SET-1

Series ONS

MATHEMATICS

Paper & Solution

Code: 65/1/N

Time: 3 Hrs.

Max. Marks: 100

General Instructions:

(i) ***All*** *questions are* ***compulsory****.*

(ii) Please check that this question paper contains ***26*** *questions.*

(iii) Questions ***1 - 6*** *in* ***Section A*** *are very short-answer type questions carrying* ***1*** *mark each.*

(w) Questions ***7 - 19*** *in* ***Section B*** *are long-answer* ***I*** *type questions carrying* ***4*** *marks each.*

(v) Questions ***20 - 26*** *in Section C are long-answer* ***II*** *type questions carrying* ***6*** *marks each.*

(vi) Please write down the serial number of the question before attempting it.

SECTION – A

Question numbers 1 to 6 carry 1 mark each.

1. If $A=\begin{pmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{pmatrix}$, find a satisfying $0 < \alpha < \frac{\Pi}{2}$ when $A+A^T=\sqrt{2}\,I_2$; where A^T is transpose of A.

Solution:

Finding $A^T=\begin{pmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{pmatrix}$

Getting $\alpha=\frac{\pi}{4}$ or $45°$

2. If A is a 3×3 matrix and $|3A|=k|A|$, then write the value of k.

Solution:

k = 27

3. For what values of k, the system of linear equations

$x+y+z=2$

$2x+y-z=3$

$3x+2y+kz=4$

has a unique solution ?

Solution:

For a unique solution

$$\begin{vmatrix}1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k\end{vmatrix}\neq 0$$

$\Rightarrow k\neq 0$

4. Write the sum of intercepts cut off by the plane $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)-5=0$ on the three axes.

Solution:

Getting equation as $\frac{x}{5/2}+\frac{y}{5}+\frac{z}{-5}=1$

Sum of intercepts $\frac{5}{2}+5-5=\frac{5}{2}$

5. Find λ and μ if

$\left(\hat{i}+3\hat{j}+9\hat{k}\right)\times\left(3\hat{i}-\lambda\hat{j}+\mu\hat{k}\right)=\vec{o}.$

Solution:

Getting $\lambda=-9$ and $\mu=27$

6. If $\vec{a}=4\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+\hat{k},$ then find a unit vector parallel to the vector $\vec{a}+\vec{b}.$

Solution:

$\vec{a}+\vec{b}=6\hat{i}-3\hat{j}+2\hat{k}$

Unit vector parallel to $\vec{a}+\vec{b}$ is $\frac{1}{7}(6\hat{i}-3\hat{j}+2\hat{k})$

SECTION – B

Question numbers 7 to 19 carry 4 marks each.

7. Solve for x : $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x.$

OR

Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\frac{4x}{1-4x^2}\right)=\tan^{-1}2x;|2x|<\frac{1}{\sqrt{3.}}$

Solution:

$\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$

$\Rightarrow\quad \tan^{-1}\left(\frac{2x}{2-x^2}\right)=\tan^{-1}\left(\frac{2x}{1+3x^2}\right)$

$\frac{2x}{2-x^2}=\frac{2x}{1+3x^2}$

$2x\,(1+3x^2-2+x^2)=0$

$x=0,\ \frac{1}{2},-\frac{1}{2}$

OR

Let $2x=\tan\theta$

L. H. S $=\tan-1\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)-\tan^{-1}\left(\frac{2\tan\theta}{1-\tan 2\theta}\right)$

$= \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 2\theta)$

$= 3\theta - 2\theta$

$= \theta$ or $\tan^{-1} 2x$

$\therefore$ L. H. S = R. H. S

8. A typist charges ₹ 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are ₹ 180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only ₹ 2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy? Which values are reflected in this problem?

Solution:

Getting matrix equation as $\begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}\begin{pmatrix} \text{E} \\ \text{H} \end{pmatrix} = \begin{pmatrix} 145 \\ 180 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} \text{E} \\ \text{H} \end{pmatrix} = \begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}^{-1}\begin{pmatrix} 145 \\ 180 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} \text{E} \\ \text{H} \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$

$\Rightarrow$ E = 10, H = 15

The poor boy was charged ₹ 65 less

Value: Helping the poor

9. If $f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x} & , x < o \\ 2 & , x = o \\ \dfrac{\sqrt{1+bx} - 1}{x} & , x > o \end{cases}$ is continuous at $x = o$, then find the values of a and b.

