AcF305:
International Financial and Risk Management
Week 8

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Outline of Lecture 8

- Essential reading: Chapter 19 of Sercu (2009).
- Topics: Cost of international capital – InCAPM
  - Can we use local NPV techniques to determine the value of a foreign investment project? Why is it important to find out whether home country and foreign country are integrated?
  - What is the capital asset pricing model (CAPM)? How is the model derived? What is its intuition? How is the market portfolio related to the CAPM?
  - Why do we need an international CAPM? How can we implement this model to derive a firm’s cost of capital?

Value of an International Project

- In previous courses, you should have learned how to determine the NPV of a local (= produces HC cash flows) investment project.
  - *Rule:* Discount expected cash flows at the owners' opportunity cost of capital.
- Contrast this with a foreign project, generating FC cash flows: How would we determine its value? *A priori*, there are two options:
  1. Translate expected FC cash flows into HC using the expected spot rate, i.e. $E(\tilde{C}_T^* * \tilde{S}_T) = E(\tilde{C}_T)$, and discount at local discount rate. But remember $E(\tilde{C}_T^* * \tilde{S}_T) \neq E(\tilde{C}_T^*) * E(\tilde{S}_T)$
  2. Discount expected FC cash flows at foreign discount rate and translate FC value into HC at spot rate prevailing today.

What is the cost of capital?

- It is the opportunity cost from investing in a project. In other words, it establishes what would be the interest that one could obtain from investing in a similar project with equal risk

- If markets are integrated (e.g. no trading frictions and barriers), then the cost of capital no longer follows the standard CAPM you know from prior courses

Example of a NPV Calculation

- Assume you can invest into a local project, with:
    1. Initial investment equal to 10,000.
    2. Expected cash flows of 5,000 over the next four years.
- The NPV is:

$$-10{,}000+\frac{5{,}000}{(1+r)}+\frac{5{,}000}{(1+r)^2}+\frac{5{,}000}{(1+r)^3}+\frac{5{,}000}{(1+r)^4}$$

- Assume that r = 10%, then NPV=5,849.32
- *r is* the (opportunity) cost of capital

International valuation

- To compute the expected cost of capital or expected return on an asset we need to know its exposure to risk factors: market and currency factors.

- If investors can hold foreign assets, it is no longer acceptable to use a CAPM-equation with its benchmark portfolio of the local stock index

  – The local index ignores foreign assets, which could well be good investment opportunities
  – Local assets, are owned by foreigners

International valuation

- When investors put their money away from their home countries in international projects, they are exposed to exchange rate risk

- What is the appropriate benchmark

  - Foreign market portfolio?
  - Local market portfolio?
  - A combination of both?

- How should the expected rate of return be adjusted to account for exchange rate risk?

International valuation III

- In summary, feasible approaches to determine the value of an international project can be determined through the following tree:

markets integrated? =
- YES: use approach (1) or (2) → apply InCAPM
- NO: **must** use approach (1) → apply local CAPM

- Why can capital budgeting be done from the perspective of a local or foreign investor when markets are integrated?
  - In an integrated market, investors from different countries use the same cost of capital (once prices and cashflows are transformed into the same currency). Otherwise, arbitrage opportunities would exist.

- Therefore, in an integrated market, home country investors and host country investors will agree in the value of the project => Same price and same cost of capital

Segmented Markets

- When markets are segmented, the foreign discount rates are different from local discount rates; we must first translate and then discount:

$$\underbrace{E(\tilde{C}^*)}_{\text{expectation in FC}} \longrightarrow \underbrace{E(\tilde{C}^*\tilde{S})}_{\text{expectation in HC}} \longrightarrow \text{discount } E(\tilde{C}^*\tilde{S})$$

  with, **importantly**: $E(\tilde{C}^*\tilde{S}) = E(\tilde{C}^*)E(\tilde{S}) + \text{cov}(\tilde{C}^*, \tilde{S})$

