?

Ans. The total number of ways available for selecting a shirt is $3 + 2 = 5$.

This brings us to the first fundamental principle of counting, the addition principle, which can be stated as follows.

Definition of Addition Principle.

If one event can occur in m ways and the second event with no common outcomes can occur in n ways, then either of the two events can occur in $m + n$ ways.

Ex. 1 : A restaurant offers five types of cold drinks and three types of hot drinks. If a customer wants to order a drink, how many choices does the customer have?

Solution : Since the restaurant offers five cold drinks, the customer has five ways of selecting a cold drink. Similarly, since the restaurant offers three types of hot drinks, the customer has three ways of selecting a hot drink. Finally, since the customer wants to select only one drink, there are 5 + 3 = 8 choices for the customer.

Consider one more example where the addition principle is used. Consider an experiment of drawing a card from a pack of 52 cards. What is the number of ways in which the drawn card is a spade or a club?

How can we find this number? Do we think in the same way as follows?

In the given situation, any one of the following two operations can be performed.

(i) Select a spade card out of the 13 spade cards in the deck.

(ii) Select a club card out of the 13 club card in the deck.

Q.1 In how many ways can the first operation be performed?

Ans. The first operation can be performed in m = 13 ways.

Q.2 In how many ways can the second operation be performed?

Ans. The second operation can be performed in n = 13 ways.

Q.3 Is it possible to perform both the operations?

Ans. No. Only one of the two operations can be performed in the experiment.

Q.4 How many ways are then available for selecting a card that satisfies the specified condition (that the selected card is a spade or a club)?

Ans. There are m + n = 13 + 13 = 26 ways of selecting a card which is a spade or a club.

6.1.2 MULTIPLICATION PRINCIPLE

Now, consider the following situation. An Ice Cream is served either in a cup or in a cone. Also, Ice Cream is available in three flavours: vanilla, chocolate, and strawberry. This information can be represented in the form of a tree diagram as follows.

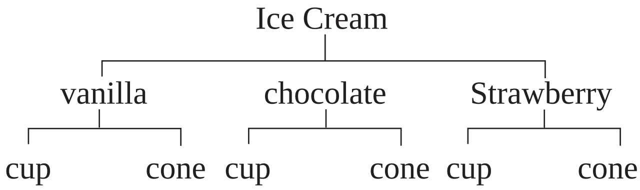

Fig. 6.3

The tree diagram in above figure shows the six different ways in which Ice Cream can be served. Can we think of a different tree diagram to represent the same information? Do we think that the following tree diagram also represents the same information?

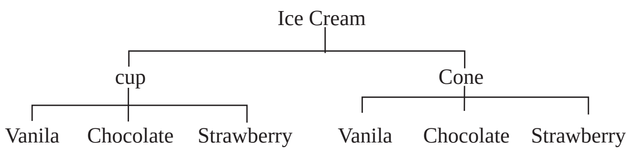

Fig. 6.4

Note that both the tree diagrams show that there are six possible outcomes, that is, six ways to serve Ice Cream. What do we understand from these two tree diagrams about counting the possible outcomes? Do we think as follows?

When the final decision depends on two decisions, say D_1 and D_2, to be taken one after the other, first determine the number of outcomes of D_1 and D_2 separately. Suppose D_1 has n_1 outcomes and D_2 has n_2 outcomes. If D_1 and D_2 are decisions that are taken one after the other, then the total number of possible decisions is obtained by multiplication, so that there are $n_1 \times n_2$ possible outcomes, as it can also be observed in the two tree diagrams in Fig.6.3 and Fig. 6.4.

In the example of Ice Cream, the final count of six is obtained by multiplying 3 (number of flavours) by 2 (number of serving options), in either order.

If there are more than two decisions, then we continue to multiply the number of options in one decision after another. This method of multiplication is relevant when these decisions are to be taken one after another.

Ex. 2 : Samadhan Bhojanalay offers a thali that has five items: roti, rice, vegetable, dal, and pickle. Following choices are available and one option is to be selected for each item.

Roti: tawa roti, chapati, tandoor roti

Rice: plain rice, steamed rice, jeera rice, dal khichadi

Vegetable: dum aloo, paneer masala, mixed veg

Dal: dal fry, dal tadka, kadhi

Pickle: lemon, mango, mirchi

How many different menus (that is, combinations) are possible?

Solution : Since one option is to be selected for each item, the number of different possible thali choices are identified as follows.

Roti 3, Rice 4, Vegetable 3, Dal 3, Pickle 3, and hence the total number of different possible thali menus is $3 \times 4 \times 3 \times 3 \times 3 = 324$. Do we think it necessary to write down all the 324 choices in order to count them?

Ex. 3 : A company decides to label each of its different products with a code that consists of two letters followed by three digits. How many different products can be labeled in this way?

Solution : Since the first two characters in a label are letters, they can be formed in $26 \times 26 = 676$ ways. The next three characters are digits and can be formed in $10 \times 10 \times 10 = 1000$ ways. The total number of distinct labels is, by the multiplication principle, given by $676 \times 1000 = 6,76,000$.

Ex. 4 : Dhaniram went to buy a car. The showroom has two body styles (sedan and hatchback), five different colors (black, red, green, blue, and white), and three different model standard model, (GL), sports model, (SS), and luxury model, (SL).

(i) How many different choices does Dhaniram have? (Activity)

(ii) If he is told that black color is not available for a sedan, then how many different choices remain available to Dhaniram? (Activity)

Now, consider the situation where Govinda has four shirts (white, yellow, pink, and green) and three pants (black, red, and maroon). Govinda wants to wear a shirt and a pant to attend a function. How many ways does he have for selecting his dress? Let us think over this problem and try to help Govinda in his selection.

Do we proceed in the same direction as shown by the following argument?

Let us denote the four shirts by four letters W, Y, P, and G. Similarly, let us denote the three pants by three letters B, R, and M. Then the possible selections for Govinda can be listed as follows. (W, B), (W, R), (W, M), (Y, B), (Y, R), (Y, M), (P, B), (P, R), (P, M), (G, B), (G, R), (G, M). The list shows that Govinda has a total of 12 choices for selecting his dress. This finding can be stated in the following form. Govinda has m = 4 shirts and n = 3 pants. Since he wants to select one shirt and one pant, he has m x n = 4 x 3 = 12 ways to make the selection.

All these examples bring us to the multiplication principle, the second fundamental principle of counting.

Definition of Multiplication Principle

If one operation can be carried out in m ways, and the second operation can be carried out in n ways, then both the operations can be carried out in m x n ways.

Consider one more example for better understanding of the multiplication principle.

Ex. 1 : A card is drawn from a deck of 52 cards and its colour and suit are noted. The card is then put back in the deck and a second card is drawn from the deck and its colour and suit are also noted.

Can we use this information to design the problem of selecting a club card and a spade card in the two draws? After studying the problem carefully, can we also obtain its solution? Do we get something similar to what follows?

Define the first event as obtaining a club card on the first draw and second event as obtaining a spade card on the second draw. Note that the first card is drawn from the deck of 52 cards. The second card is also drawn from the deck of 52 cards because the first card is put back in the deck after noting its colour and suit. The first event can occur in m = 13 ways because there are 13 club cards in the deck of 52 cards. The second event can also occur in n = 13 ways because there are 13 spade cards in the deck. The total number of ways in which one club card and one spade card are drawn, when the drawn card is put back before the second draw, is then given by

m x n = 13 x 13 = 169.

Let us try to solve the following examples for practice.

Ex. 2 : Suresh has 4 pencils and 2 erasers. He wants to take one pencil and one eraser for the examination. Can we find the number of ways in which he can select a pencil and an eraser?

Ex. 3 : Sunil has 4 ball pens of one company and 3 ball pens of another company. In how many ways can he select a ball pen?

In the above examples, can we decide when to use the addition principle and when to use the multiplication principle? Can we give reasons? What are answers in the above examples?

Remark: The addition and multiplication principles can be extended from 2 to any finite number of activities, experiments, events, or operations. For example, the two principles can be stated as follows for three activities.

Addition Principle. If three operations, when conducted simultaneously and independently of one another, can be performed in m, n, and r ways, respectively, then any one of them can be performed in m + n + r different ways.

Multiplication Principle. If an experiment consists of three activities, where activity one has m possible outcomes, activity two has n possible outcomes, and possibility three has r possible outcomes, then the total number of different possible outcomes of carrying out all the 3 activities is m x n x r.

Let us now solve some more examples using these principles.

Solved Examples

Ex. 1 : Consider the action of drawing a card from a pack of 52 cards. Find the number of ways of selecting a spade or a club card.

Solution:

When a card is drawn from the pack of 52 cards, there are two possible events as described below.

(i) One of the 13 club cards is selected.

(ii) One of the 13 spade cards is selected.

Q.1 In how many ways can the first event occur?

Ans. The first event can occur in m = 13 ways.