Solution:

L.H.L = a + 3

R.H.L = b/2

f(x) is continuous at x = 0. So, a + 3 = 2 = b/2

$\Rightarrow$ a = –1 and b = 4

10. If $x\cos(a+y) = \cos y$ then prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a+y)}{\sin a}$.

Hence show that $\sin a \dfrac{d^2y}{dx^2} + \sin 2(a+y)\dfrac{dy}{dx} = o$.

OR

Find $\dfrac{dy}{dx}$ if $y = \sin^{-1}\dfrac{6x - 4\sqrt{1-4x^2}}{5}$

Solution:

$$\frac{dx}{dy} = \frac{\sin a}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

$$\frac{d^2y}{dx^2} = \frac{-2\cos(a+y)\sin(a+y)}{\sin a}\frac{dy}{dx}$$

$$= \frac{-\sin 2(a+y)}{\sin a}\frac{dy}{dx}$$

$$\Rightarrow \sin a\frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0$$

OR

Let $2x = \sin\theta$

$$\therefore y = \sin^{-1}\left(\frac{6x - \sqrt{1-4x^2}}{5}\right)$$

$$= \sin^{-1}\left(\frac{3}{5}\sin\theta - \frac{4}{5}\cos\theta\right)$$

$$= \sin^{-1}(\cos\alpha\sin\alpha - \sin\alpha\cos\alpha) \qquad [\cos\alpha = \frac{3}{5}; \sin\alpha = \frac{4}{5}]$$

$$= \sin^{-1}(\sin(\theta - \alpha))$$

$$= \theta - \alpha$$

$$= \sin^{-1}(2x) - \alpha$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$

11. Find the equation of tangents to the curve $y = x^3 + 2x - 4$, which are perpendicular to line $x + 14y + 3 = 0$.

Solution:

Slope of the tangent = $3x^2 + 2 = 14$

Points of contact (2, 8) and (–2, –16)

Equations of tangent

14x – y – 20 = 0 and 14x – y + 12 = 0

12. Find: $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}dx$

OR

Find : $\int \frac{x^2 + x + 1}{(x^2+1)(x+2)}dx$

Solution:

Let 2x = t

$$I=\frac{1}{2}\int\frac{(t-5)}{(t-3)^3}e^t dt$$

$$=\frac{1}{2}\int\left[\frac{1}{(t-3)^2}-\frac{2}{(t-3)^3}\right]e^t\ dt$$

$$=\frac{1}{2}\frac{1}{(t-3)^2}e^t+C=\frac{1}{2}\frac{1}{(2x-3)^2}e^{2x}+C$$

OR

Writing $\dfrac{x^2+x+1}{(x^2+1)(x+2)}=\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x+2}$

$$\Rightarrow A=\frac{2}{5},\ B=\frac{1}{5},\ C=\frac{3}{5}$$

$$\therefore I=\frac{1}{5}\int\frac{2x}{x^2+1}dx+\frac{1}{5}\int\frac{dx}{x^2+1}+\frac{3}{5}\int\frac{dx}{x+2}$$

$$\Rightarrow I=\frac{1}{5}\log\left|x^2+1\right|+\frac{1}{5}\tan^{-1}x+\frac{3}{5}\log\left|x+2\right|+C$$

13. Evaluate: $\displaystyle\int_{-2}^{2}\frac{x^2}{1+5^x}\,dx.$

Solution:

Using property: $\displaystyle\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$$I=\int_{-2}^{2}\left(\frac{x^2}{1+5^x}\right)dx=\int_{-2}^{2}\left(\frac{x^2}{1+5^{-x}}\right)dx$$

$$2I=\int_{-2}^{2}x^2dx$$

$$2\text{I}=\frac{16}{3}\text{ or I}=\frac{8}{3}$$

14. Find : $\displaystyle\int(x+3)\sqrt{3-4x-x^2}\,dx.$

Solution:

Writing x + 3 = A (–4 – 2x) + B

$$\Rightarrow A=-\frac{1}{2},\ B=1$$

$$\therefore\ I=-\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}\,dx+\int\sqrt{\left(\sqrt{7}\right)^2-(x+2)^2}\,dx$$

$$I=-\frac{1}{3}(3-4-x^2)^{3/2}+\frac{x+2}{2}\sqrt{3-4x-x^2}+\frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}}+C$$

15. Find the particular solution of differential equation: $\frac{dy}{dx} = -\frac{x + y\cos x}{1+\sin x}$ given that y=1 when $x = 0$.

