

CBSE Class 12 physics
Important Questions
Chapter 1
Electric Charges and Fields

1 Mark Questions

1. Show does the force between two point charges change if the dielectric constant of the medium in which they are kept increase?

Ans. Since K

$$= \frac{FV}{FM} = \frac{\text{force between the charges in vacuum}}{\text{force between two charges in medium}}$$

$$\Rightarrow F_m = \frac{FV}{k}$$

\Rightarrow if k increases, F_m decreases.

2. A charged rod P attracts rod R where as P repels another charged rod Q. What type of force is developed between Q and R?

Ans. Suppose rod P be negatively charged since it attracts rod R

\Rightarrow R is positively charged since it repels rod Q

\Rightarrow Q is negatively charged. So force between Q and R is attractive in nature.

3. Which physical quantity has its S.I unit (1) Cm (2) N/C

Ans. (1) Electric dipole moment (2) Electric field Intensity

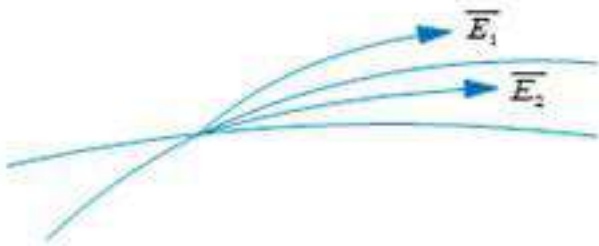
4. Define one coulomb?

Ans. Charge on a body is said to be 1 coulomb if two charges experiences a force of repulsion or attraction of $9 \times 10^9 \text{ N}$ when they are separated by a distance of 1 m.

2 Mark Questions

1. A free proton and a free electron are placed in a uniform field. Which of the two experience greater force and greater acceleration?

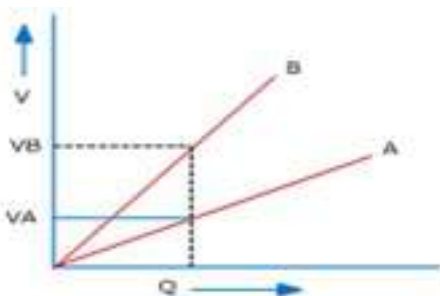
Ans. As $F = q E$ and $a = F/m$ as charge on both e^- and proton are equal and opposite in nature, so force on them would be equal but as mass of proton is more than that of electron, so acceleration of electron would be more.



2. No two electric lines of force can intersect each other? Why?

Ans. Two electric lines of force never intersect each other because if they intersect then at the point of intersection there will be two tangents which is not possible as the two tangents represents two directions for electric field lines.

3. The graph shows the variation of voltage V across the plates of two capacitors A and B versus increase of charge Q stored on them. Which of the two capacitors have higher capacitance? Give reason for your answer?



Ans. Since $C = Q/V$

For a given charge Q

$$C \propto \frac{1}{V}$$

and since $V_A < V_B$

$$\therefore C_A > C_B$$

4. An electric dipole when held at 30° with respect to a uniform electric field of 10^4 N/C experienced a Torque of $9 \times 10^{-26} \text{ Nm}$. Calculate dipole moment of the dipole?

Ans. Given

$$\theta = 30^\circ$$

$$\tau = 9 \times 10^{-26} \text{ Nm}$$

$$E = 10^4 \text{ N/c}$$

$$\bar{P} = ?$$

$$\tau \text{ (Torque)} = PE \sin \theta$$

$$P = \frac{\tau}{E \sin \theta}$$

$$\Rightarrow P = \frac{9 \times 10^{-26}}{10^4 \times \sin 30^\circ} = \frac{9 \times 10^{-26} \times 10^{-4}}{1/2}$$

$$\Rightarrow P = 18 \times 10^{-30} \text{ Cm}$$

5. (a) Explain the meaning of the statement 'electric charge of a body is quantised'.

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

Ans. (a) Electric charge of a body is quantized. This means that only integral (1, 2, ..., n)

number of electrons can be transferred from one body to the other. Charges are not transferred in fraction. Hence, a body possesses total charge only in integral multiples of electric charge.

(b) In macroscopic or large scale charges, the charges used are huge as compared to the magnitude of electric charge. Hence, quantization of electric charge is of no use on macroscopic scale. Therefore, it is ignored and it is considered that electric charge is continuous.

6. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

Ans. Rubbing produces charges of equal magnitude but of opposite nature on the two bodies because charges are created in pairs. This phenomenon of charging is called charging by friction. The net charge on the system of two rubbed bodies is zero. This is because equal amount of opposite charges annihilates each other. When a glass rod is rubbed with a silk cloth, opposite natured charges appear on both the bodies. This phenomenon is in consistence with the law of conservation of energy. A similar phenomenon is observed with many other pairs of bodies.

7. (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?

(b) Explain why two field lines never cross each other at any point?

Ans. (a) An electrostatic field line is a continuous curve because a charge experiences a continuous force when traced in an electrostatic field. The field line cannot have sudden breaks because the charge moves continuously and does not jump from one point to the other.

(b) If two field lines cross each other at a point, then electric field intensity will show two directions at that point. This is not possible. Hence, two field lines never cross each other.

8. An electric dipole with dipole moment $4 \times 10^{-9} \text{ Cm}$ is aligned at 30° with the direction of a uniform electric field of magnitude $5 \times 10^4 \text{ N C}^{-1}$. Calculate the magnitude of the torque acting on the dipole.

Ans. Electric dipole moment, $p = 4 \times 10^{-9} \text{ Cm}$

Angle made by p with a uniform electric field, $\theta = 30^\circ$

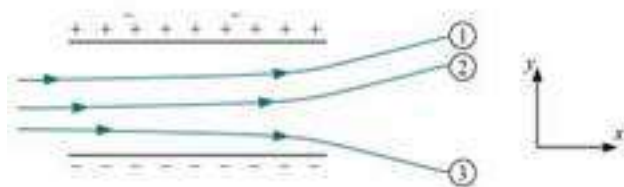
Electric field, $E = 5 \times 10^4 \text{ N C}^{-1}$

Torque acting on the dipole is given by the relation,

$$\begin{aligned} T &= pE \sin \theta \\ &= 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30 \\ &= 20 \times 10^{-5} \times \frac{1}{2} \\ &= 10^{-4} \text{ Nm} \end{aligned}$$

Therefore, the magnitude of the torque acting on the dipole is 10^{-4} N m .

9. Figure 1.33 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Ans. Opposite charges attract each other and same charges repel each other. It can be observed that particles 1 and 2 both move towards the positively charged plate and repel away from the negatively charged plate. Hence, these two particles are negatively charged. It can also be observed that particle 3 moves towards the negatively charged plate and repels away from the positively charged plate. Hence, particle 3 is positively charged.

The charge to mass ratio (emf) is directly proportional to the displacement or amount of deflection for a given velocity. Since the deflection of particle 3 is the maximum, it has the highest charge to mass ratio.

10. What is the net flux of the uniform electric field of Exercise 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Ans. All the faces of a cube are parallel to the coordinate axes. Therefore, the number of field lines entering the cube is equal to the number of field lines piercing out of the cube. As a result, net flux through the cube is zero.

11. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is $8.0 \times 10^3 \text{ N m}^2 / \text{C}$. (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?

Ans. (a) Net outward flux through the surface of the box, $\Phi = 8.0 \times 10^3 \text{ N m}^2 / \text{C}$

For a body containing net charge q , flux is given by the relation,

$$\phi = \frac{q}{\epsilon_0}$$

ϵ_0 = Permittivity of free space

$$= 8.854 \times 10^{-12} \text{ N}^{-1} \text{C}^2 \text{ m}^{-2} \quad q = \epsilon_0 \Phi$$

$$= 8.854 \times 10^{-12} \times 8.0 \times 10^3$$

$$= 7.08 \times 10^{-8}$$

$$= 0.07 \mu C$$

Therefore, the net charge inside the box is $0.07 \mu C$.

(b) No

Net flux piercing out through a body depends on the net charge contained in the body. If net flux is zero, then it can be inferred that net charge inside the body is zero. The body may have equal amount of positive and negative charges.

3 Mark Questions

1. A particle of mass m and charge q is released from rest in a uniform electric field of intensity E . calculate the kinetic energy it attains after moving a distance s between the plates?

