

## 8. LINEAR INEQUALITIES



### Let's Study

- Linear inequality.
- Solution of linear inequality.
- Graphical representation of solution of linear inequality in one variable.
- Graphical solution of linear inequality of two variable.
- Solution of system of linear inequalities in two variables.



### Let's Recall

#### Overview

In earlier classes, we have studied equations in one variable and two variables and also solved some statement problems by translating them in the form of equations. Now a natural question arises: 'Is it always possible to translate a statement problem in the form of an equation?' For example, the height of all the students in the class is less than 170 cm. The classroom can occupy at most 60 benches. Here we get certain statements involving signs '<' (less than), '>' (greater than),  $\leq$  (less than or equal) and  $\geq$  (greater than or equal) which are known as inequalities.

In this chapter, we will study linear inequalities in one and two variables. The study of inequalities is very useful in solving problems in the field of Science, Mathematics, Statistics, Optimisation problems, Economics, Psychology, etc.



### Let's Learn

#### 8.1 Linear inequality :

Let us consider the following situations:

- (i) Ramesh goes to market with ₹ 100 to buy chocolates. The price of one chocolate is ₹ 15. If he buys  $x$  chocolates, then the total amount spent by him is ₹  $15x$ . Since he has to buy chocolates in whole numbers only, he may not be able to spend the entire amount of ₹ 100. (Why?)

As ₹  $15x$  cannot exceed ₹ 100, ₹  $15x$  must be less than Rs. 100.

That is  $15x < 100$  ..... (1)

The statement (1) is not an equation as it does not involve the sign of equality.

- (ii) Radhika has ₹ 200 and wants to buy some notebooks and pens. The cost of one notebook is Rs. 40 and that of a pen is ₹ 20. In this case, if Radhika buys  $x$  notebooks and  $y$  pens, then she spends ₹  $(40x + 20y)$ .

Therefore  $40x + 20y \leq 200$  ..... (2)

**Definition:** A statement involving real numbers or two algebraic expressions and symbols '>', '<', ' $\leq$ ', ' $\geq$ ' is called an inequality or inequation. Statements (1) and (2) above are inequalities.

#### Some more examples :

- (i)  $9 > 5$ ,  $x < 5$ ,  $x + y < 5$ ,  $x^2 - x + 3 \leq 5$
- (ii) Inequalities which involve linear expression are called linear inequalities.

For example,  $x > 3$ ,  $y > 5$ ,  $x - y < 2$

(iii) An inequality may contain more than one variable and it can be linear, quadratic or cubic etc. For example,  $3x - 2 < 0$  is a linear inequality in one variable,  $2x + 3y < 4$  is a linear inequality in two variables and  $x^2 - 2x + 3 \geq 0$  is a quadratic inequality in one variable.

(iv) Inequalities involving the symbol ' $>$ ' or ' $<$ ' are called **strict inequalities**.

For example,  $3x - y > 5$ ,  $x < 3$ .

(v) Inequalities involving the symbol ' $\leq$ ' or ' $\geq$ ' are called **slack inequalities**.

For example,  $2x + y \leq 4$ ,  $3x - 5y \geq 6$

Some more examples of inequalities are:

$$ax + b < 0 \quad \dots\dots\dots (3)$$

$$ax + b > 0 \quad \dots\dots\dots (4)$$

$$ax + b \leq 0 \quad \dots\dots\dots (5)$$

$$ax + b \geq 0 \quad \dots\dots\dots (6)$$

$$ax + by < c \quad \dots\dots\dots (7)$$

$$ax + by > c \quad \dots\dots\dots (8)$$

$$ax + by \leq c \quad \dots\dots\dots (9)$$

$$ax + by \geq c \quad \dots\dots\dots (10)$$

$$ax^2 + bx + c \leq 0 \quad \dots\dots\dots (11)$$

$$ax^2 + bx + c > 0 \quad \dots\dots\dots (12)$$

Here (3), (4), (7), (8) and (12) are strict inequalities, while (5), (6), (9), (10) and (11) are slack inequalities. Inequalities from (3) to (6) are linear inequalities in one variable  $x$  when  $a \neq 0$ , while inequalities from (7) to (10) are linear inequalities in two variables  $x$  and  $y$  when  $a \neq 0$ ,  $b \neq 0$ . Inequalities (11) and (12) are quadratic inequalities in one variable  $x$  when  $a \neq 0$ . In this Chapter, we shall study linear inequalities in one and two variables only.

## 8.2 Solution of linear inequality :

The value(s) of the variable(s) making the inequality a true statement are called solutions of inequality. The set of all **solutions of an inequality**

is called the **solution set** of the inequality. For example,  $x - 2 \geq 0$  has infinite solutions as all real values of  $x$  greater than or equal to two make it a true statement. The inequality  $x^2 + 4 \leq 0$  has no solution in **R** as no real value of  $x$  makes it a true statement.

### An inequality can be solved by

- (i) Adding (or subtracting) the same expression to (from) both sides without changing the sign of inequality.
- (ii) Multiplying (or dividing) both sides by the same positive quantity without changing the sign of inequality.

