# 7. Wave Optics

**Can you recall?**

1. What does the formation of shadows tell you about the propagation of light?
2. What are laws of reflection and refraction?
3. What are electromagnetic waves?
4. What is the range of frequencies of visible light?
5. What is meant by the phase at a point along the path of a wave?

## 7.1 Introduction:

In earlier standards we have learnt that light travels in a straight line while travelling through a uniform and homogeneous medium. The path of light is called a ray of light. On encountering an interface with another medium, a ray of light gets reflected or refracted, changes its direction and moves along another straight line. The reflection is such that (i) the incident ray, the reflected ray and the normal to the boundary surface at the point of incidence are in the same plane and (ii) the angle of incidence, i.e., the angle between the incident ray and the normal to the reflecting surface, is equal to the angle of reflection, i.e., the angle between the reflected ray and the normal. For refraction of light while travelling from medium 1 to medium 2, the laws are (i) the incident ray, the refracted ray and the normal to the boundary between the two media at the point of incidence are in the same plane and (ii) the angle of incidence $i$, and the angle of refraction $r$, are related by $n_1 \sin i = n_2 \sin r$, where, $n_1$ and $n_2$ are the absolute refractive indices of medium 1 and medium 2 respectively.

We have also learnt about the reflection of light produced by spherical mirrors and refraction of light through prisms and curved surfaces of lenses. The position and nature of the image (whether real or virtual) depend on the position of objects and the focal length of the mirror or lens.

## 7.2 Nature of Light

### 7.2.1 Corpuscular Nature:

The formation of shadows as well as images by mirrors and lenses has been understood by considering rectilinear motion of light rays. This fact led R. Descartes (1596-1650) to propose a particle nature of light in the year 1636. Newton (1642-1726) developed this concept further and proposed that light is made up of particles, i.e., corpuscles which are hard, elastic and massless. A source of light emits these corpuscles which travel along straight lines in the absence of any external force. When the light corpuscles strike a reflecting surface, they undergo elastic collisions and as a result follow the laws of reflection. During refraction, it is the difference in the attractive force between the corpuscles and the particles of the medium that causes a change in the direction of the corpuscles. A denser medium exerts a larger attractive force on light corpuscles to accelerate them along the normal to the boundary. Thus, Newton's theory predicted that the speed of light in denser medium would be higher than that in a rarer medium. This contradicts the experimental observation. In this theory, light of different colours corresponds to corpuscles of different sizes. Newton performed several experiments in optics and could explain their results based on his theory. The study of optical phenomena under the assumption that it travels in a straight line as a ray is called *ray optics* or *geometrical optics* as geometry is used in this study. The laws of reflection and refraction and the formation of images that we studied in earlier standards fall under this category.

### 7.2.2 Wave Nature:

To circumvent the difficulties in corpuscular theory, it was proposed by the Dutch physicist C. Huygens (1629-1695) in the year 1668, that light is a wave. Huygens assumed light to be a wave caused by vibrations of the particles of the medium. As light could also travel in vacuum, he assumed that a hypothetical medium, called ether is present everywhere including in vacuum. Note that this ether is not the substance (ether gas) that we come across in chemistry. There was however, no evidence to prove its existence and thus, it was difficult to accept the concept.

In the nineteenth century, certain new phenomena of light namely, interference, diffraction and polarization were discovered. These could not be explained based on corpuscular theory and needed wave theory for their explanation. Huygens' theory could not only explain the new phenomena but could also explain the laws of reflection and refraction, as well as the formation of images by mirrors and lenses. It was then accepted as the correct theory of light. Wave theory showed that if the speed of light waves in denser medium is smaller than that in rarer medium then light bends towards normal. Thus, wave nature of light could explain all the visual effects exhibited by light. The branch of optics which uses wave nature of light to explain the optical phenomena is called *wave optics*.

In this chapter we are going to study wave optics and learn how the laws of reflection and refraction can be explained assuming the wave nature of light. We will also learn about the phenomena of interference, diffraction and polarization and their explanation based on wave optics. The reason why geometrical optics works in case of formation of shadows, reflection and refraction is that the wavelength of light is much smaller than the reflecting/ refracting surfaces as well as the shadow causing objects that one encounters in laboratory or in day-to-day life.

In XI$^{th}$ Std we have learnt Maxwell's equations which suggested that light is an electromagnetic wave. As all waves known till Maxwell's time needed a medium to propagate, Maxwell invoked the all-pervading hypothetical medium ether. The existence of radio waves and their speed being same as that of visible light, were experimentally verified by H. Hertz later in the nineteenth century. Michelson and Morley performed several experiments to detect ether but obtained negative result. The hypothesized ether was never detected, and its existence and necessity was ruled out by Albert Einstein (1879-1955) when he proposed the special theory of relativity in the year 1905, based on a revolutionary concept of constancy of velocity of light.

### 7.2.3 Dual Nature of Light:

In the early twentieth century, it was accepted that light has a dual nature. It can exhibit particle nature as well as wave nature under different situations. Particles of light are called photons. We will learn more about it in Chapter 14.

## 7.3 Light as a Wave:

Light is an electromagnetic wave. These waves are transverse in nature and consist of tiny oscillating electric and magnetic fields which are perpendicular to each other and to the direction of propagation of the wave. These waves do not require any material medium for propagation and can even travel through vacuum. The speed of light in a material medium (v) depends on the refractive index of the medium ($n$) which, in turn, depends on permeability and permittivity of the medium. The refractive index is equal to the ratio of the speed of light in vacuum ($c$) to the speed of light in the medium (v). The refractive index of vacuum is 1 and that of air can be approximated to be 1.

**Do you know?**

It was shown by Einstein in his special theory of relativity that the speed of light ($c$) does not depend on the velocity of the source of light or the observer. He showed that no object or information can travel faster than the speed of light in vacuum which is 300,000 km/s.

Electromagnetic waves can have wavelengths ranging from very small, smaller than a femtometre ($10^{-15}$ m), to very large, larger than a kilometre. The electromagneic waves are classified as $\gamma$-rays, X-rays, ultraviolet, visible, infrared, microwave and radio waves, in the increasing order of wavelength. Visible light comprises wavelengths in the range of 400-700 nm. Waves of different wavelengths in the visible range are perceived by our eyes as different colours, with violet having the shortest and red having the longest wavelength. White light is a mixture of waves of different wavelengths. The refractive index of a medium depends on the wavelength of the incident light. Because of this, for the same angle of incidence, the angle of refraction is different for different colours (except for normal incidence), therefore, the colours present in the white light get separated on passing through a transparent medium. This is the reason for formation of a spectrum and of a rainbow.

## 7.4 Huygens' Theory:

### 7.4.1 Primary and Secondary Sources of Light:

We see several sources of light around us, e.g., the Sun, moon, stars, light bulb, etc. These can be classified into primary and secondary sources of light. *Primary sources* are sources that emit light of their own, because of (i) their high temperature (examples: the Sun, the stars, objects heated to high temperatures, flame of any kind, etc), (ii) the effect of current being passed through them (examples: tube light, TV, etc.), and (iii) chemical or nuclear reactions (examples: firecrackers, nuclear energy generators). Light originates in these sources.

*Secondary sources* are those sources which do not produce light of their own but receive light from some other source and either reflect or scatter it around. Examples include the moon, the planets, objects like humans, animals, plants, etc., which we see due to reflected light. Majority of the sources that we see in our daily life are secondary sources. Most secondary sources are extended sources as can be seen from the examples above.

### 7.4.2 Wavefront:

We have seen that when we drop a stone in water, surface waves, commonly known as ripples, are generated which travel outwards from the point, say O, where the stone touches water. Water particles along the path of the wave move up and down, perpendicular to the water surface. The phase of the wave at a point is defined by the state of motion of the particle at that point as well as the distance of the point from the source (see Chapter 8 in XI$^{th}$ Std book). Two particles are in phase, i.e., have the same phase, if their state of motion is the same, i.e., if they have the same velocity and displacement perpendicular to the water surface and if they are at the same distance from the source. As the waves are travelling symmetrically in all directions along the water surface, all particles along the circumference of a circle with centre at O will have the same phase. The locus of all points having the same phase at a given instant of time is called a *wavefront*. Thus, a wavefront is the locus of all points where waves starting simultaneously from O reach at a given instant of time. In case of water waves the wavefronts are circles centred at O. The direction of propagation of the wave is perpendicular to the wavefronts, i.e., along the radii of the circle. The speed with which the wavefronts move is the speed of the wave. Water waves are two dimensional (along a surface) waves.

Three dimensional waves like the sound waves produced by a source of sound, or light waves produced by a light source, travel in all directions away from the source and propagate in three dimensions. Such a wave is called a *spherical wave*. In these cases, the wavefronts are surfaces passing through all points having the same distance from the source and having the same phase. Thus, in these cases, they are spheres centred on the source say at O, the cross sections of which are as shown in Fig.7.1 (a). The spheres, are wavefronts with the source at their centre. The arrows are perpendicular to the spherical surfaces and show the direction of propagation of the waves. These arrows represent the rays of light that we have considered in earlier study of optics. The wavefronts shown in the figure correspond to a diverging beam of light. We can similarly have wavefronts corresponding to a converging beam of light. Such wavefronts can be produced after passing through a lens.

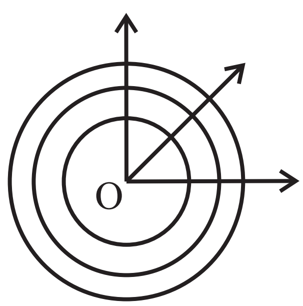



**Fig.7.1 (a): Spherical wavefronts corresponding to diverging beam of light. These are spherical waves. The source is at O.**

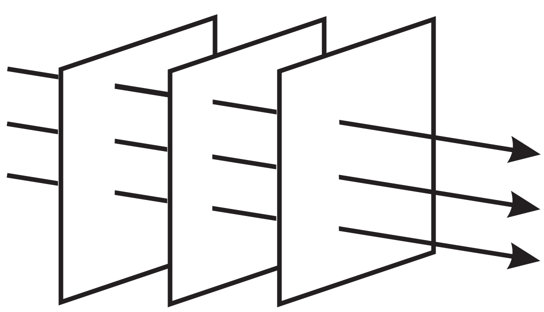


**Fig.7.1(b): Plane wavefronts corresponding to parallel beam of light. These are plane waves.**

Let us now consider a spherical wavefront which has travelled a large distance away from the source. If we take a small portion of this wavefront, it will appear to be a plane surface (just like the surface of the earth around us appears to be flat to us) with the direction of propagation perpendicular to it. In such a case the wave is called a *plane wave*. Wavefronts for a plane wave are shown in Fig.7.1 (b) where the arrows (rays) which are now parallel corresponding to a parallel beam of light, represent the direction of propagation of the wave. If the source of light is linear (along a line) the wave fronts will be cylindrical.

### 7.4.3 Huygens' Principle:

Huygens had assumed light to be a wave, similar to the mechanical wave like the water wave or sound wave, propagating in ether. Accordingly the particles of ether oscillate due to the propagation of a light wave. He put forth a principle which makes it possible to determine the shape of a wavefront at any time $t$, given its shape at an earlier time. This principle can be stated as *"Each point on a wavefront acts as a secondary source of light emitting secondary light waves called wavelets in all directions which travel with the speed of light in the medium. The new wavefront can be obtained by taking the envelope of these secondary wavelets travelling in the forward direction and is thus, the envelope of the secondary wavelets in forward direction. The wavelets travelling in the backward direction are ineffective".*

Given a wavefront at time $t$ = 0 say, we can determine the shape and position of a wavefront at a later time $t = T$ using Huygens' principle. Let us first consider a plane wavefront AB (corresponding to parallel rays), at time $t$ = 0 crosssection of which is shown in Fig.7.2 (a). According to Huygens' principle, each point on this wavefront will act as a secondary source of light and will emit spherical wavelets as shown in the figure. We have shown only the wavelets travelling in the forward direction (direction of propagation of light) as the backward travelling wavelets are supposed to be ineffective. The wavelets will be in the form of hemispheres and at a later time $t = T$, the radius of the hemispheres will be v$T$ where v is the speed of light. The wavefront at time $T$ will be the envelope of all these hemispherical wavelets and will be a plane A′ B′ as shown in the figure. Similarly, the position of a spherical wavefront at time $t = 0$ is shown as AB in Fig.7.2 (b). The wavelets emitted in the forward direction by points on AB will be hemispheres as shown in the figure. At time $T$ the radius of these spheres will be v$T$ and their envelope will be a spherical surface A′ B′ as shown.

