UC Berkeley CS61C : Machine Structures

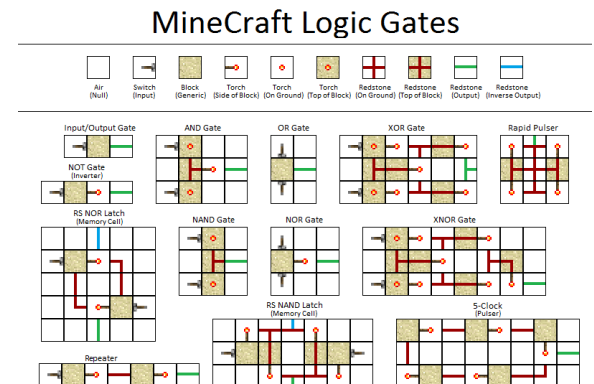## Lecture 26 – Combinational Logic Blocks

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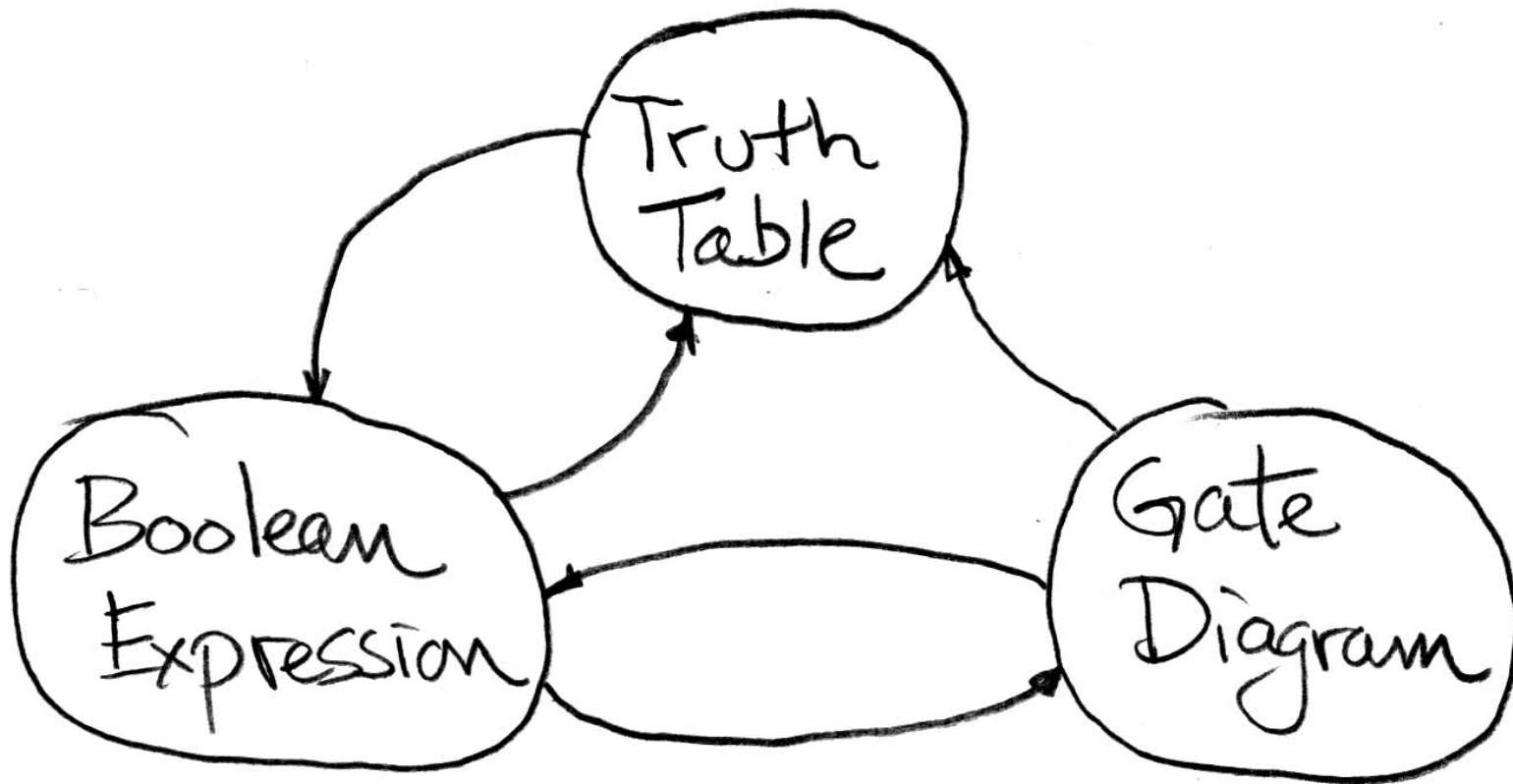
MineCraft Logic Gates ⇒
Using redstone circuits, which are "structures that can be built to active or control mechamisms", you can create all the logic gates using MineCraft.

