ACF305:
International Financial and Risk Management
Week 5

Outline of Lecture 5

- Essential reading: Chapters 6 & 7 of Sercu (2009).
- Topics:
  - **Futures**: What is the difference between forwards and futures?
  - **Using Futures in Risk Management – Hedging with futures**: How can I hedge with futures? What is a minimum variance hedge?
  - **Swaps**: What is a swap contract?

Problems with Forward Contracts

- The two main problems of forwards are default risk and a lack of secondary markets (= illiquidity).
  - Default risk problem can be alleviated by restricting access to forward markets (refusing dubious customers, asking for collateral, etc.).
  - However, while premature settlement might be possible, this is a question of negotiation (with bank), *not* a built-in right.
    → Almost all forwards remain outstanding until maturity.
- Posting a margin to the bank can reduce default risk.
  - **Note**: A margin is not a payment to the bank, i.e. the margin belongs to the customer and can only be seized by the bank when the customer defaults.

Solutions

- The magnitude of the margin differs depending on the settlement system, with two possible systems shown below:

  - **Variable collateral**: Bank only asks for small initial margin, but demands more margin when value of contract becomes negative.

  - **Daily re-contracting**: Gains and losses are settled at the end of each day and a new forward contract (with new $F_{t,T}$) is signed between the parties.

- These two systems are equivalent – See next slide

Variable Collateral & Daily Re-contracting in Action

Data	Variable collateral	Daily re-contracting
Time 0:	*S buys USD 1m forward at T = 3 at $F_{0,3}$ = 40m.*	
$F_{0,3} = 40$		
Time 1:	*Value forward now equals (38−40)/(1.02) = −1.961m*	
$F_{1,3} = 38$ $r_{1,3} = 2\%$	Agent posts margin to bank of 1.961m.	Agent cancels the old contract by paying 1.961m to the bank and gets a new contract at $F_{1,3}$.
Time 2:	*Value forward now (36−40)/1.01 = −3.960m*	*Value forward now (36−38)/1.01 = −1.980m*
$F_{2,3} = 36$ $r_{2,3} = 1\%$	Agent increases margin to 3.960m.	Agent cancels the old contract by paying 1.980m to the bank and gets a new contract at $F_{2,3}$.
Time 3:		
$F_{3,3} = 34$	Agent pays the promised 40m to the bank and gets back her deposit over 3.960m.	Agent pays the promised 36m for the USD 1m.

- **Note**: The total paid (adjusted for the time value of money) is identical.
  - Variable collateral: 40m at time t=3.
  - Daily re-contracting: 36m (at t=3) + $\underbrace{\text{2m (at } t\text{=2)}}_{1.980*1.01}$ + $\underbrace{\text{2m (at } t\text{=1)}}_{1.961*1.02}$ = 40m

Introduction to Futures Contracts I

- Forward contracts can deal with default risk problems, yet they cannot deal with illiquidity problems → Futures contracts can!

- *Definition of futures contract:*

    1. Initial value of the contract equals zero.

    2. Stipulates delivery of a known number of FC units at time T.

    3. Stipulates a HC payment for the FC (i.e. $f_{0,T}$), *to be paid later*.

Introduction to Futures Contracts I

- Only difference to forward: the timing of the payment.
  - During the life of the contract, agents pay $f_{t,T} - f_{t-1,T}$ via marking-to-market.
  - At the end of the contract, agents pay S_T to buy FC.
- Marking-to-market is a 'primitive' version of daily re-contracting, where the discounting is omitted:

price; r	40; r=0.03	38; r=0.02	36; r=0.01	34; r=0.00
futures	–	$38 - 40 = -2.000$	$36 - 38 = -2.000$	$34 - 36 = -2.000$ and then buy at 34
fwd, mk2mk	–	$\frac{38-40}{1.02} = -1.961$	$\frac{36-38}{1.01} = -1.980$	buy at 36

Introduction to Futures Contracts II

- Why are the intermediate future *mk2mk* payments not discounted?
    1. In the mid 1800s when futures were invented, maturities were short and interest rates low → low impact.

