Math 214 – Multivariable Calculus
Taylor Series

George Peschke

Department of Mathematical and Statistical Sciences
University of Alberta

Lecture 10

Definition — Taylor series / MacLaurin series

Suppose a function f is infinitely often differentiable at $c \in \mathbb{R}$. The Taylor series of f about c is

$$T(x) := \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n.$$

If $c = 0$, this is also called the MacLaurin series of f.

Question

Is $f(x) = T(x)$ on some open interval $]c-\varepsilon, c+\varepsilon[$?

Answer

Yes! - for many important functions f, but not always.

Will concentrate on examples where $T(x) = f(x)$.

Example Exponential function is given by its MacLaurin series

Show that, for all $x \in \mathbb{R}$, e^x satisfies:

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

Example cosh is given by its MacLaurin series

Recall: the even part of e^x is

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

Use this to find the Taylor series of cosh about 0.

Example sin is given by its MacLaurin series

Find the MacLaurin series of $f(x) = \sin x$ and show that it converges to sin everywhere.

Theorem — Derivative / antiderivative of a power series

Suppose

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$$

has positive radius of convergence R. Then, for all $x \in]c-R, c+R[$:

1. $$f'(x) = \sum_{n=0}^{\infty} n a_n (x-c)^{n-1}$$

2. $$\int f(x)\, dx = \sum_{n=0}^{\infty} \frac{1}{n+1} a_n (x-c)^{n+1} + C$$

Example cos is given by its MacLaurin series

Find the MacLaurin series of $u(x) = \cos x$ and show that $Tu(x) = \cos x$.

Example

Find the MacLaurin series of $f(x) = 1/(1-x)$ and its antiderivative.

Example

Find an antiderivative for $f(x) = e^{-x^2}$.

Next slide

Go on to 'n-dimensional space'