However, if both sides of inequality are multiplied (or divided) by the same negative quantity, the sign of inequality is reversed, that is ' $>$ ' changes into ' $<$ ' and vice versa.

### Representation of solution of linear inequality in one variable on the number line:

- (i) If the inequality involves ' $\leq$ ' or ' $\geq$ ', we draw rigid circle ( $\bullet$ ) on the number line to indicate that the number corresponding to the filled circle is included in the solution set.
- (ii) If the inequality involves ' $>$ ' or ' $<$ ', we draw hollow circle ( $\circ$ ) on the number line to indicate that the number corresponding to the open circle is excluded from the solution set.

### 8.3 Graphical representation of solution of linear inequality in one variable :

- (a) (i) If the inequality involves ' $\leq$ ' or ' $\geq$ ', we draw the graph of the line as a thick line to indicate that the points on this line are included in the solution set.
- (ii) If the inequality involves ' $>$ ' or ' $<$ ', we draw the graph of the line as dotted line to indicate that the points on the line are excluded from the solution set.

[Interval concept studied in Sets and Relations]

- (b) Solution of a linear inequality in one variable can be represented on the numberline as well as in the plane. The solution of a linear inequality in two variables of the type  $ax + by > c$ ,  $ax + by \geq c$ ,  $ax + by < c$  or  $ax + by \leq c$  ( $a \neq 0$  and  $b \neq 0$ ) can be represented in the plane only.
- (c) Two or more inequalities taken together form a system of inequalities. Solutions of a system of inequalities are common to all inequalities in the system.

### Important results

- (a) If  $a, b \in \mathbf{R}$  and  $b \neq 0$  then
- (i)  $ab > 0$  or  $\frac{a}{b} > 0$  implies  $a$  and  $b$  both have the same sign.
- (ii)  $ab < 0$  or  $\frac{a}{b} < 0$  implies  $a$  and  $b$  have opposite signs.
- (b) If  $a$  is any positive real number, that is,  $a > 0$ , then
- (i)  $|x| < a$  if and only if  $-a < x < a$
- (ii)  $|x| \leq a$  if and only if  $-a \leq x \leq a$
- (iii)  $|x| > a$  if and only if  $x < -a$  or  $x > a$
- (iv)  $|x| \geq a$  if and only if  $x \leq -a$  or  $x \geq a$

### SOLVED EXAMPLES

#### Ex. 1 :

Solve the inequality  $3x - 5 < x + 7$  and represent the solutions on the number line when (i)  $x$  is a natural number (ii)  $x$  is a whole number (iii)  $x$  is a real number

**Solution:** We have  $3x - 5 < x + 7$

$$\therefore 3x < x + 12 \text{ (Adding 5 to both sides)}$$

$$\therefore 2x < 12 \text{ (Subtracting } x \text{ from both sides)}$$

$$\therefore x < 6 \text{ (Dividing by 2 on both sides)}$$

- (i) Solution set is  $\{1, 2, 3, 4, 5\}$

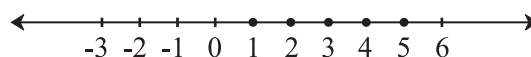


Fig. 8.1

- (ii) Solution set is  $\{0, 1, 2, 3, 4, 5\}$

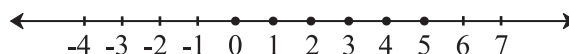


Fig. 8.2

- (iii) Solution set is  $\{x | x \in \mathbf{R} \text{ and } x < 6\} = (-\infty, 6)$ , that is, any real number less than 6.

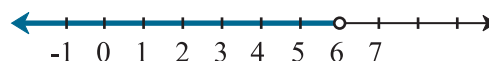


Fig. 8.3.

This is an infinite and unbounded set.

**Ex. 2 :** Solve  $\frac{x-3}{x+3} > 2$  and represent the solutions on the number line.

**Solution:** We have  $\frac{x-3}{x+3} > 2$

$$\frac{x-3}{x+3} > 2 \text{ subtracting 2 from both sides}$$

$$\frac{x-3-2(x+3)}{x+3} > 0 \text{ by simplification}$$

$$\frac{x-3-2x-6}{x+3} > 0$$

$$\frac{-x-9}{x+3} > 0$$

$$\frac{-(x+9)}{x+3} > 0$$

$$\frac{(x+9)}{x+3} < 0 \text{ multiplying by } -1$$

$$\text{Either } x+9 > 0 \text{ and } x+3 < 0$$

$$\text{or } x+9 < 0 \text{ and } x+3 > 0$$

$$\text{Either } x > -9 \text{ and } x < -3$$

$$\text{or } x < -9 \text{ and } x > -3$$

$x < -9$  and  $x > -3$  is impossible

$x > -9$  and  $x < -3$  i.e.  $-9 < x < -3$

i.e.  $x \in (-9, -3)$ . This solution set is bounded.