MineCraft Logic Gates

`minecraft.gamepedia.com/Tutorials/Basic_Logic_Gates`

Review

- **Use this table and techniques we learned to transform from 1 to another**
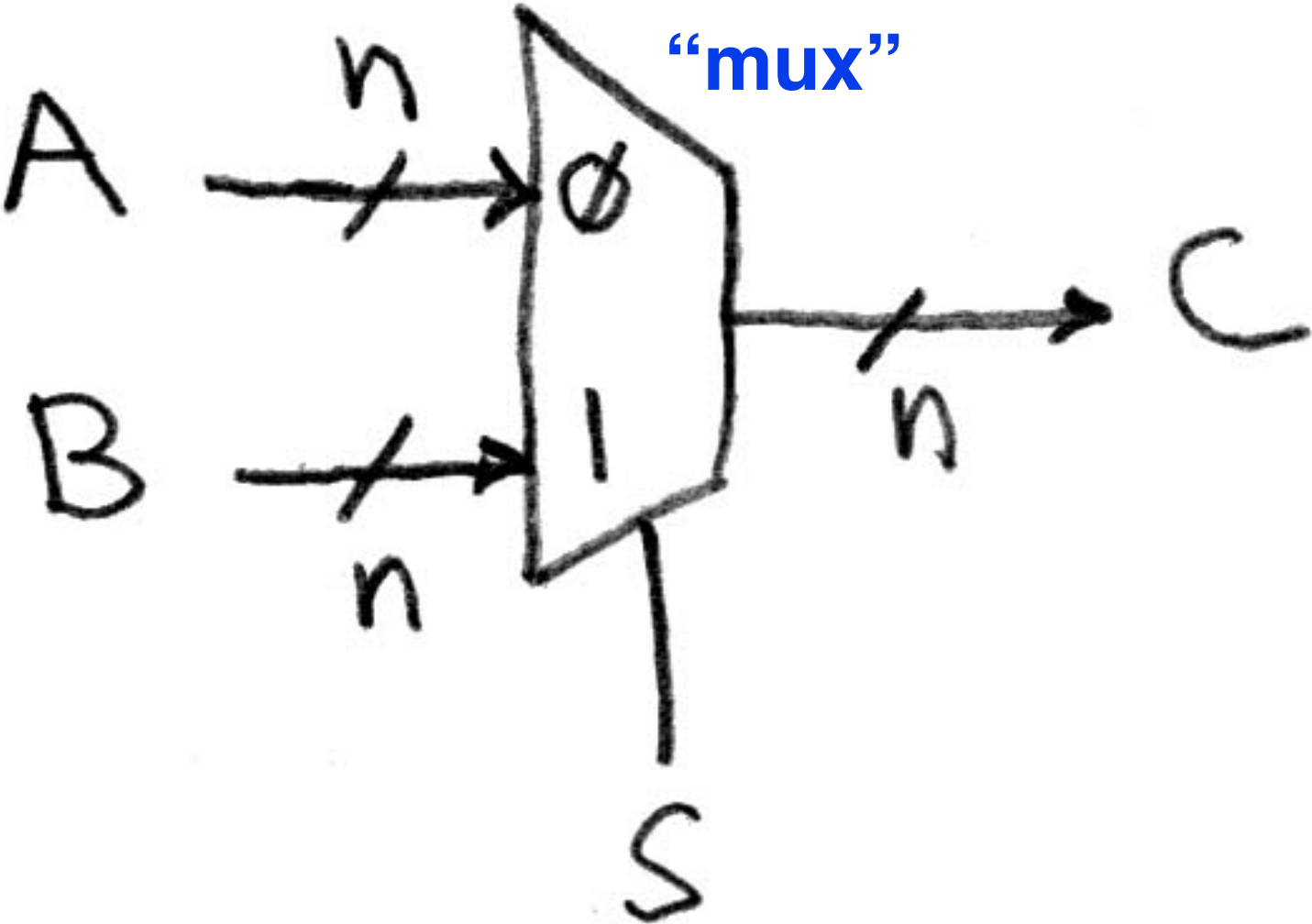
Today

