4 Time Series

Let's Study

- Uses of time series analysis.
- Components of a time series.
  - Secular Trend
  - Seasonal Variation
  - Cyclical Variation
  - Irregular Variation
- Mathematical Models
  - Additive Model
  - Multiplicative Model
- Measurement of Secular Trend
  - Graphical Method
  - Method of Moving Averages
  - Method of Least Squares

Introduction

A manufacturing company wants to predict demand for its product for next year to make a production plan. An investor wants to know fluctuations in share prices so that he can decide if he should purchase or sell certain shares. These and many other situations involve a variable that changes with time. A variable observed over a period of time is called a time series. Analysis of time series is useful in understanding the patterns of changes in the variable over time. Let us now define a time series.

Definition

Time Series is a sequence of observations made on a variable at regular time intervals over a specified period of time.

Data collected arbitrarily or irregularly does not form a time series. Time series analysis involves the use of statistical methods to analyze time series data in order to extract meaningful statistics and understand important characteristics of the observed data.

Time Series Analysis helps us to understand the underlying forces leading to a particular pattern in the time series and helps us in monitoring and forecasting data with help of appropriate statistical models.

Analysis of time series data requires maintaining records of values of the variable over time.

Some examples from day-to-day life may give a better idea of time series.

1. Monthly, quarterly, or yearly production of an industrial product.
2. Yearly GDP (Gross Domestic Product) of a country.
3. Monthly sales in a departmental store.
4. Weekly prices of vegetables.
5. Daily closing price of a share at a stock exchange.
6. Hourly temperature of a city recorded by the Meteorological Department.

4.1 Uses of Time Series Analysis

The main objective of time series analysis is to understand, interpret and assess chronological changes in values of a variable in the past, so that reliable predictions can be made about its future values. For example, the government may be interested in predicting population growth in near future for planning its welfare schemes, the agricultural ministry may be interested in predicting annual crop yield before declaring the MSP (minimum support price) of agricultural produce or an industrialist may be interested in predicting the weekly demand for his product for making the production schedule. Following are considered to be some of the important uses of time series ananlysis.

1. It is useful for studying the past behaviour of a variable.

   In a time series, the past observations on a variable are arranged in an orderly manner over a period of time. By simple observation of such a series, one can understand the nature of changes that have taken place in values of the variable during the course of time. Further, by applying appropriate technique of analysis to the series, one can study the general tendency of the variable in addition to seasonal changes, cyclical changes, and irregular or accidental changes in values of the variable.

2. It is useful for forecasting future behaviour of a variable.

   Analysis of a time series reveals the nature of changes in the value of a variable during the course of times. This can be useful in forecasting the future values of the variable. Thus, with the help of observations on an appropriate time series, future plans can be made relating to certain matters like purchase, production, sales, etc. This is how a planned economy makes plans for the future development on the basis of time series analysis of the relevant data.

3. It is useful in evaluating the performance.

   Evaluation of the actual performances in comparison with predetermined targets is necessary to judge efficiency of the work. For example, the achievements of Five-Year Plans are evaluated by determining the annual rate of growth in the gross national product. Similarly, the national policy of controlling inflation and price rises is evaluated with the help of different price indices. All these are made possible by analysis of time series of the relevant variables.

4. It is useful in making a comparative study.

   A comparative study of data relating to two or more periods, regions, or industries reveals a lot of valuable information that can guide management in taking a proper course of action. A time series itself provides a scientific basis for making comparisons between two or more related sets of data. Note that data are arranged chronologically in such a series, and the effects of its various components are gradually isolated, analyzed, and interpreted.

4.2 Components of Time Series

A graphical representation of time series data shows continuous changes in its values over time, giving an impression of fluctuating nature of data. A close look of the graph, however, reveals that the fluctuations are not totally arbitrary, and a part of these fluctuations has a steady behavior and can be related to time. This part is the systematic part of the time series and the remaining part is non systematic or irregular. The systematic part is further divided in the following broad categories: (i) secular trend (T), (ii) seasonal variation (S), and (iii) cyclical variation (C). The non systematic part is also called (iv) irregular variation (I). Every time series has some or all of these components. Of course, only the systematic components of a time series are useful in forecasting its future values.

We now discuss the four components of a time series in detail.

4.2.1. Secular Trend (T)

The secular trend is the long term pattern of a time series. The secular trend can be positive or negative depending on whether the time series exhibits an increasing long term pattern or a decreasing long term pattern. The secular trend shows a smooth and regular long term movement of the time series. The secular trend does not include short term fluctuations, but only consists of a steady movement over a long period of time. It is the movement that the series would take if there are no seasonal, cyclical or irregular variations. It is the effect of factors that are more or less constant for a long time or that change very gradually and slowly over time.

If a time series does not show an increasing or decreasing pattern, then the series is stationary around the mean.

SOLVE EXAMPLES

1. The following table shows annual sales (in lakh Rs.) of a departmental store for years 2011 to 2018.

Year	2011	2012	2013	2014
Sales	26.2	28.9	33.7	32.1
Year	2015	2016	2017	2018
Sales	39.8	38.7	45.4	44.6

The following graph shows the above time series.

