PARABOLA

LEVEL-I

*1. The parametric equation of the parabola is $x = t^2 + 1$, $y = 2t + 1$. The equation of its directrix is

(A) $x = 0$ (B) $x + 1 = 0$
(C) $y = 0$ (D) none of these

*2. The tangents to the parabola $y^2 = 4x$ at the points (1, 2) and (4, 4) meets on the line

(A) $x = 3$ (B) $x + y = 4$
(C) $y = 3$ (D) none of these

3. Normal at point to the parabola $y^2 = 8x$, where abscissa is equal to ordinate, will meet the parabola again at a point

(A) (12, –18) (B) (–12, 18)
(C) (–18, 12) (D) (18, –12)

4. If the tangents to the parabola $y^2 = 4ax$ at the points (x_1, y_1) and (x_2, y_2) meet at the point (x_3, y_3) then

(A) $y_3 = \sqrt{y_1 y_2}$ (B) $2y_3 = y_1 + y_2$
(C) $\frac{2}{y_3} = \frac{1}{y_1} + \frac{1}{y_2}$ (D) none of these

5. If tangents at A and B on the parabola $y^2 = 4ax$ intersect at the point C, then ordinates of A, C and B are

(A) always in A.P. (B) always in G.P.
(C) always in H.P. (D) none of these

6. The point P on the parabola $y^2 = 4ax$ for which |PR – PQ| is maximum, where R $\equiv$ (– a, 0), Q $\equiv$ (0, a), is

(A) (a, 2a) (B) (a, -2a)
(C) (4a, 4a) (D) (4a, -4a)

*7. The point (1, 2) is one extremity of focal chord of parabola $y^2 = 4x$. The length of this focal chord is

(A) 2 (B) 4
(C) 6 (D) none of these

8. If normals at two points of a parabola $y^2 = 4ax$ intersect on the curve, then the product of ordinates is

(A) $2a^2$ (B) $4a^2$
(C) $6a^2$ (D) $8a^2$

9. If AFB is a focal chord of the parabola $y^2 = 4ax$ and AF = 4, FB = 5, then the latus-rectum of the parabola is equal to

(A) $\frac{80}{9}$ (B) $\frac{9}{80}$
(C) 9 (D) 80

10. The length of the chord of the parabola $x^2 = 4y$ passing through the vertex and having slope $\cot\alpha$ is
(A) $4\cos\alpha \cdot \mathrm{cosec}^2\alpha$ (B) $4 \tan\alpha \sec\alpha$
(C) $4\sin\alpha . \sec^2\alpha$ (D) none of these

11. The straight line $y = mx + c$ touches the parabola $y^2 = 4a(x + a)$ if
(A) $c = am - \frac{a}{m}$ (B) $c = m - \frac{a}{m}$
(C) $c = am + \frac{a}{m}$ (D) none of these

*12. The equation of the tangent to the parabola $y^2 = 16x$ inclined at an angle of 60^0 to x-axis is
(A) $3x - \sqrt{3}y + 4 = 0$ (B) $3x + \sqrt{3}y + 4 = 0$
(C) $3x - y + 4 = 0$ (D) none of these

*13. For all parabolas $x^2 + 4x + 4y + 16 = 0$, the equations of the axis and the directrix are given by
(A) $x + 2 = 0, y - 2 = 0$ (B) $x - 2 = 0, y + 2 = 0$
(C) $x + 2 = 0, y + 2 = 0$ (D) none of these

*14. If (4, 0) is the vertex and y-axis the directrix of a parabola, then its focus is
(A) (8, 0) (B) (4, 0)
(C) (0, 8) (D) (0, 4)

15. The slope of the normal at the point $(at^2, 2at)$ of the parabola $y^2 = 4ax$ is
(A) $\frac{1}{t}$ (B) t
(C) $-t$ (D) $-\frac{1}{t}$

*16. If ASB is a focal chord of a parabola such that AS = 2 and SB = 4, then the latus rectum of the parabola is
(A) $\frac{8}{3}$ (B) $\frac{16}{3}$
(C) $\frac{25}{3}$ (D) none of these

