

## CHAPTER 1

# RELATIONS AND FUNCTIONS

### VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If  $A$  is the set of students of a school then write, which of following relations are. (Universal, Empty or neither of the two).

$$R_1 = \{(a, b) : a, b \text{ are ages of students and } |a - b| \geq 0\}$$

$$R_2 = \{(a, b) : a, b \text{ are weights of students, and } |a - b| < 0\}$$

$$R_3 = \{(a, b) : a, b \text{ are students studying in same class}\}$$

2. Is the relation  $R$  in the set  $A = \{1, 2, 3, 4, 5\}$  defined as  $R = \{(a, b) : b = a + 1\}$  reflexive?
3. If  $R$ , is a relation in set  $N$  given by

$$R = \{(a, b) : a = b - 3, b > 5\},$$

then does elements  $(5, 7) \in R$ ?

4. If  $f : \{1, 3\} \rightarrow \{1, 2, 5\}$  and  $g : \{1, 2, 5\} \rightarrow \{1, 2, 3, 4\}$  be given by

$$f = \{(1, 2), (3, 5)\}, g = \{(1, 3), (2, 3), (5, 1)\}$$

Write down  $\text{gof}$ .

5. Let  $g, f : R \rightarrow R$  be defined by

$$g(x) = \frac{x+2}{3}, f(x) = 3x - 2. \text{ Write } \text{fog}.$$

6. If  $f : R \rightarrow R$  defined by

$$f(x) = \frac{2x-1}{5}$$

be an invertible function, write  $f^{-1}(x)$ .

7. If  $f(x) = \frac{x}{x+1} \forall x \neq -1$ , Write  $fo f(x)$ .

8. Let  $*$  is a Binary operation defined on  $R$ , then if

$$(i) \quad a * b = a + b + ab, \text{ write } 3 * 2$$

(ii)  $a * b = \frac{(a+b)^2}{3}$ , Write  $(2 * 3) * 4$ .

9. If  $n(A) = n(B) = 3$ , Then how many bijective functions from  $A$  to  $B$  can be formed?
10. If  $f(x) = x + 1$ ,  $g(x) = x - 1$ , Then  $(g \circ f)(3) = ?$
11. Is  $f : N \rightarrow N$  given by  $f(x) = x^2$  is one-one? Give reason.
12. If  $f : R \rightarrow A$ , given by  

$$f(x) = x^2 - 2x + 2$$
is onto function, find set  $A$ .
13. If  $f : A \rightarrow B$  is bijective function such that  $n(A) = 10$ , then  $n(B) = ?$
14. If  $n(A) = 5$ , then write the number of one-one functions from  $A$  to  $A$ .
15.  $R = \{(a, b) : a, b \in N, a \neq b \text{ and } a \text{ divides } b\}$ . Is  $R$  reflexive? Give reason?
16. Is  $f : R \rightarrow R$ , given by  $f(x) = |x - 1|$  is one-one? Give reason?
17.  $f : R \rightarrow B$  given by  $f(x) = \sin x$  is onto function, then write set  $B$ .
18. If  $f(x) = \log\left(\frac{1+x}{1-x}\right)$ , show that  $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$ .
19. If  $*$  is a binary operation on set  $Q$  of rational numbers given by  $a * b = \frac{ab}{5}$  then write the identity element in  $Q$ .
20. If  $*$  is Binary operation on  $N$  defined by  $a * b = a + ab \forall a, b \in N$ . Write the identity element in  $N$  if it exists.