- Daily settlement payments: made through accounts with brokers and a clearing house
    - The **initial margin** is the amount of money that has to be put into the account when the futures contract is signed. Normally this is a **small fraction of the total value of the contract**.
        - The idea behind this requirement is that the margin should cover virtually all of the one-day loss, which then reduces the incentive to default. This also limits the loss to the clearing house in the event of a default.
    - When amount of money falls below **maintenance margin**, a **margin call** is issued, requesting that the margin is brought back to the initial margin.

- Mk2mk reduces the buyer's or seller's incentive to default on his obligation, i.e. he can only avoid one day of losses.

Organization & Structure of Futures Markets

- Futures are traded on organized exchanges with specific rules and with an active secondary market (examples: EUREX, LIFFE).

- Futures contracts are standardized:
  - Traders can only buy a multiple of a specific currency amount, say GBP 62,500 or EUR 125,000.

- Futures contracts are never between individuals A and B, but always between an individual and the clearing house (small fee).

- **As a result of all above**: Futures contracts are far more liquid than forward contracts

Hedging with Futures – The Major Problems

- Higher liquidity makes hedging with futures attractive.

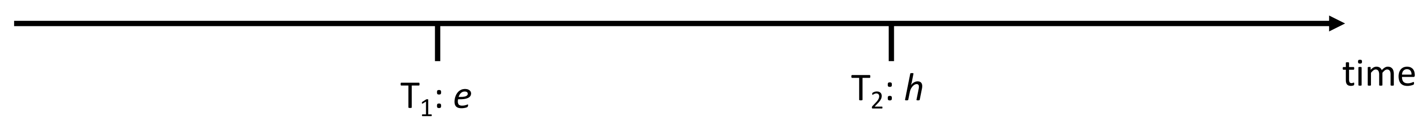
- However, several problems arise with hedging in the futures market:
    1. Contract size is fixed → unlikely to match the exposure position.

    2. Expiration date is fixed → unlikely to match the date of the exposure.

    3. Choice of underlying assets in futures markets is limited → the FC one wishes to hedge may not have a futures contract.

Hedging with Futures – Timeline of events

- *Assume:*
  1. One unit of FC *e* (“exposure”) to be received at time T_1.
  2. A futures contract for a related currency *h* (“hedge”) with expiration date T_2.
  3. The size of the futures contract is one unit of FC.



Reminder: cashflows from futures contracts

- We can compute the cashflows of a futures contract at any point in time
  - Imagine a futures contract that starts today (t), and has a maturity at time T_2. The cashflows at time T_1<T_2 are:
    - **Sum of all daily cashflows up to time T_1**, this is, the cumulative daily cashflows

- Cashflows from futures sales are:

  ➢ $\left(\boldsymbol{f_{t,T_2}} - \boldsymbol{f_{T_1,T_2}}\right) = \sum_{j=1}^{T_1}\left(f_{t+j-1,T_2} - f_{t+j,T_2}\right)$

- Cashflows from futures purchases are (the opposite):

  ➢ $\left(\boldsymbol{f_{T_1,T_2}} - \boldsymbol{f_{t,T_2}}\right) = \sum_{j=1}^{T_1}\left(f_{t+j,T_2} - f_{t+j-1,T_2}\right)$

Hedging with forward contracts

- The hedged cashflow at time T_1 of FC 1:
  - Cashflow in HC of FC at $T_1 \equiv \tilde{S}_{T1}$
  - Cashflow in HC of forward sale at $T_1 \equiv 1 \times (F_{t,T1} - \tilde{S}_{T1})$
  - **Combination** of cashflows $\equiv \tilde{S}_{T1} - 1 \times (\tilde{S}_{T1} - F_{t,T1}) = F_{t,T1}$

Therefore, what you receive at T_1 is:

HC value of FC 1 + HC value of forward sale

Hedging with forward contracts

- The hedged cashflow at time T_1 of FC 1 with forward contract at T_2:
  - Cashflow in HC of FC at $T_1 \equiv \tilde{S}_{T1}$
  - Cashflow in HC at T_1 of forward sale at $T_{2\,;}\, T_1 \equiv 1 \times \dfrac{(F_{t,T2} - \tilde{F}_{T1,T2})}{1 + r_{T1,T2}}$
  - Combination of cashflows $T_1 \equiv \tilde{S}_{T1} - 1 \times \dfrac{(\tilde{F}_{T1,T2} - F_{t,T2})}{1 + r_{T1,T2}}$

Therefore, what you receive at T_1 is:

HC value of FC 1 + HC market value of forward sale

NOTE: The cashflow from forward sale requires to cancel the contract by buying forward.