Q.2 In how many ways can the second event occur?

Ans. The second event can occur in n = 13 ways.

Q.3 How many different ways are available to select a spade card or a club card?

Ans. Using addition principle the number of ways of selecting a spade card or a club card is m + n = 26.

Ex. 2 : Madhavi goes to a shop where she finds three different types of bread, four different types of cheese, three different types of butter, and five different types of jam.

If she can make a sandwich using only one item of each category, how many different ways does she have to prepare a sandwich?

Solution:

Madhavi can select bread of any one of the three types in three ways. After selecting bread, she can select cheese in four ways, butter in 3 ways, and jam in five ways.

By the principle of multiplication, the total number of ways Madhavi can make her sandwich is 3 x 4 x 3 x 5 = 180.

Ex. 3 : Five flags are available and they can be arranged on a vertical staff. Find the number of signals that can be created by arranging at least two flags on the staff, one below another.

Solution :

First, let us note that a signal can consist of 2 flags, 3 flags, 4 flags or 5 flags.

Next, let us find the different number of signals that can be created in each of these cases.

The number of signals using two flags is same as the number of different ways of filling two positions using five flags. Since a flag, once used, cannot be used again, the multiplication principle gives the number of different two flag signals to be 5 x 4 = 20.

Similarly, the number of different three flag signals is same as the number of different ways of filling three positions, one after another, using 5 flags. By the multiplication principle, this number is 5 x 4 x 3 = 60.

Using the same argument, the number of different four–flag signals is 5 x 4 x 3 x 2 = 120. Finally, the number of different five–flag signals is 5 x 4 x 3 x 2 x 1 = 120.

Putting all these numbers together, the total number of possible different signals is

$20 + 60 + 120 + 120 = 320.$

Ex. 4 : There are 6 different routes from city A to city B and 8 different routs from city B to city C. Find the number of different routes if a person wants to travel from city A to city C via city B.

Solution:

A person can choose any one of the 6 different routes to travel from city A to city B. Therefore, he can travel from city A to city B in m = 6 different ways. After choosing one of these ways, he can travel from city B to city C in n = 8 ways.

Using the multiplication principle, the total number of ways of travelling from city A to city C via city B is $m \times n = 6 \times 8 = 48$.

Ex. 5 : Suppose 5 chocoloates are to be distributed among 4 children and there is no condition on how many chocolates a child can get (including zero.) How many different ways are possible for doing so?

Solution:

The first chocolate can be given to any of the four children. Therefore, there are four different ways of giving the first chocalate. Similarly, the second chocolate can be given in four different ways, and similarly for each of the remaining chocalates. The multiplication principle then gives the total number of different ways as

$$4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024.$$

Ex. 6 : How many even numbers can be formed using the digits 2, 3, 7, 8 so that the number formed is less than 1000?

Solution:

The condition that the number formed from the given digits is less that 1000 means that this number can have upto 3 digits. Let us therefore consider the three cases separately, namely one digit numbers, two digit numbers and three–digit numbers.

Since the number must be even, the one–digit number can be only 2 or 8. Hence, there are two ways of forming a one digit even number.

Since the required number is even, the units place of a two–digit number must be either 2 or 8. The ten's place can be filled with any of the four given digits. Therefore, there are 4x2=8 ways of forming a two–digit even number.

Finally,since the required number is even, the units place can be filled in two ways, ten's place can be filled in four ways, and hundred's place can also be filled in four ways. The number of ways of forming a three digit even number is 4x4x2=32

The addition principle finally gives the total number of ways of forming an even number less than 1000 using digits 2,3,7,8 is 2+8+32=42.

EXERCISE 6.1

1. A teacher wants to select the class monitor in a class of 30 boys and 20 girls. In how many ways can he select a student if the monitor can be a boy or a girls?
2. In question 1, in how many ways can the monitor be selected if the monitor must be a boy? What is the answer if the monitor must be a girl?
3. A Signal is generated from 2 flags by putting one flag above the other. If 4 flags of different colours are available, how many different signals can be generated?

4. How many two letter words can be formed using letters from the word SPACE, when repetition of letters (i) is allowed, (ii) is not allowed?
5. How many three–digit numbers can be formed from the digits 0, 1, 3, 5, 6 if repetitions of digits (i) are allowed, (ii) are not allowed?
6. How many three–digit numbers can be formed using the digits 2, 3,4,5,6 if digits can be repeated?
7. A letter lock has 3 rings and each ring has 5 letters. Determine the maximum number of false trials that may be required to open the lock.
8. In a test that has 5 true/false questions, no student has got all corrcet answers and no sequence of answers is repeated. What is the maximum number of students for this to be possible?
9. How many numbers between 100 and 1000 have 4 in the units place?
10. How many number between 100 and 1000 have the digit 7 exactly once?
11. How many four digit numbers will not exceed 7432 if they are formed using the digits 2,3,4,7 without repetition?
12. If numbers are formed using digits 2,3,4,5,6 without repetition, how many of them will exceed 400?
13. How many numbers formed with the digits 0,1,2,5,7,8 will fall between 13 and 1000 if digits can be repeated?
14. A school has three gates and four staircases from the first floor to the second floor. How many ways does a student have to go from outside the school to his classroom on the second floor?
15. How many five–digit numbers formed using the digit 0,1,2,3,4,5 are divisible by 3 if digits are not repeated?

We may have noticed in the examples on both the addition principle and the multiplication principle that the two or more activities or operations need not be performed in any fixed order. For example, in the example of Ice Cream, it does not matter if we select the flavour of Ice Cream first and then the serving method or vice versa. The total number of choices is still 6. Similarly, in the example of Raju selecting a shirt, it does not matter whether he makes the list of red shirts first or green shirts first. The number of ways for Raju to select a shirt is still 5. This conclusion is formally known as the third fundamental principle of counting, the invariance principle of counting.

6.2. FACTORIAL FUNCTION.

The theory of permutations and combinations uses a mathematical function known as the factorial function. Let us first understand the factorial function, which is defined for a natural number.

Definition: For a natural number n, the factorial of n, written as $n!$ and read as "n factorial", is the product of natural numbers from 1 upto n.

That is, $1 \times 2 \times 3 \ldots\ldots\ldots\ldots\ldots\ldots (n-1) \times n$

Note: The factorial function can also be defined as the product of the first n natural numbers from n to 1.

That is, $n! = n \times (n-1) \times \ldots\ldots \times 3 \times 2 \times 1$

For this, we must identify all natural numbers from 1 to the given natural number n, and then take their product. For example, if $n = 5$ then we get 1, 2, 3, 4, 5 as the first five natural numbers and the product of these natural numbers as $5 \times 4 \times 3 \times 2 \times 1 = 120$

This means that "5 factorial is equal to 120."

Can we extend the concept of product of the first 5 natural numbers to the product of the first n natural numbers? In other words, we must think of $n \times (n-1) \times (n-2) \times .....\times 3 \times 2 \times 1$, where n is the specified natural number.

Let us study the following examples.

Factorial 1 = 1, 1! = 1

Factorial 2 = $2 \times 1 = 2$, $2! = 2 \times 1 = 2$

Factorial 3 = $3 \times 2 \times 1 = 6$, $3! = 3 \times 2 \times 1 = 6$

Factorial

$4 = 4 \times 3 \times 2 \times 1 = 24$, $4! = 4 \times 3 \times 2 \times 1 = 24$

and so on.

Note: Factorial is defined for 0 (even though 0 is not a natural number) as $0! = 1$.

Note: It may be interesting to note how the multiplication principle of counting is used to obtain factorial of a natural number. Suppose we want to write the first n natural number in different possible orders. How many ways do we have to do this? It is clear that there are n numbers and they will require n positions. The first position can be filled with any of the n numbers and hence there are n ways of selecting a number for the first position. The second position can be filled with any of the remaining $n-1$ numbers, and hence there are $n-1$ ways of selecting a number for the 2^{nd} position. Similarly, there are $(n-2)$ ways of selecting a number for the third position. Continuing in this way, we can see that the total number of ways for writing the first n natural numbers is given by n!.

6.2.1 Properties of the factorial function.

- For any positive integer n, $n! = n \times (n-1)!$
- For any positive integer $n > 1$,
- $n! = n \times (n-1) \times (n-2)!$
- For any positive integer $n > 2$,
- $n! = n \times (n-1) \times (n-2) \times (n-3)!$
- (For any positive integers m and n, $(m+n)!$ is always divisible by m! and n!)

It is important to remember the following results.

- For any natural numbers m and n, $(m \times n)! \neq m! \times n!$ (Verify)
- For any natural numbers m and n,

  $(m+n)! \neq m! + n!$
- For any natural numbers m and n with $m > n$, $(m-n)! \neq m! - n!$
- For any natural numbers m and n such that m is divisible by n, $(m \div n)! \neq m! \div n!$

Ex. 1 : Find the value of 7!

