ANSWERS

1. Mathematical Logic

Exercise 1.1

1) (i) Statement, F (ii) Not statement (iii) Not statement
(iv) Statement, T (v) Not statement (vi) Statement, T
(vii) Not statement (viii) Statement, T (ix) Not statement
(x) Statement, T (xi) Statement, F (xii) Not statement
(xiii) Statement, T (xiv) Statement, T (xv) Not statement

2) (i) $p \wedge q$ (ii) $p \vee q$ (iii) $p \leftrightarrow q$
(iv) $\sim p \wedge \sim q$ (v) $p \rightarrow q$ (vi) $p \leftrightarrow q$
(vii) $\sim p \wedge q$

3) (i) F (ii) F (iii) F
(iv) T (v) T (vi) T
(vii) T

4) (i) T (ii) T (iii) F
(iv) T (v) F (vi) F
(vii) T (viii) T

5) (i) Tirupati is not in Andhra Pradesh.
(ii) 3 is a root of the equation $x^2 + 3x - 18 = 0$.
(iii) $\sqrt{2}$ is not a rational number.
(iv) Polygon ABCDE is not a pentagon.
(v) $7 + 3 \leq 5$

Exercise 1.2

1) (i) TTFT (ii) FFFF (iii) TTFT FFFT
(iv) FTTTTTTT (v) FFFF (vi) TFFT
(vii) TTTT (viii) TTTTTTTT (ix) FTTTTTTT
(x) TFTFTTFF

3) (i) Tautology (ii) Tautology (iii) Contingency
(iv) Contingency (v) Tautology (vi) Contingency
(vii) Contingency (viii) Contingency (ix) Contingency
(x) Contradiction

Exercise 1.3

1) (i) T (ii) T (iii) F
(iv) F (v) T (vi) T

2) (i) $p \wedge (q \vee r)$ (ii) $p \vee (q \vee r)$ (iii) $(p \wedge q) \vee (r \wedge s)$
(iv) $p \vee \sim q$ (v) $(\sim p \wedge q) \vee (\sim r \vee s)$
(vi) $\sim p \vee (\sim q \vee (p \wedge q) \vee \sim r)$
(vii) $[\sim(p \wedge q) \vee [p \wedge \sim (q \vee \sim s)]$
(viii) $t \vee \{ p \vee (q \wedge r)\}$
(ix) $\sim p \wedge (q \vee r) \vee c$
(x) $(p \wedge q) \wedge t$

3) (i) $x + 8 \leq 11$ and $y - 3 \neq 6$
(ii) $11 \geq 15$ and $25 \leq 20$
(iii) Quadrilateral is a square but not rhombus or quadrilateral is a rhombus but not a square.
(iv) It is not cold or not raining.
(v) It is raining and we will not go or not play football.
(vi) $\sqrt{2}$ is not a rational number.
(vii) Some natural numbers are not whole numbers.
(viii) $\exists\, n \in N$, $n^2 + n + 2$ is not divisible by 4.
(ix) $\forall\, x \in N$, $x - 17 \geq 20$.

4) (i) Converse :If $x^2 < y^2$ then $x < y$
Inverse :If $x \geq y$ then $x^2 \geq y^2$.
Contrapositive :If $x^2 \geq y^2$ then $x \geq y$.
(ii) Converse :If a family becomes literate then the woman in it is literate.
Inverse :If the woman in the family is not literate then the family does not become literate.
Contrapositive :If a family does not become literate then the woman in the family is not literate.
(iii) Converse :If pressure increases then surface area decreases.
Inverse :If surface area does not decrease then pressure does not increase.
Contrapositive :If pressure does not increase then surface area does not decrease.
(iv) Converse :If current decreases then voltage increases.
Inverse :If voltage does not increase then current does not decrease.
Contrapositive :If current does not decrease then voltage does not increase.

Exercise 1.4

1) (i) $\sim q \wedge \sim p$ (ii) $\sim p \vee q$ (iii) $\sim p \wedge q$
(iv) $(\sim p \wedge q) \vee \sim r$ (v) $p \wedge (\sim p \wedge q)$ (vi) $(p \wedge q) \wedge (\sim p \wedge q)$
(vii) $(p \vee \sim q) \wedge (\sim p \vee q)$ (viii) $(p \wedge q) \wedge (\sim p \vee q)$

2) (i) A man is not a judge or he is honest.
(ii) 2 is not rational number or $\sqrt{2}$ is irrational number.
(iii) $f(2) \neq 0$ or $f(x)$ is divisible by $(x - 2)$.

Exercise 1.5

1) i) $p \vee (q \wedge r)$ 11111000

ii) $(\sim p \wedge q) \vee (p \wedge \sim q)$ 0110

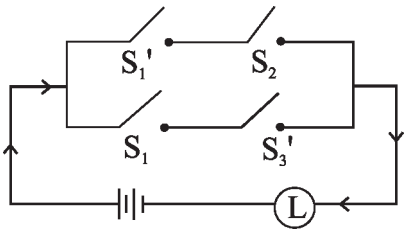
iii) $[(p \wedge (\sim q \vee r)] \vee [\sim q \wedge \sim r]$ 10110001

iv) $(p \vee q) \wedge \sim r \wedge (\sim p \vee r)$ 01000100

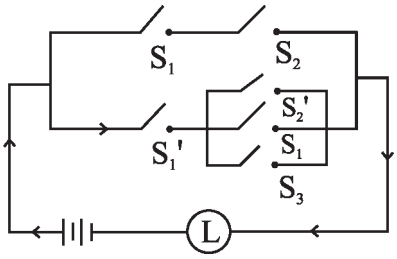
v) $[p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$ 1101

vi) $(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$ 11101000

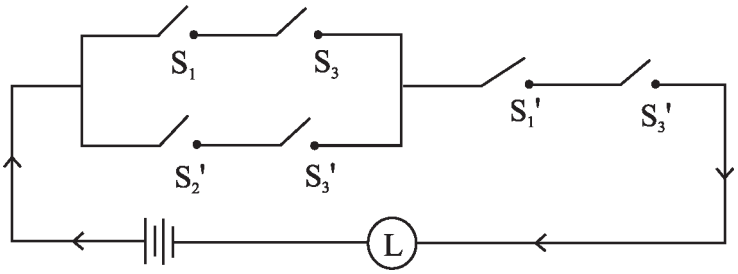
2) (i)



