CBSE-XII-2015 EXAMINATION

PHYSICS

Paper & Solution

Code: 55/2/A

Time: 3 Hrs.

Max. Marks: 70

General Instructions:

(i) All questions are compulsory. There are **26** questions in all.
(ii) This question paper has **five** sections : Section A, Section B, Section C, Section D, Section E.
(iii) Section A contain **five** questions of **one** mark each. Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains **one** value based question of **four** marks and Section E contains **three** questions of **five** marks each.
(iv) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all the three questions of five marks weightage. You have to attempt only one of the choices in such questions.
(v) You may use the following values of physical constants wherever necessary :

$c = 3\times10^{8}\ m/s$

$h = 6.63\times10^{-34}\,Js$

$e = 1.6\times10^{-19}\,C$

$\mu_0 = 4\pi\times10^{-7}\,T\ mA^{-1}$

$\varepsilon_0 = 8.854\times10^{-12}\,C^2N^{-1}m^{-2}$

$\dfrac{1}{4\pi\varepsilon_0} = 9\times10^{9}\ N\ m^2\ C^{-2}$

$m_e = 9.1\times10^{-31}\,kg$

Mass of Neutrons = $1.675\times10^{-27}\,kg$

Mass of proton = $1.673\times10^{-27}\,kg$

Avogadro's number = 6.023×10^{23} per gram mole

Boltzmann constant = $1.38\times10^{-23}\,JK^{-1}$

Section A

1. What is the function of a band pass filter used in a modulator for obtaining AM signal ?

Solution:

The output produced by square law device is passed to band pass filter which rejects the dc and the sinusoids of frequencies wm, 2wm and 2wc and retains the frequencies wc, wc – wm and wc + wm. The output of band pass filter is an AM wave.

2. A planar loop of rectangular shape is moved within the region of a uniform magnetic field acting perpendicular to its plane. What is the direction and magnitude of the current induced in it ?

Solution:

The magnetic flux linked with a circuit is not changing with time so there will be no induced current in the loop.

3. When light travels from an optically denser medium to a rarer medium, why does the critical angle of incidence depend on the colour of light ?
Solution:
The critical angle for a given pair of medium is given by

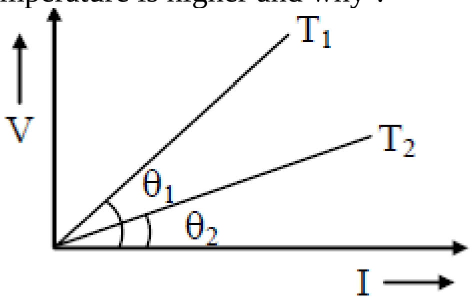
$$Sin\, i_c = \frac{1}{\mu}$$

where μ is refractive index of denser medium. The refractive index of a medium depends on the wavelength by the relation

$$\mu = A + \frac{B}{\lambda^2}$$

and each colour of light is associated with specific wavelength, so as the wavelength of light increases critical angle increases.

4. V-I graph for a metallic wire at two different temperature T1 and T2 is as shown in the figure. Which of the
two temperature is higher and why ?



Solution:
The slope of V-I graph gives the resistance of the metallic wire and the slope is higher at temperature T1 and we know that on increasing the temperature of metallic wire resistance of the wire increases, so T1 temperature is higher.

5. Define dielectric constant of a medium. What is its S.I. unit ?

Solution:
Dielectric constant (or relative permittivity) of a dielectric is the ratio of the absolute permittivity of a medium to the absolute permittivity of free space.

$$K = \frac{\varepsilon}{\varepsilon_0}$$

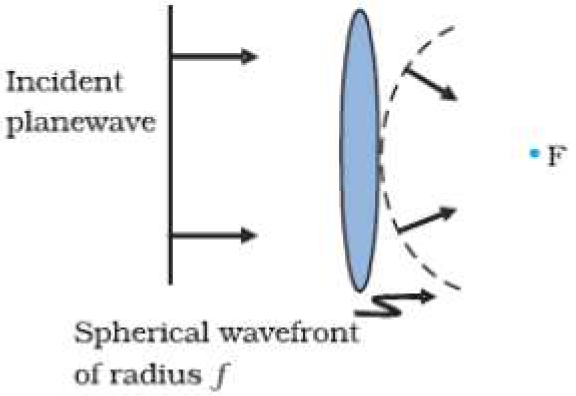
It is unit less quantity.

Section B

6. Define a wave front. Using 'Huygens' principle, draw the shape of a refracted wave front, when a plane wave is incident on a convex lens.

or

(a) When a wave is propagating from a rarer to a denser medium, which characteristic of the wave does not change and why ?
(b) What is the ratio of the velocity of the wave in the two media of refractive indices μ1 and μ2 ?
Solution:
The locus of points, which oscillate in phase is called a wave front; thus a wave front is defined as a surface of constant phase.



In the given figure we consider a plane wave incident on a thin convex lens, the emerging wave front is spherical and converges to the point F which is known as the focus.

OR

(a) Frequency of a wave does not change when the wave is propagating from a rarer to a denser medium because frequency (ν) is given by the relation

$$\nu = \frac{v}{\lambda}$$

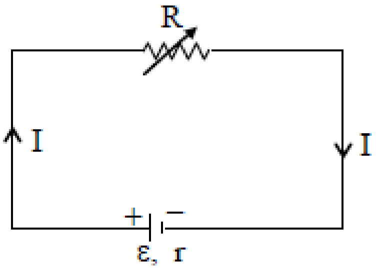
As the medium changes velocity (v) and wavelength (λ) changes such that ratio remains constant.

(b) $\frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$

Here v1 and v2 are the velocity of the wave in medium 1 and medium 2 and μ1 and μ2 are the refractive index of medium 1 and medium 2.

7. A variable resistor R is connected across a cell of emf ε and internal resistance r as shown in the figure. Draw a plot showing the variation of

(i) Terminal voltage V and (ii) the current I, as a function of R.

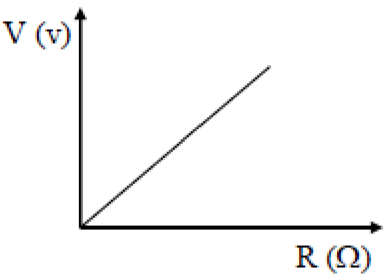


Solution:

(i) Terminal voltage across a cell as a function of R

As resistance R increases current (I) in the circuit decreases and terminal voltage (V) increases.

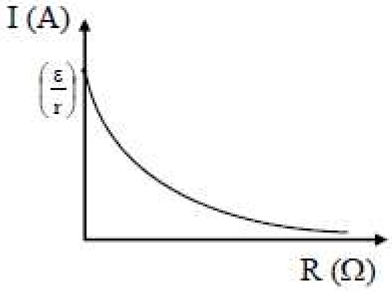
We know V = ε – Ir ; Where ε is emf of the cell



(ii) Current I as a function of R.

The current across a cell is given by $I = \frac{\varepsilon}{R+r}$

When R increases I decreases

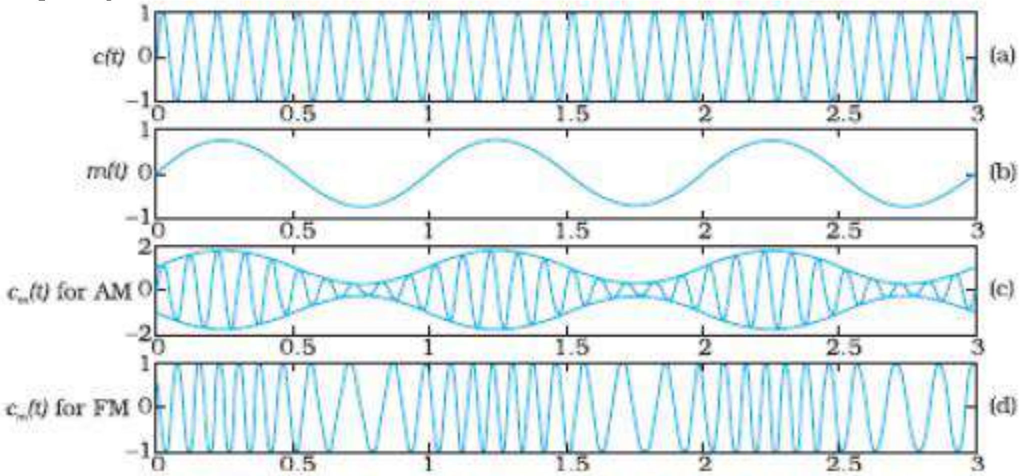


8. Differentiate between amplitude modulated (AM) and frequency modulated (FM) waves by drawing suitable diagrams. Why is FM signal preferred over AM signal ?