17. The normal to the parabola $y^2 = 8x$ at (2, 4) meets the parabola again at
(A) (18, 12) (B) (18, –12)
(C) (–18, 12) (D) none of these

*18. The value of k for which the line $x + y + 1 = 0$ touches the parabola $y^2 = kx$ is
(A) –4 (B) 4
(C) 2 (D) –2

*20. The equation of directrix of the parabola $x^2 + 4x + 4y + 8 = 0$ is
(A) $y = -1$ (B) $y = 1$

(C) $y = 0$ (D) $y = \frac{3}{2}$

21. The area of the triangle formed by the tangent and the normal to the parabola $y^2 = 4ax$ both drawn at the same end of the latus rectum and the axis of the parabola is
(A) $2\sqrt{2}\,a^2$ (B) $2a^2$
(C) $4a^2$ (D) none of these

22. If two normals at P and Q of a parabola $y^2 = 4ax$ intersect at a third point R on the curve, then the product of ordinates of P and Q is
(A) $8a^2$ (B) $4a^2$
(C) $2a^2$ (D) none of these

23. The length of the subnormal to the parabola $y^2 = 4ax$ at any point is equal to
(A) $a\sqrt{2}$ (B) $2\sqrt{2}\,a$
(C) $a/\sqrt{2}$ (D) $2a$

*24. The number of tangents to the parabola $y^2 = 8x$ through (2, 1) is
(A) 0 (B) 1
(C) 2 (D) none of these

*25. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of k is
(A) $\frac{1}{8}$ (B) 8
(C) 4 (D) $\frac{1}{4}$

*26. If the point P (4, – 2) is one end of the focal chord PQ of the parabola $y^2 = x$, then the slope of the tangent at Q is
(A) $-\frac{1}{4}$ (B) $\frac{1}{4}$
(C) 4 (D) – 4

*27. The equation of the parabola whose vertex and focus lie on the x– axis at distances a and a_1 from the origin respectively, is
(A) $y^2 = 4(a_1 - a)x$ (B) $y^2 = 4(a_1 - a)(x - a)$
(C) $y^2 = 4(a_1 - a)(x - a_1)$ (D) none of these

*28. If (2, 0) is the vertex and y– axis the directrix of the parabola, then the focus is
(A) (2, 0) (B) (– 2, 0)
(C) (4, 0) (D) (– 4, 0)

29. If the normals at t_1 and t_2 meets on the parabola then
(A) $t_2 = -t_1 - \frac{2}{t_1}$ (B) $t_1t_2 = 2$
(C) $t_1 t_2 = -1$ (D) none of these

*30. The graph represented by the equations $x = \sin^2 t$, $y = 2 \cos t$ is
(A) parabola (B) circle
(C) hyperbola (D) none of these

31. If $y = -4$ is the directrix and $(-2, -1)$ the vertex of a parabola then its focus is at ………………………………………………….

32. The condition that the line $\frac{x}{a} + \frac{y}{b} = 1$ be a normal to the parabola $y^2 = 4px$ is …………………………………………………….

33. If k = ……………., the line $y = 2x + k$ is normal to the parabola $y^2 = 4x$ at …………….

34. The value of k for which the equation $x^2 + y^2 + 2kxy + 2x + 4y + 3 = 0$ represents a parabola are ………………………………………………..

35. The point of intersection of the tangents of the parabola $y^2 = 4x$ at the points, where the parameter t has the value 1 and 2 are
(A) (3, 8) (B) (4, 5)
(C) (2, 3) (D) (4, 6)

36. If the line $y = x + k$ is a normal to the parabola $y^2 = 4x$ then k can have the value
(A) $2\sqrt{2}$ (B) 4
(C) –3 (D) 3

37. The tangents from the origin to the parabola $y^2 + 4 = 4x$ inclined of
(A) $\pi/6$ (B) $\pi/4$
(C) $\pi/3$ (D) $\pi/2$

38. Normal at point to the parabola $y^2 = 4ax$ where abscissa is equal to ordinate, will meet the parabola again at a point
(A) (6a, – 9a) (B) (–6a, 9a)
(C) (–9a, 6a) (D) (9a, – 6a)