Hedging strategy with one future

- In the two previous slides we where selling 1 unit of FC forward. We **now sell 1 unit with a futures contract**

- But now we are going to assume that we don't have a futures contract for the currency that we are exposed to (*e*) and therefore we use an alternative foreign currency as our hedge (*h*):

E.g. Our home country is UK, therefore GBP is our HC

a) Assume in six months we receive one Euro (*e*)

a) We don't have futures contract for use, hence we use Dollars (*h*)

b) The futures contract has maturity in one year.

c) But we want to know the cashflows from that contract in six month

Hedging strategy with one future

- In the two previous slides we where selling 1 unit of FC forward. We **now sell 1 unit with a futures contract**
- The hedged cash flow at time T_1 will be:
  - Cashflow in HC of FC at $T_1 \equiv \tilde{S}_{T1}^{(e)}$
  - Cashflow in HC at T_1 of future sale at $T_{2;}$ $T_1 \equiv (f_{t,T2}^{(h)} - \tilde{f}_{T1,T2}^{(h)})$
  - **Combination** of cashflows $T_1 \equiv \tilde{S}_{T1}^{(e)} - (\tilde{f}_{T1,T2}^{(h)} - f_{t,T2}^{(h)})$

= <u>HC value of long FC 1</u> + <u>HC value of 1 futures sale</u>,

Hedging strategy

- In the two previous slides we where selling 1 unit of FC forward. We now sell β units
- The hedged cash flow at time T_1 will be:
  - (1) Cashflow in HC of FC at T_1 = $\tilde{S}^{(e)}_{T1}$
  - (2) Cashflow in HC at T_1 of future sale at T_2 = $\beta \times (f^{(h)}_{t,T2} - \tilde{f}^{(h)}_{T1,T2})$
  - (3) Combination of cashflows T_1 = $\underbrace{\widetilde{S}^{(e)}_{T_1}}_{\text{Spot cashflow}} - \beta * \underbrace{\left(\widetilde{f}^{(h)}_{T_1,T_2} - f^{(h)}_{t,T_2}\right)}_{\text{Futures contract cashflow}}$

= HC value of long FC 1 + HC value of short β futures,

= (3) = VALUE in (1) + VALUE (2)

where β is the number of future contracts.

Hedging strategy: the method

- Choose β so as to minimize Var(cash flow at T_1).
- The well-known solution: $\beta = \mathrm{cov}\left(\tilde{f}^{(h)}_{T_1,T_2}, \tilde{S}^{(e)}_{T_1}\right) / \mathrm{var}\left(\tilde{f}^{(h)}_{T_1,T_2}\right)$
- How do we estimate parameter **β**?
  - We estimate **β** using past data.
  - To compute the covariance, I use past data of the contemporaneous spot rates and futures rates with maturity (T_2-T_1) months

Example

- We are at time t, and sign a futures contract to sell β units of GBP in one year for 1.3 dollars per unit. Future rate is $f_{t,T2} = 1.3$.
- We receive one euro in $T_1 = 3$ months.
- The number of USD that we receive in T_1 is $\mathbf{S_{T1}}$
- How many USD/GBP future contracts do I sell to reduce the risk of $\mathbf{S_{T1}}$**?**
  - **Minimize risk/variance of combined cashflows to determine** β

- Estimate covariance between future rates with $(T_2 - T_1) = 9$ months maturity and spot rates. Then, the variance of the same future rates.