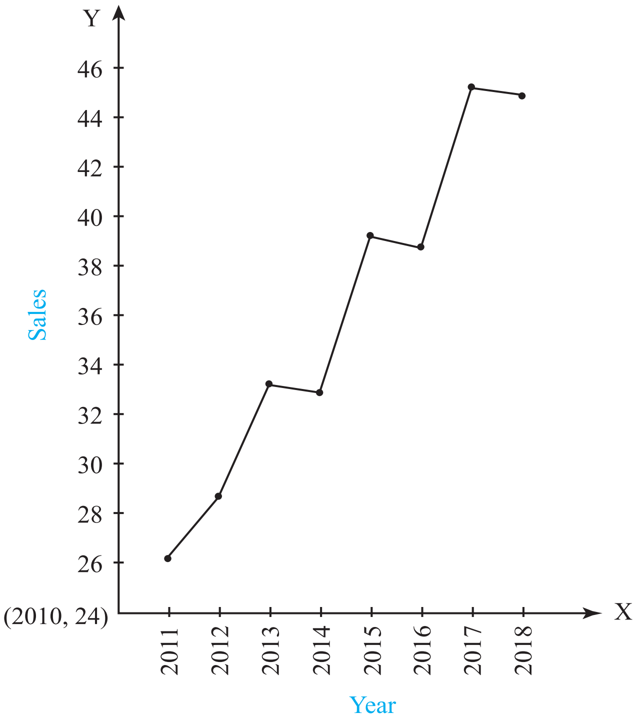

Fig. 4.1

There are ups and downs in the graph, but the time series shows an **upward trend** in long run.

2. The following table shows production (in '000 tonnes) of a commodity during years 2001-2008.

Year	2001	2002	2003	2004
Production	50.0	36.5	43.0	44.5
Year	2005	2006	2007	2008
Production	38.9	38.1	32.6	33.7

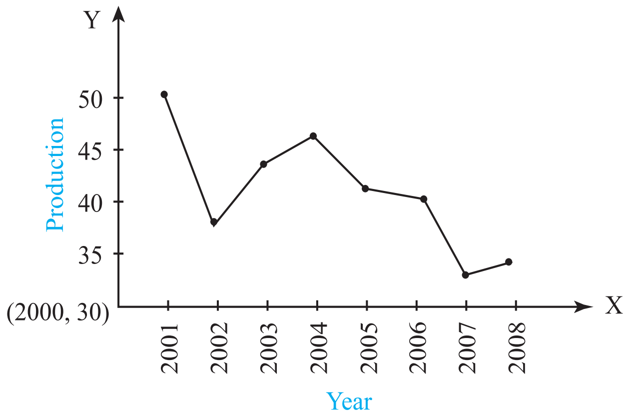

Fig. 4.2

The above graph shows a **downward trend.**

4.2.2 Seasonal Variation (S)

Many time series related to financial, economic, and business activities consist of monthly or quarterly data. It is observed very often that these time series exhibit seasonal variation in the sense that similar patterns are repeated from year to year. Seasonal variation is the component of a time series that involves patterns of change within a year that repeat from year to year.

Several commodities show seasonal fluctuations in their demand. Warm clothes and woolen products have a market during the winter season. Fans, coolers, cold drinks and ice creams are in great demand during summer. Umbrellas and raincoats are in great demand during the rainy season. Different festivals are associated with different commodities and every festival season is associated with an increase in demand for related commodities. For example, clothes and firecrackers are in great demand during Diwali. Most of the seasonal variations in demand reflect changes in climatic conditions or customs and habits of people.

All the above examples have one year as the period of seasonal variation. However, the period of seasonal variation can be a month, a week, a day, or even an hour, depending on the nature of available data. For example, cash withdrawals in a bank show seasonal variation among the days of a month, the number of books borrowed by readers from a library show seasonal variation according to days of a week, passenger traffic at a railway station has seasonal variation during hours of a day, and the temperature recorded in a city exhibits seasonal variation over hours of a day, in addition to seasonal variation with changing seasons in a year.

Seasonal variation is measured with help of seasonal indices, which are useful for short term forecasting. Such short term forecasts are useful for a departmental store in planning its inventory according to months of a year. A bank manager can use such short term forecasts in managing cash flow on different days of a week or a month.

3. The following table shows quarterly sales (in lakh Rs.) of woolen garments in four consecutive years.

Year	I				II			
Quarter	1	2	3	4	1	2	3	4
Sales	11	8	16	28	19	17	32	38
Year	III				IV			
Quarter	1	2	3	4	1	2	3	4
Sales	33	23	39	52	41	37	44	58