Ans. Since $F = qE$

$$\therefore a = \frac{F}{m} = \frac{qE}{m} \quad \text{---1}$$

Using third equation of motion

$$v^2 - u^2 = 2as$$

Initially charged particle is at rest $\therefore u = 0$

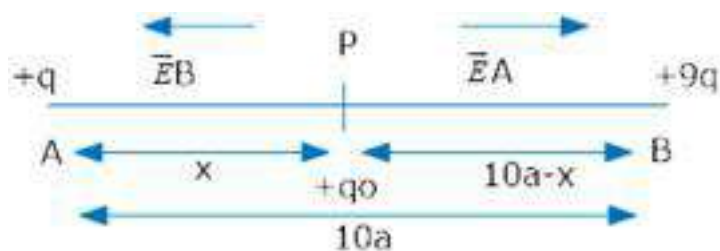
$$\Rightarrow v^2 = 2as$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} m (2as) = mas \quad \text{.....2}$$

Substituting 1 in eq. 2

$$KE = m \times \frac{qE}{m} \times s$$

$$KE = qES$$



2. Two point charges $+q$ and $+9q$ are separated by a distance of $10a$. Find the point on the line joining the two charges where electric field is zero?

Ans. Let P be the pt where test charge ($+q_0$) is present then electric field at pt. P will be zero if Field at pt. P due to $+q$ = field at pt. P due to $+9q$ -----1

$$\vec{E} \Rightarrow E_A = \frac{K(+q)}{x^2} E_B = \frac{K(+9q)}{(10a-x)^2}$$

Substituting in eq. 1

$$\frac{K(+q)}{x^2} = \frac{K(+9q)}{(10a-x)^2}$$

$$(10a-x)^2 = 9x^2 \Rightarrow 10a-x = 3x$$

$$10a = 4x \Rightarrow x = \frac{10}{4}a$$

$$x = 2.5a \text{ from charge } (+q)$$

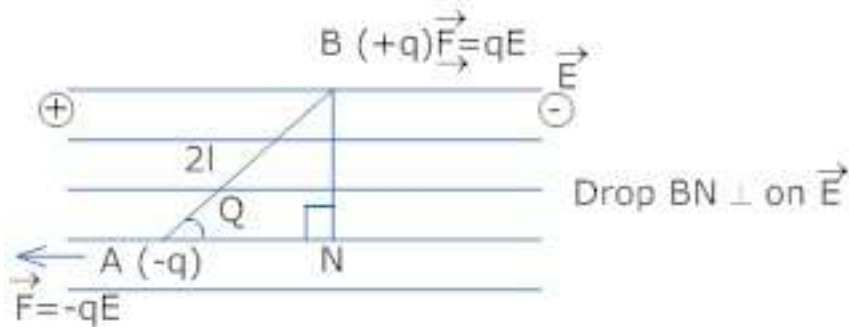
Or

$$10a - x = 10a - 2.5a = 7.5a \text{ from charge } (+9q)$$

3. Define the term dipole moment \vec{P} of an electric dipole indicating its direction. Write its S.I unit. An electric dipole is placed in a uniform electric field \vec{E} . Deduce the expression for the Torque acting on it.

Ans. Electric dipole moment is defined as the product of the magnitude of either charge and the length of dipole. Its direction is from -ve to +ve charge.

$$\vec{P} = q(\vec{2l}) \text{ Its S.I. unit is coulomb meter (Cm)}$$



Consider a dipole placed in uniform electric field and makes an angle (θ) with the electric field (\vec{E}). Since two forces act on the charges constituting an electric dipole which are equal and opposite in direction, thus a torque acts on the dipole which makes the dipole rotate.

And Torque $\tau = \text{Force} \times \perp \text{ distance}$

Here force (F) = qE

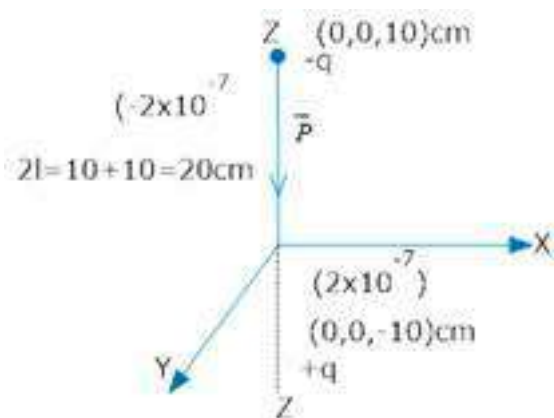
$$\text{And } \frac{BN}{AB} = \sin \theta \Rightarrow BN = AB \sin \theta = 2l \sin \theta$$

$$(\tau) = qE \times 2l \sin \theta$$

$$(\tau) = PE \sin \theta \quad (\because \vec{P} = q(\vec{2l}))$$

$$\vec{\tau} = \vec{P} \times \vec{E}$$

In vector form



4. A sphere of radius r_1 encloses a charge Q . If there is another concentric sphere S_2 of radius r_2 ($r_2 > r_1$) and there is no additional charge between S_1 and S_2 . Find the ratio of electric flux through S_1 and S_2 ?

Ans. $\theta = q/\epsilon_0$ (where θ = electric flux)

$$\theta_{s1} = \frac{Q}{\epsilon_0}$$

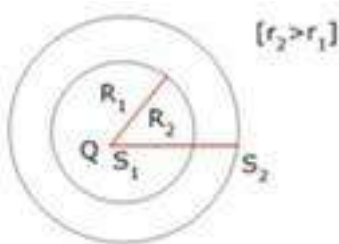
For sphere S_2

$$\theta_{s2} = \frac{Q}{\epsilon_0} \text{ (since no additional charge is given)}$$

$$\text{So } \frac{\theta_{s1}}{\theta_{s2}} = \frac{Q/\epsilon_0}{Q/\epsilon_0} = 1:1$$

$$\text{So } \theta_{s1}:\theta_{s2} = 1:1$$

5. Electric charge is uniformly distributed on the surface of a spherical balloon. Show how electric intensity and electric potential vary (a) on the surface (b) inside and (c) outside.

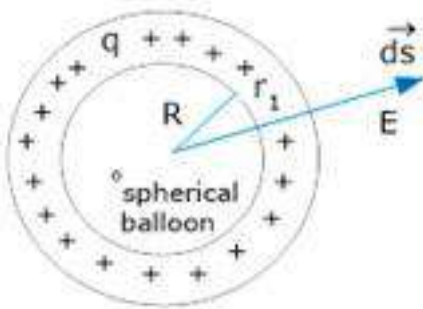


Ans. Electric field intensity on the surface of a shell

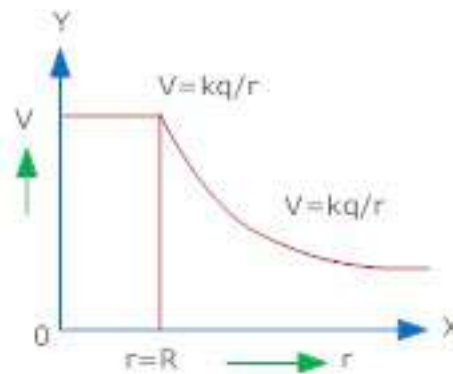
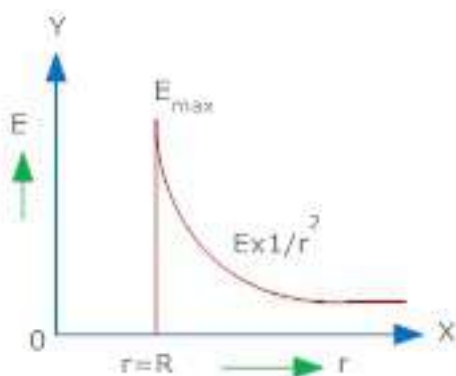
$$E = \sigma / \epsilon_0 \text{ \& } V = Kq/R$$

$$\text{Inside } E = 0 \text{ \& } V = Kq/R$$

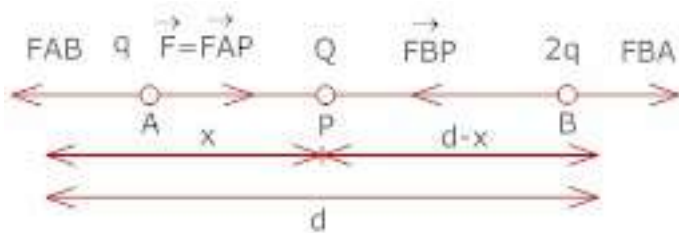
$$\text{Outside } E = \frac{\sigma}{\epsilon_0} \frac{R^2}{r^2} \text{ \& } V = Kq/r$$



Graphically



6. Two point electric charges of value q and $2q$ are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and $2q$ is zero. Calculate the position of charge Q in terms of q and d .



