LCD

1. $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ is

(A) 2 (B) –2 (C) 1/2 (D) –1/2

2. f(x) = $\begin{cases} ax^2 + bx + c, & |x| > 1 \\ x+1, & |x| \le 1 \end{cases}$. If f(x) is continuous for all values of x, then;

(A) b = 1, a + c = 0 (B) b = 0, a + c = 2
(C) b = 1, a + c = 1 (D) none of these

3. The equation of the tangent to the curve f (x) = 1 + e^{-2x} where it cuts the line y = 2 is

(A) x + 2y = 2 (B) 2x + y = 2
(C) x – 2y = 1 (D) x – 2y + 2 = 0

4. $\lim_{x\to\infty} \frac{x+\sin x}{x-\sin x}$ = ……………………………………………………….

5. $\lim_{x\to\infty}\left(\frac{x+3}{1+x}\right)^{x+3}$ = ………………………………………………………..

6. $\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$, then a = …………………………..b = ………………………

7. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$ is equal to

(A) π (B) 1/4
(C) 1/2 (D) 1

8 $\lim_{x\to\infty} \frac{\sqrt{x^2-1}}{2x+1}$ is equal to

(A) 1 (B) 0
(C) -1 (D) 1/2

9. If f(x) = $(1 - x^n)^{1/n}$, 0 < x < 1, n being an odd positive integer and h(x) = f(f(x)), then $h'\left(\frac{1}{2}\right)$ is equal to

(A) 2^n (B) 2
(C) n. 2^{n-1} (D) 1

10 Among $\lim_{x\to 0} \sec^{-1}\left(\frac{x}{\sin x}\right)$ …. (1)

and $\lim_{x\to 0} \sec^{-1}\left(\frac{\sin x}{x}\right)$ …. (2)

(A) (1) exists, (2) does not exist (B) (1) does not exist, (2) exists
(C) both (1) and (2) exist (D) neither (1) nor (2) exists

11 A function f (x) is defined as f (x) = $\begin{cases} x^2 - 3x + a, & x < 1 \\ -2, & x = 1 \\ bx + 3, & x > 1 \end{cases}$

What are the values of a and b respectively such that f (x) is continuous at x = 1.

(A) 1, –2 (B) 0, –5 (C) –1, 0 (D) 2, –3

12 Given a function f(x) continuous $\forall$ x $\in$R such that $\lim\limits_{x\to 0}\left[f(x)+\log\left(1-\frac{1}{e^{f(x)}}\right)-\log(f(x))\right]$ = 0,

then f(0) is

(A) 0 (B) 1 (C) 2 (D) 3

13 The value of $\lim\limits_{x\to\infty} x\cos\left(\frac{\pi}{4x}\right)\sin\left(\frac{\pi}{4x}\right)$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 1 (D) π

14 The value of $\lim\limits_{x\to 0}\frac{a^x - b^x}{x}$ is

(A) $\log_e\left(\frac{a}{b}\right)$ (B) $\log_e\left(\frac{b}{a}\right)$ (C) $\log_e$ (ab) (D) none of these

15 If f (x) = $\begin{cases} mx + 1, & x \le \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at x = $\frac{\pi}{2}$, then

(A) m = 1, n = 0 (B) m = $\frac{n\pi}{2}$ + 1 (C) n = $\frac{m\pi}{2}$ (D) m = n = $\frac{\pi}{2}$

16. The value of $\lim\limits_{x\to\infty}\frac{\sqrt{1+x^4}-(1+x^2)}{x^2}$ is equal to

(A) 0 (B) –1
(C) 2 (D) 1

17. $\lim\limits_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x - x}$ is equal to

(A) 1 (B) e
(C) –1 (D) 0

18. The function f (x) = $\frac{\tan(\pi-[x-\pi])}{1+[x]^2}$, where [.] denotes greatest integer function, is

(A) discontinuous at some x
(B) continuous at all x, but f′ (x) does not exist for some x
(C) f′ (x) exists for all x
(D) none of these

19 If the function f(x) = $\begin{cases} Ax - B, & x \le 1 \\ 3x, & 1 < x < 2 \\ Bx^2 - A, & x \ge 2 \end{cases}$ be continuous at x = 1 and discontinuous at x = 2, then

(a) A = 3 + B, B $\neq$ 3 (b) A = 3 + B, B = 3 (c) A = 3 + B (d) none of these

41. If $(x) = \begin{cases} ax^2 + b & , \ x \le 1 \\ bx^2 + ax + c & , \ x > 1 \end{cases}$, $b \ne 0$. Then f(x) is continuous and differentiable at $x = 1$ if

(a) $c = 0, a = 2b$ (b) $a = b, c \in R$
(c) $a = b, c = 0$ (d) $a = b, c \ne 0$.

