UNIVERSITY OF WARWICK

Summer Examination 2014

INVESTMENT MANAGEMENT

TIME ALLOWED: 2 HOURS

Answer any **THREE** questions. Where a question has more than one part, each part carries equal weight unless otherwise specified.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on **EACH** answer book you use.

Make any additional assumptions that you feel are necessary to answer the questions but state these assumptions clearly and explicitly.

Silent pocket calculators that are not capable of text storage and retrieval are permitted, but their instruction booklets are not permitted.

THIS IS A CLOSED BOOK EXAMINATION

Turn Over

QUESTION 1

a) Define what is meant by the beta of a share.

[10%]

b) You have two estimates of the beta of a share listed on the London stock market. Both estimates are calculated by regressing returns on the share against returns on the FTSE –100 index over the last five years. One is computed using daily returns, while the other is computed using weekly returns. The first estimate is 0.72, and the second is 0.95. What explanations might there be for the difference?

[15%]

c) Over the year 2012, the average daily return on British Airways (BA) shares was 0.20% with a standard deviation of 1.77%. The average daily return on British Petroleum (BP) shares was -0.07% with a standard deviation of 1.18%. The correlation between the daily returns on the two shares was -1. The returns were measures over trading days, of which there were 252 in the year.

i) Sketch two graphs showing, respectively, the average return and standard deviation of returns of a portfolio comprising a mixture of BA and BP shares as a function of the proportion of holding in BA. Show in your graphs what happens if the portfolio can hold short positions in either share. Ensure that your measures of return and risk are annualized.

[20%]

ii) What must be the value of the risk-free rate of borrowing? **[15%]**

d) A portfolio is equally split between 10 different stocks, each of which has a beta of 1 and the same correlation with the market portfolio. The idiosyncratic component of returns is uncorrelated across stocks. The annualized volatility of the market is 15%, and of the portfolio is 17%.

i) What is the volatility of each stock? [20%]

ii) What is the correlation between each stock and the market? [15%]

iii) What is the correlation between one stock and another? [5%]

i) What is the volatility of each stock?

$$\sigma_P^2 = \frac{1}{N}\sigma^2 + \frac{N-1}{N}\rho\sigma^2$$

Solve for Sigma.

0.17^2(sigma_p^2) **= [10*sigma^2+10*9*cov]/10^2**

Calculating Cov:

The return on each share can be written as:

$$r_A = \alpha_A + r_M + \varepsilon_A$$
$$r_B = \alpha_B + r_M + \varepsilon_B$$

Where the α's are constant, r_M is the market return and the ε's are the idiosyncratic risks.

Hence, Cov (r_A,r_B) = Cov(r_M, r_M) = {sigma_m^2 } = 0.15^2

⇨ **Sigma = 0.294**

ii) What is the correlation between each stock and the market?
1 = corr_m*(0.294/0.15) => corr_m = 0.51

iii) What is the correlation between one stock and another?

Corr = 0.15^2/0.294^2 {cov/sigma*sigma} = 0.260

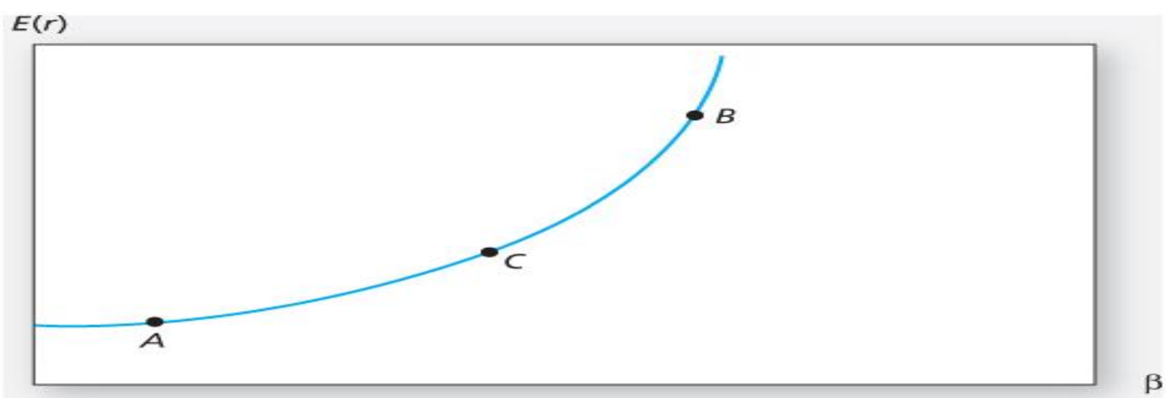
[Total marks 100%]

QUESTION 2

a) Compare the assumptions underlying Capital Asset Pricing Model and the Arbitrage Pricing Model?