- *Example:*

	State of the economy			
	Boom: C^*=150	Slump: C^*=100	Prob(S)	E($\tilde{C}$|S)
S_T=1.2	p = 0.15; C=180	p=0.35; C=120	0.50	138
S_T=0.8	p = 0.35; C=120	p= 0.15; C= 80	0.50	108
Prob(C^*)	p = 0.50	p = 0.50		

$$E(\tilde{S}) = (0.50 * 1.2) + (0.50 * 0.8) = 1.00$$
$$E(\tilde{C}^*) = (0.50 * 150) + (0.50 * 100) = 125$$

  yet, even though $E(\tilde{S}) * E(\tilde{C}^*)$ = 125, we can see that:

$$E(\tilde{S}\tilde{C}^*) = (0.15 * 180) + (0.35 * 120) + (0.35 * 120) + (0.15 * 80) = 123$$

- Then discount these expectations using the single-country CAPM.

Assumptions and Essentials of the CAPM

- In a segmented market, the value of an international project will be determined by the single-country CAPM.
- Some assumptions and essentials of this model are:
  - Investors rank portfolios based on expected return and variance.
  - The return on a portfolio can be written as:

    $$\tilde{r}_p - r_0 = \sum_{j=1}^{N} x_j(\tilde{r}_j - r_0)$$

    where r_j , r_p and r_0 are the nominal return of stock j, portfolio p and the risk-free asset and x_j is the weight invested into stock j.
  - A combination of risk-free rate and stock will have:

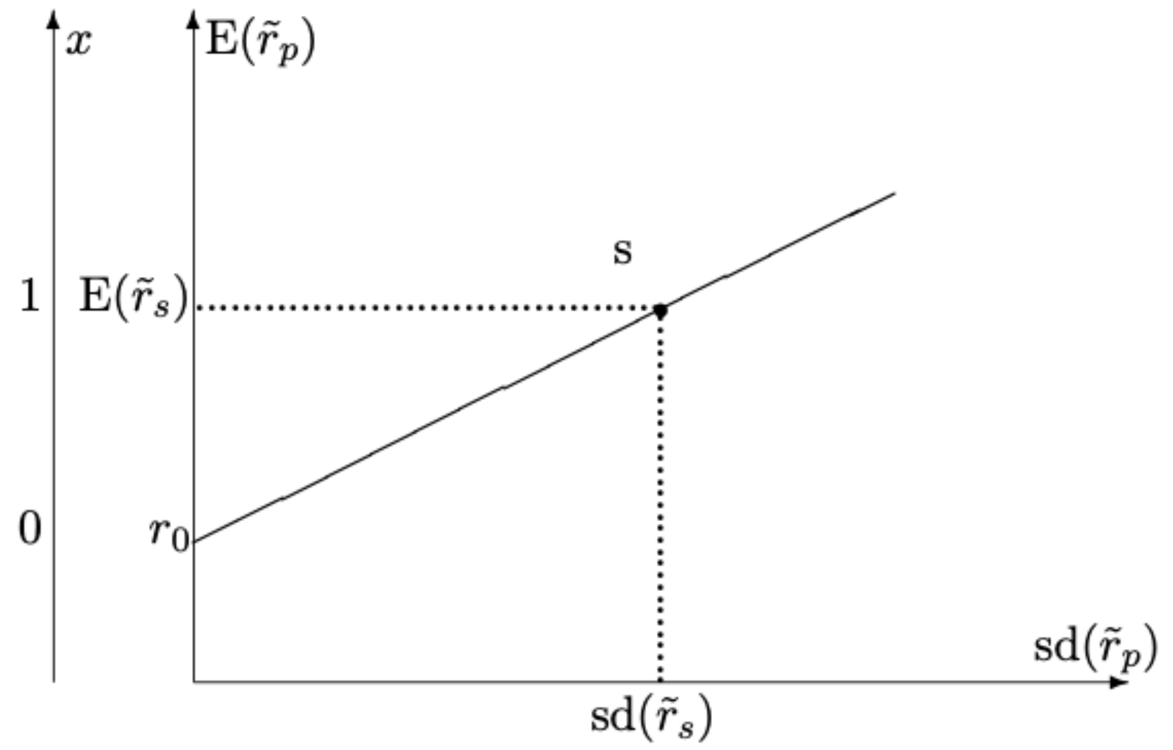
    $$\tilde{r}_p = x\tilde{r}_s + (1-x)r_0 = r_0 + x(\tilde{r}_s - r_0) \rightarrow \begin{cases} E(\tilde{r}_p) = r_0 + xE[\tilde{r}_s - r_0], \\ sd(\tilde{r}_p) = |x|\, sd(\tilde{r}_s) \end{cases}$$