42. If $f(x) = x^3$ sgn x, then
(a) f is derivable at x = 0 (b) f is continuous, but not derivable at x = 0
(c) LHD at x = 0 is 1 (d) RHD at x = 0 is 0.

43. If $f(x) = (x - x_0)\,\phi(x)$ and $\phi(x)$ is continuous at x = 0, then $f'(x_0)$ is equal to
(a) $\phi'(x_0)$ (b) $\phi(x_0)$
(c) $x_0\,\phi(x_0)$ (d) none of these.

44 If $f(x) = \begin{cases} \frac{\sin[x]}{[x]} & \text{for } [x] \ne 0 \\ 0 & \text{for } [x] = 0 \end{cases}$ where [x] denotes greatest integer function, then $\lim_{x \to 0} f(x) =$
(A) 1 (B) 0
(C) -1 (D) doesn't exist

45. If the function $f(x) = \begin{cases} \frac{\sin(2x)^2}{x^2} + e^{-x} & , x \ne 0 \\ k & , x = 0 \end{cases}$ is continuous, then k is

(A) 2 (B) 3
(C) 4 (D) 5.

46. For a function y = f(x), $\frac{dy}{dx} = (x-1)(x+2)$. Find the point of local maximum and minimum for the function y = f(x).
....

47. Find the function y = f(x) for the above function if it is given that y = 2 at x = 0.
....

48. The value of derivative of f (x) = |x –1| + |x –3| at x = 2 is
(A) –2 (B) 0
(C) 2 (D) not defined

49. The function f (x) = |sin x| –1 is
(A) continuous everywhere (B) not differentiable at $x = \frac{\pi}{3}$
(C) differentiable at x = 0 (D) differentiable everywhere

50. Let $f(x) = \begin{cases} 3x - 4, & 0 \le x \le 2 \\ 2x + \lambda, & 2 < x \le 3 \end{cases}$, if f (x) is continuous at x = 2, then λ is
(A) –1 (B) –2
(C) 2 (D) none of these

51. The number of points at which the function $f(x) = \frac{x}{\log|x|}$ is discontinuous is
(A) 1 (B) 2

(C) 3 (D) 4

52. The number of values of x $x \in [0, 2]$ at which the real function $f(x) = |x - 1/2| + |x - 1| + \tan x$ is not differentiable is

(A) 2 (B) 3

(C) 1 (D) 0

LEVEL–II

1. The function $(x^2-1)\left|x^2-3x+2\right|+\cos(|x|)$ is not differentiable at

(A) –1 (B) 0 (C) 1 (D) 2

2. For $x \in R$, $\lim\limits_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is

(A) e (B) e^{-1} (C) e^{-5} (D) e^5

3. $\lim\limits_{x\to\frac{\pi}{2}}\left[\frac{6\cos x}{2x-\pi}\right]$, where [.] denotes the greatest integer function, is equal to;

(A) - 3 (B) - 4 (C) -2 (D) none of these

4. Let f(x) = $(\tan x)^{\frac{1}{x-\frac{\pi}{4}}}$ $\forall$ x $\in$ (0, π/2) ~ {π/4}, then the value of f(π/4} such that f(x) becomes continuous at x = $\frac{\pi}{4}$ is equal to;

(A) e (B) √e (C) $\frac{1}{\sqrt{e}}$ (D) e^2

5. Let f(x)=[5+3 sinx] $\forall$ x $\in$ R. Then total number of points of discontinuity of f(x) in [0, π] is equal to;

(A) 5 (B) 6 (C) 7 (D) 4

6. f(x) = $\sin^{-1}$(sinx), x $\in$ [-2π, 2π]. Total number of critical points of f(x) is ;

(A) 3 (B) 4 (C) 5 (D) 2

7. If the line ax + by + c = 0 is normal to the curve x y + 5 = 0 then

(A) a > 0 , b > 0 (B) b > 0 , a < 0

(C) a < 0 , b < 0 (D) b < 0 , a > 0

8. The maximum value of f(x) = |x ln x| in x∈(0,1) is;

(A) 1/e (B) e

(C) 1 (D) none of these

9. f(x) = $3x^3 + 4e^x - k$ is always increasing then value of k =

(A) 2 (B) –4/9

(C) 4/9 (D) all of these

10. $\lim\limits_{x\to 2}\{[2-x]+[x-2]-x\}$ is

(A) 0 (B) 3

(C) –3 (D) does not exist

11. Let f (x) be a twice differentiable function and f′′ (0) = 2 then $\lim\limits_{x\to 2}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is

