

REASONING ABOUT PROBABILITY





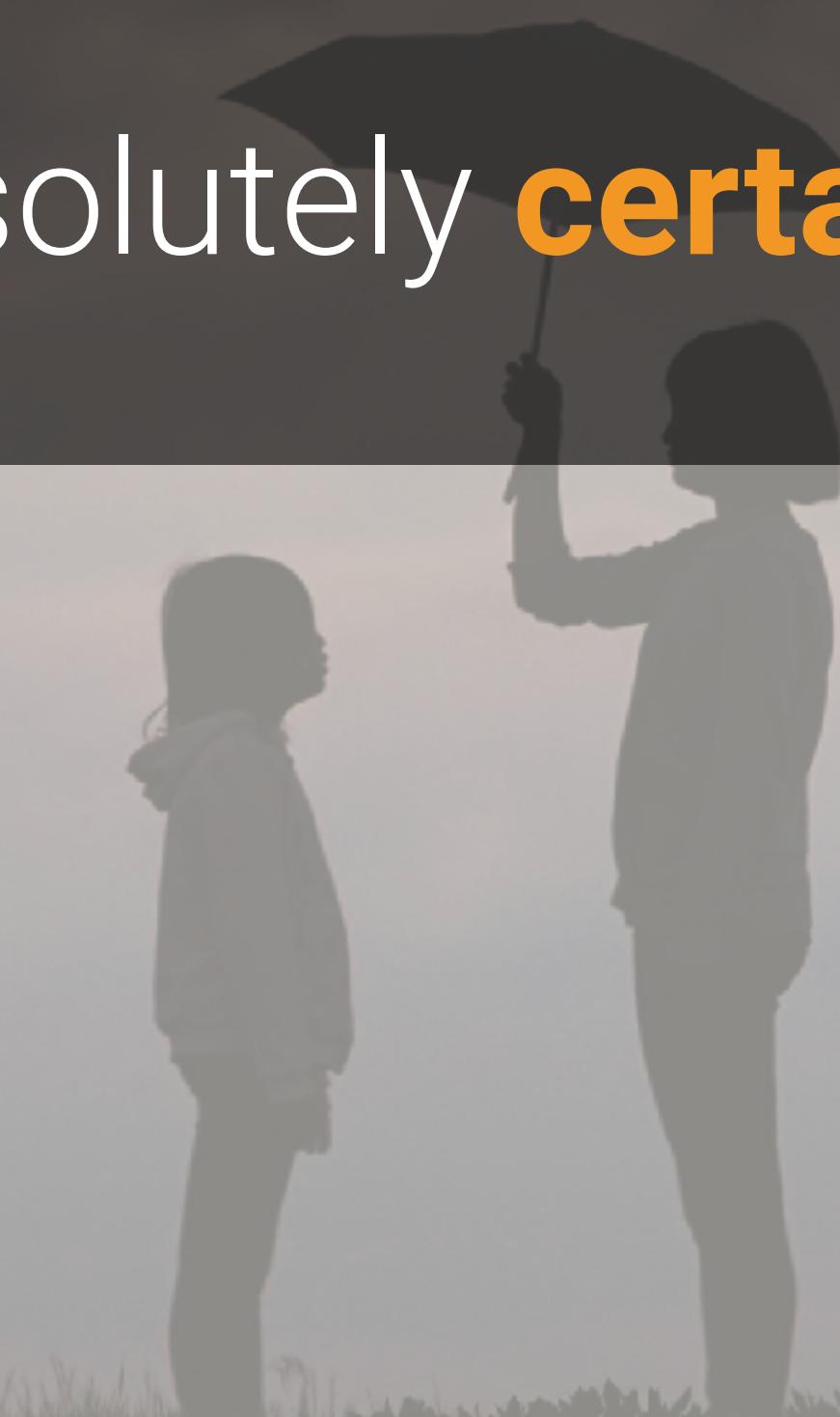


$P(\text{"Rain"}) = 0$

absolutely **certain** that it won't rain

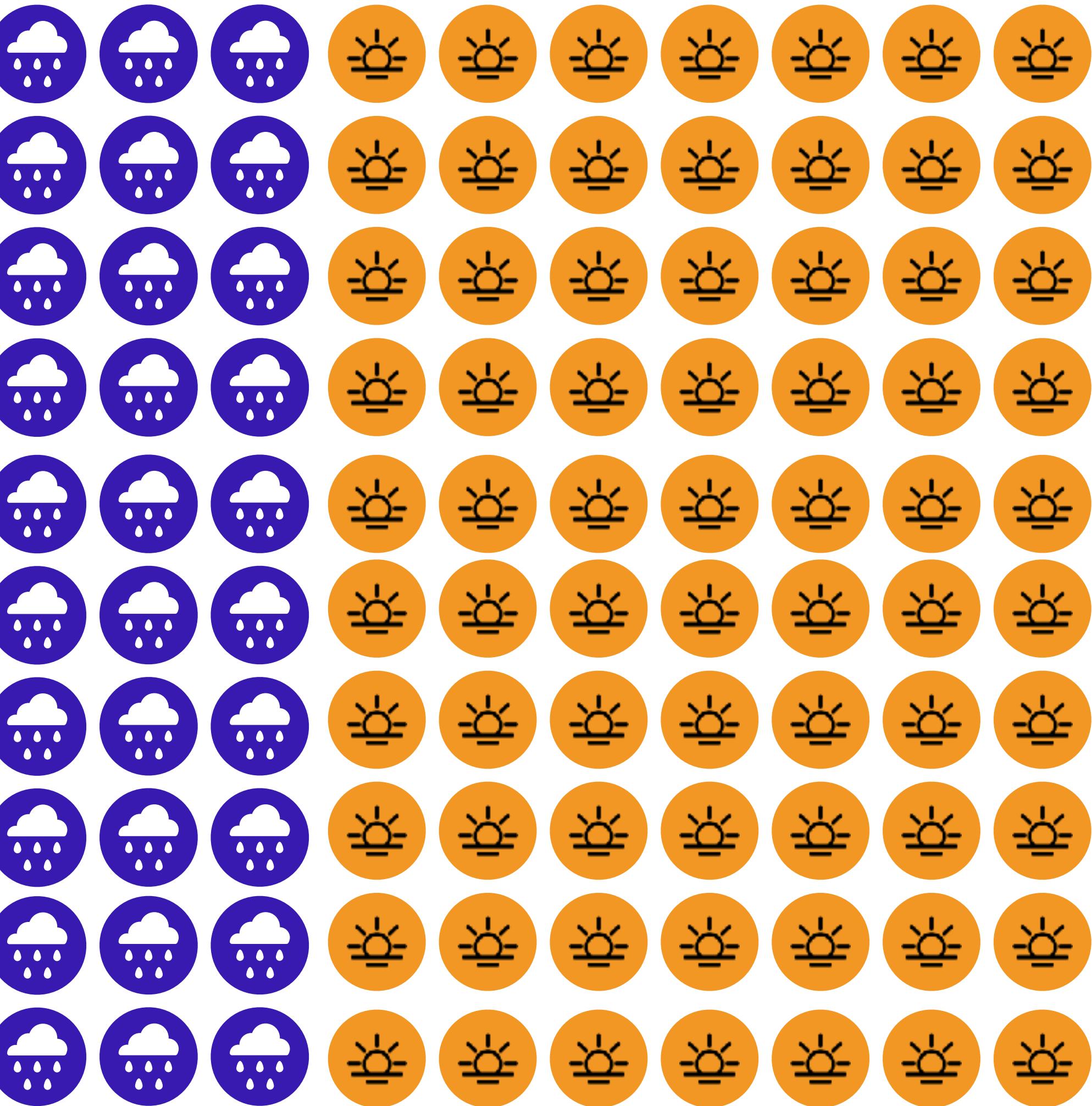
$P(\text{"Rain"}) = 1$

absolutely **certain** that it rains



$P(\text{Rain}) = 0.3$

$P(\neg \text{Rain}) = 0.8$



I observe evidence: 

$$P(\text{Rain}) = 0.3$$



$$P(\text{Clouds} \mid \text{Rain}) = 0.8$$

$$P(\neg \text{Rain}) = 0.8$$

$$P(\text{Clouds} \mid \neg \text{Rain}) = 0.2$$

DO I CHANGE MY GUESS?