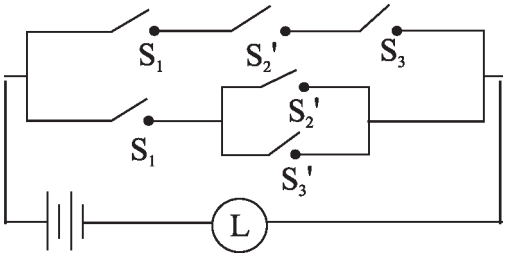
(ii)



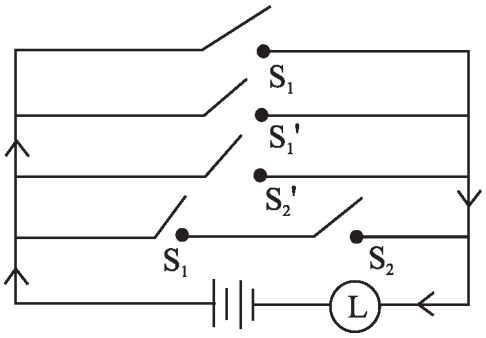
(iii)



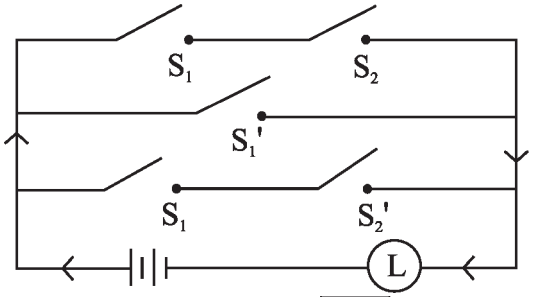
(iv)



(v)



(vi)



4) (i) $(p \vee \sim q) \vee (\sim p \wedge q)$ 1111

The lamp will glow irrespective of the status of the switches.

(ii) $[p \vee (\sim p \wedge \sim q)] \vee (p \wedge q)$ 1101

The lamp will not glow when switch S_1 is OFF and S_2 is ON otherwise it will glow.

(iii) $[p \vee \sim q \vee \sim r)] \wedge [(p \vee (q \wedge \sim r)]$ 11110000

The lamp will glow if S_1 is ON and any status of S_2.

5) (i) P

(ii) $\sim p \vee \sim q$

(iii) P

(iv) $(q \wedge r) \vee p$

Miscelleanous Exercise - 1

1)

i	ii	iii	iv	v	vi	vii
B	A	C	B	A	D	C

2) (i) Statement , T (ii) Statement, T (iii) Statement, F (iv) Not a statement
(v) Statement, T (vi) Statement, T

3) (i) T(ii) F (iii) T (iv) T(v) T (vi) F

4) (i) T(ii) F (iii) T (iv) F

5) (i) $\exists\, n \in$ N such that $n+7 \leq 6$

(ii) $\forall\, x \in$ A, $x + 9 > 15$ on x A, $\forall\, x + 9 > 15$.

(iii) No triangle is an equilateral triangle.

6) (i)

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

(ii)

(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)
p	q	~p	~q	$p \wedge q$	$\sim (p \wedge q)$	$\sim p \vee \sim q$	(vi) $\leftrightarrow$ (vii)
T	T	F	F	T	F	F	T
T	T	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	T	T	T	F	T	T	T

(iii)

(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)
p	q	~p	~q	$\sim p \wedge \sim q$	$\sim (\sim p \wedge \sim q)$	$\sim (\sim p \wedge \sim q) \wedge q$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

(iv)

(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(IX)
p	q	r	$p \wedge q$	$(p \wedge q) \vee q$	$\sim r$	$\sim r \vee$ (iv)	(v) $\wedge$ (vii)
T	T	T	T	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	F	F	F
T	F	F	F	F	T	T	F
F	T	T	F	T	F	F	F
F	T	F	F	F	T	T	F
F	F	T	F	T	F	F	F
F	F	F	F	F	T	T	F

(v)

(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	(IX)
p	q	r	$\sim p$	$\sim p \wedge q$	$q \rightarrow r$	$p \rightarrow r$)	(i) $\wedge$ (vi)	(viii $\rightarrow$ vii)
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	T	T	F	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

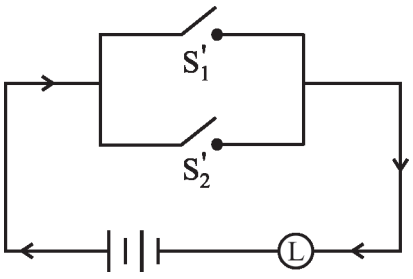
7) (i) Tautology (ii) Contradiction (iii) Contradiction (iv) Tautology
(v) Tautology (vi) Tautology (vii) Contingency (viii) Tautology

8) (i) T, T (ii) T, F (iii) T, F or F, T or F, F

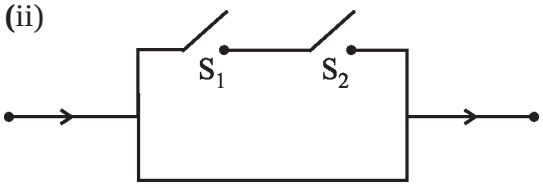
11) (i) $\sim q \wedge (\sim p \vee r)$ (ii) $\sim p \vee (\sim q \wedge \sim r)$ (iii) $(p \wedge \sim q) \vee r$ (iv) $(p \vee \sim q) \wedge (\sim p \vee q)$