(A) 6 (B) 1

(C) 12 (D) 3

12 Let h (x) = f (x) –$\{f(x)\}^2$ + $\{f(x)\}^3$ for all real values of x then

(A) h is ↑ whenever f (x) is ↓ (B) h is ↑ whenever f(x) is ↓ 0

(C) h is ↓ whenever f is ↓ (D) nothing can be said in general

13. Let $f'(x) > 0$, $g'(x) < 0$ for all $x \in R$, then
(A) $f\{g(x)\} > f\{g(x+1)\}$
(B) $f\{g(x)\} > f\{g(x-1)\}$
(C) $g\{f(x)\} >< g\{f(x+1)\}$
(D) $g\{f(x)\} > g\{f(x-1)\}$

14. $\lim\limits_{x\to\infty} \frac{\ln[x]}{l+x}$ = ………………………………………………………….. [.] G. I. F

15. $\lim\limits_{n\to\infty} \left(3^n + 5^n + 7^n\right)^{\frac{1}{n}}$ = ……………………………………………

16. If α, β are the roots of $ax^2 + bx + c = 0$ then $\lim\limits_{x\to\alpha} \frac{1-\cos\left(ax^2+bx+c\right)}{(x-\alpha)^2}$ = ………………………

17. $\lim\limits_{x\to 1}\left(1 - x + [x-1] + [1-x]\right)$ = ………………………………………………………………

18. $f(x) = \sin^{-1}(\cos x)$ then points of nondifferentiability between $[0, 2\pi]$ = ……………………..

19. Let $f(x+y) = f(x) \cdot f(y)$ for all x & y, if $f(5) = 2$ and $f'(0) = 3$, then $f'(5)$ = ……………………

20. $f(x) = \begin{cases} \frac{a\,|x^2 - x - 2|}{2 + x - x^2}, & x < 2 \\ b, & x = 2 \\ \frac{x-[x]}{x-2}, & x > 2 \end{cases}$ (where [.] denotes the greatest integer function). If f(x) is continuous at x = 2, then
(A) a = 1, b = 2
(B) a = 1, b = 1
(C) a = 0, b = 1
(D) a = 2, b = 1

21. Let $f(x) = \begin{cases} -1, & x \le 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$ and g(x) sinx + cosx, then points of discontinuity of f{g(x)} in (0, 2π) is
(A) $\left\{\frac{\pi}{2}, \frac{3\pi}{4}\right\}$
(B) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$
(C) $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$
(D) $\left\{\frac{5\pi}{4}, \frac{7\pi}{3}\right\}$

22. If α and β are the roots at $ax^2 + bx + c = 0$ then $\lim\limits_{x\to\alpha}\left(1 + ax^2 + bx + c\right)^{1/(x-\alpha)}$ is
(A) $a(\alpha - \beta)$
(B) $\ln|a(\alpha - \beta)|$
(C) $e^{a(\alpha - \beta)}$
(D) $e^{a|\alpha - \beta|}$

23. $\lim\limits_{x\to\pi/4} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$ is equal to

(A) $1/\sqrt{2}$ (B) 1/2

(C) $\frac{1}{2\sqrt{2}}$ (D) 1

24. The function f(x) = $[x]^2 - [x^2]$ where [y] denotes the greatest integer less than or equal to y), is discontinuous at

(A) all integers (B) all integers except 0 and 1

(C) all integers except 0 (D) all integers except 1

25. If the derivative of f(x) w.r. t x is $\frac{\frac{1}{2}-\sin^2 x}{f(x)}$, then f(x) is a periodic function with period

(A) π (B) 2 π

(C) $\pi/2$ (D) none of these.