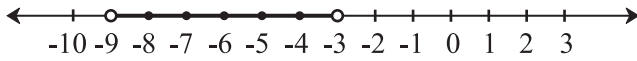


Fig. 8.4

**Example 3:** Solve  $|4x - 3| \leq 9$  and represent the solutions on the number line.

**Solution:** We have  $|4x - 3| \leq 9$

$$-9 \leq 4x - 3 \leq 9$$

$$-9 \leq 4x - 3 \text{ and } 4x - 3 \leq 9$$

$$-6 \leq 4x \text{ and } 4x \leq 12$$

$-\frac{3}{2} \leq x$  and  $x \leq 3$  i.e.  $x \in [-\frac{3}{2}, 3]$  this solution set is bounded but infinite.

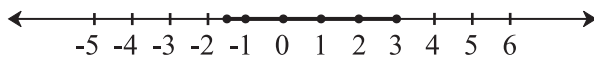


Fig. 8.5

**Ex. 4 :** Solve  $1 \leq |x - 2| \leq 3$  and represent the solutions on the number line.

**Solution:** We have  $1 \leq |x - 2| \leq 3$

$$|x - 2| \geq 1 \text{ and } |x - 2| \leq 3$$

$$x - 2 \leq -1 \text{ or } x - 2 \geq 1$$

$$\text{and } -3 \leq x - 2 \leq 3$$

$$x \leq 1 \text{ or } x \geq 3 \text{ and } -1 \leq x \leq 5$$

$$x \in (-\infty, 1] \text{ or } x \in [3, \infty) \text{ and } x \in [-1, 5]$$

$$x \in (-\infty, 1] \cup [3, \infty) \text{ and } x \in [-1, 5]$$

$x \in [-1, 1] \cup [3, 5]$  this solution set is bounded and infinite.

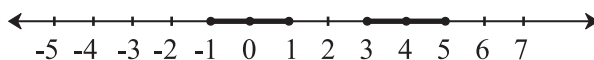


Fig. 8.6

**Ex. 5 :** The cost and revenue functions of a product are given by

$C(x) = 20x + 4000$  and  $R(x) = 60x + 2000$ , respectively, where  $x$  is the number of items produced and sold. How many items must be sold to gain profit?

**Solution:** We have, Profit = Revenue - Cost  
 $= R(x) - C(x)$

$$= (60x + 2000) - (20x + 4000)$$

$$= 40x - 2000$$

To earn profit,  $40x - 2000 > 0$

$$x > 50$$

Hence, the manufacturer must sell more than 50 items to gain profit.

**Ex. 6 :** Solve  $7x + 3 < 5x + 9$ . Show the graph of the solutions on number line.

**Solution:** We have  $7x + 3 < 5x + 9$  or

$$2x < 6 \text{ or } x < 3$$

The graph of the solutions is shown in Fig8.7

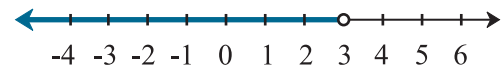


Fig. 8.7

**Ex. 7 :** Solve  $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$ . Show the graph of the solutions on number line.

**Solution:** We have  $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

$$\therefore \frac{3x-4}{2} \geq \frac{x+1-4}{4}$$

$$\therefore \frac{3x-4}{2} \geq \frac{x-3}{4}$$

$$\therefore 2(3x-4) \geq (x-3)$$

$$\therefore 6x-8 \geq x-3$$

$$\therefore 5x \geq 5$$

$$\therefore x \geq 1$$

$$x \in [1, \infty)$$



**Fig. 8.8**

The graph of solution is shown in Fig. 8.8

**Ex. 8 :** The length of a rectangle is three times the breadth. If the perimeter of the rectangle is at least 160 cm, then find the possible breadths of the rectangle.

**Solution:** If  $x$  cm is the breadth, then length is  $3x$

$$\therefore \text{Perimeter} = 2(l + b) \geq 160$$

$$2(3x + x) \geq 160$$

$$x \geq 20$$

$$\therefore \text{breadth} \geq 20 \text{ cm}$$

### EXERCISE 8.1

- Write the inequations that represent the interval and state whether the interval is bounded or unbounded.

(i)  $[-4, 7/3]$  (ii)  $(0, 0.9]$  (iii)  $(-\infty, \infty)$

(iv)  $[5, \infty)$  (v)  $(-11, -2)$  (vi)  $(-\infty, 3)$

- Solve the following inequations

(i)  $3x - 36 > 0$  (ii)  $7x - 25 \leq -4$

(iii)  $0 < \frac{x-5}{4} < 3$  (iv)  $|7x - 4| < 10$

- Sketch the graph which represents the solution set for the following inequations.

(i)  $x > 5$  (ii)  $x \geq 5$

(iii)  $x < 3$  (iv)  $x \leq 3$

(v)  $-4 < x < 3$  (vi)  $-2 \leq x < 4$

(vii)  $-3 \leq x \leq 1$  (viii)  $|x| < 4$

(ix)  $|x| \geq 3.5$

- Solve the inequations.

