AcF305:

International Financial and Risk Management

Week 6

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Outline of Lecture 6

- Essential reading: Chapter 8 of Sercu (2009).
- Topics:
  - Reminder: What are options? How are call and put options different?
  - What are institutional features of the options market?
  - What are important arbitrage relationships in the options market? How can we derive the put-call parity from these arbitrage relations?
  - How can we use options to hedge an exposure we have in a foreign currency? How can we use an option to speculate on the value of a FC?

Reminder of the Definition of Option Contracts

- So far studied: symmetric contracts, e.g. forwards or futures.
  - Example long forward: $\text{Value}_{FC} \uparrow (\downarrow) \leftrightarrow \text{Value}_{forward} \uparrow (\downarrow)$.

- Options have asymmetric payoffs, e.g. gain on the upside (e.g. when $\text{Value}_{FC} \uparrow$) & don't lose on the downside.

- Some definitions:
  - **Call** (**put**) option gives holder the "right" to **buy** (**sell**) a specified number of the underlying asset at a specified price at or before the maturity date.
  - At maturity → **European-style**; up until maturity → **American-style**.
  - Specified price: often referred to as strike price.
  - Underlying asset: stock, index, currency, interest rate, etc.

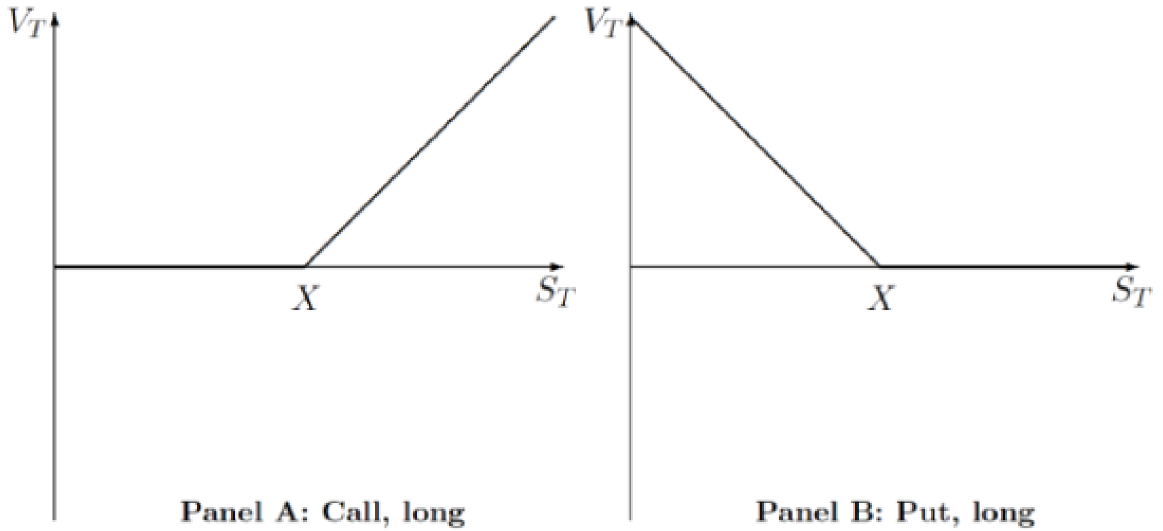
Reminder of the Definition of Option Contracts

- An agent will only exercise his right at maturity if this is beneficial
  - You only buy using the call option if this is cheaper than buying in the market (at the spot rate S_T)
  - You only sell using the put option if this gives you more money than selling in the market (at the spot rate S_T).