26. $\lim_{x\to 0}\left[(\min(y^2+2y+7))\frac{\sin x}{x}\right]$ = ? (where [.] denotes greatest integer function)

(A) 4 (B) 5

(C) 6 (D) none of these

27. $\lim_{x\to 0}\left[100\frac{\tan x}{x}\right]$ = ? (where [.] denotes greatest integer function)

(A) 100 (B) 99

(C) 101 (D) 0

28. If f (x) = |cos 2x|, then $f'\left(\frac{\pi}{4}+0\right)$ is equal to

(A) 2 (B) 0

(C) –2 (D) doesn't exist

29. $\lim_{x\to \pi/2}(\sin x)^{1/\cos x}$ =

(A) 0 (B) e

(C) 1 (D) doesn't exist

30. $\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$ equals to

(A) $\frac{1}{2}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{16}$

31. $\lim_{x\to \pi/4}(2-\tan x)^{\frac{1}{\ln(\tan x)}}$ equals to

(A) e (B) 1 (C)0 (D) e^{-1}

32. $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$ equals to

(A) 0 (B) π (C) $-\pi$ (D) not exist

33. f (x) = max {x, x^3},then the number of points where f (x) is not differentiable, are

(A) 1 (B) 2 (C) 3 (D) 4

34. $\lim_{x\to 2}\frac{\tan(e^{x-2}-1)}{\ln(x-1)}$

(A) 2 (B) -2 (C) 1 (D) –1

35. The function defined by f (x) = $\begin{cases}\frac{\sin x^2}{x}, & x \neq 0\\ 0, & x = 0\end{cases}$ is

(A) continuos and derivable at x = 0 (B) neither continuous nor derivable at x = 0
(C) continuous but not derivable at x = 0 (D) none of these

36. $\lim_{x\to 0}(1+\tan^2\sqrt{x})^{1/2x}$ is equal to

(A) 1 (B) 0 (C) $e^{1/2}$ (D) $e^{-1/2}$

37. The left hand derivative of f (x) = [x] sin (πx) at x = k, k is an integer is

(A) $(-1)^k(k-1)\pi$ (B) $(-1)^{k-1}(k-1)\pi$ (C) $(-1)^k k\pi$ (D) $(-1)^{k-1}k\pi$

38. $\lim_{x\to y}\frac{x^y-y^x}{x^x-y^y}$ is

(A) $\frac{\log ey}{1-\log y}$ (B) $\frac{1-\log y}{\log ey}$ (C) $\frac{1-\log y}{1+\log y}$ (D) (1 – log y) log ey

39. $\lim_{x\to 0}(\sin x)^x$ is

(A) 1 (B) 0 (C) ∞ (D) does not exist

40 If f(x) is a continuous function $\forall$ x $\in$ R and the range of f(x)=$(2,\sqrt{26})$ and g(x) = $\left[\frac{f(x)}{a}\right]$ is continuous $\forall$ x $\in$ R ([.] denotes the greatest integer function), then the least positive integral value of a is

(A) 2 (B) 3 (C) 6 (D) 5

41 Let $f(x)=\lim_{n\to\infty}\frac{1}{\left(\frac{3}{\pi}\tan^{-1}2x\right)^{2n}+5}$. then the set of values of x for which f(x) = 0, is

(A) $|2x| > \sqrt{3}$ (B) $|(2x)| < \sqrt{3}$ (C) $|2x| \geq \sqrt{3}$ (D) $|2x| \leq \sqrt{3}$

42 If f (x) = $\frac{\sin(e^{x-2}-1)}{\log(x-1)}$, then $\lim_{x\to 2}f(x)$ is equal to

(A) –2 (B) –1 (C) 0 (D) 1

43 If f (x) = $\begin{cases}\frac{\log(1+ax)-\log(1-bx)}{x}, & x \neq 0\\ k, & x = 0\end{cases}$ and f (x) is continuous at x = 0, the value of k is

(A) a – b (B) a + b (C) log a + log b (D) none of these

44 The expression of $\frac{dy}{dx}$ of the function y = $a^{x^{a^{x\cdots\infty}}}$ is

(A) $\frac{y^2}{x(1-y\log x)}$ (B) $\frac{y^2\log y}{x(1-y\log x)}$ (C) $\frac{y^2\log y}{x(1-y\log x\log y)}$ (D) $\frac{y^2\log y}{x(1+y\log x\log y)}$

45 The value of $\lim_{x\to\infty} \frac{x^3 \sin(1/x) - 2x^2}{1+3x^2}$ is

(A) 0 (B) – 1/3 (C) –1 (D) – 2/3

46 $\lim_{x\to 2} \frac{\tan(e^{x-2}-1)}{\ln(x-1)}$

(A) 2 (B) -2 (C) 1 (D) –1

47 Let f(x) = $\begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$. Then $f(x) . \sin x$ is

(A) differentiable at x = 0 (B) continuous at x = 0

(C) not continuous at x =0 (D) none of these

48 The function f(x) = $\frac{\cos x - \sin x}{\sin 4x}$ is not defined at x = $\pi/4$. The value which should be assigned to f at x = $\pi/4$, so that it is continuous there, is

