

# AcF302: Corporate Finance

## Week 14 Workshop

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TRIPLE-ACCREDITED, WORLD-RANKED



## Question 1:

For each of the following statements, explain whether the statement is true or false:

(1) In-the-money real options should be exercised immediately.

False

Starting positive NPV projects immediately may not always be optimal if the you can generate higher value by waiting.

(2) Out-of-the-money real options have value.

True

A project that has a negative NPV today (i.e., out-of-the-money) may end up having a positive NPV in the future if circumstances change (e.g., lower investment cost, higher cash inflows, or lower discount rate).

(3) The smaller the cost of waiting, the more attractive the option to delay becomes.

True

The forgone cash flow (or dividend in the case of a financial call) is an important determinant of the option value. The higher the cash flows you give up by waiting the less attractive the delay option becomes.

(4) The value of a company's real option to delay is directly affected by fluctuations in the company's stock price.

**False**

The value of a company's real option to delay is directly affected by fluctuations in the cash flows of the underlying investment.

## Question 2:

Your company is considering a new project at a cost of \$12 million. The project may begin today or in exactly one year. You expect the project to generate \$1,500,000 in free cash flow the first year if you begin the project today. Free cash flow is expected to grow at a rate of 3% per year. The risk-free rate is 4%. The appropriate cost of capital for this investment is 11%. The standard deviation of the project's cash flows is 30%.

Should you start the project today?

The value of the project if we started today is:

$$V_{Today} = \frac{\$1,500,000}{11\% - 3\%} = \$18,750,000$$

$$NPV = V_{today} - \text{Initial Cost} = \$18,750,000 - \$12,000,000 = \$6,750,000$$

## The value of waiting one year to start the project:

$$C = S \times N(d_1) - PV(K) \times N(d_2)$$

$$V_{Today} = \frac{\$1,500,000}{11\% - 3\%} = \$18,750,000$$

The current value of the project without the “dividend” that will be missed is

$$S^x = S - PV(Div) = \$18,750,000 - \frac{\$1,500,000}{1.11} = \$17,398,649$$

The present value of the cost to begin the project in one year is

$$PV(K) = \frac{\$12,000,000}{1.04} = \$11,538,462$$



$$d_1 = \frac{\ln[S/PV(K)]}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln[17,398,649/11,538,462]}{0.30\sqrt{1}} + \frac{0.30\sqrt{1}}{2} = 1.519$$

$$d_2 = 1.519 - (0.30\sqrt{1}) = 1.219$$

$$C = \$17,398,649 \times 0.9357 - \$11,538,462 \times 0.8888 = \$6,024,531$$

The value of waiting one year to start the project is **\$6,024,531**

Cumulative probability  $N(d)$  that a normally distributed variable will be less than  $d$  standard deviations above the mean.

$d$	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952

Note: For example, if  $d = .22$ ,  $N(d) = .5871$  (i.e., there is a .5871 probability that a normally distributed variable will be less than .22 standard deviations above the mean).

**Should you begin the project today or wait one year?**

The NPV of starting the project today is: **\$6,750,000**

The value of waiting one year to start the project is **\$6,024,531**

**Thus, it is optimal to begin the project today rather than wait.**

### Question 3:

Your firm is thinking of making an investment. If you invest today, the project will generate \$9 million in free cash flow at the end of the year and will have a continuation value of either \$150 million (if the economy improves) or \$46 million (if the economy does not improve). If you wait until next year to invest, you will lose the opportunity to make \$9 million in free cash flow, but you will know exactly what the continuation value of the investment will be. Suppose the risk-free rate is 6%, and the risk-neutral probability that the economy improves is 41%. Assume the cost of investing is the same this year or next year.

a) If the cost of investing is \$79 million, should you do so today, or wait until next year to decide?

Investing today:

$$V_0 = \frac{(0.41 \times 150 + 0.59 \times 46) + 9}{1.06} = \$92.11 \text{ million}$$

$$NPV (\text{Invest Today}) = 92.11 - 79 = \$13.11 \text{ million}$$

Waiting 1 year:

If you wait, you will only invest if the economy is in a good state, since otherwise the NPV of investing would be negative (\$46 < \$79).

$$NPV (\text{Wait}) = \frac{[0.41 \times (150 - 79)] + (0.59 \times 0)}{1.06} = \$27.46 \text{ million}$$