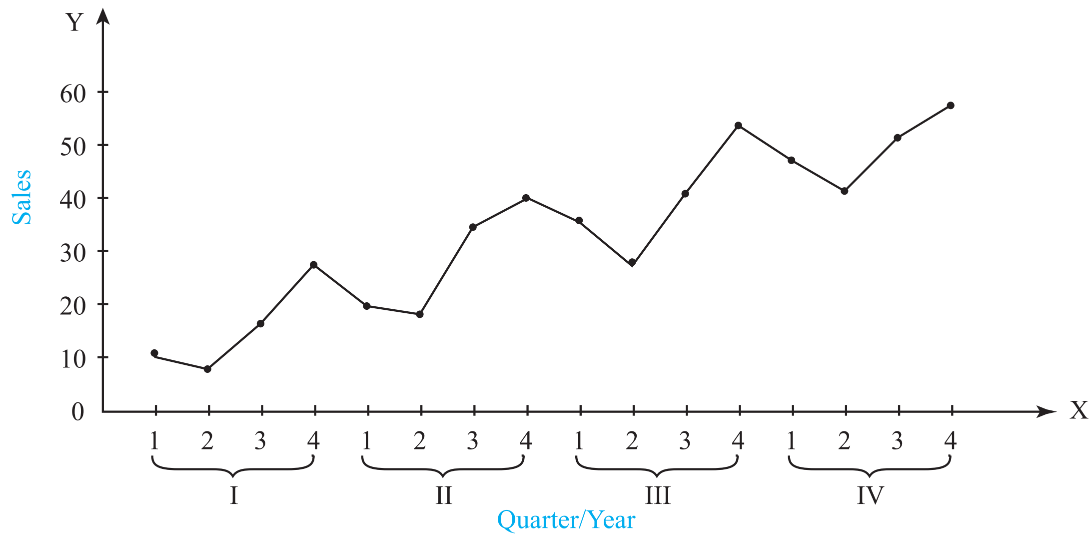

Fig. 4.3

Figure 4.3 shows a pattern that is repeated year after year. The values are lowest (in the year) in second quarter and highest (in the year) in fourth quarter of every year. Although the overall graph of the time series in Figure 4.3 shows an increasing trend, the seasonal variation within every year is very clearly visible in the graph.

4.2.3 Cyclical Variation (C)

Cyclical variation is a long term oscillatory movement in values of a time series. Cyclical variation occurs over a long period, usually several years, if seasonal variation occurs within a year. One complete round of oscillation is called a cycle. Cyclical variations need not be periodic in the sense that the length of a cycle or the magnitude of variation within a cycle can change from one cycle to another.

Cyclical variations are observed in almost all time series related to economic or business activities, where a cycle is known as a business cycle or trade cycle. Recurring ups and downs in a business are the main causes of cyclical variation.

A typical business cycle consists of the following four phases: (i) prosperity, (ii) recession, (iii) depression, (iv) recovery. Figure 4.4 depicts these four phases of a business cycle, where every phase changes to the next phase gradually in the order mentioned above.

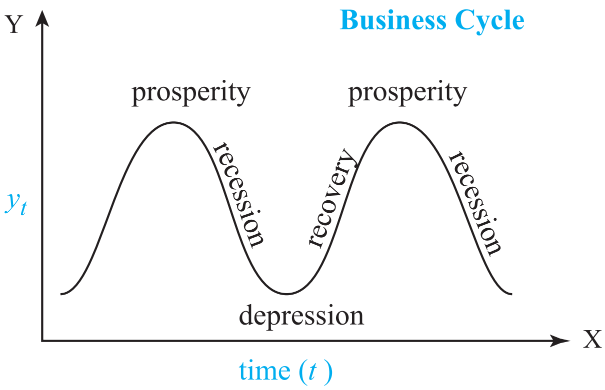

Fig. 4.4

Cyclical variations can consist of a period of 5 years, 10 years, or even longer duration. The period often changes from one cycle to another. Cyclical variation may be attributed to internal organizational factors such as purchase and inventory policies or external factors such as financial market conditions and government policies.

4.2.4 Irregular Variation (I)

Irregular variations are unexpected variations in time series caused by unforeseen events that can include natural disasters like floods or famines, political events line strikes or agitations, or international events like wars or others conflicts. As the name suggests, irregular variations do not follow any patterns and are, therefore, totally unpredictable. For this reason, irregular variation are also known as unexplained or unaccounted variations.

4.3 Mathematical Models of Time Series

Let X_t denote the value of the variable at time t. The time series is denoted by the collection of values, $\{X_t, t = 0, 1, \ldots, T\}$ where T is the total duration of observation. There are two standard mathematical models for time series based on the four components mentioned earlier, namely, secular trend (T), seasonal variation (S), cyclical variation (C), and irregular variation (I).

4.3.1 Additive Model

The additive model assumes that the value X_t at time t is the sum of the four components at time t. Thus,

$$X_t = T_t + S_t + C_t + I_t$$

The additive model assumes that the four components of the time series are independent of one another. It is also important to remember that all the four components in the additive model must be measured in the same unit of measurement. The magnitude of the seasonal variation does not depend on the value of the time series in the additive model. In other words, the magnitude of the seasonal variation does no change as the series goes up or down.

The assumption of independence of the components is often not realistic. In such situations, the multiplicative model can be used.

4.3.2 Multiplicative Model

The multiplicative model that the value X_t at the time t is obtained by multiplication of the four components at time t. That is,

$$X_t = T_t \times S_t \times C_t \times I_t$$

The multiplicative model does not assume independence of the four components of the series and is, therefore, more realistic. Values of the trend are expressed in units of measurements and other components are expressed as percentage or relative values, and hence are free from units of measurements.

It is recommended to choose the multiplicative model when the magnitude of the seasonal variation in the data depends on the magnitude of the data. In other words, the magnitude of the seasonal variation increases as the data values increase, and decreases as the data values decrease.

4.4 Measurement of Secular Trend

4.4.1 Method of Freehand Curve (Graphical Method)

In this method, a graph is drawn for the given time series by plotting X_t (on Y-axis) against t (on X-axis). Then a free hand smooth curve is plotted on the same graph to indicate the general trend.