12) (i) $(p \wedge q) \vee \sim p \vee (p \wedge \sim q)$ $\boxed{1111}$ (ii) $(p \vee q) \wedge (p \vee r)$ $\boxed{11111000}$

13) (i)

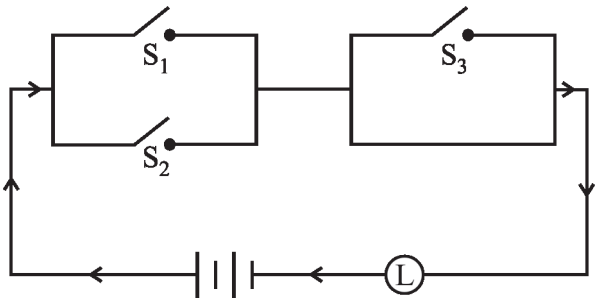


(ii)



14) (i) Logically equivalent (ii) Logically equivalent

15)

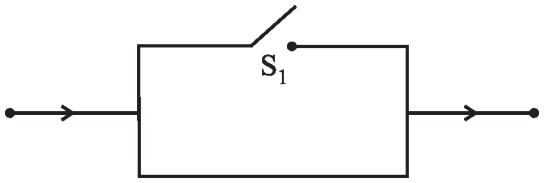


16) Current always flows

17) $(p \vee \sim q \vee \sim r) \wedge [p \vee (q \wedge r)]$

1 1 1 1 0 0 0 0 which is same as p.

Hence we can conclude that the given switching circuit is equivalent to a simple circuit with only one switch S_1.



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2. Matrices

Exercise 2.1

1) $\begin{bmatrix} -1 & 3 \\ 1 & 0 \end{bmatrix}$

2) $\begin{bmatrix} -1 & -6 & -1 \\ 2 & 5 & 4 \end{bmatrix}$

3) $A \sim \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix}$ $B \sim \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix}$ The new matrices are equal.

4) $\begin{bmatrix} -2 & 4 & -7 \\ 2 & 6 & 8 \end{bmatrix}$

5) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 9 & 9 & 21 \end{bmatrix}$

6) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 9 & 9 & 21 \end{bmatrix}$ $\therefore$ The transformations are commutative.

7) $\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$

Exercise 2.2

1) (i) 4, 3, –2, –1.

(ii) –3, –12, 6, –1, 3, 2, –11, –9, 1.

2) (i) $\begin{bmatrix} -1 & -4 \\ -3 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} -11 & -10 & -6 \\ 6 & -5 & 3 \\ -2 & -7 & 1 \end{bmatrix}$

3) (i) $\begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} -3 & -1 & -11 \\ -12 & 3 & -9 \\ 6 & 2 & 1 \end{bmatrix}$

5) (i) $\frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$ (ii) $\frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

(iii) $-\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$ (iv) $-\frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$

6) (i) $-\frac{1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

(iii) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 4 & 3 & 1 \\ \frac{5}{3} & \frac{3}{2} & \frac{1}{2} \end{bmatrix}$ (iv) $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Miscelleanous Exercise - 2(A)

1) Using $C_1 - 2C_2$, $C_1 + 3C_3$ and $C_2 - 3C_3$, We get the required result.

2) Using $R_1 - R_2$, $R_3 - R_2$, $-R_2$, $R_1 - R_2$, $R_3 - R_2$, $-R_3$, $R_1 - R_3$, $R_2 - R_3$, we get the required result.

(There can be another sequence of the transformations.)

3) The invertible matrices are (i), (iii), (v), (vi),(vii) and not invertible matrices are (ii), (iv)(viii) and (ix).

4) AB = $\begin{bmatrix} 6 & -3 \\ -4 & 1 \end{bmatrix}$ and it is invertible.

5) $A^{-1} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}$ $A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

6) (i) $X = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$

7) (i) $\frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$ (ii) $-\frac{1}{3}\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$

(iii) $\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$ (iv) $\frac{1}{29}\begin{bmatrix} 7 & 3 \\ -5 & 2 \end{bmatrix}$

(v) $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$ (vi) $\begin{bmatrix} 7 & -10 \\ 2 & -3 \end{bmatrix}$

(vii) $-\frac{1}{25}\begin{bmatrix} 10 & 0 & -15 \\ -5 & -5 & 0 \\ -10 & 5 & 10 \end{bmatrix}$ (viii) $\frac{1}{25}\begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$

(ix) $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ (x) $\begin{bmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$

8) $A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

9) $AB = \begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix}, (AB)^{-1} = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ $B^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

11) $X = \begin{bmatrix} \frac{4}{5} & 1 \\ \frac{2}{5} & 1 \end{bmatrix}$ 12) $X = -\frac{1}{3}\begin{bmatrix} 1 \\ 7 \\ -6 \end{bmatrix}$

13) $X = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

14) $\begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$

15) $-\frac{1}{6}\begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$

16) $\frac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$

17) $\frac{1}{6}\begin{bmatrix} 4 & -2 & 2 \\ -3 & 0 & 3 \\ 2 & 2 & -2 \end{bmatrix}$

18) $\begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$

19) Hint : Use the definition of the co-factors and the value of the determinant by considering.