### SHORT ANSWER TYPE QUESTIONS (4 Marks)

21. Check the following functions for one-one and onto.

(a)  $f : R \rightarrow R$ ,  $f(x) = \frac{2x-3}{7}$

(b)  $f : R \rightarrow R$ ,  $f(x) = |x + 1|$

(c)  $f : R - \{2\} \rightarrow R$ ,  $f(x) = \frac{3x-1}{x-2}$

(d)  $f : R \rightarrow [-1, 1], f(x) = \sin^2 x$

22. Consider the binary operation  $*$  on the set  $\{1, 2, 3, 4, 5\}$  defined by  $a * b = \text{H.C.F. of } a \text{ and } b$ . Write the operation table for the operation  $*$ .
23. Let  $f : R - \left\{ \frac{-4}{3} \right\} \rightarrow R - \left\{ \frac{4}{3} \right\}$  be a function given by  $f(x) = \frac{4x}{3x+4}$ . Show that  $f$  is invertible with  $f^{-1}(x) = \frac{4x}{4-3x}$ .
24. Let  $R$  be the relation on set  $A = \{x : x \in Z, 0 \leq x \leq 10\}$  given by  $R = \{(a, b) : (a - b) \text{ is multiple of } 4\}$ , is an equivalence relation. Also, write all elements related to 4.
25. Show that function  $f : A \rightarrow B$  defined as  $f(x) = \frac{3x+4}{5x-7}$  where  $A = R - \left\{ \frac{7}{5} \right\}, B = R - \left\{ \frac{3}{5} \right\}$  is invertible and hence find  $f^{-1}$ .
26. Let  $*$  be a binary operation on  $Q$ . Such that  $a * b = a + b - ab$ .
- (i) Prove that  $*$  is commutative and associative.
  - (ii) Find identify element of  $*$  in  $Q$  (if it exists).
27. If  $*$  is a binary operation defined on  $R - \{0\}$  defined by  $a * b = \frac{2a}{b^2}$ , then check  $*$  for commutativity and associativity.
28. If  $A = N \times N$  and binary operation  $*$  is defined on  $A$  as  $(a, b) * (c, d) = (ac, bd)$ .
- (i) Check  $*$  for commutativity and associativity.
  - (ii) Find the identity element for  $*$  in  $A$  (If it exists).
29. Show that the relation  $R$  defined by  $(a, b) R(c, d) \Leftrightarrow a + d = b + c$  on the set  $N \times N$  is an equivalence relation.
30. Let  $*$  be a binary operation on set  $Q$  defined by  $a * b = \frac{ab}{4}$ , show that
- (i) 4 is the identity element of  $*$  on  $Q$ .

(ii) Every non zero element of  $Q$  is invertible with

$$a^{-1} = \frac{16}{a}, \quad a \in Q - \{0\}.$$

31. Show that  $f: R_+ \rightarrow R_+$  defined by  $f(x) = \frac{1}{2x}$  is bijective where  $R_+$  is the set of all non-zero positive real numbers.
32. Consider  $f: R_+ \rightarrow [-5, \infty)$  given by  $f(x) = 9x^2 + 6x - 5$  show that  $f$  is invertible with  $f^{-1} = \frac{\sqrt{x+6}-1}{3}$ .
33. If  $*$  is a binary operation on  $R$  defined by  $a * b = a + b + ab$ . Prove that  $*$  is commutative and associative. Find the identity element. Also show that every element of  $R$  is invertible except  $-1$ .
34. If  $f, g: R \rightarrow R$  defined by  $f(x) = x^2 - x$  and  $g(x) = x + 1$  find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ . Are they equal?
35.  $f: [1, \infty) \rightarrow [2, \infty)$  is given by  $f(x) = x + \frac{1}{x}$ , find  $f^{-1}(x)$ .
36.  $f: R \rightarrow R, g: R \rightarrow R$  given by  $f(x) = [x], g(x) = |x|$  then find

$$(f \circ g)\left(\frac{-2}{3}\right) \text{ and } (g \circ f)\left(\frac{-2}{3}\right).$$

## ANSWERS

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1.  $R_1$  : is universal relation.  
 $R_2$  : is empty relation.  
 $R_3$  : is neither universal nor empty.
2. No,  $R$  is not reflexive.
3.  $(5, 7) \notin R$
4.  $g \circ f = \{(1, 3), (3, 1)\}$
5.  $(f \circ g)(x) = x \quad \forall \quad x \in R$

6.  $f^{-1}(x) = \frac{5x+1}{2}$
7.  $(f \circ f)(x) = \frac{x}{2x+1}, x \neq -\frac{1}{2}$
8. (i)  $3 * 2 = 11$   
(ii)  $\frac{1369}{27}$
9. 6
10. 3
11. Yes,  $f$  is one-one  $\because \forall x_1, x_2 \in N \Rightarrow x_1^2 = x_2^2$ .
12.  $A = [1, \infty)$  because  $R_f = [1, \infty)$
13.  $n(B) = 10$
14. 120.
15. No,  $R$  is not reflexive  $\because (a, a) \notin R \forall a \in N$
16.  $f$  is not one-one functions  
 $\because f(3) = f(-1) = 2$   
 $3 \neq -1$  i.e. distinct element has same images.
17.  $B = [-1, 1]$
19.  $e = 5$
20. Identity element does not exist.
21. (a) Bijective  
(b) Neither one-one nor onto.  
(c) One-one, but not onto.  
(d) Neither one-one nor onto.

22.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

24. Elements related to 4 are 0, 4, 8.

25.  $f^{-1}(x) = \frac{7x+4}{5x-3}$

26. 0 is the identity element.

27. Neither commutative nor associative.

28. (i) Commutative and associative.

(ii) (1, 1) is identity in  $N \times N$

33. 0 is the identity element.

34.  $(f \circ g)(x) = x^2 + x$

$(g \circ f)(x) = x^2 - x + 1$

Clearly, they are unequal.

35.  $f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2}$

36.  $(f \circ g)\left(\frac{-2}{3}\right) = 0$

$(g \circ f)\left(\frac{-2}{3}\right) = 1$