Solution: $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

(Check)

Ex. 2 : Find $(7-3)!$ and $7! - 3!$

Solution: $(7-3)! = 4! = 4 \times 3 \times 2 \times 1 = 24$.

We have already found $7! = 5040$.

Also, $3! = 3 \times 2 \times 1 = 6$.

$\therefore \quad 7! - 3! = 5040 - 6 = 5034$.

Therefore, we can conclude that $(7-3)! \neq 7! - 3!$

Ex. 3 : Find 8!, 4!, and $\frac{8}{4}!$

Solution: Note that $8! = 8 \times 7! = 8 \times 5040 = 40320$. Similarly, $4! = 4 \times 3 \times 2 \times 1 = 24$, and

$$\frac{8}{4}! = 2! = 2 \times 1 = 2.$$

$$\text{Also} = \frac{8!}{4!} = 1680$$

It is then clear that $\frac{8}{4}! \neq \frac{8!}{4!}$

Ex. 4 : Find values of

(i) $\frac{9!}{(8-3)!}$

(ii) $\frac{9!}{8!-3!}$

Solution: Using an earlier result, we have

$9! = 9 \times 8! = 9 \times 40320 = 362880.$

Similarly, $(8-3)! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$
Then we find the following values.

(i) $\frac{9!}{(8-3)!} = \frac{362880}{120} = 3024$

(ii) $\frac{9!}{8!-3!} = \frac{362880}{40320-6} = \frac{362880}{40314}$

Ex 5 : Show that $\frac{12!}{5!7!} + \frac{12!}{6!6!} = \frac{13!}{6!7!}$

Solution:

$$\begin{aligned}
\textbf{L. H. S.} &= \frac{12!}{5!7!} + \frac{12!}{6!6!} \\
&= 12!\left[\frac{1}{5!\times 7\times 6!} + \frac{1}{5!\times 6\times 6!}\right] \\
&= \frac{12!}{5!6!}\left[\frac{1}{7} + \frac{1}{6}\right] \\
&= \frac{12!}{5!6!}\left[\frac{13}{6\times 7}\right] \\
&= \frac{12!\times 13}{(5!\times 6)\times(6!\times 7)} \\
&= \frac{13!}{6!\times 7!} \\
&= \text{R. H. S.}
\end{aligned}$$

EXERCISE 6.2

1. Evaluate:

   (i) $8!$ (ii) $6!$

   (iii) $8! - 6!$ (iv) $(8 - 6)!$

2. Compute:

   (i) $\frac{12!}{6!}$ (ii) $\left(\frac{12}{6}\right)!$

   (iii) $(3 \times 2)!$ (iv) $3! \times 2!$

3. Compute

   (i) $\frac{9!}{3!\,6!}$ (ii) $\frac{6!-4!}{4!}$

   (iii) $\frac{8!}{6!-4!}$ (iv) $\frac{8!}{(6-4)!}$

4. Write in terms of factorials

   (i) $5 \times 6 \times 7 \times 8 \times 9 \times 10$

   (ii) $3 \times 6 \times 9 \times 12 \times 15$

   (iii) $6 \times 7 \times 8 \times 9$

   (iv) $5 \times 10 \times 15 \times 20 \times 25$

5. Evaluate : $\frac{n!}{r!(n-r)!}$ for

   (i) n = 8, r = 6 (ii) n = 12, r = 12,

6. Find n if

   (i) $\frac{n}{8!} = \frac{3}{6!} + \frac{1}{4!}$ (ii) $\frac{n}{6!} = \frac{4}{8!} + \frac{3}{6!}$

   (iii) $\frac{1}{n!} = \frac{1}{4!} - \frac{4}{5!}$

7. Find n if

   (i) $(n+1)! = 42 \times (n-1)!$

(ii) $(n+3)! = 110 \times (n+1)!$

8. Find n if:

(i) $\dfrac{n!}{3!(n-3)!} : \dfrac{n!}{5!(n-5)!} = 5:3$

(ii) $\dfrac{n!}{3!(n-5)!} : \dfrac{n!}{5!(n-7)!} = 10:3$

9. Find n if:

(i) $\dfrac{(17-n)!}{(14-n)!} = 5!$ (ii) $\dfrac{(15-n)!}{(13-n)!} = 12$

10. Find n if:

$$\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 24:1$$

11. Show that

$$\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)}$$

12. Show that

$$\frac{9!}{3!6!} + \frac{9!}{4!5!} = \frac{10!}{4!6!}$$

13. Find the value of :

(i) $\dfrac{8! + 5(4!)}{4! - 12}$ (ii) $\dfrac{5(26!) + (27!)}{4(27!) - 8(26!)}$

14. Show that

$$\frac{(2n)!}{n!} = 2^n (2n-1)(2n-3)...5.3.1$$

Let's Learn

We are now ready to learn the two most popular methods of counting: permutations and combinations. Permutations are ordered arrangements of some or all of the given objects, whereas combinations are selections of a specified number of objects from a given set of objects. Note that in the following discussion, objects can be persons, things, numbers, letters, symbols, or anything else.

6.3. PERMUTATIONS:

A common question in most of the word problems is to form different words, that may or may not have any meaning, using some or all of the letters in a given collection of letters. For example, consider the problem of finding the number of different words that can be formed by rearranging letters in the word "EXPAND". Another problem is about the number of different words that can be formed using any two letters in the word "EXPAND". The first question can be answered by writing down different rearrangements of the letters in the word "EXPAND" and then counting these rearrangements. The second question can be answered by first selecting any two letters from the word "EXPAND" and then arranging them in different possible ways. More precisely, the second question can also be answered by actually writing down all the possible two–letter words using the letters in the word "EXPAND". This gives us the following list: EX, XE, EP, PE, EA, AE, EN, NE, ED, DE, XP, PX, XA, AX, XN, NX, XD, DX, PA, AP, PN, NP, PD, DP, AN, NA, AD, DA, ND, DN. We find from this list that 30 different two–letter words can be made from the letters of the word "EXPAND". We note that the words XA and AX are different even though they contain the same letters. This is so because the order of the letters in the word is important. Such rearrangements of objects in different orders are called their permutations.

A permutation is formally defined as follows.

Definition (Permutation): A permutation is an ordered arrangement of some or all objects in a given group of objects.

The number of distinct permutations of r distinct objects chosen from a given collection of n distinct objects is denoted by nPr, ${}^{n}P_{r}$, or P(n,r).

6.3.1 Permutations when all objects are distinct:

Theorem 1. The number of permutations of n distinct objects taken r at a time, without repetitions, is $n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$.

Proof. The number of ways of arranging n distinct objects taken r at a time without repetitions is same as the number of ways r places can be filled using n objets.

For this, consider the following table.

Place	1st	2nd	3rd	...	r–2nd	r–1st	rth
Number of ways	n	n–1	n–2	...	[n–(r–3)]	[n–(r–2)]	[n–(r–1)]

Table 6.1

The table shows that the first place can be filled with any of the n objects. As the result, there are n ways of filling the the first place. After putting one object in the first place, only n–1 objects are available because repetitions are not allowed. Therefore, the second place can be filled in n–1 ways. After putting two distinct objects in the first two places, only n–2 objects are available for the third place, so that the third place can filled in n–2 ways.

Continuing in this way, after putting r–1 objects in the first r–1 places, the number of available objects is [n – (r–1)] for the r^{th} place. Hence, the r^{th} place can be filled in [n – (r–1)] ways.

Now, using the multiplication principle of counting, the total number of ways of filling r places using n distinct objects is given by the product:

$$n \times (n-1)\times(n-2)\times\ldots\times([n-(r-2)]\times[n-(r-1)]$$

$$= n \times(n-1)\times(n-2)\times\ldots\times(n-r+2)\times(n-r+1)$$

Hence ${}^{n}P_{r} = n\times(n-1)\times(n-2)\times\ldots\times(n-r+2)\times(n-r+1)$

If we multiply and divide this product by $(n-r)\times(n-r-1)\times\ldots\times 3\times 2\times 1$,

we find that

$${}^{n}\mathrm{Pr} = n\times(n-1)\times\ldots\times(n-r+1)\times\frac{(n-r)\times(n-r-1)\times\ldots\times3\times2\times1}{(n-r)\times(n-r-1)\times\ldots\times3\times2\times1}$$

$$=\frac{n\times(n-1)\times(n-2)\times\ldots\times(n-r+1)\times(n-r)\times(n-r-1)\times\ldots\times3\times2\times(n-r)\times(n-r-1)\times\ldots\times3\times2\times1}{(n-r)\times(n-r-1)\times\ldots\times3\times2\times1}$$

$$= \frac{n!}{(n-r)!}$$

We have thus found that

$${}^{n}\mathrm{P}_{r} = \frac{n!}{(n-r)!}, \text{ for } r \le n$$

The particular interesting case is when, all n objects are taken at a time. In this case, we obtain, by formula

$${}^{n}\mathrm{P}_{n} = n\times(n-1)\times(n-2)\times\ldots\times[n-(n-1)]$$

$$= n\times(n-1)\times(n-2)\times\ldots\times1$$

$= n!$

Alternatively, from the above formula, we obtain

$${}^{n}P_{n} = \frac{n!}{(n-n)!}$$

$$= \frac{n!}{0!}$$

$= n!$, since $0!=1$, by an earlier result.