*39. If the focus of the parabola is (–2, 1) and the directrix has the equation $x + y = 3$ then the vertex is
(A) (0, 3) (B) (–1, 1/2)
(C) (–1, 2) (D) (2, –1)

40. The locus of the point from which tangents to a parabola are at right angles is a
(A) straight line (B) pair of straight lines
(C) circle (D) none

41. Given the two ends of the latus rectum, the maximum number of parabolas that can be drawn is
(A) 1 (B) 2

(C) 0 (D) infinite

*42. The Cartesian equation of the curve whose parametric equations are $x = t^2 + 2t + 3$ and $y = t + 1$ is

(A) $y = (x-1)^2 + 2(y-1) + 3$ (B) $x = (y-1)^2 + 2(y-1) + 5$

(C) $x = y^2 + 2$ (D) None of these

*43. If line $y = 2x + \frac{1}{4}$ is tangent to $y^2 = 4ax$, then a is equal to

(A) 1/ 2 (B) 1

(C) 2 (D) None of these

44. The shortest distance between the parabola $y^2 = 4x$ and the circle $x^2 + y^2 + 6x - 12y + 20 = 0$ is

(A) $4\sqrt{2} - 5$ (B) 0

(C) $3\sqrt{2} + 5$ (D) 1

45. The equation $(13x - 1)^2 + (13y - 1)^2 = k(5x - 12y + 1)^2$ will represent a parabola if

(A) k = 2 (B) k = 81

(C) k = 169 (D) k =1

*46. If l, m be the lengths of segments of any focal chord of a parabola $y^2 = 4ax$ then length of semi–latus rectum is

(A) $\frac{l+m}{2}$ (B) $\frac{lm}{l+m}$

(C) $\frac{2lm}{l+m}$ (D) $\sqrt{lm}$

47. The normal chord of a parabola $y^2 = 4ax$ at a point whose ordinate is equal to abscissa subtends a right angle at the

(A) focus (B) vertex

(C) end of the latus rectum (D) none of these

48. If a tangent to the parabola $y^2 = ax$ makes an angle of 45°with x – axis, its point of contact will be

(A) (a/2, a/4) (B) (-a/2, a/4)

(C) (a/4, a/2) (D) (-a/4, a/2)

49. The triangle formed by the tangents to a parabola $y^2 = 4ax$ at the ends of the latus rectum and the double ordinate through the focus is

(A) equilateral (B) isosceles

(C) right angled isosceles (D) depends on a

50. The equation $\lambda x^2 + 4xy + y^2 + \lambda x + 3y + 2 = 0$ represents a parabola if λ is

(A) –4 (B) 4

(C) 0 (D) none of these

LEVEL-II

1. From point P two tangents are drawn from it to the parabola $y^2 = 4x$ such that the slope of one tangent is three times the slope of the other. The locus of P is
(A) straight line (B) circle
(C) parabola (D) none of these

*2. The chord AB of the parabola $y^2 = 4ax$ cuts the axis of the parabola at C. If $A = (at_1^2,\ 2at_1)$, $B = (at_2^2,\ 2at_2)$ and AC : AB = 1: 3 , then
(A) $t_2 = 2t_1$ (B) $t_2 + 2t_1 = 0$
(C) $t_1 + 2t_2 = 0$ (D) none of these

3. If the normals drawn at the end points of a variable chord PQ of the parabola $y^2 = 4ax$ intersect at parabola, then the locus of the point of intersection of the tangent drawn at the points P and Q is
(A) $x + a = 0$ (B) $x – 2a = 0$
(C) $y^2 - 4x + 6 = 0$ (D) none of these

4. If the normals at the end points of a variable chord PQ of the parabola $y^2 – 4y – 2x = 0$ are perpendicular, then the tangents at P and Q will intersect at
(A) $x + y = 3$ (B) $3x – 7 = 0$
(C) $y+3 = 0$ (D) $2x + 5 = 0$

*5. The number of focal chord(s) of length 4/7 in the parabola $7y^2 = 8x$ is
(A) 1 (B) zero
(C) infinite (D) none of these .