Solution:

Writing linear equation $\frac{dy}{dx} + \frac{\cos x}{1+\sin x} y = -\frac{x}{1+\sin x}$

$I.F = e^{\int \frac{\cos x}{1+\sin x} dx} = 1+\sin x$

General solution is: y(1 + sin x) $= -\frac{x^2}{2} + C$

Particular solution is: y (1 + sin x) $= 1 - \frac{x^2}{2}$

16. Find the particular solution of the differential equation

$2y\, e^{x/y}\, dx + \left(y - 2x\, e^{x/y}\right) dy = 0$

given that $x = 0$ when $y = 1$.

Solution:

$\frac{dx}{dy} = \frac{2xe^v - y}{2ye^v}$

$\frac{x}{y} = v$, then $\frac{dx}{dy} = v + y\frac{dv}{dy}$

$v + y\frac{dv}{dy} = \frac{2vye^v - y}{2ye^v}$

$2\int e^v dv = -\int \frac{dy}{y}$

General solution is: $2e^v = -\log|y| + C$ or $2e^{x/y} = -\log|y| + C$

Particular solution is: $2e^{x/y} + \log|y| = 2$

17. Show that the four points $A(4, 5, 1), B(0, -1, -1), C(3, 9, 4)$ and $D(-4, 4, 4)$ are coplanar.

Solution:

$\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}, \overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}, \overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$

For 4 points to be coplanar, $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

i.e., $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$

= – 4(12 + 3) + 6(–3 + 24) – 2(1 + 32)

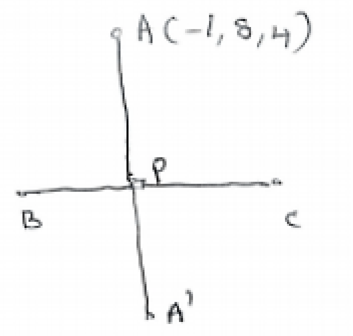
= – 60 + 126 – 66 = 0 which is true

Hence, points are coplanar.

18. Find the coordinates of the foot of perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$. hence find the image of the point A in the line BC.

Solution:

Equation of line BC: $\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} = r$



General point on BC: (2r, –2r –1, –4r + 3)
$\Rightarrow$ d.r.'s of AP: (2r + 1, –2r – 9, –4r – 1)
As $AP \perp BC \Rightarrow r = -1$
$\Rightarrow$ Co-ordinates of P: (– 2, 1, 7)
Hence, coordinates of Image of A: (– 3, – 6, 10)

19. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

OR

A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

Solution:

Let E_1 and E_2 be the events of drawing bag X and bag Y respectively.

Then, $P(E_1) = P(E_2) = \frac{1}{2}$

Let A be the event of drawing one white and one black ball from any one of the bag without replacement. Then,

$$\Rightarrow P(A/E_1) = \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} = \frac{16}{30}$$

$$P(A/E_2) = \frac{3}{6} \times \frac{3}{5} + \frac{3}{6} \times \frac{3}{5} = \frac{18}{30}$$

Using Bayes' Theorem, we have

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{18}{30}}{\frac{1}{2}\times\frac{16}{30}+\frac{1}{2}\times\frac{18}{30}} = \frac{9}{17}$$

OR

Let A_i and B_i be the events of throwing 10 by A and B in the respective ith turn. Then,

$$P(A_i) = P(B_i) = \frac{1}{12} \text{ and } P(\overline{A_i}) = P(\overline{B_i}) = \frac{11}{12}$$

Probability of winning A, when A starts first

$$= \frac{1}{12} + \left(\frac{11}{12}\right)^2 \frac{1}{12} + \left(\frac{11}{12}\right)^4 \frac{1}{12} + ...$$

$$= \frac{1/12}{1-(11/12)^2}$$

$$= \frac{12}{23}$$

Probability of winning of B = $1 - P(A) = 1 - \frac{12}{23} = \frac{11}{23}$

SECTION – C

Question numbers 20 to 26 carry 6 marks each.

20. Three numbers are selected at random (without replacement) from first six positive integers. Let X denotes the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean and variance of the distribution.

