QUESTION 1

a. Define what is meant by the beta of a share. **[5%]**

b. Do betas of a share estimated using intraday, daily and monthly returns generally differ? Explain briefly. **[15%]**

c. Explain the assumptions and implications of the "One Fund Separation" theorem. **[20%]**

a) You have been researching a AAA (ie very low risk of default) 10-year corporate bond which you believe is trading cheaply. You expect the bond to have a return of 7.5% p.a., while the comparable 10-year Treasury bond has an expected return of 6%. The annual volatility of the corporate bond is 6.5% and that of the Treasury bond is 6%; the correlation between them is practically 0.95. You can borrow or lend short term at the Treasury bill rate of 5.4%.

(i) Supposing that you want to construct a levered portfolio consisting of the corporate bond and the treasury bond, design the portfolio that has an expected return of 40% and minimum risk. **[40%]**

(ii) What is the volatility of the optimal portfolio? **[10%]**

(iii) If the correlation between the corporate and treasury bonds turns out to be 0.8 rather than the 0.95 you had assumed, would you expect the volatility of the portfolio to be different from your answer in (ii)? Explain briefly. **[10%]**

Solution:

If you invest a fraction c of your portfolio in the corporate bond, and a fraction t in the Treasury bond the expected return on the portfolio is:

$E[r_p] = 7.5\% \times c + 6\% \times t + 5.4\% \times (1 - t - c) = (5.4 + 2.1c + 0.6t)\%$

Using the standard formula for the variance of a portfolio, and noting that the risk of T-bills is zero, the variance of the portfolio is:

$Var[r_p] = (6.5\%)^2c^2 + 2 \times 0.95 \times 6.5\% \times 6\% \times ct + (6\%)^2t^2$

$= (42.25c^2 + 74.1ct + 36t^2)\%\%$

To get a return of 40%, need:

$E[r_p] = 40\%$ so $5.4 + 2.1c + 0.6t = 40$, and

$t = (40 - 5.4 - 2.1c)/0.6 = 57.667 - 3.5c$.

Substituting for *t* in the variance gives a variance (times 10000) of:

$\text{Var}[r_p] = 42.25c^2 + 74.1c(57.667 - 3.5c) + 36(57.667 - 3.5c)^2$

$= 223.9c^2 - 10259c + 119720$

Differentiating, this is minimised when $c = 10259/(2\text{x}223.9) = 22.91$.

So $t = 57.667 - 3.5c = -22.52$. Thus for every £1m of initial capital, the portfolio goes long £22.9m of the corporate bond, short £22.5m of the matching ten year bond, and puts £0.6m in cash.

(ii) What is the volatility of the portfolio?

Substituting back, this gives

$\text{Var}[r_p] = 223.9(22.91)^2 - 10259(22.91) + 119720\%\%$

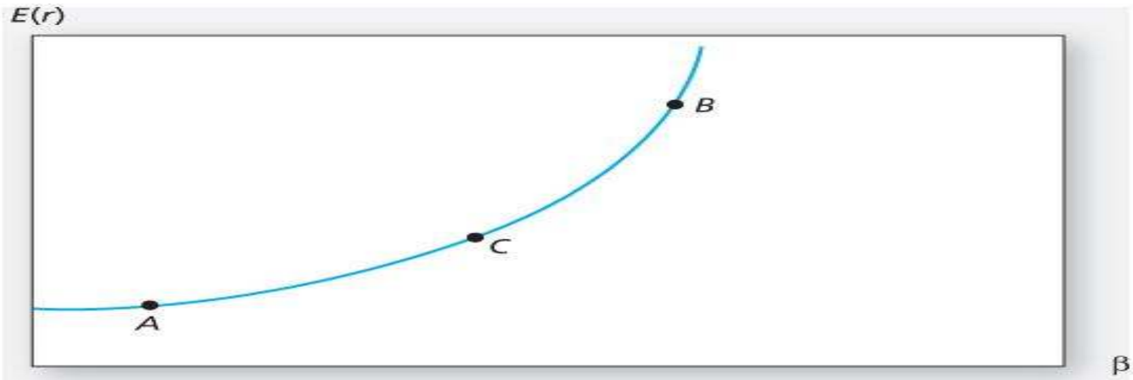
so the standard deviation - the square root of the variance – is 47%. That means that the standard deviation of annual gains or losses relative to the mean is £0.47m on every £1m of capital invested.