6. The equation of common tangent touching the circle $x^2 – 4x + y^2 = 0$ and the parabola $y^2 = 4x$ is
(A) $\sqrt{2}\, y = 2x + 1$ (B) $\sqrt{2}\, y = –(x + 2)$
(C) $\sqrt{2}\, y = x + 2$ (D) none of these

7. Three normals to the parabola $y^2 = x$ are drawn through a point (c, 0) then
(A) $c = \frac{1}{4}$ (B) $c = \frac{1}{2}$
(C) $c > \frac{1}{2}$ (D) none of these

8. Tangents are drawn from (-2, 0) to $y^2 = 8x$, radius of circle(s) that would touch these tangents and the corresponding chord of contact, can be equal to,
(A) $4\left(\sqrt{2}+1\right)$ (B) $4\left(\sqrt{2}-1\right)$
(C) $8\sqrt{2}$ (D) None of these.

9. The coordinates of the point on the parabola $y = x^2 + 7x + 2$, which is nearest to the straight line $y = 3x – 3$ are
(A) (-2, -8) (B) (1, 10)
(C) (2, 20) (D) (-1, -4)

10. The equation of the common tangent to the parabola $y^2 = 32x$ and $x^2 = 108y$ is
(A) $x = 0$ (B) $2x + 3y + 36 = 0$
(C) $2x - 3y - 36 = 0$ (D) $2x - 3y + 36 = 0$

11. The locus of the middle points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex is ………………………

12. Three normals are drawn from a point (c, 0) to the parabola $y^2 = x$. One normal is always the x-axis. the value of c for which the other two normals are perpendicular to each other is ……………………………..

13. Three distinct normals are drawn from a point to a parabola. The ordinates of the foot of two normals are –1 and 3 on the parabola. The ordinate of the foot of third normal is………………………………..

14. If two of the three feet of normals drawn from a point to the parabola $y^2 = 4x$ be (1, 2) and (1, –2) then the third foot is
(A) $(2, 2\sqrt{2})$ (B) $(2, -2\sqrt{2})$
(C) (0, 0) (D) none

15. Let $y^2 = 4ax$ be a parabola and $x^2 + y^2 + 2bx = 0$ be a circle. Then condition on a and b so that parabola and circle touch each other externally is
(A) $ab > 0$ (B) $ab < 0$
(C) $ab < -1$ (D) none of these

*16. The parametric coordinates of any point on the parabola $y^2 = x$ can be
(A) $(\sin^2\theta, \sin\theta)$ (B) $(\cos^2\theta, \cos\theta)$
(C) $(\sec^2\theta, \sec\theta)$ (D) $(\tan^2\theta, \tan\theta)$

*17. Slope of tangent to $x^2 = 4y$ from (-1, -1) can be
(A) $\frac{-1+\sqrt{5}}{2}$ (B) $\frac{-1-\sqrt{5}}{2}$
(C) $\frac{1-\sqrt{5}}{2}$ (D) $\frac{1+\sqrt{5}}{2}$

18. A line ℓ passing through the focus of the parabola $y^2 = 4ax$, intersects the parabola in two distinct points. Slope of the line ℓ is
(A) any real number (B) greater than –1 and less than 1
(C) less than –1 or greater than 1 (D) none of these

19. The length of the common chord of the curves
$y^2 - 4x - 4 = 0$ and $4x^2 + 9y^2 - 36 = 0$ is
(A) $2\sqrt{3}$ units (B) $3\sqrt{2}$ units
(C) 4 units (D) 6 units

20. $\sqrt{x} + \sqrt{y} = \sqrt{a}$ represents
(A) a part of parabola (B) ellipse
(C) Hyperbola (D) Line segment

21. A line through the focus of parabola $y^2 = 4(x - 2)$ having slope 'm' meets the curve in distinct real points, then exhaustive set of values of 'm' is;
(A) $m \in (-1, 1)$
(B) $m \in (-2, 2)$
(C) $m \in (-\infty, \infty)$
(D) none of these