[25%]

b) Suppose the relation between expected return and beta is as shown below:



i) Is there an arbitrage opportunity? Justify your answer. **[20%]**

ii) Some researchers have examined the relationship between average returns on diversified portfolios and the β and β^2 of those portfolios. What should they have discovered about the effect of β^2 on portfolio return?

[10%]

i) A long position in a portfolio (P) comprised of Portfolios A and B will offer an expected return-beta tradeoff lying on a straight line between points A and B. Therefore, we can choose weights such that $\beta_P = \beta_C$ but with expected return higher than that of Portfolio C. Hence, combining P with a short position in C will create an arbitrage portfolio with zero investment, zero beta, and positive rate of return.

ii) The argument in part (a) leads to the proposition that the coefficient of β^2 must be zero in order to preclude arbitrage opportunities.

c) Assume that security returns are generated by a standard single-index model. The risk-free rate is 2%. Suppose that there are three securities A, B, and C, characterized by the following data:

Security	β_i	$E(R_i)$	$\sigma(e_i)$
A	0.8	10%	25%
B	1.0	12%	10%
C	1.2	14%	20%

i) If market volatility is 20%, calculate the variance of returns of securities A, B and C. **[10%]**

ii) Now assume that there are an infinite number of assets with return characteristics identical to those of A, B and C, respectively. If one forms a well-diversified portfolio of type A securities, what will the mean and variance of the portfolio's excess returns? What about portfolios composed only of type B or C stocks? **[10%]**

iii) Is there an arbitrage opportunity in this market? What is it? Analyze the opportunity graphically. **[10%]**

i) $\sigma^2 = \beta^2\sigma_M^2 + \sigma^2(e)$

$$\sigma_A^2 = (0.8^2 \times 20^2) + 25^2 = 881$$

$$\sigma_B^2 = (1.0^2 \times 20^2) + 10^2 = 500$$

$$\sigma_C^2 = (1.2^2 \times 20^2) + 20^2 = 976$$

ii) If there are an infinite number of assets with identical characteristics, then a well-diversified portfolio of each type will have only systematic risk since the non-systematic risk will approach zero with large n:

Well-Diversified σ_A^2 □ 256

Well-Diversified σ_B^2 □ 400

Well-Diversified σ_C^2 □ 576

The mean will equal that of the individual (identical) stocks.

iii) There is no arbitrage opportunity because the well-diversified portfolios all plot on the security market line (SML). Because they are fairly priced, there is no arbitrage.

d) A portfolio manager is using the CAPM for making investment recommendations to her clients. Her research department has developed the information shown below:

	Forecast Return	Standard Deviation	Beta
Stock X	14.0%	36%	0.8
Stock Y	17.0%	25%	1.5
Market Index	14.0%	15%	
Risk-free rate	5.0%		

Identify and justify which stock would be more appropriate for an investor who wants to

i) Add this stock to a well-diversified equity portfolio.

ii) Hold this stock as a single-stock portfolio

[20%]

	Expected Return	**Alpha**
Stock X	**$5\% + 0.8 \times (14\% - 5\%) = 12.2\%$**	**$14.0\% - 12.2\% = 1.8\%$**
Stock Y	**$5\% + 1.5 \times (14\% - 5\%) = 18.5\%$**	**$17.0\% - 18.5\% =$**

i. Kay should recommend Stock X because of its positive alpha, compared to Stock Y, which has a negative alpha. In graphical terms, the expected return/risk profile for Stock X plots above the security market line (SML), while the profile for Stock Y plots below the SML. Also, depending on the individual risk preferences of Kay's clients, the lower beta for Stock X may have a beneficial effect on overall portfolio risk.

ii. Kay should recommend Stock Y because it has higher forecasted return and lower standard deviation than Stock X. The respective Sharpe ratios for Stocks X and Y and

the market index are:

Stock X: (14% – 5%)/36% = 0.25

Stock Y: (17% – 5%)/25% = 0.48

Market index: (14% – 5%)/15% = 0.60

The market index has an even more attractive Sharpe ratio than either of the individual stocks, but, given the choice between Stock X and Stock Y, Stock Y is the superior alternative.

