CHAPTER 10

VECTORS

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. What are the horizontal and vertical components of a vector $\vec{a}$ of magnitude 5 making an angle of 150° with the direction of x-axis.

2. What is $a \in R$ such that $\left|a\vec{x}\right| = 1$, where $\vec{x} = \hat{i} - 2\hat{j} + 2\hat{k}$?

3. When is $\left|\vec{x} + \vec{y}\right| = \left|\vec{x}\right| + \left|\vec{y}\right|$?

4. What is the area of a parallelogram whose sides are given by $2\hat{i} - \hat{j}$ and $\hat{i} + 5\hat{k}$?

5. What is the angle between $\vec{a}$ *and* $\vec{b}$, If
$$\vec{a} \cdot \vec{b} = 3 \text{ and } \left|\vec{a} \times \vec{b}\right| = 3\sqrt{3}.$$

6. Write a unit vector which makes an angle of $\frac{\pi}{4}$ with x-axis and $\frac{\pi}{3}$ with z-axis and an acute angle with y-axis.

7. If A is the point (4, 5) and vector $\overrightarrow{AB}$ has components 2 and 6 along x-axis and y-axis respectively then write point B.

8. What is the point of trisection of PQ nearer to P if positions of P and Q are $3\hat{i} + 3\hat{j} - 4\hat{k}$ and $9\hat{i} + 8\hat{j} - 10\hat{k}$ respectively?

9. Write the vector in the direction of $2\hat{i} + 3\hat{j} + 2\sqrt{3}\hat{k}$, whose magnitude is 10 units.

10. What are the direction cosines of a vector equiangular with co-ordinate axes?

11. What is the angle which the vector $3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the x-axis?

12. Write a unit vector perpendicular to both the vectors

$$3\hat{i} - 2\hat{j} + \hat{k} \text{ and } - 2\hat{i} + \hat{j} - 2\hat{k}.$$

13. What is the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$?

14. If $|\vec{a}| = 2$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{a} \perp \vec{b}$, what is the value of $|\vec{a} + \vec{b}|$?

15. For what value of λ, $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ is perpendicular to $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$?

16. What is $|\vec{a}|$, if $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 3$ and $2|\vec{b}| = |\vec{a}|$?

17. What is the angle between $\vec{a}$ and $\vec{b}$, if $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$?

18. In a parallelogram $ABCD$, $\overrightarrow{AB} = 2\hat{i} - \hat{j} + 4\hat{k}$ *and* $\overrightarrow{AC} = \hat{i} + \hat{j} + 4\hat{k}$. What is the length of side BC?

19. What is the area of a parallelogram whose diagonals are given by vectors $2\hat{i} + \hat{j} - 2\hat{k}$ and $-\hat{i} + 2\hat{k}$?

20. Find $|\vec{x}|$ if for a unit vector $\hat{a}$, $(\vec{x} - \hat{a}).(\vec{x} + \hat{a}) = 12$.

21. If $\vec{a}$ and $\vec{b}$ are two unit vectors and $\vec{a} + \vec{b}$ is also a unit vector then what is the angle between $\vec{a}$ and $\vec{b}$?

22. If $\hat{i}, \hat{j}, \hat{k}$ are the usual three mutually perpendicular unit vectors then what is the value of $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{i} \times \hat{k}) + \vec{k}.(\hat{j} \times \hat{i})$?

23. What is the angle between $\vec{x}$ and $\vec{y}$ if $\vec{x}.\vec{y} = |\vec{x} \times \vec{y}|$?

24. Write a unit vector in *xy*-plane, making an angle of 30° with the +ve direction of *x*–axis.

25. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors with $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then what is the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$?

26. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left(\vec{a} + 2\vec{b}\right)$ is perpendicular to $\left(5\vec{a} - 4\vec{b}\right)$, then what is the angle between $\vec{a}$ and $\vec{b}$?

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

27. If ABCDEF is a regular hexagon then using triangle law of addition prove that :

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3\overrightarrow{AD} = 6\overrightarrow{AO}$$

O being the centre of hexagon.

28. Points *L, M, N* divides the sides *BC, CA, AB* of a ΔABC in the ratios 1 : 4, 3 : 2, 3 : 7 respectively. Prove that $\overrightarrow{AL} + \overrightarrow{BM} + \overrightarrow{CN}$ is a vector parallel to $\overrightarrow{CK}$ where *K* divides *AB* in ratio 1 : 3.

29. The scalar product of vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ *and* $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ.

30. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude. Show that $\vec{a} + \vec{b} + \vec{c}$ makes equal angles with $\vec{a}$, $\vec{b}$ and $\vec{c}$ with each angle as $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

31. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} + 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

32. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

33. If $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the angle between $\vec{a}$ and $\vec{b}$.

34. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = 3\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - \hat{k}$, find a vector $\vec{d}$ which is perpendicular to $\vec{a}$ and $\vec{b}$ and $\vec{c} . \vec{d} = 1$.

35. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ are the given vectors then find a vector $\vec{b}$ satisfying the equation $\vec{a} \times \vec{b} = \vec{c}$, $\vec{a} . \vec{b} = 3$.

36. Find a unit vector perpendicular to plane *ABC,* when position vectors of *A, B, C* are $3\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} - \hat{j} - 3\hat{k}$ and $4\hat{i} - 3\hat{j} + \hat{k}$ respectively.

37. For any two vector, show that $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$.

38. Evaluate $\left(\vec{a} \times \hat{i}\right)^2 + \left(\vec{a} \times \hat{j}\right)^2 + \left(\vec{a} \times \hat{k}\right)^2$.