22. If $(y + b) = m_1 (x + a)$ and $(y + b) = m_2 (x + a)$ be tangents of $y^2 = 4ax$ then;
(A) $m_1 + m_2 = 0$
(B) $m_1 m_2 = 0$
(C) $m_1 m_2 = -1$
(D) $m_1 = -m_2 - \frac{2}{m_2}$

*23. A tangent to the parabola $x^2 = 4ay$ is inclined at an angle $\frac{\pi}{6}$ with the x-axis, then coordinates of point of contact is;
(A) $\left(3a, 2a\sqrt{3}\right)$
(B) $\left(\frac{a}{3}, \frac{2a}{\sqrt{3}}\right)$
(C) $\left(\frac{a}{3}, -\frac{2a}{\sqrt{3}}\right)$
(D) $\left(\frac{2a}{\sqrt{3}}, \frac{a}{3}\right)$

24. The length of focal chord of the parabola $y^2 = 4ax$ at a distance b from the vertex is c then
(A) $2a^2 = bc$
(B) $a^3 = b^2c$
(C) $ac = b^2$
(D) $b^2c = 4a^3$

LEVEL-III

1. The circle drawn with variable chord x + ay – 5 = 0 (a being a parameter) of the parabola $y^2 = 20x$ as diameter will always touch the line
(A) x + 5 = 0 (B) y + 5 = 0
(C) x + y + 5 = 0 (D) x – y + 5 = 0

2. The set of points on the axis of the parabola $2((x-1)^2 + (y-1)^2) = (x+y)^2$, from which 3 distinct normals can be drawn to the parabola, is the set of points (h, k) lying on the axis of the parabola such that
(A) h > 3 (B) h > 3/2
(C) k > 3/2 (D) k > 3

3. Radius of the circle passing through the origin and touching the parabola $y^2 = 4x$ at (1, 2)
(A) 5/6 (B) $5\sqrt{2}/6$
(C) $5/\sqrt{2}$ (D) none of these

4. If the parabola y = f (x), having axis parallel to y-axis, touches the line y = x at (1, 1) then;
(A) $2f'(0) + f(0) = 1$ (B) $2f(0) + f'(0) = 1$
(C) $2f(0) - f'(0) = 1$ (D) $2f'(0) - f(0) = 1$

*5. The length of latus rectum of the parabola whose focus is (a sin2θ, a cos2θ) and directrix is the line y = a, is
(A) $\left|4a\cos^2\theta\right|$ (B) $\left|4a\sin^2\theta\right|$
(C) $\left|4a\cos 2\theta\right|$ (D) $\left|4a\sin 2\theta\right|$

6. Chord AB of the parabola $y^2 = 4ax$ subtends a right angle at the origin. Point of intersection of tangents drawn to parabola at 'A' and 'B' lie on the line -
(A) x + 2a = 0 (B) y + 2a = 0
(C) x + 4a = 0 (D) y + 4a = 0

7. A circle is drawn to pass through the extremities of the latus rectum of the parabola $y^2 = 8x$. It is given that this circle also touches the directrix of the parabola. Radius of this circle is equal to
(A) 2 (B) $\sqrt{21}$
(C) 8 (D) 4

8. The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts the parabola $x^2 = 4ay$ at points $P_i(x_i, y_i)$, i = 1, 2, 3, 4; then
(A) $\sum y_i = 0$ (B) $\sum x_i = 0$
(C) $\sum y_i = -4(f+2a)$ (D) $\sum x_i = -2(g+2a)$

9. Maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ can be equal to

(A) 3 (B) 4

(C) 6 (D) 5

10. Maximum distance between the curves $y^2 = x - 1$ and $x^2 = y - 1$ is equal to

(A) $\frac{3\sqrt{2}}{4}$ (B) $\frac{5\sqrt{2}}{4}$

(C) $\frac{7\sqrt{2}}{4}$ (D) $\frac{\sqrt{2}}{4}$

11. Sides of an equilateral triangle ABC touch the parabola $y^2 = 4x$, then points A, B and C lie on

(A) $y^2 = 3(x+a)^2 + 4ax$ (B) $y^2 = \frac{(x+a)^2}{3} + 4ax$

(C) $x^2 = \frac{(y+a)^2}{3} + 4ay$ (D) $x^2 = 3(y+a)^2 + 4ay$

12. Length of the latus rectum of the parabola whose parametric equation is :
$x = t^2 + t + 1$; $y = t^2 - t + 1$, where t $\in$ R, is equal to