This method is simple and does not require any mathematical calculation. But, in this method, different researchers may draw different trend lines for the same set of data. Forecasting using this method is therefore risky if the person drawing free hand curve is not efficient and experienced. On the other hand, this method is quite flexible and can be used for all types of trends, linear as well as non – linear, and involves minimum amount of work.

SOLVED EXAMPLES

1. Fit a trend line to the following data using the graphical method.

Year	Number of crimes ('000)	Year	Number of crimes ('000)
1981	40	1987	43
1982	42	1988	46
1983	43	1989	47
1984	42	1990	45
1985	44	1991	46
1986	44		

Solution:

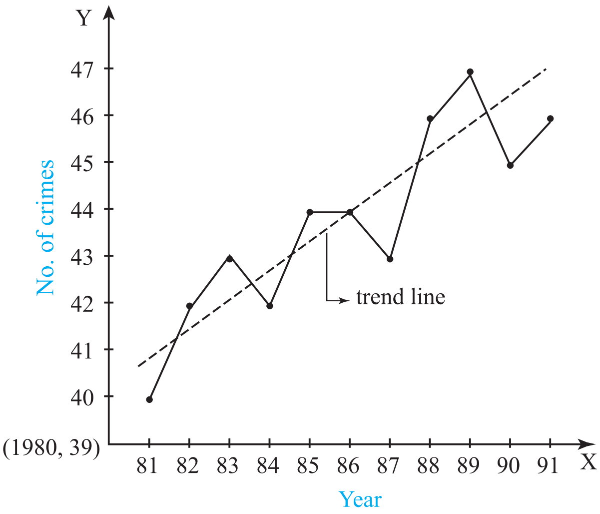

Fig. 4.5

2. The publisher of a magazine wants to determine the rate of increase in the number of subscribers. The following table shows the subscription information for eight consecutive years.

Year	1976	1977	1978	1979
No. of subscribers (in millions)	12	11	19	17
Year	1980	1981	1982	1983
No. of subscribers (in millions)	19	18	20	23

Fit a trend line by the graphical method.

Solution:

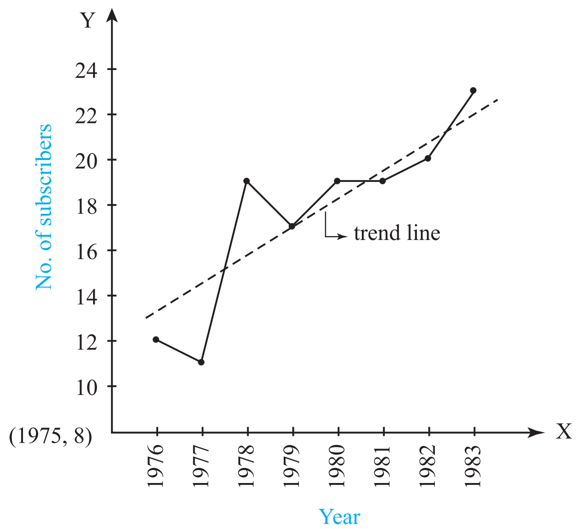

Fig. 4.6

4.4.2 Method of moving Averages

The moving average of period k of a time series forms a time series of arithmetic means of k successive observations from the original time series. The method begins with the first k observations and finds the arithmetic mean of these k observations. The next step leaves the first observation and includes observation number $k + 1$ and finds the arithmetic mean of these k observations. This process continues till the average of the last k observations is found. In other words, the method of moving averages finds the following.

First moving average $= \dfrac{X_1 + X_2 + ... + X_k}{k}$

Second moving average $= \dfrac{X_2 + X_3 + ... + X_{k+1}}{k}$

Third moving average $= \dfrac{X_3 + X_4 + ... + X_{k+2}}{k}$

and so on.

Each of these averages is written against the time point that is the middle term in the sum. As a result, when k is an odd integer, moving average values correspond to observed values of the given time series. On the other hand, when k is an even integer, the moving averages fall mid-way between two observed values of the given time series. In this case, a subsequent two-unit moving average is calculated to make the resulting moving average values correspond to observed values of the given time series.

A moving average with an appropriate period smooths out cyclical variations from the given time series and provides a good estimate of the trend. Cyclical fluctuations with a uniform period and a uniform amplitude can be completely eliminated by taking the period of moving averages that is equal to or a multiple of the period of the cycles as long as the trend is linear.

The method of moving averages is flexible in the sense that even if a few observations are added to the given series, the moving averages calculated earlier are not affected and remain unchanged. However, the method of moving averages does not provide a mathematical equation for the time series and hence cannot be used for the purpose of forecasting. Another drawback of the method of moving averages is that some of the trend values at each end of the given series cannot be estimated by this method.

SOLVED EXAMPLES

3. The following table shows gross capital formation (in crore Rs) for years 1966 to 1975.

Year	1966	1967	1968	1969	1970
Gross Capital Formation	19.3	20.9	17.8	16.1	17.6
Year	1971	1972	1973	1974	1975
Gross Capital Formation	17.8	18.3	17.3	21.4	19.3

(i) Obtain trend values using 5-yearly moving averages.

(ii) Plot the original time series and trend values obtained in (i) on the same graph.

