CBSE Class 12 physics

Practice Questions

Chapter 1

Electric Charges and Fields

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1 Mark Questions

1. Show does the force between two point charges change if the dielectric constant of the medium in which they are kept increase?

Ans. Since K

$= \frac{FV}{FM} = \frac{force \text{ between the charges in vaccuum}}{\text{force between two charges in medium}}$

$\Rightarrow Fm = \frac{FV}{k}$

$\Rightarrow$ if k increases, Fm decreases.

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2. A charged rod P attracts rod R where as P repels another charged rod Q. What type of force is developed between Q and R?

Ans. Suppose rod P be negatively charged since it attracts rod R

$\Rightarrow$ R is positively charged since it repels rod Q

$\Rightarrow$ Q is negatively charged. So force between Q and R is attractive in nature.

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3. Which physical quantity has its S.I unit (1) Cm (2) N/C

Ans. (1) Electric dipole moment (2) Electric field Intensity

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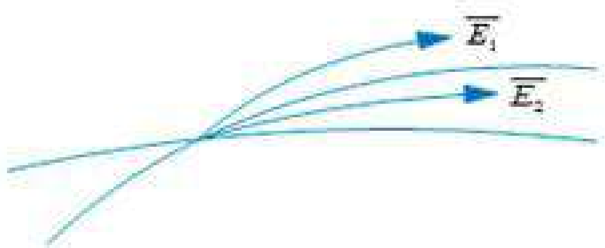
4. Define one coulomb?

Ans. Charge on a body is said to be 1 coulomb if two charges experiences a force of repulsion or attraction of $9 \times 10^{9} N$ when they are separated by a distance of 1 m.

2 Mark Questions

1. A free proton and a free electron are placed in a uniform field. Which of the two experience greater force and greater acceleration?

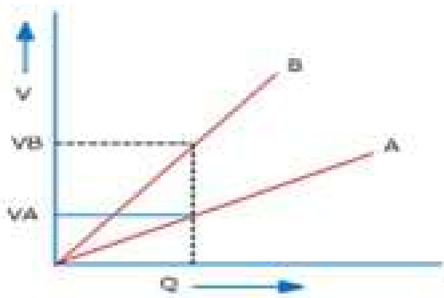
Ans. As F = q E and a = F/m as charge on both e-1 and proton are equal and opposite in nature, so force on them would be equal but as mass of proton is more than that of electron, so acceleration of electron would be more.



2. No two electric lines of force can intersect each other? Why?

Ans. Two electric lines of force never intersect each other because if they intersect then at the point of intersection there will be two tangents which is not possible as the two tangents represents two directions for electric field lines.

3. The graph shows the variation of voltage V across the plates of two capacitors A and B versus increase of charge Q stored on them. Which of the two capacitors have higher capacitance? Give reason for your answer?



Ans. Since C=Q/V

For a given charge Q

$C \propto \frac{1}{V}$

and since $V_A < V_B$

$\because C_A > C_B$

4. An electric dipole when held at 30^0 with respect to a uniform electric field of $10^4 \, N/C$ experienced a Torque of $9\times10^{-26} \, Nm$. Calculate dipole moment of the dipole?

Ans. Given

$\theta = 30^0$

$\tau = 9\times10^{-26}$ Nm

$E = 10^4$ N/c

$\vec{P} = ?$

τ (Torque) = PE $\sin\theta$

$P = \frac{\tau}{E\sin\theta}$

$\Rightarrow P = \frac{9\times10^{-26}}{10^4\times\sin 30^0} = \frac{9\times10^{-26}\times10^{-4}}{1/2}$

$\Rightarrow P = 18\times10^{-30} \, Cm$

5. (a) Explain the meaning of the statement 'electric charge of a body is quantised'.

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

Ans. (a) Electric charge of a body is quantized. This means that only integral (1, 2, …., *n*) number of electrons can be transferred from one body to the other. Charges are not transferred in fraction. Hence, a body possesses total charge only in integral multiples of electric charge.

(b) In macroscopic or large scale charges, the charges used are huge as compared to the magnitude of electric charge. Hence, quantization of electric charge is of no use on macroscopic scale. Therefore, it is ignored and it is considered that electric charge is continuous.

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6. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

Ans. Rubbing produces charges of equal magnitude but of opposite nature on the two bodies because charges are created in pairs. This phenomenon of charging is called charging by friction. The net charge on the system of two rubbed bodies is zero. This is because equal amount of opposite charges annihilates each other. When a glass rod is rubbed with a silk cloth, opposite natured charges appear on both the bodies. This phenomenon is in consistence with the law of conservation of energy. A similar phenomenon is observed with many other pairs of bodies.

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7. (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?

(b) Explain why two field lines never cross each other at any point?

Ans. (a) An electrostatic field line is a continuous curve because a charge experiences a continuous force when traced in an electrostatic field. The field line cannot have sudden breaks because the charge moves continuously and does not jump from one point to the other.

(b) If two field lines cross each other at a point, then electric field intensity will show two directions at that point. This is not possible. Hence, two field lines never cross each other.

8. An electric dipole with dipole moment 4×10^{-9} Cm is aligned at $30°$ with the direction of a uniform electric field of magnitude 5×10^{4} $N\ C^{-1}$. Calculate the magnitude of the torque acting on the dipole.

Ans. Electric dipole moment, $p = 4 \times 10^{-9}\ Cm$

Angle made by *p* with a uniform electric field, $\theta = 30°$

Electric field, $E = 5 \times 10^{4}\ N\ C^{-1}$

Torque acting on the dipole is given by the relation,

$$\mathrm{T} = pEsin\theta$$

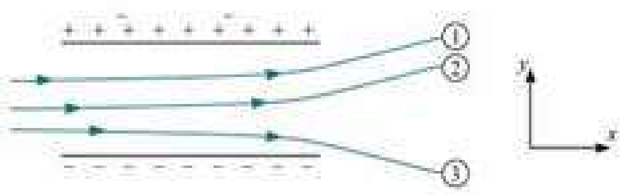
$$= 4\times10^{-9}\times5\times10^{4}\times\sin 30$$

$$= 20\times10^{-5}\times\frac{1}{2}$$

$$= 10^{-4}\,Nm$$

Therefore, the magnitude of the torque acting on the dipole is 10^{-4} N m.

9. Figure 1.33 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Ans. Opposite charges attract each other and same charges repel each other. It can be observed that particles 1 and 2 both move towards the positively charged plate and repel away from the negatively charged plate. Hence, these two particles are negatively charged. It can also be observed that particle 3 moves towards the negatively charged plate and repels away from the positively charged plate. Hence, particle 3 is positively charged.

The charge to mass ratio (emf) is directly proportional to the displacement or amount of deflection for a given velocity. Since the deflection of particle 3 is the maximum, it has the highest charge to mass ratio.

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10. What is the net flux of the uniform electric field of Exercise 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Ans. All the faces of a cube are parallel to the coordinate axes. Therefore, the number of field lines entering the cube is equal to the number of field lines piercing out of the cube. As a result, net flux through the cube is zero.

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11. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is $8.0 \times 10^3 \text{ N m}^2/\text{C}$. (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?

Ans. (a) Net outward flux through the surface of the box, $\Phi = 8.0 \times 10^3 \ N\ m^2/C$

For a body containing net charge *q*, flux is given by the relation,

$$\phi = \frac{q}{\epsilon_0}$$

ϵ_0 = Permittivity of free space

$= 8.854 \times 10^{-12} \ N^{-1} C^2 \ m^{-2}$ $q = \epsilon_0 \Phi$

$= 8.854 \times 10^{-12} \times 8.0 \times 10^3$

$= 7.08 \times 10^{-8}$

= 0.07 μC

Therefore, the net charge inside the box is 0.07 μC .

(b) No

Net flux piercing out through a body depends on the net charge contained in the body. If net flux is zero, then it can be inferred that net charge inside the body is zero. The body may have equal amount of positive and negative charges.

3 Mark Questions

1. A particle of mass m and charge q is released form rest in a uniform electric field of intensity E. calculate the kinetic energy it attains after moving a distances between the plates?

Ans. Since F = qE

$\therefore\ a = \frac{F}{m} = \frac{qE}{m}$ ---1

Using third equation of motion

$v^2 - u^2 = 2as$

Initially charged particle is at rest $\therefore$ u = o

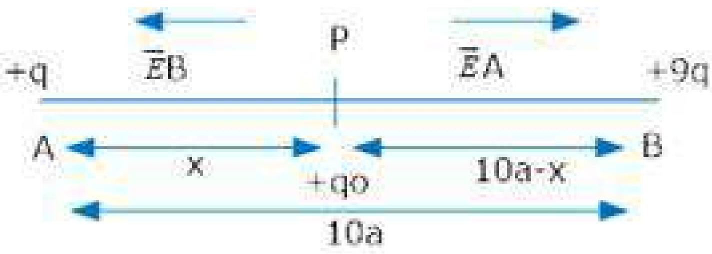
$\Rightarrow v^2 = 2as$

$KE = \frac{1}{2}\ mv^2 = \frac{1}{\cancel{2}} m\ (\cancel{2}\ as) = m\,as$.............2

Substituting 1 in eq. 2

$KE = \cancel{m} \times \frac{qE}{\cancel{m}} \times S$

$KE = qES$



2. Two point charges +q and +9q are separated by a distance of 10 a. Find the point on the line joining the two changes where electric field is zero?

Ans. Let P be the pt where test charge (+qo) is present then electric field at pt. P will be zero if Field at pt. P due to +q = field at p+. P due to + 9q-----------1

$$\vec{E} \Rightarrow E\vec{A} = \frac{K(+q)}{x^2}\ E\vec{B} = \frac{K(+9q)}{(10a-x)^2}$$

Substituting in eq. 1

$$\frac{\cancel{K}(+\cancel{q})}{x^2} = \frac{\cancel{K}(+9\cancel{q})}{(10a-x)^2}$$

$$(10a-x)^2 = 9x^2 \Rightarrow 10a - x = 3x$$

$$10a = 4x \quad \Rightarrow \quad x = \frac{10}{4}9$$

$x = 2.5$ a from change (+q)

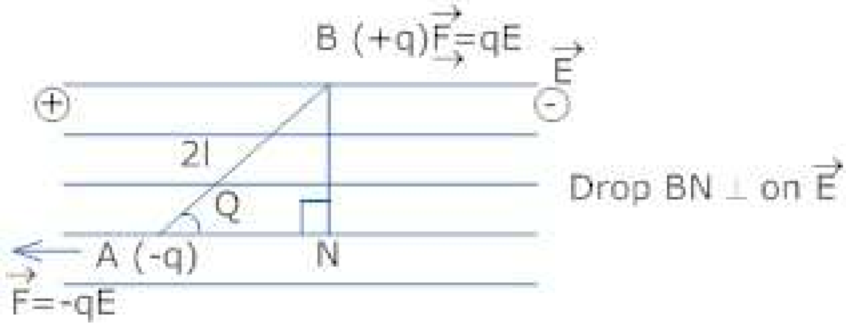
Or

$10\ a - x = 10a - 2.5a = 7.5$ a from change (+9q)

3. Define the term dipole moment $\vec{P}$ of an electric dipole indicating its direction. Write its S.I unit. An electric dipole is placed in a uniform electric field $\vec{E}$. Deduce the expression for the Torque acting on it.

Ans. Electric dipole moment is defined as the product of the magnitude of either charge and the length of dipole. Its direction is from –ve to +ve charge.

$\overrightarrow{P} = q(\overrightarrow{2l})$ Its S.I. unit is coulomb meter (Cm)



Consider a dipole placed in uniform electric field and makes an angle (θ) with the electric field $\left(\overrightarrow{E}\right)$ Since two forces acts on the charges constituting an electric dipole which are equal and opposite in direction, thus a torque acts on the dipole which makes the dipole rotate.

And Torque $\tau = Ethier\ force \times \perp distance$

Here force (F) = qE

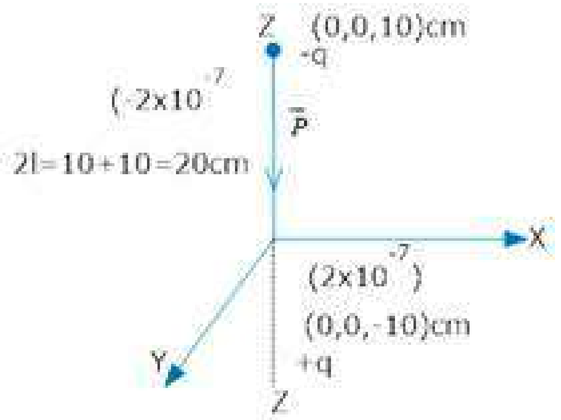
$$And\ \frac{BN}{AB} = \sin\theta \Rightarrow BN = AB \sin\theta = 2\ell \sin\theta$$

$$(\tau) = qE \times 2sin$$

(τ) = PE Sin θ $(\because \overrightarrow{P} = q(\overrightarrow{2l}))$

$$\vec{\tau} = \overrightarrow{P} \times \overrightarrow{E}$$

In vector form



4. A sphere of radius r_1 encloses a change Q. If there is another concentric sphere S_2 of radius $r_2\,(r_2 > r_1)$ and there is no additional change between $S_1 and\ S_2$. Find the ratio of electric flux through $S_1 and\ S_2$?

Ans. θ = q/$\in$o (where θ=electric flux)

$$\theta_{S1} = \frac{Q}{Eo}$$

For sphere S_2

$$\theta_{S2} = \frac{Q}{\in o} \text{ (since no additional charge is given)}$$

$$\text{So } \frac{\theta_{S1}}{\theta_{S2}} = \frac{Q/\notin o}{Q/\notin o} = 1:1$$

$$So\ \theta_{s1}:\theta_{s2} = 1:1$$

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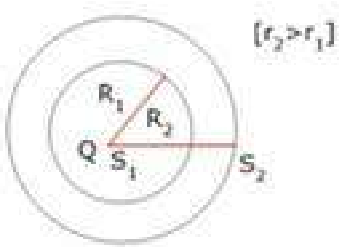
5. Electric charge is uniformly distributed on the surface of a spherical balloon. Show how electric intensity and electric potential vary (a) on the surface (b) inside and (c) outside.

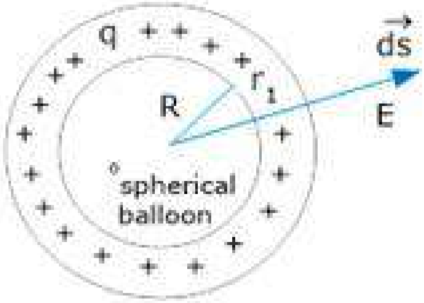


Ans. Electric field intensity on the surface of a shell

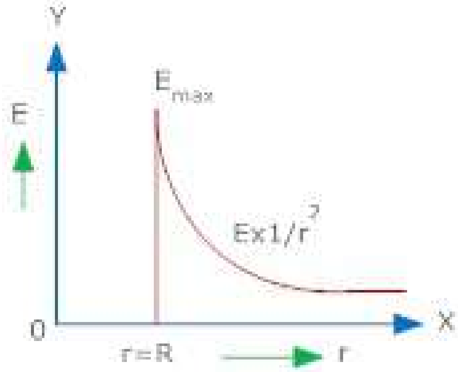
E = σ /$\in$o& V = Kq/R

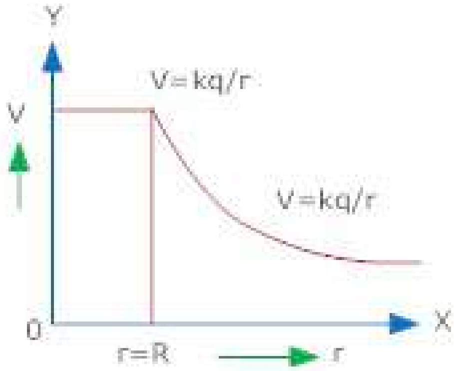
Inside E = o& V = Kq/R

Outside E = $\dfrac{\sigma}{\in o}\dfrac{R^2}{r^2}$ & V = Kq/r

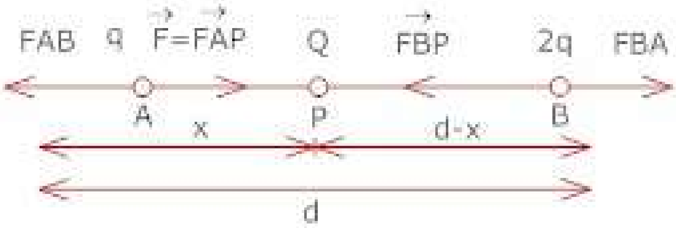


Graphically





6. Two point electric charges of value q and 2q are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and 2q is zero. Calculate the position of charge Q in terms of q and d.



Ans. Net force on charge q and 2q will be zero if the third charge is negative (i.e. of opposite sign) and q and 2q are positive, Force on change q will be zero if

$$\left|\overrightarrow{FAB}\right| = \left|\overrightarrow{FAP}\right|$$

$$\frac{\cancel{K}\cancel{q}(2q)}{d^2} = \frac{\cancel{K}\cancel{q}(Q)}{x^2}$$

$$\frac{Q}{q} = \frac{2x^2}{d^2} ----(1)$$

Force on charge 2q to be zero

if $\left|F\vec{B}A\right| = \left|F\vec{B}P\right|$

$$\frac{\cancel{K}\cancel{q}(\cancel{2}q)}{d^2} = \frac{\cancel{K}(\cancel{2}\,\cancel{q})Q}{(d-x)^2}$$

$$\frac{Q}{q} = \frac{(d-x)^2}{d^2} ----(2)$$

comparing equation 1 and 2

$$\frac{2x^2}{\cancel{d^2}} = \frac{(d-x)^2}{\cancel{d^2}}$$

$$x^2 = \frac{(d-x)^2}{2}$$

$$x^2 = \frac{(d-x)^2}{(\sqrt{2})^2}$$

$$\Rightarrow x = \frac{d-x}{\sqrt{2}} \, or \Gamma 2x + x = d$$

$$x(\sqrt{2}+1) = d$$

$$\Rightarrow x = \frac{d}{\sqrt{2}+1}$$

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7. What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?

Ans. Repulsive force of magnitude $6 \times 10^{-3}\ N$

Charge on the first sphere, $q_1 = 2\times10^{-7}\ C$

Charge on the second sphere, $q_2 = 3 \times 10^{-7}\ C$

Distance between the spheres, r = 30 cm = 0.3 m

Electrostatic force between the spheres is given by the relation,

$$F = \frac{q_1 q_2}{4\pi \epsilon_0 r^2}$$

Where, $\in 0$ = Permittivity of free space

$$s\ \frac{1}{4\pi\epsilon_0} = 9\times10^9\ Nm^2C^2$$

$$F = \frac{9\times10^9\times2\times10^{-7}\times3\times10^{-7}}{(0.3)^2} = 6\times10^{-3}\ N$$

Hence, force between the two small charged spheres is $6\times10^{-3}\ N$. The charges are of same nature. Hence, force between them will be repulsive.

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8. The electrostatic force on a small sphere of charge $0.4\ \mu C$ due to another small sphere of charge - $0.8\ \mu C$ in air is 0.2 N. (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

Ans. (a) Electrostatic force on the first sphere, F = 0.2 N

Charge on this sphere, q^1 = $0.4\ \mu C = 0.4\times10^{-6}\ C$

Charge on the second sphere, q^2 = - $0.8\ \mu C = -0.8\times10^{-6}\ C$

Electrostatic force between the spheres is given by the relation,

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\quad And\ \frac{1}{4\pi\epsilon_0} = 9\times10^9\ Nm^2C^{-2}$$

Where, ϵ_0 = Permittivity of free space

$$And, \frac{1}{4\pi\epsilon_0} = 9\times10^9 Nm^{-2}C^{-2}$$

$$r^2 = \frac{q_1 q_2}{4\pi\epsilon_0 F}$$

$$= 144\times10^{-4}$$

$$r = \sqrt{144\times10^{-4}} = 0.12m$$

The distance between the two spheres is 0.12m.

(b) Both the spheres attract each other with the same force. Therefore, the force on the second sphere due to the first is 0.2N.

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9. A polythene piece rubbed with wool is found to have a negative charge of $3\times10^{-7}\ C$

(a) Estimate the number of electrons transferred (from which to which?)

(b) Is there a transfer of mass from wool to polythene?

Ans. (a) When polythene is rubbed against wool, a number of electrons get transferred from wool to polythene. Hence, wool becomes positively charged and polythene becomes negatively charged.

Amount of charge on the polythene piece, $q = -3\times10^{-7}\ C$

Amount of charge on an electron, $e = -1.6\times10^{-19}\ C$

Number of electrons transferred from wool to polythene = *n*

n can be calculated using the relation,

$$q = ne$$

$n = \frac{q}{e}$

$= \frac{-3\times 10^{-7}}{-1.6\times 10^{-19}}$

$= 1.87 \times 10^{12}$

Therefore, the number of electrons transferred from wool to polythene is 1.87×1012 .

(b) Yes.

There is a transfer of mass taking place. This is because an electron has mass,

$me = 9.1 \times 10^{-3}\ kg$

Total mass transferred to polythene from wool,

$m = me \times n$

$= 9.1\times 10^{-31} \times 1.85 \times 10^{12}$

$= 1.706 \times 10^{-18}\ kg$

Hence, a negligible amount of mass is transferred from wool to polythene.

10.Consider a uniform electric field $E = 3 \times 10^{3}\ \hat{i}N / C$. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the *yz* plane? (b) What is the flux through the same square if the normal to its plane makes a $60°$ angle with the *x*-axis?

Ans. **(a)** Electric field intensity, $\vec{E} = 3 \times 10^{3}\ \hat{i}N / C$

Magnitude of electric field intensity, $|\vec{E}| = 3 \times 10^{3}\ N / C$

Side of the square, *s* = 10 cm = 0.1 m

Area of the square, $A = s^{2} = 0.01\ m2$

The plane of the square is parallel to the *y*-*z* plane. Hence, angle between the unit vector normal to the plane and electric field, $\theta = 0°$

Flux (Φ) through the plane is given by the relation,

$$\Phi = |\overline{E}| A \cos\theta$$

$$= 3 \times 10^3 \times 0.01 \times cos 0°$$

$$= 30 \ N \ m^2 / C$$

(b) Plane makes an angle of 60° with the *x*-axis. Hence, θ = 60°

Flux, $\Phi = |\overline{E}| A \cos\theta$

$$= 3 \times 10^3 \times 0.01 \times cos 60°$$

$$= 30 \times \frac{1}{2}$$

$$= 15 \ N \ m^2 / C$$

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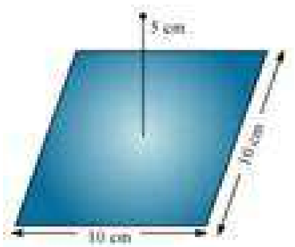
11. A point charge +10 μC is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. 1.34. What is the magnitude of the electric flux through the square? (*Hint:* Think of the square as one face of a cube with edge 10 cm.)

Ans. The square can be considered as one face of a cube of edge 10 cm with a centre where charge *q* is placed. According to Gauss's theorem for a cube, total electric flux is through all its six faces.



$$\phi_{Total} = \frac{q}{\epsilon_0}$$

Hence, electric flux through one face of the cube i.e., through the square, $\phi = \frac{\phi_{Total}}{6}$

$$= \frac{1}{6} \frac{q}{\in_0}$$

Where,

$\in_0$ = Permittivity of free space

$$= 8.854 \times 10^{-12} N^{-1} C^2 m^{-2}$$

q = 10 μC

$$= 10 \times 10^{-6} C$$

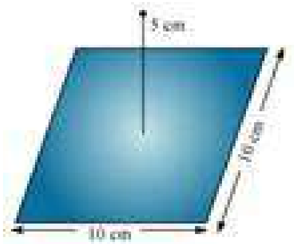
$$\therefore \phi = \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.854 \times 10^{-12}}$$

= $1.88 \times 10^5 N m^2 C^{-1}$

Therefore, electric flux through the square is $1.88 \times 10^5 N m^2 C^{-1}$.

12..A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

Ans. The square can be considered as one face of a cube of edge 10 cm with a centre where charge q is placed. According to Gauss's theorem for a cube, total electric flux is through all its six faces.



$$\phi_{Total} = \frac{q}{\in_0}$$

Hence, electric flux through one face of the cube i.e., through the square, $\phi = \frac{\phi_{Total}}{6}$

$= \frac{1}{6}\frac{q}{\in_0}$

Where,

$\in_0$ = Permittivity of free space

$= 8.854 \times 10^{-12}\ N^{-1}C^2\ m^{-2}$

q = 10 μC

$= 10 \times 10^{-6}\ C$

$\therefore \phi = \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.854 \times 10^{-12}}$

= $1.88 \times 10^5\ N\ m^2\ C^{-1}$

Therefore, electric flux through the square is $1.88 \times 10^5\ N\ m^2\ C^{-1}$.

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13. A point charge causes an electric flux of $-1.0 \times 10^3\ \mathrm{Nm^2/C}$ to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

Ans. (a) Electric flux, $\Phi = -1.0 \times 10^3\ N\ m^2/C$

Radius of the Gaussian surface,

r = 10.0 cm

Electric flux piercing out through a surface depends on the net charge enclosed inside a body. It does not depend on the size of the body. If the radius of the Gaussian surface is doubled, then the flux passing through the surface remains the same i.e., $-10^3\ Nm^2/C$.

(b) Electric flux is given by the relation,

$$\phi = \frac{q}{\epsilon_0}$$

Where,

q = Net charge enclosed by the spherical surface

ϵ_0 = Permittivity of free space = $8.854 \times 10^{-12} N^{-1} C^2 \; m^{-2}$

$$\therefore q = \phi \epsilon_0$$

$$= -1.0 \times 10^3 \times 8.854 \times 10^{-12}$$

$$= -8.854 \times 10^{-9} \; C$$

= - 8.854 nC

Therefore, the value of the point charge is –8.854 nC.

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14. A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is $1.5 \times 10^3 \; N/C$ and points radially inward, what is the net charge on the sphere?

Ans. Electric field intensity (E) at a distance (d) from the centre of a sphere containing net charge q is given by the relation,

$$E = \frac{q}{4\pi \epsilon_0 d^2}$$

Where,

q = Net charge = $1.5 \times 10^3 \; N/C$

d = Distance from the centre = 20 cm = 0.2 m

ϵ_0 = Permittivity of free space

And, $\frac{1}{4\pi \in_0}$ = 9 × 109 N m^2 C - 2

$\therefore q = E(4\pi \in_0)d^2$

$= \frac{1.5\times10^3\times(0.2)^2}{9\times10^9}$

$= 6.67\times10^9\ C$ = 6.67 nC

Therefore, the net charge on the sphere is 6.67 nC.

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15. A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC /m^2. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

Ans. (a) Diameter of the sphere, *d* = 2.4 m

Radius of the sphere, *r* = 1.2 m

Surface charge density, σ = 80.0 $\mu C / m^2 = 80\times10^{-6}\ C / m^2$

Total charge on the surface of the sphere,

Q = Charge density × Surface area

= $\sigma\times4\pi r^2$

$= 80\times10^{-6}\times4\times3.14\times(1.2)^2 = 1.447\times10^{-3}\ C$

Therefore, the charge on the sphere is $1.447\times10^{-3}\ C$.

(b) Total electric flux (ϕ_{Total}) leaving out the surface of a sphere containing net charge *Q* is given by the relation,

$\phi_{Total} = \frac{Q}{\in_0}$

Where,

$\in_0$ = Permittivity of free space

$= 8.854 \times 10^{-12}\ N^{-1} C^2\ m^{-2}$

$Q = 1.447 \times 10^{-3}\ C$

$$\phi_{Total} = \frac{1.44 \times 10^{-3}}{8.854 \times 10^{-12}}$$

$= 1.63 \times 10^8\ N\ C^{-1}\ m^2$

Therefore, the total electric flux leaving the surface of the sphere is $1.63 \times 10^8\ N\ C^{-1}\ m^2$.

---

16. An infinite line charge produces a field of $9 \times 10^4\ \mathrm{N/C}$ at a distance of 2 cm. Calculate the linear charge density.

Ans. Electric field produced by the infinite line charges at a distance *d* having linear charge density λ is given by the relation,

$$E = \frac{\lambda}{2\pi \in_0 d}$$

$\lambda = 2\pi \in_0 de$

Where,

d = 2 cm = 0.02 m

$E = 9 \times 10^4\ N / C$

$\in_0$ = Permittivity of free space

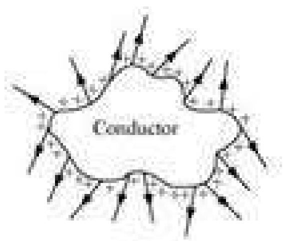
$$\frac{1}{4\pi \in_0} = 9 \times 10^9\ N\ m^2\ C^{-2}$$

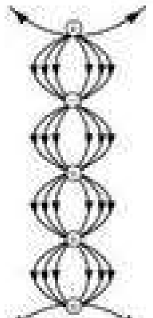
$$\pi = \frac{0.02 \times 9 \times 10^4}{2 \times 9 \times 10^9}$$

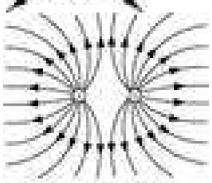
$$= 10\mu C / m$$

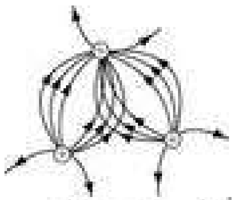
Therefore, the linear charge density is $= 10\mu C / m$.

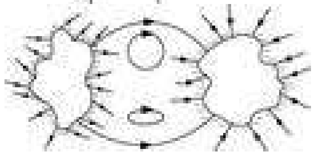
17. Which among the curves shown in Fig. 1.35 cannot possibly represent electrostatic field lines?

(a) 

(b) 

(c) 

(d) 

(e) 

Ans. (a) The field lines showed in (a) do not represent electrostatic field lines because field lines must be normal to the surface of the conductor.

(b) The field lines showed in (b) do not represent electrostatic field lines because the field lines cannot emerge from a negative charge and cannot terminate at a positive charge.

(c) The field lines showed in (c) represent electrostatic field lines. This is because the field lines emerge from the positive charges and repel each other.

(d) The field lines showed in (d) do not represent electrostatic field lines because the field lines should not intersect each other.

(e) The field lines showed in (e) do not represent electrostatic field lines because closed loops are not formed in the area between the field lines.

18. Suppose that the particle in Exercise in 1.33 is an electron projected with velocity $v_x = 2.0\times10^6\ \mathrm{m\ s^{-1}}$. If *E* between the plates separated by 0.5 cm is $9.1\times10^2\ \mathrm{N/C}$, where will the electron strike the upper plate? (| *e* | = $1.6\times10^{-19}\ \mathrm{C}$, $m_e = 9.1\times10^{-31}\ \mathrm{kg}$.)

Ans. Velocity of the particle, $v_x = 2.0\times10^6\ m/s$

Separation of the two plates, *d* = 0.5 cm = 0.005 m

Electric field between the two plates, *E* $= 9.1\times10^2\ N/C$

Charge on an electron, *q* = $1.6\times10^{-19}\ C$

Mass of an electron, *m*e = $9.1\times10^{-31}\ kg$

Let the electron strike the upper plate at the end of plate *L*, when deflection is *s*.

Therefore,

$$s = \frac{qEL^2}{2mv_x^2}$$

$$L = \sqrt{\frac{2dmv_x^2}{qE}}$$

$$= \sqrt{\frac{2\times0.005\times9.1\times10^{-31}\times(2.0\times10^6)^2}{1.6\times10-^{10}\times9.1\times10^2}}$$

$$= \sqrt{0.025\times10^{-2}} = \sqrt{2.5\times10^{-4}}$$

$$= 1.6\times10^{-2}$$

$$= 1.6cm$$

Therefore, the electron will strike the upper plate after travelling 1.6 cm.

5 Mark Questions

1. (1) The electric field $\vec{E}$ due to a point change at any point near to it is defined as:

$\vec{E} = Lim \frac{\vec{F}}{q}$ q → c where q is the test charge and $\vec{F}$ is the force acting on it. What is the significance of lim $q \rightarrow o$ in this expression?

(2) Two charges each $2 \times 10^{-7} C$ but opposite in sign forms a system. These charges are located at points A (0,0, -10) cm and B (0,0, +10) cm respectively. What is the total charge and electric dipole moment of the system?

Ans. (1) The Significance of writing lim $q \rightarrow o$ means the test charge should be vanishingly small so that it should not disturb the presence of source charge.

(2) (i) Total charge of the system

= $2 \times 10^{-7} + \left(-2 \times 10^{-7}\right)$

= zero. $\vec{P}$

(ii) $\vec{P} = q \times 2\vec{l}$

$P = 2 \times 10^{-7} \times 20 \times 10^{-2}$

$P = 4 \times 10^{-8} cm$

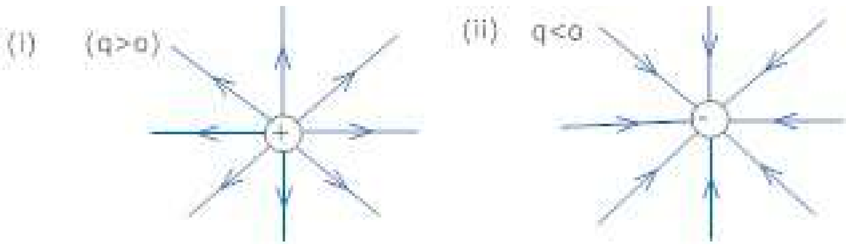
Direction of Dipole moment -Along negative z-axis.

---

2. (a) Sketch electric lines of force due to (i) isolated positive change (ie q>0) and (ii) isolated negative change (ie q<0)

(b) Two point changes q and –q are placed at a distance 2a apart. Calculate the electric field at a point P situated at a distance r along the perpendicular bisector of the line joining the charges. What is the field when r >> a?

Ans.

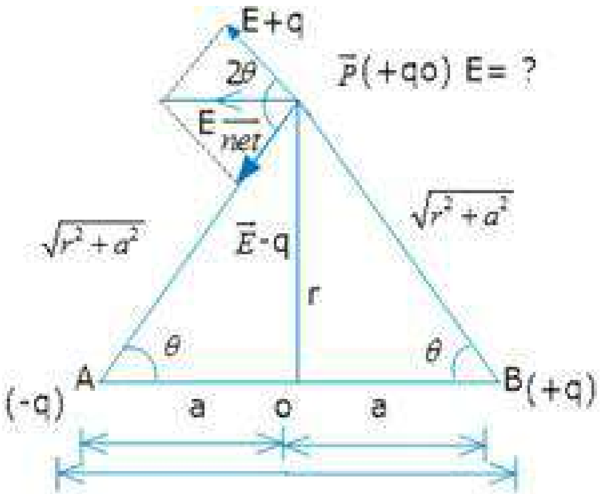


(b)

$$\left|\vec{E}+q\right| = \frac{Kq}{(r^2+a^2)}$$

$$\left|\vec{E}-q\right| = \frac{Kq}{(r^2+a^2)}$$

$$Since\ \left|\vec{E}+q\right| = \left|\vec{E}-q\right|$$



$$\left|\vec{E}net\right| = \sqrt{E+_q{}^2 + E_{-q}{}^2 + 2E_{+q}\,E_{-q}\ \cos 2\theta}$$

$$\left|\vec{E}net\right| = \sqrt{2E+_q{}^2 + 2E_{+q}{}^2 \cos 2\theta}$$

$$\left|\vec{E}net\right| = \sqrt{2E+_q{}^2 (1+\cos 2\theta)}$$

$$\left|\overrightarrow{Enet}\right| = \sqrt{2E+_q^{\ 2}\,2\cos^2\theta} = \sqrt{4E+_q^{\ 2}\ \cos^2\theta}$$

$$\left|\overrightarrow{Enet}\right| = 2E+\cos\theta\ \cos\ \theta = \frac{a}{\sqrt{r^2+a^2}}$$

$$\left|\overrightarrow{Enet}\right| = 2E+\frac{a}{\sqrt{r^2+a^2}}$$

$$\left|\overrightarrow{Enet}\right| = 2\frac{kq}{r^2+a^2}\frac{a}{\sqrt{r^2+a^2}}$$

$$\left|\overrightarrow{Enet}\right| = \frac{k2aq}{(r^2+a^2)^{3/2}} = \frac{K\overrightarrow{P}}{(r^2+a^2)^{3/2}}$$

For r >>> a (a can be neglected)

$\overrightarrow{Enet}$ can be calculated by using a parallelogram law of vector addition. Enet = $\frac{KP}{r^3}$

---

3. (a) What is an equi-potential surface? Show that the electric field is always directed perpendicular to an equi-potential surface.

(b) Derive an expression for the potential at a point along the axial line of a short electric dipole?

Ans. (a) The surface which has same potential through out is called an equipotential surface.

$$Since\ dw = \overrightarrow{F}.d\overrightarrow{x}$$

$$dw = (-qoE).d\overrightarrow{x}$$

(force on the test chage qo $\vec{F} = q\ o\ \overrightarrow{E}$)

Since work done is moving a test charge along an equipotential surface is always zero.

$\Rightarrow$ -qo $\vec{E}.\vec{dx} = 0$

Or

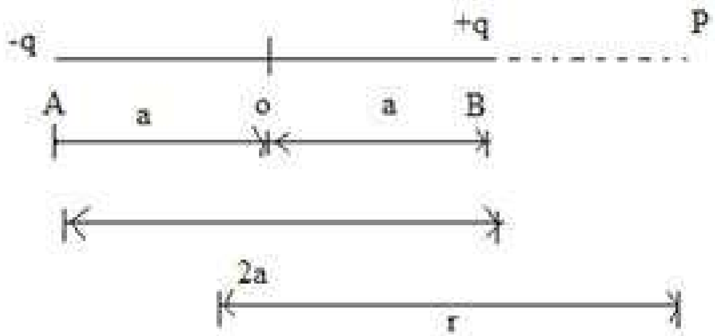
$\vec{E}.\vec{dx} = 0$

$\Rightarrow E \perp \vec{d}x$

(b) Consider an electric dipole of dipole length 2a and point P on the axial line such that OP= r

where O is the center of the dipole.

Electric Potential at point P due to the dipole



$$V = V_{PA} + V_{PB}$$

$$V = \frac{K(-q)}{(r+a)} + \frac{K(+q)}{(r-a)}$$

$$V = Kq\left[\frac{1}{r-a} - \frac{1}{r+a}\right]$$

$$V = Kq\left[\frac{(r+a)-(r-a)}{(r-a)(r+a)}\right]$$

$$V = Kq\left[\frac{\cancel{r}+a-\cancel{r}+a}{r^2-a^2}\right]$$

$$V = Kq\frac{(2a)}{r^2 - a^2} \qquad (\because \overrightarrow{P} = 2aq)$$

$$V = \frac{KP}{r^2 - a^2}$$

For a short electric dipole (a) can be neglected

$$\Rightarrow V = \frac{KP}{r^2}$$

---

4. Check that the ratio $ke_2 / G\ m_e m_p$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Ans. The given ratio is $\frac{ke^2}{Gm_c m_p}$.

Where,

G = Gravitational constant

Its unit is $N\ m^2\ kg^{-2}$

$m_e\ and m_p$ = Masses of electron and proton.

Their unit is kg.

e = Electric charge.

Its unit is C.

$\in 0$ = Permittivity of free space

Its unit is $Nm^2\ C^{-2}$

$$Therefore, unit\ of\ the\ given\ ratio\ \frac{ke^2}{Gm_c m_p}$$

$$= \frac{[Nm^2C^{+2}][C^{+2}]}{[Nm^2kg^{-2}][kg][kg]}$$

$$= M^0L^0T^0$$

Hence, the given ratio is dimensionless.

$e = 1.6 \times 10^{-19} C$

G = $6.67 \times 10^{-11} Nkg^{-2}$

$me = 9.1 \times 10^{-31} kg$

$mp = 1.66 \times 10^{-27}\ kg$

Hence, the numerical value of the given ratio is

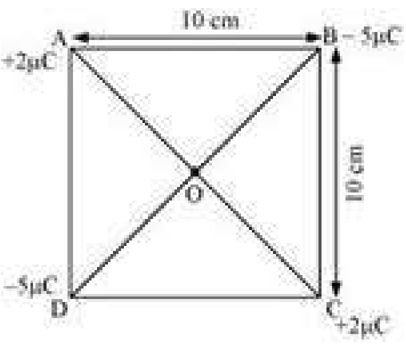
$$\frac{ke^2}{Gm_em_p} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.1 \times 10^{-3} \times 1.67 \times 10^{-22}} \approx 2.3 \times 10^{39}$$

This is the ratio of electric force to the gravitational force between a proton and an electron, keeping distance between them constant.

---

5. Four point charges qA = $2\ \mu$C, $q\mathbf{B}$ = $-5\ \mu$C, $q\mathbf{C}$ = $2\ \mu$C, and $q\mathbf{D}$ = $-5\ \mu$C are located at the corners of a square ABCD of side 10 cms. What is the force on a charge of 1 μC placed at the centre of the square?

Ans. The given figure shows a square of side 10 cm with four charges placed at its corners. O is the centre of the square.



Where,

(Sides) $AB = BC = CD = AD = 10\ cm$

(Diagonals) $AC = BD = 10\sqrt{2}cm$

$AO = OC = DO = OB\ = 5\sqrt{2}cm$ s

A charge of amount 1μ C is plasced at point O.

Force of repulsion between charges placed at corner A and centre O is equal in magnitude but opposite in direction relative to the force of repulsion between the charges placed at corner C and centre O. Hence, they will cancel each other. Similarly, force of attraction between charges placed at corner B and centre O is equal in magnitude but opposite in direction relative to the force of attraction between the charges placed at corner D and centre O. Hence, they will also cancel each other. Therefore, net force caused by the four charges placed at the corner of the square on 1 μ C charge at centre O is zero.

---

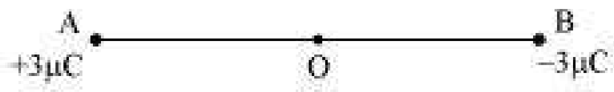
6. Two point charges $qA = 3\ \mu$ C and $qB = -3\ \mu\ C$ are located 20 cm apart in vacuum.

(a) What is the electric field at the midpoint O of the line AB joining the two charges?

(b) If a negative test charge of magnitude $1.5 \times 10^{-9}\ C$ is placed at this point, what is the force experienced by the test charge?

Ans. (a) The situation is represented in the given figure. O is the mid-point of line AB.

Distance between the two charges, $AB\ =\ 20\ cm$



$\therefore AO\ =\ OB\ =\ 10\ cm$

Net electric field at point $O\ = E$

Electric field at point O caused by + $3\mu C$ charge,

$E_1 = \dfrac{3\times10^{-6}}{4\pi\in_0 (\mathrm{AO})^2} = \dfrac{3\times10^{-6}}{4\pi\in_0 \left(10\times10^{-2}\right)^2} N/C$ along OB

Where,

$\in_0$ = Permittivity of free space

$\dfrac{1}{4\pi\in_0} = 9\times10^9 Nm^2C^{-2}$

Magnitude of electric field at point O caused by - $3\mu C$ charge,

$E_2 = \dfrac{-3\times10^{-6}}{4\pi\in_0 (OB)^2} = \dfrac{3\times10^{-6}}{4\pi\in_0 (10\times10^{-2})} N/C$ along OB

$\therefore E = E_1 + E_2$

$= 2\times\left[\left(9\times10^9\right)\times\dfrac{3\times10^{-6}}{(10\times10^{-2})^2}\right]$ [since the values $E_1 and\, E_2$ of are same, the value is multiplied with2]

$= 5.4\times10^6\ N/C\ along\ OB$

Therefore, the electric field at mid-point O is $5.4\times10^6\ NC^{-1}\ along\ OB$

(b) A test charge of amount $1.5\ \times\ 10^{-9}\ C$ is placed at mid-point O.

$q = 1.5\ \times\ 10^{-9}\ C$

Force experienced by the test charge = *F*

$\therefore F\ = qE$

$=\ 1.5\times10^{-9}\times\ 5.4\ \times\ 10^6$

$=\ 8.1\ \times\ 10^{-3}\ N$

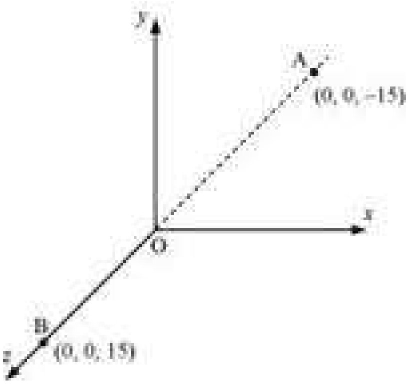
The force is directed along line OA. This is because the negative test charge is repelled by the charge placed at point B but attracted towards point A.

Therefore, the force experienced by the test charge is $8.1 \times 10^{-3}\,N$ along OA.

---

7. A system has two charges $q_A = 2.5 \times 10^{-7}$C and qB $= -2.5 \times 10^{-7}$ C located at points A: $(0,\ 0,\ -15\ \text{cm})$and $\text{B}:\ (0,\ 0,\ +15\ \text{cm})$, respectively. What are the total charge and electric dipole moment of the system?

Ans. Both the charges can be located in a coordinate frame of reference as shown in the given figure.



At A, amount of charge, $q_A = 2.5 \times 10^{-7} C$

At B, amount of charge, $q_B = -2.5 \times 10^{-7}\ C$

Total charge of the system,

$$q = q_A + q_B$$

$$= 2.510^{7}\ C - 2.510^{-7}\ C$$

$$= 0$$

Distance between two charges at points A and B,

$$d = 15 + 15 = 30\ cm = 0.3\ m$$

Electric dipole moment of the system is given by,

$$p = q_A \times d = q_B \times d$$

$= 2.5\times10^{-7}\times0.3$

$= 7.5\times10^{-8}\ Cm$ along positive *z*-axis

Therefore, the electric dipole moment of the system is $7.5\times10^{-8}C$ m along positive *z*-axis.

---

8. (a) Two insulated charged copper spheres A and B have their centers separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is $6.5\times10^{-7}\ C$ s? The radii of A and B are negligible compared to the distance of separation.

(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Ans. (a) Charge on sphere A, q_A =Charge on sphere $B, q_B = 6.5\times10^{-7}\ C$

Distance between the spheres, $r = 50\ cm = 0.5\ m$

Force of repulsion between the two spheres,

$$F = \frac{q_A q_B}{4\pi\epsilon_0 r^2}.$$

Where,

$\epsilon_0 =$ Free space permittivity

$$\frac{1}{4\pi\epsilon_0} = 9\times10^{9}\ N\ m^2\ C^{-2}$$

$$\therefore F = \frac{9\times10^{9}\times(6.5\times10^{-7})^2}{(0.5)^2}$$

$$= 1.52\times10^{-2}\ N$$

Therefore, the force between the two spheres is $1.52\times10^{-2}\ N$

(b) After doubling the charge, charge on sphere A, q_A = Charge on sphere $B, q_B = 2 \times 6.5 \times 10^{-7}\ C = 1.3 \times 10^{-6}\ C$ The distance between the spheres is halved.

$$\therefore r = \frac{0.5}{2} = 0.25 m$$

Force of repulsion between the two spheres,

$$F = \frac{q_A q_B}{4\pi \in_0 r^2}$$

$$= \frac{9\times10^9 \times1.3\times10^{-6}\times1.3\times10^{-6}}{(0.25)^2}$$

$$= 16 \times 1.52 \times 10^{-2}$$

$= 0.243\ N$ s

Therefore, the force between the two spheres is $0.243\ N$

---

9. Suppose the spheres A and B in Exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Ans. Distance between the spheres, $A \text{ and } B, r = 0.5\ m$

Initially, the charge on each sphere, $q = 6.5 \times 10^{-7}\ C$

When sphere A is touched with an uncharged sphere C, amount of charge from A will transfer to sphere

C. Hence, charge on each of the spheres, A and C, $\frac{q}{2}$.

When sphere C with charge $\frac{q}{2}$ is brought in contact with sphere B with charge *q*, total charges on the system will divide into two equal halves given as,

$$\frac{\frac{q}{2}+q}{2}=\frac{3q}{4}$$

Each sphere will share each half. Hence, charge on each of the spheres, C and B, is $\frac{3q}{4}$.

Force of repulsion between sphere A having charge $\frac{q}{2}$ and sphere B having charge

$$\frac{3q}{4}=\frac{\frac{q}{2}\times\frac{3q}{4}}{4\pi\in_0 r^2}=\frac{3q^2}{8\times4\pi\in_0 r^2}$$

$$=9\times10^9\times\frac{3\times(6.5\times10^{-7})^2}{8\times(0.5)^2}$$

$$=5.703.\times10^{-3}N$$

Therefore, the force of attraction between the two spheres is $5.703\times10^{-3}N$.

---

10. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0\times10^{-22}\ \mathrm{C/m^2}$. What is E: (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates?

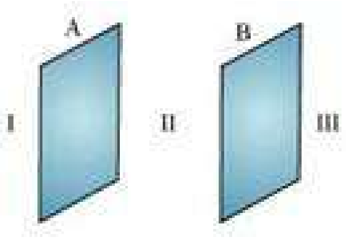
Ans. The situation is represented in the following figure.

A and B are two parallel plates close to each other. Outer region of plate A is labelled as **I**, outer region of plate B is labelled as **III**, and the region between the plates, A and B, is labelled as **II.**

Charge density of plate A, $\sigma=17.0\times10^{-22}\ C/m^2$

Charge density of plate B, $\sigma = -17.0\times10^{-22}\ C/m^2$

In the regions, **I** and **III**, electric field *E* is zero. This is because charge is not enclosed by the respective plates.



Electric field *E* in region **II** is given by the relation,

$$E\frac{\sigma}{\epsilon_0}$$

Where,

ϵ_0 =Permittivity of free space = $8.854\times10^{-12}\ N^{-1}C^2\ m^{-2}$

$$\therefore E = \frac{17.0\times10^{-22}}{8.854\times10^{-22}}$$

$$= 1.92\times10^{-10}\ N/C$$

Therefore, electric field between the plates is $= 1.92\times10^{-10}\ N/C$.

---

11. An oil drop of 12 excess electrons is held stationary under a constant electric field of $2.55\times10^4\ \mathrm{N\ C^{-1}}$ in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. $\left(g = 9.81\ \mathrm{m\ s^{-2}}; e = 1.60\times10^{-19}\ \mathrm{C}\right)$.

Ans. Excess electrons on an oil drop, *n* = 12

Electric field intensity, *E* = $2.55\times10^4\ \mathrm{N\ C^{-1}}$

Density of oil, $p = 1.26\ gm/cm^3 = 1.26\times10^3\ kg/m^3$

Acceleration due to gravity, g $= 9.81\ m\ s^{-2}$

Charge on an electron, $e = 1.6 \times 10^{-19}\ C$

Radius of the oil drop = r

Force (F) due to electric field E is equal to the weight of the oil drop (W)

$F = W$

$Eq = mg$

$Ene = \frac{4}{3}\pi r^3 p \times g$

Where,

q = Net charge on the oil drop = ne

m = Mass of the oil drop

= Volume of the oil drop × Density of oil

$= \frac{4}{3}\pi r^3 \times p$

$\therefore r = 3\sqrt{\frac{3Ene}{4\pi pg}}$

$= 3\sqrt{\frac{3 \times 2.55 \times 10^4 \times 12 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times 1.26 \times 10^3 \times 9.81}}$

$= 3\sqrt{946.09 \times 10^{-21}}$

$= 9.82 \times 10^{-7} m$

$| E|$

$= 9.82 \times 10^{-4}\ mm$

Therefore, the radius of the oil drop is $= 9.82 \times 10^{-4}\ mm.$

12. In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive *z*-direction, at the rate of $105\ NC^{-1}$ per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative *z*-direction?

Ans. Dipole moment of the system, $p = q \times dl = -10^{-7}\ C\ m$ Rate of increase of electric field per unit length,

$$\frac{de}{dl} = 10^{+5} NC^{-1}$$

Force (F) experienced by the system is given by the relation,

$F = qE$

$$F = q\frac{dE}{dl} \times dl$$

$$= p \times \frac{dE}{dl}$$

$$= -10^{-7} \times 10^{-5}$$

$$= -10^{-2}\ N$$

The force is $-10^{-2}\ N$ in the negative z-direction i.e., opposite to the direction of electric field. Hence, the angle between electric field and dipole moment is 180°.

Torque (τ) is given by the relation,

(τ) = pE sin180°

= 0

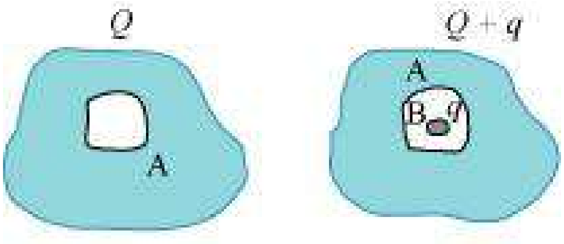
Therefore, the torque experienced by the system is zero.

13. (a) A conductor A with a cavity as shown in Fig. 1.36(a) is given a charge *Q*. Show that the entire charge must appear on the outer surface of the conductor. (b) Another conductor B with charge *q* is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is *Q* + *q* [Fig. 1.36(b)]. (c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.

Ans. (a) Let us consider a Gaussian surface that is lying wholly within a conductor and enclosing the cavity. The electric field intensity E inside the charged conductor is zero.

Let q is the charge inside the conductor and ϵ_0 is the permittivity of free space.

According to Gauss's law,



(a) (b)

Flux, $\phi = \overline{E}.\overline{ds} = \dfrac{q}{\epsilon 0}$

Here, $E = 0$

$$E = \frac{17.0\times10^{-22}}{8.854\times10^{-22}}$$

$$\therefore \epsilon_0 = 0$$

$$\therefore q = 0$$

Therefore, charge inside the conductor is zero.

The entire charge Q appears on the outer surface of the conductor.

(b) The outer surface of conductor A has a charge of amount Q. Another conductor B having charge $+q$ is kept inside conductor A and it is insulated from A. Hence, a charge of amount $-q$

will be induced in the inner surface of conductor A and +*q* is induced on the outer surface of conductor A. Therefore, total charge on the outer surface of conductor A is *Q* + *q*.

(c) A sensitive instrument can be shielded from the strong electrostatic field in its environment by enclosing it fully inside a metallic surface. A closed metallic body acts as an electrostatic shield.

---

14. A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\left[\frac{\sigma}{2\epsilon_0}\right]\hat{n}$, where $\hat{n}$ is the unit vector in the outward normal direction, and σ is the surface charge density near the hole.

Ans. Let us consider a conductor with a cavity or a hole. Electric field inside the cavity is zero.Let *E* is the electric field just outside the conductor, *q* is the electric charge, σ is the charge density, and ϵ_0 is the permittivity of free space.

Charge $|q| = \bar{\sigma} \times \bar{d}$

According to Gauss's law,

$$\phi = \bar{E}.\bar{d} = \frac{|q|}{\epsilon_0}$$

$$Eds = \frac{\bar{\sigma} \times \bar{d}}{\epsilon_0}$$

$$\therefore E = \frac{\sigma}{\epsilon_0}\hat{n}$$

Therefore, the electric field just outside the conductor is $\frac{\sigma}{\epsilon_0}\hat{n}$. This field is a superposition of field due to the cavity E^1 and the field due to the rest of the charged conductor E^1. These fields are equal and opposite inside the conductor, and equal in magnitude and direction outside the conductor.

$$\therefore E^1 + E^1 = E$$

$$E^1 = \frac{E}{2}$$

$$= \frac{\sigma}{2 \epsilon_0} \hat{n}$$

Therefore, the field due to the rest of the conductor is $\frac{\sigma}{\epsilon_0} \hat{n}$.

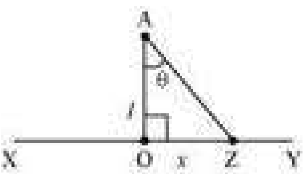
Hence, proved.

---

15. Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law. [*Hint:* Use Coulomb's law directly and evaluate the necessary integral.]

Ans. Take a long thin wire XY (as shown in the figure) of uniform linear charge density λ .

Consider a point A at a perpendicular distance *l* from the mid-point O of the wire, as shown in the following figure.



Let *E* be the electric field at point A due to the wire, XY.

Consider a small length element *dx* on the wire section with OZ = *x*

Let *q* be the charge on this piece.

$$\therefore q = \lambda dx$$

Electric field due to the piece,

$$de = \frac{1}{4\pi \in_0} \frac{\lambda dx}{(AZ)^2}$$

$$However, AZ = \sqrt{(1^2 + x^2}$$

$$\therefore de = \frac{\lambda dx}{4\pi \in_0 (l^2 + x^1}$$

The electric field is resolved into two rectangular components. $dE \cos\theta$ is the perpendicular component and $dE \sin\theta$ is the parallel component.

When the whole wire is considered, the component $dE \sin\theta$ is cancelled.

Only the perpendicular component $dE \cos\theta$ affects point A.

Hence, effective electric field at point A due to the element *dx* is *dE*1.

$$\therefore dE_1 = \frac{\lambda dx \cos\theta}{4\pi \in_0 (x^2 + l^2)} \qquad ...(1)$$

$$In \triangle AZO,$$

$$\tan\theta = \frac{x}{l}$$

$$x = l \tan\theta \qquad ...(2)$$

On differentiating equation (2), we obtain

$$\frac{dx}{d\theta} = l \sin^2\theta$$

$$dx = l \sin^2 d\theta \quad .....(3)$$

From equation (2),

$$x^2 + l^2 = l^2 + \tan^2\theta$$

$$\therefore l^2(1 + \tan^2\theta) = l^2 \sec^2\theta$$

$$x^2 + l^2 = l^2 \sin^2 \theta \quad ......(4)$$

Putting equations (3) and (4) in equation (1), we obtain

$$\therefore dE_1 = \frac{\lambda l \sec^2 d\theta}{4\pi \in_0 l^2 \sec^2 \theta} \times \cos\theta$$

$$\therefore dE_1 = \frac{\lambda \cos\theta d\theta}{4\pi \in_0 l} \quad ....(5)$$

The wire is so long that tends from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$.

By integrating equation (5), we obtain the value of field E_1 as,

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dE_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda}{4\pi \in_0} \cos\theta d\theta$$

$$E_1 = \frac{\lambda}{4\pi \in_0 l} \times 2$$

$$E_1 = \frac{\lambda}{2\pi \in_0 l}$$

Therefore, the electric field due to long wire is $\frac{\lambda}{2\pi \in_0 l}$

---

16. It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge $\left(+\frac{1}{2}\right)$ *e*, and the 'down' quark (denoted by d) of charge (-1/3) *e*, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

Ans.A proton has three quarks. Let there be *n* up quarks in a proton, each having a charge of $+\left(\frac{2}{3}e\right)$.

Charge due to *n* up quarks$=\left(\frac{2}{3}e\right)n$

Number of down quarks in a proton = 3 – *n*

Each down quark has a charge of $-\frac{1}{3}e$.

Charge due to (3 – *n*) down quarks $=\left(-\frac{1}{3}e\right)(3-n)$

Total charge on a proton = + *e*

$$\therefore e=\left(\frac{2}{3}e\right)n+\left(-\frac{1}{3}e\right)(3-n)$$

$$e=\left(\frac{2ne}{3}\right)-e+\frac{ne}{3}$$

$$2e=ne$$

$$n=2$$

Number of up quarks in a proton, *n* = 2

Number of down quarks in a proton = 3 – *n* = 3 – 2 = 1

Therefore, a proton can be represented as $'uud'$.

A neutron also has three quarks. Let there be *n* up quarks in a neutron, each having a charge of $+\frac{2}{3}e$.

Charge on a neutron due to *n* up quarks $=\left(+\frac{3}{2}e\right)n$

Number of down quarks is 3 - n, each having a charge of $=\left(-\frac{3}{2}\right)e$.

Charge on a neutron due to $(3-n)$ down quarks = $\left(-\frac{1}{3}e\right)(3-n)$

Total charge on a neutron = 0

$$0=\left(\frac{2}{3}e\right)n+\left(-\frac{1}{3}e\right)(3-n)$$

$$0=\frac{2}{3}en-e+\frac{ne}{3}$$

$$e=ne$$

$$n=1$$

Number of up quarks in a neutron, n = 1

Number of down quarks in a neutron = 3 - n = 2

Therefore, a neutron can be represented as 'udd'.

---

17. (a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where E = 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Ans. A proton has three quarks. Let there be n up quarks in a proton, each having a charge of $+\left(\frac{2}{3}e\right)$.

Charge due to n up quarks $=\left(\frac{2}{3}e\right)n$

Number of down quarks in a proton = 3 – n

Each down quark has a charge of $-\frac{1}{3}e$.

Charge due to (3 – n) down quarks $=\left(-\frac{1}{3}e\right)(3-n)$

Total charge on a proton = + e

$$\therefore e=\left(\frac{2}{3}e\right)n+\left(-\frac{1}{3}e\right)(3-n)$$

$$e=\left(\frac{2ne}{3}\right)-e+\frac{ne}{3}$$

$$2e=ne$$

$$n=2$$

Number of up quarks in a proton, n = 2

Number of down quarks in a proton = 3 – n = 3 – 2 = 1

Therefore, a proton can be represented as $'uud'$.

A neutron also has three quarks. Let there be n up quarks in a neutron, each having a charge of $+\frac{2}{3}e$.

Charge on a neutron due to n up quarks $=\left(+\frac{3}{2}e\right)n$

Number of down quarks is 3 - n, each having a charge of $=\left(-\frac{3}{2}\right)e$.

Charge on a neutron due to $(3-n)$ down quarks = $\left(-\frac{1}{3}e\right)(3-n)$

Total charge on a neutron = 0

$$0=\left(\frac{2}{3}e\right)n+\left(-\frac{1}{3}e\right)(3-n)$$

$$0=\frac{2}{3}en-e+\frac{ne}{3}$$

$$e=ne$$

$$n=1$$

Number of up quarks in a neutron, n = 1

Number of down quarks in a neutron = 3 - n = 2

Therefore, a neutron can be represented as $'udd'$.

---

18. A particle of mass *m* and charge (- *q*) enters the region between the two charged plates initially moving along *x*-axis with speed *vx* (like particle 1 in Fig. 1.33). The length of plate is *L* and an uniform electric field *E* is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2\ /\ \left(2m^{v2,}\right)$.

Compare this motion with motion of a projectile in gravitational field discussed in Section 4.10 of Class XI Textbook of Physics.

Ans. Charge on a particle of mass *m* = - *q*

Velocity of the particle = *vx*

Length of the plates =*L*

Magnitude of the uniform electric field between the plates = *E*

Mechanical force, *F* = Mass (*m*) × Acceleration (*a*)

$$a=\frac{F}{m}$$

$$However, electric\ force, F=qE$$

Therefore, acceleration, $= \frac{qE}{m}$ ...(1)

Time taken by the particle to cross the field of length *L* is given by,

$$t = \frac{Length\ of\ the\ plate}{Velocity\ of\ the} = \frac{L}{v_x} \quad ....(2)$$

In the vertical direction, initial velocity, *u* = 0

According to the third equation of motion, vertical deflection *s* of the particle can be obtained as,

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2$$

$$s = \frac{qEL^2}{2mV_x^{\ 2}} \quad ...(3)$$

Hence, vertical deflection of the particle at the far edge of the plate is

$qEL^2 / \left(2mv_x^{\ 2}\right)$. This is similar to the motion of horizontal projectiles under gravity.

CBSE Class 12 physics

Important Questions

Chapter 2

Electrostatic Potential and Capacitance

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1 Mark Questions

1. Why does the electric field inside a dielectric decrease when it is placed in an external electric field?

Ans. The electric field, inside a dielectric decrease when it is placed is an external electric field due to polarisation as it creates an internal electric field opposite to external electric field inside a dielectric due to which net electric field gets reduced.

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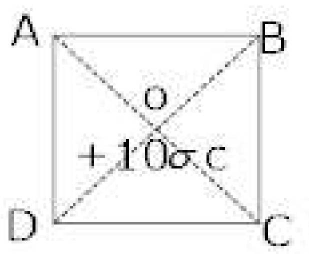
2. What is the work done in moving a 2 μ C point change from corner A to corner B of a square ABCD when a 10 μ C charge exist at the center of the square?

Ans. Since pt. A & B are at the same distance from the pt. O

VA = VB

$\rightarrow$ Work done = Zero.



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3. Force of attraction between two point electric charges placed at a distance d in a medium is F. What distance apart should these be kept in the same medium, so that force between them becomes F/3?

Ans. If two point changes are $q_1 and\ q_2$ separated by distance d

$$F = \frac{Kq_1q_2}{d^2}$$

Suppose if force becomes F/3 let the distance be x

$$\frac{F}{3} = \frac{Kq_1q_2}{x^2}$$

$$\Rightarrow \frac{Kq_1q_2}{3d^2} = \frac{kq_1q_2}{x^2}$$

$$x^2 = 3d^2$$

$\Rightarrow$ x = $\sqrt{3}$ d

---

4. The distance of the field point on the equatorial plane of a small electric dipole is halved. By what factor will the electric filed due to the dipole change?

Ans. Since $E \propto \frac{1}{r^3}$

$$\therefore E \propto \frac{1}{(r/2)^3} \Rightarrow E \propto 8/r^3$$

$\Rightarrow$ Electric field becomes eight times

---

5. The Plates of a charged capacitor are connected by a voltmeter. If the plates of the capacitor are moved further apart. What will be the effect on the reading of the voltmeter?

Ans. Since $C = \frac{A \in o}{d} \Rightarrow C \propto \frac{1}{d}$

which means if distance increases, capacitance decreases.

Since $V = \frac{Q}{C}$ and charge on the capacitor is constant.

Hence reading of the voltmeter increases.

6. What happens to the capacitance of a capacitor when a dielectric slab is placed between its plates?

Ans. The introduction of dielectric in a capacitor reduces the effective charge on plate and hence increases the capacitance.

2 Mark Questions

1. Show mathematically that the potential at a point on the equatorial line of an electric dipole is Zero?

Ans. Electric potential at point P doe to the dipole

$$V = Vp_A + Vp_B$$

V = $\frac{K(-q)}{r} + \frac{K(+q)}{r}$

V=o

2. A parallel plate capacitor with air between the plates has a capacitance of $8\,pF\ \left(1pF = 10^{-12}F\right)$. What will be the capacitance if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant 6?

Ans. For air C_o = $\frac{A\in_o}{d}$

Co = $8\times pF = 8\times10^{-12}F$

$$\therefore \frac{AC_o}{d} = 8\times10^{-12}$$

Now d' = d/2 and K = 6

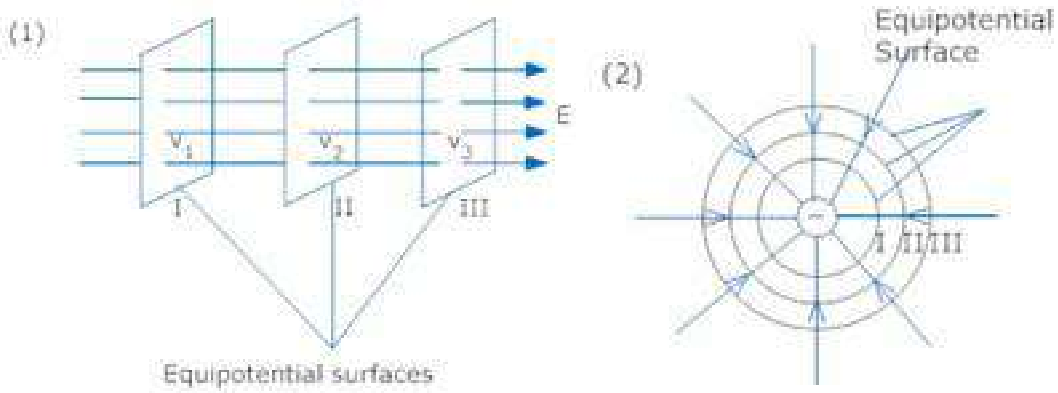
$$\Rightarrow C' = \frac{AC_o}{d'} \times K$$

$$C' = \frac{AC_o}{d'} \times 2 \times K = 8 \times 10^{-12} \times 2 \times 6$$

$$C' = 96 \times 10^{-12} pF$$

3. Draw one equipotential surfaces (1) Due to uniform electric field (2) For a point charge (q < o)?

Ans.



4. If the amount of electric flux entering and leaving a closed surface are ϕ_1 and ϕ_2 respectively. What is the electric charge inside the surface?

Ans. Net flux = ϕ_2 - ϕ_1

Since $\phi = q / \in_0$

$$Q = (\phi_2 - \phi_1) \in_0$$

$\Rightarrow$ Q = $\phi \in_0$

5. A steam of electrons travelling with speed $\upsilon \frac{m}{s}$ at right angles to a uniform electric field E is deflected in a circular path of radius r. Prove that $\frac{e}{m} = \frac{v^2}{rE}$?

Ans. The path of the electron traveling with velocity $v\frac{m}{s}$ at right angles of $\vec{E}$ is circular.

$\therefore$ It requires a centripetal force $f = \frac{mv^2}{r}$

which is provided by an electrostatic force f = eE

$$\Rightarrow \quad eE = \frac{mv^2}{r}$$

$\frac{e}{m} = \frac{v^2}{Er}$ b

---

6. The distance between the plates of a parallel plate capacitor is d. A metal plate of thickness $\left(\frac{d}{2}\right)$ is placed between the plates. What will be the effect on the capacitance?

Ans. For air $C = \frac{A \in o}{d}$

Thickness t = $\frac{d}{2}$ only when $k = \infty$

$$Co = \frac{\in oA}{d}$$

$$C_{metal} = \frac{\in oA}{(d-t)}$$

$$= \frac{\in oA}{\left(d - \frac{d}{2}\right)}$$

$$= 2 \in oA$$

$C_{matel} = 2Co$, Hence capacitance will get doubled.

7. Two charges $2\mu C$ and $-2\ \mu C$ are placed at points A and B 6 cm apart.

1. Identify an equipotential surface of the system.

2. What is the direction of the electric field at every point on this surface?

Ans. The situation is represented in the given figure.

An equipotential surface is the plane on which total potential is zero everywhere. This plane is normal to line AB. The plane is located at the mid-point of line AB because the magnitude of charges is the same.

1. The direction of the electric field at every point on this surface is normal to the plane in the direction of AB.

---

8. In a Van de Graaff type generator a spherical metal shell is to be a 15×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is $5 \times 10^7\ Vm^{-1}$. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)

Ans. Potential difference, V = 15×10^6 V

Dielectric strength of the surrounding gas = $5 \times 10^7 V/m$

Electric field intensity, E = Dielectric strength = $5 \times 10^7 V/m$

Minimum radius of the spherical shell required for the purpose is given by,

$$r = \frac{V}{E}$$

$$= \frac{15 \times 10^6}{5 \times 10^7} = 0.3m = 30cm$$

Hence, the minimum radius of the spherical shell required is 30 cm.

---

9. A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

Ans. According to Gauss's law, the electric field between a sphere and a shell is determined by the charge q_1 on a small sphere. Hence, the potential difference, V, between the sphere and the shell is independent of charge q_2. For positive charge q_1, potential difference V is always positive.

---

10. Describe schematically the equipotential surfaces corresponding to

1. a constant electric field in the z-direction,

2. a field that uniformly increases in magnitude but remains in a constant (say, z) direction,

3. a single positive charge at the origin, and

4. a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Ans. 1. Equidistant planes parallel to the x-y plane are the equipotential surfaces.

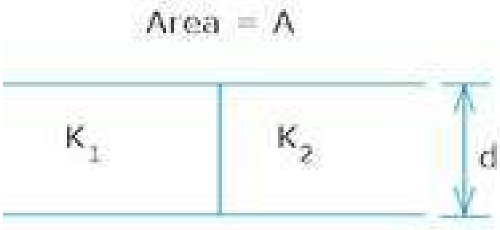
2. Planes parallel to the x-y plane are the equipotential surfaces with the exception that when the planes get closer, the field increases.

3. Concentric spheres centered at the origin are equipotential surfaces.

4. A periodically varying shape near the given grid is the equipotential surface. This shape gradually reaches the shape of planes parallel to the grid at a larger distance.

3 Mark Questions

1.Two dielectric slabs of dielectric constant $K_1 and\ K_2$ are filled in between the two plates, each of area A, of the parallel plate capacitor as shown in the figure. Find the net capacitance of the capacitor? Area of each plate = $\frac{A}{2}$



Ans. Here the two capacitors are in parallel

$\therefore$ Net capacitance $C = C_1 + C_2$

$$C_1 = \frac{K_1 \in_o A/2}{d} = \frac{K_1 \in_o A}{2d}$$

$$C_2 = \frac{K_1 \in_o A/2}{2d} = \frac{K_2 \in_o A}{2d}$$

$$\Rightarrow C = \frac{K_1 \in_o A}{2d} + \frac{K_2 \in_o A}{2d}$$

$$C = \frac{\in_o A}{2d}(K_1 + K_2)$$

---

2. Prove that the energy stored in a parallel plate capacitor is given by $\frac{1}{2}CV^2$?

Ans. Suppose a capacitor is connected to a battery and it supplies small amount of change dq at constant potential V, then small amount of work done by the battery is given by

dw = Vdq

$\Rightarrow$ dw = qc/dq (Since q = CV)

Total work done where capacitor is fully changed to q.

$$\int dw = W = \int_0^q q/c\, dq \Rightarrow \quad W = \frac{1}{C}\int_o^q q dq \Rightarrow \quad W = \frac{q^2}{2C} = \frac{C^2V^2}{2C}$$

$$W = \frac{1}{2}CV^2$$

This work done is stored in the capacitor in the form of electrostatic potential energy.

$$\Rightarrow W = U = \frac{1}{2}CV^2$$

---

3. State Gauss's Theorem in electrostatics? Using this theorem define an expression for the field intensity due to an infinite plane sheet of change of charge density $\sigma C/m^2$?

Ans. Gauss's Theorem states that electric flux through a closed surface enclosing a charge q in vacuum is $1/\in_o$ times the magnitude of the charge enclosed

Is $\phi = q/\in_o$

Consider a charge is distributed over an infinite sheet of area S having surface change density $\sigma C/m^2$.

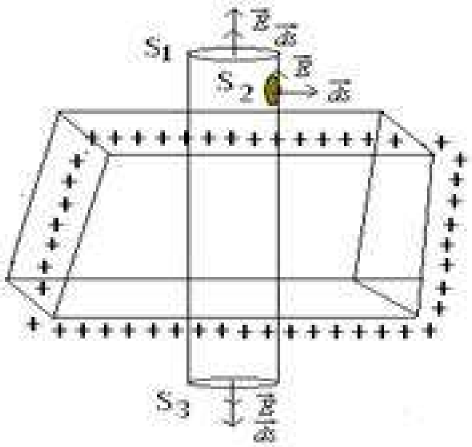
To enclose the charge on sheet an imaginary Gaussian surface cylindrical in shape is assumed and it is divided into three sections $S_1,\ S_2,\ \&\ S_3$

According to Gauss's theorem

$$\phi = q/\in_o = \frac{\sigma S}{\in_o} \text{...........}(1)$$

We know $\phi = \int E.ds$

For the given surface $\phi = \int_{S_1} \vec{E}.\vec{ds} + \int_{S_2} \vec{E}.\vec{ds} + \int_{S_3} \vec{E}.\vec{dsq}$



$$\Rightarrow \phi = \int_{S_1} E.ds\cos o^{\circ} + \int_{S_3} Eds\cos o^{\circ} +$$

$$\left(\because \theta = 90^{\circ}\ for S_2 \therefore \int_{S_2} \vec{E}.\vec{ds} = o \right)$$

$$\phi = E\int_{S_1} ds + E\int_{S_2} ds$$

$$\phi = E\left[\int_{o}^{S} ds + \int_{o}^{S} ds \right]$$

$$\phi = E.2S\ldots\ldots\ldots(2)$$

Combined eq. (1) & (2)

$$E = \frac{\sigma}{2\epsilon_o}$$

$$E.2S = \frac{\sigma S}{\epsilon_o}$$

4. Derive an expression for the total work done in rotating an electric dipole through an angle θ in a uniform electric field?

Ans. We know τ = PE sin θ

If an electric dipole is rotated through an angled θ against the torque acting on it, then small amount of work done is

dw = τ dθ = PE sin θdθ

For rotating through on angle θ, from 90°

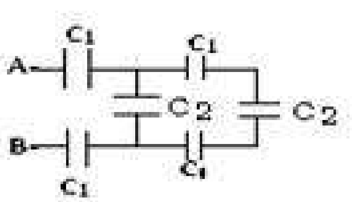
$$w = \int_{90^{\circ}}^{\theta} PE \sin\theta$$

$$w = PE\left|-\cos\theta\right|_{90}^{\theta}$$

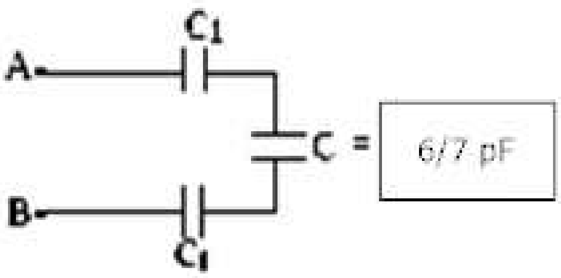
w = - PE cos θ

---

5. If $C_1 = 3pF$ and $C_2 = 2pF$, calculate the equivalent capacitance of the given network between points A & B?



Ans. Since C_1, $C_2 and$ C_1 are in series



$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_1}$$

$$\Rightarrow \frac{1}{C} = \frac{1}{3} + \frac{1}{2} + \frac{1}{3}$$

$$\frac{1}{C} = \frac{2+3+2}{6}$$

$$C = \frac{6}{7}\, pF$$

C_2 and C are in series

$$\Rightarrow C' = \frac{2}{1} + \frac{6}{7} = \frac{14+6}{7} = \frac{20}{7}\, pF$$

Here $C_1,\ C' \ and\ C_1$ are in series

$$\therefore \frac{1}{Cnet} = \frac{1}{C_1} + \frac{1}{C'} + \frac{1}{C_1}$$

$$\frac{1}{Cnet} = \frac{1}{3} + \frac{7}{20} + \frac{1}{3}$$

$$\frac{1}{Cnet} = \frac{20+21+20}{60} = \frac{61}{60}$$

$\Rightarrow$ Cnet = 60/61 pF

6. Prove that energy stored per unit volume in a capacitor is given by $\frac{1}{2} \in_o E^2$, where E is the electric field of the capacitor?

Ans. We know capacitance of a parallel plate capacitor, $C = \frac{A \in_o}{d}$, electric filed in between the plates $E = \frac{\sigma}{\epsilon_o}$ where σ is the surface charge density of the plates.

Energy stored per unit volume = $\frac{Energy\ stored}{Volume}$

Energy stored per unit volume = $\frac{\frac{1}{2}\frac{q^2}{C}}{Ad}$ (Volume of the capacitor = Ad)

$$= \frac{\frac{1}{2}\times\frac{(\sigma A)^2}{\frac{A\epsilon_0}{d}}}{Ad} = \frac{\frac{1}{2}\times E^2 \epsilon_o^{\ 2} A^2 d}{A^2 \epsilon_o d}$$

$\Rightarrow$ Energy stored/volume = $\frac{1}{2}\epsilon_o E^2$ Hence proved

---

7. Keeping the voltage of the charging source constant. What would be the percentage change in the energy stored in a parallel plate capacitor if the separation between its plates were to be decreased by 10%?

Ans. $U = \frac{1}{2}CV^2$

For parallel plate $U = \frac{1}{2}\ \frac{A\in o}{d}V^2$

When d' = d – 10% of d= 0.9 d

Then $U' = \frac{1}{2}\ \frac{A\in o}{0.9d}V^2$

Change in energy = U'-U = $\frac{1}{2}\frac{AEo}{d}V^2(\frac{1}{0.9}-1)$

$$U' - U = U\left(\frac{0.1}{0.9}\right) = U/9$$

% change = $\dfrac{U' - U}{U} \times 100\ \% = \dfrac{U}{9} \times \dfrac{1}{U} \times 100\% = \dfrac{100\%}{9}$

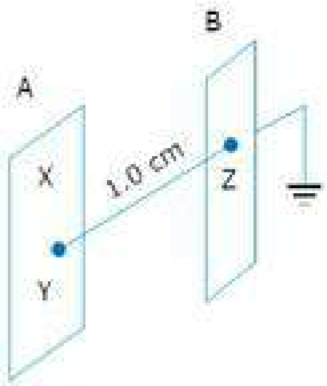
% change = 11.1%

---

8. Two identical plane metallic surfaces A and B are kept parallel to each other in air separated by a distance of 1.0 cm as shown in the figure. Surface A is given a positive potential of 10V and the outer surface of B is earthed.

(a) What is the magnitude and direction of uniform electric field between point Y and Z? What is the work done in moving a change of 20 μc from point X to Y?

(b) Can we have non-zero electric potential in the space, where electric field strength is zero?



Ans. (a)Since $E = \dfrac{dv}{dr} = \dfrac{10V}{1 \times 10^{-2}} = 1000 \dfrac{V}{M}$

(ii) Since surface A is an equipotential surface ie $\Delta V = 0$

$\therefore$ Work done from X to Y = Zero .

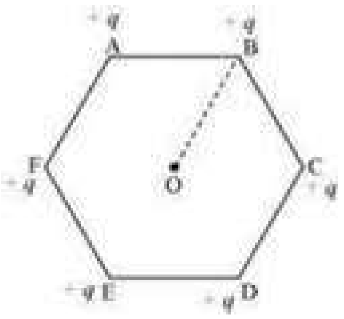
(b) $E = \dfrac{-dV}{dr}$ if E=0

$\dfrac{dV}{dr} = 0 \Rightarrow dv = 0$ Or V= constant (non zero)

So we can have non-zero electric potential, where electric field is zero.

9. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

Ans. The given figure shows six equal amount of charges, q, at the vertices of a regular hexagon.



Where,

Charge, $q = 5\,\mu C = 5 \times 10^{-6} C$

Side of the hexagon, l = AB = BC = CD = DE = EF = FA = 10 cm

Distance of each vertex from centre O, d = 10 cm

Electric potential at point O,

$$V = \frac{6 \times q}{4\pi \epsilon_0 d}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 NC^{-2} m^{-2}$$

$$\therefore V = \frac{6 \times 9 \times 10^9 \times 5 \times 10^{-6}}{0.1}$$

$$= 2.7 \times 10^6 V$$

Therefore, the potential at the centre of the hexagon is $2.7 \times 10^6 V$.

10. A spherical conductor of radius 12 cm has a charge of $1.6 \times 10^{-7} C$ distributed uniformly on its surface. What is the electric field

1. Inside the sphere

2. Just outside the sphere

3. At a point 18 cm from the centre of the sphere?

Ans. Radius of the spherical conductor, r = 12 cm = 0.12 m

Charge is uniformly distributed over the conductor, $q = 1.6 \times 10^{-7} C$

Electric field inside a spherical conductor is zero. This is because if there is field inside the conductor, then charges will move to neutralize it.

1. Electric field E just outside the conductor is given by the relation,

$$E = \frac{q}{4\pi \epsilon_0 r^2}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \, N m^2 C^{-2}$$

$$\therefore E = \frac{1.6 \times 10^{-7} \times 9 \times 10^{-9}}{(0.12)^2}$$

$$= 10^5 \, NC^{-1}$$

Therefore, the electric field just outside the sphere is $10^5 \, NC^{-1}$.

2. Electric field at a point 18 m from the centre of the sphere = E_1

Distance of the point from the centre, d = 18 cm = 0.18 m

$$E_1 = \frac{q}{4\pi \epsilon_0 d^2}$$

$$= \frac{9\times10^9 \times1.6\times10^{-7}}{(18\times10^{-2}}$$

$$= 4.4\times10^4 N/C$$

Therefore, the electric field at a point 18 cm from the centre of the sphere is $4.4\times10^4 N/C$.

---

11. Three capacitors each of capacitance 9 pF are connected in series.

1. What is the total capacitance of the combination?

2. What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Ans. Capacitance of each of the three capacitors, C = 9 pF

Equivalent capacitance (C') of the combination of the capacitors is given by the relation,

$$\frac{1}{C^1} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$= \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{9}$$

$$\therefore C' = 3\mu F$$

Therefore, total capacitance of the combination is $3\mu F$.

1. Supply voltage, V = 100 V

Potential difference (V') across each capacitor is equal to one-third of the supply voltage.

$$\therefore V' = \frac{V}{3} = \frac{120}{3} = 40V$$

Therefore, the potential difference across each capacitor is 40 V.

12. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.

1. What is the total capacitance of the combination?

2. Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Ans. Capacitances of the given capacitors are

$$C_1 = 2pF$$

$$C_2 = 3pF$$

$$C_3 = 4pF$$

For the parallel combination of the capacitors, equivalent capacitor C' is given by the algebraic sum,

$$C' = 2+3+9pF$$

Therefore, total capacitance of the combination is 9 pF.

1. Supply voltage, V = 100 V

The voltage through all the three capacitors is same = V = 100 V

Charge on a capacitor of capacitance C and potential difference V is given by the relation,

$$q = vc.....(i)$$

For C = 2 pF,

$$Charge = VC = 100\times 2 = 200\ pc = 2\times 10^{-10}\,C$$

For C = 3 pF,

$$Charge = VC = 100\times 3 = 300\ pc = 3\times 10^{-10}\,C$$

For C = 4 pF,

$$Charge = VC = 100\times 4 = 200\ pc = 4\times 10^{-10}\,C$$

13. In a parallel plate capacitor with air between the plates, each plate has an area of $6\times10^{-3}\,m^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Ans. Area of each plate of the parallel plate capacitor, A $= 6\times10^{-3}\,m^2$

Distance between the plates, d = 3 mm $= 3\times10^{-3}\,m$

Supply voltage, V = 100 V

Capacitance C of a parallel plate capacitor is given by,

Where, $C = \frac{\epsilon_0 A}{d}$

ϵ_0 = Permittivity of free space

$= 8.854\times10^{-12}\,N^{-1}m^{-2}C^{-2}$

$$\therefore C = \frac{8.854\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$$

$= 17.71\times10^{-12}\,F$

$= 17.71\,pF$

Potential V is related with the charge q and capacitance C as

$$V = \frac{q}{C}$$

$\therefore q = VC$

$= 1.771\times10^{-9}\,C$

$= 100\times17.71\times10^{-12}$

Therefore, capacitance of the capacitor is 17.71 pF and charge on each plate is 1.771×10^{-9}.

14. Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

1. While the voltage supply remained connected.

2. After the supply was disconnected.

Ans. Dielectric constant of the mica sheet, k = 6

1. Initial capacitance, $C = 1.771\times10^{-11}$

New capaci tan ce, $C' = kC = 6\times1.771\times10^{-11} = 106pF$

Supply voltage, V = 100 V

New charge $q^1 = C^1V = 6\times1.7717\times10^{-9} = 1.06\times10^{-8}C$ s

Potential across the plates remains 100 V.

2. Dielectric constant, k = 6

Initial capacitance , $C = 1.771\times10^{-11}F$

New capacitance $C^1 = kC = 6\times1.771\times10^{-11} = 106pF$

If supply voltage is removed, then there will be no effect on the amount of charge in the plates.

Charge = $= 1.771\times10^{-9}C$

Potential across the plates is given by,

$$\therefore V' = \frac{q}{C^1}$$

$$= \frac{1.771\times10^{-9}}{106\times10^{-12}}$$

$$= 16.7V$$

15. A 12 pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

Ans. Capacitor of the capacitance, $C = 12pF = 12 \times 10^{-12} F$

Potential difference, V = 50 V

Electrostatic energy stored in the capacitor is given by the relation,

$$E_B \frac{q_2}{4\pi \in_0 (BZ)^2}$$

$$\cos\theta = \frac{0.10}{0.18} = \frac{5}{9} = 0.5556$$

$$\theta = \cos^{-1}{}^{0} 0.5556 = 56.25$$

$$\therefore 2\theta = 112.5^0$$

$$\cos 2\theta = -038$$

$$\overrightarrow{E_2} - \overrightarrow{E_1} = \frac{\sigma}{2 \in_0} \hat{n} + \frac{\sigma}{2 \in_0} \hat{n} = \frac{\sigma}{\in_0} \hat{n}$$

$$E(2\pi dL) = \frac{\lambda L}{\in_0}$$

$$= \frac{q_1 q_2}{4\pi \in_0 d_1} - 27.2\, eV$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{1.06 \times 10^{-10}} - 27.2\, eV$$

$$= 21.73 \times 10^{-10} J - 27.2eV$$

$$= 13.58eV - 27.2eV$$

$= 13.6 eV$

$$E = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2$$

$$= 1.5 \times 10^{-8} J$$

Therefore, the electrostatic energy stored in the capacitor is $1.5 \times 10^{-8} J$.

---

16. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Ans. Capacitance of the capacitor, C = 600 pF Potential difference, V = 200 V

Electrostatic energy stored in the capacitor is given by,

$$E = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times (600 \times 10^{-12}) \times (200)^2$$

$$= 1.2 \times 10^{-5} J$$

If supply is disconnected from the capacitor and another capacitor of capacitance C = 600 pF is connected to it, then equivalent capacitance (C') of the combination is given by,

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C}$$

$$= \frac{1}{600} + \frac{1}{600} = \frac{2}{600} = \frac{1}{300}$$

$$\therefore C' = 300 pF$$

New electrostatic energy can be calculated as

$$E^1 = \frac{1}{2} \times C^1 \times V^2$$

$$= \frac{1}{2} \times 300 \times (200)^2$$

$$= 0.6 \times 10^{-5} J$$

Loss in electrostatic energy = $E - E'$

$$= 1.2 \times 10^{-5} - 0.6 \times 10^{-5}$$

$$= 0.6 \times 10^{-5}$$

$$= 6 \times 10^{-6} J$$

Therefore, the electrostatic energy lost in the process is $6 \times 10^{-6} J$.

---

17. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge

1. A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

2. Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

Ans. (a) Charge placed at the centre of a shell is +q. Hence, a charge of magnitude –q will be induced to the inner surface of the shell. Therefore, total charge on the inner surface of the shell is –q.

Surface charge density at the inner surface of the shell is given by the relation,

$$\sigma_1 = \frac{Total\ charge}{Outer\ surgece\ area} = \frac{-q}{4\pi r_1^2} \ldots (i)$$

$$\sigma_2 = \frac{Total\ charge}{Outer\ surgece\ area} = \frac{Q+q}{4\pi r_2^2} \ldots (ii)$$

A charge of +q is induced on the outer surface of the shell. A charge of magnitude Q is placed on the outer surface of the shell. Therefore, total charge on the outer surface of the shell is Q + q. Surface charge density at the outer surface of the shell,

1. Yes

The electric field intensity inside a cavity is zero, even if the shell is not spherical and has any irregular shape. Take a closed loop such that a part of it is inside the cavity

along a field line while the rest is inside the conductor. Net work done by the field in carrying a test charge over a closed loop is zero because the field inside the conductor is zero. Hence, electric field is zero, whatever is the shape.

---

18. If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H^+_2. In the ground state of an H^+_2, the two protons are separated by roughly 1.5 $\overset{0}{A}$, and the electron is roughly 1 $\overset{0}{A}$ from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Ans.

The system of two protons and one electron is represented in the given figure.

Charge on proton 1, $q_1 = 1.6 \times 10^{-19} C$

Charge on proton 2, $q_2 = 1.6 \times 10^{-19} C$

Charge on electron, $q_3 = -1.6 \times 10^{-19} C$

Distance between protons 1 and 2, $d_1 = 1.5 \times 10^{-10}\ m$

Distance between proton 1 and electron, $d_2 = 1 \times 10^{-10}\ m$

Distance between proton 2 and electron, $d_3 = 1 \times 10^{-10}\ m$

The potential energy at infinity is zero.

Potential energy of the system,

$$V = \frac{q_1 q_2}{4\pi \in_0 d_1} + \frac{q_2 q_3}{4\pi \in_0 d_3} + \frac{q_3 q_1}{4\pi \in_0 d_2}$$

Substituting $\dfrac{1}{4\pi \in_0 d} = 9 \times 10^9\ Nm^2C^{-2}{}_2$

$$V = \frac{9 \times 10^9 \times 10^{-19} \times 10^{-19}}{10^{-10}} \left[-(16)^2 + \frac{(1.6)^2}{1.5} + -(1.6)^2 \right]$$

$$= -30.7 \times 10^{-19} J$$

$$= -19.2 eV$$

Therefore, the potential energy of the system is $-19.2\ eV$.

---

19. Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Ans. Let a be the radius of a sphere A, QA be the charge on the sphere, and CA be the capacitance of the sphere. Let b be the radius of a sphere B, QB be the charge on the sphere, and CB be the capacitance of the sphere. Since the two spheres are connected with a wire, their potential (V) will become equal.

Let EAbe the electric field of sphere A and EB be the electric field of sphere B. Therefore, their ratio,

$$\frac{E_A}{E_B} = \frac{Q_A}{4\pi \in_0 \times a_2} \times \frac{b^2 4\pi \in_0}{Q_B}$$

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{b^2}{a^2} \quad ...(1)$$

However $\frac{Q_A}{Q_B} = \frac{C_A V}{C_B V}$

And $\frac{C_A}{C_B} = \frac{a}{b}$

$$\therefore \frac{Q_A}{Q_B} = \frac{q}{b} \quad ...(2)$$

Putting the value of (2) in (1), we obtain

$$\therefore \frac{E_A}{E_B} = \frac{a}{b}\frac{b^2}{a^2} = \frac{b}{a}$$

Therefore, the ratio of electric fields at the surface is $\frac{b}{a}$.

---

20. What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realize from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

Ans. Capacitance of a parallel capacitor, V = 2 F

Distance between the two plates, d = 0.5 cm = $0.5 \times 10^{-2}\ m$ Capacitance of a parallel plate capacitor is given by the relation,

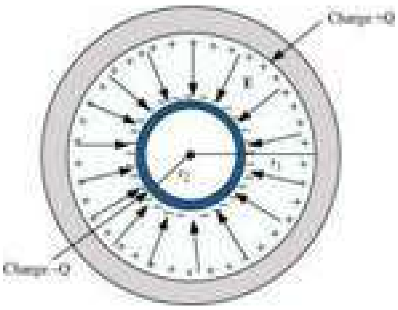
Where,

$\in_0$ = Permittivity of free space = $= 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$

$$\therefore A = \frac{2 \times 0.5 \times 10^{-2}}{8.85 \times 10^{-12}}$$
$$= 1130 km^2$$

Hence, the area of the plates is too large. To avoid this situation, the capacitance is taken in the range of μF .