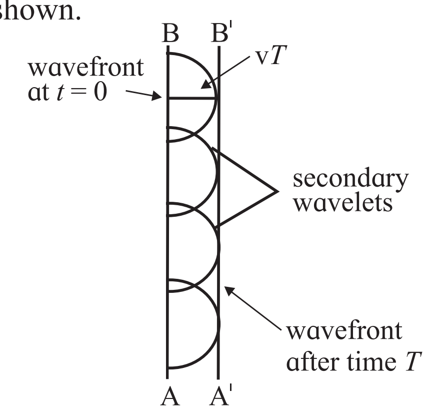



**Fig.7.2 (a): Progress of a plane wavefront.**

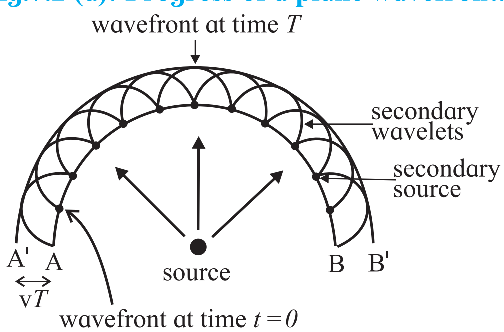



**Fig.7.2 (b): Progress of a spherical wavefront.**

Huygens' theory is an empirical theory. There is no reason why the backward travelling waves will not be effective. The theory was accepted just because it explained various optical phenomena as we will see next.

## 7.5 Reflection of Light at a Plane Surface

Let us consider a plane wavefront AB perpendicular to the plane of the paper, incident at an angle $i$ with the normal to a plane reflecting surface (mirror) MN which is also perpendicular to the plane of the paper as shown in Fig.7.3. The figure shows a cross section of the setup. RA and QB show the direction of incidence. Let us assume that the incident wavefront AB touches the reflecting surface at A at time $t = 0$. The point B will touch the reflecting surface at C after a time $t = T$. Between time $t = 0$ and $T$, different points along the incident wavefront reach the reflecting surface successively and secondary wavelets will start propagating in the form of hemispheres from those points in succession. For reflection, the hemispheres to be considered are on the same side of the mirror. The wavelet emitted by point A will have a radius v$T$ at time $T$. The radius of the wavelet emitted by C will be zero at that time. The radii of the wavelets emitted by points between A and C will gradually decrease from v$T$ to 0. The envelope of these wavelets forms the reflected wavefront. This is shown by EC which is the common tangent to the reflected wavelet originating from A and other secondary wavelets emitted by points between A and C.

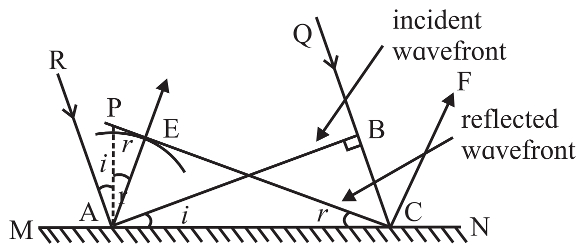



**Fig.7.3: Reflection at a plane surface.**

Obviously, AE = BC = v$T$, the distance travelled by light in the same medium in same time. The arrow AE shows the direction of propagation of the reflected wave. The normal to MN at A is shown by AP, the angle of incidence ∠RAP = $i$. As RA and AP are perpendicular to AB and AC respectively, ∠BAC is also equal to $i$. The triangles ABC and AEC are right angled triangles and have common hypotenuse (AC) and one equal side (AE = BC). Hence, the two triangles are congruent and we have,

$\angle ACE = \angle BAC = i$. --- (7.1)

The angle of reflection is $\angle PAE = r$. As AE is perpendicular to CE and AP is perpendicular to AC,

$\angle ACE = \angle PAE = r$. --- (7.2)

Eqs. (7.1) and (7.2) give us $i = r$ which is the law of reflection.

It is also clear from the figure that the incident ray, normal and the reflected ray are in the same plane which is the plane of the paper. This is the other law of reflection.

Let us assume the rays, RA and QC to be coming from the extremities of the object, i.e., AB is the size of the object. The distance between the corresponding reflected rays AE and CF will be same as AB as can be seen from the congruent triangles, ABC and AEC. Thus, the size of the object in the reflected image will be same as the actual size of the object.

Let us assume A and B to be the right and left sides of the object respectively as it looks into the mirror. After reflection, the right side, at A is seen at E and the left side at B is seen at C. The right side has now become left side and vice-versa as the reflected wavefront comes out of the mirror. This is called lateral reversal. Below we will see that lateral reversal does not occur during refraction at a plane surface.

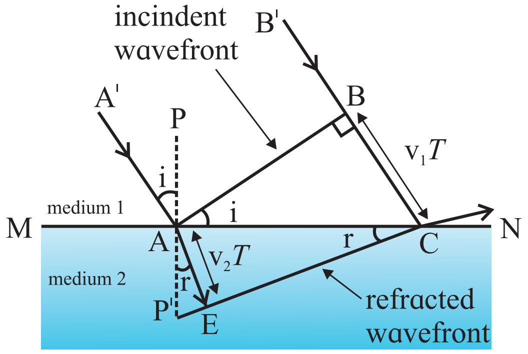



**Fig.7.4: Refraction of light.**

## 7.6. Refraction of Light at a Plane Boundary Between Two Media :

Consider a wavefront AB, incident on a plane boundary MN, separating two uniform and optically transparent media as shown in Fig 7.4. At time $t = 0$, A has just reached the boundary surface, while B reaches the surface at C at a later time $t = T$. Let the speed of light be $v_1$ in medium 1 and $v_2$ in medium 2. Thus, BC = $v_1 T$. At time $t = T$, the radius (AE) of the secondary wavelet emitted from A will be $v_2 T$. The refracted wavefront will be the envelope of wavelets successively emitted by all the points between A and C between time $t = 0$ and $t = T$. CE is the tangent to the secondary wavelet emitted from A. It is also the common tangent to all the secondary wavelets emitted by points between A and C. The normal to the boundary at A is shown by PP′.

$\angle A'AP = \angle BAC$ = the angle of incidence = $i$

$\angle P'AE = \angle ACE$ = angle of refraction = $r$

From $\Delta ABC$,

$\sin i = v_1 T / AC$ --- (7.3)

From $\Delta AEC$,

$\sin r = v_2 T / AC$ --- (7.4)

From Eqs. (7.3) and (7.4) we get

$\sin i / \sin r = v_1 / v_2 = (c/v_2)/(c/v_1) = n_2/n_1$,

$n_1 \sin i = n_2 \sin r$, --- (7.5)

Here, $n_1$ and $n_2$ are the absolute refractive indices of media 1 and 2 respectively. Eq. (7.5) is the law of refraction and is also called the Snell's law. Also, it is clear from the figure that the incident and refracted rays and the normal to the boundary surface are in the same plane. If $v_1 > v_2$, i.e., $n_1 < n_2$. Then $i > r$. Thus, during oblique incidence, the refracted ray will bend towards the normal while going from an optically rarer (smaller refractive index) to an optically denser (higher refractive index) medium. While entering an optically rarer medium form a denser medium, the refracted ray will bend away from the normal.

The refracted image will not be inverted as can be seen from the diagram. Also, except for normal incidence, the image seems to be bent (broken) below the boundary surface as the rays change their direction on crossing the surface. At normal incidence, the rays travel along the same direction and there is no breaking of the image.

### Dependence of Wavelength on the Refractive Index of the Medium:

Consider monochromatic light incident normally on a boundary between a rarer medium and a denser medium as shown in Fig. 7.5 (a). The boundary between the two surfaces is shown by PQ. The three successive wavefronts AB, CD and EF are separated by a distance $\lambda_1$, which is the wavelength of light in the first medium. After refraction, the three wavefronts are indicated by A′B′, C′D′ and E′F′. Assuming the second medium to be denser, the speed of light will be smaller in that medium and hence, the wavefronts will move slower and will be able to cover less distance than that covered in the same time in the first medium. They will therefore be more closely spaced than in the first medium. The distance between any two wavefronts is $\lambda_2$, equal to the wavelength of light in the second medium. Thus, $\lambda_2$ will be smaller than the wavelength in the first medium. We can easily find the relation between $\lambda_1$ and $\lambda_2$ as follows.

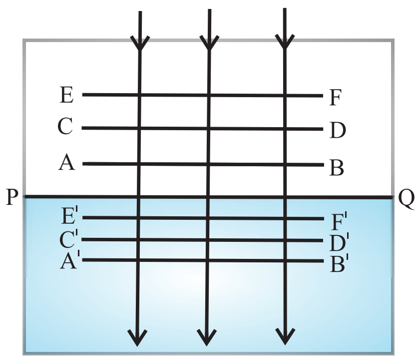



**Fig.7.5 (a): Change in wavelength of light while going from one medium to another for normal incidence.**

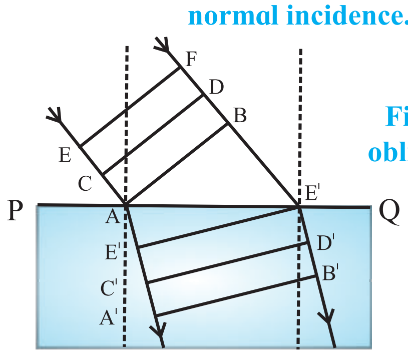



**Fig.7.5 (b): For oblique incidence.**

Let the wavefront AB reach the boundary surface PQ at time $t = 0$ and the next wavefront CD which is at a distance of $\lambda_1$ from AB, reach PQ at time $t = T$. As the speed of the wave is $v_1$ in medium 1 and $T$ is the time period in which the distance $\lambda_1$ is covered by the wavefront , we can write

$$T = \lambda_1 / v_1 \quad \text{--- (7.6)}$$

In medium 2, the distance travelled by the wavefront in time $T$ will be $\lambda_2$. The relation between these two quantities will be given by

$$T = \lambda_2 / v_2 \quad \text{--- (7.7)}$$

Eq. (7.6) and Eq. (7.7) show that the velocity in a medium is proportional to the wavelength in that medium and give

$$\lambda_2 = \lambda_1 v_2 / v_1 = \lambda_1 n_1 / n_2 \quad \text{--- (7.8)}$$

If medium 1 is vacuum where the wavelength of light is $\lambda_0$ and $n$ is the refractive index of medium 2, then $\lambda$, the wavelength of light in medium 2, can be written as

$$\lambda = \lambda_0 v_2 / c = \lambda_0 / n \quad \text{--- (7.9)}$$

The ratio of the frequencies $\nu_1$ and $\nu_2$, of the wave in the two media can be written, using Eq. (7.8) as,

$$\nu_1 / \nu_2 = (v_1/\lambda_1)/(v_2/\lambda_2) = 1 \quad \text{--- (7.10)}$$

This illustrates what we have assumed above by taking same $T$ in equations 7.6 and 7.7, that the frequency of a wave remains unchanged while going from one medium to another. Similar analysis goes through if the wave is incident at an angle as shown in Fig.7.5 (b).

**Remember this**

The frequency of a wave is its fundamental property and does not change while going from one medium to another. The speed and the wavelength of a wave do change and are inversely proportional to the relative refractive index of the second medium with respect to the first.

### 7.7 Polarization:

We know that light is an electromagnetic wave and that its electric ($E$) and magnetic ($B$) field vectors are perpendicular to each other and to the direction of propagation. We also know that light is emitted by atoms. Thus, when one atom emits a wave along the $x$-axis say, its electric field may be along the $y$-axis and magnetic field will be along the $z$-axis. However, if another atom in the source emits a wave travelling along the $x$-axis, it is not necessary that its electric field be along the $y$-axis. It can be along any direction in the $y$-$z$ plane and the magnetic field will be along a direction perpendicular to it. Thus, in general, the electric fields of waves emitted along the $x$-axis by a light source like the Sun, stars or a light bulb will be in all possible directions in the $y$-$z$ plane and the corresponding magnetic fields will be perpendicular to their electric fields. Such light is called *unpolarized* light and is represented by double headed arrows (showing the directions of electric field) in a plane perpendicular to the direction of propagation. This is shown in Fig.7.6 (a) for a light beam travelling perpendicular to the plane of the paper. On the other hand, if somehow light is constrained so that its electric field is restricted along one particular direction, then it is called *plane polarized* light. This is shown in Fig.7.6 (b) for a light beam travelling perpendicular to the plane of the paper.

