

CHAPTER 8

APPLICATIONS OF INTEGRALS

LONG ANSWER TYPE QUESTIONS (6 MARKS)

- Find the area enclosed by circle $x^2 + y^2 = a^2$.
- Find the area of region bounded by $\{(x, y) : |x - 1| \leq y \leq \sqrt{25 - x^2}\}$.
- Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- Find the area of region in the first quadrant enclosed by x -axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.
- Find the area of region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$
- Prove that the curve $y = x^2$ and, $x = y^2$ divide the square bounded by $x = 0, y = 0, x = 1, y = 1$ into three equal parts.
- Find smaller of the two areas enclosed between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line
 $bx + ay = ab$.
- Find the common area bounded by the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.
- Using integration, find the area of the region bounded by the triangle whose vertices are
(a) $(-1, 0), (1, 3)$ and $(3, 2)$ (b) $(-2, 2), (0, 5)$ and $(3, 2)$
- Using integration, find the area bounded by the lines.
(i) $x + 2y = 2, y - x = 1$ and $2x + y - 7 = 0$
(ii) $y = 4x + 5, y = 5 - x$ and $4y - x = 5$.
- Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.
- Find the area of the region bounded by
 $y = |x - 1|$ and $y = 1$.

13. Find the area enclosed by the curve $y = \sin x$ between $x = 0$ and $x = \frac{3\pi}{2}$ and x -axis.
14. Find the area bounded by semi circle $y = \sqrt{25 - x^2}$ and x -axis.
15. Find area of region given by $\{(x, y) : x^2 \leq y \leq |x|\}$.
16. Find area of smaller region bounded by ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and straight line $2x + 3y = 6$.
17. Find the area of region bounded by the curve $x^2 = 4y$ and line $x = 4y - 2$.
18. Using integration find the area of region in first quadrant enclosed by x -axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.
19. Find smaller of two areas bounded by the curve $y = |x|$ and $x^2 + y^2 = 8$.
20. Find the area lying above x -axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.
21. Using integration, find the area enclosed by the curve $y = \cos x$, $y = \sin x$ and x -axis in the interval $\left(0, \frac{\pi}{2}\right)$.
22. Sketch the graph $y = |x - 5|$. Evaluate $\int_0^6 |x - 5| dx$.
23. Find area enclosed between the curves, $y = 4x$ and $x^2 = 6y$.
24. Using integration, find the area of the following region :

$$\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\}$$

ANSWERS

1. πa^2 sq. units.
2. $\left(25\frac{\pi}{4} - \frac{1}{2}\right)$ sq. units.
3. πab sq. units
4. $(4\pi - 8)$ sq. units
5. $\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{8} \sin^{-1}\left(\frac{1}{3}\right)$ sq. units
7. $\frac{(\pi - 2)ab}{4}$ sq. units
8. $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ sq. units
9. (a) 4 sq. units (b) 2 sq. units
10. (a) 6 sq. unit **[Hint.** Coordinate of vertices are (0, 1) (2, 3) (4, - 1)]

(b) $\frac{15}{2}$ sq. **Hint :** Coordinate of vertices are $(-1, 1)$ $(0, 5)$ $(3, 2)$

11. $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq. units

12. 1 sq. units

13. 3 sq. units

14. $\frac{25}{2} \pi$ sq. units

15. $\frac{1}{3}$ sq. units

16. $\frac{3}{2}(\pi - 2)$ sq. units

17. $\frac{9}{8}$ sq. units

18. $\frac{\pi}{3}$ sq. unit

19. 2π sq. unit.

20. $\frac{4}{3}(8 + 3\pi)$ sq. units

21. $(2 - \sqrt{2})$ sq. units.

22. 13 sq. units.

23. 8 sq. units.

24. $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq. units