21.A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig. 2.36).



Show that the capacitance of a spherical capacitor is given by

$C = \dfrac{4\pi \in_0 r_1 r_2}{r_1 - r_2}$ **where r_1 and r_2 are the radii of outer and inner spheres, respectively.**

Ans. Radius of the outer shell = r_1

Radius of the inner shell = r_2

The inner surface of the outer shell has charge +Q.

$$V = \frac{Q}{4\pi \in_0 r_2} - \frac{Q}{4\pi \in_0 r_1}$$

The outer surface of the inner shell has induced charge $-Q$. Potential difference between the two shells is given by,

Where,

$\in_0$ = Permittivity of free space

$$V = \frac{Q}{4\pi \in_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= \frac{Q(r_1 - r_2)}{4\pi \in_0 r_1 r_2}$$

Capacitance of the given system is given by,

$$C \frac{charge(Q)}{potential\ difference(V)}$$

$$= \frac{4\pi \in_0 r_1 r_2}{r_1 - r_2}$$

Hence, proved.

---

22. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC . Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

Ans. Length of a co-axial cylinder, l = 15 cm = 0.15 m

Radius of outer cylinder, r_1 = 1.5 cm = 0.015 m

Radius of inner cylinder, r_2 = 1.4 cm = 0.014 m

Charge on the inner cylinder, q = 3.5 $\mu C = 3.5 \times 10^{-6}$ C

Capacitance of a co-axil cylinder of radii r_1 and r_2 is given by the relation,

$$C = \frac{2\pi \in_0 1}{\log_2 \frac{r_1}{r_2}}$$

Where,

$\in_0$ = Permittivity of free space = $8.85\times10^{-12}N^{-1}m^{-2}C^2$

$$\therefore C=\frac{2\pi\times8.85\times10^{-12}\times0.15}{2.30261\log_{10}\left(\frac{0.15}{0.14}\right)}$$

$$=\frac{2\pi\times8.85\times10^{-12}\times0.15}{2.3026\times0.0299}=1.2\times10^{-10}F$$

Potential difference of the inner cylinder is given by,

$$V=\frac{q}{C}$$

$$=\frac{3.5\times10-6}{1.2\times10^{-10}}=2.92\times10^4V$$

---

23. A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about $10^7\ \mathrm{Vm}^{-1}$. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

Ans. Potential rating of a parallel plate capacitor, V = 1 kV = 1000 V

Dielectric constant of a material, $\in_r$ =3 Dielectric strength = $10^7\ \mathrm{Vm}$

For safety, the field intensity never exceeds 10% of the dielectric strength. Hence, electric field intensity, E = 10% of 10^7 = $10^6V/m$

Capacitance of the parallel plate capacitor, C = 50 pF = $50\times10^{-12}F$

Distance between the plates is given by,

$$d = \frac{V}{E}$$

$$= \frac{1000}{10^6} = 10^{-3}\, m$$

Capacitance is given by the relation,

$$C = \frac{\epsilon_0 \epsilon_1\, A}{d}$$

Where,

A = Area of each plate

ϵ_0 = Permittivity of free space = $8.85 \times 10^{-12} N^{-1} C^2 m^{-2}$

$$\therefore A = \frac{CD}{\epsilon_0 \epsilon_r}$$

$$= \frac{50 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 3} \approx 19 cm^2$$

Hence, the area of each plate is about 19 cm^2 .

5 Mark Questions

1. (a) Define dielectric constant in terms of the capacitance of a capacitor? On what factor does the capacitance of a parallel plate capacitor with dielectric depend?

(b) Find the ratio of the potential differences that must be applied across the

(1) parallel

(2) Series combination of two identical capacitors so that the energy stored in the two cases becomes the same.

Ans. (a) Dielectric constant is defined as the ratio of capacitance of a capacitor when the dielectric is filled in between the plates to the capacitance of a capacitor when there is vaccuum in between the plates

In K = $\frac{Cm}{Co}\frac{\text{Capacitance of a capcitor when dielectric is in between the plates}}{\text{Capacitance of a capcitor with vaccuum in between the plates.}}$

Capacitance of a parallel plate capacitor with dielectric depends on the following factors

$$C_m = \frac{KA\in_o}{d}$$

(1) Area of the plates

(2) Distance between the plates

(3) Dielectric constant of the dielectric between the plates

(b) Let the capacitance of each capacitor be C

Cp = C + C = 2C (in parallel)

$C_S = \frac{C\times C}{C+C} = \frac{C}{2}$ (in series)

Let $Vp \ and \ V_S$ be the values of potential difference

This $Up = \frac{1}{2}CpVp^2 = \frac{1}{2} \times 2C \times Vp^2 = CVp^2$

$$Us = \frac{1}{2}CsVs^2 = \frac{1}{2} \times \frac{C}{2} \times Vs^2 = \frac{CVs^2}{4}$$

But $U_P = U_S$ (given)

$$CVp^2 = \frac{CVs^2}{4}$$

$$\frac{Vp^2}{Vs^2} = 1/4$$

$\Rightarrow$ Vp : Vs = 1 : 2

---

2. (a) An air filled capacitor is given a charge of 2 μ C raising its potential to 200 V. If on inserting a dielectric medium, its potential falls to 50 V, what is the dielectric constant of the medium?

(b) A conducting stab of thickness 't' is introduced without touching between the

plates of a parallel plate capacitor separated by a distance d (t<d). Derive an

expression for the capacitance of a capacitor?

Ans. (a) $K = \frac{V}{V^1} = \frac{200}{50} = 4$ (Where V = 200 V for air filled capacitor $V^1 = 50$ after insertion of a dielectric)

(b) For a parallel plate capacitor when air/vaccuum is in between the plates $C_o = \frac{A \in^o}{d}$

Since electric field inside a conducting stab is zero, hence electric field exist only between the space (d-t) $\Rightarrow V = E_0(d-t)$

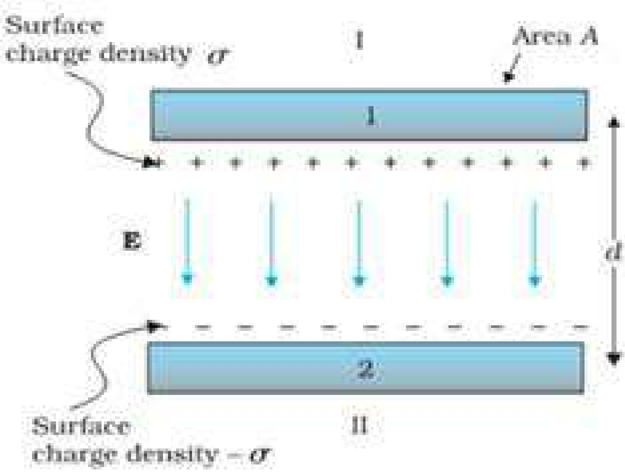
Where E_o is the electric field exist between the plates?

And $E_o = \dfrac{\sigma}{\in_o} = \dfrac{q}{A \in_o}$

Where A is the area of each plates

$$\Rightarrow V = \frac{q}{A \in_o}(d-t)$$

Hence capacitor of a parallel plate capacitor



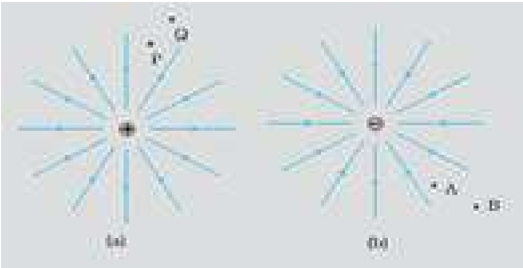
$$C = \frac{q}{V} = \frac{q}{q(d-t)} A \in_o$$

$$C = \frac{A \in_o}{d-t}$$

$$C = \frac{A \in_o}{d(1-t/d)}$$

$$C = \frac{C_o}{\left(1-\dfrac{t}{d}\right)}$$

3. Figure (a) and (b) shows the field lines of a single positive and negative changes respectively



(a) Give the signs of the potential difference: $Vp - Vq$ and V_B -- V_A

(b) Give the sign of the work done by the field in moving a small positive change from Q to P.

(c) Give the sign of the work done by the field in moving a small negative change from B to A.

Ans. (a) We know $V \;\alpha\; \frac{1}{r}$

$Vp > Vq \Rightarrow Vp\text{-- }Vq\text{-- Positive}$

$VA < V_B \Rightarrow V_B\text{--}V_A\text{-- Positive}$

Because V_B is less negative then V_A.

(b) In moving a positive change form Q to P work has to be done against the electric field so it is negative.

(c) In moving a negative change form B to A work is done along the same direction of the field so it is positive.

---

4. With the help of a labelled diagram, explain the principle, construction and working of a vandegraff generator. Mention its applications?

Ans. Vandegraff generator is a device which is capable of producing a high potential of the order of million volts.

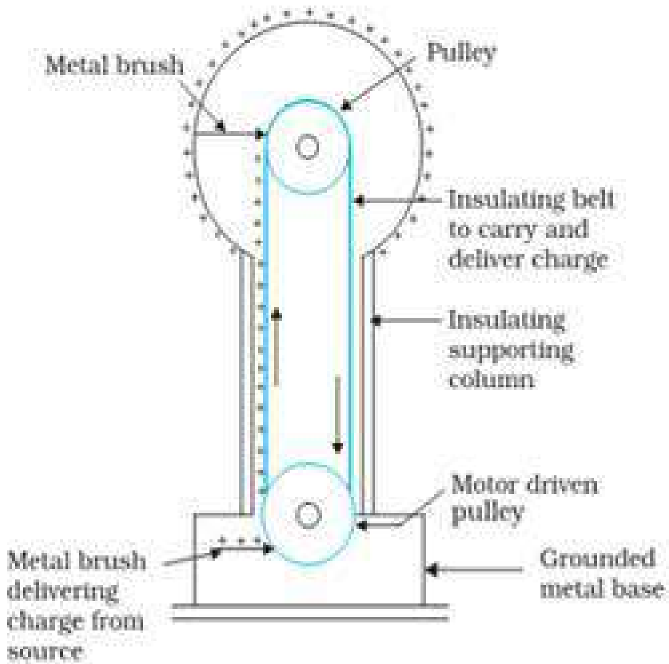
Principle (1) The charge always resides on the outer surface of hollow conductor.

(2) The electric discharge in air or gas takes place readily at the pointed ends of the conductors.

Construction: - It consist of a large hollow metallic sphere S mounted on two insulating columns A and B and an endless belt made up of rubber which is running over two pulleys $P_1 a$ with the help of an electric motor. $B_1 and\ B_2$ are two sharp metallic brushes. The lower brush B_1 is given a positive potential by high tension battery and is called a spray brush while the upper brush B_2 connected to the inner part of the Sphere S.

Working: - When brush B_1 is given a high positive potential then it produces ions, due to the action of sharp points. Thus the positive ions so produced get sprayed on the belt due to repulsion between positive ions and the positive charge on brush B_1. Then it is carried upward by the moving belt. The pointed end of B_2 just touches the belt collects the positive change and make it move to the outer surface of the sphere S. This process continues and the potential of the shell rises to several million volts.

Applications – Particles like proton, Deutrons, α -- particles etc are accelerated to high speeds and energies.



5. Two charges $5\times10^{-8}C$ and $-3\times10^{-8}C$ are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

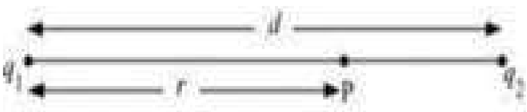
Ans. There are two charges,

$q_1 = 5\times10^{-8}C$

$q_2 = -3\times10^{-8}C$

Distance between the two charges, d = 16 cm = 0.16 m

Consider a point P on the line joining the two charges, as shown in the given figure.



r = Distance of point P from charge q_1

Let the electric potential (V) at point P be zero.

Potential at point P is the sum of potentials caused by charges q1 and q2 respectively.

$$\therefore V = \frac{q_1}{4\pi\in_0 r} + \frac{q2}{4\pi\in_0 (d-r)} \quad ...(i)$$

Where,

$\in_0$ = Permittivity of free space

For V = 0, equation (i) reduces to

$$\frac{q_1}{4\pi\in_0 r} = -\frac{q2}{4\pi\in_0 (d-r)}$$

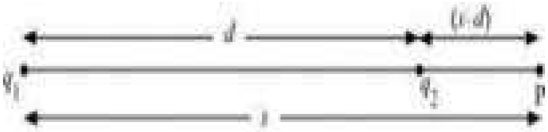
$$\frac{q_1}{r} = \frac{-q_2}{d-r}$$

$$\frac{5\times10^{-8}}{r} = -\frac{(-3\times10^{-8})}{(0.16-r)}$$

$$\frac{0.16}{r} = \frac{8}{5}$$

$$\therefore r = 0.1m = 10cm$$

Therefore, the potential is zero at a distance of 10 cm from the positive charge between the charges.



Suppose point P is outside the system of two charges at a distance s from the negative charge, where potential is zero, as shown in the following figure.

For this arrangement, potential is given by,

$$V = \frac{q_1}{4\pi\in_0 s} + \frac{q_2}{4\pi\in_0 (s-d)} \qquad ...(ii)$$

For V = 0, equation (ii) reduces to

$$\frac{q_1}{4\pi\in_0 s} = -\frac{q_2}{4\pi\in_0 (s-d)}$$

$$\frac{q_1}{s} = \frac{-q_2}{s-d}$$

$$\frac{5\times10^{-8}}{s} = -\frac{(-3\times10^{-8})}{(s-0.16)}$$

$$1-\frac{0.16}{s} = \frac{3}{5}$$

$$\frac{0.16}{s} = \frac{2}{5}$$

$\therefore s = 0.4m = 40cm$

Therefore, the potential is zero at a distance of 40 cm from the positive charge outside the system of charges.

---

6. A parallel plate capacitor with air between the plates has a capacitance of 8 pF ($1pF = 10^{-12} F)$. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Ans. Capacitance between the parallel plates of the capacitor, C = 8 pF

Initially, distance between the parallel plates was d and it was filled with air. Dielectric constant of air, k = 1

Capacitance, C, is given by the formula,

$$C = \frac{k \epsilon_0 A}{d}$$

$$= \frac{\epsilon_0 A}{d} \qquad ...(i)$$

Where,

A = Area of each plate

ϵ_0 = Permittivity of free space

If distance between the plates is reduced to half, then new distance, $d^{-TM} = \frac{d}{2}$ Dielectric constant of the substance filled in between the plates, k' = 6 Hence, capacitance of the capacitor becomes

$$C^1 = \frac{k^1 \epsilon_0 A}{d} = \frac{6 \epsilon_0 A}{\frac{d}{2}}$$ ....(ii)

Taking ratios of equations (i) and (ii), we obtain

$$C' = 2\times 6C$$

$$= 12C$$

$$= 12\times 8 = 96pF$$

Therefore, the capacitance between the plates is 96 pF.

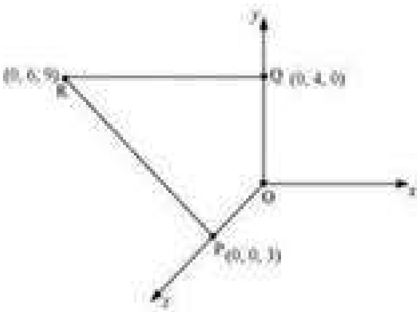
---

7. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of $-2\times 10^{-9}C$ from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

Ans. Charge located at the origin, q = 8 mC $= 8\times 10^{-3}C$

Magnitude of a small charge, which is taken from a point P to point R to point Q, $q_1 = -2\times 10^{-9}C$

All the points are represented in the given figure.



Point P is at a distance, $d_1 = 3\ cm$, from the origin along z-axis. Point Q is at a distance, $d_2 = 4\ cm$, from the origin along y-axis.

Potential at point P, $V_1 = \dfrac{q}{4\pi \in_0 \times d_2}$

Potential at point Q, $V_2 = \dfrac{q}{4\pi \in_0 \times d_2}$

Work done (W) by the electrostatic force is independent of the path.

$$\therefore W = q_1 [V_2 - V_1]$$

$$= q_1 \left[\frac{q}{4\pi \in_0 d_2} - \frac{q}{4\pi \in_0 d_2} \right]$$

$$= \frac{qq_1}{4\pi \in_0} \left[\frac{1}{d_2} - \frac{1}{d_1} \right] \qquad ...(i)$$

$$where, \frac{1}{4\pi \in_0} = 9 \times 10^9 \, N m^2 C^{-2}$$

$$\therefore W = 9 \times 10^9 \times 8 \times 10^{-3} \times (-2 \times 10^{-9}) \left[\frac{1}{0.04} - \frac{1}{0.03} \right]$$

$$= -144 \times 10^{-3} \times \left(\frac{-25}{3} \right)$$

$$= 1.27 \, J$$

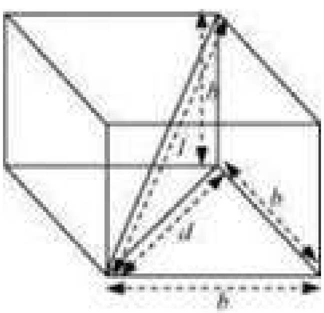
Therefore, work done during the process is 1.27 J.

---

8. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Ans.Length of the side of a cube = b

Charge at each of its vertices = q



A cube of side b is shown in the following figure.

d = Diagonal of one of the six faces of the cube

$$d^2 = \sqrt{b^2 + b^2} = \sqrt{2b^2}$$

$$d = b\sqrt{2}$$

l = Length of the diagonal of the cube

$$l^2 = \sqrt{d^2 + b^2}$$

$$= \sqrt{\left(\sqrt{2b}\right)^2 + b^2}$$

$$= \sqrt{2b^2 + b^2} = \sqrt{3b^2}$$

$$l = b\sqrt{3}$$

$$r = \frac{l}{2} = \frac{b\sqrt{3}}{2}$$

Is the difference between the centre of the cube and one of the eight vertices

The electric potential (V) at the centre of the cube is due to the presence of eight charges at the vertices.

$$V = \frac{8q}{4\pi \in_0}$$

$$= \frac{8q}{4\pi \in_0 \left(b\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{4q}{\sqrt{3}\pi \in_0 b}$$

$$\frac{4q}{\sqrt{3}\pi \epsilon_0 b}$$

Therefore, the potential at the centre of the cube is .

The electric field at the centre of the cube, due to the eight charges, gets cancelled. This is because the charges are distributed symmetrically with respect to the centre of the cube. Hence, the electric field is zero at the centre.

---

9. Two tiny spheres carrying charges 1.5 μC and $2.5\mu C$ are located 30 cm apart. Find the potential and electric field:

1. at the mid-point of the line joining the two charges, and

2. at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

Ans.

Two charges placed at points A and B are represented in the given figure. O is the mid- point of the line joining the two charges.

Magnitude of charge located at A, q1 = 1. $5\mu C$

Magnitude of charge located at B, q2 = 2. $5\mu C$

Distance between the two charges, d = 30 cm = 0.3 m

1. Let V_1 and E_1 are the electric potential and electric field respectively at O.

V_1 = Potential due to charge at A + Potential due to charge at B

$$V_1 = \frac{q_1}{4\pi \epsilon_0 \left(\frac{d}{2}\right)} + \frac{q_2}{4\pi \epsilon_0 \left(\frac{d}{2}\right)} = \frac{1}{4\pi \epsilon_0 \left(\frac{d}{2}\right)}(q_1 + q_2)$$

Where,

$\in_0$ = Permittivity of free space

$$\frac{1}{4\pi\in_0} = 9\times10^9 NC^2m^{-2}$$

$$\therefore v_1 = \frac{9\times10^9\times10^{-6}}{\left(\frac{0.30}{2}\right)}(2.5+1.5)$$

$$= 2.4\times10^5 V$$

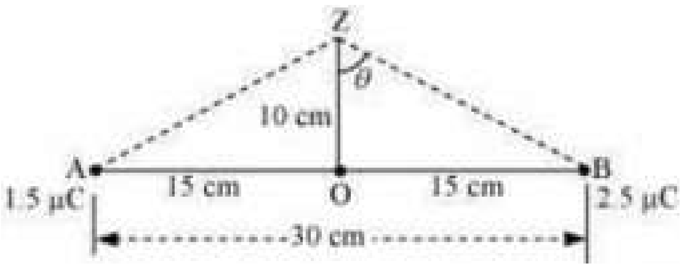
E_1 = Electric field due to q_2 *Electric field due to* q_1

$$= \frac{q_2}{4\pi\in_0\left(\frac{d}{2}\right)^2} - \frac{q_1}{4\pi\in_0\left(\frac{d}{2}\right)^2}$$

$$= \frac{9\times10^9}{\left(\frac{0.30}{2}\right)^2}\times10^6\times(2.5-1.5)$$

$$= 4\times10^5 Vm^{-1}$$

Therefore, the potential at mid-point is $2.4\times 10^5 V$ and the electric field at mid-point is $4\times10^5 V\ m^{-1}$. The field is directed from the larger charge to the smaller charge.



1. Consider a point Z such that normal distance OZ = 10 cm = 0.1 m, as shown in the following figure.

V_2 *and* E_2 are the electric potential and electric field respectively at Z. It can be observed from the figure that distance,

BZ=AZ= $\sqrt{(0.1)^2+(0.15)^2}=0.18m$

V_2 = Electric potential due to A + Electric Potential due to B

$$=\frac{q_1}{4\pi\in_0(AZ)}+\frac{q_1}{4\pi\in_0(BZ)}$$

$$=\frac{9\times10^9\times10^{-6}}{0.18}(1.5+2.5)$$

$$=2\times10^5V$$

Electric field due to q at Z,

$$E_A\frac{q_1}{4\pi\in_0(AZ)}$$

$$=\frac{9\times10^9\times1.5\times10^{-6}}{0.18}$$

$$=0.416\times10^6V/m$$

Electric field due to q2 at Z,

$$E_B\frac{q_2}{4\pi\in_0(BZ)^2}$$

$$=\frac{9\times10^9\times2.5\times10^{-6}}{(0.18)^2}$$

$$=0.69\times10^6Vm^{-1}$$

The resultant field intensity at Z,

$$E = \sqrt{E^2{}_A + E_B{}^2 + 2E_A E_B \cos 2\theta}$$

Where, 2θ is the angle, $\angle AZB$

From the figure, we obtain

$$\cos\theta = \frac{0.10}{0.18} = \frac{5}{9} = 0.5556$$

$$\theta = \cos^{-1}{}^{0}0.5556 = 56.25$$

$$\therefore 2\theta = 112.5^0$$

$$\cos 2\theta = -038$$

$$E\sqrt{(0.416\times10^{6})^2 \times 2\times0.416\times0.69\times10^{12}\times(-0.38)}$$

$$= 6.6\times10^5\, Vm^{-4}$$

Therefore, the potential at a point 10 cm (perpendicular to the mid-point) is $2.0\times10^5\, V$ and electric field is

$$= 6.6\times10^5\, Vm^{-4}\cdot$$

---

10. Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by $\left(\overline{E_2} - \overline{E}_1\right).\hat{n} = \frac{\sigma}{\epsilon_0}$ Where $\hat{n}$ is a unit vector normal to the surface at a point and Ïƒ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is $\hat{n} / \epsilon_0$

1. Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [Hint: For (a), use Gauss's law. For, (b) use the fact that work done by electrostatic field on a closed loop is zero.]

Ans. Electric field on one side of a charged body is E_1 and electric field on the other side of the same body is E_2. If infinite plane charged body has a uniform thickness, then electric field due to one surface of the charged body is given by,

$$\overline{E}_1 = -\frac{\sigma}{2\epsilon_0}\hat{n} \ldots\ldots\ldots\ldots\ldots(i)$$

Where,

$\hat{n}$ = Unit vector normal to the surface at a point Ïƒ = Surface charge density at that point

Electric field due to the other surface of the charged body,

$$\overrightarrow{E_2} = -\frac{\sigma}{2\epsilon_0}\hat{n} \qquad \ldots(ii)$$

Electric field at any point due to the two surfaces,

$$\overrightarrow{E_2} - \overrightarrow{E_1} = \frac{\sigma}{2\epsilon_0}\hat{n} + \frac{\sigma}{2\epsilon_0}\hat{n} = \frac{\sigma}{\epsilon_0}\hat{n}$$

$$(\overrightarrow{E_2} - \overrightarrow{E_1}).\hat{n} = \frac{\sigma}{\epsilon_0} \qquad \ldots(iii)$$

$$\overrightarrow{E_1}$$

Since inside a closed conductor, $\overline{E_1}$ = 0,

$$\therefore \overline{E} = \overline{E}_2 = -\frac{\sigma}{2\epsilon_0}\hat{n}$$

$$\frac{\sigma}{\epsilon_0}\hat{n}$$

Therefore, the electric field just outside the conductor is.

1. When a charged particle is moved from one point to the other on a closed loop, the work done by the electrostatic field is zero. Hence, the tangential component of electrostatic field is continuous from one side of a charged surface to the other.

11. A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Ans. Charge density of the long charged cylinder of length L and radius r is Î».

Another cylinder of same length surrounds the pervious cylinder. The radius of this cylinder is R.

Let E be the electric field produced in the space between the two cylinders.

Electric flux through the Gaussian surface is given by Gauss's theorem as,

$$\phi = E\left(2\pi d\right)L$$

Where, d = Distance of a point from the common axis of the cylinders Let q be the total charge on the cylinder.

It can be written as

$$\therefore \phi = E(2\pi dL) = \frac{q}{\epsilon_0}$$

Where,

q = Charge on the inner sphere of the outer cylinder

ϵ_0 = Permittivity of free space

$$E(2\pi dL) = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 d}$$

$$= \frac{\lambda}{2\pi \epsilon_0 d}$$

Therefore, the electric field in the space between the two cylinders is .

12.In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 $\overset{0}{A}$.

1. Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

2. What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

3. What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 $\overset{0}{A}$ separation?

Ans. The distance between electron-proton of a hydrogen atom, $d = 0.53 A$

Charge on an electron, $q_1 = 1.6 \times 10^{-19} C$

Charge on a proton, $q_2 = +1.6 \times 10^{-19} C$

1. Potential at infinity is zero.

$$= 0 - \frac{q_1 q_2}{4\pi \in_0 d}$$

Potential energy of the system, p-e = Potential energy at infinity -Potential energy at distance, d

Where,

$\in_0$ is the permittivity of free space

$$\frac{1}{4\pi \in_0} = 9 \times 10^9 Nm^2C^{-2}$$

$$= 0 - \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{0.53 \times 10^{10}}$$

$\therefore$ Potential energygy $= -43.7\times10^{-19}J$

Since $1.6\times10^{-19}J = 1eV$,

$= -43.7\times10^{-19}$

$\therefore$ Potential energy $= \dfrac{-43.7\times10^{-19}}{1.6\times10^{-19}}$

$= -27.2eV$

Therefore, the potential energy of the system is $-27.2eV$.

2. Kinetic energy is half of the magnitude of potential energy.

Kinetic energy $= \dfrac{1}{2}\times(-27.2) = 13.6\,eV$

Total energy = $13.6\times\ \ 27.2$ = 13.6 eV

Therefore, the minimum work required to free the electron is 13.6 eV.

3. When zero of potential energy is taken, $d_1 = 1.06A$

$\therefore$ Potential energy of the system = Potential energy at d_1 – Potential energy at d

$= \dfrac{q_1q_2}{4\pi\in_0 d_1} - 27.2eV$

$= \dfrac{9\times10^9\times(1.6\times10^{-19})^2}{1.06\times10^{-10}} - 27.2\,eV$

$= 21.73\times10^{-10}J - 27.2eV$

$= 13.58eV - 27.2eV$

$= 13.6eV$

---

13. Two charges $-\mathbf{q}$ and $+\mathbf{q}$ are located at points $(0,\ 0,\ \text{-a})$ and $(0,\ 0,\ \text{a})$,

respectively.

1. What is the electrostatic potential at the points?

2. Obtain the dependence of potential on the distance r of a point from the origin when r/a >> 1.

3. How much work is done in moving a small test charge from the point (5, 0, 0) to $(-7, 0, 0)$ along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

Ans. (1) Zero at both the points

Charge $-q$ *is located at* $(0, 0, -a)$ and charge + q is located at (0, 0, a). Hence, they form a dipole. Point (0, 0, z) is on the axis of this dipole and point (x, y, 0) is normal to the axis of the dipole. Hence, electrostatic potential at point (x, y, 0) is zero. Electrostatic potential at point (0, 0, z) is given by,

$$V = \frac{1}{4\pi \epsilon_0}\left(\frac{q}{z-a}\right) + \frac{1}{4\pi \epsilon_0}$$

$$= \frac{q(z+a-z+a)}{4\pi \epsilon_0 (z^2 - a^2)}$$

$$= \frac{2qa}{4\pi \epsilon_0 (z^2 - a^2)}$$

$$= \frac{p}{4\pi \epsilon_0 (z^2 - a^2)}$$

Where,

ϵ_0 = Permittivity of free space

p = Dipole moment of the system of two charges = 2qa

2. Distance r is much greater than half of the distance between the two charges. Hence, the potential (V) at a distance r is inversely proportional to square of the distance

i.e., $4\alpha\frac{1}{r^2}$

3. Zero

The answer does not change if the path of the test is not along the x-axis.

A test charge is moved from point (5, 0, 0) to point (-7, 0, 0) along the x-axis. Electrostatic potential (V_1) at point (5, 0, 0) is given by,

$$V_1 = \frac{-q}{4\pi \in_0}\frac{1}{\sqrt{(5-0)^2+(-a)^2}} + \frac{q}{4\pi \in_0}\frac{1}{(5-0)^2+a^2}$$

$$= \frac{-q}{4\pi \in_0 \sqrt{25^2+a^2}} + \frac{q}{4\pi \in_0 \sqrt{25+a^2}}$$

$$= 0$$

Electrostatic potential, V_2, at point (- 7, 0, 0) is given by,

$$V_2 = \frac{-q}{4\pi \in_0}\frac{1}{\sqrt{(-7)^2+(-a)^2}} + \frac{q}{4\pi \in_0}\frac{1}{\sqrt{(-7)^2+(a)^2}}$$

$$= \frac{-q}{4\pi \in_0 \sqrt{49+a^2}} + \frac{q}{4\pi \in_0}\frac{1}{\sqrt{49+a^2}}$$

$$= 0$$

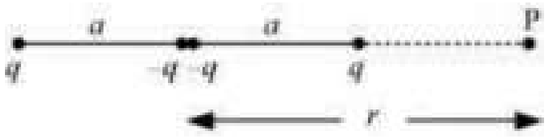
Hence, no work is done in moving a small test charge from point (5, 0, 0) to point (âˆ’7, 0, 0) along the x-axis.

The answer does not change because work done by the electrostatic field in moving a test charge between the two points is independent of the path connecting the two points.

---

14. Figure 2.34 shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on r for r/a >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e.,

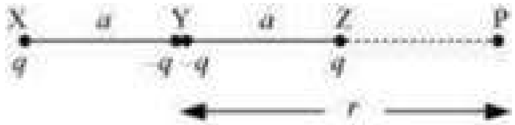
a single charge).



Ans. Four charges of same magnitude are placed at points X, Y, Y, and Z respectively, as shown in the following figure.

A point is located at P, which is r distance away from point Y. The system of charges forms an electric quadrupole.

It can be considered that the system of the electric quadrupole has three charges.



Charge +q placed at point X

Charge $-2q$ placed at point Y

Charge +q placed at point Z

XY = YZ = a

YP = r

PX = r + a

PZ = r - a

Electrostatic potential caused by the system of three charges at point P is given by,

$$V = \frac{1}{4\pi \in_0}\left[\frac{q}{XP} - \frac{2q}{YP} + \frac{q}{ZP}\right]$$

$$= \frac{1}{4\pi \in_0}\left[\frac{q}{r+a} - \frac{2q}{r} + \frac{q}{r-a}\right]$$

$$= \frac{q}{4\pi \in_0}\left[\frac{r(r-a) - 2(r+a)(r-a) + r(r+a)}{r(r+a)(r-a)}\right]$$

$$= \frac{q}{4\pi \in_0}\left[\frac{r^2 - ra - 2r^2 + 2a^2 + r^2 + ra}{r(r^2 - a^2)}\right]$$

$$= \frac{q}{4\pi \in_0}\left[\frac{2a^2}{r(r^2 - a^2)}\right]$$

$$= \frac{2qa^2}{4\pi \in_0 r^3\left(1 - \frac{a^2}{r^2}\right)}$$

Since $\frac{r}{a} >> 1$,

$$\therefore \frac{r}{a} >> 1$$

$\frac{r^2}{a^2}$ is taken as negligible.

$$\therefore V = \frac{2qa^2}{4\pi \in_0 r^3}$$

It can be inferred that potential, $V \propto \frac{1}{r^3}$

However, it is known that for a dipole, $V \propto \frac{1}{r^2}$

And, for a monopole, $V \propto \frac{1}{r}$

---

15. An electrical technician requires a capacitance of $2\mu F$ in a circuit across a potential difference of 1 kV. A large number of 1 2μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Ans. Total required capacitance, C = $2\mu F$ Potential difference, V = 1 kV = 1000 V

Capacitance of each capacitor, C_1 = $1\mu F$

Each capacitor can withstand a potential difference, V_1 = 400 V

$$\frac{1000}{400}=2.5$$

Suppose a number of capacitors are connected in series and these series circuits are connected in parallel (row) to each other. The potential difference across each row must be 1000 V and potential difference across each capacitor must be 400 V. Hence, number of capacitors in each row is given as

Hence, there are three capacitors in each row. Capacitance of each row $=\frac{1}{1+1+1}=\frac{1}{3}\,uF$

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$+.......n terms

$$=\frac{n}{3}$$

However, capacitance of the ciruit is given as 2 μF .

$$\therefore \frac{n}{3}=2$$

N=6

Let there are n rows, each having three capacitors, which are connected in parallel. Hence, equivalent capacitance of the circuit is given as

Hence, 6 rows of three capacitors are present in the circuit. A minimum of 6×3 i.e., 18 capacitors are required for the given arrangement.

16.Obtain the equivalent capacitance of the network in Fig. 2.35. For a 300 V supply, determine the charge and voltage across each capacitor.

Ans. Capacitance of capacitor C_1 is 100 pF.

Capacitance of capacitor C_2 is 200 pF.

Capacitance of capacitor C_3 is 200 pF.

Capacitance of capacitor C_4 is 100 pF.

Supply potential, V = 300 V

Capacitors C_2 and C_3 are connected in series. Let their equivalent capacitance be C^1

$$\therefore \frac{1}{C^1} = \frac{1}{200} + \frac{1}{200} = \frac{2}{200}$$

C^1 =100pF

Capacitors $C_1 and\ C'$ are in parallel. Let their equivalent capacitance be C^n

$$\therefore C^n = C^1 + C_1$$

=100+100=200pF

$C^n and C_4$ are connected in series. Let their equivalent capacitance be C.

$$\therefore \frac{1}{C} = \frac{1}{C^n} + \frac{1}{C_4}$$

$$= \frac{1}{200} + \frac{1}{100} = \frac{2+1}{200}$$

$$C = \frac{200}{3} pF$$

Hence, the equivalent capacitance of the circuit is $\frac{200}{3} pF$ Potential difference across

$C'' = V''$

Potential difference across C_4 = V_4

$$\therefore V^5 + V_4 = V = 300V$$

Charge on C_1 is given by,

Q_4 = CV

$$= \frac{200}{3} \times 10^{-12} \times 300$$

$$= 2 \times 10^{-8} C$$

$$\therefore V_4 = \frac{Q_4}{C_4}$$

$$= \frac{2 \times 10^{-8}}{100 \times 10^{-12}} = 200V$$

$\therefore$ voltage across C_1 is given by,

$$V_1 = V - V_4$$

=300-200=100V

$$Q_1 = C_1 V_1$$

$$= 100 \times 10^{-12} \times 100$$

$$= 10^{-8}$$

Hence, potential difference, V_1, across C_1 is 100 V. Charge on C_1 is given by,

C_2 and C_3 having same capacitances have a potential difference of 100 V together. Since

C_2 and C_3 are in series, the potential difference across C_2 and C_3 is given by,

V_2 = V_3 = 50 V

Therefore, charge on C_2 is given by,

$$Q_2 = C_2 V_2$$

$= 200 \times 10^{-12} \times 50$

$= 10^{-8} C$

And charge on C_3 is given by,

$Q_3 = C_3 V_3$

$= 200 \times 10^{-12} \times 50$

$= 10^{-8} C$

$\frac{200}{3}$ pF with

Hence, the equivalent capacitance of the given circuit is

$Q_1 = 10^{-8} C, V_1 = 100V$

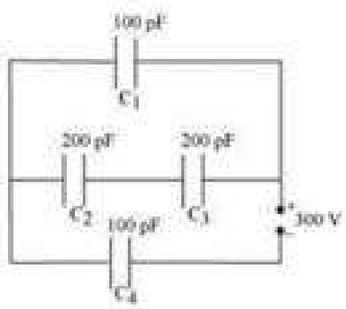
$Q_2 = 10^{-8} C, V_2 = 50V$

$Q_3 = 10^{-8} C, V_3 = 50V$

$Q_4 = 2 \times 10^{-8} C \ V_4 = 200V$

---

17. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.



1. How much electrostatic energy is stored by the capacitor?

2. View this energy as stored in the electrostatic field between the plates, and obtain the

energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

Ans. Area of the plates of a parallel plate capacitor, A = 90 cm^2 = $90\times10^{-4}\ m^2$

Distance between the plates, d = 2.5 mm = $2.5\times10^{-3}\ m$

Potential difference across the plates, V = 400 V

1. Capacitance of the capacitor is given by the relation,

$$C=\frac{\epsilon_0 A}{d}$$

Electrostatic energy stored in the capacitor is given by the relation, $E_1=\frac{1}{2}CV^2$

$$=\frac{1}{2}\frac{\epsilon_0 A}{d}V^2$$

Where,

ϵ_0 = Permittivity of free space = $8.85\times10^{-12}C^2N^{-1}m^{-2}$

$$\therefore E_1=\frac{1\times8.85\times10^{-12}\times90\times10^{-4}\times(400)^2}{2\times2.5\times10^{-3}}$$

$$=2.55\times10^{-6}J$$

Hence, the electrostatic energy stored by the capacitor is $2.55\times10^{-6}J$

2. Volume of the given capacitor,

$$V^1=A\times d$$

$$=90\times10^{-4}\times25\times10^{-3}$$

$$=2.25\times10^{-4}m^3$$

Energy stored in the capacitor per unit volume is given by,

$$u = \frac{E_1}{V^1}$$

$$= \frac{2.55 \times 10^{-6}}{2.25 \times 10^{-4}} = 0.113 Jm^{-3}$$

Again, u= $\frac{E_1}{V^1}$

Where,

$\frac{V}{d}$ = Electric intensity = E

$$\therefore u = \frac{1}{2} \in_0 E^2$$

---

18. A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

Ans. Capacitance of a charged capacitor, $C_1 = 4 \mu F = 4 \times 10^{-6} F$ Supply voltage, V_1 = 200 V Electrostatic energy stored in C_1 is given by,

$$E_1 = \frac{1}{2} C_1 V_1^2$$

$$= \frac{1}{2} \times 4 \times 10^{-6} \times (200)^2$$

$$= 8 \times 10^{-2} J$$

$$C_2 = 2uF = 2 \times 10^{-6} \text{F}$$

Capacitance of an uncharged capacitor,

When C_2 is connected to the circuit, the potential acquired by it is V_2.

$$\therefore V_2(C_1+C_2)=C_1V_1$$

$$V_2\times(4+2)\times10^{-6}=4\times10^{-6}\times200$$

$$V_2=\frac{400}{3}V$$

According to the conservation of charge, initial charge on capacitor C_1 is equal to the final charge on capacitors, C_1 and C_2.

Electrostatic energy for the combination of two capacitors is given by,

$$E_2=\frac{1}{2}(C_1+C_2)V_2^2$$

$$=\frac{1}{2}(2+4)\times10^{-6}\times\left(\frac{400}{3}\right)^2$$

$$=5.33\times10^{-2}J$$

Hence, amount of electrostatic energy lost by capacitor C_1

---

19. Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $\left(\frac{1}{2}\right)$ QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor $\frac{1}{2}$.

Ans. Let F be the force applied to separate the plates of a parallel plate capacitor by a distance of $\Delta\times$. Hence, work done by the force to do so $= F\Delta\times$

As a result, the potential energy of the capacitor increases by an amount given as $uA\Delta\times$.

Where,

u = Energy density

A = Area of each plate

d = Distance between the plates

V = Potential difference across the plates

The work done will be equal to the increase in the potential energy i.e.,

$$F \triangle x = uA \triangle x$$

$$F = uA = \left(\frac{1}{2} \epsilon_0 E^2\right) A$$

Electric intensity is given by,

$$E = \frac{V}{d}$$

$$\therefore F = \frac{1}{2} \epsilon_0 \left(\frac{V}{d}\right) EA = \frac{1}{2}\left(\epsilon_0 A \frac{V}{d}\right) E$$

However, capacitance, $C = \frac{\epsilon_0 A}{d}$

$$\therefore F = \frac{1}{2}(CV)E$$

Charge on the capacitor is given by,

Q = CV

$$\therefore F = \frac{1}{2} QE$$

The physical origin of the factor, $\frac{1}{2}$, in the force formula lies in the fact that just outside the conductor, field is E and inside it is zero. Hence, it is the average value, $\frac{E}{2}$, of the field that contributes to the force.

20. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 µC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

1. Determine the capacitance of the capacitor.

2. What is the potential of the inner sphere?

3. Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Ans. Radius of the inner sphere, r_2 = 12 cm = 0.12 m Radius of the outer sphere, r_1 = 13 cm = 0.13 m Charge on the inner sphere, $q = 2.5\mu C = 2.5\times10^{-6}$

Dielectric constant of a liquid, $\in_r = 32$

(a) Capacitance of the capacitor is given by the relatioon,

$$C = \frac{4\pi \in_0 r_1 r_2}{r_1 - r_2}$$

Where,

$\in_0$ = Permittivity of free space = $8.85\times10^{-12} C^2 N^{-1} m^{-2}$

$$V = \frac{1}{4\pi \in_0} 9\times10^9 Nm^2C^{-2}$$

$$\therefore C = \frac{32\times0.12\times0.13}{9\times10^9\times(0.13-.12)}$$

$$\approx 5.5\times10^{-9} F$$

Hence, the capacitance of the capacitor is approximately.

1. Potential of the inner sphere is given by,

$$V = \frac{q}{C}$$

$$= \frac{2.5\times10^{-6}}{5.5\times10^{-9}} = 4.5\times10^{2} V$$

Hence, the potential of the inner sphere is $4.5\times10^{2} V$.

2. Radius of an isolated sphere, r = $12\times\ 10^{-2}\ m$

3. Capacitance of the sphere is given by the relation,

$$C^1 - 4\pi \in_0 r$$

$$= 4\pi\times8.85\times10^{-12}\times12\times10^{-12}$$

$$= 1.33\times10^{-11} F$$

The capacitance of the isolated sphere is less in comparison to the concentric spheres. This is because the outer sphere of the concentric spheres is earthed. Hence, the potential difference is less and the capacitance is more than the isolated sphere.

---

21. Answer carefully:

1. Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by Q_1 $Q_2 / 4\pi\epsilon_0 r^2$, where r is the distance between their centres?

2. If Columb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?

3. A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?

4. What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

1. We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?

2. What meaning would you give to the capacitance of a single conductor?

3. Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

Ans. 1. The force between two conducting spheres is not exactly given by the expression $Q_2 / 4\pi\varepsilon_0 r^2$, because there is a non-uniform charge distribution on the spheres.

2. Gauss's law will not be true, if Coulomb's law involved $1/r^3$ dependence, instead of $1/r^2$, on r.

3. Yes, If a small test charge is released at rest at a point in an electrostatic field configuration, then it will travel along the field lines passing through the point, only if the field lines are straight. This is because the field lines give the direction of acceleration and not of velocity.

1. Whenever the electron completes an orbit, either circular or elliptical, the work done by the field of a nucleus is zero.

2. No Electric field is discontinuous across the surface of a charged conductor. However, electric potential is continuous.

1. The capacitance of a single conductor is considered as a parallel plate capacitor with one of its two plates at infinity.

2. Water has an unsymmetrical space as compared to mica. Since it has a permanent dipole moment, it has a greater dielectric constant than mica.

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22. Answer the following:
1. The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)

2. A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area $1\,m^2$. Will he get an electric shock if he touches the metal sheet next morning?

3. The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

4. What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning? (Hint: The earth has an electric field of about $100\ Vm^{-1}$ at its surface in the downward direction, corresponding to a surface charge density = $-10^{-9}C\ m^{-2}$. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a

whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)

Ans. We do not get an electric shock as we step out of our house because the original equipotential surfaces of open air changes, keeping our body and the ground at the same potential.
1. Yes, the man will get an electric shock if he touches the metal slab next morning. The steady discharging current in the atmosphere charges up the aluminum sheet. As a result, its voltage rises gradually. The raise in the voltage depends on the capacitance of the capacitor formed by the aluminium slab and the ground.
2. The occurrence of thunderstorms and lightning charges the atmosphere continuously. Hence, even with the presence of discharging current of 1800 A, the atmosphere is not discharged completely. The two opposing currents are in equilibrium and the atmosphere remains electrically neutral.
3. During lightning and thunderstorm, light energy, heat energy, and sound energy are dissipated in the atmosphere.

CBSE Class 12 physics

Important Questions

Chapter 3

Current Electricit

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1 Mark Questions

1. If the temperature of a good conductor decreases, how does the relaxation time of electrons in the conductor change?

Ans. We know $\rho = \frac{m}{ne^2 \tau}$

When temperature decreases, collision decreases and thus relaxation time increases which in turn decreases the resistivity.

---

2. If potential difference V applied across a conductor is increased to 2V, how will the drift velocity of the electron change?

Ans. $Vd = \frac{eE\tau}{m}$

$$Vd = \frac{eV\tau}{\ell m}$$

$\therefore$ Double the P.D means drift velocity gets doubled.

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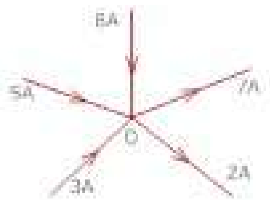
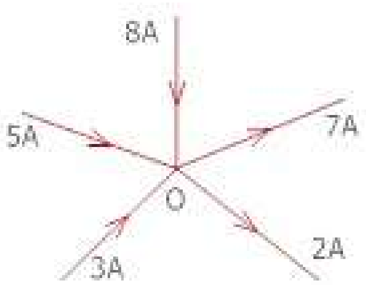
3. What is the value of current I at O in the adjoining circuit?



Ans. i = 5 + 3 – 2 – 7 + 8

i = 16 - 9

i = 7A



4. **State one condition for maximum current to be drawn from the cell ?**

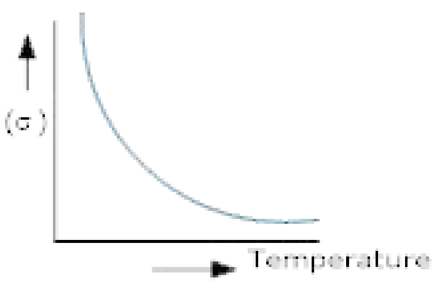
Ans. Since $I = \dfrac{E}{R + r}$ for maximum current, internal resistance should be Zero.

5. **Resistivites of copper, silver and manganin are $1.7\times10^{-8}m$, $1.0\times10^{-8}m$ and $44\times10^{-8}m$. respectively which of these is the best conductor ?**

Ans. For a particular length and area of cross-section, The resistance is directly proportionate to, specific resistance .

$\therefore$ silver is the best conductor because its specific resistance is less.

6. **Draw the graph showing the variation of conductivity with temperature for a metallic conductor?**



Ans. The conductivity decreases with the increase in temperature.

7. **If a wire is stretched to double of its length. What will be its new resistivity?**

Ans. No change in its resistivity because resistivity depends only on the nature of the material.

---

8. Name any one material having a small value of temperature coefficient of resistance. Write one use of this material?

Ans. Nichrome, an alloy has small value of temperature coefficient of resistance. It is used for making standard resistance coil.

---

9. Two wires A and B are of the same metal and of same length have their areas of cross section in the ratio 2:1 if the same potential difference is applied across each wire in turn, what will be the ratio of current flowing in A & B?

Ans. Since R $= \frac{1}{A}$

If area are in the ratio 2:1 resistance will be in the ratio 1:2.

And I = $\frac{V}{R}$ => $I = \frac{1}{R}$

$\therefore$ current will be in the ratio 2:1

2 Mark Questions

1. Two electric bulbs A and B are marked 220V, 40 w and 220V, 60 W respectively. Which one has a higher resistance?

Ans. We know $R \quad \frac{V}{p}^{\,2}$

For Bulb A, $R_1 = \frac{(220)^2}{40} = 1210\Omega$

For Bulb B $R_2 = \frac{(220)^2}{60} = 806.67\Omega$

Bulb A has higher resistance because its power is less.

---

2. A Carbon resistor has three strips of red colour and a gold strip. What is the value of resistor? What is its tolerance?

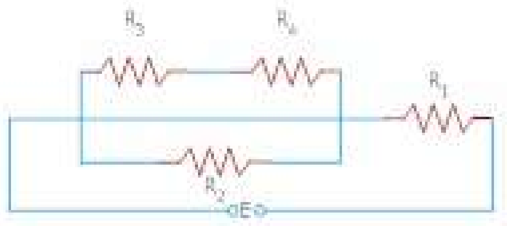
Ans. R R R Gold

$(\ 22\times10^{2}\ )^{+}5\ \%$ Value of the Resistor = 2200 Ω

Tolerance = $\pm$5 %

---

3. Determine the voltage drop across the resistor R_1 in the circuit given below with E= 60V, $R_1 = 18\Omega$, $R_2 = 10\Omega$ $R_3 = 5\Omega$ and $R_4 = 10\Omega$?



Ans. R_3 & R_4 are in series

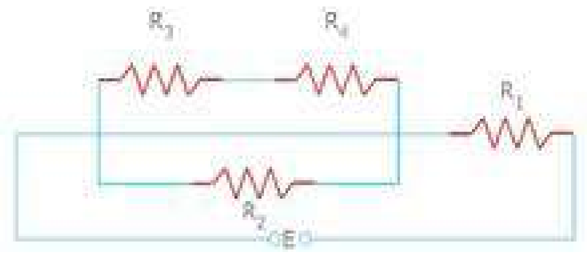
= $R^1 = 5 + 10 = 15\ \Omega$

Now R_1 and R_2 are parallel

$$\therefore \frac{1}{R''} = \frac{1}{R^1} + \frac{1}{R_2}$$

$$\frac{1}{R''} = \frac{1}{15} + \frac{1}{10} = \frac{4+6}{60} = \frac{10}{60}$$

$$R'' = \frac{60}{10} = 6\,\Omega$$



Now R_1 and R^{11} are series

Rnet = $R'' + R_1$

$\Rightarrow$ Rnet = 6+18 = 24 Ω

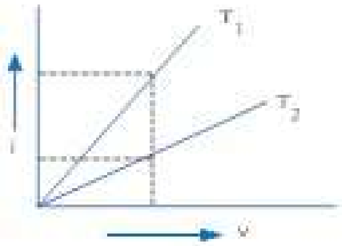
I= $\frac{V}{R}$ = $\frac{60}{24}$ $Ampere$

Now voltage drop across

$$R_1 = IR_1 = \frac{60}{24} \times 18$$

$$V = 45 \text{ Volts}$$

4. Two heated wires of same dimensions are first connected in series and then it's parallel to a source of supply. What will be the ratio of heat produced in the two cases?



Ans. $H = I^2\ Rt\ \left(\because\ I = \frac{V}{R}\right)$ Let Resistance of each wire =R

$$H = \frac{V^2}{R^{\cancel{2}}} \times \cancel{R} \times t$$

$$H = \frac{V^2}{R} t$$

$$\Rightarrow H \alpha \frac{1}{R}$$

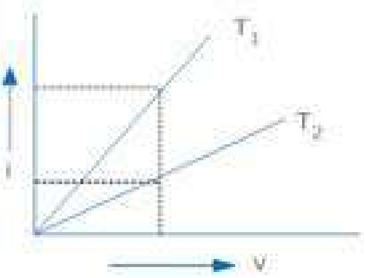
$$\frac{Hseries}{Hparallel} = \frac{Rparallel}{Rseries}$$

$$= \frac{\left(\frac{1}{R}+\frac{1}{R}\right)^{-\frac{1}{R}}}{R+R} \frac{R/2}{2R} = \frac{R}{2R \times 2} = \frac{1}{4}$$

---

5. V.I graph for a metallic wire at two different temperatures T_1 and T_2 is shown in figure. Which of these two temperatures is higher and why?

Ans. Slope $\frac{i}{V} = \frac{1}{R}$



= Smaller the slope larger is the resistance and since resistance increases with the increases in temperature for metals. Slope is small for T_2

T_2 temperature is higher

6.A set of n-identical resistors, each of resistance R ohm when connected in series have an effective resistance of X ohm and when the resistors are connected in parallel the effective resistance is Y ohm. Find the relation between R, X and Y ?

Ans. n – resistors connected in series

X = nR -----1)

n – Resistors connected in parallel

Y = $\frac{R}{n}$ ------2)

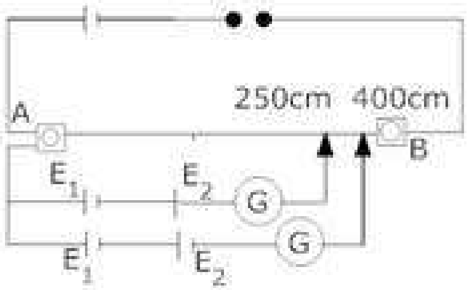
Multiply eg. (1) & (2)

XY = $\cancel{n}R \times \frac{R}{\cancel{n}}$

XY = R^2 $\boxed{R = \sqrt{XY}}$

---

7. Show the resistance of a conductor is given by $R = \frac{ml}{ne^2\tau A}$



Ans. For a conductor of length l and area A if (E_2) electric field is applied, Then the digit velocity of electrons is given by

$$vd = \frac{eE}{m}\tau$$

Since I = neAvd

$$I = neA\left(\frac{eE}{m}\tau\right)$$

$$I = neA\left(\frac{eV}{ml}\tau\right)\ (\because E = v/\ell)$$

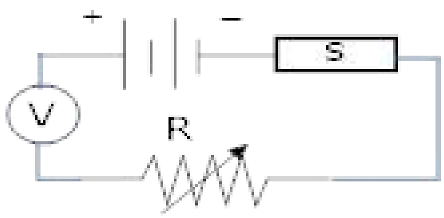
$$\frac{V}{I} = \frac{ml}{ne^2A\zeta}$$

$$R = \frac{m}{ne^2\tau}\left(\frac{l}{A}\right)\ (\because V/I = R)$$

$$or\ R = \frac{ml}{ne^2\tau A}$$

---

8. Figure shows a piece of pure semiconductor S in series with a variable resistor Rand a source of constant voltage V. Would you increase and decrease the value of R to keep the reading of ammeter (A) constant, when semiconductor S is heated ? Give reasons.



Ans. Resistance of a semi conductor decreases on increasing the temperature, so in order to increase the temperature, s is heated and in order to maintain the ammeter current constant total resistance is the above circuit should remain unchanged, hence value of r has to be increased.

---

9. Why is constantan or manganin used for making standard resistors?

Ans. The alloys such as constantan or manganin are used for making standard resistors because their resistivities are high and has low temperature coefficient of resistance.

---

10. What are ohmic and non-ohmic resistors? Give one example of each?

Ans. A resistor which obey ohm's law are called ohmic resistors for eg -> metals

A resistor which do not obey ohm's law are called non-ohmic resistors .eg -> semiconductor diode , transistor etc.

---

11. The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω , what is the maximum current that can be drawn from the battery?

Ans. Emf of the battery, *E*= 12 V

Internal resistance of the battery, *r*= 0.4 Ω

Maximum current drawn from the battery can be calculated as:

The maximum current drawn from the given battery is 30 A.

---

12. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to

63.0 cm, what is the emf of the second cell?

Ans. Emf of the cell, $E_1 = 1.25V$

Balance point of the potentiometer, $I_1 = 35cm$

The cell is replaced by another cell of emf E_2.

New balance point of the potentiometer, $I_2 = 63cm$

The balance condition id given by the relation,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$E_2 = E_1 \times \frac{l_2}{l_1}$$

$$= 1.25 \times \frac{63}{35} = 2.25V$$

Therefore, emf of the second cell is 2.25 V.

---

13. What conclusion can you draw from the following observations on a resistor made of alloy manganin?

CURRENT	VOLTAGE	CURRENT	VOLTAGE
0.2	3.94	3	59.2
0.4	7.87	4	78.8
0.6	11.8	5	98.6
0.8	15.7	6	118.5
1.0	19.7	7	138.2
2.0	39.7	8	158.0

Ans. It can be inferred from the given table that the ratio of voltage with current is a constant, which is equal to 19.7. Hence, manganin is an ohmic conductor i.e., the alloy obeys Ohm’s law. According to Ohm’s law, the ratio of voltage with current is the resistance of the conductor. Hence, the resistance of manganin is 19.7 Ω.

3 Mark Questions

1. What happens to the resistance of the wire when its length is increased to twice its original length?

Ans. $R = s\frac{\ell}{A} = s\left(\frac{\ell}{\pi r^2}\right)$

Now $\ell^1 = 2\ell$ and radius becomes r^1

Since volume of the wire remains the same

$\therefore \quad \pi r^2 \ell = \pi r'^2 \ell 1 = \pi r'^2 \, 2\ell$

$r'^2 = r^2/2$

$\therefore$ New Resistance

$R' = P\left(\frac{\ell'}{\pi r'^2}\right)$

$R' = P\left(\frac{2\ell}{\pi r^2 / 2}\right)$

$R' = 4\rho \left(\frac{\ell}{\pi r^2}\right)$

$\therefore$ New Resistance becomes four times.

---

2. Mark the direction of current in the circuit as per kirchoff's first rule. What is the value of main current in the shown network?

Ans. R_2 *and* R_3 are in series

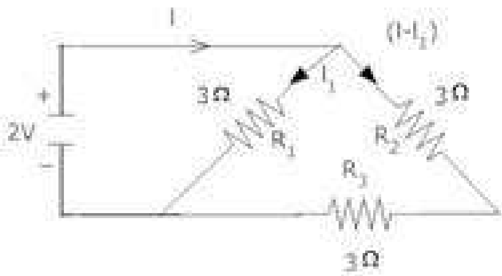
R = 3+3=6Ω

R *and* R_1 are in parallel

$$\frac{1}{R_{net}}=\frac{1}{R}+\frac{1}{R}$$

$$\frac{1}{Rnet}=\frac{1}{6}+\frac{1}{3}=\frac{3+6}{18}$$

$$Rnet=\frac{18}{9}=2\Omega$$



Net Current

I=$\frac{V}{R}$

I= $\frac{2}{2}$ =1A

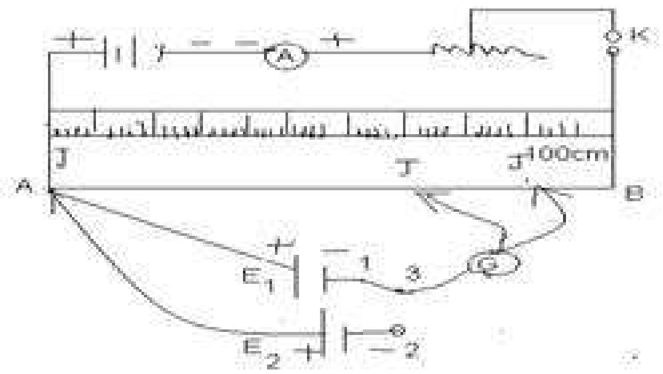
3.(a) Why do we prefer potentiometer to measure the emf of cell than a voltmeter?

(b) With suitable circuit diagram, show how emfs of 2 cells can be compared using a

potentiometer?

Ans. (a) Since potentiometer is based on null method i.e. it draws no current from the cell therefore potentiometer is preferred to measure the emf of a cell than a voltmeter because emf of a cell is equal to terminal potential difference when no current flows from the cell.

(b) Potentiometer works on the principle that when a constant current flows through the wire of Uniform area of cross- section then



(Condition – close the switch and 3 such that E_1 comes in the circuit)

P.D. across AJ is $V_{AJ} \ \alpha \ l_1$

Since no current flows between E_1 and V_{AJ}

= V_{AJ} = El

= $E_1 \ \alpha \ l_1 \Rightarrow E_1 = kl_1$ ----(1)

Close the switch 2 and 3 , cell E_2 comes in the circuit and balance point is obtained of J_1

= Since no current flows because A and J_1 are at same potential then V_{AJ} 1 = E2

= V_{AJ1} = E_2 = Kl_2 ----(2)

Comparing eg. (1) and (2)

$$\frac{E_1}{E_2} = \frac{\cancel{K}l_1}{\cancel{K}l_2}$$

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

4. Potential difference V is applied across the ends of copper wire of length (l) and diameter D. What is the effect on drift velocity of electrons if

(1) V is doubled

(2) l is doubled

(3) D is doubled

Ans.

(1) Since V_d

$= V_d = \frac{I}{neA} = \frac{V}{R(neA)}$

$$= \frac{V}{\left(\mathrm{P}\frac{\ell}{\cancel{A}}\right)(ne\cancel{A})} = \frac{V}{nep\ell}$$

V is doubled, drift velocity gets doubled.

(2) If l is doubled, drift velocity gets halved.

(3) Since V of is independent of D, drift velocity remains unchanged.

5. What is drift velocity? Derive expression for drift velocity of electrons in a good conductor in terms of relaxation time of electrons?

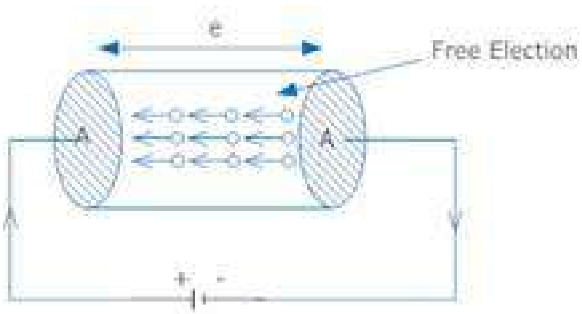
Ans. If is defined as the average velocity with which free electrons gets drifted in a direction opposite to that of electric field

If m is the mass of the electron and e be the charge of electron

Then on application of the electric field E, acceleration acquired by the electron is

$$a = \frac{eE}{m}$$

first eg of motion $v = u + a t$



since average initial velocity

u = O V= υ d

t = τ

(relaxation time)

=>vd = a τ

$$vd = \frac{e\,E\,\tau}{m}$$

wheree is the change on electron

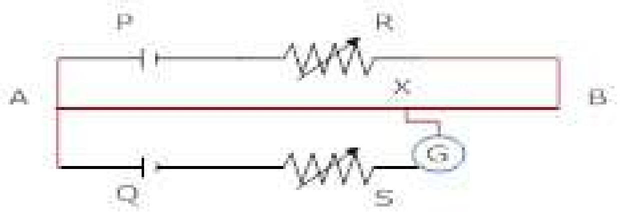
E os the electric field intensity

τ is the relaxation time

m is the mass of electron.

---

6. The potentiometer circuit shown, the balance (null) point is at X. State with reason, where the balance point will be shifted when



(1) Resistance R is increased, keeping all parameters unchanged.

(2) Resistance S is increased, keeping R constant.

(3) Cell P is replaced by another cell whose emf is lower than that of cell Q.

Ans. (a) When resistance R is increased, the current through potentiometer wire AB will decrease, hence potential difference across A will decrease, so balance point shifts towards B.

(b) When resistance S is increased terminal potential difference of the battery will decrease, so balance point will be obtained at smaller length and hence shifts towards A.

(c) When cell P is replaced by another cell whose emf is lower than that of cell Q, the P.D. across AB will be less than that of emfQ so balance point will not be obtained.

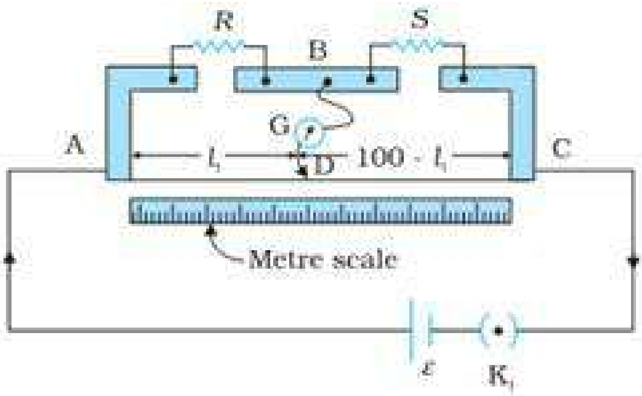
---

7. **(a) Using the principle of wheat stone bridge describe the method to determine the specific resistance of a wire in the laboratory. Draw the circuit diagram and write the formula used ?**

In a whetstone bridge experiment, a student by mistake, connects key (k) in place of galvanometer and galvanometer (G) in place of Key (K). What will be the change in the deflection of the bridge.

Ans. (a) Close the Key (k) and jockey is moved along the wire till a certain point B is reached where galvanometer shows no deflection. Then the bridge is said to be balanced.

If Rcm is the resistance per can length of the wire then.



$$\frac{R}{X} = \frac{l\ \text{Rcm}}{(100-l)Rcm}$$

$$X = \frac{\text{R}\ (100\text{-}\ell)}{\ell}$$

Since $P = \frac{XA}{\ell^1}$ Where ℓ^1 is the length of the wire.

$$P = \frac{R(100-\ell)A}{\ell\ (\ell')}$$

(b) When the bridge is balanced, there will be no current in key, therefore constant current flows through the galvanometer and hence no change in deflection on pressing the key.

8. Two primary cells of emf's $E_1 and\ E_2$ are connected to the potentiometer wire AB as shown in the figure if the balancing length for the two combinations of the cells are 250 cm and 400 cm. find the ratio of $E_1 and\ E_2$.

Ans. $E_1 -\ E_2 = K \times 250$ ----(1)

$E_1 + E_2 =\ K \times 400$ ----(2)

Adding eg. (1) &(2)

$2E_1$ = 250K+ 400K

$2E_1$ = 250K + 400

$2E_1$ = 650K

$E_1 = \dfrac{650}{2}K$

E_1 = 325 K -----(3)

Subtracting eg. (1) & (2)

E_2 = 75K

$\therefore \dfrac{E_1}{E_2} = \dfrac{325K}{75K}$

=> $\dfrac{E_1}{E_2} = 4.33$

---

9. Explain with the help of a circuit diagram, how the value of an unknown resistance can be determined using a wheat stone bridge?

Ans. Here P , Q , R are known resistance and X is an unknown resistance. Applying Kirchhoff's law for closed path ABDA .

$I_1P + I_3G - I_2R = 0$ -----(1)

For closed path BCDB

$(I_1 - I_3)\ Q - (I_2 + I_3) - I_3G$ -----(2)

Now the bridge is said to be balanced when

no current flows through the galvanometer

$\Rightarrow$ Ig = 0

$\therefore$ Eg. (1) & (2) becomes

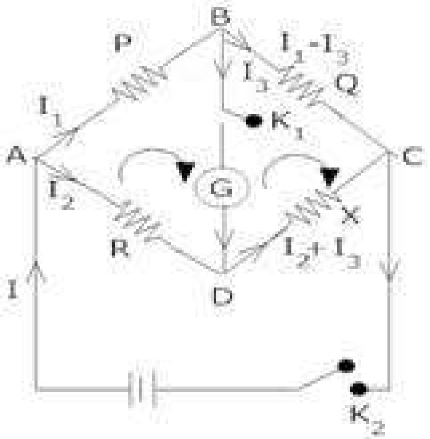
$I_1P = I_2R$

$$\frac{I_1}{I_2} = \frac{R}{P} \text{ -----(3)}$$

$$I_1 Q = I_2 X$$

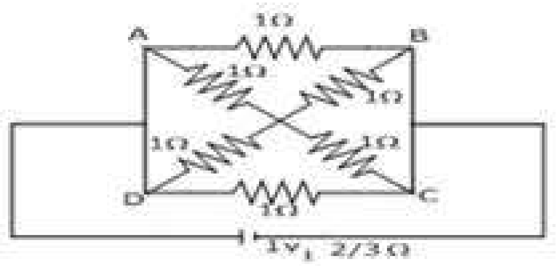
$$\frac{I_1}{I_2} = \frac{X}{Q} \text{ -----(4)}$$

Equating (3) & (4)



$$\frac{R}{P} = \frac{X}{Q} \Rightarrow X = \frac{RQ}{P}$$

10. Find the current drawn from a cell of emf IV and internal resistance 2/3 Ω connected to the network shown in the figure. E = 1v r = 2/3 Ω



Ans. $\frac{I}{R_1} = \frac{1}{1} + \frac{1}{1}$

$$\frac{1}{R_1} = 2$$

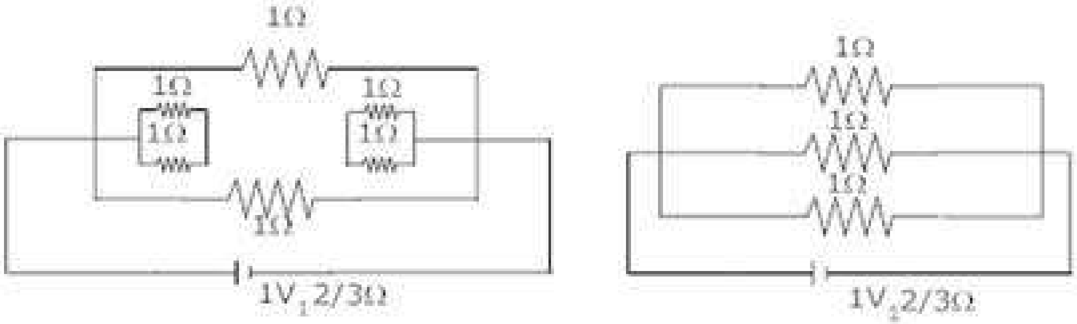
$\Rightarrow R_1 = \frac{1}{2}$

$R_2 = \frac{1}{2}$

$R = R_1 + R_2$

$\Rightarrow R = \frac{1}{2} + \frac{1}{2}$

$\Rightarrow R = 1\,\Omega$



Now 1 Ω , R and 1 Ω are in parallel

$$\Rightarrow \frac{1}{Rnet} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1}$$

$$\frac{1}{Rnet} = \frac{3}{1} \Rightarrow \text{Rnet} = \frac{3}{1}\Omega$$

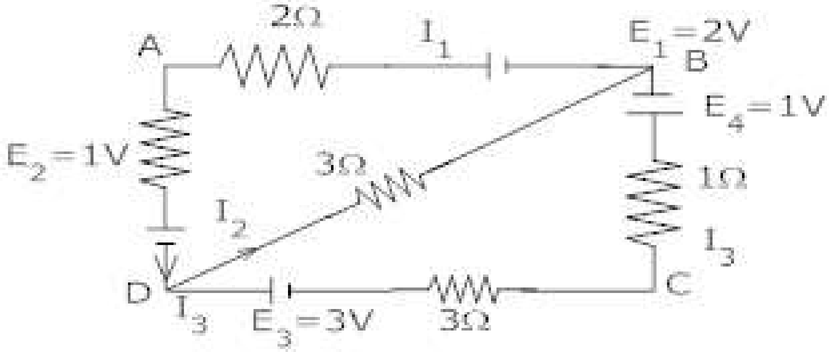
$$I = \frac{E}{R+r} = \frac{1}{\frac{3}{1} + \frac{3}{3}} = \frac{3}{3} = 1A$$

I=1A

---

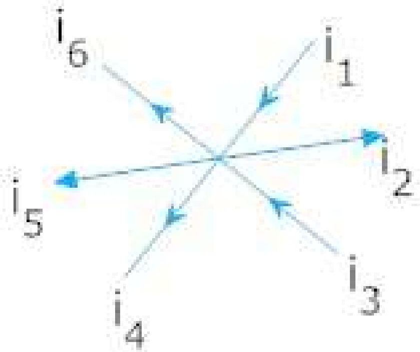
11. (a) State and explain kirchoff's law?

(b) In the network shown, find the values of current I_1, I_2 and I_3.



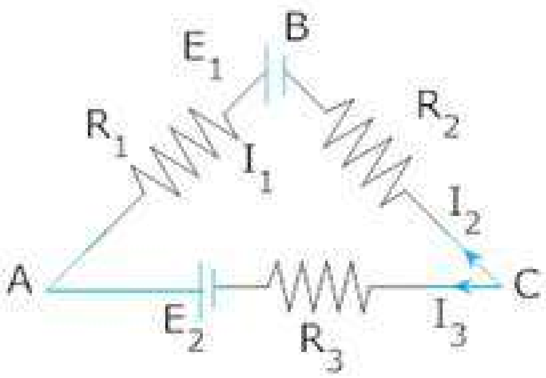
Ans. (a) Kirchoff's first law – it states that the algebraic sum of the currents meeting

at a point in an electrical circuit is always zero.



$=> i_1 - i_2 + i_3 - i_4 - i_5 - i_6 = 0$

Kirhoff's second law – it states that in any closed part of an electrical circuit, the algebraic sum of emf & is equal to the algebraic sum of the products of resistances and current flowing through them for eg. For closed path ABCA



$R_1 I_1 - R_3 I_3 + R_2 I_2 - E_1 + E_2 = 0$

Or $\boxed{E_1 - E_2 = R_1 I_1 - R_3 I_3 + R_2 I_2}$

(b) Applying kirchoff's law at point -D

$$I_1 = I_2 + I_3$$

For closed path ABDA

$$2I_1 + 1 - 2 + I_1 + 3I_2 = 0$$

$$3I_1 + 3I_2 - 1 = 0$$

$3I_1 + 3I_2 = 1$ -----(2)

For closed path DBCD

$$3I_2 - 1 - I_3 - 3I_3 + 3 = 0$$

$$3I_2 - 4I_3 + 2 = 0$$

Or $4I_3 - 3I_2 - 2 = 0$

$4I_3 - 3I_2 = 2$ -----(3)

Solving eg. (1) , (2) & (3)

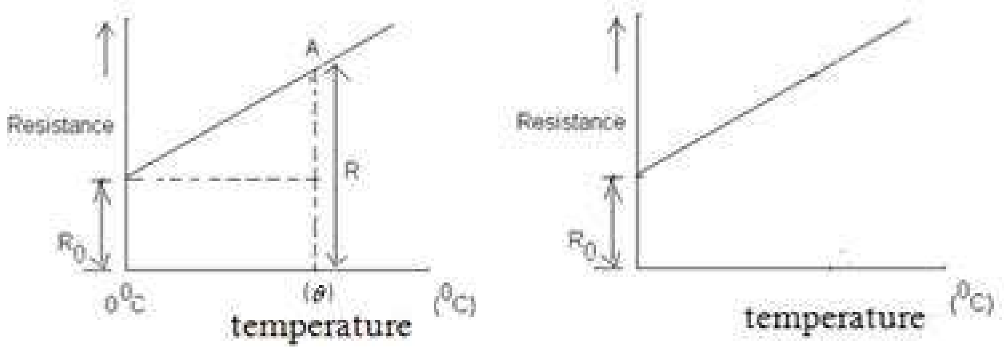
I1 = $\frac{13}{33}A$, $I_2 = \frac{-2}{33}A$ and $I_3 = \frac{5}{11}A$

---

12. The variation of resistance of a metallic conductor with temperature is given in figure.

(a) Calculate the temperature coefficient of resistance from the graph.

(b) State why the resistance of the conductor increases with the rise in temperature.



Ans. (a) Temperature coefficient of Resistance

$$\alpha = \frac{R - R_0}{R_0 \theta}$$

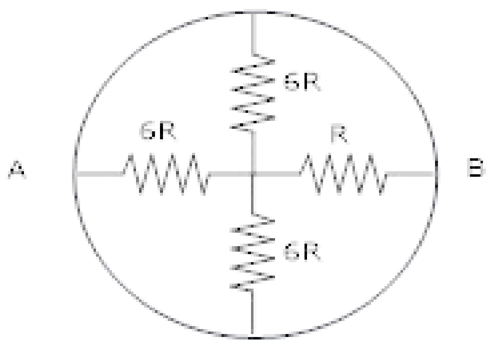
Where R is the resistance of the conductor and Ω is the temperature corresponding to pt.A

(b) Since R = $p\frac{l}{A} = \frac{m}{ne^2\tau}\left(\frac{t}{A}\right)$ P=Resistivity

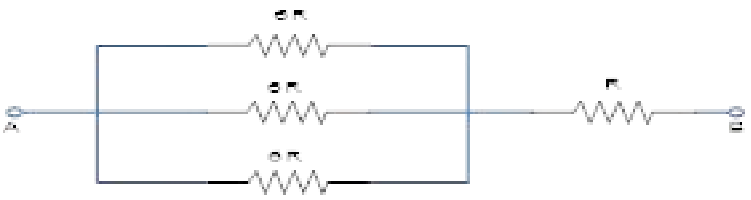
When temperature increases, no of collisions increases average relaxation time decreases, hence resistance Increases.

---

13. A circle ring having negligible resistance is used to connect four resistors of resistances 6R , 6R , 6R and R as shown in the figure. Find the equivalent resistance. between points A & B



Ans.



6R , 6R and 6R are in parallel

$$\frac{1}{R_s} = \frac{1}{6R} + \frac{1}{6R} + \frac{1}{6R}$$

$$\frac{1}{R_s} = \frac{3}{6R}$$

$R_S = \dfrac{\cancel{6}R}{\cancel{3}}$ 2R

R_S =2R => 2R and R are in series

$\therefore R_{net} = 2R+R$

$R_{net} = 3R$

---

14. A battery of emf E and internal resistance r sends a current $I_1 and\ I_2$, when connected to an external resistance of $R_1 \ and \ R_2$ respectively. Find the emf. and internal resistance of the battery?

Ans. $I_1 = \dfrac{E}{R_1 + r} \Rightarrow E = I_1(R_1 + r)$ ---(1)

Similarly $E = I_2(R_2 + r)$ ---(2)

From (1) & (2)

$I_1(R_1 + r) = I_2(R_2 + r)$

$$I_2r - I_1r = I_1R_1 - I_2R_2$$

$$r\left(I_2 - I_1\right) = I_1R_1 - I_2R_2$$

$$\boxed{r = \frac{I_1R_1 - I_2R_2}{I_2 - I_1}}$$

Emf. $\left(E\right) = I_1\left(R_1 + r\right)$

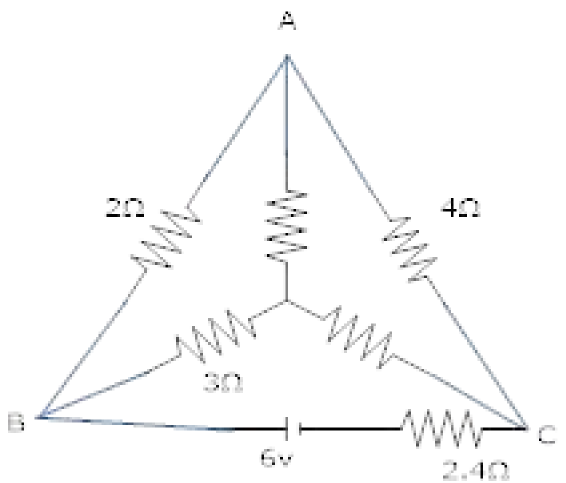
$$E = I_1\left[R_1 + \frac{I_1R_1 - I_2R_2}{I_2 - I_1}\right]$$

$$E = I_1\left[\frac{I_2R_1 - I_1\cancel{R_1} + \cancel{I_1}R_1 - I_2R_2}{I_2 - I_1}\right]$$

$$E = \frac{I_1I_2\left(R_1 - R_2\right)}{I_2 - I_1}$$

---

15. Find the value of unknown resistance X in the circuit shown in the figure if no current flows through the section AO. Also calculate the current drawn by the circuit from the battery of emf. 6v and negligible internal resistance.



Ans. As no current flows through AO then the circuit is said to be balanced wheat Stone bridge.

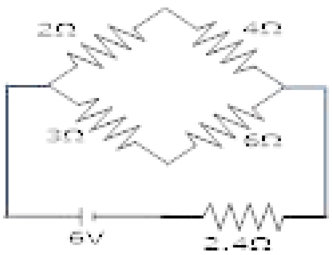
$$\frac{2}{4}=\frac{3}{X}$$

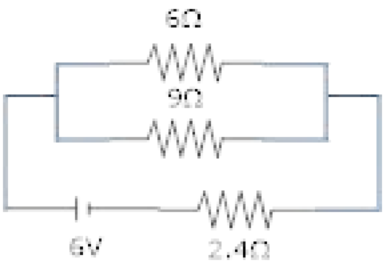
X = $\frac{12}{2}=6$

X = 6 Ω

Since in branch AO , I=0

$\therefore$ Resistance of 10 Ω between A and O is ineffective and the circuit reduce to





2 Ω and 4Ω are in series 3Ω and 6Ω are in series

6 Ω and 9 Ω are in parallel

$$\therefore \frac{1}{R_p}=\frac{1}{6}+\frac{1}{9}=\frac{9+6}{54}=\frac{15}{54}$$

R_p = $\frac{54}{15}\Omega$

R_p and 2.4 Ω are in parallel

Reff = 2.4 + $\frac{54}{15}$

Reff = $\frac{24}{10}+\frac{54}{15}=\frac{360+540}{150}=\frac{\cancel{900}^{18\,6}}{\cancel{150}_{3}}$

$\mathrm{R}eff = 6\,\Omega$

Current I = $\left(\frac{V}{R}=\frac{6}{6}=1\right)$

=> I=1A

---

16. (a) Obtain ohm's law from the expression for electrical conductivity.

(b) A cylindrical wire is stretched to increase its length by 10% calculate the

percentage increase in resistance?

Ans. (a) We know I = neAvd

J = $\frac{I}{A}=$ nevd

Vol = $\frac{cE\tau}{m}$

=> J = $\frac{ne^2E\tau}{m}$

Since J = σ E $\quad \therefore\ \sigma=\frac{T}{E}=\frac{ne^2\tau}{m}$

Let l and A be the length and area of the write.

I = JA

I = $\frac{ne^2E\tau}{m}\times A\ \left(\therefore E={}^{v}\!/_{e}\right)$

=> I = $\frac{ne^2 v\tau}{m\ell} A$ => V = $\left(\frac{m}{ne^2\tau}\right)\left(\frac{\ell}{A}\right) I$

$V = RI$

=> R = $\frac{\ell}{A}$ where $\rho = \frac{m}{ne^2\tau}$ (specific resistance of a wire)

(b) $\ell^1 = \ell + \frac{10}{100}\ell = 1.1\ell$ $\frac{\ell^1}{\ell} = 1.1$

Since volume of the wire remains the same

Al = $A^1\ell^1$ $\frac{A^1}{A} = \frac{\ell}{\ell}$

Since R = $\rho\frac{\ell}{A}$ and $R^1 = \rho \frac{\ell^1}{A^1}$

$\therefore \frac{R^1}{R} = \frac{\ell^1}{A^1} \times \frac{A}{\ell} = \frac{\ell^1}{\ell} \times \frac{\ell^1}{\ell} = \left(\frac{\ell^1}{\ell}\right)^2$

$\frac{R^1}{R} = (1.1)^2 = 1.21$

$\therefore$ Percentage increase in Resistance is

$\frac{R^1 - R}{R} \times 100 = 21\%$

---

17. The current I flows through a wire of radius r and the free electron drift with a velocity v_d what is the drift velocity of electrons through a wire of same material but having double the radius, when a current of 2I flows through it?

Ans. I = ne A vd

$\Rightarrow$ vd = $\frac{I}{neA} = \frac{I}{ne\pi r^2}$ (1)

If vd' is the drift velocity of electrons in the second wire

Vd' = $\frac{I'}{nA'e} \Rightarrow vd' = \frac{2I}{n4\pi r^2 e} = \frac{1}{2}\left(\frac{I}{n\pi r^2 e}\right)$ -----(2)

From eq . (1) & (2)

$$vd' = \frac{vd}{2}$$

---

18. Three identical cells, each of emf. 2v and unknown internal resistance are connected in parallel .This combination is connected to a 5ohm resister. If the terminal voltage across the cell is 1.5volt. What is the internal resistance of each cell .hence define internal resistance of a cell?

Ans. E = 2v, V=1.5v, R = 5 Ω

Total internal resistance = $\frac{r}{3}$

Since r = $\left(\frac{E}{V} - 1\right)R$

$$\frac{r}{3} = \left(\frac{2}{1.5} - 1\right)5$$

$$\frac{r}{3} = \left(\frac{2 - 1.5}{1.5}\right)5$$

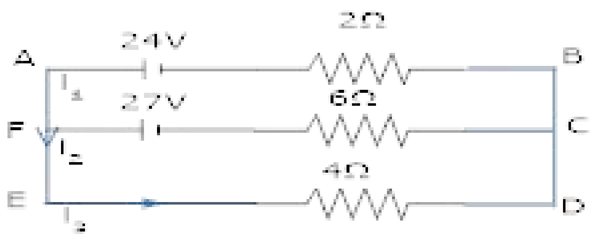
$$r = 1\cancel{5}\left(\frac{0.5}{\cancel{1.5}}\right)$$

$$r = 5 0hm$$

The resistance offered by the electrolyte of the cell, when the electric current flows through it

, is called as internal resistance of a cell.

19. Using Kirchhoff's law, determine the current I_1, I_2 and I_3 for the network shown.



Ans. Applying junction rule at point F

$I_1 = I_2 + I_3$ ----(1)

Loop rule for BAFCB

$2I_1 + 6I_2 - 24 + 27 = 0$

$2I_1 + 6I_2 + 3 = 0$ b ----(2)

Loop rule for FCDEF

$27 + 6I_2 - 4I_3 = 0$ ----(3)

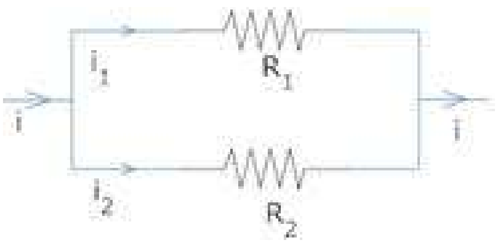
solving eg . (1) , (2) & (3) we get

$I_1 = 3A$, $I_2 = -1.5A$, $I_3 = 4.5A$

20. Show that when a current is divided between two resistances in accordance with kirchoff's laws, the heat provided is minimum?

Ans. Consider two resistance R_1 and R_2 in parallel and i_1 and i_2 be the current. Using kirchoff's first law

$i = i_1 + i_2$ ----(1)



kirchoff's second law

$$i_1R_1 - i_2R_2 = 0$$

$$\frac{i_1}{i_2} = \frac{R_2}{R_1} \quad \text{--------(2)}$$

Heat produced in the circuit in t second is

$$H = i_1^2R_1t + i^2{}_2R_2t$$

$H = i^2{}_1R_1t + (i - i_1)^2 R_2t$ (using eg.(1))

If the heat produced is minimum then $\frac{dH}{di_1} = 0$

$\therefore$ 0= $2i_1R_1t + 2(i\text{-}i_1)(\text{-}1) R_2t$

$\cancel{2}(i\text{-}i_1) R_2 \cancel{t} = \cancel{2}i_1 R_1 \cancel{t}$

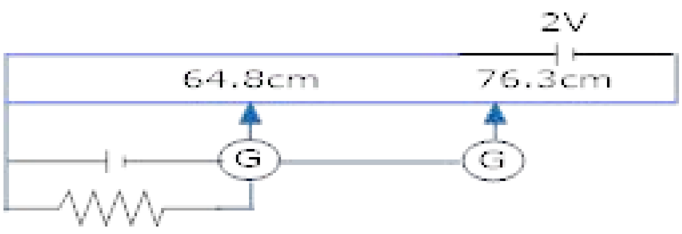
$$(i - i_1) R_2 = i_1R_1$$

$\frac{i_1}{i_2} = \frac{R_2}{R_1}$ This is in accordance with kirchoff's law.

---

21. (a) Define emf. of a cell? On what factors does it depend?

(b) Figure below shows a 2.0v potentiometer used for the determination of internal resistance of a 1.5v cell. The balance point of the cell in open circuit is 76.3cm. When a resistance of 9.5 Ω is used in external circuit of the cell the balance point shifts to 64.8cm length of the potentiometer. Determine the internal resistance of the cell.



Ans. (a) It is defined as the potential difference between the two electrodes of the cell in open Circuit (when no current is drawn) It depends on the following factors

(i) Nature of Electrodes

(ii) Nature and concentration of the Electrolytes

(iii) Temperature of the cell.

(b) Internal resistance of the cell.

r = R $\left(\frac{\ell 1-\ell_2}{\ell_2}\right)$

Here $\ell_1 = 76.3cm$

$\ell_2 = 64.8cm$

R = 9.5 Ω

=> r = 9.5 $\left(\frac{76.3-64.8}{64.8}\right)$

$r = 1.68\Omega$

---

22. A battery of emf 10 V and internal resistance 3 Ω is connected resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?

Ans. Emf of the battery, E=10V

Internal resistance of the battery ,r =3 Ω

Current in the circuit, I=0.5A

Resistance of the resistor$= R$

The relation for current using Ohm's law is,

$$I = \frac{E}{R+r}$$

$$R + r = \frac{E}{I}$$

$$= \frac{10}{0.5} = 20\Omega$$

$$\therefore R = 20 - 3 = 17\Omega$$

Terminal voltage of the resistor=V

According to Ohm's law,

V=IR

$$= 0.5 \times 17$$

$$= 8.5V$$

Therefore, the resistance of the resistor is 17Ω and the terminal voltage is 8.5v.

---

23. (a) Three resistors 1Ω, $2\ \Omega$, $and\ 3\ \Omega$ are combined in series. What is the total resistance of the combination?

(b) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.

Ans.(a) Three resistors of resistances $1\Omega, 2\Omega\ and\ 3\Omega$ are combined in series. Total resistance of the combination is given by the algebraic sum of individual resistances.

Total resistance=$1 + 2 + 3 = 6\Omega$

(b) Current flowing through the circuit=I

Emf of the battery, E=12V

Total resistance of the circuit, R= 6Ω

The relation for current using Ohm's law is,

$$I = \frac{E}{R}$$

$$= \frac{12}{6} = 2\text{A}$$

Potential drop across 1Ω resistor = V_1

For Ohm's law, the value of V_1 can be obtained as

$$V_1 = 2 \times 1 = 2V \ldots\ldots\ldots(i)$$

Potential drop across 2Ω resistor= V_2

Again, from Ohm's law, the value of V_2 can be obtained as

$$V_2 = 2 \times 2 = 4V \ldots\ldots\ldots(ii)$$

Potential drop across 3Ω resistor= V_3

Again, from Ohm's law,the value of V_3 can be obtained as

$$V_3 = 2 \times 3 = 6V \ldots\ldots\ldots(iii)$$

Therefore, the potential drop across $1\Omega, 2\Omega\ and\ 3\Omega$ resistors are 2V, 4V, and 6V respectively.

---

24. At room temperature $(27.0\ ^{\circ}C)$ the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is $1.70 \times 10^{-4}\ ^{0}C^{-1}$

Ans. Room temperature, T=27 ^0C

Resistance of the heating element at $T_1, R_1 = 117\Omega$

Temperature co-efficient of the material of the filament,

$\alpha = 1.70 \times 10^{-4}\ ^0C^{-1}$

α is given by the relation,

$$\alpha = \frac{R_1 - R}{R(T_1 - T)}$$

$$T_1 - T = \frac{R_1 - R}{R\alpha}$$

$$T_1 - 27 = \frac{117 - 100}{100\left(1.7\times10^{-4}\right)}$$

$$T_1 - 27 = 1000$$

Therefore, at 1027 ^0C, the resistance of the element is 117 Ω.

---

25. A negligibly small current is passed through a wire of length 15 m and uniform cross section $= 6.0\times10^{-7}m^2$,and its resistance is measured to be 5.0 Ω . What is the material at the temperature of the experiment?

Ans. Resistivity of material can be calculated as:

Length of the wire, l=15m

Area of cross-section of the wire, a $= 6.0\times10^{-7}m^2$

Resistance of the material of the wire, R $= 5.0\Omega$

Resistivity of the material of the wire=p

Resistance is related with the resistivity as:

$$R = p\frac{1}{A}$$

$$p = \frac{RA}{l}$$

$$= \frac{5\times 6\times 10^{-7}}{15}$$

$$= 2\times 10^{-7}\,\Omega m$$

Therefore, the resistivity of the material is $2\times 10^{-7}\,\Omega m$.

---

26. A silver wire has a resistance of 2.1 Ω at $27.5\ ^{\circ}C$, and a resistance of 2.7 Ω at $100\ ^{\circ}C$. Determine the temperature coefficient of resistivity of silver.

Ans. Temperature, $T_1 = 27.5^0\,C$

Resistance of the silver wire at $T_1, R_1 = 2.1\Omega$

Temperature, $T_2 = 1000^0\,C$

Resistance of the silver wire at $T_2, R_2 = 2.7\Omega$

Temperature coefficient if silver= α

It is related with temperature and resistance as

$$\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$$

$$= \frac{2.7 - 2.1}{2.1(100 - 27.5)}$$

$$= 0.0039\,^0C^{-1}$$

Therefore, the temperature coefficient of silver is 0.0039 $^0C^{-1}$.

27. A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds toa steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0 °C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $=1.7\times10^{-4}\,{}^0C^{-1}$.