$A = [a_{ij}]_{3 \times 3}$

20) $X = \frac{1}{6}\begin{bmatrix} 4 & 4 & 2 \\ 11 & 8 & -5 \\ 10 & 10 & 2 \end{bmatrix}$

Exercise 2.3

1) (i) 0, 1 (ii) 3, 1 (iii) Not solvable

2) (i) 4, –3 (ii) $\frac{1}{2}, \frac{1}{2}$ (iii) 1, 2 (iv) 2,–3

3) Rs. 5 for a pencil Rs. 8 for a pen and Rs.8 for an eraser.

4) The numbers are 1, –2, 3.

5) The cost price of one T.V. set is Rs.3000 and of one V.C.R. is Rs. 13,000.
The selling price of one T.V.Set is Rs.4000 and that of V.C.R. is Rs. 13,500.

Miscellaneous exercise - 2 (B)

I)

1	2	3	4	5	6	7	8	9	10	11	12
A	B	D	B	B	B	B	A	B	B	B	D

II) 1) (i) $-\frac{5}{11}, \frac{12}{11}$ (ii) $2-\frac{4}{a}, 0, -1+\frac{4}{a}$

(iii) $x = 3, y = 2, z = -2$ (iv) $x = 2, y = -3$

(v) $x = \frac{5}{2}, y = \frac{3}{2}, z = -2,$

2) (i) 1, 1, 1 (ii) $\frac{1}{3}, \frac{2}{3}, 1$

(iii) 1, 2, 1 (iv) 1, 2, 3

(v) 3, 2, 1 (vi) -1, 1, 2

3) The numbers are 1, 2, 3

4) Cost of a pencil, a pen and a book is respectively Rs.10, Rs.20 and Rs.25.

5) The costs are $3, \frac{5}{3}, \frac{4}{3}$

6) The numbers are 1, –1, 2

7) 1750, 1500, 1750

8) Maths Rs.150, Phy. Rs.30, Chem. Rs. 30

3. Trigonometric Functions

Exercise 3.1

1) (i) $\frac{\pi}{3}, \frac{5\pi}{3}$ (ii) $\frac{\pi}{6}, \frac{11\pi}{6}$ (iii) $\frac{\pi}{6}, \frac{7\pi}{6}$ (iv) $0, \pi$

2) (i) $\frac{7\pi}{6}, \frac{11}{6}$ (ii) $\frac{3\pi}{4}, \frac{7\pi}{4}$ (iii) $\frac{4\pi}{3}, \frac{5\pi}{3}$

3) (i) $n\pi + (-1)^n \frac{\pi}{6}, n \in z$ (ii) $2n\pi \pm \frac{\pi}{6}, n \in z$

(iii) $n\pi + \frac{\pi}{6}, n \in z$ (iv) $n\pi + \frac{\pi}{2}, n \in z$

4) (i) $2n\pi \pm \frac{\pi}{4}, n \in z$ (ii) $n\pi + (-1)^n \frac{5\pi}{4}, n \in z$ (iii) $n\pi + \frac{3\pi}{4}, n \in z$

5) (i) $\frac{n\pi}{2} + (-1)^n \frac{\pi}{12}, n \in z$ (ii) $\frac{3n\pi}{2} + \frac{\pi}{2}, n \in z$ (iii) $\frac{n\pi}{4} + \frac{3\pi}{16}, n \in z$

6) (i) $n\pi \pm \frac{\pi}{6}, n \in z$ (ii) $n\pi \pm \frac{\pi}{6}, n \in z$ (iii) $\frac{n\pi}{3}, n \in z$

7) (i) $n\pi, n \in z$ (ii) $n\pi \text{ or } n\pi \pm \frac{\pi}{3}, n \in Z$

(iii) $2n\pi$ or $2n\pi \pm \frac{\pi}{2}, n \in z$

8) (i) and iv) have solutions (ii) and (iii) do not have solutions

Exercise 3.2

1) (i) $(1,1)$ (ii) $(0,4)$ (iii) $\left(-\frac{3}{4\sqrt{2}}, \frac{3}{4\sqrt{2}}\right)$ (iv) $\left(\frac{1}{4}, \frac{\sqrt{3}}{4}\right)$

2) (i) $\left(2, \frac{\pi}{4}\right)$ (ii) $\left(\frac{1}{2}, \frac{\pi}{2}\right)$ (iii) $\left(2, \frac{5\pi}{3}\right)$ (iv) $\left(3, \frac{\pi}{3}\right)$

3) (i) $2 : \sqrt{6} : 1+\sqrt{3}$

10) (i) $\frac{4}{5}$ (ii) $\frac{1}{\sqrt{10}}$ (iii) $\frac{3}{\sqrt{10}}$ (iv) $\frac{1}{3}$ (v) 216 (vi) $\frac{3}{5}$

Exercise 3.3

1) (i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{6}$ (iii) $-\frac{\pi}{4}$ (iv) $-\frac{\pi}{3}$ (v) $\frac{\pi}{4}$ (vi) $\frac{2\pi}{3}$

2) (i) $\frac{3\pi}{4}$ (ii) $\frac{2\pi}{3}$ (iii) $-\frac{\pi}{3}$ (iv) $-\frac{\pi}{12}$

Miscellaneous exercise - 3

I)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
B	A	A	A	D	C	A	B	A	C	B	D	A	B	D	A	B	A	B	B

II) i) $\left\{\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}$ ii) $\left\{\frac{3\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{15\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}$

2) (i) $\left\{\frac{5\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

(ii) $\left\{\frac{3\pi}{20}, \frac{7\pi}{20}, \frac{11\pi}{20}, \frac{15\pi}{20}, \frac{19\pi}{20}, \frac{23\pi}{20}, \frac{27\pi}{20}, \frac{31\pi}{20}, \frac{35\pi}{20}, \frac{39\pi}{20}\right\}$

