REASONING ABOUT PROBABILITY


P("Rain") = 0 absolutely **certain** that it won't rain
P("Rain") = 1 absolutely **certain** that it rains

P(Rain) = 0.3

P(¬Rain) = 0.8

I observe evidence: 

P(Rain) = 0.3

P(¬Rain) = 0.8

P(Clouds | Rain) = 0.8

P(Clouds | ¬Rain) = 0.2


DO I CHANGE MY GUESS?

I observe evidence: 

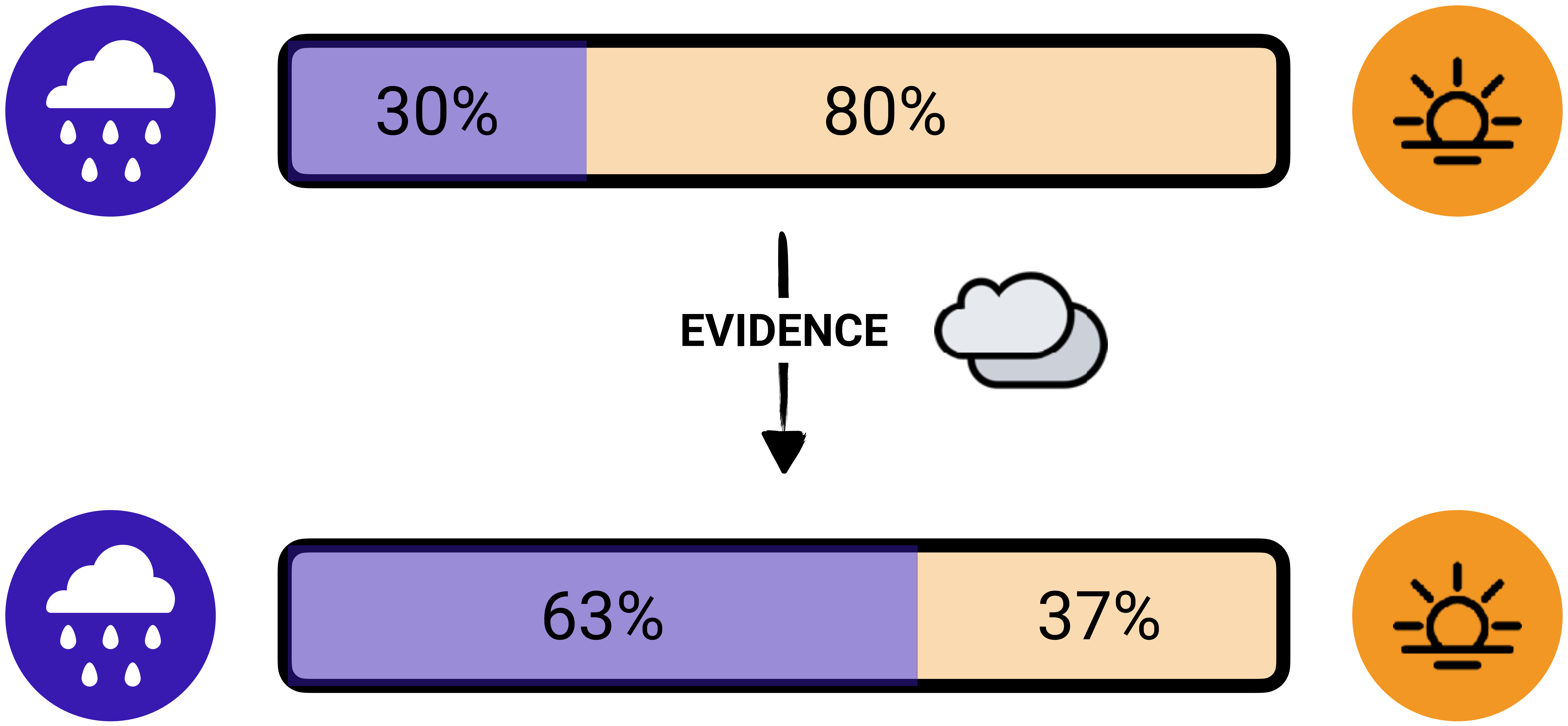
$P(\text{Rain} \mid \text{Clouds}) = \dfrac{24}{24 + 14} \approx 0.63$