I observe evidence: 

$$P(\text{Rain} \mid \text{Clouds}) = \frac{24}{24 + 14} \approx 0.63$$

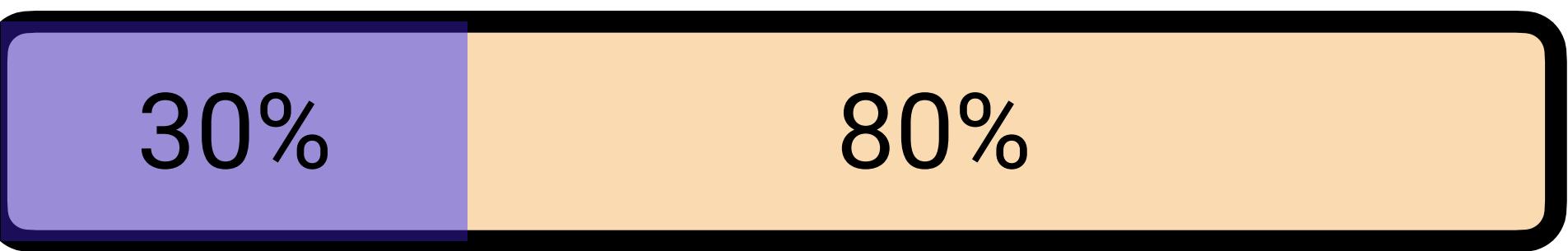
$$P(\text{Clouds} \mid \text{Rain}) = 0.8$$

$$P(\text{Rain}) = 0.3$$

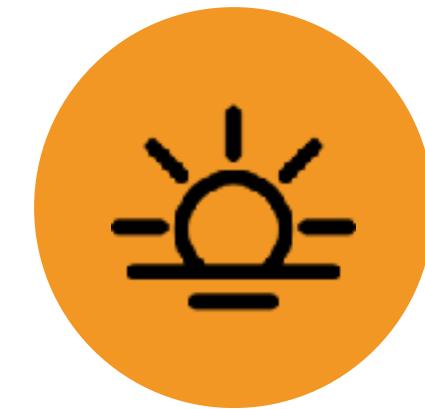
$$P(\neg \text{Rain}) = 0.8$$



$$P(\text{Clouds} \mid \neg \text{Rain}) = 0.2$$



EVIDENCE
↓



When to use Bayes' theorem

You have a
hypothesis:



It's gonna rain

You have observed
evidence:



There are dark clouds

You want to
know:

$$P(\text{H} | \text{E})$$

$$P(\text{hypothesis} \text{ given } \text{evidence})$$

“Prior” → $P(\text{ Hypothesis }) = 30 / 100 = 0.3$



“Prior” → $P(\text{ Hypothesis }) = 30 / 100 = 0.3$

“Likelihood”
 $P(\text{ Evidence } | \text{ H })$
= 0.8



“Prior” → $P(\text{Hypothesis}) = 30 / 100 = 0.3$

“Likelihood”
 $P(\text{Evidence} | \text{H}) = 0.8$



\neg means “not”
 $P(E | \neg H) = 0.2$

$$P(H | E) = \frac{P(H) + P(E | H)}{P(H)P(E | H) + P(\neg H)P(E | \neg H)}$$

“Prior” “Likelihood”

$$P(H) = 0.3$$

$$P(E | H) = 0.8$$



$$P(\neg H) = 0.7$$

$$P(E | \neg H) = 0.2$$

$$P(H|E) = \frac{P(H) P(E|H)}{P(E)} = \frac{P(H) P(E|H)}{P(H)P(E|H) + P(\neg H)P(E|\neg H)}$$