Solution :

Table 4.3

Year	X_t	5-yearly moving total	5-yearly moving averages (trend value)
1966	19.3	-	-
1967	20.9	-	-
1968	17.8	91.7	18.34
1969	16.1	90.2	18.04
1970	17.6	87.6	17.52
1971	17.8	87.1	17.42
1972	18.3	92.4	18.48
1973	17.3	94.1	18.82
1974	21.4	-	-
1975	19.3	-	-

Note that 5-yearly average is not available for the first 2 years and last 2 years.

The following graph shows the original time series and the trend values obtained in Table 4.3.

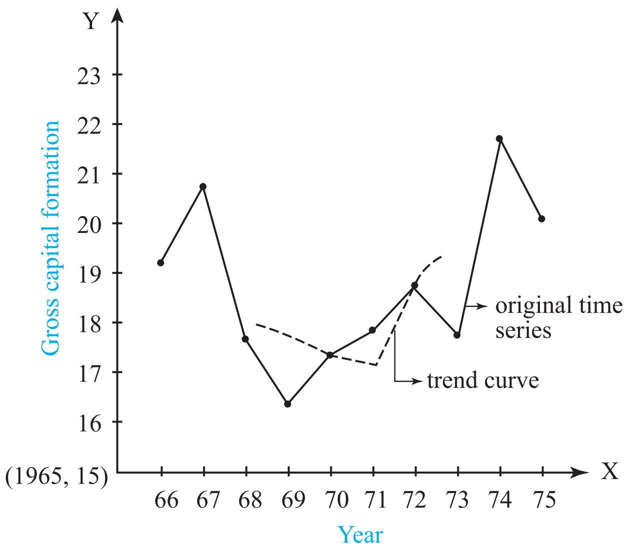

Fig. 4.7

4. Obtain 4-yearly centered moving averages for the following time series.

Year	1987	1988	1989	1990	1991
Annual sales (in lakh Rs.)	3.6	4.3	4.3	3.4	4.4
Year	1992	1993	1994	1995	
Annual sales (in lakh Rs.)	5.2	3.8	4.9	5.4	

Solution:

Year	X_i	4-yearly moving total	4-yearly moving average	2-unit moving total	4-yearly centred moving average (trend value)
1987	3.6				
1988	4.3				
		15.6	3.9		
1989	4.3			8	4
		16.4	4.1		
1990	3.4			8.425	4.2125
		17.3	4.325		
1991	4.4			8.525	4.2625
		16.8	4.2		
1992	5.2			8.775	4.3875
		18.3	4.575		
1993	3.8			9.4	4.7
		19.3	4.825		
1994	4.9				
1995	5.4				

Note: Entries in the third and fourth columns are *between* tabulated time periods, while entries in fifth and sixth columns are *in front of tabulated* time periods.

4.4.3 Method of Least Squares

This is the most objective and perhaps the best method of determining trend in a given time series. The method begins with selection of an appropriate form of trend equation and then proceeds with estimation of the unknown constants in this equation. It is a common practice to choose a polynomial of a suitable degree and then to determine its unknown coefficients by the method of least squares. The choice of the degree of polynomial is often based on the graphical representation of the given data.

In a linear trend, the equation is given by

$$X_t = a + bt$$

The method of least squares involves solving the following set of linear equations, commonly known as normal equations.

$$\sum X_t = na + b\sum t$$

$$\sum tX_t = a\sum t + b\sum t^2$$

Where n is the number of the time periods for which data is available, whereas,

$\sum X_t$, $\sum tX_t$, $\sum t$ and $\sum t^2$ are obtained

from the data. The least squares estimates of a and b are obtained by solving the two equations in the two unknowns, namely a and b. The required equation of the trend line is the obtained by substituting these estimates in equation $X_t = a + bt$

SOLVED EXAMPLES

5. Fit a trend line by the method of least squares to the time series in Example 1 (Section 4.4.1). Also, obtain the trend value for the number of crimes in the year 1993.

Solution:

Let the trend line be represented by the equation $y_t = a + bt$ Calculations can be made simpler by transforming from t to u using the formula

$$u = \frac{t - \text{middle } t \text{ value}}{h} \quad \text{......... if } n \text{ is odd}$$

$$= \frac{t - \text{mean of two middle } t \text{ values}}{h} \quad \text{......... if } n \text{ is even}$$

Where h is the difference between successive t values.

In the given problem, $n = 11$ (odd), middle t value is 1986, and $h = 1$.

$\therefore$ we use the transformation

$$u = \frac{t - 1986}{1} = t - 1986.$$

The equation of the trend line then becomes

$$y_t = a' + b'u$$

The two normal equations are then given by

$$\sum y_t = na' + b' \sum u \; ,$$

$$\sum uy_t = a' \sum u + b' \sum u^2$$

We obtain $\sum u$, $\sum u^2$, $\sum y_t$ and $\sum uy_t$ from the following table.