Solution:

When the amplitude of carrier wave is changed in accordance with the intensity of the signal, it is called amplitude modulation.

When the frequency of carrier wave is changed in accordance with the intensity of the signal, it is called frequency modulation.



Modulation of a carrier wave: (a) a sinusoidal carrier wave; (b) a modulating signal; (c) amplitude modulation; (d) frequency modulation

FM signal is preferred over AM signal because

(i) Various electrical machines and noises cause amplitude disturbance in the transmission of amplitude modulated wave. This makes the reception noisy. So, there is a need for Frequency Modulation which can reduce the noise factor.

(ii) Fidelity or audio quality of amplitude modulated transmission is poor. This type of transmission is also not good for musical programmes. There is a need to eliminate amplitude-sensitive noise. This is possible if we eliminate amplitude variation. In other words, there is a need to keep the amplitude of the carrier constant. This is precisely what we do in frequency modulation.

9. Determine the distance of closest approach when an alpha particle of kinetic energy 4.5 MeV strikes a nucleus of Z = 80, stops and reverses its direction.

Solution:

At the distance of nearest approach

$PE = KE$

$$\frac{k(ze)(2e)}{r_0} = 4.5\,MeV = 4.5\times10^6\times1.6\times10^{-19}\,J$$

$$r_0 = \frac{k(ze)(2e)}{4.5\times1.6\times10^{-13}}$$

$$= \frac{9\times10^9\times(80)\times2\times(1.6\times10^{-19})^2}{4.5\times1.6\times10^{-13}} = 51.2\times10^{-15}\,m$$

10. When the electron orbiting in hydrogen atom in its ground state moves to the third excited state, show how the de Broglie wavelength associated with it would be affected.

Solution:

The velocity of a electron in a hydrogen atom is given by the relation

$$v_n = \frac{e^2}{2n\varepsilon_0 n}\ so\ v_n \propto \frac{1}{n}\ ...(i)$$

and the de Broglie wavelength associated with it is $\lambda = \frac{h}{p} = \frac{h}{mv}$

So $\lambda \propto \frac{1}{v_n}$ ...(ii)

using equation (i) and (ii) $\lambda \propto n$

So when electron jump from n = to n = 4 level

$$\frac{\lambda_1}{\lambda_2} = \frac{n_1}{n_2} = \frac{1}{4}$$

$$\lambda_2 = 4\lambda_1$$

So wavelength increases four times.

Section C

11. In Young's double slit experiment, the two slits are separated by a distance of 1.5 mm and the screen is placed 1 m away from the plane of the slits. A beam of light consisting of two wavelengths 650 nm and 520 nm is used to obtain interference fringes. Find

(a) the distance of the third bright fringe for λ = 520 nm on the screen from the central maximum.

(b) the least distance from the central maximum where the bright fringes due to both the wavelengths coincide.

Solution:

(a). Third bright fringe for $\lambda1$ = 520 nm is given by

$$x_3 = \frac{3\lambda D}{d} = \frac{3\times520\times10^{-9}\times1}{1.5\times10^{-3}} = 1.04\times10^{-3}\,m$$

$$= 1.04\,mm$$

(b) Let n1 bright band of $\lambda1$ = 520 nm coincides with n2 bright band of $\lambda2$ = 650 nm

So $\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$

$$n_1\lambda_1 = n_2\lambda_2$$

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{650}{520} = \frac{5}{4}$$

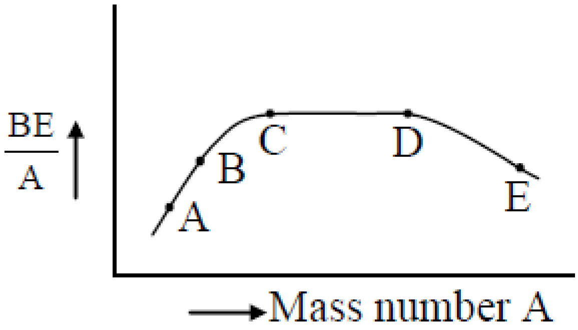
so the least distance from the central maximum where the bright fringes due to both the wavelengths coincide is

$$x = \frac{n_1 \lambda_1 D}{d} = \frac{5 \times 520 \times 10^{-9} \times 1}{1.5 \times 10^{-3}}$$

$$= 1.73 \times 10^{-3} m$$

$$= 1.73\, mm$$

12. (a) The figure shows the plot of binding energy (BE) per nucleon as a function of mass number A. The letters A, B, C, D and E represent the positions of typical nuclei on the curve. Point out, giving reasons, the two processes (in terms of A, B, C, D and E), one of which can occur due to nuclear fission and the other due to nuclear fusion.



(b) Identify the nature of the radioactive radiations emitted in each step of the decay process given below.

$${}^{A}_{Z}X \to {}^{A-4}_{Z-1}Y \to {}^{A-4}_{Z-1}W$$

Solution:

(a) The nuclei at A and B undergo nuclear fusion as their binding energy per nucleon is small and they are less stable so they fuse with other nuclei to become stable. The nuclei at E undergo nuclear fission as its binding energy per nucleon is less it splits into two or more lighter nuclei and become stable.

(b) ${}^{A}_{Z}X \to {}^{A-4}_{Z-2}Y \to {}^{A-4}_{Z-1}W$

An alpha particle $\left({}^{4}_{2}He\right)$ is emitted in the first reaction as atomic mass of Y is reduced by 4 and atomic number is reduced by 2. An electron $\left({}^{0}_{-1}e\right)$ is emitted in the second reaction as atomic mass of W remains the same and atomic number is increased by 1.

13. Name the three different modes of propagation in a communication system. State briefly why do the electromagnetic waves with frequency range from a few MHz upto 30 MHz can reflect back to the earth. What happens when the frequency range exceeds this limit?

Solution:

The three different modes of propagation in a communication system are

(1) Ground wave

(2) Sky wave

(3) Space wave

In the frequency range from a few MHz up to 30 to 40 MHz, long distance communication can be achieved by ionospheric reflection of radio waves back towards the earth. This mode of propagation is called sky wave propagation and is used by short wave broadcast services. The ionosphere is so called because of the presence of a large number of ions or charged particles. It extends from a height of ~ 65 Km to about 400 Km above the earth's surface. Ionisation occurs due to the absorption of the ultraviolet and other high-energy radiation coming from the sun by air molecules.

The ionosphere is further subdivided into several layers. The degree of ionisation varies with the height. The density of atmosphere decreases with height. At great heights the solar radiation is intense but there are few molecules to be ionised. Close to the earth, even though the molecular concentration is very high, the radiation intensity is low so that the ionisation is again low. However, at some intermediate heights, there occurs a peak of ionisation density.

The ionospheric layer acts as a reflector for a certain range of frequencies (3 to 30 MHz). Electromagnetic waves of frequencies higher than 30 MHz penetrate the ionosphere and escape. The phenomenon of bending of em waves so that they are diverted towards the earth is similar to total internal reflection in optics

14. Define the terms "stopping potential' and 'threshold frequency' in relation to photoelectric effect. How does one determine these physical quantities using Einstein's equation ?