Ans. Supply voltage, V=230V

Initial current drawn= $I_1 = 3.2\text{A}$

Initial resistance= R_1, which is given by the relation,

$$R_1 = \frac{V}{I}$$

$$= \frac{230}{3.2} = 71.87\Omega$$

Steady state value of the current, $I_2 = 2.8\text{A}$

Resistance at the steady state= R_2, which is given as

$$R_1 = \frac{230}{2.8} = 82.14\Omega$$

Temperature co-efficient of nichrome, $\alpha = 1.70\times10^{-4}\,{}^0C^{-1}$

Initial temperature of nichrome, $T_1 = 27.0^0 C$

Study state temperature reached by nichrome= T_2

T_2 can be obtained by the relation for α,

$$\alpha = \frac{R_2 - R_1}{R_1\left(T_2 - T_1\right)}$$

$$T_2 - 27^0 C = \frac{82.41 - 71.87}{71.87\times1.7\times10^{-4}} = 840.5$$

$$T_2 = 840.5 + 27 = 867.5\,^0C$$

Therefore, the steady temperature of the heating element is $867.5^0 C$

28. A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

Ans. Emf of the storage battery, *E*

Internal resistance of the battery, r= 0.5Ω

DC supply voltage, *Vr*= 120 V

Resistance of the resistor, $R = 15.5\ \Omega$

Effective voltage in the circuit = V^1

*Rr*is connected to the storage battery in series. Hence, it can be written as

$$V_1 = V - E$$

$$V_1 = 120 - 8 = 112\ V$$

Current flowing in the circuit can be calculated as:

Voltage across resistor R is given by the product , IR= 7 $\times$ 15.5 = 108.5 V

DC supply voltage = Terminal voltage of battery + Voltage drop across R

Terminal voltage of battery = 120-108.5 = 11.5 VA series resistor in a charging circuit limits the current drawn from the external source. The current will be

extremely high in its absence. This is very dangerous.

29. The number density of free electrons in a copper conductor estimated in Example is $8.5 \times 10^{28} m^{-3}$. How long does an electron take to drift from one end of a wire 3.0 m

long to its other end? The area of cross section of the wire is $2\times10^{28}\,m^{-3}$ and it is carrying a current of 3.0 A.

Ans. Number density of free electrons in a copper conductor, n= $8.5\times10^{28}\,m^{-3}$

Length of the copper wire, l= $3.0\,m$

Area of cross-section of the wire, $A=2.0\times10^{-6}\,m^{2}$

Current carried by the wire, I= $3.0\,A$, Which is given by the relation,

$I=nAeVd$

Where,

$e=$ Electric charge= $1.6\times10^{-19}\,C$

$V_d=$ Drift velocity $=\frac{Length\ of\ the\ wire}{Time\ taken\ to\ cover}$

$I=nAe\frac{l}{t}$

$t=\frac{nAel}{I}$

$=\frac{3\times8.5\times2\times10^{-6}\times1.6\times10^{-19}}{3.0}$

$=2.7\times10^{4}\,s$

Therefore, the time taken by an electron to drift form one end of wire to the other is $2.7\times10^{4}\,s.$

---

30. The earth's surface has a negative surface charge density of $10^{-9}\,Cm^{-2}$. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to the low conductivity of the lower atmosphere) in a current of only 1800 A over the entire globe. If there were no mechanism of sustaining atmospheric electric

field, how much time (roughly) would be required to neutralise the earth's surface? (This never happens in practice because there is a mechanism to replenish electric charges, namely the continual thunderstorms and lightning in different part of the globe [Radius of earth = $6.37\times10^{6}\,m.$ **])**

Ans. Surface charge density of the earth, $\delta = 10^{-9}C\ \ m^{-2}$

Current over the entire globe, $I = 1800\text{A}$

Radius of the earth, r $= 6.37\times10^{6}m.$

Surface area of the earth,

$$A = 4\pi r^{2}$$

$$= 4\pi(6.37106)^{2}$$

$$= 5.091014\ m^{2}$$

Charge on the earth surface,

$$q = \sigma\times A$$

$$= 10^{-9}\times\ 5.09\times10^{14}$$

$$= 5.09\times10^{5}\ \ C$$

Time taken to neutralize the earth's surface = tCurrent,

$$I = \frac{q}{t}$$

$$t = \frac{q}{I}$$

$$= \frac{5.09\times10^{5}}{1800} = 282.77s$$

Therefore, the time taken to neutralize the earth's surface is 282.77s.

31. Choose the correct alternative:

(a) Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.

(b) Alloys usually have much (lower/higher) temperature coefficients of resistance than pure metals.

(c) The resistivity of the alloy manganin is nearly independent of/increases rapidly with increase of temperature.

(d) The resistivity of a typical insulator (e.g., amber) is greater than that of a metal by a factor of the order of $\left(10^{22}/10^{3}\right)$.

Ans. (a) Alloys of metals usually have greater resistivity than that of their constituent metals.

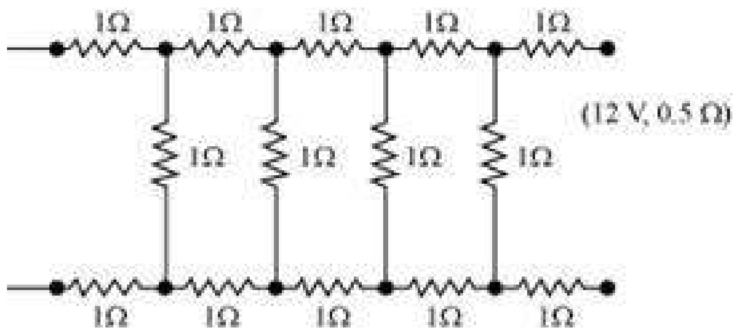
(b) Alloys usually have lower temperature coefficients of resistance than pure metals.

(c) The resistivity of the alloy, manganin, is nearly independent of increase of temperature.

(d) The resistivity of a typical insulator is greater than that of a metal by a factor of the order of 10^{22} .

---

32. Determine the current drawn from a 12 V supply with internal resistance 0.5 Ω by the infinite network shown in Fig 3.32.Each resistor has 1Ω resistance.



Ans. The resistance of each resistor connected in the given circuit, $R = 1\Omega$

Equivalent resistance of the given circuit $= R'$

The network is infinite, Hence, equivalent resistance is given by the relation,

$$\therefore R' = 2 + \frac{R'}{(R+1)}$$

$$(R)^2 - 2R - 2 = 0$$

$$R' = \frac{2 \pm \sqrt{4+8}}{2}$$

$$= \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$$

Negative value of R' cannot be accepted. Hence, equivalent resistance,

$$R' = (1+\sqrt{3}) = 1 + 1.73 = 2.73\Omega$$

Internal resistance of circuit is 0.5Ω

Hence, total resistance of the given circuit $= 2.73 + 0.5 = 3.23\ \Omega$

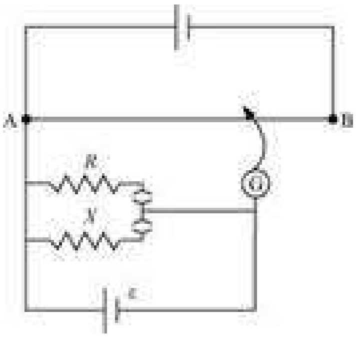
Supply Voltage, $V = 12V$

According to Ohm; s Law, current drawn from the source is given by the ratio,

$$\frac{12}{3.23} = 3.72\text{A}$$

---

33. Figure 3.34 shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor *R* = 10.0 Ω is found to be 58.3 cm, while that with the unknown resistance *X* is 68.5 cm. Determine the value of *X*. What might you do if you failed to find a balance point with the given cell of emf ε ?



Ans. Resistance of the standard resistor, R = 10.0 Ω

Balance point for this resistance, $l_1 = 58.3\ cm$

Current in the potentiometer wire = i

Hence, potential drop across R, $E_1 = iR$

Resistance of the unknown resistor = X

Balance point for this resistor, $l_2 = 68.5\ cm$

Hence, potential drop across X, $E_2 = iX$

The relation connecting emf and balance point is,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$\frac{iR}{iX} = \frac{l_1}{l_2}$$

$$X = \frac{l_1}{l_2} \times R$$

$$= \frac{68.5}{58.3} \times 10 = 11.749\Omega$$

Therefore, the value of the unknown resistance, X, is 11.75 Ω.

If we fail to find a balance point with the given cell of emf, ε, then the potential drop across

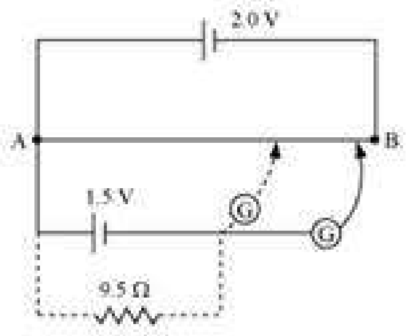
*R*and *X* must be reduced by putting a resistance in series with it. Only if the potential drop across *R*or *X* is smaller than the potential drop across the potentiometer wire AB, a balance point is obtained.

---

34. Figure shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.

Ans. Internal resistance of the cell= r

Balance point of the cell in open circuit, $l_1 = 76.3cm$



An external resistance (R) is connected to the circuit with $R = 9.5\Omega$

New balance point of the circuit, $l_2 = 64.8cm$

Current flowing through the circuit= I

The relation connecting resistance and emf is,

$$r = \left(\frac{l_2 - l_1}{l_2}\right) R$$

$$= \frac{76.3 - 64.8}{64.8} \times 9.5 = 1.68\Omega$$

Therefore, the internal resistance of the cell is 1.68Ω .

5 Mark Questions

1. (a) Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination?

(b) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.

Ans. (a) There are three resistors of resistances,

$R_1 = 2$, $R_2 = 4$, $and\ R_3 = 5$

They are connected in parallel. Hence, total resistance(R) of the combination is given by,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{10+5+4}{20} = \frac{19}{20}$$

$$\therefore R = \frac{20}{19}\Omega$$

Therefore, total resistance of the combination is $\frac{20}{19}\Omega$.

(b) Emf of the battery, V=20V

Current (I_1) flowing through resistor R_1 is given by,

$$I_1 = \frac{V}{R_1}$$

$$= \frac{20}{2} = 10\text{A}$$

Current (I_2) flowing through resistor R_2 is given by,

$$I_1 = \frac{V}{R_2}$$

$$= \frac{20}{4} = 5\text{A}$$

Current (I_3) flowing through resistor R_3 is given by,

$$I_3 = \frac{V}{R_3}$$

$$= \frac{20}{5} = 4\text{A}$$

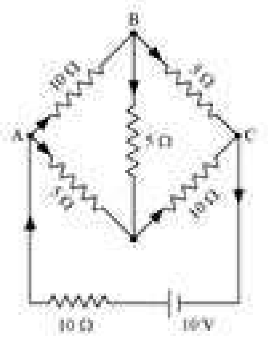
Total current, $I = I_1 + I_2 + I_3 = 10 + 5 + 4 = 19\text{A}$

Therefore, the current through each resister is 10A, 5A, and 4A respectively and the total current is 19A.

---

2. Determine the current in each branch of the network shown in fig 3.30:

Ans. Current flowing through various branches of the circuit is represented in the given figure.



$I_1 =$ Current flowing through the outer circuit

$I_2 =$ Current flowing through branch AB

$I_3 =$ Current flowing through branc $\mathrm{A}D$

$I_2 - I_4 =$ Current flowing through branch $\mathrm{B}C$

$I_3 + I_4 =$ Current flowing through branch CD

$I_4 =$ Current flowing through branch BD

For the closed circuit ABDA, potential is zero i.e.,

$$10I_2 + 5I_4 - 5I_3 = 0$$

$$2I_2 + I_4 - I_3 = 0$$

$$I_3 = 2I_2 + I_4 \ \ldots\ (1)$$

For the closed circuit BCDB, potential is zero i.e.,

$$5(I_2 - I_4) - 10(I_3 + I_4) - 5I_4 = 0$$

$$5I_2 + 5I_4 - 10I_3 - 10I_4 - 5I_4 = 0$$

$$5I2 - 10I_3 - 20I_4 = 0$$

$$I_2 = 2I_3 + 4I_4 \ \ldots\ (2)$$

For the closed circuit ABCFEA, potential is zero i.e.,

$$-10 + 10\ (I_1) + 10(I_2) + 5(I_2 - I_4) = 0$$

$$10 = 15I_2 + 10I_1 - 5I_4$$

$$3I_2 + 2I_1 - I_4 = 2 \ \ldots \ (3)$$

From equations (1) and (2), we obtain

$$I_3 = 2(2I_3 + 4I_4) + I_4$$

$$I_3 = 4I_3 + 8I_4 + I_4$$

$$-3I_3 = 9I_4$$

$$-3I_4 = +I_3 \ \ldots \ (4)$$

Putting equation (4) in equation (1), we obtain

$$I_3 = 2I_2 + I_4$$

$$-4I_4 = 2I_2$$

$$I_2 = -\ 2I_4 \ \ldots \ (5)$$

It is evident from the given figure that,

$$I_1 = I_3 + I_2 \ \ldots \ (6)$$

Putting equation (6) in equation (1), we obtain

$$3I_2 + 2(I_3 + I_2) - I_4 = \ 2$$

$$5I_2 + 2I_3 - I_4 \ = \ 2 \ \ldots \ (7)$$

Putting equations (4) and (5) in equation (7), we obtain

$$5(-2\,I_4) + 2(-\ 3I_4) - I_4 = 2$$

$$10I_4 - 6I_4 - I_4 = 2$$

$$17I_4 = -2$$

Equation (4) reduces to

$$I_3 = -3(I_4)$$

$$= -3\left(\frac{-2}{17}\right) = \frac{6}{17}\text{ A}$$

$$I_2 = -2(I_4)$$

$$= -2\left(\frac{-2}{17}\right) = \frac{4}{17}\text{ A}$$

$$I_2 - I_4 = \frac{4}{17} - \left(\frac{-2}{17}\right) = \frac{6}{17}\text{ A}$$

$$I_3 + I_4 = \frac{6}{17} + \left(\frac{-2}{17}\right) = \frac{4}{17}\text{ A}$$

$$I_1 = I_3 + I_2$$

$$= \frac{6}{17} + \frac{4}{17} = \frac{10}{17}\text{ A}$$

Therefore, current in branch $in\ AB = \frac{4}{17}\text{ A}$

In branch $BC = \frac{6}{17}\text{ A}$

In branch $CD = \frac{-4}{17}\text{ A}$

In branch $AD = \frac{6}{17}\text{ A}$

In branch $BD = \frac{-2}{17}$ A

Therefore total current = $\frac{4}{17} + \frac{6}{17} + \frac{-4}{17} + \frac{6}{17} + \frac{-2}{17} = \frac{10}{17}$ A

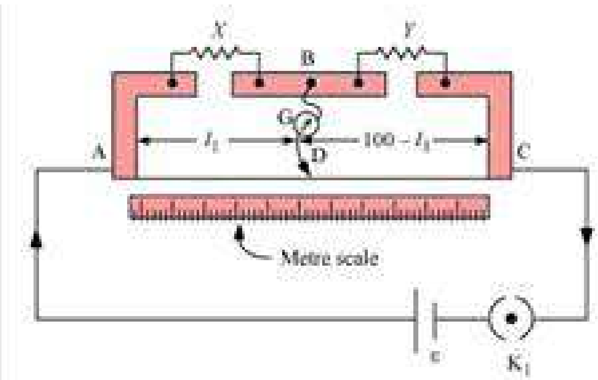
---

3. (a) In a metre bridge [Fig. 3.27], the balance point is found to be at 39.5 cm from the end when the resistor *Y* is of 12.5 Ω.

(b) Determine the balance point of the bridge above if X and Y are interchanged.

(c) What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?

Ans. A metre bridge with resistors *X* and *Y* is represented in the given figure.



(a) Balance point form end $A, l_1 = 39.5 cm$

Resistance of the resistor $Y = 12.5 \Omega$

Condition for the balance is given as,

$$\frac{X}{Y} = \frac{100 - l_1}{l_1}$$

$$X = \frac{100 - 39.5}{39.5} \times 12.5 = 8.2 \Omega$$

Therefore, the resistance of resistor *X* is 8.2Ω.

The connection between resistors in a Wheatstone or metre bridge is made of thick copper strips to minimize the resistance, which is not taken into consideration in the bridge formula.

If *X* and *Y* are interchanged, $then\, l_1 \;and\; 100-l_1$ get interchanged.

The balance point of the bridge will be $100-l_1$ from A.

$100-l_1$ = 100 − 39.5 = 60.5 cm

Therefore, the balance point is 60.5 cm from A.

When the galvanometer and cell are interchanged at the balance point of the bridge, the galvanometer will show no deflection. Hence, no current would flow through the galvanometer.

---

4. (a) Six lead-acid type of secondary cells each of emf 2.0 V and internal resistance 0.015 Ω are joined in series to provide a supply to a resistance of 8.5 Ω. What are the current drawn from the supply and its terminal voltage?

(b) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car?

Ans. (a) Number of secondary cells, *n*

Emf of each secondary cell, *E* $= 2.0V$

Internal resistance of each cell, r= 0.015 Ω

series resistor is connected to the combination of cells.

Resistance of the resistor, *R* = 8.5 Ω

Current drawn from the supply=I, which is given by the relation,

$$I = \frac{nE}{R+nr}$$

$$= \frac{6 \times 2}{8.5 + 6 \times 0.015}$$

$$= \frac{12}{8.59} = 1.39\text{A}$$

Terminal voltage, $V = IR = 1.39 \times 8.5 = 11.87$ A

Therefore, the current drawn from the supply is 1.39 A and terminal voltage is 11.87 A.

After a long use, emf of the secondary cell, E=1.9V

Internal resistance of the cell, r= 380 Ω

Maximum current $E = \frac{E}{r} = \frac{1.9}{380} = 0.005\text{A}$

Therefore, the maximum current drawn from the cell is 0.005 A. Since a large current is required to start the motor of a car, the cell cannot be used to start a motor.

---

5. Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables. ($\rho_{Al} = 2.63 \times 10^{-8}\ \Omega\text{m}$, $\rho_{Cu} = 1.72 \times 10^{-8}\ \Omega\text{m}$, Relative density of Al = 2.7, of Cu = 8.9.)

Ans. Resistivity of aluminium, $\rho_{Al} = 2.63 \times 10^{-8}\ m$

Relative density of aluminium, $d_1 = 2.7$

Let l_1 be the length of aluminium wire and m_1 be its mass.

Resistance of the aluminium wire = R_1

Area of cross-section of the aluminium wire = A_1

Resistivity of copper, $\rho_{Cu} = 1.72 \times 10^{-8}\ m$

Relative density of copper, $d_2 = 8.9$

Let l_2 be the length of copper wire and m_2 be its mass.

Resistance of the copper wire = R_2

Area of cross-section of the copper wire = A_2

The two relations can be written as

$$R_1 = \rho_1 \frac{l_1}{A_1} \quad ......(1)$$

$$R_2 = \rho_2 \frac{l_2}{A_2} \quad ......(2)$$

It is given that,

$$R_1 = R_2$$

$$\therefore \rho_1 = \frac{l_1}{A_1} = \rho_2 = \frac{l_2}{A_2}$$

And,

$$l_2 = l_2$$

$$\therefore \frac{\rho_1}{A_1} = \frac{\rho_2}{A_2}$$

$$\frac{A_1}{A_2} = \frac{\rho_1}{\rho_2}$$

$$= \frac{2.63 \times 10^{-8}}{1.72 \times 10^{-8}} = \frac{2.63}{1.72}$$

Mass of the aluminium wire,

$$m_1 = Volume \times Density$$

$$= A_1 l_1 \times d_1 = A_1 l_1 d_1 \quad ......(3)$$

Mass of the copper wire,

$$m_2 = Volume \times Density$$

$$= A_2 l_2 \times d_2 = A_2 l_2 d_2 \quad ......(4)$$

Dividing equation (3) by equation (4), we obtain

$$\frac{m_1}{m_2} = \frac{A_1 l_1 d_1}{A_2 l_2 d_2}$$

For $l_1 = l_2$,

$$\frac{m_1}{m_2} = \frac{A_1 l_1}{A_2 l_2}$$

For $\frac{A_1}{A_2} = \frac{2.63}{1.72}$,

$$\frac{m_1}{m_2} = \frac{2.63}{1.72} \times \frac{2.7}{8.9} = 0.46$$

It can be inferred from this ratio that m_1 is less than m_2. Hence, aluminium is lighter than copper.

Since aluminium is lighter, it is preferred for overhead power cables over copper.

---

6. Answer the following questions:

[a] A steady current flows in a metallic conductor of non-uniform cross section of non-uniform cross section. Which of these quantities is constant along the conductor: current, current density, electric field, drift speed?

[b] Is Ohm's law universally applicable for all conducting elements?

If not, give examples of elements which do not obey Ohm's law.

[c] A low voltage supply from which one needs high currents must have very low resistance. Why?

[d] A high tension (HT) supply of, say, 6 kV must have a very large internal resistance

Ans.

[a] When a steady current flows in a metallic conductor of non-uniform cross section, the current flowing through the conductor is constant. Current density, electric field, and drift speed are inversely proportional to the area of cross-section. Therefore, they are not constant.

[b]No, Ohm's law is not universally applicable for all conducting elements. Vacuum diode semi-conductor is a non-ohmic conductor. Ohm's law is not valid for it.

[c] According to Ohm's law, the relation for the potential is V=IR

Voltage (*V*) is directly proportional to current (I).

R is the internal resistance of the source,

$$I = \frac{V}{R}$$

If *V* is low, then *R* must be very low, so that high current can be drawn from the source.

[d] In order to prohibit the current from exceeding the safety limit, a high tension supply must have a very large internal resistance. If the internal resistance is not large, then the current drawn can exceed the safety limits in case of a short circuit.

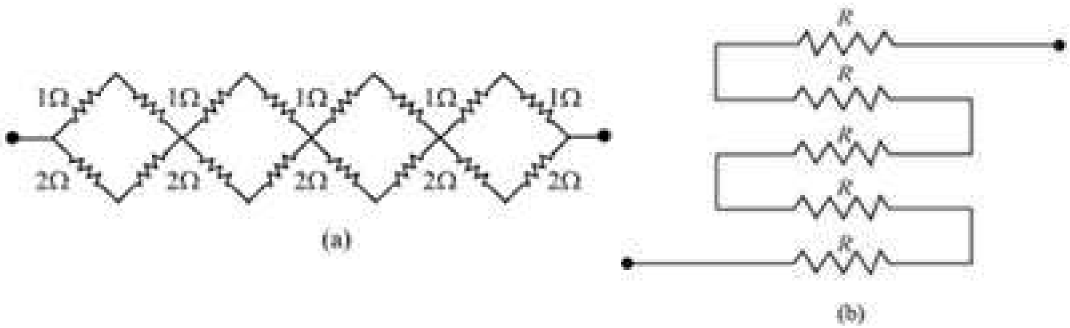
---

7. [a] Given *n* resistors each of resistance R, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?

[b] Given the resistances of $1\,\Omega,\ 2\,\Omega,\ 3\,\Omega,$ how will be combine them to get an equivalent resistance of

$(i)\ (11/3)\ \Omega, (ii)\ (11/5)\ \Omega, (iii)\ 6\,\Omega,\ (iv)(6/11)\ \Omega?$

[c] Determine the equivalent resistance of networks shown in fig.3.31.



Ans. (a) Total number of resistors= n

Resistance of each resistor $= R$

(i) When n resistors are connected in series, effective resistance R_1 is the maximum, given by the product nR.

Hence, maximum resistance of the combination, $R_1 = nR$

(ii) When n resistors are connected in parallel, the effective resistance (R_2) is the minimum, given by the ration $\frac{R}{n}$.

Hence, minimum resistance of the combination, $R_2 = \frac{R}{n}$

(iii) The ration of the maximum to the minimum resistance is,

$$\frac{R_1}{R_2} = \frac{nR}{R} = n^2$$

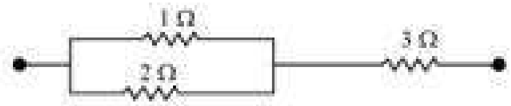
(b) The resistance of the given resistors is,

$$R_1 = 1\Omega, R_2 = 2\Omega, R_3 = 3\Omega$$

$i.$ Equivalent resistance, $R' = \frac{11}{3}\Omega$

Consider the following combination of the resistors.

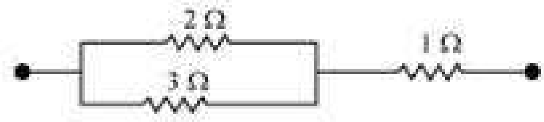
Equivalent resistance of the circuit is given by,



$$R' = \frac{2\times1}{2+1} + 3 = \frac{2}{3} + 3 = \frac{11}{3}\Omega$$

$ii.$ Equivalent resistance, $R' = \frac{11}{5}\Omega$

Equivalent resistance of the circuit is given by,



$$R' = \frac{2\times3}{2+3} + 1 = \frac{6}{5} + 1 = \frac{11}{5}\Omega$$

$iii.$ Equivalent resistance, $R' = 6\Omega$

Consider the series combination of the resistors, as shown in the given circuit.

Equivalent resistance of the circuit is given by the sum,

$$R' = 1 + 2 + 3 = 6\Omega$$

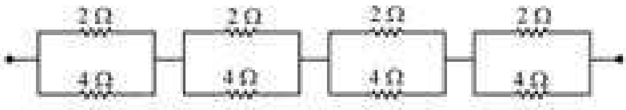
Consider the series combination of the resistors, as shown in the given ciruit. Equivalent resistance of the circuit is given by,

$$R' = \frac{1\times2\times3}{1\times+2\times3+3\times1} = \frac{6}{11}\Omega$$

(c) (i) It can be observed form the given circuit that in the first small loop, two resistors of resistance 1Ω each are connected in series.

Hence, their equivalent resistance= $(1+1)=2\Omega$

It can also be observed that two resistors of resistance 2Ω each are connected in series.



Hence, their equivalent resistance $=(2+2)=4\Omega$

Therefore, the circuit can be redrawn as

It can be observed that 2Ω and 4Ω resistors are connected in parallel in all the four loops.

Hence, equivalent resistance (R') of each loop is given by,

$$R'=\frac{2\times 4}{2+4}=\frac{8}{6}=\frac{4}{3}\Omega$$

The circuit reduces to

All the four resistors are connected in series.

Hence, equivalent resistance of the given circuit is $\frac{4}{3}\times 4=\frac{16}{3}\Omega$

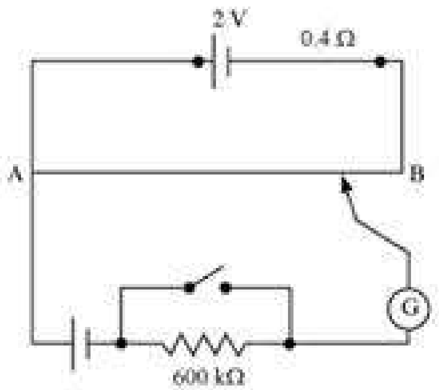
(iii) It can be observed from the given circuit that five resistors of resistance R each are connected in series.

Hence, equivalent resistance of the circuit $=R+R+R+R+R \ =5R$

---

8. Figure shows a potentiometer with a cell of 2.0 V and internal resistance 0.40 Ω maintaining a potential drop across the resistor wire AB. A standard cell which

maintains a constant emf of 1.02 V (for very moderate currents up to a few mA) gives a balance point at 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replace by a cell of unknown emf and the balance point found similarly, turns out to be at 82.3 cm length of the wire.



[a] What is the value ε ?

[b] What purpose does the high resistance of 600 $k\Omega$ have?

[c] Is the balance point affected by this high resistance?

[d] Is the balance point affected by the internal resistance of the driver cell?

[e] Would the method work in the above situation if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V?

[f] Would the circuit work well for determining an extremely small emf, say of the order of a few mV (such as the typical emf of a thermo-couple)? If not, how will you modify the circuit?

Ans. [a] Constant emf of the given standard cell, $E_1 = 1.02V$

Balance point on the wire, $l_1 = 67.3cm$

A cell of unknown emf, ε , replaced the standard cell. Therefore, new balance on the wire, $l = 82.3cm$

The relation connecting emf and balance point is,

$$\frac{E_1}{l_1} = \frac{\varepsilon}{l}$$

$$\varepsilon = \frac{l}{l_1} \times E_1$$

$$= \frac{82.3}{67.3} \times 1.02 = 1.247V$$

The value of unknown emf is $1.247V$.

[b] The purpose of using the high resistance of 600 $k\Omega$ is to reduce the current through the galvanometer when the movable contact is far from the balance point.

[c] The balance point is not affected by the presence of high resistance.

[d] The point is not affected by the internal resistance of the driver cell.

[e] The method would not work if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V. This is because if the emf of the driver cell of the potentiometer is less than the emf of the other cell, then there would be no balance point on the wire.

[f] The circuit would not work well for determining an extremely small emf. As the circuit would be unstable, the balance point would be close to end A. Hence, there would be a large percentage of error.

Modification: The given circuit can be modified if a series resistance is connected with the wire AB. The potential drop across AB is slightly greater than the emf measured. The percentage error would be small.

CBSE Class 12 physics
Important Questions
Chapter 4
Moving Charges and Magnetism

---

1 Mark Questions

1. State two properties of the material of the wire used for suspension of the coil in a moving coil galvanometer?

Ans. (a) Non-Brittle conductor

(b) Restoring Torque per unit Twist should be small.

---

2. What will be the path of a charged particle moving along the direction of a uniform magnetic field?

Ans. The path of a charged particle will be a straight line path as no force acts on the particle.

---

3. Two wires of equal lengths are bent in the form of two loops. One of the loop is square shaped whereas the other loop is circular. These are suspended in a uniform magnetic field and the same current is passed through them. Which loop will experience greater torque? Give reasons?

Ans. since $\tau = NIAB$

Since Area of – circular loops is more Than of a square loop

$\Rightarrow$ Torque experienced by a circular loop is greater.

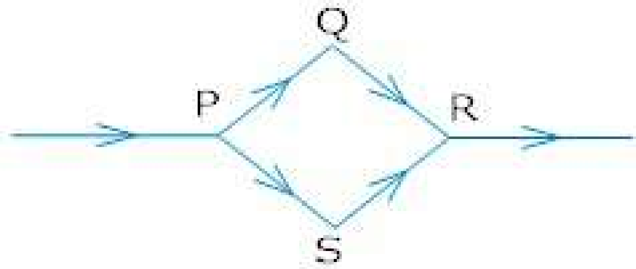
---

4. A cyclotron is not suitable to accelerate electron. Why?

Ans. A cyclotron is not suitable to accelerate electron because its mass is less due to which they gain speed and step out of the dee immediately.

2 Mark Questions

1. A steady current flows in the network shown in the figure. What will be the magnetic field at the centre of the network?



Ans. Zero, because magnetic field at the centre of the loop is just equal and opposite i.e. magnetic field due 1- PQR is equal and opposite to that of PSR.

---

2. An $\propto$ - particle and a proton are moving in the plane of paper in a region where there is uniform magnetic field B directed normal to the plane of paper. If two particles have equal linear momenta, what will be the ratio of the radii of their trajectories in the field?

Ans. Since radius of the path (R) = $\frac{mv}{Bq}$

=> $R \propto \frac{1}{q}$

=> $\frac{R\propto}{RP} = \frac{qp}{q\propto} = \frac{e}{2e} = \frac{1}{2}$

=> $R\propto$: Rp = 1:2.

3. Give one difference each between diamagnetic and ferromagnetic substances. Give one example of each?

Ans. Diamagnetic substances are weakly repelled by a magnet eg. Gold.

Ferromagnetic materials are strongly attracted by a magnet eg. Iron.

---

4. Write the expression for the force acting on a charged particle of charge q moving with velocity is in the presence of magnetic field B. Show that in the presence of this force.

(a) The K.E. of the particle does not change.

(b) Its instantaneous power is zero.

Ans. Since F = q ($\vec{v} \times \vec{B}$)

(a) Since direction of force is perpendicular to the plane containing $(\vec{v} \times \vec{B}$)

=> w = Fs cos θ $(= 90^0)$

w = Fs $cos 90^0$ = 0

=> KE = 0

$\therefore$ KE will not – change

(b) since p = Fvcos θ = Fv $cos 90^0$ = 0

=> Instantaneous power is also zero.

---

5. An electron of kinetic energy 25KeV moves perpendicular to the direction of a uniform magnetic field of 0.2 millitesla calculate the time period of rotation of the electron in the magnetic field?

Ans. B = 0.2 T = 0.2$\times$ 10^{-3}T

Time Period T = $\frac{2\pi M}{QB}$

$$T = \frac{2\times3.14\times9.1\times10^{-31}}{1.6\times10^{-17}\times0.2\times10^{-3}}$$

$T = 1.787\times\ 10^{-7}$ second

---

6. It is desired to pass only 10% of the current through a galvanometer of resistance 90 Ω. How much shunt resistance be connected across the galvanometer?

Ans. IG = 10% of I = $\frac{10I}{100}$ $\quad G = 90\Omega$

S = $\frac{IgG}{I - Ig} = \frac{\frac{1\cancel{0}I}{1\cancel{0}\cancel{0}}\times9\cancel{0}}{I - \frac{1\cancel{0}I}{10\cancel{0}}}$

S = $\frac{9I}{\frac{10I - I}{10}}$

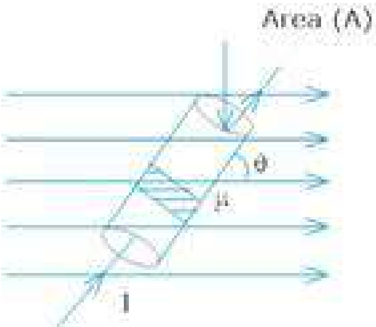
S = $\frac{90\cancel{I}}{90\cancel{I}} = 10$

=> $s = 10\Omega$

3 Mark Questions

1. Derive an expression for the force acting on a current carrying conductor placed in a uniform magnetic field Name the rule which gives the direction of the force. Write the condition for which this force will have (1) maximum (2) minimum value?

Ans. A conductor is placed in a uniform magnetic field $\vec{B}$ which makes and angle θ with $\vec{B}$. Let I current flows through the conductor.



If n is the no. of electrons per unit volume of the conductor, then Total no. of electrons in small current element dℓ = nAdl

=> θ = Ne

=> θ = nAdl e

$\vec{f}$ be the force experienced by each electron

$\vec{f}$ = e $(\vec{vd} \times \vec{B})$

Force experienced by small current element

$\vec{dF} = \text{neAdl}\ (\vec{vd} \times \vec{B})$

dF = neAvd dl B sinθ

(I = neAvd)

=> df = IdlBsin θ

Hence total force experienced

F = $\int_0^l dF = \int_0^l IdlB \sin\theta$

F = IBl $\sin\theta$

In vector form $\vec{F}$ = I $(\vec{l} \times \vec{B})$

(a) Force will be maximum when $\theta = 90^0$

(b) Force will be minimum when $\theta = 0^0$

---

2. A straight wire carries a current of 10A. An electron moving at $10^7 m/s$ is at distance 2.0 cm from the wire. Find the force acting on the electron if its velocity is directed towards the wire?

Ans. Here I = 10A

$V = 10^7 m/s$

R = 2.0 cm = $2 \times 10^{-2} m$

Force acting on moving electron (F) = qVB $\sin\theta$

$$\Rightarrow B = \frac{\mu 0}{4\pi}\frac{2I}{r}$$

$B = \frac{10^{-7} \times \cancel{2} \times 10}{\cancel{2} \times 10^{-2}} = 10\text{-}4$ tesla and $\perp$ to the plane of paper and directed downwards.

Now $F = 1.6 \times 10^{-19} \times 10^7 \times 10^{-4} \sin 90^0$

$F = 1.6 \times 10^{-16}$ Newton.

3. State Biot- Savarts law. Derive an expression for magnetic field at the centre of a circular coil of n-turns carrying current – I?

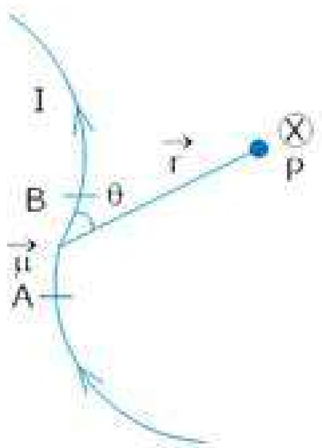
Ans. Biot – Savart law states that the magnetic field db due to a current element $\overrightarrow{dl}$ at any point is

ie dB $\propto$ I

dB $\propto$ dl

dB $\propto$ $\sin\theta$

dB $\propto$ $\frac{1}{r^2}$

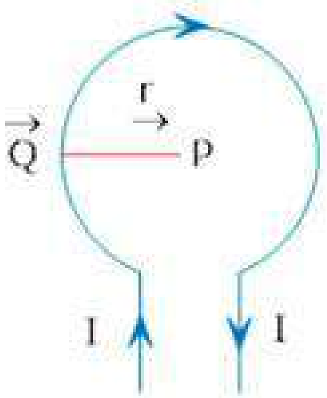


Combining all we get

dB $\propto$ $\frac{Idl\sin\theta}{r^2}$

$$dB = \frac{\mu 0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

Consider a circular loop of radius r carrying a current I.



Since dl $\perp \vec{r}$

=> $\theta = 90^0$

Applying Biot Savart law

dB = $\frac{\mu 0}{4\pi} \frac{Idl \sin 90^0}{r^2}$

For entire closed circular loop

B = $\int_0^{2\pi r} \frac{\mu 0}{4\pi} \frac{Idl \sin 90^0}{r^2}$

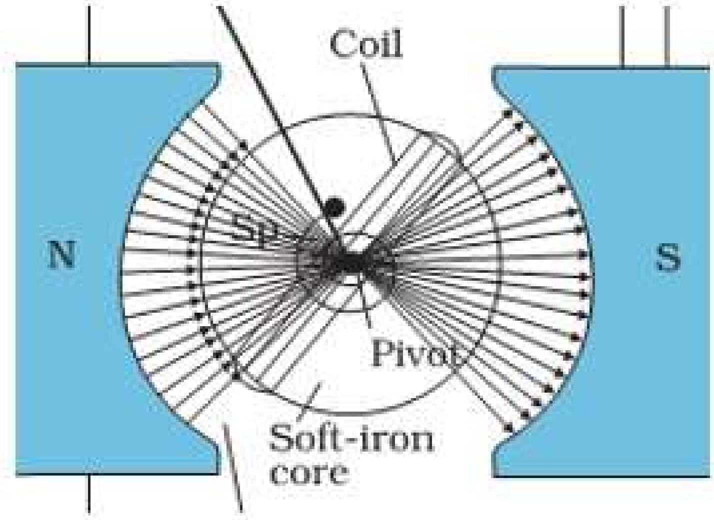
B = $\frac{\mu 0}{4\pi} \frac{I}{r^2} \int_0^{2\pi r} dl = \frac{\mu o}{4\pi} \frac{I}{r^2} \times 2\pi r$

For n turns of a coil $B = \frac{\mu 0}{4\pi} \frac{2\pi nI}{r}$

---

4. What is radial magnetic field? How it is obtained in moving coil galvanometer?

Ans. A radial magnetic field is one in which plane of the coil always lies in the direction of the magnetic field. It can be obtained by



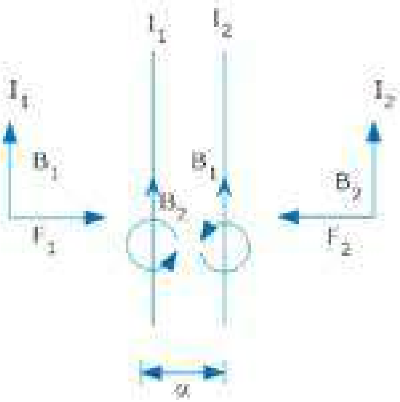
(a) Properly cutting the pole pieces concave in shape.

(b) Placing soft iron cylindrical core between the pole pieces.

---

5. Two straight parallel current carrying conductors are kept at a distanced r from each other in air. The direction for current in both the conductor is same. Find the magnitude and direction of the force between them. Hence define one ampere?

Ans. Consider two parallel conductors carrying or current $-I_1$ & I_2 and is separated by a distance 'd'.



Magnetic field due to current I_1 at any paint on conductor (2) is

$$B_1 = \frac{\mu 0}{4\pi}\frac{2I1}{d} \quad \text{----(1)}$$

($\perp$ to the plane & Downwards $(\times)$)

Since current carrying conductor is placed at right angles to the magnetic field

=> F = BI l $sin\ 90^0$

F = B I l

=> Force experienced per unit length of conductor ----(2)

$Is\ F_2 = B_1 I_2 1$

$F_2 = \frac{\mu 0}{4\pi} \frac{2I_1\ I_2}{d}$ ----(2)

Fleming's left hand Rule says F_2 is directed towards conductor (1)

Similarly $F_1 = \frac{\mu 0}{4\pi} \frac{2I_1\ I_2}{d}$ (Directed Towards conductor (2))

Since $F_1 and\ F_2$ are equal and opposite so two parallel current carrying conductor attract each other.

Since F = $\frac{\mu 0}{4\pi}\left(\frac{2I_1\ I_2}{d}\right)$

If $I_1 = I_2$ = 1A d = 1m

$F = 2\times10^{-7}\ m.$

Thus one ampere is that current which is flowing in two infinitely long parallel conductors separated by a distance of 1 meter in vacuum and experiences a force of $F = 2\times10^{-7}\ m.$ on each meter of the other wire.

---

6. A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field B at the centre of the coil?

Ans. Number of turns on the circular coil, *n* = 100

Radius of each turn, *r* = 8.0 cm = 0.08 m

Current flowing in the coil, I = 0.4 A

Magnitude of the magnetic field at the centre of the coil is given by the relation,

$$|B| = \frac{\mu_0}{4\pi}\frac{2\pi n I}{r}$$

Where,

$$B = \frac{4\pi\times10^{-7}\times2\times35}{4\pi\times0.2}$$

$$= 3.5\times10^{-5}T$$

μ_0 = Permeability of free space

= $4\pi \times 10^{-7}$ T m A^{-1}

$$|B| = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{2\pi\times100\times0.4}{0.08}$$

$$= 3.14\times10^{-4}T$$

Hence, the magnitude of the magnetic field is $= 3.14\times10^{-4}T$.

---

7. A long straight wire carries a current of 35 A. What is the magnitude of the field B at a point 20 cm from the wire?

Ans. Current in the wire, I = 35 A

Distance of a point from the wire, r = 20 cm = 0.2 m

Magnitude of the magnetic field at this point is given as:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

Where,

μ_0 = Permeability of free space = $= 4n\times10^{-7} TmA^{-1}$

$$B = \frac{4\pi\times10^{-7}\times2\times35}{4\pi\times0.2}$$

$$= 3.5\times10^{-5} T$$

Hence, the magnitude of the magnetic field at a point 20 cm from the wire is $= 3.5\times10^{-5} T$

---

8. A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of B at a point 2.5 m east of the wire.

Ans. Current in the wire, I = 50 A

A point is 2.5 m away from the East of the wire.

$\therefore$ Magnitude of the distance of the point from the wire, r = 2.5 m.

Magnitude of the magnetic field at that point is given by the relation, $B = \frac{\mu_0 2I}{4\pi r}$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} T\ m\ A^{-1}$

$$B = \frac{4\pi\times10^{-7}\times2\times50}{4\pi\times2.5}$$

$$= 4\times10^{-6} T$$

The point is located normal to the wire length at a distance of 2.5 m. The direction of the current in the wire is vertically downward. Hence, according to the Maxwell's right hand thumb rule, the direction of the magnetic field at the given point is vertically upward.

---

9. A horizontal overhead power line carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

Ans. Current in the power line, I = 90 A

Point is located below the power line at distance, r = 1.5 m

Hence, magnetic field at that point is given by the relation,

$$B = \frac{\mu_0 2I}{4\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} T\ m\ A^{-1}$

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 50}{4\pi \times 1.5}$$

$$1.2 \times 10^{-5} T$$

The current is flowing from East to West. The point is below the power line. Hence, according to Maxwell's right hand thumb rule, the direction of the magnetic field is towards the South.

---

10. What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30º with the direction of a uniform magnetic field of 0.15 T?

Ans. Current in the wire, I = 8 A

Magnitude of the uniform magnetic field, B = 0.15 T

Angle between the wire and magnetic field, θ = 30°.

Magnetic force per unit length on the wire is given as:

$f = BI \sin\theta$

= $0.15 \times 8 \times 1 \times sin 30°$

= $0.6\ N\ m^{-1}$

Hence, the magnetic force per unit length on the wire is $0.6\ N\ m^{-1}$.

---

11. A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

Ans. Length of the wire, l = 3 cm = 0.03 m

Current flowing in the wire, I = 10 A

Magnetic field, B = 0.27 T

Angle between the current and magnetic field, θ = 90°

Magnetic force exerted on the wire is given as:

$F = BIl \sin\theta$

$= 0.27 \times 10 \times 0.03\ sin 90°$

$= 8.1 \times 10^{-2} N$

Hence, the magnetic force on the wire is $8.1 \times 10^{-2} N$. The direction of the force can be obtained from Fleming's left hand rule.

---

12. Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

Ans. Current flowing in wire A, $I_A = 8.0\ A$

Current flowing in wire B, $I_B = 5.0\ A$

Distance between the two wires, r = 4.0 cm = 0.04 m

Length of a section of wire A, l = 10 cm = 0.1 m

Force exerted on length l due to the magnetic field is given as:

$$B = \frac{\mu_0 2 I_A I_B l}{4\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} T\ m\ A^{-1}$

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 8 \times 5 \times 0.1}{4\pi \times 0.04}$$

$$= 2 \times 10^{-5} N$$

The magnitude of force is $= 2 \times 10^{-5} N$. This is an attractive force normal to A towards B because the direction of the currents in the wires is the same.

---

13. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of B inside the solenoid near its centre.

Ans. Length of the solenoid, l = 80 cm = 0.8 m

There are five layers of windings of 400 turns each on the solenoid.

$\therefore$ Total number of turns on the solenoid, N = 5 × 400 = 2000

Diameter of the solenoid, D = 1.8 cm = 0.018 m

Current carried by the solenoid, I = 8.0 A

Magnitude of the magnetic field inside the solenoid near its centre is given by the relation,

$$B = \frac{\mu_0 N I}{l}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} T\ m\ A^{-1}$

$$B = \frac{4\pi\times10^{-7}\times2000\times8}{0.8}$$

$$=8\pi\times10^{-3}$$

$$= 2.512\times10^{-2}T$$

Hence, the magnitude of the magnetic field inside the solenoid near its centre is $= 2.512\times10^{-2}T$

---

14. A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?

Ans. Length of a side of the square coil, l = 10 cm = 0.1 m

Current flowing in the coil, I = 12 A

Number of turns on the coil, n = 20

Angle made by the plane of the coil with magnetic field, $\theta = 30^\circ$

Strength of magnetic field, B = 0.80 T

Magnitude of the magnetic torque experienced by the coil in the magnetic field is given by the relation,

T = $n\,BIA\sin\theta$

Where,

A = Area of the square coil

$$\Rightarrow l\times l = 0.1\times0.1 = 0.01\ m^2$$

$$\therefore T = 20\times0.8\times12\times0.01\times sin30^\circ$$

= 0.96 N m

Hence, the magnitude of the torque experienced by the coil is 0.96 N m.

15. (a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of $60°$ with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

(b) Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

Ans. (a) Number of turns on the circular coil, n = 30

Radius of the coil, r = 8.0 cm = 0.08 m

Area of the coil $= \pi r^2 = \pi(0.08)^2 = 0.0201\, m^2$

Current flowing in the coil, I = 6.0 A

Magnetic field strength, B = 1 T

Angle between the field lines and normal with the coil surface, $\theta = 60°$

The coil experiences a torque in the magnetic field. Hence, it turns. The counter torque applied to prevent the coil from turning is given by the relation,

$T = nIBA\sin\theta$ … (*i*)

= $30 \times 6 \times 1 \times 0.0201 \times \sin 60°$

= 3.133 N m

(b) It can be inferred from relation (*i*) that the magnitude of the applied torque is not dependent on the shape of the coil. It depends on the area of the coil. Hence, the answer would not change if the circular coil in the above case is replaced by a planar coil of some irregular shape that encloses the same area.

16. A magnetic field of 100 G $(1\ G = 10^{-4}\ T)$ is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about $10^{-3}\ m^2$. The maximum current-carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic.

Ans. Magnetic field strength, B = 100 $G = 100\times10^{-4}\ T$

Number of turns per unit length, $n = 1000\ turns\ m^{-1}$

Current flowing in the coil, I = 15 A

Permeability of free space, $\mu_0 = 4\pi\times10^{-7}\ T\ m\ A^{-1}$

Magnetic field is given by the relation, $B = \mu_0 nI$

$$\therefore nI = \frac{B}{\mu_0}$$

$$= \frac{100\times10^{-4}}{4\pi\times10^{-7}} = 7957.74$$

$$\approx 8000\ A/m$$

If the length of the coil is taken as 50 cm, radius 4 cm, number of turns 400, and current 10 A, then these values are not unique for the given purpose. There is always a possibility of some adjustments with limits.

---

17. A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm, around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid, (b) inside the core of the toroid, and (c) in the empty space surrounded by the toroid.

Ans. Inner radius of the toroid, $r_1 = 25\ cm$ = 0.25 m

Outer radius of the toroid, $r_2 = 26\ cm$ = 0.26 m

Number of turns on the coil, N = 3500

Current in the coil, I = 11 A

(a) Magnetic field outside a toroid is zero. It is non-zero only inside the core of a toroid.

(b) Magnetic field inside the core of a toroid is given by the relation,

$$B = \frac{\mu_0 N I}{l}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} T\ m\ A^{-1}$

l = length of toroid

$$= 2\pi\left[\frac{r_1 + r_2}{2}\right]$$

$$= \pi(0.25 + 0.26)$$

$$= 0.51\pi$$

$$\therefore B = \frac{4\pi \times 10^{-7} \times 3500 \times 11}{0.51\pi}$$

$$\approx 3.0 \times 10^{-2} T$$

(c) Magnetic field in the empty space surrounded by the toroid is zero.

---

18. Answer the following questions:

(a) A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?

(b) A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?

(c) An electron travelling west to east enters a chamber having a uniform electrostatic field in north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight line path.

Ans.(a) The initial velocity of the particle is either parallel or anti-parallel to the magnetic field. Hence, it travels along a straight path without suffering any deflection in the field.

(b) Yes, the final speed of the charged particle will be equal to its initial speed. This is because magnetic force can change the direction of velocity, but not its magnitude.

(c) An electron travelling from West to East enters a chamber having a uniform electrostatic field in the North-South direction. This moving electron can remain undeflected if the electric force acting on it is equal and opposite of magnetic field. Magnetic force is directed towards the South. According to Fleming's left hand rule, magnetic field should be applied in a vertically downward direction.

---

19. A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

(b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before? (Ignore the mass of the wires.) $g = 9.8\ m\,s^{-2}$.

Ans. Length of the rod, l = 0.45 m

Mass suspended by the wires, m = 60 g = $60 \times 10^{-3}\ kg$

Acceleration due to gravity, $g = 9.8\ m\,s^{-2}$

Current in the rod flowing through the wire, I = 5 A

(a) Magnetic field (B) is equal and opposite to the weight of the wire i.e., BIl + mg

$$\therefore B = \frac{mg}{Il}$$

$$= \frac{60 \times 10^{-3} \times 9.8}{5 \times 0.45} = 0.26T$$

A horizontal magnetic field of 0.26 T normal to the length of the conductor should be set up in order to get zero tension in the wire. The magnetic field should be such that Fleming's left hand rule gives an upward magnetic force.

(b) If the direction of the current is revered, then the force due to magnetic field and the weight of the wire acts in a vertically downward direction.

∴Total tension in the wire = $BIl + mg$

$$0.26 \times 5 \times 0.45 + \left(60 \times 10^{-3}\right) \times 9.8$$

= 1.176 N

---

20. The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart? Is the force attractive or repulsive?

Ans. Current in both wires, I = 300 A

Distance between the wires, r = 1.5 cm = 0.015 m

Length of the two wires, l = 70 cm = 0.7 m

Force between the two wires is given by the relation,

$$F = \frac{\mu_0 I^2}{2\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} T\ m\ A^{-1}$

$$\therefore F = \frac{4\pi \times 10^{-7} \times (300)^2}{2\pi \times 0.015}$$

=1.2 N/m

Since the direction of the current in the wires is opposite, a repulsive force exists between them.

---

21. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

(a) total torque on the coil,

(b) total force on the coil,

(c) average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area $10^{-5}\ m^2$, and the free electron density in copper is given to be about $10^{29}\ m^{-3}$.)

Ans. Number of turns on the circular coil, n = 20

Radius of the coil, r = 10 cm = 0.1 m

Magnetic field strength, B = 0.10 T

Current in the coil, I = 5.0 A

(a) The total torque on the coil is zero because the field is uniform.

(b) The total force on the coil is zero because the field is uniform.

(c) Cross-sectional area of copper coil, A = $10^{-5}\ m^2$

Number of free electrons per cubic meter in copper, N = $10^{29}\ m^{-3}$

Charge on the electron, $e = 1.6 \times 10^{-19}\ C$

Magnetic force, $F = Bev_d$

Where,

v_d = Drift velocity of electrons $= \frac{I}{NeA}$

$\therefore F = \frac{BeI}{NeA}$

$= \frac{0.10 \times 5.0}{10^{29} \times 10^{-5}}$

$5 \times 10^{-25} N.$

Hence, the average force on each electron is $5 \times 10^{-25} N.$

---

22. galvanometer coil has a resistance of 12 Ω and the metre shows full scale deflection for a current of 3 mA. How will you convert the metre into a voltmeter of range 0 to 18 V?

Ans. Resistance of the galvanometer coil, *G* = 12 Ω

Current for which there is full scale deflection, I_g = 3 mA = $3 \times 10^{-3} A$

Range of the voltmeter is 0, which needs to be converted to 18 V.

$\therefore V = 18V$

Let a resistor of resistance *R* be connected in series with the galvanometer to convert it into a voltmeter. This resistance is given as:

$R = \frac{V}{I_g} - G$

$= \frac{18}{3 \times 10^{-3}} - 12 = 6000 - 12$

$= 5988\,\Omega$

Hence, a resistor of resistance $5988\,\Omega$ is to be connected in series with the galvanometer.

---

23. A galvanometer coil has a resistance of 15 Ω and the metre shows full scale deflection for a current of 4 mA. How will you convert the metre into an ammeter of range 0 to 6 A?

Ans. Resistance of the galvanometer coil, *G* = 15 Ω

Current for which the galvanometer shows full scale deflection,

I_g = 4 mA = $4 \times 10^{-3}\ A$

Range of the ammeter is 0, which needs to be converted to 6 A.

$\therefore$ Current, *I* = 6 A

A shunt resistor of resistance *S* is to be connected in parallel with the galvanometer to convert it into an ammeter. The value of *S* is given as:

$$S = \frac{I_g G}{I - I_g}$$

$$= \frac{4 \times 10^{-3} \times 15}{6 - 4 \times 10^{-3}}$$

$$S = \frac{6 \times 10^{-2}}{6 - 0.004} = \frac{0.06}{5.996}$$

$$\approx 0.01\,\Omega = 10\,m\Omega$$

Hence, a $10\,m\Omega$ shunt resistor is to be connected in parallel with the galvanometer.

5 Mark Questions

1. (a) What is cyclotron? Explain its working principle?

(b) A cyclotron's oscillator frequency is 10MHz what should be the operating magnetic field for accelerating protons? If radius of its dees is 20cm, what is the K.E. of the proton beam produced by the accelerator? ($e = 1.6 \times 10^{-19} c$**,** $mp = 1.6 \times 10^{-27} kg$**,** $1Mev = 1.602 \times 10^{-13} J$ **)?**

Ans. (a) It is a device used to accelerate charged particles like protons, deuterons, $\propto$ -particle etc.

It is based on the principle that a charged particle can be accelerated to very high energies by making it pass through a moderate electric field a number of times and applying a strong magnetic field at the same time.

(b) v = 10MHz = 10×10^6 Hz

$e = 1.6 \times 10^{-19} c$

$mp = 1.6 \times 10^{-27} kg$

r = 20cm = $20 \times 10^{-2} m$

KE = $\dfrac{q^2 B^2 r^2}{2m}$

Using $v = \dfrac{qB}{2\pi m}$

$$B = \frac{2\pi m V}{q}$$

$$B = \frac{2\times3.14\times1.6\times10^{-27}\times10^{7}}{1.6\times10^{-19}}$$

$$B = 0.628\ T$$

$$KE = \frac{\left(1.6\times10^{-19}\right)^2\times(0.66)^2\times(0.2)^2}{2\times1.67\times10^{-27}}$$

$$KE = 13.35\times 10^{-13}\ \text{J}$$

$$1.602\times10^{-13}\,Joules = 1\text{MeV}$$

Since $\Rightarrow 12.02\times 10^{-13}\text{J has } \frac{12.02\times10^{-13}}{1.602\times10^{-13}}MeV$

$$KE = 8.3MeV$$

---

2. (a) Draw a labelled diagram of a moving coil galvanometer. Prove that in a radial magnetic field, the deflection of the coil is directly proportional to the current flowing in the coil.

(b) A galvanometer can be converted into a voltmeter to measure upto

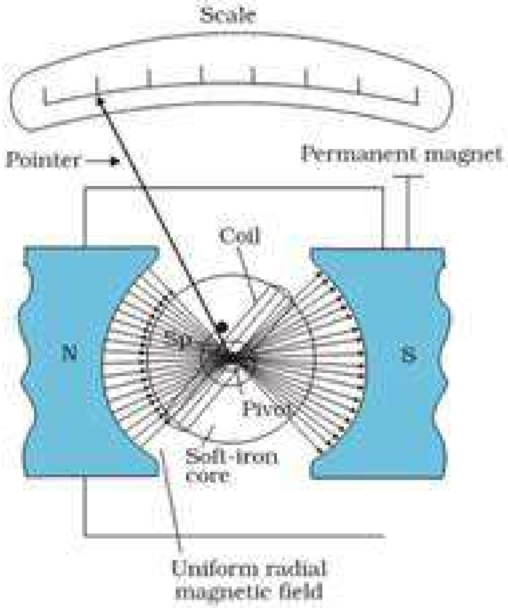
(i) V volt by connecting a resistance R_1 series with the coil

(ii) $V/2$ volt by connecting a resistance R_2 in series with coil Find R in terms of $R_1\,and\ R_2$ required to convert – it into a voltmeter that can read upto '2v' volt.

Ans. (a) When a current I is passed through a coil two equal and opposite forces acts on the arms of a coil to form a couple which exerts a Torque on the coil.

=> $\tau = \text{NIAB}\sin\theta$

If θ = $90^0\left(sin90^0 = 1\right)$



θ is the angle made by the normal to the plane of coil with B

τ = NIAB ----(1)

This is called as deflecting torque

As the coil deflected the spring is twisted and a restoring torque per unit twist then the restoring torque for the deflecting & is given by

τ' = k ϕ ----(2)

In equilibrium

Deflecting Torgue=Restoring Torgue

NIAB = Kϕ

I = $\frac{K\phi}{NAB}\phi$

I = Gϕ where G = $\frac{K}{NAB}$ (galvanometer constant)

=> $I \propto \phi$

Thus deflection of the coil is directly proportional to the current flowing in the coil.

(b) We know Ig = $\frac{V}{R+R_G}$

=> Ig = $\frac{V}{R_1+R_G}$ -----(1)

And Ig = $\frac{\frac{v}{2}}{R_2+R_G}$ -------- (2)

Equating (1) & (2)

$$\frac{V}{R_1+R_G}=\frac{\frac{v}{2}}{R_2+R_G}$$

$$Ie\ R_1+R_G = 2\left(R_2+R_G\right)$$

$$R_G = -2R_2 + R_1$$

For conversion Ig = $\frac{2V}{R+R_G}$

=> Ig $\frac{V}{R1+R_G}=\frac{2V}{R+R_G}$

$$Ig = 2R_1 + 2R_G = R + R_G$$

$$R = 2R_1 + R_G$$

$$R = 2R_1 + R_1 - 2R_2$$

$$R = 3R_1 - 2R_2$$

---

3. Two moving coil meters, M_1 and M_2 have the following particulars:

$R_1 = 10\Omega$, $N_1 = 30$,

$A_1 = \mathbf{3.6 \times 10^{-3}\, m^2}$, $B_1 = 0.25\ T$

$R_2 = 14\,\Omega$, $N_2 = 42$,

$A_2 = \mathbf{1.8 \times 10^{-3}\, m^2}$, $B_2 = 0.50\ T$

(The spring constants are identical for the two meters).

Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of $M_2\ and\ M_1$.

Ans. For moving coil meter M_1:

Resistance, $R_1 = 10\Omega$

Number of turns, $N_1 = 30$,

Area of cross-section, $A_1 = 3.6 \times 10^{-3}\, m^2$

Magnetic field strength, $B_1 = 0.25\ T$

Spring constant $K_1 = K$

For moving coil meter M_2:

Resistance, $R_2 = 14\,\Omega$

Number of turns, $N_2 = 42$,

Area of cross-section, $A_2 = 1.8 \times 10^{-3}\, m^2$

Magnetic field strength, $B_2 = 0.50\ T$

Spring constant, $K_2 = K$

(a) Current sensitivity of M_1 is given as:

$$I_{s1} = \frac{N_1 B_1 A_1}{K_1}$$

And, current sensitivity of M_2 is given as:

$$I_{s1} = \frac{N_2 B_2 A_2}{K_2}$$

$\therefore$ Ratio $\frac{I_{s2}}{I_{s1}} = \frac{N_2 B_2 A_2 K_1}{K_2 N_1 B_1 A_1}$

$$= \frac{42 \times 0.5 \times 1.8 \times 10^{-3} \times K}{K \times 30 \times 0.25 \times 3.6 \times 10^{-3}} = 1.4$$

Hence, the ratio of current sensitivity of $M_2\, to\, M_1$ is 1.4.

(b) Voltage sensitivity for M_2 is given as:

$$V_{s2} = \frac{N_2 B_2 A_2}{K_2 R_2}$$

And, voltage sensitivity for M_1 is given as:

---

4. In a chamber, a uniform magnetic field of 6.5 G $(1\ G = 10^{-4}\ T)$ is maintained. An electron is shot into the field with a speed of $4.8 \times 10^6\ \mathrm{m\ s^{-1}}$ normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. (*e* = $1.6 \times 10^{-19}\ \mathrm{C}$, *m*$_e$= $9.1 \times 10^{-31}\ \mathrm{kg}$)

Ans. Magnetic field strength, *B* = 6.5 G = $6.5 \times 10^{-4}\ T$

Speed of the electron, *v* = $4.8 \times 10^6\ \mathrm{m/s}$

Charge on the electron, *e* = $1.6 \times 10^{-19}\ \mathrm{C}$

Mass of the electron, $m_e = 9.1 \times 10^{-31}\ \mathrm{kg}$

Angle between the shot electron and magnetic field, $\theta = 90^\circ$

Magnetic force exerted on the electron in the magnetic field is given as:

$F = evB \sin\theta$

This force provides centripetal force to the moving electron. Hence, the electron starts moving in a circular path of radius *r*.

Hence, centripetal force exerted on the electron,

$$F_c = \frac{mv^2}{r}$$

In equilibrium, the centripetal force exerted on the electron is equal to the magnetic force i.e.,

$$F_c = F$$

$$\frac{mv^2}{r} = evB \sin\theta$$

$$r = \frac{mv}{B e \sin\theta}$$

$$= \frac{9.1\times10^{-31}\times4.8\times10^{6}}{6.5\times10^{-4}\times1.6\times10^{-19}\times\sin 90^\circ}$$

$$4.2\times10^{-2} m$$

= 4.2 cm

Hence, the radius of the circular orbit of the electron is 4.2 cm.

---

5. In Exercise 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.

Ans. Magnetic field strength, $B = 6.5\times10^{-4} T$

Charge of the electron, $e = 1.6 \times 10^{-19}\,C$

Mass of the electron, $m_e = 9.1 \times 10^{-31}\,kg$

Velocity of the electron, $v = 4.8 \times 10^{6}\,m/s$

Radius of the orbit, r = 4.2 cm = 0.042 m

Frequency of revolution of the electron = ν

Angular frequency of the electron = ω = 2nv

Velocity of the electron is related to the angular frequency as:

$v = r\omega$

In the circular orbit, the magnetic force on the electron is balanced by the centripetal force. Hence, we can write:

$$evB = \frac{mv^2}{r}$$

$$eB = \frac{m}{r}(r\omega) = \frac{m}{r}(r2\pi\nu)$$

$$\nu = \frac{Be}{2\pi m}$$

This expression for frequency is independent of the speed of the electron.

On substituting the known values in this expression, we get the frequency as:

$$\nu = \frac{6.5 \times 10^{-4} \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$$

$$= 18.2 \times 10^{6}\,Hz$$

$$\approx 18\,MHz$$

Hence, the frequency of the electron is around 18 MHz and is independent of the speed of the electron.

---

6. Two concentric circular coils X and Y of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

Ans. Radius of coil X, $r_1 = 16\ cm$ = 0.16 m

Radius of coil Y, $r_2 = 10\ cm$ = 0.1 m

Number of turns of on coil X, $n_1 = 20$

Number of turns of on coil Y, $n_2 = 25$

Current in coil X, $I_1 = 16\ A$

Current in coil Y, $I_2 = 18\ A$

Magnetic field due to coil X at their centre is given by the relation,

$$B = \frac{\mu_0 n_1 I_1}{2r_1}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} T\ m\ A^{-1}$

$$\therefore B_1 = \frac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 0.16}$$

$= 4\pi \times 10^{-4} T$ (towards East)

Magnetic field due to coil Y at their centre is given by the relation, $B = \frac{\mu_0 n_2 I_2}{2r_2}$

$$B_2 = \frac{4\pi\times10^{-7}\times25\times18}{2\times0.10}$$

$=9\pi\times10^{-4}T$ (towards East)

Hence, net magnetic field can be obtained as:

$$B = B_2 - B_1$$

$$= 9\pi\times10^{-4} - 4\pi\times10^{-4}$$

$$= 5\pi\times10^{-4}T$$

$$= 1.57\times10^{-3}T\left(towards\ west\right)$$

---

7. For a circular coil of radius *R* and *N* turns carrying current *I*, the magnitude of the magnetic field at a point on its axis at a distance *x* from its centre is given by,

$$B = \frac{\mu_0 IR^2 N}{2\left(x^2+R^2\right)^{\frac{3}{2}}}$$

(a) Show that this reduces to the familiar result for field at the centre of the coil.

(b) Consider two parallel co-axial circular coils of equal radius *R*, and number of turns *N*, carrying equal currents in the same direction, and separated by a distance *R*. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to *R*, and is given by, $B = 0.72 - \frac{\mu_0 BNI}{R}$ **,**

approximately. [Such an arrangement to produce a nearly uniform magnetic field over a small region is known as *Helmholtz coils*.]

Ans. Radius of circular coil = *R*

Number of turns on the coil = *N*

Current in the coil = *I*

Magnetic field at a point on its axis at distance *x* is given by the relation,

$$B = \frac{\mu_0 I R^2 N}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

Where,

μ_0 = Permeability of free space

(a) If the magnetic field at the centre of the coil is considered, then $x = 0$.

$$\therefore B = \frac{\mu_0 I R^2 N}{2R^3} = \frac{\mu_0 I N}{2R}$$

This is the familiar result for magnetic field at the centre of the coil.

(b) Radii of two parallel co-axial circular coils = R

Number of turns on each coil = N

Current in both coils = I

Distance between both the coils = R

Let us consider point Q at distance d from the centre.

Then, one coil is at a distance of $\frac{R}{2} + d$ from point Q.

$\therefore$ Magnetic field at point Q is given as:

$$B_1 = \frac{\mu_0 N I R^2}{2\left[\left(\frac{R}{2} + d\right)^2 + R^2\right]^{\frac{3}{2}}}$$

Also, the other coil is at a distance of $\frac{R}{2} - d$ from point Q.

∴ Magnetic field due to this coil is given as:

$$B_1 = \frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}-d\right)^2 + R^2\right]^{\frac{3}{2}}}$$

Total magnetic field, $B = B_1 + B_2$

$$= \frac{\mu_0 IR^2}{2}\left[\left\{\left(\frac{R}{2}-d\right)^2 + R^2\right\}^{-\frac{3}{2}} + \left\{\left(\frac{R}{2}+d\right)^2 + R^2\right\}^{-\frac{3}{2}}\right]$$

$$= \frac{\mu_0 IR^2}{2}\left[\left\{\frac{5R^2}{4} + d^2 - Rd\right\}^{-\frac{3}{2}} + \left\{\frac{5R^2}{4} + d^2 + Rd\right\}^{-\frac{3}{2}}\right]$$

$$= \frac{\mu_0 IR^2}{2} \times \left(\frac{5R^2}{4}\right)^{-\frac{3}{2}}\left[\left(1 + \frac{4}{5}\frac{d^2}{R^2} - \frac{4}{5}\frac{d}{R}\right)^{-\frac{3}{2}} + \left(1 + \frac{4}{5}\frac{d^2}{R^2} + \frac{4}{5}\frac{d}{R}\right)^{-\frac{3}{2}}\right]$$

For d << R, neglecting the factor $\frac{d^2}{R^2}$, we get:

$$\approx \frac{\mu_0 IR^2}{2} \times \left(\frac{5R^2}{4}\right)^{-\frac{3}{2}} \times \left[\left(1 - \frac{4d}{5R}\right)^{-\frac{3}{2}} + \left(1 + \frac{4d}{5R}\right)^{-\frac{3}{2}}\right]$$

$$\approx \frac{\mu_0 IR^2 N}{2R^3} \times \left(\frac{4}{5}\right)^{-\frac{3}{2}} \times \left[1 - \frac{6d}{5R} + 1\frac{6d}{5R}\right]$$

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 IN}{R} = 0.72\left(\frac{\mu_0 IN}{R}\right)$$

Hence, it is proved that the field on the axis around the mid-point between the coils is uniform.

8. An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30º with the initial velocity.

Ans. Magnetic field strength, *B* = 0.15 T

Charge on the electron, $e = 1.6 \times 10^{-19}\ C$

Mass of the electron, $m = 9.1 \times 10^{-31}\ kg$

Potential difference, *V* = 2.0 $kV = 2 \times 10^{3}\ V$

Thus, kinetic energy of the electron = eV

$$\Rightarrow eV = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2eV}{m}} \quad \text{......(1)}$$

Where,

v = velocity of the electron

(a) Magnetic force on the electron provides the required centripetal force of the electron. Hence, the electron traces a circular path of radius *r*.

Magnetic force on the electron is given by the relation,

B ev

Centripetal force $= \frac{mv^2}{r}$

$$\therefore Bev = \frac{mv^2}{r}$$

$r = \dfrac{mv}{Be}$ ......(2)

From equations (1) and (2), we get

$$r = \frac{m}{Be}\left[\frac{2eV}{m}\right]^{\frac{1}{2}}$$

$$= \frac{9.1\times10^{-31}}{0.15\times1.6\times10^{-19}}\times\left(\frac{2\times1.6\times10^{-19}\times2\times10^{3}}{9.1\times10^{-31}}\right)^{\frac{1}{2}}$$

$$= 100.55\times10^{-5}$$

$$= 1.01\times10^{-3}\,m$$

= 1 mm

Hence, the electron has a circular trajectory of radius 1.0 mm normal to the magnetic field.

(b) When the field makes an angle $\theta\, of\;\, 30^\circ$ with initial velocity, the initial velocity will be,

$$v_1 = v\sin\theta$$

From equation (2), we can write the expression for new radius as: $r_1 = \dfrac{mv_1}{Be}$

$$= \frac{mv\sin\theta}{Be}$$

$$= \frac{9.1\times10^{-31}}{0.15\times1.6\times10^{-19}}\times\left(\frac{2\times1.6\times10^{-19}\times2\times10^{3}}{9\times10^{-31}}\right)^{\frac{1}{2}}\times\sin30^\circ$$

$$= 0.5\times10^{-3}\,m$$

= 0.5 mm

Hence, the electron has a helical trajectory of radius 0.5 mm along the magnetic field direction.

---

9. A magnetic field set up using Helmholtz coils (described in Exercise 4.16) is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is $9.0\times10^{-5}\ \text{V m}^{-1}$, make a simple guess as to what the beam contains. Why is the answer not unique?

Ans. Magnetic field, *B* = 0.75 T

Accelerating voltage, *V* = 15 $kV = 15\times10^{3}\,V$

Electrostatic field, *E* = $9.0\times10^{-5}\ \text{V m}^{-1}$

Mass of the electron = *m*

Charge of the electron = *e*

Velocity of the electron = *v*

Kinetic energy of the electron = *eV*

$$\Rightarrow \frac{1}{2}mv^2 = eV$$

$$\therefore \frac{e}{m} = \frac{v^2}{2V} \quad \text{...... (1)}$$

Since the particle remains unelected by electric and magnetic fields, we can infer that the electric field is balancing the magnetic field.

$$\therefore eE = evB$$

$$v = \frac{E}{B} \quad \text{........(2)}$$

Putting equation (2) in equation (1), we get

$$\frac{e}{m}=\frac{1}{2}\frac{\left(\frac{E}{B}\right)^2}{V}=\frac{E^2}{2VB^2}$$

$$=\frac{\left(9.0\times10^5\right)^2}{2\times15000\times(0.75)^2}$$

$$=4.8\times10^7\ C/kg$$

This value of specific charge *e*/*m* is equal to the value of deuteron or deuterium ions. This is not a unique answer. Other possible answers are He^{++}, Li^{++} , etc.

---

10. A uniform magnetic field of 1.5 T exists in a cylindrical region of radius10.0 cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,

(a) the wire intersects the axis,

(b) the wire is turned from N-S to northeast-northwest direction,

(c) the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm?

Ans. Magnetic field strength, B = 1.5 T

Radius of the cylindrical region, r = 10 cm = 0.1 m

Current in the wire passing through the cylindrical region, I = 7 A

(a) If the wire intersects the axis, then the length of the wire is the diameter of the cylindrical region.

Thus, l = 2r = 0.2 m

Angle between magnetic field and current, $\theta = 90°$

Magnetic force acting on the wire is given by the relation,

$F = BIl \sin\theta$

$= 1.5 \times 7 \times 0.2 \times \sin 90°$

= 2.1 N

Hence, a force of 2.1 N acts on the wire in a vertically downward direction.

(b) New length of the wire after turning it to the Northeast-Northwest direction can be given as:

$$l_1 = \frac{l}{\sin\theta}$$

Angle between magnetic field and current, $\theta = 45°$

Force on the wire,

$F = BIl_1 \sin\theta$

$= BIl$

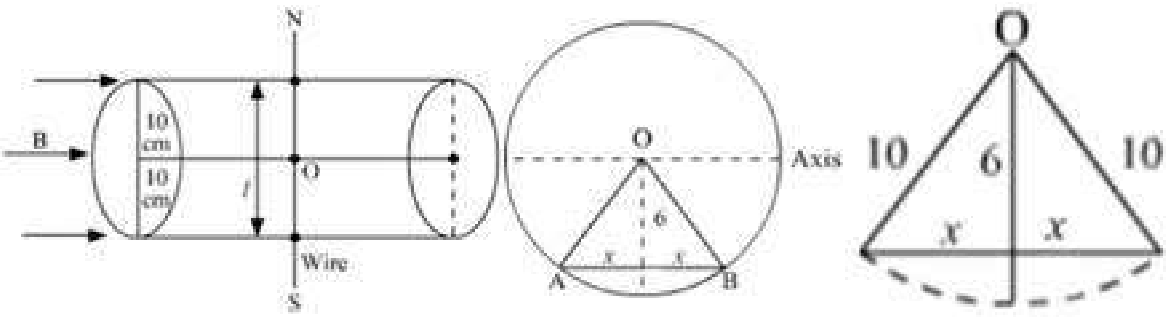
$= 1.5 \times 7 \times 0.2$

=2.1 N

Hence, a force of 2.1 N acts vertically downward on the wire. This is independent of angle θ because $l \sin\theta$ is fixed.

(c) The wire is lowered from the axis by distance, $d = 6.0$ cm

Suppose wire is passing perpendicularly to the axis of cylindrical magnetic field then lowering 6 cm means displacing the wire 6 cm from its initial position towards to end of cross sectional area.



$x = \sqrt{10^2 - 6^2}$

= 8cm

Thus the length of wire in magnetic field will be 16 cm as AB= L =2x =16 cm

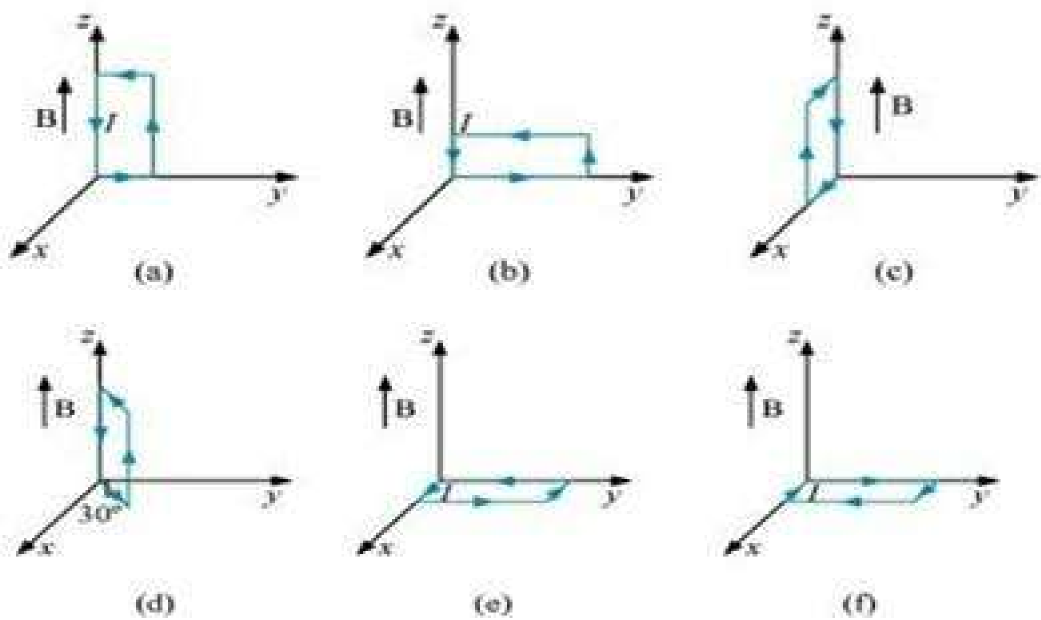
Now the force,

$F = iLB \; sin90^\circ$ as the wire will be perpendicular to the magnetic field.

$F = 7 \times 0.16 \times 1.5 = 1.68\ N$ The direction will be given by right hand curl rule or screw rule i.e. vertically downwards.

---

11. A uniform magnetic field of 3000 G is established along the positive *z*-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in Fig. 4.28? What is the force on each case? Which case corresponds to stable equilibrium?



Ans. Magnetic field strength, B = 3000 G = $3000\times10^{-4}\,T$ = 0.3 T

Length of the rectangular loop, l = 10 cm

Width of the rectangular loop, b = 5 cm

Area of the loop,

$$A = l\times b = 10\times5 = 50\ cm^2 = 50\times10^{-4}\,m^2$$

Current in the loop, I = 12 A

Now, taking the anti-clockwise direction of the current as positive and vise-versa:

(a) Torque, $\bar{\tau} = I\,\bar{A}\times\bar{B}$

From the given figure, it can be observed that A is normal to the y-z plane and B is directed along the z-axis.

$$\therefore \tau = 12\times\left(50\times10^{-4}\right)\hat{j}\times0.3\hat{k}$$

$$1.8\times10^{-2}\,\hat{j}\,Nm$$

The torque is $1.8\times10^{-2}\,Nm$ along the negative y-direction. The force on the loop is zero because the angle between A and B is zero.

(b) This case is similar to case (a). Hence, the answer is the same as (a).

(c) Torque $\tau = I\,\bar{A}\times\bar{B}$

From the given figure, it can be observed that A is normal to the x-z plane and B is directed along the z-axis.

$$\therefore \tau = -12\times\left(50\times10^{-4}\right)\hat{j}\times0.3\hat{k}$$

$$1.8\times10^{-2}\hat{i}\,Nm$$

The torque is $1.8\times10^{-2}\,Nm$ along the negative x direction and the force is zero.

(d) Magnitude of torque is given as:

$|\tau| = IAB$

$= 12 \times 50 \times 10^{-4} \times 0.3$

$1.8 \times 10^{-2}\ Nm$

Torque is $1.8 \times 10^{-2}\ Nm$ at an angle of $240°$ with positive *x* direction. The force is zero.

(e) Torque $\tau = I\bar{A} \times \bar{B}$

$= \left(50 \times 10^{-4} \times 12\right)\hat{k} \times 0.3\hat{k}$

= 0

Hence, the torque is zero. The force is also zero.

(f) Torque $\tau = I\bar{A} \times \bar{B}$

$= \left(50 \times 10^{-4} \times 12\right)\hat{k} \times 0.3\hat{k}$

= 0

Hence, the torque is zero. The force is also zero.

In case (e), the direction of $I\bar{A}$ and $\bar{B}$ is the same and the angle between them is zero. If displaced, they come back to an equilibrium. Hence, its equilibrium is stable.

Whereas, in case (f), the direction of $I\bar{A}$ and $\bar{B}$ is opposite. The angle between them is 180°. If disturbed, it does not come back to its original position. Hence, its equilibrium is unstable.

---

12. A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with

appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? $g = 9.8\ m\ s^{-2}$

Ans. Length of the solenoid, *L* = 60 cm = 0.6 m

Radius of the solenoid, *r* = 4.0 cm = 0.04 m

It is given that there are 3 layers of windings of 300 turns each.

$\therefore$ Total number of turns, $n = 3 \times 300$ = 900

Length of the wire, *l* = 2 cm = 0.02 m

Mass of the wire, *m* = 2.5 g = $2.5 \times 10^{-3}\ kg$

Current flowing through the wire, *i* = 6 A

Acceleration due to gravity, $g = 9.8\ m/s^2$

Magnetic field produced inside the solenoid, $B = \frac{\mu_0 nI}{L}$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} T\ m\ A^{-1}$

I = Current flowing through the windings of the solenoid

Magnetic force is given by the relation, F = Bil

$= \frac{\mu_0 nI}{L} il$

Also, the force on the wire is equal to the weight of the wire.

$\therefore mg = \frac{\mu_0 nIil}{L}$

$I = \frac{mgL}{\mu_0 nil}$

$$= \frac{2.5 \times 10^{-3} \times 9.8 \times 0.6}{4\pi \times 10^{-7} \times 900 \times 0.02 \times 6} = 108 A$$

Hence, the current flowing through the solenoid is 108 A.

CBSE Class 12 physics

Important Questions

Chapter 5

Magnetism and Matter

1 Mark Questions

1. How does the intensity of magnetization of a paramagnetic material vary with increasing applied magnetic field?

Ans. Intensity of magnetization increases with the increase in applied magnetic field.

---

2. An iron bar magnet is heated to 1000^0C and then cooled in a magnetic field free space. Will it retain magnetism?

Ans. Curie temperature of iron is about 770^0C but when it is heated to a very higher temperature magnetism of iron further gets lost and it will not retain

magnetism.

---

3. How will the magnetic field intensity at the centre of a circular wire carrying current change, if the current through the wire is doubled and radius of the coil is halved?

Ans. Since B = $\frac{\mu_o}{4\pi}\frac{2\pi I}{r}$

$$B' = \frac{\mu_o}{4\pi}\frac{2\pi(2I)}{r/2}$$

$$B' = 4\left(\frac{\mu_o}{4\pi}\frac{2\pi I}{r}\right)$$

B'= 4B

4. Can neutrons be accelerated in a cyclotron? Why?

Ans. No, neutrons cannot be accelerated in a cyclotron because neutron is neutral and cyclotron can accelerate only charged particles.

---

5. What type of magnetic material is used in making permanent magnets?

Ans. Material having high coercivity is used in making permanent magnets.

---

6. Which physical quantity has the unit wb / m^2 ? Is it a scalar or a vector quantity?

Ans. Magnetic field. It is a vector quantity.

2 Mark Questions

1. A bar magnet of magnetic moment M is aligned parallel to the direction of a uniform magnetic field B. What is the work done to turn the magnet, so as the align its magnetic moment?

(i) Opposite to the field direction

(ii) Normal to the field direction?

Ans. Since work done W = MB $\left(\cos\theta_1 - \cos\theta_2\right)$

(i) $\theta_1 = 0^\circ$ and $\theta_2 = 180^\circ$

$\rightarrow W = MB(\cos 0 - \cos 180^\circ)$

W = MB [1- (-1)]

W = 2MB

(ii) $\theta_1 = 0^\circ$ and $\theta_2 = 90^\circ$

W = MB $\left(cos\ 0 - cos\ 90^\circ\right)$

W = MB

2. An electron in the ground state of hydrogen atom is revolving in anti - clock wise direction in a circular orbit. The atom is placed normal to the electron orbit makes an angle of 30^{0} in the magnetic field. Find the torque experienced by the orbiting electron?

Ans. Magnetic moment associated with electron M = $\dfrac{eh}{4\pi m_e}$

$$\theta = 30^\circ$$

$$and \quad \tau = MB\sin\theta$$

$$\tau = \frac{eh}{4\pi m_e}B \times \sin 30^\circ = \frac{eh}{4\pi m_e}B \times \frac{1}{2}$$

$$\tau = \frac{ehB}{8\pi m_e}$$

---

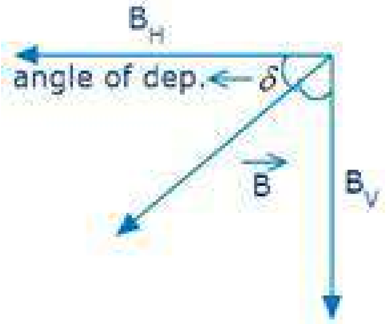
3. Define angle of dip. Deduce the relation connecting angle of dip and horizontal component of earth's total magnetic field with the horizontal direction.

Ans. $\dfrac{BH}{B} = \cos\delta$

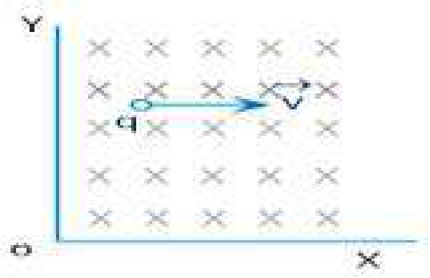
$$\frac{BV}{B} = \sin\delta$$

$$\Rightarrow \frac{\sin\delta}{\cos\delta} = \frac{BV}{B} \times \frac{B}{BH}$$

$$Tan\delta = \frac{BV}{BH}$$



4. A point change +q is moving with speed v perpendicular to the magnetic field B as shown in the figure. What should be the magnitude and direction of the applied electric field so that the net force acting on the charge is zero?



Ans. Force on the charge due to magnetic field = qVB sin θ

Since $\vec{B}$ is $\perp$ to the plane of paper and in words

$\therefore$ F = qVB $sin\ 90^\circ$

F = qVB (along OY)

Force on the charge due to electric field

F = qE

Net force on change is zero if $\Rightarrow$ qE = qVB

E = VB

(along YO)

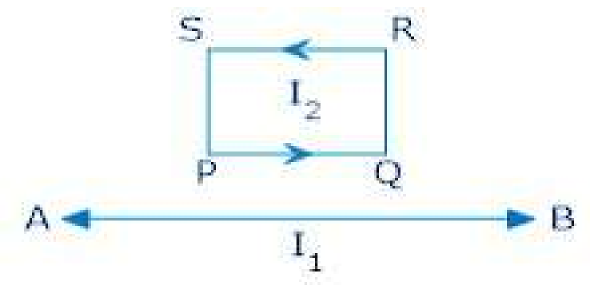
---

5. The energy of a charged particle moving in a uniform magnetic field does not change. Why?

Ans. The force on a charged particle in a uniform magnetic field always acts in a direction perpendicular to the motion of the charge. Since work done by the magnetic field on the charge is zero, hence energy of the charged particle will not change.

---

6. In the figure, straight wire AB is fixed; white the loop is free to move under the influence of the electric currents flowing in them. In which direction does the loop begin to move? Justify.



Ans. Since current in AB and arm PQ are in same direction therefore wire will attract the arm PQ with a force (say F_1)

But repels the arm RS with a force (say F_2)

Sine arm PQ is closer to the wire AB

$F_1 > F_2$ i.e. the loop will move towards the wire.

---

7. State two factors by which voltage sensitivity of a moving coil galvanometer can be increased?

Ans. Voltage sensitivity = $\frac{nBA}{kR}$

It can be increased by

(1) increasing B using powerful magnets

(2) decreasing k by using phosphor borne strip

---

8. What is the magnetic moment associated with a coil of 1 turns, area of cross- section $10^{-4} m^2$ carrying a current of 2A?

Ans. m = NIA

$m = 1\times10^{-4}\times2$

$m = 2\times10^{-4} Am^2$.

---

9. A Rowland ring of mean radius 15 cm has 3500 turns of wire wound on a

ferromagnetic core of relative permeability 800. What is the magnetic field B in the core for a magnetising current of 1.2 A?

Ans. Mean radius of a Rowland ring, r= 15 cm = 0.15 m

Number of turns on a ferromagnetic core, N= 3500

Relative permeability of the core material, $\mu_r = 800$

Magnetising current, I= 1.2 A

The magnetic field is given by the relation:

$$B = \frac{\mu_r \mu_0 IN}{2\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} Tm A^{-1}$

$$B = \frac{800 \times 4\pi \times 10^{-7} \times 1.2 \times 3500}{2\pi \times 0.15}$$

$= 4.48T$

Therefore, the magnetic field is n the core is 4.48 T.

---

10. At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

Ans. Angle of declination, $\theta = 12^\circ$

Angle of dip, $\delta = 60^0$

Horizontal component of earth's magnetic field, BH= 0.16 G

Earth's magnetic field at the given location = *B*

We can relate *B* and *BH* as:

$$B_H = B\cos\delta$$

$$\therefore B = \frac{B_H}{\cos\delta}$$

$$= \frac{0.16}{\cos 60^0} = 0.32G$$

Earth's magnetic field lies in the vertical plane, $12°$ West of the geographic meridian, making an angle of 60°(upward) with the horizontal direction. Its magnitude is 0.32ss G.

---

11. A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at $22°$ with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine the magnitude of the earth's magnetic field at the place.

Ans. Horizontal component of earth's magnetic field, *BH*= 0.35 G

Angle made by the needle with the horizontal plane = Angle of dip = $\delta = 22^0$

Earth's magnetic field strength = *B*

We can relate *B* and *BH* as:

$$B_H = B\cos\theta\delta$$

$$\therefore B = \frac{B_H}{\cos\delta}$$

$$= \frac{0.35}{\cos 22^0} = 0.377G$$

Hence, the strength of earth's magnetic field at the given location is 0.377 G.

12. If the solenoid in Exercise 5.5 is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of $30°$ with the direction of applied field?

Ans. Magnetic field strength, *B* = 0.25 T

Magnetic moment, *M*= $0.6\ T^{-1}$

The angle θ, between the axis of the solenoid and the direction of the applied field is $30°$.

Therefore, the torque acting on the solenoid is given as:

$\tau = MB\sin\theta$

$= 0.6 \times 0.25 \sin 30^0$

$= 705 \times 10^{-2}\ J$

---

13. A closely wound solenoid of 800 turns and area of cross section $2.5 \times 10^{-4}\mathrm{m}^2$ carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?

Ans. Number of turns in the solenoid, *n* = 800

Area of cross-section, *A*= $2.5 \times 10^{-4} m^2$

Current in the solenoid, *I*= 3.0 A

A current-carrying solenoid behaves as a bar magnet because a magnetic field develops along its axis, i.e., along its length.

The magnetic moment associated with the given current-carrying solenoid is calculated as:

M= *n I A*

= $800 \times 3 \times 2.5 \times 10^{-4}$

= $0.6\ J\ T^{-1}$

14. A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to $4.5 \times 10^{-2} J$. What is the magnitude of magnetic moment of the magnet?

Ans. Magnetic field strength, *B*= 0.25 T

Torque on the bar magnet, *T*= $4.5 \times 10^{-2} J$

Angle between the bar magnet and the external magnetic field, $\theta = 30^\circ$

Torque is related to magnetic moment (*M*) as:

T= *MB* sin θ

$$\therefore M = \frac{T}{\mathbf{B} \sin\theta}$$

$$= \frac{4.4 \times 10^{-2}}{0.25 \times \sin 30^0} = 0.36 JT^{-1}$$

Hence, the magnetic moment of the magnet is $0.36\ J\ T^{-1}$.

3 Mark Questions

1. A short bar magnet of magnetic moment 0.9 J/T is placed with its axis at 60^{o} to a uniform magnetic field. It experiences a torque of 0.063 Nm. (i) calculate the strength of the magnetic field and (ii) what orientation of the bar magnet corresponds to the equilibrium position in the magnetic field?

Ans. (i) Since $\tau = MB\sin\theta$

Here $\theta = 60^{o}$

$\tau = 0.063Nm$

$M = 0.9\ J/T$

$$\Rightarrow B = \frac{\tau}{M\sin\theta} = \frac{0.063}{0.9\times\sin 60^{o}}$$

$\Rightarrow B = 0.081T$

(ii) The magnet will be in stable equilibrium in the magnetic field if $\tau = 0$

$\Rightarrow MB\sin\theta = 0 \Rightarrow \theta = 0^{o}$

i.e When magnet aligns itself parallel to the field

---

2. A beam of electrons is moving with a velocity of $3\times10^{6} m/s$ and carries a current of 1 μ A.

(a) How many electrons per second pass a given point?

(b) How many electrons are in 1m of the beam?

(c) What is the total force on all the electrons in 1m of the beam if it passes through the field of $0.1NA^{-1}m^{-1}$?

Ans. $\upsilon = 3\times10^{6}\,m/A$

$I = 1\mu A = 1\times10^{-6}A$

(a) $n = \dfrac{I}{q} = \dfrac{10^{-6}}{1.6\times10^{-19}C} = 6.25\times10^{12}$

(b) Electrons traverse a distance of $3\times10^{6}\,m$ in 1 s

$\therefore$ No. of electrons in 1 meter of the beam

$= \dfrac{6.25\times10^{12}}{3\times10^{6}} = 2.08\times10^{6}\,m^{-1}$

(c) Force on 1 meter of the beam of electrons

---

3. What is the main function of soft iron core used in a moving coil galvanometer? A galvanometer gives full deflection for Ig. Can it be converted into an ammeter of range I < Ig?

Ans. Soft iron core is used the moving coil galvanometer because it increases the strength of the magnetic field thus increases the sensitivity of the galvanometer.

