

PART II	(SECOND AND FINAL YEAR)	
ACCOUNTING A	ND FINANCE	
AcF 321	INVESTMENTS ( <u>Mock Exam</u> )	(2 hours + 15 minutes reading time)
Answer any <b>THRE</b>	EE questions.	

 Suppose the index model for stock A is estimated from excess returns with the following results:

$$r_A = 1\% + 1.0 \, r_M + e_A$$
  
 $\sigma_M = 10\%$ 

# Co-efficient of Determination

$$R\_Square_A = 0.10;$$

What is the standard deviation of stock A?

[6 marks]

Answer:

The standard deviation of each stock can be derived from the following equation for  $R\_Square$ :

$$R\_Square_i^{\square} = \frac{\beta_i^2 \sigma_M^2}{\sigma_i^2} = \frac{\text{Explained variance}}{\text{Total variance}}$$

Therefore:

$$\sigma_A^2 = \frac{\beta_A^2 \sigma_M^2}{R\_Square_A^{:::}} = \frac{1.0^2 \times .10^2}{0.20} = 0.1$$

$$\sigma_A = .316 = 31.60\%$$

b. State the assumptions and implications of the two-fund separation theorem

[5 marks]

c. Consider two stocks A and B. Both have an expected return of 10%, their standard deviations are 18% and 16% respectively, and their correlation is 0.35. Finally, the risk-free rate of return is 3%. Assuming all the assumptions of the one-fund theorem hold true,

### Required:

 Identify the composition of the minimum variance portfolio, and calculate its expected return and standard deviation.

[12 marks]

ii. Identify the composition of the tangency portfolio, and calculate its expected return and standard deviation.

[4 marks]

iii. Calculate the volatility of the efficient portfolio that would be expected to generate 12.1%

[6 marks]

Answer:

i. If a proportion  $\alpha$  is put into stock A and (1-  $\alpha$  is put into stock B) the volatility would be:

$$\sqrt{\alpha^2 \sigma_A^2 + (1 - \alpha^2) \sigma_B^2 + 2 \sigma_{A,B} \alpha (1 - \alpha)}$$

$$\sqrt{\alpha^2 \cdot 0.0324 + 0.0256 \sigma_R^2 + 0.02016 \alpha (1 - \alpha)}$$

The term inside the square root sign is 0.03784  $\alpha$   $^2$  - 0.03104  $\alpha$  + 0.0256. Differentiating with respect to  $\alpha$  this gives 0.07568  $\alpha$  - 0.03104. The volatility is minimized by setting the derivative equal to zero, so  $\alpha$  = 3104/7568 = 0.41. So, to get minimum risk, it is necessary to put 41% of the money into stock A.

It has an expected return of 10% and a volatility of 13.9%.

- ii. Since both stocks have the same mean, the minimum variance portfolio is also the tangency portfolio. It also has an expected return of 10% and a volatility of 13.9%.
- iii. The efficient portfolio would be obtained by combining the risk-free security with the tangency portfolio [3 marks]

The composition of the required portfolio would be obtained as follows:

$$\delta(R_F) + (1 - \delta)R_{Tangency} = 12.1\%$$
;  $\delta = -0.30$ ;  $\sigma = 1.3 * \sigma_{Tangency} = 18.07\%$ 

- a. An analyst is interested in testing whether momentum risk is priced in addition to the market factor (Market) that is considered in CAPM. The analyst has decided to employ the Fama-McBeth two-stage procedure.
  - i. Explain what is momentum in stock returns. Ensure to briefly discuss any empirical evidence in the literature regarding momentum in returns.

[5 marks]

ii. Briefly explain how you would construct a zero-investment portfolio to mimic the momentum factor.

[4 marks]

- iii. Explain the first-stage of the regression analysis you would employ.

  [5 marks]
- iv. What would the dependent and independent variables in the second stage of the analysis be? [5 marks]
- v. What would the results be in a CAPM world? [6 marks]
- c. Explain how the 'Joint Hypothesis' problem would affect your interpretation of the empirical evidence relating to market efficiency.