Solution:

Stopping potential : For a particular frequency of incident radiation, the minimum negative (retarding) potential V_0 given to the anode plate for which the photocurrent stops or becomes zero is called the cut-off or stopping potential. Threshold frequency : There exists a certain minimum cut-off frequency ν_0 for which the stopping potential is zero and below ν_0 the electron emission is not possible.

This cut-off frequency is known as threshold frequency ν_0, which is different for different metal. In photoelectric effect, an electron absorbs a quantum of energy ($h\nu$) of radiation. If this quantum of energy absorbed by electron exceeds the minimum energy required to come out of the metal surface by electron, the kinetic energy of the emitted electron is

$K = h\nu - \varphi$ ... (1)

where, φ is the minimum energy for electron to come out of the metal, and is different for different electrons in the metal. The maximum kinetic energy of photo electrons is given by

$K_{max} = h\nu - \varphi 0$ ... (2)

where, $\varphi 0$ – work function or least value of φ equation (2) is known as Einstein's photoelectric equation.

Explanation of photoelectric effect with the help of Einstein's photoelectric equation

(i) According to equation (2), K_{max} depends linearly on ν, and is independent of intensity of radiation. This happens because, here, photoelectric effect arises from the absorption of a single quantum of radiation by a single electron. The intensity of radiation (that is proportional to the number of energy quanta per unit area per unit time) is irrelevant to this basic process.

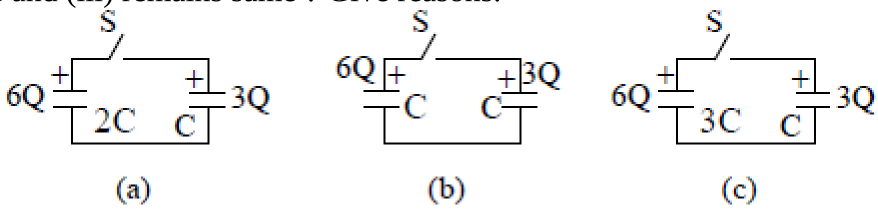
(ii) Since K_{max} must be non-negative, equation (2) implies that photoelectric emission is possible only if $h\nu > \varphi 0$

or $\nu > \nu_0$, where $\nu_0 = V_0 = \frac{\phi_0}{h}$

Thus, there exists a threshold frequency $\nu_0 = V_0 = \frac{\phi_0}{h}$ exists, below which photoelectric emission is not possible, and is independent of intensity.

(iii) As intensity of radiation is proportional to the number of energy quanta per unit area per unit time. The greater the number of energy quanta available, the greater is the number of electrons absorbing the energy quanta and therefore, the number of electrons coming out of the metal (for $\nu > \nu_0$) is more and so is photoelectric current.

15. Three circuits, each consisting of a switch 'S' and two capacitors, are initially charged, as shown in the figure. After the switch has been closed, in which circuit will the charge on the left-hand capacitor (i) increase, (ii) decrease and (iii) remains same ? Give reasons.

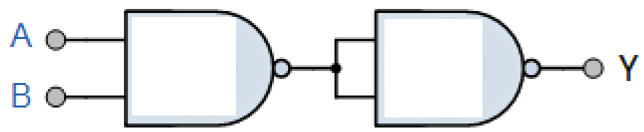


Solution:
When charged capacitors are connected to each other then the charge will flow from the capacitor with higher potential towards the capacitor with lower potential untill a common potential is reached.
(a) In fig. (a) the potential of both the capacitor is same so the charge on left hand capacitor remains the same
(b) In fig. (b) the potential of left hand capacitor is high so charge from 6Q to 3Q. Therefore charge on left hand capacitor will decrease.
(c) In fig. (c) the potential of left hand capacitor is low so charge will flow from 3Q to 6Q. Therefore charge on left hand capacitor will increase.

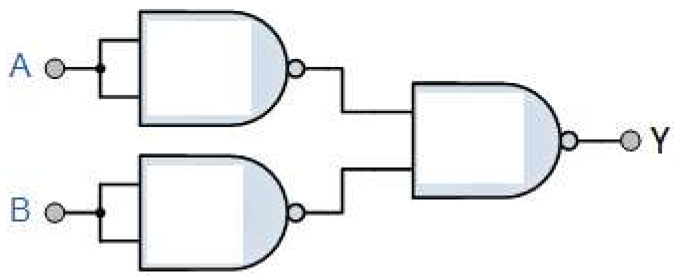
16. The outputs of two NOT gates are fed to a NOR gate. Draw the logic circuit of the combination of gates. Write its truth table. Identify the gate equivalent to this circuit.

OR

You are given circuits (a) and (b) as shown in the figures, which consists of NAND gates. Identify the logic operation carried out by the two. Write the truth tables for each. Identify the gates equivalent to the tow circuits.

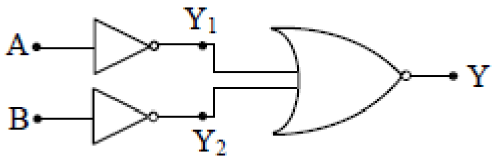


a)



b)

Solution:



$Y_1 = \overline{A}$

$Y_2 = \overline{B}$

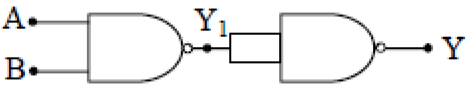
$Y = \overline{Y_1 + Y_2} = \overline{\overline{A} + \overline{B}} = \overline{\overline{A}}.\overline{\overline{B}} = A.B$

The equivalent gate is AND gate

Truth table

A	B	Y_1	Y_2	Y
0	0	1	1	0
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

OR

(a) 

Y_1 = **Error! Objects cannot be created from editing field codes.**

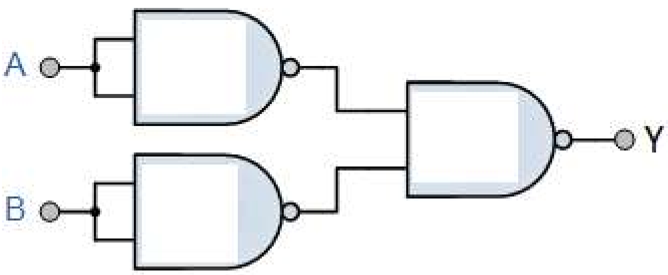
$Y = \overline{(\overline{A.B}).(\overline{A.B})} = (\overline{\overline{A.B}}).(\overline{\overline{A.B}}) = (A.B) + (A.B)$

$= A.B$

The equivalent gate is AND Gate

Truth table

A	B	Y_1	Y
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1



(b)