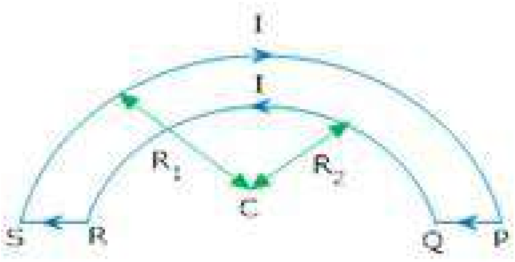
We know S = $\dfrac{GIg}{I-Ig}$

For I < Ig, S becomes negative

Hence it cannot be converted into an ammeter of range I < Ig.

---

4. Two wires loops PQRSP formed by joining two semicircular wires of radii $R_1 and\ R_2$ carries a current I as shown in the figure. What is the direction of the magnetic induction at the centre C.?



Total torque on the coil

Ans. Magnetic field due to semicircle QR at C. is

$$B_1 = \frac{1}{2}\frac{\mu 0}{4\pi}\frac{2\pi I}{R_1}$$

Magnetic field due to semicircle is at C is

$$B_2 = \frac{1}{2}\frac{\mu 0}{4\pi}\frac{2\pi I}{R_2}$$

Net field $B = B_1 - B_2$

$$B = \frac{1}{\cancel{2}}\frac{\mu 0}{\cancel{4}\pi}\cancel{2}\pi \mathrm{I}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$B = \frac{\mu 0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

---

5. A circular coil is placed in uniform magnetic field of strength 0.10T normal to the plane of coil. If current in the coil is 5.0A. Find.

(a) Total torque on the coil

(b) Total force on the coil

(c) Average force on each electron due to magnetic field

(The coil is made of copper wire of cross- sectional area $10^{-5}\,m^2$ and free electron density in copper is $10^{29}\,m^{-3}$)

Ans. (a) B = 0.10T

$\theta = 0^0$ (Normal to plane of the coil)

I = 5.0 A, Area = $10^{-5}\,m^2$ n = $10^{29}\,m^{-3}$

$\tau = MB\sin\theta$

$\tau = MB\sin 0^0 = 0$

(b) Total force on the coil = 0 Newton

(c) Fav = q $(\overrightarrow{vd} \times \overrightarrow{B})$

(I = neAVd)

Fav = $\frac{q I}{n \cancel{e} A} \times B$

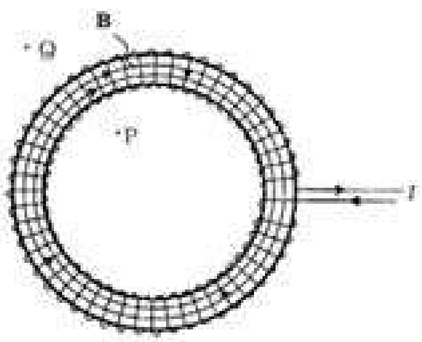
Fav = $\frac{IB}{nA} = \frac{5 \times 0.10}{10^{29} \times 10^{-5}}$

$Fav = 5 \times 10^{-25}\,N$

---

6. Using Ampere's circuital law, derive an expression for magnetic field along the axis of a Toroidal solenoid?

Ans. If n be the no, of turns per unit length I be the current flowing through the Toroid



Then from Ampere's circuital law

$$\oint \vec{B} \cdot \vec{d\ell} = \mu_0 \times \text{total current flowing in the toroid}$$

$$\oint \vec{B} \cdot \vec{d\ell} = \mu_0 \left(2\pi r n I\right)$$

$$\int_0^{2\pi r} B d\ell \cos 0^0 = \mu_0 \left(2\pi r n I\right)$$

$$B \int_0^{2\pi r} d\ell = \mu_0 \left(2\pi r n I\right)$$

$$B . \cancel{2}\cancel{\pi}\cancel{r} = \mu_0 \left(\cancel{2}\cancel{\pi}\cancel{r} n I\right)$$

$$\mathrm{B} = \mu o n I$$

---

7. A short bar magnet of magnetic moment m = $0.32\ \mathrm{JT}^{-1}$ is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (a) stable, and (b) unstable equilibrium? What is the potential energy of the magnet in each case?

Ans. Moment of the bar magnet, *M*= $0.32\ J\,T^{-1}$

External magnetic field, *B*= 0.15 T

(a) The bar magnet is aligned along the magnetic field. This system is considered as being in stable equilibrium. Hence, the angle θ, between the bar magnet and the magnetic field is $0°$.

Potential energy of the system $= -MB\cos\theta$

$$= -0.32 \times 0.15 \cos 0^0$$

$$= -4.8 \times 10^{-2} J$$

(b) The bar magnet is oriented 180°to the magnetic field. Hence, it is in unstable equilibrium.

$$\theta = 180°$$

Potential energy = – *MB* $cos\theta$

$= 0.32 \times 0.15 \cos 180^0$

$= 4.8 \times 10^{-2} J$

---

8. A closely wound solenoid of 2000 turns and area of cross-section $1.6 \times 10^{-4} m^2$, carrying a current of 4.0 A, is suspended through its centre allowing it to turn in a horizontal plane.

(a) What is the magnetic moment associated with the solenoid?

(b) What is the force and torque on the solenoid if a uniform horizontal magnetic field of $7.5 \times 10 - 2T$ is set up at an angle of $30°$ with the axis of the solenoid?

Ans. Number of turns on the solenoid, *n* = 2000

Area of cross-section of the solenoid, *A*= $1.6 \times 10^{-4} m^2$

Current in the solenoid, *I*= 4 A

(a) The magnetic moment along the axis of the solenoid is calculated as:

M= *nAI*

= $2000 \times 1.6 \times 10^{-4} \times 4$

= 1.28 A m^2

(b) Magnetic field, *B* = $7.5 \times 10^{-2} T$

Angle between the magnetic field and the axis of the solenoid, $\theta = 30°$

Torque, $\tau = MB \sin \theta$

$= 1.28 \times 7.5 \times 10^{-2} \sin 30^0$

$= 4.8 \times 10^{-2} Nm$

Since the magnetic field is uniform, the force on the solenoid is zero. The torque on the solenoid is $4.8 \times 10^{-2} Nm$ s

9. A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0×10^{-2} T . The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of $2.0\ s^{-1}$. What is the moment of inertia of the coil about its axis of rotation?

Ans. Number of turns in the circular coil, *N*= 16

Radius of the coil, *r*= 10 cm = 0.1 m

Cross-section of the coil, *A*= $\pi r2 = \pi \times (0.1)^2 m^2$

Current in the coil, *I*= 0.75 A

Magnetic field strength, *B*= $5.0 \times 10^{-2} T$

Frequency of oscillations of the coil, *v*= $2.0\ s^{-1}$

∴Magnetic moment, *M*= *NIA* $= NI\pi r^2$

= $16\times0.75\times\pi\times(0.1)^2$

= $0.377\ J\ T^{-1}$

Frequency is given by the relation:

$$v=\frac{1}{2\pi}\sqrt{\frac{MB}{1}}$$

Where,

I= Moment of inertia of the coil

$$\therefore 1=\frac{MB}{4\pi^2 v^2}$$

$$=\frac{0.377\times5\times10^{-2}}{4\pi^2\times(2)^2}$$

$= 1.19 \times 10^{-4} Kgm^2$

Hence, the moment of inertia of the coil about its axis of rotation is $1.19 \times 10^{-4} Kgm^2$

---

10. A short bar magnet has a magnetic moment of $0.48\ J\ T^{-1}$. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (a) the axis, (b) the equatorial lines (normal bisector) of the magnet.

Ans. Magnetic moment of the bar magnet, *M*= $0.48\ J\ T^{-1}$

(a) Distance, *d* = 10 cm = 0.1 m

The magnetic field at distance *d*, from the centre of the magnet on the axis is given by the relation:

$$B = \frac{\mu_0 2M}{4\pi d^3}$$ ss

Where,

μ_0 = Permeability of free space = $\mu_0 \ 4\pi \times 10^{-7} Tm A^{-1}$

$$\therefore B = \frac{4\pi \times 10^{-7} \times 2 \times 0.48}{4\pi \times (01)^1}$$

$= 0.96 \times 10^{-4} T = 0.96G$

The magnetic field is along the S - N direction.

(b) The magnetic field at a distance of 10 cm (i.e., *d*= 0.1 m) on the equatorial line of the magnet is given as:

$$B = \frac{\mu_0 M}{4\pi \times d^3}$$

$= 0.48G$

The magnetic field is along the N - S direction.

---

11. A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null-point (i.e., 14 cm) from the centre of the magnet? (At *null points*, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field.)

Ans. Earth's magnetic field at the given place, *H*= 0.36 G

The magnetic field at a distance *d*, on the axis of the magnet is given as:

$$B_1 = \frac{\mu_0 M}{4\pi \times d^3} = H \quad \text{......(i)}$$

Where,

μ_0 = Permeability of free space

M= Magnetic moment

The magnetic field at the same distance *d*, on the equatorial line of the magnet is given as:

$$B_2 = \frac{\mu_0 M}{4\pi \times d^3} = \frac{H}{2} \quad \text{[Using equation (i)]}$$

Total magnetic field, $B = B_1 + B_2$

$$= H + \frac{H}{2}$$

$$= 0.36 + 0.18 = 0.54G$$

Hence, the magnetic field is 0.54 G in the direction of earth's magnetic field.

---

12. A long straight horizontal cable carries a current of 2.5 A in the direction 10° south

of west to 10°north of east. The magnetic meridian of the place happens to be 10° west of the geographic meridian. The earth's magnetic field at the location is 0.33 G, and the angle of dip is zero. Locate the line of neutral points (ignore the thickness of the cable). (At *neutral points*, magnetic field due to a current-carrying cable is equal and opposite to the horizontal component of earth's magnetic field.)

Ans. Current in the wire, I= 2.5 A

Angle of dip at the given location on earth, $\delta = 0^\circ$

Earth's magnetic field, H= 0.33 G = $0.33 \times 10^{-4} T$

The horizontal component of earth's magnetic field is given as:

H_H = H cos δ

$$= 0.33 \times 10^{-4} \times \cos 0^0 = 0.33 \times 10^{-4} T$$

The magnetic field at the neutral point at a distance $R$from the cable is given by the relation:

$$H_H = \frac{\mu_0 1}{2\pi R}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} TmA^{-1}$

$$\therefore R = \frac{\mu_0 1}{2\pi H_H}$$

$$= \frac{4\pi \times 10^{-7} \times 2.5}{2\pi \times 0.33 \times 10^{-4}} = 15.15 \times 10^{-3} m = 1.51 cm$$

Hence, a set of neutral points parallel to and above the cable are located at a normal distance of 1.51 cm.

---

13. A compass needle free to turn in a horizontal plane is placed at the centre of circular coil of 30 turns and radius 12 cm. The coil is in a vertical plane making an angle

of 45° with the magnetic meridian. When the current in the coil is 0.35 A, the needle points west to east.

(a) Determine the horizontal component of the earth's magnetic field at the location.

(b) The current in the coil is reversed, and the coil is rotated about its vertical axis by an angle of 90º in the anticlockwise sense looking from above. Predict the direction of the needle. Take the magnetic declination at the places to be zero.

Ans. Number of turns in the circular coil, *N*= 30

Radius of the circular coil, *r*= 12 cm = 0.12 m

Current in the coil, *I*= 0.35 A

Angle of dip, $\delta' = 45^\circ$

(a) The magnetic field due to current *I*, at a distance *r*, is given as:

$$B = \frac{\mu_0 2\pi r N I}{4\pi r}$$ s

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} Tm A^{-1}$

$$B = \frac{4\pi \times 10^{-7} \times 2\pi \times 30 \times 0.35}{4\pi \times 0.12}$$ s

= $5.49 \times 10^{-5} T$

The compass needle points from West to East. Hence, the horizontal component of earth's magnetic field is given as:

$$B_h = B\sin\delta = 5.49 \times 10^{-5} \sin 45^\circ = 3.88 \times 10^{-5} T = 0.388\ G$$

(b) When the current in the coil is reversed and the coil is rotated about its vertical axis by an angle of 90°, the needle will reverse its original direction. In this case, the needle will point from East to West.

14. A magnetic dipole is under the influence of two magnetic fields. The angle between the field directions is $60°$, and one of the fields has a magnitude of 1.2×10^{-2} T. If the dipole comes to stable equilibrium at an angle of $15°$ with this field, what is the magnitude of the other field?

Ans. Magnitude of one of the magnetic fields, $B_1 = 1.2 \times 10^{-2} T$

Magnitude of the other magnetic field = B_2

Angle between the two fields, $\theta = 60°$

At stable equilibrium, the angle between the dipole and field B_1, $\theta_1 = 15°$

Angle between the dipole and field B_2, $\theta_2 = \theta - \theta_1$ = $60° - 15° = 45°$

At rotational equilibrium, the torques between both the fields must balance each other.

∴ Torque due to field $B_1 = Torque\ due\ to\ field B_2$

$$MB_1 sin\theta_1 = MB_2 sin\theta_2$$

Where,

M= Magnetic moment of the dipole

$$\therefore B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2}$$

$$= \frac{1.2 \times 10^{-2} \times \sin 15^0}{\sin 45^0} = 4.39 \times 10^{-3} T$$ s

Hence, the magnitude of the other magnetic field is $4.39 \times 10^{-3} T$.

---

15. The magnetic moment vectors μ_s and μ_l associated with the intrinsic spin angular momentum S and orbital angular momentum l, respectively, of an electron are predicted by quantum theory (and verified experimentally to a high accuracy) to be given by:

$\mu_s = -(e/m)S_z$, $\mu_l = -(e/2m)_l$ **Which of these relations is in accordance with the result expected *classically*? Outline the derivation of the classical result.**

Ans. The magnetic moment associated with the orbital angular momentum is valid with the classical mechanics.

The magnetic moment associated with the orbital angular (l) momentum is given as μ_l

For current *i* and area of cross-section *A*, we have the relation:

Magnetic moment

$$\mu_l = \mathrm{IA} = (e - t)\pi r^2 .....(i)$$

Where,

e = Charge of the electron

r = Radius of the circular orbit

T = Time taken to complete one rotation around the circular orbit of radius r

Angular momentum, $I = mvr = mr(2nr/t)......(ii)$

Where,

M = Mass of the electron

V = Velocity of the electorn

Dividing equation (1) by equation (2),we get:

$$\mu_l / l = -e/2m$$

Therefore of the two relations, $\mu_l / l = -(e/2m)I$ is in accordance with class physics.

5 Mark Questions

1. A particle of mass m and charge q moving with a uniform speed v normal to a uniform magnetic field B describes a circular path of radius & Derive expressions for (1) Radius of the circular path (2) time period of revolution (3) Kinetic energy of the particle?

Ans. A particle of mass (m) and change (q) moving with velocity v normal to $\vec{B}$ describes a circular path if

$$\frac{mv^2}{r} = qBv\sin\theta \quad (\therefore \theta = 90^0)$$

$$\Rightarrow \frac{mv^{\cancel{2}}}{r} = q\cancel{v}B$$

$$\Rightarrow \boxed{r = \frac{mv}{Bq}} ----(1)$$

Since Time period of Revolution

During circular path = $\frac{\text{Circumference of circle}}{velocity}$

$$\Rightarrow T = \frac{2\pi r}{v} \quad (\therefore v = \frac{Bqr}{m} \text{ from eg.(1)})$$

$$\Rightarrow T = \frac{2\pi \cancel{r} \times m}{Bq\cancel{r}}$$

$$T = \frac{2\pi m}{Bq} \text{----}(2)$$

Kinetic energy K.E = $\frac{1}{2}$ mv^2

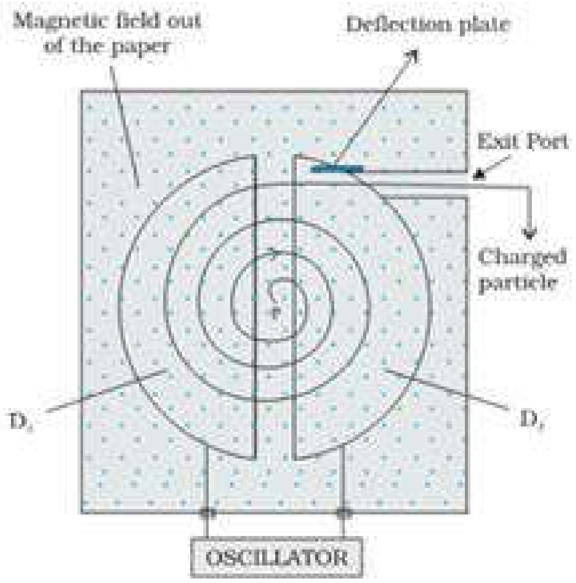
=> KE = $\frac{1}{2} m \left(\frac{Bqr}{m}\right)^2$

$$KE = \frac{B^2 q^2 r^2}{2m} ---(3)$$

---

2. Write an expression for the force experienced by the charged particle moving in a uniform magnetic field B With the help of labeled diagram explain the working of cyclotron? Show that cyclotron frequency does not depend upon the speed of the particle?

Ans. Force experienced by the charged particle moving at right angles to uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ is given by $\vec{F}$ = q ($\vec{v} \times \vec{B}$) Initially Dee D_1 is negatively charged and Dee D_2 is positively charged so, the positive ion will get accelerated towards Dee D_1 since the magnetic field is uniform and acting at right angles to the plane of the Dees so the ion completes a circular path in D_1 when ions comes out into the gap, polarity of the Dee’s gets reversed used the ion is further accelerated towards Dee D_2 with greater speed and cover a bigger semicircular path. This process is separated time and again and the speed of the ion becomes faster till it reaches the periphery of the dees where it is brought out by means of a deflecting plate and is made to bombard the target.



Since F = qVBsin90^0 provides the necessary centripetal force to the ion to cover a circular path so we can say $\frac{mv^{\cancel{2}}}{r} = q\cancel{v}B$

=> r = $\frac{mv}{Bq}$ ----- (1)

Time period = $\frac{2\pi r}{v} = \frac{2\pi \cancel{v} m}{Bq\cancel{v}} = \frac{2\pi m}{Bq}$

V = $\frac{1}{T} = \frac{Bq}{2\pi m}$

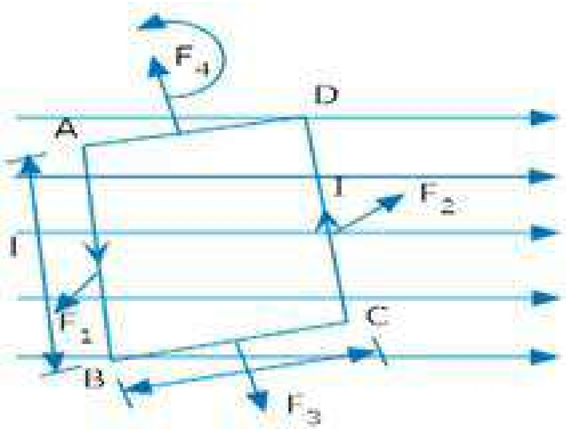
$\Rightarrow$ frequency is independent of velocity

---

3. (a) Obtain an expression for the torque acting on a current carrying circular loop.

(b) What is the maximum torque on a galvanometer coil 5 cm $\times$ 12 cm of 600 turns when carrying a current of 10^{-5} A. in a field where flux density is $0.10\ Wb\ /\ m^2$?

Ans. ABCD is a rectangular loop of length (L), breadth (b) and area (A). Let I be the Current flowing in the anti clockwise direction. Let θ be the angle between the normal to the loop and magnetic field $\overrightarrow{B}$



Force acting on arm AB of the loop

$$\vec{F}_1 = I(\vec{L} \times \vec{B})(outwards)$$

Force on arm CD

$$\vec{F}_2 = I(\vec{L} \times \vec{B})(inwards)$$

Force on arm BC

$$\vec{F}_3 = I(\vec{b} \times \vec{B})(downwards)$$

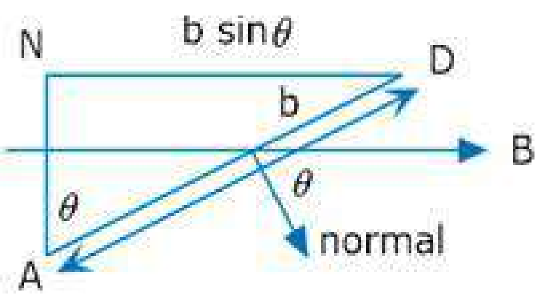
Force on arm DA

$$\vec{F}_4 = I(\vec{b} \times \vec{B})(upwards)$$

Since $F_3 and\ F_4$ are equal and opposite and also acts along the same line, hence they cancel each other.

$F_1 and\ F_2$ are also equal and opposite but their line of action is different, so they form a couple and makes the rectangular loop rotate anti clockwise.

Thus τ = either force $\times \perp$ distance



$$\tau = I(\overline{L} \times \overline{B}) \times DN$$

$$\tau = I(\overline{L} \times \overline{B}) \times b \sin \theta$$

$$\tau = I\ LB \sin 90^o\, b \sin \theta$$

$$\tau = I \text{ A B } \sin\theta$$

For loop of N turns

$$\tau = NIAB \sin \theta$$

$$\tau = MB \sin \theta \left(\therefore M = NIA \right)$$

$$\vec{\tau} = \overrightarrow{M} \times \overrightarrow{B}$$

Where M is magnetic moment of the loop.

$$\tau = NIAB \sin \theta$$

Torque will be maximum when θ = 90^o

$$\Rightarrow \tau_{max} = NIAB \ \ (\therefore Sin 90^o = 1)$$

$$\tau_{max} = 600 \times 10^{-5} \times (0.10)(60 \times 10^{-4})$$

$$\tau_{max} = 3.6 \times 10^{-6} Nm$$

---

4. The current sensitivity of a moving coil galvanometer increases by 20% when its resistance is increased by a factor of two. Calculate by what factor, the voltage sensitivity changes?

Ans. Current sensitivity $\dfrac{\alpha}{I} = \dfrac{nBA}{k} ------(i)$

Voltage sensitivity $\dfrac{\alpha}{V} = \dfrac{nBA}{kR} ------(ii)$

Resistance of a galvanometer increases when n and A are changed

Given R' = 2R

Then n = n' and A = A'

New current sensitivity

$$\frac{\alpha'}{I'}=\frac{n'A'B}{k}------(iii)$$

New voltage sensitivity

$$\frac{\alpha'}{V}=\frac{\alpha'}{I'R'}=\frac{n'A'B}{2kR}-----(iv)$$

$$Since\frac{\alpha'}{I'}=\frac{120}{100}\frac{\alpha}{I}------(v)$$

From (i) and (iii)

$$\frac{n'A'B}{R}=\frac{\alpha}{I}\frac{120}{100}$$

$$\frac{n'A'\cancel{B}}{\cancel{k}}=\frac{nA\cancel{B}}{\cancel{k}}\frac{120}{100}$$

n’A’= $\frac{6}{5}nA$

Using equation (iv)

$$\frac{\alpha'}{V}=\frac{6}{5}\frac{nAB}{2kR}$$

$$\frac{\alpha'}{V}=\frac{3nAB}{5kR}$$

$$\frac{\alpha'}{V}=\frac{3}{5}\frac{\alpha}{V}$$

Thus voltage sensitivity decreases by a factor of $\frac{3}{5}$.

5. (a) Show how a moving coil galvanometer can be converted into an ammeter?

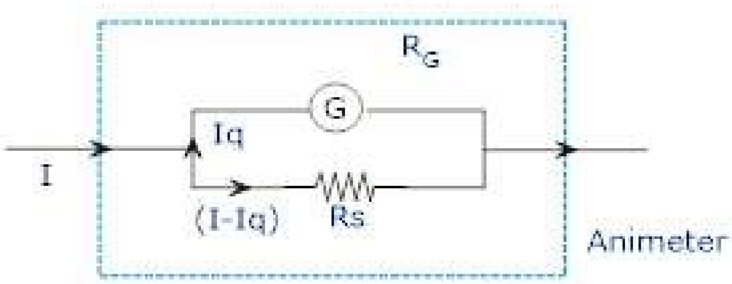
(b) A galvanometer has a resistance 30Ω and gives a full scale deflection for a current of 2mA. How much resistance in what way must be connected to convert into?

(1) An ammeter of range 0.3A

(2) A voltammeter of range 0.2V.

Ans. (a) A galvanometer can be converted into an ammeter by connecting a low resistance called shunt parallel to the galvanometer.

Since G and R_S are in parallel voltage across then is same $IgR_G = (I - Ig)\ R_S$



$$\Rightarrow Rs = \left(\frac{Ig}{I - Ig}\right) R_G$$

(b) (1) I = 0.3A G = 30Ω Ig = 2mA = $2 \times 10^{-3} A$

Sheent (S) = $\frac{IgG}{I - Ig}$

$$S = \frac{2\times10^{-3}\times30}{(0.3 - 2\times10^{-3})}$$

S = 0.2Ω

(2) G = 30π, Ig = 2mA = $2\times10^{-3} A$, V = 0.2V

Shunt Resistance (R) $\left(\frac{V}{Ig}-G\right)$

$$R=\left(\frac{0.2}{2\times10^{-3}}-30\right)$$

R = 70 Ω

---

6. A monoenergetic (18 keV) electron beam initially in the horizontal direction is subjected to a horizontal magnetic field of 0.04 G normal to the initial direction. Estimate the up or down deflection of the beam over a distance of 30 cm (*me*= $9.11\times10^{-19}C$). [Note: *Data* in this exercise are so chosen that the answer will give you an idea of the effect of earth's magnetic field on the motion of the electron beam from the electron gun to the screen in a TV set.]

Ans. Energy of an electron beam, *E*= 18 keV = $18\times10^{3}eV$

Charge on an electron, *e*= $1.6\times10^{-19}C$

E= $18\times10^{3}\times1.6\times10^{-19}J$

Magnetic field, *B* = 0.04 G

Mass of an electron, *me*= $9.11\times10^{-19}kg$

Distance up to which the electron beam travels, *d* = 30 cm = 0.3 m

We can write the kinetic energy of the electron beam as:

$$E=\frac{1}{2}mv^{2}$$

$$v=\sqrt{\frac{2E}{m}}$$

$$=\sqrt{\frac{2\times18\times10^{3}\times1.6\times10^{-19}\times10^{-15}}{9.11\times10^{-31}}}$$

$= 0.795 \times 10^{8}\, m/s$

The electron beam deflects along a circular path of radius, *r*.

The force due to the magnetic field balances the centripetal force of the path.

$$BeV = \frac{mv^2}{r}$$

$$\times r = \frac{mv}{Be}$$

$$= \frac{9.11 \times 10^{-31} \times 0.795 \times 10^{8}}{0.4 \times 10^{-4} \times 1.6 \times 10^{-19}} = 11.3m$$

Let the up and down deflection of the electron beam be $x = r(1 - \cos\theta)$

Where,

θ= Angle of declination

$$\sin\theta = \frac{d}{r}$$

$$= \frac{0.3}{11.3}$$

$$\theta = \sin^{-1}\frac{0.3}{11.3} = 1.521^{0}$$

$$and\ x = 11.3(1 - \cos 10521^{0})$$

$$= 0.0039m = 3.9mm$$

Therefore, the up and down deflection of the beam is 3.9 mm.

---

7. A sample of paramagnetic salt contains 2.0×10^{24} atomic dipoles each of dipole moment 1.5×10^{-23} J T^{-1} . The sample is placed under a homogeneous magnetic field

of 0.64 T, and cooled to a temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K? (Assume Curie's law)

Ans. Number of atomic dipoles, *n*= 2.0×10^{24}

Dipole moment of each atomic dipole, *M*= $1.5 \times 10^{-23} J\,T^{-1}$

When the magnetic field, B_1 = 0.64 T

The sample is cooled to a temperature, T_1 = 4.2°K

Total dipole moment of the atomic dipole, $M_{tot} = n \times M$

= $2 \times 10^{24} \times 1.5 \times 10^{-23}$

= $30\,J\,T^{-1}$

Magnetic saturation is achieved at 15%.

Hence, effective dipole moment, $M_1 = \frac{15}{100} \times 30 = 4.5 JT^{-1}$

When the magnetic field, B_2 = 0.98 T

Temperature, T_2 = 2.8°K

Its total dipole moment = M_2

According to Curie's law, we have the ratio of two magnetic dipoles as:

$$\frac{M_2}{M_1} = \frac{B_2}{B_1} \times \frac{T_1}{T_2}$$

$$\therefore M_2 = \frac{B_2 T_1 M_1}{B_1 T_2}$$

$$= \frac{0.98 \times 4.2 \times 4.5}{2.8 \times 0.64} = 10.336 JT^{-1}$$

Therefore, $10.336JT^{-1}$ is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K.

---

8. A telephone cable at a place has four long straight horizontal wires carrying a current of 1.0 A in the same direction east to west. The earth's magnetic field at the place is 0.39 G, and the angle of dip is $35°$. The magnetic declination is nearly zero. What are the resultant magnetic fields at points 4.0 cm below the cable?

Ans. Number of horizontal wires in the telephone cable, *n*= 4

Current in each wire, I = 1.0 A

Earth's magnetic field at a location, *H*= 0.39 G = $0.39 \times 10^{-4} T$

Angle of dip at the location, $\delta = 35°$

Angle of declination, $\theta \sim 0^0$

For a point 4 cm below the cable:

Distance, *r* = 4 cm = 0.04 m

The horizontal component of earth's magnetic field can be written as:

$\mathrm{H}_h = \mathrm{H} cos\delta - \mathrm{B}$ Where,

B= Magnetic field at 4 cm due to current *I* in the four wires

$4 = \times \frac{\mu_0 1}{2\pi r}$

μ_0 = Permeability of free space = $4\pi \times 10^{-7} Tm A^{-1} \times B = 4 \times \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.04}$

= $0.2 \times 10^{-4} T$ = 0.2 G

$\therefore H_h = 0.39 \cos 35° - 0.2$

= $0.39 \times 0.819 - 0.2 \approx 0.12G$

The vertical component of earth's magnetic field is given as:

$Hv = H\sin\delta$

$= 0.39 \ sin \ 35° = 0.22 \ G$

The angle made by the field with its horizontal component is given as:

$$\theta = \tan^{-1}\frac{H_V}{H_v}$$

$$= \tan^{-1}\frac{0.22}{0.12} = 61.39^0$$

The resultant field at the point is given as:

$$H_1 = \sqrt{(H_v)^2 + (H_h)^2}$$

$$= \sqrt{(0.22)^2 + (0.12)^2} = 0.25G \text{ s}$$

For a point 4 cm above the cable:

Horizontal component of earth's magnetic field:

$$H_h = Hcos\delta + B$$

= 0.39 cos 35° + 0.2 = 0.52 G

Vertical component of earth's magnetic field:

$\mathrm{H}_v = \mathrm{H}sin\delta$ = 0.39

$sin \ 35° = 0.22 \ G$

Angle, $\theta \tan^{-1}\frac{H_V}{H_v} = \tan^{-1}\frac{0.22}{0.52} = 22.9°$

And resultant field:

$$H_2 = \sqrt{(H_v)^2 + (H_h)^2}$$

9. Answer the following questions:

(a) Explain qualitatively on the basis of domain picture the irreversibility in the magnetisation curve of a ferromagnet.

(b) The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy?

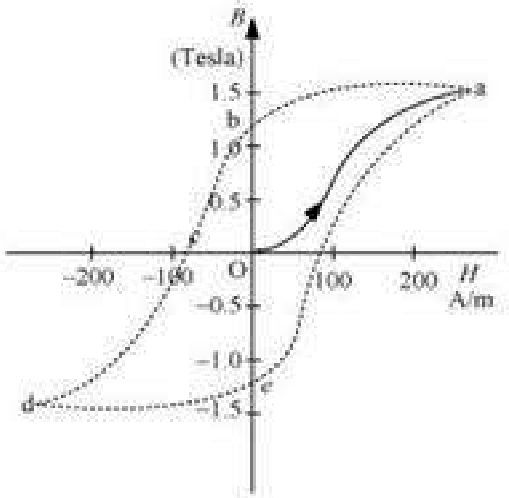
(c) 'A system displaying a hysteresis loop such as a ferromagnet, is a device for storing memory?' Explain the meaning of this statement.

(d) What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player, or for building 'memory stores' in a modern computer?

(e) A certain region of space is to be shielded from magnetic fields.

Suggest a method.

Ans. The hysteresis curve (*B-H* curve) of a ferromagnetic material is shown in the following figure.



(a) It can be observed from the given curve that magnetisation persists even when the external field is removed. This reflects the irreversibility of a ferromagnet.

(b) The dissipated heat energy is directly proportional to the area of a hysteresis loop. A carbon steel piece has a greater hysteresis curve area. Hence, it dissipates greater heat energy.

(c) The value of magnetisation is memory or record of hysteresis loop cycles of magnetisation. These bits of information correspond to the cycle of magnetisation. Hysteresis loops can be used for storing information.

(d) Ceramic is used for coating magnetic tapes in cassette players and for building memory stores in modern computers.

(e) A certain region of space can be shielded from magnetic fields if it is surrounded by soft iron rings. In such arrangements, the magnetic lines are drawn out of the region.

---

10. Answer the following questions:

(a) Why does a paramagnetic sample display greater magnetisation (for the same magnetising field) when cooled?

(b) Why is diamagnetism, in contrast, almost independent of temperature?

(c) If a to roid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty?

(d) Is the permeability of a ferromagnetic material independent of the magnetic field? If not, is it more for lower or higher fields?

(e) Magnetic field lines are always nearly normal to the surface of a ferromagnetic at every point. (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point.) Why?

(f) Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetization of a ferromagnet?

Ans. (a) Owing to therandom thermal motion of molecules, the alignments of dipoles get disrupted at high temperatures. On cooling, this disruption is reduced. Hence, a paramagnetic sample displays greater magnetisation when cooled.

(b) The induced dipole moment in a diamagnetic substance is always opposite to the magnetising field. Hence, the internal motion of the atoms (which is related to the temperature) does not affect the diamagnetism of a material.

(c) Bismuth is a diamagnetic substance. Hence, a toroid with a bismuth core has a magnetic field slightly greater than a toroid whose core is empty.

(d) The permeability of ferromagnetic materials is not independent of the applied magnetic field. It is greater for a lower field and vice versa.

(e) The permeability of a ferromagnetic material is not less than one. It is always greater than one. Hence, magnetic field lines are always nearly normal to the surface of such materials at every point.

(f) The maximum possible magnetisation of a paramagnetic sample can be of the same order of magnitude as the magnetisation of a ferromagnet. This requires high magnetising fields for saturation.

---

11. A short bar magnet of magnetic moment 5.25×10^{-2} J T $^{-1}$ is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with earth's field on s

(a) its normal bisector and (b) its axis. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distances involved.

Ans. Magnetic moment of the bar magnet, *M*= $5.25 \times 10^{-2} J T^{-1}$

Magnitude of earth's magnetic field at a place, *H*= 0.42 G = $0.42 \times 10^{-4} T$

(a) The magnetic field at a distance *R* from the centre of the magnet on the normal bisector is given by the relation:

$$B = \frac{\mu_0 M}{4\pi R^3}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} Tm A^{-1}$

When the resultant field is inclined at $45°$ with earth's field, *B*= *H*

$$\therefore = \frac{\mu_0 M}{4\pi R^3} = H = 0.42\times10^{-4}$$

$$R^3 = \frac{\mu_0 M}{0.42\times10^{-4}\times4\pi}$$

$$= \frac{4\pi\times10^{-7}\times5.25\times10^{-2}}{4\pi\times0.42\times10^{-4}} = 12.5\times10^{-5}$$

$$\therefore R = 0.05m = 5cm$$

(b) The magnetic field at a distanced R' from the centre of the magnet on its axis is given as:

$$B^1 = \frac{\mu_0 2M}{4\pi R^3}$$

The resultant field is inclined at $45°$ with earth's field.

$$\therefore B' = H$$

$$\frac{\mu_0 2M}{4\pi(R')^3} = H$$

$$(R')^3 = \frac{\mu_0 2M}{4\pi\times H}$$

$$= \frac{4\pi\times10^{-7}\times2\times5.25\times10^{-2}}{4\pi\times0.42\times10^{-4}} = 25\times10^{-5}$$

$$\therefore R' = 0.063m = 6.3cm$$

---

12. If the bar magnet in exercise 5.13 is turned around by $180°$, where will the new null points be located?

Ans. The magnetic field on the axis of the magnet at a distance *d*1= 14 cm, can be written as:

$$B_1 = \frac{\mu_0 2M}{4\pi(d_1)^3}$$ ...(i)

Where,

M = Magnetic moment

μ_0 = Permeability of free space

H= Horizontal component of the magnetic field at d_1

If the bar magnet is turned through $180°$, then the neutral point will lie on the equatorial line.

Hence, the magnetic field at a distance d_2, on the equatorial line of the magnet can be written as:

$$B_2 = \frac{\mu_0 2M}{4\pi(d_2)^3} = H$$ ......(2)

Equating equations (1) and (2), we get:

$$\frac{2}{(d_1)^3} = \frac{1}{(d_2)^3}$$

$$\left(\frac{d_2}{d_1}\right)^3 = \frac{1}{2}$$ s

$$\therefore d_2 = d_1 \times \left(\frac{1}{2}\right)^{\frac{1}{3}}$$

$$= 14 \times 0.794 = 11.1cm$$

The new null points will be located 11.1 cm on the normal bisector.

13. A bar magnet of magnetic moment $1.5\ J\ T^{-1}$ lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment: (i) normal to the field direction, (ii) opposite to the field direction?

(b) What is the torque on the magnet in cases (i) and (ii)?

Ans. (a) Magnetic moment, *M*= $1.5\ J\ T^{-1}$

Magnetic field strength, *B*= 0.22 T

(i) Initial angle between the axis and the magnetic field, $\theta_1 = 0°$

= 0° Final angle between the axis and the magnetic field, $\theta_2 = 90°$

The work required to make the magnetic moment normal to the direction of magnetic field is given as:

$W = -MB(\cos\theta_0 - \cos\theta_1)$

$= -1.5 \times 0.22(\cos 90^0 - \cos 0^0)$

$= -0.33(0-1)$

$= 0.33\ J$

(ii) Initial angle between the axis and the magnetic field, $\theta_1 = 0°$

Final angle between the axis and the magnetic field, $\theta_2 = 180°$

The work required to make the magnetic moment opposite to the direction of magnetic field is given as:

$W = -MB(\cos\theta_2 - \cos\theta_1)$

$= -1.5 \times 0.22(\cos 180^0 - \cos 0^0)$

$= -0.33(-1-1)$

$= 0.66 J$

(b) For case (i): $\theta = \theta_2 = 90^0$

$\therefore$Torque, $\tau = MB \sin\theta$

$= 1.5 \times 0.22 \sin 90^0$

$= 0.33 J$

For case (ii): $\theta = \theta_2 = 180^0$

$\therefore$ Torque, $\tau = MB \sin\theta$ ss

$= MB \sin 180^0 = 0 J$

---

14. Answer the following questions regarding earth's magnetism:

(a) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

(b) The angle of dip at a location in southern India is about $18°$.

Would you expect a greater or smaller dip angle in Britain?

(c) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?

(d) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole?

(e) The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8×10^{22} J T $^{-1}$ located at its centre. Check the order of magnitude of this number in some way.

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing

possible at all?

Ans. (a) The three independent quantities conventionally used for specifying earth's magnetic field are:

(i) Magnetic declination,

(ii) Angle of dip, and

(iii) Horizontal component of earth's magnetic field

(b) The angle of dip at a point depends on how far the point is located with respect to the North Pole or the South Pole. The angle of dip would be greater in Britain (it is about 70°) than in southern India because the location of Britain on the globe is closer to the magnetic North Pole.

(c) It is hypothetically considered that a huge bar magnet is dipped inside earth with its north pole near the geographic South Pole and its south pole near the geographic North Pole.

Magnetic field lines emanate from a magnetic north pole and terminate at a magnetic south pole. Hence, in a map depicting earth's magnetic field lines, the field lines at Melbourne, Australia would seem to come out of the ground.

(d) If a compass is located on the geomagnetic North Pole or South Pole, then the compass will be free to move in the horizontal plane while earth's field is exactly vertical to the magnetic poles. In such a case, the compass can point in any direction.

(e) Magnetic moment, *M*= $8 \times 10^{22}\ J\ T^{-1}$

Radius of earth, *r*= $6.4 \times 10^{6}\ m$

Magnetic field strength,

$$B = \frac{800 \times 4\pi \times 10^{-7} \times 1.2 \times 3500}{2\pi \times 0.15} = 4.48T$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ TmA$

$$\therefore \frac{4\pi\times10^{-7}\times8\times10^{22}}{4\pi\times(6.4\times10^{6})^{3}}=0.3G$$

This quantity is of the order of magnitude of the observed field on earth.

(f) Yes, there are several local poles on earth's surface oriented in different directions. A magnetised mineral deposit is an example of a local N-S pole.

---

15. Answer the following questions:

(a) The earth's magnetic field varies from point to point in space.

Does it also change with time? If so, on what time scale does it change appreciably?

(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e., the source of energy) to sustain these currents?

(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

(e) The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion?

(f) Interstellar space has an extremely weak magnetic field of the order of 10-12 T. Can such a weak field be of any significant consequence? Explain.

[Note: Exercise 5.2 is meant mainly to arouse your curiosity. Answers to some questions above are tentative or unknown. Brief answers wherever possible are given at the end. For details, you should consult a good text on geomagnetism.]

Ans. (a) Earth's magnetic field changes with time. It takes a few hundred years to change by an appreciable amount. The variation in earth's magnetic field with the time cannot be neglected.

(b) Earth's core contains molten iron. This form of iron is not ferromagnetic. Hence, this is not considered as a source of earth's magnetism.

(c) The radioactivity in earth's interior is the source of energy that sustains the currents in the outer conducting regions of earth's core. These charged currents are considered to be responsible for earth's magnetism.

(d) Earth reversed the direction of its field several times during its history of 4 to 5 billion years. These magnetic fields got weakly recorded in rocks during their solidification. One can get clues about the geomagnetic history from the analysis of this rock magnetism.

(e) Earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km) because of the presence of the ionosphere. In this region, earth's field gets modified because of the field of single ions. While in motion, these ions produce the magnetic field associated with them.

(f) An extremely weak magnetic field can bend charged particles moving in a circle. This may not be noticeable for a large radius path. With reference to the gigantic interstellar space, the deflection can affect the passage of charged particles.

CBSE Class 12 physics
Important Questions
Chapter 6
Electromagnetic Induction

---

1 Mark Questions

1. A metallic wire coil is stationary in a non – uniform magnetic field. What is the emf. Induced in the coil?

Ans. NO emf is induced in the coil as there is no change in the magnetic flux linked with the secondary coil.

---

2. Why does metallic piece becomes very hot when it is surrounded by a coil carrying high frequency (H.F) alternating current?

Ans. When a metallic piece is surrounded by a coil carrying high frequency (H.F) alternating current, it becomes hot because eddy currents are produced which in turn produces joule's heating effect.

---

3. An electrical element X when connected to an alternating voltage source has current through it leading the voltage by $\frac{\pi}{2}$ radian. Identify X and write expression for its reactance?

Ans. X is a purely capacitive circuit

$$X_c = \frac{1}{2\pi\upsilon c} = \frac{1}{wC}$$

---

4. A transformer steps up 220 V to2200 V. What is the transformation ratio?

Ans. $K = \frac{Ns}{Np} = \frac{Es}{Ep} = \frac{2200}{220} = 10$

5. The induced emf is also called back emf. Why?

Ans. it is because induced emf produced in a circuit always opposes the cause which produces it.

---

6. Why the oscillations of a copper disc in a magnetic field are lightly damped?

Ans. Copper disc oscillates because of the production of eddy currents which opposes its oscillating motion and as a result the motion gets damped.

7.A metallic wire coil is stationary in a non – uniform magnetic field. What is the emf. Induced in the coil?

Ans.NO emf is induced in the coil as there is no change in the magnetic flux linked with the secondary coil.

2 Mark Questions

1. IF the rate of change of current of 2A/s induces an emf of 1OmV in a solenoid. What is the self-inductance of the solenoid?

Ans. $L = \frac{\in}{dI/dt} = \frac{10\times10^{-3}}{2} = 5\times10^{-3}\,Henry$

$\Rightarrow L = 5\times10^{-3}\,H$

---

2. A circular copper disc. 10 cm in radius rotates at a speed of 2π rad/s about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0.2T acts perpendicular to the disc.

1) Calculate the potential difference developed between the axis of the disc and the rim.

2) What is the induced current if the resistant of the disc is 2Ω?

Ans. (1) Radius = 10cm, B = 0.2T w = 2 π rad/s

$$\in = \frac{1}{2} Bwr^2$$

$$\in = \frac{1}{2} \times 0.2 \times 2\pi \times (0.1)^2$$

$$\in = 0.00628 \ volts$$

$$I = \frac{\in}{R} = \frac{0.0628}{2}$$

I = 0.0314 A

---

3. An ideal inductor consumes no electric power in a.c. circuit. Explain?

Ans. P = E rms I rms cos ϕ

But for an ideal inductor $\phi = \frac{\pi}{2}$

$$\Rightarrow \cos\phi = \cos\frac{\pi}{2} = 0$$

$\Rightarrow$ P=0

---

4. Capacitor blocks d.c. why?

Ans. The capacitive reactance

$$X_C = \frac{1}{wC} = \frac{1}{2\pi Vc}$$

For d.c. υ = 0

$$\Rightarrow X_C = \infty$$

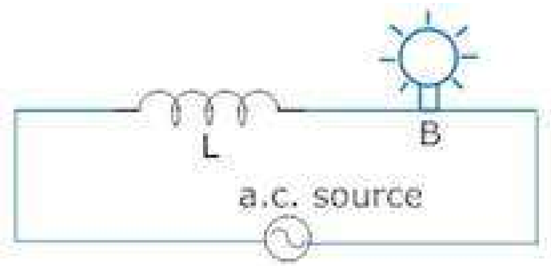
Since capacitor offers infinite resistance to the flow of d.c. so d.c. cannot pass through the capacitor.

5. Why is the emf zero, when maximum number of magnetic lines of force pass through the coil?

Ans. The magnetic flux will be maximum in the vertical position of the coil. But as the coil rotates $\frac{d\phi}{dt} = 0$

Hence produced emf $\in = \frac{d\phi}{dt} = 0$

6. An inductor L of reactance X_L is connected in series with a bulb B to an a.c. source as shown in the figure.



Briefly explain how does the brightness of the bulb change when

(a) Number of turns of the inductor is reduced.

(b) A capacitor of reactance $X_C = X_L$ is included in series in the same circuit.

Ans. (a) Since Z = $\sqrt{R^2 + X_L^2}$

When number of turns of the inductor gets reduced X_L and Z decreases and in turn current increases

Hence the bulb will grow more brightly

(b) When capacitor is included in the circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

But $X_L = X_C$ (given)

$\Rightarrow$ Z = R (minimum)

Hence brightness of the bulb will become maximum.

7. A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of $5 \times 10^{-4} T$ and the dip angle is $30°$.

Ans. Speed of the jet plane, *v* = 1800 km/h = 500 m/s

Wing span of jet plane, *l* = 25 m

Earth's magnetic field strength, *B* = $5.0 \times 10^{-4}\ T$

Angle of dip, $\delta = 30^0$ s

Vertical component of Earth's magnetic field,

BV = *B* sin δ

= $5 \times 10 - 4\ sin 30°$

= $2.5 \times 10 - 4\ T$

Voltage difference between the ends of the wing can be calculated as:

$e = (B_v) \times l \times v$

= $2.5 \times 10^{-4} \times 25 \times 500$

= 3.125 V

Hence, the voltage difference developed between the ends of the wings is 3.125 V.

---

8. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

Ans. Mutual inductance of a pair of coils, μ = 1.5 H

Initial current, I_1 = 0 A

Final current I_2 = 20 A

Change in current, $dI = 1_2 - 1_1 = 20 - 0 = 20A$

Induced emf, $e = \frac{d\phi}{dt}$ ...(i)

Where $d\phi$ is the change in the flux linkages with the coil.

Emf is related with mutual inductance as:

Equating equations (1) and (2), we get

$$\frac{d\phi}{dt} = \mu \frac{dI}{dt}$$

$$d\phi = 1.5 \times (20)$$

$$= 30Wb$$

Hence, the change in the flux linkage is 30 Wb.

---

9. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1} , at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2} .

(a) What is the instantaneous value of the emf induced in the wire?

(b) What is the direction of the emf?

(c) Which end of the wire is at the higher electrical potential?

Ans. Length of the wire, l = 10 m

Falling speed of the wire, v = 5.0 m/s

Magnetic field strength, B = $0.3 \times 10^{-4}\ Wb\ m^{-2}$

(a) Emf induced in the wire,

$e = Blv$

$$= 0.3 \times 10^{-4} \times 5 \times 10$$

$= 1.5 \times 10^{-3} V$

(b) Using Fleming's right hand rule, its can be inferred that the direction of the induced emf is from West to East.

(c) The eastern end of the wire is at a higher potential.

---

10. A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Ans. Length of the rod, l = 1 m

Angular frequency, ω = 400 rad/s

Magnetic field strength, B = 0.5 T

One end of the rod has zero linear velocity, while the other end has a linear velocity of $l_{(i)}$.

Average linear velocity of the rod, $v = \frac{1_{(i)} + 0}{2} = \frac{l_{(i)}}{2}$

Emf developed between the centre and the ring,

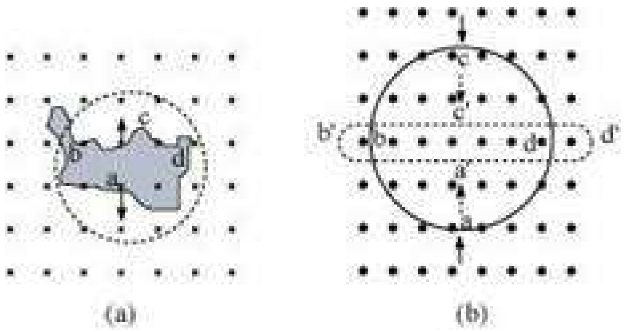
$$e = Blv = Bl\left(\frac{l_{(i)}}{2}\right) = \frac{Bl^2(i)}{2}$$

$$= \frac{0.5 \times (1)^2 \times 400}{2} = 100V$$

Hence, the emf developed between the centre and the ring is 100 V.

---

11. Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6.19:



(a) A wire of irregular shape turning into a circular shape;

(b) A circular loop being deformed into a narrow straight wire.

Ans. According to Lenz's law, the direction of the induced *emf* is such that it tends to produce a current that opposes the change in the magnetic flux that produced it.

(a) When the shape of the wire changes, the flux piercing through the unit surface area increases. As a result, the induced current produces an opposing flux. Hence, the induced current flows along adcb.

(b) When the shape of a circular loop is deformed into a narrow straight wire, the flux piercing the surface decreases. Hence, the induced current flows along

$e = \dfrac{4\pi\times10^{-7}\times50\times0.1\times10}{2\pi\times0.2}$ $e = 5\times10^{-5}V$ $a'd'c'b'$.

3 Mark Questions

1. How is the mutual inductance of a pair of coils affected when

(1) Separation between the coils is increased.

(2) The number of turns of each coil is increased.

(3) A thin iron sheet is placed between two coils, other factors remaining the same. Explain answer in each case.

Ans. (1) When the Separation between the coils is increased, the flux linked with the secondary coils decreases, hence mutual induction decreases.

(2) Since m = $\frac{\mu o N_1 N_2 A}{l}$, so when $N_1 and\ N_2$ increases, mutual induction increase.

(3) Mutual induction will increase because

$M \propto \mu r$ (Relative permeability of material)

---

2. Distinguish between resistances, reactance and impedance of an a.c. circuit?

Ans.

	Resistance	Reactance	Impedance
1	Opposition offered by the resistor to the flow of current	Opposition offered by the inductor or capacitor to the flow of current	Opposition offered by the combination of resistor, inductor or capacitor
2	It is independent of the frequency of the source.	It depends on the frequency of the source	It depends on the frequency of the source

3. A sinusoidal voltage V = 200 sin 314t is applied to a resistor of 10 Ω resistance. Calculate

(1) rms value of the voltage

(2) rms value of the current

(3) Power dissipated as heat in watt.

Ans. V = 200 sin 314t

V = Vo sin wt

Vo = 200V, w = 314 rad/s.

R = 10 Ω

(1) V rms = $\sqrt{2}Vo$

V rms = $\sqrt{2}\times$200 = 282.8 V

(2) $I \text{ rms} = \frac{V \text{ rms}}{R} = \frac{282.8}{10}$

I rms = 28.28 A

(3) Since circuit is purely resistive

$\therefore \phi = 0$

$\Rightarrow P = EvIv\cos\phi$

$P = EvIv$

$P = 282.8 \times 28.28$

P = 7.998 watt

---

4. Obtain an expression for the self inductance of a long solenoid? Hence define one

Henry?

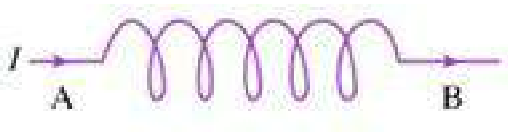
Ans. Consider a long solenoid of area A through which current I is flowing

Let N be the total number of turns in the solenoid

Total flux ϕ = NBA

Here = B = $\mu on I$

Where n is no. of turns per unit length of the solenoid



N = nl

$$\Rightarrow \phi = nl \times \mu on IA$$

$$\phi = \mu on^2 AlI -----(1)$$

$$Also \ \phi = LI ------------(2)$$

Equation (1) & (2)

$$\mu on^2 Al\cancel{I} = L\cancel{I}$$

$$L = \mu on^2 Al$$

$$\Rightarrow L = \frac{\mu o N^2 A}{l} \text{ [n = N/}\ell\text{]}$$

One Henry – if current is changing at a rate of 1 A/s in a coil induces an emf. of 1 volt in it then the inductance of the coil is one henry.

---

5. A conducting rod rotates with angular speed w with one end at the centre and other end at circumference of a circular metallic ring of radius R, about an axis passing through the centre of the coil perpendicular to the plane of the coil A constant magnetic field B parallel to the axis is present everywhere. Show that the emf. between

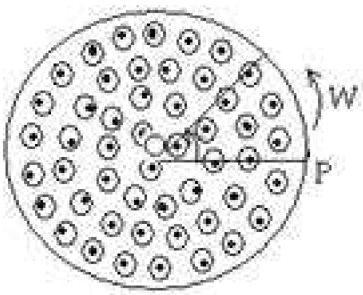
the centre and the metallic ring is $\frac{1}{2}BwR^2$.

Ans. Consider a circular loop connect the centre with point P with a resistor.

The potential difference across the resistor = induced emf.

$\in = B\times$ Rate of change of area of loop.

If the resistor QP is rotated with angular velocity w and turns by an angle θ in time t then



Area swept $A=\frac{1}{2}\times R\times R\theta$

$$A=\frac{1}{2}R^2\theta$$

$$\phi = BA\cos\theta^\circ = BA$$

$$\varphi = B\times\frac{1}{2}R^2\theta$$

$$\in = \frac{d\phi}{dt}=\frac{d}{dt}\left(\frac{1}{2}BR^2\theta\right)=\frac{1}{2}BR^2\left(\frac{d\theta}{dt}\right)$$

$$\in = \frac{1}{2}BwR^2$$

---

6. (a) At a very high frequency of a.c., capacitor behaves as a conductor. Why?

(b) Draw the graph showing the variation of reactance of

(i) A capacitor

(ii) An inductor with a frequency of an a.c. circuit.

Ans. (a) $X_C = \frac{1}{2\pi\upsilon c}$

For a.c. when $\upsilon \to \infty X_C = 0$

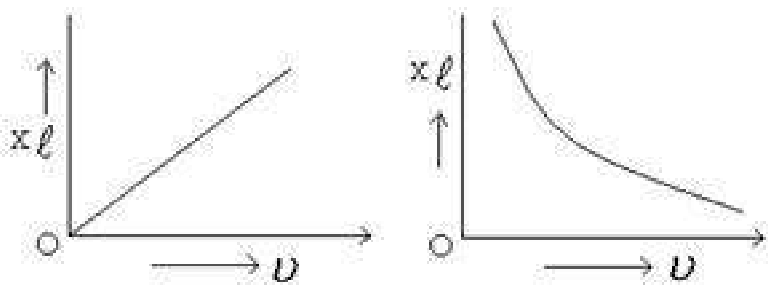
Thus at a very high frequency of a.c. capacitor behaves as a conductor

(b) $X_C = \frac{1}{2\pi\upsilon c}$

$\Rightarrow X_C \propto \frac{1}{\upsilon}$

$X_L = WL = 2\pi\upsilon L$

$XL \propto \upsilon$



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7. Calculate the current drawn by the primary of a transformer which steps down 200 V to 20 V to operate a device of resistance 20 Ω . Assume the efficiency of the transformer to be 80%?

Ans. η = 80% Ep = 200V Es = 20V Z = 20 Ω

$$Is = \frac{Es}{Z} = \frac{20}{20} = 1A$$

$$Now\ \eta = \frac{EsIs}{EpIp}$$

$$\eta = \frac{80}{100} = \frac{20 \times 1}{200 \times Ip}$$

$$\Rightarrow Ip = \frac{2000}{80 \times 200}$$

Ip = 0.125 A

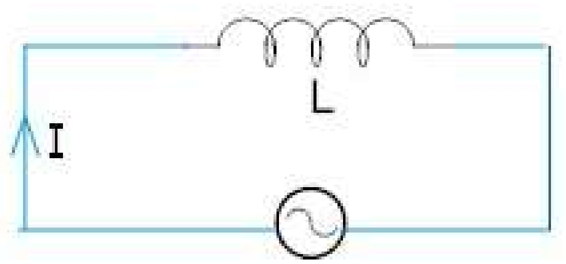
---

8. An a.c. voltage E = Eo sin wt is applied across an inductance L. obtain the expression for current I?

Ans.E = Eo sin wt (Given)

Emf produced across L = $\frac{-LdI}{dt}$

Total emf of the circuit = $E + \left(\frac{-LdI}{dt}\right)$



Since there is no circuit element across which potential drop may occur

$$\therefore E + \left(\frac{-LdI}{dt}\right) = 0$$

$$\Rightarrow E = \frac{LdI}{dt}$$

$$dI = \frac{E}{L}dt$$

$$dI.\frac{Eo}{L}\sin wt\ \text{dt}$$

Integrating

$$I=\frac{Eo}{L}\int\sin wt\ \text{dt}$$

$$I=\frac{Eo}{L}\left(\frac{-\cos wt}{w}\right)$$

$$I=\frac{-Eo}{wL}\cos wt$$

$$I=\frac{Eo}{wL}\sin\left(wt-\frac{\pi}{2}\right)$$

$$I=\frac{Eo}{X_L}\sin\left(wt-\frac{\pi}{2}\right)$$

$where\,\frac{Eo}{X_L}=Io$ (peak vlue of current)

$\Rightarrow$ I = Io sin $\left(wt-\frac{\pi}{2}\right)$

---

9. A series circuit with L = 0.12H, C = 0.48mF and R = 25 Ω is connected to a 220V variable frequency supply. At what frequency is the circuit current maximum?

Ans.L = 0.12H, C = 0.48mF = $0.48\times10^{-3}F$

R = 25 Ω Ev = 220V

$$Iv=\frac{Ev}{\sqrt{R^2+(X_L-X_C)^2}}$$

In the circuit when I is maximum, R will be minimum

$$\Rightarrow X_L = X_C$$

$$I = \frac{Ev}{R}$$

$$and\ \mathrm{f} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{0.12\times 0.48\times 10^{-3}}}$$

f = 21 Hz

---

10. A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is $1\ \mathrm{cm\ s^{-1}}$ in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

Ans. Length of the rectangular wire, *l* = 8 cm = 0.08 m

Width of the rectangular wire, *b* = 2 cm = 0.02 m

Hence, area of the rectangular loop,

A = *lb*

$$= 0.08\times 0.02$$

$$= 16\times 10 - 4\, m^2$$

Magnetic field strength, *B* = 0.3 T

Velocity of the loop, *v* = 1 cm/s = 0.01 m/s

(a) Emf developed in the loop is given as:

e = *Blv*

$$= 0.3 \times 0.08 \times 0.01 = 2.4 \times 10^{-4}\ V$$

Time taken to travel along the width. ts

$$\frac{Dis\tan ce travelled}{Velocity} = \frac{b}{v} = \frac{0.02}{0.01} = 2s$$

Hence, the induced voltage is $2.4 \times 10^{-4}\ V$ which lasts for 2 s.

(b) Emf developed, *e* = *Bbv*

= $0.3 \times 0.02 \times 0.01 = 0.6 \times 10^{-4} V$

Time taken to travel along the length,

t= $\frac{Dis\tan ce travelled}{Velocity} = \frac{1}{v} = \frac{0.08}{0.01} = 8s$

Hence, the induced voltage is $0.6 \times 10^{-4}\ V$ which lasts for 8 s.

---

11. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.

Ans. Initial current, $I_1 = 5.0\ A$

Final current, $I_2 = 0.0\ A$

Change in current, $dI = I_1 - I_2 = 5A$

Time taken for the change, *t* = 0.1

Average emf, *e* = 200 V

For self-inductance (*L)* of the coil, we have the relation for average emf as:

e = *L* $\frac{di}{dt}$ s

$$L = \frac{r}{\left(\frac{di}{dt}\right)}$$

$d\phi$

$$= \frac{200}{\frac{5}{0.1}} = 4H$$

$$L = \frac{\varepsilon}{\left(\frac{di}{dt}\right)} \text{s}$$

Hence, the self-induction of the coil is 4 H.

---

12. Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of $0.02\ \mathrm{T\ s^{-1}}$. If the cut is joined and the loop has a resistance of 1.6 Ω how much power is dissipated by the loop as heat? What is the source of this power?

Ans. Sides of the rectangular loop are 8 cm and 2 cm.

Hence, area of the rectangular wire loop,

$$A = length \times width$$

$$= 8 \times 2 = 16 cm^2$$

$$= 16 \times 10 - 4\, m^2$$

Initial value of the magnetic field, $B' = 0.3T$

Rate of decrease of the magnetic field, $\frac{dB}{dt} = 0.02T/s$

Emf developed in the loop is given as:

$$e = \frac{d\phi}{dt}$$

Where,

$d\phi$ = Change in flux through the loop area

= AB

$$\therefore e = \frac{d(AB)}{dt} = \frac{AdB}{dt}$$

$$= 16 \times 10^{-4} \times 0.02 = 0.32 \therefore 10^{-4} V$$

Resistance of the loop, R = 1.6 Ω

The current induced in the loop is given as:

$$i = \frac{r}{R}$$

$$= \frac{0.32 \times 10^{-4}}{1.6} = 2 \times 10^{-5} A$$

Power dissipated in the loop in the form of heat is given as:

$$P = i^2 R$$

$$= (2 \times 10^{-5})^2 \times 1.6$$

$$= 6.4 \times 10^{-10} W$$

The source of this heat loss is an external agent, which is responsible for changing the magnetic field with time.

---

13. An air-cored solenoid with length 30 cm, area of cross-section $25\ \text{cm}^2$ and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Ans. Length of the solenoid, l = 30 cm = 0.3 m

Area of cross-section, A = $25\ cm^2 = 25 \times 10^{-4}\ m^2$

Number of turns on the solenoid, N = 500

Current in the solenoid, I = 2.5 A

Current flows for time, t = 10^{-3} s

Average back emf, $e = \frac{d\phi}{dt}$ ...(1)

Where,

$d\phi$ = Change in flux

= NAB ......(2)

Where,

B = Magnetic field strength

$= \mu_0 \frac{NI}{l}$ ...(3)

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ T\ m\ A^{-1}$

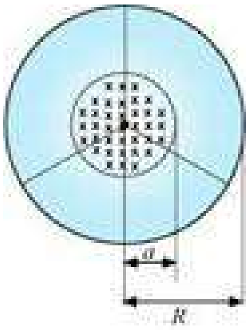
Using equations (2) and (3) in equation (1), we get

$$e = \frac{\mu_0 N^2 IA}{lt}$$

$$= \frac{4\pi \times 10^{-7} \times (500)^2 \times 2.5 \times 25 \times 10^{-4}}{0.3 \times 10^{-3}} = 6.5V$$

Hence, the average back emf induced in the solenoid is 6.5 V.

14. A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass *M* and radius *R*. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig. 6.22). A uniform magnetic field extends over a circular region within the rim. It is given by,



$\mathbf{B} = -\mathbf{B}_0\ \mathbf{k}\ (r \leq a; a < R)$ **= 0 (otherwise) What is the angular velocity of the wheel after the field is suddenly switched off?**

Ans. Line charge per unit length

$$= \lambda = \frac{Total\ charge}{Length} = \frac{Q}{2\pi r}$$

Where,

r = Distance of the point within the wheel

Mass of the wheel = *M*

Radius of the wheel = *R*

Magnetic field, $B = -\overline{B} = B_0\ \hat{k}$

At distance *r*, the magnetic force is balanced by the centripetal force i.e.,

$$BQv = \frac{Mv^2}{r}$$

Where

V=linear velocity of the wheel

$$\therefore B2\pi r\lambda r^2 = \frac{Mv}{r}$$

$$v = \frac{B2\pi r\lambda r^2}{M}$$

$$BQv = \frac{Mv^2}{r}$$

$\therefore$ Angular velocity, $(i) = \frac{v}{R} = \frac{B2\pi r\lambda r^2}{M}$

$For\ r \leq and\ a < R,\ we\ get:$

$$(i) = -\frac{2B_0 a^2 \lambda}{MR} k$$

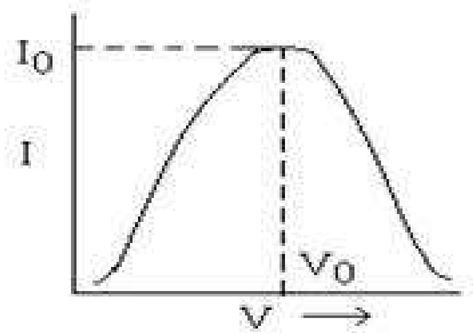
5 Mark Questions

1. (a) State the condition under which the phenomenon of resonance occurs in a series LCR circuit. Plot a graph showing the variation of current with frequency of a.c. sources in a series LCR circuit.

(b) Show that in a series LCR circuit connected to an a.c. source exhibits resonance at its natural frequency equal to $\frac{1}{\sqrt{LC}}$?

Ans. (a) In a series LCR circuit

Resonance occurs when $X_L = X_C$.



The variation of current with frequency of a.c. source in series LCR circuit

(b) Electrical resonance takes place in a series LCR circuit when circuit allows maximum alternating current for which

$X_L = X_C$.

Impedance Z = $\sqrt{R^2 + (X_L - X_C)^2}$

$$I = \frac{E}{Z} - \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

For electrical resonance

$X_L = X_C$.

$$wL = \frac{1}{wC} \quad or \quad w^2 = \frac{1}{LC}$$

$$w = \frac{1}{\sqrt{LC}}$$

Where w is the natural frequency of the circuit.

---

2. In a step up transformer, transformation ratio is 100. The primary voltage is 200 V and input is 1000 watt. The number of turns in primary is 100. Calculate

(1) Number of turns in the secondary

(2) Current in the primary

(3) The voltage across the secondary

(4) Current in the secondary

(5) Write the formula for transformation ratio?

Ans. (1) k = 100, Ep = 200V

EpIp = 1000 W, Np = 100

$$K = 100 = \frac{Ns}{Np}$$

$$\Rightarrow Ns = 100 \times Np$$

$$Ns = 100 \times 100$$

NS = 10,000

(2) EpIp = 1000W

$$Ip = \frac{1000}{Ep}$$

$$Ip = \frac{1000}{200} = 5A$$

Ip = 5A

(3) $\frac{Es}{Ep} = \frac{Ns}{Np}$

$$\therefore Es = Ep \times \frac{Ns}{Np}$$

$$Es = 200 \times 100$$

Es = 20000 Volt

(4) $\frac{Es}{Ep} = \frac{Ip}{Is}$

$$Is = \frac{IpEp}{Es}$$

$$Is = \frac{1000}{20000} = \frac{1}{20}$$

Is = 0.05 A

(5) For step up Trans former k > 1 $K = \frac{Ns}{Np}$

---

3. Drive an expression for the average power consumed in a.c. series LCR circuit. Hence define power factor?

Ans. For an a.c. series circuit

E = Eo sin wt

And I = Io sin (wt + ϕ)

Where ϕ is the phase angle by which current leads the emf.

Now using dw = EIdt

dw = (Eo sin wt) (Io sin (wt + ϕ))dt

dw = EoIo sin wt (sin wt cos ϕ + cos wt sin ϕ)dt

dw = EoIo (sin^2 wt cos ϕ + sin wt cos wt sin ϕ)dt

$$\left[\because \sin 2wt = 2 \sin wt \cos wt\right]$$

$$\Rightarrow dw = EoIo\left(\frac{1-\cos 2wt}{2}\cos\phi + \frac{\sin 2wt}{2}\sin\phi\right)$$

$$\left[\therefore \sin^2 wt = \frac{1-\cos 2wt}{2}\right]$$

$$dw = \frac{EoIo}{2}(\cos\phi - \cos\phi\cos 2wt + \sin\phi\sin not)dt$$

Integrating within limits t = o to t = T

$$W = \frac{EoIo}{2}\left[\cos\phi\int_O^T dt - \cos\phi\int_O^T \cos 2wtdt + \sin\phi\int_O^T \sin 2wtdt\right]$$

$$\Rightarrow W = \frac{EoIo}{2}\cos\phi\int_O^T dt\left[\because \int_O^T \sin 2wtdt = \int_O^T \cos 2wtdt = 0\right]$$

$$W = \frac{EoIo}{2}T\cos\phi$$

Hence average power consumed in a.c circuit is given by

$$Pav = \frac{W}{T} = \frac{EoIo}{2}\cos\phi$$

Pav = EvIv cos ϕ -----(1)

Power factor – In the above expression

Cos ϕ is termed as power factor

When cos ϕ = 1 ϕ = 0^{o}

It means circuit is purely resistive and Pav = EvIv

When cos ϕ = 0 ϕ = 90^{o}

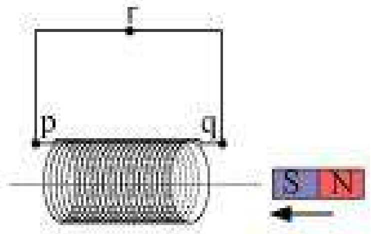
It means circuit is purely capacitive or inductive.

Pav = 0

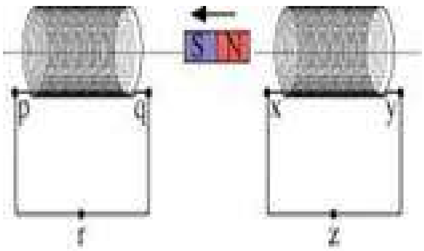
Hence

4. Predict the direction of induced current in the situations described by the following Figs. 6.18(a) to (f).

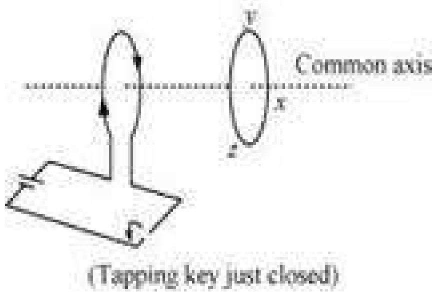
(a)



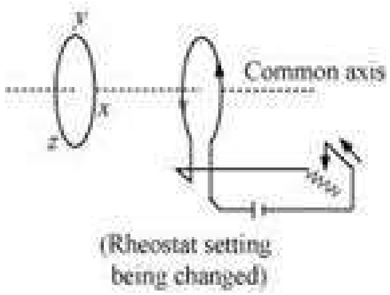
(b)



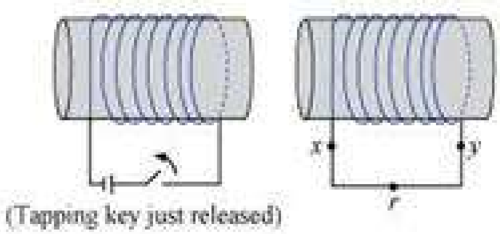
(c)



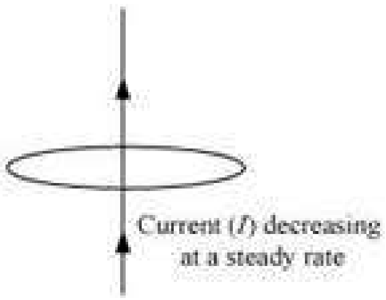
(d)



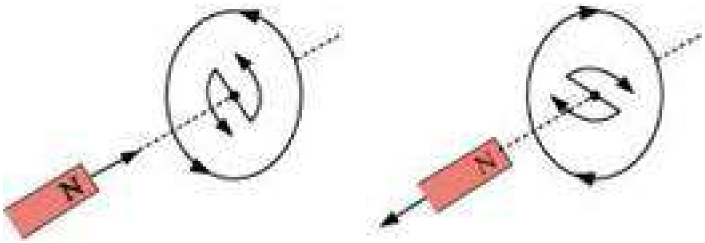
(e)



(f)



Ans. The direction of the induced current in a closed loop is given by Lenz's law. The given pairs of figures show the direction of the induced current when the North pole of a bar magnet is moved towards and away from a closed loop respectively. Using Lenz's rule, the direction of the induced current in the given situations can be predicted as follows:



(a) The direction of the induced current is along **qrpq**.

(b) The direction of the induced current is along **prqp**.

(c) The direction of the induced current is along ***yzxy***.

(d) The direction of the induced current is along ***zyxz***.

(e) The direction of the induced current is along ***xryx***.

(f) No current is induced since the field lines are lying in the plane of the closed loop.

5. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

Ans. Number of turns on the solenoid = 15 turns/cm = 1500 turns/m

Number of turns per unit length, *n* = 1500 turns

The solenoid has a small loop of area, *A* = $2.0\ cm^2 = 2 \times 10 m^2$

Current carried by the solenoid changes from 2 A to 4 A.

∴ Change in current in the solenoid, *di* = 4 - 2 = 2 A

Change in time, *dt* = 0.1 s

Induced *emf* in the solenoid is given by Faraday's law as:

$e = \frac{d\phi}{dt}$ ......(i)

Where,

ϕ = Induced flux through the small loop

= *BA* ...(*ii*)

B = Magnetic field

= $\mu_0 ni$ ...(iii)

μ_0 = Permeability of free space

$= 4\pi \times 10^{-7}\ H/m$

Hence, equation (*i*) reduces to:

$$e = \frac{d}{dt}\left(\frac{di}{dt}\right)$$

$$= A\mu_0 n \times \left(\frac{di}{dt}\right)$$

$$= 2\times10^{-4}\times4\pi\times10^{-7}\times1500\times\frac{2}{0.1}$$

$$= 7.54\times10^{-6} V$$

Hence, the induced voltage in the loop is $7.54\times10^{-6} V$

---

6. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 $\mathbf{rad\ s^{-1}}$ in a uniform horizontal magnetic field of magnitude $\mathbf{3.0\times10^{-2}\ T}$. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10Ω , calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Ans. Max induced *emf* = 0.603 V

Average induced *emf* = 0 V

Max current in the coil = 0.0603 A

Average power loss = 0.018 W

(Power comes from the external rotor)

Radius of the circular coil, r = 8 cm = 0.08 m

Area of the coil, $A = \pi r^2 = \pi \times (0.08)^2\ m^2$

Number of turns on the coil, N = 20

Angular speed, ω = 50 rad/s

Magnetic field strength, B = $3 \times 10^{-2}\ T$

Resistance of the loop, R = 10 Ω

Maximum induced *emf* is given as:

$$e = N\omega AB$$

$$= 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2}$$ = 0.603 V

The maximum *emf* induced in the coil is 0.603 V.

Over a full cycle, the average *emf* induced in the coil is zero.

Maximum current is given as:

$$I = \frac{e}{R}$$

$$= \frac{0.603}{10} = 0.0603 A$$

Average power loss due to joule heating:

$$P = \frac{el}{2}$$

$$= \frac{0.603 \times 0.0603}{2} = 0.018W$$

The current induced in the coil produces a torque opposing the rotation of the coil. The rotor is an external agent. It must supply a torque to counter this torque in order to keep the coil rotating uniformly. Hence, dissipated power comes from the external rotor.

---

7. A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of 8 cm s^{-1} in the positive *x*-direction in an environment containing a magnetic field in the positive *z*-direction. The field is neither uniform in space nor constant in time. It has a gradient of $10^{-3} \text{ T cm}^{-1}$ along the negative *x*-direction (that is it increases by $10^{-3} \text{ T cm}^{-1}$ as one moves in the negative *x*-direction), and it is decreasing in time at the rate of 10^{-3} T s^{-1} . Determine the direction and magnitude of the induced current in the loop if its resistance is 4.50 Ω .

Ans. Side of the square loop, *s* = 12 cm = 0.12 m

Area of the square loop, $A = 0.12 \times 0.12 \;\; = 0.0144 \; m^2$

Velocity of the loop, *v* = 8 cm/s = 0.08 m/s

Gradient of the magnetic field along negative *x*-direction,

$$\frac{dB}{dx} = 10^{-3} Tcm^{-1} = 10^{-1} Tm^{-1}$$

And, rate of decrease of the magnetic field,

$$\frac{dB}{dx} = 10^{-3} Ts^{-1}$$

Resistance of the loop, $R = 4.5 m\Omega = 4.5 \times 10^{-3} \Omega$

Rate of change of the magnetic flux due to the motion of the loop in a non-uniform magnetic field is given as:

$$\frac{d\phi}{dt} = A \times \frac{dB}{dx} \times v$$

$$= 144 \times 10^{-4} m^2 \times 10^{-1} \times 0.08$$

$$= 11.52 \times 10^{-5} Tm^2 s^{-1}$$

Rate of change of the flux due to explicit time variation in field *B* is given as:

$$\frac{d\phi'}{dt} = A \times \frac{dB}{dx}$$

$$= 144 \times 10^{-4} \times 10^{-3}$$

$$= 1.44 \times 10^{-5} Tm^2 s^{-1}$$

Since the rate of change of the flux is the induced *emf*, the total induced *emf* in the loop can be calculated as:

$$e = 144 \times 10^{-5} + 11.52 \times 10^{-5}$$

$$= 12.96 \times 10^{-5} V$$

∴Induced current, $i = \frac{e}{R}$ s

s$= \frac{12.96 \times 10^{-5}}{1.5 \times 10^{-3}}$

$$i = 2.88 \times 10^{-2} A$$

Hence, the direction of the induced current is such that there is an increase in the flux through the loop along positive z-direction.

---

8. It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area $2\ \mathrm{cm}^2$ with 25 closely wound turns, is

positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is 0.50 Ω . Estimate the field strength of magnet.

Ans. Area of the small flat search coil, *A* = $2\ cm^2 = 2 \times 10^{-4}\ m^2$

Number of turns on the coil, *N* = 25

Total charge flowing in the coil, *Q* = 7.5 mC = $7.5 \times 10^{-3}\ C$

Total resistance of the coil and galvanometer, *R* = 0.50 Ω

Induced current in the coil,

$$I = \frac{Induced\ emf\,(e)}{R} \qquad ...(i)$$

Induced emf is given as:

$$e = -N\frac{d\phi}{dt} \qquad ...(2)$$

Where,

$d\phi$ = Charge in flux

Combining equations (1) and (2), we get

$$1 = -\frac{N\frac{d\phi}{dt}}{R}$$

$$Idt = -\frac{N}{R}d\phi \qquad ...(3)$$

Initial flux through the coil, ϕ_i = *BA*

Where,

B = Magnetic field strength

Final flux through the coil, $\phi_f = 0$

Integrating equation (3) on both sides, we have

$$\int Idt = \frac{-N}{R}\int_{\phi_i}^{\phi_f} d\phi$$

But total charge, $Q = \int Idt$.

$$\therefore Q = \frac{-N}{R-}(\phi_f - \phi_i) = \frac{-N}{R}(-\phi_i) = +\frac{N\phi_i}{R}$$

$$Q = \frac{NBA}{R}$$

$$\therefore B = \frac{QR}{NA}$$

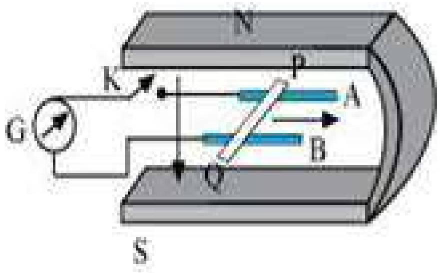
$$= \frac{7.5\times10^{-3}\times0.5}{25\times2\times10^{-4}} = 0.75T$$

Hence, the field strength of the magnet is 0.75 T.

---

9. Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, *B* = 0.50 T, resistance of the closed loop containing the rod = 9.0 mΩ . Assume the field to be uniform.

(a) Suppose K is open and the rod is moved with a speed of 12 cm s^{-1} in the direction shown. Give the polarity and magnitude of the induced emf.



(b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

(c) With K open and the rod moving uniformly, there is *no* experience magnetic force due to the motion of the rod. Explain.

(d) What is the retarding force on the rod when K is closed?

(e) How much power is required (by an external agent) to keep the rod moving at the same speed (=$12\ \text{cm s}^{-1}$) when K is closed? How much power is required when K is open?

(f) How much power is dissipated as heat in the closed circuit? What is the source of this power?

(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

Ans. Length of the rod, l = 15 cm = 0.15 m

Magnetic field strength, B = 0.50 T

Resistance of the closed loop, R = 9 $\Omega = 9 \times 10^{-3}\ \Omega$

(a) Induced emf = 9 mV; polarity of the induced emf is such that end P shows positive while end Q shows negative ends.

Speed of the rod, v = 12 cm/s = 0.12 m/s

Induced emf is given as:

$e = Bvl$

$= 0.5 \times 0.12 \times 0.15$

$= 9 \times 10^{-3}$ v

= 9 mV

The polarity of the induced emf is such that end *P* shows positive while end *Q* shows negative ends.

(b) Yes; when key K is closed, excess charge is maintained by the continuous flow of current.

When key K is open, there is excess charge built up at both ends of the rods.

When key K is closed, excess charge is maintained by the continuous flow of current.

(c) Magnetic force is cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rod.

There is no net force on the electrons in rod PQ when key K is open and the rod is moving uniformly. This is because magnetic force is cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rods.

(d) Retarding force exerted on the rod, *F* = *IBl*

Where,

I = Current flowing through the rod

$$= \frac{e}{R} = \frac{9 \times 10^{-3}}{9 \times 10^{-3}} = 1A$$

$$\therefore F = 1 \times 0.5 \times 0.15$$

$$= 75 \times 10^{-3} N$$

(e) 9 mW; no power is expended when key K is open.

Speed of the rod, *v* = 12 cm/s = 0.12 m/s

Hence, power is given as:

$P = Fv$

$= 75 \times 10^{-3} \times 0.12$

$= 9 \times 10^{-3} W$

$= 9\, mW$

When key K is open, no power is expended.

(f) 9 mW; power is provided by an external agent.

Power dissipated as heat = *I*2 *R*

$= (1)2 \times 9 \times 10^{-3}$

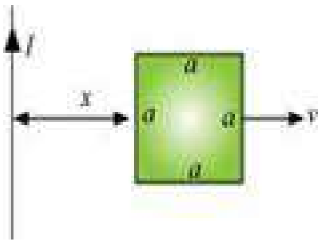
= 9 mW

The source of this power is an external agent.

(g) Zero In this case, no emf is induced in the coil because the motion of the rod does not cut across the field lines.

---

10. (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side *a* as shown in Fig. 6.21.

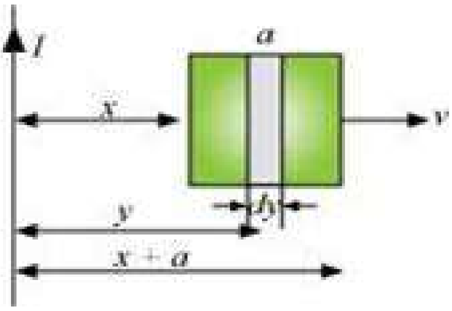


(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, *v* = 10 m/s.

Calculate the induced emf in the loop at the instant when *x* = 0.2 m.

Take *a* = 0.1 m and assume that the loop has a large resistance.

Ans.



(a) Take a small element *dy* in the loop at a distance *y* from the long straight wire (as shown in the given figure).

Magnetic flux associated with element $dy, d\phi = BdA$

Where,

dA = Area of element *dy* = *a dy*

B = Magnetic field at distance *y*

$= \frac{\mu_0 I}{2\pi y}$ s

I = Current in the wire

μ_0 = Permeability of free space = $4\pi \times 10^{-7} T\, m\, A^{-1}$

$\therefore d\phi = \frac{\mu_0 I a}{2\pi} \frac{dy}{y}$

$\phi = \frac{\mu_0 I a}{2\pi} \int \frac{dy}{y}$

y tends from *x* to a+x.s

$\therefore \phi = \frac{\mu_0 I a}{2\pi} \int_x^{a+x} \frac{dy}{y}$

$= \frac{\mu_0 I a}{2\pi} [\log_e y]_x^{a+x}$

$$= \frac{\mu_0 Ia}{2\pi} \log_e \left(\frac{a+x}{x} \right)$$

For mutual inductance M, the flux is given as:

$$\phi = mI$$

$$\therefore mI = \frac{\mu_0 Ia}{2\pi} \log \left(\frac{a}{x} + 1 \right)$$

$$m = \frac{\mu_0 Ia}{2\pi} \log \left(\frac{a}{x} + 1 \right)$$

(b) Emf induced in the loop, *e* = *B'av* $\left(\frac{\mu_0 I}{2\pi} \right) av$ Given,

I = 50 A

x = 0.2 m

a = 0.1 m

v = 10 m/s

$$BQv = \frac{Mv^2}{r}$$

$$\therefore B 2\pi r \lambda r^2 = \frac{Mv}{r}$$

$$v = \frac{B 2\pi r \lambda r^2}{M}$$

$$For\ r \leq and\ a < R, we\ get:$$

$$(i) = -\frac{2B_0 a^2 \lambda}{MR} k$$

$$e = 5 \times 10^{-5} V$$

CBSE Class 12 physics

Practice Questions

Chapter 7

Alternating Current

---

1 Mark Questions

1. Power factor of an a.c. circuit is 0.5. What will be the phase difference between voltage and current in the circuit?

Ans.

$\cos\phi = 0.5 = \frac{1}{2}$

$\phi = 60^\circ$

Hence the phase difference is 60°

---

2. Weber is the unit of which physical quantity? Hence define it?

Ans.weber is the SI unit of magnetic flux. The weber is the magnetic flux that, linking a circuit of one turn, would produce in it an electromotive force of 1 volt if it were reduced to zero at a uniform rate in 1 second.

---

3. Two identical loops, one of copper and another of aluminum are rotated with the same speed in the same magnetic field. In which case, the induced

(a) emf.

(b) current will be more and why?

Ans. The induced emf will be same in both the loops but induced current will be more in copper loop because its resistance is less.

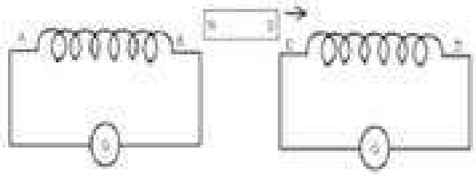
---

4. A transformer cannot be used to step up d.c. voltage?

Ans. When a d.c voltage source is applied across the primary of the transformer,the current in the primary coil remains constant. Hence there is no change in the magnetic flux linked with the secondary. Therefore the voltage across the secondary coil is zero.Thus a transformer can't step up dc voltage.

2 Mark Questions

1. Magnet is moved in the direction indicated by an arrow between two coil AB and CD as shown in the figure. Suggest the direction of current in each coil.



Ans. (1) Total impedance of the circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

(2) $f = \dfrac{1}{2\pi\sqrt{LC}}$

---

2. How does the self induction of a coil change when?

(1) The number of turns in a coil is decreased

(2) An iron rod is introduced into it. Justify.

Ans. L = 2H, C = 32 $\mu F = 32\times10^{-6}$ F, R = 10 Ω

$$wr = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times32\times10^{-6}}}$$

Wr = 125 rad/sec.

ϕ – Value of the circuit

$$\phi = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32\times10^{-6}}}$$

$\Rightarrow \phi$ =25

3. A coil of inductance L, a capacitor of capacitance C and a resistor of resistance R are all put in series with an alternating source of emf E = (Eo sin wt). Write an expression for the

(1) Total impedance of the circuit

(2) Frequency of the source emf for which the current carrying circuit will show resonance.

Ans. (1) Total impedance of the circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

(2) $f = \dfrac{1}{2\pi\sqrt{LC}}$

---

4. Obtain the resonant frequency w_r of series LCR circuit with = L =2 H, C = 32 μ F and R = 10 Ω. What is the ϕ – value of this circuit?

Ans. L = 2H, C = 32 μ $F = 32 \times 10^{-6}$ F, R = 10 Ω

$$wr = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}}$$

Wr = 125 rad/sec.

ϕ – Value of the circuit

$$\phi = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}}$$

$\Rightarrow \phi$ =25

5. A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Ans. Resistance of the resistor, R = 100 Ω

Supply voltage, V = 220 V

Frequency, ν = 50 Hz

(a) The rms value of current in the circuit is given as:

$$I = \frac{V}{R}$$

$$= \frac{220}{100} = 2.20A$$

(b) The net power consumed over a full cycle is given as:

$P = VI$

$$= 220 \times 2.2 = 484\ W$$

---

6. (a) The peak voltage of an ac supply is 300 V. What is the rms voltage?

(b) The rms value of current in an ac circuit is 10 A. What is the peak current?

Ans. (a) Peak voltage of the ac supply, V_0 = 300 V

Rms voltage is given as:

$$V = \frac{V_0}{\sqrt{2}}$$

$$= \frac{300}{\sqrt{2}} = 212.1V$$

(b) The rms value of current is given as:

I = 10 A

Now, peak current is given as:

$$I_0 = \sqrt{2}I$$

$$= 10\sqrt{2} = 14.1A$$

---

7. A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

Ans. Inductance of inductor, L = 44 mH = $44 \times 10^{-3}\ H$

Supply voltage, V = 220 V

Frequency, ν = 50 Hz

Angular frequency, $\omega = 2\pi\nu$

Inductive reactance, XL = $X_L = \omega L = 2\pi\nu L = 2\pi \times 50 \times 44 \times 10^{-3}\ \Omega$

Rms value of current is given as: $I = \dfrac{V}{X_L}$ s

$$= \frac{200}{2\pi \times 50 \times 44 \times 10^{-3}} = 15.92A$$

Hence, the rms value of current in the circuit is 15.92 A.

---

8. A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

Ans. Capacitance of capacitor, C = 60 μF = $60 \times 10^{-6}\ F$

Supply voltage, V = 110 V

Frequency, ν = 60 Hz

Angular frequency, $\omega = 2\pi\nu$

Capacitive reactance $X_c = \dfrac{1}{\omega C}$

$$= \frac{1}{2\pi\nu C}$$

$$= \frac{1}{2\times 3.14\times 60\times 60\times 10^{-6}}\,\Omega^{-1}$$

Rms value of current is given as:

$$I = \frac{V}{X_c}$$

$$= 110\times 2\times 3.14\times 60\times 10^{-6}\times 60 = 2.49A$$

Hence, the rms value of current is 2.49 A.

---

9. Obtain the resonant frequency ωr of a series *LCR* circuit with $L = 2.0\,H, C = 32\,\mu F$ and *R* = 10 Ω . What is the *Q*-value of this circuit?

Ans. Inductance, L = 2.0 H

Capacitance, C = 32 μF = $32\times 10^{-6}\,F$

Resistance, R = 10 Ω

Resonant frequency is given by the relation,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{2\times32\times10^{-6}}} = \frac{1}{8\times10^{-3}} = 125\,s^{-1}$$

Now, Q-value of the circuit is given as:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{10}\sqrt{\frac{2}{32\times10^{-6}}} = \frac{1}{10}\times\frac{1}{4\times10^{3}} = 25$$

Hence, the Q-Value of this circuit is 25.

---

10. A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Ans. Capacitance, C = 30 μF = $30\times10^{-6}F$

Inductance, L = 27 mH = $27\times10^{-3}H$

Angular frequency is given as:

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= 1.11\times10^{3}\,rad/s$$

Hence, the angular frequency of free oscillations of the circuit is $1.11\times10^{3}\,rad/s.$

---

11. Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at later time?

Ans. Capacitance of the capacitor, C = 30 μF = $30\times10^{-6}\,F$

Inductance of the inductor, L = 27 mH = $27\times10^{-3}\,H$

Charge on the capacitor, Q = 6 mC = $6\times10^{-3}\,C$

Total energy stored in the capacitor can be calculated by the relation,

$$E = \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{1}{2} \times \frac{(6\times10^{-3})^2}{30\times10^{-6}}$$

$$= \frac{6}{10} = 0.6J$$

Total energy at a later time will remain the same because energy is shared between the capacitor and the inductor.

---

12. At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is $100\ m^3\ s^{-1}$. If the turbine generator efficiency is 60% estimate the electric power available from the plant (g= $9.8\ m\ s^{-2}$) .

Ans. Height of water pressure head, h = 300 m

Volume of water flow per second, V = 100 m3/s

Efficiency of turbine generator, n = 60% = 0.6

Acceleration due to gravity, g = $9.8\ m/s^2$

Density of water, p = $10^3\ kg/m^3$

Electric power available from the plant = $\eta \times hpgV$

= $0.6\times300\times10^3\times9.8\times100$

= $176.4\times10^6\ W$

= 176.4 MW

---

13. A power transmission line feeds input power at 2300 V to a stepdown transformer with its primary windings having 4000 turns. What should be the number of turns in

the secondary in order to get output power at 230 V?

Ans. Input voltage, V_1 = 2300

Number of turns in primary coil, n_1 = 4000

Output voltage, V_2 = 230 V

Number of turns in secondary coil = n_2

Voltage is related to the number of turns as:

$$\frac{V_1}{V_2} = \frac{n_1}{n_2}$$

$$\frac{2300}{230} = \frac{4000}{n_2}$$

$$n_2 = \frac{4000 \times 230}{2300} = 400$$

Hence, there are 400 turns in the second winding.

3 Mark Questions

1. A variable frequency 230V alternating voltage source is connected across a series combination of L = 5H, C = 80 μ F and R = 40 Ω . Calculate

(a) Angular frequency of the source which drives the circuit in resonance

(b) Impedance of the circuit

(c) Amplitude of current at resonance.