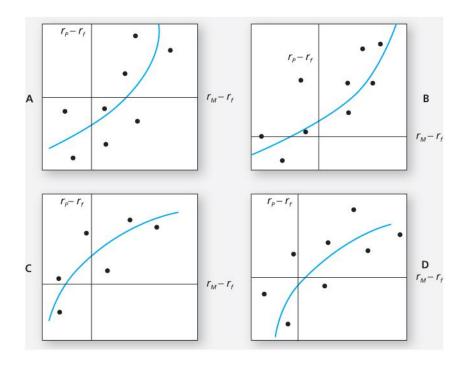
[6 marks]

a. Show that combining an actively managed portfolio with a holding of risk-free bonds will not change the actively managed portfolio's Treynor measure.

[6 marks]

b. Evaluate the market timing and security selection abilities of four managers whose performances are plotted in the accompanying diagrams.

[12 marks]



c. Consider the two (excess return,  $r_p$ - $r_i$ ) index-model regression results for stocks A and B. The risk-free rate over the period was 6%, and the market's average return was 14%. Performance is measured using an index model regression on excess returns. Which stock is the best choice under the following circumstances?

	Stock A	Stock B
Index Model regression estimates	$1\%+1.2(r_m-r_f)$	$2\% + .8(r_m - r_f)$
R-square	0.576	0.436
Residual standard deviation $\sigma$ (e )	10.30%	19.10%
Standard deviation of excess returns	21.60%	24.90%

(i) This is the only risky asset to be held by the investor

[5 marks]

(ii) This stock will be mixed with the rest of the investor's portfolio, currently composed solely of holdings in the market index fund.

[5 marks]

(iii) This is one of many stocks that the investor is analyzing to form an actively managed stock portfolio

[5 marks]

Solution:

		Stock A	Stock B
(i)	Alpha = regression intercept	1.0%	2.0%
(ii)	Information ratio = $\frac{\alpha_P}{\sigma(e_P)}$	0.0971	0.1047
(iii)	*Sharpe measure = $\frac{r_P - r_f}{\sigma_P}$	0.4907	0.3373
(iv)	**Treynor measure = $\frac{r_P - r_f}{\beta_P}$	8.833	10.500

<sup>\*</sup> To compute the Sharpe measure, note that for each stock,  $(r_P - r_f)$  can be computed from the right-hand side of the regression equation, using the assumed parameters  $r_M = 14\%$  and  $r_f = 6\%$ . The standard deviation of each stock's returns is given in the problem.

- (i) If this is the only risky asset held by the investor, then Sharpe's measure is the appropriate measure. Since the Sharpe measure is higher for Stock A, then A is the best choice.
- (ii) If the stock is mixed with the market index fund, then the contribution to the overall Sharpe measure is determined by the appraisal ratio; therefore, Stock B is preferred
- (iii) If the stock is one of many stocks, then Treynor's measure is the appropriate measure, and Stock B is preferred.

<sup>\*\*</sup> The beta to use for the Treynor measure is the slope coefficient of the regression equation presented in the problem.

a. Firm XYZ is required to make a \$10M payment in 2 years and another \$10M payment in 4 years. The yield curve is flat at 10% with semi-annual compounding. Firm XYZ wants to form a portfolio using a 1-year floating rate note (FRN) and a 5-year U.S. strip to fund the payments.

## Required:

- What is the present value of the liabilities? [4 marks]
- ii. What is the (modified) duration of the liabilities? [5 marks]
- iii. What are the (modified) durations of the assets? [4 marks]
- iv. How much of each bond must the portfolio contain for it to still be able to fund the payments after a small shift in the yield curve? [7 marks]

### Solution:

- i. The value of the liabilities is given by: 10M/[1+0.1/2]2 + 10M/[1+0.1/2]6 = 8.22M + 6.78M = 15M
- ii. The duration of the liabilities is given by: 2\*[8.22/15] + 4\*[6.78/15] = 2.90 years. Modified Duration = 2.90 years/(1+0.10/2) = 2.77 years.
- The (Modified) duration of the FRN is practically 0 and the duration of the 5-year iii. strip is 5/(1+0.10/2).
- Let A1 be the portfolio's dollar investment in the FRN and A2 be the portfolio's dollar iv. investment in the 5-year strips.