(iii) You set up the strategy. The correlation turns out to be 0.8 rather than the 0.95 you had assumed. What would you expect the volatility of the portfolio to be?

Substituting into the formula for volatility gives a portfolio volatility of 90.8%.

QUESTION 2

a) Suppose the relation between expected return and beta is as shown below:



i) According to CAPM, is there an arbitrage opportunity? Justify your answer. **[15%]**

ii) Some researchers have examined the relationship between average returns on diversified portfolios and the β and β^2 of those portfolios. What should they have discovered about the effect of β^2 on portfolio return?

[10%]

i) A long position in a portfolio (P) comprised of Portfolios A and B will offer an expected return-beta tradeoff lying on a straight line between points A and B. Therefore, we can choose weights such that $\beta_P = \beta_C$ but with expected return higher than that of Portfolio C. Hence, combining P with a short position in C will create an arbitrage portfolio with zero investment, zero beta, and positive rate of return.

ii) The argument in part (a) leads to the proposition that the coefficient of β^2 must be zero in order to preclude arbitrage opportunities.

b) Discuss the empirical performance of CAPM (as presented in Fama and French, 2004). Ensure to also explain the main reasons for such a performance.

[20%]

c) Assume that the stock market returns have the market index as a common factor, and that all stocks in the economy have a beta of 1 on the market index. Firm-specific returns all have a standard deviation of 30%. Suppose that an analyst studies 20 stocks, and finds that one-half have an alpha of 2%, and the other half an alpha of -2%. Suppose the analyst buys \$1 million of an equally weighted portfolio of positive alphas, and shorts \$1 million of an equally weighted portfolio of the negative alpha stocks.

i) What is the expected profit (in dollars) and standard deviation of the analyst's profit?

[20%]

ii) How does your answer change if the analyst examines 100 stocks instead of 20 stocks?

[15%]

Answer:

i) Shorting an equally-weighted portfolio of the ten negative-alpha stocks and investing the proceeds in an equally-weighted portfolio of the ten positive-alpha stocks eliminates the market exposure and creates a zero-investment portfolio. Denoting the systematic market factor as RM , the expected dollar return is (noting that the expectation of non-systematic risk, e, is zero):

$\$1,000,000 \times [0.02 + (1.0 \times RM)] - \$1,000,000 \times [(-0.02) + (1.0 \times RM)]$

$= \$1,000,000 \times 0.04 = \$40,000$

The sensitivity of the payoff of this portfolio to the market factor is zero because the exposures of the positive alpha and negative alpha stocks cancel out. Thus, the systematic component of total risk is also zero. The variance of the analyst's profit is not zero, however, since this portfolio is not well diversified.

For n = 20 stocks (i.e., long 10 stocks and short 10 stocks) the investor will have a $100,000 position (either long or short) in each stock. Net market exposure is zero, but firm-specific risk has not been fully diversified. The variance of dollar returns from the positions in the 20 stocks is:

$20 \times [(100{,}000 \times 0.30)^2] = 18{,}000{,}000{,}000$

The standard deviation of dollar returns is $134,164.

ii) if n = 100 stocks (50 stocks long and 50 stocks short), the investor will have a $20,000 position in each stock, and the variance of dollar returns is:

100 × [(20,000 × 0.30)2] = 3,600,000,000

The standard deviation of dollar returns is $60,000.

Notice that, when the number of stocks increases by a factor of 5 (i.e., from 20 to 100), standard deviation decreases by a factor of = 2.23607 (from $134,164 to $60,000).

d) Describe the Fama-French 3 factor model. Ensure to explain the risk factors and the factor loadings associated with the model.