Therefore, it is optimal to wait because  $NPV(\text{wait}) > NPV(\text{today})$

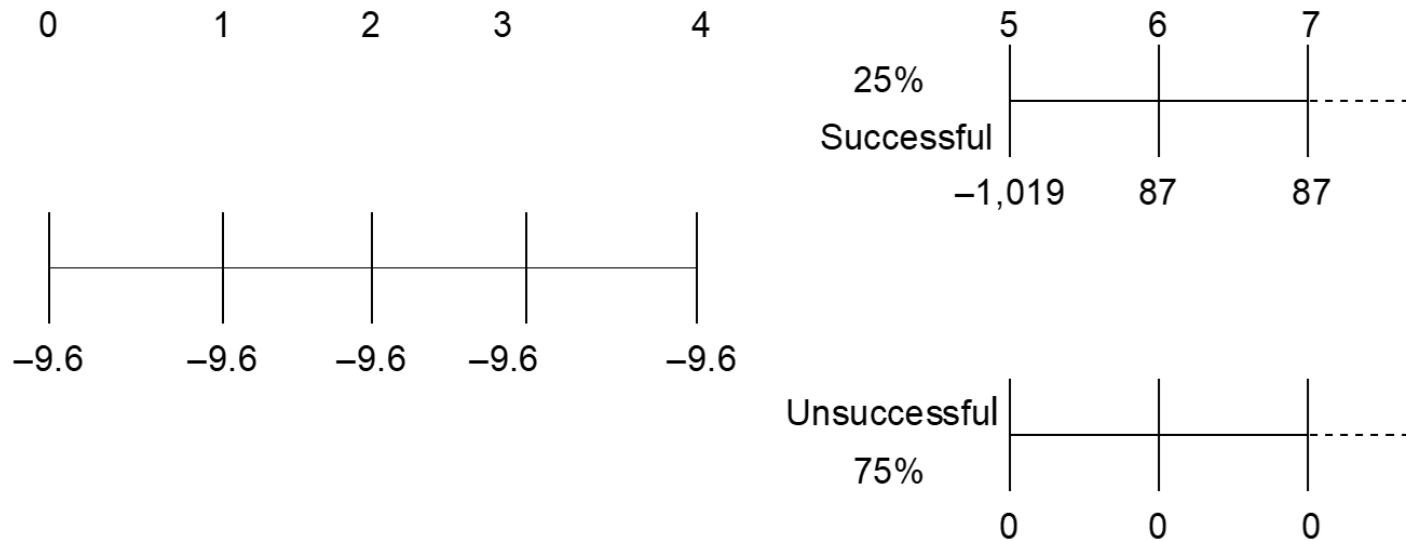
## Question 4:

Your R&D division has just synthesized a material that will superconduct electricity at room temperature; you have given the go ahead to try to produce this material commercially. It will take five years to find out whether the material is commercially viable, and you estimate that the probability of success is 25%. Development will cost \$9.6 million per year, paid at the **beginning** of each year.

If development is successful and you decide to produce the material, the factory will be built immediately. It will cost \$1019 million to put in place and will generate profits of \$87 million at the end of every year in perpetuity. Assume that the current five-year risk-free interest rate is 9.7% per year, and the yield on a perpetual risk-free bond will be 11.6%, 10.4%, 7.8%, or 4.5% in five years. Assume that the risk-neutral probability of each possible rate is the same.

a) What is the value of this project in year 5?

b) What is the value of this project today? Should you undertake the project?



If development is successful, then the NPV (at time 5) of producing the wire is:

$$\text{NPV}_5(r) = \frac{87}{r} - 1019$$

This is negative for  $r > 8.5\%$ , so the wire will only be produced if the rates are 7.8% or 4.5%:

$$\text{NPV}_5(7.8\%) = \frac{87}{0.078} - 1019 = 96.38$$

$$\text{NPV}_5(4.5\%) = \frac{87}{0.045} - 1019 = 914.33$$

So, the expected value of the growth opportunity at time 5 if development is successful is:

$$EV_5 = 0.25(96.38) + 0.25(914.33) = 252.68 \longrightarrow \text{The 0.25 in the equation is the 25\% probability of each rate happening}$$

There is a 25% chance of success so the expected value at time 5 of the investment opportunity is:

$$V_5 = 252.68(25\%) = \$63.17 \longrightarrow \text{The 25\% here is the probability of development being successful}$$

The NPV of the development opportunity at time 0 is therefore

$$NPV = -9.6 - \frac{9.6}{0.097} \left( 1 - \frac{1}{1.097^4} \right) + \frac{63.17}{1.097^5} = -\$0.47$$

Thus, the project should not be undertaken.



## Question 5

- Bianchi has come up with a new mountain bike prototype and is ready to go ahead with pilot production and test marketing.
- The pilot production and test marketing phase will last for **one year** and cost **£500,000**.
- Your management team believes that there is a **50% chance that the test marketing will be successful** and that there will be sufficient demand for the new mountain bike.
- If the test-marketing phase is successful, then Bianchi will **invest £3 million** in year one to build a plant.

- If the test marketing is **successful**, the plant will **generate** expected annual after tax cash flows of **£400,000 in perpetuity** beginning in year two.
- If the test marketing is **not successful**, Bianchi can still go ahead and build the new plant, but the expected annual after tax cash flows would be **only £200,000 in perpetuity** beginning in year two.
- Bianchi has the option to stop the project at any time and **sell the prototype** mountain bike to an overseas competitor for **£300,000**.
- Bianchi's cost of capital is **10%**.

a) Assuming that Bianchi can sell the prototype in year one for \$300,000, what is the NPV of the Mountain Bike Project?