Ans. Power factor of circuit A

$$\cos \phi A = \frac{R}{\sqrt{R^2 + X_L{}^2}} = \frac{R}{\sqrt{R^2 + 9R^2}}$$

$$\cos \phi A = \frac{\cancel{R}}{\sqrt{10}\cancel{R}}$$

$$\cos \phi A = \frac{1}{\sqrt{10}} \text{ ----(1)}$$

Power factor of circuit B

$$\cos \phi B = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + (3R - R)^2}}$$

$$\cos \phi B = \frac{R}{\sqrt{R^2 + 4R^2}} = \frac{R}{\sqrt{5R^2}}$$

$$\cos \phi B = \frac{1}{\sqrt{5}} \text{ ----(2)}$$

$$\frac{\cos \phi B}{\cos \phi A} = \frac{\frac{1}{\sqrt{5}}}{\frac{1}{\sqrt{10}}}$$

$$\cos \phi B : \cos \phi A = \sqrt{2}$$

---

2. Show that in the free oscillations of an LC circuit, the sum of the energies stored in the capacitor and the inductor is constant in time?

Ans. Energy stored in capacitor

$$= \frac{1}{2}\frac{q_2}{C} \; (\because q = q0\cos wt)$$

$$= \frac{1}{2} q0^2 \cos^2 wt \quad \text{----(1)}$$

Energy stored in an inductor = $\frac{1}{2}LI^2$

$$\Rightarrow \text{ E} = \frac{1}{2}L\left(\frac{dq}{dt}\right)^2 \left(I = Io\cos wt \because I = \frac{dq}{dt}\right)$$

$$E = \frac{1}{2}L\left[\frac{d}{dt}\left(q_o \cos wt\right)\right]^2$$

$$E = \frac{1}{2}Lq_0^2 w^2 \sin^2 wt$$

$$E = \frac{q_0^2}{2}\sin^2 wt\cancel{L}\left(\frac{1}{\cancel{L}C}\right)\left(\because w^2 = \frac{1}{LC}\right)$$

$$E = \frac{q_0^2}{2C}\sin^2 wt$$

Combining (1) & (2) to get total amount of energy

$$E = \frac{q_o^{\;2}}{2C}\left[\sin^2 wt + \cos^2 wt\right]$$

(Constant) $E = \frac{q_o^{\;2}}{2C}$

3. Define mutual inductance? What is its S.I. unit? Write the expression for the mutual inductance between a pair of circular coils of radius r and R (R > r).

Ans. It is defined as the phenomenon of inducing emf in a coil due to the rate of change of current in a nearby coil. Its S.I. unit is Henry (H).

Let two coaxial concentric

coils of radio r and R (R > r)

be placed in air. If current

I_2 flows through R, the magnetic flux gets linked up with secondary coil (coils of radius r) & is given by

$$\phi s = BA = \left(\frac{\mu_o I_2}{2R}\right)\left(\pi r^2\right)$$

$$\phi s = \frac{\mu_o \pi r^2 I_2}{2R} \quad \text{----(1)}$$

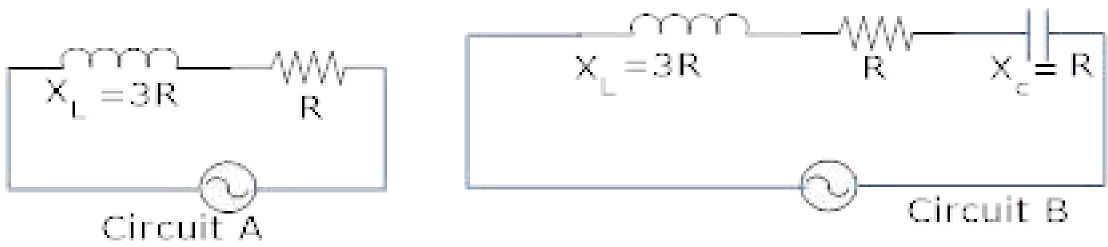
$$Also\ \ \phi s = MI_2 \quad \text{----(2)}$$

Combining equation (1) & (2)

$$MI_2 = \frac{\mu_o \pi r^2 I_2}{2R}$$

$$M = \frac{\mu_o \pi r^2}{2R}$$

4. Figure shows two electric circuits A and B. calculate the ratio of power factor of the circuit B to the power factor of the circuit A?



Ans. Power factor of circuit A

$$\cos\phi A = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{R^2 + 9R^2}}$$

$$\cos\phi A = \frac{\cancel{R}}{\sqrt{10}\cancel{R}}$$

$$\cos\phi A = \frac{1}{\sqrt{10}} \text{ -----(1)}$$

Power factor of circuit B

$$\cos\phi B = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + (3R - R)^2}}$$

$$\cos\phi B = \frac{R}{\sqrt{R^2 + 4R^2}} = \frac{R}{\sqrt{5R^2}}$$

$$\cos\phi B = \frac{1}{\sqrt{5}} \text{ ----(2)}$$

$$\frac{\cos\phi B}{\cos\phi A} = \frac{\frac{1}{\sqrt{5}}}{\frac{1}{\sqrt{10}}}$$

$$\cos\phi B : \cos\phi A = \sqrt{2}$$

5. A horizontal straight wire l0 m long is extending along east and west and is falling with a speed of 5.0 m/s at right angles to the horizontal component of the earth's magnetic field of strength $0.30 \times 10^{-4} wb/m^2$.

(a) What is the instantaneous value of the emf induced in the wire?

(b) What is the direction of the emf?

(c) Which end of the wire is at the higher potential?

Ans. length of the wire, l =10m

υ = 5.0m/s.

$B_H = 0.30 \times 10^{-4} wb/m^2$

(a) Induced emf $E = B_H l \upsilon$

$E = 0.30 \times 10^{-4} \times 10 \times 5$

$E = 1.5 \times 10^{-3} V$

(b) Induced emf sets up from west to east.

(c) The potential will be more for eastern end.

6. A circular coil of N turns and radius r is kept normal to a magnetic field, given by B = Bo cos wt. Deduce an expression for emf. Induced in the coil. State the rule which helps to detect the direction of induced current.

Ans. B = Bo cos wt (given)

$$E = \frac{-Nd\phi}{dt}$$

$$E = \frac{-NAd}{dt}(Bo\cos wt)$$

$$E = -NABo(-\sin wt)(wt)$$

$$E = NABo(-\sin wt)(w)$$

$$E = NABow\sin wt$$

$$(A = \pi R^2)$$

$$E = NBo\pi R^2 w\ sin\ wt$$

Lenz’s law is used to fuid the direction of induced emf. It states the direction of induced emf is opposite to the cause producing the induced emf.

---

7. In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.

Ans. In the inductive circuit,

Rms value of current, I = 15.92 A

Rms value of voltage, V = 220 V

Hence, the net power absorbed can be obtained by the relation,

$P = VI\cos\phi$

Where,

ϕ = Phase difference between V and I

For a pure inductive circuit, the phase difference between alternating voltage and current is $90°$ i.e., $\phi = 90°$.

Hence, $P = 0$ i.e., the net power is zero.

In the capacitive circuit,

Rms value of current, $I = 2.49$ A

Rms value of voltage, $V = 110$ V

Hence, the net power absorbed can ve obtained as:

$P = VI \cos \phi$

For a pure capacitive circuit, the phase difference between alternating voltage and current is $90°$ i.e., $\phi = 90°$.

Hence, $P = 0$ i.e., the net power is zero.

---

8. A series *LCR* circuit with *R* = 20 Ω , *L* = 1.5 H and *C* = 35 μF is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Ans. At resonance, the frequency of the supply power equals the natural frequency of the given *LCR* circuit.

Resistance, $R = 20\ \Omega$

Inductance, $L = 1.5$ H

Capacitance, $C = 35\ \mu F = 30 \times 10^{-6}\ F$

AC supply voltage to the *LCR* circuit, $V = 200$ V

Impedance of the circuit is given by the relation,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

At resonance, $\omega L = \frac{1}{\omega C}$s

$\therefore Z = R = 20\Omega$

Current in the circuit can be calculated as:

$$I = \frac{V}{Z}$$

$$= \frac{200}{20} = 10A$$

Hence, the average power transferred to the circuit in one complete cycle= *VI*

= 200 × 10 = 2000 W.

---

9. A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its *LC* circuit has an effective inductance of 200 μH , what must be the range of its variable capacitor?

[*Hint:* For tuning, the natural frequency i.e., the frequency of free oscillations of the *LC* circuit should be equal to the frequency of the radio wave.]

Ans. The range of frequency (ν) of a radio is 800 kHz to 1200 kHz.

Lower tuning frequency, ν_1 = 800 kHz = $800 \times 10^3\ Hz$

Upper tuning frequency, ν_2 = 1200 kHz = 1200×10^3 Hz

Effective inductance of circuit *L* = 200 $\nu = 200 \times 10^2 H$

Capacitance of variable capacitor for ν_1 is given as:

$C1 = \dfrac{1}{(i)_1{}^2 L}$

Where,

ω_1 *= Angular frequency for capacitor C1*

$= 2\pi V_1 = 2\pi \times 800 \times 10^3\, radS^{-1}$

$$\therefore C_1 = \frac{1}{(2\pi \times 800 \times 10^3)^2 \times 200 \times 10^{-6}}$$

$= 1.9809 \times 10^{-10}\, F = 198.1 pF$

Capacitance of variable capacitor for v_2 *,*

$C2 = \dfrac{1}{(i)^2{}_2 L}$

Where,

ω_2 *= Angular frequency for capacitor* C_2

$= 2\pi v_2 = 2\pi \times 1200 \times 10^3\, rad\, S^{-1}$

$$\therefore C_2 = \frac{1}{(2\pi \times 1200 \times 10^3)^2 \times 200 \times 10^{-6}}$$

$= 88.04 pF$ *s*

Hence, the range of the variable capacitor is from 88.04 pF to 198.1 pF.

10. Obtain the resonant frequency and Q-factor of a series LCR circuit with L = 3.0 H, C = 27 μF , and R = 7.4 Ω . It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Ans. *Inductance, L = 3.0 H*

Capacitance, C = 27 μF = $27 \times 10^{-6} F$

Resistance, R = 7.4 Ω

At resonance, angular frequency of the source for the given LCR series circuit is given as:

$$(i)_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}} = \frac{10^3}{9} = 111.11 rad\, S^{-1}$$

Q-factor of the series:

$$= \frac{111.11 \times 3}{7.4} = 45.0446$$

To improve the sharpness of the resonance by reducing its full width at half maximum' by a factor of 2 without changing ω_r *, we need to reduce R to half i.e.,*

Resistance = $\frac{R}{2} = \frac{7.4}{2} = 3.7\Omega$

5 Mark Questions

1. (a) Why is electric power generally transmitted over long distances at high a.c. voltage?

(b) An a.c. generator consist of a coil of 50 turns, area $2.5m^2$ rotating at an angular speed of 60 rad/s in uniform magnetic field of B = 0.3 T between two fixed pole pieces. Given R = 500 Ω .

(i) Find the maximum current drawn from the generator?

(ii) What will be the orientation of the coil wrt. B to have max and zero magnetic flux?

Would the generator work if the coils were stationary and instead the pole pieces rotated together with the same speed?

Ans. (a) Electric power is transmitted over long distances at high a.c. voltage so that small current flows through the transmission line because it reduces the power loss $\left(I^2R\right)$.

(b) n = 50, A = $2.5m^2$, w = 60 rad/s B = 0.3 T R = 500 Ω

(i) Eo = nABw

$$Eo = 50 \times 2.5 \times 0.3 \times 60$$

Eo = 2250 V

$$\Rightarrow Io = \frac{Eo}{R} = \frac{2250}{500}$$

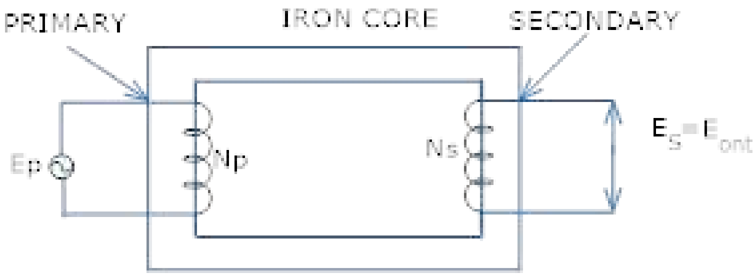
Io = 4.5 A

(ii) Magnetic flux will be maximum if the coil is in the vertical position and it will be zero when the coil is in the horizontal position.

(iii) The generator will work whenever there is relative motion between the coil and magnet.

2. Explain with the help of labeled diagram, the principle construction and working of a transformer?

Ans. Principle – A transformer coverts low a.c. voltage to high a.c. voltage or vice – versa. It is based on the principle of mutual induction i.e. emf is induced in a coil when a changing current is produced in the neighboring coil



<u>Construction</u> –It consist of two coils wounded on a soft iron core. One of the coils called the primary is connected to an a.c. source. The other coil called the secondary is connected to the load.

<u>Working</u> – When an alternating emf is applied across the primary coil the input voltage keeps on changing with time due to which magnetic flux through the primary coil changes. This changing magnetic flux gets linked up with the secondary coil also which in turn produces induced emf in the secondary coil.

$$Es = Ns\frac{d\phi s}{dt} - - - - (1)$$

$$Ep = Np\frac{d\phi p}{dt} - - - - (2)$$

If all the magnetic flux generated in the primary coil gets linked up with the secondary coil

i.e. $\phi s = \phi p$

Then ef. (1) & (2) becomes

$$\frac{Es}{Ep} = \frac{Ns}{Np}$$

$$Es = \frac{Ns}{Np} Ep \text{ ----(3)}$$

$\frac{Ns}{Np} = K$ Is called transformation ratio

k > 1 for step up transformer

k < 1 for step down transformer

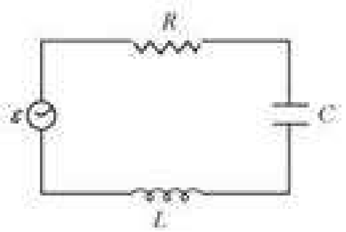
if there is no loss of energy

EsIs = EpIp

$$\frac{Es}{Ep} = \frac{Ip}{Is}$$

---

3. Figure 7.21 shows a series *LCR* circuit connected to a variable frequency 230 V source. *L* = 5.0 H, *C* = 80 μF , *R* = 40 Ω



(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the *LC* combination is zero at the resonating frequency.

Ans. Inductance of the inductor, *L* = 5.0 H

Capacitance of the capacitor, C = 80 μH = $80 \times 10^{-6}\ F$

Resistance of the resistor, R = 40 Ω

Potential of the variable voltage source, V = 230 V

(a) Resonance angular frequency is given as:

$$\omega_R = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \frac{10^3}{20} = 50\, rad/s$$

Hence, the circuit will come in resonance for a source frequency of 50 rad/s.

(b) Impedance of the circuit is given by the relation,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

At resonance,

$$\omega L = \frac{1}{\omega C}$$

$$\therefore Z = R = 40\Omega$$

Amplitude of the current at the resonating frequency is given as: $I_0 = \frac{V_0}{Z}$

Where,

V_0 = Peak voltage

$$= \sqrt{2V}$$

$$\therefore I_0 = \frac{\sqrt{2}V}{Z}$$

$$= \frac{\sqrt{2} \times 230}{40} = 8.13$$

Hence, at resonance, the impedance of the circuit is 40 Ω and the amplitude of the current is 8.13 A.

(c) Rms potential drop across the inductor,

$$(V_L)rms = I \times \omega_R L$$

Where,

I = rms current

$$= \frac{I_0}{\sqrt{2}} = \frac{\sqrt{2}V}{\sqrt{2}Z} = \frac{230}{40} A$$

$$\therefore (V_{L)})_{ms} = \frac{230}{40} \times 50 \times 5 = 1437.5V$$

Potential drop across the capacitor,

$$(V_{L)})_{ms} = I \times \frac{1}{(i)_R C}$$

$$= \frac{230}{40} \times \frac{1}{50 \times 80 \times 10^{-6}} = 1437.5V$$

Potential drop across the resistor,

(VR)rms = IR

$$= \frac{230}{40} \times 40 = 230 \text{ V}$$

Potential drop across the LC combination,

$$V_{LC} = I\left((i)_R L - \frac{1}{(i)_R C}\right)$$

At resonance, $(i)_R L - \dfrac{1}{(i)_R C}$

$\therefore VLC = 0$

Hence, it is proved that the potential drop across the *LC* combination is zero at resonating frequency.

---

4. An *LC* circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be *t* = 0.

(a) What is the total energy stored initially? Is it conserved during *LC* oscillations?

(b) What is the natural frequency of the circuit?

(c) At what time is the energy stored

(i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)?

(d) At what times is the total energy shared equally between the inductor and the capacitor?

(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Ans. Inductance of the inductor, *L* = 20 mH = $20 \times 10^{-3}\ H$

Capacitance of the capacitor, *C* = 50 μF = $50 \times 10^{-6}\ F$

Initial charge on the capacitor, *Q* = 10 mC = $10 \times 10^{-3}\ C$

(a) Total energy stored initially in the circuit is given as:

$$E = \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{(10\times10^{-3})^2}{2\times50\times10^{-6}} = 1J$$

Hence, the total energy stored in the *LC* circuit will be conserved because there is no resistor connected in the circuit.

(b) Natural frequency of the circuit is given by the relation,

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{20\times10^{-3}\times50\times10^{-6}}}$$

$$= \frac{10^3}{2\pi} = 159.24Hz$$

Natural angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

Hence, the natural frequency of the circuit is 103 rad/s.

(c) (i) For time period ($T = \frac{1}{V} = \frac{1}{159.24} = 6.28ms$), total charge on the capacitor at time *t*,

$$Q' = Q\cos\frac{2\pi}{T}t$$

For energy stored is electrical, we can write *Q*' = *Q*.

Hence, it can be inferred that the energy stored in the capacitor is completely electrical at time,

$t = 0, \frac{T}{2}, T, \frac{3T}{2}, ...$

(ii) Magnetic energy is the maximum when electrical energy, *Q*"² is equal to 0.

Hence, it can be inferred that the energy stored in the capacitor is completely magnetic at time,

$$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots\ldots$$

(d) Q^1 = Charge on the capacitor when total energy is equally shared between the capacitor and the inductor at time *t*.

When total energy is equally shared between the inductor and capacitor, the energy stored in the capacitor = $\frac{1}{2}$ (maximum energy).

$$\Rightarrow \frac{1}{2}\cdot\frac{(Q^1)^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{1}{4}\frac{Q^2}{C}$$

$$Q^1 = \frac{Q}{\sqrt{2}}$$

$$Q^1 = Q\cos\frac{2\pi}{T}t.$$

$$\frac{Q}{\sqrt{2}} = Q\cos\frac{2\pi}{T}t$$

$$\cos\frac{2\pi}{T}t = \frac{1}{\sqrt{2}} = \cos(2n+1)\frac{\pi}{4};$$

$$where\, n = 0, 1, 2, \ldots$$

$$t = (2n+1)\frac{T}{8}$$

Hence, total energy is equally shared between the inductor and the capacity at time,

$$t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}\ldots\ldots$$

(e) If a resistor is inserted in the circuit, then total initial energy is dissipated as heat energy in the circuit. The resistance damps out the *LC* oscillation.

---

5. A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz ac supply.

(a) What is the maximum current in the coil?

(b) What is the time lag between the voltage maximum and the current maximum?

Ans. Inductance of the inductor, *L* = 0.50 H

Resistance of the resistor, *R* = 100 Ω

Potential of the supply voltage, *V* = 240 V

Frequency of the supply, ν = 50 Hz

(a) Peak voltage is given as:

$$V_0 = \sqrt{2}V$$

$$= \sqrt{2} \times 240 = 339.41V$$

Angular frequency of the supply,

$$\omega = 2\pi\nu$$

$$= 2\pi \times 50 = 100\ \pi\ rad / s$$

Maximum current in the circuit is given as:

$$I_0 = \frac{V_0}{\sqrt{R^2 + (i)^2 L^2}}$$

$$= \frac{339.41}{\sqrt{(100)^2 + (100\pi)^2 (0.50)^2}} = 1.82A$$

(b) Equation for voltage is given as:

$V = V_0 \ cos\omega t$ Equation for current is given as:

$I = I_0 \ cos \ \left(\omega t - \Phi\right)$ Where,

Φ = Phase difference between voltage and current

At time, *t* = 0.

V = V_0 (voltage is maximum)

$$For \omega t - \Phi = \ 0 \ i.e., \ at \ time \ t = \frac{\Phi}{\omega} \ ,$$

I = I_0 (current is maximum)

Hence, the time lag between maximum voltage and maximum current is $t = \frac{\phi}{(i)}$.

Now, phase angle *Φ* is given by the relation,

$$\tan \phi = \frac{(i) L}{R}$$

$$= \frac{2\pi \times 50 \times 0.5}{100} = 1.57$$

$$\phi = 57.5^0 = \frac{57.5\pi}{180} rad$$

$$(i) \ t = \frac{57.5\pi}{180}$$

$$1 = \frac{57.5}{180 \times 2\pi \times 50}$$

$$= 3.19 \times 10^{-3} S$$

$$= 3.2 ms$$

Hence, the time lag between maximum voltage and maximum current is 3.2 ms.

---

6. Obtain the answers (a) to (b) in Exercise 7.13 if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Ans. Inductance of the inductor,
L = 0.5 Hz

Resistance of the resistor, R = 100 Ω

Potential of the supply voltages,
V = 240 V

Frequency of the supply,
ν = 10 kHz $= 10^4\ Hz$

Angular frequency,

$$\omega = 2\pi\nu = 2\pi \times 10^4\ rad/s$$

(a) Peak voltage,

$$V_0 = \sqrt{2} \times V = 240\sqrt{2}\,V$$

Maximum current,

$$I_0 \frac{V_0}{\sqrt{R^R + \omega^2 L^2}}$$

$$= \frac{240\sqrt{2}}{\sqrt{(100)^2 + (2\pi \times 10^4)^2 \times (0.50)^2}} = 1.1 \times 10^{-2} A$$

(b) For phase difference Φ, we have the relation:

$$\tan\phi = \frac{\omega L}{R}$$

$$\frac{2\pi \times 10^4 \times 0.5}{100} = 100\pi$$

$$\phi = 89.82^0 = \frac{89.82\pi}{180} rad$$

$$\omega t = \frac{89.82\pi}{180}$$

$$t = \frac{89.82\pi}{180 \times 2\pi \times 10^4} = 25us$$

It can be observed that *I*0 is very small in this case. Hence, at high frequencies, the inductor amounts to an open circuit.

In a dc circuit, after a steady state is achieved, ω = 0. Hence, inductor L behaves like a pure conducting object.

---

7. A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.

(a) What is the maximum current in the circuit?

(b) What is the time lag between the current maximum and the voltage maximum?

Ans. Capacitance of the capacitor, *C* = 100 μF = 100×10^{-6} F

Resistance of the resistor, *R* = 40 Ω

Supply voltage, *V* = 110 V

(a) Frequency of oscillations, ν = 60 Hz

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 60 rad/s$

For a *RC* circuit, we have the relation for impedance as:

$$Z = R^2 + \frac{1}{\omega^2 C^2} \text{s}$$

Peak voltage, $V_0 = V\sqrt{2} = 110\sqrt{2}V$

Maximum current is given as:

$$I_0 = \frac{V_0}{Z}$$

$$= \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$= \frac{100\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{1600 + \frac{1}{(120\pi)^2}}} = 3.24A$$

(b) In a capacitor circuit, the voltage lags behind the current by a phase angle of *Φ*. This angle is given by the relation:

$$\frac{1}{\omega L} - \omega C = 0$$

$$\therefore \tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR}$$

$$= \frac{1}{12\pi \times 10^4 \times 40} = 0.6635$$

$$\phi = \tan^{-1}(0.6635) = 33.56$$

$$= \frac{33.56\pi}{180} rad$$

$$\therefore Time lag = \frac{\phi}{(i)}$$

$$= \frac{33.56\pi}{180 \times 120\pi} = 1.55 \times 10^{-3} S = 1.55 ms$$

Hence, the time lag between maximum current and maximum voltage is 1.55 ms.

---

8. Obtain the answers to (a) and (b) in Exercise 7.15 if the circuit is connected to a 110 V, 12 kHz supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behavior with that of a capacitor in a dc circuit after the steady state.

Ans. Capacitance of the capacitor,

$C = 100\ \mu F = 100 \times 10^{-6}\ F$

Resistance of the resistor,

$R = 40\ \Omega$

Supply voltage, $V = 110$ V

Frequency of the supply,

$\nu = 12$ kHz $= 12 \times 10^{3}$ Hz

Angular Frequency,

$$\omega = 2\pi\nu = 2 \times \pi \times 12 \times 10^{3}03$$

$$= 24\pi \times 10^{3}\ rad/s$$

Peak voltage,

$$V_0 = V\sqrt{2} = 110\sqrt{2}V$$

Maximum current,

$$I_0 - \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(24\pi \times 10^3 \times 100 \times 10^{-6})^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{1600 + \left(\frac{10}{24\pi}\right)^2}} = 3.9A$$

For an *RC* circuit, the voltage lags behind the current by a phase angle of Φ given as:

$$\tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR}$$

$$= \frac{1}{24\pi \times 10^3 \times 100 \times 10^{-6} \times 40}$$

$$\tan\phi = \frac{1}{96\pi}$$

$$\therefore \phi = 0.2^0$$

$$= \frac{0.2\pi}{180} rad$$

$$\therefore timelag = \frac{\phi}{(i)}$$

Hence, Φ tends to become zero at high frequencies. At a high frequency, capacitor C acts as a conductor.

In a dc circuit, after the steady state is achieved, ω = 0. Hence, capacitor C amounts to an open circuit.

---

9. Keeping the source frequency equal to the resonating frequency of the series *LCR* circuit, if the three elements, *L*, *C* and *R* are arranged in parallel, show that the total current in the parallel *LCR* circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency.

Ans. An inductor (*L*), a capacitor (*C*), and a resistor (*R*) is connected in parallel with each other in a circuit where,

L = 5.0 H

C = 80 μF = $80 \times 10^{-6}\ F$

R = 40 Ω

Potential of the voltage source, *V* = 230 V

Impedance (Z) of the given parallel *LCR* circuit is given as:

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Where,

ω = Angular frequency

At resonance, $\frac{1}{\omega L} - \omega C = 0$

$$\therefore \omega = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50\, rad/s$$

Hence, the magnitude of *Z* is the maximum at 50 rad/s. As a result, the total current is minimum.

Rms current flowing through inductor L is given as:

$$I_L = \frac{V}{\omega L}$$

$$= \frac{230}{50 \times 5} = 0.92 A$$

Rms current flowing through capacitor C is given as:

$$I_C = \frac{V}{\frac{1}{\omega C}} = \omega CV$$

$$= 50 \times 80 \times 10^{-6} \times 230 = 0.92A$$

Rms current flowing through resistor R is given as:

$$I_R = \frac{V}{R}$$

$$= \frac{230}{40} = 5.75A$$

---

10. A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.

(a) Obtain the current amplitude and rms values.

(b) Obtain the rms values of potential drops across each element.

(c) What is the average power transferred to the inductor?

(d) What is the average power transferred to the capacitor?

(e) What is the total average power absorbed by the circuit? ['Average' implies 'averaged over one cycle'.]

Ans. Inductance,
L = 80 mH = 80×10^{-3} H

Capacitance,
$C = 60\ \mu F = 60 \times 10^{-6}\ F$

Supply voltage, V = 230 V

Frequency, ν = 50 Hz

Angular frequency,
$\omega = 2\pi\nu = 100\pi\ rad/s$

Peak voltage, *V*0= $V\sqrt{2} = 230\sqrt{2}V$

(a) Maximum current is given as:

$$I_0 = \frac{V_0}{\left((i) L - \frac{1}{(i)C} \right)}$$

$$= \frac{230\sqrt{3}}{\left(10\pi \times 80 \times 10^{-3} - \frac{1}{100\pi \times 60 \times 10^{-6}} \right)}$$

$$= \frac{230\sqrt{2}}{\left(8\pi - \frac{1000}{6\pi} \right)} = -11.63A$$

The negative sign appears because $(i) L < \frac{1}{(i)C}$

Amplitude of maximum current, $\left| I_0 \right| = 11.63A$

Hence, rms value of current, $I = \frac{I_0}{\sqrt{2}} = \frac{-11.63}{\sqrt{2}} = -8.22A$

(b) Potential difference across the inductor,

$$V_L = I \times \omega L$$

$$= 8.22 \times 100\pi \times 80 \times 10^{-3}$$

= 206.61 V

Potential difference across the capacitor,

$$V_c = 1 \times \frac{1}{(i) C}$$

$$= 8.22 \times \frac{1}{100\pi \times 60 \times 10^{-6}} = 436.3V$$

(c) Average power consumed by the inductor is zero as actual voltage leads the current by $\frac{\pi}{2}$

.

(d) Average power consumed by the capacitor is zero as voltage lags current by $\frac{\pi}{2}$.

(e) The total power absorbed (averaged over one cycle) is zero.

---

11. Suppose the circuit in Exercise 7.18 has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

Ans. Average power transferred to the resistor = 788.44 W

Average power transferred to the capacitor = 0 W

Total power absorbed by the circuit = 788.44 W

Inductance of inductor, L = 80 mH = 80×10^{-3} H

Capacitance of capacitor, C = 60 μF = 60×10^{-6} F

Resistance of resistor, R = 15 Ω

Potential of voltage supply, V = 230 V

Frequency of signal, ν = 50 Hz

Angular frequency of signal, $\omega = 2\pi \times (50) = 100\pi \ rad/s$

The elements are connected in series to each other. Hence, impedance of the circuit is given as:

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(15)^2 + \left(100\pi(80 \times 10^{-3}) - \frac{1}{(100\pi \times 60 \times 10^{-6})}\right)^2}$$

$$= \sqrt{(15)^2 + (25.12 - 53.08)^2} 31.728 \Omega$$

Current flowing in the circuit,

$$I = \frac{V}{Z} = \frac{230}{31.728} = 7.25 A \quad I = \frac{V}{Z} = \frac{230}{31.728} = 7.25 A$$

Average power transferred to resistance is given as:

$P_R = I^2 R$ = $(7.25)^2 \times 15$ = 788.44 W

Average power transferred to capacitor, *PC* = Average power transferred to inductor, *PL* = 0

Total power absorbed by the circuit:

= *PR* + *PC* + *PL*

= 788.44 + 0 + 0 = 788.44 W

Hence, the total power absorbed by the circuit is 788.44 W.

---

12. A series *LCR* circuit with *L* = 0.12 H, *C* = 480 nF, *R* = 23 Ω is connected to a 230 V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum. Obtain this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum. Obtain the value of this maximum power.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(d) What is the *Q*-factor of the given circuit?

Ans. Inductance, *L* = 0.12 H

Capacitance, *C* = 480 nF = $480 \times 10^{-9}\ F$

Resistance, *R* = 23 Ω

Supply voltage, V = 230 V

Peak voltage is given as:

V_0 = $\sqrt{2} \times 230$ = 325.22 V

(a) Current flowing in the circuit is given by the relation,

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left((1)L - \frac{1}{(1)C}\right)^2}}$$

Where,

I_0 = maximum at resonance

At resonance, we have

$$(1)_R L - \frac{1}{(1)_R C} = 0$$

Where,

ωR = Resonance angular frequency

$$\therefore (1)_R = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}} = 4166.67\, rad / S$$

∴Resonant frequency,

$$\nu_R = \frac{(i)_R}{2\pi} = \frac{4166.67}{2 \times 3.14} = 663.48 Hz$$

And, maximum current

$$(I_0)_{Max} = \frac{V_0}{R} = \frac{325.22}{23} = 14.14 A$$

(b) Maximum average power absorbed by the circuit is given as:

$$(P_{av})_{Max} = \frac{1}{2}(I_0)_{Max}^{\ 2} R$$

$$= \frac{1}{2} \times (14.14)^2 \times 23 = 2299.3W$$

Hence, resonant frequency (V_R) is $663.48 Hz$

(c) The power transferred to the circuit is half the power at resonant frequency.

Frequencies at which power transferred is half, = $(1)_R \pm \Delta(1)$

$$= 2\pi\left(v_R \pm \Delta v\right)$$

Where,

$$\Delta(i) = \frac{R}{2L}$$

$$= \frac{23}{2 \times 0.12} = 95.83\, rad/s$$

Hence, change in frequency,

$$\Delta v = \frac{1}{2\pi}\Delta(1) = \frac{95.83}{2\pi} = 15.26 Hz$$

$$\therefore v_R + \Delta v = 66348 + 15.26 = 678.74 Hz$$

And, $v_R - \Delta v = 663.48 - 15.26 = 648.22 Hz$

Hence, at 648.22 Hz and 678.74 Hz frequencies, the power transferred is half.

At these frequencies, current amplitude can be given as:

$$I' = \frac{1}{\sqrt{2}} \times (I_0)_{Max}$$

$$= \frac{14.14}{\sqrt{2}} = 10A$$

(d) Q-factor of the given circuit can be obtained using the relation,

$$Q = \frac{(i)_R L}{R}$$

$$= \frac{4166.67 \times 0.12}{23} = 21.74 \text{ s}$$

Hence, the Q-factor of the given circuit is 21.74.

---

13. Answer the following questions:

(a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?

(b) A capacitor is used in the primary circuit of an induction coil.

(c) An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across *C* and the ac signal across *L*.

(d) A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.

(e) Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?

Ans. (a) Yes; the statement is not true for rms voltage

It is true that in any ac circuit, the applied voltage is equal to the average sum of the instantaneous voltages across the series elements of the circuit. However, this is not true for rms voltage because voltages across different elements may not be in phase.

(b) High induced voltage is used to charge the capacitor. A capacitor is used in the primary circuit of an induction coil. This is because when the circuit is broken, a high induced voltage is used to charge the capacitor to avoid sparks.

(c) The dc signal will appear across capacitor *C* because for dc signals, the impedance of an inductor (*L*) is negligible while the impedance of a capacitor (*C*) is very high (almost infinite). Hence, a dc signal appears across *C*. For an ac signal of high frequency, the impedance of *L* is high and that of *C* is very low. Hence, an ac signal of high frequency appears across *L*.

(d) If an iron core is inserted in the choke coil (which is in series with a lamp connected to the ac line), then the lamp will glow dimly. This is because the choke coil and the iron core increase the impedance of the circuit.

(e) A choke coil is needed in the use of fluorescent tubes with ac mains because it reduces the voltage across the tube without wasting much power. An ordinary resistor cannot be used instead of a choke coil for this purpose because it wastes power in the form of heat.

---

14. A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets power from the line through a 4000–220 V step-down transformer at a sub-station in the town.

(a) Estimate the line power loss in the form sof heat.

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(c) Characterise the step up transformer at the plant.

Ans. Total electric power required,
P = 800 kW = 800×10^3 W

Supply voltage, V = 220 V

Voltage at which electric plant is generating power, V' = 440 V

Distance between the town and power generating station, d = 15 km

Resistance of the two wire lines carrying power = 0.5 Ω /km

Total resistance of the wires,
R = (15 + 15)0.5 = 15 Ω

A step-down transformer of rating 4000 - 220 V is used in the sub-station.

Input voltage, V_1 = 4000 V

Output voltage, V = 220 V

Rms current in the wire lines is given as:

$$I = \frac{P}{V_1}$$

$$= \frac{800 \times 10^3}{4000} = 200 A$$

(a) Line power loss = $I^2 R$

= $(200)^2 \times 15$

= $600 \times 103\ W$

= 600 kW

(b) Assuming that the power loss is negligible due to the leakage of the current:

Total power supplied by the plant
= 800 kW + 600 kW

= 1400 kW

(c) Voltage drop in the power line
= *IR* = 200 × 15 = 3000 V

Hence, total voltage transmitted from the plant
= 3000 + 4000

= 7000 V

Also, the power generated is 440 V.

Hence, the rating of the step-up transformer situated at the power plant is 440 V - 7000 V.

15. Do the same exercise as above with the replacement of the earlier transformer by a 40,000-220 V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?

Ans. The rating of a step-down transformer is 40000 V - 220 V.

Input voltage, V_1 = 40000 V

Output voltage, V_2 = 220 V

Total electric power required,
P = 800 kW = 800×10^3 W

Source potential, *V* = 220 V

Voltage at which the electric plant generates power, *V*' = 440 V

Distance between the town and power generating station, *d* = 15 km

Resistance of the two wire lines carrying power = 0.5 Ω /km

Total resistance of the wire lines,
R = (15 + 15)0.5 = 15 Ω

P = $V_1 I$

Rms current in the wire line is given as:

$$I = \frac{P}{V_1}$$

$$= \frac{800 \times 10^3}{40000} = 20A$$

(a) Line power loss = $I_2 R$

= $(20)^2 \times 15$

= 6 kW

(b) Assuming that the power loss is negligible due to the leakage of current.

Hence, power supplied by the plant
= 800 kW + 6kW = 806 kW

(c) Voltage drop in the power line
= *IR* = 20×15 = 300 V

Hence, voltage that is transmitted by the power plant

= 300 + 40000 = 40300 V

The power is being generated in the plant at 440 V.

Hence, the rating of the step-up transformer needed at the plant is

440 V – 40300 V.

Hence, power loss during transmission
= $\frac{600}{1400} \times 100 = 42.8\,\%$

In the previous exercise, the power loss due to the same reason is $\frac{6}{806} \times 100 = 0.744\,\%$. Since the power loss is less for a high voltage transmission, high voltage transmissions are preferred for this purpose.

CBSE Class 12 physics

Practice Questions

Chapter 8

Electromagnetic Waves

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1 Mark Questions

1. The charging current for a capacitor is 0.25A. What is the displacement current across its plates?

Ans. Displacement current remains the same as charging current and is equal to 0.25A.

---

2. Write the following radiations in a descending order of frequencies: red light, x – rays, microwaves, radio waves

Ans. X – rays, Red light, Microwaves and Radio waves.

---

3. How does the frequency of a beam of ultraviolet light change, when it goes from air into glass?

Ans. There is no effect on the frequency of ultraviolet light.

---

4. What is the ratio of speed of gamma rays and radio waves in vacuum?

Ans. One.

---

5. It is necessary to use satellites for long distance TV transmission. Why?

Ans. Television signals are not reflected back by the layer of atmosphere called ionosphere thus TV signals from air earth station are reflected back to the earth

by means of an artificial satellite

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6. What is the role of ozone layer in the atmosphere?

Ans. It absorbs all the harmful ultraviolet radiations thus protecting us from reaching the dangerous effects of uv radiations.

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7. What is the nature of waves used in radar?

Ans. Microwaves are used in Radar.

---

8. What physical quantity is the same for X-rays of wavelength 10-10 m, red light of wavelength 6800 $\mathring{A}$ and radio waves of wavelength 500 m?

Ans. The speed of light (3×10^8 m/s) in a vacuum is the same for all wavelengths. It is independent of the wavelength in the vacuum.

2 Mark Questions

1. Write the application of Infra-red radiations?

Ans. (1) infra-red radiations are used to take photographs under foggy conditions.

(2) Infra-red radiations are used in revealing the secret writings on the ancient walls.

---

2. Which constituent radiation of the electromagnetic spectrum is used?

(1) To photograph internal parts of human body.

(2) For air aircraft navigation

Ans. (1) X -Rays

(2) Microwaves

---

3. Electric field in a plane electromagnetic wave is given by

$$E_z = 60 \sin\left(\frac{10^3 x}{2} + \left(10^{11}\right)\frac{3t}{2}\right) V/m.$$

(a) Write an expression for the magnetic field

(b) What is the magnitude of wavelength and frequency of the wave?

Ans. (a) $C = \frac{E_O}{B_O}$

$$B_O = \frac{E_O}{C} = \frac{60}{3\times10^8}$$

$$B_O = 2\times10^{-7} T$$

Since magnetic field and electric field are $\perp$ to each other

$$By = 2\times10^{-7} T \sin\left(\frac{10^3}{2}x + \left(10^{11}\right)\frac{3t}{2}\right) \text{----(1)}$$

Compare e.g. (1) with standard equation

By = B_O sin $2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)$

$$\lambda = 4\pi\times10^{-3} m$$

$$Also\ 2\pi\frac{1}{T} = (10)^{11}\frac{3}{2}$$

$$\frac{1}{T} = \upsilon = \frac{3\times10^{11}}{2\times2\pi}$$

$$\upsilon = \frac{3}{4\pi}\times10^{11} Hz$$

4. IF the earth did not have atmosphere would its average surface temperature be higher or lower than what it is now?

Ans. The infra-red radiations get trapped inside the earth's atmosphere due to green house effect which makes the earth warm. Therefore average temperature of the earth would have been low.

---

5. Sky waves are not used in transmitting TV signals, Why? Suggest two methods by which range of TV transmission can be increased?

Ans. Sky waves are not used in transmitting TV signals as they are not reflected by the ionosphere.

Methods of increasing range of TV transmission

(1) Tall antenna

(2) Geostationary satellites

---

6. "Greater the height of a TV transmitting antenna, greater is its coverage." Explain.

Ans. Since d = $\sqrt{2hR}$

If height is increased distance upto which TV coverage can be done will increases.

---

7. A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

Ans. The electromagnetic wave travels in a vacuum along the z-direction. The electric field (E) and the magnetic field (H) are in the x-y plane. They are mutually perpendicular.

Frequency of the wave, ν = 30 MHz = $30 \times 10^6 \; s^{-1}$

Speed of light in a vacuum, c = 3×10^8 m/s

Wavelength of a wave is given as:

$$\lambda = \frac{c}{\nu}$$

$$= \frac{3 \times 10^8}{30 \times 10^6} = 10\,m$$

---

8. A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

Ans. A radio can tune to minimum frequency, ν_1 = 7.5 MHz= $7.5 \times 10^6 \; Hz$

Maximum frequency, ν_2 = 12 MHz = $12 \times 10^6 \; Hz$
Speed of light, c = 3×10^8 m/s

Corresponding wavelength for ν_1 can be calculated as:

$$\lambda_1 = \frac{c}{\nu_1}$$

$$= \frac{3 \times 10^8}{7.5 \times 10^6} = 40\,m$$

Corresponding wavelength for ν_2 can be calculated as:

$$\lambda_2 = \frac{c}{\nu_2}$$

$$= \frac{3 \times 10^8}{12 \times 10^6} = 25\,m$$

Thus, the wavelength band of the radio is 40 m to 25 m.

9. A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

Ans. The frequency of an electromagnetic wave produced by the oscillator is the same as that of a charged particle oscillating about its mean position i.e., 10^9 Hz.

---

10. The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is B_0 = 510 nT. What is the amplitude of the electric field part of the wave?

Ans. Amplitude of magnetic field of an electromagnetic wave in a vacuum,

*B*0 = 510 nT = $510 \times 10^{-9}\ T$

Speed of light in a vacuum, *c* = $3 \times 10^8\ m/s$

Amplitude of electric field of the electromagnetic wave is given by the relation,

$E =_c B_0$

= $3 \times 108 \times 510 \times 10^{-9}$ = 153 N/C

Therefore, the electric field part of the wave is 153 N/C.

3 Mark Questions

1. In a plane electromagnetic wave, the electric field oscillates sinusoid ally with a frequency of $2\times10^{10}\,Hz$ and amplitude 48V/m.

(a) What is the wavelength of the em. wave?

(b) Calculate the amplitude of the oscillating magnetic field.

(c) Calculate average energy density of the electromagnetic field of the wave?

Ans. (a) V = $2\times10^{10}\,Hz$

E_O = 48 V/m

$$\lambda=\frac{C}{V}=\frac{3\times10^{8}}{2\times10^{10}}$$

$$\lambda=1.5\times10^{-2}m$$

(b) $E_O = cB_O$

$$B_O=\frac{E_O}{C}=\frac{48}{3\times10^{8}}$$

$$B_O=16\times10^{-8}\text{ Tesla}$$

(c) Energy density

$$U=\frac{1}{2}\epsilon_O E^2$$

$$U=\frac{1}{2}8.85\times10^{-12}\times48\times48$$

$$U=1\times10^{-8}J/m^3$$

2. Find the wavelength of electromagnetic waves of frequency $6\times10^{12}\ Hz$ in free space. Give two applications of the type of wave.

Ans. V = $6\times10^{12}\ Hz$

Using $\lambda = \frac{c}{v}$

$$\lambda = \frac{3\times10^{8}}{6\times10^{12}}$$

$$\lambda = 5\times10^{-5}\ m$$

These are infra-red radiations

<u>**Applications**</u>

(1) It keeps the earth warm.

(2) Infra-red lamps are used to treat muscular strains.

---

3. A plane monochromatic wave lies in the visible region. It is represented by the sinusoidal variation with time by the following components of electric field

Ex = 0, Ey = 4 sin $\left[\frac{2\pi}{\lambda}(x-vt)\right]$, **Ez = 0**

Where $\upsilon = 5\times10^{14}\ Hg$ **And** λ **is the wavelength of light.**

(a) What is the direction of propagation of the wave?

(b) What is its amplitude?

(c) Compute the component of magnetic field?

Ans. (a) The direction of propagation of wave is along + x – axis.

(b) Amplitude = 4 units

(c) Component of magnetic of field

$$Bz = \frac{Eo}{C} = \frac{4}{3\times10^{8}}$$

$$Bz = 1.33\times10^{-8}\,Tesla$$

---

4. Write the characteristics of em waves? Write the expression for velocity of electromagnetic waves in terms of permittivity and permeability of the medium?

Ans. Characteristics of em waves

(1) It travels in free space with speed of light $c = 3\times10^{8}\,m/s.$

(2) Electromagnetic waves are transverse in nature.

Velocity of em waves in vacuum C

$$= \frac{1}{\sqrt{\mu_o \epsilon_o}}$$

---

5. The electric field of a plane electromagnetic wave in vacuum is represented by.

Ex = 0, Ey = 0.5 cos $\left[2\pi\times10^{8}\left(t-x/c\right)\right]$ **and Ez = 0**

(a) What is the direction of propagation of electromagnetic wave?

(b) Determine the wavelength of the wave?

(c) Compute the component of associated magnetic field?

Ans. (a) The equation Ey = 0.5 cos $\left[2\pi\times10^{8}\left(t-x/c\right)\right]$

Represents wave is propagating along + x – axis

(b) Comparing equation with the standard one

Ey = E_O cos w $(t - x / c)$

$$w = 2\pi \times 10^8$$

$$\cancel{2}\cancel{\pi}\upsilon = \cancel{2}\cancel{\pi} \times 10^8$$

$$\upsilon = 10^8$$

$$\Rightarrow \lambda = \frac{c}{v} = \frac{3 \times 10^8}{10^8}$$

$$\lambda = 3m$$

(c) Associated magnetic field is $\perp$ to electric field and the direction of propagation. Since wave is propagating along x – axis, electric field is along, y – axis

Thus, magnetic field is along z – axis

$$\Rightarrow Bx = 0, By = 0$$

$$Bz = B_o \cos[2\pi \times 10^8 (t - x / x)]$$

$$Bz = \frac{E_o}{c} \cos 2\pi \times 10^8 (t - x / x)]$$

---

6. Find the wavelength of electromagnetic waves of frequency $5 \times 10^{19}\ Hz$ in free space. Give its two applications.

Ans. Using $C = \lambda\upsilon$

$$\upsilon = 5 \times 10^{19} \text{Hz}$$

$$\lambda = \frac{3 \times 10^8}{5 \times 10^{19}}$$

$\lambda = 0.6 \times 10^{-11} = 6 \times 10^{-12} m$

These are Gamma Rays.

Applications

(1) These rays are used to get information regarding atomic structure.

(2) They have very high penetrating power so they are used for detection purpose

---

7. (1) State the condition under which a microwave oven heats up food items containing water molecules most efficiently?

(2) Name the radiations which are next to these radiations in em. Spectrum having (a) Shorter wavelength (b) Longer wavelength

Ans. (1) Frequency of the microwaves must be equal to the resonant frequency of the water molecules present in the food item.

(2) (a) visible light

(b) Microwaves

---

8. The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula *E* = $h\nu$ (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

Ans. Energy of a photon is given as:

$$E = h\nu = \frac{hc}{\lambda}$$

Where,

h = Planck's constant = $6.6 \times 10^{-34}\ Js$

c = Speed of light = 3×10^8 m/s

λ = Wavelength of radiation

$$\therefore E = \frac{6.6\times10^{-34}\times3\times10^{8}}{\lambda} = \frac{19.8\times10^{-26}}{\lambda}$$

$$= \frac{19.8\times10^{-26}}{\lambda\times1.6\times10^{-19}} = \frac{12.375 t 10^{-7}}{\lambda} eV$$

The given table lists the photon energies for different parts of an electromagnetic spectrum for different λ .

λ (m)	E (eV)
103	12.375×10^{-10}
1	12.375×10^{-7}
10^{-3}	12.375×10^{-4}
10^{-6}	12.375×10^{-1}
10^{-8}	12.375×10^{1}
10^{-10}	12.375×10^{3}
10^{-12}	12.375×10^{5}

The photon energies for the different parts of the spectrum of a source indicate the spacing of the relevant energy levels of the source.

---

9. About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation

(a) at a distance of 1 m from the bulb?

(b) at a distance of 10 m?

Assume that the radiation is emitted isotropic ally and neglect reflection.

Ans. Power rating of bulb, P = 100 W

It is given that about 5% of its power is converted into visible radiation.

∴ Power of visible radiation,

$$P' = \frac{5}{100} \times 100 = 5W$$

Hence, the power of visible radiation is 5W.

(a) Distance of a point from the bulb, *d* = 1 m Hence, intensity of radiation at that point is given as:

$$1 = \frac{P'}{4\pi d^2}$$

$$= \frac{5}{4\pi(1)^2} = 0.398W / m^2$$

(b) Distance of a point from the bulb, d_1 = 10 m Hence, intensity of radiation at that point is given as:

$$1 = \frac{P'}{4\pi(d_1)^2}$$

$$= \frac{5}{4\pi(10)^2} = 0.00398W / m^2$$

---

10. Use the formula λ_m *T*= 0.29 cm K to obtain the characteristic temperature ranges for different parts of the electromagnetic spectrum. What do the numbers that you obtain tell you?

Ans. A body at a particular temperature produces a continuous spectrum of wavelengths. In case of a black body, the wavelength corresponding to maximum intensity of radiation is given according to Planck's law. It can be given by the relation,

$$\lambda_m = \frac{0.29}{T} cm K$$

Where,

λ_m = maximum wavelength

T = temperature

Thus, the temperature for different wavelengths can be obtained as:

For $\lambda_m = 10^{-4}\ cm$;

$$T = \frac{0.29}{10^{-4}} = 2900\ ^{o}K$$

For $\lambda_m = 5\ \times 10^{-5}\ cm$;

$$T = \frac{0.29}{5 \times 10^{-5}} = 5800\ ^{o}K$$

For $\lambda_m = 10^{-6}$ cm;

$T = \dfrac{0.29}{10^{-6}} = 290000\ ^{o}K$ and so on.

The numbers obtained tell us that temperature ranges are required for obtaining radiations in different parts of an electromagnetic spectrum. As the wavelength decreases, the corresponding temperature increases.

5 Mark Questions

1. Given below are some famous numbers associated with electromagnetic radiations in different contexts in physics. State the part of the electromagnetic spectrum to which each belongs.

(a) 21 cm (wavelength emitted by atomic hydrogen in interstellar space).

(b) 1057 MHz (frequency of radiation arising from two close energy levels in hydrogen; known as Lamb shift).

(c) 2.7 K [temperature associated with the isotropic radiation filling all space-thought to be a relic of the 'big-bang' origin of the universe].

(d) 5890 $\overset{0}{A}$ - 5896 $\overset{0}{A}$ [double lines of sodium]

(e) 14.4 keV [energy of a particular transition in 57Fe nucleus associated with a famous high resolution spectroscopic method

(Mossbauer spectroscopy)].

Ans. (a) Radio waves; it belongs to the short wavelength end of the electromagnetic spectrum.

(b) Radio waves; it belongs to the short wavelength end.

(c) Temperature, $T = 2.7\ °K$

λ_m is given by Planck's law as:

$$\lambda_m = \frac{0.29}{2.7} = 0.11\,cm$$

This wavelength corresponds to microwaves.

(d) This is the yellow light of the visible spectrum.

(e) Transition energy is given by the relation,

$E = h\nu$

Where,

h = Planck's constant = $6.6 \times 10^{-34}\ Js$

ν = Frequency of radiation

Energy, E = 14.4 K eV

$$\therefore \nu = \frac{E}{h}$$

$$= \frac{14.4 \times 10^3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$= 3.4 \times 10^{18} \ Hz$$

This corresponds to X-rays.

---

2. Answer the following questions:

(a) Long distance radio broadcasts use short-wave bands. Why?

(b) It is necessary to use satellites for long distance TV transmission. Why?

(c) Optical and radio telescopes are built on the ground but X-ray astronomy is possible only from satellites orbiting the earth. Why?

(d) The small ozone layer on top of the stratosphere is crucial for human survival. Why?

(e) If the earth did not have an atmosphere, would its average surface temperature be higher or lower than what it is now?

(f) Some scientists have predicted that a global nuclear war on the earth would be followed by a severe 'nuclear winter' with a devastating effect on life on earth. What might be the basis of this prediction?

Ans. (a) Long distance radio broadcasts use shortwave bands because only these bands can be refracted by the ionosphere.

(b) It is necessary to use satellites for long distance TV transmissions because television signals are of high frequencies and high energies. Thus, these signals are not reflected by the ionosphere. Hence, satellites are helpful in reflecting TV signals. Also, they help in long distance TV transmissions.

(c) With reference to X-ray astronomy, X-rays are absorbed by the atmosphere. However, visible and radio waves can penetrate it. Hence, optical and radio telescopes are built on the ground, while X-ray astronomy is possible only with the help of satellites orbiting the Earth.

(d) The small ozone layer on the top of the atmosphere is crucial for human survival because it absorbs harmful ultraviolet radiations present in sunlight and prevents it from reaching the Earth's surface.

(e) In the absence of an atmosphere, there would be no greenhouse effect on the surface of the Earth. As a result, the temperature of the Earth would decrease rapidly, making it chilly and difficult for human survival.

(f) A global nuclear war on the surface of the Earth would have disastrous consequences. Post-nuclear war, the Earth will experience severe winter as the war will produce clouds of smoke that would cover maximum parts of the sky, thereby preventing solar light form reaching the atmosphere. Also, it will lead to the depletion of the ozone layer.

---

3. Suppose that the electric field part of an electromagnetic wave in vacuum is E = {(3.1 N/C) cos [(1.8 rad/m) *y* + (5.4×10^6 rad/s)*t*]} $\hat{i}$ **.**

(a) What is the direction of propagation?

(b) What is the wavelength λ **?**

(c) What is the frequency ν **?**

(d) What is the amplitude of the magnetic field part of the wave?

(e) Write an expression for the magnetic field part of the wave.

Ans. (a) From the given electric field vector, it can be inferred that the electric field is directed along the negative *x* direction. Hence, the direction of motion is along the negative *y* direction i.e., $-\hat{j}$.

(b) It is given that,

$$\vec{E} = 3.1 N/C \cos\left[\left(1.8 rad/m\right) y + \left(5.4 \times 10^{8} rad/s\right) t\right] \hat{i}$$ ...(i)

The general equation for the electric field vector in the positive *x* direction can be written as:

$$\vec{E} = E_0 \sin\left(kx - (i)t\right) - \hat{j}$$ ...(2)

On comparing equations (1) and (2), we get

Electric field amplitude, E_0 = 3.1 N/C

Angular frequency, $\omega = 5.4 \times 10^{8}\ rad/s$

Wave number, *k* = 1.8 rad/m

Wavelength, $\lambda = \frac{2\pi}{1.8}$ = 3.490 m

(c) Frequency of wave is given as:

$$\lambda = \frac{2\pi}{1.8}$$

$$= \frac{5.4 \times 10^{8}}{2\pi} = 8.6 \times 10^{7}\ Hz$$

(d) Magnetic field strength is given as:

$$B_0 = \frac{E_0}{c}$$

Where,

c = Speed of light = $3 \times 10^8\ m/s$

$$\therefore B_0 = \frac{3.1}{3 \times 10^8} = 1.03 \times 10^{-7}\ T$$

(e) On observing the given vector field, it can be observed that the magnetic field vector is directed along the negative z direction. Hence, the general equation for the magnetic field vector is written as:

$$\vec{B} = B_0 \cos\left(ky + (i)t\right)\hat{k}$$

$$= \left\{\left(1.03 \times 10^{-7}\ T\right)\cos\left[\left(1.8 rad/m\right)y + \left(5.4Kt10^6 rad/s\right)t\right]\right\}\hat{k}$$

---

4. In a plane electromagnetic wave, the electric field oscillates sinusoid ally at a frequency of 2.0×10^{10} Hz and amplitude 48 V m^{-1} .

(a) What is the wavelength of the wave?

(b) What is the amplitude of the oscillating magnetic field?

(c) Show that the average energy density of the E field equals the average energy density of the B field. [c = 3×10^8 m s^{-1}.]

Ans. Frequency of the electromagnetic wave, $\nu = 2.0 \times 10^{10}\ Hz$

Electric field amplitude, $E_0 = 48\ V\ m^{-1}$

Speed of light, c = $3 \times 10^8\ m/s$

(a) Wavelength of a wave is given as:

$$\lambda = \frac{c}{\nu}$$

$$= \frac{3\times10^{8}}{2\times10^{10}} = 0.015m$$

(b) Magnetic field strength is given as:

$$B_0 = \frac{E_0}{c}$$

$$= \frac{48}{3\times10^{8}} = 1.6\times10^{-7}T$$

(c) Energy density of the electric field is given as:

$$U_E = \frac{1}{2}\in_0 E^2$$

And, energy density of the magnetic field is given as:

$$U_B = \frac{1}{2\mu_0}B^2$$

Where,

$\in_0$ = Permittivity of free space

μ_0 = Permeability of free space

We have the relation connecting *E* and *B* as:

E = *cB* … (1)

Where,

$$c = \frac{1}{\sqrt{\in_0 \mu_0}} \text{ … (2)}$$

Putting equation (2) in equation (1), we get

$$E = \frac{1}{\sqrt{\epsilon_0 \ \mu_0}} B$$

Squaring both sides, we get

$$E^2 = \frac{1}{\sqrt{\epsilon_0 \ \mu_0}} B^2$$

$$\epsilon_0 \ E^2 = \frac{B^2}{\mu_0}$$

$$\frac{1}{2} \epsilon_0 \ E^2 = \frac{1}{2} \frac{B^2}{\mu_0}$$

$$\Rightarrow U_E = U_B$$

---

5. Suppose that the electric field amplitude of an electromagnetic wave is E_0 = 120 N/C and that its frequency is ν = 50.0 MHz. (a) Determine, B_0 , $\omega, k, and\ \lambda$. (b) Find expressions for E and B.

Ans. Electric field amplitude, E_0 = 120 N/C

Frequency of source, ν = 50.0 MHz = $50 \times 10^6 \ Hz$

Speed of light, *c* = 3×10^8 m/s

(a) Magnitude of magnetic field strength is given as:

$$B_0 \frac{E_0}{c}$$

$$= \frac{120}{3 \times 10^8}$$

$$= 4 \times 10^{-7} T = 400 \, nT$$

Angular frequency of source is given as:

$$\omega = 2\pi v = 2\pi \times 50 \times 10^6$$

$$= 3.14 \times 108 \ rad / s$$

Propagation constant is given as:

$$k = \frac{(i)}{c}$$

$$= \frac{3.14 \times 10^8}{3 \times 10^8} = 1.05 rad / m$$

Wavelength of wave is given as:

$$\lambda = \frac{c}{v}$$

$$= \frac{3 \times 10^8}{50 \times 10^6} = 6.0 \, m$$

(b) Suppose the wave is propagating in the positive *x* direction. Then, the electric field vector will be in the positive *y* direction and the magnetic field vector will be in the positive *z* direction. This is because all three vectors are mutually perpendicular.

Equation of electric field vector is given as:

$$\overline{E} = E_0 \sin\left(kx - (i)t\right) j$$

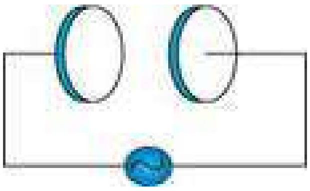
$$= 120 \sin\left[1.05x - 3.14 \times 10^8 t\right] j$$

And, magnetic field vector is given as:

$$\overrightarrow{B} = B_0 \sin\left(kx - (t)t\right) \hat{k}$$

$$\overrightarrow{B} = \left(4 \times 10^{-7}\right) \sin \left[1.05x - 3.14 \times 10^8 t\right] \hat{k}$$

6. A parallel plate capacitor (Fig. 8.7) made of circular plates each of radius *R* = 6.0 cm has a capacitance *C* = 100 pF. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad s^{-1}.



(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.

Ans. Radius of each circular plate,
R = 6.0 cm = 0.06 m

Capacitance of a parallel plate capacitor,
C = 100 pF = $100 \times 10^{-12}\ F$

Supply voltage, V = 230 V

Angular frequency, ω = 300 rad s^{-1}

(a) Rms value of conduction current, $I = \dfrac{V}{X_c}$

Where,

XC = Capacitive reactance

$$= \frac{1}{\omega C}$$

$$\therefore I = V \times \omega C$$

$= 230 \times 300 \times 100 \times 10^{-12}$

$= 6.9 \times 10^{-6} A$

= 6.9 μA

Hence, the rms value of conduction current is 6.9 μA

(b) Yes, conduction current is equal to displacement current.

(c) Magnetic field is given as:

$$B = \frac{\mu_o r}{2\pi R^2} I_o$$

Where,

μ_0 = Free space permeability $= 4\pi \times 10^{-7} NA^{-2}$

*I*0 = Maximum value of current = $\sqrt{2} I$

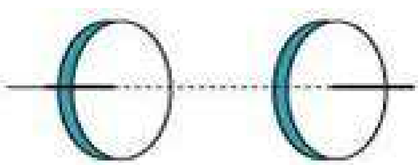
r = Distance between the plates from the axis = 3.0 cm = 0.03 m

$$\therefore B = \frac{4\pi \times 10^{-7} \times 0.03 \times \sqrt{2} \times 6.9 \times 10^{-6}}{2\pi > 0(06\ )^2}$$

= $1.63 \times 10^{-11}\ T$ Hence, the magnetic field at that point is $1.63 \times 10^{-11}\ T$.

---

7. Figure 8.6 shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.



(a) Calculate the capacitance and the rate of charge of potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

Ans. Radius of each circular plate,
r = 12 cm = 0.12 m

Distance between the plates,
d = 5 cm = 0.05 m

Charging current,
I = 0.15 A

Permittivity of free space,
$\varepsilon_0 = 8.85 \times 10^{-12}\ C2\ N^{-1}\ m^{-2}$

(a) Capacitance between the two plates is given by the relation,

$$C = \frac{\varepsilon_0 A}{d}\ \lambda = \frac{c}{v} = \frac{3\times10^8}{30\times10^6} = 10m$$

Where,

A = Area of each plate $= \pi r^2$

$$C = \frac{\varepsilon_0 \pi r^2}{d}$$

$$= \frac{8.85\times10\pi^{-12}\times0k2(\quad)^2}{0.05}$$

$$-8.0032\times10^{-12}F = 80.032pF$$

Charge on each plate, *q* = *CV*

Where,

V = Potential difference across the plates

Differentiation on both sides with respect to time (*t*) gives:

$$\frac{dq}{dt} = C\frac{dV}{dt}$$

$$But, \frac{dq}{dt} = current\,(I)$$

$$\therefore \frac{dV}{dt} = \frac{1}{C}$$

$$\Rightarrow \frac{0.15}{80.032 \times 10^{-12}} = 1.87 \times 10^{9} V/s$$

Therefore, the change in potential difference between the plates is 1.87×10^{9} V/s.

(b) The displacement current across the plates is the same as the conduction current. Hence, the displacement current, *id* is 0.15 A.

(c) Yes

Kirchhoff's first rule is valid at each plate of the capacitor provided that we take the sum of conduction and displacement for current.

CBSE Class 12 physics

Practice Questions

Chapter 9

Ray Optics and Optical

---

1 Mark Questions

1. A person is standing before a concave mirror cannot see his image, unless he is beyond the centre of curvature? Why?

Ans. When man stands beyond focus is i.e. between focus and centre of curvature, his real and inverted image is formed beyond C is beyond him and thus he cannot see the image. But when he stands beyond C, image is formed between focus and centre of curvature is in front of him and thus he is able to see his image.

---

2. For what angle of incidence, the lateral shift produced by a parallel sided glass plate is maximum?

Ans. We know

$$d = \frac{t}{\cos r}\sin(90^\circ - r) \quad (when\ \text{Li} = 90^\circ)$$

$$d = \frac{t}{\cos r}\cos r$$

D =t

Lateral shift is maximum

---

3. You read a newspaper, because of the light if reflects. Then why do you not see even a faint image of yourself in the newspaper?

Ans. The image is produced due to regular reflection of light but when we read a newspaper, because of diffused (irregular) reflection of light we are not able to see even a faint image.

4. A substance has critical angle of 45° for yellow light what is its refractive index?

Ans. $\mu = \dfrac{1}{\sin C}$

$$\mu = \frac{1}{\sin 45^\circ} = \frac{1}{\frac{1}{\sqrt{2}}}$$

$$\mu = \sqrt{2}$$

---

5. An object is placed between the pole and focus of a concave mirror produces a virtual and enlarged image. Justify using mirror formula?

Ans. $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$v = \frac{uf}{u-f}$$

For a concave mirror

$$f = -ve$$

$$u = -ve$$

Given V < f so ϑ is positive, hence image is virtual.

Now magnification $m = \dfrac{v}{u}$

$$\text{since } v > \text{o and u} < \text{o}$$

$\therefore m = +ve$, Hence enlarged image is produce

---

6. A converging and diverging lens of equal focal lengths are placed coaxially in contact. Find the focal length and power of the combination?

Ans. $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$

For converging lens $f_1=+f$

For diverging lens $f_2=-f$

$$\Rightarrow \frac{1}{F}=\frac{1}{f}-\frac{1}{f}$$
$$\Rightarrow F=\frac{1}{0}\Rightarrow \infty=F$$

$$Now\ \mathrm{P}=\frac{1}{F}=\frac{1}{\infty}=0$$

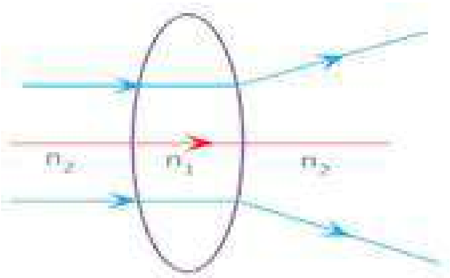
P=0

Hence

---

7. The refractive index of a material of a convex lens is n_1 it is immersed in a medium of refractive index n_2. A parallel beam of light is incident on thelens. Trace the path of the emergent rays when $n_2 > n_1$.

Ans. When $n_2 > n_1$ then the convex lens behaves as a concave



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8. In a telescope the focal length of the objective and the eye piece are 60cm and 5cm respectively. What is? (1) Its magnification power (2) Tube length

Ans. magnification $M=\frac{-fo}{fe}=\frac{-60}{5}=-12$

$L = fo + fe$

Tube length $L = 60 + 5 = 65cm.$

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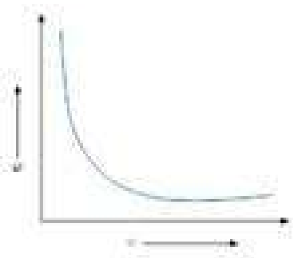
9. Show the variation of u and υ in case of a convex mirror?

Ans.



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10. Two lenses having focal length $f_1 and\ f_2$ are placed coaxially at a distance x from each other. What is the focal length of the combination?

Ans. $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$:

---

11. Does short-sightedness (myopia) or long-sightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation? If not, what might cause these defects of vision?

Ans. A myopic or hypermetropic person can also possess the normal ability of accommodation of the eye-lens. Myopia occurs when the eye-balls get elongated from front to back. Hypermetropia occurs when the eye-balls get shortened. When the eye-lens loses its ability of accommodation, the defect is called presbyopia.

2 Mark Questions

1. What are optical fibres? Give their one use?

Ans. Optical fibres consist of thin and long strands of fine quality glass or quartz coated with a thin layer of material of refractive index less than the refractive index of strands. They work on the principle of total internal reflection so they do not suffer any loss.

Uses

The optical fibres are used in medical investigations i.e. one can examine the inside view of stomach and intestine by a method called endoscopy.

---

2. How the focal lengths of a lens change with increase in the wavelength of the light?

Ans. $\delta = A(\mu - 1)$

$$\delta \propto \mu \ \sin ce \mu \propto \frac{1}{\lambda 2}$$

i.e. when wavelength increases $\mu -$ decreases and according to

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ focal length increases.

---

3. Show with a ray diagram, how an image is produced in a total reflecting prism?

Ans. the two rays from the object PQ undergoes total internal reflection firstly at the face AB and then at BC forming the find image $P'Q'$ (real and inverted image)

---

4. The radii of the curvature of the two spherical surfaces which is a lens of required focal length are not same. It forms image of an object. The surfaces of the lens facing the object and the image are inter-changed. Will the position of the image change?

Ans. As we know

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When $R_1 and\ R_2$ gets interchanged focal length of the lens remains the same hence position of the image will not change.

---

5. A thin converging lens has focal length (f) when illuminated by violet light. State with reason how the focal length of the lens will change if violet light is replaced by red light

Ans. Since

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

n for violet is more that n for red colour hence focal length of the lens will decreases when violet light is replaced by red light.

---

6. Thin prism of angle 60° gives a deviation of 30°. What is the refractive index of material of the prism?

Ans. $n = \dfrac{\sin \dfrac{A + Sm}{2}}{\sin \dfrac{A}{2}}$

$$= \frac{\sin\left(\dfrac{60 + 30}{2}\right)}{\sin \dfrac{60}{2}} = \frac{\sin 45^\circ}{\sin 30^\circ}$$

n =1.41

7. Although the surfaces of a goggle lens are curved it does not have any power. Why?

Ans. The two surface of the goggle lens are parallel i.e. one surface convex and the other concave thus the power of the two surfaces and equal but of opposite sign.

$$p = p_1 + p_2 = p + (-p) = 0$$

---

8. A ray of light in incident normally on one face of the prism of apex angle 30^o and refractive index $\sqrt{2}$. Find the angle of deviation for the ray of light?

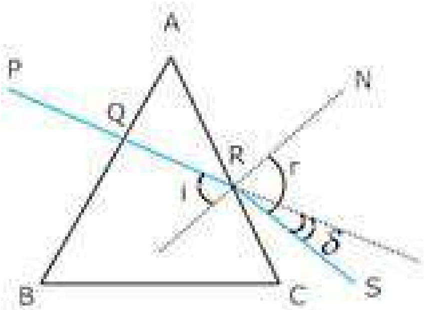
Ans. When ray PQ falls normally on AB then if goes straight at QR (no refraction)

$$n = \sqrt{2}$$

$$A = 30^o$$

$$i = 30^o$$

Applying Snell's law for face AC



$$n = \frac{\sin r}{\sin 30^o}$$

$$\sin r = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ r = 45^o

Now angle of deviation

$$\delta = r - i$$

$\delta = 45^\circ - 30^\circ$

$\delta = 15^\circ$

9. Following data was recorded for values of object distance and corresponding values of image distance in the experiment on study of real image formation by a convex lens of power +5 D. one of three observation is incorrect. Identify and give reason?

S. No.(u)123456

Object distance (v)**253035455055**

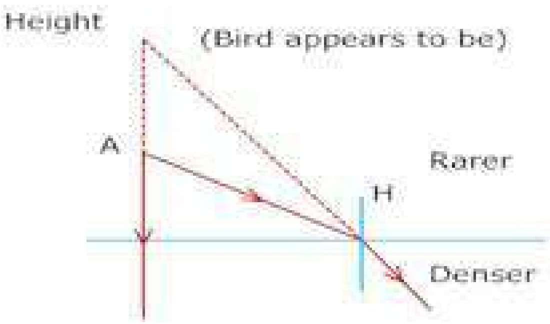
Image distance976137353230

Ans. $f = \frac{1}{P} = \frac{1}{5} = 0.20m$

Observation (3) is incorrect because both object and the image here lies between f and 2f.

10. A bird flying high in the air appears to be higher than in reality. Explain why?

Ans. Bird flying in air is in the rarer medium and if we see it from denser medium than light form bird refract towards the normal thus appears to come from the higher point. i.e. Apparent height > Real height (BIRD APPEARS TO BE)



11. What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

Ans. Focal length of the convex lens, f_1 = 30 cm

Focal length of the concave lens, f_2 = - 20 cm

Focal length of the system of lenses = f

The equivalent focal length of a system of two lenses in contact is given as:

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\frac{1}{f}=\frac{1}{30}-\frac{1}{20}=\frac{2-3}{60}=-\frac{1}{60}$$

$$\therefore f=-60cm$$

Hence, the focal length of the combination of lenses is 60 cm. The negative sign indicates that the system of lenses acts as a diverging lens.

---

12. The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

Ans. Distance between the object and the image, d= 3 m

Maximum focal length of the convex lens = f_{max}

For real images, the maximum focal length is given as:

$$f_{max}=\frac{d}{4}$$

$$=\frac{3}{4}=0.75m$$

Hence, for the required purpose, the maximum possible focal length of the convex lens is 0.75 m.

13. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

Ans. Distance between the image (screen) and the object, *D*= 90 cm

Distance between two locations of the convex lens, *d* = 20 cm

Focal length of the lens = *f*

Focal length is related to *d* and *D* as:

$$f = \frac{D^2 - d^2}{4D}$$

$$= \frac{(90)^2 - (20)^2}{4 \times 90} = \frac{770}{36} = 21.39\, cm$$

Therefore, the focal length of the convex lens is 21.39 cm.

---

14. You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will

(a) deviate a pencil of white light without much dispersion,

(b) disperse (and displace) a pencil of white light without much deviation.

Ans.(a) Place the two prisms beside each other. Make sure that their bases are on the opposite sides of the incident white light, with their faces touching each other. When the white light is incident on the first prism, it will get dispersed. When this dispersed light is incident on the second prism, it will recombine and white light will emerge from the combination of the two prisms.

(b) Take the system of the two prisms as suggested in answer **(a)**. Adjust (increase) the angle of the flint-glass-prism so that the deviations due to the combination of the prisms become equal. This combination will disperse the pencil of white light without much deviation.

15. A myopic person has been using spectacles of power -1.0 dioptre for distant vision. During old age he also needs to use separate reading glass of power + 2.0 dioptres. Explain what may have happened.

Ans. The power of the spectacles used by the myopic person, *P*= - 1.0 D

Focal length of the spectacles,

$$f = \frac{1}{P} = \frac{1}{-1\times10^{-2}} = -100cm$$

Hence, the far point of the person is 100 cm. He might have a normal near point of 25 cm. When he uses the spectacles, the objects placed at infinity produce virtual images at 100 cm. He uses the ability of accommodation of the eye-lens to see the objects placed between 100 cm and 25 cm.

During old age, the person uses reading glasses of power, $p' = +2D$

The ability of accommodation is lost in old age. This defect is called presbyopia. As a result, he is unable to see clearly the objects placed at 25 cm.

---

16. A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

Ans. In the given case, the person is able to see vertical lines more distinctly than horizontal lines. This means that the refracting system (cornea and eye-lens) of the eye is not working in the same way in different planes. This defect is called astigmatism. The person's eye has enough curvature in the vertical plane. However, the curvature in the horizontal plane is insufficient. Hence, sharp images of the vertical lines are formed on the retina, but horizontal lines appear blurred. This defect can be corrected by using cylindrical lenses.

---

17. A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25 cm)?

Ans. Focal length of the objective lens, f_o = 140 cm

Focal length of the eyepiece, f_e = 5 cm

Least distance of distinct vision, *d* = 25 cm

(a) When the telescope is in normal adjustment, its magnifying power is given as:

$$m = \frac{f_o}{f_o}$$

$$= \frac{140}{5} = 28$$

(b) When the final image is formed at *d*, the magnifying power of the telescope is given as:

$$\frac{f_o}{f_o}\left[1 + \frac{f_e}{d}\right]$$

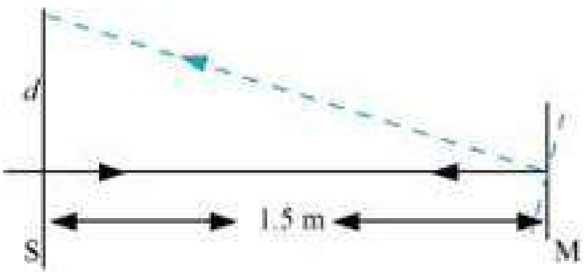
$$= \frac{140}{5}\left[1 + \frac{5}{25}\right]$$

$$= 28\left[1 + 0.2\right]$$

$$= 28 \times 1.2 = 33.6$$

---

18. Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Fig. 9.36. A current in the coil produces a deflection of 3.5°of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?



Ans. Angle of deflection, $\theta = 3.5°$

Distance of the screen from the mirror, *D*= 1.5 m

The reflected rays get deflected by an amount twice the angle of deflection i.e., $2\theta = 7.0°$

The displacement (*d*) of the reflected spot of light on the screen is given as:

$$\tan 2\theta = \frac{d}{1.5}$$

$$\therefore d = 1.5 \times \tan 7^0 = 0.184\,m = 18.4\,cm$$

Hence, the displacement of the reflected spot of light is 18.4 cm.

3 Mark Questions

1. Find the radius of curvature of the convex surface of a plane convex lens, whose focal length is 0.3m and the refractive index of the material of the lens is 1.5?

Ans. $\mu = 1.5$ $\quad$ f=0.3m

For plane convex lens

$f \quad R_1 = R$

$R_2 = -\infty$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{0.3} = (1.5 - 1)\left(\frac{1}{R} + \frac{1}{\infty}\right)$$

$$\Rightarrow \left(\frac{1}{R}\right)0.5 = \frac{1}{0.3}$$

R = 0.15 m

2. Show that the limiting value of the angle of prism is twice its critical angle? Hence define critical angle?

Ans. Angle of the prism $(A) = r_1 + r_2$

For limiting $A_{max} = (r_1)_{max} + (r_2)_{max}$

(Maximum)

Value of angle of prism for $(r_1)_{max}$ means $i = 90^\circ$

But when $i = 90^\circ$ $(r_1)_{max}$ = C

$A_{max} = C + C$

A_{max} = 2C

The angle of incidence for which angle of refraction is 90^o is called critical angle.

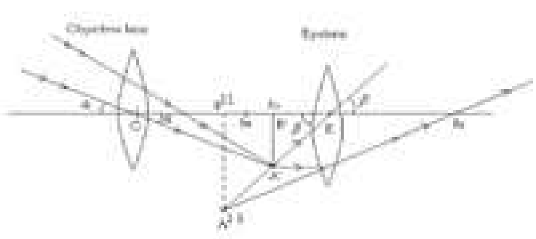
---

3. Draw a labeled diagram of telescope when the image is formed at the least distance of distinct vision? Hence derive the expression for its magnifying power?

Ans. magnifying power

$= \frac{\text{angle subtended by the image at the eye}}{\text{angle subtended by the object at the eye}}$

$\text{MP} = \frac{\tan\beta}{\tan\alpha} = \frac{\beta}{\alpha}$ (since angles are very small)



$$\tan\beta \frac{A'B'}{B'E} \ and \ \tan\alpha = \frac{A'B'}{B'O}$$

$$MP = \frac{A'B'}{B'E} \times \frac{A'B'}{B'O}$$

$$MP = \frac{B'O}{B'E} = \frac{fo}{-\upsilon e}$$

$$MP = \frac{-fo}{\upsilon e} \text{ ----(i)}$$

For eye piece

$$\frac{1}{\upsilon} - \frac{1}{\upsilon} = \frac{1}{fe}$$

$$-\frac{1}{D} - \frac{1}{-\upsilon e} = \frac{1}{fe}$$

Multiply by D

$$-1 + \frac{D}{\upsilon e} = \frac{D}{fe}$$

$$\frac{D}{ve} = \frac{D}{fe} + 1$$

$$\frac{1}{\upsilon e} = \frac{1}{fe} + \frac{1}{D} = \frac{1}{fe}\left(1 + \frac{fe}{D}\right)$$

Substituting in e.g. (i)

$$MP = \frac{-fo}{fe}\left(1 + \frac{fe}{D}\right)$$

4. Drive the expression for the angle of deviation for a ray of light passing through an equilateral prism of refracting angle A?

Ans. At the surface AB

$$\delta_1 = i - r_1$$

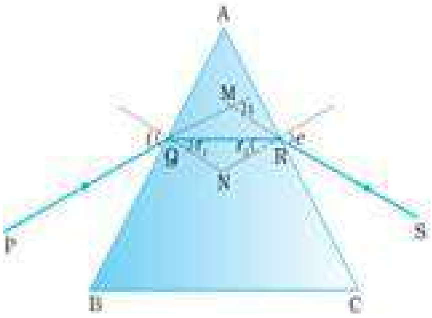
At the surface AC

$$\delta_2 = e - r_2$$

$$Thus \quad \delta = \delta_1 + \delta_2$$

$$\delta = i + e - (r_1 + r_2) \text{ ----(1)}$$

In quadrilateral AQOR



$$\angle A + \angle Q + \angle O + \angle R = 360^\circ$$

$$\Rightarrow \angle A + \angle O = 180^\circ \text{ ----(2)}$$

Now in ΔQOR

$$\angle Q + \angle O + \angle R = 180^\circ$$

Or

$$r_1 + r_2 + \angle O = 180^\circ \text{ ----(3)}$$

From (2) and (3)

$r_1 + r = A$ ----(4)

Substituting equation (4) in equation (1)

δ = i + e – A

Or

A + δ = i + e

---

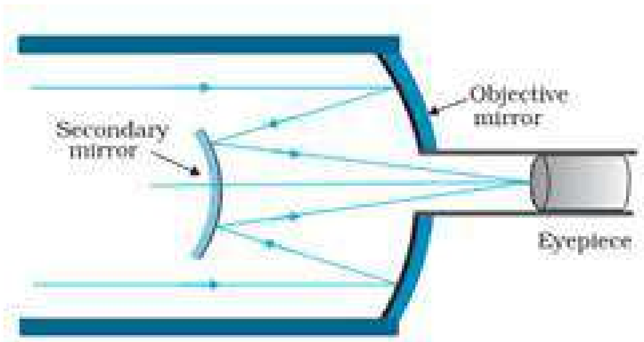
5. Draw a ray diagram to illustrate image formation by a Newtonian type reflecting telescope? Hence state two advantages of it over refracting type telescopes?

Ans. Advantages

(1) The image formed in reflecting type telescope is free from chromatic aberrations

(2) The image formed is very bright due to its large light gathering power.

NEWTONIAN TELESCOPE (REFLECTING TYPE)



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6. The magnifying power of an astronomical telescope in the normal adjustment position is 100. The distance between the objective and the eye piece is 101cm. calculate the focal length of the objective and the eye piece.

Ans. $fo + fe = 101$ ----(1)

$$M = \left|\frac{fo}{fe}\right| = 100$$

fo = 100 fe ----(2)

Substituting equation. (2) in equation (1)

fe + 100fe = 101

101fe = 101

fe = 1cm

Substituting fe in equation (2)

$$fo = 100 \times 1$$

$$fo = 100cm$$

---

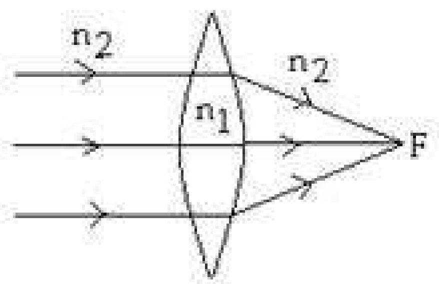
7. A convex lens made up of refractive index n_1 is kept in a medium of refractive index n_2. Parallel rays of light are incident on the lens. Complete the path of rays of light emerging from the convex lens if

(1) $n_1 > n_2$

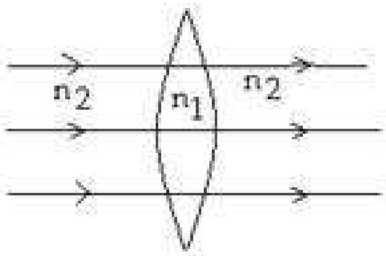
(2) $n_1 = n_2$

(3) $n_1 < n_2$

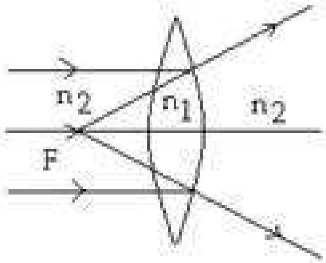
Ans. (1) When $n_1 > n_2$ the lens behaves as a convex lens.



(2) When $n_1 = n_2$ the lens behaves as a plane plate so no refraction takes place



(3) When $n_1 < n_2$ the lens behave as a convex lens



---

8. Derive the relation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

Where f_1 and f_2 are focal lengths of two thin lenses and F is the focal length of the combination in contact.

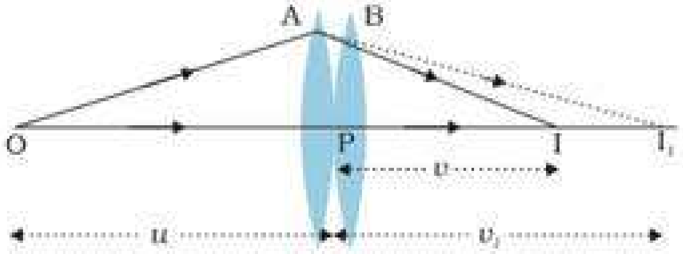
Ans. Consider two thin lenses in contact having focal length f_1 and f_2 For the first lens

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} ------(1)$$

For the second lens I_1 acts as an object which forms the final image I

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} -----(2)$$

Adding equation (1) & (2)



$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{v}-\frac{1}{u}+\frac{1}{v_1}-\frac{1}{v_1}$$

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{v}-\frac{1}{u}$$

Using lens formula

$$\left(\because \frac{1}{v}-\frac{1}{u}=\frac{1}{F}\right)$$

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{F}$$

For n no. of thin lenses is contact

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots\ldots.$$

---

9. A convex lens has a focal length 0.2m and made of glass is immersed in water $(\mu=1.33)$ find the change in focal length of the lens?

Ans. Fair = 0.2m, a μ g = 1.50

$$\frac{1}{fair}=(a\mu g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{0.2} = (1.50 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$or \ \frac{1}{R_1} - \frac{1}{R_2} = 10$$ ----(1)

Now $w\mu g = \frac{a\mu g}{a\mu w} = \frac{1.50}{1.33} = 1.128$

$$\Rightarrow \frac{1}{fn} = (w\mu g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Where too is the focal length of the lens when immersed in water?

$$\frac{1}{fw} = (1.128 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{fw} = (0.128) = \times 10 = 1.28$$

$$\Rightarrow fw = \frac{1}{1.28}$$

$$fw \ = \ 0.78m$$

---

10. A reflecting type telescope has a concave reflector of radius of curvature 120cm. calculate the focal length of eye piece to achieve a magnification of 20?

Ans. M = 20

R = 120cm (fro concave reflector)

$$fo = \frac{R}{2} = \frac{-120}{2} = -60cm$$

$$M = \frac{fo}{fe} \Rightarrow \frac{-60}{fe} = -20$$

$\Rightarrow$ fe = 3cm

---

11. Show that a convex lens produces an N time magnified image, when the object distances from the lens have magnitude $\left(f \pm \frac{f}{N}\right)$. Here f is the magnitude of the focal length of the lens. Hence find two values of object distance, for which value of u convex lens of power 2.5 D will produce an image that is four times as large as the object?

Ans. Magnifying power

$$m = \frac{f}{v + f}$$

For real image m = -N

$$-N = \frac{f}{u + f} \Rightarrow u + f = \frac{-f}{N}$$

Or

$$u = -\left(f + \frac{f}{N}\right) \text{ ----(1)}$$

For virtual image m =N

$$N = \frac{fe}{u + f}$$

$$u + f = -f + \frac{f}{N}$$

Or

$$u = -\left(f \pm \frac{f}{N}\right) \text{ ----(2)}$$

From equation (1) & (2) we can say that magnification produced by a lens can be N if u = $u = -\left(f \pm \frac{f}{N}\right)$

Now power of a lens = 2.5 D

$$\therefore f = \frac{1}{p} = \frac{1}{2.5} m$$

$$f = \frac{1}{2.5} \times 100 = 40 cm$$

$$m = \pm 4$$

$\Rightarrow$ m equation (1)

$$\pm 4 = \frac{4}{u + 40}$$

$$u + 40 = \pm 10$$

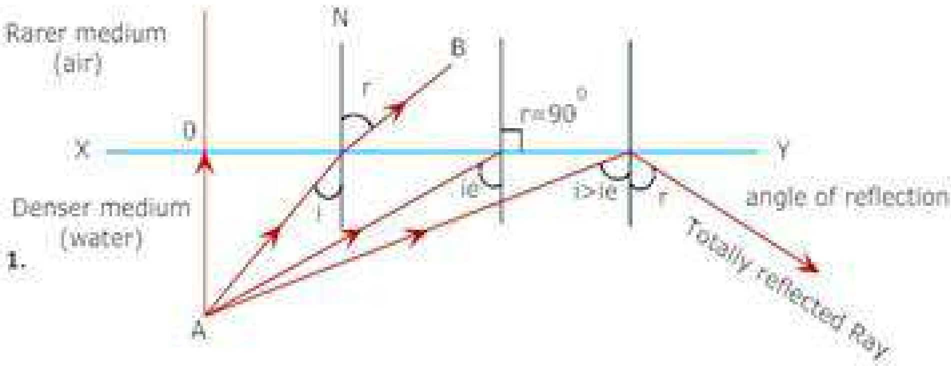
$$or\ u = -40 \pm 10$$

u = 30cm or -50cm

---

12. Define total infernal reflection of light? Hence write two advantages of total reflecting prisms over a plane mirror?

Ans. The phenomenon of reflection of light when a ray of light traveling from a denser medium is sent back to the same denser medium provided the angle of incidence is greater than the angle called critical angle is called total internal reflection.



<u>Advantages</u>

1. It does require silvering

2. Multiple reflections do not take place in a reflecting prism due to this; only one image is formed, which is very bright.

---

13. An equi–convex lens of radius of curvature R is cut into two equal parts by a vertical plane, so it becomes a plano-convex lens. If f is the focal length of equi–convex lens, then what will be focal length of the plano –convex lens?

Ans. We know

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

For equi –convex lens $R_1=-R_2=R$

$$\frac{1}{f}=(n-1)\left(\frac{1}{R}-\frac{1}{-R}\right)$$

$$\frac{1}{f}=(n-1)\left(\frac{1}{R}+\frac{1}{-R}\right)$$

$$\frac{1}{f}=\frac{2(n-1)}{R} \quad \text{----(1)}$$

For plano convex lens

$R_1 = R \ and \ R_2 = \infty$

$$\frac{1}{f'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\frac{1}{f'} = \frac{(n-1)}{R} \quad \text{----(2)}$$

From (1) & (2)

$$\frac{f'}{f} = 2$$

Or

f' = 2f

---

14. A converging lens of focal length 6.25cm is used as a magnifying glass if near point of the observer is 25cm from the eye and the lens is held close to the eye. Calculate (1) Distance of object from the lens. (2) Angular magnification and (3) Angular magnification when final image is formed at infinity.

Ans. (1) $\frac{1}{f} = \frac{1}{\upsilon} - \frac{1}{u}$

$f = 6.24cm$

$\vartheta = -25cm$

$$\frac{1}{u} = \frac{-1}{25} - \frac{1}{6.25} = \frac{-1}{5}$$

u = -5cm

(2) $m = 1 + \frac{D}{F} = 1 + \frac{25}{6.25}$

m = 5cm

$m = \frac{D}{f}$

$m = \frac{25}{6.25} = 4$

$\Rightarrow$ m = 4

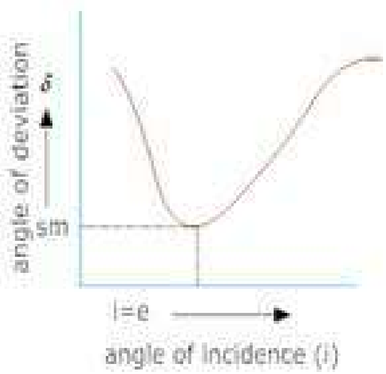
---

15. Draw a graph to show that variation of angle of deviation Dm with that of angle of incidence i for a monochromatic ray of light passing through a glass prism of refracting angle A. hence deduce the relation?

$\mu = \sin\left(\frac{A + \delta m}{2}\right) \Big/ \sin\frac{A}{2}$

Ans. For the minimum deviation position

$i = \frac{\delta m + A}{2}$



$\angle i = \angle e$

$r_1 = r_2 = r$ (Say)

We know $\angle i = \angle e = \delta + A ----(1)$

Also $r_1 + r_2$ = A

Or 2r = A

$$r = \frac{A}{2}$$

Applying minimum deviation condition is equation. (1)

2i = $\delta m + A$

$$i = \frac{\delta m + A}{2}$$

Applying Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

Or

$$\mu = \frac{\sin\left(\frac{A + \delta m}{2}\right)}{\sin\frac{A}{2}}$$

16. An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Ans. Size of the object, h_1 = 3 cm

Object distance, *u*= - 14 cm

Focal length of the concave lens, *f* = - 21 cm

Image distance = *v*

According to the lens formula, we have the relation:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=-\frac{1}{21}-\frac{1}{14}=\frac{-2-3}{42}=\frac{-5}{42}$$

$$\therefore v=-\frac{42}{5}=-8.4cm$$

Hence, the image is formed on the other side of the lens, 8.4 cm away from it. The negative sign shows that the image is erect and virtual.

The magnification of the image is given as:

$$m=\frac{\text{Im}\,age\;height\left(h_2\right)}{Object\;height\left(h_1\right)}=\frac{v}{u}$$

$$\therefore h_2=\frac{-8.4}{-14}\times 3=0.6\times 3=1.8cm$$

Hence, the height of the image is 1.8 cm.

If the object is moved further away from the lens, then the virtual image will move toward the focus of the lens, but not beyond it. The size of the image will decrease with the increase in the object distance.

17. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, and (b) a concave lens of focal length 16 cm?

Ans. In the given situation, the object is virtual and the image formed is real.