- **Data Multiplexors**

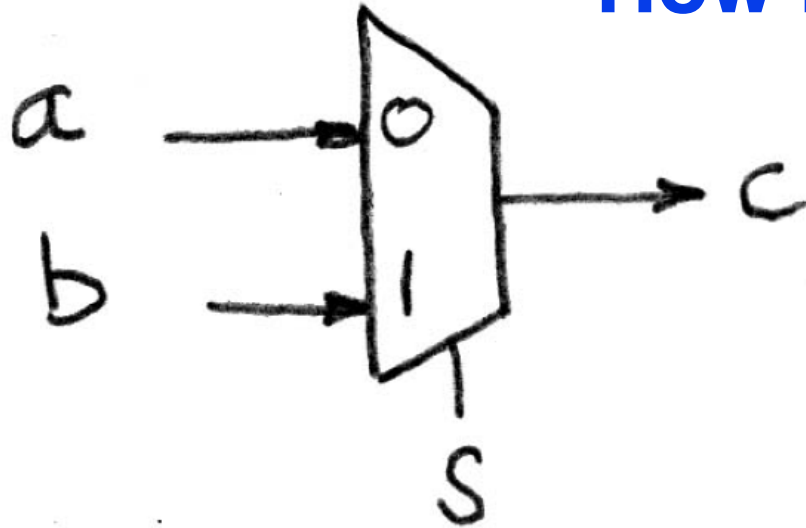- **Arithmetic and Logic Unit**

- **Adder/Subtractor**

Data Multiplexor (here 2-to-1, n-bit-wide)
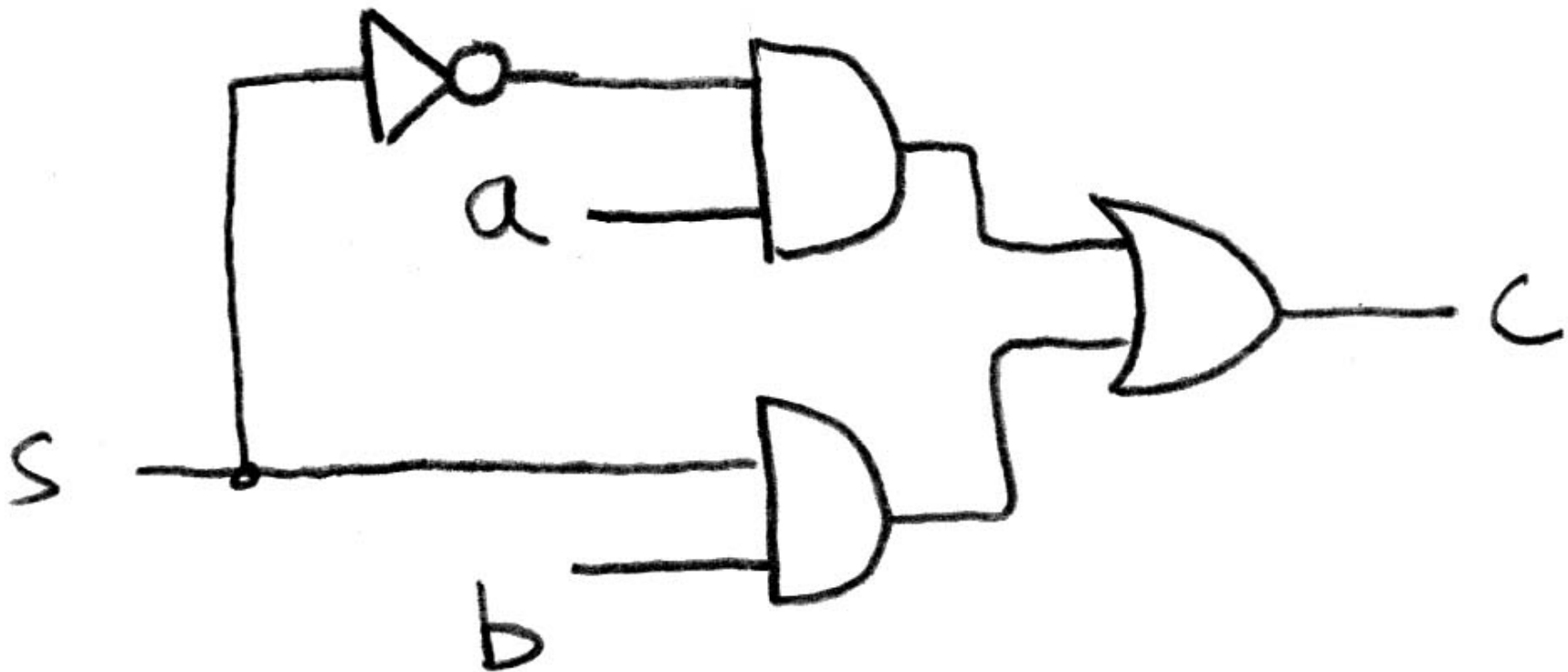

"mux"

N instances of 1-bit-wide mux

How many rows in TT?

$$c = \bar{s}a\bar{b} + \bar{s}ab + s\bar{a}b + sab$$
$$= \bar{s}(a\bar{b} + ab) + s(\bar{a}b + ab)$$
$$= \bar{s}(a(\bar{b} + b)) + s((\bar{a} + a)b)$$
$$= \bar{s}(a(1) + s((1)b)$$
$$= \bar{s}a + sb$$

How do we build a 1-bit-wide mux?

$$\overline{s}a + sb$$

4-to-1 Multiplexor?