$Y_1 = \overline{A.A} = \overline{A}$

$Y_2 = \overline{B.B} = \overline{B}$

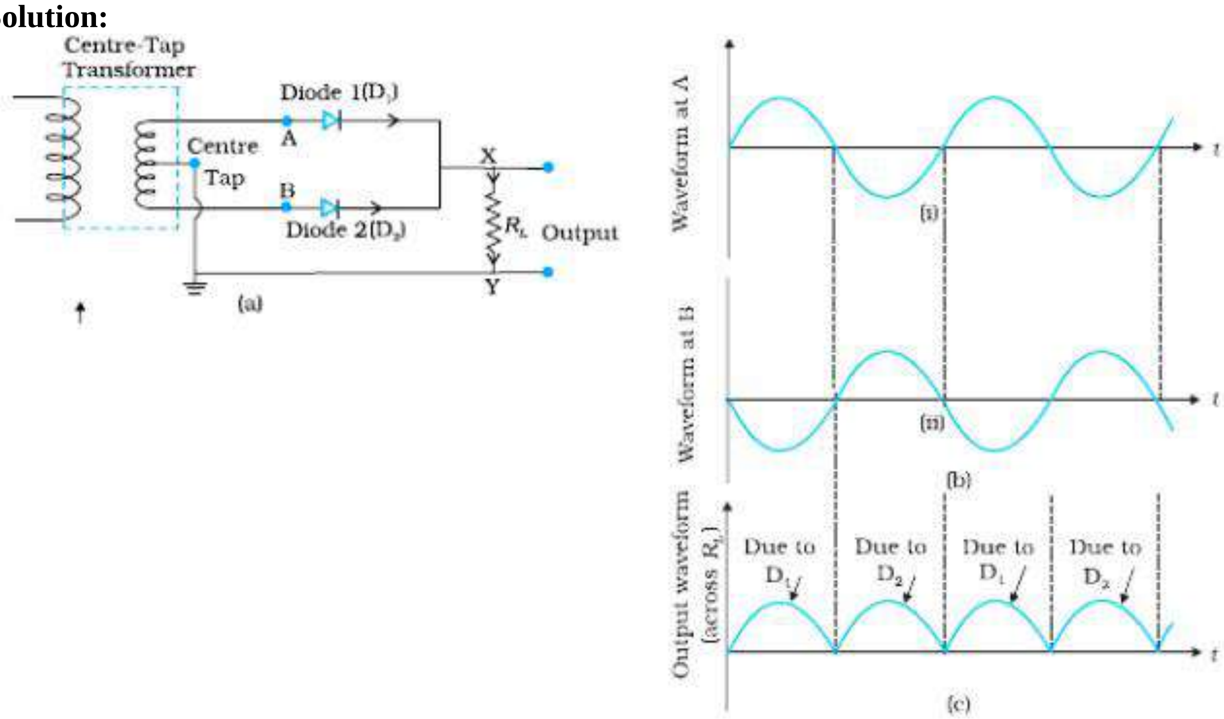
$Y = \overline{Y_1.Y_2} = \overline{\overline{A}.\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

The equivalent gate is OR gate
Truth table

A	B	Y1	Y2	Y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

17. With the help of a circuit diagram, explain the working of a junction diode as a full wave rectifier. Draw its input and output waveforms. Which characteristic property makes the junction diode suitable for rectification?

Solution:



(a) A Full-wave rectifier circuit; (b) Input wave forms given to the diode D1 at A and to the diode D2 at B; (c) Output waveform across the load RL connected in the full-wave rectifier circuit.

The circuit using two diodes, shown in Fig.(a), gives output rectified voltage corresponding to both the positive as well as negative half of the ac cycle. Hence, it is known as full-wave rectifier.
Here the p-side of the two diodes are connected to the ends of the secondary of the transformer. The n-side of the diodes are connected together, and the output is taken between this common point of diodes and the midpoint of the secondary of the transformer. So for a full-wave rectifier the secondary of the transformer is provided with a centre tapping and so it is called centre-tap transformer.

As can be seen from Fig.(c) the voltage rectified by each diode is only half the total secondary voltage. Each diode rectifies only for half the cycle, but the two do so for alternate cycles. Thus, the output between their common terminals and the centretap of the transformer becomes a full-wave rectifier output. Suppose the

input voltage to A with respect to the centre tap at any instant is positive. It is clear that, at that instant, voltage at B being out of phase will be negative as shown in Fig.(b). So, diode D1 gets forward biased and conducts (while D2 being reverse biased is not conducting). Hence, during this positive half cycle we get an output current (and a output voltage across the load resistor RL) as shown in Fig.(c). In the course of the ac cycle when the voltage at A becomes negative with respect to centre tap, the voltage at B would be positive. In this part of the cycle diode D_1 would not conduct but diode D_2 would, giving an output current and output voltage (across R_L) during the negative half cycle of the input ac. Thus, we get output voltage during both the positive as well as the negative half of the cycle.
The diode under forward biased offers negligible resistance so it will conduct while under reverse biased it offers very high resistance so it will not conduct. Therefore it is a unidirectional device which conducts only in one direction. This characteristic property makes the junction diode suitable for rectification

18. A potential difference V is applied across a conductor of length L and diameter D. How is the drift velocity, v_d, of charge carriers in the conductor affected when (i) V is halved, (ii) L is doubled and (iii) D is halved ? justify your answer in each case.
Solution
We know drift velocity is given by

$|v_d| = \frac{e\tau}{m}E$ Also $E = \frac{V}{L}$

So $|v_d| = \frac{e\tau}{m}\left(\frac{V}{L}\right)$

(i) When V is halved drift velocity (v_d) gets halved
(ii) When L is doubled drift velocity (v_d) gets halved
(iii) When D is halved drift velocity (v_d) remains same.

19. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its 'dees' is 60 cm, calculate the kinetic energy (in MeV) of the proton beam produced by the accelerator.
Solution
Frequency of oscillators $(v) = 10\ MH_Z = 10_7\ H_Z$

Mass of proton, $m = 1.67 \times 10^{-27}\ kg$

Charge of proton $= 1.6 \times 10^{-19}\ C$

Operating magnetic field is given by the relation

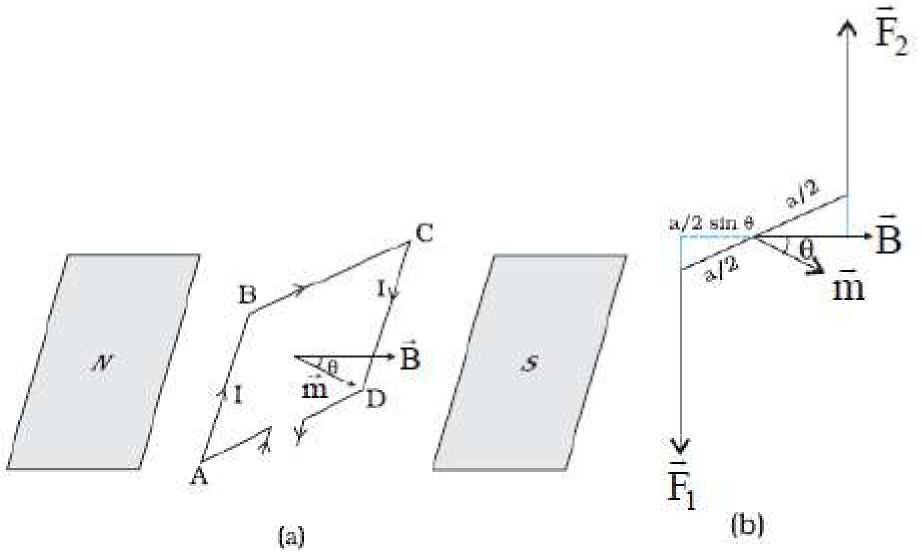
$$B = \frac{2\pi mv}{q} = \frac{2 \times 3.14 \times 1.67 \times 10^{-27} \times 10^{7}}{1.6 \times 10^{-19}}$$

$= 0.65\,T$

Radius of dees = 60 cm = 0.6 m

$$KE = \frac{q^2B^2r^2}{2m} = \frac{(1.6 \times 10^{-19})^2 (0.65)^2 (0.6)^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-13}} Mev$$

20. Deduce the expression for the torque $\vec{\tau}$ acting on a planar loop of area $\vec{A}$ and carrying current I placed in a uniform magnetic field $\vec{B}$. If the loop is free to rotate, what would be its orientation in stable equilibrium ?
Solution:



(a) The area vector of the loop ABCD makes an arbitrary angle θ with the magnetic field.
(b) Top view of the loop. The forces F_1 and F_2 acting on the arms AB and CD are indicated.
We consider the case when the plane of the loop, is not along the magnetic field, but makes an angle θ with it. Fig.(a) illustrates this general case. The forces on the arms BC and DA are equal, opposite, and act along the axis of the coil, which connects the centres of mass of BC and DA. Being collinear along the axis they cancel each other, resulting in no net force or torque. The forces on arms AB and CD are F_1 and F_2. They too are equal and opposite, with magnitude,
$F_1 = F_2 = I\,b\,B$
But they are not collinear! This results in a couple Fig.(b) is a view of the arrangement from the AD end and it illustrates these two forces constituting a couple. The magnitude of the torque on the loop is,

$$\tau = F_1 \frac{a}{2} \sin\theta + F_2 \frac{a}{2} \sin\theta$$

$$= I\, ab\, B \sin\theta$$

$= I\,A\,B \sin\theta$ ... (i)
As $\theta \to 0,$ the perpendicular distance between the forces of the couple also approaches zero. This makes the forces collinear and the net force and torque zero. The torques in above equation can be expressed as vector product of the magnetic moment of the coil and the magnetic field. We define the magnetic moment of the current loop as,
$m = I\,A$
where the direction of the area vector A is given by the right-hand thumb rule and is directed into the plane of the paper in Fig.(a). Then as the angle between m and B is θ, equation (i) can be expressed by one expression

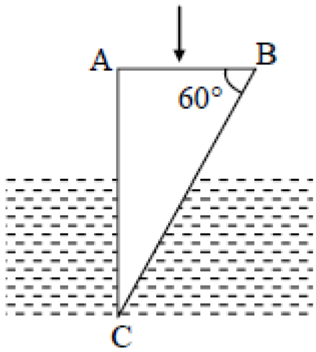
$$\vec{\tau} = \vec{m} \times \vec{B}$$

we see that the torque $\vec{\tau}$ vanishes when $\vec{m}$ is either parallel or antiparallel to the magnetic field $\vec{B}$ This indicates a state of equilibrium as there is no torque on the coil (this also applies to any object with a magnetic moment $\vec{m}$). When $\vec{m}$ and $\vec{B}$ re parallel the equilibrium is a stable one.

21. How are electromagnetic waves produced ? What is the source of the energy carried by a propagating electromagnetic wave ?
Identify the electromagnetic radiations used
(i) In remote switches of household electronic device; and
(ii) as diagnostic tool in medicine

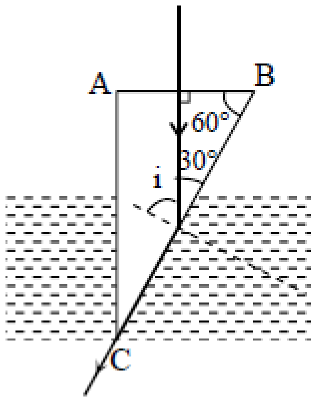
Solution:
According to Maxwell's theory accelerated charges radiate electromagnetic waves. Consider a charge oscillating with some frequency. (An oscillating charge is an example of accelerating charge.) This produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn, is a source of oscillating electric field, and so on. The oscillating electric and magnetic fields thus regenerate each other, so to speak, as the wave propagates through the space. The frequency of the electromagnetic wave naturally equals the frequency of oscillation of the charge. The energy associated with the propagating wave comes at the expense of the energy of the source – the accelerated charge.
The electromagnetic radiation used are :
(1) Infrared rays
(2) X-rays

22. (a) A ray of light is incident normally on the face AB of a right angled glass prism of refractive index ${}_a\mu_g = 1.5$. The prism is partly immersed in a liquid of unknown refractive index. Find the value of refractive index of the liquid so that the ray grazes along the face BC after refraction through the prism.



(b) Trace the path of the rays if it were incident normally on the face AC.
Solution:

(a)



Given the refractive index of prism μg = 1.5 the ray will grazes along the face BC when the angle of incidence i is equal to the critical angle for the glass and liquid interface here i = 60° (from the fig.)

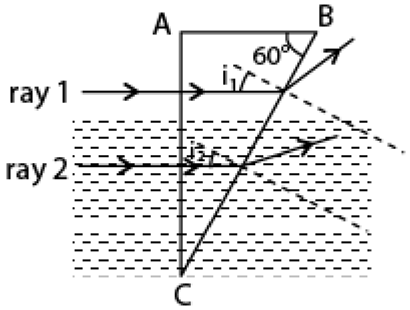
and sin i $= \dfrac{\mu_\ell}{\mu_g}$

$\mu_\ell = u_g \sin i$

$\mu_\ell = 1.5 \sin 60^\circ$

$\mu_\ell = 1.3$

So the refractive index of the liquid is 1.3



The angle of incidence for ray 1 and ray 2 on face BC is equal to 30°
For ray 1
The critical angle for glass-air interface is

$$\sin i_c = \frac{\mu_a}{\mu_g} = \frac{1}{1.5} = .66$$

$$i_c = 41^\circ$$

Now since the angle of incidence is smaller than the critical angle refraction will take place as shown in figure
For ray 2
the critical angle for glass-liquid interface is

$$\sin i_c = \frac{\mu_\ell}{\mu_g} = \frac{1.3}{1.5} = 0.866$$

$$i_c = 60^\circ$$

Now since the angle of incidence is smaller than the critical angle refraction will take place as shown in figure

Section D

23. Sunita and her friends visited an exhibition . The policeman asked them to pass through a metal detector. Sunita's friends were initially scared of it. Sunita, however, explained to them the purpose and working of the metal detector.
Answer the following questions :
(a) On what principle does a metal detector work ?
(b) Why does the detector emit sound when a person carrying any metallic object walks through it ?
(c) State any two qualities which Sunita displayed while explaining the purpose of walking through the detector.
Solution:
(a) The metal detector works on the principle of resonance in ac circuit.
(b) When Sunita's friend is passed through a metal detector, her friend in fact, passing through a coil of many turns. The coil is connected to a capacitor tuned so that the circuit is in resonance. When she walk through the metal in her pocket, the impedance of the circuit changes-resulting in significant change in current in the circuit. The change in current is detected and the electric circuitry causes a sound to be emitted as an alarm.
(c) Curiosity, power of observation.

Section E

24. (a) State Faraday's law of electromagnetic induction .
(b) Explain, with the help of a suitable example, how we can show that Lenz's law is a consequence of the principle of conservation of energy.

(c) Use the expression for Lorentz force acting on the charge carriers of a conductor to obtain the expression for the induced emf across the conductor of length l moving with velocity v through a magnetic field B acting perpendicular to its length.

OR

(a) Using phasor diagram, derive the expression for the current flowing in an ideal inductor connected to an a.c. source of voltage, $v = v_0 \sin \omega t$. Hence plot graphs showing variation of (i) applied voltage and (ii)the current as a function of ωt.

(b) Derive an expression for the average power dissipated in a series LCR circuit.

Solution

(a) Faraday gave laws for relating induced emf to the flux. These are given as under :

(i) Whenever there is a change of magnetic flux through a circuit, there will be an induced emf and this will last as long as the change persists.

(ii) The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit.

Mathematically, the induced emf is given by

$$\varepsilon = -\frac{d\phi_B}{dt}$$

(b) Lenz's law states that the polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it. The negative sign shown in equation $\varepsilon = -\frac{d\phi_B}{dt}$ represents this effect.

Conservation of energy :

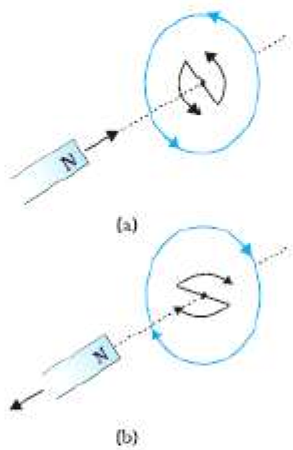
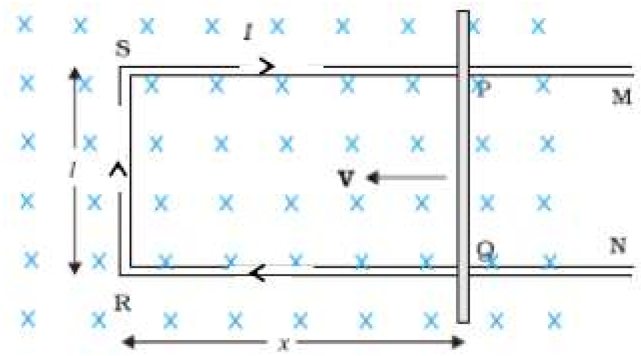


Illustration of Lenz's law.