Year t	y_t	u	u^2	uy_t	Trend Value
1981	40	−5	25	−200	41.0457
1982	42	−4	16	−168	41.6002
1983	43	−3	9	−129	42.1547
1984	42	−2	4	−84	42.7092
1985	44	−1	1	−44	43.2637
1986	44	0	0	0	43.8182
1987	43	1	1	43	44.3727
1988	46	2	4	92	44.9272
1989	47	3	9	141	45.4817
1990	45	4	16	180	46.0362
1991	46	5	25	230	46.5907
Total	482	0	110	61	

The normal equations are

$482 = 11a' + b'(0)$; $61 = a'(0)\ 110\ b'$

$$\therefore a' = \frac{482}{11} = 43.8182 \; ; \; b' = \frac{61}{110} = 0.5545$$

The equation of the trend line is then given by

$y_t = 43.8182 + (0.5545)\, u$, where $u = t - 1986$

Trend values computed using this result are given in the last column of the table.

For the year 1993, $x = 7$

$y = 43.8182 + 0.5545(7)$

$= 43.8182 + 3.8815 = 47.6997$

6. Fit a trend line by the method of least squares to the time series in Example 2 (Section 4.4.1). Also obtain the trend value for the number of subscribers in the year 1984.

Solution :

Let the equation of the trend line, if the n is even

$$y_t = a + bt$$

In the given problem, $n = 8$ (even), two middle t – values are 1979 and 1980, and $h = 1$.

We use the transformation

$$u = \frac{t - 1979.5}{\frac{1}{2}}$$

$$= 2t - 3959.$$

The equation of the trend line then becomes

$$y_t = a' + b'u$$

The two normal equations are

$$\sum y_t = na' + b' \sum u \; , \qquad (1)$$

$$\sum uy_t = a' \sum u + b' \sum u^2 \qquad (2)$$

We obtain $\sum u$, $\sum u^2$, $\sum y_t$ and $\sum uy_t$ from the following table.

Table 4.2

Year t	y_t	u	u^2	uy_t	Trend Value
1976	12	−7	49	−84	12.3336
1977	11	−5	25	−55	13.774
1978	19	−3	9	−57	15.2144
1979	17	−1	1	−17	16.6548
1980	19	1	1	19	18.0952
1981	18	3	9	54	19.5356
1982	20	5	25	100	20.976
1983	23	7	49	161	22.4164
Total	139	0	168	121	

From (1) and (2),

$$139 = 8a' + b'(0) \qquad (3)$$

$$121 = a'(0) + b'(168) \qquad (4)$$

From (3), $a' = 17.375$.

From (4), $b' = 0.7202$.

The equation of the trend line is then given by

$$y_t = 17.375 + (0.7202)u,$$

where $u = 2t - 3959$.

Trend values computed using this result are given in the last column of **Table 4.2**.

Estimation of the trend value for the year 1984 is obtained as shown below.

For $t = 1984$, $u = 2(1984) - 3959 = 9$.

And hence

$$\widehat{y_t} = 17.375 + (0.7202)(9)$$

$$= 23.8568 \text{ (in millions)}$$

EXERCISE 4.1

1. The following data gives the production of bleaching powder (in '000 tonnes) for the years 1962 to 1972.

Year	1962	1963	1964	1965	1966	
Production	0	0	1	1	4	
Year	1967	1968	1969	1970	1971	1972
Production	2	4	9	7	10	8

Fit a trend line by graphical method to the above data.

2. Use the method of least squares to fit a trend line to the data in Problem 1 above. Also, obtain the trend value for the year 1975.

3. Obtain the trend line for the above data using 5 yearly moving averages.

4. The following table shows the index of industrial production for the period from 1976 to 1985, using the year 1976 as the base year.

Year	1976	1977	1978	1979	1980
Index	0	2	3	3	2
Year	1981	1982	1983	1984	1985
Index	4	5	6	7	10

Fit a trend line to the above data by graphical method.

5. Fit a trend line to the data in Problem 4 above by the method of least squares. Also, obtain the trend value for the index of industrial production for the year 1987.

6. Obtain the trend values for the data in problem 4 using 4-yearly centered moving averages.

7. The following table gives the production of steel (in millions of tonnes) for years 1976 to 1986.

Year	1976	1977	1978	1979	1980	1981
Production	0	4	4	2	6	8
Year	1982	1983	1984	1985	1986	
Production	5	9	4	10	10	

Fit a trend line to the above data by the graphical method.

8. Fit a trend line to the data in Problem 7 by the method of least squares. Also, obtain the trend value for the year 1990.

9. Obtain the trend values for the above data using 3-yearly moving averages.

10. The following table shows the production of gasoline in U.S.A. for the years 1962 to 1976.

Year	Production (million Barrels)	Year	Production (million Barrels)
1962	0	1970	6
1963	0	1971	7
1964	1	1972	8
1965	1	1973	9
1966	2	1974	8
1967	3	1975	9
1968	4	1976	10
1969	5		

(i) Obtain trend values for the above data using 5-yearly moving averages.

(ii) Plot the original time series and trend values obtained above on the same graph.