P(Rain) = 0.3

P(¬Rain) = 0.8

P(Clouds | Rain) = 0.8

P(Clouds | ¬Rain) = 0.2



30%
80%
EVIDENCE
63%
37%

When to use Bayes' theorem

You have a hypothesis:

It's gonna rain

You have observed evidence:

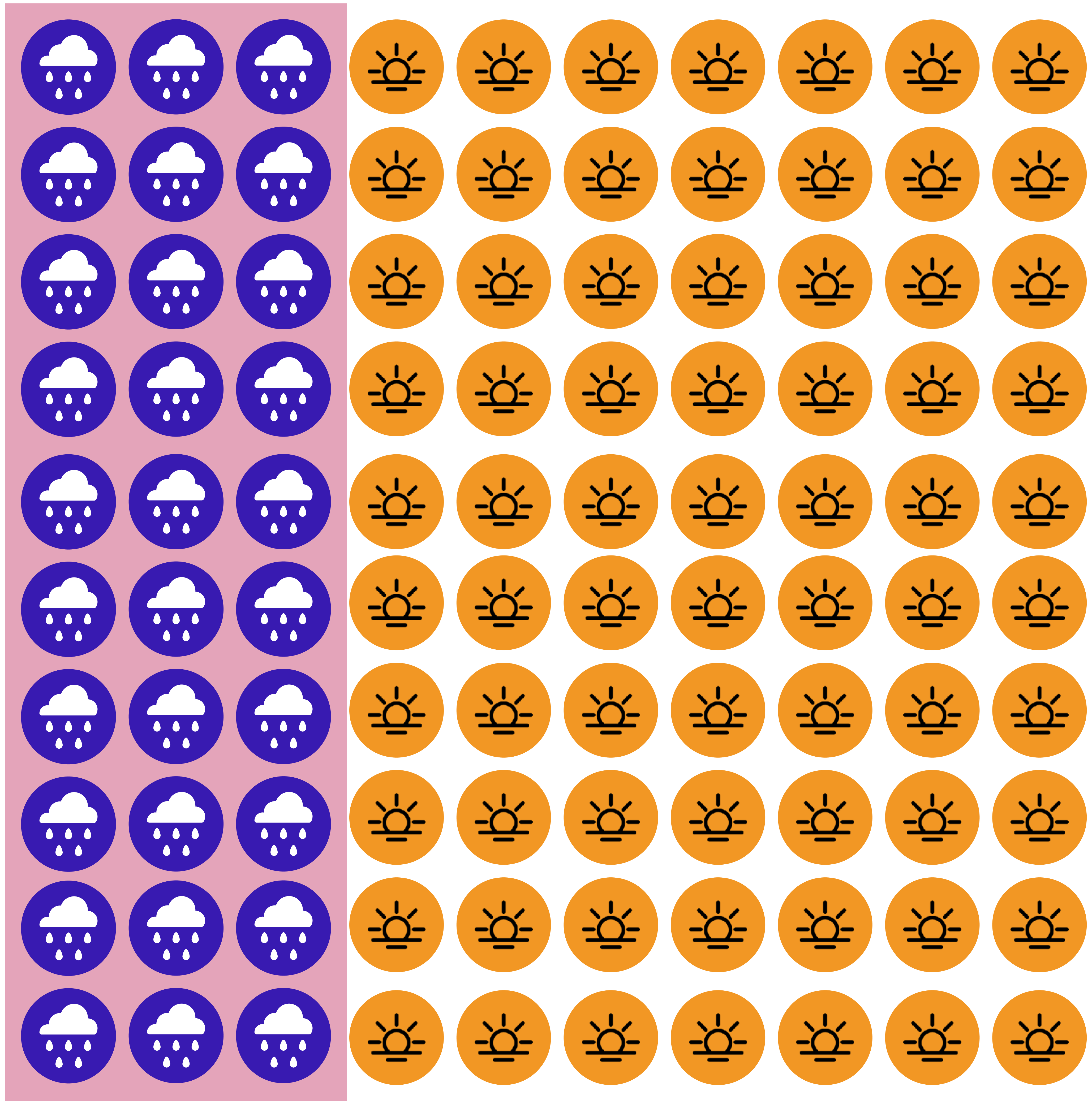
There are dark clouds

You want to know:

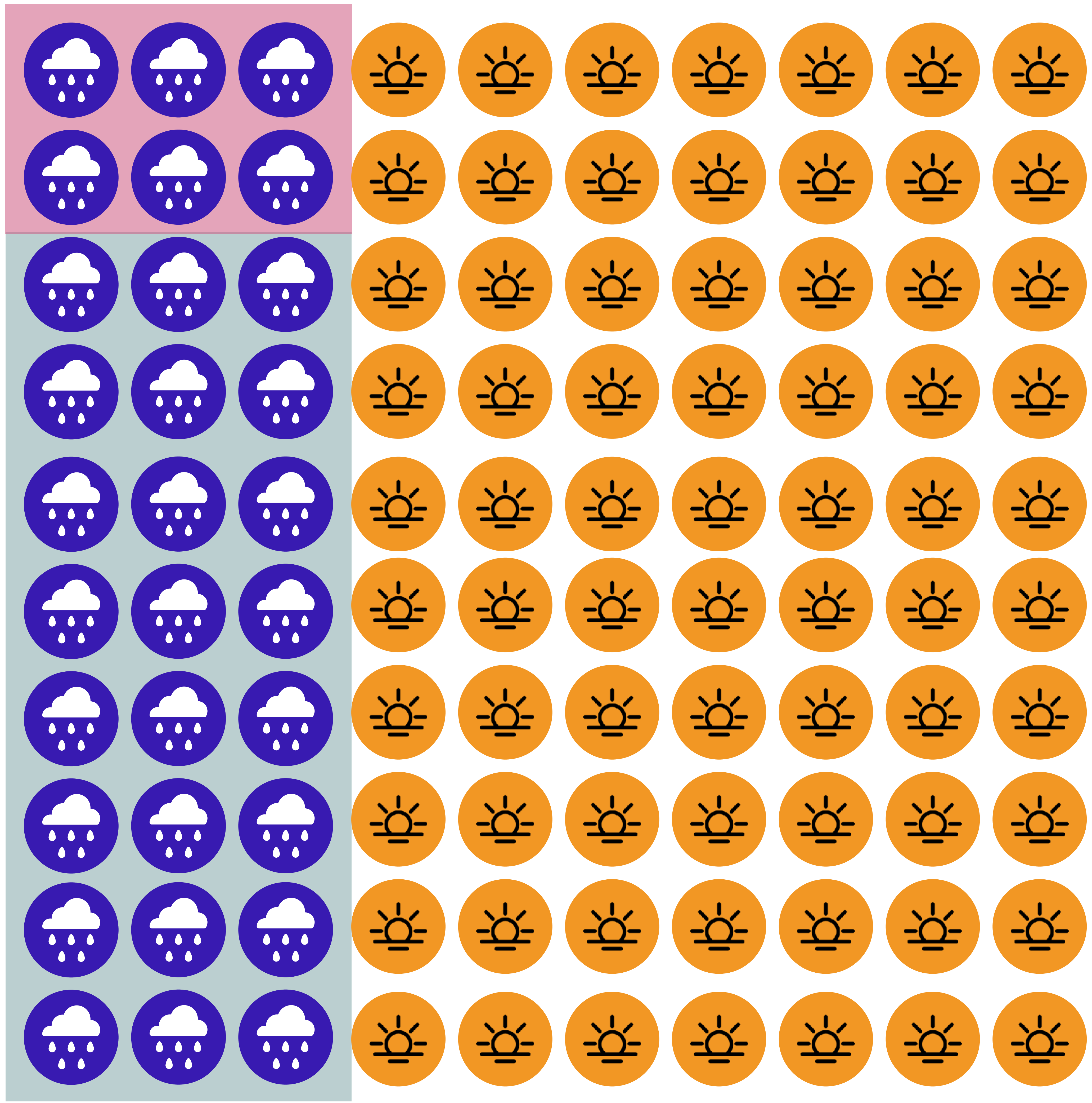
$$P(\,H \mid E\,)$$

$$P\left(\begin{array}{c}\text{hypothesis}\\ \text{given}\\ \text{evidence}\end{array}\right)$$

"Prior" → $P(\text{Hypothesis}) = 30 / 100 = 0.3$



“Prior” ⟶ $P($ Hypothesis $) = 30 / 100 = 0.3$



“Likelihood”