Suppose that the induced current was in the direction opposite to the one depicted in Fig.(a). In that case, the South-pole due to the induced current will face the approaching North-pole of the magnet. The bar magnet will then be attracted towards the coil at an ever increasing acceleration. A gentle push on the magnet will initiate the process and its velocity and kinetic energy will continuously increase without expending any energy. If this can happen, one could construct a perpetual-motion machine by a suitable arrangement. This violates the law of conservation of energy and hence can not happen. Now consider the correct case shown in Fig.(a). In this situation, the bar magnet experiences a repulsive force due to the induced current. Therefore, a person has to do work in moving the magnet. Where does the energy spent by the person go? This energy is dissipated by Joule heating produced by the induced current.

(c)



The arm PQ is moved to the left side, thus decreasing the area of the rectangular loop. This movement induces a current *I* as shown.

Let us consider a straight conductor moving in a uniform and time independent magnetic field. Figure shows a rectangular conductor PQRS in which the conducto r PQ is free to move. The rod PQ is moved towards the left with a constant velocity v as shown in the figure. Assume that there is no loss of energy due to friction. PQRS forms a closed circuit enclosing an area that changes as PQ moves. It is placed in a uniform magnetic field B which is perpendicular to the plane of this system. If the length RQ=x and RS $=\ell$ the magnetic flux ϕ_B enclosed by the loop PQRS will be

$$\phi_B = B\ell x$$

Since x is changing with time, the rate of change of flux φB will induce an emf given by:

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d\left(B\ell x\right)}{dt}$$

$$= -B\ell\frac{dx}{dt} = B\ell v$$

where we have used dx/dt = –v which is the speed of the conductor PQ. The induced emf $B\ell v$ is called motional emf. Thus, we are able to produce induced emf by moving a conductor instead of varying the magnetic field, that is, by changing the magnetic flux enclosed by the circuit. It is also possible to explain the motional emf expression by invoking the Lorentz force acting on the free charge carriers of conductor PQ. Consider any arbitrary charge q in the conductor PQ. When the rod moves with speed v, the charge will also be moving with speed v in the magnetic field B. The Lorentz force on this charge is qvB in magnitude, and its direction is towards Q. All charges experience the same force, in magnitude and direction, irrespective of their position in the rod PQ.

The work done in moving the charge from P to Q is,

$$W = qvB\ell$$

Since emf is the work done per unit charge,

$$\varepsilon = \frac{W}{q}$$

$$= B\ell v$$

This equation gives emf induced across the rod PQ

The total force on the charge at P is given by

$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

OR

(a)

An ac source connected to an inductor.
Fig. (a)

Figure (a)shows an ac source connected to an inductor. Usually, inductors have appreciable resistance in their windings, but we shall assume that this inductor has negligible resistance. Thus, the circuit is a purely inductive ac circuit. Let the voltage across the source be $V = V_m \sin \omega t$. Using the Kirchhoff's loop rule, $\sum \varepsilon(t) = 0$, and since there is no resistor in the circuit,

$$v - L\frac{di}{dt} = 0 \quad ...(i)$$

where the second term is the self-induced Faraday emf in the inductor; and L is the self-inductance of the inductor. The negative sign follows from Lenz's law.

From equation (i) we have

$$\frac{di}{dt} = \frac{v}{L} = \frac{v_m}{L}\sin \omega t ...(ii)$$

Equation (ii) implies that the equation for i(t), the current as a function of time, must be such that its slope di/dt is a sinusoidally varying quantity, with the same phase as the source voltage and an amplitude given by vm/L. To obtain the current, we integrate di/dt with respect to time:

$$\int \frac{di}{dt}dt = \frac{v_m}{L}\int \sin(\omega t)dt$$

and get

$$i = -\frac{v_m}{\omega L}\cos(\omega t) + cons\tan t$$

The integration constant has the dimension of current and is time independent. Since the source has an emf which oscillates symmetrically about zero, the current it sustains also oscillates symmetrically about zero, so that no constant or time-independent component of the current exists. Therefore, the integration constant is zero.

Using

$$-\cos(\omega t) = \sin\left(\omega t - \frac{\pi}{2}\right), \text{ we have}$$

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Where $i_m = \frac{v_m}{\omega L}$ is the amplitude of the current. The quantity ωL is analogous to the resistance and is called inductive reactance, denoted by X_L :

$$X_L = \omega L$$

The amplitude of the current is, then

$$i_m = \frac{v_m}{x_L}$$

The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω). The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit. The inductive reactance is directly proportional to the inductance and to the frequency of the current.

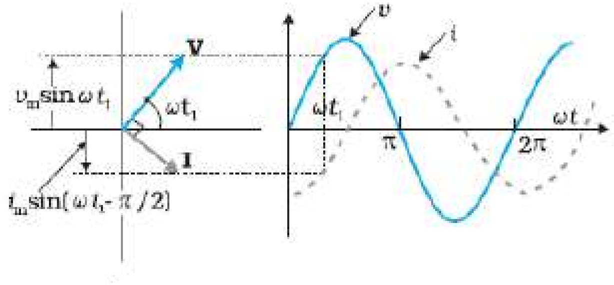


Fig. (b) A Phasor diagram for the circuit in Fig.(a)
Fig. (c) Graph of v and i versus ωt.

(b) We have seen that a voltage $v = v_m \sin\omega t$ applied to a series RLC circuit drives a current in the circuit given by $i = i_m \sin(\omega t + \phi)$ where

$$i_m = \frac{v_m}{z} \text{ and } \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Therefore, the instantaneous power p supplied by the source is

$$p = vi = (v_m \sin\omega t) \times [i_m \sin(\omega t + \phi)]$$

$$= \frac{v_m i_m}{2}[\cos\phi - \cos(2\omega t + \phi)]$$

The average power over a cycle is given by the average of the two terms in R.H.S. of above equation. It is only the second term which is time-dependent. Its average is zero (the positive half of the cosine cancels the negative half). Therefore,

$$P = \frac{v_m i_m}{2}\cos\phi = \frac{v_m}{\sqrt{2}}\frac{i_m}{\sqrt{2}}\cos\phi$$

$$= V\ I\ \cos\phi$$

This can also be written as

$$P = I^2 Z \cos\phi$$

So, the average power dissipated depends not only on the voltage and current but also on the cosine of the phase angle ϕ between them. The quantity $\cos\phi$ is called the power factor.

25. (a) State Gauss's law in electrostatics. Show, with the help of a suitable example along with the figure, that the outward flux due to a point charge 'q'. in vacuum within a closed surface, is independent of its size or shape and is given by q/ε_0

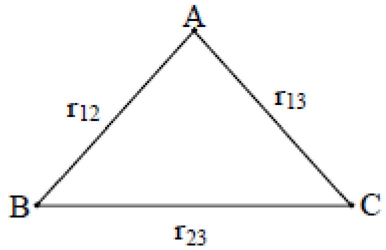
(b) Two parallel uniformly charged infinite plane sheets, '1' and '2' , have charge densities $+\sigma$ and -2σ respectively. Give the magnitude and direction of the net electric field at a point.

(i) in between the two sheets and

(ii) outside near the sheet '1'.

OR

(a) Define electrostatic potential at a point. Write its S.I. unit. Three point charges q1, q2 and q3 are kept respectively at points A, B and C as shown in the figure, Derive the expression for the electrostatic potential energy of the system.

