Probability Experiment

- Let us consider one example where there is a bag of multiple balls comprising both Blue and red balls(3 B, 2 R). Suppose if we want to find out the probability of getting four red balls when a person tried selecting the ball four times from the bag. Each time she picks up the ball from the bag, notes down what is the color of the ball and she would again place the ball back to the bag. She continues this activity four times and each time notes down the color of the ball she picked.

- What is the probability that she picked red ball each time i.e. all the picked up balls being red color.

- We can theoretically find out this probability or we can do this test practically to find out the real probability.

- Let us do this test practically. We have arranged some 75 people to do this test. Each person picked up the ball from the bag four times. Each time they would place the ball back to the bag after selecting and again picks up the ball i.e the total number of balls available in the bag is going to be same each time you pick a ball.

Out of 75 people who took the test, 2 persons picked up all 0 red balls(All blue balls), 12 people picked up 1 red ball, 26 people picked up 2 red balls, 25 people picked up 3 red balls and 10 people picked up 4 red balls.

So probability would be as follows:

P(0) = 2/75 = 0.027
P(1) = 12/75 = 0.16
P(2) = 26/75= 0.347
P(3) = 25/75 = 0.33
P(4) = 10/75 = 0.133

Where as the theoretical result are as follows:

P(0) = 2/5*2/5*2/5*2/5 = 0.0256
P(1) = 4*3/5*2/5*2/5*2/5 = 0.1536
P(2) = 6*3/5*3/5*2/5*2/5= 0.3456
P(3) = 4* 3/5*3/5*3/5*2/5 = 0.3456
P(4) = 1* 3/5*3/5*3/5*3/5 = 0.1296

We can see that theoretical result is almost same to the practical result with the sufficient number of tests(75).
If we have less number of practicaltests say 10, then the result would be far from the theoretical result

EXPECTED VALUE OF MONEY WON AFTER 1 GAME

X can take two values: +150 and -10

EXPECTED VALUE OF MONEY WON AFTER 1 GAME

X can take two values: +150 and -10

P(X = +150) = P(4 red balls) = 0.133

P(X = -10) = P(0, 1, 2 or 3 red balls) = 0.027 + 0.160 + 0.347 + 0.333 = 0.867

EXPECTED VALUE OF MONEY WON AFTER 1 GAME

X can take two values: +150 and -10

P(X = +150) = P(4 red balls) = 0.133

P(X = -10) = P(0, 1, 2 or 3 red balls) = 0.027 + 0.160 + 0.347 + 0.333 = 0.867

So, EV = (150 x 0.133) + (-10 x 0.867) = +11.28

Probability Distribution

x	P(x)
0	0.027
1	0.160
2	0.347
3	0.333
4	0.133

Figure 5 - Tabular Form of Probability Distribution

Expected Value

$$EV(X) = x_1 * P(X = x_1) + x_2 * P(X = x_2) + \ldots\ldots\ldots\ldots\ldots\ldots + x_n * P(X = x_n)$$

Another way of writing this is

$$EV(X) = \sum_{i=1}^{i=n} x_i * P(X = x_i)$$

- **Random Variable (X)** : A **random variable**, usually written **X**, is a **variable** whose possible values are numerical outcomes of a **random** phenomenon. There are two types of **random variables**, discrete and continuous

- **Probability Distribution**: A probability distribution is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range. This range will be between the minimum and maximum statistically possible values

- **Expected Value (EV)**: calculated by multiplying each of the possible outcomes by the likelihood each outcome will occur, and summing all of those values

Lets say probability of getting the red ball is denoted by p, then the probability of getting blue ball becomes 1-p.

P(x=4) = P * P * P * P = P^4
P(x=3) = P * P * P * (1-P) = 4 P^3 (1-p)
.
.
.
Goes on

PROBABILITY DISTRIBUTION FOR GENERAL PROBABILITY p

x	$P(X=x)$
0	$(1-p)^4$
1	$4p(1-p)^3$
2	$6p^2(1-p)^2$
3	$4p^3(1-p)$
4	p^4

BINOMIAL PROBABILITY DISTRIBUTION

x	P(X=x)
0	${}^{n}C_{0}(p)^{0}(1-p)^{n}$
1	${}^{n}C_{1}(p)^{1}(1-p)^{n-1}$
2	${}^{n}C_{2}(p)^{2}(1-p)^{n-2}$
3	${}^{n}C_{3}(p)^{3}(1-p)^{n-3}$
.	.
.	.
.	.
.	.
n	${}^{n}C_{n}(p)^{n}(1-p)^{0}$

So, the formula for finding **binomial probability is** given by –

$$P(X=r) = {}^{n}C_{r}(p)^{r}(1-p)^{n-r}$$

Where n is no. of trials, p is probability of success and r is no. of successes after n trials.

However there are some conditions that need to be followed in order for us to be able to apply the formula.

1.Total number of trials is fixed at n

2.Each trial is binary, i.e., has only two possible outcomes - success or failure

3.Probability of success is same in all trials, denoted by p