(A) 0 (B) 1 (C) -1 (D) none of these

49 $\mathrm{Lim}_{x\to\infty} \frac{\ln x - [x]}{[x]}$, ([.] denotes the greatest integer function)

(A) has value –1 (B) has value 0 (B) has value 1 (D) does not exist

50. The function $\frac{e^{\tan x} - 1}{e^{\tan x} + 1}$ is discontinuous

(A) at $n\pi$, $n \in$ I (B) at (2n+1) $\frac{\pi}{2}$, $n \in$ I (C) No where (D) Every where

51 If a, b, c, d are positive, then $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}$ =

(A) $e^{d/b}$ (B) $e^{c/a}$

(C) $e^{(c+d)/(a+b)}$ (D) e

52 The length of the largest interval in which the function 3 sin x – 4 $\sin^3$ x is increasing, is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$

(C) $\frac{3\pi}{2}$ (D) π

53. The interval in which f (x) = $e^{|x^2-6x+8|}$ increases, is

(A) $(-\infty, 2) \cup (3, 4)$ (B) R

(C) $(2, 3) \cup (4, \infty)$ (D) (2, 4)

54. If x + |y| = 2y, then y as a function of x is

(A) continuous but not differentiable at x = 0 (B) continuous and differentiable at x = 0

(C) differentiable for all x (D) none of these

55. If y= a log |x| + bx^2 + x has its extremum values at x = –1 and x = 2, then

(A) a = 2, b = –1 (B) a = 2, b = –1/2

(C) a = –1/2, b = 1/2 (D) none of these

56. The points of extremum of the function $\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)dt$, is/are

(A) $x = 0$ (B) $x = 1/2$
(C) $x = -2$ (D) $x = \pm 1$

57. Let $f(x) = x^{n+1} + a.\, x^n$, where 'a' is a positive real number. Then x = 0 is a point of
(A) local minimum for any integer n (B) local maximum for any integer n
(C) local minimum if n is an even integer (D) local minimum if n is an odd integer

58. Least natural number 'a' for which $x + ax^{-2} > 2 \ \forall \ x \in (0, \infty)$ is
(A) 1 (B) 2 (C) 5 (D) none of these

59. Let $f(x) = \lim\limits_{n\to\infty} (\sin x)^{2n}$, then f is
(a) continuous at $x = \pi/2$, (b) discontinuous at $x = \pi/2$
(c) discontinuous at $x = -\pi/2$
(d) discontinuous at infinite number of points.

60. Let $f(x) = \begin{cases} x^n \sin\dfrac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, then f(x) is continuous, but not differentiable at x = 0, if
(a) $n \in (0, 1]$ (b) $n \in [1, \infty)$ (c) $n \in (-\infty, 0)$ (d) n= 0

61. If $f(x) = \{|x| - |x-1|\}^2$, then $f'(x)$ equals
(a) 0 for all x (b) $2\{|x| - |x-1|\}$
(c) $\begin{cases} 0 \text{ for } x < 0 \text{ and for } x > 1 \\ 4(2x-1) \text{ for } 0 < x < 1 \end{cases}$ (d) $\begin{cases} 0 \text{ for } x < 0 \\ 4(2x-1) \text{ for } x > 0 \end{cases}$

62. If the function $f(x) = \begin{cases} (1+|\sin x|)^{\frac{a}{|\sin x|}} & , \quad \dfrac{-\pi}{6} < x < 0 \\ b & , \quad x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}} & , \quad 0 < x < \dfrac{\pi}{6}, \end{cases}$ is continuous at x = 0, then
(a) $a = \log_e^b, \ a = \dfrac{2}{3}$ (b) $\log_e^a, \ a = \dfrac{2}{3}$
(c) $a = \log_e^b, \ b = 2$ (d) none of these

63. The function $f(x) = \begin{cases} \sin\left(\dfrac{\pi x}{2}\right) & , \quad x < 1 \\ |2x-3|[x] & , \quad x \geq 1 \end{cases}$
(a) is continuous at x = 1 (b) is differentiable at x = 1
(c) is continuous but not differentiable at x = 1 (d) none of these

64. The value of p for which the function f(x) = $\begin{cases} \dfrac{(4^x-1)^3}{\sin\left(\dfrac{x}{p}\right)\log\left\{1+\dfrac{x^2}{3}\right\}} & , x \neq 0 \\ 12(\log 4)^3 & , x = 0 \end{cases}$ is continuous at x = 0 is