(i)  $5x + 7 > 4 - 2x$  (ii)  $3x + 1 \geq 6x - 4$

(iii)  $4 - 2x < 3(3 - x)$  (iv)  $\frac{3}{4}x - 6 \leq x - 7$

(v)  $-8 \leq -(3x - 5) < 13$

(vi)  $-1 < 3 - \frac{x}{5} \leq 1$

(vii)  $2|4 - 5x| \geq 9$  (viii)  $|2x + 7| \leq 25$

(ix)  $2|x + 3| > 1$  (x)  $\frac{x+5}{x-3} < 0$

(xi)  $\frac{x-2}{x+5} > 0$

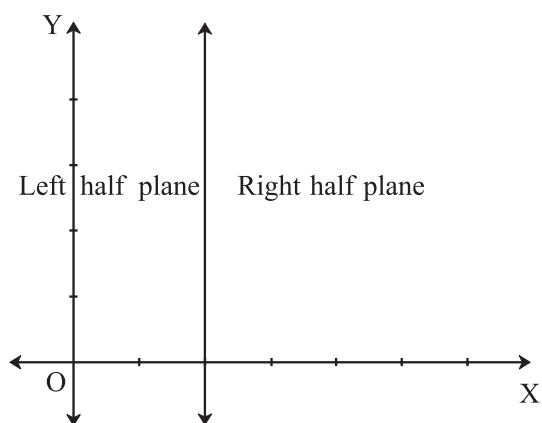
- Rajiv obtained 70 and 75 marks in first two unit tests. Find the minimum marks he should get in the third test to have an average of at least 60 marks.
- To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade 'A' in the course.
- Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.
- Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.
- The longest side of a triangle is twice the shortest side and the third side is 2cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm then find the minimum length of the shortest side.



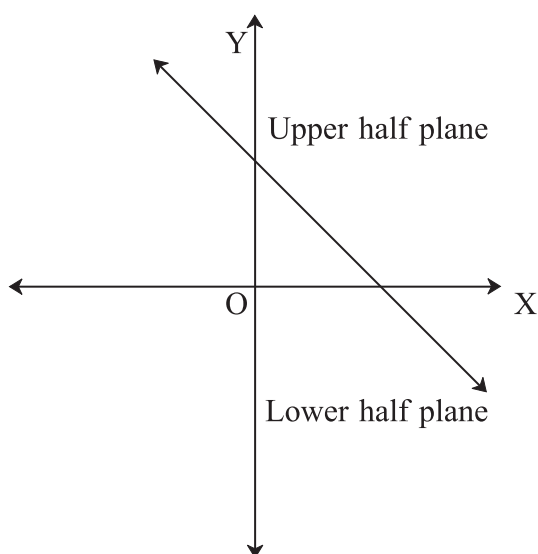
### Let's Learn

### 8.4 Graphical solution of linear inequality of two variable :

A line divides the Cartesian plane into two parts. Each part is known as a half plane. A vertical line will divide the plane in left and right half planes and a non-vertical line will divide the plane into lower and upper half planes.



**Fig. 8.9**



**Fig. 8.10**

A point in the Cartesian plane will either lie on a line or will lie in either of the half planes I or II. We shall now examine the relationship, if any, of the points in the plane and the inequalities

$ax + by < c$  or  $ax + by > c$ .

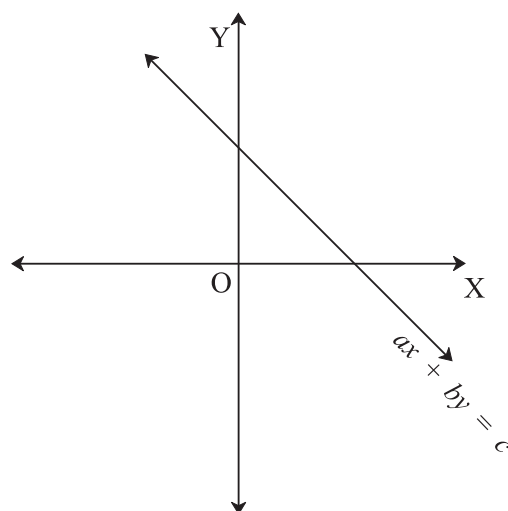
Let us consider the line  $ax + by = c$ ,  $a \neq 0$  and  $b \neq 0$  ..... (1)

There are three possibilities namely:

(i)  $ax + by = c$                       (ii)  $ax + by > c$

(iii)  $ax + by < c$ .

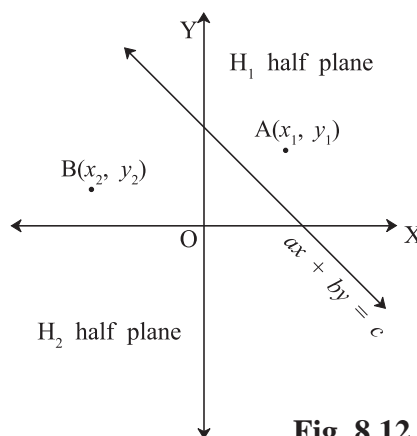
In case (i), verify that all points  $(x, y)$  satisfying (1) lie on the line it represents and conversely.



**Fig. 8.11**

In cases (ii) and (iii) if any point in one of half planes (say  $H_1$  plane) satisfies the inequation then every point of that half plane ( $H_1$  plane) satisfies the same inequation. Therefore every point of that half plane (say  $H_1$  plane) is the solution of the inequation. Therefore  $H_1$  half plane is the solution set of the inequation.