6.3.5 Properties of Permutations

(i) $^nP_n = n!$

(ii) $^nP_0 = 1$

(iii) $^nP_1 = n$

(iv) $^nP_r = n\,{}^{(n-1)}P_{(r-1)}$

$= n(n-1)\,{}^{(n-2)}P_{(r-2)}$

$= n(n-1)(n-2)\,{}^{(n-3)}P_{(r-3)} = \ldots$

(vi) $\dfrac{^np_r}{^np_{r-1}} = n-r+1$

SOLVED EXAMPLES

Ex. 1 : Find the value of $^4P_2$ and interpret the answer.

Solution: $^4P_2 = \dfrac{4!}{(4-2)!} = \dfrac{4!}{2!} = \dfrac{4\times 3\times 2!}{2!}$

$= 4 \times 3 = 12.$

Interpretation four different objects can be arranged linearly in 12 ways when taken 2 at a time Suppose P_1, P_2, P_3, P_4 are the four objects. The permutations of the four objects, taken two at a time are given by (P_1,P_2), (P_1,P_3), (P_1,P_4), (P_2,P_1), (P_2,P_3), (P_2,P_4), (P_3,P_1), (P_3,P_2), (P_3,P_4), (P_4,P_1), (P_4,P_2), (P_4,P_3).

Ex. 2 : How many different ways are there to arrange letters of the word "WORLD? How many of these arrangements begin with the letter R? How many arrangements can be made taking three letters at a time?

Solution: The word WORLD has 5 letters W, O, R, L, D. These can be arranged among themselves in $^5P_5$=5!=120 different ways.

If an arrangement begins with R, the remaining four letters can be arranged in $^4P_4$=4!=24 ways.

The number of arrangements of 5 letters, taken three at a time, is $^5P_3 = \dfrac{5!}{2!} = 60$

Ex. 3 : How many two digit numbers can be formed from the digits 2,4,5,6,7 if no digit is repeated?

Solution: Every arrangement of digits gives a different number. Therefore, the problem is to find the number of arrangements of five digits taken two at a time This is given by $^5P_2 = \dfrac{5!}{3!} = 20$

Alternatively, a two digit number has one digit in the unit's place and one digit in the ten's place.

The ten's place can be filled with any of the five given digits in five different ways.

Since a digit cannot be repeated, the unit's place can be filled using any of the remaining four digits in four different ways.

The multiplication principle then gives the required number as 5 × 4 = 20

Ex. 4 : How many numbers can be formed with the digits 3,4,6,7,8 taken all at a time? Find the sum of all such numbers.

Solution: The five digits can be arranged in $^5P_5$ = 5! = 120 ways.

Now, consider any one of the five given digits, say 3. suppose the digit 3 is in the unit's place. The other four digits can be arranged in $^4P_4$ = 4! = 24 ways to form numbers that have 3 in the unit's place. This shows that 24 of the 120 numbers have 3 in the unit's place.

Similarly, each of the other four digits is in the unit's place in 24 of the 120 numbers.

The sum of the digits in the units place among all 120 numbers is 24 (3+4+6+7+8) = 24×28 = 672.

Similary, the sum of the digits in the ten's place among all 120 numbers is 672. The same is also the sum of the digits in each place among all 120 numbers.

The required sum is then given by 672 (1+10+100+1000+10000)

= 672 × 11111 = 74,66,592

Ex. 5 : A teacher has 2 different books on English, 3 different books on physics, and 4 different books on Mathematics. These books are to be placed in a shelf so that all books on any one subjects are together. How many different ways are there to do this?

Solution: First, let us consider all books on each subjects to be one book, so that there are three books, say E,P,M. These three can be arranged in ${}^3P_3$ = 3! = 6 different ways. Now, in each of these ways, the 2 books on English can be arranged in ${}^2P_2$ = 2! = 2 different ways the 3 books on physics can be arranged in ${}^3P_3$ =3! = 6 different ways, and the four books on Mathematics can be arranged in ${}^4P_4$ = 4! = 24 different ways.

The required number of arrangments is then given by 6×2×6×24 = 1728

Ex. 6 : Find n if ${}^nP_5 = 42 \times {}^nP_3$

Solution: We are given that ${}^nP_5 = 42 \times {}^nP_3$

That is, $\frac{n!}{(n-5)!} = 42\frac{n!}{(n-3)!}$

$$\therefore \frac{n!}{(n-5)!} = 42\frac{n!}{(n-3)(n-4)(n-5)!}$$

$$\therefore (n-3)(n-4) = 42$$

$$\therefore n^2 - 7n - 30 = 0$$

$$\therefore (n-10)(n+3) = 0$$

$$\therefore n = 10 \text{ or } n = -3$$

Since n cannot be negative, n=10

6.3.2 Permutations when repetitions are allowed:

We now consider problems of arranging n distinct objects taken r at a time when repetitions are allowed.

Theorem 2. The number of permutations of n distinct objects taken r at a time, when repetitions are allowed, is same as the number of ways of filling r places using n distinct objects when repetitions are allowed. Consider the following table.

Place	1^{st}	2^{nd}	3^{rd}	...	$(r-2)^{th}$	$(r-1)^{th}$	r^{th}
Number of ways	n	n	n	...	n	n	n

Table 6.2

The first place can be filled in n ways because it can be filled with any of the n distinct objects. After putting one object in the first place, the number of distinct objects available for filling the second place is still n because repetitions are allowed. Therefore, the second place can also be filled in n ways. Similarly, after putting two objects in the first two places, the third place can be filled in n ways since repetitions are allowed and hence all the n distinct objects are still available. Continuing in this way, we notice that every place up to the r^{th} can be filled in n ways because repetitions are allowed.

As the result of the above argument, it can be concluded that the number of permutations of n distinct objects taken r at a time, when repetitions are allowed, is given by

n×n×....×n(r times) = n^r

Remarks:

1. The number of permutations of n distinct objects taken all at a time, when m specified objects always come together, is $m! \times (n-m+1)!$.

2. The number of permutations of n distinct objects, taken all at a time, when m specified objects are never together is $(n-m)(n-m+1)!$

3. The number of permutations of n distinct objects taken r at a time, when a specified object is always to be included, in each arrangement is.

   $r \times {}^{(n-1)}P_{(r-1)}$

4. The number of permutations of n distinct objects taken r at a time, when a specified object is not to be included in any permutation, is ${}^{(n-1)}P_r$

SOLVED EXAMPLES

Ex. 1 : It is required to arrange 8 books on a shelf. Find the number of ways to do this if two specified books are

(i) always together

(ii) never together.

Solution :

(i) The number of ways in which two specified books are always together is $2\times(8-1)!=2\times7! = 10080$

(ii) The number of ways in which two specified books are never together is $(8-2)\ (8-1)! = 6\times7! = 30240$

Ex. 2 : In how many ways can 7 examination papers be arranged so that papers 6 and 7 are never together?

Solution : The number of ways in which any two papers are never together is.

$(7-2)\ (7-1)! = 5\times6! = 3600$

Ex. 3 : A family of 3 brothers and 5 sisters is to be arranged for a photograph in such a ways that.

(i) all brothers sit together.

(ii) no two brothers sit together.

Solution :

(i) Since all 3 brothers are together, treat them as one person, so that there are 5+1=6 persons. the number of arranging them is

${}^{6}P_{6} = 6! = 720.$

Once this is done, the three brothers can be arranged among themselves in ${}^{3}P_{3} = 3! = 6$ ways.

The Total number of arrangements is then given by $6!\times3! = 4320$.

(ii) 5 Sister can be arranged among themselves in ${}^{5}P_{5} = 5! = 120$ ways. consider the following arrangement.

$* \ S_1 * S_2 * S_3 * S_4 * S_5 *$

Where * indicates a position where one brother can be placed so that no two brothers are together. Since there are 6 positions and 3 brothers, the numbers of arrangements is

$${}^{6}P_{3} = \frac{6!}{3!} = 120$$

The required number of arrangements is then given by

${}^{5}P_{5} \times {}^{6}P_{3} = 120 \times 120$

$= 14400.$

It is possible that some of the n objects are always kept together in a permutation problem.

EXERCISE 6.3

1. Find n if ${}^{n}P_6 : {}^{n}P_3 = 120:1$

2. Find m and n if ${}^{(m+n)}P_2 = 56$ and ${}^{m-n}P_2 = 12$

3. Find r if ${}^{12}P_{r-2} : {}^{11}P_{r-1} = 3:14$

4. Show that $(n+1)\ {}^{n}P_r = (n-r+1)\ {}^{(n+1)}P_r$

5. How many 4 letter words can be formed using letters in the word MADHURI if (i) letters can be repeated? (ii) letters cannot be repeated?