How can we get polarized light? There are certain types of materials which allow

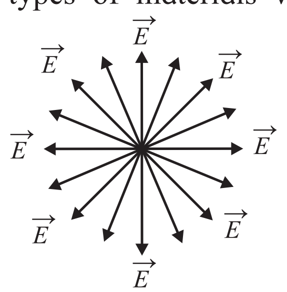



**Fig. 7.6 (a): Unpolarized light coming towards us or going away from us.**


**Fig. 7.6 (b): Polarized light coming towards us or going away from us with electric field along the horizontal direction.**

only those light waves which have their electric field along a particular direction to pass through. These materials are called polarizers. A polaroid is a kind of synthetic plastic sheet which is used as polarizer. The particular direction along which the electric field of the emergent wave is oriented is called the polarizing axis of the polarizer. This is shown in Fig.7.7 (a). Thus, when unpolarized light passes through a polarizer, the emergent light is plane polarized. The plane ABCD containing the electric field vectors of plane polarized light is called the *plane of vibration* while the plane perpendicular to the plane of the vibration and the electric field vector is called *plane of polarization.*

If the polarized light is made to pass through another polarizer with its polarizing axis perpendicular to the polarizing axis of the first polarizer, no light can emerge on the other side as the second polarizer would allow only waves having electric field parallel to its polarizing axis to pass through. On the other hand, if the polarizing axis of the second polarizer makes an angle smaller than $90^{o}$ with that of the first polarizer, then the component of the electric field of the polarized light along the direction of the polarizing axis of the second polarizer can pass through. The intensity of light will reduce after passing through each polarizer. We will mathematically calculate this below. We have used the electric field to explain the phenomenon of polarization, however, it could also be explained in the same manner using the magnetic field.

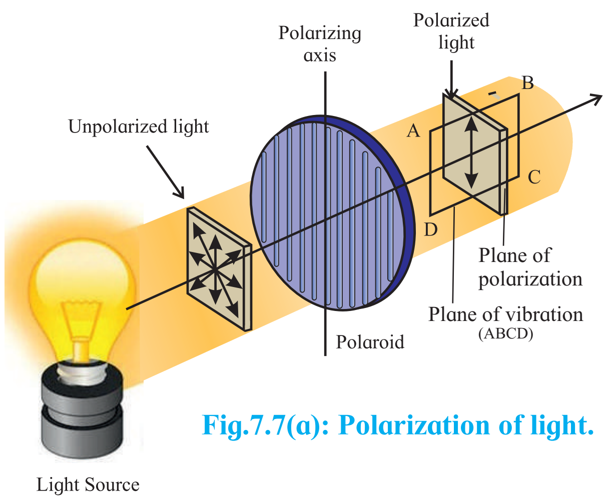



**Fig.7.7(a): Polarization of light.**

Consider an unpolarized wave having angular frequency $\omega$ and wave vector $k$ $(=2\pi/\lambda)$, travelling along the $x$-direction. The magnitude of its electric field is given by $E = E_0 \sin\left(kx - \omega t\right), E_0$ being the amplitude of the wave (see Chapter 13 of the XI[th] Std book). The intensity of the wave will be proportional to $|E_0|^2$. The direction of the electric field can be anywhere in the *y-z* plane we will consider the passage of this wave through two polarizers as shown in Fig. 7.7 (b). Let us consider a particular wave having its electric field at an angle $\phi$ to the axis of the first polarizer. The component $E_0 \cos\phi$ will pass through the first polarizer while the normal component $E_0 \sin\phi$ will be obstructed. The intensity of this particular wave after passing through the polarizer will be proportional to the square of its amplitude, i.e., to $|E_0 \cos\phi|^2$. For unpolarized incident wave, $\phi$ can have all values from 0 to 180°. Thus, to get the intensity of the plane polarized wave emerging from the first polarizer, we have to average $|E_0 \cos\phi|^2$ over all values of $\phi$ between 0 and 180°. The value of the average of $\cos^2\phi$ is ½. Hence, the intensity of the wave will be proportional

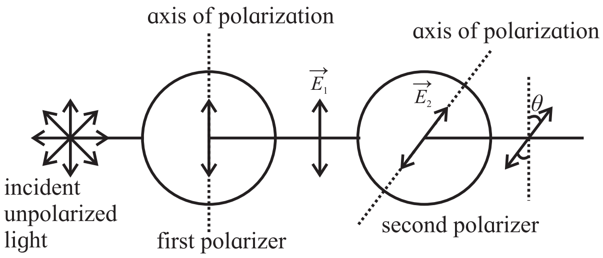



**Fig.7.7(b): Unpolarized light passing through two polarizers.**

to $\frac{1}{2}|E_0|^2$, i.e., *the intensity of an unpolarized wave reduces by half after passing through a polarizer.*

Let us now consider the linearly polarized wave emerging from first polarizer. Let us assume that the polarized wave has its electric field ($\vec{E}_1$) along the *y*-direction as shown in the figure. We can write the electric field as

$$\vec{E}_1 = \hat{j}E_{10}\sin(kx - \omega t), \qquad \text{--- (7.11)}$$

where, $E_{10}$ is the amplitude of this polarized wave. The intensity of the polarized wave is given by

$$I_1 \propto |E_{10}|^2 \qquad \text{--- (7.12)}$$

Now if this wave passes through second polarizer whose polarization axis makes an angle $\theta$ with the *y*-direction, only the component $E_{10}\cos\theta$ will pass through. Thus, the amplitude of the wave which passes through (say $E_{20}$) is now $E_{10}\cos\theta$ and its intensity $I_2$ will be

$$I_2 \propto |E_{20}|^2$$

$$I_2 \propto |E_{10}|^2 \cos^2\theta, \text{ or } I_2 = I_1 \cos^2\theta. \qquad \text{--- (7.13)}$$

This is known as *Malus' law* after E. L. Malus (1775-1812) who discovered the law experimentally. *Malus' law gives the intensity of a linearly polarized wave after it passes through a polarizer.*

Note that $\theta$ is the angle between the axes of polarization of the two polarizers. If $\theta$ is equal to zero, i.e., the polarization axes of the two polarizers are parallel, the intensity does not change while passing through the second polarizer. If $\theta$ is 90°, $\cos^2\theta$ is 0 and no light emerges from the second polarizer. $\theta = 0°$ and 90° are known as parallel and cross settings of the two polarizers.

**Remember this**

Only transverse waves can be polarized while longitudinal waves cannot be polarized. In transverse waves, the oscillations can be along any direction in a plane which is perpendicular to the direction of propagation of the wave. By restricting the oscillations to be along only one direction in this plane, we get a plane polarized wave. For longitudinal waves, e.g., the sound wave, the particles of the medium oscillate only along one direction, which is the direction of propagation of the wave, so there is nothing to restrict.

**Example 7.1:** Unpolarized light of intensity $I_0$, is made to pass through three polarizers $P_1$, $P_2$ and $P_3$ successively. The polarization axis of $P_2$ makes an angle of $\theta_1$ with that of $P_1$, while that of $P_3$ makes an angle $\theta_2$ with that of the $P_2$. What will be the intensity of light coming out of $P_3$?

**Solution:** The first polarizer, $P_1$ will polarize the incident unpolarized light. The intensity after passing through this polarizer will be $I_1 = I_0/2$ as discussed above. Let us assume that the amplitude of the electric field after passing through $P_1$ is $E_{10}$. While passing through $P_2$, a component of the electric field, $E_{20} = E_{10}\cos\theta_1$ will be able to pass through. Thus, the intensity of light coming out of $P_2$ will be $I_2 = (I_1\cos^2\theta_1) = (I_0\cos^2\theta_1)/2$. While passing through $P_3$, a component $E_{30} = E_{20}\cos\theta_2$ will pass through. Thus, the intensity of light coming out of $P_3$ will be $I_3 = (I_0\cos^2\theta_1\cos^2\theta_2)/2$

### 7.7.1 Polarization by Reflection: Brewster's Law:

When light is incident at an angle on a boundary between two transparent media having refractive indices $n_1$ and $n_2$, part of it gets refracted and the rest gets reflected. Let us consider unpolarized light incident from medium of refractive index $n_1$ on such a boundary perpendicular to the plane of the paper, as shown in Fig.7.8.

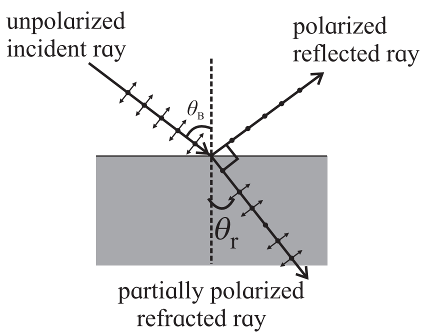



**Fig. 7.8: Polarization by reflection.**

The incident wave is unpolarized. Its electric field which is in the plane perpendicular to the direction of incidence, is resolved into two components, one parallel to the plane of the paper, shown by double arrows and the other perpendicular to the plane of the paper shown by dots. Both have equal magnitude. In general, the reflected and refracted rays do not have equal magnitudes of the two components and hence are partially polarized. It was experimentally discovered by D. Brewster in 1812 that for a particular angle of incidence $\theta_B$ (shown in the figure), the reflected wave is completely plane polarized with its electric field perpendicular to the plane of the paper while the refracted wave is partially polarized. This particular angle of incidence is called the Brewster's angle. For this angle of incidence, the refracted and reflected rays are perpendicular to each other. From the figure, for angle of refraction $\theta_r$ we have,

$$\theta_B + \theta_r = 90^o \qquad \text{--- (7.14)}$$

From law of refraction we have,

$n_1 \sin\theta_B = n_2 \sin\theta_r$. This with Eq.(7.14) gives

$n_1 \sin\theta_B = n_2 \sin(90-\theta_B)$, giving

$\frac{n_2}{n_1} = \tan\theta_B$, or

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) \qquad \text{--- (7.15)}$$

This is known as *Brewster's law*.

The phenomena of polarization by reflection is used to cut out glare from the reflecting surfaces using special sunglasses. Sunglasses are fitted with polaroids which reduce the intensity of the partially or fully polarized reflected light coming to the eyes from reflecting surfaces. As seen above, the intensity of Sunlight or light coming from artificial sources which is completely unpolarized is also reduced to half by the polaroid. This phenomenon of polarization by reflection works only for nonmetallic surfaces.

**Use your brain power**

**What will you observe if**

1. you look at an unpolarized source of light through a polarizer?
2. you look at the source through two polarizers and rotate one of them around the path of light for one full rotation?
3. instead of rotating only one of the polaroid, you rotate both polaroids simultaneously in the same direction?

**Example 7.2:** For what angle of incidence will light incident on a bucket filled with liquid having refractive index 1.5 be completely polarized after reflection?

**Solution:** The reflected light will be completely polarized when the angle of incidence is equal to the Brewster's angle which is given by $\theta_B = \tan^{-1}\frac{n_2}{n_1}$ , where $n_1$ and $n_2$ are refractive indices of the first and the second medium respectively. In this case, $n_1 = 1$ and $n_2 = 1.5$.

Thus, the required angle of incidence = Brewster's angle = $\tan^{-1}\frac{1.5}{1} = 56.31^{\circ}$

### 7.7.2 Polarization by Scattering:

When Sunlight strikes air molecules or dust particles in the atmosphere, it changes its direction. This is called scattering. We see the sky as blue because of this scattering as blue light is preferentially scattered. If there were no scattering, the sky would appear dark to us as long as we do not directly look at the Sun and we could see stars even during the day. When Sunlight is scattered, it gets partially polarized in a way similar to the reflected light seen above (Fig. 7.8). The degree of polarization depends on the angle of scattering, i.e., the angle between the direction of the light incident on the molecule or dust particle and the direction of the scattered light. If this angle is 90°, the scattered light is plane polarized. Thus, the scattered light reaching us from different directions in the sky is polarized to different degrees.

**Can you tell?**

1. If you look at the sky in a particular direction through a polaroid and rotate the polaroid around that direction what will you see?
2. Why does the sky appear to be blue while the clouds appear white?