Payoff Diagrams of Long Call & Put Options

X=0.50

Figure 8.1: **Expiry Value of Calls and Puts, Long**



Panel A: Call, long **Panel B: Put, long**

If $S_T =$	0.45	0.46	0.47	0.48	0.49	0.50	0.51	0.52	0.53	0.54	0.55
then $C_T =$	0	0	0	0	0	0	0.01	0.02	0.03	0.04	0.05
and $P_T =$	0.05	0.04	0.03	0.02	0.01	0	0	0	0	0	0

$$\tilde{C}_T = \begin{cases} \tilde{S}_T - X & \text{if } \tilde{S}_T > X \\ 0 & \text{otherwise} \end{cases} = Max(\tilde{S}_T - X, 0) = (\tilde{S}_T - X)_+$$

$$\tilde{P}_T = \begin{cases} X - \tilde{S}_T & \text{if } X > \tilde{S}_T \\ 0 & \text{otherwise} \end{cases} = Max(X - \tilde{S}_T, 0) = (X - \tilde{S}_T)_+$$

Early Exercise & Option Premium

- American-style options can be exercised early (at $\tau < T$). When would a rational agent perform an early exercise?

  Condition 1 The immediate payoff (e.g. $S_\tau - X$ for call) must be positive → same as at maturity.

  Condition 2 The option value at τ (= market price) is lower than the immediate payoff – (ARBITRAGE)

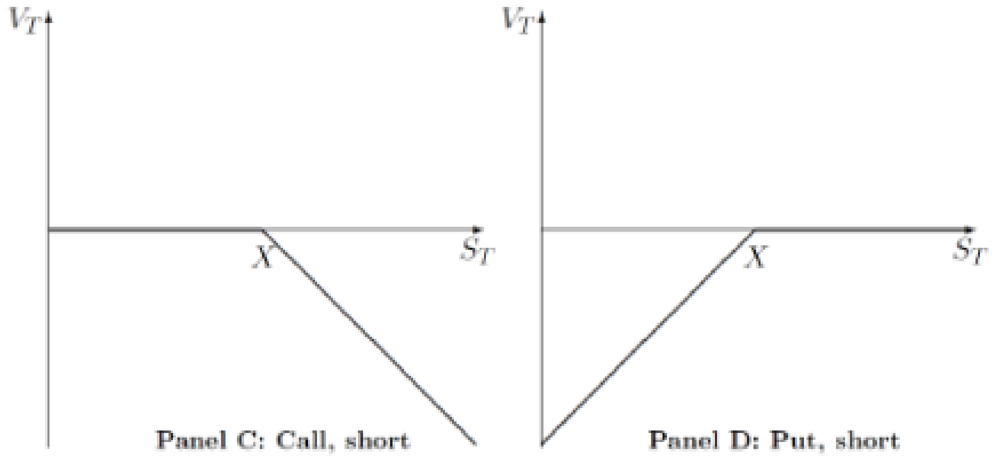
Early Exercise & Option Premium

- Purchasing an option is like taking out insurance.
  - *Example:* A knows he will have to buy CHF at T: he buys a call option to insure that the price is no higher than the strike.
  - *Example:* B knows she will have to sell CHF at T: she buys a put option to insure that the price is no lower than the strike.
- As insurance is costly, the option buyer needs to pay a **price (= premium)** to the option seller (= writer).

Option writers sell options in exchange of a premium from buyers.

Payoff Diagrams of Short Call & Put Options

Figure 8.2: **Expiry Value of Calls and Puts, Short**



- Combined with long positions, it is possible to create a forward's payoff.
  - Replicate a long forward contract with rate X: buy one call with strike price X and sell one put with strike price X.
  - Replicate a short forward contract with rate X: buy one put option with strike price X and sell one call with strike price X.
- → yields put-call parity (discussed later).

Jargon

- *Moneyness:* Relates to the immediate payoff of an option (even if it cannot be realized, as for European-style options).

   - **At-the-money**: spot price is equal to strike price.
   - **In-the-money**: immediate exercise generates a positive cash flow.
   - **Out-of-the-money**: immediate exercise generates a negative cash flow.

- **Intrinsic value**: reveals the immediate payoff of an option.

   - *Example call:* $(S_t - X)_+$.
   - Only difference to maturity payment: S_t instead of S_T.

- **Note**: Even deep out-of-the-money options will have a strictly positive value (= market price).