39. If $\hat{a}$ and $\hat{b}$ are unit vector inclined at an angle θ than prove that :

(i) $\sin\dfrac{\theta}{2} = \dfrac{1}{2}|\hat{a} - \hat{b}|$. (ii) $\tan\dfrac{\theta}{2} = \left|\dfrac{\hat{a} - \hat{b}}{\hat{a} + \hat{b}}\right|$.

40. For any two vectors, show that $|\vec{a} \times \vec{b}| = \sqrt{a^2b^2 - \left(\vec{a} \cdot \vec{b}\right)^2}$.

41. $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{j}$ and $\vec{c} = x\hat{i} + (x - 2)\hat{j} - \hat{k}$. If $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then find the value of *x*.

42. Prove that angle between any two diagonals of *a* cube is $\cos^{-1}\left(\dfrac{1}{3}\right)$.

43. Let $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}\cdot\hat{b} = \hat{a}\cdot\hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\dfrac{\pi}{6}$, then prove that $\hat{a} = \pm 2\left(\hat{b} \times \hat{c}\right)$.

44. Prove that the normal vector to the plane containing three points with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ lies in the direction of vector $\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}$.

45. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices A, B, C of a triangle ABC then show that the area of ΔABC is $\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$.

46. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then prove that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$ provided $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

47. Dot product of a vector with vectors $\hat{i}+\hat{j}-3\hat{k}$, $\hat{i}+3\hat{j}-2\hat{k}$ and $2\hat{i}+\hat{j}+4\hat{k}$ is 0, 5 and 8 respectively. Find the vectors.

48. If $\vec{a}=5\hat{i}-\hat{j}+7\hat{k}$, $\hat{b}=\hat{i}-\hat{j}-\lambda\hat{k}$, find λ such that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal.

49. Let $\vec{a}$ and $\vec{b}$ be vectors such that $\left|\vec{a}\right|=\left|\vec{b}\right|=\left|\vec{a}-\vec{b}\right|=1$, then find $\left|\vec{a}+\vec{b}\right|$.

50. If $\left|\vec{a}\right|=2$, $\left|\vec{b}\right|=5$ and $\vec{a}\times\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$, find the value of $\vec{a}\cdot\vec{b}$.

51. $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{b}\times\vec{c}=\vec{a}$ and $\vec{a}\times\vec{b}=\vec{c}$. Prove that $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular to each other and $\left|\vec{b}\right|=1$, $\left|\vec{c}\right|=\left|\vec{a}\right|$.

52. If $\vec{a}=2\hat{i}-3\hat{j}$, $\vec{b}=\hat{i}+\hat{j}-\hat{k}$ *and* $\vec{c}=3\hat{i}-\hat{k}$ find $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$.

53. Find volume of parallelepiped whose coterminous edges are given by vectors $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$, and $\vec{c}=3\hat{i}-\hat{j}+2\hat{k}$.

54. Find the value of λ such that $\vec{a}=\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=\lambda\hat{i}-\hat{j}+\lambda\hat{k}$ are coplanar.

55. Show that the four points (–1, 4, –3), (3, 2, –5) (–3, 8, –5) and (–3, 2, 1) are coplanar.

56. For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, prove that

$$\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = 2\left[\vec{a} \ \vec{b} \ \vec{c}\right]$$

57. For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, prove that $\vec{a}-\vec{b}$, $\vec{b}-\vec{c}$ and $\vec{c}-\vec{a}$ are coplanar.

ANSWERS

1. $-\dfrac{5\sqrt{3}}{2}, \dfrac{5}{2}$.

2. $a = \pm\dfrac{1}{3}$

3. $\vec{x}$ and $\vec{y}$ are like parallel vectors.

4. $\sqrt{126}$ sq units.

5. $\dfrac{\pi}{3}$

6. $\dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{2}\hat{j} + \dfrac{1}{2}\hat{k}$

7. (6, 11)

8. $\left(5, \dfrac{14}{3}, -6\right)$

9. $4\hat{i} + 6\hat{j} + 4\sqrt{3}\,\hat{k}$.

10. $\pm\dfrac{1}{\sqrt{3}}, \pm\dfrac{1}{\sqrt{3}}, \pm\dfrac{1}{\sqrt{3}}$.

11. $\cos^{-1}\left(\dfrac{3}{7}\right)$.

12. $\dfrac{3\hat{i} + 4\hat{j} - \hat{k}}{\sqrt{26}}$.

13. 0

14. 4

15. –9

16. 2

17. $\dfrac{\pi}{2}$.

18. $\sqrt{5}$

19. $\dfrac{3}{2}$ sq. units.

20. $\sqrt{13}$

21. $\frac{2\pi}{3}$

22. -1

23. $\frac{\pi}{4}$

24. $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

25. $-\frac{3}{2}$

26. $\frac{\pi}{3}$

29. $\lambda = 1$

31. $\overrightarrow{\beta} = \left(\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + \left(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}\right)$.

33. $60°$

34. $\frac{1}{4}\hat{i} + \frac{1}{4}\hat{j} + \frac{3}{4}\hat{k}$.

35. $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$.

36. $\frac{-1}{\sqrt{165}}\left(10\hat{i} + 7\hat{j} - 4\hat{k}\right)$.

38. $2\left|\overrightarrow{a}\right|^2$

41. $x = -2$

47. $\hat{i} + 2\hat{j} + \hat{k}$

48. $\pm\sqrt{73}$

49. $\sqrt{3}$

50. $\frac{91}{10}$

52. 4

53. 37

54. $\lambda = 1$