How many rows in TT?
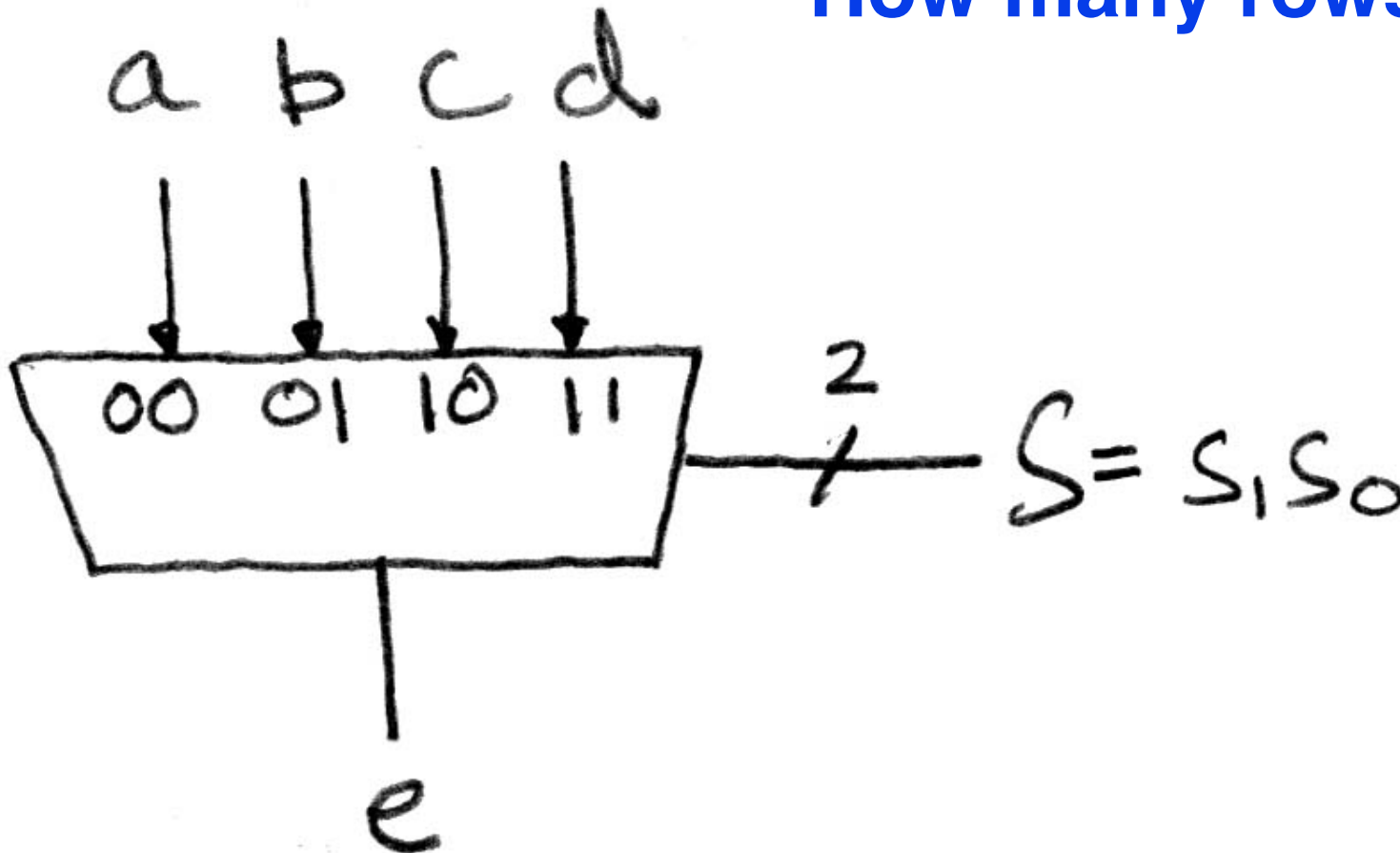
$$e = \overline{s_1}\,\overline{s_0}\,a + \overline{s_1}\,s_0\,b + s_1\,\overline{s_0}\,c + s_1\,s_0\,d$$

Is there any other way to do it?