Let's Remember

- A time series is a set of observations made at regular intervals of time and therefore arranged chronologically (that is, according to time).
- A time series has the following four components:

  (i) Secular trend

  (ii) Seasonal variation

  (iii) Cyclical variation

  (iv) Irregular variation
- Secular trend of a time series is its smooth and regular long-term movement.
- Seasonal variation involves patterns of change within a year that are repeated from year to year. Seasonal variations often reflect changes in underlying climatic conditions, or customs and habits of people.
- Cyclical variation is a long-term oscillatory movement that occurs over a long period of time, mostly more than two years or more.
- A standard business cycle consists of the following four phases:

  (i) Prosperity (ii) Decline

  (iii) Depression (iv) Recovery
- Irregular variations are caused by either random factors or unforeseen events like floods, famines, earthquakes, strikes, wars, etc.
- Additive Model:

  $$X_t = T_t + S_t + C_t + I_t$$

  where X_t is the value of the variable X at time t and T_t, S_t, C_t and I_t are secular trend, seasonal variation, cyclical variation, and irregular variation at time t, respectively.
- Multiplicative Model:

  $$X_t = T_t \times S_t \times C_t \times I_t$$

  where the notation is same as in the additive model.
- Secular trend can be measured using

  (i) Graphical method

  (ii) Method of moving averages

  (iii) Method of least squares
- The moving averages of period k of a time series form a new series of arithmetic means, each of k successive observations of the given time series.
- Method of least squares involves solving the following two normal equations.

  $$\sum X_t = na + b\sum t$$

  $$\sum tX_t = a\sum t + b\sum t^2$$

MISCELLANEOUS EXERCISE - 4

I) Choose the correct alternative.

1. Which of the following can't be a component of a time series?

   (a) Seasonality (b) Cyclical

   (c) Trend (d) Mean

2. The first step in time series analysis is to
   (a) Perform regression calculations
   (b) Calculate a moving average
   (c) Plot the data on a graph
   (d) Identify seasonal variation

3. Time-series analysis is based on the assumption that
   (a) Random error terms are normally distributed.
   (b) The variable to be forecast and other independent variables are correlated.
   (c) Past patterns in the variable to be forecast will continue unchanged into the future.
   (d) The data do not exhibit a trend.

4. Moving averages are useful in identifying
   (a) Seasonal component
   (b) Irregular component
   (c) Trend component
   (d) Cyclical component

5. We can use regression line for past data to forecast future data. We then use the line which
   (a) Minimizes the sum of squared deviations of past data from the line
   (b) Minimizes the sum of deviations of past data from the line.
   (c) Maximizes the sum of squared deviations of past data from the line
   (d) Maximizes the sum of deviations of past data from the line.

6. Which of the following is a major problem for forecasting, especially when using the method of least squares?
   (a) The past cannot be known
   (b) The future is not entirely certain
   (c) The future exactly follows the patterns of the past
   (d) The future may not follow the patterns of the past

7. An overall upward or downward pattern in an annual time series would be contained in which component of the times series
   (a) Trend (b) Cyclical
   (c) Irregular (d) Seasonal

8. The following trend line equation was developed for annual sales from 1984 to 1990 with 1984 as base or zero year.
   $Y1 = 500 + 60X$ (in 1000 Rs). The estimated sales for 1984 (in 1000 Rs) is:
   (a) Rs 500 (b) Rs 560
   (c) Rs 1,040 (d) Rs 1,100

9. What is a disadvantage of the graphical method of determining a trend line?
   (a) Provides quick approximations
   (b) Is subject to human error
   (c) Provides accurate forecasts
   (d) Is too difficult to calculate

10. Which component of time series refers to erratic time series movements that follow no recognizable or regular pattern.
   (a) Trend (b) Seasonal
   (c) Cyclical (d) Irregular

II) Fill in the blanks

1. ___ components of time series is indicated by a smooth line.
2. ___ component of time series is indicated by periodic variation year after year.
3. ___ component of time series is indicated by a long wave spanning two or more years.
4. ___ component of time series is indicated by up and down movements without any pattern.
5. Addictive models of time series ___ independence of its components.
6. Multiplicative models of time series ___ independence of its components.
7. The simplest method of measuring trend of time series is _____
8. The method of measuring trend of time series using only averages is ___

9. The complicated but efficient method of measuring trend of time series is ____.

10. The graph of time series clearly shows ____ of it is monotone.

III) State whether each of the following is True or False.

1. The secular trend component of time series represents irregular variations.
2. Seasonal variation can be observed over several years.
3. Cyclical variation can occur several times in a year.
4. Irregular variation is not a random component of time series.
5. Additive model of time series does not require the assumption of independence of its components.
6. Multiplicative model of time series does not require the assumption of independence of its components.
7. Graphical method of finding trend is very complicated and involves several calculations.
8. Moving average method of finding trend is very complicated and involves several calculations.
9. Least squares method of finding trend is very simple and does not involve any calculations.
10. All the three methods of measuring trend will always give the same results.

IV) Solve the following problems.

1. The following table shows the production of pig-iron and ferro-alloys ('000 metric tonnes)

Year	1974	1975	1976	1977	1978
Production	0	4	9	9	8
Year	1979	1980	1981	1982	
Production	5	4	8	10	

Fit a trend line to the above data by graphical method.

2. Fit a trend line to the data in Problem IV (1) by the method of least squares.
3. Obtain trend values for data in Problem IV (1) using 5-yearly moving averages.
4. Following table shows the amount of sugar production (in lac tonnes) for the years 1971 to 1982.

Year	Production	Year	Production
1971	1	1977	3
1972	0	1978	6
1973	1	1979	5
1974	2	1980	1
1975	3	1981	4
1976	2	1982	10

Fit a trend line to the above data by graphical method.