Ans. Net force on charge q and $2q$ will be zero if the third charge is negative (i.e. of opposite sign) and q and $2q$ are positive, Force on charge q will be zero if

$$|F_{AB}| = |F_{AP}|$$

$$\frac{Kq(2q)}{d^2} = \frac{Kq(Q)}{x^2}$$

$$\frac{Q}{q} = \frac{2x^2}{d^2} \text{-----(1)}$$

Force on charge 2q to be zero

$$\text{if } |F_{\overline{B}A}| = |F_{\overline{B}P}|$$

$$\frac{Kq(2q)}{d^2} = \frac{K(2q)Q}{(d-x)^2}$$

$$\frac{Q}{q} = \frac{(d-x)^2}{d^2} \text{-----(2)}$$

comparing equation 1 and 2

$$\frac{2x^2}{d^2} = \frac{(d-x)^2}{d^2}$$

$$x^2 = \frac{(d-x)^2}{2}$$

$$x^2 = \frac{(d-x)^2}{(\sqrt{2})^2}$$

$$\Rightarrow x = \frac{d-x}{\sqrt{2}} \text{ or } 2x+x=d$$

$$x(\sqrt{2}+1)=d$$

$$\Rightarrow x = \frac{d}{\sqrt{2}+1}$$

7. What is the force between two small charged spheres having charges of $2 \times 10^{-7} \text{ C}$ and $3 \times 10^{-7} \text{ C}$ placed 30 cm apart in air?

Ans. Repulsive force of magnitude $6 \times 10^{-3} \text{ N}$

Charge on the first sphere, $q_1 = 2 \times 10^{-7} \text{ C}$

Charge on the second sphere, $q_2 = 3 \times 10^{-7} \text{ C}$

Distance between the spheres, $r = 30 \text{ cm} = 0.3 \text{ m}$

Electrostatic force between the spheres is given by the relation,

$$F = \frac{q_1 q_2}{4\pi \epsilon_0 r^2}$$

Where, ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{C}^{-2}$$

$$F = \frac{9 \times 10^9 \times 2 \times 10^{-7} \times 3 \times 10^{-7}}{(0.3)^2} = 6 \times 10^{-3} \text{ N}$$

Hence, force between the two small charged spheres is $6 \times 10^{-3} \text{ N}$. The charges are of same nature. Hence, force between them will be repulsive.

8. The electrostatic force on a small sphere of charge $0.4 \mu\text{C}$ due to another small sphere of charge $-0.8 \mu\text{C}$ in air is 0.2 N . (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

Ans. (a) Electrostatic force on the first sphere, $F = 0.2 \text{ N}$

Charge on this sphere, $q^1 = 0.4 \mu\text{C} = 0.4 \times 10^{-6} \text{ C}$

Charge on the second sphere, $q^2 = -0.8 \mu\text{C} = -0.8 \times 10^{-6} \text{ C}$

Electrostatic force between the spheres is given by the relation,

$$F = \frac{q_1 q_2}{4\pi \epsilon_0 r^2} \text{ And } \frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{C}^{-2}$$

Where, ϵ_0 = Permittivity of free space

$$\text{And, } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^{-2}\text{C}^{-2}$$

$$r^2 = \frac{q_1 q_2}{4\pi\epsilon_0 F}$$

$$= 144 \times 10^{-4}$$

$$r = \sqrt{144 \times 10^{-4}} = 0.12 \text{ m}$$

The distance between the two spheres is 0.12m.

(b) Both the spheres attract each other with the same force. Therefore, the force on the second sphere due to the first is 0.2N.

9. A polythene piece rubbed with wool is found to have a negative charge of $3 \times 10^{-7} \text{ C}$

(a) Estimate the number of electrons transferred (from which to which?)

(b) Is there a transfer of mass from wool to polythene?

Ans. (a) When polythene is rubbed against wool, a number of electrons get transferred from wool to polythene. Hence, wool becomes positively charged and polythene becomes negatively charged.

Amount of charge on the polythene piece, $q = - 3 \times 10^{-7} \text{ C}$

Amount of charge on an electron, $e = - 1.6 \times 10^{-19} \text{ C}$

Number of electrons transferred from wool to polythene = n

n can be calculated using the relation,

$$q = ne$$

$$\begin{aligned}
 n &= \frac{q}{e} \\
 &= \frac{-3 \times 10^{-7}}{-1.6 \times 10^{-19}} \\
 &= 1.87 \times 10^{12}
 \end{aligned}$$

Therefore, the number of electrons transferred from wool to polythene is 1.87×10^{12} .

(b) Yes.

There is a transfer of mass taking place. This is because an electron has mass,

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

Total mass transferred to polythene from wool,

$$\begin{aligned}
 m &= m_e \times n \\
 &= 9.1 \times 10^{-31} \times 1.87 \times 10^{12} \\
 &= 1.706 \times 10^{-18} \text{ kg}
 \end{aligned}$$

Hence, a negligible amount of mass is transferred from wool to polythene.

10. Consider a uniform electric field $\vec{E} = 3 \times 10^3 \hat{i} \text{ N/C}$. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis?

Ans. (a) Electric field intensity, $\vec{E} = 3 \times 10^3 \hat{i} \text{ N/C}$

Magnitude of electric field intensity, $|\vec{E}| = 3 \times 10^3 \text{ N/C}$

Side of the square, $s = 10 \text{ cm} = 0.1 \text{ m}$

Area of the square, $A = s^2 = 0.01 \text{ m}^2$

The plane of the square is parallel to the y - z plane. Hence, angle between the unit vector normal to the plane and electric field, $\theta = 0^\circ$

Flux (Φ) through the plane is given by the relation,

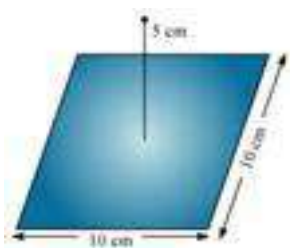
$$\begin{aligned}\Phi &= |\vec{E}| A \cos \theta \\ &= 3 \times 10^3 \times 0.01 \times \cos 0^\circ \\ &= 30 \text{ N m}^2 / \text{C}\end{aligned}$$

(b) Plane makes an angle of 60° with the x -axis. Hence, $\theta = 60^\circ$

$$\begin{aligned}\text{Flux, } \Phi &= |\vec{E}| A \cos \theta \\ &= 3 \times 10^3 \times 0.01 \times \cos 60^\circ \\ &= 30 \times \frac{1}{2} \\ &= 15 \text{ N m}^2 / \text{C}\end{aligned}$$

11. A point charge $+10 \mu\text{C}$ is at a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. 1.34. What is the magnitude of the electric flux through the square? (*Hint: Think of the square as one face of a cube with edge 10 cm.*)

Ans. The square can be considered as one face of a cube of edge 10 cm with a centre where charge q is placed. According to Gauss's theorem for a cube, total electric flux is through all its six faces.



$$\phi_{\text{Total}} = \frac{q}{\epsilon_0}$$

Hence, electric flux through one face of the cube i.e., through the square, $\phi = \frac{\phi_{Total}}{6}$

$$= \frac{1}{6} \frac{q}{\epsilon_0}$$

Where,

ϵ_0 = Permittivity of free space

$$= 8.854 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$$

$$q = 10 \mu\text{C}$$

$$= 10 \times 10^{-6} \text{ C}$$

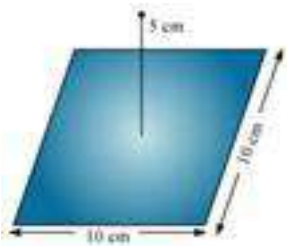
$$\therefore \phi = \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.854 \times 10^{-12}}$$

$$= 1.88 \times 10^5 \text{ N m}^2 \text{ C}^{-1}$$

Therefore, electric flux through the square is $1.88 \times 10^5 \text{ N m}^2 \text{ C}^{-1}$.