(a) 1
(b) 2
(c) 3
(d) 4

65. $\lim_{x\to\infty}\left(\frac{1}{1-n^2}+\frac{2}{1-n^2}+.......+\frac{n}{1-n^2}\right)$ is equal to

(A) 0
(B) –1/2
(C) 1/2
(D) none of these

66 $\lim_{x\to 2}(-1)^{[x]}$, where [x] is the greatest integer function, is equal to

(A) 1
(B) –1
(C) ± 1
(D) doesn't exist

67 $\lim_{x\to\infty}\left(\frac{x}{2+x}\right)^{2x} =$

(A) e^{-4}
(B) e^{-6}
(C) e^{-2}
(D) none of these

68 If f (x) = [x sin π x] { where [x] denotes greatest integer function}, then f (x) is

(A) continuous at x = 0
(B) continuous in (-1, 0)
(C) differentiable at x = 1
(D) differentiable in (-1, 1)

69 In order that function f (x) = $(x + 1)^{\cot x}$ is continuous at x = 0, f (0) must be defined as

(A) 0
(B) e
(C) 1/e
(D) none of these

70 $\lim_{x\to\infty}\left(\frac{x^n}{e^x}\right)$ = 0, (n is integer), for

(A) no value of n
(B) all value of n
(C) only negative value of n
(D) only positive value of n

71 $\lim_{n\to\infty}(4^n+5^n)^{\frac{1}{n}}$ is equal to

(A) 4
(B) 5
(C) e
(D) none of these

72 $\lim_{x\to\frac{\pi}{2}}\left[\sin^{-1}(\sin x)\right]$ equals, where [.] denotes the greatest integer function

(A) $\frac{\pi}{2}$
(B) 0
(C) 1
(D) does not exist

73 The value of derivative of f (x) = |x –1| + |x –3| at x = 2 is

(A) –2 (B) 0

(C) 2 (D) not defined

74 The number of points where the function $f(x) = x^2 - 1 + |\ln|x||$ is not differentiable is

(A) 1 (B) 2

(C) 3 (D) none of these

75 f (x) is a continuous function and takes only rational values. If f (0) = 3, then f (2) equals

(A) 5 (B) 0

(C) 1 (D) none of these

76 $\lim\limits_{x\to 0}\dfrac{|x+1|+|x-1|-2}{x}$ is equal to

(A) 1 (B) −1 (C) 2 (D) 0

77. $\lim\limits_{x\to 0}\dfrac{x.2^x - x}{1-\cos x}$ is equal to

(A) log2 (B) $\dfrac{1}{2}$ log2 (C) 2 log2 (D) none of these

78. If f(x) = $\left(\dfrac{x^2+5x+3}{x^2+x+3}\right)^x$. Then $\lim\limits_{x\to\infty} f(x)$ is

(A) e^4 (B) e^3 (C) e^2 (D) None of these

79. $\lim\limits_{x\to 0}\dfrac{\sin^{-1}x - \tan^{-1}x}{x^3}$ is equal to

(A) 1/2 (B) 2 (C) −1/2 (D) None of these

80. $\lim\limits_{x\to 0}\dfrac{(a+x)^2\sin(a+x) - a^2\sin a}{x}$ is equal to

(A) a^2 cosa + a sina (B) a^2 cosa + 2a sina (C) $2a^2$ cosa + a cosa (D) None of these

81. Let f : R → R is a differentiable function and f(1) = 4. Then the value of $\lim\limits_{x\to 1}\int_4^{f(x)}\dfrac{2t}{x-1}dt$ is

(A) 8 f′(1) (B) 4f′(1) (D) 2f′(1) (D) None of these

82. If f(x) = $\begin{cases} x-3 & x<0 \\ x^2-3x+2, & x\ge 0 \end{cases}$ and g(x) = f(|x|) + |f(x)| , then g(x) is continuous at

(A) R − {0} (B) R^+

(C) R − {1, 2} (D) R − {0, 1, 2}

83. The value the limit $\lim\limits_{x\to 0}\dfrac{\left(e^{a/x}-e^{-a/x}\right)}{\left(e^{a/x}+e^{-a/x}\right)}$, a > 0 is

(A) 0 (B) 1

(C) infinity (D) does not exist

84. The number of points where g(f(x)) is discontinuous given that g(x) = $\frac{1}{x^2+x-1}$ and f(x) = $\frac{1}{x-3}$ is

(A) 1 (B) 2

(C) 3 (D) 4

85. The value of $\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$ is

(A) e^2 (B) e^3

(C) e^5 (D) none of these

86. The number of points at which the function f(x) = |x − 0.5| + |x − 1| + tanx does not have a derivative in the interval (0, 2) is