## 7.8 Interference:

We have learnt about the superposition of waves in Chapter 6. According to this principle, when two or more waves overlap, the resultant displacement of a particle of the medium, at a given point is the sum of the displacements of the particle produced by individual waves, as if each wave is the only one which is present. Because of this, particles in the medium present where the crests (or troughs) of the two waves coincide will have larger displacements, while for particles present where the crest of one wave coincides with the trough of the other, the displacement will be minimum. If the amplitudes of the two waves are equal, then for the first set of particles, the displacement will be twice the amplitude of the individual wave, and for the second set of particles, the displacement will be zero. Thus, the intensity of the wave which is proportional to the square of the amplitude of the wave, will be nonuniform, being larger at some places and smaller at others. This is called *interference*.

Interference is shown in Fig.7.9 for water waves. $S_1$ and $S_2$ are sources of water waves of the same wavelength and amplitude, and are in phase with each other, i.e., at any given instant of time, the phases of the waves emitted by both sources are equal. The crests are shown by continuous circles while the troughs are shown by dotted circles. Points where the crests of one wave coincide with the crests of another wave and where the troughs of one wave coincide with the troughs of another wave are shown by blue dots. At these points the displacement is maximum and is twice that for each wave. These are points of *constructive interference*. The points where the crests of one wave coinside with the troughs of another are shown by red dots. The displacement is zero at these points. These are points of *destructive interference.* Thus, along some straight lines radially diverging from the midpoint of $S_1S_2$, there is constructive interference (along the radial lines connecting the blue dots) and destructive interference (along the radial lines connecting the red dots). Interference had been observed in the case of water waves and sound waves. It was observed for light waves in the laboratory for the first time by Thomas Young (1773-1829) in the year 1801. As noted above, this was the first proof of the wave nature of light. We will discuss this below.

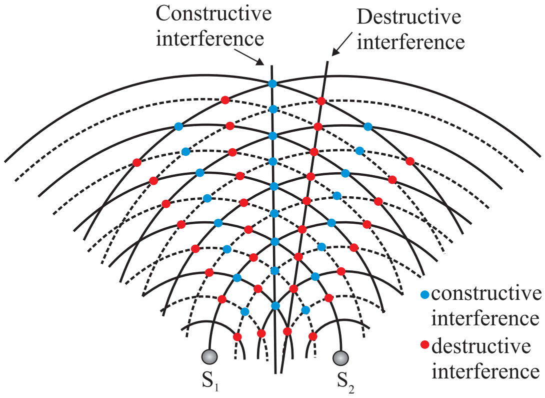



**Fig.7.9: Interference for water waves.**

### 7.8.1 Coherent Sources of Light:

Two sources which emit waves of the same frequency having a constant phase difference, independent of time, are called coherent sources. At any given point in space, at every instant of time, there are light waves from multiple sources overlapping one another. These must be interfering and we should be able to see interference all around us at all time. However, we see no interference pattern. This is because different sources emit waves of different frequencies and even if they emit waves of the same frequency, they are not in phase. Thus, the interference pattern changes every instant of time and no pattern is sustained over a significant length of time for us to see.

For interference to be seen over sustained periods,we need two sources of light which emit waves of the same frequency and the waves emitted by them are in phase or have a constant phase difference between them, i.e. we need coherent sources of light. This criterion cannot be satisfied by two independent primary sources as they emit waves independently and there need not be a constant phase relation between them. Thus, to obtain sustained interference pattern one usually obtains two secondary sources from the same primary source as is done in Young's double slit experiment below.

### 7.8.2 Young's Double Slit Experiment:

In this experiment, a plane wavefront is made to fall on an opaque screen AB having two similar narrow slits $S_1$ and $S_2$. The plane wavefront can be either obtained by placing a linear source S far away from the screen or by placing it at the focus of a convex lens kept close to AB. The rays coming out of the lens will be parallel rays and the wavefront will be a plane wave front. Figure 7.10 shows a cross section of the experimental set up and the slits have their lengths perpendicular to the plane of the paper. For better results, the slits should be about 2-4 mm apart from each other. An observing screen PQ is placed behind AB. For simplicity we assume that the slits $S_1$ and $S_2$ are equidistant from the S so that the wavefronts starting from S and reaching the $S_1$ and $S_2$ at every instant of time are in phase.

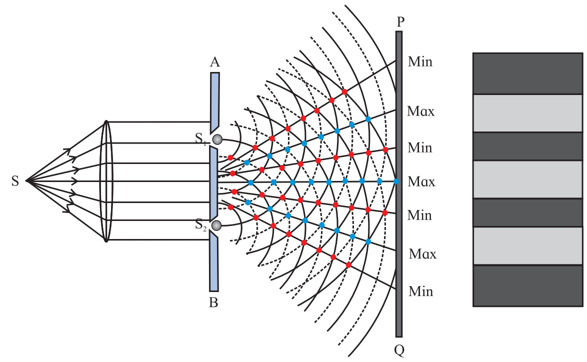



**Fig. 7.10: Young's double slit experiment.**

When the rays fall on $S_1$ and $S_2$, the two slits act as secondary sources of light emitting cylindrical wavelets (with axis along the slit length) to the right of AB. The two secondary sources emit waves in phase with each other at all times (as the waves falling on them are in phase coming from a plane wavefront).

The crests/troughs of the secondary wavelets superpose as shown in the figure and interfere constructively along straight lines joining the blue dots as shown. The point where these lines meet the screen have high intensity and are bright. The images of the slits are also perpendicular to the plane of the paper. The image plane, rotated through 90° is shown on the right side of the figure. Midway between these lines are lines joining the red dots along which the crest of one wave coincides with the trough of the other causing zero intensity and producing dark images of the slits on the screen PQ. These are the dark regions on the screen as shown on the right. The dark and bright regions are called fringes and the whole pattern is called interference pattern.

Let us now determine the positions and intensities of the fringes mathematically. Let us take the direction of propagation of the plane waves incident on AB as *x*-axis. The screen is along the *y-z* plane, *y*-axis being along the plane of the paper. The origin of the axes can be taken to be O, the central point of $S_1 S_2$ as shown in Fig.7.11 which shows the *x-y* cross section of the experimental setup. Let the distance between the two slits $S_1$ and $S_2$ be *d* and that between O and O′ be *D*. For better results *D* should be about a metre for the slit separation mentioned above.

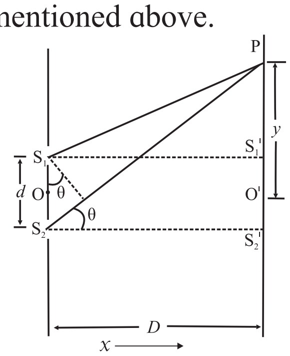



**Fig.7.11: Geometry of the double slit experiment.**

The point O′ on the screen is equidistant from $S_1$ and $S_2$.Thus, the distances travelled by the wavelets starting from $S_1$ and $S_2$ to reach O′ will be equal. The two waves will be in phase at O′, resulting in constructive interference. Thus, there will a bright spot at O′ and a bright fringe at the centre of the screen, perpendicular to the plane of the paper as shown at the right in the Fig. 7.10.

Now let us determine the positions of other bright fringes on the screen. Consider any point P on the screen. The two wavelets from $S_1$ and $S_2$ travel different distances to reach P and so the phases of the waves reaching P will not be the same. If the path difference ($\Delta l$) between $S_1P$ and $S_2P$ is an integral multiple of $\lambda$, the two waves arriving there will interfere constructively producing a bright fringe at P. If the path difference between $S_1P$ and $S_2P$ is half integral multiple of $\lambda$, there will be destructive interference and a dark fringe will be located at P.

Considering triangles $S_1S_1'P$ and $S_2S_2'P$, we can write

$$(S_2P)^2 - (S_1P)^2$$

$$= \left\{ D^2 + \left( y + \frac{d}{2} \right)^2 \right\} - \left\{ D^2 + \left( y - \frac{d}{2} \right)^2 \right\}$$

$$= 2\,yd, \text{giving,}$$

$S_2P - S_1P = \Delta l = 2yd / (S_2P + S_1P)$,

For $d/D << 1$, we can write $S_2P + S_1P \approx 2\,D$, giving

$$\Delta l = 2\frac{yd}{2D} = y\frac{d}{D}$$

Thus, the condition for constructive interference at P can be written as

$$\Delta l = y_n \frac{d}{D} = n\lambda. \qquad \text{--- (7.16)}$$

$y_n$ being the position (*y*-coordinate) of $n^{th}$ bright fringe ($n = 0, \pm1, \pm2, \ldots$). It is given by

$$y_n = n\,\lambda\,D/d. \qquad \text{--- (7.17)}$$

Similarly, the position of $n^{th}$ ($n = \pm1, \pm2, \ldots$) dark fringe (destructive interference) is given by

$$S_2P - S_1P = \Delta l = y_n \frac{d}{D} = \left( n - \frac{1}{2} \right)\lambda, \text{ giving}$$

$$y_n = (n-1/2)\,\lambda\,D/d. \qquad \text{--- (7.18)}$$

The distance between any two successive dark or any two successive bright fringes $(y_{n+1} - y_n)$ is equal. This is called the *fringe width* and is given by,

Fringe width $= W = \Delta y = y_{n+1} - y_n$

$$W = \lambda\,D/d \qquad \text{--- (7.19)}$$

Thus, both dark and bright fringes are equidistant and have equal widths.

We can also write Eq.(7.16) to Eq.(7.19) in terms of phase difference between the two waves as follows.

The relation between path difference ($\Delta l$) and phase difference $\Delta\phi$ is given by,

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta l \qquad \text{--- (7.20)}$$

Thus, the phase difference between the two waves reaching P, from $S_1$ and $S_2$ is given by,

$$\Delta\phi = y\frac{d}{D}\left(\frac{2\pi}{\lambda}\right). \qquad \text{--- (7.21)}$$

The condition for constructive interference in terms of phase difference is given by

$$\Delta\phi_n = n\,2\pi\,, \qquad n = 0, \pm1, \pm2, \ldots \text{---(7.22)}$$

The condition for destructive interference in terms of phase difference is given by

$$\Delta\phi_n = \left(n - \frac{1}{2}\right)2\pi, \qquad n = \pm1, \pm2, \ldots \text{---(7.23)}$$

**Remember this**

- For the interference pattern to be clearly visible on the screen, the distance ($D$) between the slits and the screen should be much larger than the distance ($d$) between two slits, i.e., $D>>d$. We have already used this condition while deriving Eq.(7.16).
- Conditions given by Eq.(7.16-7.19) and hence the locations of the fringes are derived assuming that the two sources $S_1$ and $S_2$ are in phase. If there is a nonzero phase difference between them it should be added appropriately. This will shift the entire fringe pattern but will not change the fringe widths.

**Intensity distribution:**

As there is constructive interference at the centre of a bright fringe, the amplitude of the wave is twice that of the original wave incident on AB and the intensity, $I$, being proportional to the square of the amplitude, is four times the intensity of the incident wave $I_0$ say. At the centres of the dark fringes, the intensity is zero. At in between points, the intensity gradually changes from zero to $4I_0$ and vice versa.

Thus, the interference fringes are equally bright and equally spaced. This is however, valid only in the limit of vanishing widths of the slits. For wide slits, the waves reaching a given point on the screen from different points along a single slit differ in path lengths travelled and the intensity pattern changes resulting in a blurred interference pattern with poor contrast.

We can calculate the intensity at a point P on the screen where the phase difference between the two waves is $\phi$, as follows.

We can write the equations of the two waves coming from $S_1$ and $S_2$, at the point P on the screen as

$E_1 = E_0 \sin\,\omega t$ and $E_2 = E_0 \sin\,(\omega t + \phi)$

The incident intensity $I_0 = |E_0|^2$

The resultant electric field at P will be given by $E = E_0 \sin\,\omega t\; + E_0 \sin\,(\omega t + \phi) = 2\,E_0 \cos\,(\phi/2)\;\sin(\omega t + \phi/2)$. The amplitude of the wave is $2\,E_0 \cos(\phi/2)$.

Thus, the intensity at P will be proportional to $|2\,E_0 \cos\,(\phi/2)|^2$ and it will be equal to $4I_0 \cos^2(\phi/2)$.

Let us consider the case when the amplitudes of the waves coming from the two slits are different, $E_{10}$ and $E_{20}$, say. In this case, the intensities of the bright and dark bands will be different from what is discussed above. At the centre of the bright fringes, the amplitude will be $E_{10} + E_{20}$ and hence the intensity will be proportional to $|E_{10} + E_{20}|^2$, while at the centres of the dark fringes, the intensities will not be zero but will be proportional to $|E_{10} - E_{20}|^2$.