12..A point charge of $2.0 \mu\text{C}$ is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

Ans. The square can be considered as one face of a cube of edge 10 cm with a centre where charge q is placed. According to Gauss's theorem for a cube, total electric flux is through all its six faces.



$$\phi_{Total} = \frac{q}{\epsilon_0}$$

Hence, electric flux through one face of the cube i.e., through the square, $\phi = \frac{\phi_{Total}}{6}$

$$= \frac{1}{6} \frac{q}{\epsilon_0}$$

Where,

ϵ_0 = Permittivity of free space

$$= 8.854 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$$

$$q = 10 \mu\text{C}$$

$$= 10 \times 10^{-6} \text{ C}$$

$$\therefore \phi = \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.854 \times 10^{-12}}$$

$$= 1.88 \times 10^5 \text{ N m}^2 \text{ C}^{-1}$$

Therefore, electric flux through the square is $1.88 \times 10^5 \text{ N m}^2 \text{ C}^{-1}$.

13. A point charge causes an electric flux of $-1.0 \times 10^3 \text{ Nm}^2 / \text{C}$ to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

Ans. (a) Electric flux, $\Phi = -1.0 \times 10^3 \text{ N m}^2 / \text{C}$

Radius of the Gaussian surface,

$$r = 10.0 \text{ cm}$$

Electric flux piercing out through a surface depends on the net charge enclosed inside a body. It does not depend on the size of the body. If the radius of the Gaussian surface is doubled, then the flux passing through the surface remains the same i.e., $-10^3 \text{ Nm}^2 / \text{C}$.

(b) Electric flux is given by the relation,

$$\phi = \frac{q}{\epsilon_0}$$

Where,

q = Net charge enclosed by the spherical surface

ϵ_0 = Permittivity of free space = $8.854 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$

$$\therefore q = \phi \epsilon_0$$

$$= -1.0 \times 10^3 \times 8.854 \times 10^{-12}$$

$$= -8.854 \times 10^{-9} \text{ C}$$

$$= -8.854 \text{ nC}$$

Therefore, the value of the point charge is -8.854 nC .

14. A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is $1.5 \times 10^3 \text{ N/C}$ and points radially inward, what is the net charge on the sphere?

Ans. Electric field intensity (E) at a distance (d) from the centre of a sphere containing net charge q is given by the relation,

$$E = \frac{q}{4\pi\epsilon_0 d^2}$$

Where,

$$q = \text{Net charge} = 1.5 \times 10^3 \text{ N/C}$$

$$d = \text{Distance from the centre} = 20 \text{ cm} = 0.2 \text{ m}$$

ϵ_0 = Permittivity of free space

And, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

$$\therefore q = E(4\pi\epsilon_0)d^2$$

$$= \frac{1.5 \times 10^3 \times (0.2)^2}{9 \times 10^9}$$

$$= 6.67 \times 10^{-9} \text{ C} = 6.67 \text{ nC}$$

Therefore, the net charge on the sphere is 6.67 nC.

15. A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of $80.0 \mu\text{C} / \text{m}^2$. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

Ans. (a) Diameter of the sphere, $d = 2.4 \text{ m}$

Radius of the sphere, $r = 1.2 \text{ m}$

Surface charge density, $\sigma = 80.0 \mu\text{C} / \text{m}^2 = 80 \times 10^{-6} \text{ C} / \text{m}^2$

Total charge on the surface of the sphere,

$Q = \text{Charge density} \times \text{Surface area}$

$$= \sigma \times 4\pi r^2$$

$$= 80 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2 = 1.447 \times 10^{-3} \text{ C}$$

Therefore, the charge on the sphere is $1.447 \times 10^{-3} \text{ C}$.

(b) Total electric flux (ϕ_{Total}) leaving out the surface of a sphere containing net charge Q is given by the relation,

$$\phi_{\text{Total}} = \frac{Q}{\epsilon_0}$$

Where,

ϵ_0 = Permittivity of free space

$$= 8.854 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$$

$$Q = 1.447 \times 10^{-3} \text{ C}$$

$$\phi_{\text{Total}} = \frac{1.44 \times 10^{-3}}{8.854 \times 10^{-12}}$$

$$= 1.63 \times 10^8 \text{ N C}^{-1} \text{ m}^2$$

Therefore, the total electric flux leaving the surface of the sphere is $1.63 \times 10^8 \text{ N C}^{-1} \text{ m}^2$.

16. An infinite line charge produces a field of $9 \times 10^4 \text{ N/C}$ at a distance of 2 cm. Calculate the linear charge density.

Ans. Electric field produced by the infinite line charges at a distance d having linear charge density λ is given by the relation,

$$E = \frac{\lambda}{2\pi\epsilon_0 d}$$

$$\lambda = 2\pi\epsilon_0 dE$$

Where,

$$d = 2 \text{ cm} = 0.02 \text{ m}$$

$$E = 9 \times 10^4 \text{ N/C}$$

ϵ_0 = Permittivity of free space

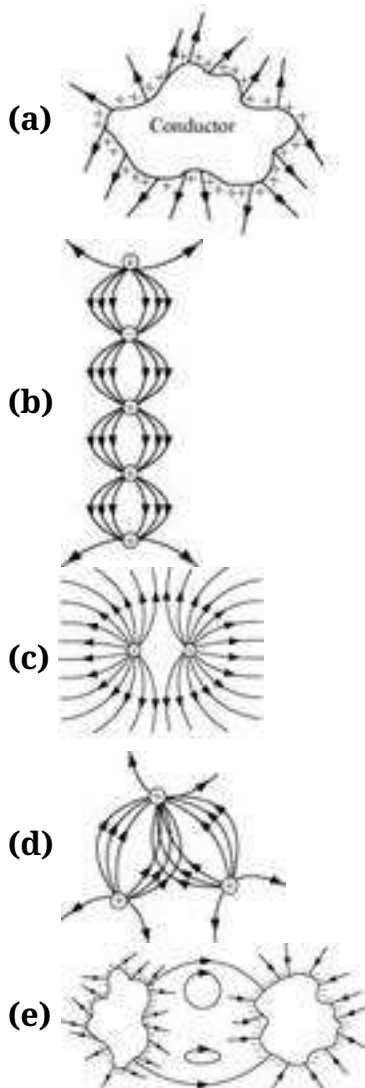
$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\pi = \frac{0.02 \times 9 \times 10^4}{2 \times 9 \times 10^9}$$

$$= 10 \mu\text{C} / \text{m}$$

Therefore, the linear charge density is $= 10 \mu\text{C} / \text{m}$.

17. Which among the curves shown in Fig. 1.35 cannot possibly represent electrostatic field lines?



Ans. (a) The field lines showed in (a) do not represent electrostatic field lines because field lines must be normal to the surface of the conductor.

(b) The field lines showed in (b) do not represent electrostatic field lines because the field lines cannot emerge from a negative charge and cannot terminate at a positive charge.

(c) The field lines showed in (c) represent electrostatic field lines. This is because the field

lines emerge from the positive charges and repel each other.

(d) The field lines showed in (d) do not represent electrostatic field lines because the field lines should not intersect each other.

(e) The field lines showed in (e) do not represent electrostatic field lines because closed loops are not formed in the area between the field lines.

18. Suppose that the particle in Exercise in 1.33 is an electron projected with velocity $v_x = 2.0 \times 10^6 \text{ m s}^{-1}$. If E between the plates separated by 0.5 cm is $9.1 \times 10^2 \text{ N/C}$, where will the electron strike the upper plate? ($|e| = 1.6 \times 10^{-19} \text{ C}$, $m_e = 9.1 \times 10^{-31} \text{ kg}$.)

Ans. Velocity of the particle, $v_x = 2.0 \times 10^6 \text{ m/s}$

Separation of the two plates, $d = 0.5 \text{ cm} = 0.005 \text{ m}$

Electric field between the two plates, $E = 9.1 \times 10^2 \text{ N/C}$

Charge on an electron, $q = 1.6 \times 10^{-19} \text{ C}$

Mass of an electron, $m_e = 9.1 \times 10^{-31} \text{ kg}$

Let the electron strike the upper plate at the end of plate L , when deflection is s .

Therefore,

$$\begin{aligned} s &= \frac{qEL^2}{2mv_x^2} \\ L &= \sqrt{\frac{2dmv_x^2}{qE}} \\ &= \sqrt{\frac{2 \times 0.005 \times 9.1 \times 10^{-31} \times (2.0 \times 10^6)^2}{1.6 \times 10^{-19} \times 9.1 \times 10^2}} \\ &= \sqrt{0.025 \times 10^{-2}} = \sqrt{2.5 \times 10^{-4}} \\ &= 1.6 \times 10^{-2} \\ &= 1.6 \text{ cm} \end{aligned}$$

Therefore, the electron will strike the upper plate after travelling 1.6 cm.