[20%]

QUESTION 3

a) Explain how the different 'limits to arbitrage' would affect your interpretation of the empirical evidence relating to market efficiency. **[10%]**

b) Explain Synchronization Risk. Ensure to discuss how evidence presented in Brunnermeier and Nagel (2004) relate to it. **[15%]**

c) Explain the Grossman-Stiglitz paradox. **[10%]**

d) A portfolio *P* is invested in two assets *A* and *B*, with the remainder being invested in the market *M*. The diversifiable components of *A* and *B* are uncorrelated with each other. Over the last year, you have the following statistics, where *T* denotes T-bills in which the portfolio was not invested:

	A	*B*	*M*	*T*
Return	8.8%	9.6%	9.0%	4.4%
Beta	0.90	1.10	1.00	
Standard Deviation	22.0%	25.4%	20.0%	
Portfolio weight	14.0%	7.5%	78.5%	

i. compute the Jensen measure for each asset and for the portfolio as a whole **[5%]**

ii. Calculate the idiosyncratic standard deviation for both *A* and *B*. **[10%]**

iii. Calculate the portfolio's beta, total standard deviation and idiosyncratic standard deviation. **[15%]**

iv. Calculate the Sharpe ratio of the portfolio and of the market. **[10%]**

	A	*B*	*M*	*P*	*T*
Return	8.8%	9.6%	9.0%	9.02%	4.4%
Beta	0.90	1.10	1.00	0.99	
Standard Deviation	22.0%	25.4%	20.0%	20.0%	
Systematic risk	18.0%	22.0%	20.0%	19.9%	
Idiosyncratic risk	12.6%	12.7%	0.0%	2.0%	
Portfolio weight	14.0%	7.5%	78.5%		
Jensen	0.26%	0.14%	0.00%	0.05%	
Sharpe	0.200	0.205	0.230	0.231	

i. The portfolio return and beta are simply the weighted average of its components. The Jensen measure can then be computed.

ii. To get the idiosyncratic risk of *A* and *B*, first compute their market risk (beta times the standard deviation of the market) and then compute the idiosyncratic risk by noting that market variance plus idiosyncratic variance equals total variance.

iii. The portfolio's beta has already been calculated. Its market risk is its beta times the standard deviation of the market. Its idiosyncratic variance is the weighted sum of the idiosyncratic variances of its components.

iv. The Jensen measures of both shares are positive, showing that they would improve portfolio performance if added to a holding of the market portfolio. They also have Sharpe measures that are lower than the market showing that neither held alone would be better than the market. The relatively small exposure to A and B does improve the Sharpe ratio of the portfolio. In fact, use of solver shows that this the portfolio that has the maximum possible Sharpe ratio.

e) An equity fund has £100m under management at the beginning of the year. The following table shows the cash inflows (positive) and cash outflows (negative) that occur at the end of each of the two years.

	Year 1	Year 2
Cash Flow (£m)	20	0
Fund value *after* addition of cash flow (£m)	135	120

Calculate the time-weighted and value-weighted rates of return on the fund. Under what circumstances would each measure be an appropriate estimate of an average rate of return?

[15%]

Answer:

	Year 1	Year 2	
Cash Flow (£m)	20	0	
Fund value after addition of cash flow (£m)	135	120	
Annual Returns	15.0%	-11.1%	
	Portfolio		
Time weighted return	1.1%		
Cash flow	100	20	-120
Value weighted return	0.0%		

f) Explain market-timing of portfolio returns. How would you test for it?