Hint: NCAA tourney!
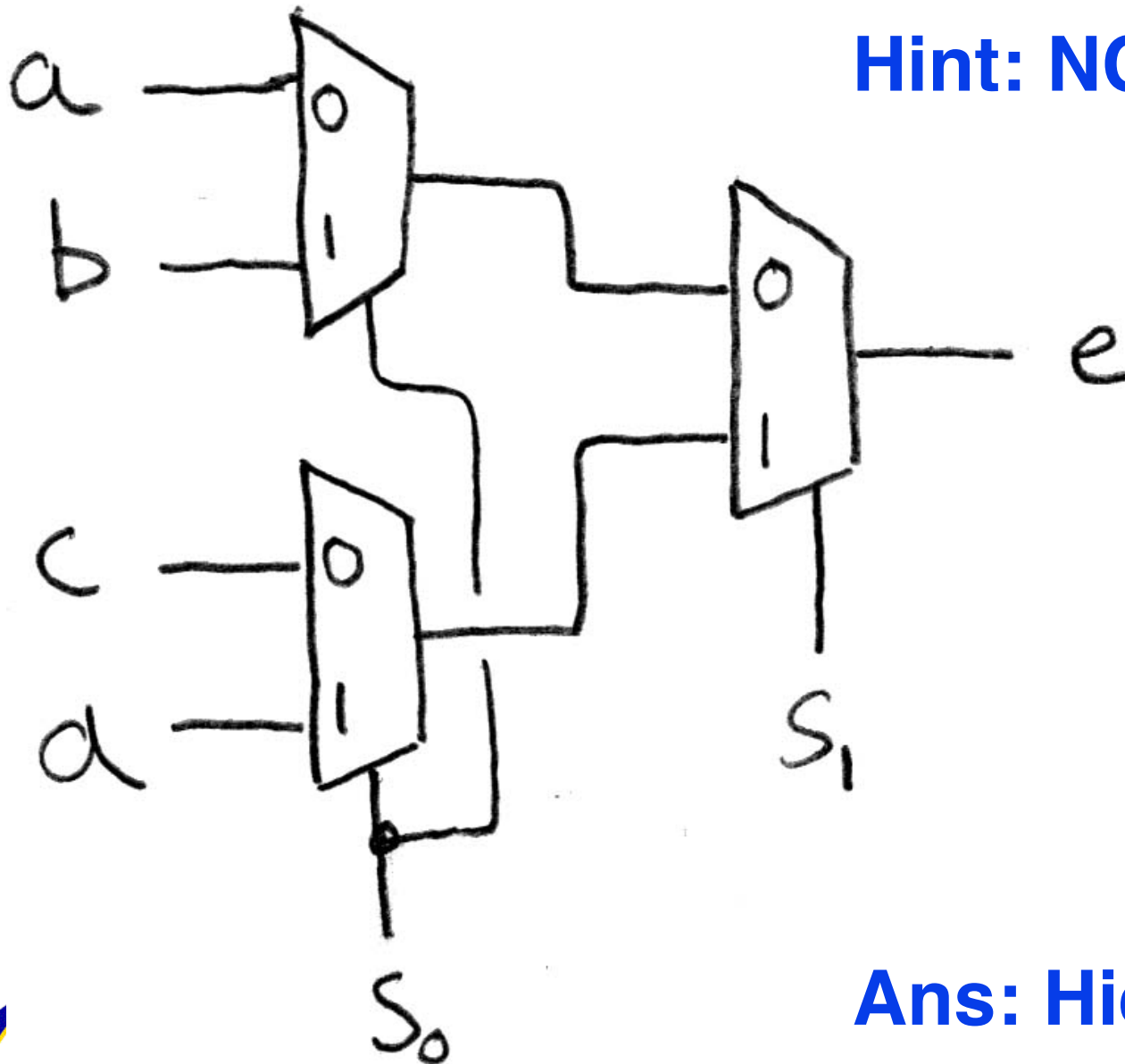
Ans: Hierarchically!

Arithmetic and Logic Unit

- **Most processors contain a special logic block called "Arithmetic and Logic Unit" (ALU)**

- **We'll show you an easy one that does ADD, SUB, bitwise AND, bitwise OR**
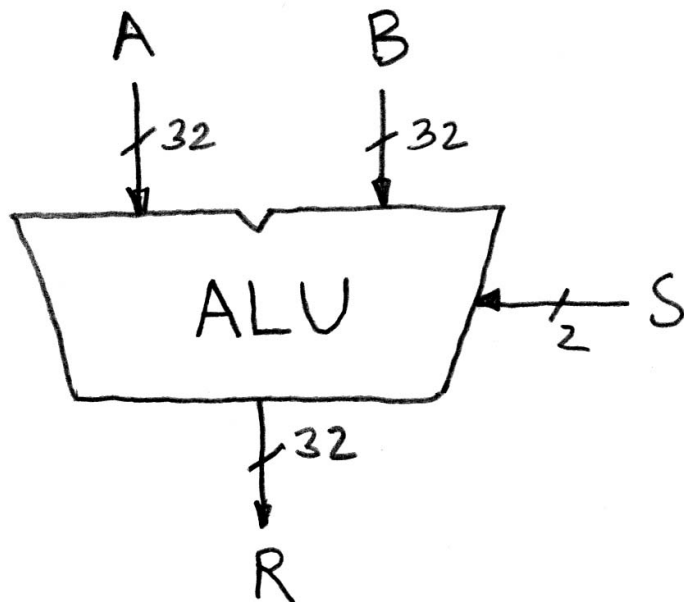
when S=00, R=A+B
when S=01, R=A-B
when S=10, R=A AND B
when S=11, R=A OR B