6. Determine the number of arrangements of letters of the word ALGORITHM if.
   (i) vowels are always together.
   (ii) no two vowels are together.
   (iii) consonants are at even positions.
   (iv) O is first and T is last.

7. In a group photograph, 6 teachers are in the first row and 18 students are in the second row. There are 12 boys and 6 girls among the students. If the middle position is reserved for the principal and if no two girls are together, find the number of arrangements.

8. Find the number of ways in which letters of the word HISTORY can be arranged if
   (i) Y and T are together
   (ii) Y is next to T.

9. Find the number of arrangements of the letters in the word BERMUDA so that consonents and vowels are in the same relative positions.

10. Find the number of ways of 4–digit numbers that can be formed using the digits 1,2,4,5,6,8 if
   (i) digits can be repeated
   (ii) digits cannot be repeated

11. How many numbers can be formed using the digits 0,1,2,3,4,5 without repetition so that resulting numbers are between 100 and 1000?

12. Find the number of ways of 6–digit numbers using the digits 3,4,5,6,7,8 without repetition. How many of these numbers are
   (i) divisible by 5?
   (ii) not divisible by 5?

13. A code word is formed by two distinct English letters followed by two non–zero distinct digits. Find the number of such code words. Also, find the number of such code words that end with an even digit.

14. Find the number of ways in which 5 letters can be posted in 3 post boxes if any number of letters can be posted in a post box.

15. Find the number of arranging 11 distinct objects taken 4 at a time so that a specified object.
   (i) always occurs
   (ii) never occurs.

6.3.3 Permutations when all objects are not distinct

Consider the problem of rearranging letters in the words like GOOD, INDIA, GEOLOGY, MATHEMATICS, or PHILOSOPHY, where some letter occur more than once and hence all letters are not distinct.

Can we solve this problem? If we think so, how can we claim that we have obtained the right answer? The following result can provide us with a justification to our claim.

Theorem. Consider a collection of n objects, not all distinct, where n_1 objects are of one type and the remaining $n-n_1$ are distinct. The number of permutations of these n objects taken all at a time is $\dfrac{n!}{n_1!}$ provided $n_1 \le n$.

Remarks.

1. The number of permutations of n objects, not all distinct, where n_1 objects are of one type and n_2 objects are of a second type, taken all at a time is

$$\frac{n!}{n_1!n_2!} \text{ provided } n_1 + n_2 \leq n.\ .$$

2. The number of permutations of n objects, not all distinct, where n_i objects are of type i, i=1, 2, …, k, taken all at a time is $\frac{n!}{n_1!n_2!...n_k!}$ provided $n_1+ n_2+...n_k \leq n$.

SOLVED EXAMPLES

Ex. 1 : Find the number of distinct words that can be formed from the letters of the word "COMMON".

Solution : The given word "COMMON" has six letters, but two letters, namely "O" and "M" are repeated twice each. Therefore, the required number is

$$\frac{6!}{2!2!} = 180$$

Ex. 2 : A coin is tossed eight times. In how many ways can we obtain

(a) 5 heads and 3 tails?

(b) at least 6 heads?

(c) at most 3 tails?

Solution : (a) $\frac{8!}{5!3!}$

(b) $\frac{8!}{6!2!} + \frac{8!}{7!} + \frac{8!}{8!}$

$= 28 + 8 + 1 = 37$

(c) $\frac{8!}{8!} + \frac{8!}{7!} + \frac{8!}{2!6!} + \frac{8!}{3!5!}$

$= 1 + 8 + 28 + 56 = 93$

Ex. 3 : How many distinct 8 digit numbers can be formed from digits in the number 62270208?

Solution : The given number has 8 digits. The digit 0 occurs twice and 2 occurs three times in this number. Therefore, the required number is

$$\frac{8!}{2!3!} = 3360$$

EXERCISE 6.4

1. Find the number of permutations of letters in each of the following words.
   (i) DIVYA
   (ii) SHANTARAM
   (iii) REPRESENT
   (iv) COMBINE

2. You have 2 identical books on English, 3 identical books on Hindi, and 4 identical books on Mathematics. Find the number of distinct ways of arranging them on a shelf .

3. A coin is tossed 8 times. In how many ways can we obtain. (i) 4 heads and 4 tails? (ii) at least 6 heads?

4. A bag has 5 red, 4 blue, and 4 green marbles. If all are drawn one by one and their colours are recorded, how many different arrangements can be found?

5. Find the number of ways of arranging letters of the word MATHE MATICAL How many of these arrangements have all vowels together?

6. Find the number of different arrangements of letters in the word MAHARASHTRA. How many of these arrangements have (a) letters M and T never together? (b) all vowels together?

7. How many different words are formed if the letters R is used thrice and letters S and T are used twice each?

8. Find the number of arrangements of letters in the word MUMBAI so that the letter B is always next to A.

9. Find the number of arrangements of letters in the word CONSTITUTION that begin and end with N.

10. Find the number of different ways of arranging letters in the word ARRANGE. How many of these arrangements the two R's and two A's are not together?

11. How many distinct 5 digit numbers can be formed using the digits 3,2,3,2,4,5

12. Find the number of distinct numbers formed using the digits 3,4,5,6,7,8,9, so that odd positions are occupied by odd digits.

13. How many different 6–digit numbers can be formed using digits in the number 659942? How many of them are divisible by 2?

14. Find the number of district words formed from letters in the word INDIAN. How many of them have the two N's togethers?

15. Find the number of different ways of arranging letters in the word PLATOON if. (i) the two O's are never together. (ii) consonents and vowels occupy alternate positions.

6.3.4 Circular Permutations:

The arrangements of objects considered so far have been on a straight line. Sometimes arrangements are to be made around a circle, as would be required for a round table dinner or a round table meeting. Consider the following problem. Ramesh, Suresh, and Dinesh are to be seated for dinner at a round table. How many different ways can they be seated? Is this number the same as the number of ways they can be seated if they are to be seated in a row?

Before attempting to answer these questions, consider the following arrangements.

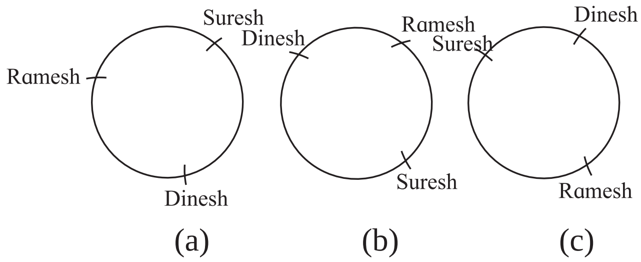

Fig. 6.5

Can we claim that the seating arrangements in Fig. 6.5 (b) and Fig. 6.5 (c) are different from the seating arrangement in Fig. 6.5 (a)? How can we justify our answer?

We may find that our answer to the question is negative. That is, the seating arrangements in Fig, 6.5(b) and Fig. 6.5(c) are not different from the seating arrangement in Fig. 6.5(a). This is justified by the fact that the relative positions of the three friends are the same in all the three seating arrangements. The only difference between the three seating arrangements is that the seats of the three friends have been rotated (either clockwise or anticlockwise) to obtain one arrangement from another.

If we want to have different relative positions of the three friends, we must consider the following two seating arrangements.

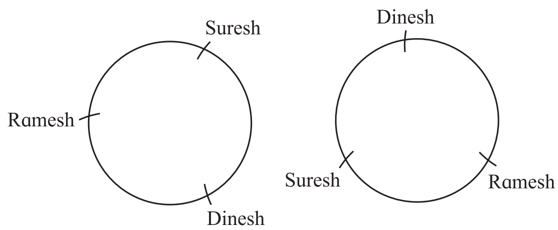

Fig. 6.6

In general, if n distinct objects are to be arranged around a circle, then n different linear arrangements coincide with a single circular arrangement. Since there are n! different linear arrangements of n distinct objects taken all at a time, the number of circular arrangements of n distinct objects taken all at a time is given by

$$\frac{n!}{n} = (n-1)!$$

Remarks:

1. It may be easy to understand the above formula if we arrange the n distinct objects as follows. A circular arrangement does not have a fixed starting point, and hence rotations of any one arrangement appear to be the same. In order to fix the starting point, fix the position of one of the n distinct objects, and then arrange the remaining n–1 distinct objects. Now, we have the problem of arranging n–1 distinct objects taken all at a time, the number of such arrangements is (n–1)!

2. If clockwise and anticlockwise arrangements that are mirror images of each other are not to be considered different, then the number of circular arrangements of n distinct objects is $\frac{(n-1)!}{2}$

Now consider the situation where m of the n objects are alike and cannot be distinguished from one another. The following result gives the formula for the number of circular arrangements of these n objects.