  - ... while that of two stocks (with $x_2 = 1 - x_1$) implies:

    $$\tilde{r}_p = x_1\tilde{r}_1 + (1-x_1)\tilde{r}_2 \rightarrow \begin{cases} E(\tilde{r}_p) = E(\tilde{r}_1) + x_1[E(\tilde{r}_1) - E(\tilde{r}_2)], \\ sd(\tilde{r}_p) = (x_1^2 \operatorname{var}(\tilde{r}_1) + 2x_1x_2 \operatorname{cov}(\tilde{r}_1, \tilde{r}_2) + x_2^2 \operatorname{var}(\tilde{r}_2))^{0.5} \end{cases}$$

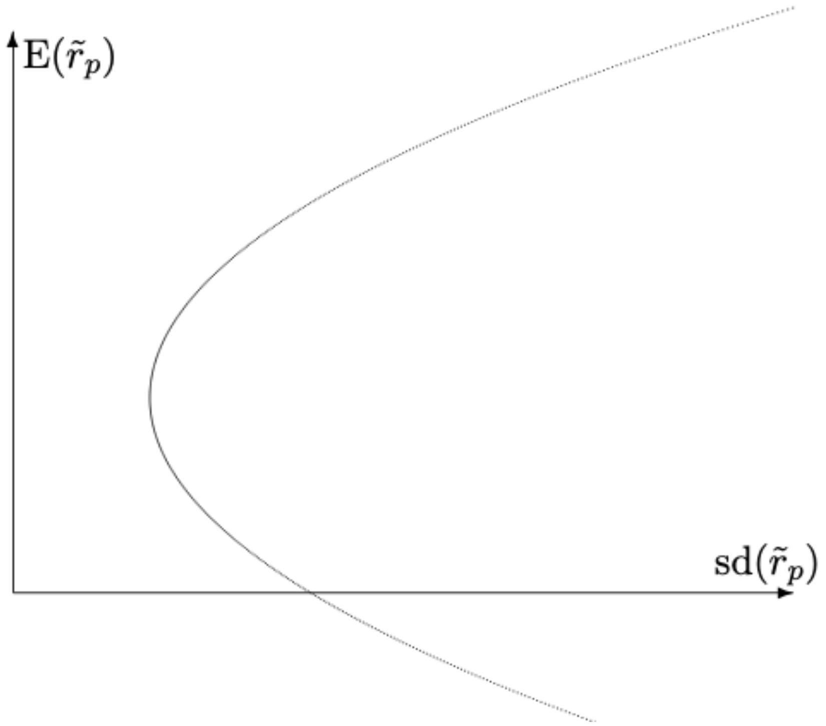
Relation between Risk-free Asset and Stocks