Our simple ALU
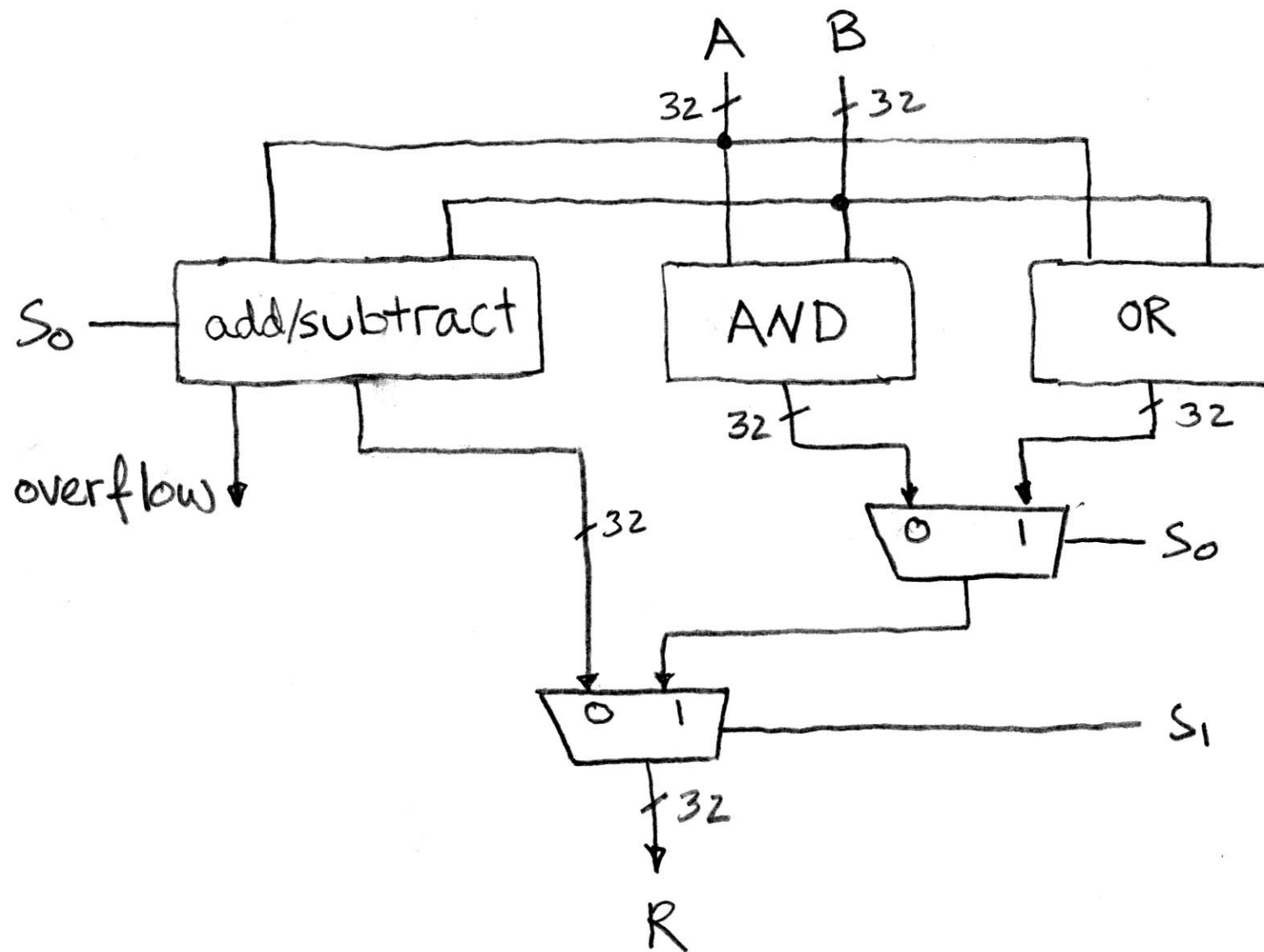
Administrivia

- **How did you find project 2?**

  a) **Hated it**

  b) **Disliked it**

  c) **Neutral**

  d) **Liked it**

  e) **Loved it**

Adder/Subtracter Design -- how?

- **Truth-table, then determine canonical form, then minimize and implement as we've seen before**

- **Look at breaking the problem down into smaller pieces that we can cascade or hierarchically layer**

Adder/Subtracter – One-bit adder LSB...

$$a_3 \quad a_2 \quad a_1 \quad \boxed{a_0}$$
$$+ \quad b_3 \quad b_2 \quad b_1 \quad b_0$$
$$\overline{s_3 \quad s_2 \quad s_1 \quad s_0}$$

a_0	b_0	s_0	c_1
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s_0 =$$
$$c_1 =$$

Adder/Subtracter – One-bit adder (1/2)...

a_i	b_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$
\begin{array}{cccc}
 & a_3 & a_2 & \boxed{a_1} & a_0 \\
+ & b_3 & b_2 & \boxed{b_1} & b_0 \\
\hline
 & s_3 & s_2 & \boxed{s_1} & s_0
\end{array}
$$