1. The region containing all the solutions of an inequality is called the solution region.
2. In order to identify the half plane represented by an inequality, it is just sufficient to take any point  $(a, b)$  (not on the line) and check which inequality it satisfies. If it satisfies, then the inequality represents the half plane. Colour the region which contains the point. For convenience, the point  $(0, 0)$  is preferred.



**Fig. 8.12**



3. If an inequality is of the type  $ax + by \geq c$  or  $ax + by \leq c$ , then the points on the line  $ax + by = c$  are also included in the solution region. So draw a dark line in the solution region.
4. If an inequality is of the form  $ax + by > c$  or  $ax + by < c$ , then the points on the line  $ax + by = c$  are not to be included in the solution region. So draw a broken or dotted line in the solution region.

### Consider linear inequalities in two variables x and y :

$$40x + 20y \leq 120 \text{ ..... (1)}$$

Let us now solve this inequality keeping in mind that  $x$  and  $y$  can be only whole numbers. In this case, we find the pairs of values of  $x$  and  $y$ , which make the statement (1) true. In fact, the set of such pairs will be the solution set of the inequality (1).

To start with, let  $x = 0$ . Then L.H.S. of (1) is

$$40x + 20y \leq 120$$

$$(0) + 20y \leq 120.$$

Thus, we have  $20y \leq 120$  or  $y \leq 6$  ..... (2)

For  $x = 0$ , the corresponding values of  $y$  can be 0, 1, 2, 3, 4, 5, 6 only. In this case, the solutions of (1) are (0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5) and (0, 6). Similarly, other solutions of (1), when  $x = 1, 2$  and 3 are: (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (2, 0), (2, 1), (2, 2), (3, 0)

This is shown in Fig 8.13

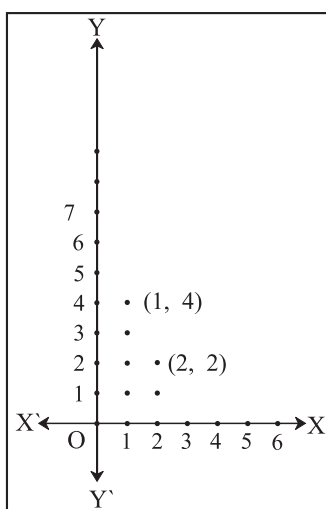


Fig. 8.13

Let us now extend the domain of  $x$  and  $y$  from whole numbers to real numbers, and see what will be the solutions of (1) in this case.

You will see that the graphical method of solution will be very convenient in this case. For this purpose, let us consider the (corresponding) equation and draw its graph.

$$40x + 20y = 120 \text{ ..... (3)}$$

In order to draw the graph of the inequality (1), we take one point say (0, 0), in half plane I and check whether values of  $x$  and  $y$  satisfy the inequality or not.

We observe that  $x = 0, y = 0$  satisfy the inequality. Thus, we say that the half plane I is the graph (Fig 8.14) of the inequality. Since the points on the line also satisfy the inequality (1) above, the line is also a part of the graph.

Thus, the graph of the given inequality is half plane I including the line itself. Clearly half plane II is not the part of the graph. Hence, solutions of inequality (1) will consist of all the points of its graph (half plane I including the line).

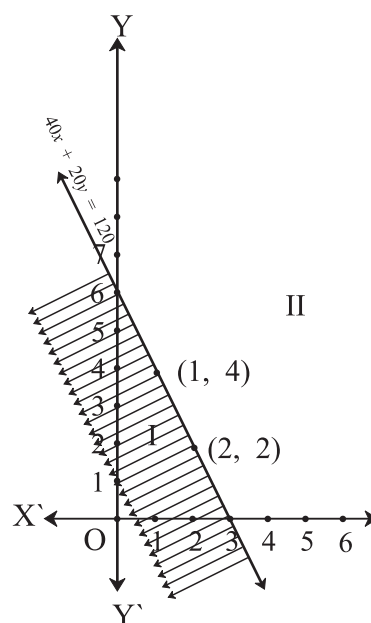


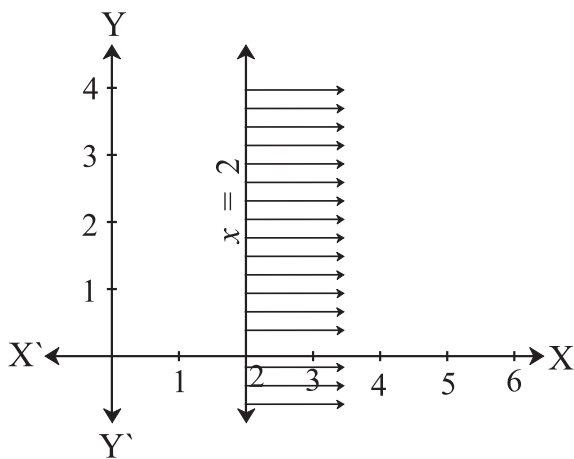
Fig. 8.14

We shall now consider some examples to explain the above procedure for solving a linear inequality involving two variables.