Object distance, u = +12 cm

(a) Focal length of the convex lens, f= 20 cm

Image distance = v

According to the lens formula, we have the relation:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{12}=\frac{1}{20}$$

$$\frac{1}{v}=\frac{1}{20}+\frac{1}{12}=\frac{3+5}{60}=\frac{8}{60}$$

$$\therefore v=\frac{60}{8}=7.5\,cm$$

Hence, the image is formed 7.5 cm away from the lens, toward its right.

(b) Focal length of the concave lens, f= –16 cm

Image distance = v

According to the lens formula, we have the relation:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=-\frac{1}{16}+\frac{1}{12}=\frac{-3+4}{48}=\frac{1}{48}$$

$$\therefore v=48cm$$

Hence, the image is formed 48 cm away from the lens, toward its right.

---

18. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20cm?

Ans. Refractive index of glass, $\mu=1.55$

Focal length of the double-convex lens, f = 20 cm

Radius of curvature of one face of the lens = R_1

Radius of curvature of the other face of the lens = R_2

Radius of curvature of the double-convex lens =R

$\therefore R_1 = R \, and \, R_2 = -R$

The value of *R* can be calculated as:

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} + \frac{1}{R}\right]$$

$$\frac{1}{20} = 0.55 \times \frac{2}{R}$$

$$\therefore R = 0.55 \times 2 \times 20 = 22cm$$

Hence, the radius of curvature of the double-convex lens is 22 cm.

---

19. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Ans. Focal length of the objective lens, f_o = 144 cm

Focal length of the eyepiece, f_e = 6.0 cm

The magnifying power of the telescope is given as:

$$m = \frac{f_0}{f_0}$$

$$= \frac{144}{6} = 24$$

The separation between the objective lens and the eyepiece is calculated as:

$$f_0 + f_0$$

$$= 144 + 6 = 150cm$$

Hence, the magnifying power of the telescope is 24 and the separation between the objective lens and the eyepiece is 150 cm.

20. (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?

(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is $3.48\times10^6\,\mathrm{m}$, and the radius of lunar orbit is $3.8\times10^8\,\mathrm{m}$.

Ans. Focal length of the objective lens,

f_o = 15 m = 15×10^2 cm

Focal length of the eyepiece,

f_e = 1.0 cm

(a) The angular magnification of a telescope is given as:

$$\alpha = \frac{f_0}{f_0}$$

$$= \frac{15\times10^2}{1.0} = 1500$$

Hence, the angular magnification of the given refracting telescope is 1500.

(b) Diameter of the moon,

d= $3.48\times10^6\ m$

Radius of the lunar orbit,

r0= 3.8×10^8 m

Let d' be the diameter of the image of the moon formed by the objective lens.

The angle subtended by the diameter of the moon is equal to the angle subtended by the image.

$$\frac{d}{r_0} = \frac{d'}{d_0}$$

$$\frac{3.48 \times 10^6}{3.8 \times 10^8} = \frac{d'}{15}$$

$$\therefore d' = \frac{3.48}{3.8} \times 10^{-2} \times 15$$

$$= 13.74 \times 10^{-2}\, m = 13.74\, cm$$

Hence, the diameter of the moon's image formed by the objective lens is 13.74 cm

---

21. A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

Ans. Actual depth of the pin, *d*= 15 cm

Apparent dept of the pin = d'

Refractive index of glass, $\mu = 1.5$

Ratio of actual depth to the apparent depth is equal to the refractive index of glass, i.e.

$$\mu = \frac{d}{d'}$$

$$\therefore d' = \frac{d}{\mu}$$

$$= \frac{15}{1.5} = 10\, cm$$

The distance at which the pin appears to be raised = $d' - d$

=15-10=5 cm

For a small angle of incidence, this distance does not depend upon the location of the slab.

---

22. A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.

(a) What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass?

(b) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

Ans. (a) Focal length of the magnifying glass, f = 5 cm

Least distance of distance vision, d = 25 cm

Closest object distance = u

Image distance, v= - d= - 25 cm

According to the lens formula, we have:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{-25} - \frac{1}{5} = \frac{-5-1}{25} = \frac{-6}{25}$$

$$\therefore u = -\frac{25}{6} = -4.167\,cm$$

Hence, the closest distance at which the person can read the book is 4.167 cm.

For the object at the farthest distant (u'), the image distance $(v') = \infty$

According to the lens formula, we have:

$$\frac{1}{f} = \frac{1}{v'} - \frac{1}{u'}$$

$$\frac{1}{u'} = \frac{1}{\infty} - \frac{1}{5} = -\frac{1}{5}$$

$$\therefore u' = -5 cm$$

Hence, the farthest distance at which the person can read the book is
5 cm.

(b) Maximum angular magnification is given by the relation:

$$\alpha_{\max} = \frac{d}{|u|}$$

$$= \frac{25}{\frac{25}{6}} = 6$$

Minimum angular magnification is given by the relation:

$$\mu_2 \ \alpha_{\min} = \frac{d}{|u'|}$$

$$= \frac{25}{5} = 5$$

23. A card sheet divided into squares each of size 1 mm2 is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 9 cm) held close to the eye.

(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) What is the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Ans. (a) Area of each square, *A*= 1 mm2

Object distance, *u*= - 9 cm

Focal length of a converging lens, *f* = 10 cm

For image distance *v*, the lens formula can be written as:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} + \frac{1}{9}$$

$$\frac{1}{v} = -\frac{1}{90}$$

$$\therefore v = -90cm$$

Magnification, $m = \frac{v}{u}$

$$= \frac{-90}{-9} = 10$$

∴Area of each square in the virtual image = (10)2*A*

= 102 ×1 = 100 mm^2

= 1 cm^2

(b) Magnifying power of the lens $= \frac{d}{|u|} = \frac{25}{9} = 2.8$

(c) The magnification in **(a)** is not the same as the magnifying power in **(b)**.

The magnification magnitude is $\left(\left|\frac{v}{u}\right|\right)$ and the magnifying power is $\left(\left|\frac{d}{u}\right|\right)$.

The two quantities will be equal when the image is formed at the near point (25 cm).

---

24. (a) At what distance should the lens be held from the figure in Exercise 9.29 in order to view the squares distinctly with the maximum possible magnifying power?

(b) What is the magnification in this case?

(c) Is the magnification equal to the magnifying power in this case? Explain.

Ans. (a) The maximum possible magnification is obtained when the image is formed at the near point (*d*= 25 cm).

Image distance, *v*= - *d*= - 25 cm

Focal length, *f*= 10 cm

Object distance = *u*

According to the lens formula, we have:

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$\frac{1}{u}=\frac{1}{v}-\frac{1}{f}$$

$$=\frac{1}{-25}-\frac{1}{10}=\frac{-2-5}{50}=-\frac{7}{50}$$

$$\therefore u=-\frac{50}{7}=-7.14\,cm$$

Hence, to view the squares distinctly, the lens should be kept 7.14 cm away from them.

(b) Magnification =

$$\left|\frac{v}{u}\right|=\frac{25}{\frac{50}{7}}=3.5$$

(c) Magnifying power =

$$\frac{d}{u}=\frac{25}{\frac{50}{7}}=3.5$$

Since the image is formed at the near point (25 cm), the magnifying power is equal to the magnitude of magnification.

---

25. What should be the distance between the object in Exercise 9.30 and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm2. Would you be able to see the squares distinctly with your eyes very close to the magnifier?

[*Note:* Exercises 9.29 to 9.31 will help you clearly understand the difference between magnification in absolute size and the angular magnification (or magnifying power) of an instrument.]

Ans. Area of the virtual image of each square, $A=6.25\ mm^2$

Area of each square, $A0 = 1\ mm^2$

Hence, the linear magnification of the object can be calculated as:

$$m = \sqrt{\frac{A}{A_0}}$$

$$= \sqrt{\frac{6.25}{1}} = 2.5$$

But m = $\frac{imagr\ dis\tan ce(v)}{Object\ dis\tan ce(u)}$

$$\therefore v = mu$$

$= 2.5u$ ...(1)

Focal length of the magnifying glass, *f*= 10 cm

According to the lens formula, we have the relation:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{2.5u} - \frac{1}{u} = \frac{1}{u}\left(\frac{1}{2.5} - \frac{1}{1}\right) = \frac{1}{u}\left(\frac{1-2.5}{2.5}\right)$$

$$\therefore u = -\frac{1.5\times 10}{2.5} = -6\ cm$$

$$and\ v = 2.5u$$

$$= 2.5 \times 6 = -15cm$$

The virtual image is formed at a distance of 15 cm, which is less than the near point (i.e., 25 cm) of a normal eye. Hence, it cannot be seen by the eyes distinctly.

---

26. (a) For the telescope described in Exercise 9.34 (a), what is the separation between the objective lens and the eyepiece?

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c) What is the height of the final image of the tower if it is formed at 25 cm?

Ans. Focal length of the objective lens, f_o = 140 cm

Focal length of the eyepiece, f_e = 5 cm

(a) In normal adjustment, the separation between the objective lens and the eyepiece $= f_o + f_e = 140 + 5 = 145 cm$

(b) Height of the tower, h1= 100 m

Distance of the tower (object) from the telescope, u= 3 km = 3000 m

The angle subtended by the tower at the telescope is given as:

$$\theta = \frac{h_1}{u}$$

$$= \frac{100}{3000} = \frac{1}{30} rad$$

The angle subtended by the image produced by the objective lens is given as:

$$\theta = \frac{h_2}{f_o} = \frac{h_2}{140} rad$$

Where,

h_2 = Height of the image of the tower formed by the objective lens

$$\frac{1}{30} = \frac{h_2}{140}$$

$$\therefore h_2 = \frac{140}{30} = 4.7 cm$$

Therefore, the objective lens forms a 4.7 cm tall image of the tower.

(c) Image is formed at a distance, *d*= 25 cm. The magnification of the eyepiece is given by the relation:

$$m = 1 + \frac{d}{f_e}$$

$$= 1 + \frac{25}{5} = 1 + 5 = 6$$

Height of the final image $= mh_2 = 6 \times 4.7 = 28.2cm$

Hence, the height of the final image of the tower is 28.2 cm.

5 Mark Questions

1. Prove that $\frac{n_2}{v} = \frac{n_1}{v} = \frac{n_2 - n_1}{R}$

When refraction occurs of a convex spherical refracting surface and the ray travels from rarer to denser medium.

Ans. From $\Delta AOC\ i = \alpha + \gamma$ ----(i)

Similarly form $\Delta AIC\ \gamma = \gamma + \beta$

$r = \gamma - \beta$ ----(ii)

From $\Delta ANO \tan \alpha = \frac{h}{NO}$

$$\Delta ANI \tan \beta = \frac{h}{NI}$$

$$\Delta ANC \tan \gamma = \frac{h}{NC}$$

Same aperture of the spherical surface is small so point N lies close to P and since angles α, β *and* γ are very small $\therefore \tan\alpha \approx \alpha, \tan\beta \approx \beta$ and $\tan\gamma \approx \gamma$

$$\Rightarrow \alpha = \frac{h}{PO}, \beta = \frac{h}{PI}$$

and $\gamma = \dfrac{h}{PC}$

Applying sign conventional

$$\alpha = \frac{h}{-U}, \beta = \frac{h}{\upsilon}$$

and $\gamma = \dfrac{h}{R}$

Substituting these values in e.g. (1) & (2)

$$i = \frac{h}{U} + \frac{h}{R}$$

$$r = \frac{h}{R} - \frac{h}{\upsilon}$$

According to snell's law

$$n_{21} = \frac{n_2}{n_1} = \frac{\sin i}{\sin r} = \frac{i}{r}$$

(Since angles are very small)

$$\frac{n_2}{n_1} = \frac{i}{r}$$

$$n_2 r = n_1 i \quad or \quad n_2\left(\frac{+h}{R} - \frac{h}{\upsilon}\right) = n_1\left(\frac{-h}{U} + \frac{h}{R}\right)$$

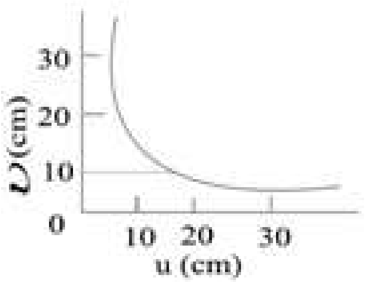
$$\frac{n_2}{R}-\frac{n_2}{\upsilon}=\frac{-n_1}{U}+\frac{n_1}{R}$$

$$\frac{n_2}{R}-\frac{n_2}{\upsilon}=\frac{-n_1}{U}+\frac{n_1}{R}$$

$$\frac{n_2-n_1}{R}=\frac{n_2}{\upsilon}-\frac{n_1}{U}$$

---

2. A lens forms a real image of an object. The distance of the object. From the lens is U cm and the distance of the image from the lens is υ cm. The given graph shows the variation of υ and U



(a) What is the nature of the lens?

(b) Using the graph find the focal length of the lens?

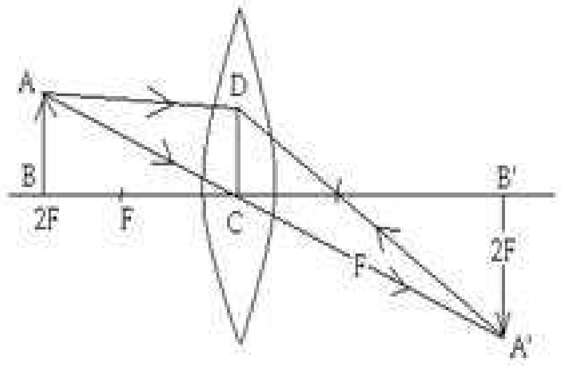
(c) Draw a ray diagram to show the formation of image of same size as that of object in case of converging lens hence derive lens equation?

Ans. (a) convex lens

(b) $\frac{1}{\upsilon}-\frac{1}{U}=\frac{1}{f}$

$When \quad U \to \infty$

$\frac{1}{\upsilon}=\frac{1}{f}$ i.e $\upsilon = f$



In the given graph f = 10cm.

(c) $\Delta ABC \text{ and } A'B'C \text{ are similar}$

$$\therefore \frac{A'B'}{AB} = \frac{B'C}{BC} ----(1)$$

ΔDCF and $A'B'F$ are similar

$$\therefore \frac{A'B'}{DC} = \frac{B'F}{FC}$$

$$\Rightarrow \frac{A'B'}{DC} = \frac{B'F}{FC} = \frac{A'B'}{AB} \text{ ----(2) } (\because DC = AB)$$

Combining equation (1) & (2)

$$\frac{B'C}{BC} = \frac{B'F}{BF}$$

Using sign conventions

$B'C = +\upsilon$

$BC = -U$

$B'F = B'C = FC$

$B'F = +\upsilon - f$

$FC = +f$

$$\Rightarrow \frac{\upsilon}{-U} = \frac{\upsilon - f}{f}$$

$\upsilon f = -\upsilon U + fU$

Divide by U υ f

$$\frac{1}{U} = \frac{-1}{f} + \frac{1}{\upsilon}$$

Or

$$\frac{1}{f} = \frac{1}{\upsilon} - \frac{1}{U}$$

Hence derived

---

3. By stating sign conventions and assumptions used derive the relation between $u, \upsilon \; and \; f$ in case of a concave mirror?

Ans. Sign conventions

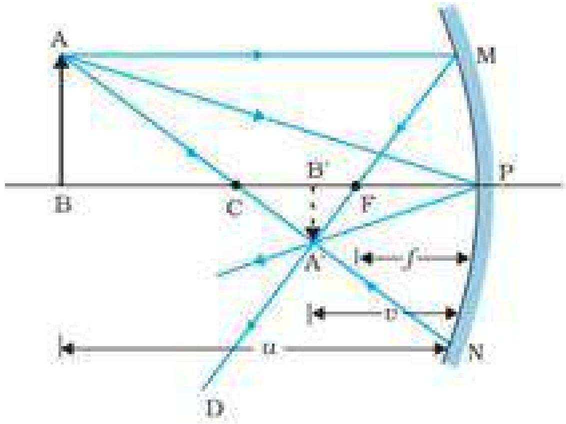
(1) All distances are measured from the pole of the mirror.

(2) Distance measured in the direction of incident light is positive and those measured in the direction opposite to the incident light are negative.

(3) Height measured upwards is positive and height measured downwards is negative.

Assumptions

(1) Aperture of the spherical mirror is considered to be very small.



$In\ \Delta A'B'F\ and\ MDF$

$\angle A'FB' = \angle MFD$ ($Vertically$ opp.$\angle$'s.)

$\angle D = \angle B'$ (each $90°$)

$\Rightarrow$ Remaining angles equal

$\therefore$ A'B'F and MDF are similar

$$\therefore \frac{A'B'}{MD} = \frac{B'F}{FD}$$

$or\ \frac{A'B'}{AB} = \frac{B'F}{FP}$ ----(1) ($\therefore$ Dlies closetol AB + MD)

Similarly $\Delta APB\ \ A'PB'$ and are also similar

$\therefore \frac{A'B'}{AB} = \frac{B'P}{FP}$ ----(2)

Combining equation (1) & (2) and using sign conventions

$$\frac{B'F}{FP} = \frac{B'P}{BP} \begin{pmatrix} \because B'F = B'P - FP = -v + f \\ FP = -f \quad BP = -u \\ B'P = -v \end{pmatrix}$$

$$\Rightarrow \frac{-v+f}{-f} = \frac{-v}{-u}$$

$$or \ (-v+f)u = -fv$$

$$or \ -vu + fu = -fv$$

Divide by we uvf get

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

---

4. (a) A person looking at a mesh of crossed wires is able to see the vertical lines more distinctly than the horizontal wires. What is the effect due to? How is such a defect of vision corrected?

(b) A man with normal near point (25cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5cm.

(i) What is the closest and the farthest distance at which he can read the book when viewing through the magnifying glass ?

(ii) What is the maximum and minimum angular magnificent (magnifying power) possible using the above simple microscope?

Ans. (a) It is due to the defect called astigmatisms and is caused due to irregular surface of cornea and curvature of the eye lens is different in different planes. This type of defect can be corrected using cylindrical lens.

(b) (i) Here f = 5cm $v = -2.5cm$

For closest point $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{5} = \frac{1}{-2.5} - \frac{1}{v}$ and thus $\frac{-1}{u} = \frac{1}{5} + \frac{1}{-2.5}$

$$\frac{-1}{u}=\frac{6}{25}$$

$\Rightarrow$ u = -4.2cm

A for farthest point f = 5cm $\quad v=-\infty$

$\Rightarrow$ using $\frac{1}{5}=\frac{1}{-\infty}-\frac{1}{v}$

u = -5cm

(b) (ii) Angular magnification

$$m=\frac{D}{|v|}=\frac{25}{4.2}=6$$

Maximum angular magnification

$$m=\frac{25}{5}\Rightarrow m=5$$

Minimum angular magnification

---

5. Four double convex lens with following specification are available.

	LensFocal	lengthAperture
A	100cm	10cm
B	100cm	5cm
C	10cm	2cm
D	5cm	2cm

(a) Which of the given four lenses should be selected as objective and eyepiece to construct an astronomical telescope and why? What will be the magnifying power and length of the tube of this telescope?

(b) An object is seen with the help of a simple microscope, firstly in red light and then is blue light. Will the magnification be same in both the cases? Why?

Ans. (a) objective of the telescope should be of large aperture as it has to gather maximum light and should be of large focal length to have maximum magnification.

Hence lens A is selected as objective and lens D as eyepiece of small aperture and small focal length.

$$M.P = \left|\frac{fo}{fe}\right| = \frac{100}{5} = 20$$

M.P. = 20

(b) L = fo + fe

L = 100 + 5

L = 10.5 cm

$$M.P = 1 + \frac{D}{f}$$

$$f\,\text{Red} > f\,\text{Blue}$$

$$M_{RED} < M_{Blue}$$

$\therefore$ When red light as replaced by blue light, magnifying power increases.

---

6. A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Ans. Size of the candle, h= 2.5 cm

Image size = h'

Object distance, u= - 27 cm

Radius of curvature of the concave mirror, R= - 36 cm

$$\therefore h' = -\frac{v}{u} \times h$$

Focal length of the concave mirror,

$$= -\left(\frac{-54}{-27}\right) \times 2.5 = -5\,cm$$

Image distance = *v*

The image distance can be obtained using the mirror formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$-\frac{1}{-18} - \frac{1}{-27} = \frac{-3+2}{54} = -\frac{1}{54}$$

$$\therefore v = -54\,cm$$

Therefore, the screen should be placed 54 cm away from the mirror to obtain a sharp image.

The magnification of the image is given as:

$$m = \frac{h'}{h} = -\frac{v}{u}$$

$$\therefore h' = -\frac{v}{u} \times h$$

$$= -\left(\frac{-54}{-27}\right) \times 2.5 = -5\,cm$$

The height of the candle's image is 5 cm. The negative sign indicates that the image is inverted and real.

If the candle is moved closer to the mirror, then the screen will have to be moved away from the mirror in order to obtain the image.

7. A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

Ans. Height of the needle, h_1 = 4.5 cm

Object distance, *u* = – 12 cm

Focal length of the convex mirror, *f*= 15 cm

Image distance = *v*

The value of *v* can be obtained using the mirror formula:

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$\frac{1}{u}=\frac{1}{v}-\frac{1}{f}$$

$$=\frac{1}{-25}-\frac{1}{5}=\frac{-5-1}{25}=\frac{-6}{25}$$

$$\therefore u=-\frac{25}{6}=-4.167\,cm$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$=\frac{1}{15}+\frac{1}{12}=\frac{4+5}{60}=\frac{9}{60}$$

$$\therefore v=\frac{60}{9}=6.7\,cm$$

Hence, the image of the needle is 6.7 cm away from the mirror. Also, it is on the other side of the mirror. The image size is given by the magnification formula:

$$m = \frac{h_2}{h_1} = -\frac{v}{u}$$

$$\therefore h_2 = -\frac{v}{u} \times h_1$$

$$= \frac{-6.7}{-12} \times 4.5 = +2.5\,cm$$

Hence, magnification of the image, $m = \frac{h_2}{h_1} = \frac{2.5}{4.5} = 0.56$

The height of the image is 2.5 cm. The positive sign indicates that the image is erect, virtual, and diminished.

If the needle is moved farther from the mirror, the image will also move away from the mirror, and the size of the image will reduce gradually.

---

8. A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Ans. Actual depth of the needle in water, h_1 = 12.5 cm

Apparent depth of the needle in water, h_2 = 9.4 cm

Refractive index of water = μ

The value of μ can be obtained as follows:

$$\mu = \frac{h_1}{h_2}$$

$$= \frac{12.5}{9.4} \approx 1.33$$

Hence, the refractive index of water is about 1.33.

Water is replaced by a liquid of refractive index, $\mu' = 1.63$

The actual depth of the needle remains the same, but its apparent depth changes. Let *y* be the new apparent depth of the needle. Hence, we can write the relation:

$$\mu' = \frac{h_1}{y}$$

$$\therefore y = \frac{h_1}{\mu'}$$

$$= \frac{12.5}{1.63} = 7.67\ cm$$

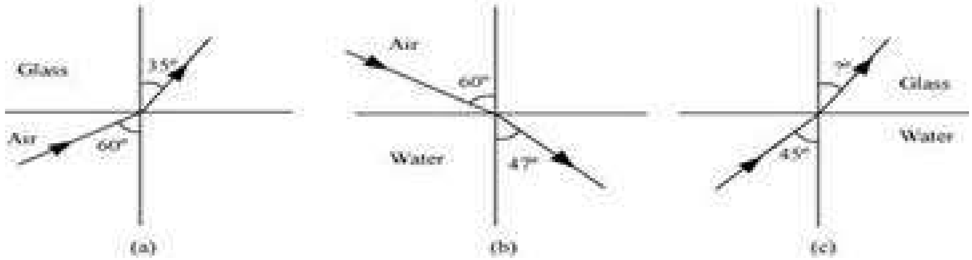
Hence, the new apparent depth of the needle is 7.67 cm. It is less than $h2$. Therefore, to focus the needle again, the microscope should be moved up.

∴Distance by which the microscope should be moved up = 9.4 - 7.67

= 1.73 cm

---

9. Figures 9.34 (a) and (b) show refraction of a ray in air incident at $60°$ with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is $45°$ with the normal to a water-glass interface [Fig. 9.34(c)].



Ans. As per the given figure, for the glass - air interface:

Angle of incidence, *i* = $60°$

Angle of refraction, *r*= $35°$

The relative refractive index of glass with respect to air is given by Snell's law as:

$$= mh_2 = 6 \times 4.7 = 28.2cm$$

$$\mu_g^a = \frac{\sin i}{\sin r}$$

$$= \frac{\sin 60^0}{\sin 35^0} = \frac{0.8660}{0.5736} = 1.51 \text{ ...(1)}$$

$$\mu_g^a = \frac{\sin i}{\sin r}$$

$$= \frac{\sin 60^0}{\sin 35^0} = \frac{0.8660}{0.5736} = 1.51 \text{ ...(i)}$$

As per the given figure, for the air - water interface:

Angle of incidence, *i*= $60°$

Angle of refraction, *r*= $47°$

The relative refractive index of water with respect to air is given by Snell's law as:

$$\mu_g^a = \frac{\sin i}{\sin r}$$

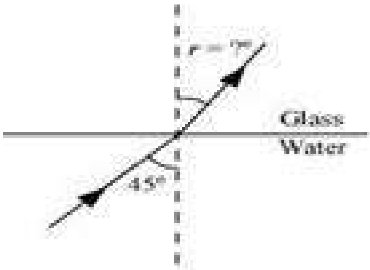
$$= \frac{\sin 60}{\sin 47} = \frac{0.8660}{0.7314} = 1.184$$ ...(2)

Using (1) and (2), the relative refractive index of glass with respect to water can be obtained as:

$$\mu_g^w = \frac{\mu_g^a}{\mu_w^a}$$

$$= \frac{1.51}{1.184} = 1.275$$

The following figure shows the situation involving the glass - water interface.



Angle of incidence, *i*= $45°$

Angle of refraction = *r*

From Snell's law, *r*can be calculated as:

$$\frac{\sin i}{\sin r} = \mu_g^w$$

$$\frac{\sin 45^0}{\sin r} = 1.275$$

$$\sin r = \frac{\frac{1}{\sqrt{2}}}{1.275} = 0.5546$$

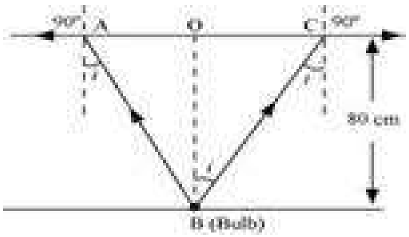
$$\therefore r = \sin^{-1}(0.5546) = 38.67^0$$

Hence, the angle of refraction at the water - glass interface is $38.68°$.

---

10. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

Ans.



Actual depth of the bulb in water, d_1 = 80 cm = 0.8 m

Refractive index of water, $\mu = 1.33$

The given situation is shown in the following figure:

Where,

i= Angle of incidence

r= Angle of refraction = $90°$

Since the bulb is a point source, the emergent light can be considered as a circle of radius,

$$R = \frac{AC}{2} = AO = OB$$

Using Snell' law, we can write the relation for the refractive index of water as:

$$\mu = \frac{\sin r}{\sin i}$$

$$1.33 = \frac{\sin 90^0}{\sin i}$$

$$\therefore i = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75^0$$

Using the given figure, we have the relation:

$$\tan i = \frac{OC}{OB} = \frac{R}{d_1}$$

$$\therefore R = tan\ 48.75° \times 0.8 = 0.91\ m$$

∴ Area of the surface of water

$= \pi R^2 = \pi(0.91)^2 = 2.61\ m^2$

Hence, the area of the surface of water through which the light from the bulb can emerge is approximately 2.61 m^2 .

---

11. A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be $40°$. What is the refractive index of the material of the prism? The refracting angle of the prism is $60°$. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

Ans. Angle of minimum deviation, δ_m = $40°$

Angle of the prism, *A* = $60°$

Refractive index of water, μ= 1.33

Refractive index of the material of the prism = μ'

The angle of deviation is related to refractive index (μ') as:

$$\mu' = \frac{\sin\frac{(A+\delta_m)}{2}}{\sin\frac{A}{2}}$$

$$= \frac{\sin\frac{(60^0+40^0)}{2}}{\sin\frac{60^0}{2}} = \frac{\sin 50^0}{\sin 30^0} = 1.532$$

Hence, the refractive index of the material of the prism is 1.532.

Since the prism is placed in water, let $\delta_m{}'$ be the new angle of minimum deviation for the same prism.

The refractive index of glass with respect to water is given by the relation:

$$\mu_g^a = \frac{\mu'}{\mu} = \frac{\frac{\left(A+\delta_m^{'}\right)}{2}}{\sin\frac{A}{2}}$$

$$\sin\frac{\left(A+\delta_m{}'\right)}{2} = \frac{\mu'}{\mu}\sin\frac{A}{2}$$

$$\sin\frac{\left(A+\delta_m{}'\right)}{2} = \frac{1.532}{1.33}\times\sin\frac{60^0}{2} = 0.5759$$

$$\frac{\left(A+\delta_m{}'\right)}{2} = \sin^{-1}0.5759 = 35.16^0$$

$$60^0 + \delta_m{}' = 70.32^0$$

$$\therefore \delta_m{}' = 70.32^0 - 60^0 = 10.32^0$$

Hence, the new minimum angle of deviation is $10.32°$.

---

12. A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the

objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?

Ans. Focal length of the objective lens, f_1 = 2.0 cm

Focal length of the eyepiece, f_2 = 6.25 cm

Distance between the objective lens and the eyepiece, *d*= 15 cm

(a) Least distance of distinct vision, $d' = 25 cm$

∴Image distance for the eyepiece, v_2 = - 25 cm

Object distance for the eyepiece = u_2

According to the lens formula, we have the relation:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{u_2} = \frac{1}{v_2} - \frac{1}{f_2}$$

$$= \frac{1}{-25} - \frac{1}{6.25} = \frac{-1-4}{25} = \frac{-5}{25}$$

$$\therefore u_2 = -5\, cm$$

Image distance for the objective lens, $v_1 = d + u_2 = 15 - 5 = 10\, cm$

Object distance for the objective lens = u_2

According to the lens formula, we have the relation:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1}$$

$$= \frac{1}{10} - \frac{1}{2} = \frac{1-5}{10} = \frac{-4}{10}$$

$$\therefore u_1 = -2.5\,cm$$

Magnitude of the object distance, $|u_1|$= 2.5 cm

The magnifying power of a compound microscope is given by the relation:

$$m = \frac{v_1}{|u_1|}\left(1 + \frac{d'}{f_2}\right)$$

$$= \frac{10}{2.5}\left(1 + \frac{25}{6.25}\right) = 4(1+4) = 20$$

Hence, the magnifying power of the microscope is 20.

(b) The final image is formed at infinity.

∴Image distance for the eyepiece, $v_2 = \infty$

Object distance for the eyepiece = u_2

According to the lens formula, we have the relation:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{\infty} - \frac{1}{u_2} = \frac{1}{6.25}$$

$$\therefore u_2 = -6.25\,cm$$

Image distance for the objective lens, $v_1 = d + ul2 = 15 - 6.25 = 8.75\,cm$

Object distance for the objective lens = u_1

According to the lens formula, we have the relation:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1}$$

$$= \frac{1}{8.75} - \frac{1}{2.0} = \frac{2 - 8.75}{17.5}$$

$$\therefore u_1 = -\frac{17.5}{6.75} = -2.59\, cm$$

Magnitude of the object distance, $|u_1|$ = 2.59 cm

The magnifying power of a compound microscope is given by the relation:

$$m = \frac{v_1}{|u_1|}\left(\frac{d'}{|u_2|}\right)$$

$$= \frac{8.75}{2.59} \times \frac{25}{6.25} = 13.51$$

Hence, the magnifying power of the microscope is 13.51.

---

13. A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope,

Ans. Focal length of the objective lens, f_o = 8 mm = 0.8 cm

Focal length of the eyepiece, f_e = 2.5 cm

Object distance for the objective lens, u_o = - 9.0 mm = - 0.9 cm

Least distance of distant vision, *d* = 25 cm

Image distance for the eyepiece, v_e = - *d*= - 25 cm

Object distance for the eyepiece = u_e

Using the lens formula, we can obtain the value of u_e as:

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= \frac{1}{-25} - \frac{1}{2.5} = \frac{-1-10}{25} = \frac{-11}{25}$$

$$\therefore u_e = -\frac{25}{11} = -2.27\,cm$$

We can also obtain the value of the image distance for the objective lens (v_0) using the lens formula.

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

$$\frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0}$$

$$= \frac{1}{0.8} - \frac{1}{0.9} = \frac{0.9-0.8}{0.72} = \frac{0.1}{0.72}$$

$$\therefore v_0 = 7.2cm$$

The distance between the objective lens and the eyepiece $= |u_e| + v_0$

$= 2.27 + 7.2$

$= 9.47\,cm$

The magnifying power of the microscope is calculated as:

$$\frac{v_0}{|u_0|}\left(1+\frac{d}{f_e}\right)$$

$$= \frac{7.2}{0.9}\left(1+\frac{25}{2.5}\right) = 8(1+10) = 88$$

Hence, the magnifying power of the microscope is 88.

---

14. Use the mirror equation to deduce that:

(a) an object placed between *f* and 2*f* of a concave mirror produces a real image beyond 2*f*.

(b) a convex mirror always produces a virtual image independent of the location of the object.

(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

[*Note:* This exercise helps you deduce algebraically properties of

images that one obtains from explicit ray diagrams.]

Ans. (a) For a concave mirror, the focal length (f)is negative.

$\therefore f < 0$

When the object is placed on the left side of the mirror, the object distance (u)is negative.

$\therefore u < 0$

For image distance *v*, we can write the lens formula as:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u} \qquad \text{...(i)}$$

The object lies between *f* and 2*f*.

$\therefore 2f < u < f$ ($\because$ u and f are negative)

$$\frac{1}{2f}>\frac{1}{u}>\frac{1}{f}$$

$$-\frac{1}{2f}<-\frac{1}{u}<-\frac{1}{f}$$

$$\frac{1}{f}-\frac{1}{2f}<\frac{1}{f}-\frac{1}{u}<0 \quad \text{...(2)}$$

Using equation (1), we get:

$$\frac{1}{2f}<\frac{1}{v}<0$$

$\therefore \frac{1}{v}$ is negative, i.e., *v*is negative.

$$\frac{1}{2f}<\frac{1}{v}$$

$$2f>v$$

$$-v>-2f$$

Therefore, the image lies beyond 2*f*.

(b) For a convex mirror, the focal length (*f*) is positive.

$\therefore f > 0$

When the object is placed on the left side of the mirror, the object distance (*u*) is negative.

$\therefore u < 0$

For image distance *v*, we have the mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

Using equation (2), we can conclude that:

$$\frac{1}{v} < 0$$

$$v > 0$$

Thus, the image is formed on the back side of the mirror.

Hence, a convex mirror always produces a virtual image, regardless of the object distance.

(c) For a convex mirror, the focal length (*f*) is positive.

$\therefore f > 0$

When the object is placed on the left side of the mirror, the object distance (*u*) is negative,

$\therefore u < 0$

For image distance *v*, we have the mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

But we have u <0

$$\therefore \frac{1}{v} > \frac{1}{f}$$

$$v < f$$

Hence, the image formed is diminished and is located between the focus (*f*) and the pole.

(d) For a concave mirror, the focal length (*f*) is negative.

$$\therefore f < 0$$

When the object is placed on the left side of the mirror, the object distance (*u*) is negative.

$$\therefore u < 0$$

It is placed between the focus (*f*) and the pole.

$$\therefore f > u > 0$$

$$\frac{1}{f} < \frac{1}{u} < 0$$

$$\frac{1}{f} - \frac{1}{u} < 0$$

For image distance *v*, we have the mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\therefore \frac{1}{v} < 0$$

$$v > 0$$

The image is formed on the right side of the mirror. Hence, it is a virtual image.

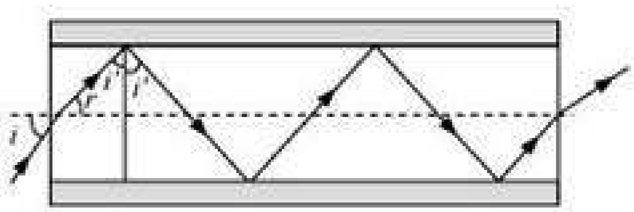
For u < 0 and v > 0, we can write:

$$\frac{1}{u} > \frac{1}{v}$$

$$v > u$$

---

15. (a) Figure 9.35 shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68.



The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.

(b) What is the answer if there is no outer covering of the pipe?

Ans.(a) Refractive index of the glass fibre, $\mu_1 = 1.68$

Refractive index of the outer covering of the pipe, μ_2 = 1.44

Angle of incidence = i

Angle of refraction = r

Angle of incidence at the interface = i'

The refractive index (μ) of the inner core - outer core interface is given as:

$$\mu = \frac{\mu_2}{\mu_1} = \frac{1}{\sin i'}$$

$$\sin i' = \frac{\mu_1}{\mu_2}$$

$$= \frac{1.44}{1.68} = 0.8571$$

$$\therefore i' = 59^0$$

For the critical angle, total internal reflection (TIR) takes place only when $i > i'$, i.e., $i > 59°$

Maximum angle of reflection, $r_{max} = 90^0 - i^{-1} = 90^0 - 59^0 - 31^0$

Let, i_{max} be the maximum angle of incidence.

The refractive index at the air - glass interface, $\mu_1 = 1.68$

We have the relation for the maximum angles of incidence and reflection as:

$$\mu_1 = \frac{\sin i_{max}}{\sin r_{max}}$$

$$\sin i_{max} = \mu_1 \sin r_{max}$$

$$= 1.68 \sin 31^0$$

$$= 1.68 \times 0.5150$$

$$= 0.8652$$

$$\therefore i_{max} = \sin^{-1} 0.8652 \approx 60^0$$

Thus, all the rays incident at angles lying in the range $0 < i < 60°$ will suffer total internal reflection.

(b) If the outer covering of the pipe is not present, then:

Refractive index of the outer pipe, μ_1 = Refractive index of air =1

For the angle of incidence $i = 90°$, we can write Snell's law at the air - pipe interface as:

$$\frac{\sin i}{\sin r} = \mu_2 = 1.68$$

$$\sin r = \frac{\sin 90^0}{1.68} = \frac{1}{1.68}$$

$$r = \sin^{-1}(0.5952)$$

$$= 36.5^0$$

$$\therefore i' = 90^0 - 36.5^0 = 53.5^0$$

Since i' > r, All incident rays will suffer total internal reflection

---

16.Answer the following questions:

(a) You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.

(b) A virtual image, we always say, cannot be caught on a screen.

Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?

(c) A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?

(d) Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?

(e) The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

Ans.(a) Yes

Plane and convex mirrors can produce real images as well. If the object is virtual, i.e., if the light rays converging at a point behind a plane mirror (or a convex mirror) are reflected to a point on a screen placed in front of the mirror, then a real image will be formed.

(b) No

A virtual image is formed when light rays diverge. The convex lens of the eye causes these divergent rays to converge at the retina. In this case, the virtual image serves as an object for the lens to produce a real image.

(c) The diver is in the water and the fisherman is on land (i.e., in air). Water is a denser medium than air. It is given that the diver is viewing the fisherman. This indicates that the light rays are travelling from a denser medium to a rarer medium. Hence, the refracted rays will move away from the normal. As a result, the fisherman will appear to be taller.

(d) Yes; Decrease

The apparent depth of a tank of water changes when viewed obliquely. This is because light bends on travelling from one medium to another. The apparent depth of the tank when viewed obliquely is less than the near-normal viewing.

(e) Yes

The refractive index of diamond (2.42) is more than that of ordinary glass (1.5). The critical angle for diamond is less than that for glass. A diamond cutter uses a large angle of incidence to ensure that the light entering the diamond is totally reflected from its faces. This is the reason for the sparkling effect of a diamond.

17.(a) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?

(b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.

Ans. Focal length of the convex lens, f_1 = 30 cm

Focal length of the concave lens, f_2 = - 20 cm

Distance between the two lenses, d= 8.0 cm

(a) When the parallel beam of light is incident on the convex lens first:

According to the lens formula, we have:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

Where,

u_1 = Object distance = ∞

v_1= Image distance

$$\frac{1}{v_1} = \frac{1}{30} - \frac{1}{\infty} = \frac{1}{30}$$

$$\therefore v_1 = 30\,cm$$

The image will act as a virtual object for the concave lens.

Applying lens formula to the concave lens, we have:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

Where,

u_2 = Object distance

= (30 - d) = 30 - 8 = 22 cm

v_2 = Image distance

$$\frac{1}{v_2} = \frac{1}{22} - \frac{2}{20} = \frac{10-11}{220} = \frac{-1}{220}$$

$$\therefore v_2 = -220\,cm$$

The parallel incident beam appears to diverge from a point that is $\left(220 - \frac{d}{2} = 220 - 4\right) 216\,cm$ from the centre of the combination of the two lenses.

(ii) When the parallel beam of light is incident, from the left, on the concave lens first:

According to the lens formula, we have:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2}$$

Where,

u_2 = Object distance = $-\infty$

v_2 = Image distance

$$\frac{1}{v_2} = \frac{1}{-20} + \frac{1}{-\infty} = -\frac{1}{20}$$

$$\therefore v_2 = -20\,cm$$

The image will act as a real object for the convex lens.

Applying lens formula to the convex lens, we have:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

Where,

u_1 = Object distance

= - (20 + *d*) = - (20 + 8) = - 28 cm

v_1 = Image distance

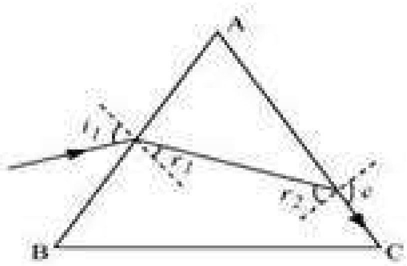
$$\frac{1}{v_1} = \frac{1}{30} + \frac{1}{-28} = \frac{14-15}{420} = \frac{-1}{420}$$

$$\therefore v_2 = -420\,cm$$

Hence, the parallel incident beam appear to diverge

18.At what angle should a ray of light be incident on the face of a prism of refracting angle $60°$ so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Ans. The incident, refracted, and emergent rays associated with a glass prism ABC are shown in the given figure.



Angle of prism, $\angle A = 60°$

Refractive index of the prism, μ= 1.524

i_1 = Incident angle

r_1 = Refracted angle

r_2 = Angle of incidence at the face AC

e= Emergent angle = $90°$

According to Snell's law, for face AC, we can have:

$$\frac{\sin e}{\sin r_2} = \mu$$

$$\sin r_2 = \frac{1}{\mu} \times \sin 90^0$$

$$= \frac{1}{1.524} = 0.6562$$

$$\therefore r_2 = \sin^{-1} 0.6562 \approx 41^0$$

It is clear from the figure that angle $A = r_1 + r_2$

$$\therefore r_1 = A - r_2 = 60 - 41 = 19^0$$

According to Snell's law, we have the relation:

$$\mu = \frac{\sin i_1}{\sin r_1}$$

$$\sin i_1 = \mu \sin r_1$$

$$= 1.524 \times \sin 19^0 = 0.496$$

$$\therefore i_1 = 29.75^0$$

Hence, the angle of incidence is $29.75°$.

19.For a normal eye, the far point is at infinity and the near point of distinct vision is about 25cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.

Ans. Least distance of distinct vision, *d*= 25 cm

Far point of a normal eye, $d' = \infty$

Converging power of the cornea, $P_c = 40D$

Least converging power of the eye-lens, $P_e = 20D$

To see the objects at infinity, the eye uses its least converging power.

Power of the eye-lens, $P = P_c + P_e$ = 40 + 20 = 60 D

Power of the eye-lens is given as:

$$P = \frac{1}{Focal\ length\ of\ the\ eye\ lens\ (f)}$$

$$f = \frac{1}{P}$$

$$= \frac{1}{60D}$$

$$= \frac{100}{60} = \frac{5}{3} cm$$

To focus an object at the near point, object distance (*u*)= - *d*= - 25 cm

Focal length of the eye-lens = Distance between the cornea and the retina

= Image distance

Hence, image distance, $v = \frac{5}{3} cm$

According to the lens formula, we can write:

$$\frac{1}{f'} = \frac{1}{v} - \frac{1}{u}$$

Where,

f' = Focal length

$$\frac{1}{f'} = \frac{3}{5} + \frac{1}{25} = \frac{15+1}{25} = \frac{16}{25} cm^{-1}$$

$$power, P' = \frac{1}{f'} \times 100$$

$$= \frac{16}{25} \times 100 = 64D$$

∴Power of the eye-lens = 64 - 40 = 24 D

Hence, the range of accommodation of the eye-lens is from 20 D to 24 D.

20.Answer the following questions:

(a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?

(b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?

(c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?

(d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

(e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?

Ans.(a) Though the image size is bigger than the object, the angular size of the image is equal to the angular size of the object. A magnifying glass helps one see the objects placed closer than the least distance of distinct vision (i.e., 25 cm). A closer object causes a larger angular size. A magnifying glass provides angular magnification. Without magnification, the object cannot be placed closer to the eye. With magnification, the object can be placed much closer to the eye.

(b) Yes, the angular magnification changes. When the distance between the eye and a magnifying glass is increased, the angular magnification decreases a little. This is because the angle subtended at the eye is slightly less than the angle subtended at the lens. Image distance does not have any effect on angular magnification.

(c) The focal length of a convex lens cannot be decreased by a greater amount. This is because making lenses having very small focal lengths is not easy. Spherical and chromatic aberrations are produced by a convex lens having a very small focal length.

(d) The angular magnification produced by the eyepiece of a compound microscope is

$$\left[\left(\frac{25}{f_0}\right)+1\right]$$

Where,

f_e = Focal length of the eyepiece

It can be inferred that if f_e is small, then angular magnification of the eyepiece will be large.

The angular magnification of the objective lens of a compound microscope is given as

$$\frac{1}{\left(|u_o| f_o\right)}$$

Where,

u_0 = Object distance for the objective lens

f_0 = Focal length of the objective

The magnification is large when u_0 > f_0. In the case of a microscope, the object is kept close to the objective lens. Hence, the object distance is very little. Since u_0 is small, f_0 will be even smaller. Therefore, f_0 and f_0 are both small in the given condition.

(e) When we place our eyes too close to the eyepiece of a compound microscope, we are unable to collect much refracted light. As a result, the field of view decreases substantially. Hence, the clarity of the image gets blurred.

The best position of the eye for viewing through a compound microscope is at the eye-ring attached to the eyepiece. The precise location of the eye depends on the separation between the objective lens and the eyepiece.

21.An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?

Ans. Focal length of the objective lens, f_0 = 1.25 cm

Focal length of the eyepiece, f_e = 5 cm

Least distance of distinct vision, *d*= 25 cm

Angular magnification of the compound microscope = 30X

Total magnifying power of the compound microscope, *m*= 30

The angular magnification of the eyepiece is given by the relation:

$$m_e = \left(1 + \frac{d}{f_e}\right)$$

$$= \left(1 + \frac{25}{5}\right) = 6$$

The angular magnification of the objective lens (*m*o) is related to *m*eas:

$m_e\, m_e = m$

$$m_0 = \frac{m}{m_e}$$

$$= \frac{30}{6} = 5$$

We also have the relation:

$$m_0 = \frac{\text{image distace for the objective lens } (v_0)}{\text{Object distance for the objective lens } (-u_0)}$$

$$5 = \frac{v_0}{-u_0}$$

$$\therefore v_0 = -5u_0$$

Applying the lens formula for the objective lens:

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

$$\frac{1}{1.25} = \frac{1}{-5u_0} - \frac{1}{u_0} = \frac{-6}{5u_0}$$

$$\therefore u_0 = \frac{-6}{5} \times 1.25 = -1.5\ cm$$

$$And\ v_0 = -5u_0$$

$$= -5 \times (-1.5) = 7.5cm$$

The object should be placed 1.5 cm away from the objective lens to obtain the desired magnification.

Applying the lens formula for the eyepiece:

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

Where,

v_e = Image distance for the eyepiece = - *d*= - 25 cm

u_e = Object distance for the eyepiece

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= \frac{-1}{25} - \frac{1}{5} = -\frac{6}{25}$$

$$\therefore u_e = -4.17\,cm$$

Separation between the objective lens and the eyepiece $= |u_e| + |v_0|$

=4.17+7.5

=11.67 cm

Therefore, the separation between the objective lens and the eyepiece should be 11.67 cm.

22.An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?

Ans. Focal length of the objective lens, f_0 = 1.25 cm

Focal length of the eyepiece, f_e = 5 cm

Least distance of distinct vision, *d*= 25 cm

Angular magnification of the compound microscope = 30X

Total magnifying power of the compound microscope, *m*= 30

The angular magnification of the eyepiece is given by the relation:

$$m_e \left(1 + \frac{d}{f_e}\right)$$

$$= \left(1 + \frac{25}{5}\right) = 6$$

The angular magnification of the objective lens (*m*o) is related to *m*eas:

$m_o\ m_e = mm$

$$m_o = \frac{m}{m_e}$$

$$= \frac{30}{6} = 5$$

$$m_o = \frac{image\ distance\ for\ the\ objective\ lens(v_o)}{Object\ distance\ for\ the\ objective\ lens(-u_o)}$$

$$5 = \frac{v_o}{-u_o}$$

$$\therefore v_o = -5u_o$$

Applying the lens formula for the objective lens:

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\frac{1}{1.25} = \frac{1}{-5u_o} - \frac{1}{u_o} = \frac{-6}{5u_o}$$

$$\therefore u_o = \frac{-6}{5} \times 1.25 = -1.5\ cm$$

And $v_o = -5u_o$

$$= -5 \times (-1.5) = 7.5cm$$

The object should be placed 1.5 cm away from the objective lens to obtain the desired magnification.

Applying the lens formula for the eyepiece:

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

Where,

v_e = Image distance for the eyepiece = - *d*= - 25 cm

u_e = Object distance for the eyepiece

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= \frac{-1}{25} - \frac{1}{5} = -\frac{6}{25}$$

$$\therefore u_e = -4.17\, cm$$

Separation between the objective lens and the eyepiece $= |u_e| + |v_o|$

=4.17+7.5

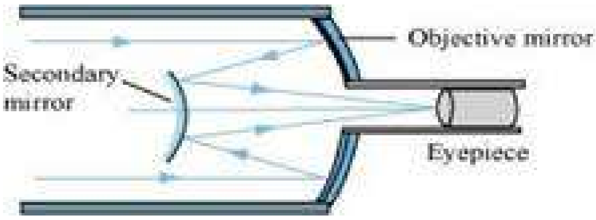
=11.67 cm

Therefore, the separation between the objective lens and the eyepiece should be 11.67 cm.

23.A Cassegrain telescope uses two mirrors as shown in Fig. 9.33. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?

Ans. The following figure shows a Cassegrain telescope consisting of a concave mirror and a convex mirror.

Distance between the objective mirror and the secondary mirror, *d*= 20 mm



Radius of curvature of the objective mirror, $R_1 = 220\ mm$

Hence, focal length of the objective mirror, $f_1 = \frac{R_1}{2} = 110\,mm$

Radius of curvature of the secondary mirror, *R*1 = 140 mm

Hence, focal length of the secondary mirror, $f_2 = \frac{R_2}{2} = \frac{140}{2} = 70mm$

The image of an object placed at infinity, formed by the objective mirror, will act as a virtual object for the secondary mirror.

Hence, the virtual object distance for the secondary mirror, $u = f_1 - d$

=110-20

=90 mm

Applying the mirror formula for the secondary mirror, we can calculate image distance (*v*)as:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_2}$$

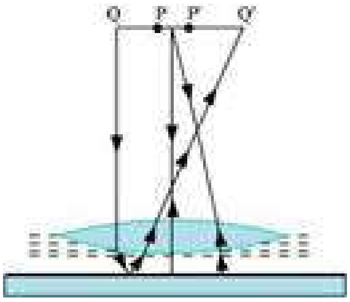
$$\frac{1}{v} = \frac{1}{f_2} - \frac{1}{u}$$

$$= \frac{1}{70} - \frac{1}{90} = \frac{9-7}{630} = \frac{2}{630}$$

$$\therefore v = \frac{630}{2} = 315mm$$

Hence, the final image will be formed 315 mm away from the secondary mirror.

24.Figure 9.37 shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?



Ans. Focal length of the convex lens, f_1 = 30 cm

The liquid acts as a mirror. Focal length of the liquid = f_2

Focal length of the system (convex lens + liquid), f = 45 cm

For a pair of optical systems placed in contact, the equivalent focal length is given as:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$$

$$= \frac{1}{45} - \frac{1}{30} = -\frac{1}{90}$$

$$\therefore f_2 = -90cm$$

Let the refractive index of the lens be μ_1 and the radius of curvature of one surface be *R*. Hence, the radius of curvature of the other surface is - *R*.

R can be obtained using the relation:

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\frac{1}{30} = (1.5 - 1)\left(\frac{2}{R}\right)$$

$$\therefore R = \frac{30}{0.5 \times 2} = 30cm$$

Let μ_2 be the refractive index of the liquid.

Radius of curvature of the liquid on the side of the plane mirror = ∞

Radius of curvature of the liquid on the side of the lens, *R*= - 30 cm

The value of μ_2 can be calculated using the relation:

$$\frac{1}{f_2} = (\mu_2 - 1)\left[\frac{1}{-R} - \frac{1}{\infty}\right]$$

$$\frac{-1}{90} = (\mu_2 - 1)\left[\frac{1}{+30} - 0\right]$$

$$\mu_2 - 1 = \frac{1}{3}$$

$$\therefore \mu_2 = \frac{4}{3} = 1.33$$

CBSE Class 12 physics

Practice Questions

Chapter 10

Wave Optics

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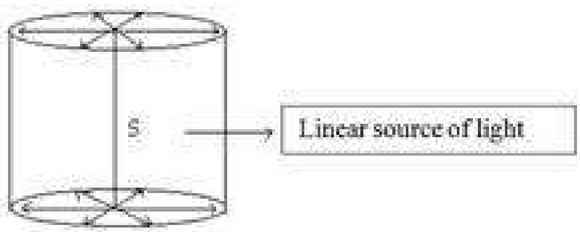
1 Mark Questions

1. Draw a diagram to show cylindrical wave front?

Ans.



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2. A light wave enters from air to glass. How will the following be affected:

(i) Energy of the wave

(ii) Frequency of the wave:

Ans. (1) A part of light is reflected back into the air. Thus energy of the wave will be lower in the glass.

(2) Frequency of the wave remains unchanged.

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3. What is the Brewster angle for air to glass transition? $(\mu g = 1.5)$

Ans. $\mu = \tan ip$

$1.5 = \tan ip$

$\Rightarrow ip = \tan^{-1}(1.5)$

---

4. What is the shape of the wave front when light is diverging from a point source?

Ans. Spherical

---

5. State the conditions that must be satisfied for two light sources to be coherent?

Ans. (1) They must emit waves continuously of same wavelengths.

(2) The phase difference between the waves must be zero or constant

---

6. In young's double slit experiment. The distance between the slits is halved, what change in the fringe width will take place?

Ans. $\beta = \frac{\lambda D}{d}$ $when\ d' = \frac{d}{2}$

$$\therefore \beta' = \frac{2\lambda D}{d}$$

$$\beta' = 2\beta$$

2 Mark Questions

1.Obtain an expression for the ratio of intensities at maxima and minima in an interference pattern.

Ans. Suppose $a_1 and\ a_2$ be the amplitudes and $I_1 and\ I_2$ the intensities of light waves which interfere each other

$$\text{Intensity } \alpha \text{ (Amplitude)}^2$$

$$\frac{I_1}{I_2} = \frac{a_1}{a_2}$$

After interference (applying superposition principle)

Amplitude at maxima = $a_1 + a_2$

Amplitude at minima = $a_1 - a_2$

$$\frac{\text{Imax}}{\text{Imin}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

$$\frac{\text{Imax}}{\text{Imin}} = \frac{\left(\frac{a_1}{a_2} + 1\right)^2}{\left(\frac{a_1}{a_2} - 1\right)^2} = \left(\frac{r+1}{r-1}\right)^2$$

$where\ \text{r} = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}$ = amplitude ratio of two waves.

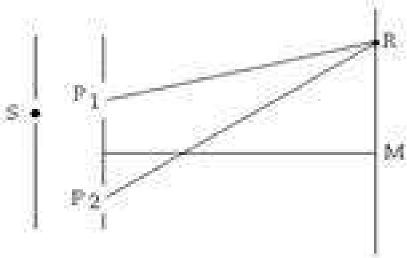
2.A slit S is illuminated by a monochromatic source of light to give two coherent sources $P_1 and\ P_2$. These given bright and dark bands on a screen. At a point R, on the screen, there is a dark fringe. What relation must exist between the lengths $P_1R\ and\ P_2R$?

Ans. There will be a dark fringe at point R When path difference

$$= P_2R - P_1R$$

.

Where λ is the wavelength of the light and n = 0, 1, 2, 3 -----------



3.In young's double slit experiment how is the fringe width change when

(a) Light of smaller frequency is used

(b) Distance between the slits is decreased?

Ans. $\beta = \frac{D\lambda}{d}$

If light of smaller frequency is of higher wavelength is used the fringe width will increase.

(b) If distance between the slits is decreased

i.e $\beta \alpha \frac{1}{d}$. Fringe width will increase.

4.Write two points of difference between interference and diffraction?

Ans.

S.	Interference	Diffraction
1	Interference occurs due to superposition of light coming from two coherent sources.	It is due to the superposition of the waves coming from different parts of the same wave front.
2	All bright fringes are of equal intensity	The intensity of bright fringes decreases with increasing distance from the central bright fringes.

5.Consider interference between two sources of intensities I and 4I. What will be the intensity at points where phase differences is $(1)\frac{\pi}{2}$ $(2)\pi$

Ans. $I = a^2 + b^2 + 2ab\cos\phi$

Where a and b are amplitudes of two coherent waves having phase difference of ϕ .

Here $a^2 = I$, $b^2 = 4I$

I = I + 4I + 2 $\sqrt{I}\sqrt{4I}\cos\phi$

I = 5I + 4I cos ϕ

(i) $When\phi = \frac{\pi}{2}$

$I = 5I + 4I\cos\frac{\pi}{2}$

I = 5I

(ii) Why $\phi = \pi$

I = 5I + 4I cos ϕ

I = 5I – 4I

I = I

6.Can white light produce interference? What is the nature?

Ans. White light produces interference but due to different colour present in white light interference pattern overlaps the central bright fringe for all the colours is at the position, so its colour is white. The white central bright fringe is surrounded by few coloured rings.

7.(a) The refractive index of glass is 1.5. What is the speed of light in glass? Speed of light in vacuum is 3.0×10^8 m s^{-1})

(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

Ans.(a) Refractive index of glass, μ = 1.5

Speed of light, c = 3×10^8 m/s

Speed of light in glass is given by the relation,

$$v = \frac{c}{\lambda}$$

$$\frac{3 \times 10^8}{1.5} = 2 \times 10^8 m/s$$

Hence, the speed of light in glass is 2×10^8 m/s.

(b) The speed of light in glass is not independent of the colour of light.

The refractive index of a violet component of white light is greater than the refractive index of a red component. Hence, the speed of violet light is less than the speed of red light in glass. Hence, violet light travels slower than red light in a glass prism.

8.What is the Brewster angle for air to glass transition? (Refractive index of glass = 1.5.)

Ans. Refractive index of glass, $\mu = 1.5$

Brewster angle = θ

Brewster angle is related to refractive index as:

$$\tan\theta = \mu$$

$$\theta = \tan^{-1}(1.5) = 56.31^0$$

Therefore, the Brewster angle for air to glass transition is $56.31°$.

9.Estimate the distance for which ray optics is good approximation for an aperture of 4 mm and wavelength 400 nm.

Ans. Fresnel's distance (*ZF*) is the distance for which the ray optics is a good approximation. It is given by the relation,

$$Z_F = \frac{a^2}{\lambda}$$

Where,

Aperture width, a = 4 mm = $4 \times 10^{-3}\ m$

Wavelength of light, λ = 400 nm = $400 \times 10^{-9}\ m$ $Z_F = \frac{(4 \times 10^{-3})^2}{400 \times 10^{-9}} = 40m$

Therefore, the distance for which the ray optics is a good approximation is 40 m.

10.Let us list some of the factors, which could possibly influence the speed of wave propagation:

(i) Nature of the source.

(ii) Direction of propagation.

(iii) Motion of the source and/or observer.

(iv) Wave length.

(v) Intensity of the wave. On which of these factors, if any, does

(a) The speed of light in vacuum,

(b) The speed of light in a medium (say, glass or water), depend?

Ans.(a) The speed of light in a vacuum i.e., $3 \times 10^8\ m/s$ (approximately) is a universal constant. It is not affected by the motion of the source, the observer, or both. Hence, the given factor does not affect the speed of light in a vacuum.

(b) Out of the listed factors, the speed of light in a medium depends on the wavelength of light in that medium.

---

11. For sound waves, the Doppler formula for frequency shift differs slightly between the two situations: (i) source at rest; observer moving, and (ii) source moving; observer at rest. The exact Doppler formulas for the case of light waves in vacuum are, however, strictly identical for these situations. Explain why this should be so. Would you expect the formulas to be strictly identical for the two situations in case of light travelling in a medium?

Ans. No Sound waves can propagate only through a medium. The two given situations are not scientifically identical because the motion of an observer relative to a medium is different in the two situations. Hence, the Doppler formulas for the two situations cannot be the same.

In case of light waves, sound can travel in a vacuum. In a vacuum, the above two cases are identical because the speed of light is independent of the motion of the observer and the motion of the source. When light travels in a medium, the above two cases are not identical because the speed of light depends on the wavelength of the medium.

---

12. In double-slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is $0.1°$. What is the spacing between the two slits?

Ans. Wavelength of light used, λ = 6000 nm = $600 \times 10^{-9}\ m$

Angular width of fringe, $\theta = 0.1^0 = 0.1 \times \frac{\lambda}{180} = \frac{3.14}{1800} rad$

Angular width of a fringe is related to slit spacing (*d*) as:

$$\theta = \frac{\lambda}{d}$$

$$d = \frac{\lambda}{\theta}$$

$$= \frac{600 \times 10^{-9}}{\frac{3.14}{1800}} = 3.44 \times 10^{-4}\, m$$

Therefore, the spacing between the slits is $3.44 \times 10^{-4}\, m$.

13.In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of $n\lambda / a$. Justify this by suitably dividing the slit to bring out the cancellation.

Ans. Consider that a single slit of width *d* is divided into *n* smaller slits.

$\therefore$ Width of each slit, $d' = \frac{d}{n}$

Angle of diffraction is given by the relation,

$$\theta = \frac{\frac{d}{d}\lambda}{d} = \frac{\lambda}{d}$$

Now, each of these infinitesimally small slit sends zero intensity in direction θ. Hence, the combination of these slits will give zero intensity.

14.Answer the following questions:

(a) When a low flying aircraft passes overhead, we sometimes notice

a slight shaking of the picture on our TV screen. Suggest a possible explanation.

(b) As you have learnt in the text, the principle of linear superposition of wave displacement is basic to understanding intensity distributions in diffraction and interference patterns. What is the justification of this principle?

Ans.(a) Weak radar signals sent by a low flying aircraft can interfere with the TV signals received by the antenna. As a result, the TV signals may get distorted. Hence, when a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen.

(b) The principle of linear superposition of wave displacement is essential to our understanding of intensity distributions and interference patterns. This is because superposition follows from the linear character of a differential equation that governs wave motion. If y_1 and y_2 are the solutions of the second order wave equation, then any linear combination of y_1 and y_2 will also be the solution of the wave equation.

3 Mark Questions

1. State Brewster law? Using this law prove that, at the polarizing angle of incidence, the reflected and transmitted rays are perpendicular to each other?

Ans. according to Brewster law the longest of the angle of polarization for transparent medium is equal to the refractive index of the medium.

$i.e\ \mu = \tan i_p$

Proof. Using Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

$$When\ \mathrm{i} = \mathrm{i}_p \quad \mu = \frac{\sin i_p}{\sin r_p}$$ ---------(1)

$$Also = \tan i_p = \frac{\sin i_p}{\sin i_p}$$ -------(2)

$$from\ (1)\,\&\,(2)$$

$$\frac{\sin i_p}{\sin r_p} = \frac{\sin i_p}{\cos i_p}$$

$$\sin r_p = \cos i_p$$

$$\sin r_p = \sin(90^\circ - i_p)$$

$$\Rightarrow rp + i_p = 90^\circ$$

2. In a single slit experiment, how is the angular width of central bright fringe maximum changed when

1) The slit width increased

2) The distance between the slit and the screen is increased.

3) Light of smaller wavelength is used.

Ans. In single slit diffraction

$$\beta = \frac{2D\lambda}{d}$$

(a) When slit width'd' is increased. β decreases

(b) When 'D' is increased, width of central bright fringe will become maximum i.e

increase.

(c) When light of smaller wavelength is used, the width of central bright maximum

decrease.

---

3. In a young's double slit experiment, the slit are repeated at 0.24mm. The screen is 1.2m away from the slits. The fringe width is 0.3cm calculate the wavelength of light used in the experiment?

Ans. $\beta = 0.3cm = 3.0 \times 10^{-3}\, m$

$D = 1.2m$

$d = 0.24mm = 2.4 \times 10^{-4}\, m$

$$\beta = \frac{D\lambda}{d}$$

$$\Rightarrow \lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{3.0\times10^{-3}\times2.4\times10^{-4}}{1.2}$$

$$\lambda = 6.0\times10^{-7}\,m$$

---

4. Two coherent sources whose intensity ratio is 81:1 produce interference fringes. Calculate the ratio of intensity of maxima and minima in the interference pattern?

Ans $\frac{I_1}{I_2} = \frac{81}{1}$

Intensity α $(\text{Amplitude})^2$

$$\frac{a_1}{a_2} = \sqrt{\frac{81}{1}} = \frac{9}{1} = r$$

$$\frac{\mathrm{I_{max}}}{\mathrm{I_{min}}} = \frac{(r+1)^2}{(r-1)^2} = \left(\frac{9+1}{9-1}\right)^2 = \left(\frac{10}{8}\right)^2$$

$$\frac{\mathrm{I_{max}}}{\mathrm{I_{min}}} = \frac{100}{64}$$

$$\frac{\mathrm{I_{max}}}{\mathrm{I_{min}}} = \frac{25}{16}$$

$$I_{max} : I_{min} = 25:16$$

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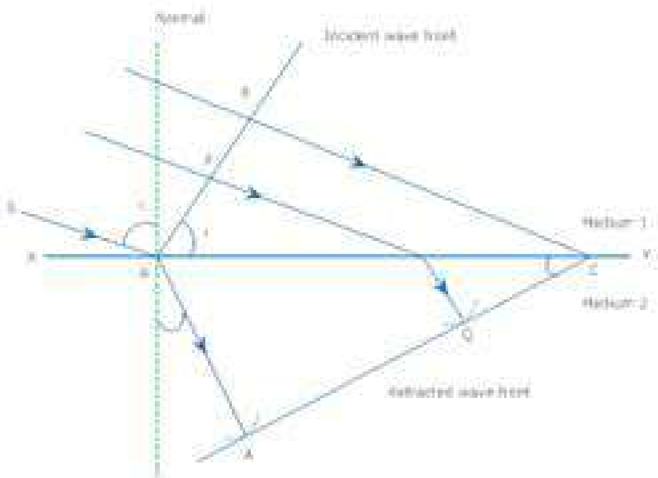
5. Using Huygens's principle deduce the laws of refraction?

Ans. According to Huygens's theory each point on AB given rise to new wave fronts give taken by the wavelets to reach from



$P \text{ to } Q$

$t = \frac{PQ}{\vartheta^1} + \frac{OQ}{\vartheta^2}$ ------(1)

$In\ PAO$

$\sin i = \frac{PO}{AO}$

$PO = AO \sin i$

$\sin r = \frac{OQ}{OC}$

$OQ = OC \sin r$

Substituting in equation (1)

$$t = \frac{AO \sin i}{\upsilon_1} + \frac{(AC - AO)\sin r}{\upsilon_2}$$

$$t = AO\left(\frac{\sin i}{\upsilon_1} - \frac{\sin r}{\upsilon_2}\right) + \frac{AC \sin r}{\upsilon_2}$$

Since time is independent of equation

$\therefore$ Term containing AO must be zero.

$$i.e\ \frac{\sin i}{v_1} - \frac{\sin r}{v_2} = O \Rightarrow \frac{\sin i}{v_1} = \frac{\sin r}{v_2}$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \mu \left(\because \mu = \frac{C}{v} \right)$$

Hence proved Snell's law

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6. A young's double slit experiment using light of wavelength 400 nm, interference fringes of width to 600nm, and the separation between the slits is halved. If one wants the observed fringe width on the screen to be the same in the two cases,find the ratio of the distance between the screen and the plane of the interfering

Ans. Let D_1 be the distance between the screen and the sources, when light of wavelength 400nm is used.

$$\beta = \frac{D\alpha}{d}$$

$$X = \frac{D_1 \times 400 \times 10^{-9}}{d}$$

In order to obtain the same fringe width

$$\frac{D_2 \times 600 \times 10^{-9}}{d} = \times \text{-------(2)}$$

From equation (1) and (2)

$$\frac{D_1}{D_2} = 1.5$$

$$\frac{D_1}{D_2} = \frac{600 \times 10^{-9}}{400 \times 10^{-9}}$$

sources in the two arrangements.

7. In young's double slit experiment while using a source of light of wavelength $5000A^o$, the fringe width obtained is 0.6cm. If the distance between the slit and the screen is reduced to half, calculate the new fringe width?

Ans. $\lambda = 5000A^o = 5\times10^{-7}\,m$

$$\beta = 0.6cm = 0.6\times10^{-2}m$$

$$\beta = \frac{\lambda D}{d}$$

$$\frac{\beta}{\lambda} = \frac{D}{d} \Rightarrow \frac{D}{d} = \frac{0.6\times10^{-2}}{5\times10^{-7}}$$

$\frac{D}{d} = 1.2\times10^{4}$ --------(1)

$$\text{New Distance } D' = \frac{D}{d}$$

New fringe width $\beta' = \frac{\lambda D'}{d} = \frac{\lambda D'}{2d}$

$$\beta' = \frac{5\times10^{-7}\times1.2\times10^{4}}{2}$$

$$\beta' = 3\times10^{-3}m$$

8. What is polarization of light? What type of waves show the property of polarization? Name any two methods to produce plane polarized light

Ans. The phenomenon of restricting the vibrations of a light vector in a particular direction in a plane perpendicular to the direction of propagation of light is called polarisation of light. Transverse waves show the property of polarisaiton.

Two methods to produce plane polarised light

(1)Polarisation by Reflection

(2) Polarization by scattering

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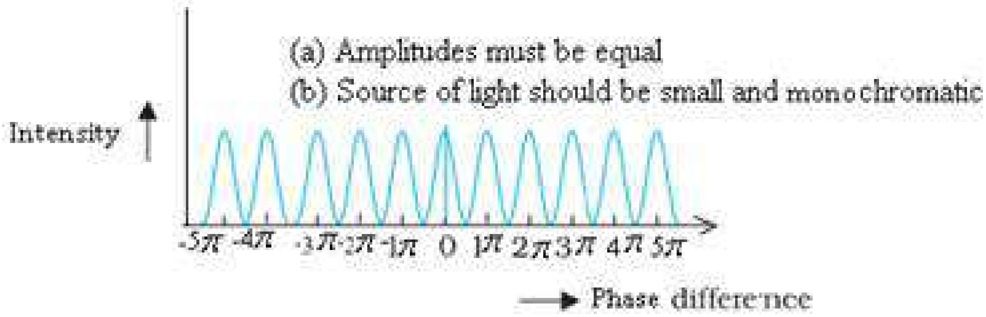
9. Draw the curve depicting, variation of intensity in the interference pattern in young's double slit experiment. State conditions for obtaining sustained interference of light?

Ans.



Conditions for sustained interference of light

(1) Two sources must be coherent sources of light.

(2)Two sources should exist light waves continuously. Intensity monochromatic

---

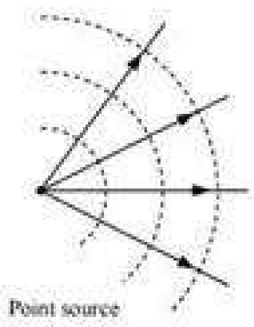
10. What is the shape of the wave front in each of the following cases:

(a) Light diverging from a point source.

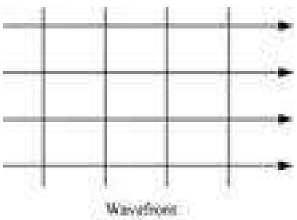
(b) Light emerging out of a convex lens when a point source is placed at its focus.

(c) The portion of the wave front of light from a distant star intercepted by the Earth.

Ans. (a) The shape of the wave front in case of a light diverging from a point source is spherical. The wavefront emanating from a point source is shown in the given figure.



(b) The shape of the wavefront in case of a light emerging out of a convex lens when a point source is placed at its focus is a parallel grid. This is shown in the given figure.



(c) The portion of the wavefront of light from a distant star intercepted by the Earth is a plane.

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11. In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.

Ans. Distance between the slits, d = 0.28 mm = 0.28×10^{-3} m

Distance between the slits and the screen, D = 1.4 m

Distance between the central fringe and the fourth (n = 4) fringe,

u = 1.2 cm = 1.2×10^{-2} m

In case of a constructive interference, we have the relation for the distance between the two fringes as:

$$u = n\lambda \frac{D}{d}$$

Where,

n = Order of fringes = 4

λ = Wavelength of light used

$$\therefore \lambda = \frac{ud}{nD}$$

$$= \frac{1.2 \times 10^{-2} \times 0.28 \times 10^{-3}}{4 \times 1.4}$$

$$= 6 \times 10^{-7}$$

$$= 600nm$$

Hence, the wavelength of the light is 600 nm.

---

12. In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3.

Ans. Distance of the screen from the slits, D = 1 m

Wavelength of light used, $\lambda_1 = 600nm$

Angular width of the fringe in $air, \theta_1 = 0.2^0$

Angular width of the fringe in water = θ_2

Refractive index of water, $\mu = \frac{4}{3}$

Refractive index is related to angular width as:

$$\mu = \frac{\theta_1}{\theta_2}$$

$$\theta_2 = \frac{3}{4}\theta_1$$

$$\frac{3}{4} \times 0.2 = 0.15$$

Therefore, the angular width of the fringe in water will reduce to $0.15°$.

---

13. Light of wavelength 5000 $\overset{0}{\mathrm{A}}$ falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

Ans. Wavelength of incident light, λ = 5000 $\overset{0}{\mathrm{A}}$ ¦ = 5000×10^{-10} m

Speed of light, $c = 3 \times 10^8\ m$

Frequency of incident light is given by the relation,

$$V = \frac{c}{\lambda}$$

$$= \frac{3 \times 10^8}{5000 \times 10^{-10}} = 6 \times 10^{14} Hz$$

The wavelength and frequency of incident light is the same as that of reflected ray. Hence, the wavelength of reflected light is 5000 $\overset{0}{\mathrm{A}}$ ¦ and its frequency is $6 \times 10^{14}\ Hz$.

When reflected ray is normal to incident ray, the sum of the angle of incidence, $\angle i$ and angle of reflection, $\angle r$ is $90°$.

According to the law of reflection, the angle of incidence is always equal to the angle of reflection. Hence, we can write the sum as:

$$\angle i + \angle r = 90$$

$\angle i + \angle i = 90$

$$\angle i = \frac{90}{2} = 45^0$$

Therefore, the angle of incidence for the given condition is $45°$.

---

14. The 6563 $\overset{0}{\mathrm{A}}$ H_0 line emitted by hydrogen in a star is found to be red shifted by 15 $\overset{0}{\mathrm{A}}$ ¦. Estimate the speed with which the star is receding from the Earth.

Ans. Wavelength of H_0 line emitted by hydrogen,

λ = 6563 $\overset{0}{\mathrm{A}}$ ¦

= $6563 \times 10^{-10}\ m.$

Star's red-shift, $\left(\lambda' - \lambda\right) = 15\overset{0}{A} = 15 \times 10^{-10} m$

Speed of light, $c = 3 \times 10^8\ m/s$

Let the velocity of the star receding away from the Earth be *v*.

The red shift is related with velocity as:

$$\lambda' - \lambda = \frac{v}{c}\lambda$$

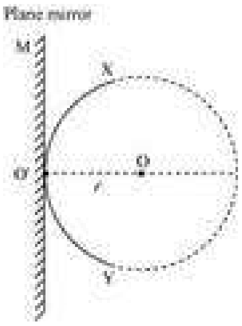
$$v = \frac{c}{\lambda} \times \left(\lambda' - \lambda\right)$$

$$= \frac{3 \times 10^8 \times 15 \times 10^{-10}}{6563 \times 10^{-10}} = 6.87 \times 10^5\ m/s$$

Therefore, the speed with which the star is receding away from the Earth is $6.87 \times 10^5\ m/s.$

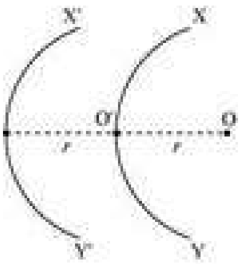
15. You have learnt in the text how Huygens' principle leads to the laws of reflection and refraction. Use the same principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the object distance from the mirror.

Ans. Let an object at O be placed in front of a plane mirror MO' at a distance *r* (as shown in the given figure).



A circle is drawn from the centre (O) such that it just touches the plane mirror at point O'. According to Huygens' Principle, XY is the wavefront of incident light.

If the mirror is absent, then a similar wavefront X'Y' (as XY) would form behind O' at distance *r* (as shown in the given figure).



$X'Y'$ can be considered as a virtual reflected ray for the plane mirror. Hence, a point object placed in front of the plane mirror produces a virtual image whose distance from the mirror is equal to the object distance (*r*).

16. A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit.

Ans. Wavelength of light beam, λ = 500 nm = $500 \times 10^{-9}\ m$ Distance of the screen from the slit, *D* = 1 m

For first minima, *n* = 1

Distance between the slits = *d*

Distance of the first minimum from the centre of the screen can be obtained as:

x = 2.5 mm = $2.5 \times 10^{-3}\ m$

It is related to the order of minima as:

$$n\lambda = x\frac{d}{D}$$

$$d = \frac{n\lambda D}{x}$$

$$\frac{1\times 500\times 10^{-9}\times 1}{2.5\times 10^{-3}} = 2\times 10^{-4}\, m = 0.2mm$$

Therefore, the width of the slits is 0.2 mm.

5 Mark Questions

1.(a) State Huygens's principle for constructing wavefronts?

(b) Using Huygens's principle deduce the laws of reflection of light?

(c) What changes in diffraction pattern of a single slit will you observe. when the monochromatic source of light is replaced by a source of white light?

Ans.(a) According to Huygens's principle

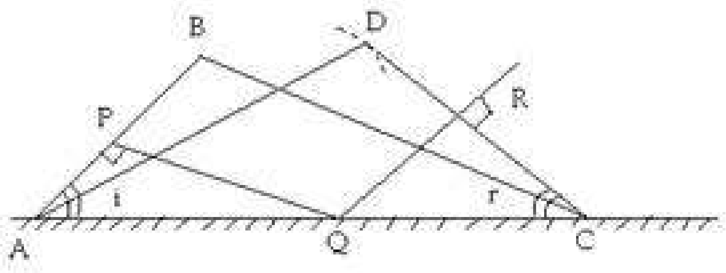
(1) Each source of light spreads waves in all directions.

(2) Each point on the wavefront give rise to new disturbance which produces secondary wavelets which travels with the speed of light.

(3) Only forward envelope which encloses the tangent gives the new position of wavefront.

(4) Rays are always perpendicular is the wavefront.

(b) A plane wave front AB incident at A hence every point on AB give rise to new waves. Time taken by the ray to reach from P to R



$$t = \frac{PQ}{v} + \frac{QR}{v} ------(1)$$

$$In\ \Delta PAQ \sin i = \frac{PQ}{AQ}$$

$PQ = AQ \sin i$

$In\ \Delta RCQ \sin r = \dfrac{QR}{QC}$

$QR = QC \sin r$

$Substituting\ in\ equation$ (1)

$$t = \frac{AQ \sin i}{v} + \frac{QC \sin r}{v}$$

$$t = \frac{AQ \sin i}{v} + \frac{\left(AC - AQ\right) \sin r}{v}$$

$$t = \frac{AQ \sin i}{v} + \frac{QC \sin r}{v} - \frac{AQ \sin r}{v}$$

$$\frac{AQ\left(\sin i - \sin r\right)}{v} + \frac{AC \sin r}{v}$$

Since all the secondary wavelets takes the same time to go form the incident wavefront to the reflected wavefront so it must be independent of Q

i.e sin i– sin r = O

sin i = sin r

or i = r $\rightarrow$ law of Reflection of light

(c) (1) The diffracted light consists of different colours.

(2) It results in overlapping of different colours.

2.(a) Coloured spectrum is seen, when we look through a muslin cloth. Why?

(b) What changes in diffraction pattern of a single slit will you observe. when the monochromatic source of light is replaced by a source of white light?

Ans. (a) Muslin cloth consist of very fine threads which acts as fine slits and when light pass through it, light gets diffracted giving rise to a coloured spectrum.

(b) (i) Diffracted lights consist of different colours.

(ii) It results in overlapping of different colours.

3.A slit of width 'a' is illuminated by light of wavelength $6000\,A^{o}$. For what value of 'a' will the :-

(i) First maximum fall at an angle of diffraction of 30^{o} ?

(ii) First minimum fall at an angle of diffraction 30^{o} ?

Ans. $\lambda = 6000A^{o} = 6000\times10^{-10}\text{m}$

θ 1=30°, m=1

(1) For first maximum

$$\sin Q_m = \frac{\left(m+\frac{1}{2}\right)\lambda}{a}$$

$$\sin Q_1 = \frac{3\lambda}{2a}$$

$$or\ \text{a} = \frac{3\lambda}{2\sin\theta_1} = \frac{3\times6\times10^{-7}}{2\times\sin30^{o}}$$

(2) For first minimum

$$\sin Q_m = \frac{m\lambda}{a}$$

$$or \sin Q_1 = \frac{\lambda}{a}$$

$$a = \frac{\lambda}{\sin \theta_1}$$

$$a = \frac{6 \times 10^{-7}}{\sin 30^\circ}$$

$$A = 1.2 \times 10^{-6} m$$

4.(a) Derive all expression for the fringe width in young's double slit experiment?

(b) If the two slits in young's double slit experiment have width ratio 4:1, deduce the ratio of intensity of maxima and minima in the interference pattern?

Ans. Path difference between

$S_1P \; and \; S_2P$

$\Delta x = S_2P - S_1P$ --------(A)

$In \; \Delta S_2BP$

$$(S_2P) = \left[(S^2B)^2 + (PB^2)\right]^{\frac{1}{2}}$$

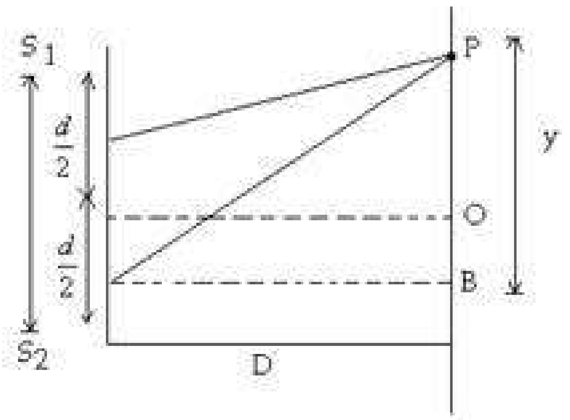
$$S_2P = D\left[1 + \frac{\left(y + \frac{d}{2}\right)}{D^2}\right]^{\frac{1}{2}}$$ -------(1)

Using Binomial theorem expand equation. (1) and neglect higher terms

$$S_2P = D + \frac{\left(y + \frac{d}{2}\right)^2}{2D}$$

Similarity $S_1P = D + \frac{\left(y - \frac{d}{2}\right)^2}{2D}$ ------(2)

Substituting equation (1) & (2) in equation (A)



$$\Delta x = \frac{\left(y + \frac{d}{2}\right)^2 - \left(y - \frac{d}{2}\right)^2}{2D}$$

$$\Delta x = \frac{y^2 + \frac{d^2}{4} + yd - y^2 - \frac{d^2}{4} + yd}{2D}$$

$$\Delta x = \frac{2yd}{2D}$$

$$\Delta x = \frac{yd}{D}$$

For bright fringes

Path difference = $x\lambda$

$$x\lambda = \frac{yd}{D}$$

$$i.e\ y = \frac{x\lambda D}{d}$$

$$\text{form} = 1 \quad y_1 = \frac{\lambda D}{d}$$

$$n = 2 \quad y_2 = \frac{\lambda D}{d}$$

For fringe width

$$\beta = y_2 - y_1$$

$$\beta = \frac{\lambda d}{d}$$

(b) $\frac{a_1^2}{a_2^2} = \frac{w_1}{w_2} = \frac{4}{1}$

$$\frac{a_1}{a_2} = \frac{2}{1}$$

$$or\ a_1 = 2a_2$$

Using $\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$

$$\frac{I_{max}}{I_{min}} = \frac{(2a_1 + a_2)^2}{(2a_1 - a_2)^2} = \left(\frac{3a_2}{a_2}\right)^2$$

$$\frac{I_{max}}{I_{min}} = \frac{9}{1}$$

5.Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected, and (b) refracted light? Refractive index of water is 1.33.

Ans. Wavelength of incident monochromatic light,

λ = 589 nm = $589 \times 10^{-9}\ m$

Speed of light in air, c = 3 × 108 m/s

Refractive index of water, μ = 1.33

(a) The ray will reflect back in the same medium as that of incident ray. Hence, the wavelength, speed, and frequency of the reflected ray will be the same as that of the incident ray.

Frequency of light is given by the relation,

$$V = \frac{c}{\lambda}$$

$$= \frac{3\times10^{8}}{5000\times10^{-10}} = 6\times10^{14}\,Hz$$

$$\angle i + \angle r = 90^{0}$$

$$\angle i + \angle i = 90^{0}$$

$$Z_F = \frac{\left(4\times10^{-3}\right)^2}{400\times10^{-9}} = 40m$$

$$\left(\lambda' - \lambda\right) = 15\overset{0}{A} = 15\times10^{-10}\,m$$

$$\lambda' - \lambda = \frac{v}{c}\lambda$$

$$v = \frac{c}{\lambda}\times\left(\lambda' - \lambda\right)$$

$$\frac{v}{c} = \frac{\sin i}{\sin r} = \mu$$

$$\theta = 0.1^0 = 0.1 \times \frac{\lambda}{180} = \frac{3.14}{1800} rad$$

$$d = \frac{\lambda}{\theta}$$

$$= \frac{600 \times 10^{-9}}{\frac{3.14}{1800}} = 3.44 \times 10^{-4} m$$

$$\therefore \lambda = \frac{a^2}{Z_P}$$

$$n\lambda = x\frac{d}{D}$$

$$d = \frac{n\lambda D}{x}$$

$$\theta = \frac{\frac{d}{d}\lambda}{d} = \frac{\lambda}{d}$$

$$= \frac{3 \times 10^8}{589 \times 10^{-9}}$$

$$= 5.09 \times 10^{14} Hz$$

Hence, the speed, frequency, and wavelength of the reflected light are 3×10^8 m/s, 5.09×10^{14} Hz, and 589 nm respectively.

(b) Frequency of light does not depend on the property of the medium in which it is travelling. Hence, the frequency of the refracted ray in water will be equal to the frequency of the incident or reflected light in air.

$\therefore$ Refracted frequency, ν = 5.09×10^{14} Hz

Speed of light in water is related to the refractive index of water as:

$$V = \frac{c}{\mu}$$

$$V = \frac{3 \times 10^8}{1.33} = 2.26 \times 10^8 \, m/s$$

Wavelength of light in water is given by the relation,

$$\lambda = \frac{V}{\nu}$$

$$= \frac{2.26 \times 10^8}{5.09 \times 10^{14}}$$

$$= 444.007 \times 10^{-9} \, m$$

$$= 444.01 nm$$

Hence, the speed, frequency, and wavelength of refracted light are $2.26 \times 10^8 \; m/s$, $444.01 nm$, and $5.09 \times 10^{14} \; Hz$ respectively.

6.In Young's double-slit experiment using monochromatic light of wavelength λ , the intensity of light at a point on the screen where path difference is λ , is *K* units. What is the intensity of light at a point where path difference is λ /3?

Ans. Let I_1 and I_2 be the intensity of the two light waves. Their resultant intensities can be obtained as:

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

Where,

ϕ = Phase difference between the two waves

For monochromatic light waves,

$$I_1 = I_2$$

$$\therefore I' = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

$$= 2I_1 + 2I_1 \cos\phi$$

Phase difference = $\frac{2\pi}{\lambda} \times path\ difference$

Since path difference = λ ,

Phase difference, $\phi = 2\pi$

$$\therefore I' = 2I_1 + 2I_1 = 4I_1$$

Given,

I' = *K*

$$\therefore I' = \frac{K}{4} \quad \dots(1)$$

When path difference $= \frac{\lambda}{3}$,

Phase difference, $\phi = \frac{2\pi}{3}$

Hence, resultant intensity, $I_R' = I_1 + I_1 + 2\sqrt{I_1 I_1} \cos\frac{2\pi}{3}$

$$= 2I_1 + 2I_1\left(-\frac{1}{2}\right) = I_1$$

Using equation (1), we can write:

$$I_R = I_1 = \frac{K}{4}$$

Hence, the intensity of light at a point where the path difference is $\frac{\lambda}{3}$ is $\frac{K}{4}$ units.

---

7. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide? $\lambda_1 = 600nm$

Ans. Wavelength of the light beam, $\therefore x = 3\times 650\frac{D}{d} = 1950\left(\frac{D}{d}\right)nm$

$$n\lambda_2 = (n-1)\lambda_1$$

$$\lambda_1 = 650nm$$

$$\therefore n = 5$$

Wavelength of another light beam, $\lambda_2 = 520nm$

Distance of the slits from the screen = D

Distance between the two slits = d

(a) Distance of the nth bright fringe on the screen from the central maximum is given by the relation,

$$x = n\lambda_1\left(\frac{D}{d}\right)$$

For third bright fringe. N=3

$$\therefore x = 3 \times 650 \frac{D}{d} = 1950 \left(\frac{D}{d} \right) nm$$

(b) Let the *n*th bright fringe due to wavelength λ_2 and (*n* - 1)th bright fringe due to wavelength λ_1 coincide on the screen. We can equate the conditions for bright fringes as:

$$n\lambda_2 = (n-1)\lambda 1$$

520n=650n-650

650=130n

$$\therefore n = 5$$

Hence, the least distance from the central maximum can be obtained by the relation:

$$x = \lambda_2 \frac{D}{d}$$

$$= 5 \times 520 \frac{D}{d} = 260 \frac{D}{d} nm$$

Note: The value of *d* and *D* are not given in the question.

8.Explain how Corpuscular theory predicts the speed of light in a medium, say, water, to be greater than the speed of light in vacuum. Is the prediction confirmed by experimental determination of the speed of light in water? If not, which alternative picture of light is consistent with experiment?

Ans. No; Wave theory

Newton's corpuscular theory of light states that when light corpuscles strike the interface of two media from a rarer (air) to a denser (water) medium, the particles experience forces of attraction normal to the surface. Hence, the normal component of velocity increases while the component along the surface remains unchanged.

Hence, we can write the expression:

C sin i=v sin r

C sin i=v sin r

Where,

i = Angle of incidence

r = Angle of reflection

c = Velocity of light in air

v = Velocity of light in water

We have the relation for relative refractive index of water with respect to air as:

$$\mu = \frac{v}{c}$$

Hence, equation (i) reduces to

$$\frac{v}{c} = \frac{\sin i}{\sin r} = \mu$$

But, $\mu > 1$

Hence, it can be inferred from equation (ii) that $v > c$. This is not possible since this prediction is opposite to the experimental results of $c > v$.

The wave picture of light is consistent with the experimental results.

9.Answer the following questions: (a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band?

(b) In what way is diffraction from each slit related to the interference pattern in a double-slit experiment?

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a

bright spot is seen at the centre of the shadow of the obstacle. Explain why?

(d) Two students are separated by a 7 m partition wall in a room 10 m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily.

(e) Ray optics is based on the assumption that light travels in a straight line. Diffraction effects (observed when light propagates through small apertures/slits or around small obstacles) disprove this assumption. Yet the ray optics assumption is so commonly used in understanding location and several other properties of images in optical instruments. What is the justification?

Ans.(a) In a single slit diffraction experiment, if the width of the slit is made double the original width, then the size of the central diffraction band reduces to half and the intensity of the central diffraction band increases up to four times.

(b) The interference pattern in a double-slit experiment is modulated by diffraction from each slit. The pattern is the result of the interference of the diffracted wave from each slit.

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. This is because light waves are diffracted from the edge of the circular obstacle, which interferes constructively at the centre of the shadow. This constructive interference produces a bright spot.

(d) Bending of waves by obstacles by a large angle is possible when the size of the obstacle is comparable to the wavelength of the waves.

On the one hand, the wavelength of the light waves is too small in comparison to the size of the obstacle. Thus, the diffraction angle will be very small. Hence, the students are unable to see each other. On the other hand, the size of the wall is comparable to the wavelength of the sound waves. Thus, the bending of the waves takes place at a large angle. Hence, the students are able to hear each other.

(e) The justification is that in ordinary optical instruments, the size of the aperture involved is much larger than the wavelength of the light used.

10.Two towers on top of two hills are 40 km apart. The line joining them passes 50 m above a hill halfway between the towers. What is the longest wavelength of radio waves, which can be sent between the towers without appreciable diffraction effects?

Ans. Distance between the towers, *d* = 40 km

Height of the line joining the hills, *d* = 50 m.

Thus, the radial spread of the radio waves should not exceed 50 km.

Since the hill is located halfway between the towers, Fresnel's distance can be obtained as:

Z_P = 20 km = 2×10^4 m

Aperture can be taken as:

a = *d* = 50 m

Fresnel's distance is given by the relation,

$$Z_P = \frac{a^2}{\lambda}$$

Where,

λ = Wavelength of radio waves

$$\therefore \lambda = \frac{a^2}{Z_P}$$

$$= \frac{(50)^2}{2 \times 10^4} = 1250 \times 10^{-4} = .1250m = 12.5cm$$

Therefore, the wavelength of the radio waves is 12.5 cm.