5 Mark Questions

1. (1) The electric field \vec{E} due to a point charge at any point near to it is defined as:

$$\vec{E} = \lim_{q \rightarrow 0} \frac{\vec{F}}{q} \text{ where } q \text{ is the test charge and } \vec{F} \text{ is the force acting on it. What is}$$

the significance of $\lim_{q \rightarrow 0}$ in this expression?

(2) Two charges each $2 \times 10^{-7} \text{ C}$ but opposite in sign forms a system. These charges are located at points A (0,0, -10) cm and B (0,0, +10) cm respectively. What is the total charge and electric dipole moment of the system?

Ans. (1) The Significance of writing $\lim_{q \rightarrow 0}$ means the test charge should be vanishingly small so that it should not disturb the presence of source charge.

(2) (i) Total charge of the system

$$= 2 \times 10^{-7} + (-2 \times 10^{-7})$$

$$= \text{zero. } \vec{P}$$

$$(ii) \vec{P} = q \times 2\vec{l}$$

$$P = 2 \times 10^{-7} \times 20 \times 10^{-2}$$

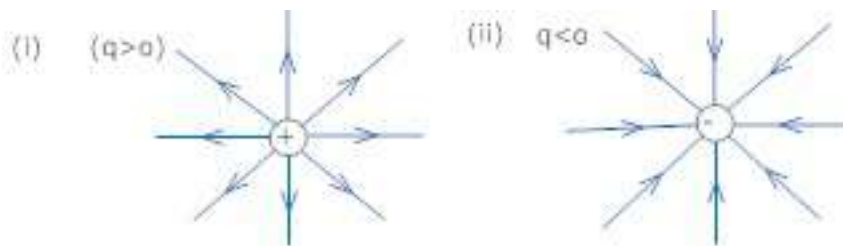
$$P = 4 \times 10^{-8} \text{ cm}$$

Direction of Dipole moment -Along negative z-axis.

2. (a) Sketch electric lines of force due to (i) isolated positive charge (ie $q > 0$) and (ii) isolated negative charge (ie $q < 0$)

(b) Two point charges q and $-q$ are placed at a distance $2a$ apart. Calculate the electric field at a point P situated at a distance r along the perpendicular bisector of the line joining the charges. What is the field when $r \gg a$?

Ans.

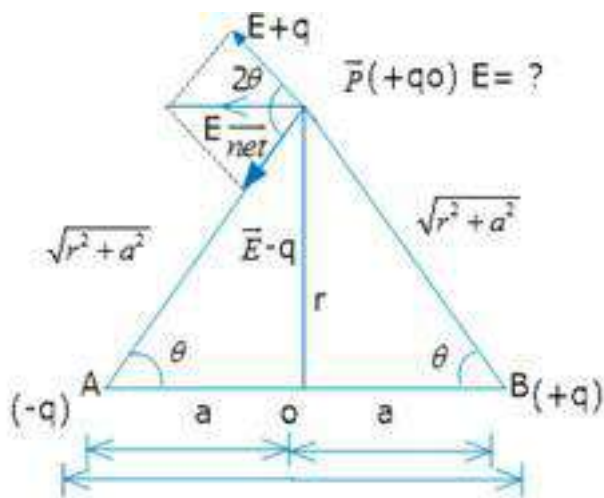


(b)

$$|\vec{E} + q| = \frac{Kq}{(r^2 + a^2)}$$

$$|\vec{E} - q| = \frac{Kq}{(r^2 + a^2)}$$

Since $|\vec{E} + q| = |\vec{E} - q|$



$$|\vec{E}_{net}| = \sqrt{E_{+q}^2 + E_{-q}^2 + 2E_{+q} E_{-q} \cos 2\theta}$$

$$|\vec{E}_{net}| = \sqrt{2E_{+q}^2 + 2E_{+q}^2 \cos 2\theta}$$

$$|\vec{E}_{net}| = \sqrt{2E_{+q}^2 (1 + \cos 2\theta)}$$

$$|\vec{E}_{net}| = \sqrt{2E + \frac{a^2}{r^2} 2 \cos^2 \theta} = \sqrt{4E + \frac{a^2}{r^2}} \cos^2 \theta$$

$$|\vec{E}_{net}| = 2E + \frac{a^2}{r^2} \cos^2 \theta = \frac{a^2}{r^2 + a^2}$$

$$|\vec{E}_{net}| = 2E + \frac{a^2}{r^2 + a^2}$$

$$|\vec{E}_{net}| = 2 \frac{kq}{r^2 + a^2} \frac{a^2}{r^2 + a^2}$$

$$|\vec{E}_{net}| = \frac{k2aq}{(r^2 + a^2)^{3/2}} = \frac{K\vec{P}}{(r^2 + a^2)^{3/2}}$$

For $r \gg a$ (a can be neglected)

\vec{E}_{net} can be calculated by using a parallelogram law of vector addition. $E_{net} = \frac{KP}{r^3}$

3. (a) What is an equi-potential surface? Show that the electric field is always directed perpendicular to an equi-potential surface.

(b) Derive an expression for the potential at a point along the axial line of a short electric dipole?

Ans. (a) The surface which has same potential through out is called an equipotential surface.

$$\text{Since } dw = \vec{F} \cdot d\vec{x}$$

$$dw = (-q_0 E) \cdot d\vec{x}$$

$$(\text{force on the test charge } q_0 \vec{F} = q_0 \vec{E})$$

Since work done in moving a test charge along an equipotential surface is always zero.

$$\Rightarrow -q \oint \vec{E} \cdot d\vec{x} = 0$$

Or

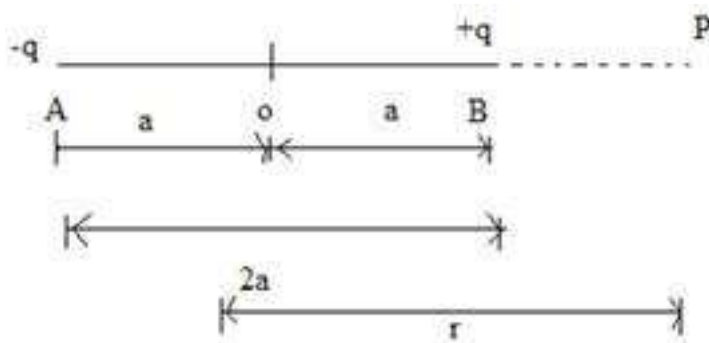
$$\vec{E} \cdot d\vec{x} = 0$$

$$\Rightarrow E \perp d\vec{x}$$

(b) Consider an electric dipole of dipole length $2a$ and point P on the axial line such that $OP = r$

where O is the center of the dipole.

Electric Potential at point P due to the dipole



$$V = V_{PA} + V_{PB}$$

$$V = \frac{K(-q)}{(r+a)} + \frac{K(+q)}{(r-a)}$$

$$V = Kq \left[\frac{1}{r-a} - \frac{1}{r+a} \right]$$

$$V = Kq \left[\frac{(r+a) - (r-a)}{(r-a)(r+a)} \right]$$

$$V = Kq \left[\frac{r+a - r+a}{r^2 - a^2} \right]$$

$$V = Kq \frac{(2a)}{r^2 - a^2} \quad (\because \bar{P} = 2aq)$$

$$V = \frac{KP}{r^2 - a^2}$$

For a short electric dipole (a) can be neglected

$$\Rightarrow V = \frac{KP}{r^2}$$

4. Check that the ratio $ke^2 / G m_e m_p$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Ans. The given ratio is $\frac{ke^2}{Gm_e m_p}$.

Where,

G = Gravitational constant

Its unit is $N m^2 kg^{-2}$

m_e and m_p = Masses of electron and proton.

Their unit is kg.

e = Electric charge.

Its unit is C.