[10%]

QUESTION 4

a) What is the difference between the duration of a bond and its maturity? Give an example of a bond that has a very short duration and a very long maturity. **[10%]**

b) Explain the difference between a bond's dirty and clean price **[10%]**

c) Prices, coupon rates (paid annually) and maturities of three comparable Treasury bonds, as of May 16, 2018, are listed below:

Bond	*Price($)*
6% of 05/15/2019	102.00
6% of 05/15/2020	100.00
10% of 05/15/2021	105.00

i. Calculate all spot rates implicit in these prices. **[15%]**

ii. What is the expected one-year spot rate after one year assuming that the expectations theory of term structure is correct? **[10%]**

iii. Based on the spot rates you have calculated, is there an arbitrage opportunity if a 12% coupon (paid annually) bond maturing in 3 years is priced at $115? Explain. **[15%]**

Answer:

i)

0.5- year bond:

$$102 = \frac{106}{1 + R_{0,0,1}}$$

$$\Rightarrow R_{0,0,1} = 3.92\%\text{(Annualized)}$$

1- year Bond:

$$100 = \frac{6}{1 + R_{0,0,1}} + \frac{106}{(1 + R_{0,0,2})^2}$$

$$\text{We know } R_{0,0,1} \text{ from (a)}$$

$$\Rightarrow R_{0,0,2} = 6.06\%$$

1.5 - year bond:

$$105 = \frac{10}{1 + R_{0,0,1}} + \frac{10}{(1 + R_{0,0,2})^2} + \frac{110}{(1 + R_{0,0,3})^3}$$

$$\text{We know } R_{0,0,1} \text{ and } R_{0,0,2} \text{ from (a) and (b)}$$

$$\Rightarrow R_{0,0,3} = 8.35\%$$

Next calculate all the relevant forward rates.

ii) Expected one-year spot rate after one year = forward rate between year 1 and 2

iii)
$$\text{Price} = \frac{12}{(1 + R_{0,0,1})} + \frac{12}{(1 + R_{0,0,2})^2} + \frac{112}{(1 + R_{0,0,3})^3}$$

$$\Rightarrow Price = 110.27$$

d) A 12.75-year maturity zero-coupon bond selling at an YTM of 8% has convexity of 150.3 and modified duration of 11.81 years. A 30-year maturity 6% coupon bond making annual coupon payments also selling at an YTM of 8% has nearly identical duration – 11.79 – but considerably higher convexity of 231.2.

   i. Suppose YTM on both bonds increases to 9%. What will be actual percentage capital loss on each bond? What percentage capital loss would be predicted (using duration and convexity)? **[15%]**

   ii. Repeat part (a), but this time assume YTM decreases to 7%. **[15%]**

   iii. Do you think it would be possible for two bonds to be priced initially at the same YTM? Explain. **[10%]**

Answer:

i. The price of the zero coupon bond ($1,000 face value) selling at a yield to maturity of 8% is $374.84 and the price of the coupon bond is $774.84

At a YTM of 9% the actual price of the zero coupon bond is $333.28 and the actual price of the coupon bond is $691.79

<u>Zero coupon bond:</u>

$$\text{Actual \% loss} = \frac{\$333.28 - \$374.84}{\$374.84} = -0.1109 = 11.09\% \text{ loss}$$

The percentage loss predicted by the duration-with-convexity rule is:

$$\text{Predicted \% loss} = [(-11.81) \times 0.01] + [0.5 \times 150.3 \times 0.01^2] = -0.1106 = 11.06\% \text{ loss}$$

<u>Coupon bond:</u>

$$\text{Actual \% loss} = \frac{\$691.79 - \$774.84}{\$774.84} = -0.1072 = 10.72\% \text{ loss}$$

The percentage loss predicted by the duration-with-convexity rule is:

$$\text{Predicted \% loss} = [(-11.79) \times 0.01] + [0.5 \times 231.2 \times 0.01^2] = -0.1063 = 10.63\% \text{ loss}$$

ii. Now assume yield to maturity falls to 7%. The price of the zero increases to $422.04, and the price of the coupon bond increases to $875.91

<u>Zero coupon bond:</u>

$$\text{Actual \% gain} = \frac{\$422.04 - \$374.84}{\$374.84} = 0.1259 = 12.59\% \text{ gain}$$

The percentage gain predicted by the duration-with-convexity rule is:

Predicted % gain $= [(-11.81)\times(-0.01)] + [0.5\times 150.3\times 0.01^2] = 0.1256 = 12.56\%$ gain