$$s_i \quad = $$

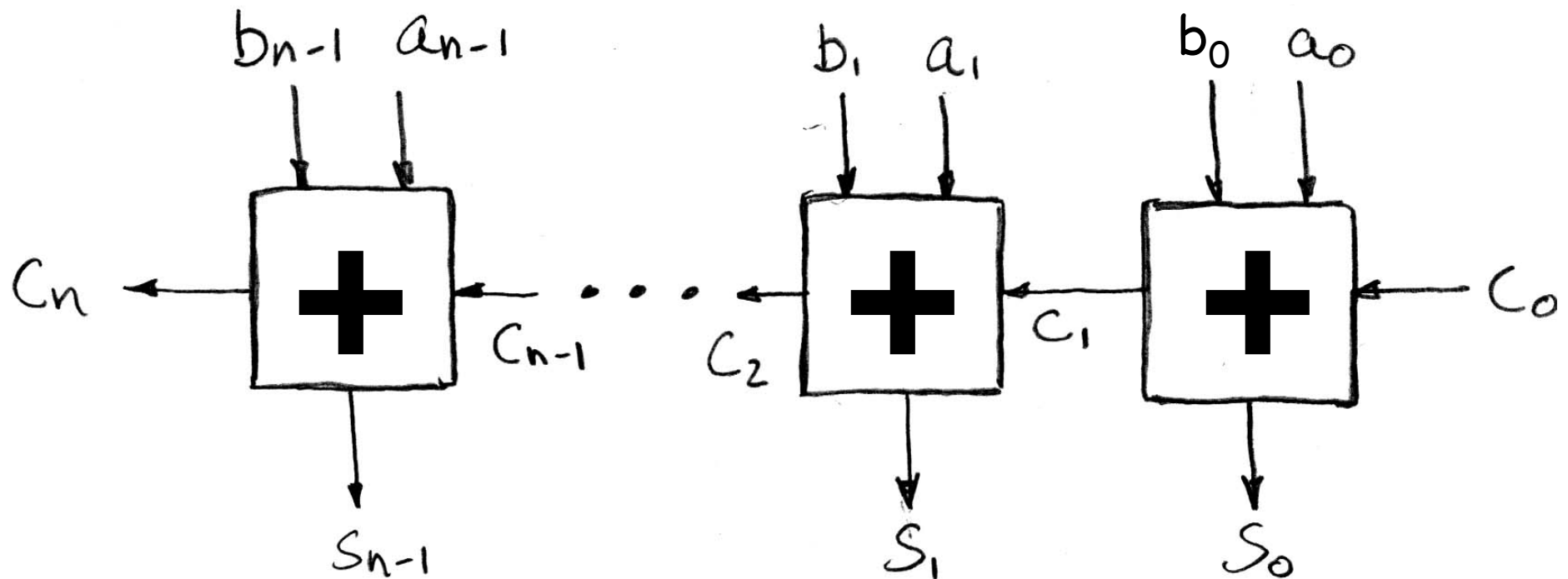$$c_{i+1} \quad = $$

Adder/Subtracter – One-bit adder (2/2)...

$$s_i = \text{XOR}(a_i, b_i, c_i)$$
$$c_{i+1} = \text{MAJ}(a_i, b_i, c_i) = a_i b_i + a_i c_i + b_i c_i$$

N 1-bit adders ⇒ 1 N-bit adder

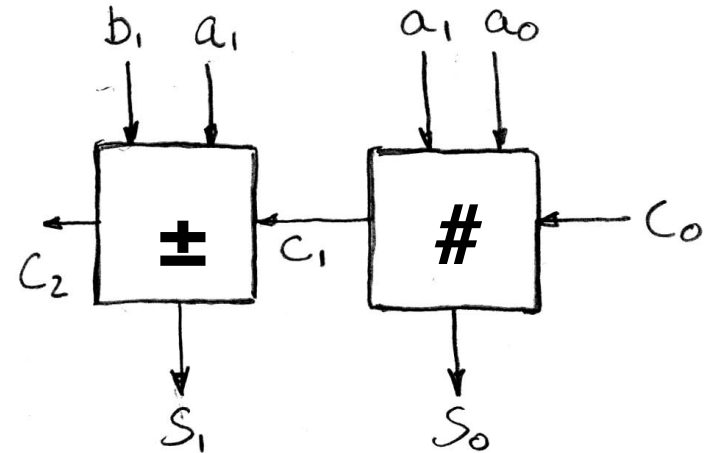## What about overflow?
Overflow = c_n?

What about overflow?

- **Consider a 2-bit signed # & overflow:**
    - `10 = -2 + -2 or -1`
    - `11 = -1 + -2 only`
    - `00 = 0 NOTHING!`
    - `01 = 1 + 1 only`

- **Highest adder**

    - C_1 = Carry-in = C_{in}, C_2 = Carry-out = C_{out}

    - No C_{out} or $C_{in} \Rightarrow$ NO overflow!

    - C_{in}, and $C_{out} \Rightarrow$ NO overflow!

    - C_{in}, but no $C_{out} \Rightarrow$ A,B both > 0, overflow!

    - C_{out}, but no $C_{in} \Rightarrow$ A,B both < 0, overflow!

What op?

What about overflow?