Figure 19.1: **Combinations of risky stock portfolio s and asset 0**



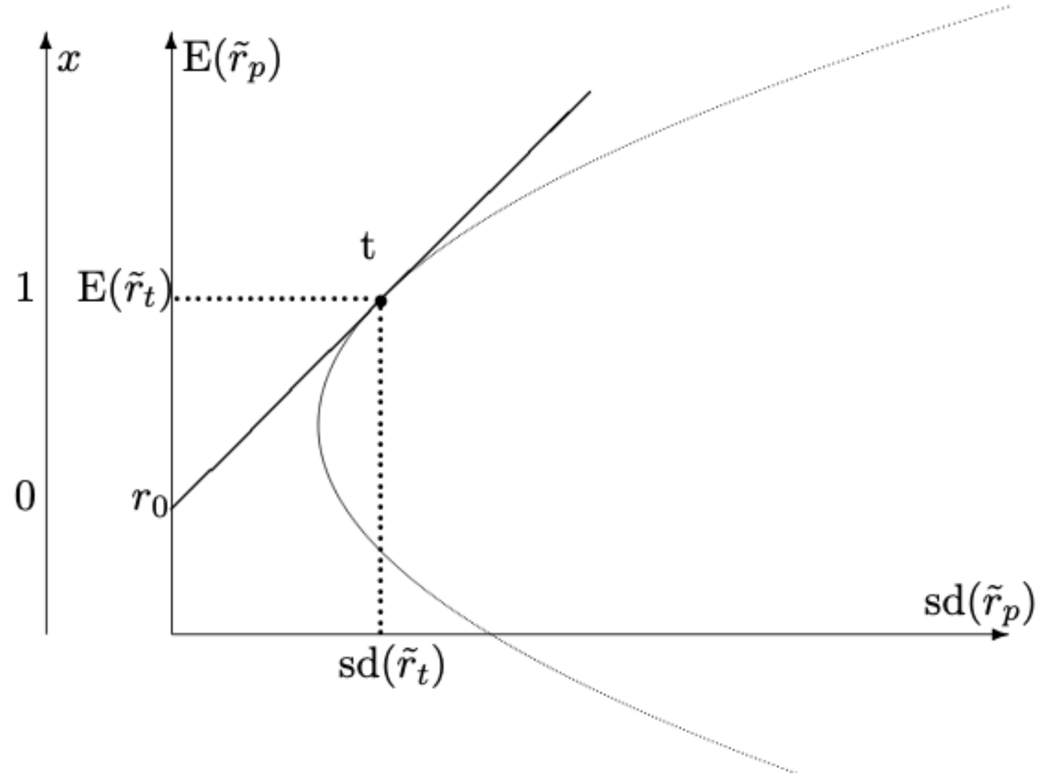
Relation between Risk-free Asset and Stocks

Figure 19.2: **The risk-return bound with just risky assets**



Relation between Risk-free Asset and Stocks

Figure 19.3: **Efficient Portfolios & the Tangency Portfolio**



Forming an Optimal (Efficient) Portfolio

- An investor wants to form an optimal portfolio, i.e. he wants to maximize expected return and minimize variance.
  - **Expected portfolio return:** $E(\tilde{r}_p) = r_0 + \sum_{j=1}^{N} x_j E(\tilde{r}_j - r_0)$
  - **Portfolio variance:** $\mathrm{var}(\tilde{r}_p) = \sum_{j=1}^{N} x_j \sum_{k=1}^{N} x_k \,\mathrm{cov}(\tilde{r}_j, \tilde{r}_k)$

- To this end, the investor must determine the extra benefit (= $E(\tilde{r}_p)$) and extra cost (= $\mathrm{var}(\tilde{r}_p)$) of investing *just a little bit more* into stock *j*.

Forming an Optimal (Efficient) Portfolio

- *Example with two stocks:*

$$\left\{\begin{array}{ll} \text{extra benefit:} & \dfrac{\partial E(\tilde{r}_p - r_0)}{\partial x_k} = E(\tilde{r}_k - r_0) \\ \text{extra cost:} & \dfrac{\partial \operatorname{var}(\tilde{r}_p)}{\partial x_k} \propto \operatorname{cov}(\tilde{r}_k, \tilde{r}_p) \end{array}\right\} \longleftarrow \text{taking partial derivatives}$$

- **Rule**: In an optimal portfolio, the extra benefit-over-extra cost ratio must be equal across all stocks.

  - *In practice:* Investors can identify an optimal portfolio from computing the ratio of all stock's expected excess return over their covariance with the portfolio; if all ratios are equal: portfolio is optimal.

Optimal Portfolios & Investors' Risk Aversion

- More rigorously:

Identification of an Optimal Portfolio:
$$\frac{E(\tilde{r}_j - r_0)}{\text{cov}(\tilde{r}_j, \tilde{r}_p)} = \lambda, \text{ for all risky assets } j = 1, 2, 3, \ldots, N$$

where λ equals an investor's relative risk aversion.