CBSE Class 12 physics

Practice Questions

Chapter 11

Dual Nature of Radiation and Matter

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1 Mark Questions

1.Calculate the energy associated in eV with a photon of wavelength $4000 A^{o}$?

Ans. $E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}}$

$E = 4.95\times10^{-19} J$

$E = \frac{4.95\times10^{-19} J}{1.6\times10^{-19}} eV$

E = 3.09 eV

2.Mention one physical process for the release of electron from the surface of a metal?

Ans. Photoelectric emission.

3.The maximum kinetic energy of photoelectron is 2.8 eV. What is the value of stopping potential?

Ans. $\frac{1}{2}mv^2 = eVo = 2.8eV$

$\Rightarrow Vo = 2.8V$

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4. Calculate the threshold frequency of photon for photoelectric emission from a metal of work function 0.1eV?

Ans. $\phi_o = hv_o$

$$v_o = \frac{\phi_o}{h} = \frac{0.1eV}{6.6\times10^{-34} Js}$$

$$v_o = \frac{0.1\times1.6\times10^{-19} J}{6.6\times10^{-34} Js}$$

$$v_o = 2.4\times10^{-13} \Delta^{-1}$$

5.Ultraviolet light is incident on two photosensitive materials having work function ϕ_1 and ϕ_2 (ϕ_1 > ϕ_2). In which of the case will K.E. of emitted electrons be greater? Why?

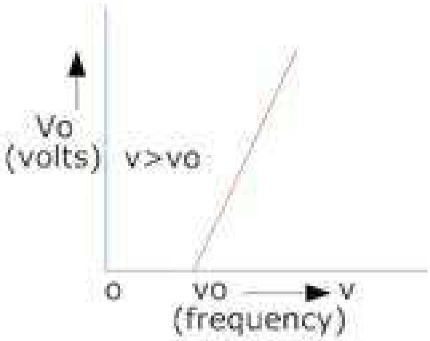
Ans. $hv = \phi_o + K.E.$

If ϕ_1 > ϕ_2 thus K.E. will be more for second surface whose work function is less.

6.Show graphically how the stopping potential for a given photosensitive surface varies with the frequency of incident radiations?

Ans. v_o - threshold of frequency

or cut off frequency



7. How does the stopping potential applied to a photocell change if the distance between the light source and the cathode of the cell is doubled?

Ans. Stopping potential does not depend on the intensity of the light source which changes due to the change in distance from the light source.

8.On what factor does the retarding potential of a photocell depend?

Ans. It depends upon the frequency of the incident light

9.Electron and proton are moving with same speed, which will have more wavelength?

Ans. Since $\lambda \alpha \frac{1}{\sqrt{m}}$ so electron being lighter will have more wavelengths

2 Mark Questions

1. Derive an expression for debroglie wavelength of an electron?

Ans. If a beam of electrons traveling through a potential difference of V volt, the electron acquires kinetic energy.

$$\frac{1}{2}mv^2 = eV$$

Multiply by m

$$m^2v^2 = 2meV$$

$$\text{Now } \lambda = \frac{h}{mv}$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{2meV}} \ (\because eV = E)$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{2mE}} \ since\ m,\ e,\ h\ are\ constant$$

$$\therefore \lambda = \frac{12.27}{\sqrt{V}}\ A^\circ$$

2. Light of wavelength $2000 A^{o}$ falls on an aluminum surface. In aluminum 4.2 eV are required to remove an electron. What is the kinetic energy of (a) fastest (b) the slowest photoelectron?

Ans. $\lambda = 2000 A^{o} = 22 \times 10^{-7} m$

$\phi_o = 4.2 eV$

(a) $K.E_{\max} = \frac{1}{2} m V_{\max}^2 = hv - \phi o$

$$\frac{1}{2} m V_{\max}^2 = \frac{hc}{\lambda} - \phi_o = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7}} - 4.2$$

$$\frac{1}{2} m V_{\max}^2 = 6.2 - 4.2 = 2 eV$$

This is the K.E of the fastest electron

(b) Zero

---

3. An electromagnetic wave of wavelength λ is incident on a photosensitive surface of negligible work function. If the photoelectrons emitted from this surface have de-Broglie wavelength λ_1. Prove that $\lambda = \left(\frac{2mc}{h}\right) \lambda_1^2$.

Ans. $hv = \phi + K.E.$

$$\frac{hv}{\lambda} = o + K.E.$$

$$K.E. = \frac{hc}{\lambda}$$

Using $\lambda_1 = \frac{h}{\sqrt{2mK.E.}}$

$$\lambda_1 = \frac{h}{\sqrt{2m\frac{hc}{\lambda}}} = \sqrt{\frac{h\lambda}{2mc}}$$

Squaring we get

$$\lambda_1^2 = \frac{h\lambda}{2mc}$$

Or

$$\lambda = \left(\frac{2mc}{n}\right)\lambda_1^2$$

---

4. It is difficult to remove a free electron from copper than from sodium? Why?

Ans. since $\phi_o = \frac{hc}{\lambda_o}$

Where λ_o is the threshold wavelength

Since $\lambda_o Na > \lambda_o Cu$

$\therefore$ Work function for copper is greater and it becomes difficult to remove a free electron from copper.

---

5. Obtain the expression for the maximum kinetic energy of the electrons emitted from a metal surface in terms of the frequency of the incident radiation and the threshold frequency?

Ans. $K.E_{\max} = hv - w$

W is the threshold energy or work function depends upon threshold frequency v_o

$w = hv_o$

$$or\ \frac{1}{2}m\upsilon^2_{max} = hv - hv_o$$

Or

$$hv = \frac{1}{2}m\upsilon^2_{max} + hv_o$$

---

6. For a given K.E. which of the following has the smallest de–broglie wavelength: electron, proton, $\alpha-\text{particle}$?

Ans. Debroglie wavelength

$$\lambda = \frac{h}{m\upsilon}$$

$$\text{and m } \upsilon = \sqrt{2mE}$$

When E is energy $\Rightarrow \lambda = \frac{h}{\sqrt{2mE}}$

Comparing masses we get mass of $\alpha-\text{particle}$ is more; hence wavelength of alpha particle is minimum.

---

7. Photoelectrons are emitted with a maximum speed of 7×10^5m/s from a surface when light of frequency 8×10^{14}Hz is incident on it. Find the threshold frequency for this surface?

Ans. $h\left(v - v_o\right) = \frac{1}{2}m\upsilon^2_{max}$

$$v_o = \frac{v - m\upsilon^2_{max}}{2h}$$

$$v_o = 8\times10^{14} - \frac{9.1\times10^{-31}\times(7\times10^5)^2}{2\times6.63\times10^{-34}}$$

$$v_o = 4.64\times10^{14}\,Hz$$

8. Is photoelectric emission possible at all frequencies? Give reason for your answer?

Ans. No, photoelectric emission is not possible at all frequencies because it is possible only if radiation energy is greater than work function $\omega = h\nu_o$ of the emitter.

---

9. Assume that the frequency of the radiation incident on a metal plate is greater than its threshold frequency. How will the following change, if the incident radiation is doubled? (1) Kinetic energy of electrons

(2) Photoelectric current

Ans.(1) If the frequency of the incident radiation is doubled $h\nu - h\nu_o$ is increased, hence kinetic energy is increased.

(2) If the frequency of the incident radiation is doubled there will be no change in the number of photoectrons i.e. photoelectronic current.

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10. Why are de – broglie waves associated with a moving football is not visible?

Ans. The wavelength of a wave associated with a moving football is extremely small, which cannot be detected.

Since $\lambda = \frac{h}{mv}$

---

11. By how much would the stopping potential for a given photosensitive surface go up if the frequency of the incident radiations were to be increased from 4$\times$ 10^{15} Hz to 8 $\times$ 10^{15} Hz? ($h = 6.4\times10^{-3}4Js$, $e = 1.6\times10^{-19}C$, $c = 3\times10^{8}m/s$ **)**

Ans. Stopping potential $V_o \alpha \nu$

$$\Rightarrow \frac{V_{o2}}{V_{01}} = \frac{\nu_2}{\nu_1} = \frac{8\times10^{15}}{4\times10^{15}} = 2$$

$$\Rightarrow V_{o2} = 2V_{01}$$

12. Work function of Na is 2.3eV. Does sodium show photoelectric emission for light on

the velocity of photoelectrons?

Ans. Since $\frac{1}{2}mv^2 \alpha \frac{1}{\lambda}$

$\therefore$ Velocity of photoelectrons increases with the decrease in the wavelength of the incident light.

---

13. An electron and an alpha particle have the same debroglie wavelength associated with them? How are their kinetic energies related to each other?

Ans. $\frac{1}{2}mv^2 = qV$

and P $= \sqrt{2mE} = \sqrt{2mqV}$

$$P(\alpha) = \sqrt{2 \times 4m_p} \times 2q_pV - -(1) \begin{pmatrix} \because m\alpha = 4mp \\ q\alpha = 2qp \end{pmatrix}$$

Dividing equation (1) and (2)

$$\frac{P}{P_{proton}} = \frac{\sqrt{2 \times 4m_p \times 2q_pV}}{\sqrt{2m_pq_pV}} = \frac{\sqrt{8}}{1}$$

$$Since\ \lambda\alpha \frac{1}{P}$$

$$\Rightarrow \frac{\lambda\alpha}{\lambda P} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

$$\lambda\alpha : \lambda P = 1 : 2\sqrt{2}$$

14. An $\alpha - particle$ and a proton are accelerated from rest through same potential difference V. find the ratio of de–broglie wavelength associated with them?

Ans. $\phi_o = leV = 1\times1.6\times10^{-19} Joules$

$\lambda = 3000A^o = 3000\times10^{-10} m$

$or\ \lambda = 3\times10^{-7} m$

$$E = hv = \frac{hc}{\lambda}$$

$$E = \frac{6.6\times10^{-34}\times3\times10^{8}}{3\times10^{-7}}$$

$E = 6.625\ \times\ 10^{-19} Joules$

Now kinetic energy $(K.E.) = \frac{1}{2}m\vartheta^2 = hv - \phi_o$

$$\frac{1}{2}m\upsilon^2 = 6.625\times10^{-19} - 1.6\times10^{-19} J$$

$$\frac{1}{2}m\upsilon^2 = 5.025\times10^{-19}$$

$$\Rightarrow \upsilon^2 = \frac{2\times5.025\times10^{-19}}{9.1\times10^{-31}}$$

$$or\ \ \upsilon = \sqrt{\frac{2\times5.025\times10^{-19}}{9.1\times10^{-31}}}$$

$\upsilon = 1\times10^{6} m/s$

15.The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

Ans. Photoelectric cut-off voltage, V_0 = 1.5 V

The maximum kinetic energy of the emitted photoelectrons is given as:

$K_e = eV_0$

Where,

e= Charge on an electron $= 1.6 \times 10^{-19}$ C

$\therefore K_e = 1.6 \times 10^{-19} \times 1.5$

$= 2.4 \times 10^{-19}\ J$

Therefore, the maximum kinetic energy of the photoelectrons emitted in the given experiment is 2.4×10^{-19} J.

16.The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cutoff voltage for the photoelectric emission.

Ans. Threshold frequency of the metal, $\nu_0 = 3.3 \times 10^{14}\ Hz$

Frequency of light incident on the metal, $\nu = 8.2 \times 10^{14}\ Hz$

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Cut-off voltage for the photoelectric emission from the metal = V_0

The equation for the cut-off energy is given as:

$eV_0 = h(\nu - \nu_0)$

$$V_0 = \frac{h(\nu - \nu_0)}{e}$$

$$= \frac{6.626\times10^{-34}\times\left(8.2\times10^{14}-3.3\times10^{14}\right)}{1.6\times10^{-19}} = 2.0292V$$

Therefore, the cut-off voltage for the photoelectric emission is 2.0292 V

17.The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Ans. No

Work function of the metal, $\phi_0 = 4.2eV$

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Wavelength of the incident radiation, λ = 330 nm $= 330\times 10^{-9}$ m

Speed of light, $c = 3 \times 10^{8}$ m/s

The energy of the incident photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626\times10^{-34}\times3\times10^{8}}{330\times10^{-9}} = 6.0\times10^{-19}J$$

$$= \frac{6.0\times10^{-19}}{1.6\times10^{-19}} = 3.76eV$$

It can be observed that the energy of the incident radiation is less than the work function of the metal. Hence, no photoelectric emission will take place.

3 Mark Questions

1.The following table gives the values of work functions for a few sensitive metals.

S. No.	Metal	Work function(eV)
1.	Na	1.92
2.	K	2.15
3.	Mo	4.17

If each of these metals is exposed to radiations of wavelength 3300nm, which of these will not exit photoelectrons and why?

Ans. That material will not emit photoelectrons whose work function is greater than the energy of the incident radiation.

$$E=\frac{hc}{\lambda}=\frac{6.6\times10^{-34}\times3\times10^{8}}{33\times10^{-8}}$$

$$E=6.20\times10^{-19}\ \text{Joules}$$

$$\Rightarrow E=\frac{6.20\times10^{-19}}{1.6\times10^{-19}eV}$$

E = 3.76 eV

Hence work function of M_o is (4.17eV) which is greater than the energy of the incident radiation (= 3.76 eV) so M_o will not emit photoelectrons.

2.Define threshold wavelength for photoelectric effect? Debroglie wavelength associated with an electron associated through a potential difference V is λ ? What will be the new wavelength when the accelerating potential is increase to 4V?

Ans. The maximum wavelength of radiation needed to cause photoelectric emission is known as threshold wavelength.

$$\lambda = \frac{12.27}{\sqrt{V}} A^{\circ}$$

$$\lambda' = \frac{12.27}{\sqrt{4V}} A^{\circ} = \frac{12.27}{2\sqrt{V}} A^{\circ}$$

$$\frac{\lambda'}{\lambda} = \frac{1}{2}$$

Or

$$\lambda' = \frac{\lambda}{2}$$

---

3. An electron has kinetic energy equal to 100eV. Calculate (1) momentum (2) speed (3) Debroglie wavelength of the electron.

Ans. $\frac{1}{2}mv^2 = 100eV$

$$\frac{1}{2}mv^2 = 100\times1.6\times10^{-19} J$$

$$\frac{1}{2}mv^2 = 1.6\times10^{-17} J ---(1)$$

Multiply by m

$$\frac{1}{2}m^2v^2 = 1.6\times10^{-17}$$

$$m^2v^2 = 2\times1.6\times10^{-17}$$

(1) (Momentum) $P = m\upsilon = \sqrt{2\times1.6\times10^{-7}}$

$P = 5.40\times10^{-24}\,Kgm/s$

(2) Speed $\frac{P}{m}$

$\upsilon = \frac{5.40\times10^{-24}}{9.1\times10^{-31}}$

$\upsilon = 5.93\times10^{6}\,m/s$

(3) Debroglie wavelength $\lambda = \frac{h}{mV}$

$= \frac{6.6\times10^{-34}}{5.40\times10^{-24}}$

$\Rightarrow \lambda = 1.23\,A^{\circ}$

---

4. (a) Define photoelectric work function? What is its unit?

(b) In a plot of photoelectric current versus anode potential, how does

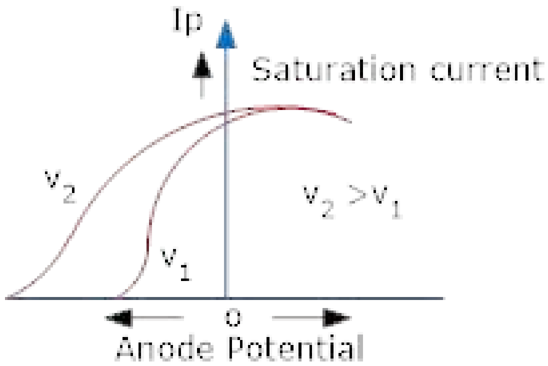
(i) Saturation current varies with anode potential for incident radiations of different frequencies but same intensity?

(ii) The stopping potential varies for incident radiations of different intensities but same frequency.

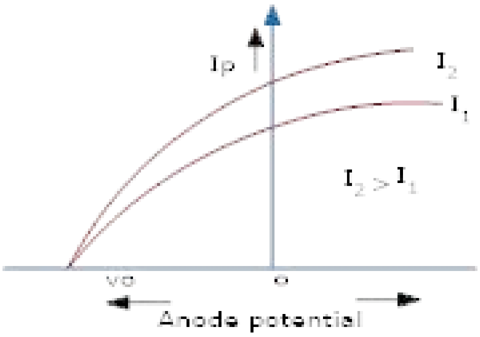
(iii) Photoelectric current vary for different intensities but same frequency of radiations? Justify your answer in each case?

Ans. (a) The minimum amount of energy required to take out an electron from the surface of metal. It is measured in electron volt (eV).

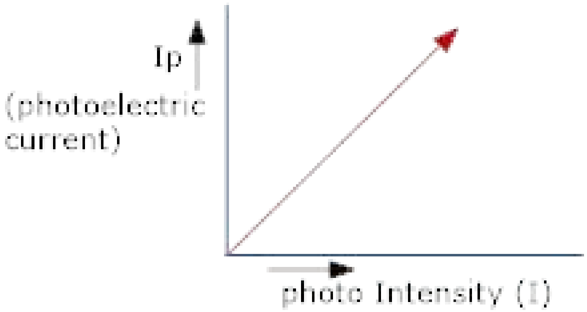
(b) (i) Saturation current depends only on the intensity of incident radiation but is independent of the frequency of incident radiation.



(ii) Stopping potential does not depend on the intensity of incident radiations.



(iii) Photoelectric current is directly proportional to the intensity of incident radiations, provided the given frequency is greater than the threshold frequency.



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5. Photoelectric work function of a metal is 1eV. Light of wavelength $3000A^o$ falls on it. What is the velocity of the effected photoelectron?

Ans. $\lambda = \frac{h}{mv}$ or $mv = \frac{h}{\lambda}$

$$\Rightarrow K.E(E') = \frac{h^2}{2m\lambda^2} ---(2)$$

$$K.E = \frac{P^2}{2m}$$

$$\Rightarrow \frac{(K.E.)electron}{(K.E.)alpha} = \frac{m\alpha}{me}\left(\begin{array}{c}\because \lambda = \frac{h}{P} \\ \text{is same}\end{array}\right)$$

6.The wavelength λ of a photon and debroglie wavelength of an electron have the same value. Show that the energy of the photon is $\frac{2\lambda mc}{h}$ times the kinetic energy of electron where m, c, and h have their usual meanings?

Ans. Energy of a photon $E = h\nu = \frac{hc}{\lambda} ------(1)$

Kinetic energy $E' = \frac{1}{2}m\upsilon^2 = \frac{m^2\upsilon^2}{2m} = \frac{(mv)^2}{2m}$ of an electron

But de-broglie wavelength of an electron is given by

$$\lambda = \frac{h}{m\upsilon} \text{ or } m\upsilon = \frac{h}{\lambda}$$

$$\Rightarrow K.E(E') = \frac{h^2}{2m\lambda^2} --- (2)$$

Dividing (1) by (2)

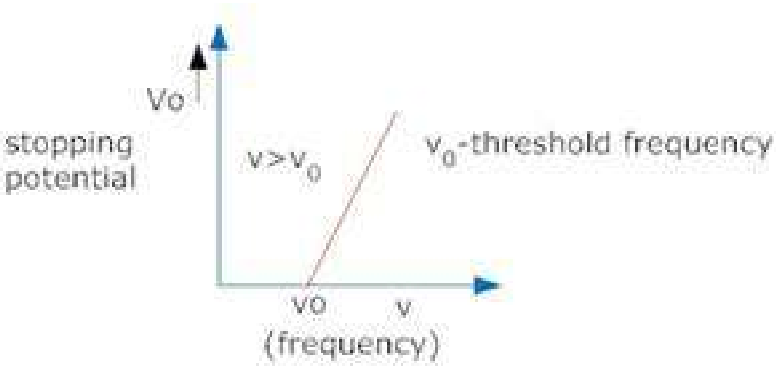
$$\frac{E}{E'} = \frac{hc}{\lambda} \times \frac{2m\lambda^2}{h^2} = \frac{2m\lambda c}{h}$$

$$E = \left(\frac{2m\lambda c}{h}E'\right)$$

7.Draw a graph showing the variation of stopping potential with frequency of the incident radiations. What does the slope of the line with the frequency axis indicate. Hence define threshold frequency?

Ans. Slope of the graph $= \dfrac{\Delta v_o}{\Delta v}$

Einstein photoelectric equation



$$eV_o = hv - \phi_o - - - - - - (1)$$

Differentiating equation (1)

$$e\Delta V_o = h\Delta v$$

$$\frac{\Delta V_o}{\Delta v} = \frac{h}{e}$$

Thus slope is equal to the ratio of planck's constant to the charge on electron.

Threshold frequency – The minimum values of frequency of the incident light below which photoelectric emission is not possible is called as threshold frequency.

8.Find the

(a) maximum frequency, and

(b) minimum wavelength of X-rays produced by 30 kV electrons.

Ans. Potential of the electrons, *V*= 30 kV $= 3\times10^4$ V

Hence, energy of the electrons, $E = 3\times10^4$ eV

Where,

e= Charge on an electron = $1.6\times10^{-19}C$

(a) Maximum frequency produced by the X-rays = ν

The energy of the electrons is given by the relation:

$E = h\nu$

Where,

h= Planck's constant $= 6.626\times10^{-34}$ Js

$\therefore \nu =$

Hence, the maximum frequency of X-rays produced is 7.24×10^{18} Hz.

(b) The minimum wavelength produced by the X-rays is given as:

$$\lambda = \frac{c}{\nu}$$

$$= \frac{3\times10^8}{7.24\times10^{18}} = 4.14\times10^{-11}m = 0.0414 nm$$

Hence, the minimum wavelength of X-rays produced is 0.0414 nm.

---

9. The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the

(a) maximum kinetic energy of the emitted electrons,

(b) Stopping potential, and

(c) maximum speed of the emitted photoelectrons?

Ans. Work function of caesium metal, $\phi_0 = 2.14\,eV$

Frequency of light, $v = 6.0 \times 10^{14}\,Hz$

(a) The maximum kinetic energy is given by the photoelectric effect as:

$$K = hv - \phi_0$$

Where,

h= Planck's constant = $6.626 \times 10-34Js$

$$\therefore K = \frac{6.626 \times 10^{34} \times 6 \times 10^{14}}{1.6 \times 10^{-19}} - 2.14$$

$$= 2.485 - 2.140 = 0.345ev$$

Hence, the maximum kinetic energy of the emitted electrons is
0.345 eV.

(b) For stopping potential V_0, we can write the equation for kinetic energy as:

$$K = eV_0$$

$$\therefore V_0 = \frac{K}{e}$$

$$= \frac{0.345 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.345V$$

Hence, the stopping potential of the material is 0.345 V.

(c) Maximum speed of the emitted photoelectrons = *v*

Hence, the relation for kinetic energy can be written as:

$$K = \frac{1}{2}mv^2$$

Where,

m= Mass of an electron $= 9.1\times10^{-31}$ kg

$$v^2 = \frac{2k}{m}$$

$$= \frac{2\times0.345\times1.6\times10^{-19}}{9.1\times10^{-31}} = 0.1104\times10^{12}$$

$$\therefore v = 3.323\times10^{5}\ m/s = 332.3 km/s$$

Hence, the maximum speed of the emitted photoelectrons is
332.3 km/s.

10.The energy flux of sunlight reaching the surface of the earth is $1.388\times10^{3}\ \mathbf{W/m^2}$. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

Ans. Energy flux of sunlight reaching the surface of earth, $1.388\times10^{3}\ \mathbf{W/m^2}$

Hence, power of sunlight per square metre, $P = 1.388\times10^{3}$ W

Speed of light, $c = 3\times10^{8}$ m/s

Planck's constant, $h = 6.626\times10^{-34}$ Js

Average wavelength of photons present in sunlight, $\lambda = 550 nm$

$$= 550\times10^{-9}\ m$$

Number of photons per square metre incident on earth per second = n

Hence, the equation for power can be written as:

$$P = nE$$

$$\therefore n = \frac{P}{E} = \frac{P\lambda}{hc}$$

$$= \frac{1.388 \times 10^3 \times 550 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^8} = 3.84 \times 10^{21}\ photons / m^2 / S$$

Therefore, every second, 3.84×10^{21} photons are incident per square metre on earth.

---

11. In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.

Ans. The slope of the cut-off voltage (*V*) versus frequency (ν) of an incident light is given as:

$$\frac{V}{\nu} = 4.12 \times 10^{-15} Vs$$

V is related to frequency by the equation:

$$h\nu = eV$$

Where,

e= Charge on an electron $= 1.6 \times 10^{-19}$ C

h= Planck's constant

$$\therefore h = e \times \frac{V}{\nu}$$

$$= 1.6 \times 10^{-19} \times 4.12 \times 10^{-15} = 6.592 \times 10^{-34} js$$

Therefore, the value of Planck's constant is $6.592 \times 10^{-34} js$

12.A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?

Ans. Power of the sodium lamp, *P*= 100 W

Wavelength of the emitted sodium light, λ = 589 nm $= 589\times10^{-9}$ m

Planck's constant, $h = 6.626 \times10^{-34}$ Js

Speed of light, $c = 3 \times10^{8}$ m/s

(a) The energy per photon associated with the sodium light is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626\times10^{-34}\times3\times10^{8}}{589\times10^{-9}}$$

$$= \frac{3.37\times10^{-19}}{1.6\times10^{-19}} = 2.11eV$$

(b) Number of photons delivered to the sphere = *n*

The equation for power can be written as:

$$P = nE$$

$$\therefore n = \frac{P}{E}$$

$$= \frac{100}{3.37\times10^{-19}} = 2.96\times10^{20}\ photons\ /\ s$$

Therefore, every second, 2.96×10^{20} photons are delivered to the sphere.

5 Mark Questions

1.Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

(a) Find the energy and momentum of each photon in the light beam,

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

Ans. Wavelength of the monochromatic light, λ = 632.8 nm $= 632.8 \times 10^{-9}$ m

Power emitted by the laser, *P*= 9.42 mW $= 9.42 \times 10^{-3}$ W

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Speed of light, $c = 3 \times 10^{8}$ m/s

Mass of a hydrogen atom, $m = 1.66 \times 10^{-27}$ kg

(a) The energy of each photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{632.8 \times 10^{-9}} = 3.141 \times 10^{-19} J$$

The momentum of each photon is given as:

$$P = \frac{h}{\lambda}$$

$$= \frac{6.626 \times 10^{-34}}{632.8} = 1.047 \times 10^{-27}\, kg\, ms^{-1}$$

(b) Number of photons arriving per second, at a target irradiated by the beam = n

Assume that the beam has a uniform cross-section that is less than the target area.

Hence, the equation for power can be written as:

$$P = \frac{h}{\lambda}$$

$$\therefore n = \frac{P}{E}$$

$$= \frac{9.42 \times 10^{-3}}{3.141 \times 10^{-19}} \approx 3 \times 10^{16}\, photon / s$$

(c) Momentum of the hydrogen atom is the same as the momentum of the photon,

$$p = 1.047 \times 10^{-27}\, kg\, ms^{-1}$$

Momentum is given as:

$$p = mv$$

Where,

v= Speed of the hydrogen atom

$$\therefore v = \frac{p}{m}$$

$$= \frac{1.047 \times 10^{-27}}{1.66 \times 10^{-27}} = 0.621 m / s$$

2.Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^{5} m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Ans. Frequency of the incident photon, $v = 488nm = 488\times10^{-9}\ m$

Maximum speed of the electrons, $v = 6.0\ \times10^{5}$ m/s

Planck's constant, $h = 6.626\ \times10^{-34}$ Js

Mass of an electron, $m = 9.1\ \times10^{-31}$ kg

For threshold frequency v_0, the relation for kinetic energy is written as:

$$\frac{1}{2}mv = h(v - v_0)$$

$$v_0 = v = \frac{mv_2}{2h}$$

$$= 7.21\times10^{14} - \frac{\left(9.1\times10^{-31}\right)\times\left(6\times10^{5}\right)^2}{2\times\left(6.626\times10^{-34}\right)}$$

$$= 7.21\times10^{14} - 2.472\times10^{14}$$

$$= 4.738\times10^{14}\ Hz$$

Therefore, the threshold frequency for the photoemission of electrons is $4.738\times10^{14}\ Hz$ s

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3. Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

Ans. Wavelength of light produced by the argon laser, λ = 488 nm

$=488 \times 10^{-9}$ m

Stopping potential of the photoelectrons, V_0 = 0.38 V

1eV $=1.6\times10^{-19}$ J

$$\therefore V_0 = \frac{0.38}{1.6\times10^{-19}} eV$$

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Charge on an electron, *e*= 1.6 × 10 - 19C

Speed of light, *c*= 3 × 10 m/s

From Einstein's photoelectric effect, we have the relation involving the work function ϕ_0 of the material of the emitter as:

$$eV_0 = \frac{hc}{\lambda} - \phi_0$$

$$\phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times488\times10^{-9}} - \frac{1.6\times10^{-19}\times0.38}{1.6\times10^{-19}}$$

$$= 2.54 - 0.38 = 2.16\,eV$$

Therefore, the material with which the emitter is made has the work function of 2.16 eV.

4.Calculate the

(a) momentum, and

(b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Ans. Potential difference, *V*= 56 V

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Mass of an electron, $m = 9.1 \times 10^{-31}$ kg

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

(a) At equilibrium, the kinetic energy of each electron is equal to the accelerating potential, i.e., we can write the relation for velocity (v) of each electron as:

$$\frac{1}{2}MV^2 = eV$$

$$v^2 = \frac{2eV}{m}$$

$$\therefore v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 56}{9.1 \times 10^{-31}}}$$

$$= \sqrt{19.69 \times 10^{12}} = 4.44 \times 10^6 \, m/s$$

The momentum of each accelerated electron is given as:

$p = mv$

$= 9.1 \times 10^{-31} \times 4.44 \times 10^6$

$= 4.04 \times 10^{-24}$ kg $m\,s^{-1}$.Therefore, the momentum of each electron is 4.04×10^{-24} kg$m\,s^{-1}$.

(b) De Broglie wavelength of an electron accelerating through a potential *V*, is given by the relation:

$$\lambda = \frac{12.27}{\sqrt{V}} \overset{0}{A}$$

$$= \frac{12.27}{\sqrt{56}} \times 10^{-10} \, m$$

$= 0.1639\,nm$

Therefore, the de Broglie wavelength of each electron is 0.1639 nm.

5.What is the

(a) momentum,

(b) speed, and

(c) de Broglie wavelength of an electron with kinetic energy of 120 eV.

Ans. Kinetic energy of the electron, *Ek*= 120 eV

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Mass of an electron, $m = 9.1 \times 10^{-31}$ kg

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

(a) For the electron, we can write the relation for kinetic energy as:

$$E_k = \frac{1}{2}mv^2$$

Where,

v= Speed of the electron

$$\therefore v^2 = \sqrt{\frac{2eE_k}{m}}$$

$$= \sqrt{\frac{2\times1.6\times10^{-19}\times120}{9.1\times10^{-31}}}$$

$$= \sqrt{42.198\times10^{12}} = 6.496\times10^6\ m/s$$

Momentum of the electron, *p*= *mv*

$=9.1 \times 10^{-31} \times 6.496 \times 10^{6}$

$= 5.91 \times 10^{-24}$ kg $m\,s^{-1}$

Therefore, the momentum of the electron is 5.91×10^{-24} kg m s^{-1}.

(b) Speed of the electron, *v* $=6.496 \times 10^{6}$ m/s

(c) De Broglie wavelength of an electron having a momentum *p*, is given as:

$$\lambda = \frac{h}{p}$$

$$= \frac{6.6\times10^{-34}}{5.91\times10^{-24}} = 1.116\times10^{-10}\,m$$

$$= 0.112\,nm$$

Therefore, the de Broglie wavelength of the electron is 0.112 nm.

6.The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

(a) an electron, and

(b) a neutron, would have the same de Broglie wavelength.

Ans. Wavelength of light of a sodium line, λ = 589 nm $=589 \times 10^{-9}$ m

Mass of an electron, *me* $=9.1 \times 10^{-31}$ kg

Mass of a neutron, *mn*= 1.66×10^{-27} kg

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

(a) For the kinetic energy *K*, of an electron accelerating with a velocity *v*, we have the relation:

$$K = \frac{1}{2} m_e v^2 \text{.....(1)}$$

We have the relation for de Broglie wavelength as:

$$\lambda = \frac{h}{m_e v}$$

$$\therefore v^2 = \frac{h^2}{\lambda^2 {m_e}^2} \text{.............(2)}$$

Substituting equation (2) in equation (1), we get the relation:

$$K = \frac{1}{2} \frac{m_e h^2}{\lambda^2 {m_e}^2} = \frac{h^2}{2\lambda m_e} \text{.......(3)}$$

$$= \frac{(6.6 \times 10^{-34})^2}{2 \times (589 \times 10^{-9})^2 \times 9.1 \times 10^{-31}}$$

$$= 6.9 \times 10^{-25} J$$

$$= \frac{6.9 \times 10^{-25}}{1.6 \times 10^{-19}} = 4.31 \times 10^{-6} eV = 4.31 \mu eV$$

Hence, the kinetic energy of the electron is 6.9×10^{-25} J or 4.31 μeV.

(b) Using equation (3), we can write the relation for the kinetic energy of the neutron as:

$$\frac{h^2}{2\lambda^2 m_n}$$

$$= \frac{\left(6.6 \times 10^{-34}\right)^2}{2 \times \left(589 \times 10^{-9}\right) \times 1.66 \times 10^{-27}}$$

$$= 3.78 \times 10^{-28} J$$

$$= \frac{3.78\times10^{-28}}{1.6\times10^{-19}} = 2.36\times10^{-9}eV = 2.36neV$$

Hence, the kinetic energy of the neutron is 3.78×10^{-28} J or 2.36 neV.

7.What is the de Broglie wavelength of

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,

(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and

(c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

Ans.(a) Mass of the bullet, *m*= 0.040 kg

Speed of the bullet, *v*= 1.0 km/s = 1000 m/s

Planck's constant, *h*= 6.6×10^{-34} Js

De Broglie wavelength of the bullet is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6\times10^{-34}}{0.040\times1000} = 1.65\times10^{-35}\,m$$

(b) Mass of the ball, *m*= 0.060 kg

Speed of the ball, *v*= 1.0 m/s

De Broglie wavelength of the ball is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6\times10^{-34}}{0.060\times1} = 1.1\times10^{-32}\,m$$

(c) Mass of the dust particle, $m = 1 \times 10^{-9}$ kg

Speed of the dust particle, v= 2.2 m/s

De Broglie wavelength of the dust particle is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6\times10^{-34}}{2.2\times10^{-9}} = 3.0\times10^{-25}\,m\text{ s}$$

---

8. An electron and a photon each have a wavelength of 1.00 nm. Find

(a) their momenta,

(b) the energy of the photon, and

(c) the kinetic energy of electron.

Ans. Wavelength of an electron (λ_e) and a photon (λ_p), $\lambda_e = \lambda_p = \lambda = 1nm$

$= 1 \times 10^{-9}$m

Planck's constant, h= 6.63×10^{-34} Js

(a) The momentum of an elementary particle is given by de Broglie relation:

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda}$$

It is clear that momentum depends only on the wavelength of the particle. Since the wavelengths of an electron and a photon are equal, both have an equal momentum.

$$\therefore p = \frac{6.63\times10^{-34}}{1\times10^{-9}} = 6.63\times10^{-25}\,kg\,ms^{-1}$$

(b) The energy of a photon is given by the relation:

$$E = \frac{hc}{\lambda}$$

Where,

Speed of light, *c*= 3×10^{8} m/s

$$\therefore E = \frac{6.63\times10^{-34}\times3\times10^{8}}{1\times10^{-9}\times1.6\times10^{-19}}$$

$$= 1243.1 eV = 1.243 keV$$

Therefore, the energy of the photon is 1.243 keV.

(c) The kinetic energy (*K*) of an electron having momentum *p*,is given by the relation:

$$K = \frac{1}{2}\frac{p^2}{m}$$

Where,

m= Mass of the electron = 9.1×10^{-31} kg

p = $6.63 \times 10^{-25} kg\ m\ s^{-1}$

$$\therefore K = \frac{1}{2}\times\frac{\left(6.63\times10^{-25}\right)^2}{9.1\times10^{-31}} = 2.415\times10^{-19}\,J$$

$$= \frac{2.415\times10^{-19}}{1.6\times10^{-19}} = 1.51 eV$$

Hence, the kinetic energy of the electron is 1.51 eV.

9.(a) For what kinetic energy of a neutron will the associated de Broglie wavelength be $1.40 \times 10-10$ m ?

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of (3/2) *kT* at 300 K.

Ans.(a) De Broglie wavelength of the neutron, $\lambda = 1.40 \times 10^{-10} m$

Mass of a neutron, *mn*= 1.66×10^{-27} kg

Planck's constant, *h*= 6.6×10^{-34} Js

Kinetic energy (*K*) and velocity (*v*) are related as:

$$K = \frac{1}{2} m_n v^2 \ldots (1)$$

De Broglie wavelength (λ) and velocity (*v*) are related as:

$$\lambda = \frac{h}{m_n v}$$

Using equation (2) in equation (1), we get:

$$K = \frac{1}{2} \frac{m_n h^2}{\lambda^2 m_n^2} = \frac{h^2}{2\lambda^2 m_n}$$

$$= \frac{\left(6.63 \times 10^{-34}\right)^2}{2 \times \left(1.40 \times 10^{-10}\right)^2 \times 1.66 \times 10^{-27}} = 6.75 \times 10^{21} J$$

$$= \frac{3.75 \times 10^{-21}}{1.6 \times 10^{-19}} = 4.219 \times 10^{-2} eV$$

Hence, the kinetic energy of the neutron is 6.75×10^{-21}J or 4.219×10^{-2} eV.

(b) Temperature of the neutron, *T*= 300 K

Boltzmann constant, k= 1.38×10^{-23} kg $m^2 s^{-2} K^{-1}$

Average kinetic energy of the neutron:

$$K' = \frac{3}{2} kT$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21} J$$

The relation for the de Broglie wavelength is given as:

$$\lambda' = \frac{h}{\sqrt{2K' m_n}}$$

Where

$$m_n = 1.66 \times 10^{-27} kg$$

$$h = 6.6 \times 10^{-34} Js$$

$$K' = 6.75 \times 10^{-21} J$$

$$\therefore \lambda' = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.21 \times 10^{-21} \times 1.66 \times 10^{-27}}} = 1.46 \times 10^{-10} m = 0.146 nm$$

Therefore, the de Broglie wavelength of the neutron is 0.146 nm.

10.Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Ans. The momentum of a photon having energy (hv) is given as:

$$p = \frac{hv}{c} = \frac{h}{v}$$

$$\lambda = \frac{h}{p}$$

Where,

λ = Wavelength of the electromagnetic radiation

c= Speed of light

h= Planck's constant

De Broglie wavelength of the photon is given as:

$$\lambda = \frac{h}{mv}$$

But p=mv

$$\therefore \lambda = \frac{h}{p}$$

Where,

m= Mass of the photon

v= Velocity of the photon

Hence, it can be inferred from equations (*i*) and (*ii*) that the wavelength of the electromagnetic radiation is equal to the de Broglie wavelength of the photon.

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11. What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

Ans. Temperature of the nitrogen molecule, *T*= 300 K

Atomic mass of nitrogen = 14.0076 u

Hence, mass of the nitrogen molecule, $m = 2 \times 14.0076$ = 28.0152 u

But 1 u = 1.66×10^{-27} kg

$\therefore m$= $28.0152 \times 1.66 \times 10^{-27}$ kg

Planck's constant, h= 6.63×10^{-34} Js

Boltzmann constant, k= $1.38 \times 10^{-23} J K^{-1}$

We have the expression that relates mean kinetic energy $\left(\frac{3}{2}kT\right)$ of the nitrogen molecule with the root mean square speed (V_{ms}) as:

$$\frac{1}{2}mv^2_{ms} = \frac{3}{2}kT$$

$$V_{ms} = \sqrt{\frac{3kT}{m}}$$

Hence, the de Broglie wavelength of the nitrogen molecule is given as:

$$\lambda = \frac{h}{mv_{ms}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{3\times28.0152\times1.66\times10^{-27}\times1.38\times10^{-23}\times300}}$$

$$= 0.028\times10^{-9}\,m$$

$$= 0.028\,nm$$

Therefore, the de Broglie wavelength of the nitrogen molecule is 0.028 nm.

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12. (a) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons. The *specific charge* of the electron, i.e., its *e/m* is given to be $1.76\times10^{11}\,\mathrm{C\,kg^{-1}}$.

(b) Use the same formula you employ in (a) to obtain electron speed for an collector potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified?

Ans.(a) Potential difference across the evacuated tube, *V*= 500 V

Specific charge of an electron, *e/m*= $1.76\times 10^{11}\ C\ kg^{-1}$

The speed of each emitted electron is given by the relation for kinetic energy as:

$$KE=\frac{1}{2}mv^2=eV$$

$$\therefore v=\left(\frac{2eV}{m}\right)^{\frac{1}{2}}=\left(2V\times\frac{e}{m}\right)^{\frac{1}{2}}$$

$$=\left(2\times 50\times 1.76\times 10^{11}\right)^{\frac{1}{2}}=1.327\times 10^{7}\ m/s$$

Therefore, the speed of each emitted electron is $1.327\times 10^{7}\ m/s$

(b) Potential of the anode, *V*= 10 MV = 10×10^{6} V

The speed of each electron is given as:

$$v=\left(2V\times\frac{e}{m}\right)^{\frac{1}{2}}$$

$$=\left(2\times 10^{7}\times 1.76\times 10^{11}\right)^{\frac{1}{2}}$$

$$=1.88\times 10^{9}\ m/s$$

This result is wrong because nothing can move faster than light. In the above formula, the expression $\left(mv^2/2\right)$ for energy can only be used in the non-relativistic limit, i.e., for $v << c$.

For very high speed problems, relativistic equations must be considered for solving them. In the relativistic limit, the total energy is given as:

E= mc^2

Where,

m = Relativistic mass

$$= m_0\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

$m0$= Mass of the particle at rest

Kinetic energy is given as:

$K = mc^2 - m_0c^2$

13.(a) A mono-energetic electron beam with electron speed of $5.20\times10^6\,\mathrm{m\ s^{-1}}$ is subject to a magnetic field of $1.30\times10^{-4}\,\mathrm{T}$ normal to the beam velocity. What is the radius of the circle traced by the beam, given *e/m* for electron equals $1.76\times10^{11}\,\mathrm{C\ kg^{-1}}$.

(b) Is the formula you employ in (a) valid for calculating radius of the path of a 20 MeV electron beam? If not, in what way is it modified?

[Note: Exercises 11.20(b) and 11.21(b) take you to relativistic mechanics which is beyond the scope of this book. They have been inserted here simply to emphasise the point that the formulas you use in part (a) of the exercises are not valid at very high speeds or energies. See answers at the end to know what 'very high speed or energy' means.]

Ans.(a) Speed of an electron, v= 5.20×10^6 m/s

Magnetic field experienced by the electron, B= $1.30\times10^{-4}T$

Specific charge of an electron, e/m= $1.76\times10^{11}\,C\,kg^{-1}$

Where,

e= Charge on the electron = $1.6\times10^{-19}C$

m= Mass of the electron = $9.1\times10^{-31}kg^{-1}$

The force exerted on the electron is given as:

$$F = \left|\vec{v} \times \vec{B}\right|$$

$$= evB \sin\theta$$

θ = Angle between the magnetic field and the beam velocity

The magnetic field is normal to the direction of beam.

$$\therefore \theta = 90^0$$

$$F = evB$$

The beam traces a circular path of radius, *r*. It is the magnetic field, due to its bending nature, that provides the centripetal force $\left(F = \frac{mv^2}{r}\right)^{\frac{1}{2}}$ for the beam.

Hence, equation (1) reduces to:

$$evB = \frac{mv^2}{r}$$

$$\therefore r = \frac{mv}{eB} = \frac{v}{\left(\frac{e}{m}\right)B}$$

$$= \frac{5.20 \times 10^6}{\left(1.76 \times 10^{11}\right) \times 1.30 \times 10^{-4}} = 0.227m22.7cm$$

Therefore, the radius of the circular path is 22.7 cm.

(b) Energy of the electron beam, $E = 20\ MeV = 20 \times 10^6 \times 1.6 \times 10^{-19}\ J$

The energy of the electron is given as:

$$E = \frac{1}{2}mv^2$$

$$\therefore v = \left(\frac{2E}{m}\right)^{\frac{1}{2}}$$

$$= \sqrt{\frac{2 \times 20 \times 10^6 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 2.652 \times 10^9 \ m/s$$

This result is incorrect because nothing can move faster than light. In the above formula, the expression $\left(mv^2 / 2\right)$ for energy can only be used in the non-relativistic limit, i.e., for *v* << *c*

When very high speeds are concerned, the relativistic domain comes into consideration.

In the relativistic domain, mass is given as:

$$m = m_0 \left[1 - \frac{v^2}{c^2}\right]^{\frac{1}{2}} \text{s}$$

Where,

m_0 = Mass of the particle at rest

Hence, the radius of the circular path is given as:

$$r = mv / eB$$

$$= \frac{m_0 v}{eB\sqrt{\frac{c^2 - V^2}{c^2}}}$$

14.An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure $(\sim 10^{-2}\ mm$ of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path can be viewed because the gas ions in the path focus the beam by attracting electrons, and emitting light by electron capture; this method is known as the 'fine beam tube' method. Determine *e*/*m* from the data.

Ans. Potential of an anode, *V* = 100 V

Magnetic field experienced by the electrons, *B*= $2.83 \times 10^{-4} T$

Radius of the circular orbit *r*= 12.0 cm = $12.0 \times 10^{-2} m$

Mass of each electron = *m*

Charge on each electron = *e*

Velocity of each electron = *v*

The energy of each electron is equal to its kinetic energy, i.e.,

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2eV}{m}$$

It is the magnetic field, due to its bending nature, that provides the centripetal force $\left(F = \frac{mv^2}{r}\right)$ for the beam. Hence, we can write:

Centripetal force = Magnetic force

$$\frac{mv^2}{r} = evB$$

$$eB = \frac{mv}{r}$$

$$v = \frac{eBr}{m}$$

Putting the value of *v* in equation (1), we get:

$$\frac{2eV}{m} = \frac{e^2B^2r^2}{m^2}$$

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

$$= \frac{2 \times 100}{\left(2.83 \times 10^{-4}\right)^2 \times \left(12 \times 10^{-2}\right)} = 1.73 \times 10^{11}\, Ckg^{-1}$$

Therefore, the specific charge ratio (*e/m*) is $1.73 \times 10^{11}\, Ckg^{-1}$

15.(a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 A° .What is the maximum energy of a photon in the radiation?

(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Ans.(a) Wavelength produced by an X-ray tube, $\lambda = 0.45\, \overset{0}{A} = 0.45 \times 10^{-10}\, m$

Planck's constant, $h = 6.626 \times 10^{-34}\, Js$

Speed of light, $c = 3 \times 10^{8}\, m/s$

The maximum energy of a photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{0.45 \times 10^{-10} \times 1.6 \times 10^{19}}$$

$$= 27.6 \times 10^{3}\, eV = 27.6 keV$$

Therefore, the maximum energy of an X-ray photon is 27.6 keV.

(b) Accelerating voltage provides energy to the electrons for producing X-rays. To get an X-ray of 27.6 keV, the incident electrons must possess at least 27.6 keV of kinetic electric energy. Hence, an accelerating voltage of the order of 30 keV is required for producing X-rays.

16.In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2 BeV into two Y rays of equal energy. What is the wavelength associated with each

γ **-ray? (** $1\text{BeV} = 10^9\text{eV}$ **)**

Ans. Total energy of two γ -rays:

E= 10. 2 BeV

= $10.2 \times 10^9 eV$

= $10.2 \times 10^9 \times 1.6 \times 10^{-10} J$

Hence, the energy of each γ -ray:

$$E' = \frac{E}{2}$$

$$= \frac{10.2 \times 1.6 \times 10^{-10}}{2} = 8.16 \times 10^{-10} J$$

Planck's constant, $h = 60626 \times 10^{-34} Js$

Speed of light, $c = 3 \times 10^8 \ m/s$

Energy is related to wavelength as:

$$E' = \frac{hc}{\lambda}$$

$$\therefore \lambda = \frac{hc}{E'}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{8.16 \times 10^{-10}} = 2.436 \times 10^{-16} m$$

Therefore, the wavelength associated with each γ -ray is $2.436 \times 10^{-16} m$

17.Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about photons! The second

number tells you why our eye can never 'count photons', even in barely detectable light.

(a) The number of photons emitted per second by a Medium wave transmitter of 10 kW power, emitting radiowaves of wavelength 500 m.

(b) The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive $(\sim 10^{-10}\,\mathrm{W\ m^{-2}})$. Take the area of the pupil to be about 0.4 cm2, and the average frequency of white light to be about $6\times 10^{14}\,\mathrm{Hz}$.

Ans.(a) Power of the medium wave transmitter, *P*= 10 kW = 104 W = 104J/s

Hence, energy emitted by the transmitter per second, *E*= 104

Wavelength of the radio wave, λ = 500 m

The energy of the wave is given as:

$$E_1 = \frac{hc}{\lambda}$$

Where,

h= Planck's constant = $6.6\times 10^{-34}\,Js$

c= Speed of light = $3\times 108 m/s$

$$\therefore E_1 = \frac{6.6\times10^{-34}\times3\times10^{8}}{500} = 3.96\times10^{-28}\,J$$

Let *n* be the number of photons emitted by the transmitter.

∴*nE*1= *E*

$$n = \frac{E}{E_1}$$

$$= \frac{10^4}{3.96\times10^{-28}} = 2.525\times10^{31}$$

$$\approx 3\times10^{31}$$

The energy ($E1$) of a radio photon is very less, but the number of photons (n) emitted per second in a radio wave is very large.

The existence of a minimum quantum of energy can be ignored and the total energy of a radio wave can be treated as being continuous.

(b) Intensity of light perceived by the human eye, $I = 10^{-10} W\,m^{-2}$

Area of a pupil, $A = 0.4\ cm^2$ = 0.4 × 10 - 4m2

Frequency of white light, $\nu = 6 \times 10^{14} Hz$

The energy emitted by a photon is given as:

$E = h\nu$

Where,

h= Planck's constant = $6.6 \times 10^{-34} Js$

$$\therefore E = 6.6\times10^{-34}\times6\times10^{14}$$

$$= 3.96 \times 10^{-19} J$$

Let n be the total number of photons falling per second, per unit area of the pupil.

The total energy per unit for n falling photons is given as:

$$E = n\times 3.96 \times 10^{-19} J\,s^{-1}m^{-2}$$

The energy per unit area per second is the intensity of light.

$$\therefore E = I$$

$$n\times 3.96 \times 10^{-19} = 10^{-10}$$

$$n = \frac{10^{-10}}{3.96 \times 10^{-19}} \text{ s}$$

$= 2.52 \times 10^{8} m^{2} s^{-1}$

The total number of photons entering the pupil per second is given as:

$nA = n \times A$

$= 2.52 \times 10^{8} \times 0.4 \times 10^{-4}$

$= 1.008 \times 10^{4} s^{-1}$

This number is not as large as the one found in problem **(a)**, but it is large enough for the human eye to never see the individual photons.

18.Ultraviolet light of wavelength 2271 $\overset{\circ}{\text{A}}$ ¦ from a 100 W mercury source irradiates a photo-cell made of molybdenum metal. If the stopping potential is -1.3 V, estimate the work function of the metal. How would the photo-cell respond to a high intensity $(\sim 10^{5} W\ m^{-2})$ red light of wavelength 6328 $\overset{\circ}{\text{A}}$ ¦ produced by a He-Ne laser?

Ans. Wavelength of ultraviolet light, λ = 2271 $\overset{\circ}{\text{A}}$ ¦ = $2271 \times 10^{-10} m$

Stopping potential of the metal, *V*0= 1.3 V

Planck's constant, $h = 6.6 \times 10^{-34} J$

Charge on an electron, *e*= $1.6 \times 10^{-19} C$

Work function of the metal = ϕ_0

Frequency of light = ν

We have the photo-energy relation from the photoelectric effect as:

$\phi_0 = h\nu - eV_0$

$$= \frac{hc}{\lambda} = eV_0$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{2271\times10^{-10}} - 1.6\times10^{-19}\times1.3$$

$$= 8.72\times10^{-19} - 2.08\times10^{-19}$$

$$= 6.64\times10^{-19}\,J$$

$$= \frac{6.64\times10^{-19}}{1.6\times10^{-19}} = 4.15eV$$

Let v_0 be the threshold frequency of the metal.

$\therefore \phi_0 = hv_0$

$$v_0 = \frac{\phi_0}{h}$$

$$= \frac{6.64\times10^{-19}}{6.6\times10^{-34}} = 1.006\times10^{15}\,Hz$$

Wavelength of red light, $\lambda_r = 6328\,\overset{0}{A} = 6328\times10^{-10}m$

$\therefore$Frequency of red light, $v_r = \frac{c}{\lambda_r}$

$$= \frac{3\times10^{8}}{6328\times10^{-10}} = 4.74\times10^{14}\,Hz$$

Since $v_0 > v_r$, the photocell will not respond to the red light produced by the laser.

19.Monochromatic radiation of wavelength 640.2 nm $\left(1\text{nm} = 10^{-9}\text{m}\right)$ from a neon lamp irradiates photosensitive material made of caesium on tungsten. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source and its 427.2

nm line irradiates the same photo-cell. Predict the new stopping voltage.

Ans. Wavelength of the monochromatic radiation, λ = 640.2 nm

= $640.2 \times 10^{-9} m$

Stopping potential of the neon lamp, V_0 = 0.54 V

Charge on an electron, *e*= $1.6 \times 10^{-19} C$

Planck's constant, *h* = $6.6 \times 10^{-34} Js$

Let ϕ_0 be the work function and ν be the frequency of emitted light.

We have the photo-energy relation from the photoelectric effect as:

$$ev_0 = h\nu - \phi_0$$

$$\phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{640.2\times10^{-9}} - 1.6\times10^{-19}0\times0.54$$

$$= 3.093\times10^{-19} - 0.864\times10^{-19}$$

$$= 2.229\times10^{-19} J$$

$$= \frac{2.229\times10^{-19}}{1.6\times10^{-19}} = 1.39eV$$

Wavelength of the radiation emitted from an iron source, λ = 427.2 nm

= $427.2 \times 10^{-9} m$

Let V'_0 be the new stopping potential. Hence, photo-energy is given as:

$$eV_0' = \frac{hc}{\lambda'} - \phi_0$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{427.2\times10^{-9}} - 2.229\times10^{-19}$$

$$= 4.63\times10^{-19} - 2.229\times10^{-19}$$

$$= 2.401\times10^{-19}\,J$$

$$= \frac{2.401\times10^{-19}}{1.6\times10^{-19}} = 1.5eV$$

Hence, the new stopping potential is 1.50 eV.

20.A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used:

$$\lambda_1 = 3650\,\text{Å},\ \lambda_2 = 4047\,\text{Å},\ \lambda_3 = 4358\,\text{Å},\ \lambda_4 = 5461\,\text{Å},\ \lambda_5 = 6907\,\text{Å},$$

The stopping voltages, respectively, were measured to be:

$$V_{01} = 1.28\text{ V},\ V_{02} = 0.95\text{ V},\ V_{03} = 0.74\text{ V},\ V_{04} = 0.16\text{ V},\ V_{05} = 0\text{ V}$$

Determine the value of Planck's constant *h*, the threshold frequency and work function for the material.

[*Note:* You will notice that to get *h* from the data, you will need to know *e* (which you can take to be 1.6×10^{-19} C). Experiments of this kind on Na, Li, K, etc. were performed by Millikan, who, using his own value of *e* (from the oil-drop experiment) confirmed Einstein's photoelectric equation and at the same time gave an independent estimate of the value of *h*.]

Ans. Einstein's photoelectric equation is given as:

$$eV_0 = hv - \phi_0$$

$$V_0 = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

Where, V_0 = Stopping potential

h= Planck's constant

e= Charge on an electron

ν = Frequency of radiation

ϕ_0 = Work function of a material

It can be concluded from equation (1) that potential V_0 is directly proportional to frequency ν .

Frequency is also given by the relation:

$$\nu = \frac{Speed\ of\ light(c)}{Wavelength(\lambda)}$$

This relation can be used to obtain the frequencies of the various lines of the given wavelengths.

$$\nu_1 = \frac{c}{\lambda_1} = \frac{3\times10^8}{3650\times10^{-10}} = 8.219\times10^{14}\,Hz$$

$$E^2 = p^2c^2 + m_0^2c^4$$

$$= \left(6.6\times10^{-19}\times3\times10^8\right) + \left(0.8176\times10^{-13}\right)$$

$$= 392.04\times10^{-22} + 0.6685\times10^{-26}$$

$$\approx 392.04\times10^{-22}$$

$$\therefore E = 1.98\times10^{-10}\,J$$

$$=\frac{1.98\times10^{-10}}{1.6\times10^{-19}}$$

$$=1.24\times10^{9}\,eV=1.24\,BeV$$

$$0.7268\times10^{-10}m$$

$$\lambda=\frac{h}{\sqrt{2mE}}$$

$$=\frac{Atomic\,weight}{N_A}$$

$$=\frac{4}{6.023\times10^{23}}$$
$$=6.64\times10^{-24}g$$

$$=6.64\times10^{-27}kg$$

$$\therefore\lambda=\frac{h}{\sqrt{3mkT}}$$

$$=\frac{6.6\times10^{-34}}{3\times6.64\times10^{-27}\times1.38\times10^{-23}\times300}$$

$$\frac{V}{N}=\frac{kT}{P}$$

$$r=\left(\frac{V}{N}\right)^{\frac{1}{3}}=\left(\frac{kT}{P}\right)^{\frac{1}{3}}$$

$$=\left[\frac{1.38\times10^{-23}\times300}{1.01\times10^{5}}\right]^{\frac{1}{3}}$$

$$=3.35\times10^{-9}m$$

$$\therefore\lambda=\frac{6.6\times10^{-34}}{\sqrt{3\times9.11\times10^{-31}\times1.38\times10^{-23}\times300}}$$

$$\frac{1}{2}$$

$$\approx 6.2\times10^{-9}\,m$$

$$\left(c = \sqrt{2V\left(\frac{e}{n}\right)}\right)$$

$$\left(v = Br\left(\frac{e}{m}\right)\right)$$

$$v_G = \frac{dv}{dk}$$

$$= \frac{dv}{d\left(\frac{1}{\lambda}\right)} = \frac{dE}{dp} = \frac{d\left(\frac{p^2}{2m}\right)}{dp} = \frac{p}{m}$$

$$v_3 = \frac{c}{\lambda_3} = \frac{3\times10^8}{4358\times10^{-10}} = 6.884\times10^{14}\,Hz$$

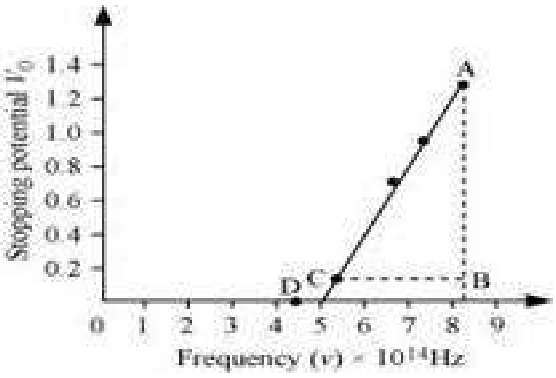
$$v_4 = \frac{c}{\lambda_4} = \frac{3\times10^8}{5461\times10^{-10}} = 5.493\times10^{14}\,Hz$$

$$v_5 = \frac{c}{\lambda_5} = \frac{3\times10^8}{6907\times10^{-10}} = 4.343\times10^{14}\,Hz$$

The given quantities can be listed in tabular form as:

Frequency × 1014 Hz	8.219	7.412	6.884	5.493	4.343
Stopping potential *V*0	1.28	0.95	0.74	0.16	0

The following figure shows a graph between $v and V_0$.



It can be observed that the obtained curve is a straight line. It intersects the $\nu - axis$ at 5×10^{14} Hz, which is the threshold frequency (ν_0) of the material. Point D corresponds to a frequency less than the threshold frequency. Hence, there is no photoelectric emission for the λ_5 line, and therefore, no stopping voltage is required to stop the current.

Slope of the straight line = $\frac{AB}{CB} = \frac{1.28 - 0.16}{(8.214 - 5.493) \times 10^{14}}$

From equation (1), the slope $\frac{h}{e}$ can be written as:

$$\frac{h}{e} = \frac{1.28 - 0.16}{(8.214 - 5.493) \times 10^{14}}$$

$$\therefore h = \frac{1.12 \times 1.6 \times 10^{19}}{2.726 \times 10^{14}} = 6.573 \times 10^{-34} \; Js$$

The work function of the metal is given as:

$$\phi_0 = h\nu_0$$

$$= 6.573 \times 10^{-34} \times 5 \times 10^{14}$$

$$= 3.286 \times 10^{-19} \; J$$

$$= \frac{3.286 \times 10^{-19}}{1.6 \times 10^{-18}} = 2.054 eV$$

21.The work function for the following metals is given:

Na: 2.75 eV; K: 2.30 eV; Mo: 4.17 eV; Ni: 5.15 eV. Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 $\mathring{A}$ ¦ from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?

Ans. Mo and Ni will not show photoelectric emission in both cases

Wavelength for a radiation, λ = 3300 $\mathring{A} = 300 \times 10^{-10} m$

Speed of light, *c*= $3 \times 10^8 m/s$

Planck's constant, *h*= 6.6×10^{-34} Js

The energy of incident radiation is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3300 \times 10^{-10}} = 6 \times 10^{-19} J$$

$$= \frac{6 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.158 eV$$

It can be observed that the energy of the incident radiation is greater than the work function of Na and K only. It is less for Mo and Ni. Hence, Mo and Ni will not show photoelectric emission.

If the source of light is brought near the photocells and placed 50 cm away from them, then the intensity of radiation will increase. This does not affect the energy of the radiation. Hence, the result will be the same as before. However, the photoelectrons emitted from Na and K will increase in proportion to intensity.

22.Light of intensity 10^{-5} W m^{-2} falls on a sodium photo-cell of surface area $2\ cm^2$. Assuming that the top 5 layers of sodium absorb the incident energy, estimate time required for photoelectric emission in the wave-picture of radiation. The work function

for the metal is given to be about 2 eV. What is the implication of your answer?

Ans. Intensity of incident light, *I*= $10^{-5} W\ m^{-2}$

Surface area of a sodium photocell, *A*= $2\ cm^2 = 2 \times 10^{-4} m^2$

Incident power of the light, $P = I \times A$

$= 10^{-5} \times 2 \times 10^{-4} = 2 \times 10^{-9} W$

Work function of the metal, ϕ_0 = 2 eV

$= 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19} J$

Number of layers of sodium that absorbs the incident energy, *n*= 5

We know that the effective atomic area of a sodium atom, $A_e\ is\ 10^{-20} m^2$.

Hence, the number of conduction electrons in *n*layers is given as:

$$n' = n \times \frac{A}{A_e}$$

$$= 5 \times \frac{2 \times 10^{-4}}{10^{-20}} = 10^{17}$$

The incident power is uniformly absorbed by all the electrons continuously. Hence, the amount of energy absorbed per second per electron is:

$$E = \frac{P}{n'}$$

$$= \frac{2 \times 10^{-9}}{10^{17}} = 2 \times 10^{-26} J/s$$

Time required for photoelectric emission:

$$t = \frac{\phi_0}{E}$$

$$=\frac{3.2\times10^{-19}}{2\times10^{-26}}=1.6\times10^{7}s\approx0.507\ years$$

The time required for the photoelectric emission is nearly half a year, which is not practical. Hence, the wave picture is in disagreement with the given experiment.

23.Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy? (For quantitative comparison, take the wavelength of the probe equal to 1 $\overset{0}{\mathrm{A}}$ ¦, which is of the order of inter-atomic spacing in the lattice) $\left(m_e = \mathbf{9.11\times10^{-31}kg}\right)$ **.**

Ans. An X-ray probe has a greater energy than an electron probe for the same wavelength.

Wavelength of light emitted from the probe, $\lambda = 1\overset{0}{\mathrm{A}} = 10^{-10}m$

Mass of an electron, *me*= $9.11\times10^{-31}kg$

Planck's constant, *h*= $6.6\times10^{-34}Js$

Charge on an electron, *e*= $1.6\times10^{-19}C$

The kinetic energy of the electron is given as:

$$E=\frac{1}{2}m_ev^2$$

$m_ev=\sqrt{2Em_e}$ s

Where, *v*= Velocity of the electron

m_ev = Momentum (*p*) of the electron

According to the de Broglie principle, the de Broglie wavelength is given as:

$$\lambda=\frac{h}{P}=\frac{h}{m_ev}=\frac{h}{\sqrt{2Em_e}}$$

$$\therefore E = \frac{h^2}{2\lambda^2 m_e}$$

$$= \frac{\left(6.6\times10^{-34}\right)^2}{2\times\left(10^{-10}\right)^2\times9.11\times10^{-31}} = 2.39\times10^{-17} J$$

$$= \frac{2.39\times10^{-17}}{1.6\times10^{-19}} = 149.375 eV$$

Energy of a photon, $E' = \frac{hc}{\lambda e} eV$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{10^{-10}\times1.6\times10^{-19}}$$

$$= 12.375\times10^{3}\ eV = 12.375\ keV$$

Hence, a photon has a greater energy than an electron for the same wavelength.

24.(a) Obtain the de Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in Exercise 11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain. $\left(mn = \mathbf{1.675\times10^{-27} kg}\right)$

(b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature $\left(27\ ^{\circ}C\right)$**. Hence explain why a fast neutron beam needs to be the rmalised with the environment before it can be used for neutron diffraction experiments.**

Ans.(a) De Broglie wavelength = $2.327\times10^{-12} m$; neutron is not suitable for the diffraction experiment

Kinetic energy of the neutron, K= 150 eV

$$= 150 \times 1.6 \times 10^{-19}$$

$= 2.4 \times 10^{-17} J$

Mass of a neutron, *mn*= $1.675 \times 10^{-27} kg$

The kinetic energy of the neutron is given by the relation:

$$K = \frac{1}{2} m_n v^2$$

$$m_n v = \sqrt{2Km_n}$$

Where, *v* = Velocity of the neutron

$m_n v$ = Momentum of the neutron

De-Broglie wavelength of the neutron is given as:

$$\lambda = \frac{h}{m_e v} = \frac{h}{\sqrt{2Km_n}}$$

It is clear that wavelength is inversely proportional to the square root of mass.

Hence, wavelength decreases with increase in mass and vice versa.

$= 2.327 \times 10^{-12} m$

It is given in the previous problem that the inter-atomic spacing of a crystal is about $1 \overset{0}{A}$, i.e., $10^{-10} m$. Hence, the inter-atomic spacing is about a hundred times greater. Hence, a neutron beam of energy 150 eV is not suitable for diffraction experiments.

(b) De Broglie wavelength = $1.447 \times 10^{-10} m$

Room temperature, $T = 27°C$ = 27 + 273 = 300 K

The average kinetic energy of the neutron is given as:

$$E = \frac{3}{2} kT$$

Where, k = Boltzmann constant = $1.38 \times 10^{-23} J\ mol^{-1} K^{-1}$

The wavelength of the neutron is given as:

$$\lambda = \frac{h}{\sqrt{2m_n E}} = \frac{h}{\sqrt{3m_n kT}}$$

$$= 1.447 \times 10^{-10}\ m$$

This wavelength is comparable to the inter-atomic spacing of a crystal. Hence, the high-energy neutron beam should first be the rmalised, before using it for diffraction.

25.An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Ans. Electrons are accelerated by a voltage, V= 50 kV = $1.38 \times 10^{-23} J\ mol^{-1} K^{-1}$

Charge on an electron, e= $1.6 \times 10^{-19} C$

Mass of an electron, me= $9.11 \times 10^{-31} kg$

Wavelength of yellow light = $5.9 \times 10^{-7} m$

The kinetic energy of the electron is given as:

$E = eV$

$$= 1.6 \times 10^{-19} \times 50 \times 10^{3}$$

$$= 8 \times 10^{-15} J$$

De Broglie wavelength is given by the relation:

$$\lambda = \frac{h}{\sqrt{2m_n E}}$$

$$= \frac{6.6\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times8\times10^{-15}}} = 5.467\times10^{-12}\,m$$

This wavelength is nearly 105 times less than the wavelength of yellow light.

The resolving power of a microscope is inversely proportional to the wavelength of light used. Thus, the resolving power of an electron microscope is nearly 105times that of an optical microscope.

26.The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length-scale of 10-15m or less. This structure was first probed in early 1970's using high energy electron beams produced by a linear accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV.)

Ans. Wavelength of a proton or a neutron, $\lambda \approx 10^{-15} m$

Rest mass energy of an electron:

$$m_0c^2 = 0.511\,MeV$$

$= 0.511 \times 10^{6}\times 1.6 \times 10^{-19}$ Planck's constant, *h*= 6.6 × 10 - 34Js

$$= 0.8176 \times 10^{-13}\,J$$

Speed of light, *c*= $3 \times 10^{8}\,m/s$

The momentum of a proton or a neutron is given as:

$$P = \frac{h}{\lambda}$$

$$= \frac{6.6\times10^{-34}}{10^{-15}} = 6.6\times10^{-19}\,kg\,m/s$$

The relativistic relation for energy (*E*) is given as:

$$E^2 = p^2c^2 + m_0^2c^4$$

$$= \left(6.6\times10^{-19}\times3\times10^{8}\right)+\left(0.8176\times10^{-13}\right)$$

$$= 392.04\times10^{-22} + 0.6685\times10^{-26}$$

$$\approx 392.04\times10^{-22}$$

$$\therefore E = 1.98\times10^{-10}\,J$$

$$= \frac{1.98\times10^{-10}}{1.6\times10^{-19}}$$

$$= 1.24\times10^{9}\,eV = 1.24\,BeV$$

Thus, the electron energy emitted from the accelerator at Stanford, USA might be of the order of 1.24 BeV.

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27. Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature $(27\ ^\circ\mathrm{C})$ and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.

Ans. De Broglie wavelength associated with He atom = $0.7268\times10^{-10}\,m$

Room temperature, *T*= $27^\circ C$ = 27 + 273 = 300 K

Atmospheric pressure, *P*= 1 atm = $1.01\times10^{5}\,Pa$

Atomic weight of a He atom = 4

Avogadro's number, NA= 6.023×10^{23}

Boltzmann constant, *k*= $1.38\times10^{-23}\,J\,mol^{-1}K^{-1}$

Average energy of a gas at temperature *T*, is given as:

$$E = \frac{3}{2}kT$$

De Broglie wavelength is given by the relation:

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Where, m= Mass of a He atom

$$= \frac{Atomic\,weight}{N_A}$$

$$= \frac{4}{6.023 \times 10^{23}}$$

$$= 6.64 \times 10^{-24}\,g = 6.64 \times 10^{-27}\,kg$$

$$\therefore \lambda = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.6 \times 10^{-34}}{3 \times 6.64 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}$$

$$= 0.7268 \times 10^{-10}\,m$$

We have the ideal gas formula:

$PV = RT$

$PV = kNT$

$$\frac{V}{N} = \frac{kT}{P}$$

Where, V= Volume of the gas

N= Number of moles of the gas

Mean separation between two atoms of the gas is given by the relation:

$$r = \left(\frac{V}{N}\right)^{\frac{1}{3}} = \left(\frac{kT}{P}\right)^{\frac{1}{3}}$$

$$= \left[\frac{1.38\times10^{-23}\times300}{1.01\times10^{5}}\right]^{\frac{1}{3}}$$

$$= 3.35\times10^{-9}\ m$$

Hence, the mean separation between the atoms is much greater than the de Broglie wavelength.

28.Compute the typical de Broglie wavelength of an electron in a metal at $27\ ^\circ C$ and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m.

[*Note:* Exercises 11.35 and 11.36 reveal that while the wave-packets associated with gaseous molecules under ordinary conditions are non-overlapping, the electron wave-packets in a metal strongly overlap with one another. This suggests that whereas molecules in an ordinary gas can be distinguished apart, electrons in a metal cannot be distinguished apart from one another. This in distinguish ability has many fundamental implications which you will explore in more advanced Physics courses.]

Ans. Temperature, $T = 27°C$ = 27 + 273 = 300 K

Mean separation between two electrons, *r*= $2\times10^{-10} m$

De Broglie wavelength of an electron is given as:

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

Where, *h*= Planck's constant = $6.6\times10^{-34} Js$

m= Mass of an electron = $9.11\times10^{-31} kg$

k= Boltzmann constant = $1.38\times10^{-23} J\ mol^{-1}K^{-1}$

$$\therefore \lambda = \frac{6.6\times10^{-34}}{\sqrt{3\times9.11\times10^{-31}\times1.38\times10^{-23}\times300}}$$

$$\approx 6.2\times10^{-9}\,m$$

Hence, the de Broglie wavelength is much greater than the given inter-electron separation.

29.Answer the following questions:

(a) Quarks inside protons and neutrons are thought to carry fractional charges [(+2/3)*e*; (- 1/3)*e*]. Why do they not show up in Millikan's oil-drop experiment?

(b) What is so special about the combination *e/m*? Why do we not simply talk of *e* and *m* separately?

(c) Why should gases be insulators at ordinary pressures and start conducting at very low pressures?

(d) Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?

(e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations:

$$E = h\nu \;, p = \frac{h}{\lambda}$$

But while the value of λ is physically significant, the value of ν (and therefore, the value of the phase speed $\lambda\nu$) has no physical significance. Why?

Ans.(a) Quarks inside protons and neutrons carry fractional charges. This is because nuclear force increases extremely if they are pulled apart. Therefore, fractional charges may exist in nature; observable charges are still the integral multiple of an electrical charge.

(b) The basic relations for electric field and magnetic field are

$\left(eV = \frac{1}{2}mv^2\right) and \left(eBv = \frac{mv^2r}{r}\right)$ respectively

These relations include *e*(electric charge), *v*(velocity), *m*(mass), *V*(potential), *r*(radius), and *B*(magnetic field). These relations give the value of velocity of an electron as $\left(c = \sqrt{2V\left(\frac{e}{n}\right)}\right)$ and

$\left(v = Br\left(\frac{e}{m}\right)\right)$ respectively.

It can be observed from these relations that the dynamics of an electron is determined not by *e* and *m* separately, but by the ratio e/*m*.

(c) At atmospheric pressure, the ions of gases have no chance of reaching their respective electrons because of collision and recombination with other gas molecules. Hence, gases are insulators at atmospheric pressure. At low pressures, ions have a chance of reaching their respective electrodes and constitute a current. Hence, they conduct electricity at these pressures.

(d) The work function of a metal is the minimum energy required for a conduction electron to get out of the metal surface. All the electrons in an atom do not have the same energy level. When a ray having some photon energy is incident on a metal surface, the electrons come out from different levels with different energies. Hence, these emitted electrons show different energy distributions.

(e) The absolute value of energy of a particle is arbitrary within the additive constant. Hence, wavelength (λ) is significant, but the frequency (v) associated with an electron has no direct physical significance.

Therefore, the product $v\lambda$ (phase speed) has no physical significance.

Group speed is given as:

$$v_G = \frac{dv}{dk}$$

$$= \frac{d\nu}{d\left(\frac{1}{\lambda}\right)} = \frac{dE}{dp} = \frac{d\left(\frac{p^2}{2m}\right)}{dp} = \frac{p}{m}$$

This quantity has a physical meaning.

CBSE Class 12 physics

Important Questions

Chapter 12

Atoms

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1 Mark Questions

1. Name the series of hydrogen spectrum lying in ultraviolet and visible region?

Ans. Lyman series in ultraviolet region and Balmer series in visible region.

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2. What is Bohr's quantisation condition for the angular momentum of an electron in the second orbit?

Ans. Since $L = \frac{nh}{2\pi}$ where $n = 2$

$$L = \frac{2h}{2\pi}$$

$$\Rightarrow L = \frac{h}{\pi}$$

2 Mark Questions

1. Define Bohr's radius?

Ans. The radius of the first orbit of hydrogen atom is called Bohr's radius.

It is equal to $5.29 \times 10^{-11} m = 0.53 A^{o}$.

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2. State the limitations of Bohr's atomic model?

Ans. (1) It does not give any indication regarding the arrangement and distribution of electrons in on atom.

(2) It could not account for the wave nature of electrons.

3. Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?

Ans. In the alpha-particle scattering experiment, if a thin sheet of solid hydrogen is used in place of a gold foil, then the scattering angle would not be large enough. This is because the mass of hydrogen ($1.67 \times 10^{-27}\ kg$) is less than the mass of incident α-particles ($6.64 \times 10^{-27}\ kg$). Thus, the mass of the scattering particle is more than the target nucleus (hydrogen). As a result, the α-particles would not bounce back if solid hydrogen is used in the α-particle scattering experiment.

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4. The ground state energy of hydrogen atom is -13.6 eV. What are the kinetic and potential energies of the electron in this state?

Ans. Ground state energy of hydrogen atom, *E* = - 13.6 eV

This is the total energy of a hydrogen atom. Kinetic energy is equal to the negative of the total energy.

Kinetic energy = - *E* = - (- 13.6) = 13.6 eV

Potential energy is equal to the negative of two times of kinetic energy.

Potential energy = $-\ 2\times(13.6)$ = - 27 .2 eV

5. If Bohr's quantisation postulate (angular momentum $= nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Ans. We never speak of quantization of orbits of planets around the Sun because the angular momentum associated with planetary motion is largely relative to the value of Planck's constant *(h)*. The angular momentum of the Earth in its orbit is of the order of 1070*h*. This leads to a very high value of quantum levels *n* of the order of 1070. For large values of *n*, successive energies and angular momenta are relatively very small. Hence, the quantum levels for planetary motion are considered continuous.

3 Mark Questions

1. The half life period of a radioactive substance is 30 days. What is the time for $\frac{3}{4}th$ of its original mass to disintegrate?

Ans. $\frac{N}{N_o} = \left(\frac{1}{2}\right)\frac{t}{T}$

Here $N = N_o - \frac{3}{4}N_o$

$N = \frac{1}{4}N_o$

$\Rightarrow \frac{1}{4} = \left(\frac{1}{2}\right)\frac{t}{30}$

Or $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)\frac{t}{30}$

$\frac{t}{30} = 2$

Or

t = 60 days

---

2. How many α and β-particles **are emitted when** $90\overset{232}{Th}$ **changes to** $82\overset{208}{Pb}$

Ans. $90\overset{232}{Th} \rightarrow 82\overset{208}{Pb} + 2\overset{4}{He} + y\ {}_{-1}^{o}e$

According to low of conservation of atomic number and mass number

90 = 82 + 2x – y

2x – y = 8 ----------- (1)

232 = 208 + 4x

$\Rightarrow$ x = 6 ----------- (2)

From (1) & (2)

2 (6) – y = 8

12 – 8 = y

Or

y = 4

---

3. Binding energies of $_{8}^{16}O$ and $_{17}^{35}Cl$ are 127.35 MeV and 289.3 MeV respectively.

Which of the two nuclei are more stable?

Ans. Stability of a nucleus is proportional to binding energy per nucleon

B.E / nucleon of

$$_{8}^{16}O = \frac{127.35}{8} = 15.82 MeV / nucleon$$

B.E / nucleon of

$$_{17}^{35}Cl = \frac{289.3}{17} = 17.02 MeV / nucleon$$

$\therefore$ $_{17}^{35}Cl$ Is more stable than $_{8}^{16}O$

4. What is the shortest wavelength present in the Paschen series of spectral lines?

Ans. Rydberg's formula is given as:

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19}[\frac{1}{n_1^2} - \frac{1}{n_2^2}]$$

Where,

h = Planck's constant = $6.6 \times 10^{-34}\ Js$

c = Speed of light = $3 \times 10^{8}\ m/s$

(n_1 and n_2 are integers)

The shortest wavelength present in the Paschen series of the spectral lines is given for values n_1 = 3 and $n_2 = \infty$.

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19}\left[\frac{1}{(3)^2} - \frac{1}{(\infty)^2}\right]$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8} \times 9}{21.76 \times 10^{-19}}$$

$$= 8.189 \times 10^{7}\ m$$

$$= 818.9\ nm$$

---

5. A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?

Ans. Separation of two energy levels in an atom,

E = 2.3 eV

$$= 2.3 \times 1.6 \times 10^{-19}$$

$= 3.68 \times 10^{-19}\ J$

Let ν be the frequency of radiation emitted when the atom transits from the upper level to the lower level.

We have the relation for energy as:

$E = h\nu$

Where,

h = Planck's constant $= 6.62 \times 10^{-4}\ Js$

$$\therefore \nu = \frac{E}{h}$$

$$= \frac{3.68 \times 10^{-19}}{6.62 \times 10^{-32}} = 6.62 \times 10^{-32} = 5.55 \times 10^{14}$$

Hence, the frequency of the radiation is 5.6×10^{14} Hz.

---

6. The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the *n* = 2 and *n* =3 orbits?

Ans. The radius of the innermost orbit of a hydrogen atom, $r_1 = 5.3 \times 10^{-11}\ m.$

Let r_2 be the radius of the orbit at n = 2. It is related to the radius of the innermost orbit as:

$$r_2 = (n)^2 r_1$$

$$= 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10} m$$

For n = 3, we can write the corresponding electron radius as:

$$r_3 = (n)^2 r_1$$

$$= 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10} m$$

Hence, the radii of an electron for n = 2 and n = 3 orbits are $2.12\times10^{-10} m \text{ and } 4.77\times10^{-10} m$ respectively.

---

7. In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^{4} m / s. (Mass of earth = 6.0×10^{24} kg.)

Ans. Radius of the orbit of the Earth around the Sun, r = $1.5 \times 10^{11}\ m$

Orbital speed of the Earth, $v = 3 \times 10^{4}\ m/s$

Mass of the Earth, m = $6.0 \times 10^{24}\ kg$

According to Bohr's model, angular momentum is quantized and given as:

$$mvr = \frac{nh}{2\pi}$$

Where,

h = Planck's constant = $6.62 \times 10^{-34}\ Js$

n = Quantum number

$$\therefore n = \frac{mvr2\pi}{h}$$

$$= \frac{2\pi\times6\times10^{24}\times3\times10^{4}\times1.5\times10^{11}}{6.62\times10^{-34}}$$

$$= 25.61\times10^{73} = 2.6\times10^{74}$$

Hence, the quanta number that characterizes the Earth' revolution is 2.6×10^{74}.

8. The total energy of an electron in the first excited state of the hydrogen atom is about -3.4 eV.

(a) What is the kinetic energy of the electron in this state?

(b) What is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Ans.(a) Total energy of the electron, $E = -3.4$ eV

Kinetic energy of the electron is equal to the negative of the total energy.

$\Rightarrow K = -E$

$= -(-3.4) = +3.4$ eV

Hence, the kinetic energy of the electron in the given state is +3.4 eV.

(b) Potential energy (U) of the electron is equal to the negative of twice of its kinetic energy.

$\Rightarrow U = -2K$

$= -2 \times 3.4 = -6.8\ eV$

Hence, the potential energy of the electron in the given state is - 6.8 eV.

(c) The potential energy of a system depends on the reference point taken. Here, the potential energy of the reference point is taken as zero. If the reference point is changed, then the value of the potential energy of the system also changes. Since total energy is the sum of kinetic and potential energies, total energy of the system will also change.

5 Mark Questions

1. THE total energy of and electron in the first excited state of hydrogen atom is -3.4 eV. Calculate

(1) K.E. of the electron in this state.

(2) P.E. of the electron in this state and

(3) Which of the answer would change if zero of PE is changed? Justify your answer?

Ans. (i) K.E = -E = 3.4eV

(ii) P.E = 02 $\times$ K.E

P.E = 02 $\times$ 3.4

(iii) If the zero of the P.E is changed, K.E remains unchanged but the P.E will change, hence total energy will change.

---

2. Prove that the speed of election in the ground sate of hydrogen atom is equal to the speed of electron in the first excited state of hydrogen like Li^{++} atom.

Ans. $\upsilon_n = \frac{2\pi Ke^2}{nh}$

For ground state of hydrogen atom x = 1 $\upsilon_1 = \frac{2\pi Ke^2}{h}$

From hydrogen like atom $\left(\upsilon_n\right)_\mu = \frac{Z\times 2\pi Ke^2}{nh}$ --------(1)

For Li^{++} atom z = 3 n = 2

$$\Rightarrow (\upsilon_n)_{Li^{++}} = \frac{2\times 2\pi K e^2}{2h}$$

$$(\upsilon_n)_{Li^{++}} = \frac{2\pi K e^2}{h} \text{ ------(2)}$$

Hence from (1) and (2)

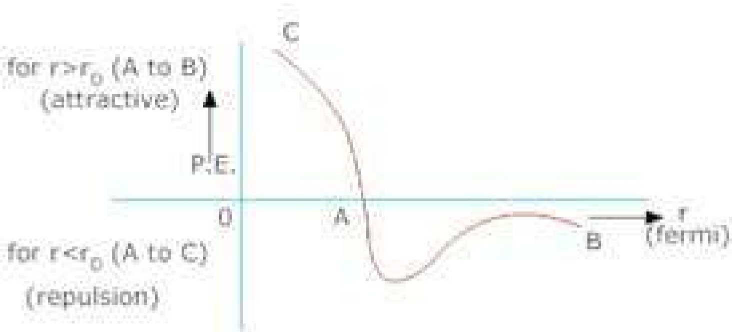
$$(\upsilon_n)_H = (\upsilon_n)_{Li^{++}}$$

---

3. Draw a graph showing variation of potential energy of a pair of nucleon as a function of their separation indicate the region in which the nuclear force is (a) Attractive (b) Repulsive. Also write two characteristics features which distinguish it from the coulomb's force.

Ans.(i) Nuclear forces are charge independent.

(ii) They are non – central forces.



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4. Choose the correct alternative from the clues given at the end of the each statement:

(a) The size of the atom in Thomson's model is .... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

(b) In the ground state of .... electrons are in stable equilibrium, while in ....

electrons always experience a net force.

(Thomson's model/ Rutherford's model.)

(c) A *classical* atom based on .... is doomed to collapse.

(Thomson's model/ Rutherford's model.)

(d) An atom has a nearly continuous mass distribution in a .... but has a highly non-uniform mass distribution in ....

(Thomson's model/ Rutherford's model.)

(e) The positively charged part of the atom possesses most of the mass in .... (Rutherford's model/both the models.)

Ans.(a) The sizes of the atoms taken in Thomson's model and Rutherford's model have the same order of magnitude.

(b) In the ground state of Thomson's model, the electrons are in stable equilibrium. However, in Rutherford's model, the electrons always experience a net force.

(c) A *classical* atom based on Rutherford's model is doomed to collapse.

(d) An atom has a nearly continuous mass distribution in Thomson's model, but has a highly non-uniform mass distribution in Rutherford's model.

(e) The positively charged part of the atom possesses most of the mass in both the models.

---

5. A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of the photon.

Ans. For ground level, n_1 = 1

Let E_1 be the energy of this level. It is known that E_1 is related with n_1 as:

$$E_1 = \frac{-13.6}{n_1^2}$$

$$= \frac{-13.6}{1^2} = -13.6\, eV$$

The atom is excited to a higher level, n_2 = 4.

Let E_2 be the energy of this level.

$$\therefore E_2 = \frac{-13.6}{n_2^2} eV$$

$$= \frac{-13.6}{4^2} = -13.6\, eV$$

The amount of energy absorbed by the photon is given as:

E = E_2 - E_1

$$= \frac{-13.6}{16} - \left(-\frac{13.6}{1}\right)$$

$$= \frac{13.6 \times 15}{16} eV$$

$$= \frac{13.6 \times 15}{16} \times 1.6 \times 10^{-19} = 2.04 \times 10^{-18} J$$

For a photon of wavelength λ, the expression of energy is written as:

$$E = \frac{hc}{\lambda}$$

Where,

h = Planck's constant = 6.6×10^{-34} Js

c = Speed of light = 3×10^8 m/s

$$\therefore \lambda = \frac{hc}{E}$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{2.04\times10^{-18}}$$

$$= 9.7\times10^{-8}m = 97nm$$

And, frequency of a photon is given by the relation,

$$\nu = \frac{c}{\lambda}$$

$$= \frac{3\times10^{8}}{9.7\times10^{-8}} \approx 3.1\times10^{15}\,Hz$$

Hence, the wavelength of the photon is 97 nm while the frequency is $3.1\times10^{15}\,Hz$.

---

6. (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the *n* = 1, 2, and 3 levels. (b) Calculate the orbital period in each of these levels.

Ans. (a) Let v_1 be the orbital speed of the electron in a hydrogen atom in the ground state level, n_1 = 1. For charge (*e)* of an electron, v_1 is given by the relation,

$$v_1 = \frac{e^2}{n_1 4\pi\epsilon_0\left(\frac{h}{2\pi}\right)} = \frac{e^2}{2\epsilon_0 h}$$

Where,

e = $1.6\times10^{-19}\,C$

∈0 = Permittivity of free space = $8.85\times10^{-12}N^{-1}C^2m^{-2}$

h = Planck's constant = $6.62\times10^{-34}\,Js$

$$\therefore v_1 = \frac{\left(1.6\times10^{-19}\right)^2}{2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$= 0.0218\times10^{8} = 2.18\times10^{6}\,m/s$$

For level n_2 = 2, we can write the relation for the corresponding orbital speed as:

$$v_1 = \frac{e^2}{n_2 2 \in_0 h}$$

$$= \frac{\left(1.6\times10^{-19}\right)^2}{2\times2\times8.85\times10^{-22} \therefore 6.62\times10^{-34}} \text{ s}$$

$$= 1.09\times10^6 \, m/s$$

And, for n_3 = 3, we can write the relation for the corresponding orbital speed as:

$$v_3 = \frac{e^2}{n_2 2 \in_0 h}$$

$$= \frac{\left(1.6\times10^{-19}\right)^2}{3\times2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$= 7.27\times10^6 \, m/s$$

Hence, the speed of the electron in a hydrogen atom in n = 1, n=2, and n=3 is 2.18×10^6 m/s, $1.09\times10^6 \; m/s$, $7.27 \times 10^5 \; m/s$ respectively.

(b) Let *T*1 be the orbital period of the electron when it is in level *n*1 = 1.

Orbital period is related to orbital speed as:

$$T_1 = \frac{2\pi r_1}{v_1}$$

Where,

r_1 = Radius of the orbit

$$= \frac{n_1^2 h^2 \in_0}{\pi m e^2}$$

h = Planck's constant = $6.62 \times 10^{-34}\ Js$

e = Charge on an electron = $1.6 \times 10^{-19}\ C$

∈0 = Permittivity of free space = $8.85 \times 10^{-12}\ N^{-1}\ C^2\ m^{-2}$

m = Mass of an electron = $9.1 \times 10^{-31}\ kg$

$$\therefore T_1 = \frac{2\pi r_1}{v_1}$$

$$= \frac{2\pi \times (1)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{2.18 \times 10^6 \times \pi \times 9.1 \times 10^{31} \times \left(1.6 \times 10^{-19}\right)^2}$$

$$= 15.27 \times 10^{-17} = 1.527 \times 10^{-16}$$

For level n_2 = 2, we can write the period as:

$$T_2 = \frac{2\pi r_2}{v_2}$$

Where,

r_2 = Radius of the electron in n_2 = 2

$$= \frac{(n_2)^2\ h^2\ \in_0}{\pi m e^2}$$

$$\therefore T_2 = \frac{2\pi \times (2)^2}{v_2}$$

$$= \frac{2\pi \times (2)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{1.09 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2}$$

$$= 1.22 \times 10^{-15}$$

And, for level $n_3 = 3$, we can write the period as:

$$T_3 = \frac{2\pi r_2}{v_3}$$

Where,

$$r_3 = Radius\ of\ the\ electron\ \ in n_3 = 3\ \frac{(n_3)^2 h^2 \in_0}{\pi m e^2}$$

$$\therefore T_3 = \frac{2\pi r_3}{v_3}$$

$$\frac{(n_3)^2 h^2 \in_0}{\pi m e^2}$$

$$= \frac{2\pi \times (3)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{7.27 \times 10^5 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$= 4.12 \times 10^{-15}$$

$$\therefore T_3 = \frac{2\pi r_3}{v_3}$$

Hence, the orbital period in each of these levels is $1.52 \times 10^{-16}\ s,\ 1.22 \times 10^{-15}\ s,$ $and\ 4.12 \times 10^{-15} s$ respectively.

Q9. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Ans. It is given that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5 eV. Also, the energy of the gaseous hydrogen in its ground state at room temperature is - 13.6 eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes - 13.6 + 12.5 eV i.e., - 1.1 eV.

Orbital energy is related to orbit level (*n)* as:

$$E = \frac{-13.6}{(n)^2} eV$$

For n = 3, $E = \frac{-13.6}{9} = -1.5 eV$

This energy is approximately equal to the energy of gaseous hydrogen. It can be concluded that the electron has jumped from n = 1 to n = 3 level.

During its de-excitation, the electrons can jump from n = 3 to n = 1 directly, which forms a line of the Lyman series of the hydrogen spectrum.

We have the relation for wave number for Lyman series as:

$$\frac{1}{\lambda} = R_y \left(\frac{1}{l^2} - \frac{1}{n^2} \right)$$

Where,

Ry = Rydberg constant = $1.097 \times 107\ m^{-1}$

λ = Wavelength of radiation emitted by the transition of the electron

For n = 3, we can obtain λ as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{l^2} - \frac{1}{3^2} \right)$$

$$= 1.097 \times 10^7 \left(1 - \frac{1}{9} \right) = 1.097 \times 10^7 \times \frac{8}{9}$$

$$\lambda = \frac{9}{8 \times 1.097 \times 10^7} = 102.55 nm$$

If the electron jumps from n = 2 to n = 1, then the wavelength of the radiation is given as:

$$\frac{1}{\lambda}=1.097\times10^{7}\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

$$=1.097\times10^{7}\left(1-\frac{1}{4}\right)=1.097\times10^{7}\times\frac{3}{4}$$

$$\lambda=\frac{4}{1.097\times10^{7}\times3}=121.54nm$$

If the transition takes place from n = 3 to n = 2, then the wavelength of the radiation is given as:

$$\frac{1}{\lambda}=1.097\times10^{7}\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$=1.097\times10^{7}\left(\frac{1}{4}-\frac{1}{9}\right)=1.097\times10^{7}\times\frac{5}{36}$$

$$\lambda=\frac{36}{5\times1.097\times10^{7}}=656.33nm$$

This radiation corresponds to the Balmer series of the hydrogen spectrum. Hence, in Lyman series, two wavelengths i.e., 102.5 nm and 121.5 nm are emitted. And in the Balmer series, one wavelength i.e., 656.33 nm is emitted.