(A) 8 (B) 4

(C) 2 (D) $\sqrt{2}$

13. A circle having its centre at (2, 3) is cut orthogonally by the parabola $y^2 = 4x$. The possible intersection point of these curves can be

(A) (1, 2) or (3, $2\sqrt{3}$) (B) (1, 2) or (4, 4)

(C) (9, 6) or (3, $2\sqrt{3}$) (D) None

14. The vertex of the parabola $(x+y-1)^2 = 2(x-y+2)$ is

(A) (2, –1) (B) $\left(-\frac{13}{4}, \frac{17}{4}\right)$

(C) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (D) $\left(\frac{19}{8}, \frac{35}{8}\right)$

15. The axis of the parabola $(x+y-1)^2 = 2(x-y+2)$ is

(A) y = x + 2 (B) x – y = 1

(C) x + y = 2 (D) x + y = 1

16. The line x + y = a touches the parabola $y = x - x^2$ and

$f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$, $g\left(\frac{5}{4}\right) = 1$, $b = g(f(x))$, then `

(A) a = b (B) a = 2b
(C) a + b = 0 (D) a + 2b = 0

17. The co-ordinates of the point on the parabola $y^2 = 8x$, which is at minimum distance from the circle $x^2 + (y+6)^2 = 1$ are
(A) (2, 4) (B) (–2, 4)
(C) (–2, –4) (D) (2, –4)

18. If three normals can be drawn to the parabola $y^2 = x$ from the point (C, 0), then the two normals other than the axis of the parabola are perpendicular to each other if C =
(A) $\frac{3}{4}$ (B) $\frac{4}{3}$
(C) $-\frac{3}{4}$ (D) $-\frac{4}{3}$

19. If $f(x) = \frac{1}{1-x}$ and $\alpha, \ \beta(\alpha > \beta)$ be the values of x, where f(f(x)) is not defined, then a ray of light parallel to the axis of the parabola $y^2 = 4x$ after reflection from the internal surface of the parabola will necessarily pass through the point
(A) (α, α) (B) (α, β)
(C) (β, β) (D) None

*20. If t_1 and t_2 be the ends of a focal chord of the parabola $y^2 = 4ax$, then the equation $t_1x^2 + ax + t_2 = 0$ has
(A) imaginary roots, (B) both roots positive
(C) one positive and one negative roots (D) both roots negative

ANSWERS

LEVEL –I

1.	A	2.	C	3.	D	4.	B
5.	A	6.	A	7.	B	8.	D
9.	A	10.	A	11.	C	12.	A
13.	C	14.	A	15.	C	16.	B
17.	B	18.	B	20.	C	21.	C
22.	A	23.	D	24.	A	25.	C
26.	C	27.	B	28.	C	29.	B
30.	A	31.	(-2, 2)	32.	$a^3b = 2pba^2 + pb^3$		
33.	-12, (4, -4)	34.	± 1	35.	C	36.	C
37.	B	38.	D	39.	C	40.	A
41.	B	42.	C	43.	A	44.	A
45.	D	46.	C	47.	A	48.	C
49.	C	50.	B				

LEVEL –II

1.	C	2.	B	3.	B	4.	D
5.	B	6.	D	7.	C	8.	B
9.	A	10.	B	11.	$y^2 – 2ax + 8a^2 = 0$		
12.	$c = \frac{3}{4}$						
13.	-2	14.	C	15.	B	16.	D
17.	A, B	18.	D	19.	C	20.	A
21.	D	22.	C	23.	D	24.	D

LEVEL –III

1.	A	2.	A, B, C, D	3.	C	4.	B
5.	B	6.	C	7.	D	8.	B
9.	D	10.	A	11.	A	12.	D
13.	B	14.	C	15.	D	16.	A
17.	D	18.	A	19.	B	20.	C