### 7.8.3 Conditions for Obtaining Well Defined and Steady Interference Pattern:

The following conditions have to be satisfied for the interference pattern to be steady and clearly visible. Some of these have already been mentioned above. However, we list them all here for completeness.

1. **The two sources of light should be coherent**. This is the essential condition for getting sustained interference pattern. As we have seen, the waves emitted by two coherent sources are always in phase or have a constant phase difference between them at all times. If the phases and phase difference vary with time, the positions of maxima and minima will also change with time and the interference pattern will not be steady. For this reason, it is preferred that the two secondary sources used in the interference experiment are derived from a single original source as was shown in Fig.7.10.

**Example 7.3:** Plane wavefront of light of wavelength 5500 Å is incident on two slits in a screen perpendicular to the direction of light rays. If the total separation of 10 bright fringes on a screen 2 m away is 2 cm, find the distance between the slits.

**Solution:** Given: $\lambda$ = 5500 Å, $D$ = 2 m and Distance between 10 fringes = 2 cm

= 0.02 m.

The fringe width W= 0.02/10 = 0.002 m

W= $\lambda D/d$

$\therefore d$ = 5500 x $10^{-10}$ x 2 / 0.002

= 5.5 x $10^{-4}$ m = 0.055 cm

**Example 7.4:** In a Young's double slit experiment, the difference in path lengths between the rays starting from the two slits $S_1$ and $S_2$ and reaching a point A on the screen is 0.0075 mm and reaching another point B on the screen on the other side of the central fringe is 0.0015 mm. How many bright and dark fringes are observed between A and B if the wavelength of light used is 6000 Å?

**Solution:** The path difference at a point P on the screen at a distance $y$ from the centre is given by $\Delta l = y\dfrac{d}{D}$,

where $d$ and $D$ are the distances between the slits and between the wall containing the slits and the screen respectively. Thus, we are given,

$\Delta l_A = y_A \dfrac{d}{D} = 0.0075$ mm and $\Delta l_B = y_B \dfrac{d}{D}$ = 0.0015 mm, giving

$y_A = 0.0075\, D/d$ and $y_B = 0.0015\, D/d$ mm

Here, $y_A$ and $y_B$ are the distances of points A and B from the centre of the screen.

Thus, the distance between the points A and B is $y_A + y_B = 0.009\, D/d$ mm.

The width of a bright or dark fringe (i.e., the distance between two bright or two dark fringes) is given by W = $\lambda D/d$. Thus, there will be (0.009 $D/d$)/W = 0.009/ $\lambda$ = 0.009/(6000 x $10^{-7}$) = 15 bright fringes between A and B (including the central one) and 14 dark fringes in between the bright fringes.

**Alternate solution:**

$$\left(\frac{\text{path difference}}{\lambda}\right)_A = \frac{75\times10^{-7}\,\text{m}}{6\times10^{-7}\,\text{m}} = 12.5$$

$$\therefore (\text{path difference})_A = 12.5\lambda = \left(13-\frac{1}{2}\right)\lambda$$

Thus, point A is at the centre of 13th dark band on one side.

Similarly,

$$(\text{path difference})_B = \frac{15}{6}\lambda = 2.5\lambda = \left(3-\frac{1}{2}\right)\lambda$$

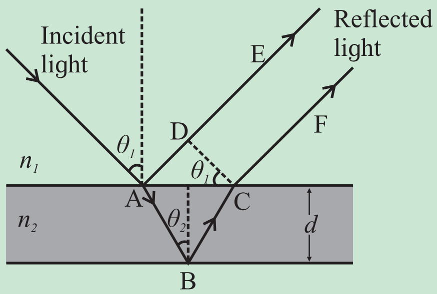
Thus, point B is at the centre of the 3rd dark band on the other side.

Thus, between A and B there will be 12 + 2 + central bright = 15 bright bands.

Also excluding the bands at A and B, there will be 12 + 2 = 14 dark bands between A and B.

**Do you know?**

Several phenomena that we come across in our day to day life are caused by interference and diffraction of light. These are the brilliant colours of soap bubbles as well as those seen in a thin oil film on the surface of water, the bright colours of butterflies and peacocks etc. Most of these colours are not due to pigments which absorb specific colours but are due to interference of light




waves that are reflected by different layers.

**Interference due to thin films:**

The brilliant colours of soap bubbles and thin oil films on the surface of water are due to interference of light waves reflected from the upper and lower surfaces of the film. The two rays have a path difference which depends on the point on the film that is being viewed. This is shown in the figure.

The incident wave gets partially reflected from upper surface as shown by the ray AE. Rest of the light gets refracted and travels along AB. At B it again gets partially reflected and travels along BC. At C it refracts into air and travels along CF. The parallel rays AE and CF have a phase difference due to their different path lengths in different media. As can be seen from the figure, the phase difference depends on the angle of incidence $\theta_1$, i.e., the angle of incidence at the top surface, which is the angle of viewing, and also on the wavelength of the light as the refractive index of the material of the thin film depends on it. The two rays AE and CF interfere, producing maxima and minima for different colours at different angles of viewing. One sees different colours when the film is viewed at different angles.

As the reflection at A is from the denser boundary, there is an additional phase difference of $\pi$ radians (or an additional path difference $\lambda/2$). This should be taken into account for mathematical analysis.

2. **The two sources of light must be monochromatic.** As can be seen from the condition for bright and dark fringes, the position of these fringes as well as the width of the fringes (Eq. (7.17), (7.18) and (7.19)) depend on the wavelength of light and the fringes of different colours are not coincident. The resultant pattern contains coloured, overlapping bands. (*In fact, original Young's experiment was with pin holes* (*not slits*) *and for sunlight, producing coloured interference pattern with central point as white*).
3. **The two interfering waves must have the same amplitude**. Only if the amplitudes are equal, the intensity of dark fringes (destructive interference) is zero and the contrast between bright and dark fringes will be maximum.
4. **The separation between the two slits must be small in comparison to the distance between the plane containing the slits and the observing screen.** This is necessary as only in this case, the width of the fringes will be sufficiently large to be measurable (see Eq.(7.19)) and the fringes are well separated and can be clearly seen.
5. **The two slits should be narrow.** If the slits are broad, the distances from different points along any one of the slits to a given point on the screen are significantly different and therefore, the waves coming through the same slit will interfere among themselves, causing blurring of the interference pattern.
6. **The two waves should be in the same state of polarization.** This is necessary only if polarized light is used for the experiment. The explanation of this condition is beyond the scope of this book.

### 7.8.4 Methods for Obtaining Coherent Sources:

In Young's double slit experiment, we obtained two coherent sources by making the light from a single source pass through two narrow slits. There are other ways to get two coherent sources. We will discuss some of those here.

**i) Lloyd's mirror:** This is an extensively used device. The light from a source is made to fall at a grazing angle on a plane mirror as shown in Fig.7.12. Some of the light falls directly on the screen as shown by the blue lines in the figure and some light falls after reflection, as shown by red lines. The reflected light appears to come from a virtual source and so we get two sources. They are derived from a single source and hence are coherent. They interfere and an interference pattern is obtained as shown in the figure. Note that even though we have shown the direct and reflected rays by blue and red lines, the light is monochromatic having a single wavelength.

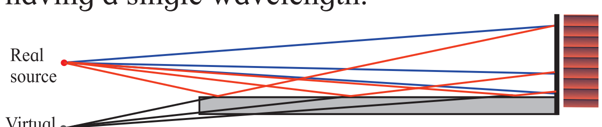



**Fig.7.12: Lloyd's mirror.**

**ii) Fresnel biprism:** A biprism is a prism with vertex angle of nearly 180°. It can be considered to be made up of two prisms with very small refracting angle ranging from 30′ to 1°, joined at their bases. In experimental arrangement, the refracting edge of the biprism is kept parallel to the length of the slit. Monochromatic light from a source is made to pass through a narrow slit S as shown in Fig.7.13 and fall on the biprism.

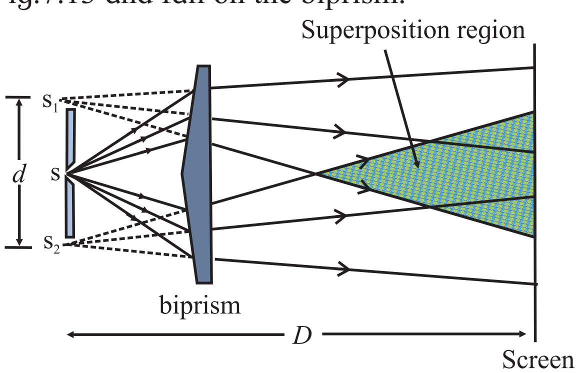
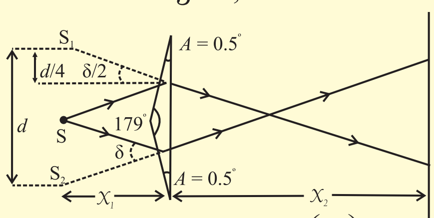



**Fig. 7.13: Fresnel Biprism.**

The two halves of the biprism form virtual images $S_1$ and $S_2$. These are coherent sources having obtained from a single secondary source S. The two waves coming from $S_1$ and $S_2$ interfere and form interference fringes like that in Young's double slit experiment in the shaded region shown in the figure.

**Example 7.5:** An isosceles prism of refracting angle $179^0$ and refractive index 1.5 is used as a biprism by keeping it 10 cm away from a slit, the edge of the biprism being parallel to the slit. The slit is illuminated by a light of wavelength 500 nm and the screen is 90 cm away from the biprism. Calculate the location of the centre of 20th dark band and the path difference at this location.

**Solution:** From the figure,




$$\tan\left(\frac{\delta}{2}\right) \cong \left(\frac{\delta}{2}\right)^c = \frac{\left(\frac{d}{4}\right)}{x_1}$$

and for a thin prism, $\delta = A(\mu - 1)$

$$\therefore d = 2\delta x_1 = 2A(\mu - 1)x_1$$

$$= 2\left(0.5 \times \frac{\pi}{180}\right)(1.5 - 1)10$$

$$= \frac{\pi}{36}\,\text{cm}$$

$$\lambda = 500\,\text{nm} = 5 \times 10^{-7}\,\text{m},$$

$$L = 10\,\text{cm} + 90\,\text{cm} = 1\,\text{m}$$

Distance of 20th dark band from the central bright band, $X_{20D} = \left(20 - \frac{1}{2}\right)W$

$$\therefore X_{20D} = (19.5)\frac{\lambda L}{d}$$

$$= \frac{19.5 \times \left(5 \times 10^{-7}\right) \times 1}{\left(\frac{\pi \times 10^{-2}}{36}\right)}$$

$$= \frac{19.5 \times 180 \times 10^{-5}}{\pi}$$

$$= 0.1117\ \text{m}$$

### 7.8.5 Optical Path:

The phase of a light wave having angular frequency $\omega$ and wave vector $k$, travelling in vacuum along the $x$ direction is given by $(kx\text{-}\omega t)$ (Eq.(7.11)). Remember that wave vector $k$ and the angular frequency $\omega$ are related as $k = \omega/\text{v}$, v being the speed of the wave, which is $c$ in vacuum. If the light wave travels a distance $\Delta x$, its phase changes by $\Delta\phi = k\Delta x = \omega\Delta x/\text{v}$. If the wave is travelling in vacuum $k = \omega/c$ and $\Delta\phi = \omega\Delta x/c$. In case the wave is travelling in a medium having a refractive index $n$, then its wave vector $k'$ and angular frequency are related by $k' = \omega/\text{v} = \omega/(c/n)$, v being the speed of the wave in this medium. Thus, if the wave travels a distance $\Delta x$ in this medium, the phase difference generated will be

$$\Delta\phi' = k'\Delta x = \omega n\,\Delta x/c = \omega\,\Delta x'/c, \quad \text{--- (7.24)}$$

where, $\Delta x' = n\,\Delta x$. --- (7.25)

Thus, when a wave travels a distance $\Delta x$ through a medium having refractive index of $n$, its phase changes by the same amount as it would if the wave had travelled a distance $n\,\Delta x$ in vacuum. We can say that a path length of $\Delta x$ in a medium of refractive index $n$ is equivalent to a path length of $n\,\Delta x$ in vacuum. $n\,\Delta x$ is called the *optical path* travelled by a wave. Thus, optical path through a medium is the effective path travelled by light in vacuum to generate the same phase difference. In vacuum, the optical path is equal to the actual path travelled as $n = 1$.

