ACF305
International Financial and Risk Management

Week 10 tutorial

Question 1

Consider the following covariance matrix and expected return vector for assets 1, 2 and 3:

$$V=\begin{bmatrix} 0.01 & 0.002 & 0.001 \\ 0.002 & 0.0025 & 0.003 \\ 0.001 & 0.003 & 0.01 \end{bmatrix} \qquad E(r)=\begin{bmatrix} 0.033 \\ 0.0195 \\ 0.025 \end{bmatrix}$$

d. Is the portfolio considered last week (with weights for assets j=0,…,3 equal to [0.2, 0.4, 0.2, 0.2]) efficient?

e. Are the following portfolios efficient?

   i. Weights (0.7, 0.1, 0.1, 0.1) for assets j=0,…,3.

   ii. Weights (0.6, 0.2, 0.1, 0.1) for assets j=0,…,3.

f. What is the portfolio held by an investor with risk-aversion measure λ=2.5?

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g. Assume that there are no outside bills (no external supply /demand for risk-free asset), that is, all risk-free lending and borrowing is among investors. Therefore, the average investor holds only risky assets. What is the portfolio composition? What is the average investor's risk-aversion measure λ?

Solution

d. Subtract the risk-free rate from each asset's expected return. Then divide this number by the covariance with the portfolio computed in the former sub-question (for the three assets, the covariances were: 0.0046, 0.0019 and 0.003, respectively). The calculation for asset 1 is: $(0.033-0.01)/0.0046 = 5$. You should find that each ratio is equal to 5 – the coefficient of relative risk aversion. As a result, yes, this portfolio is efficient.

Solution

e. The first choice cannot be optimal. The optimal portfolio is unique and its weight in the first asset is always two times its weights in the second and third asset. This is not the case for the first portfolio weights, which are equal across the three assets. However, this is the case for the second set of weights, which means that the second set is optimal. The higher weight in the risk-free assets shows that this portfolio would be held by a more risk-averse investor than the portfolio in (d).

Solution

f. When computing the benefit-to-risk ratios, the investment weights never influence the numerator, which is simply the asset's expected excess return. However, they do play an important role in the denominator, i.e. they influence the covariance with the portfolio. In fact, doubling the weights will double the covariance with the portfolio. We have seen in (d) that an investor with risk aversion equal to 5 holds portfolio [0.2,0.4,0.2,0.2]. As a result, an investor with risk aversion equal to 2.5 would choose portfolio [−0.6,0.8,0.4,0.4].

Solution

g. Simply scale up the weights in the risky assets until they sum up to one. In (d), the weights sum up to 0.80. Therefore, multiplying with 1/0.80 will do the job. The average investor's portfolio weights will then be equal to 0.50, 0.25 and 0.25, with a zero weight in the risk-free asset.

Question 2

Suppose that your assistant has run a market-model regression for a company that produces sophisticated drilling machines, and finds the following results (*t*-statistics in parentheses):

$$r_j = \alpha + \beta r_m + \gamma s + e_j,$$

$$\underset{(0.52)}{r_j = 0.002} + \underset{(1.25)}{0.56 r_m} + \underset{(2.06)}{4.25 s} + e_j.$$

Your assistant remarks that, as the estimated beta is insignificant, the true beta is zero. The exposure, in contrast, is significant, and must be equal to the estimated coefficient. How do you react?

Solution

You cannot simply conclude that $\beta = 0$. The low t-statistic says that, on the basis of only the sample information, it is possible that the true beta is zero. But you know more than a computer or calculator (which evaluate only the sample information) and would not expect the true beta for a highly cyclical sector (machine tools) to be zero, or even much below unity. Thus, the estimate 0.56 is probably better than the conjecture $\beta = 0$. An industry β would be more reliable.

Solution

The high t-statistic for the exposure (γ) means that one can reject, beyond what most statisticians would call a reasonable doubt, the hypothesis that the true exposure is zero. However, from a purely statistical point of view, the true exposure could still be 0.5, or 0.75, or 1. In fact, with $\sigma = 4.25/2.06 = 2.063$, anything in the range of +/- 4.04 (1.96 x 2.063) is statistically acceptable. However, your common sense tells you that true exposures are unlikely to exceed unity. Thus, the estimated beta may be erring on the downward side, and the estimated γ almost surely errs on the upward side.

Question 3

Suppose that the world beta for a German stock (in euro) equals 1.5, and its exposures to the dollar, the yen, and the pound are 0.3, 0.2, and 0.1, respectively.

a. What is the best replicating portfolio if you can invest in a world-market index fund, as well as in dollars, yens, pounds, and euros?
b. What additional information is needed to identify the cost of capital?

Solution

a. $x_w = 1.5$, $x_{USD} = 0.3$, $x_{JPY} = 0.2$, $x_{GBP} = 0.1$, and $x_{EUR} = 1 - 1.5 - 0.3 - 0.2 - 0.1 = -1.1$.

b. The expected excess returns on each of these assets (including, for the currencies, the risk-free rates) would be needed to identify the cost of capital.

Question 4

Suppose that there are two countries, the US (which is the foreign country) and Canada. The exposure of the company XUS, in terms of USD, is estimated as follows:

$$r^*_{XUS} = 0.12 + 0.30 s_{USD/CAD} + \varepsilon.$$

What is the company's exposure in terms of CAD?

Solution

Over short periods, the percentage change in the CAD/USD rate is approximately equal to the negative of the change in the USD/CAD exchange rate.

Thus, -1 percent rise in the USD/CAD value means -1 percent drop in the CAD/USD rate, and on average -0.30 percent drop in the stocks USD price.

Thus, the total effect on the CAD return from XUS of a 1 percent rise of the USD is 1 - 0.30 = 0.70:

$$r_{XUS} = a + 0.7s_{CAD/USD} + \varepsilon$$