$P($ Evidence | H $)$
$= 0.8$

"Prior" → $P($ Hypothesis $) = 30 / 100 = 0.3$

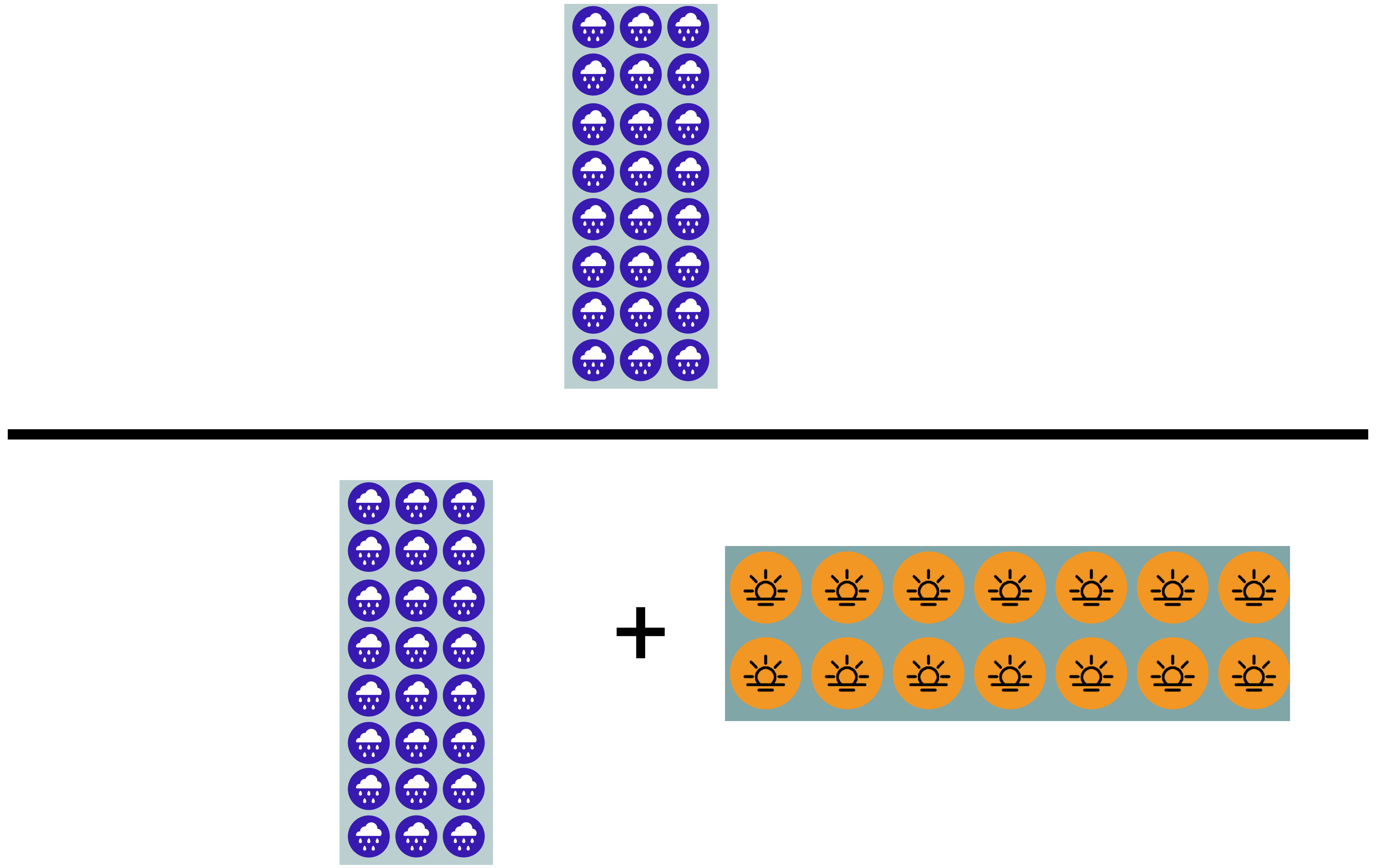
"Likelihood"

$P($ Evidence $|$ H $)$
$= 0.8$

¬ means "not"