(iii) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

3) (i) and (ii) have solution, (iii) and iv) do not have solutions

4) (i) $n\pi + \frac{2\pi}{3}, n \in z$ (ii) $n\pi \pm \frac{\pi}{3}, n \in z$ (iii) $(2n+1)\pi$ or $2n\pi + \frac{\pi}{2}, n \in z$

iv) $2n\pi \pm \frac{\pi}{2}, n \in z$

10) $c = \sqrt{6}, A = 105^0, B = 15^0$

19) (i) $\frac{3\pi}{5}$ (ii) $\frac{\pi}{6}$

26) $\frac{\pi}{4}$ **27)** $\frac{1}{\sqrt{3}}$

28) 0 **29)** $\frac{1}{6}$

♦♦♦♦♦

4. Pair of Straight Lines

Exercise 4.1

1) (i) $6x^2 + xy - y^2 = 0$

(ii) $x^2 - xy - 6y^2 + x + 7y - 2 = 0$

(iii) $xy - 3x - 2y + 6 = 0$

(iv) $6x^2 - 7xy - 3y^2 - 3x + 32y - 45 = 0$

(v) $3x^2 + 11xy + 6y^2 - 16x - 13y + 5 = 0$

2) (i) $y = 0, 7x + 3y = 0$

(ii) $\sqrt{5}\,x - 3y = 0, \sqrt{5}\,x + 3y = 0,$

(iii) $x = 0, x - 4y = 0$

(iv) $3x + 2y = 0, x - 4y = 0$

(v) $3x + \sqrt{3}\,y = 0, x = \sqrt{3}\,y = 0$

(vi) $(\text{cosec}\,\alpha - \cot\alpha)x + y - 0, (\text{cosec}\,\alpha + \cot\alpha)x - y = 0$

(vii) $(\sec\alpha - \tan\alpha)x + y = 0, (\sec\alpha + \tan\alpha)x - y = 0$

3) (i) $3x^2 + 8xy + 5y^2 = 0$

(ii) $x^2 + 2xy - 5y^2 = 0$

(iii) $x^2 - xy = 0$

(iv) $4xy + 3y^2 = 0$

4) (i) -2 (ii) ± 2 (iii) 12

5) (i) $25a + 16b = 40h$ (ii) $9a + 6h + b = 0$

6) $ap^2 + 2hpq + bq^2 = 0$

7) $3x^2 - y^2 = 0$

Exercise 4.2

3) k = 4 **4)** i) 30° ii) $\tan^{-1}\left(\frac{3}{5}\right)$ iii) 45° iv) 60°

5) $23x^2 + 48xy + 3y^2 = 0$ **7)** $x^2 - 3y^2 = 0$

Exercise 4.3

1) (i) $2x^2 + 3xy - 9y^2 - 5x - 24y - 7 = 0$ (ii) $x^2 + xy - y^2 - x - 8y - 11 = 0$

2) $h^2 - ab = -1 < 0$

3) $2x - 3y + 4 = 0$ and $x + y - 5 = 0$ are separate equations of lines.

4) $2x - y + 3 = 0$ and $x + y - 1 = 0$ are separate equations. $\theta = \tan^{-1}(3)$.

5) (i) $x - y - 3 = 0, x - 2y - 4 = 0$ (ii) $2x - y + 4 = 0, 5x + 3y - 1 = 0$

6) (i) –12 (ii) 15 (iii) –6

7) $p = -3, q = -8$

8) $p = 8, q = 1$

9) $36x^2 - 25xy - 252x + 350y - 784 = 0$

10) $7x - 8y = 0$

11) (1, 0)

Miscellaneous exercise - 4

I.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
B	B	B	A	D	D	A	B	B	B	C	C	D	D

II. 1) (i) $x^2 - y^2 = 0$ (ii) $2x^2 + 3xy + y^2 - 7x - 4y + 3 = 0$ (iii) $6x^2 - 5xy + y^2 = 0$

(iv) $3x^2 - y^2 = 0$ (v) $xy - 2x - y + 2 = 0$ (vi) $xy - 2x - 3y + 6 = 0$

(vii) $8x^2 + 2xy - 3y^2 + 12x + 14y - 8 = 0$ (viii) $2x^2 + 2xy - y^2 = 0$

(ix) $x^2 - 81 = 0$ (x) $x^2 - 2xy - 2x + 6y - 3 = 0$ (xi) $2x^2 - 7xy + 3y^2 = 0$

3) (i) $2x - 3y = 0, 3x + 2y = 0$ (ii) $x - 2y = 0, x + 2y = 0$

(iii) $\sqrt{3}x + y = 0, \sqrt{3}x - y = 0$ (iv) $(\sqrt{3} - 1)x + y = 0, (\sqrt{3} + 1)x - y = 0$

4) (i) $5x^2 + 4xy - y^2 = 0$ (ii) $9x^2 - 3xy - 2y^2 = 0$ (iii) $x^2 + xy - y^2 = 0$

5) (i) 0 (ii) –1 (iii) 1 (iv) 8 (v) 1 (vi) 6 (vii) 5

6) $3x^2 + 2xy - 3y^2 = 0$

7) $x^2 - 3y^2 = 0$

8) $\frac{50}{\sqrt{3}}$

10) $x^2 - 2xy - y^2 = 0$

11) –4

13) (i) 0° (ii) $\tan^{-1}(3)$ (iii) $\tan^{-1}(3)$

14) $x^2 - 3y^2 = 0$

18) Area = $\sqrt{3}$ sq. unit, Perimeter = 6 unit

22) $e = 0$ or $bd = ae$ **26)** $a = 1, c = 0$.

5. Vectors

Exercise 5.1

1) 25

2) (i) $2\bar{a} - 2\bar{b}$ (ii) $\bar{a} + \bar{b}$ (iii) $\bar{b} - \bar{a}$

3) $\overrightarrow{OC} = 2\bar{a} + 2\bar{b}$, $\overrightarrow{OD} = -3\bar{a} + 2\bar{b}$, $\overrightarrow{OE} = -2\bar{a} + \bar{b}$

5) Vectors do not form a triangle.