   - While early exercise has zero value, option could still become profitable later on.

Option value = Intrinsic value + Time value.

Institutional Features of Option Markets

- Options are available both in over-the-counter markets (OTC options) and on organized exchanges (traded options).
- **Traded options**:
  - Organized secondary market with clearing house as guarantor.
  - Standardized in many aspects: (1) expiration dates, (2) contract sizes and (3) exercise prices.
  - *Example USD/EUR options:*
    - Expiration dates: 1-3 months (all months); 3, 6 and 9 months (Mar, Jun, Sep, Dec); 3 years (Sep).
    - Contract size: USD 10,000.
- **OTC options**:
  - Sold by financial institutions, price quoted two-ways.
  - Like forward contracts: tailor-made.

Arbitrage Relationships for Options I

- *Assume:*
  1. American-style option premia: C_t^{Am} and P_t^{Am}
  2. European-style option premia: C_t and P_t

Relation 1: Option prices are non-negative

$$C_t \geq 0, C_t^{Am} \geq 0 \quad \text{and} \quad P_t \geq 0, P_t^{Am} \geq 0$$

- **Explanation**: The payoff of a degenerate option is nonnegative, so buy infinite amount if price is negative.

Relation 2: American options are worth no less than European options

$$C_t^{Am} \geq C_t \geq 0 \quad \text{and} \quad P_t^{Am} \geq P_t \geq 0$$

- **Explanation**: American option = European option + right to exercise before maturity.

Arbitrage Relationships for Options II

Relation 3: A European call is more valuable than a forward purchase

$$C_t \geq \underbrace{\frac{S_t}{1+r^*_{t,T}} - \frac{X}{1+r_{t,T}}}_{\text{forward value}}$$

- **Explanation**: at maturity, the payoffs are:
  - Call option: $Max(S_T - X, 0)$, where X = strike price.
  - Long forward: $S_T - X$, where X = forward rate.
- *Example:*
  - If $X = 43.785$, $S_t = 48$ and $r_{t,T} = r^*_{t,T} = 2\%$, then the value of forward with $X = 43.785$: $F_{t,T} = 48/(1.02) - 43.785/(1.02) = 4.13235$.
  - **Assume**: $C_t = 3.5$ (observed market price).
  - Arbitrage strategy: buy call and sell forward.

Relation 1, 2 and 3 (together):

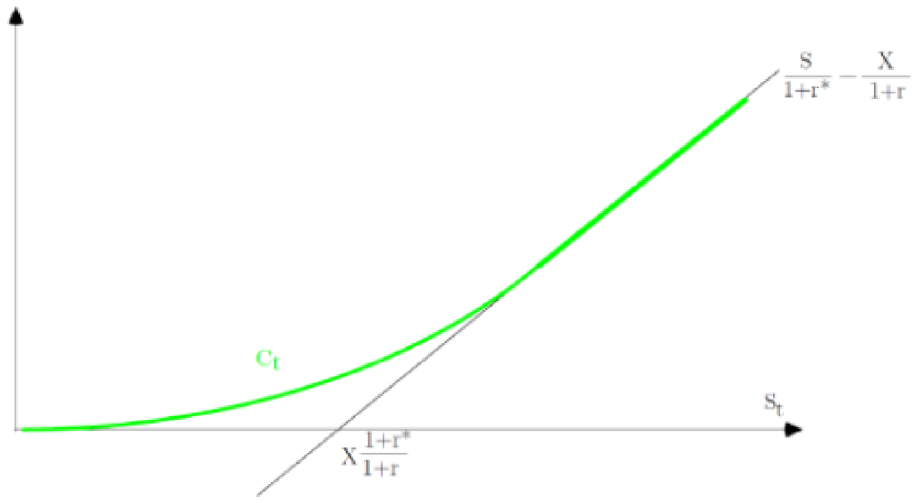
$$C_t^{Am} \geq C_t \geq Max\left(\frac{S_t}{1+r^*_{t,T}} - \frac{X}{1+r_{t,T}}, 0\right)$$