Optimal Portfolios & Investors' Risk Aversion

- *Example:* Assume the following data are known:

	$E(\tilde{r}_j - r)$	(co)variances	
Asset 1	0.092	$\mathrm{cov}(\tilde{r}_1, \tilde{r}_1) = 0.04$	$\mathrm{cov}(\tilde{r}_1, \tilde{r}_2) = 0.05$
Asset 2	0.148	$\mathrm{cov}(\tilde{r}_2, \tilde{r}_1) = 0.05$	$\mathrm{cov}(\tilde{r}_2, \tilde{r}_2) = 0.09$

- Check whether the portfolio x_1 = 0.40 and x_2 = 0.60 is efficient via the following steps:
  1. Compute each stock's covariance with the portfolio; use the formula:

     $$\mathrm{cov}(\tilde{r}_j,\tilde{r}_p) = \mathrm{cov}(\tilde{r}_j, x_1\tilde{r}_1 + x_2\tilde{r}_2) = x_1\,\mathrm{cov}(\tilde{r}_j,\tilde{r}_1) + x_2\,\mathrm{cov}(\tilde{r}_j,\tilde{r}_2)$$

     $$\mathrm{cov}(\tilde{r}_1,\tilde{r}_p) = 0.40\times 0.04 + 0.60\times 0.05 = 0.046;\ \mathrm{cov}(\tilde{r}_2,\tilde{r}_p) = 0.40\times 0.05 + 0.60\times 0.09 = 0.074$$

  2. Calculate the ratio of expected excess return over covariance for each stock:

     $$\lambda_1 = 0.092/0.046 = 2;\ \lambda_2 = 0.148/0.074 = 2$$

- An investor with risk aversion equal to 2 will hold this portfolio.

More Examples of Efficient Portfolios

- *Example:* What will an investor with a different risk aversion do?
  - Assume x_1 = 0.20 and x_2 = 0.30, with the remainder invested into the risk-free asset.
  - Is this also an efficient portfolio? [YES/NO]
    Where on the efficient frontier, if at all, would this investor be? [further to the right/left than before]
- As this investor is more risk-averse than our previous investor, his lambda coefficient is higher than before:

$$\lambda = \underbrace{\frac{0.092}{0.023}}_{stock\,1} = \underbrace{\frac{0.148}{0.037}}_{stock\,2} = 4$$

- As lambdas are equal across stocks (=4), the lambda of the portfolio will also be equal to all the stocks' lambdas:

$$\text{Relative risk aversion} = \lambda = \frac{E(\tilde{r}_p - r_0)}{\text{var}(\tilde{r}_p)}$$

which implies that an investor's relative risk aversion can be measured by the portfolio that the investor holds.

Capital Asset Pricing Model

- Assumptions:
  1. Homogeneous opportunities (equal access to the same assets)
  2. Homogeneous expectations (investors use the same estimates of assets' expected returns and of their variances).
- An implication: All investors hold the same equity market portfolio and therefore, for all risky stocks $j = 1, 2, 3, \ldots, N$:

$$\frac{E(\tilde{r}_j - r_0)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)} = \underbrace{\lambda_m = \frac{E(\tilde{r}_m - r_0)}{\text{var}(\tilde{r}_m)}}_{\textit{from last slide}}$$

$$\Leftrightarrow \quad E(\tilde{r}_j - r_0) = \frac{E(\tilde{r}_m - r_0)}{\text{var}(\tilde{r}_m)} \text{cov}(\tilde{r}_j, \tilde{r}_m) = \beta_{j,m} E(\tilde{r}_m - r_0)$$

- Beta is a measure of a stock's relative risk, i.e. its co-movement with the market.
  - If $\beta_{j,m} = 0$, an asset's expected return is the risk-free rate.
  - If $\beta_{j,m} > 0$, an asset's expected return contains a risk premium.

Why the InCAPM must differ from the CAPM

- When markets are integrated, the market portfolio must contain the stocks of all countries into which investors can invest (e.g. world market)

- However, there are other differences to the standard CAPM, for example
  - A U.K. investor cares about his wealth in £.
  - When our U.K. investor invests into, say, the U.S., he is not interested in the \$-return, but in the £-return, which is:

    $$r_{Sj}^{£} \approx r_{Sj}^{\$} + r_{\$}$$

    $$= \text{gain/loss}_{\text{U.S. stock}} + \text{gain/loss}_{\text{currency}}$$
  - When differences in inflation rates do not offset changes in the exchange rate, **real U.S. and U.K. returns will differ**, violating the homogeneous expectations assumption.