Solution:

The variate X takes values 3, 4, 5, and 6

$$P(X=3) = \frac{1}{20};\ P(X=4) = \frac{3}{20};\ P(X=5) = \frac{6}{20};\ P(X=6) = \frac{10}{20};$$

Probability distribution is:

X	3	4	5	6
P(X)	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$

Mean $= \Sigma XP(X) = \frac{105}{20} = \frac{21}{4}$

Variance $\Sigma X^2 P(X) - (\Sigma X\ P(X))^2 = \frac{63}{80}$

21. Let $A = R \times R$ and * be a binary operation on A defined by $(a,b)*(c,d) = (a+c, b+d)$

Show that * is commutative and associative. Find the identity element for * on A. Also find the inverse of every element (a, b) € A.

Solution:

Proving * is commutative
Proving * is associative
Getting identity element as (0, 0)
Getting inverse of (a, b) as (–a, –b)

22. Prove that $y = \dfrac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ on $\left[0, \dfrac{\pi}{2}\right]$.

OR

Show that semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$.

Solution:

Getting $\dfrac{dy}{d\theta} = \dfrac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$

Equating $\dfrac{dy}{d\theta}$ to 0 and getting critical point as $\cos\theta = 0$ *i.e.*, $\theta = \dfrac{\pi}{2}$

For all $\theta, 0 \le \theta \le \dfrac{\pi}{2}, \dfrac{dy}{d\theta} \ge 0$

Hence, y is an increasing function of θ on $\left[0, \dfrac{\pi}{2}\right]$

OR

Writing $V = \dfrac{1}{3}\pi r^2 h = \dfrac{\pi}{2} l^3 \sin^2\theta\cos\theta$

Getting $\dfrac{dV}{d\theta} = \dfrac{\pi}{2} l^3 [2\sin\theta\cos^2\theta - \sin^3\theta]$

For maxima and minima, $\dfrac{dV}{d\theta} = 0$

$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{3}}$ or $\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

Getting $\dfrac{d^2V}{d\theta^2}$ negative

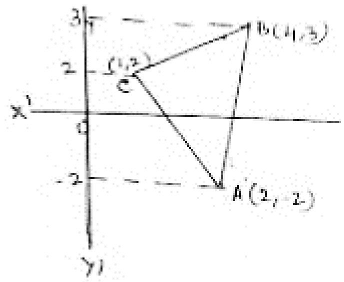
Hence, volume of the cone is maximum when semi-vertical angle is $\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

23. Using the method of integration, find the area of the triangular region whose vertices are $(2,-2),(4,3)$ and $(1,2)$.

Solution:

Writing equations of three sides in terms of y as

$$x_{AB}=\frac{2}{5}(y+2)+2;\ x_{BC}=3(y-3)+4;\ x_{AC}=\frac{-1}{4}(y+2)+2$$



Area $\int_{-2}^{3}\left(\frac{2}{5}(y+2)+2\right)dy-\int_{-2}^{2}\left(-\frac{1}{4}(y+2)+2\right)dy-\int_{2}^{3}\left(3(y-3)+4\right)dy$

$$=\left[\frac{2}{10}(y+2)^2+2y\right]_{-2}^{3}-\left[-\frac{1}{8}(y+2)^2+2y\right]_{-2}^{2}-\left[\frac{3}{2}(y-3)^2+4y\right]_{2}^{3}$$

$$=15-6-\frac{5}{2}\text{ or }\frac{13}{2}$$

24. Find the equation of the plane which contains the line of intersection of the planes

$\vec{r}.(\hat{i}-2\hat{j}+3\hat{k})-4=0$ and

$\vec{r}.(-2\hat{i}+\hat{j}+\hat{k})+5=0$

and whose intercept on x-axis is equal to that of on y-axis.