Arbitrage and the Value of a Call Contract

- **Question:** How do you make a sure profit if the relation below doesn't hold?

$$C_t \geq \frac{S_t}{1+r^*_{t,T}} - \frac{X}{1+r_{t,T}}$$

- **The 3 relations all together**:



Arbitrage Relationships for Options III

Relation 4: An American call is at least as valuable as its intrinsic value
$C_t^{Am} \geq Max(S_t - X, 0) \equiv$ intrinsic value

- **Explanation**: If at time t, $C_t^{Am} \leq Max(S_t - X, 0)$, an agent can exercise and immediately buy back the call; profit: $Max(S_t - X, 0) - C_t^{Am}$.
- Similar relations to (3) and (4) exist for put options:

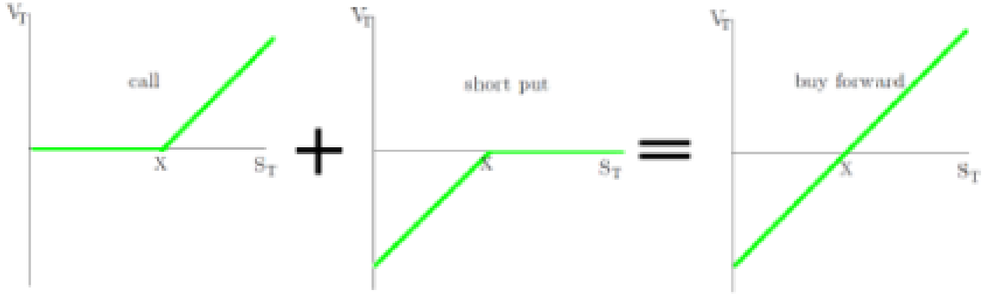
Relation 5 $\quad P_t \geq \dfrac{X}{1+r_{t,T}} - \dfrac{S_t}{1+r_{t,T}^{*}}$

Relation 6 $\quad P_t^{Am} \geq Max(X - S_t, 0) \equiv$ intrinsic value

Put-Call Parity for European Options

- The prices of European call and put options with the same strike price X are also related:
  - Buy one European call option & sell one European put option:
  - At maturity:

$$\underbrace{Max\left(\tilde{S}_T - X, 0\right)}_{\text{long call}} - \underbrace{Max\left(X - \tilde{S}_T, 0\right)}_{\text{short put}} = \begin{cases} \tilde{S}_T - X & \text{if } \tilde{S}_T > X \\ -\left(X - \tilde{S}_T\right) & \text{if } X > \tilde{S}_T \end{cases}$$



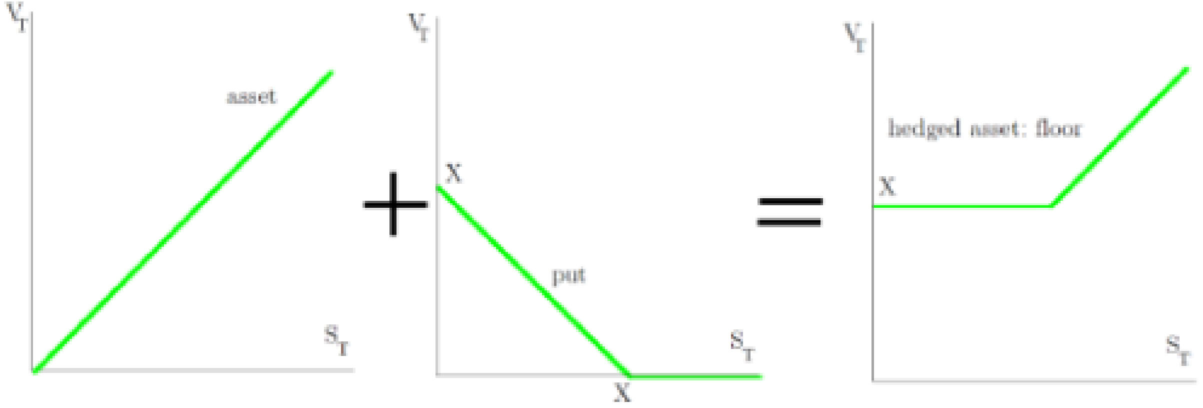
- **Arbitrage**: Instruments with the same payoff must have the same price. Law of one price