$P($ E $|$ ¬H $) = 0.2$

$P(H \mid E) =$

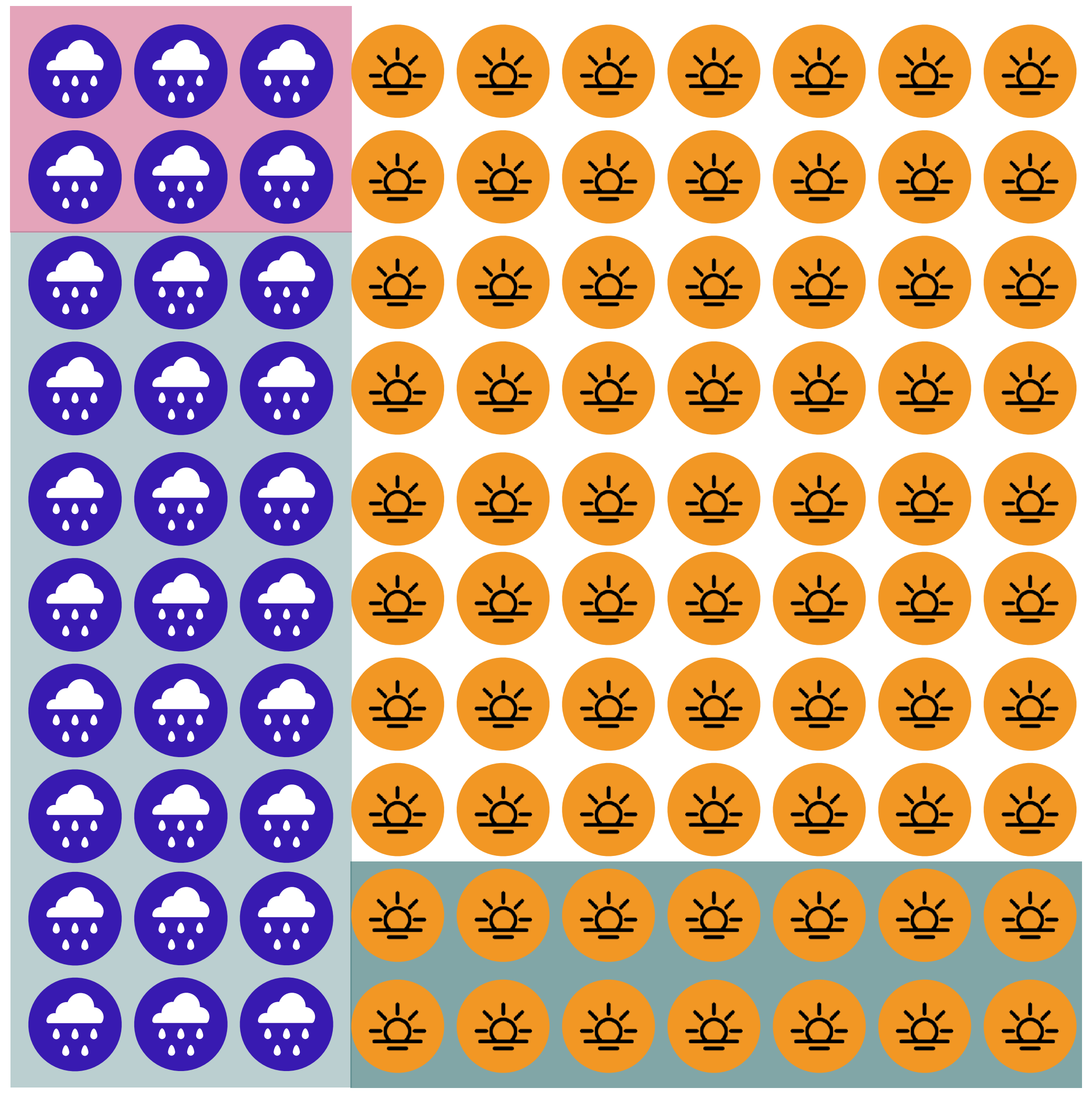


"Prior" **"Likelihood"**

$$= \frac{P(H)\;P(E \mid H)}{P(H)\,P(E \mid H) + P(\neg H)\,P(E \mid \neg H)}$$

$P(H) = 0.3$

$P(E \mid H) = 0.8$