(A) 1 (B) 2 (C) 3 (D) 4

87. Let f(x + y) = f(x) f(y) $\forall$ x, y $\in$ R. Suppose that f(3) = 3 and f′(0) = 11 then f′(3) is given by

(A) 22 (B) 44 (C) 28 (D) 33

88. The function f(x) = $\begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ then which of the following is not true

(A) continuous at x = 1 (B) continuous at x = 3

(C) differentiable at x = 1 (D) differentiable at x = 3

89. The function f(x) = max{1 − x, 1 + x, 2}, x $\in$ (−∞, ∞) is

(A) differentiable at all points

(B) differentiable at all points except at x = 1 and x = −1

(C) continuous at all points except at x = 1 and x = −1, where it is discontinuous

(D) None of these

90. Let f(x) = [$\tan^2$x] where [.] is greatest integer function then

(A) $\lim_{x\to 0} f(x)$ does not exist (B) f(x) is continuous at x = 0

(C) f(x) is not differentiable x = 0 (D) f′(0) = 1

LEVEL–III

1. The number of critical points of f (x) = max (sin x , cos x) for $x \in (0 , 2\pi)$

(A) 2 (B) 5
(C) 3 (D) non

2. If $f(x) = \int_0^x (t+1)\,(e^t - 1)\,(t-2)\,(t+4)\,dt$ then f (x) would assume the local minima at;

(A) x = - 4 (B) x = 0
(C) x = -1 (D) x = 2.

3. Let f (x) = [cos x + sin x], $0 < x < 2\pi$ where [x] denotes the greatest integer less than or equal to x. The number of points of discontinuity of f (x) is

(A) 6 (B) 5
(C) 4 (D) 3

4. $\lim\limits_{x\to -1^+} \dfrac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$ ……………………………………………………………………….

5. $f(x) = \dfrac{1}{\log|x|}$ is discontinuous at x = ……………………………………………….

6. The value of the limit

$\lim\limits_{x\to 0}\left\{1^{1/\sin^2 x} + 2^{1/\sin^2 x} + ... + n^{1/\sin^2 x}\right\}^{\sin^2 x}$

(A) ∞ (B) 0
(C) $\dfrac{n(n+1)}{2}$ (D) n

7. $\lim\limits_{x\to 0} \dfrac{\cos(\sin x) - \cos x}{x^4}$ is equal to

(A) 1/5 (B) 1/6
(C) 1/4 (D) ½

8. If $\tan^{-1}(x + h) = \tan^{-1}(x) + (h\ \sin y)(\sin y) - (h\ \sin y)^2 \cdot \dfrac{\sin 2y}{2} + (h\ \sin y)^3 \cdot \dfrac{\sin 3y}{3} + \ . .. .,$ where $x \in (0, 1)$, $y \in (\pi/4, \pi/2)$, then

(A) $y = \tan^{-1}x$ (B) $y = \sin^{-1}x$
(C) $y = \cot^{-1}x$ (D) $y = \cos^{-1}x$

9. The value of $\lim\limits_{|x|\to\infty} \cos[\tan^{-1}(\sin(\tan^{-1} x))]$ is equal to

(A) -1 (B) $\sqrt{2}$ (C) $-\dfrac{1}{\sqrt{2}}$ (D) $\dfrac{1}{\sqrt{2}}$

10. If $\lim\limits_{x\to 0} \int_0^x \dfrac{t^2 dt}{(x - \sin x)\sqrt{a+t}} = 1$, then the value of a is

(A) 4 (B) 2 (C) 1 (D) none of these

12 For some g, let f(x) = x(x+3) $e^{g(x)}$ be a continuous function. If there exists only one point x = d such that f′(d) = 0, then

(A) d < -3 (B) d > 0 (C) $-3 \le d \le 0$ (D) -3 <d < 0

13 $\lim_{n\to\infty}\left[1-\ln\left(1+\frac{1}{n}\right)^{n-1}\right]$ is equal to

(A) 0 (B) 1 (C) e (D) none of these

14 The value of $\lim_{x\to\infty}\frac{x^n+nx^{n-1}+1}{e^{[x]}}$, $n \in I$ is

(A) 1 (B) 0 (C) n (D) n(n –1)

15 Given a function f(x) continuous $\forall\ x \in R$ such that $\lim_{x\to 0}\left[f(x)+\log\left(1-\frac{1}{e^{f(x)}}\right)-\log(f(x))\right]$ = 0, then f(0) is

(A) 0 (B) 1 (C) 2 (D) 3

16 Let R be the set of real numbers and f : R $\to$ R be such that for all x and y in R $| f(x) - f(y) | \le | x - y |^7$. Then f (x) is.