6) $\bar{c} = \frac{1}{2}\bar{a} + \frac{1}{2}\bar{b}$. $\bar{d} = \frac{1}{2}\bar{b} - \frac{1}{2}\bar{a}$.

7) $\frac{7}{\sqrt{5}}\hat{i} - \frac{14}{\sqrt{5}}\hat{j}$

8) (a) 6 (b) 4 (c) 2 (d) $2\sqrt{10}$ (e) $2\sqrt{13}$ (f) $2\sqrt{5}$

9) (a) $x = -3$, $y = 4$, $z = 5$ (b) (0, 1, 6)

10) $\frac{\sqrt{3}}{2}$ sq. units

11) Terminal Point is (3, 1, 7)

13) $q = \frac{5}{2}$

14) Non coplanar

15) $\bar{r} = 2\bar{a} + 2\bar{b} - 3\bar{c}$

Exercise 5.2

1) (i) $\frac{1}{5}$ (–11, 4, –9) (ii) (–19, 8, –21)

2) M(6, –1, 5)

3) (i) C divides externally in the ration 3:1. (ii) $p = 9$, $q = 2$.

6) 15 : 4 and 10 : 9 respectively

9) C ≡ (–2, 0, 2)

10) OP : PD = 3 : 2

11) $\sqrt{107}$

12) G ≡ (4, –3, 2)

Exercise 5.3

1) $\pm\left(\frac{2}{\sqrt{17}}\hat{i} + \frac{2}{\sqrt{17}}\hat{j} + \frac{3}{\sqrt{17}}\hat{k}\right)$

6) (i) Parallel (ii) Orthogonal (iii) Orthogonal (iv) Neither parallel nor orthogonal

7) $\angle P = 45^\circ$

8) (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$

10) $\frac{\pi}{3}$

11) 0, $\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

12) $\frac{2}{11}, \frac{-6}{11}, \frac{9}{11}$

13) (0, 5, 7) or (8, -3, 3)

14) –1, 1, 2 or 1, 2, 3.

Exercise 5.4

1) $-4\hat{i}+10\hat{j}+22\hat{k}$

2) $\pm\left(\frac{2}{3}\hat{i}-\frac{2}{3}\hat{j}+\frac{1}{3}\hat{k}\right)$

3) 60^{o}

4) $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

5) (i) ± 6 ii) 1.6

7) $6i + 12j + 6k$

8) $\sqrt{146}$ sq. units

10) $\sqrt{42}$ sq. units

12) $\bar{b}=\frac{1}{3}\left(5\hat{i}+2\hat{j}+2\hat{k}\right)$

13) $2\hat{j}+\hat{k}$

14) $\frac{3\pi}{4}$

16) i) –3, 5, 11 ii) 4, –4, 4

17) $\left(\frac{-8}{5}, \frac{16}{5}, \frac{24}{5}\right)$

Exercise 5.5

1) 110

2) 23 cubic units

3) $p = 2$

6) (i) –12 ii) 16

7) $\frac{16}{3}$ cubic units

9) (i) $6\hat{i}+3\hat{j}-6\hat{k}$ (ii) $-2\hat{i}+4\hat{j}$

Not same ; as $\bar{a}\times(\bar{b}\times\bar{c})$ lies in the plane of $\bar{b}$ and $\bar{c}$ whereas $(\bar{a}\times\bar{b})\times\bar{c}$ lies in the plane of $\bar{a}$ and $\bar{b}$.

Miscellaneous exercise - 5

I.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
C	B	B	B	A	D	C	A	B	B	A	B	A	A	A	B	C	B	A	A

II. 1) (i) $\bar{b}-\frac{1}{2}\bar{a}$ (ii) $\bar{b}-3\bar{a}$ (iii) $\frac{3}{2}\bar{a}-\bar{b}$ (iv) $2\bar{a}-\bar{b}$

2) $-\frac{1}{2}\bar{a}-\frac{1}{2}\bar{b}+\bar{c}$

4) $\overrightarrow{AB}=-2\hat{i}+5\hat{j}+\hat{k}$ and $\overrightarrow{AD}=4\hat{i}-2\hat{j}+3\hat{k}$

5) 3

6) $\sqrt{2}$

7) (i) Right angled triangle ii) Isosceles triangle

8) (i) $2j\pm2\sqrt{3}\,\hat{k}$ ii) $\pm5\sqrt{2}\,\hat{i}+5\sqrt{2}\,\hat{k}$

9) $\frac{1}{\sqrt{17}}(3i+2j+2k)$ and $\frac{1}{\sqrt{21}}(-i-2j+4k)$

11) $\pm\frac{1}{\sqrt{17}}(i+4j)$

12) $\hat{i}+4\hat{j}-4\hat{k}=1(2\hat{i}-\hat{j}+3\hat{k})+2(\hat{i}-2\hat{j}+4\,\hat{k})+3(-\hat{i}+3\hat{j}-5\hat{k})$

14) $7(\hat{i}+\hat{j}+\hat{k})$

15) (–4, 9, 6)

20) OP : PD = 3 : 2

21) $3\hat{i}+2\hat{k}$

22) $-\frac{3}{2}$

24) $\bar{a}_1=6\hat{i}+2\hat{k}$ and $\bar{a}_2=-\hat{i}-2\hat{j}+3\hat{k}$

25) $\pm\left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right)$

26) $\cos\theta=\frac{7}{5\sqrt{2}}$

27) $\cos\alpha=\frac{2}{3},\cos\beta=\frac{1}{3}$ *and* $\cos\gamma=\frac{2}{3}$ $\cos\alpha=\frac{1}{3},\ \cos\beta=\cos\gamma=\frac{2}{3}$

28) $2\hat{i}-\hat{j}$

30) $\cos^{-1}\left(\frac{1}{6}\right)$

31) $\left(\frac{19}{9}, \frac{28}{9}, \frac{41}{9}\right)$

33) $\frac{bc\hat{i}+ac\hat{j}+ab\hat{k}}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}$ and $\text{area} = \frac{1}{2}\sqrt{b^2c^2+a^2c^2+a^2b^2}$

34) a) meaningful, scalar b) meaningless c) meaningful, vector
d) meaningless e) meaningless f) meaningful, scalar
g) meaningless h) meaningful, vector i) meaningful, scalar
j) meaningful scalar k) meaningless l) meaningless

36) (i) No ii) No iii) Yes

37) $\sqrt{286}$ sq. units.