When a stock is held as a single stock portfolio, standard deviation is the relevant risk measure. For such a portfolio, beta as a risk measure is irrelevant. Although holding a single asset is not a typically recommended investment strategy, some investors may hold what is essentially a single-asset portfolio when they hold the stock of their employer company. For such investors, the relevance of standard deviation versus beta is an important issue.

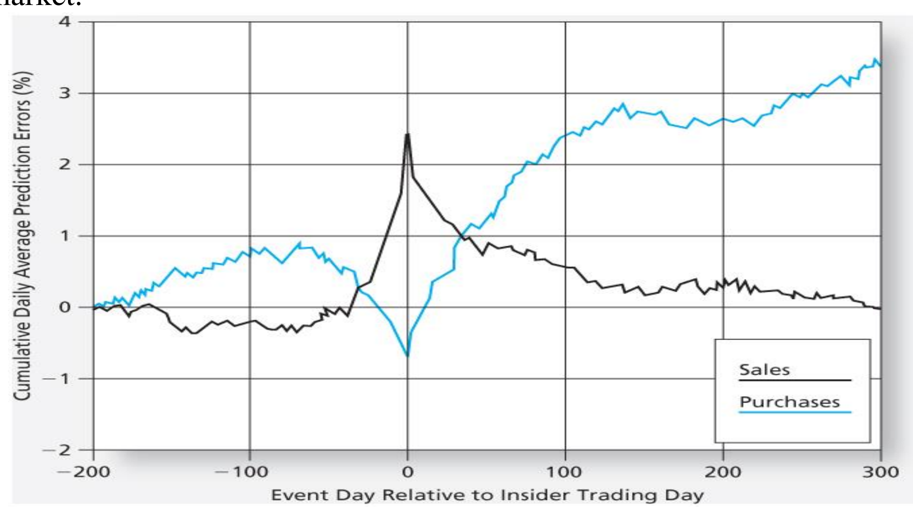
[Total marks 100%]

QUESTION 3

a) In the CAPM, investors are assumed to be mean-variance optimizers. Explain why this leads them to seek portfolios with the maximum possible expected Sharpe ratio.

[20%]

b) The following graph plots cumulative abnormal returns of stocks surrounding insider trading in the equity market.



i) How do you interpret the graph? How does the presented evidence relate to the Grossman-Stiglitz paradox **[20%]**

ii) Based on the evidence presented in the graph, comment on the validity of imposing restrictions on insider trading. **[10%]**

c) Consider the two (excess return, r_p-r_f) index-model regression results for stocks *AB* and *BC*. The risk-free rate over the period was 6%, and the market's average return was 14%. Performance is measured using an index model regression on excess returns.

	Stock *A*	Stock *B*
Index model regression estimates	$1\% + 1.2(r_M - r_f)$	$2\% + .8(r_M - r_f)$
R-square	.576	.436
Residual standard deviation, $\sigma(e)$	10.3%	19.1%
Standard deviation of excess returns	21.6%	24.9%

Which stock is the best choice under the following circumstances?

i) This is the only risky asset to be held by the investor.

ii) This stock will be mixed with the rest of the investor's portfolio, currently composed solely of holdings in the market index fund.

iii) This is one of many stocks that the investor is analyzing to form an actively managed stock portfolio

[30%]

		Stock AB	**Stock BC**
(i)	**Alpha = regression intercept**	**1.0%**	**2.0%**
(ii)	**Information ratio =** $\dfrac{\alpha_P}{\sigma(e_P)}$	**0.0971**	**0.1047**
(iii)	***Sharpe measure =** $\dfrac{r_P - r_f}{\sigma_P}$	**0.4907**	**0.3373**
(iv)	****Treynor measure =** $\dfrac{r_P - r_f}{\beta_P}$	**8.833**	**10.500**

*** To compute the Sharpe measure, note that for each stock, ($r_P - r_f$) can be computed from the right-hand side of the regression equation, using the assumed parameters $r_M = 14\%$ and $r_f = 6\%$. The standard deviation of each stock's returns is given in the problem.**

**** The beta to use for the Treynor measure is the slope coefficient of the regression equation presented in the problem.**

(i) If this is the only risky asset held by the investor, then Sharpe's measure is the appropriate measure. Since the Sharpe measure is higher for Stock AB, then AB is the best choice

(ii) If the stock is mixed with the market index fund, then the contribution to the overall Sharpe measure is determined by the appraisal ratio; therefore, Stock BC is preferred.