Optical path in a medium can also be defined as the corresponding path in vacuum that the light travels in the same time as it takes in the given medium.

Now, time $= \dfrac{\text{distance}}{\text{speed}}$ $\quad \therefore t = \dfrac{d_{\text{medium}}}{\text{v}_{\text{medium}}} = \dfrac{d_{\text{vacuum}}}{\text{v}_{\text{vacuum}}}$

$$\therefore \text{Optical path} = d_{\text{vacuum}} = \frac{\text{v}_{\text{vacuum}}}{\text{v}_{\text{medium}}} \times d_{\text{medium}}$$

$$= n \times d_{\text{medium}}$$

Thus, a distance $d$ travelled in a medium of refractive index $n$ introduces a path difference $= nd - d = d(n-1)$ over a ray travelling equal distance through vacuum.

Two waves interfere constructively when their optical path lengths are equal or differ by integral multiples of the wavelength.

If we introduce a transparent plate of thickness $t$ and refractive index $n$ in front of slit $S_1$ (Fig. 7.11), then the optical path travelled by the wave along $S_1P$ is higher than that travelled by the wave along $S_1P$ in absence of the plate by $(n\text{-}1)t$. Thus, the optical path lengths and therefore, the phases of the rays reaching the midpoint O′ from $S_1$ and $S_2$ will not be equal. The path lengths will be equal at a point different than O′ and so the bright fringe will not occur at O′ but at a different point where the two optical path lengths are equal. The dark and bright fringes will be situated symmetrically on both sides of this central fringe. Thus, the whole interference pattern will shift in one direction.

**Example 7.6:** What must be the thickness of a thin film which, when kept near one of the slits shifts the central fringe by 5 mm for incident light of wavelength 5400 Å in Young's double slit interference experiment? The refractive index of the material of the film is 1.1 and the fringe width is 0.5 mm.

**Solution:** Given $\lambda = 5400$ Å, the refractive index of the material of the film = 1.1 and the shift of the central bright fringe = 5 mm.

Let $t$ be the thickness of the film and P be the point on the screen where the central fringe has shifted. Due to the film kept in front of slit $S_1$ say, the optical path travelled by the light passing through it increases by $t$ (1.1-1)= 0.1$t$. Thus, the optical paths between the two beams passing through the two slits are not equal at the midpoint of the screen but are equal at the point P, 5 mm away from the centre. At this point the distance travelled by light from the other slit $S_2$ to the screen is larger than that from $S_1$ by $0.1t$.

The difference in distances $S_2P - S_1P = yd/D = y\lambda/W$, where $y$ is the distance along the screen = 5 mm = 0.005 m and $W$ is given to be 0.5 mm = 0.0005 m.

This has to be equal to the difference in optical paths introduced by the film.

Thus, $0.1t = 0.005 \times 5400 \times 10^{-10}/0.0005$.

$t = 5.4 \times 10^{-5}$m = 0.054 mm

## 7.9 Diffraction of Light:

We know that shadows are formed when path of light is blocked by an opaque obstacle. Entire geometrical optics is based on the rectilinear propagation of light. However, as discussed earlier, the phenomenon exhibited by light such as interference can only be explained by considering the wave nature of light. Diffraction is another such phenomenon. In certain experiments, light is seen to bend around edges of obstacles in its path and enter into regions where shadows are expected on the basis of geometrical optics. This, so called bending of light around objects, is called the phenomenon of diffraction and is common to all waves. We are very familiar with the fact that sound waves travel around obstacles as we can hear someone talking even though there are obstacles, e.g. a wall, placed between us and the person who is talking. As we will see below, light actually does not bend around edges in diffraction, but is able to reach the shadow region due to the emission by the secondary sources of light on the edge of the obstacle. The phenomenon of diffraction is intimately related to that of interference. Diffraction is essentially the interference of many waves rather than two which we have encountered in interference. Also, diffraction is noticeable only when the size of the obstacle or slits is of the order of the wavelength of light. As diffraction is a wave phenomenon, it is applicable to sound waves as well. As wavelengths of sound waves are larger, diffraction of sound is easier to observe.

### 7.9.1 Fresnel and Fraunhofer Diffraction:

Diffraction can be classified into two types depending on the distances involved in the experimental setup.

1. **Fraunhofer diffraction:** If the distances between the primary source of light, the obstacle/slit causing diffraction and the screen for viewing the diffraction pattern are very large, the diffraction is called Fraunhofer diffraction. In this case, the wavefront incident on the obstacle can be considered to be a plane wavefront. For this, we generally place the source of light at the focus of a convex lens so that a plane wavefront is incident on the obstacle and another convex lens is used on the other side of the obstacle to make the pattern visible on the screen. Figure 7.14 shows this arrangement schematically.
2. **Fresnel diffraction:** In this case, the distances are much smaller and the incident wavefront is either cylindrical or spherical depending on the source. A lens is not required to observe the diffraction pattern on the screen.

### 7.9.2 Experimental set up for Fraunhofer diffraction:

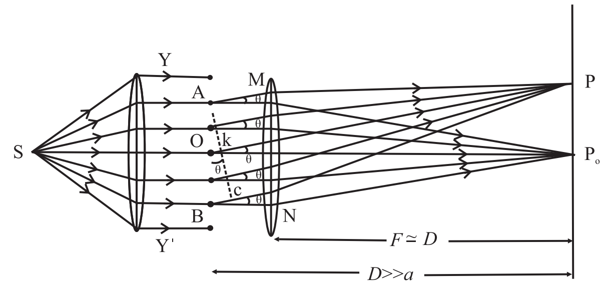



Fig. 7.14: Set up for fraunhofer diffraction .

Figure 7.14 shows a monochromatic source of light S at the focus of a converging lens. Ignoring aberrations, the emerging beam will consist of plane parallel rays resulting in plane wavefronts. These are incident on the diffracting element such as a slit, a circular aperture, a double slit, a grating, etc. Emerging beam is incident on another converging lens that focuses the beam on a screen.

In the case of a circular aperture, S is a point source and the lenses are bi-convex. For linear elements like slits, grating, etc., the source is linear and the lenses are cylindrical in shape so that the focussed image is also linear. In either case, a plane wavefront (as if the source is at infinity) approaches and leaves the diffracting element. This is as per the requirements of Fraunhofer diffraction.

### 7.9.3 Fraunhofer Diffraction at a Single Slit:

Figure 7.14 shows the cross section of a plane wavefront YY′ incident on a single slit of width AB. The centre of the slit is at point O. As the width of the slit is in the plane of the paper, its length is perpendicular to the paper. The slit can be imagined to be divided into a number of extremely thin slits (or slit elements). The moment the plane wavefront reaches the slit, each slit element becomes the secondary source of cylindrical wavefronts responsible for diffraction in all possible directions. A cylindrical lens (with its axis parallel to the slit) kept next to the slit converges the emergent beams on to the screen kept at the focus. The distance $D$ between the slit and the screen is practically the focal length $F$ of the lens. For all practical set ups, the width $a$ of the slit is of the order of $10^{-4}$ to $10^{-3}$ m and distance $D$ is of the order of 10 m, i.e., $D >> a$.

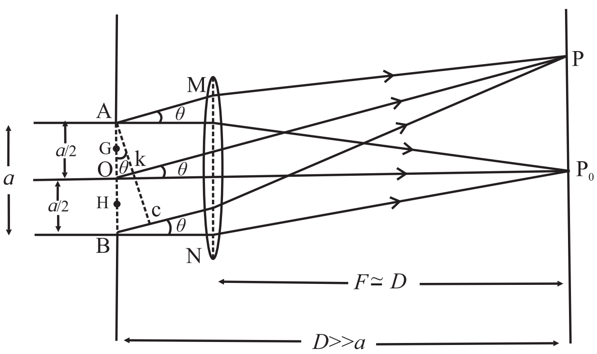



Fig. 7.15: Fraunhofer diffraction .

*Location of minima and maxima:* Figure 7.15 is a part of figure 7.14. It shows two sets of parallel rays originating at the slit elements. The central and symmetric beam focuses at the central point (line) $P_0$, directly in front of O, the center of the slit.

Rays parallel to the axis from all points from A to B are focused at the central point $P_0$. Thus, all these rays must be having equal optical paths. Hence all these arrive at $P_0$ with the same phase, and thereby produce constructive interference at $P_0$.

In the case of point P at angular position $\theta$ the optical paths of the rays from A to C till point P are equal as AC is normal to AM. From this points onwards, there is no path difference or phase difference between these rays. Hence, the paths of rays between AB and AC are responsible for the net path difference or phase difference at point P. The path difference between extreme rays is BC $= a\sin\theta$ . Let this be equal to $\lambda$.

Let K be the midpoint of AC. $\therefore$ OK $= \dfrac{1}{2}$BC $= \dfrac{\lambda}{2}$

Thus, the path difference between AP and OP is $\lambda/2$. As a result, these two rays (waves) produce destructive interference at P. Now consider any pair of points equidistant respectively from A and O, such as G and H, separated by distance $a/2$ along the slit. Rays (waves) from any such pair will have a path difference of $\lambda/2$ at P. Thus, all such pairs of points between AO and OB will produce destructive interference at P. This makes point P to be the first minimum.

This discussion can be extended to points on the screen having path differences $2\lambda$, $3\lambda$, … $n\lambda$ between the extreme rays reaching them and it can be seen that these points will be dark and hence will be positions of dark fringes.

Again, the same logic is applicable for points on the other side of $P_0$.

Hence, at the location of $n^{\text{th}}$ minima, the path difference between the extreme rays

$$a\sin\theta = \pm n\lambda \qquad \text{--- (7.26)}$$

If we assume the maxima to be in between the respective minima, we can write the path difference between the extreme rays at the $n^{\text{th}}$ maxima as

$$a\sin\theta = \pm\left(n+\frac{1}{2}\right)\lambda \qquad \text{--- (7.27)}$$

**Do you know?**

- Why did we use $\lambda$ as the path difference between the waves originating from extreme points for the first minima?
- On receiving energy from two waves, at the position of the first dark fringe, the path difference between the two waves must be $\lambda/2$. In the case of a single slit, a point on the screen receives waves from all the points on the slit. For the point on the screen to be dark, there must be a path difference of $\lambda/2$ for all pairs of waves. One wave from this pair is from upper half part of the slit and the other is from the lower half e.g., points A and O, G and H, O and B, etc. Thus, the minimum path difference between the waves originating from the extreme points A and B must be $\frac{\lambda}{2}+\frac{\lambda}{2}=\lambda$.

*In reality, these are nearly midway and not exactly midway between the dark fringes.*

**Distances of minima and maxima from the central bright point,(i.e., the distances on the screen):**

Equations (7.26) and (7.27) relate the path difference at the locations of $n^{\text{th}}$ dark and $n^{\text{th}}$ bright point respectively. As described earlier, the distance $D >> a$. Hence the angle $\theta$ is very small. Thus, if it is expressed in radian, we can write, $\sin\theta \cong \tan\theta \cong \theta = \frac{y}{D}$ where $y$ is the distance of point P, on the screen from the central bright point, the screen being at a distance $D$ from the diffracting element (single slit).

Let $y_{nd}$ and $y_{nb}$ be the distances of $n^{\text{th}}$ dark point and $n^{\text{th}}$ bright point from the central bright point.

Thus, at the $n^{\text{th}}$ dark point on either side of the central bright point, using Eq. (7.26)

$$\theta_{nd} = \frac{y_{nd}}{D} = n\frac{\lambda}{a}\ ,\ y_{nd} = n\frac{\lambda D}{a} = nW \qquad \text{--- (7.28)}$$

Also, at the $n^{\text{th}}$ bright point on either side of the central bright point, using Eq. (7.27)

$$\theta_{nb} \cong \frac{y_{nb}}{D} \cong \left(n+\frac{1}{2}\right)\frac{\lambda}{a} \qquad \text{--- (7.29)}$$

$$y_{nb} \cong \left(n+\frac{1}{2}\right)\frac{\lambda D}{a} \cong \left(n+\frac{1}{2}\right)W\ ,$$

where, $W = \frac{\lambda D}{a}$ is similar to the fringe width in the interference pattern. In this case also it is the distance between consecutive bright fringes or consecutive dark fringes, except the central (zeroth) bright fringe.

***Width of the central bright fringe*:**

The central bright fringe is spread between the first dark fringes on either side. Thus, width of the central bright fringe is the distance between the centres of first dark fringe on either side.