Theorem. The number of circular arrangements of n objects, of which m objects are alike (indistinguishable), is given by $\frac{(n-1)!}{m!}$

Does this argument remind us a similar argument that we came across earlier?

Remark: The number of circular permutations of n distinct objects taken r at a time can be found under two ddifferent conditions as follows.

(a) When clockwise and anticlockwise arrangements are considered to be different, then the required number of circular arrangements is given by $\frac{{}^nP_r}{r}$

(b) When clockwise and anticlockwise arrangements are not to be considered different, then the required number of circular arrangements is given by $\frac{{}^nP_r}{2r}$

We may try to verify these two statements.

SOLVED EXAMPLES

Ex. 1 : In how many ways can 8 students be seated at a round table so that two particular students are together if

students are seated with respect to each other?

Solution : Treat the two particular students as a single student, so we have 7 students.

students are seated with respect to each other means one student is seated first and other students are seated in positions relative to him. Since there are 6 students to be seated with respect to the first student, there are 6! ways of seating them. The two students who are together can be seated in 2! = 2 distinct ways. Therefore, the required number is

$6! \times 2! = 1440$

Ex. 2 : Find the number of distinct ways of seating 6 men and 6 women around a table when

(a) there is no restriction

(b) no two women are seated next to each other.

Solution (a) : When there is no restriction, there are 12 persons to be seated around a table. The number of possible arrangements is $(12-1)! = 11!$

(b) The six men can be seated around a table in $(6-1)! = 5!$ ways. Since no two women sit next to each other, a woman must be seated between two men. There are 6 spaces between men and there are six women. Therefore, the required number is $(5!)\ (6!) = 86400$.

EXERCISE 6.5

1. In how many different ways can 8 friends sit around a table?

2. A party has 20 participants and a host. Find the number of distinct ways for the host to sit with them around a circular table. How many of these ways have two specified persons on either side of the host?

3. Delegates from 24 countries participate in a round table discussion. Find the number of seating arrangments where two specified delegates are. (i) always together, (ii) never together.

4. Find the number of ways for 15 people to sit around the table so that no two arrangements have the same neighbours.

5. A committee of 20 members sits around a table. Find the number of arrangements that have the president and the vice president together.

6. Five men, two women, and a child sit around a table. Find the number of arrangements where the child is seated (i) between the two women. (ii) between two men.

7. Eight men and six women sit around a table. How many of sitting arrangements will have no two women together?

8. Find the number of seating arrangements for 3 men and 3 women to sit around a table so that exactly two women are together.

9. Four objects in a set of ten objects are alike. Find the number of ways of arranging them in a circular order.

10. Fifteen persons sit around a table. Find the number of arrangements that have two specified persons not sitting side by side.

6.4 Combinations

We may have noticed that permutations involve ordered arrangements of objects. We shall now consider situations where the order of objects in an arrangement is not important, but only selection of objects in the arrangement is important. A selection of objects without any consideration for the order is called a combination.

Let us consider the following example. A group of 5 students, denoted by S_1, S_2, S_3, S_4, S_5, is available for selection and a team of two of these students is to be selected for a doubles game. In how many ways can the team be selected?

Let us write down all the possible team selections, so that we can count them. Do we get the following teams?

S_1 and S_2, S_1 and S_3, S_1 and S_4, $S_1$and S_5, S_2 and S_3, S_2 and S_4, S_2 and S_5, S_3 and S_4, S_3 and S_5, S_4 and S_5. Is "S_2 and S_1" a different selection from "S_1 and S_2"? Does the order of selection play any role here?

When we consider a team of two players, the team "S_1 and S_2" is same as the team "S_2 and S_1". This is so because only selection of two players

S_1 and S_2 is enough to form the team. The order in which the two players are selected is immaterial. In other words, the order of selection does not play any role in this situation. We may notice that "S_1 and S_2" and "S_2 and S_1" are two different permutations, but the same selection. The same conclusion can be drawn about all other pairs listed above. Such selections, where the order of selection is immaterial, are called combinations. Let us now have the formal definition of a combination.

Definition: Combination. A combination of a set of n distinct objects taken r at a time without repetition is an r–element subset of the n objects.

Note: The arrangement of the elements does not matter in a combination.

The number of combinations of n distinct objects taken r at a time without repetition is denoted by ${}_nC_r$, ${}^nC_r$, or C(n,r).

Let us go back to the example of selecting a team of 2 students from the group of 5. This time, however, we want to select a team of 3 students. Can we make a list of all possible selections? Let us try and confirm that these selections are as follows.

S_1, S_2 and S_3; S_1, S_2 and S_4; S_1, S_2 and S_5; S_1, S_3 and S_4; S_1, S_3 and S_5; S_1, S_4 and S_5; S_2, S_3 and S_4; S_2, S_3 and S_5; S_2, S_4 and S_5; S_3,S_4 and S_5.

What is interesting to note is the number of possible teams in both the examples is 10. In other words, what we have found is that ${}^5C_2 = {}^5C_3 = 10$. Can we explain why this happens?

Let us consider selection of a team of 3 students from the group of 5. When we select 3 students, we leave 2 students out of the team. Therefore, selecting a team of 3 students from the group of 5 is same as selecting 2 students (to left out of the team) from the group of 5.

The question that may arise now is: "Is such a relation true in general?" This question can be answered when we learn the mathematical formula for the number of combinations of n distinct objects taken r at a time without repetition. The following theorem gives this formula.

Theorem. The number of combinations of n distinct objects taken r at a time wirhout repetitions is given by ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

The relation between ${}^nP_r$ and ${}^nC_r$ is

${}^nC_r \times r! = {}^nP_r$

6.4.1 Properties of combinations.

1. $$\begin{aligned} {}^nC_{n-r} &= \frac{n!}{(n-r)![n-(n-r)]!} \\ &= \frac{n!}{(n-r)!r!} \\ &= {}^nC_r \end{aligned}$$

$\therefore {}^nC_r = {}^nC_{n-r}$ for $0 \le r \le n$

2. ${}^nC_0 = \dfrac{n!}{0!(n-0)!} = \dfrac{n!}{n!} = 1$, because 0! = 1 as stated earlier.

3. If ${}^nC_r = {}^nC_s$, then either s = r or s = n–r.

4. ${}^nC_r = \dfrac{{}^nP_r}{r!}$

5. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

6. ${}^nC_0 + {}^nC_1 + \ldots {}^nC_n = 2^n$

7. ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots$

   $= {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = 2^{(n-1)}$

8. ${}^nC_r = \dfrac{n}{r}\,{}^{n-1}C_{r-1}$

   $= \dfrac{n}{r}\,\dfrac{n-1}{r-1}\,{}^{n-2}C_{r-2} = \ldots..$

SOLVED EXAMPLES

Ex.1 : Find the values of

(i) ${}^{7}C_3$ (ii) ${}^{10}C_7$ (iii) ${}^{52}C_3$

Solution : We know that

$${}^{n}C_r = \frac{n!}{r!(n-r)!}$$

(i) $${}^{7}C_3 = \frac{7!}{3!(7-3)!}$$

$$= \frac{7!}{3!4!}$$

$$= \frac{7\times6\times5\times4!}{3\times2\times1\times4!}$$

$$= \frac{7\times6\times5}{3\times2\times1} = 35$$

(ii) $${}^{10}C_7 = \frac{10!}{7!(10-7)!}$$

$$= \frac{10!}{7!3!}$$

$$= \frac{10\times9\times8}{3\times2}$$

$$= 120$$

(iii) $${}^{52}C_3 = \frac{52!}{3!(52-3)!}$$

$$= \frac{52!}{3!49!}$$

$$= \frac{52\times51\times50}{3\times2\times1} = 22100$$

Ex.2 : Find n and r, if

$${}^{n}C_{r-1} : {}^{n}C_r : {}^{n}C_{r+1} = 14 : 8 : 3$$

Solution : We know that

$$\therefore {}^{n}C_{r-1} = \frac{n!}{(r-1)!(n-r+1)!}$$

$$\therefore {}^{n}C_r = \frac{n!}{r!(n-r)!}$$

$${}^{n}C_{r+1} = \frac{n!}{(r+1)!(n-r-1)!}$$

$$\frac{{}^{n}C_{r-1}}{{}^{n}C_r} = \frac{r}{(n-r+1)} \text{ and } \frac{{}^{n}C_r}{{}^{n}C_{r+1}} = \frac{r+1}{n-r}$$

$$\frac{{}^{n}C_{r-1}}{{}^{n}C_r} = \frac{14}{8} = \frac{7}{4} \text{ and } \frac{{}^{n}C_r}{{}^{n}C_{r+1}} = \frac{8}{3}$$

$$\therefore \frac{r}{n-r+1} = \frac{7}{4} \text{ and } \frac{r+1}{n-r} = \frac{8}{3}$$

$\therefore$ $7n - 11r + 7 = 0$ - (i)

$\therefore$ $8n - 11r - 3 = 0$ - (ii)

Solving this, we get $r = 7$ and $n = 10$.