ϵ_0 = Permittivity of free space

Its unit is $Nm^2 C^{-2}$

Therefore, unit of the given ratio $\frac{ke^2}{Gm_e m_p}$

$$= \frac{[Nm^2C^{+2}][C^{+2}]}{[Nm^2kg^{-2}][kg][kg]}$$

$$= M^0 L^0 T^0$$

Hence, the given ratio is dimensionless.

$$e = 1.6 \times 10^{-19} C$$

$$G = 6.67 \times 10^{-11} Nkg^{-2}$$

$$m_e = 9.1 \times 10^{-31} kg$$

$$m_p = 1.66 \times 10^{-27} kg$$

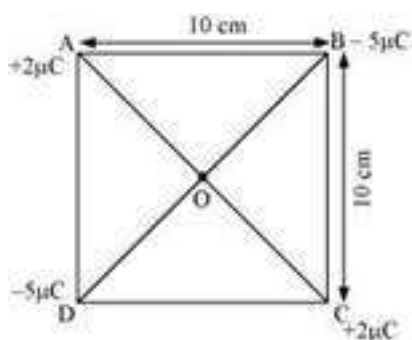
Hence, the numerical value of the given ratio is

$$\frac{ke^2}{Gm_em_p} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.67 \times 10^{-27}} \approx 2.3 \times 10^{39}$$

This is the ratio of electric force to the gravitational force between a proton and an electron, keeping distance between them constant.

5. Four point charges $q_A = 2 \mu C$, $q_B = -5 \mu C$, $q_C = 2 \mu C$, and $q_D = -5 \mu C$ are located at the corners of a square ABCD of side 10 cms. What is the force on a charge of $1 \mu C$ placed at the centre of the square?

Ans. The given figure shows a square of side 10 cm with four charges placed at its corners. O is the centre of the square.



Where,

$$\text{(Sides)} \quad AB = BC = CD = AD = 10 \text{ cm}$$

$$\text{(Diagonals)} \quad AC = BD = 10\sqrt{2} \text{ cm}$$

$$AO = OC = DO = OB = 5\sqrt{2} \text{ cm}$$

A charge of amount $1 \mu\text{C}$ is placed at point O.

Force of repulsion between charges placed at corner A and centre O is equal in magnitude but opposite in direction relative to the force of repulsion between the charges placed at corner C and centre O. Hence, they will cancel each other. Similarly, force of attraction between charges placed at corner B and centre O is equal in magnitude but opposite in direction relative to the force of attraction between the charges placed at corner D and centre O. Hence, they will also cancel each other. Therefore, net force caused by the four charges placed at the corner of the square on $1 \mu\text{C}$ charge at centre O is zero.

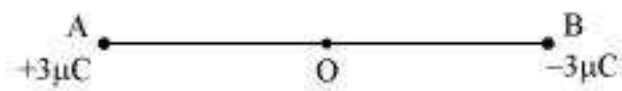
6. Two point charges $q_A = 3 \mu\text{C}$ and $q_B = -3 \mu\text{C}$ are located 20 cm apart in vacuum.

(a) What is the electric field at the midpoint O of the line AB joining the two charges?

(b) If a negative test charge of magnitude $1.5 \times 10^{-9} \text{ C}$ is placed at this point, what is the force experienced by the test charge?

Ans. (a) The situation is represented in the given figure. O is the mid-point of line AB.

Distance between the two charges, $AB = 20 \text{ cm}$



$$\therefore AO = OB = 10 \text{ cm}$$

Net electric field at point O = E

Electric field at point O caused by $+3\mu\text{C}$ charge,

$$E_1 = \frac{3 \times 10^{-6}}{4\pi \epsilon_0 (AO)^2} = \frac{3 \times 10^{-6}}{4\pi \epsilon_0 (10 \times 10^{-2})^2} \text{ N/C along OB}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

Magnitude of electric field at point O caused by $-3\mu\text{C}$ charge,

$$E_2 = \frac{-3 \times 10^{-6}}{4\pi \epsilon_0 (OB)^2} = \frac{3 \times 10^{-6}}{4\pi \epsilon_0 (10 \times 10^{-2})^2} \text{ N/C along OB}$$

$$\therefore E = E_1 + E_2$$

$$= 2 \times \left[(9 \times 10^9) \times \frac{3 \times 10^{-6}}{(10 \times 10^{-2})^2} \right] \text{ [since the values } E_1 \text{ and } E_2 \text{ of are same, the value is multiplied with 2]}$$

$$= 5.4 \times 10^6 \text{ N/C along OB}$$

Therefore, the electric field at mid-point O is $5.4 \times 10^6 \text{ NC}^{-1} \text{ along OB}$

(b) A test charge of amount $1.5 \times 10^{-9} \text{ C}$ is placed at mid-point O.

$$q = 1.5 \times 10^{-9} \text{ C}$$

Force experienced by the test charge = F

$$\therefore F = qE$$

$$= 1.5 \times 10^{-9} \times 5.4 \times 10^6$$

$$= 8.1 \times 10^{-3} \text{ N}$$

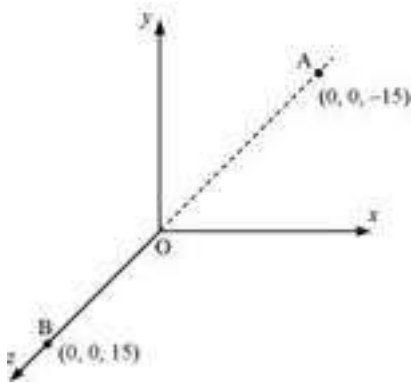
The force is directed along line OA . This is because the negative test charge is repelled by

the charge placed at point B but attracted towards point A.

Therefore, the force experienced by the test charge is $8.1 \times 10^{-3} \text{ N}$ along OA.

7. A system has two charges $q_A = 2.5 \times 10^{-7} \text{ C}$ and $q_B = -2.5 \times 10^{-7} \text{ C}$ located at points A: $(0, 0, -15 \text{ cm})$ and B: $(0, 0, +15 \text{ cm})$, respectively. What are the total charge and electric dipole moment of the system?

Ans. Both the charges can be located in a coordinate frame of reference as shown in the given figure.



At A, amount of charge, $q_A = 2.5 \times 10^{-7} \text{ C}$

At B, amount of charge, $q_B = -2.5 \times 10^{-7} \text{ C}$

Total charge of the system,

$$\begin{aligned} q &= q_A + q_B \\ &= 2.5 \times 10^{-7} \text{ C} - 2.5 \times 10^{-7} \text{ C} \\ &= 0 \end{aligned}$$

Distance between two charges at points A and B,

$$d = 15 + 15 = 30 \text{ cm} = 0.3 \text{ m}$$

Electric dipole moment of the system is given by,

$$p = q_A \times d = q_B \times d$$

$$= 2.5 \times 10^{-7} \times 0.3$$

$$= 7.5 \times 10^{-8} \text{ Cm along positive z-axis}$$

Therefore, the electric dipole moment of the system is $7.5 \times 10^{-8} \text{ C m}$ along positive z-axis.

8. (a) Two insulated charged copper spheres A and B have their centers separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is $6.5 \times 10^{-7} \text{ C}$ s? The radii of A and B are negligible compared to the distance of separation.

(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Ans. (a) Charge on sphere A, q_A = Charge on sphere B, $q_B = 6.5 \times 10^{-7} \text{ C}$

Distance between the spheres, $r = 50 \text{ cm} = 0.5 \text{ m}$

Force of repulsion between the two spheres,

$$F = \frac{q_A q_B}{4\pi \epsilon_0 r^2}.$$

Where,

ϵ_0 = Free space permittivity

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore F = \frac{9 \times 10^9 \times (6.5 \times 10^{-7})^2}{(0.5)^2}$$

$$= 1.52 \times 10^{-2} \text{ N}$$

Therefore, the force between the two spheres is $1.52 \times 10^{-2} \text{ N}$

(b) After doubling the charge, charge on sphere A, q_A = Charge on sphere

$B, q_B = 2 \times 6.5 \times 10^{-7} \text{ C} = 1.3 \times 10^{-6} \text{ C}$ The distance between the spheres is halved.

$$\therefore r = \frac{0.5}{2} = 0.25 \text{ m}$$

Force of repulsion between the two spheres,

$$\begin{aligned} F &= \frac{q_A q_B}{4\pi \epsilon_0 r^2} \\ &= \frac{9 \times 10^9 \times 1.3 \times 10^{-6} \times 1.3 \times 10^{-6}}{(0.25)^2} \\ &= 16 \times 1.52 \times 10^{-2} \\ &= 0.243 \text{ N s} \end{aligned}$$

Therefore, the force between the two spheres is 0.243 N

9. Suppose the spheres A and B in Exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Ans. Distance between the spheres, A and $B, r = 0.5 \text{ m}$

Initially, the charge on each sphere, $q = 6.5 \times 10^{-7} \text{ C}$

When sphere A is touched with an uncharged sphere C, amount of charge from A will transfer to sphere

C. Hence, charge on each of the spheres, A and C, $\frac{q}{2}$.

When sphere C with charge $\frac{q}{2}$ is brought in contact with sphere B with charge q , total charges on the system will divide into two equal halves given as,

$$\frac{\frac{q}{2} + q}{2} = \frac{3q}{4}$$

Each sphere will share each half. Hence, charge on each of the spheres, C and B, is $\frac{3q}{4}$.