Example solution:

Future rates	Spot rates		
f_(T1,T2)	**S_T1**		
10.5377	6.6501		
11.8339	11.0349		
7.7412	8.7254		
10.8622	7.9369		
10.3188	8.7147		
8.6923	7.795		
9.5664	7.8759		
10.3426	9.4897		
13.5784	9.409		
12.7694	9.4172		
		Cov(f_(T1,T2),S_T1) =	**0.9748**
		Var(f_(T1,T2))=	**3.1325**
	beta = Cov(f_(T1,T2),S_T1)/Var(f_(T1,T2)) =		**0.3112**

Understanding hedging strategies

- Consider an investor who owns a share of a fund and wants to develop a hedging strategy. The fund returns are defined in a CAPM style (we will talk more on this topic)

$$r_{Fund_{,}t} = \alpha + \beta r_{Mkt_{,}t} + \varepsilon_t$$

Where $r_{Fund_{,}t}$ is the fund excess return at time t, and $r_{Mkt_{,}t}$ is the market excess return at time t. ε_t is the idiosyncratic term, uncorrelated with $r_{Mkt_{,}t}$.

- Think of what the above equation implies.
  - A strategy that invests one unit on the fund and shorts (sell) β units of the market, gives returns that are ***uncorrelated with the main source of systematic risk***

$$\boldsymbol{r_{Hedging\ Strategy_{,}t} = r_{Fund_{,}t} - \beta r_{Mkt_{,}t} = \alpha + \varepsilon_t}$$

Hedging with Futures – Some Special Cases I

- **Case I: Perfect Match**
  - *Assume:* Futures contract expires at T_1 and is available for the desired FC.
  - Then:

$$\beta = \operatorname{cov}\left(\tilde{f}^{(h)}_{T_1,T_2}, \tilde{S}^{(e)}_{T_1}\right) / \operatorname{var}\left(\tilde{f}^{(h)}_{T_1,T_2}\right) = \operatorname{cov}\left(\tilde{f}^{(e)}_{T_1,T_1}, \tilde{S}^{(e)}_{T_1}\right) / \operatorname{var}\left(\tilde{f}^{(e)}_{T_1,T_1}\right)$$

$$= \operatorname{cov}\left(\tilde{S}^{(e)}_{T_1}, \tilde{S}^{(e)}_{T_1}\right) / \operatorname{var}\left(\tilde{S}^{(e)}_{T_1}\right) = \operatorname{var}\left(\tilde{S}^{(e)}_{T_1}\right) / \operatorname{var}\left(\tilde{S}^{(e)}_{T_1}\right) = 1$$

  - **Optimal solution**: Sell one futures contract (over FC 1).
- **Case II: Currency Mismatch (Cross-Hedge)**
  - *Assume:* Futures contract expires at T_1, yet only a related FC is available – not the desired FC.
  - Then:

$$\beta = \operatorname{cov}\left(\tilde{f}^{(h)}_{T_1,T_2}, \tilde{S}^{(e)}_{T_1}\right) / \operatorname{var}\left(\tilde{f}^{(h)}_{T_1,T_2}\right) = \operatorname{cov}\left(\tilde{f}^{(h)}_{T_1,T_1}, \tilde{S}^{(e)}_{T_1}\right) / \operatorname{var}\left(\tilde{f}^{(h)}_{T_1,T_1}\right)$$

$$= \operatorname{cov}\left(\tilde{S}^{(h)}_{T_1}, \tilde{S}^{(e)}_{T_1}\right) / \operatorname{var}\left(\tilde{S}^{(h)}_{T_1}\right)$$

$$= \textit{slope coefficient in } \tilde{S}^{(e)}_{T_1} = \alpha + \beta\, \tilde{S}^{(h)}_{T_1} + \tilde{\varepsilon}$$

Hedging with Futures – Some Special Cases II

- **Mind your stats**
  - Series are normally non-stationarity (beyond the scope of this course), and it would be meaningless to run the above regression.
  - Better to run the difference equation:
    $$\Delta S_t^{(e)} = \alpha' + \beta\, \Delta S_t^{(h)} + \tilde{\varepsilon}_t'$$
    or even better the percentage change regression:

Error in the book

$$\Delta S_t^{(e)} \frac{S_t^{(e)}}{S_t^{(e)}} = \alpha' + \beta\, \Delta S_t^{(h)} \frac{S_t^{(h)}}{S_t^{(h)}} + \tilde{\varepsilon}_t' \Leftrightarrow \tilde{s}_t^{(e)} = \alpha'' + \gamma\, \tilde{s}_t^{(h)} + \tilde{\varepsilon}_t'' \quad with \quad \boxed{\beta = \gamma \frac{S_t^{(e)}}{S_t^{(h)}}}$$