“Prior” “Likelihood”

“Posterior”

$$P(H) = 0.3$$

$$P(E|H) = 0.8$$



$$P(\neg H) = 0.7$$

$$P(E|\neg H) = 0.2$$

This is Bayes Theorem

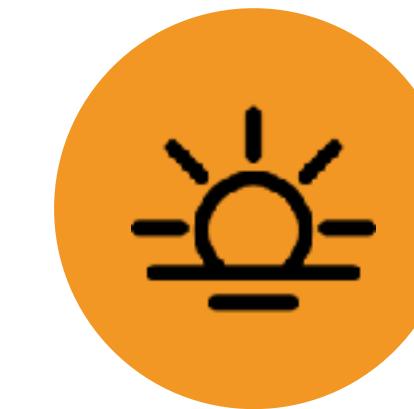
$$P(\mathbf{H} \mid \mathbf{E}) = \frac{P(\mathbf{H}) P(\mathbf{E} \mid \mathbf{H})}{P(\mathbf{E})}$$

prior: initial degree of belief in hypothesis

likelihood: the probability of the evidence given the hypothesis

posterior: degree of belief in hypothesis, after seeing evidence

The diagram illustrates the components of Bayes' Theorem. It shows the formula $P(\mathbf{H} \mid \mathbf{E}) = \frac{P(\mathbf{H}) P(\mathbf{E} \mid \mathbf{H})}{P(\mathbf{E})}$. Three arrows point from text definitions to specific terms in the formula: one arrow points from the 'prior' definition to $P(\mathbf{H})$; another arrow points from the 'likelihood' definition to $P(\mathbf{E} \mid \mathbf{H})$; and a third arrow points from the 'posterior' definition to $P(\mathbf{H} \mid \mathbf{E})$.

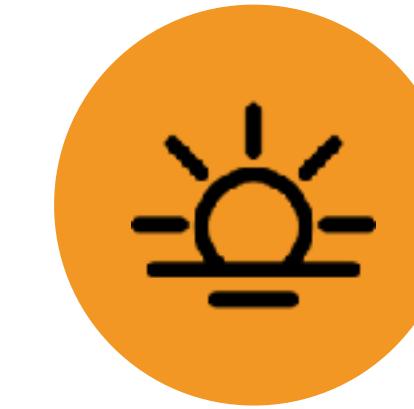
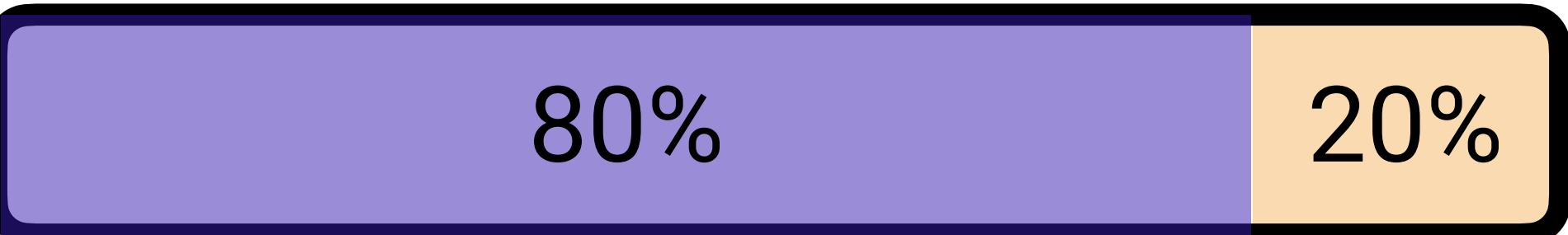


EVIDENCE

“Today’s posterior is tomorrow’s prior.”

- Lindley (1972:2)

EVIDENCE



DO I BRING MY UMBRELLA?



What is more likely?



Steve is very **shy and withdrawn**, invariably helpful but with very little interest in people or in the world of reality. A **meek and tidy soul**, he has a need for order and structure, and a passion for detail.

librarian



farmer