**Ex. 11:** Solve  $3x - 6 \geq 0$  graphically in two dimensional plane.

**Solution:** Graph of  $3x - 6 = 0$  is given in the Fig 8.15 We select a point, say  $(0, 0)$  and substituting it in given inequality, we see that:

$3(0) - 6 \geq 0$  or  $-6 \geq 0$  which is false.



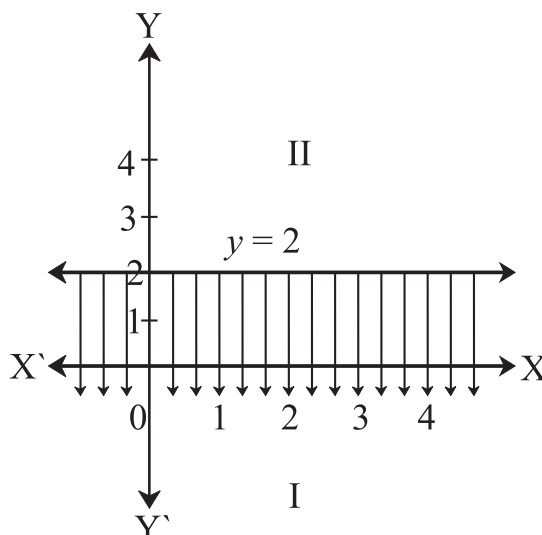
**Fig. 8.15**

Thus, the solution region is the shaded region on the right hand side of the line  $x = 2$ . Which is opposite side of point (OP)

**Ex. 12:** Solve  $y < 2$  graphically.

**Solution:** Graph of  $y = 2$  is given in the Fig 8.16 Let us select a point,  $(0, 0)$  in lower half plane I and putting  $y = 0$  in the given inequality, we see that  $0 < 2$  which is true.

Thus, the solution region is the shaded region below the line  $y = 2$ . Hence, every point below the line (excluding all the points on the line) determines the solution of the given inequality.

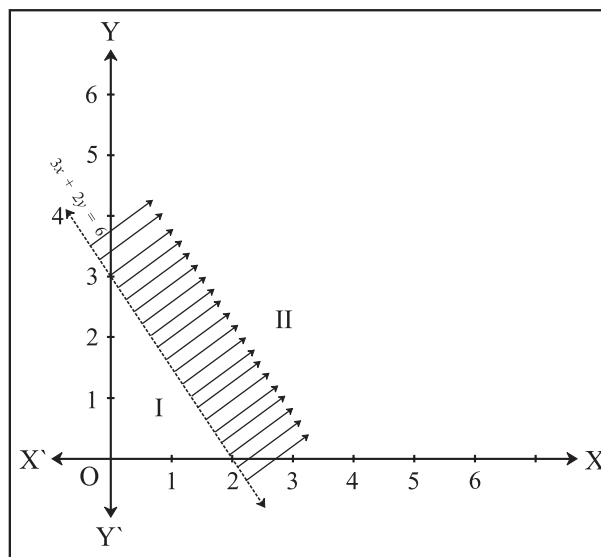


**Fig. 8.16**

**Ex. 13:** Solve  $3x + 2y > 6$  graphically.

**Solution:** Graph of  $3x + 2y = 6$  is given as dotted line in the Fig

This line divides the XY-plane in two half planes I and II. We select a point (not on the line), say  $(0, 0)$ , which lies in one of the half planes (Fig 8.17) and determine if this point satisfies the given inequality, we note that  $3(0) + 2(0) > 6$  or  $0 > 6$ , which is false.



**Fig. 8.17**

Hence, half plane I is not the solution region of the given inequality. Clearly, any point on the



line does not satisfy the given strict inequality. In other words, the shaded half plane II excluding the points on the line is the solution region of the inequality.

## EXERCISE 8.2

1. Solve the following inequations graphically in two-dimensional plane

- |                         |                                             |
|-------------------------|---------------------------------------------|
| (i) $x \leq -4$         | (ii) $y \geq 3$                             |
| (iii) $y \leq -2x$      | (iv) $y - 5x \geq 0$                        |
| (v) $x - y \geq 0$      | (vi) $2x - y \leq -2$                       |
| (vii) $4x + 5y \leq 40$ | (viii) $\frac{1}{4}x + \frac{1}{2}y \leq 1$ |

2. Mr. Rajesh has Rs. 1800 to spend on fruits for meeting. Grapes cost Rs. 150 per kg. and peaches cost Rs. 200 per kg. Formulate and solve it graphically.
3. Diet of sick person must contain at least 4000 units of vitamin. Each Unit of food F1 contains 200 units of vitamin, where as each unit of food F2 contains 100 units of vitamins. Write an inequation to fulfill sick person's requirements and represent the solution set graphically.



### Let's Learn

## 8.5 Solution of System of Linear Inequalities in Two Variables (Common solution) :

In previous Section, we have learnt how to solve linear inequality in one or two variables graphically. We will now illustrate the method for solving a system of linear inequalities in two variables graphically through some examples.