$\therefore$ Width of the central bright fringe,

$$W_c = 2y_{1d} = 2W = 2\left(\frac{\lambda D}{a}\right)$$

### 7.9.4 Comparison of Young's Double Slit Interference Pattern and Single Slit Diffraction Pattern:

For a common laboratory set up, the slits in the Young's double slit experiment are much thinner than their separation. They are usually obtained by using a biprism or a Lloyd's mirror. The separation between the slits is a few mm only. With best possible set up, we can usually see about 30 to 40 equally spaced bright and dark fringes of nearly same brightness.

The single slit used to obtain the diffraction pattern is usually of width less than 1 mm. Taken on either side, we can see around 20 to 30 fringes with central fringe being the brightest. Also, width of the central bright fringe is twice that of all the other bright fringes (in the single slit diffraction pattern). Table 7.1 gives mathematical comparison between interference and diffraction patterns and Fig. (7.16) shows corresponding ($I$ - $\theta$) graphs.

*Remember the following while using the table:*

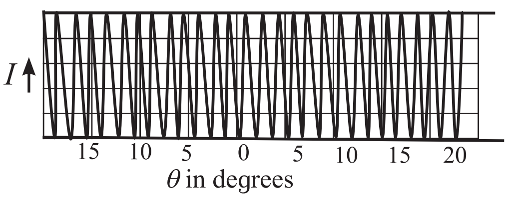
Central bright fringe is the ZEROTH bright fringe ($n = 0$), and not the first.

$d$: slit separation, $a$: slit width, $D$: Slit/s to screen separation

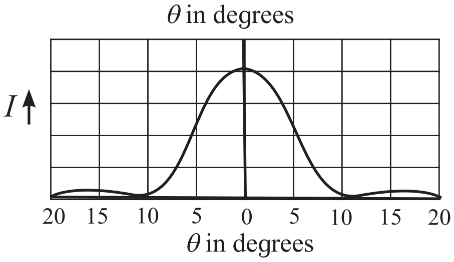
$W$: Separation between consecutive bright or dark fringes.

**Table 7.1: Comparison between interference and diffraction patterns**

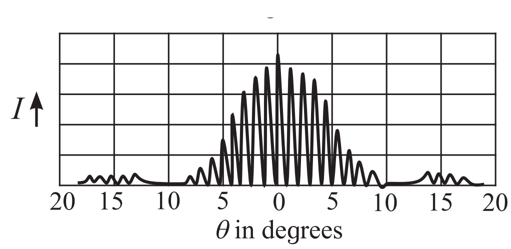
| Physical quantity | | Young's double slit interference Pattern | Single slit diffraction pattern |
|---|---|---|---|
| Fringe width $W$ | | $W = \dfrac{\lambda D}{d}$ | $W = \dfrac{\lambda D}{a}$ Except for the central bright fringe |
| For $n^{th}$ bright fringe | Phase difference, $\delta$ between extreme rays | $n(2\pi)$ | $\left(n+\dfrac{1}{2}\right)(2\pi)$ |
| | Angular position, $\theta$ | $n\left(\dfrac{\lambda}{d}\right)$ | $\left(n+\dfrac{1}{2}\right)\left(\dfrac{\lambda}{d}\right)$ |
| | Path difference, $\Delta x$ between extreme rays | $n\lambda$ | $\left(n+\dfrac{1}{2}\right)\lambda$ |
| | Distance from the central bright spot, $y$ | $n\left(\dfrac{\lambda D}{d}\right) = nW$ | $\left(n+\dfrac{1}{2}\right)\left(\dfrac{\lambda D}{a}\right) = nW$ |
| For $n^{th}$ dark fringe | Phase difference, $\delta$ between extreme rays | $\left(n-\dfrac{1}{2}\right)(2\pi)$ | $n(2\pi)$ |
| | Angular position, $\theta$ | $\left(n-\dfrac{1}{2}\right)\left(\dfrac{\lambda}{d}\right)$ | $n\left(\dfrac{\lambda}{d}\right)$ |
| | Path difference, $\Delta x$ between extreme rays | $\left(n-\dfrac{1}{2}\right)\lambda$ | $n\lambda$ |
| | Distance from the central bright spot, $y$ | $\left(n-\dfrac{1}{2}\right)\left(\dfrac{\lambda D}{d}\right) = nW$ | $n\left(\dfrac{\lambda D}{a}\right) = nW$ |




**(a)**




**(b)**




**(c)**

**Fig.7.16: Intensity $I$ distribution in (a) Young's double slit interference (b) single slit diffraction and (c) double slit diffraction.**

## Double slit diffraction pattern:

What pattern will be observed due to diffraction from two slits rather than one? In this case the pattern will be decided by the diffraction pattern of the individual slits, as well as by the interference between them. The pattern is shown in Fig.7.16 (c). There are narrow interference fringes similar to those in Young's double slit experiment, but of varying brightness and the shape of their envelope is that of the single slit diffraction pattern.

## 7.10 Resolving Power:

Diffraction effect is most significant while discussing the resolving power of an optical instrument. We use an optical instrument to see minor details of all the parts of an object or to see distinct images of different nearby objects and not only for magnification.

Consider your friend showing two of her fingers (as we do while showing a victory sign) from a distance less than 10 m. You can easily point out that she is showing two fingers, i.e., you can easily distinguish the two fingers. However, if she shows the same two fingers from a distance over 50 m, most of you will NOT be able to distinguish the two fingers, i.e., you can't definitely say whether those are two fingers or it is a single finger.

This ability to distinguish two physically separated objects as two distinct objects is known as the resolving power of an optical instrument. In the example given, it was your eye. In other words, from a distance less than 10 m, your eye is able to ***resolve*** the two fingers. From beyond 50 m, it may not possible for you to resolve the two fingers.

Resolving power of an optical instrument (an eye, a microscope, a telescope, etc.) generally depends upon the aperture (usually the diameter of the lens or mirror) and the wavelength of the light used. In general, the resolving ability of an instrument is stated in terms of the visual angle, which is the angle subtended at the eye by the two objects to be resolved (which are assumed to be point objects). It is the minimum visual angle between two objects that can be resolved by that instrument. This minimum angle is called limit of resolution. Reciprocal of the limit of resolution is called the resolving power.

### 7.10.1 Rayleigh's Criterion for Limit of Resolution (or for Resolving Power):

According to Lord Rayleigh, the ability of an optical instrument to distinguish between two closely spaced objects depends upon the diffraction patterns of the two objects (slits, point objects, stars, etc.), produced at the screen (retina, eyepiece, etc.). According to this criterion, two objects are *just resolved* when the first minimum of the diffraction pattern of one source coincides with the central maximum of the diffraction pattern of the other source, and vice versa.

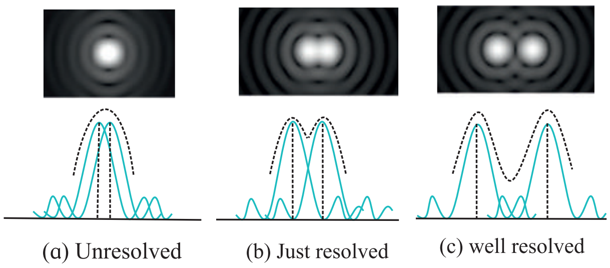



**Fig. 7.17 Rayleigh's criterion for resolution of objects.**

In Fig. 7.17(b), first minimum of the diffraction pattern of second object is coinciding with the central maximum of the first and vice versa. In such case we find the objects to be just resolved as the depression in the resultant envelopee is noticeable. For Fig 7.17(a), the depression is not noticeable and in Fig 7.17(c), it can be clearly noticed.

**(i) Two linear objects:** Consider two self luminous objects (slits) separated by some distance. As per Rayleigh's criterion, the first minimum of the diffraction pattern of one of the sources should coincide with the central maximum of the other. Graphical pattern of the diffraction by two slits at the *just resolved* condition is as shown in the lower half of the

Fig 7.17(b). Angular separation (position) of the first principal minimum is

$$d\theta = \frac{\lambda}{a} \qquad \text{--- (7.30)}$$

As this minimum coincides with the central maximum of the other, this must be the minimum angular separation between the two objects, and hence the limit of resolution of that instrument.

$$\therefore \text{Limit of resolution, } d\theta = \frac{\lambda}{a}$$

Minimum separation between the two linear objects that are just resolved, at distance $D$ from the instrument, is

$$y = D(d\theta) = \frac{D\lambda}{a} \qquad \text{--- (7.31)}$$

It is obviously the distance of the first minimum from the centre.

**(ii) Pair of Point objects:** In the case of a microscope, quite often, the objects to be viewed are similar to point objects. The diffraction pattern of such objects consists of a central bright disc surrounded by concentric rings, called Airy disc and rings as shown in Fig. 7.18 (a). Abbe was the first to study this thoroughly, and apply it to Fraunhofer diffraction.

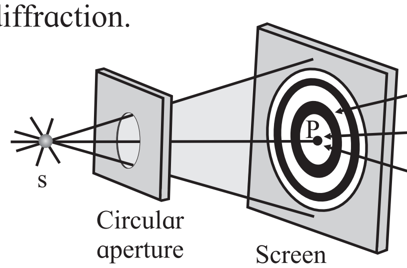



**Fig 7.18 (a): A schematic diagram showing formation of Airy disc and rings.**

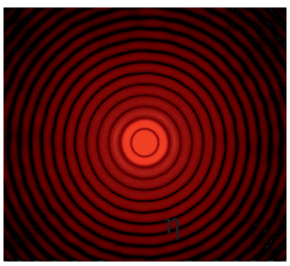


**Fig 7.18 (b): A real Airy disc obtained by using a laser.**

Figure (7.18 (b)) shows a real Airy disk formed by passing a red laser beam through a 90 μm pinhole aperture that shows several orders (rings) of diffraction.

According to Lord Rayleigh, for such objects to be just resolved, the first dark ring of the diffraction pattern of the first object should be formed at the centre of the diffraction pattern of the second, and vice versa. In other words, the minimum separation between the images on the screen is radius of the first dark ring (Fig. 7.17 (b) and Fig. 7.19 (a))

The discussion till here is applicable to any instrument such as an eye, a microscope, a telescope, etc.

### 7.10.2 Resolving Power of a Microscope:

Fig 7.19 shows two point objects O and O′ separated by a distance $a$ in front of an objective AB of a microscope. Medium between the objects and the objective is of refractive index $n$. Wavelength of the light emitted by the sources in the medium is $\lambda_n$. Angular separation between the objects, at the objective is $2\alpha$.

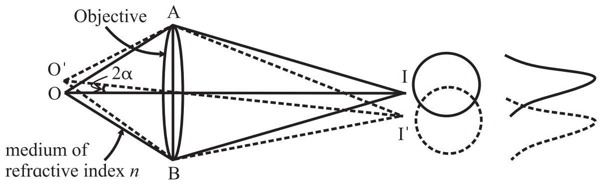



**Fig 7.19 (a) : Resolution of objects by a microscope.**

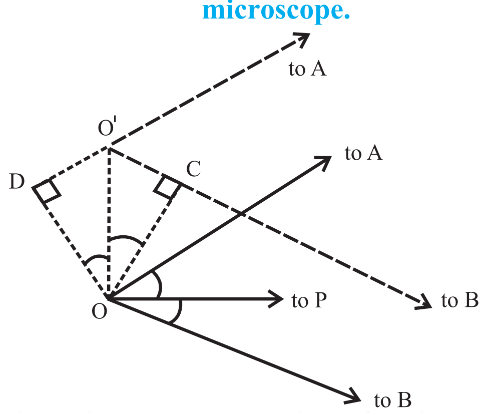



**Fig 7.19 (b): Enlarged view of region around O and O′ in Fig. 7.19 (a).**

I and I′ are centers of diffraction patterns (Airy discs and rings) due to O and O′ on the screen (effectively at infinity). According to Rayleigh's criterion the first dark ring due to O′ should coincide with I and that of O should coincide with I′. The nature of illumination at a point on the screen is decided by the effective path difference at that point. Let us consider point I where first dark ring due to O′ is located. Paths of the extreme rays reaching I from O′ are O′AI and O′BI. As point I is equidistant from A and B, the paths AI and BI are equal. Thus, the actual path difference is O′B - O′A.

