ACF305
International Financial and Risk Management

Week 9 tutorial

Question 1

The country Prince Rupert’s Land (PRL) has two companies, the Hudson Bay Company (HBC) and the Boston Tea Traders (BTT). In equilibrium, the returns of these two companies have the following distributions:

	Expected excess returns	Covariances HBC	Covariances BTT
HBC	0.11	0.04	0.01
BTT	0.08	0.01	0.02

a. Vary the weight of HBC from 0 to 1 by increments of 0.1 and compute how the portfolio covariance risks of HBC and BTT change as a function of the weights x_{hbc} and $x_{btt} = 1 - x_{hbc}$.

b. Find the optimal weights of x_{hbc} and $x_{btt} = 1 - x_{hbc}$ that equalise the ratio of expected excess stock return over stock covariance with the portfolio for the two stocks.

c. If the total value of the PRL stock market portfolio is 1,000, what are the values of HBC and BTT?

Solution

a. The covariance between the return of HBC and the return of a portfolio consisting of it and BTT can be computed from:

$$x \text{ var(HBC)} + (1 - x) \text{ cov(HBC, BTT)},$$

where x indicates the weight of HBC. Consequently, the covariance between the return of BTT and the return of a portfolio consisting of it and HBC can be computed according to:

$$x \text{ cov(HBC, BTT)} + (1 - x) \text{ var(BTT)}.$$

These covariances are shown in the table below for different investment weights.

Portfolio		Covariance with Portfolio		Relative Risk Aversion	
x (HBC)	(1–x) (BTT)	HBC	BTT	HBC	BTT
0.0	1.0	0.010	0.020	11.00	4.00
0.1	0.9	0.013	0.019	8.46	4.21
0.2	0.8	0.016	0.018	6.88	4.44
0.3	0.7	0.019	0.017	5.79	4.71
0.4	0.6	0.022	0.016	5.00	5.00
0.5	0.5	0.025	0.015	4.40	5.33
0.6	0.4	0.028	0.014	3.93	5.71
0.7	0.3	0.031	0.013	3.55	6.15
0.8	0.2	0.034	0.012	3.24	6.67
0.9	0.1	0.037	0.011	2.97	7.27
1.0	0.0	0.040	0.010	2.75	8.00

b. The optimal weights are $x = 0.40$ and $(1-x) = 0.60$, as these weights equalize the ratio of expected excess stock return over stock covariance with the portfolio for the two stocks. The relative risk aversion is the value of this ratio, which is 5.00. (The latter point will be explained in more detail during the lecture on Wednesday.)

c. If the stock market portfolio is worth 1,000 and HBC contributes 40% to this portfolio, while BTT contributes 60%, then the market value of HBC and BTT must be 400 and 600, respectively.

Question 2

Consider the following covariance matrix and expected return vector for assets 1, 2 and 3:

$$V = \begin{bmatrix} 0.01 & 0.002 & 0.001 \\ 0.002 & 0.0025 & 0.003 \\ 0.001 & 0.003 & 0.01 \end{bmatrix} \qquad E(r) = \begin{bmatrix} 0.033 \\ 0.0195 \\ 0.025 \end{bmatrix}$$

a. Compute the expected return on a portfolio with weights for assets j=0,…,3 equal to [0.2, 0.4, 0.2, 0.2], when T-bill (asset 0) yields a return of 1%. Do so directly, and then via the excess returns.
b. Compute the variance of the same portfolio.
c. Compute the covariance of the return on each asset with the total portfolio return and verify that it is weighted covariance.

Solution

a. Directly: The expected portfolio return is the weighted average of the expected returns of all assets:

$$0.2*0.01 + 0.4*0.033 + 0.2*0.0195 + 0.2*0.025 = 0.0241 = 2.41\%.$$

Via excess returns: The excess return for an asset is its expected return minus the risk-free rate. The expected portfolio return is the weighted average of the excess returns of all risky assets plus the risk-free rate:

$$[0.4*(0.033-0.01) + 0.2*(0.0195-0.01) + 0.2*(0.025-0.01)] + 0.01 = 0.0141 + 0.01 = 0.0241 = 2.41\%.$$

Solution

b. The variance of the portfolio return is the sum of the squared weights in each asset times their variances plus 2 times the weight in the first asset times the weight in the second asset times the covariance between the two assets etc. Note that we can ignore the risk-free asset, as both its variance and covariance with other assets is zero:

$$0.4^2*0.01 + 0.2^2*0.0025 + 0.2^2*0.01 + 2*0.4*0.2*0.002 + 2*0.4*0.2*0.001 + 2*0.2*0.2*0.003 = 0.00282.$$

Solution

c. The covariance of asset 1 with the total portfolio are:

$\text{cov}(r(1), r(p)) = \text{cov}(r(1), x(1)*r(1) + x(2)*r(2) + x(3)*r(3)) = x(1)*\text{cov}(r(1), r(1)) + x(2)*\text{cov}(r(1), r(2)) + x(3)*\text{cov}(r(1), r(3)) = 0.4*0.01 + 0.2*0.002 + 0.2*0.001 = 0.0046.$

Note that we can again ignore the risk-free asset, as its variance and covariances are all zero. The covariances for assets 2 and 3 with the total portfolio are calculated accordingly. For the three assets, the covariances are: 0.0046, 0.0019 and 0.003, respectively.