<u>Coupon bond</u>

Actual % gain $= \dfrac{\$875.91 - \$774.84}{\$774.84} = 0.1304 = 13.04\%$ gain

The percentage gain predicted by the duration-with-convexity rule is:

Predicted % gain $= [(-11.79)\times(-0.01)] + [0.5\times 231.2\times 0.01^2] = 0.1295 = 12.95\%$ gain

iii. The 6% coupon bond, which has higher convexity, outperforms the zero regardless of whether rates rise or fall. This can be seen to be a general property using the duration-with-convexity formula: the duration effects on the two bonds due to any change in rates are equal (since the respective durations are virtually equal), but the convexity effect, which is always positive, always favours the higher convexity bond. Thus, if the yields on the bonds change by equal amounts, as we assumed in this example, the higher convexity bond outperforms a lower convexity bond with the same duration and initial yield to maturity. Hence, this situation cannot persist. No one would be willing to buy the lower convexity bond if it always underperforms the other bond.

<u>QUESTION 5</u>

a) Explain the relation between convenience yields and the term structure of crude-oil futures prices. How is it related to "rollover risk" in the futures market, as illustrated in the case of Metallgesellschaft AG? **[30%]**

b) Suppose the 1-year futures price on a stock-index is 1,218, the stock index is currently is 1,200, the 1-year risk-free interest is 3%, and the year-end dividend that will be paid on a 1,200 investment in the market index portfolio is $15.

i) What should be the price of the contract? **[5%]**

ii) Formulate a zero-net-investment arbitrage portfolio and show that you can lock in riskless profits equal to the futures mispricing. **[10%]**

iii) Now assume that if you short sell the stocks in the market index, the proceeds of the short sale are kept with the broker, and you do not receive any interest income on the funds. Is there still an arbitrage opportunity? Explain. **[10%]**

iv) Given the short-sale rules, what is the no-arbitrage band for the stock-futures price relationship? **[20%]**

Answer:

i) From parity: $F_0 = 1{,}200 \times (1 + 0.03) - 15 = 1{,}221$

Actual F_0 is 1,218; so the futures price is 3 below the "proper" level.

ii) Buy the relatively cheap futures, sell the relatively expensive stock and lend the proceeds of the short sale:

	CF Now	CF in 1 year
Buy futures	0	$S_T - 1{,}218$
Sell shares	1,200	$-S_T - 15$
Lend $1,200	−1,200	1,236
Total	0	3

iii) If you do not receive interest on the proceeds of the short sales, then the $1200 you receive will not be invested but will simply be returned to you. The proceeds from the strategy in part (b) are now negative: an arbitrage opportunity no longer exists.

	CF Now	CF in 1 year
Buy futures	0	$S_T - 1{,}218$
Sell shares	1,200	$-S_T - 15$
Place $1,200 in margin account	−1,200	1,200
Total	0	−33

iv) If we call the original futures price F_0, then the proceeds from the long-futures, short-stock strategy are:

	CF Now	CF 1 year
Buy futures	0	$S_T - F_0$
Sell shares	1,200	$-S_T - 15$
Place $1,200 in margin account	−1,200	1,200
Total	0	$1{,}185 - F_0$

F_0 can be as low as 1,185 without giving rise to an arbitrage opportunity.

On the other hand, if F_0 is higher than the parity value (1,221), then an arbitrage opportunity (buy stocks, sell futures) will exist.

	CF Now	CF 1 year
Sell futures	0	$F_0 - S_T$
Buy shares	−1,200	$S_T + 15$
Borrow $1,200	1,200	−1,236
Total	0	$F_0 - 1{,}221$

Therefore, the no-arbitrage range is:

$$1{,}185 \leq F_0 \leq 1{,}221$$

c) Explain the concept of "time diversification" as applied to dynamic investment strategies. **[15%]**

d) Discuss the difference between future and forward contracts in terms of their settlement. Explain how this difference could affect the risks and pricing of contracts. **[10%]**