Write Poisson Distribution into Exponential Family

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Poisson Distribution:

f(k;λ) = P(X = k) = $\frac{\lambda^k e^{-\lambda}}{k!} = \frac{1}{k!} e^{\log_e \lambda^k} e^{-\lambda} = \frac{1}{k!} e^{(k \log_e \lambda) - \lambda}$
Now, force into Exponential family...
$\rightarrow T(y) = k$
$\rightarrow \zeta = \log_e \lambda$
$\rightarrow b(y) = \frac{1}{y!}$
$\rightarrow a(\zeta) = \lambda = e^{\zeta}$
$\Rightarrow P(y;\zeta) = \frac{1}{y!} e^{(y\zeta - e^{\zeta})}$