- **Case III: Maturity Mismatch (Delta-Hedge)**
  - *Assume:* Futures contract is available for desired currency, yet matures at T_2.
  - Then:
    $$\beta = \mathrm{cov}\left(\tilde{f}_{T_1,T_2}^{(h)}, \tilde{S}_{T_1}^{(e)}\right) / \mathrm{var}\left(\tilde{f}_{T_1,T_2}^{(h)}\right) = \mathrm{cov}\left(\tilde{f}_{T_1,T_2}^{(e)}, \tilde{S}_{T_1}^{(e)}\right) / \mathrm{var}\left(\tilde{f}_{T_1,T_2}^{(e)}\right)$$
  - Regress (% changes in) spot rates on (% changes in) futures rates with maturity left equal to $T_2 - T_1$.

Hedging with Futures – Size Adjustments

- *Assumption so far:* Exposure equal to one unit of FC and size of futures contract equal to one unit of FC.
- If exposure is larger, say η_{s_j}, and size of futures contract is larger, say η_{f_i}, then the optimal solution will be:

$$\text{hedge ratio} = \frac{\eta_{s_j}}{\eta_{f_i}} \beta$$

  and then round to the nearest integer.

- *Example:* A U.S. agent wants to hedge a SEK 2.17m inflow with EUR futures with a contract size of EUR 125,000. As a result, he performs the following regression:

$$\Delta S_{[USD/SEK]} = 0.003 + 0.105 \Delta f_{[USD/EUR]},$$

  with an R^2 of 0.83 and a t-statistic of 15.62.

- The hedge ratio should thus be $\frac{2{,}170{,}000}{125{,}000} * 0.105 = 1.822 \approx 2.$

Origin of Swap Contracts

- First well-known swap contract between IBM and World Bank (WB); purpose: to save transaction costs.

- The starting situation:
  - IBM wanted to replace its debt in DEM and CHF with USD debt.
  - WB wanted to borrow in DEM and CHF.

- Using a swap contract:
  1. WB issues USD debt, converts proceedings into DEM/CHF and uses them to make loans to their customers.
  2. WB then pays IBM's DEM/CHF debt obligations, while IBM pays WB's USD debt obligations → save transaction costs, defer taxes.

Subsequent Evolution of the Swap Market

- While a forward contract is like an exchange of two promissory notes (PNs), a swap contract is like an exchange of two bonds.

- A swap is a transaction where at the time of the contract's initiation the two parties agree to exchange two cashflow streams of equal present value. The deal is structured as a single contract, with a right of offset.

Fixed-for-Fixed Currency Swaps: Example I

- A Japanese company wants to borrow cheaply in JPY at 1% for seven years and then swap the loan into USD.
  - The quoted swap rates are 0.6% on JPY and 3% on USD.
  - The spot rate is JPY/USD 100.

	loan	swap		Combined
	JPY 1000 borr'd at 1%	JPY 1000 lent, at 0.6%	USD 10m borr'd at 3%	
principal at t	JPY 1000m	<JPY 1000m>	USD 10m	USD 10m
interest (p.a.)	<JPY 10m>	JPY 6m	<USD 0.3m>	<JPY 4m>& <USD 0.3m>
principal at T	<JPY 1000m>	JPY 1000m	<USD 10m>	<USD 10m>

- Sometimes, payoff scheme is not ideal. E.g. the Japanese company must make interest payments in both USD and JPY.

Summary, Homework and Additional Reading

- **In this lecture**, we dealt with:
  - Futures: Differences to forwards, organization of market.
  - Hedging with futures: The minimum variance hedge: some theoretical concerns.
  - Swaps: The development and evolution of swaps, some examples.

- **At home**, you will need to cover:
  - Carefully review how to hedge with futures contracts.

- **Additional reading**:
  - Kritzman, M. (1991), "What Practitioners Need to Know...About Regressions", *Financial Analysts Journal* 47(3), 12-15. [Optional]