(iii)) If the stock is one of many stocks, then Treynor's measure is the appropriate measure, and Stock BC is preferred.

d) Discuss the different factors that create of 'limits to arbitrage'. **[20%]**

[Total marks 100%]

QUESTION 4

a) Explain how the duration changes as the payment frequency increases. How does the duration differ from the modified duration in the limit as the payment frequency becomes infinite? **[15%]**

b) The current yield curve for default-free zero-coupon bonds are as follows:

Maturity (Years)	YTM (%)
1	10%
2	11
3	12

i) What are the implied 1-year forward rates? **[15%]**

ii) Assume that the pure expectations hypothesis of the term structure is correct. If market expectations are accurate, what will be the pure yield curve next year? **[10%]**

iii) If you purchase a 2-year zero-coupon bond now, what is the expected total rate of return over the next year? What if you purchase a 3-year zero-coupon bond? Ignore taxes. **[15%]**

iv) What should be the current price of a 3-year maturity bond with a 12% coupon rate paid annually? If you purchased it at that price, what would you total expected rate of return be over the next year? Ignore taxes. **[20%]**

i) We obtain forward rates from the following table:

Maturity	**YTM**	**Forward Rate**	**Price (for parts c, d)**
1 year	**10%**		**$1,000/1.10 = $909.09**
2 years	**11%**	**$(1.11^2/1.10) - 1 = 12.01\%$**	**$\$1{,}000/1.11^2 = \$811.62$**
3 years	**12%**	**$(1.12^3/1.11^2) - 1 = 14.03\%$**	**$\$1{,}000/1.12^3 = \$711.78$**

ii) We obtain next year's prices and yields by discounting each zero's face value at the forward rates for next year that we derived in part (a):

Maturit	**Price**	**YTM**

y		
1 year	**$1,000/1.1201 = $892.78**	**12.01%**
2 years	**$1,000/(1.1201 × 1.1403) = $782.93**	**13.02%**

Note that this year's upward sloping yield curve implies, according to the expectations hypothesis, a shift upward in next year's curve.

iii) **Next year, the 2-year zero will be a 1-year zero, and will therefore sell at a price of: $1,000/1.1201 = $892.78**

Similarly, the current 3-year zero will be a 2-year zero and will sell for: $782.93

Expected total rate of return:

2-year bond: $$\frac{\$892.78}{\$811.62} - 1 = 1.1000 - 1 = 10.00\%$$

3-year bond: $$\frac{\$782.93}{\$711.78} - 1 = 1.1000 - 1 = 10.00\%$$

iv) The current price of the bond should equal the value of each payment times the present value of $1 to be received at the "maturity" of that payment. The present value schedule can be taken directly from the prices of zero-coupon bonds calculated above.

Current price = ($120 × 0.90909) + ($120 × 0.81162) + ($1,120 × 0.71178)

= $109.0908 + $97.3944 + $797.1936 = $1,003.68

Similarly, the expected prices of zeros one year from now can be used to calculate the expected bond value at that time:

Expected price 1 year from now = ($120 × 0.89278) + ($1,120 × 0.78293)

= $107.1336 + $876.8816 = $984.02

Total expected rate of return =

$$\frac{\$120 + (\$984.02 - \$1{,}003.68)}{\$1{,}003.68} = 0.1000 = 10.00\%$$

c) You are managing a portfolio of $1 million, Your target duration is 10 years, and you can choose from two bonds: a zero-coupon bond with maturity of 5 years, and a perpetuity, each currently yielding 5%

i) How much of each bond will you hold in your portfolio?

ii) How will these fractions change next year if target duration is now 9 years?

[25%]

The duration of the perpetuity is: 1.05/0.05 = 21 years

i)Call w the weight of the zero-coupon bond. Then:

$(w \times 5) + [(1 - w) \times 21] = 10 \Rightarrow w = 11/16 = 0.6875$

Therefore, the portfolio weights would be as follows: 11/16 invested in the zero and 5/16 in the perpetuity.

ii)Next year, the zero-coupon bond will have a duration of 4 years and the perpetuity will still have a 21-year duration. To obtain the target duration of nine years, which is now the duration of the obligation, we again solve for w:

$(w \times 4) + [(1 - w) \times 21] = 9 \Rightarrow w = 12/17 = 0.7059$

So, the proportion of the portfolio invested in the zero increases to 12/17 and the proportion invested in the perpetuity falls to 5/17.