Force of repulsion between sphere A having charge $\frac{q}{2}$ and sphere B having charge

$$\begin{aligned} \frac{3q}{4} &= \frac{\frac{q}{2} \times \frac{3q}{4}}{4\pi\epsilon_0 r^2} = \frac{3q^2}{8 \times 4\pi\epsilon_0 r^2} \\ &= 9 \times 10^9 \times \frac{3 \times (6.5 \times 10^{-7})^2}{8 \times (0.5)^2} \\ &= 5.703 \times 10^{-3} \text{ N} \end{aligned}$$

Therefore, the force of attraction between the two spheres is $5.703 \times 10^{-3} \text{ N}$.

10. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-22} \text{ C/m}^2$. What is E: (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates?

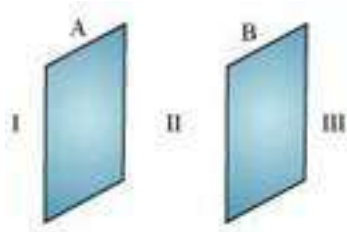
Ans. The situation is represented in the following figure.

A and B are two parallel plates close to each other. Outer region of plate A is labelled as **I**, outer region of plate B is labelled as **III**, and the region between the plates, A and B, is labelled as **II**.

Charge density of plate A, $\sigma = 17.0 \times 10^{-22} \text{ C/m}^2$

Charge density of plate B, $\sigma = -17.0 \times 10^{-22} \text{ C/m}^2$

In the regions, **I** and **III**, electric field E is zero. This is because charge is not enclosed by the respective plates.



Electric field E in region **II** is given by the relation,

$$E = \frac{\sigma}{\epsilon_0}$$

Where,

$$\epsilon_0 = \text{Permittivity of free space} = 8.854 \times 10^{-12} \text{ N}^{-1} \text{C}^2 \text{ m}^{-2}$$

$$\begin{aligned} \therefore E &= \frac{17.0 \times 10^{-22}}{8.854 \times 10^{-12}} \\ &= 1.92 \times 10^{-10} \text{ N/C} \end{aligned}$$

Therefore, electric field between the plates is $= 1.92 \times 10^{-10} \text{ N/C}$.

11. An oil drop of 12 excess electrons is held stationary under a constant electric field of $2.55 \times 10^4 \text{ N C}^{-1}$ in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3} . Estimate the radius of the drop. ($g = 9.81 \text{ m s}^{-2}$; $e = 1.60 \times 10^{-19} \text{ C}$).

Ans. Excess electrons on an oil drop, $n = 12$

Electric field intensity, $E = 2.55 \times 10^4 \text{ N C}^{-1}$

Density of oil, $\rho = 1.26 \text{ gm/cm}^3 = 1.26 \times 10^3 \text{ kg/m}^3$

Acceleration due to gravity, $g = 9.81 \text{ m s}^{-2}$

Charge on an electron, $e = 1.6 \times 10^{-19} \text{ C}$

Radius of the oil drop = r

Force (F) due to electric field E is equal to the weight of the oil drop (W)

$$F = W$$

$$Eq = mg$$

$$Ene = \frac{4}{3} \pi r^3 p \times g$$

Where,

q = Net charge on the oil drop = ne

m = Mass of the oil drop

= Volume of the oil drop \times Density of oil

$$= \frac{4}{3} \pi r^3 \times p$$

$$\therefore r = \sqrt[3]{\frac{3Ene}{4\pi pg}}$$

$$= \sqrt[3]{\frac{3 \times 2.55 \times 10^4 \times 12 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times 1.26 \times 10^3 \times 9.81}}$$

$$= \sqrt[3]{946.09 \times 10^{-21}}$$

$$= 9.82 \times 10^{-7} \text{ m}$$

$$|E|$$

$$= 9.82 \times 10^{-4} \text{ mm}$$

Therefore, the radius of the oil drop is $= 9.82 \times 10^{-4} \text{ mm}$.

12. In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of 10^5 NC^{-1} per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative z-direction?

Ans. Dipole moment of the system, $p = q \times dl = -10^{-7} \text{ C m}$ Rate of increase of electric field per unit length,

$$\frac{dE}{dl} = 10^5 \text{ NC}^{-1}$$

Force (F) experienced by the system is given by the relation,

$$F = qE$$

$$F = q \frac{dE}{dl} \times dl$$

$$= p \times \frac{dE}{dl}$$

$$= -10^{-7} \times 10^5$$

$$= -10^{-2} \text{ N}$$

The force is -10^{-2} N in the negative z-direction i.e., opposite to the direction of electric field. Hence, the angle between electric field and dipole moment is 180° .

Torque (τ) is given by the relation,

$$(\tau) = pE \sin 180^\circ$$

$$= 0$$

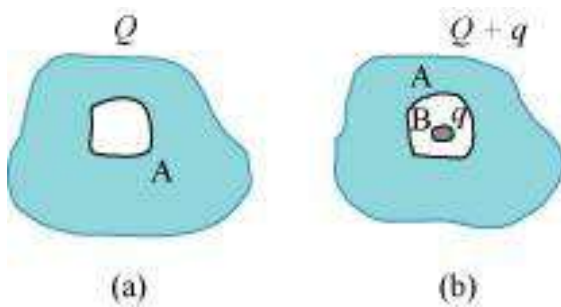
Therefore, the torque experienced by the system is zero.

13. (a) A conductor A with a cavity as shown in Fig. 1.36(a) is given a charge Q . Show that the entire charge must appear on the outer surface of the conductor. (b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is $Q + q$ [Fig. 1.36(b)]. (c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.

Ans. (a) Let us consider a Gaussian surface that is lying wholly within a conductor and enclosing the cavity. The electric field intensity E inside the charged conductor is zero.

Let q is the charge inside the conductor and ϵ_0 is the permittivity of free space.

According to Gauss's law,



$$\text{Flux, } \phi = \overline{E} \cdot \overline{ds} = \frac{q}{\epsilon_0}$$

Here, $E = 0$

$$E = \frac{17.0 \times 10^{-22}}{8.854 \times 10^{-22}}$$

$$\therefore \epsilon_0 = 0$$

$$\therefore q = 0$$

Therefore, charge inside the conductor is zero.

The entire charge Q appears on the outer surface of the conductor.

(b) The outer surface of conductor A has a charge of amount Q . Another conductor B having charge $+q$ is kept inside conductor A and it is insulated from A. Hence, a charge of amount $-q$

will be induced in the inner surface of conductor A and $+q$ is induced on the outer surface of conductor A. Therefore, total charge on the outer surface of conductor A is $Q + q$.

(c) A sensitive instrument can be shielded from the strong electrostatic field in its environment by enclosing it fully inside a metallic surface. A closed metallic body acts as an electrostatic shield.

14. A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\left[\frac{\sigma}{2\epsilon_0} \right] \hat{n}$, where \hat{n} is the unit vector in the outward normal direction, and σ is the surface charge density near the hole.

Ans. Let us consider a conductor with a cavity or a hole. Electric field inside the cavity is zero. Let E is the electric field just outside the conductor, q is the electric charge, σ is the charge density, and ϵ_0 is the permittivity of free space.

$$\text{Charge } |q| = \sigma \times \vec{d}$$

According to Gauss's law,

$$\phi = \vec{E} \cdot \vec{d} = \frac{|q|}{\epsilon_0}$$

$$E ds = \frac{\sigma \times \vec{d}}{\epsilon_0}$$

$$\therefore E = \frac{\sigma}{\epsilon_0} \hat{n}$$

Therefore, the electric field just outside the conductor is $\frac{\sigma}{\epsilon_0} \hat{n}$. This field is a superposition of field due to the cavity E^1 and the field due to the rest of the charged conductor E^1 . These fields are equal and opposite inside the conductor, and equal in magnitude and direction outside the conductor.

$$\therefore E^1 + E^1 = E$$

$$E^1 = \frac{E}{2}$$

$$= \frac{\sigma}{2\epsilon_0} \hat{n}$$

Therefore, the field due to the rest of the conductor is $\frac{\sigma}{\epsilon_0} \hat{n}$.