Relation 7: Put-call parity
$$C_t - P_t = \frac{S_t}{1 + r^*_{t,T}} - \frac{X}{1 + r_{t,T}}$$

Hedging Linear Exposures with Options

- The simplest example:
  - USCO has to sell one unit of FC at time T.
  - It hedges this exposure by buying one put option on one unit of FC with strike price HC 0.80.

at maturity: $\rightarrow \begin{cases} \text{use option to sell FC at 0.80} & \text{if } \tilde{S}_T < 0.80 \\ \text{let option expire and sell FC spot} & \text{if } 0.80 < \tilde{S}_T \end{cases}$



- Long forward contract instead: USCO always has to sell HC 0.80 for FC.
- A similar logic applies to hedging with call options (e.g. USCO has to buy one unit of FC at time T).

Hedging More Complicated Exposures with Options

- In the former example, the one unit of FC will be obtained with certainty.
- In reality, most cash inflows or outflows are risky, e.g.:
    1. International tenders (firms bidding for contracts).
    2. FC A/R with substantial default risk.
    3. Portfolio investments in a foreign country (with FC).
- Options can be used to hedge these cash flows *if they arise*.
- *Example:* USCO bids for a contract worth FC 1:

		Outcome of tender:	
Hedge strategy	$\tilde{S}_T$ outcome classes	Win the contract	Fail to win contract
sell forward	$\tilde{S}_T \geq 0.80$	$\tilde{S}_T + (0.80 - \tilde{S}_T) = 0.80$	$0 + (0.80 - \tilde{S}_T) \leq 0$
	$\tilde{S}_T < 0.80$	$\tilde{S}_T + (0.80 - \tilde{S}_T) = 0.80$	$0 + (0.80 - \tilde{S}_T) > 0$
buy a put	$\tilde{S}_T \geq 0.80$—don't exercise	$\tilde{S}_T + 0 \geq 0.80$	0
	$\tilde{S}_T < 0.80$—exercise	$\tilde{S}_T + (0.80 - \tilde{S}_T) = 0.80$	$0 + (0.80 - \tilde{S}_T) > 0$

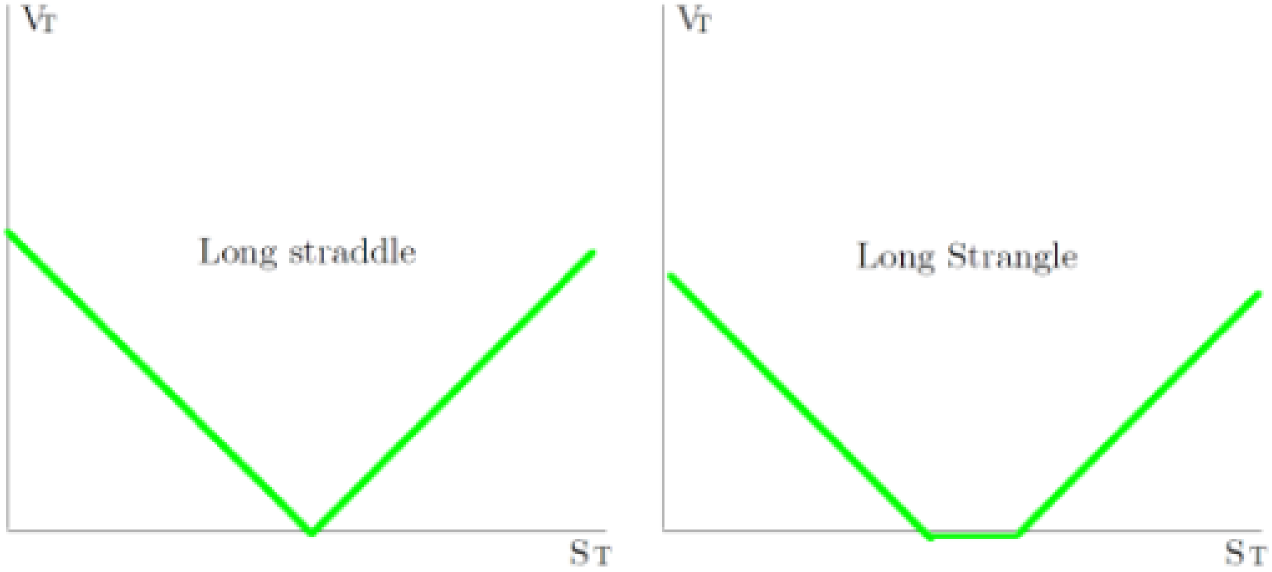
Using Options for Speculation on FX Changes