40) $a = \pm\frac{1}{\sqrt{3}}$

41) $2a^3$ cu. units.

44) 2 cubic units, $\frac{1}{3}$ cubic units

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6. Line and Plane

Exercise 6.1

1) $\bar{r} = \left(-2\hat{i}+\hat{j}+\hat{k}\right)+\lambda\left(4\hat{i}-\hat{j}+2\hat{k}\right)$

2) $\bar{r} = \left(3\hat{i}+4\hat{j}-7\hat{k}\right)+\lambda\left(3\hat{i}-5\hat{j}+8\hat{k}\right)$

3) $\bar{r} = \left(5\hat{i}+4\hat{j}+3\hat{k}\right)+\lambda\left(-3\hat{i}+4\hat{j}+2\hat{k}\right)$

4) $\bar{r} = \left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}-3\hat{k}\right)$

5) $\bar{r} = \left(-\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+\hat{k}\right)$

6) $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{1}$

7) $\frac{x-2}{-1} = \frac{y-2}{1} = \frac{z-1}{-1}$

8) $\frac{x+2}{3} = \frac{y-3}{-2} = \frac{z-4}{-2}$

9) (–11, –4, 5)

10) $\bar{r} = \left(3\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(-2\hat{i}-3\hat{j}-2\hat{k}\right)$

Exercise 6.2

1) $\sqrt{35}$

2) $(1, 2, 3)$, $\sqrt{14}$

3) $\frac{1}{\sqrt{3}}$

4) $2\sqrt{29}$

5) $2\sqrt{6}$, $(3, -4, -2)$

6) $\left(\frac{99}{53}, \frac{-187}{53}, \frac{95}{53}\right)$

7) a) do not intersect b) do not intersect

8) $\frac{9}{2}$

Miscellaneous exercise - 6A

1) $\bar{r} = \left(3\hat{i} + 4\hat{j} - 7\hat{k}\right) + \lambda\left(6\hat{i} - \hat{j} + \hat{k}\right)$

2) $\bar{r} = \left(3\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right)$

3) $\bar{r} = \left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right) + \lambda\left(3\hat{i} + 5\hat{j} + 6\hat{k}\right)$

4) $\bar{r} = \left(-5\hat{i} - 4\hat{j} - 5\hat{k}\right) + \lambda\left(3\hat{i} + 5\hat{j} + 6\hat{k}\right)$

5) $\bar{r} = \lambda\left(5\hat{i} - 2\hat{j} + 3\hat{k}\right)$

6) $x = 3$, $y = -2$

7) $\frac{x-3}{-2} = \frac{y-2}{1}; z = 1$

8) $x - 1 = y - 1 = z - 2$

9) $\frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$

10) $\bar{r} = \lambda\left(-\hat{i} + \hat{k}\right)$

11) $-\frac{10}{11}$

12) $60°$

13) $45°$

14) $45°$

15) $(2, 3, -1)$

16) i) intersect ii) intersect

17) -1

18) $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$, $\bar{r}=\left(-\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+\hat{k}\right)$

19) $\frac{2}{\sqrt{13}},\frac{3}{\sqrt{13}},0$

20) $\frac{x}{7}=\frac{y}{-12}=\frac{z}{5}$

21) $\bar{r}=\left(2\hat{j}+\frac{5}{3}\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{k}\right)$

22) (2, 0, 5), (0, 4, 1)

Exercise 6.3

1) $\bar{r}\cdot\left(2\hat{i}+\hat{j}-2\hat{k}\right)=126$

2) 1

3) $\left(\frac{18}{7},\frac{54}{7},\frac{-27}{7}\right)$

4) $\bar{r}\cdot\left(\frac{3}{13}\hat{i}+\frac{4}{13}\hat{j}+\frac{12}{13}\hat{k}\right)=6$, (i) 6 (ii) $\left(\frac{3}{13},\frac{4}{13},\frac{12}{13}\right)$

5) $\bar{r}\cdot\left(4\hat{i}+5\hat{j}+6\hat{k}\right)=15$

6) $2y + 5z = 19$

7) $z = 6$

8) $\bar{r}\cdot\left(\hat{i}\right)=1$

9) $\bar{r}\cdot\left(-4\hat{i}-\hat{j}+5\hat{k}\right)=26$

10) $5x - 2y - 3z = 38$

11) $\bar{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1$

Exercise 6.4

1) $60°$

2) $\sin^{-1}\left(\frac{5}{7\sqrt{6}}\right)$

3) $\bar{r}\cdot\left(-\hat{i}+2\hat{j}-\hat{k}\right)=7$

4) 4

5) 3

Miscellaneous exercise - 6B

I.