Ex.3 : A plane consists of n points. Find the number of straight lines and triangles that can be obtained by joining points in a plane, if

(i) no three points are collinear.

(ii) p points are collinear ($p \geq 3$).

Solution :

(i) Straight line can be drawn by joining any two points and triangle can be drawn by joining any three non-collinear points.

From n points, any two points can be selected in ${}^{n}C_2$ ways.

$\therefore$ Number of straight lines = ${}^{n}C_2$

Since no three points are collinear, any three points can be selected in ${}^{n}C_3$ ways.

$\therefore$ Number of triangles = ${}^{n}C_3$

(ii) If we are given that p-point are non-collinear, we can obtain ${}^pC_2$ straight lines and ${}^pC_3$ triangles. But we are given that p-points are collinear, therefore they form only one line and no triangle i.e. we have counted ${}^pC_2 - 1$ extra lines and ${}^pC_3$ extra triangles.

If p-points ($p \geq 3$) are collinear, then number of straight lines = ${}^nC_2 - ({}^pC_2 - 1)$ and number of triangles = ${}^nC_3 - {}^pC_3$

Ex.4 : Four cards are drawn from a pack of 52 playing cards. In how many different ways can this be done? How many selections will contain

(i) exactly one card of each suit ?

(ii) all cards of the same suit ?

(iii) all club cards ?

(iv) at least one club card ?

(v) three kings and one queen ?

(vi) three black and one red card?

Solution :

(i) A pack of 52 cards contains 4 different suits. viz. club, spade, diamond and heart. Each suit contains 13 cards. Club and spade are black coloured cards. Diamond and heart are red coloured cards i.e. pack of 52 card contains 26 black and 26 red colour cards.

From a pack of 52 cards any 4 cards can be drawn in ${}^{52}C_4 = \dfrac{52!}{4! \times 48!}$ ways.

(i) Exactly one card of each suit.

One club card can be selected in ${}^{13}C_1 = 13$ ways. One spade card can be selected in ${}^{13}C_1 = 13$ ways. One diamond card can be selected in ${}^{13}C_1 = 13$ ways. One heart card can be selected in ${}^{13}C_1 = 13$ ways.

$\therefore$ Using fundamental principal, exactly one card of each suit can be selected in $13 \times 13 \times 13 \times 13 = (13)^4$ ways.

(ii) All cards are of the same suit.

From 4 suits, any one suit can be selected in ${}^4C_1 = 4$ ways. After this is one done, any four cards from selected suit can be drawn in ${}^{13}C_4 = 715$ ways.

$\therefore$ Using fundamental principal, all 4 cards of the same suit can be drawn in $4 \times {}^{13}C_4 = 2860$ ways.

(iii) All club cards

From 13 club cards, any 4 club cards can be drawn in ${}^{13}C_4 = 715$ ways.

(iv) At least one club.

From a pack of 52 cards, any 4 cards can be drawn in ${}^{52}C_4 = \dfrac{52!}{4! \times 48!}$ ways if there is no condition.

From ${}^{52}C_4$ selections remove those selections that do not contain any club card, so that in the remaining selections we have at least one club card i.e. 4 cards are drawn from remaining 39 non-club cards and this can be done in ${}^{39}C_4$ ways.

$\therefore$ Number of selections which contain at least one club card = ${}^{52}C_4 - {}^{39}C_4$

(v) Three king cards and one queen card.

From 4 kings, any 3 king cards can be selected in ${}^4C_3 = 4$ ways and from 4 queen cards 1 queen card can be selected in ${}^4C_1 = 4$ ways.

$\therefore$ Using fundamental principle, three king cards and one queen card can be selected in $4 \times 4 = 16$ ways.

(vi) Three black cards and one red card.

From 26 black cards, any 3 black cards can be selected in ${}^{26}C_3$ ways and from 26 red cards, any one red card can be selected in ${}^{26}C_1$ ways.

$\therefore$ Using fundamental principle, three black and one red card can be selected in ${}^{26}C_3 \times {}^{26}C_1$ ways.

Ex.5 : Find n, if ${}^nC_8 = {}^nC_6$

Solution : If ${}^nC_x = {}^nC_y$

then either $x = y$ or $x = n - y$

$\therefore\ 8 = n - 6 \quad \therefore\ n = 14$

Find r,

If ${}^{16}C_4 + {}^{16}C_5 + {}^{17}C_6 + {}^{18}C_7 = {}^{19}C_r$

Solution :

$({}^{16}C_4 + {}^{16}C_5) + {}^{17}C_6 + {}^{18}C_7 = {}^{19}C_r$

$\therefore\quad ({}^{17}C_5 + {}^{17}C_6) + {}^{18}C_7 = {}^{19}C_r$

$\therefore\quad ({}^{18}C_6 + {}^{18}C_7) = {}^{19}C_r$

$\therefore\quad {}^{19}C_7 = {}^{19}C_r$

$\therefore\ r = 7$ or $r = 19 - 7 = 12$

Ex.6 : Find the difference between the maximum values of ${}^8C_r$ and ${}^{11}C_r$

Solution : Maximum values of ${}^8C_r$ occurs at

$$r = \frac{8}{2} = 4 \quad \left[r = \frac{n}{2} \text{ if n is even}\right]$$

$\therefore$ maximum values of ${}^8C_r = {}^8C_4 = 70$

Maximum values of ${}^nC_r$ occurs at

$$r = \frac{10}{2} = 5 \text{ or at } \frac{12}{2} = 6 \quad \left[\begin{array}{c} r = \frac{n-1}{2} \text{ or } r = \frac{n+1}{2} \\ \text{if n is odd} \end{array}\right]$$

$\therefore$ Maximum values of ${}^{11}C_r = {}^{11}C_5 = {}^{11}C_6 = 462$

$\therefore$ difference between the maximum values of ${}^8C_r$ and ${}^{11}C_r = {}^8C_4 - {}^{11}C_5 = 70 - 462 = -392$

OR ${}^{11}C_5 - {}^8C_4 = 462 - 70 = 392$

EXERCISE 6.6

1. Find the value of (i) ${}^{15}C_4$ (ii) ${}^{80}C_2$

   (iii) ${}^{15}C_4 + {}^{15}C_5$ (iv) ${}^{20}C_{16} - {}^{19}C_{16}$

2. Find n if

   (i) ${}^6P_2 = n\ {}^6C_2$

   (ii) ${}^{2n}C_3 : {}^nC_2 = 52 : 3$

   (iii) ${}^nC_{n-3} = 84$

3. Find r if ${}^{14}C_{2r} : {}^{10}C_{2r-4} = 143:10$

4. Find n and r if.

   (i) ${}^nP_r = 720$ and ${}^nC_{n-r} = 120$

   (ii) ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 20: 35:42$

5. If ${}^nP_r = 1814400$ and ${}^nC_r = 45$, find r

6. If ${}^nC_{r-1} = 6435$, ${}^nC_r = 5005$, ${}^nC_{r+1} = 3003$ find ${}^rC_5$.

7. Find the number of ways of drawing 9 balls from a bag that has 6 red balls, 5 green balls, and 7 blue balls so that 3 balls of every colour are drawn.

8. Find the number of ways of selecting a team of 3 boys and 2 girls from 6 boys and 4 girls.

9. After a meeting, every participant shakes hands with every other participants. If the number of handshakes is 66, find the number of participants in the meeting.

10. If 20 points are marked on a circle, how many chords can be drawn?

11. Find the number of diagonals of an n–sided polygon. In particular, find the number of diagonals when.

    (a) n = 10 (b) n = 15

    (c) n = 12

12. There are 20 straight line in a plane so that no two lines are parallel and no three lines are concurrent. Determine the number of points of intersection.

13. Ten points are plotted on a plane. Find the number of straight lines obtained by joining these points if

    (i) no three points are collinear.

    (ii) four points are collinear.

14. Find the number of triangles formed by joining 12 points if

    (i) no three points are collinear

    (ii) four points are collineas.