$P(\neg H) = 0.7$

$P(E \mid \neg H) = 0.2$

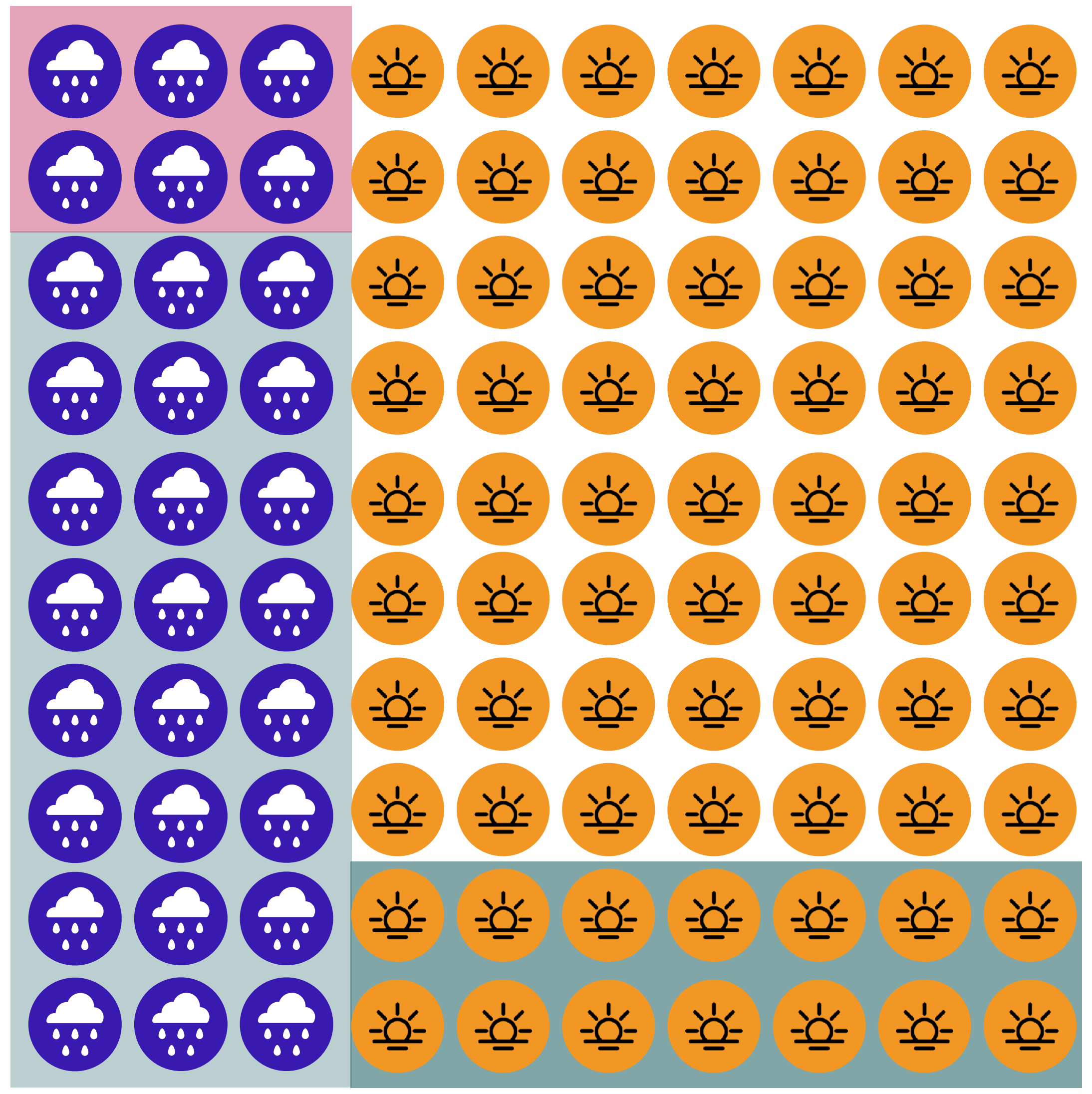
"Prior" "Likelihood"

$$P(H \mid E) = \frac{P(H)\;P(E \mid H)}{P(E)} = \frac{P(H)\;P(E \mid H)}{P(H)\,P(E \mid H) + P(\neg H)\,P(E \mid \neg H)}$$

"Posterior"