1	2	3	4	5	6	7	8	9	10
B	A	A	C	D	B	B	C	D	B

11	12	13	14	15	16	17	18	19	20
A	D	D	D	A	A	B	B	A	B

II. 1) $\bar{r}\cdot\left(2\hat{i}+\hat{j}+2\hat{k}\right)=15$

2) 1

3) (2, 3, 6)

4) i) $\frac{1}{2}$ ii) $\frac{3}{13},\frac{4}{13},\frac{12}{13}$

5) $\bar{r}\cdot\left(16\hat{i}+4\hat{k}\right)=20$

6) $y + 2 = 0$

7) $6x + 8y + 7z = 148$

8) $\bar{r}\cdot\left(\hat{i}+2\hat{j}\right)=5$

9) $\bar{r}\cdot\left(bc\hat{i}+ca\hat{j}+ab\hat{k}\right)=abc$

10) $\bar{r}\cdot\left(-3\hat{i}+3\hat{j}+4\hat{k}\right)=35$

11) $\bar{r}\cdot\left(5\hat{i}-4\hat{j}+\hat{k}\right)=0$

12) $x + y + z = 6, x - 2y + z = 0$

13) $x + y + z = 3$

14) 90°

15) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

16) $\bar{r}\cdot\left(-\hat{i}+2\hat{j}-\hat{k}\right)=7$

17) 0 unit

18) 19 units

19) $\bar{r}\cdot\left(\hat{i}-\hat{k}\right)=0$

20) $\bar{r}\cdot\left(\hat{i}-4\hat{k}\right)=-5$

21) $\bar{r}\cdot\left(\hat{k}\right)=0$

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7. Linear Programming

Exercise 7.3

1) maximize $z = 30x + 20y$ subject to $10x + 6y \leq 60$, $5x + 4y \leq 35$, $x \geq 0$, $y \geq 0$

2) maximize $z = 3x + 2y$ subject to $2x + y \geq 14$, $2x + 3y \geq 22$, $x + y \geq 1$, $x \geq 0$, $y \geq 0$

3) maximize $p = 350x + 400y$ subject to $3x + 2y \leq 120$, $2x + 5y \leq 160$, $x \geq 0$, $y \geq 0$

4) maximize $z = 10x + 15y$ subject to $2x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$, $x \geq 0$, $y \geq 0$

5) maximize $p = 13.5x + 55y$ subject to $x + 2y \leq 10$, $3x + 4y \leq 12$, $x \geq 0$, $y \geq 0$

6) maximize $z = 500x + 750y$ subject to $2x + 3y \leq 40$, $x + 4y \leq 70$, $x \geq 0$, $y \geq 0$

7) minimize $z = 4.5x + 3.5y$ subject to $4x + 6y \geq 18$, $14x + 12y \geq 28$, $7x + 8y \geq 14$, $x \geq 0$, $y \geq 0$

8) maximize $z = x_1 + x_2$ subject to $\frac{x_1}{60} + \frac{x_2}{90} \leq 1$, $5x_1 + 8x_2 \leq 600$, $x \geq 0$, $x_2 \geq 0$

9) minimize $C = 20\ x_1 + 6x_2$ s. t $x_1 > 4$, $x_2 < 2$, $x_1 + x_2 \geq 5$, $x \geq 0$, $x_2 \geq 0$.

Exercise 7.4

1) Maximum at (4, 2), 60

2) Maximum at (0, 6), maximum value = 36

3) Maximum at (4.5, 2.5), 59

4) Maximum at (2, 3), maximum value = 95

5) Maximum at (4, 5), maximum $z = 37$

6) minimum at (0, 5), 5

7) minimum at (1.5, 4), 52

8) minimum at (2, 0.5), 22.5

Miscellaneous exercise - 7

I.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	C	B	C	A	D	C	B	A	B	B	B	A	C	C

5) (i) $x_1 = 4.5$, $x_2 = 3$ max $z = 40.5$.

(ii) $x = 3$, $y = 18$ min $z = 48$.

(iii) infinite number of optimum solutions on the line 3x + 5y = 10 between $A\left(\frac{45}{16}, \frac{5}{16}\right)$ and B(0, 2).

6) (i) $x = 4, y = 3$ maximize $z = 25$.
(ii) $x = 10, y = 15$maximize $z = 1350$.
(iii) $x = 3, y = 18$ maximize $z = 48$.

7) maximize $z = 140x + 210y$ s.t. $3x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$
$x, y \geq 0$ where x = no. of tables = 3
y = no. of chairs = 9
maximize z = maximum profit = 2310

8) Maximize $z = 180x + 220y$ s.t. $6x + 4y \leq 120$, $3x + 10y \leq 180$, $x \geq 0, y \geq 0$.
Ans. $x = 10, y = 15$.

9) Minimize $z = 4x + 6y$ s.t. $x + 2y \geq 80$, $3x + y \geq 75$, $x \geq 0, y \geq 0$.
Ans. $x = 14, y = 33$.

10) Maximize $z = 2000x + 3000y$ s.t. $3x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$, $x \geq 0, y \geq 0$.
Ans. $x = 3, y = 9$.

11) Minimize $z = 800x + 640y$ s.t. $4x + 2y \geq 16$, $12x + 2y \geq 24$, $2x + 6y \geq 18$, $x \geq 0, y \geq 0$.
Ans. Minimum cost ₹3680 when $x = 3, y = 2$.

12) Maximize $z = 75x + 125y$ s.t. $4x + 2y \leq 208$, $2x + 4y \leq 152$, $x \geq 0, y \geq 0$.
Ans. $x = 44, y = 16$.

13) Maximize z $= -3x + 4y$ s.t. $x + y \leq 450$, $2x + y \leq 600$, $x \geq 0$, y ≥ 0
maximum profit = Rs. 1800 at (0, 450)

14) Maximize z $= 20x + 30y$ s.t. $2x + 2y \leq 210$, $3x + 4y \leq 300$, $x \geq 0$, y ≥ 0
maximum profit = Rs. 2400 at (30, 60)