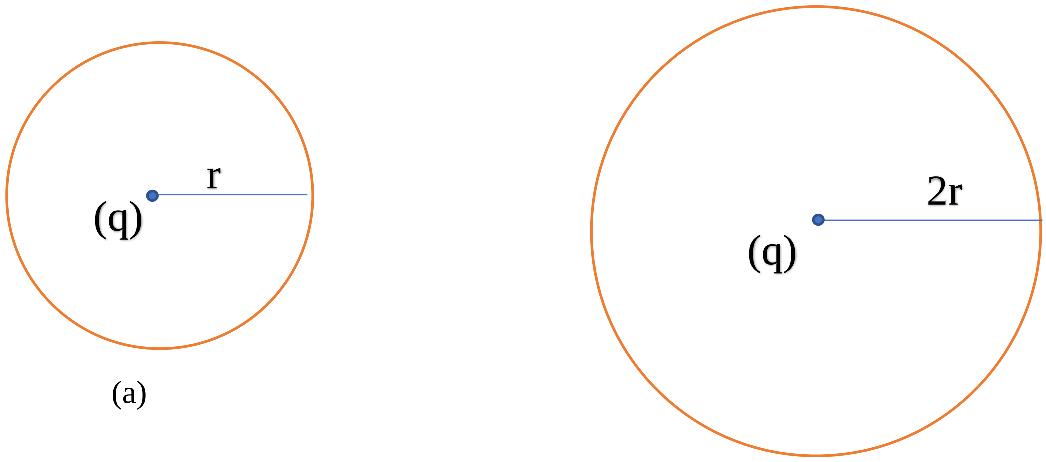


(b) Depict the equipotential surface due to
(i) an electric dipole,
(ii) two identical positive charges separated by a distance.
Solution:

(a) Statement : The electric flux linked with a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by a closed surface.
Mathematical expression :

$$\phi_E = \oint \vec{E}.d\vec{s} = \frac{1}{\varepsilon_0}(q_{net})$$

Consider two spherical surface of radius r and 2r respectively and a charge 1 is enclosed in it. According to gauss theorem the total electric flux linked with a closed surface depends on the charge enclosed in it so



(a)

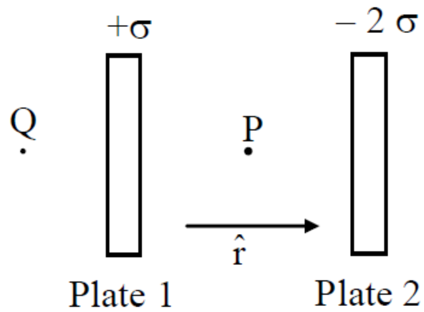
(b)

$$\phi_E = \frac{q}{\varepsilon_0}$$

and for fig (b)

$$\phi_E = \frac{q}{\varepsilon_0}$$

Which is same in both the case so it is independent of size and shape of closed surface.



Let $\hat{6}$ be the unit vector directed from left to right
Let P and Q are two points in the inner and outer region of two plates respectively charge densities on plates are $+\sigma$ *and* -2σ
(i) Electric field at point P in the inner region of the plates

$$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{r} \text{ and } \vec{E}_2 = \frac{2\sigma}{2\varepsilon_0}\hat{r}$$

$\therefore$ Net electric field in the inner region of the plates (i.e., at P) is

$$\vec{E} = \vec{E}_1 \, \vec{E}_2$$

$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{\varepsilon_0}\right)\hat{r}$$

$$\vec{E} = \frac{3\sigma}{2\varepsilon_0}\hat{r}$$

(ii) Electric field at point Q in the outer region of plate 1

$$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}(-\hat{r}) \text{ and } \vec{E}_2 = \frac{2\sigma}{2\varepsilon_0}\hat{r}$$

$\therefore$ Net electric field in the outer region of plate 1 (i.e. at Q) is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{\sigma}{\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\right)\hat{r}$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{r}$$

OR

(a) The electrostatic potential (V) at any point in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point. Its S.I. unit is Volt. The potential energy of a system of three charges q_1, q_2 and q_3 located at $\vec{r}_1, \vec{r}_2, \vec{r}_3$, respectively. To bring q1 first from infinity to $\vec{r}_1$, no work is required. Next we bring q2 from infinity to $\vec{r}_2$. As before, work done in this step is

$$q_2V_1\left(\vec{r}_2\right) = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}} \qquad (1)$$

The charges q_1 and q_2 produce a potential, which at any point P is given by

$$V_{1,2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}}\right)\text{.........(2)}$$

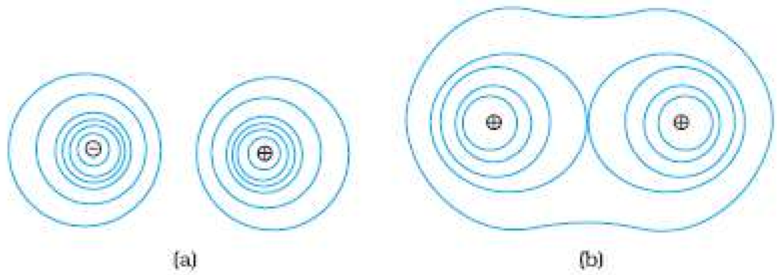
Work done next in bringing q3 from infinity to the point $\vec{r}_3$ is $q3$ times V1, 2 at

$$q_3V_{1,2}\left(\vec{r}_3\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}}\right)\text{......(3)}$$

The total work done in assembling the charges at the given locations is obtained by adding the work done in different steps [Eq. (1) and Eq. (3)] and gets stored in the form of potential energy.

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

(b) Equipotential surfaces for (a) a dipole and (b) two identical positive charges are shown in Figure.



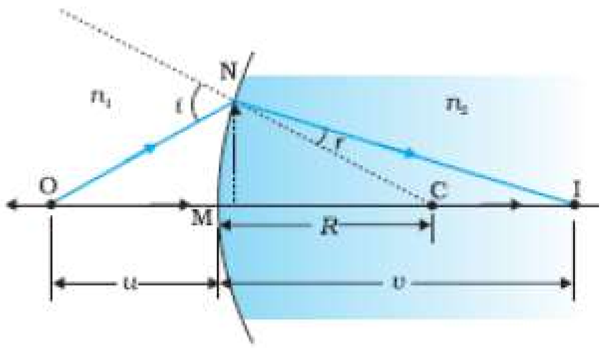
Some equipotential surfaces for (a) a dipole, (b) two identical positive charges.

26. (a) A point-object is placed on the principal axis of convex spherical surface of radius of curvature R, which separates the two media of refractive indices n1 and n2 ($n_2 > n_1$). Draw the ray diagram and deduce the relation between the distance of the object (u), distance of the image (v) and the radius of curvature (R for refraction to take place at the convex spherical surface from rarer to denser
medium.
(b) Use the above relation to obtain the condition on the position of the object and the radius of curvature in terms of n_1 and n_2 when the real image is formed

OR

(a) Draw a labelled ray diagram showing the formation of image by a compound microscope in normal adjustment. Derive the expression for its magnifying power.
(b) How does the resolving power of a microscope change when
(i) the diameter of the objective lens is decreased.
(ii) the wavelength of the incident light is increased ?
Justify your answer in each case.

Solution: (a)



Refraction at a spherical surface separating two media.