**Ex. 14:** Solve the following system of inequalities graphically

$$5x + 4y \leq 40 \quad \text{..... (1)}$$

$$x \geq 2 \quad \text{..... (2)}$$

$$y \geq 3 \quad \text{..... (3)}$$

**Solution:** Draw the graphs of the lines  $5x + 4y = 40$ ,  $x = 2$  and  $y = 3$  Note that the inequality (1) represents shaded region below the line  $5x + 4y = 40$  and inequality (2) represents the shaded region right of line  $x = 2$  but inequality (3) represents the shaded region above the line  $y = 3$ . Hence, the points in the shaded region (Fig 8.18) represent the required solutions

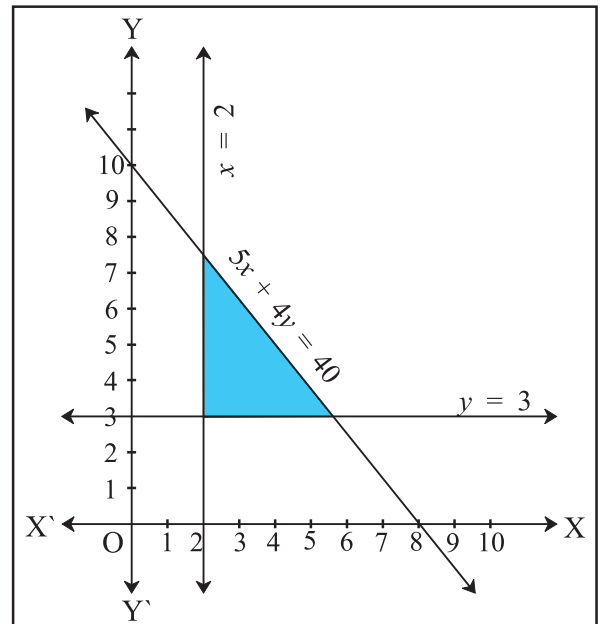


Fig. 8.18

In many practical situations involving system of inequalities, variables  $x$  and  $y$  often represent quantities that cannot have negative values; for example, number of units produced, number of articles purchased, number of hours worked, etc. Clearly, in such cases,  $x \geq 0$ ,  $y \geq 0$  and the solution region lies only in the first quadrant.

**Ex. 15:** Solve the following system of inequalities graphically.

$$8x + 3y \leq 100 \quad \text{..... (1)}$$

$$x \geq 0 \quad \text{..... (2)}$$

$$y \geq 0 \quad \text{..... (3)}$$

**Solution:** Draw the graph of the line

$$8x + 3y = 100$$

The inequality  $8x + 3y \leq 100$  represents the shaded region below the line, including the points on the line  $8x + 3y = 100$  (Fig 8.19). Since  $x \geq 0$ ,  $y \geq 0$ , all points in the shaded region, including the points on the line and the axes, represent the solution of the given system of inequalities.

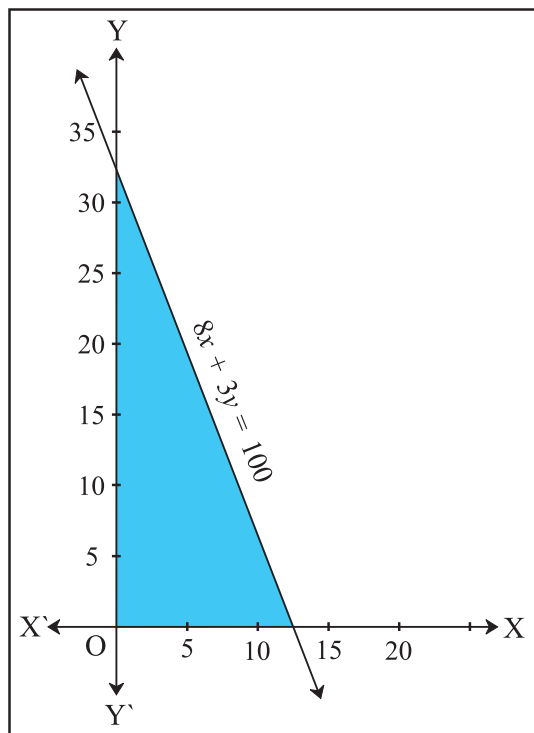


Fig. 8.19

**Ex. 16:** Solve the following system of inequalities graphically

$$x + 2y \leq 8 \quad \text{..... (1)}$$

$$2x + y \leq 8 \quad \text{..... (2)}$$

$$x \geq 0 \quad \text{..... (3)}$$

$$y \geq 0 \quad \text{..... (4)}$$

**Solution:** Draw the graphs of the lines  $x + 2y = 8$  and  $2x + y = 8$ .

The inequalities (1) and (2) represent the region below the two lines, including the points on the respective lines. Since  $x \geq 0$ ,  $y \geq 0$ , all points in the shaded region represent solutions of the given system of inequalities (Fig 8.20)

