Lecture Note

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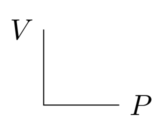
February 6, 2020

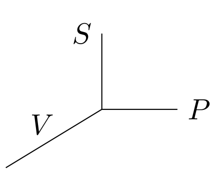
Recall

P(t) $\leftarrow$ price
V(t) $\leftarrow$ volume
S(t) $\leftarrow$ sensitivity
C(t) $\leftarrow$ cash flow
Sh(t) $\leftarrow$ sharpe ratio
$\vdots$
many more

$\binom{N}{K}$



$\binom{N}{2}$



$\binom{N}{3}$

$X_i = \frac{P_i - P_{i-1}}{P_i}, P_i = P(t_i)$, $Y_i = \frac{V_i - V_{i-1}}{V_i}, V_i = V(t_i)$

$$X = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_n & y_n \end{bmatrix}$$

Consider $\mathbf{X}^T\mathbf{X}$ and $\mathbf{X}\mathbf{X}^T \leftarrow$ We orthogonally diagonalize them.

Let $\mathbf{A_{2x2}} = \mathbf{X}^\top\mathbf{X} \Rightarrow$there exists orthogonal matrix P_{2x2} such that

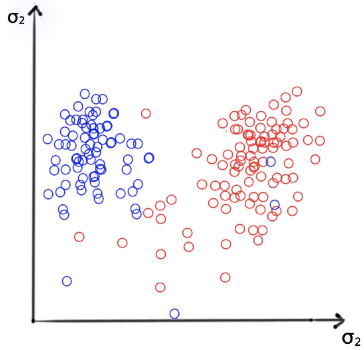
$\mathbf{P}^\top\mathbf{AP} = \mathbf{D}= \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$,

$$\mathbf{P} = (\vec{V_1}, \vec{V_2}), \vec{V_1} \perp \vec{V_2}, \|\vec{V_1}\| = \|\vec{V_2}\| = 1 \tag{1}$$

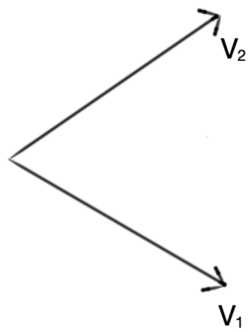
Claim: $\lambda_i \geq 0, i = 1, 2$.
Why? A is symmetric and A is non-negative definite, so $\forall \vec{u} = \mathbb{R}^2$.
Consider $\tilde{\mathbf{u}}^\top\mathbf{A}\tilde{\mathbf{u}} \geq 0$, $\tilde{\mathbf{u}}^\top\mathbf{X}^\top\mathbf{X}\tilde{\mathbf{u}} = (\mathbf{X}\tilde{\mathbf{u}})^\top(\mathbf{X}\tilde{\mathbf{u}}) = \|\mathbf{X}\tilde{\mathbf{u}}\|^2 \geq 0$.
Recall: (Theorem) A symmetric matrix is non-negative definite if and only if all its eigenvalues are non-negative (ie $\lambda_i \geq 0$).
So $\sigma_i = \sqrt{\lambda_i} \Leftrightarrow \vec{V_i}$, where σ_i are called singular values of $\mathbf{X}$.

Your HW Example

We can use daily price data for two stocks to perform analysis. To do this form a martix of the price data for the two stocks for every month. Follow the "left side" approach. Find the singular values (σ_1^k,σ_2^k). k=1,2,...



v1 = [cosθ ,sinθ] v2 = [-sinθ,cosθ]



$$\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} = e^{i\theta},\ \{e^{i\theta} | 0 \leq \Theta \leq 2\pi\} = \text{S'}$$