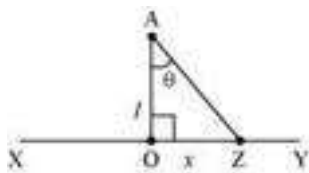
Hence, proved.

15. Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law. [Hint: Use Coulomb's law directly and evaluate the necessary integral.]

Ans. Take a long thin wire XY (as shown in the figure) of uniform linear charge density λ .



Consider a point A at a perpendicular distance l from the mid-point O of the wire, as shown in the following figure.



Let E be the electric field at point A due to the wire, XY.

Consider a small length element dx on the wire section with $OZ = x$

Let q be the charge on this piece.

$$\therefore q = \lambda dx$$

Electric field due to the piece,

$$de = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{(AZ)^2}$$

$$\text{However, } AZ = \sqrt{l^2 + x^2}$$

$$\therefore de = \frac{\lambda dx}{4\pi\epsilon_0 (l^2 + x^2)}$$

The electric field is resolved into two rectangular components. $dE \cos \theta$ is the perpendicular component and $dE \sin \theta$ is the parallel component.

When the whole wire is considered, the component $dE \sin \theta$ is cancelled.

Only the perpendicular component $dE \cos \theta$ affects point A.

Hence, effective electric field at point A due to the element dx is $dE \cos \theta$.

$$\therefore dE_{\cos \theta} = \frac{\lambda dx \cos \theta}{4\pi\epsilon_0 (x^2 + l^2)} \quad \dots(1)$$

In $\triangle AZO$,

$$\tan \theta = \frac{x}{l}$$

$$x = l \tan \theta \quad \dots(2)$$

On differentiating equation (2), we obtain

$$\frac{dx}{d\theta} = l \sec^2 \theta$$

$$dx = l \sec^2 \theta d\theta \quad \dots(3)$$

From equation (2),

$$x^2 + l^2 = l^2 + \tan^2 \theta$$

$$\therefore l^2 (1 + \tan^2 \theta) = l^2 \sec^2 \theta$$

$$x^2 + l^2 = l^2 \sin^2 \theta \quad \dots\dots(4)$$

Putting equations (3) and (4) in equation (1), we obtain

$$\therefore dE_1 = \frac{\lambda l \sec^2 \theta d\theta}{4\pi \epsilon_0 l^2 \sec^2 \theta} \times \cos \theta$$

$$\therefore dE_1 = \frac{\lambda \cos \theta d\theta}{4\pi \epsilon_0 l} \quad \dots\dots(5)$$

The wire is so long that tends from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$.

By integrating equation (5), we obtain the value of field E_1 as,

$$\frac{\pi}{2} \int_{-\frac{\pi}{2}} dE_1 = \frac{\pi}{2} \int_{-\frac{\pi}{2}} \frac{\lambda}{4\pi \epsilon_0 l} \cos \theta d\theta$$

$$E_1 = \frac{\lambda}{4\pi \epsilon_0 l} \times 2$$

$$E_1 = \frac{\lambda}{2\pi \epsilon_0 l}$$

Therefore, the electric field due to long wire is $\frac{\lambda}{2\pi \epsilon_0 l}$

16. It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge $\left(+\frac{1}{2}\right)e$, and the 'down' quark (denoted by d) of charge $(-1/3)e$, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

Ans. A proton has three quarks. Let there be n up quarks in a proton, each having a charge of $+\left(\frac{2}{3}e\right)$.

$$\text{Charge due to } n \text{ up quarks} = \left(\frac{2}{3}e\right)n$$

Number of down quarks in a proton = $3 - n$

Each down quark has a charge of $-\frac{1}{3}e$.

$$\text{Charge due to } (3 - n) \text{ down quarks} = \left(-\frac{1}{3}e\right)(3 - n)$$

Total charge on a proton = $+e$

$$\therefore e = \left(\frac{2}{3}e\right)n + \left(-\frac{1}{3}e\right)(3 - n)$$

$$e = \left(\frac{2ne}{3}\right) - e + \frac{ne}{3}$$

$$2e = ne$$

$$n = 2$$

Number of up quarks in a proton, $n = 2$

Number of down quarks in a proton = $3 - n = 3 - 2 = 1$

Therefore, a proton can be represented as ' uud '.

A neutron also has three quarks. Let there be n up quarks in a neutron, each having a charge of $+\frac{2}{3}e$.

$$\text{Charge on a neutron due to } n \text{ up quarks} = \left(+\frac{2}{3}e\right)n$$

Number of down quarks is $3 - n$, each having a charge of $= \left(-\frac{1}{3}\right)e$.

Charge on a neutron due to $(3 - n)$ down quarks $= \left(-\frac{1}{3}e\right)(3 - n)$

Total charge on a neutron $= 0$

$$0 = \left(\frac{2}{3}e\right)n + \left(-\frac{1}{3}e\right)(3 - n)$$

$$0 = \frac{2}{3}en - e + \frac{ne}{3}$$

$$e = ne$$

$$n = 1$$

Number of up quarks in a neutron, $n = 1$

Number of down quarks in a neutron $= 3 - n = 2$

Therefore, a neutron can be represented as ' udd '.

17. (a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where $E = 0$) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Ans. A proton has three quarks. Let there be n up quarks in a proton, each having a charge of $+\left(\frac{2}{3}e\right)$.

Charge due to n up quarks $= \left(\frac{2}{3}e\right)n$

Number of down quarks in a proton $= 3 - n$

Each down quark has a charge of $-\frac{1}{3}e$.

$$\text{Charge due to } (3 - n) \text{ down quarks} = \left(-\frac{1}{3}e\right)(3 - n)$$

Total charge on a proton = $+e$

$$\therefore e = \left(\frac{2}{3}e\right)n + \left(-\frac{1}{3}e\right)(3 - n)$$

$$e = \left(\frac{2ne}{3}\right) - e + \frac{ne}{3}$$

$$2e = ne$$

$$n = 2$$

Number of up quarks in a proton, $n = 2$

Number of down quarks in a proton = $3 - n = 3 - 2 = 1$

Therefore, a proton can be represented as ' uud '.

A neutron also has three quarks. Let there be n up quarks in a neutron, each having a charge of $+\frac{2}{3}e$.

$$\text{Charge on a neutron due to } n \text{ up quarks} = \left(+\frac{2}{3}e\right)n$$

Number of down quarks is $3 - n$, each having a charge of $= \left(-\frac{1}{3}e\right)$.

$$\text{Charge on a neutron due to } (3 - n) \text{ down quarks} = \left(-\frac{1}{3}e\right)(3 - n)$$

Total charge on a neutron = 0

$$0 = \left(\frac{2}{3}e\right)n + \left(-\frac{1}{3}e\right)(3-n)$$

$$0 = \frac{2}{3}en - e + \frac{ne}{3}$$

$$e = ne$$

$$n = 1$$

Number of up quarks in a neutron, $n = 1$

Number of down quarks in a neutron = $3 - n = 2$

Therefore, a neutron can be represented as '*udd*'.

18. A particle of mass m and charge ($-q$) enters the region between the two charged plates initially moving along x -axis with speed v_x (like particle 1 in Fig. 1.33). The length of plate is L and a uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $\frac{qEL^2}{2m v_x^2}$.

Compare this motion with motion of a projectile in gravitational field discussed in Section 4.10 of Class XI Textbook of Physics.

Ans. Charge on a particle of mass $m = -q$

Velocity of the particle = v_x

Length of the plates = L

Magnitude of the uniform electric field between the plates = E

Mechanical force, $F = \text{Mass } (m) \times \text{Acceleration } (a)$

$$a = \frac{F}{m}$$

However, electric force, $F = qE$

Therefore, acceleration, $= \frac{qE}{m}$... (1)

Time taken by the particle to cross the field of length L is given by,

$$t = \frac{\text{Length of the plate}}{\text{Velocity of the}} = \frac{L}{v_x} \dots (2)$$

In the vertical direction, initial velocity, $u = 0$

According to the third equation of motion, vertical deflection s of the particle can be obtained as,

$$s = ut + \frac{1}{2}at^2$$
$$s = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2$$

$$s = \frac{qEL^2}{2mV_x^2} \dots (3)$$

Hence, vertical deflection of the particle at the far edge of the plate is

$qEL^2 / (2mv_x^2)$. This is similar to the motion of horizontal projectiles under gravity.