Why the InCAPM must differ from the CAPM

– An extreme example is the return on the T-bill. Suppose that there is no inflation (nominal returns are the same as real returns)

- To a US investor, the CAD T-bill is one of the available risky assets and is therefore included in the US tangency portfolio.

- To a Canadian investor, the CAD T-bill is riskfree, and it is not part of the Canadian tangency portfolio.

Optimal Portfolio Formation in an Integrated World

• Assume a world with 2 integrated countries, say Canada (home) & the U.S. (abroad, denoted by an asterisk)

- Integrated world: Canadian and U.S. investors can both invest into Canadian and U.S. stocks → together, the world market portfolio

- Portfolio choices can be summarised as follows:

$$\begin{aligned}
&\text{Canadians choose } p \text{ such that } \mathrm{E}(\tilde{r}_j - r) = \lambda \operatorname{cov}(\tilde{r}_j, \tilde{r}_p),\\
&\text{Americans choose } p^* \text{ such that } \mathrm{E}(\tilde{r}_j^* - r^*) = \lambda \operatorname{cov}(\tilde{r}_j^*, \tilde{r}_{p^*}^*).
\end{aligned}$$

where *p* denotes portfolio and the asterisk ($*$) refers to amounts in the foreign currency (USD)

Optimal Portfolio Formation in an Integrated World

• Translated into CAD, the problem of the American investor can be written as (see technical note 19.2 in the textbook for the derivation):

$$\text{Americans choose } p^* \text{ such that } \mathrm{E}(\tilde{r}_j - r) = \lambda \mathrm{cov}(\tilde{r}_j, \tilde{r}_{p^*}) + (1-\lambda)\, \mathrm{cov}(\tilde{r}_j, \tilde{s}),$$

where $\tilde{s}$ is the percentage change in the exchange rate (CAD per USD)

Covariance with the exchange rate

- The covariance between stock returns and exchange rates is proportional to the coefficient γ from the regression model

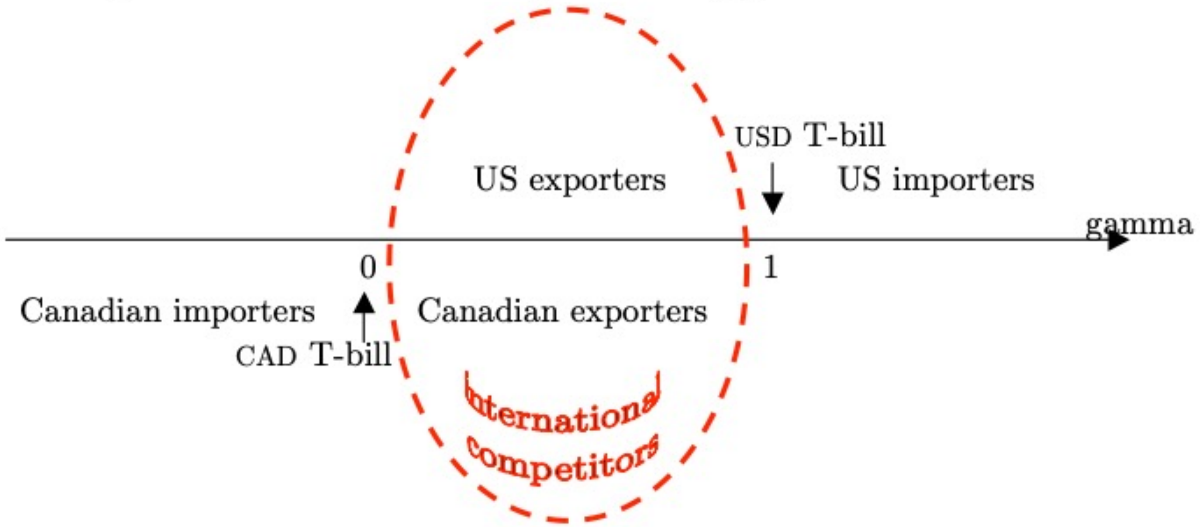
$$\tilde{r}_j = \alpha_{j,s} + \gamma_j \, \tilde{s}_{\mathrm{CAD/USD}} + \epsilon_{j,s}.$$