The dollar value of the portfolio must equal the value of the liabilities. So A1 + A2 = 15M. [1 mark]

The duration of the portfolio equals: w1D1 +(1- w1) D2

where w1 = A1/15M.

Setting the duration of the portfolio equal to the duration of the liabilities gives:

2.77 = w1 D1 + (1-w1) D2 => w1 = 0.390.

Thus.

A1 = 0.390 \* 15M = 5.85M

A2 = 15M - 5.85M = 9.14M.

b. The current yield curve for default-free zero-coupon bonds are as follows:

Maturity (Years)	YTM (%)
1	5%
2	6%
3	7%
4	8%

(i) What are the implied 1-year forward rates?

[6 marks]

(ii) Assume that the pure expectations hypothesis of the term structure is correct and that the face value of all bonds is \$1,000. If market expectations are accurate, what will be the pure yield curve next year?

[7 marks]

#### Solution:

(i) [2 marks for each of the 3 calculations]

Maturity (Years)	YTM (%)	1-year Forward Rates
1	5%	
2	6%	$((1.05)^2/1.06) - 1 = 7.01\%$
3	7%	$(1.07^3/1.06^2) - 1 = 9.03\%$
4	8%	$(1.08^4/1.07^3) - 1 = 11.06\%$

(ii) We obtain next year's prices and yields by discounting each zero's face value at the forward rates for next year that we derived in part (i).

[1 mark for year 1 and 3 marks each for year 2 and 3]

Maturity	Price	YTM
1 year 2 years	\$1,000/1.1201 = \$934.49 \$1,000/(1.1201 × 1.1403) = \$857.11	7.01% 8.01%
3 years	\$1,000/(1.07*1.09*1.11)=771.78	9.02%

#### **QUESTION 5**

a. You have been researching an AAA (ie very low risk of default) 10-year corporate bond which you believe is trading cheaply. You expect the bond to have a return of 8% per annum, while the comparable 10-year Treasury bond has an expected return of 5%. The annual volatility of the corporate bond is 5% and that of the Treasury bond is 3%; the correlation between them is 0.95. You can borrow or lend short term at the Treasury bill rate of 3%.

## Required:

(i) Supposing that you want to construct a levered portfolio consisting of the corporate bond and the treasury bond, design the portfolio that has an expected return of 30% and minimum risk.

[15 marks]

(ii) What is the volatility of the optimal portfolio?

[3 marks]

(iii) If the correlation between the corporate and treasury bonds turns out to be 0.8 rather than the 0.95 you had assumed, and the portfolio composition is as estimated in (i), would you expect the volatility of the portfolio to be different from your answer in (ii)? Explain briefly.

[5 marks]

### Solution:

(i) If you invest a fraction c of your portfolio in the corporate bond, and a fraction t in the Treasury bond the expected return on the portfolio is:

$$E[r_p] = 8\% \times c + 5\% \times t + 3\% \times (1 - t - c) = (3 + 5c + 2t)\%$$

Using the standard formula for the variance of a portfolio, and noting that the risk of T-bills is zero, the variance of the portfolio is:

$$Var[r_p] = (5\%)^2 c^2 + 2 \times 0.95 \times 5\% \times 3\% \times ct + (3\%)^2 t^2$$

To get a return of 30%, need:

$$E[r_p] = 30\%$$
 so  $3 + 5c + 2t = 30$ , and

$$t = 13.5 - 2.5c$$
.

Substituting for t in the variance gives a quadratic equation c. Differentiate it w.r.t c and set the equation to zero. This yields that the variance is minimized when c = 9.45

So t = 13.5-2.5c = -10.125. Thus for every £1m of initial capital, the portfolio goes long £9.45m of the corporate bond, short £10.125m of the matching ten year bond, and puts £1.675m in cash.

- (ii) Substituting back values of c=9.45 and t=-10.125 in the variance equation yields that the variance of the optimal portfolio is 2.37%%, or that the standard deviation is 15.39%.
- (iii) Substituting into the formula for volatility gives a portfolio volatility of 28.48%. [3 marks]
   Since the correlation has reduced, the hedge is less perfect, leading to higher volatility.
- b. Explain the five major differences between hedge funds and mutual funds.

[10 marks]