*Value if test is successful*

$$\text{Build plant: } NPV = -\$3,000,000 + \frac{\$400,000}{.10} = \$1,000,000$$

*Value if test unsuccessful*

$$\text{Build plant } NPV = -\$3,000,000 + \frac{\$200,000}{.10} = -\$1,000,000$$

If test is successful you build the plant because it has +ve NPV. If test is unsuccessful you sell the prototype because building the plant has a –ve NPV.

Don't Build (Sell Prototype)  $NPV = \$300,000$

Since  $\$300,000 > -\$1,000,000$  you should sell prototype if test is unsuccessful

$$NPV = \frac{(.5)\$1,000,000 + (.5)\$300,000}{1.10} - \$500,000 = \$90,909$$

The 0.5s here are the probabilities of success and failure of the test marketing.

b) Suppose that instead of the pilot production and test marketing, Bianchi decides to go ahead and build their manufacturing plant immediately. Assuming that the probability of high or low demand is still 50%, what is the NPV of the Mountain Bike Project in this case?

$$NPV = \frac{(.5)\$400,000 + (.5)\$200,000}{.10} - \$3,000,000 = \$0$$

c) What is the value of the option to do pilot production and test marketing?

Value of option = NPV with pilot production and test marketing  
- NPV of building today

$$= \$90,909 - \$0 = \mathbf{\$90,909}$$

d) Suppose that Bianchi does not have the ability to sell the prototype in year one. Should the company go ahead with the project in this case?

*Value if test is successful*

$$\text{Build plant: } NPV = -\$3,000,000 + \frac{\$400,000}{.10} = \$1,000,000$$

*Value if test unsuccessful*

$$\text{Build plant } NPV = -\$3,000,000 + \frac{\$200,000}{.10} = -\$1,000,000$$

Don't Build (abandon)  $NPV = \$0$

Since  $\$0 > -\$1,000,000$  you should abandon

$$NPV = \frac{(.5)\$1,000,000 + (.5)\$0}{1.10} - \$500,000 = -\$45,455$$

**Since the NPV is negative, Bianchi should not go ahead with the project.**

## Question 6:

Pharco is developing a new drug that will slow the aging process. In order to succeed, two breakthroughs are needed, one to increase the potency of the drug, and the second to eliminate toxic side effects. Research to improve the drug's potency is expected to require an upfront investment of \$10.4 million and take 2 years; the stage has a 6% chance of success. Reducing the drug's toxicity will require a \$28.9 million upfront investment, take 4 years, and has an 18% chance of success. If both efforts are successful, Pharco can sell the patent for the drug to a major drug company for \$2.03 billion. All risk is idiosyncratic, and the risk-free rate is 5.8%.

- a) What is the NPV of launching both research stages simultaneously?
- b) What is the optimal order of staging the drug development?
- c) What is the NPV with the optimal staging?

a) What is the NPV of launching both research efforts simultaneously?

The NPV of launching both research efforts simultaneously is:

$$\text{NPV} = \text{PV (future cash inflows)} - \text{PV (costs)}$$

$$\begin{aligned}\text{NPV} &= [0.06 \times 0.18 \times (\$2.03 \text{ billion} / 1.058^4)] - \$10.4 \text{ million} - \$28.9 \text{ million} \\ &= -\$21.80 \text{ million}\end{aligned}$$



## b) What is the optimal order to stage the investments?

To find the optimal order to stage the investments we can use the failure cost index:

$$\text{Failure Cost Index} = \frac{1 - PV(\text{success})}{PV(\text{investment})}$$

The failure cost index for the "potency" stage is:

$$\text{Failure Cost Index} = \frac{1 - \frac{0.06}{(1+0.058)^2}}{\$10.4 \text{ million}} = 0.0910$$

The failure cost index for the "toxicity" stage is:

$$\text{Failure Cost Index} = \frac{1 - \frac{0.18}{(1+0.058)^4}}{\$28.9 \text{ million}} = 0.0296$$

**So, work on potency first, then toxicity—higher risk and smaller investment.**

c) What is the NPV with the optimal staging?

$$\text{NPV} = \text{PV (cash inflow)} - \text{PV (costs)}$$

$$\begin{aligned} &= [\text{PV (cash flow from selling the patent)} * \text{probability that both stages are successful}] \\ &\quad - \text{PV (potency cost)} - [\text{PV (toxicity cost)} * \text{probability that potency stage is successful}] \end{aligned}$$

$$= \left( 0.06 \times 0.18 \times \frac{\$2,030 \text{ million}}{1.058^6} \right) - \$10.4 \text{ million} - \left( 0.06 \times \frac{\$28.9 \text{ million}}{1.058^2} \right) = \mathbf{\$3.68 \text{ million}}$$