(A) linear (B) constant

(C)quadratic (D) none of these.

17. Find the value of $\lim_{x\to 0}\left(\frac{1}{x^2}-\cot^2 x\right)$

(A) 2/5 (B) 2/3

(C) 1/4 (D) 1/5.

18 $\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$ is

(A) 1 (B) –1

(C) 0 (D) doesn't exist

19 Given that f (x) is a non-zero differentiable function such that f (x + y) = f (x). f (y), $\forall$ x, y $\in$ R, and f′ (0) = 1 then ln f (1) is equal to

(A) 0 (B) 1

(C) e (D) none of these

20 The largest interval where the function f (x) = $\frac{x}{1+|x|}$ is differentiable

(A) $(-\infty, \infty)$ (B) $(0, \infty)$

(C) $(-\infty, 0) \cup (0, \infty)$ (D) none of these

21 $\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{ex}{2}}{x^2}$ is equal to

(A) $\frac{11e}{24}$ (B) $-\frac{11e}{24}$ (C) $\frac{e}{24}$ (D) None of these

22 The value of the limit $\lim_{x\to0}\left(\frac{1-3^x-4^x+12^x}{\sqrt{2\cos x+7}-3}\right)$ is

(A) 0
(B) − 6(log3) (log4)
(C) 1
(D) none of these

23 Let $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$, for all x, y $\in$ R and if f(x) is differentiable, and $f'(0) = -1$, $f(0) = 1$ then the function f(x) is

(A) −x + 1
(B) x + 1
(C) $x^2 - 1$
(D) x −1

24 The points of discontinuity of the function fog where g(x) = $\frac{1}{x-1}$ and f(x) = $\frac{1}{x^2+x-2}$ are

(A) $\frac{1}{2}$, 2, 1 (B) 2, 1 (B) 2, $\frac{1}{2}$ (D) none of these

ANSWERS

LEVEL −I

1.	C	2.	B	3.	A	4.	C

5.	B	6.	C	7.	1	8.	e^2
9.	−2, −1	10.	C	11.	D	12.	B
13.	D	14.	D	15.	C	16.	A
17.	A	18.	D	19.	A	20.	B
21.	B	22.	B	23.	D	24.	A
25.	A	26.	B	27.	B	28.	C
29.	A	30.	B	31.	A	32.	C
33.	C	34.	A	35.	C	36.	B
37.	A	38.	D	39.	A	40.	A
41.	A	42.	B	43.	B	44.	D
45.	D	46.	−2, 1	47.	$f(x) = x^3/3 + x^2/2 - 2x + 2$		
48.	B						
49.	A	50.	B	51.	C	52.	A

LEVEL −II

1.	D	2.	C	3.	A	4.	D
5.	A	6.	B	7.	A, C	8.	A
9.	D	10.	C	11.	A	12.	C
13.	B, D	14.	0	15.	7	16.	$(2a\alpha + b)^2 / 2$
17.	−1	18.	0, π, 2π	19.	6	20.	B
21.	B	22.	C	23.	B	24.	B
25.	A	26.	B	27.	B	28.	A
29.	C	30.	B	31.	D	32.	C
33.	C	34.	C	35.	A	36.	C
37.	A	38.	C	39.	D	40.	C
41.	A	42.	D	43.	B	44.	A
45.	B	46.	C	47.	B	48.	D
49.	A	50.	B	51.	A	52.	B
53.	C	54.	A	55.	B	56.	D
57.	C	58.	B	59.	D	60.	B
61.	C	62.	A	63.	C	64.	D
65.	B	66.	D	67.	A	68.	A
69.	B	70.	B	71.	B	72.	C
73.	B	74.	B	75.	D	76.	D
77.	C	78.	A	79.	A	80.	B
81.	A	82.	A	83.	D	84.	C
85.	A	86.	C	87.	D	88.	D
89.	B	90.	B				

LEVEL −III

1.	C	2.	C	3.	B	4.	$1 / \sqrt{2\pi}$
5.	0, ±1	6.	D	7.	D	8.	C
9.	D	10.	A			12.	D
13.	A	14.	B	15.	A	16.	B
17.	B	18.	D	19.	B	20.	A
21.	A	22.	B	23.	A	24.	A