Figure shows the geometry of formation of image I of an object O on the principal axis of a spherical surface with centre of curvature C, and radius of curvature R. The rays are incident from a medium of refractive index n_1, to another of refractive index n_2. As before, we take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made. In particular, NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis.
We have, for small angles,

$$\tan \angle NOM = \frac{MN}{OM}$$

$$\tan \angle NCM = \frac{MN}{MC}$$

$\tan \angle NIM = \dfrac{MN}{MI}$

Now, for ΔNOC, is the exterior angle. Therefore, i $= \angle NOM + \angle NCM$

$i = \dfrac{MN}{OM} + \dfrac{MN}{MC}$.......(1)

Similarly,

$r = \angle NCM - \angle NIM$

i.e., $r = \dfrac{MN}{MC} - \dfrac{MN}{MI}$............(2)

Now, by Snell's law

$n_1 \sin i = n_2 \sin r$

or for small angles

$n_1 i = n_2 r$

Substituting i and r from Eqs.(1) and (2), we get

$\dfrac{n_1}{OM} + \dfrac{n_2}{MI} = \dfrac{n_2 - n_1}{MC}$.........(3)

Here, OM, MI and MC represent magnitudes of distances. Applying the Cartesian sign convention,

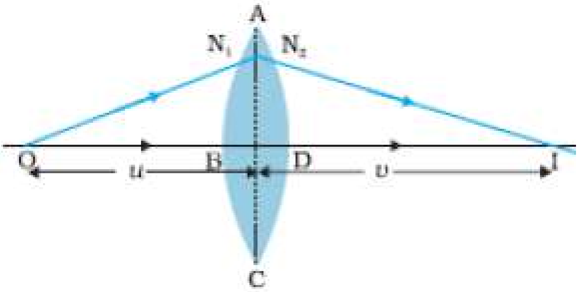
OM = – u, MI = +v, MC = +R

Substituting these in Eq.(3), we get

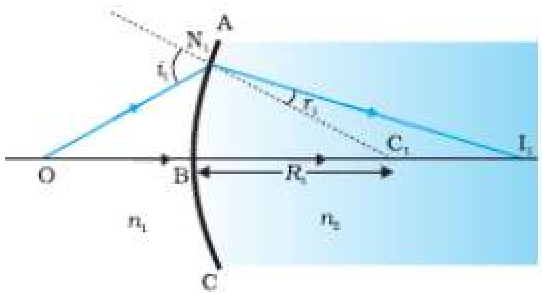
$\dfrac{n_2}{v} - \dfrac{n_1}{u} = \dfrac{n_2 - n_1}{R}$.............(4)

Equation (4) gives us a relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface. It holds for any curved spherical surface.

(b)



(a)



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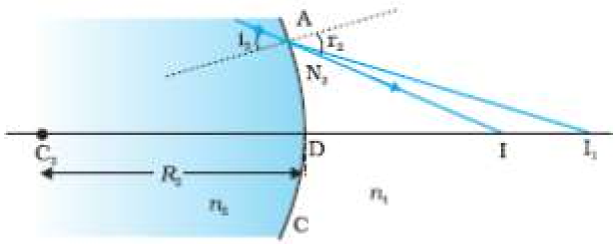


(c)

(a) The position of object, and the image formed by a double convex lens,
(b) Refraction at the first spherical surface and (c) Refraction at the second spherical surface.

Figure (a) shows the geometry of image formation by a double convex lens. The image formation can be seen in terms of two steps: (i) The first refracting surface forms the image I_1 of the object O [Fig.(b)]. The image I_1 acts as a virtual object for the second surface that forms the image at I [Fig.(c)]. Applying Equation $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ to the first interface ABC,

We get

$$\frac{n_1}{OB} + \frac{n_2}{BI_1} = \frac{n_2 - n_1}{BC_1} \quad \text{.....(1)}$$

A similar procedure applied to the second interface ADC gives,

$$\frac{n_2}{DI_1} + \frac{n_1}{DI} = \frac{n_2 - n_1}{DC_1} \quad \text{.....(2)}$$

For a thin lens, $BI_1 = DI_1$. Adding

Eqs.(1) and (2), we get

$$\frac{n_1}{OB} + \frac{n_1}{DI} = (n_2 - n_1)\left(\frac{1}{BC_1} + \frac{1}{DC_2}\right) \quad \text{.....(3)}$$

Suppose the object is at infinity, i.e., OB $\rightarrow \infty$ *and* $DI = f$, *Eq.*(3) *gives*

$$\frac{n_1}{f} = (n_2 - n_1)\left(\frac{1}{BC_1} + \frac{1}{DC_2}\right) \text{.....(4)}$$

The point where image of an object placed at infinity is formed is called the focus F, of the lens and the distance f gives its focal length. By the sign convention,

$BC_1 = + R_1$,

$DC_2 = – R_2$

So Eq.(4) can be written as

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{.......(5)} \left(\because n_{21} = \frac{n_2}{n_1}\right)$$

Equation (5) is known as the lens maker's formula. It is useful to design lenses of desired focal length using surfaces of suitable radii of curvature. Note that the formula is true for a concave lens also. In that case R_1 is negative, R_2 is positive and therefore, f is negative.

From Eqs. (3) and (4), we get

$$\frac{n_1}{OB} + \frac{n_1}{DI} = \frac{n_1}{f} \quad \text{.........(6)}$$

Again, in the thin lens approximation, B and D are both close to the optical centre of the lens. Applying the sign convention,

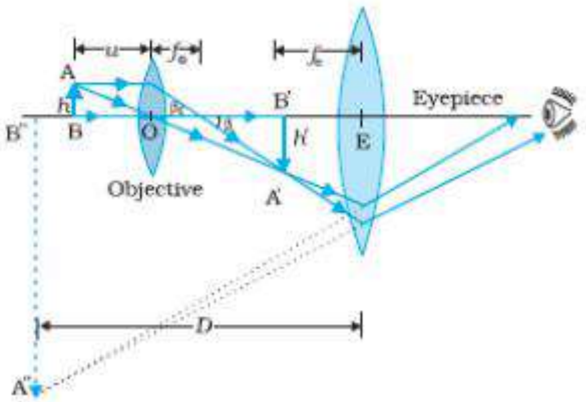
BO = – u, DI = +v, we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \ldots\ldots\ldots (7)$$

Equation (7) is the familiar thin lens formula.

OR

(a)



A simple microscope has a limited maximum magnification (≤ 9) for realistic focal lengths. For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a compound microscope. A schematic diagram of a compound microscope is shown in Fig. The lens nearest the object, called the objective, forms a real, inverted, magnified image of the object. This serves as the object for the second lens, the eyepiece, which functions essentially like a simple microscope or magnifier, produces the final image, which is enlarged and virtual. The first inverted image is thus near (at or within) the focal plane of the eyepiece, at a distance appropriate for final image formation at infinity, or a little closer for image formation at the near point. Clearly, the final image is inverted with respect to the original object. We now obtain the magnification due to a compound microscope. The ray diagram of Figure shows that the (linear) magnification due to the objective, namely h'/h, equals

$$m_0=\frac{h'}{h}=\frac{L}{f_0}$$

where we have used the result

$$\tan\beta=\left(\frac{h}{f_0}\right)=\left(\frac{h'}{L}\right)$$

Here h′ is the size of the first image, the object size being h and f_o being the focal length of the objective. The first image is formed near the focal point of the eyepiece. The distance L, i.e., the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length f_e) is called the tube length of the compound microscope. As the first inverted image is near the focal point of the eyepiece, we use the result for the simple microscope to obtain the (angular) magnification m_e due to it

equation $m=\left(1+\frac{D}{f}\right)$, when the final image is formed at the near point, is

$$m_e=\left(1+\frac{D}{f_e}\right)$$

When the final image is formed at infinity, the angular magnification due to the eyepiece is

$m_e = (D/f_e)$

Thus, the total magnification, when the image is formed at infinity, is

$$m=m_0 m_e=\left(\frac{L}{f_0}\right)\left(\frac{D}{f_e}\right)$$

Clearly, to achieve a large magnification of a small object (hence the name microscope), the objective and eyepiece should have small focal lengths.

(b) The resolving power of a microscope is given by the relation

$$RP = \frac{1}{d_{\min}} = \frac{2n\sin\beta}{1.22\lambda}$$

(i) When the diameter of the objective lens is decreased β decreases so resolving power decreases.

(ii) When the wavelength of the incident light is increased resolving power decreases.