- How are different assets exposed in CAD terms? Some examples

1. Canadian risk-free asset: not affected ⇒ zero covariance
2. U.S. risk-free asset: one-to-one affected ⇒ positive covariance
3. Canadian importer: an increase in the CAD/USD rate means bad news ⇒ negative covariance with exchange rate
4. Canadian manufacturer: an increase in the CAD/USD rate means good news ⇒ positive covariance with exchange rate
5. Also consider a U.S. exporter or an importer

Covariance with the exchange rate

Figure 19.4: **Relative exposures (γ) of various assets**



- Firms from both countries can show very different exposures to the CAD/USD exchange rate

A Quick Derivation of the InCAPM

- The two equations that determine the Canadian and U.S. market portfolios

$$\begin{aligned} \text{CDN: } \mathrm{E}(\tilde{r}_j - r) &= \lambda \operatorname{cov}(\tilde{r}_j, \tilde{r}_p), \\ \text{US: } \mathrm{E}(\tilde{r}_j - r) &= \lambda \operatorname{cov}(\tilde{r}_j, \tilde{r}_{p^*}) + (1-\lambda) \operatorname{cov}(\tilde{r}_j, \tilde{s}). \end{aligned}$$

can be aggregated into (see technical note 19.3)

$$\mathrm{E}(\tilde{r}_j - r) = \lambda \operatorname{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \operatorname{cov}(\tilde{r}_j, \tilde{s}),$$

where w refers to the world market portfolio and κ captures national invested wealths and risk aversions

- The above expression (see the final (!) technical note 19.4) leads to the InCAPM formula

A Quick Derivation of the InCAPM

- In its standard form, the International Capital Asset Pricing Model (InCAPM) is usually written as

$$\mathrm{E}(\tilde{r}_j - r) = \beta_{j,w}\mathrm{E}(\tilde{r}_w - r) + \gamma_{j,s}\mathrm{E}(\tilde{s} + r^* - r),$$

where $\beta_{j,w;s}$ and $\gamma_{j,s;w}$ are regression coefficients corresponding to the market risk and exchange rate risk, respectively, estimated from the market model with a single currency exposure

$$\tilde{r}_j = \alpha_{j,w,s} + \beta_{j,w;s}\tilde{r}_w + \gamma_{j,s;w}\tilde{s} + \tilde{\epsilon_{j;w,s}}.$$

- In your assignment, assume (to simplify computations) that all risk free rates are equal to zero.

An InCAPM in an Integrated World with N Countries

- When there are *N+ 1* integrated countries, the InCAPM can be written as:

$$E(\tilde{r}_j - r_0) = \beta_{j,w;all\ s} E(\tilde{r}_w - r_0) + \sum_{k=1}^{N} \gamma_{j,s_k;w,other\ s} E(\tilde{s}_k + r^*_{0,k} - r_0)$$

where $\beta_{j,w;all\ s}$ and $\gamma_{j,s_k;w,other\ s}$ are regression coefficients from the market model and n exposure models

- If there are 100 integrated countries, there would be 101 coefficients: this somehow seems too much of a good thing:

  – Reduce the number of slope coefficients through focusing only on the more important countries.
  – Alternatively, drop all slope coefficients, but still use the world market portfolio.

Summary, Homework and Additional Reading

- **In this lecture**, we dealt with:
  - One needs to be aware of differences regarding how the value of a local and a foreign investment project can be determined.
  - The difference between segmented and integrated markets and the reason of why this is important for capital budgeting.
  - Standard Capital Asset Pricing Model provides the basis for capital budgeting analysis
  - In the international context (with integrated markets), International CAPM needs to be applied

- **At home**, you will need to cover:
  - Take another look at the mathematical derivation of the InCAPM.
  - Solve exercises for the next workshop