[Total marks 100%]

QUESTION 5

a) Discuss the difference between future and forward contracts in terms of their settlement. Explain how this difference could affect the risks and pricing of contracts. **[10%]**

b) Consider the following information:

$r_{us} = 4\%$; $r_{uk} = 7\%$ (annual yields)
$E_0 = 2.00$ dollars per pound
$F_0 = 1.98$ dollars per pound (1-year delivery)

i) Where would lend and where would you borrow? **[5%]**
ii) Is there an arbitrage opportunity? How could you arbitrage? **[15%]**

i) Lend in the U.K. and Borrow in the U.S.

ii) Lending in the U.S. offers a 4% rate of return. Lending in the U.K. and covering interest rate risk with futures or forwards offers a rate of return of:

$$r_{US} = \left[(1 + r_{UK}) \times \frac{F_0}{E_0}\right] - 1 = \left[1.07 \times \frac{1.98}{2.00}\right] - 1 = 0.0593 = 5.93\%$$

An arbitrage strategy involves simultaneous lending (UK) and borrowing (US) with the covering of interest rate risk:

Action Now	CF in \$	Action at period-end	CF in \$
Borrow \$2.00 in U.S.	**\$2.00**	**Repay loan**	**$-\$2.00 \times 1.04$**
Convert borrowed dollars to pounds; lend £1 pound in U.K.	**–\$2.00**	**Collect repayment; exchange proceeds for dollars**	**$1.07 \times E_1$**

Sell forward £1.07 at F_0 = \$1.98	**0**	**Unwind forward**	**1.07 × (\$1.98 – E_1)**
Total	**0**	**Total**	**\$0.0386**

c) You are managing a \$50 million diversified portfolio of US equities for a client, who has suddenly become very nervous about the short term prospects for the US stock market. The beta of the portfolio is 0.92. The size of a futures contract is \$10/point. If the futures price remains at 1,422, how many S&P futures contracts would you sell to hedge market risk? If the portfolio returns are exactly as volatile as the S&P futures', estimate the change in the standard deviation of the portfolio returns because of the hedge. [**20%**]

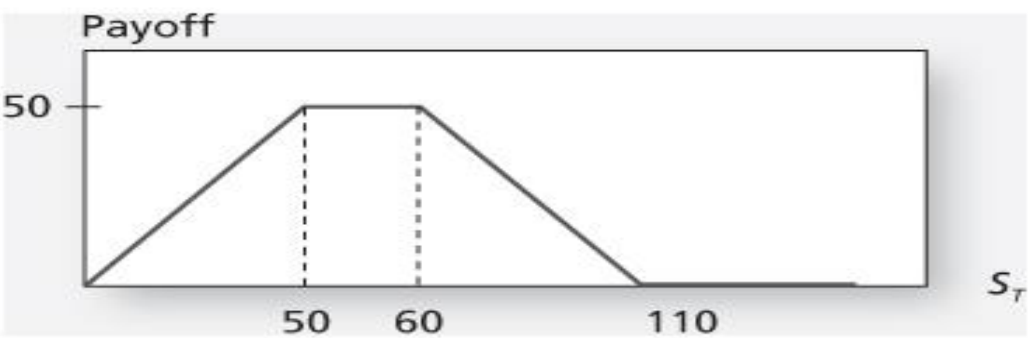
i)N = b (P/F) = 0.92*(50 million/1422)
ii) reduce total variance to {1 - row^2} of its previous value
[since sigma of the portfolio = sigma of the futures contract, row = beta]= 1-0.92^2.

d) Write a brief summary of the advantages and disadvantages of using the futures market to minimise exposure to the market relative to:

a. Simply liquidating the portfolio when the value drops by more than 5%.
b. Buying six month put options with a strike of 1351 to protect the portfolio

[20%]

e) Devise a portfolio using only call options and shares of stock with the following value (payoff) at the option expiration date. If the stock price is currently 53, what kind of bet is the investor making? **[30%]**



Buy a share of stock, write a call with X = \$50, write a call with X = \$60, and buy a call with X = \$110.

Position	**$S_T < 50$**	**$50 \le S_T \le 60$**	**$60 < S_T \le 110$**	**$S_T > 110$**
Buy stock	**S_T**	**S_T**	**S_T**	**S_T**
Write call, X = \$50	**0**	**$-(S_T - 50)$**	**$-(S_T - 50)$**	**$-(S_T - 50)$**
Write call, X = \$60	**0**	**0**	**$-(S_T - 60)$**	**$-(S_T - 60)$**
Buy call, X = \$110	**0**	**0**	**0**	**$S_T - 110$**

Total	**S_T**	**50**	**$110 - S_T$**	**0**

The investor is making a volatility bet. Profits will be highest when volatility is low and the stock price S_T is between \$50 and \$60.