- **Consider a 2-bit signed # & overflow:**

  ```
  10 = -2
  11 = -1
  00 =  0
  01 =  1
  ```
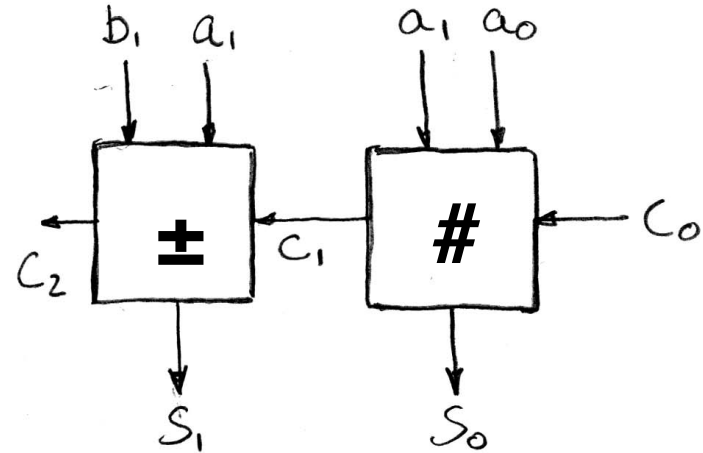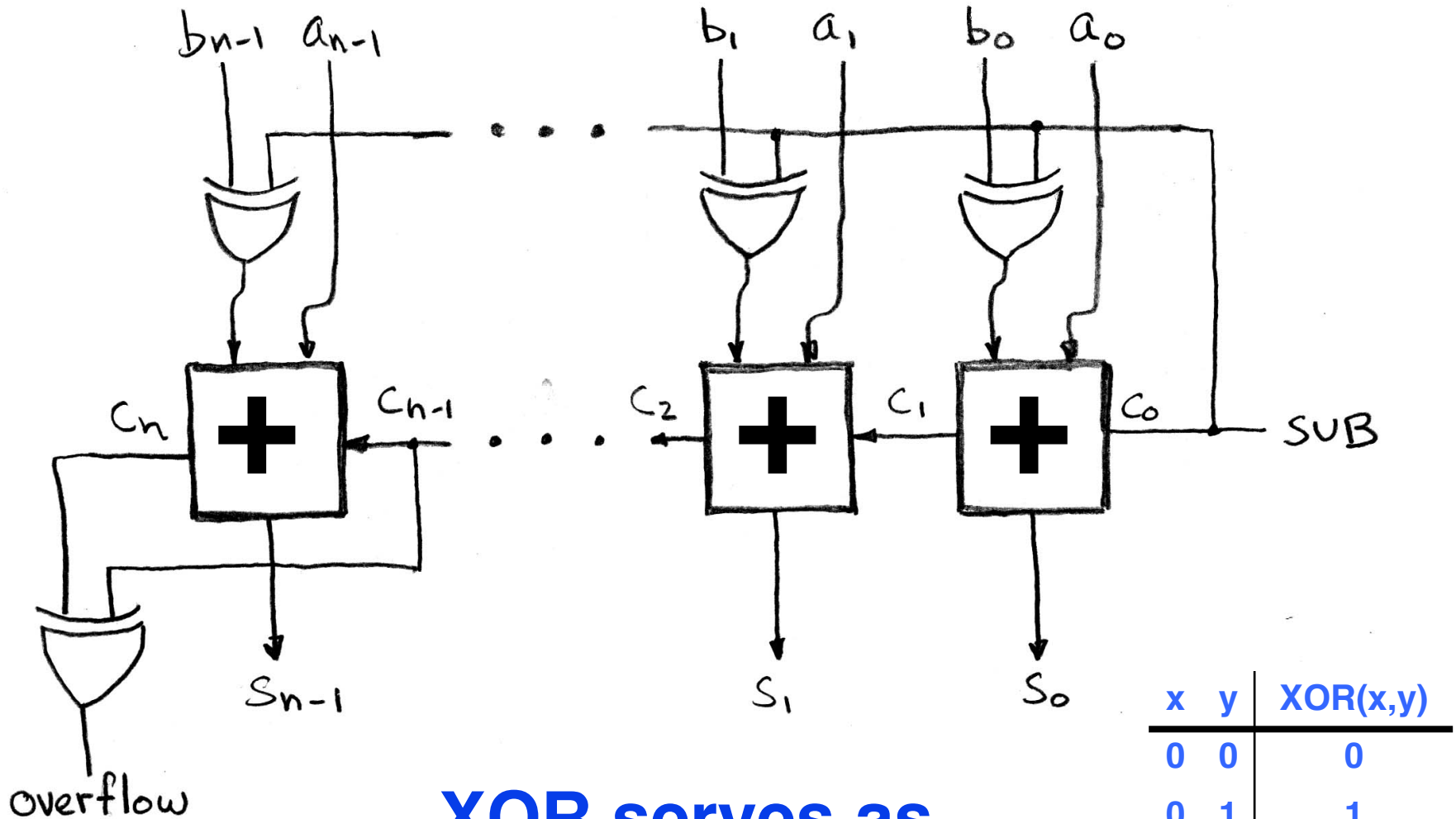

- **Overflows when…**

  - **C_{in}, but no $C_{out} \Rightarrow$ A,B both > 0, overflow!**
  - **C_{out}, but no $C_{in} \Rightarrow$ A,B both < 0, overflow!**

$$\text{overflow} = c_n \text{ XOR } c_{n-1}$$

Extremely Clever Subtractor

XOR serves as conditional inverter!

x	y	XOR(x,y)
0	0	0
0	1	1
1	0	1
1	1	0

Peer Instruction

1) **Truth table for mux with 4-bits of signals has 2^4 rows**

2) **We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl**

	1	2
a)	F	F
b)	F	T
c)	T	F
d)	T	T

Peer Instruction Answer

1) **Truth table for mux with 4-bits of signals controls 16 inputs, for a total of 20 inputs, so truth table is 2^{20} rows…FALSE**

2) **We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl … TRUE**

1) **Truth table for mux with 4-bits of signals is 2^4 rows long**

2) **We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl**

	12
a)	FF
b)	FT
c)	TF
d)	TT

"And In conclusion…"

- **Use muxes to select among input**
  - **S input bits selects 2^S inputs**
  - **Each input can be n-bits wide, indep of S**

- **Can implement muxes hierarchically**

- **ALU can be implemented using a mux**
  - **Coupled with basic block elements**

- **N-bit adder-subtractor done using N 1-bit adders with XOR gates on input**
  - **XOR serves as conditional inverter**