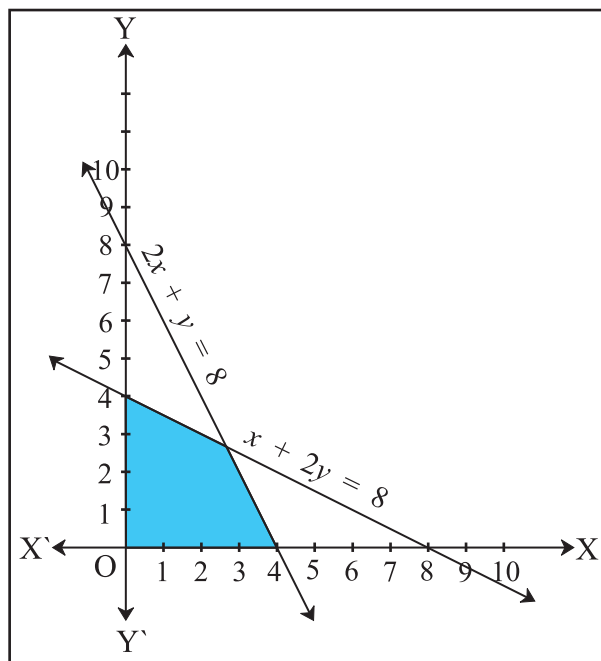


Fig. 8.20

### EXERCISE 8.3

Find the graphical solution of following system of linear inequations

1.  $x - y \leq 0$ ,  $2x - y \geq -2$
2.  $2x + 3y \geq 12$ ,  $-x + y \leq 3$ ,  $x \leq 4$ ,  $y \geq 3$
3.  $3x + 2y \leq 1800$ ,  $2x + 7y \leq 1400$ ,
4.  $0 \leq x \leq 350$ ,  $0 \leq y \leq 150$
5.  $\frac{x}{60} + \frac{y}{90} \leq 1$ ,  $\frac{x}{120} + \frac{y}{75} \leq 1$ ,  $y \geq 0$ ,  $x \geq 0$
6.  $3x + 2y \leq 24$ ,  $3x + y \geq 15$ ,  $x \geq 4$
7.  $2x + y \geq 8$ ,  $x + 2y \geq 10$ ,  $x \geq 0$ ,  $y \geq 0$



**Let's Remember**

### Inequality or inequation :

**Definition :** A statement involving real numbers or two algebraic expressions and symbols ' $>$ ', ' $<$ ', ' $\leq$ ', ' $\geq$ ' is called an inequality or inequation.

If  $a, b \in \mathbb{R}$  and  $b \neq 0$  then

(i)  $ab > 0$  or  $\frac{a}{b} > 0$  implies  $a$  and  $b$  both have the same sign.

(ii)  $ab < 0$  or  $\frac{a}{b} < 0$  implies  $a$  and  $b$  both have opposite signs.

- If  $a$  is any positive real number, that is,  $a > 0$ , then

(i)  $|x| < a$  if and only if  $-a < x < a$

(ii)  $|x| \leq a$  if and only if  $-a \leq x \leq a$

(iii)  $|x| > a$  if and only if  $x < -a$  or  $x > a$

(iv)  $|x| \geq a$  if and only if  $x \leq -a$  or  $x \geq a$

### MISCELLANEOUS EXERCISE - 8

Solve the following system of inequalities graphically

1.  $x \geq 3, y \geq 2$
2.  $3x + 2y \leq 12, x \geq 1, y \geq 2$
3.  $2x + y \geq 6, 3x + 4y < 12$
4.  $x + y \geq 4, 2x - y \leq 0$
5.  $2x - y \geq 1, x - 2y \leq -1$
6.  $x + y \leq 6, x + y \geq 4$
7.  $2x + y \geq 8, x + 2y \geq 10$
8.  $x + y \leq 9, y > x, x \geq 0$
9.  $5x + 4y \leq 20, x \geq 1, y \geq 2$
10.  $3x + 4y \leq 60, x + 3y \leq 30, x \geq 0, y \geq 0$
11.  $2x + y \geq 4, x + y \leq 3, 2x - 3y \leq 6$
12.  $x - 2y \leq 3, 3x + 4y \geq 12, x \geq 0, y \geq 1$

13.  $4x + 3y \leq 60, y \geq 2x, x \geq 3, x, y \geq 0$

14.  $3x + 2y \leq 150, x + 4y \geq 80, x \leq 15, y \geq 0, x \geq 0$

15.  $x + 2y \leq 10, x + y \geq 1, x - y \leq 0, x \geq 0, y \geq 0$

### Activity 8.1

Visit a supermarket, you have Rs.100, & you want to buy some biscuits packet & toffees. If each biscuit packet cost Rs.15 & each toffee costs Rs.8. How many biscuits packets & toffee can be purchased. Write it in the inequation form.

Find the different combinations of number of biscuit packets & toffees using inequality condition.

### Activity 8.2

Throw a die 15 times

- a) Let  $x$  be getting prime number on upper most face Find how many times  $x$  is achieved. Write in the form of inequation.
- b) If  $y$  is getting multiple of 2, what is the possibility of  $y$  ? Draw the graph for  $x < 16$  & for  $y > 3$

### Activity 8.3

You are Mr. X, have Rs.10000 to invest in shares. If you want to buy some shares of company X whose market price Rs.150 & some shares of company y with market price Rs.230, then how many maximum shares each you can buy? Formulate & draw the graph.