15. A word has 8 consonants and 3 vowels. How many distinct words can be formed if 4 consonents and 2 vowels are chosen?

EXERCISE 6.7

1. Find n if ${}^{n}C_{8} = {}^{n}C_{12}$
2. Find n if ${}^{23}C_{3n} = {}^{23}C_{2n+3}$
3. Find n if ${}^{21}C_{6n} = {}^{21}C_{n^2+5}$
4. Find n if ${}^{2n}C_{r-1} = {}^{2n}C_{r+1}$
5. Find if ${}^{n}C_{n-2}=15$
6. Find x if ${}^{n}P_{r} = x\ {}^{n}C_{r}$
7. Find r if ${}^{11}C_{4} + {}^{11}C_{5} + {}^{12}C_{6} + {}^{13}C_{7} = {}^{14}C_{r}$
8. Find the value of $\sum_{r=1}^{4} {}^{21-r}C_{4} + {}^{17}C_{5}$
9. Find the differences between the largest values in the following:

   (i) ${}^{14}C_{r} - {}^{12}C_{r}$, (ii) ${}^{13}C_{r} - {}^{8}C_{r}$, (iii) ${}^{15}C_{r} - {}^{11}C_{r}$,

10. In how many ways can a boy invite his 5 friends to a party so that at least three join the party?
11. A group consists of 9 men and 6 women. A team of 6 is to be selected. How many of possible selections will have at least 3 women?
12. A committee of 10 persons is to be formed from a group of 10 women and 8 men. How many possible committees will have at least 5 women? How many possible committee will have men in majority?
13. A question paper has two sections. section I has 5 questions and section II has 6 questions. A student must answer at least two question from each section among 6 questions he answers. How many different choices does the student have in choosing questions?
14. There are 3 wicketkeepers and 5 bowlers among 22 cricket players. A team of 11 players is to be selected so that there is exactly one wicketkeeper and at least 4 bowlers in the team. How many different teams can be formed?
15. Five students are selected from 11. How many ways can these students be selected if.

    (a) two specified students are selected?

    (b) two specified students are not selected?

Let's Remember

- **Addition principle -** If one event can occur in m ways and the second event with no common outcomes can occur out in n ways, then either of the two events can occur in $m+n$ ways.

- **Multiplication principle -** If one operation can be carried out in m ways, followed by the second operation that can be carried out in n ways, then the two operations can be carried out in $m \times n$ ways.

- **Factorial Function -** For a natural number n, the factorial of n, written as $n!$ and read as "n factorial" is the product of first n natural numbers.

  That is, $n! = 1 \times 2 \times ... \times (n-1) \times n$.

- **Permutations -** A permutation is an ordered arrangement of some or all objects in a given group of objects.

  The number of distinct permutations of r distinct objects chosen from a given collection of n distinct objects is denoted by nPr, ^nP_r, or P(n,r).

$$^nP_r = \frac{n!}{(n-r)!} \quad \text{for } r \leq n.$$

- The number of permutations of n distinct objects taken all at a time when 2 specified objects always come together, is $2 \times (n-1)!$

- The number of permutations of n distinct object taken r at a time, when a specified object is always to be included, is $r \times {}^{n-1}P_{r-1}$. The number of permutations of n distinct objects taken r at a time, when n object specified object is not be inculded in any permutation, is $^{n-1}P_r$.

- **Permutations when all objects are not distinct-** The number of permutation of n objects, not all distinct, where n_1 objects are of type i, i=1, 2, ..., k, taken all at a time is

$$\frac{n!}{n_1!n_2!........n_k!} \text{ provided } n_1 + n_2 + ... n_k \leq n.$$

Circular Permutations -

- It may be easy to understand the above formula if we arrange the n distinct object as follows. A circular arrangement does not have a fixed starting point, and hence rotations of any one arrangement appear to be the same. In order to fix the starting point, fix the position of one of the n distinct objects, and then arrange the remaining n-1 distinct objects. Now, we have the problem of arranging n-1 distinct objects taken all at a time the number of such arrangements is (n-1)!

- If clock wise and anticlockwise arrangement that are mirror image of each other are not to be considered different, then the number of circular arrangements of n distinct objects is

$$\frac{n!}{(n-r)!}$$

- The number of circular arrangement of n objects, of which m objects are alike (indistinguishable, is given by $\frac{(n-1)!}{m!}$

- When clock wise and anti clock wise arrangements are considered to be different, then the required number of circular

- arrangements is given by $\frac{^nP_r}{r}$

- When clock wise and anti clock wise arrangements are not to be considered different, then the required number of circular arrangements is given by $\frac{^nP_r}{2r}$

Properties of Permutations

- $^nP_n = n!$
- $^nP_0 = 1$
- $^nP_1 = n$
- $^nP_r = n\, {}^{n-1}P_{r-1} = n(n-1)\, {}^{(n-2)}P_{r-2}$
  $= n(n-1)(n-2)\, {}^{(n-3)}P_{r-3} = ...$
  $$\frac{^nP_r}{^nP_{r-1}} = n - r + 1$$

- **Combinations -** The arrangement of the elements does not matter in a combination. The number of combinations of n distinct objects taken r at a time without repetition is denoted by ${}^nC_r$, nCr, or C(n,r).

- $${}^nC_r = \frac{n!}{r!(n-r)!}$$
- ${}^nC_r \times r! = {}^nP_r$

Properties of combinations.

- Thus, ${}^nC_{n-r} = {}^nC_r$ for $0 \le r \le n$.
- $${}^nC_o = \frac{n!}{0!(n-0)!} = \frac{n!}{n!} = 1, \text{ Because } 0! = 1$$ as has been stated earlier.
- $${}^nC_n = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = 1,$$ again because 0! = 1
- If ${}^nC_r = {}^nC_s$, then either s = r or s = n –r.
- $${}^nC_r = \frac{{}^np_r}{r!}$$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_o + {}^nC_1 + \ldots {}^nC_n = 2^n$
- ${}^nC_o + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots$ $= 2^{n-1}$

MISCELLANEOUS EXERCISE - 6

1. Find the value of r if ${}^{56}P_{r+2} : {}^{54}P_{r-1} = 30800:1$
2. How many words can be formed by writing letters in the word CROWN in different order?
3. Find the number of words that can be formed by using all the letters in the word REMAIN If these words are written in dictionary order, what will be the 40th word?
4. Find the number of ways of distributing n balls in n cells. What will be the number of ways if each cell must be occupied?
5. Thane is the 19th station from C.S.T. If a passenger can purchase a ticket from any station to any other station, how many different ticket must be available at the booking window?
6. English alphabet has 11 symmetric letters that appear same when looked at in a mirror. These letters are A, H, I, M, O, T, U, V, W, X, and Y. How many symmentric three letter password can be formed using these letters?
7. How many numbers formed using the digits 3,2,0,4,3,2,3 exceed one million?
8. Ten students are to be selected for a project from a class of 30 students. These are 4 students who want to be together either in the project or not in the project. Find the number of possible selections.
9. A student find 7 books of his interest, but can borrow only three books. He wants to borrow chemistry part II book only if Chemistry Part I can also be borrowed. Find the number of ways he can choose three books that he can borrow.
10. 30 objects are to be divided in three groups containing 7,10,13 objects. Find the number of distinct ways for doing so.
11. A Student passes an examination if he/she secures a minimum in each of the 7 subjects. Find the number of ways a student can fail.
12. Nine friends decide to go for a picnic in two groups. One group decides to go by car and the other group decides to go by train. Find the number of different ways of doing so if there must be at least 3 friends in each group.

13. Five balls are to be placed in three boxes, where each box can contain upto five balls. Find the number of ways if no box is to remain empty.
14. A hall has 12 lamps and every lamp can be switched on independently. Find the number of ways of illuminating the hall.
15. How many quadratic equations can be formed using numbers from 0,2,4,5 as coefficients if a coefficient can be repeated in an equation.
16. How many six–digit telephone numbers can be formed if the first two digits are 45 and no digit can appear more than once?
17. A question paper has 6 questions. How many ways does a student have if he wants to solve at least one question?
18. Find the number of ways of dividing 20 objects in three groups of sizes 8,7,and 5.
19. There are 8 doctors and 4 lawyers in a panel. Find the number of ways for selecting a team of 6 if at least one doctor must be in the team.
20. Four parallel lines intersect another set of five parallel lines. Find the number of distinct parallelograms that can be formed.

Activity 6.1

(1) Bring your family photo

No. of members = ☐

No of males = ☐

No of females = ☐

If they are to be stand in a line for a photograph

(a) How many options are there?

(b) If males are together & females are together how many arrangements are there?

(c) If no two females are together then how many arrangement are there?

Activity 6.2

Note down your 10 digit mobile phone number.

(a) How many different 2 digit numbers are possible?

(b) How many 3 digits numbers with repetition are possible?

(c) How many 4 digit even numbers without repetition are possible.

Activity 6.3

1) You have a party at your place. You go to shop. Find the no. of items and their flavours.

☐ Flavours of cakes are available.

☐ Flavours of cookies are available.

☐ types of wafers are available.

☐ types of cold drinks are available.

You have to select 1 type of cake, 2 types of cookies, 2 types of wafers and 1 type of cold drink.

Find the total number of combinations.

2) Make a number card on A4 size paper for each of the digits 1, 4, 5, 6, 8, 0, 7 and laminate them.

   i) Take any 5 number cards and place them on table.

   ii) See how many 2 digit numbers you can form without repetitions.

   iii) See how many 3 digit numbers you can form without repetition.

   iv) See how many 4 digit numbers you can form without repetition.

(**Note :** Construct the different problems from such number cards)