- Options can be used to speculate on changes in the exchange rate:
  - A is bullish about the future prospects of FC → he buys a call option on the FC (and hopes that it will end up in-the-money).
  - B is bearish about the future prospects of FC → she buys a put option on the FC (and hopes that it will end up in-the-money).

- Traders can also *sell* (instead of buy) options to speculate on changes in the exchange rate:
  - A is bullish about the future prospects of FC → he sells a put option on the FC (and hopes that it will expire unexercised).
  - B is bearish about the future prospects of FC → she sells a call option on the FC (and hopes that it will expire unexercised).

Using Options for Speculation on FX Volatility

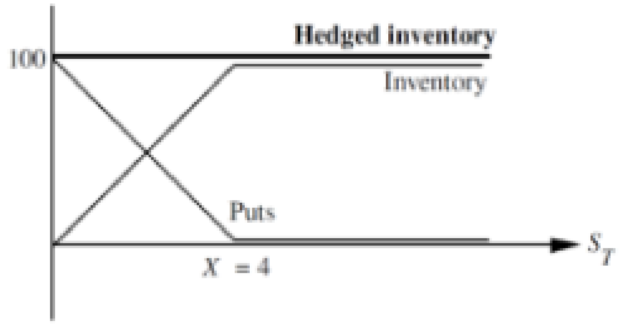
- In the former case, the speculator disagreed with the market about the expected value of the future exchange rate.
- In contrast to this, he can agree about the expected value of the exchange rate, but disagree about the volatility.
  - Essentially, the speculator believes that large movements in the exchange rate are more likely than the market believes.
- This can be exploited with a straddle or strangle strategy (buy call and put):



Hedging Non-Linear Exposures with Options

- In the former examples, the asset value was linear in the exchange rate.
  - When long FC 1, then value FC 1 increases in HC/FC.
- **BUT**: Competitive threats, price pressure or financial contracts can make the exposure non-linear:
  - *Example:* DanskWool sells wool in Denmark for DKK 100.
  - Australian competitors sell wool for AUD 25 in Australia, but enter the Danish market as soon as they can compete (in prices) with DanskWool.
  - Once Australians have entered the Danish market, DanskWool needs to match their price.

$\tilde{S}_T = ...$	3	3.2	3.4	3.6	3.8	4	4.2	4.4	4.6	4.8	5
Wool price	75	80	85	90	95	100	100	100	100	100	100
25 puts on AUD	25	20	15	10	5	0	0	0	0	0	0
wool + puts	100	100	100	100	100	100	100	100	100	100	100



  - Buying 25 puts at strike price DKK/AUD 4 hedges the exposure.

Summary, Homework and Additional Reading

- **In this lecture**, we dealt with:

  - A review of basic option concepts & a reminder of option jargon.
  - The institutional features of the options market.
  - Some important arbitrage relationships, their derivation and the concept of put-call parity.
  - The use of options for hedging or even speculation.

- **At home**, you will need to cover:
  - Carefully read the chapter from the book.