Solution:

Required equation of the plane is

$\vec{r}.[(1-2\lambda)\hat{i}+(-2+\lambda)\hat{j}+(3+\lambda)\hat{k}]=4-5\lambda$

Intercept of the plane on x-axis = Intercept of the plane on y-axis

$$\Rightarrow\frac{4-5\lambda}{1-2\lambda}=\frac{4-5\lambda}{\lambda-2}\ i.e.,\ \lambda=1,\frac{4}{5}\ \left(\text{rejecting }\lambda=\frac{4}{5}\right)$$

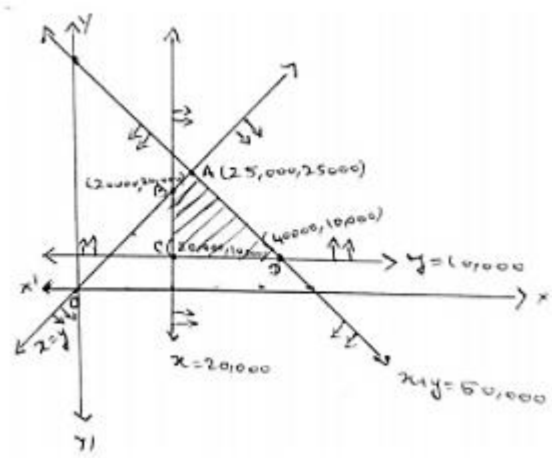
Required equation of the plane is $\vec{r}.(-\hat{i}-\hat{j}+4\hat{k})+1=0$

25. A retired person wants to invest an amount of ₹ 50,000. His broker recommends investing in two type of bonds ‘A’ and ‘B’ yielding 10% and 9% return respectively on the invested amount. He decides to invest at least ₹ 20,000 in bond ‘A’ and at least ₹ 10,000 in bond ‘B’. He also wants to invest at least as much in bond ‘A’ as in bond ‘B’. He also wants to invest at least as much in bond ‘A’ as in bond ‘B’. Solve this linear programming problem graphically to maximize his returns.

Solution:

Let the investment in bond A be ₹ x and in bond B be ₹ y

Objective function is: $Z = \frac{x}{10} + \frac{9}{100}y$



Subject to constraints

$x + y \geq 50000;\ x \geq 20,000;\ y \geq 10,000;\ x \geq y (*)$

Correct Figure

Vertices of feasible region are A, B, C, and D

Point	$Z = \frac{x}{10} + \frac{9}{100}y$	Value
A(25,000, 25000)	2500 + 2250	4750
B(20,000, 20,000)	2000 + 1800	3800
C(20,000, 10,000)	2000 + 900	2900
D(40,000, 10,000)	4000 + 900	4900

Return is maximum when ₹ 40000 are invested in Bond A and ₹ 10000 in Bond B

Maximum return is ₹ 4900

Since there are more than 3 constraints, student may be given full 6 marks even if reaches up to (*).

26. Using properties of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

OR

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = O$ find k.

Solution:

$$\Delta = \begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix}$$

$R_1 \to zR_1,\ R_2 \to xR_2,\ R_3 \to yR_3$

$$\Delta = \frac{1}{xyz}\begin{vmatrix} z(x+y)^2 & z^2x & z^2y \\ x^2z & x(z+y)^2 & x^2y \\ y^2z & y^2x & y(z+x)^2 \end{vmatrix}$$

$$= \frac{xyz}{xyz}\begin{vmatrix} (x+y)^2 & z^2 & z^2 \\ x^2 & (z+y)^2 & x^2 \\ y^2 & y^2 & (z+x)^2 \end{vmatrix}$$

$C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} (x+y)^2 - z^2 & 0 & z^2 \\ 0 & (z+y)^2 - x^2 & x^2 \\ y^2 - (z+x)^2 & y^2 - (z+x)^2 & (z+x)^2 \end{vmatrix}$$

$$= (x+y+z)^2\begin{vmatrix} x+y+z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ y-z-x & y-z-x & (z+x)^2 \end{vmatrix}$$

$R_3 \to R_3 - R_1 - R_2$ we get

$$= (x+y+z)^2\begin{vmatrix} x+y+z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ -2x & -2z & 2xz \end{vmatrix}$$

$C_1 \to C_1 + \frac{C_3}{z}, C_2 \to \frac{C_3}{x}$ we get

$$\Delta = (x+y+z)^2\begin{vmatrix} x+y & \frac{z^2}{x} & z^2 \\ \frac{x^2}{z} & z+y & x^2 \\ 0 & 0 & 2xz \end{vmatrix}$$

$= 2xyz\ (x + y + z)^3$

OR

For getting $A^2 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$

For getting $A^3 = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$

Simplifying $A^3 - 6A^2 + 7A + kI_3$ as $\begin{pmatrix} k-2 & 0 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-2 \end{pmatrix}$

Equating $\begin{pmatrix} k-2 & 0 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow k - 2 = 0$

$K = 2$