5. Fit a trend line to data in Problem 4 by the method of least squares.
6. Obtain trend values for data in Problem 4 using 4-yearly centered moving averages.
7. The percentage of girls' enrollment in total enrollment for years 1960-2005 is shown in the following table.

Year	1960	1965	1970	1975	1980
Production	0	3	3	4	4
Year	1985	1990	1995	2000	2005
Production	5	6	8	8	10

Fit a trend line to the above data by graphical method.

8. Fit a trend line to the data in Problem 7 by the method of least squares.
9. Obtain trend values for the data in Problem 7 using 4-yearly moving averages.
10. Following data shows the number of boxes of cereal sold in years 1977 to 1984.

Year	1977	1978	1979	1980
No. of boxes in ten thousands	1	0	3	8
Year	1981	1982	1983	1984
No. of boxes in ten thousands	10	4	5	8

Fit a trend line to the above data by graphical method.

11. Fit a trend line to data in Problem 10 by the method of least squares.

12. Obtain trend values for data in Problem 10 using 3-yearly moving averages.

13. Following table shows the number of traffic fatalities (in a state) resulting from drunken driving for years 1975 to 1983.

Year	1975	1976	1977	1978	1979
No. of deaths	0	6	3	8	2
Year	1980	1981	1982	1983	
No. of deaths	9	4	5	10	

Fit a trend line to the above data by graphical method.

14. Fit a trend line to data in Problem 13 by the method of least squares.

15. Obtain trend values for data in Problem 13 using 4-yearly moving averages.

16. Following table shows the all India infant mortality rates (per '000) for years 1980 to 2010.

Year	1980	1985	1990	1995
IMR	10	7	5	4
Year	2000	2005	2010	
IMR	3	1	0	

Fit a trend line to the above data by graphical method.

17. Fit a trend line to data in Problem 16 by the method of least squares.

18. Obtain trend values for data in Problem 16 using 3-yearly moving averages.

19. Following tables shows the wheat yield ('000 tonnes) in India for years 1959 to 1968.

Year	Yield	Year	Yield
1959	0	1964	0
1960	1	1965	4
1961	2	1966	1
1962	3	1967	2
1963	1	1968	10

Fit a trend line to the above data by the method of least squares.

20. Obtain trend values for data in Problem 19 using 3-yearly moving averages.

Activities

Note: You may change the origin and scale in the following problems according to your convenience.

1. Daily SENSEX index values at opening are given for fifty days in the following table. Plot a graph from the data. Find the trend graphically, using moving averages, and by the method of least squares.

Date	Index	Date	Index
1-Jan-19	36161.8	2-Jan-19	36198.13
3-Jan-19	35934.5	4-Jan-19	35590.79
7-Jan-19	35971.18	8-Jan-19	35964.62
9-Jan-19	36181.37	10-Jan-19	36258
11-Jan-19	36191.87	14-Jan-19	36113.27
15-Jan-19	35950.08	16-Jan-19	36370.74
17-Jan-19	36413.6	18-Jan-19	36417.58
21-Jan-19	36467.12	22-Jan-19	36649.92
23-Jan-19	36494.12	24-Jan-19	36146.55
25-Jan-19	36245.77	28-Jan-19	36099.62
29-Jan-19	35716.72	30-Jan-19	35819.67
31-Jan-19	35805.51	1-Feb-19	36311.74
4-Feb-19	36456.22	5-Feb-19	36573.04
6-Feb-19	36714.54	7-Feb-19	37026.56
8-Feb-19	36873.59	11-Feb-19	36585.5
12-Feb-19	36405.72	13-Feb-19	36279.63
14-Feb-19	36065.08	15-Feb-19	35985.68
18-Feb-19	35831.18	19-Feb-19	35543.24
20-Feb-19	35564.93	21-Feb-19	35837
22-Feb-19	35906.01	25-Feb-19	35983.8
26-Feb-19	35975.75	27-Feb-19	36138.83
28-Feb-19	36025.72	1-Mar-19	36018.49
5-Mar-19	36141.07	6-Mar-19	6544.86
7-Mar-19	36744.02	8-Mar-19	36753.59
8-Mar-19	36753.59	12-Mar-19	37249.65

2. Onion prices (per quintal) in a market are given for fifteen days. Plot a graph of given data. Find the trend graphically, using moving averages, and by the method of least squares.

Date	Price
24/01/2019	400
31/01/2019	650
3/2/19	650
7/2/19	700
24/02/2019	550
28/02/2019	550
5/3/19	500
7/3/19	600
14/03/2019	600
28/03/2019	600
7/4/19	800
11/4/19	801
14/04/2019	800
25/04/2019	800
2/5/19	800

3. Following table gives the number of persons injured in road accidents for 11 years. Plot a graph from the data. Find the trend graphically, using moving averages, and by the method of least squares.

Year	2006	2007	2008	2009	2010	2011
No. of injured	29955	27464	32224	32144	25400	28366
Year	2012	2013	2014	2015	2016	
No. of injured	27185	26314	24488	23825	22072	

4. Following table gives the number of road accidents due to over-speeding in Maharashtra for 9 years. Plot a graph from the data. Find the trend graphically, using moving averages, and by the method of least squares.

Year	2008	2009	2010	2011	2012
No. of accidents	38680	18090	21238	28489	27054
Year	2013	2014	2015	2016	
No. of accidents	26931	22925	24622	22071	