The region around O′ and O is highly enlarged in Fig. 17.19 (b). From this figure it can be proved that

path difference = DO′ + O′C

$$= 2\,a\sin\alpha \qquad \text{--- (7.32)}$$

**(i) Microscope with a pair of non-luminous objects (dark objects):** In common microscopy, usually self-luminous objects are not observed. Non-luminous objects are illuminated by some external source. In general, this illumination is not normal, but it is oblique. Also, majority of the objects viewed through a microscope can be considered to be point objects producing Airy rings in their diffraction patterns. Often the eye piece is filled with some transparent material. Let the wavelength of light in this material be $\lambda_n = \lambda/n$, $\lambda$ being the wavelength of light in air and $n$ is the refractive index of the medium. In such a set-up the path difference at the first dark ring is $\lambda_n$.

Thus, form Eq. (7.32) $2\,a\sin\alpha = \lambda_n = \lambda/n$

$$a = \frac{\lambda}{2n\sin\alpha} = \frac{\lambda}{2(N.A.)} \qquad \text{--- (7.33)}$$

The factor $n\sin\alpha$ is called numerical aperture (*N. A.*).

The resolving power, $R = \frac{1}{a} = \frac{2(N.A)}{\lambda}$ - (7.34)

**(ii) Microscope with self luminous point objects:** Applying Abbe's theory of Airy discs and rings to Fraunhofer diffraction due to a pair of self luminous point objects, the path difference between the extreme rays, at the first dark ring is given by 1.22 $\lambda$,

Thus, for the requirement of just resolution,

$$2\,a\sin\alpha = 1.22\lambda_n$$

$$\therefore a = \frac{1.22\lambda_n}{2\sin\alpha}$$

$$\therefore a = \frac{1.22\lambda}{2n\sin\alpha} = \frac{0.61\lambda}{n\sin\alpha} = \frac{0.61\lambda}{(N.A.)} \qquad \text{--- (7.35)}$$

The resolving power $R = \frac{1}{a} = \frac{(N.A.)}{0.61\lambda}$ --- (7.36)

For better resolution, $a$ should be minimum. This can be achieved by using an oil filled objective which provides higher value of $n$.

**Do you know?**

- The expression of resolving power of a microscope is inversely proportional to the wavelength $\lambda$ used to illuminate the object. Can we have a source with a very small wavelength compared to that of the visible light?
- You will study later in Chapter 14 that electrons exhibit wave like behaviour. By controlling the speed of the electrons, we can control their wavelength. This principle is used in Electron Microscopes. Modern electron microscopes can have a magnification $\sim 10^6$ and can resolve objects with separation < 10 nm. For ordinary optical microscopes the minimum separation is generally around $10^{-3}$ mm.

### 7.10.3 Resolving Power of a Telescope:

Telescopes are normally used to see distant stars. For us, these stars are like luminous point objects, and are far off. Thus, their diffraction patterns are Airy discs. Also, as the objects to be seen are far off, only the angular separation between the two is of importance and not the linear separation between them. Fig. 7.20 shows objective AB of a telescope receiving two sets of parallel beams from two distant objects with an angular separation $\theta$. *Resolving power of a telescope is then defined as the reciprocal of the least angular separation between the objects that are just resolved.*

According to Rayleigh's criterion, the minimum separation between the images I and I′ must be equal to the radius of the first dark Airy ring.

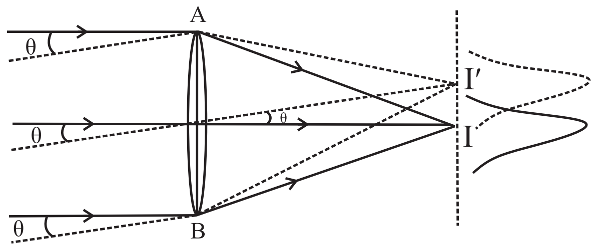



**Fig. 7.20: Resolution by a telescope.**

If Airy's theory is applied to Fraunhofer diffraction of a pair of point objects, the path difference between the extreme rays, at the first dark ring is given by 1.22 $\lambda$.

$\therefore BI' - AI' = 1.22\lambda$

If $D$ is the aperture of the telescope (diameter AB of its objective),

$$BI' - AI' \cong D \times \theta = 1.22\lambda$$

$$\therefore \theta = \frac{1.22\lambda}{D} \quad \text{and} \quad R = \frac{1}{\theta} = \frac{D}{1.22\lambda} \qquad \text{--- (7.37)}$$

Thus, to increase the resolving power of a telescope (for a given wavelength), its objective (aperture) should be as large as possible. Using a large lens invites a lot of difficulties such as aberrations, initial moulding of the lens, post launch issues such as heavy mass for changing the settings, etc. The preferred alternative is to use a front coated curved mirror as the objective. As discussed in XI$^{th}$ Std. a parabolic mirror is used in order to eliminate the spherical aberration. Again, constructing a single large mirror invites other difficulties. Recent telescopes use segmented mirrors that have a number of hexagonal segments to form a large parabolic mirror. Two largest optical telescopes under construction have mirrors of 30 m and 40 m diameters.

**Radio Telescope:** Wavelengths of radio waves are in metres. According to Eq. (7.37), if we want to have same limit of resolution as that of an optical telescope, the diameter (aperture) of the radio telescopes should be very large (at least few kilometers). Obviously, a single disc of such large diameter is impracticable. In such cases arrays of antennae spread over several kilometres are used. Giant Metre-wave Radio Telescope (GMRT) located at Narayangaon, near Pune, Maharashtra, uses such an array consisting of 30 dishes of 45 m diameter each, spread over 25 km. It is the largest distance between two of its antennae. A photograph of 5 GMRT dishes is shown in Fig. 7.21.

**Fig. 7.21 : GMRT Radio Telescope.**

The highest angular resolution achievable ranges from about 60 arcsec (arcsec is3600$^{th}$ part of a degree) at the lowest frequency of 50 MHz to about 2 arcsec at 1.4 GHz.

**Example 7.7:** A telescope has an objective of diameter 2.5 m. What is its angular resolution when it observes at 5500 Å?

**Solution:** Angular resolution, $\theta = 1.22\,\lambda/a$, $a$ being the diameter of the aperture

$= 1.22 \times 5.5 \times 10^{-7} / 2.5 = 2.684 \times 10^{-7}$ rad

$= 0.06$ arcsec

**Example 7.8:** What is the minimum distance between two objects which can be resolved by a microscope having the visual angle of $30^0$ when light of wavelength 500 nm is used?

**Solution:** According to Eq. (7.35) the minimum distance is given by

$a_{min} = 0.61\,\lambda/\sin\alpha$

$a_{min} = 0.61 \times 5.0 \times 10^{-7}/\sin 30^o = 6.1 \times 10^{-7}$ m.

**Internet my friend**

1. https://en.wikipedia.org/wiki/Wave_interference
2. http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/interfcon.html
3. http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/diffracon.html
4. https://opentextbc.ca/physicstestbook2/chapter/limits-of-resolution-the-rayleigh-criterion/

## Exercises

**1. Choose the correct option.**

i) Which of the following phenomenon proves that light is a transverse wave?

(A) reflection (B) interference
(C) diffraction (D) polarization

ii) Which property of light does not change when it travels from one medium to another?

(A) velocity (B) wavelength
(C) amplitude (D) frequency

iii) When unpolarized light is passed through a polarizer, its intensity

(A) increases (B) decreases
(C) remains unchanged
(D) depends on the orientation of the polarizer

iv) In Young's double slit experiment, the two coherent sources have different intensities. If the ratio of maximum intensity to the minimum intensity in the interference pattern produced is 25:1. What was the ratio of intensities of the two sources?

(A) 5:1 (B) 25:1 (C) 3:2 (D) 9:4

v) In Young's double slit experiment, a thin uniform sheet of glass is kept in front of the two slits, parallel to the screen having the slits. The resulting interference pattern will satisfy

(A) The interference pattern will remain unchanged
(B) The fringe width will decrease
(C) The fringe width will increase
(D) The fringes will shift.

**2. Answer in brief.**

i) What are primary and secondary sources of light?

ii) What is a wavefront? How is it related to rays of light? What is the shape of the wavefront at a point far away from the source of light?

iii) Why are multiple colours observed over a thin film of oil floating on water? Explain with the help of a diagram.

iv) In Young's double slit experiment what will we observe on the screen when white light is incident on the slits but one slit is covered with a red filter and the other with a violet filter? Give reasons for your answer.

v) Explain what is optical path length. How is it different from actual path length?

3. Derive the laws of reflection of light using Huygens' principle.

4. Derive the laws of refraction of light using Huygens' principle.

5. Explain what is meant by polarization and derive Malus' law.

6. What is Brewster's law? Derive the formula for Brewster angle.

7. Describe Young's double slit interference experiment and derive conditions for occurrence of dark and bright fringes on the screen. Define fringe width and derive a formula for it.

8. What are the conditions for obtaining good interference pattern? Give reasons.

9. What is meant by coherent sources? What are the two methods for obtaining coherent sources in the laboratory?

10. What is diffraction of light? How does it differ from interference? What are Fraunhoffer and Fresnel diffractions?

11. Derive the conditions for bright and dark fringes produced due to diffraction by a single slit.

12. Describe what is Rayleigh's criterion for resolution. Explain it for a telescope and a microscope.

13. White light consists of wavelengths from 400 nm to 700 nm. What will be the wavelength range seen when white light is passed through glass of refractive index 1.55?

[Ans: 258.1 - 451.6 nm]

14. The optical path of a ray of light of a given wavelength travelling a distance of 3 cm in flint glass having refractive

index 1.6 is same as that on travelling a distance $x$ cm through a medium having refractive index 1.25. Determine the value of $x$.

[Ans: 3.84 cm]

15. A double-slit arrangement produces interference fringes for sodium light ($\lambda = 589$ nm) that are 0.20° apart. What is the angular fringe separation if the entire arrangement is immersed in water ($n = 1.33$)?

[Ans: 0.15°]

16. In a double-slit arrangement the slits are separated by a distance equal to 100 times the wavelength of the light passing through the slits. (a) What is the angular separation in radians between the central maximum and an adjacent maximum? (b) What is the distance between these maxima on a screen 50.0 cm from the slits?

[Ans: 0.01 rad, 0.5 cm]

17. Unpolarized light with intensity $I_0$ is incident on two polaroids. The axis of the first polaroid makes an angle of $50^o$ with the vertical, and the axis of the second polaroid is horizontal. What is the intensity of the light after it has passed through the second polaroid?

[Ans: $I_0/2 \times (\cos 40^0)^2$]

18. In a biprism experiment, the fringes are observed in the focal plane of the eyepiece at a distance of 1.2 m from the slits. The distance between the central bright band and the $20^{th}$ bright band is 0.4 cm. When a convex lens is placed between the biprism and the eyepiece, 90 cm from the eyepiece, the distance between the two virtual magnified images is found to be 0.9 cm. Determine the wavelength of light used.

[Ans: 5000 Å]

19. In Fraunhoffer diffraction by a narrow slit, a screen is placed at a distance of 2 m from the lens to obtain the diffraction pattern. If the slit width is 0.2 mm and the first minimum is 5 mm on either side of the central maximum, find the wavelength of light.

[Ans: 5000 Å]

20. The intensity of the light coming from one of the slits in Young's experiment is twice the intensity of the light coming from the other slit. What will be the approximate ratio of the intensities of the bright and dark fringes in the resulting interference pattern? [Ans: 34]

21. A parallel beam of green light of wavelength 550 nm passes through a slit of width 0.4 mm. The intensity pattern of the transmitted light is seen on a screen which is 40 cm away. What is the distance between the two first order minima?

[Ans: 1.1 mm]

22. What must be the ratio of the slit width to the wavelength for a single slit to have the first diffraction minimum at 45.0°?

[Ans: 1.274]

23. Monochromatic electromagnetic radiation from a distant source passes through a slit. The diffraction pattern is observed on a screen 2.50 m from the slit. If the width of the central maximum is 6.00 mm, what is the slit width if the wavelength is (a) 500 nm (visible light); (b) 50 μm (infrared radiation); (c) 0.500 nm (X-rays)?

[Ans: 0.4167 mm, 41.67 mm, $4.167 \times 10^{-4}$ mm]

24. A star is emitting light at the wavelength of 5000 Å. Determine the limit of resolution of a telescope having an objective of diameter of 200 inch.

[Ans: $1.2 \times 10^{-7}$ rad]

25. The distance between two consecutive bright fringes in a biprism experiment using light of wavelength 6000 Å is 0.32 mm by how much will the distance change if light of wavelength 4800 Å is used?

[Ans: 0.064 mm]

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