$P(H) = 0.3$

$P(E \mid H) = 0.8$



$P(\neg H) = 0.7$

$P(E \mid \neg H) = 0.2$

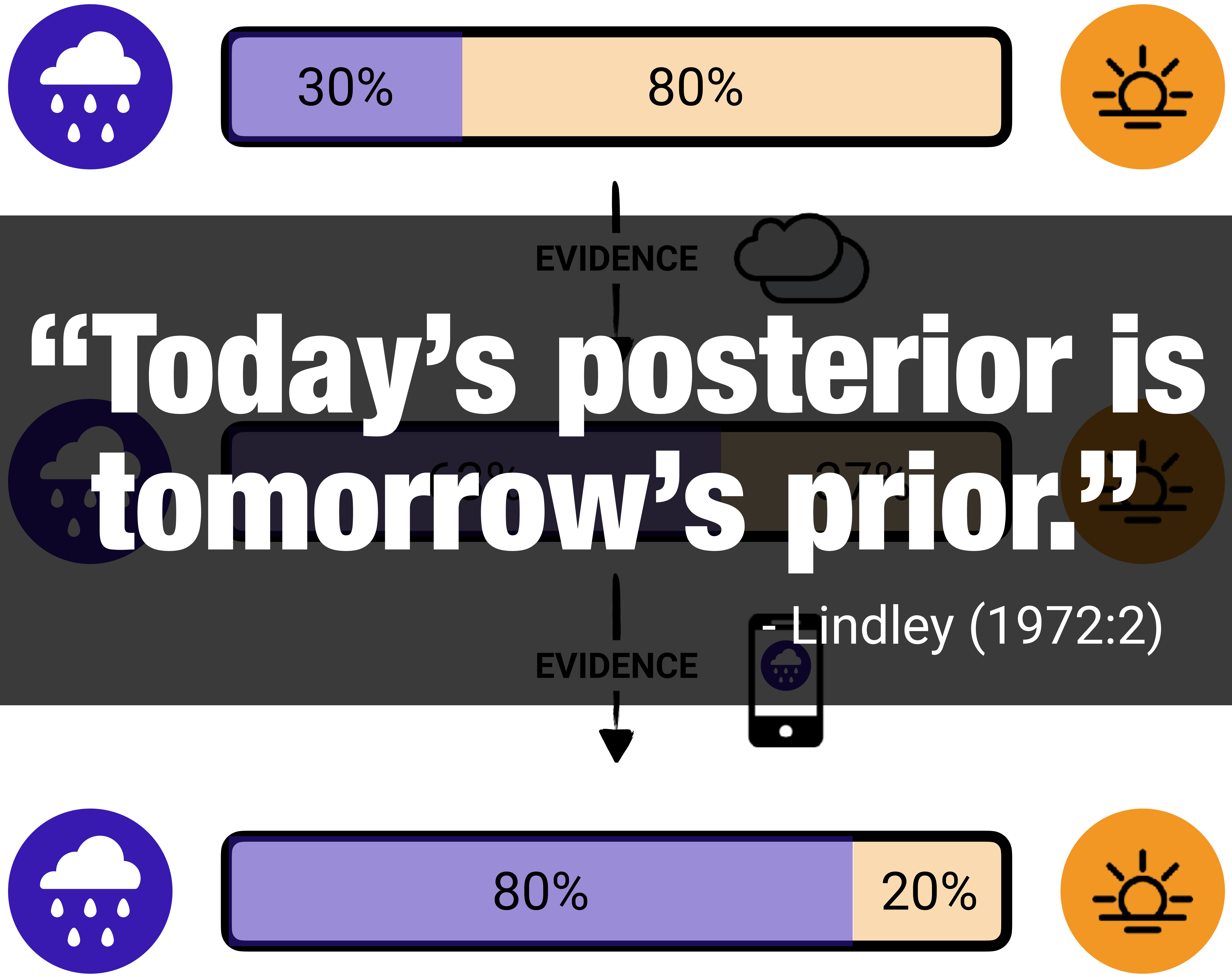
This is Bayes Theorem

prior: initial degree of belief in hypothesis

likelihood: the probability of the evidence given the hypothesis

$$P(\mathbf{H} \mid \mathbf{E}) = \frac{P(\mathbf{H})\ P(\mathbf{E} \mid \mathbf{H})}{P(\mathbf{E})}$$

posterior: degree of belief in hypothesis, after seeing evidence



"Today's posterior is tomorrow's prior."

\- Lindley (1972:2)


DO I BRING MY UMBRELLA?

What is more likely?

Steve is very **shy and withdrawn**, invariably helpful but with very little interest in people or in the world of reality. A **meek and tidy soul**, he has a need for order and structure, and a passion for detail.

librarian

farmer