10.Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

(a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(b) Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(c) Keeping other factors fixed, it is found experimentally that for small thickness t, the

number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

(d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Ans.(a) about the same

The average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model is about the same size as predicted by Rutherford's model. This is because the average angle was taken in both models.

(b) much less

The probability of scattering of α-particles at angles greater than $90°$ predicted by Thomson's model is much less than that predicted by Rutherford's model.

(c) Scattering is mainly due to single collisions. The chances of a single collision increases linearly with the number of target atoms. Since the number of target atoms increase with an increase in thickness, the collision probability depends linearly on the thickness of the target.

(d) Thomson's model

It is wrong to ignore multiple scattering in Thomson's model for the calculation of average angle of scattering of α-particles by a thin foil. This is because a single collision causes very little deflection in this model. Hence, the observed average scattering angle can be explained only by considering multiple scattering.

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11. The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-14}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Ans. Radius of the first Bohr orbit is given by the relation,

$$r_1 = \frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2} \text{.................(1)}$$

Where,

∈0 = Permittivity of free space

$h = \ Planck's \ constant \ = \ 6.63 \times 10^{-34} \ Js$

$me \ = \ Mass \ of \ an \ electron = \ 9.1 \times 10^{-31} \ kg$

$e = \ Charge \ of \ an \ electron = \ 1.9 \times 10^{-19} \ C$

$mp \ = \ Mass \ of \ a \ proton = \ 1.67 \times 10^{-27} \ kg$

r = Distance between the electron and the proton

Coulomb attraction between an electron and a proton is given as:

$$F_G = \frac{e^2}{4\pi \in_0 r^2} \text{.................(2)}$$

Gravitational force of attraction between an electron and a proton is given as:

$$F_G = \frac{Gm_p m_c}{r_2} \text{.................(3)}$$

Where,

G = Gravitational constant = $6.67 \times 10^{-11} \ N \ m^2 / kg^2$

If the electrostatic (Coulomb) force and the gravitational force between an electron and a proton are equal, then we can write:

∴*F*G = *F*C

$$\frac{Gm_p m_e}{r_2} = \frac{e^2}{4\pi \in_0 r^2}$$

$$\therefore \frac{e^2}{4\pi \in_0} = Gm_p m_e \text{............}(4)$$

Putting the value of equation (4) in equation (1), we get:

$$r_1 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_p m_e^2}$$

$$= \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{6.67\times10^{-11}\times1.67\times10^{-27}\times\left(9.1\times10^{-31}\right)^2} \approx 1.21\times10^{29}\, m$$

It is known that the universe is 156 billion light years wide or $1.5 \times 10^{27}\ m$ wide. Hence, we can conclude that the radius of the first Bohr orbit is much greater than the estimated size of the whole universe.

---

12. Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level *n* to level (*n*-1). For large *n*, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Ans. It is given that a hydrogen atom de-excites from an upper level (*n)* to a lower level (*n* - 1). We have the relation for energy $\left(E_1\right)$ of radiation at level *n* as:

$$E_1 = h\nu_1 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \times \left(\frac{1}{n^2}\right) \text{......}(i)$$

Where,

ν_1 = Frequency of radiation at level n h = plank's constant

m = mass of hydrogen atom

e = charge on an electron

$\in_0$ = Permittivity of free space

Now, the relation for energy (E_2) of radiation at level (*n* - 1) is given as:

$$E_2 = hv_2 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \times \frac{1}{(n-1)^2} \ldots\ldots(ii)$$

Where,

$$v_2 = Frequency\ of\ radiation\ at\ level\,(n-1)$$

Energy (*E*) released as a result of de-excitation:

$$E = E_2 - E_1$$

$$hv = E_2 - E_1 \ \ldots\ (iii)$$

Where,

v = Frequency of radiation emitted

Putting values from equations (i) and (ii) in equation (iii), we get:

$$v = \frac{me^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4(2n-1)}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3 n^2 (n-1)^2}$$

For large n, we can write $(2n-1) \approx 2n$ and $(n-1) \approx n$

$$\therefore \nu = \frac{me^4}{32\pi \in_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \text{..................}(iv)$$

Classical relation of frequency of revolution of an electron is given as:

$$\nu_c = \frac{v}{2\pi r} \text{ .....(v)}$$

Where,

Velocity of the electron in the nth orbit is given as:

$$v = \frac{e^2}{4\pi \in_0 \left(\frac{h}{2\pi}\right) n} \text{ ....(vi)}$$

And, radius of the nth orbit is given as:

$$r = \frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{me^2} n^2 \text{ ...(vii)}$$

Putting the values of equations (vi) and (vii) in equation (v), we get:

$$\nu_c = \frac{me^4}{32\pi^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \text{ .....(viii)}$$

Hence, the frequency of radiation emitted by the hydrogen atom is equal to its classical orbital frequency.

---

13. lassically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger

than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom $\left(\sim\ 10^{-10}\ \mathrm{m}\right)$.

(a) Construct a quantity with the dimensions of length from the fundamental constants *e*, *me*, and *c*. Determine its numerical value.

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves *c*. But energies of atoms are mostly in non-relativistic domain where *c* is not expected to play any role. This is what may have suggested Bohr to discard *c* and look for 'something else' to get the right atomic size. Now, the Planck's constant *h* had already made its appearance elsewhere. Bohr's great insight lay in recognising that *h*, *me*, and *e* will yield the right atomic size. Construct a quantity with the dimension of length from *h*, *me*, and *e* and confirm that its numerical value has indeed the correct order of magnitude.

Ans.(a) Charge on an electron, *e* = $1.6\times10^{-19}\ C$

Mass of an electron, *m*e = $9.1\times10^{-31}\ kg$

Speed of light, *c* = $3\times10^{8}\ m/s$

Let us take a quantity involving the given quantities as $\left(\dfrac{e^2}{4\pi\in_0 m_e c^2}\right)$

Where,

∈0 = Permittivity of free space

And, $\dfrac{1}{4\pi\in_0}=9\times10^{9}\ Nm^2C^{-2}$

The numerical value of the taken quantity will be:

$$\frac{1}{4\pi \in_0} \times \frac{e^2}{m_e c^2}$$

$$= 9 \times 10^9 \times \frac{\left(1.6 \times 10^{-19}\right)^2}{9.1 \times 10^{-31} \times \left(3 \times 10^8\right)^2}$$

$$= 2.81 \times 10^{-15} m$$

Hence, the numerical value of the taken quantity is much smaller than the typical size of an atom.

(b) Charge on an electron $e = 1.6 \times 10^{-19} C$

Mass of an electron, me = $9.1 \times 10^{-31}\ kg$

Planck's constant, h = $6.63 \times 10^{-34}\ Js$

Let us take a quantity involving the given quantities as $\left(\frac{e^2}{4\pi \in^0 m_e c^2}\right)$.

Where,

∈0 = Permittivity of free space

And, $\frac{1}{4\pi \in_0} = \ = 9 \times 10^9\ Nm^2 C^{-2}$

The numerical value of the taken quantity will be:

$$\frac{1}{4\pi \in_0} \times \frac{\left(1.6 \times 10^{-19}\right)^2}{9.1 \times 10^{-31} \times \left(3 \times 10^8\right)^2}$$

$$= 2.81 \times 10^{-15} m$$

Hence, the value of the quantity taken is of the order of the atomic size.

14. Obtain the first Bohr's radius and the ground state energy of a *muonic hydrogen atom* [i.e., an atom in which a negatively charged muon $\left(\mu^{-}\right)$ of mass about 207me orbits around a proton].

Ans. Mass of a negatively charged muon, $m\mu^{-} = 207 m_e$

According to Bohr's model,

Bohr radius, $r_e \propto \left(\frac{1}{m_e}\right)$

And, energy of a ground state *electronic hydrogen atom*, $E_e \propto m_e$.

We have the value of the first Bohr orbit, $r_e = 0.53 A = 0.53 \times 10^{-10} m$

Let r_μ be the radius of *muonic hydrogen atom*.

At equilibrium, we can write the relation as:

$$m_\mu r_\mu = m_e r_e$$

$$207 m_e \times r_\mu = m_e r_e$$

$$\therefore r_\mu = \frac{0.53 \times 10^{-10}}{207} = 2.56 \times 10^{-13} m$$

Hence, the value of the first Bohr radius of a *muonic hydrogen atom* is

$2.56 \times 10^{-13}\ m.$ We have,

Ee= - 13.6 eV

Take the ratio of these energies as:

$$\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu} = \frac{m_e}{207 m_e}$$

$$E_\mu = 207 E_e$$

$$= 207 \times (-13.6) = -2.81 keV$$

Hence, the ground state energy of a muonic hydrogen atom is - 2.81 keV.

CBSE Class 12 physics

Important Questions

Chapter 13

Nuclei

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1 Mark Questions

1. Complete the following nuclear reactions

(a) $4Be^{9} +_{1} H^{1} \rightarrow_{3} Li^{6} +$ ....

(b) $5B^{10} + 2\overset{4}{He} \rightarrow 7N^{13} +$ ....

Ans. (a) $4Be^{9} + 1\overset{1}{H} \rightarrow 3Li^{6} + 2\overset{4}{He}$

(b) $5B^{10} + 2\overset{4}{H}e \rightarrow 7\overset{13}{N} + on^{1}$

---

2. What is $Q - value$ of a nuclear reaction?

Ans. $Q - value =$ (Mass of reactants − Mass of products)

---

3. The wavelengths of some of the spectral lines obtained in hydrogen spectrum are $9546A^{o}$, $6463A^{o}$ and $1216A^{o}$. Which one of these wavelengths belongs to Lyman series?

Ans. $1216A^{o}$ belongs to Lyman series

---

4. Write the empirical relation for paschen series lines of hydrogen atom?

Ans. $\frac{1}{\lambda} = R\left(\frac{1}{3^{2}} - \frac{1}{ni^{2}}\right)$ where $ni = 4, 5, 6, ---$

2 Mark Questions

1. What fraction of tritium will remain after 25 years? Given half life of tritium as 12.5 years

Ans. $\frac{N}{N_o}=\left(\frac{1}{2}\right)^{t/T}=\left(\frac{1}{2}\right)^{25/12.5}$

$$\frac{N}{N_o}=\left(\frac{1}{2}\right)^{2}=1/4=0.25$$

2.Calculate the kinetic energy and potential energy of an electron in the first orbit of hydrogen atom. Given $e=1.6\times10^{-19}\,C$ **and** $r=0.53\times10^{-10}\,m$.

Ans.(i) $K.E.=\frac{e^2K}{2a_0}$

$$K.E.=\frac{(1.6\times10^{-19})^2\times9\times10^9}{2\times0.53\times10^{-10}}$$

$$K.E.=21.74\times10^{-19}\,J$$

$$K.E.=\frac{21.74\times10^{-19}}{1.6\times10^{-19}}=13.59eV$$

K.E. = 13.59eV

(ii) $P.E.=\frac{-e^2K}{r}=-2K.E.$

$$P.E.=-2\times13.59$$

P.E. = -27.18eV

3. Why is nuclear fusion not possible in laboratory?

Ans. Nuclear fusion is not possible in laboratory as it is performed in high temperature. This cannot be attained in the laboratory.

---

4. Express 16mg mass into equivalent energy in electron volt?

Ans. $E = mc^2$

= $16 \times 10^{-6} Kg(3 \times 10^8 m/s)^2$

= $16 \times 9 \times 10^{10}$ Joules

$$E = \frac{16 \times 9 \times \times 10^{10}}{1.6 \times 10^{-19}} eV$$

$$E = 9 \times 10^{30} eV$$

5.The three stable isotopes of neon: $^{20}_{10}Ne$, $^{21}_{10}Ne$ and $^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Ans. tomic mass of $^{20}_{10}Ne$, m_1 = 19.99 u

Abundance of, $\eta_1 = 90.51\%$

Atomic mass of, $m_2 = 20.99u$

Abundance of, $\eta_2 = 0.27\%$

Atomic mass of, $m_3 = 21.99\ u$

Abundance of, $\eta_3 = 9.22\%$

The average atomic mass of neon is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$= \frac{19.99 \times 90.51 \times 20.99 \times 0.27 \times 21.99 \times 9.22}{90.51 + 0.27 + 9.22}$$

$$= 20.1771u$$

---

6. From the relation $R = R_0 A^{1/3}$, where *R*0is a constant and *A* is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of *A*).

Ans. We have the expression for nuclear radius as:

$R = R_0 A^{1/3}$ Where,

*R*0= Constant.

A= Mass number of the nucleus

Nuclear matter density, $\rho = \frac{Mass\ of\ the\ nucleus}{Volume\ of\ me\ nucleus}$

Let *m* be the average mass of the nucleus.

Hence, mass of the nucleus = *mA*

$$\therefore p = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi\left(R_0 A^{\frac{1}{3}}\right)^3} = \frac{3mA}{4\pi R_0^3 \mathrm{A}} = \frac{3m}{4\pi R_0^3}$$

Hence, the nuclear matter density is independent of *A*. It is nearly constant.

3 Mark Questions

1. A neutron is absorbed by a $3\,Li$ nucleus with subsequent emission of alpha particle. Write the corresponding nuclear reaction?

Ans. $\overset{1}{o}n + 3\overset{60}{Li} \rightarrow {}_2^4He(\alpha - particle) + z\overset{A}{Y}$

$0+3=2+Z \Rightarrow Z=1$

(Conservation of Atomic Number)

$1+6=4+A$

$\Rightarrow A=3$

(Conservation of Mass Number)

$\therefore y\overset{A}{Y} = {}_1^4Y = {}_1^3H$

$\Rightarrow \overset{1}{o}n + 3\overset{6}{Li} \rightarrow {}_2^4He +_1 \overset{3}{4}$

---

2. If the activity of a radioactive substance drops to $\frac{1}{8}th$ of its initial value in 30 years, find its half life period?

Ans. $\frac{A}{A_o} = \left(\frac{1}{2}\right)^{t/T}$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{30/T}$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{30/T}$$

$$\Rightarrow \frac{30}{T} = 3$$

Or

T = 10 years

3.Show that nuclear density is in dependent of mass number A of a nucleus?

Ans. Nuclear density = $\frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{(\text{Mass of P or N}) \times A}{\text{Volume of Nucleus}}$

$$P = \frac{\left(1.6\times10^{-27}\right)A}{\frac{4}{3}\pi R^3} = \frac{\left(1.6\times10^{-27}\right)\times A}{\frac{4}{3}\pi\left(R_o A^{\frac{1}{3}}\right)^3}$$

Here $R_o = 1.2\times10^{-15} m$

$$\Rightarrow P = \frac{\left(1.6\times10^{-27}\right)\times A}{\frac{4}{3}\pi R_o{}^3 A} = \frac{1.6\times10^{-27}}{\frac{4}{3}\times3.14\times(1.2\times10^{-15})^3}$$

$$\Rightarrow P = 2.21\times10^{17} Kg/m^3$$

---

4. A radioactive nucleus undergoes a series of decay according to the scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta-} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72 respectively, what are there

number for A_4?

Ans. $^{180}_{72}A \xrightarrow{\alpha} {}^{176}_{70}A_1 \xrightarrow{\beta-} {}^{176}_{71}A_2 \xrightarrow{\alpha} {}^{172}_{69}A_3 \xrightarrow{\gamma} {}^{172}_{69}A_4$

---

5. Distinguish between isotopes and isobars. Give one example for each of the species?

Ans. The elements which have same atomic number but different mass number are called Isotopes.

For eg $\rightarrow$ $6c^{10}$ $6c^{11}$ $6c^{12}$ $6c^{14}$ (Isotopes of carbon)

Thus nuclides of different elements having same mass number but different atomic number are called isobars.

For eg $\rightarrow$ ${}^{3}_{1}\mathrm{H}$ and ${}^{3}_{2}\mathrm{He}$

${}^{7}_{3}\mathrm{Li}$ and ${}^{7}_{4}\mathrm{Be}$

---

6. A radio active nuclide decays to form a stable nuclide its half life is 3 minutes. What fractions of its 1g will remain radioactive after 9 minutes?

Ans. Suppose no. of atoms/gram = No

t = 9 minutes

$T_{1/2} = 3\,\text{minutes}$

$$\frac{N}{No} = \left(\frac{1}{2}\right)^{t/T_{1/2}} = \left(\frac{1}{2}\right)^{9/3} = \left(\frac{1}{2}\right)^{3} = 1/8$$

$\Rightarrow N = \frac{No}{8}$

$\Rightarrow$ Fraction decayed $= \frac{No - N}{No} = \frac{No - \frac{No}{8}}{No}$

$= 1 - \frac{1}{8} = \frac{7}{8} = 0.875$

$\therefore$ Fraction remain undecayed = 1 – 0.875 = 0.125

---

7. Obtain the binding energy (in MeV) of a nitrogen nucleus $^{14}_{7}N$, given $m\left(^{14}_{7}N\right)$ =14.00307 u

Ans. Atomic mass of nitrogen $\left(_{7}N^{14}\right)$, *m* = 14.00307 u

A nucleus of nitrogen $_{7}N^{14}$ contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\therefore \Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

= 7.054775 + 7.06055 - 14.00307

= 0.11236 u

$But\ 1\,u = 931.5\ MeV/c^2$

$\therefore \Delta m = 0.11236 \times 931.5\ MeV/c^2$

Hence, the binding energy of the nucleus is given as:

$$Eb = \Delta mc^2$$

Where,

c= Speed of light

$$\therefore Eb = 0.11236 \times 931.5\left(\frac{MeV}{c^2}\right)\times c^2$$

= 104.66334 MeV

Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

---

8. Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.

Ans. Nuclear radius of the gold isotope $_{79}au^{197} = R_{Au}$

Nuclear radius of the silver isotope $_{47}Ag^{107} = R_{Ag}$

Mass number of gold, A_{Au} = 197

Mass number of silver, A_{Ag} = 107

The ratio of the radii of the two nuclei is related with their mass numbers as:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{R_{Au}}{R_{Ag}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is about 1.23.

9.Suppose, we think of fission of a $^{56}_{26}Fe$ nucleus into two equal fragments, $^{28}_{13}Al$. Is the fission energetically possible? Argue by working out *Q* of the process. Given

$m\left({}_{26}^{56}Fe\right)=55.93494u$ **and** $m\left({}_{13}^{28}Al\right)=27.98191u.$.

Ans. The fission of ${}_{26}^{56}Fe$ can be given as:

${}_{13}^{56}Fe \rightarrow 2\,{}_{13}^{28}Al$

It is given that:

Atomic mass of $m\left({}_{26}^{56}Fe\right)$ = 55.93494 u

Atomic mass of $m\left({}_{13}^{28}Al\right)=27.98191u$

The *Q*-value of this nuclear reaction is given as:

$Q=\left[m\left({}_{26}^{56}Fe\right)-2m\left({}_{13}^{28}Al\right)\right]c^2$

$=\left[55.93494-2\times 27.98191\right]c^2$

$=\left(0.02888c^2\right)u$

$But 1u=931.5MeV/c^2$

$\therefore Q=-0.02888\times 931.5=-26.902MeV$

The *Q*-value of the fission is negative. Therefore, the fission is not possible energetically. For an energetically-possible fission reaction, the *Q*-value must be positive.

10.The fission properties of ${}_{94}^{234}Pu$ are very similar to those of ${}_{92}^{235}U$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure ${}_{94}^{239}Pu$ undergo fission?

Ans. Average energy released per fission of ${}_{94}^{239}Pu$, $E_{av}=180MeV$

Amount of pure ${}_{94}Pu^{239}$,*m* = 1 kg = 1000 g

NA= Avogadro number = 6.023×10^{23}

Mass number of $^{239}_{94}Pu$ = 239 g

1 mole of $_{94}Pu^{239}$ contains NAatoms.

$\therefore m$g of $_{94}Pu^{239}$ contains $\left(\frac{N_A}{Mass\ number}\times m\right) atoms$

$$=\frac{6.023\times10^{23}}{239}\times1000=2.52\times10^{24}\ atoms$$

$\therefore$Total energy released during the fission of 1 kg of $^{239}_{94}Pu$ is calculated as:

$$E=E_{av}\times2.52\times10^{24}$$

$$=180\times2.52\times10^{24}=4.536\times10^{26}\ MeV$$

Hence, $4.536\times10^{26}\ MeV$ is released if all the atoms in 1 kg of pure $_{94}Pu^{239}$ undergo fission.

---

11. Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Ans. When two deuterons collide head-on, the distance between their centres, *d* is given as:

Radius of 1stdeuteron + Radius of 2nddeuteron

Radius of a deuteron nucleus = 2 fm = $2\times10^{-15}m$

$$\therefore d=2\times10^{-15}+2\times10^{-15}=4\times10^{-15}m$$

Charge on a deuteron nucleus = Charge on an electron = *e*= $1.6\times10^{-19}C$

Potential energy of the two-deuteron system:

$$V = \frac{e^2}{4\pi \epsilon_0 d}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 Mm^2c^{-2}$$

$$\therefore V = \frac{9 \times 10 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15}} J$$

$$= \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15} \times \left(1.6 \times 10^{-19}\right)} eV$$

$$= 360\, keV$$

Hence, the height of the potential barrier of the two-deuteron system is 360 keV.

5 Mark Questions

1.The wavelength of the first member of Balmer series in the hydrogen spectrum is $6563A^o$. Calculate the wavelength of the first member of lyman series in the same spectrum.

Ans. We know $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{ni^2}\right), n = 3, 4, 5 --$

For first member ni = 3 (Balmer series)

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_1} = R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$\lambda_1 = \dfrac{36}{5R}$ --------------- (1)

For first member of Lyman series

$$\frac{1}{\lambda_1'} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{\lambda_1'} = R\left(1 - \frac{1}{4}\right)$$

Or

$\lambda_1' = \dfrac{4}{3R}$ ---------------- (2)

From (1) and (2)

$$\frac{\lambda_1'}{\lambda_1} = \frac{4}{3R} \times \frac{5R}{36}$$

$$\frac{\lambda_1'}{\lambda_1} = \frac{5}{27}\lambda_1$$

$$\lambda_1 = 6563A^o$$

$$\Rightarrow \lambda_1' = 1215.4A^o$$

2. A neutron is absorbed by a $3Li^6$ nucleus with subsequent emission of $\alpha - particle.$ Write the corresponding nuclear reaction. Calculate the energy released in this reaction.

Given mass of $3\,Li^6$ = 6.015126 a.m.u.

Mass of $2\,He^4$ = 4.000 26044 a.m.u.

Mass of neutron $\left(o\,n^1\right)$ = 1.0086654 a.m.u.

Mass of tritium $\left({}_1H^1\right)$ = 3.016049 a.m.u.

Ans. Nuclear reaction is given by

$$o\,n^1 + 3\,Li^6 \rightarrow 2\,He^4 + {}_1H^3$$

Mass of reactants $= m\left(o\,n^1\right) + m\left(3\,Li^6\right)$

=1.0086654+6.015126=7.0237914 a.m.u

Mass Defect, Δm = mass of reactant – mass of product.

Δm = 7.0237194 – 7.0186534

Δm = 0.005138 a.m.u.

Since1. a.m.u. = 931MeV

$\therefore$ Energy released = $\Delta m \times$ 931MeV

E = 0.005138 $\times$ 931

E = 4.783MeV

3.Define decay constant of a radioactive sample. Which of the following radiation α-rays, β-rays and γ-rays.

(i) Are similar to X – rays?

(ii) Are easily absorbed by matter?

Ans. Radioactive decay constant (λ) is the reciprocal of time during which the number of atoms in the radioactive substance reduced to 36.8% of the original number of atoms in it.

(i) are similar to $X - rays$

(ii) Penetration power of α-rays is less than that of β and γ-rays so γ-rays are easily absorbed by matter.

4.State radioactive decay law and hence derive the relation $N = N_o e^{-\lambda t}$ **where symbols their usual meanings.**

Ans. According to radioactive decay law the rate of disintegration of a radioactive substance at an instant is directly proportional to the number of nuclei in the radioactive substance at that time i.e.

$N = N_o e^{-\lambda t}$ Where symbols have their usual meanings

Consider a radioactive substance having No atoms initially at time (t = o). After time (t) no. of atoms left undecayed be N.

If dN is the no. of atoms decayed in tine dt then according to radioactive decay law

$$\frac{-dN}{dt} \alpha N \ or \ \frac{-dN}{dt} = \lambda N \text{------(1)}$$

Where λ is decay constant and negative sign indicates that a radioactive sample goes on decreasing with time. Equation (1) can also be written as

$$\frac{dN}{N} = -\lambda dt$$

Integrating both the sides

$\log e\, N = \lambda t + K$ -----(2)

Where K is constant of integration

When t = o, N = No

$\Rightarrow$ K = loge No

Substituting K in equation (2)

$$w/_e\, N = \log e\, No = -\lambda t \left(\because \log e\, m - \log e\, n = \log e\, {}^{m}/_{n}\right)$$

$$\log e\ \frac{N}{No} = -\lambda t$$

$$\frac{N}{No} = e^{-\lambda t}$$

$$N = Noe^{-\lambda t}$$

---

5. Define half life and decay constant of a radioactive element. Write their S.I. unit. Define expression for half life?

Ans. The time during which half of the atoms of the radioactive substance disintegrates is called half life of a radioactive substance.

We know $N = Noe^{-\lambda t}$

When $t = T\frac{1}{2}(Half\ \text{life})$

$$N = \frac{No}{2}$$

$$\Rightarrow \frac{No}{2} = No\,e^{-\lambda T ½}$$

$$\frac{1}{2} = e^{-\lambda T ½}$$

$$or\ e^{\lambda T ½} = 2$$

$$\lambda T \frac{1}{2} = \log e 2$$

$$\lambda T \frac{1}{2} = 2.303 \times \log_{10} 2$$

$$\lambda T \frac{1}{2} = 2.303 \times 0.3010$$

$$T \frac{1}{2} = \frac{0.6931}{\lambda}$$

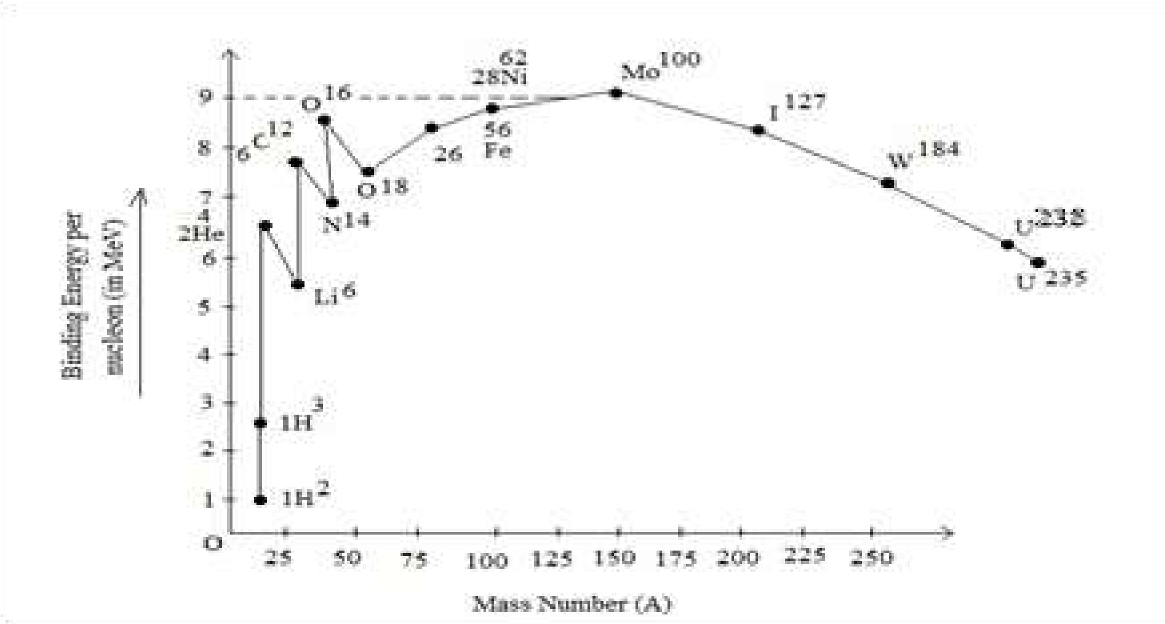
S.I. unit – second (s)

<u>Radioactive decay constant</u> (λ) is the reciprocal of the time during which the number of atoms in the radioactive substance reduces to 36.8 % of the original number of atoms in it.

$$S.I.\ unit - s^{-1} or\ min^{-1}$$

6.Draw a curve between mass number and binding energy per nucleon. Give two salient features of the curve. Hence define binding energy?

Ans. The total energy required to disintegrate the nucleus into its constituent particles is called binding energy of the nucleus.



Salient features of the curve

(1) The intermediate nuclei have large value of binding energy per nucleon, so they are most stable. (For 30 < A > 63)

(2) The binding energy per nucleon has low value for both the light and heavy nuclei. So they are unstable nuclei.

7.(a) Two stable isotopes of lithium $^{6}_{3}Li$ and $^{7}_{3}Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, $^{10}_{5}\mathrm{B}$ and $^{11}_{5}\mathrm{B}$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of $^{10}_{5}\mathrm{B}$ and $^{11}_{5}\mathrm{B}$.

Ans. (a) Mass of lithium isotope $^{6}_{3}Li$, m_1 = 6.01512 u

Mass of lithium isotope $^{7}_{3}Li$, m_2 = 7.01600 u

Abundance of ${}^{6}_{3}Li$, η_1 = 7.5%

Abundance of ${}^{7}_{3}Li$, η_2 = 92.5%

The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$= \frac{6.0512 \times 7.5 \times 7.01600 \times 92.5}{92.5 + 7.5}$$

$$= 6.940934u$$

(b) Mass of boron isotope ${}^{10}_{5}\mathrm{B}$, m_1 = 10.01294 u

Mass of boron isotope ${}^{11}_{5}\mathrm{B}$, m_2 = 11.00931 u

$$Abundance\ of\ , \eta_1 = x\%,$$

$$Abundance\ of\ , \eta_2 = (100 - x)\%$$

Atomic mass of boron, *m*= 10.811 u

The atomic mass of boron atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + 100 - x}$$

$$1081.11 = 10.01294x + 1100.931 - 11.00931x$$

$$\therefore x = \frac{19.821}{0.99637} = 19.89\%$$

And 100 - x= 80.11%

Hence, the abundance of $^{10}_{5}\mathrm{B}$ is 19.89% and that of $^{11}_{5}\mathrm{B}$ is 80.11%.

8.Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data: $m\left(^{56}_{26}Fe\right)$ **= 55.934939 u** $m\left(^{209}_{83}Bi\right)$ **= 208.980388 u**

Ans. Atomicmass of $^{56}_{26}Fe$, m_1 = 55.934939 u

$^{56}_{26}Fe$ nucleus has 26 protons and (56 - 26) = 30 neutrons

Hence, the mass defect of the nucleus, $\Delta m = 26\times m_H + 30\times m_n - m_1$

Where,

Mass of proton, $m_H = 1.007825\ u$

Mass of a neutron, $m_n = 1.008665\ u$

$\therefore\ \Delta m = 26\times 1.007825 + 30\times 1.008665 - 55.934939$

= 26.20345 + 30.25995 - 55.934939

= 0.528461 u

But 1 u = 931.5 MeV/c2

$\therefore\ \Delta m = 0.528461\times 931.5\ MeV/c^2$

The binding energy of this nucleus is given as:

$Eb_1 = \Delta mc^2$ Where,

c= Speed of light

$$\therefore Eb_1 = 0.528461 \times 931.5\left(\frac{MeV}{c_2}\right) \times c^2$$

= 492.26 MeV

Average binding energy per nucleon $= \frac{492.26}{56} = 8.79 MeV$

Atomic mass of $^{209}_{83}Bi$, m_2 = 208.980388 u

$^{209}_{83}Bi$ nucleus has 83 protons and (209 - 83) 126 neutrons.

Hence, the mass defect of this nucleus is given as:

$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$ Where,

Mass of a proton, $m_H = 1.007825\ u$

Mass of a neutron, $m_n = 1.008665\ u$

$$\therefore \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$$

= 83.649475 + 127.091790 - 208.980388

= 1.760877 u

But 1 u = 931.5 MeV/*c*2

$$\therefore \Delta m' = 1.760877 \times 931.5\ MeV / c^2$$

Hence, the binding energy of this nucleus is given as:

$$E_{b2} = \Delta m' c^2 = 1.760877 \times 931.5\left(\frac{MeV}{c^2}\right) \times c^2$$

= 1640.26 MeV

Average binding energy per nucleon = $\frac{1640}{209} = 7.848 MeV$

9.A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ atoms (of mass 62.92960 u).

Ans. Mass of a copper coin, *m*' = 3 g

Atomic mass of $_{29}Cu^{63}$ atom, *m* = 62.92960 u

The total number of $_{29}Cu^{63}$ atoms in the coin, $N = \frac{N_A \times m'}{Mass\,number}$

Where,

NA= Avogadro's number $= 6.023 \times 10^{23}\,atoms/g$ Mass number = 63 g

$\therefore N = 2.868 \times 10^{22}\,atoms$

$_{29}Cu^{63}$ nucleus has 29 protons and (63 - 29) 34 neutrons

∴Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where,

Mass of a proton, m_H= 1.007825 u

Mass of a neutron, m_n= 1.008665 u

$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$ = 0.591935 u

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22}$

$= 1.69766958 \times 10^{22} u$

$But\ 1\,u = 931.5\ MeV/c^2$

$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5\ MeV/c^2$ Hence, the binding energy of the nuclei of the coin is given as:

$$Eb = \Delta mc^2$$

$$= 1.69766958 \times 10^{22} \times 931.5 = 1.581 \times 10^{25}\ MeV$$

$$But\ 1\ MeV = 1.6 \times 10^{-13} J$$

$$Eb = 1.581 \times 10^{25} \times 1.6 \times 10^{-13} = 2.5296 \times 10^{12} J$$

This much energy is required to separate all the neutrons and protons from the given coin.

10.Write nuclear reaction equations for

(i) *α*-decay of $^{226}_{88}Ra$ **(ii) *α*-decay of** $^{242}_{94}Pu$ **(iii)** $\beta^- - decay\ of\ ^{32}_{15}P$

(iv) $\beta^- - decay\ of\ ^{210}_{83}Bi$ **(v)** $\beta^+ - decay\ of\ ^{11}_{6}C$ **(vi)** $\beta^+ - decay\ of\ ^{97}_{43}Tc$

(vii) Electron capture of $^{120}_{54}Xe$

Ans. α is a nucleus of helium $\left({}_2He^4\right)$ and β is an electron $\left(e^-\ for\ \beta^-\ and\ e^+\ for\ \beta^+\right)$. In every *α*-decay, there is a loss of 2 protons and 4 neutrons. In every $\beta^+ -$ decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every $\beta^- -$ -decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus. For the given cases, the various nuclear reactions can be written as:

$$(i)\ {}_{88}Ra^{226} \rightarrow {}_{86}Rn^{222} + {}_2He^4$$

$$(ii)\ ^{242}_{94}Pu \rightarrow\ ^{238}_{92}U + {}^4_2He$$

$$(iii)\ ^{32}_{15}P \rightarrow\ ^{32}_{16}S + e^- + \bar{v}$$

$$(iv)\ ^{210}_{83}B \rightarrow\ ^{210}_{84}PO + e^- + \bar{v}$$

$$(v)\ ^{11}_{6}C \rightarrow\ ^{11}_{5}B + e^+ + v$$

$$(vi)\,{}_{43}^{97}Tc \rightarrow {}_{42}^{97}MO + e^{+} + v$$

$$(vii)\,{}_{54}^{120}Xe + e^{+} \rightarrow {}_{53}^{120}I + v$$

11.A radioactive isotope has a half-life of *T* years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

Ans. Half-life of the radioactive isotope = *T* years

Original amount of the radioactive isotope = N_0

(a) After decay, the amount of the radioactive isotope = *N*

It is given that only 3.125% of *N*0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

$$But\,\frac{N}{N_0} = e^{-\lambda r}$$

Where,

λ = Decay constant

t= Time

$$\therefore -\lambda t = \frac{1}{32}$$

$$-\lambda t = In1 - In32$$

$$-\lambda t = 0 - 3.4567$$

$$t = \frac{3.4567}{\lambda}$$

$$\text{Since}\,\lambda = \frac{0.693}{T}$$

$$\therefore t = \frac{3.466}{\frac{0.693}{T}} \approx 5T$$

Hence, the isotope will take about 5*T*years to reduce to 3.125% of its original value.

(b) After decay, the amount of the radioactive isotope = *N*

It is given that only 1% of *N*0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

$$But\,\frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore e^{-\lambda t} = \frac{1}{100}$$

$$-\lambda t = In 1 - In 100$$

$$-\lambda t = 0 - 4.6052$$

$$t = \frac{4.6052}{\lambda}$$

Since, λ = 0.693/*T*

$$\therefore t = \frac{4.6052}{\frac{0.693}{T}} = 6.645T$$ years

Hence, the isotope will take about 6.645*T*years to reduce to 1% of its original value.

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12. The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small

proportion of radioactive $^{14}_{6}C$ present with the stable carbon isotope $^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of $^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of $^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Ans. Decay rate of living carbon-containing matter, *R*= 15 decay/min

Let N be the number of radioactive atoms present in a normal carbon- containing matter.

Half life of $^{14}_{6}C, T_{1/2} = 5730$ years,

The decay rate of the specimen obtained from the Mohenjodaro site:

R' = 9 decays/min

Let N' be the number of radioactive atoms present in the specimen during the Mohenjodaro period.

Therefore, we can relate the decay constant, λ and time, *t* as:

$$\frac{N}{N'} = \frac{R}{R'} = e^{-\lambda t}$$

$$e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$$

$$-\lambda t = \log_e \frac{3}{5} = -0.5108$$

$$\therefore t = \frac{0.5108}{\lambda}$$

$$But \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730}$$

$$\therefore t = \frac{0.5108}{\frac{0.693}{5730}} = 4223.5$$

Hence, the approximate age of the Indus-Valley civilisation is 4223.5 years.

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13. Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.

Ans. The strength of the radioactive source is given as:

$$\frac{dN}{dt} = 8.0 mCi$$

$$= 8 \times 10^{-3} \times 3.7 \times 10^{10}$$

$$= 29.6 \times 10^{7} decay / s$$

Where,

N= Required number of atoms

$$of \frac{60}{27} Co, T_{1/2} = 5.3 years$$

$$Half - life = 5.3 \times 365 \times 24 \times 60 \times 60$$

$$= 1.67 \times 10^{8} s$$

For decay constant λ , we have the rate of decay as:

$$\frac{dN}{dt} = \lambda N$$

$$= \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67 \times 10^{8}} s^{-1}$$

$$\therefore N = \frac{1}{\lambda}\frac{dN}{dt}$$

Where, $\lambda = \dfrac{29.6\times10^{7}}{\dfrac{0.693}{1.67\times10^{8}}} = 7.133\times10^{16}\, atoms$

For $_{27}Co^{60}$: $Mass\ of\ 6.023\times10^{23}$ (Avogadro's number) atoms = 60 g

∴Mass of 7.133×10^{16} atoms $= \dfrac{60\times7.133\times10^{16}}{6.023\times10^{23}} = 7.016\times10^{-6}\, g$

Hence, the amount of $_{27}Co^{60}$ necessary for the purpose is $7.106\times10-6g$.

14.The half-life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

Ans. Half life of $^{90}_{38}Sr$, $t_{1/2}$ = 28 years

$= 28\times365\times24\times60\times60$

= $8.83\times10^{8}\, s$

Mass of the isotope, *m*= 15 mg

90 g of $^{90}_{38}Sr$ atom contains 6.023×10^{23} (Avogadro's number) atoms.

Therefore, 15 mg of $^{90}_{38}Sr$ contains:

$\dfrac{6.023\times10^{23}\times15\times10^{-3}}{90}$ $i.e.\ 1.0038\times10^{20}$ number of atoms

Rate of disintegration, $\dfrac{dN}{dt} = \lambda N$

Where,

λ = Decay constant $= \dfrac{0.693}{8.83\times10^{8}} s^{-1}$

$$\therefore \frac{dN}{dt} = \frac{0.693\times1.0038\times10^{20}}{8.83\times10^{8}} = 7.878\times10^{10}\,atoms/s$$

Hence, the disintegration rate of 15 mg of the given isotope is

$7.878 \times 10^{10}\,atoms/s.$

15.Find the Q-value and the kinetic energy of the emitted *α*-particle in the α-decay of (a) $^{226}_{88}Ra$ and (b) $^{220}_{86}Rn$. Given $m\left(^{226}_{88}Ra\right)$ = 226.02540 u, $m\left(^{222}_{86}Rn\right)$ = 222.01750 u, $m\left(^{220}_{86}Rn\right)$ = 220.01137 u, $m\left(^{216}_{84}Po\right)$ = 216.00189 u.

Ans. (a) Alpha particle decay of $^{226}_{88}Ra$ emits a helium nucleus. As a result, its mass number reduces to (226 - 4) 222 and its atomic number reduces to (88 - 2) 86. This is shown in the following nuclear reaction.

$$^{226}_{88}Ra \rightarrow {}^{222}_{86}Ra + {}^{4}_{2}He$$

Q-value of

emitted *α*-particle = (Sum of initial mass - Sum of final mass) c^2

Where,

c= Speed of light

It is given that:

$$m\left(^{226}_{88}Ra\right) = 226.02540u$$

$$m\left(^{222}_{86}Rn\right) = 222.01750u$$

$$m\left(^{4}_{2}He\right) = 4.002603u$$

Q-value = [226.02540 - (222.01750 + 4.002603)] uc^2

$= 0.005297\ uc^2$

But 1 u = 931.5 MeV/c^2

$\therefore Q = 0.005297 \times 931.5 \approx 4.94\ MeV$

Kinetic energy of the α-particle $= \left(\frac{Mass\ no\ after\ decay}{Mass\ no\ before\ decay} \right) \times Q$

$= \frac{222}{226} \times 4.94 = 4.85 MeV$

(b) Alpha particle decay of ${}^{220}_{86}Rn$ is shown by the following nuclear reaction.

${}^{220}_{86}Rn \rightarrow {}^{216}_{84}Po + {}^{4}_{2}He$

It is given that:

Mass of $\left({}^{220}_{86}Rn \right)$ = 220.01137 u

Mass of $\left({}^{216}_{84}Po \right)$ = 216.00189 u

$\therefore Q$-value = $= [220.01137 - (216.00189 + 4.00260)] \times 931.5$

$\approx\ 641\ MeV$

Kinetic energy of the α-particle $\left(\frac{220-4}{220} \right) \times 6.41$

= 6.29 MeV

16.The radionuclide 11C decays according to ${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^{+} + \nu : T_{1/2} = 20.3\min$

The maximum energy of the emitted positron is 0.960 MeV.

Given the mass values: $m\left({}^{11}_{6}C \right) = 11.011434u$ **and** $m\left({}^{11}_{6}B \right) = 11.009305u$, **calculate *Q* and compare it with the maximum energy of the positron emitted**

Ans. The given nuclear reaction is:

$$^{11}_{6}C \rightarrow {}^{11}_{5}B + e^{+} + \nu$$

Half life of $^{11}_{6}C$ nuclei, $T_{1/2} = 20.3 \text{ min}$

Atomic mass of $m\left(^{11}_{6}C\right)$ = 11.011434 u

Atomic mass of $m\left(^{11}_{6}B\right) = 11.009305u$

Maximum energy possessed by the emitted positron = 0.960 MeV

The change in the *Q*-value (Δ*Q*) of the nuclear masses of the $^{11}_{6}C$ nucleus is given as:

$$\Delta Q = \left[m'\left({}_{6}C^{11}\right) - \left[m'\left(^{11}_{5}B\right) + m_e\right]\right]c^2 \text{ ....(1)}$$

Where,

$m_e =$ Mass of an electron or positron = 0.000548 u

c = Speed of light

m' = Respective nuclear masses

If atomic masses are used instead of nuclear masses, then we have to add 6 m_e in the case of ^{11}C and 5 m_e in the case of ^{11}B .

Hence, equation (1) reduces to:

$$\Delta Q = \left[m\left({}_{6}C^{11}\right) - m\left(^{11}_{5}B\right) - 2m_e\right]c^2$$

Here, $m\left({}_{6}C^{11}\right)$ and $m\left(^{11}_{5}B\right)$ are the atomic masses.

$$\therefore \Delta Q = [11.011434 - 11.009305 - 2 \times 0.000548]c^2$$

$$= \left(0.001033c^2\right) u$$

But 1 u = 931.5 Mev/ c^2

$$\therefore \Delta Q = 0.001033 \times 931.5 \approx 0.962\ MeV$$

The value of *Q* is almost comparable to the maximum energy of the emitted positron.

17.The nucleus $^{23}_{10}Ne$ decays by β^- emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m\left(^{23}_{10}Ne\right)$ = **22.994466 u**

$m\left(^{23}_{11}Na\right)$ = **22.989770 u.**

Ans. In β^- emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

β^- emission of the nucleus $^{23}_{10}Ne$ is given as:

$$^{23}_{10}Ne \rightarrow {}^{23}_{11}Na + e^- + \bar{v} + Q$$

It is given that:

Atomic mass of $m\left(^{23}_{10}Ne\right)$ = 22.994466 u

Atomic mass of $m^{23}_{11}Na$ = 22.989770 u

Mass of an electron, m_e = 0.000548 u

Q-value of the given reaction is given as:

$$Q = \left[m\left(^{23}_{10}Ne\right) - \left[m\left(^{23}_{11}Na\right) + m_e\right]\right]c_2$$

There are 10 electrons in $^{23}_{10}Ne$ and 11 electrons in $^{23}_{11}Na$. Hence, the mass of the electron is cancelled in the *Q*-value equation.

$\therefore Q = [22.994466 - 22.989770]c^2$

$= (0.004696c^2)u$

The daughter nucleus is too heavy as compared to e^- and ν^-. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the *Q*-value, i.e., 4.374 MeV.

18.The *Q* value of a nuclear reaction $A + b \rightarrow c + d$ is defined by *Q* = $[m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the *Q*-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) ${}^1_1H + {}^3_1H \rightarrow {}^2_1H + {}^2_1H$

(ii) ${}^{12}_6C + {}^{12}_6C \rightarrow {}^{20}_{10}Ne + {}^4_2He$

Atomic masses are given to be

$m\left({}^2_1H\right) = 2.014102u$

$m\left({}^3_1H\right) = 3.016049u$

$m\left({}^{12}_6C\right) = 12.000000u$

$m\left({}^{20}_{10}Ne\right) = 19.992439u$

Ans. (i) The given nuclear reaction is:

It is given that:

Atomic mass $m\left({}^1_1H\right) = 1.007825u$

Atomic mass $m\left({}^3_1H\right) = 3.016049u$

Atomic mass $m\left({}_1^2H\right) = 2.014102u$

According to the question, the *Q*-value of the reaction can be written as:

$$Q = \left[m\left({}_1^1H\right) + m\left({}_1^3H\right) - 2m\left({}_1^2H\right)\right]c^2$$

$$= \left[1.007825 + 3.016049 - 2 \times 2.014102\right]c^2$$

$$Q = (-0.00433c^2)u$$

$$But 1u = 913.5 MeV / c^2$$

$$\therefore Q = -0.00433 \times 931.5 = -4.0334 MeV$$

The negative Q-value of the reaction shows that the reaction is endothermic.

(ii) The given nuclear reaction is:

$${}_6^{12}C + {}_6^{12}C \rightarrow {}_{10}^{20}Ne + {}_2^4He$$

It is given that:

Atomic mass of $m\left({}_6^{12}C\right) = 12.0u$

Atomic mass of $m\left({}_{10}^{20}Ne\right) = 19.992439u$

Atomic mass of $m\left({}_2^4He\right) = 4.0026.3u$

The *Q*-value of this reaction is given as:

$$Q = \left[2m\left({}_6^{12}C\right) - m\left({}_{10}^{20}Ne\right) - m\left({}_2^4He\right)\right]c^2$$

$$= \left[2 \times 12.0 - 19.992439 - 4.002603\right]c^2$$

$$= (0.004958c^2)u$$

$$= 0.004958 \times 931.5 = 4.618377 MeV$$

The positive *Q*-value of the reaction shows that the reaction is exothermic.

19.A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much $^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

Ans. Half life of the fuel of the fission reactor, $t_{\frac{1}{2}} = 5\,years$

$$= 5\times365\times24\times60\times60\ s$$

We know that in the fission of 1 g of $^{235}_{92}U$ nucleus, the energy released is equal to 200 MeV.

1 mole, i.e., 235 g of $^{235}_{92}U$ contains $6.023\ \times10^{23}\,atoms$.

$\therefore$1 g $^{235}_{92}U$ contains $\dfrac{6.023\times10^{23}}{235}\,atoms$

The total energy generated per gram of $^{235}_{92}U$ is calculated as:

$$E = \frac{6.023\times10^{23}}{235}\times200MeV\ /\ g$$

$$= \frac{200\times6.023\times10^{23}\times1.6\times10^{-19}\times10^{6}}{235} = 8.20\times10^{10}J\ /\ g$$

The reactor operates only 80% of the time.

Hence, the amount of $^{235}_{92}U$ consumed in 5 years by the 1000 MW fission reactor is calculated as:

$$= \frac{5\times80\times60\times60\times365\times24\times1000\times10^{6}}{100\times8.20\times10^{10}}g$$

$$\approx 1538kg$$

$\therefore$Initial amount of ${}^{235}_{92}U = 2\times 1538 = 3076\ kg$

20.How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}He + n + 3.27MeV$.

Ans. The given fusion reaction is:

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}He + n + 3.27MeV$$

Amount of deuterium, *m*= 2 kg

1 mole, i.e., 2 g of deuterium contains $6.023\ \times 10^{23}\ atoms$.

$\therefore 2.0\ kg$ of deuterium contains $= \dfrac{6.023\times 10^{23}}{2}\times 2000\ = 6.023\times 10^{26}\ atoms$

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released.

$\therefore$Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2}\times 6.023\times 10^{26}\ MeV$$

$$= \frac{3.27}{2}\times 6.023\times 10^{26}\times 1.6\times 10^{-19}\times 10^{6}$$

$$= 1.576\times 10^{14}\ J$$

Power of the electric lamp, *P*= 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

$$\frac{1.576\times 10^{14}}{100\times 60\times 60\times 24\times 365}\approx 4.9\times 10^{4}\ years$$

21.For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K - shell, is captured by the nucleus and a neutrino is emitted). $e^+ + {}_Z^AX \rightarrow {}_{Z-1}^{A}Y + \nu$ Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice - versa.

Ans. Let the amount of energy released during the electron capture process be Q_1. The nuclear reaction can be written as:

$e^+ + {}_Z^AX \rightarrow {}_{Z-1}^{A}Y + \nu + Q_1$ ....(1)

Let the amount of energy released during the positron capture process be Q_2. The nuclear reaction can be written as:

${}_Z^AX \rightarrow {}_{Z-1}^{A}Y + e^+ + \nu + Q_2$ .....(2)

$m_N\left({}_Z^AX\right) =$ Nuclear mass of ${}_Z^AX$

$m_N\left({}_{Z-1}^{A}Y\right)$ = Nuclear mass of ${}_{Z-1}^{A}Y$

$m\left({}_Z^AX\right)$ = Atomic mass of ${}_Z^AX$

$m\left({}_{Z-1}^{A}Y\right)$ = Atomic mass of ${}_{Z-1}^{A}Y$

m_e = Mass of an electron

c = Speed of light

Q-value of the electron capture reaction is given as:

$$Q_1 = \left[m_N\left({}_Z^AX\right) + m_e - m_N\left({}_{Z-1}^{A}Y\right)\right]c^2$$

$$= \left[m\left({}_Z^AX\right) = Zm_e + m_e - m\left({}_{Z-1}^{A}Y\right) + (Z-1)m_e\right]c^2$$

$$=\left[m\left({}_{Z}^{A}X\right)-m\left({}_{Z-1}^{A}Y\right)\right]c^{2}$$ ....(3)

Q-value of the positron capture reaction is given as:

$$Q_2=\left[m\left({}_{Z}^{A}X\right)-m\left({}_{Z-1}^{A}Y\right)-2m_e\right]c^2$$

$$=\left[m\left({}_{Z}^{A}X\right)-m\left({}_{Z-1}^{A}Y\right)+(Z-1)m_e-m_e\right]c^2$$

$$=\left[m\left({}_{Z}^{A}X\right)-m\left({}_{Z-1}^{A}Y\right)-2m_e\right]c^2$$ ....(4)

It can be inferred that if $Q_2>0$, then $Q_1>0$; Also, if $Q_1>0$, it does not necessarily mean that $Q_2>0$.

In other words, this means that if β^+ emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is because the Q-value must be positive for an energetically-allowed nuclear reaction.

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22. In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}_{12}^{24}Mg$ (23.98504u), ${}_{12}^{25}Mg$ (24.98584u) and ${}_{12}^{26}Mg$ (25.98259u). The natural abundance of ${}_{12}^{24}Mg$ is 78.99% by mass. Calculate the abundances of other two isotopes.

Ans. Average atomic mass of magnesium, m= 24.312 u

Mass of magnesium isotope ${}_{12}^{24}Mg$, m_1 = 23.98504 u

Mass of magnesium isotope ${}_{12}^{25}Mg$, m_2 = 24.98584 u

Mass of magnesium isotope ${}_{12}^{26}Mg$, m_3 = 25.98259 u

Abundance of ${}_{12}^{24}Mg$, η_1 = 78.99%

Abundance of ${}_{12}^{25}Mg$, η_2 = x%

Hence, abundance of $^{26}_{12}Mg$, η_3 = 100 - *x* - 78.99% = (21.01-*x*) %

We have the relation for the average atomic mass as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$24.312 = \frac{23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 = x)}{100}$$

$$2431.2 = 1894.5783096 + 24.98584x + 545.8942159 = 25.98259x$$

$$0.99675x = 9.2725255$$

And $\therefore x \approx 9.3\%$

Hence, the abundance of $^{25}_{12}Mg$ is 9.3% and that of $^{26}_{12}Mg$ is 11.71%.

23.The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei $^{41}_{20}Ca$ and $^{27}_{13}Al$ from the following data:

$m\left(^{40}_{20}Ca\right)$ = **39.962591 u**

$m\left(^{41}_{20}Ca\right)$ = **40.962278 u**

$m\left(^{26}_{13}Al\right)$ = **25.986895 u**

$m\left(^{27}_{13}Al\right)$ = **26.981541 u**

Ans. For $^{41}_{20}Ca$: Seperation energy= $8.363007 MeV$

For $^{27}_{13}Al$: Seperation energy= $13.059 MeV$

A neutron $_0n^1$ is removed from a $\left(^{41}_{20}Ca\right)$ nucleus. The corresponding nuclear reaction can be written as: $^{41}_{20}Ca \rightarrow ^{40}_{20}Ca + ^{1}_{0}n$

It is given that:

Mass $m\left({}^{40}_{20}Ca\right)$ = 39.962591 u

Mass $m\left({}^{41}_{20}Ca\right)$ = 40.962278 u

Mass $m\left({}_{0}n^{1}\right)$ = 1.008665 u

The mass defect of this reaction is given as:

$$\Delta m = m\left({}^{401}_{20}Ca\right)+\left({}^{1}_{0}n\right)-m\left({}^{41}_{20}Ca\right)$$

$$= 39.962591 + 1.008665 - 40.962278 \; = 0.008978u$$

$$But 1u = 931.5 MeV/c^2$$

$$\therefore \Delta m = 0.008978 \times 931.5 \, MeV/c^2$$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta mc^2$$

$$= 0.008978 \times 931.8.363007 MeV$$

For ${}^{27}_{13}Al$, the neutron removal reaction can be written as:

$${}^{27}_{13}Al \rightarrow {}^{26}_{13}Al + {}^{1}_{0}n$$

It is given that:

Mass $m\left({}^{27}_{13}Al\right)$ = 26.981541 u

Mass $m\left({}^{26}_{13}Al\right)$ = 25.986895 u

The mass defect of this reaction is given as:

$$\Delta m = m\left({}^{26}_{13}Al\right)+m\left({}^{1}_{0}n\right)-m\left({}^{27}_{13}Al\right)$$

= 25.986895 + 1.008665 – 26.981541

= 0.014019 u

$= 0.014019 \times 931.5 MeV / c^2$

Hence, the energy required for neutron removal is calculated as:

$E = \Delta mc^2$

$= 0.014019 \times 931.13.059 MeV$

---

24. A source contains two phosphorous radio nuclides $^{32}_{15}P$ $(T_{1/2}$ =14.3d) and $^{32}_{15}P$ $(T_{1/2}$ = 25.3d). Initially, 10% of the decays come from $^{33}_{15}P$. How long one must wait until 90% do so?

Ans. Half life of $^{32}_{15}P$, $T_{1/2}$ = 14.3 days

Half life of $^{33}_{15}P$, $T'_{1/2}$ = 25.3 days

$^{33}_{15}P$ nucleus decay is 10% of the total amount of decay.

The source has initially 10% of $^{33}_{15}P$ nucleus and 90% of $^{32}_{15}P$ nucleus.

Suppose after *t*days, the source has 10% of $^{32}_{15}P$ nucleus and 90% of $^{33}_{15}P$ nucleus.

Initially:

Number of $^{33}_{15}P$ nucleus = *N*

Number of $^{32}_{15}P$ nucleus = 9 *N*

Finally:

Number of $^{33}_{15}P$ nucleus= $9N'$

Number of $^{32}_{15}P$ nucleus= N'

For $^{32}_{15}P$ nucleus, we can write the number ratio as:

$$\frac{N'}{9N}=\left(\frac{1}{2}\right)\frac{t}{T_{1/2}}$$

$$N'=9N(2)\frac{-t}{14.3}$$ ....(1)

For $^{33}_{15}P$, we can write the number ratio as:

$$\frac{9N'}{N}=\left(\frac{1}{2}\right)\frac{t}{T1/2}$$

$$9N'=N(2)^{\frac{-t}{25.3}}$$ ....(2)

On dividing equation (1) by equation (2), we get:

$$\frac{1}{9}=9\times 2\left(\frac{t}{25.3}-\frac{t}{14.3}\right)$$

$$\frac{1}{81}=2-\left(\frac{11t}{25.3\times 14.3}\right)$$

$$\log 1-\log 81=\frac{-11t}{25.3\times 14.3}\log 2$$

$$\frac{-11t}{25.3\times 14.3}=\frac{0-1.908}{0.301}$$

$$t=\frac{25.3\times 14.3\times 1.908}{11\times 0.301}\approx 208.5\,days$$

Hence, it will take about 208.5 days for 90% decay of $_{15}P^{33}$.

25.Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$^{223}_{88}Ra \rightarrow ^{209}_{82}Pb + ^{14}_{6}C$, $^{223}_{88}Ra \rightarrow ^{219}_{86}Rn + ^{4}_{2}He$ **Calculate the *Q*-values for these decays and determine that both are energetically allowed.**

Ans. Take a $^{14}_{6}C$ emission nuclear reaction:

$$^{223}_{88}Ra \rightarrow ^{209}_{82}Pb + ^{14}_{6}C$$

We know that:

Mass of $^{223}_{88}Ra$, m_1 = 223.01850 u

Mass of $^{209}_{82}Pb$, m_2 = 208.98107 u

Mass of $^{14}_{6}C$, m_3 = 14.00324 u

Hence, the *Q*-value of the reaction is given as:

$Q = \left(m_1 - m_2 - m_3\right)c^2$

= (223.01850 - 208.98107 - 14.00324) c^2

$= \left(0.03419c^2\right)\ u$

$But\ 1\ u = 931.5\ MeV/c^2$

$\therefore Q = 0.03419 \times 931.5$

= 31.848 MeV

Hence, the *Q*-value of the nuclear reaction is 31.848 MeV. Since the value is positive, the reaction is energetically allowed.

Now take a $^{4}_{2}He$ emission nuclear reaction:

$$^{223}_{88}Ra \rightarrow ^{229}_{86}Rn + ^{4}_{2}He$$

We know that:

Mass of $^{223}_{88}Ra$, m_1 = 223.01850

Mass of $^{219}_{82}Rn$, m_2 = 219.00948

Mass of $^{4}_{2}He$, m_3 = 4.00260

Q-value of this nuclear reaction is given as:

$$Q = (m_1 - m_2 - m_3)c^2$$

= (223.01850 - 219.00948 - 4.00260) c^2

$= (0.00642c^2)\ u = 0.00642 \times 931.5$ = 5.98 MeV

Hence, the *Q*value of the second nuclear reaction is 5.98 MeV. Since the value is positive, the reaction is energetically allowed.

26.Consider the fission of $^{238}_{92}U$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are $^{140}_{58}Ce$ and $^{99}_{44}Ru$. Calculate Q for this fission process. The relevant atomic and particle masses are

m $\left(^{238}_{92}U\right)$ =**238.05079 u**

m $\left(^{140}_{58}Ce\right)$ =**139.90543 u**

m $\left(^{99}_{44}Ru\right)$ = **98.90594 u**

Ans. In the fission of $^{238}_{92}U$, 10 β^- particles decay from the parent nucleus. The nuclear reaction can be written as:

$$^{238}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{58}Ce + ^{94}_{44}Ru + 10\,^{0}_{-1}e$$

It is given that:

Mass of a nucleus $^{238}_{92}U$ m_1 = 238.05079 u

Mass of a nucleus ${}^{140}_{58}Ce$ m_2 = 139.90543 u

Mass of a nucleus ${}^{99}_{44}Ru$, m_3 = 98.90594 u

Mass of a neutron ${}^{1}_{0}n$ m_4 = 1.008665 u

Q-value of the above equation,

$$Q = \left[m'\left({}^{238}_{92}U \right) + m\left({}^{1}_{0}n \right) - m'\left({}^{140}_{58}Ce \right) - m'\left({}^{99}_{44}Ru \right) - 10m_e \right] c^2$$

Where,

m' = Represents the corresponding atomic masses of the nuclei

$m'\left({}^{238}_{92}U \right) = m_1 - 92m_e$

$m'\left({}^{140}_{58}Ce \right) = m_2 - 58m_e$

$m'\left({}^{99}_{44}Ru \right) = m_3 - 44m_e$

$m\left({}^{1}_{0}n \right) = m_4$

$$Q = \left[m_1 - 92m_e + m_4 - m_2 + 58m_e - m_3 + 44m_e - 10m_e \right] c^2$$

$$= \left[m_1 + m_4 - m_2 - m_3 \right] c^2$$

=[238.0507+1.008665-139.90543-98.90594] c^2

$$= \left[0.247995c^2 \right] u$$

$$But 1u = 931.5 MeV / c^2$$

$$\therefore Q = 0.247995 \times 931.5 = 231.007 MeV$$

Hence, the *Q*-value of the fission process is 231.007 MeV.

27.Consider the D - T reaction (deuterium - tritium fusion) ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$

(a) Calculate the energy released in MeV in this reaction from the data:

$m\left({}_1^2H\right)$ **= 2.014102 u**

$m\left({}_1^3H\right)$ **= 3.016049 u**

(b)Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles = 2(3*kT*/2); *k* = Boltzman's constant, *T* = absolute temperature.)

Ans. (a) Take the D-T nuclear reaction: ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$

It is given that:

Mass of ${}_1^2H$, m_1 = 2.014102 u

Mass of ${}_1^3H$, m_2 = 3.016049 u

Mass of ${}_2^4He$ m_3 = 4.002603 u

Mass of ${}_0^1n$, m_4 = 1.008665 u

Q-value of the given D-T reaction is:

$$Q = \left[m_1 + m_2 - m_3 - m_4\right]c^2$$

= [2.014102 + 3.016049 - 4.002603 - 1.008665] c^2

= [0.018883 c^2] u

But 1 u = 931.5 MeV/ c^2

$\therefore Q = 0.018883 \times 931.5$ = 17.59 MeV

(b) Radius of deuterium and tritium, $r \approx 2.0$ fm = $2 \times 10^{-15} m$ Distance between the two nuclei at the moment when they touch each other, *d* = *r* + *r* = $4 \times 10^{-15} m$ Charge on the deuterium nucleus = *e*

Charge on the tritium nucleus = *e*

Hence, the repulsive potential energy between the two nuclei is given as:

$$V = \frac{e^2}{4\pi \in_0 (d)}$$

Where,

$\in_0$= Permittivity of free space

$$\frac{1}{4\pi \in_0} = 9 \times 10^9 Nm^2C^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15}} = 5.76 \times 10^{-14} J$$

$$\frac{5.76 \times 10^{-14}}{1.6 \times 10^{-19}} = 3.6 \times 10^5 eV = 360 keV$$

Hence, $5.76 \times 10^{-14} J$ or $360 keV$ of kinetic energy (KE) is needed to overcome the Coulomb repulsion between the two nuclei.

However,it is given that:

KE $= 2 \times \frac{3}{2} kT$

Where, *k*= Boltzmann constant = $1.38 \times 10^{-23} m^2\ kg\ s^{-2} K^{-1}$

T = Temperature required for triggering the reaction

$$\therefore T = \frac{KE}{3K}$$

$$= \frac{5.76\times10^{-14}}{3\times1.38\times10^{-23}} = 1.39\times10^{9}K$$

Hence, the gas must be heated to a temperature of $1.39 \times 10^{9} K$ to initiate the reaction.

---

28. Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of ^{235}U in a fission reactor.

Ans. (a) Amount of hydrogen, *m*= 1 kg = 1000 g

1 mole, i.e., 1 g of hydrogen $\left({}_{1}^{1}H\right)$ contains 6.023×10^{23} atoms.

$\therefore$1000 g of ${}_{1}^{1}H$ contains $6.023 \times 10^{23} \times 1000\ atoms.$

Within the sun, four ${}_{1}^{1}H$ nuclei combine and form one ${}_{2}^{4}H$ nucleus. In this process 26 MeV of energy is released.

Hence, the energy released from the fusion of 1 kg ${}_{1}^{1}H$ is:

$$E_1 = \frac{6.023\times10^{23}\times26\times10^{3}}{4} = 39.1495\times10^{26}MeV$$

(b) Amount of ${}_{92}^{235}U$ = 1 kg = 1000 g

1mole, i.e., 235 g of ${}_{92}^{235}U$ contains 6.023×10^{23} atoms.

$\therefore$1000 g of ${}_{92}^{235}U$ contains $\frac{6.023\times10^{23}\times1000}{235}atoms$

It is known that the amount of energy released in the fission of one atom of ${}_{92}^{235}U$ is 200 MeV.

Hence, energy released from the fission of 1 kg of ${}_{92}^{235}U$ is:

$$E_2 = \frac{6\times 10\times 1000\times 200}{235} = 5.106\times 10^{26}\,MeV$$

$$\therefore \frac{E_1}{E_1} = \frac{39.1495\times 10^{26}}{5.106\times 10^{26}} = 7.67 \approx 8$$

Therefore, the energy released in the fusion of 1 kg of hydrogen is nearly 8 times the energy released in the fission of 1 kg of uranium.

---

29. Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200MeV.

Ans. Amount of electric power to be generated, *P*= 2×10^5 MW

10% of this amount has to be obtained from nuclear power plants.

$\therefore$Amount of nuclear power, $P_1 = \frac{10}{100}\times 2\times 10^5$

= $2\times 104 MW$

= $2\times 104\times 106 J/s$

= $2\times 1010\times 60\times 60\times 24\times 365\, J/y$

Heat energy released per fission of a ^{235}U nucleus, *E*= 200 MeV

Efficiency of a reactor = 25%

Hence, the amount of energy converted into the electrical energy per fission is calculated as:

$$\frac{25}{100}\times 200 = 50 MeV$$

$$= 50 \times 1.6 \times 10^{-19} \times 10^{6} = 8 \times 10^{-12}\, J$$

Number of atoms required for fission per year:

$$\frac{2 \times 10^{10} \times 60 \times 60 \times 24 \times 365}{8 \times 10^{-12}} = 78840 \times 10^{24}\, atoms$$

1 mole, i.e., 235 g of U^{235} contains 6.023×10^{23} atoms.

∴Mass of 6.023×10^{23} atoms of U^{235} = 235 g = $235 \times 10^{-3}\, kg$

∴Mass of 78840×10^{24} atoms of U^{235}

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 78840 \times 10^{24}$$

$$= 3.076 \times 10^{4}\, kg$$

Hence, the mass of uranium needed per year is $3.076 \times 10^{4}\, kg$.

CBSE Class 12 physics

Important Questions

Chapter 14

Electronic Devices

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1 Mark Questions

1.Give the ratio of number of holes and the no. of conduction electrons in an intrinsic semiconductor.

Ans. $\frac{nh}{ne} = 1$ (As in intrinsic semiconductor ne = nh)

2.What type of impurity is added to obtain n-type semiconductor?

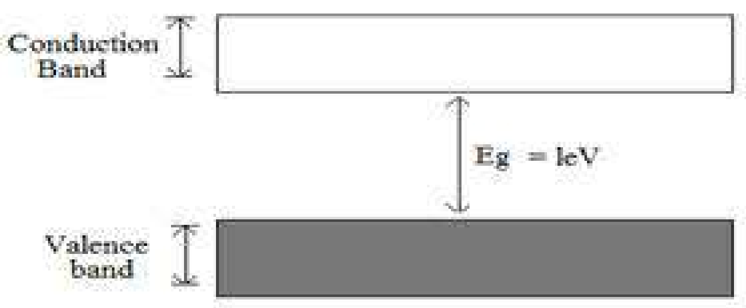
Ans. Pentavalent atoms like Arsenic (As)

3.Doping of silicon with indium leads to which type of semiconductor?

Ans. Indium is a trivalent impurity, thus doping of silicon with indium leads to p-type semiconductor.

4.Draw an energy level diagram for an intrinsic semiconductor?

Ans. In intrinsic semiconductor (ne = nh)



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5. A semiconductor has equal electron and hole concentration of $6\times10^{8}m^{-3}$. On doping

with a certain impurity electron concentration increases to $3\times10^{12}\,m^{-3}$. Identify the type of semiconductor after doping?

Ans. As ne > nh, thus resulting semiconductor is of n-type.

6.How does the energy gap of an intrinsic semiconductor vary, when doped with a trivalent impurity?

Ans. When a trivalent impurity is added to an intrinsic semiconductor, an acceptor energy level is created in the forbidden energy gap which lies above the valence band. Due to this electrons easily transformed to the acceptor energy level.

---

7. How does width of depletion layer of p.n junction diode change with decrease in reverse bias?

Ans. Decrease in reverse bias will decrease in width of the depletion layer.

---

8. Under what condition does a junction diode work as open switch?

Ans. A junction diode works an open switch when it is reverse biased.

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9. Which type of biasing gives a semiconductor diode very high resistance?

Ans. Reverse biasing

---

10. If The output of a 2-input NAND gate is fed as the input to a NOT gate

(i) name the new logic gate obtained and

(ii) write down its truth table?

Ans. Logic gate obtained is AND gate.

A	B	Y
0	0	0
0	1	0

1	0	0
1	1	1

11.Define current amplification factor in a common – emitter mode of transistor?

Ans. Ratio of small change in collection current to the small change in base current at constant collector emitter junction voltage is called current amplification factor.

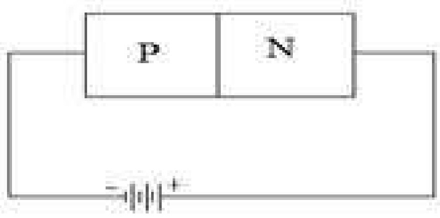
12.Why is a semiconductor damaged by a strong current?

Ans. When a strong current passes through a semiconductor large amount of heat is produced which breaks the covalent bonds in the semiconductor due to which it gets damaged.

2 Mark Questions

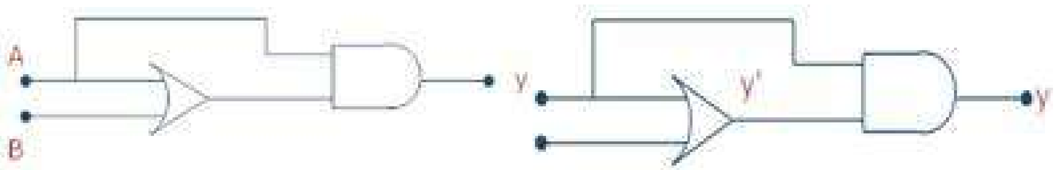
1.Draw a pn junction with reverse bias? Which biasing will make the resistance of a p-n-junction high?

Ans.



Reverse biasing will make the resistance high as it will not allow the current to pass.

2.Write the truth table for the following combination of gates?

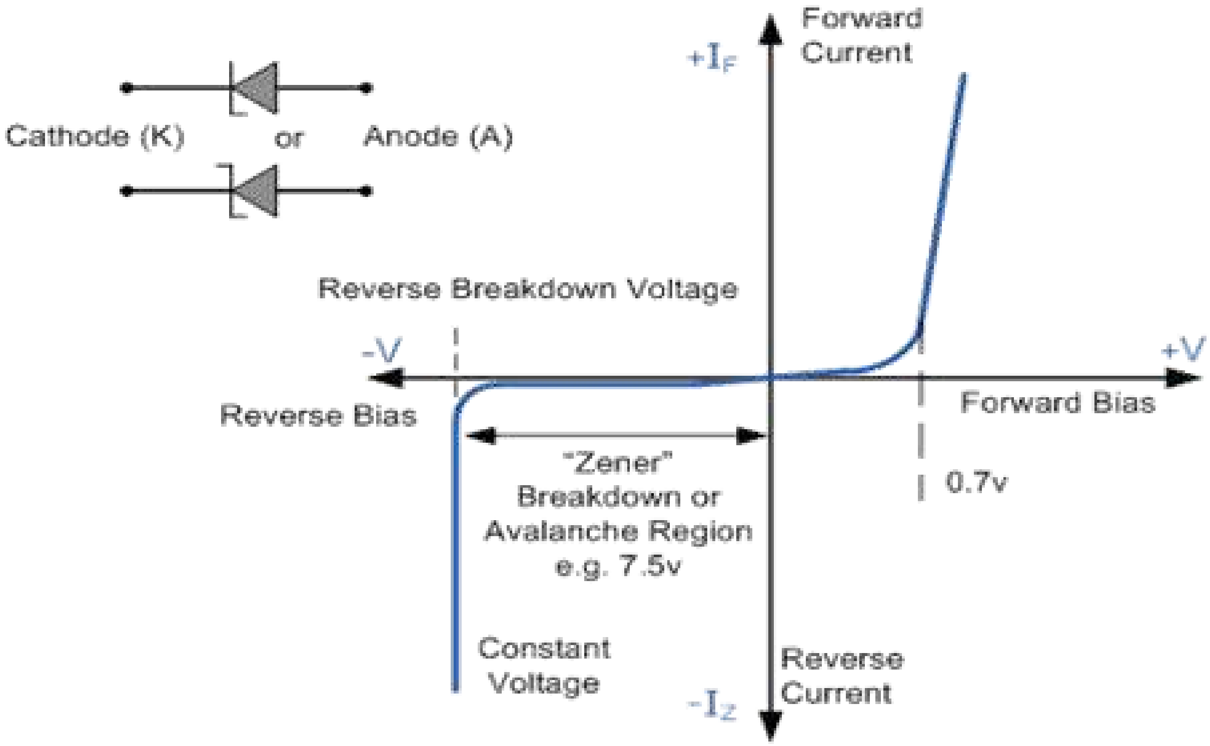


Ans.

A	B	Y’	Y
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

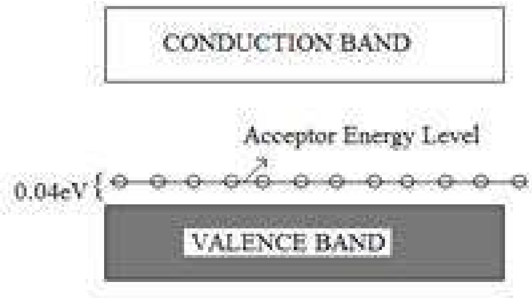
3.Draw the voltage current characteristics of a zener diode?

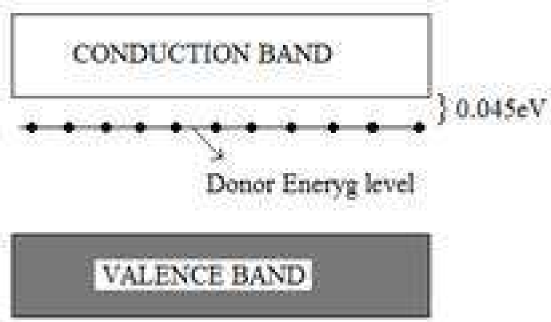
Ans.



4.For a extrinsic semiconductor, indicate on the energy band diagram the donor and acceptor levels?

Ans. N-type Extrinsic Semiconductor P-type Extrinsic Semiconductor





5.What do you mean by depletion region and potential barrier in junction diode?

Ans.A layer around the junction between p and n-sections of a junction diode where charge carriers electrons and holes are less in number is called depletion region. The potential difference created across the junction due to the diffusion of charge carriers across the junction is called potential barrier.

6.A transistor has a current gain of 30. If the collector resistance is 6kΩ, input resistance is 1kΩ, calculate its voltage gain?

Ans. Given $Rin = 1k\Omega$

$Rout = 6k\Omega$

$\therefore Rgain = \frac{6}{1} = 6$

$\therefore$ Voltage gain = current gain $\times$Rgain

Voltage gain = 30$\times$6 = 180

7. **What are the advantages and disadvantages of semiconductor devices over vacuum tubes?**

Ans. Advantages – Semiconductor devices are very small in size as compared to the vacuum tubes. It requires low voltage for their operation

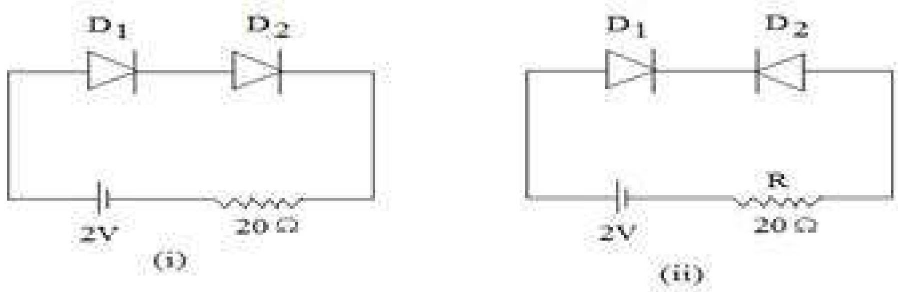
Disadvantage – Due to the rise in temperature and by applying high voltage it can be damaged.

8.The base of a transistor is lightly doped. Explain why?

Ans. In a transistor, the majority carries form emitter region moves towards the collector region through base. If base is made thick and highly doped, majority carriers will combine with the other carriers within the base and only few is collected by the collector which leads to small output collector current. Thus in order to have large output collector current, base is made thin and lightly doped.

---

9. Determine the currents through resistance R of the circuits (i) and (ii) when similar diodes $D_1 and\ D_2$ are connected as shown in the figure.



Ans. In figure (i) $D_1 and\ D_2$ are forward biased

$$\Rightarrow I = \frac{V}{R} = \frac{2}{20} = 0.1A$$

In figure (ii) D_1 is forward biased but D_2 is reverse biased due to which $D_1 and\ D_2$ offers infinite resistance

$\therefore I = 0$

10.What do you mean by hole in a circuit? Write its two characteristics?

Ans.A vacancy created in a covalent bond in a semiconductor due to the release of electron is known as hole in a semiconductor.

Characteristics of hole

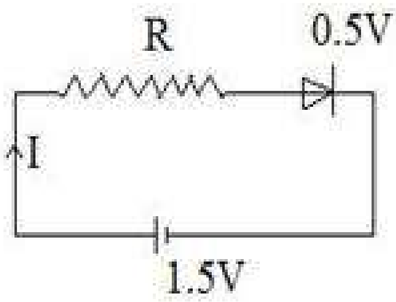
(i) Hole is equivalent to a positive electronic charge.

(ii) Mobility of hole is less than that of an electron

---

11. Diode used in the figure has a constant voltages drop at 0.5V at all currents and

a maximum power rating of 100mW. What should be the value of the resistance R, connected in series for maximum current?



Ans. $P = 100mW = 100 \times 10^{-3}$

$V = 0.5V$

$P = VI$

$$\Rightarrow I = \frac{P}{V} = \frac{100 \times 10^{-3}}{0.5}$$

I = 0.2A

For the given circuit

IR = + 0.5 – 1.5 = 0

I R = 0.5

IR=0.5-1.5

IR-1=0

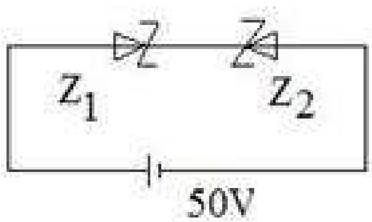
0.2 x R=1

$$R = \frac{1}{0.2} = 5\Omega$$

$$R = 5\Omega$$

---

12. Zener diode Z_1 has saturation current of 20A and reverse breakdown voltage of 100V where as the corresponding value of Z_2 are 40 μ A and 40. Find the current through the circuit?



Ans. Here Z_1 is forward biased where as Z_2 reverse biased hence Z_1 behaves as a conductor and reverse saturation current will flow from Z_2

Thus $R_{Z_2} = \dfrac{40}{40 \times 10^{-6}}$

$R_{Z_2} = 10^6 \Omega$

Now 50V will appear across Z_2 so

$I = \dfrac{50}{10^6}$

$I = 50 \times 10^{-6} A$

13. In an n-type silicon, which of the following statement is true:

(a) Electrons are majority carriers and trivalent atoms are the dopants.

(b) Electrons are minority carriers and pentavalent atoms are the dopants.

(c) Holes are minority carriers and pentavalent atoms are the dopants.

(d) Holes are majority carriers and trivalent atoms are the dopants.

Ans. The correct statement is **(c)**.

In an n-type silicon, the electrons are the majority carriers, while the holes are the minority carriers. An n-type semiconductor is obtained when pentavalent atoms, such as phosphorus, are doped in silicon atoms.

14. Which of the statements given in Exercise 14.1 is true for p-type semiconductors.

Ans. The correct statement is **(d).**

In a p-type semiconductor, the holes are the majority carriers, while the electrons are the minority carriers. A p-type semiconductor is obtained when trivalent atoms, such as aluminium, are doped in silicon atoms.

15. Carbon, silicon and germanium have four valence electrons each. These are characterised by valence and conduction bands separated by energy band gap respectively equal to $(E_g)_C$, $(E_g)_{Si}$ and $(E_g)_{Ge}$ Which of the following statements is true?

(a) $(E_g)_{Si} < (E_g)_{Ge} < (E_g)_C$

(b) $(E_g)_C < (E_g)_{Ge} > (E_g)_{Si}$

(c) $(E_g)_C > (E_g)_{Si} > (E_g)_{Ge}$

(d) $(E_g)_C = (E_g)_{Si} = (E_g)_{Ge}$

Ans. The correct statement is **(c)**.

Of the three given elements, the energy band gap of carbon is the maximum and that of germanium is the least.

The energy band gap of these elements are related as: $(E_g)_C > (E_g)_{Si} > (E_g)_{Ge}$

---

16. In an unbiased p-n junction, holes diffuse from the p-region to n-region because

(a) free electrons in the n-region attract them.

(b) they move across the junction by the potential difference.

(c) hole concentration in p-region is more as compared to n-region.

(d) All the above.

Ans. The correct statement is **(c)**.

The diffusion of charge carriers across a junction takes place from the region of higher concentration to the region of lower concentration. In this case, the p-region has greater concentration of holes than the n-region. Hence, in an unbiased p-n junction, holes diffuse from the p-region to the n-region.

---

17. When a forward bias is applied to a p-n junction, it

(a) raises the potential barrier.

(b) reduces the majority carrier current to zero.

(c) lowers the potential barrier.

(d) None of the above.

Ans. The correct statement is **(c)**.

When a forward bias is applied to a p-n junction, it lowers the value of potential barrier. In the case of a forward bias, the potential barrier opposes the applied voltage. Hence, the potential barrier across the junction gets reduced.

---

18. For transistor action, which of the following statements are correct:

(a) Base, emitter and collector regions should have similar size and doping concentrations.

(b) The base region must be very thin and lightly doped.

(c) The emitter junction is forward biased and collector junction is reverse biased.

(d) Both the emitter junction as well as the collector junction are forward biased.

Ans. The correct statement is **(b)**, **(c)**.

For a transistor action, the junction must be lightly doped so that the base region is very thin. Also, the emitter junction must be forward-biased and collector junction should be reverse-biased.

---

19. For a transistor amplifier, the voltage gain

(a) remains constant for all frequencies.

(b) is high at high and low frequencies and constant in the middle frequency range.

(c) is low at high and low frequencies and constant at mid frequencies.

(d) None of the above.

Ans. The correct statement is **(c)**.

The voltage gain of a transistor amplifier is constant at mid frequency range only. It is low at high and low frequencies.

---

20. In half-wave rectification, what is the output frequency if the input frequency is 50 Hz. What is the output frequency of a full-wave rectifier for the same input frequency.

Ans. Input frequency = 50 Hz

For a half-wave rectifier, the output frequency is equal to the input frequency.

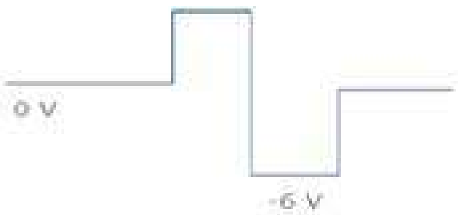
$\therefore$ Output frequency = 50 Hz

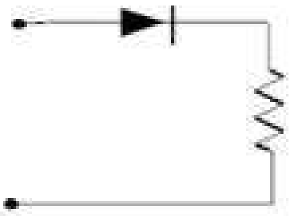
For a full-wave rectifier, the output frequency is twice the input frequency.

$\therefore$ Output frequency = 2×50 = 100 Hz

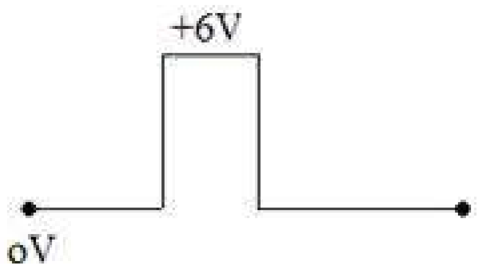
3 Mark Questions

1. What is an ideal diode? Draw the output wave form across the load resistor R, if the input waveform is as shown in the figure.





Ans. An ideal diode has zero resistance when forward biased and an infinite resistance when it is reverse biased. Output wave from is



2. With the help of a labeled circuit diagram, explain full wave rectification using junction diode. Draw input and output wave forms?

Ans. Full wave rectifier consists of two diodes and a transformer with central tap. For any half cycle of a.c. input only one diode is forward biased where as the other one is reverse biased.

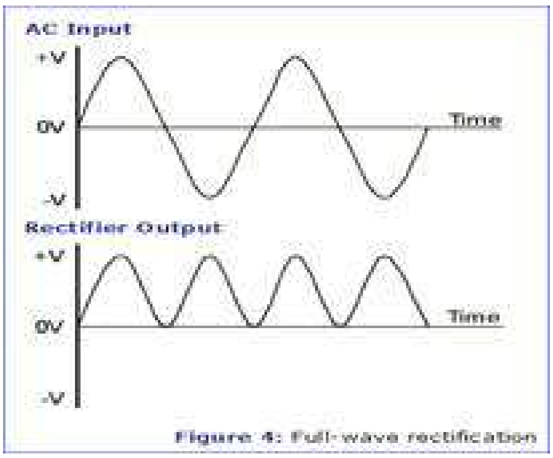
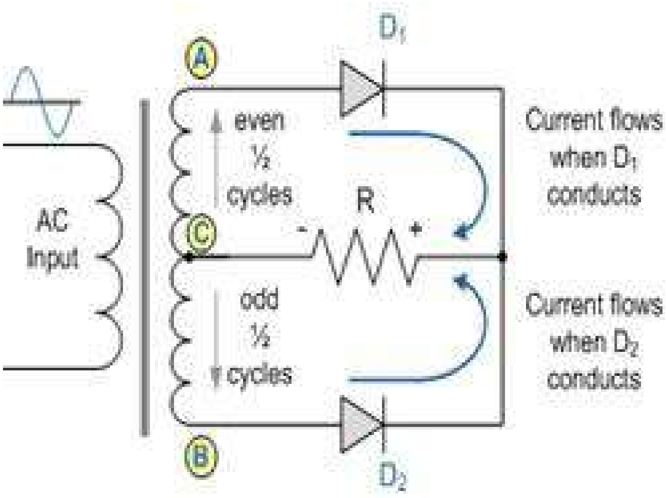
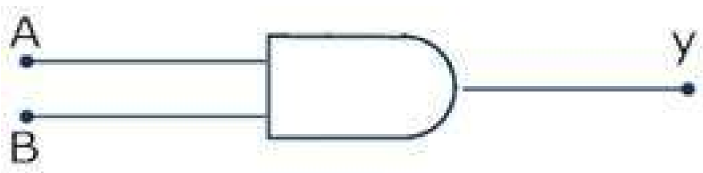


Figure 4: Full-wave rectification



Suppose for positive half of a.c. input diode D_1 is forward biased and D_2 is reverse biased, then the current will flow across D_1 where as for negative half of a.c. input diode D_2 is forward biased and the current flows across D_2. Thus for both the halves output is obtained and current flows in the same direction across load resistance R_2 and thus a.c. is converted into d.c.

3.Name the gate shown in the figure and write its truth table?



Ans. It is AND gate and its truth table is

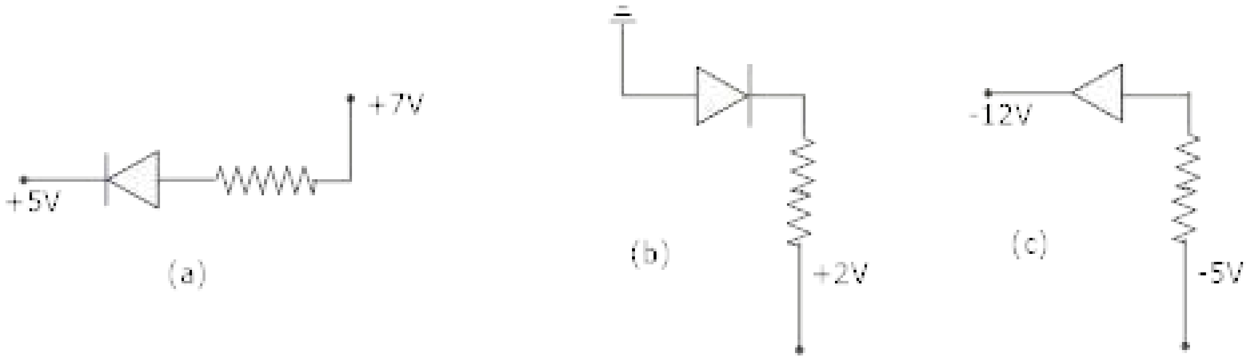
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

4. **In the following diagrams indicate which of the diodes are forward biased and which are reverse bias?**

Ans.(a) Forward Biased

(b) Reverse Biased

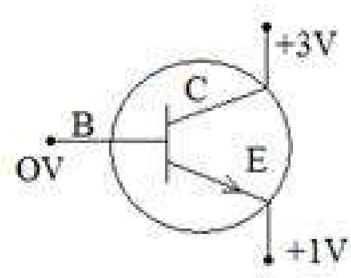
(c) forward Biased



5.In the given figure, is

(i) The emitter base

(ii) collector base forward or reverse biased? Justify.

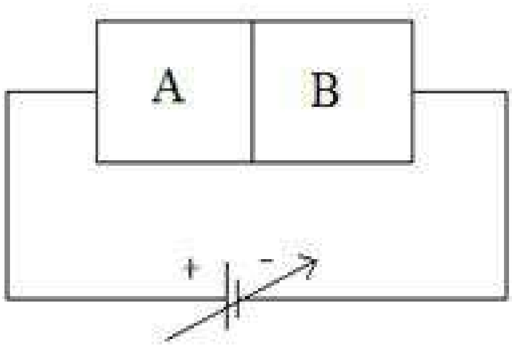


Ans. Figure shows n-p-n transistor

(i) Emitter is reversed biased because n-region is connected to higher potential.

(ii) Collector is also reversed biased because n-region of p-n junction is at higher potential than p-region.

6.Two semiconductor materials A and B shown in the figure are made by doping germanium crystal with arsenic and indium respectively. The two are joined end to end and connected to a battery as shown.

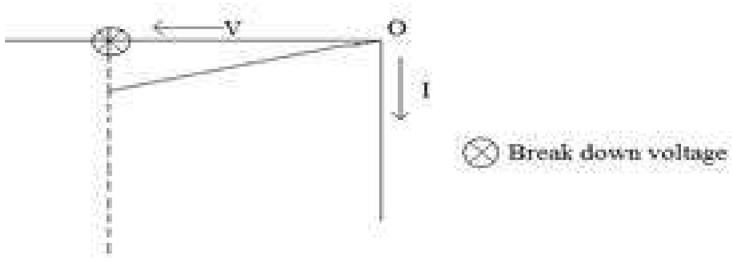


(a) Will the junction be forward biased or reverse biased? Justify

(b) Sketch a V-I graph for this arrangement

Ans. Material A is n-type as it is doped with pentavalent impurity and material B is p-type as it is doped with trivalent impurity. As a result the junction becomes reverse biased because positive terminal of the battery is connected to n-type and negative terminal to the p-type hence it is reversed biased.

V-I graph for the given circuit



---

7. Calculate emitter current for which $\beta = 100$ **and** $I_\beta = 20\mu A$ **?**

Ans. $\beta = 100$,

$I_B = 20\,\mu A = 20 \times 10^{-6} A$

$B = \dfrac{I_C}{I_B}$

$I_C = BI_B = 100 \times 20 = 2000\,\mu A$

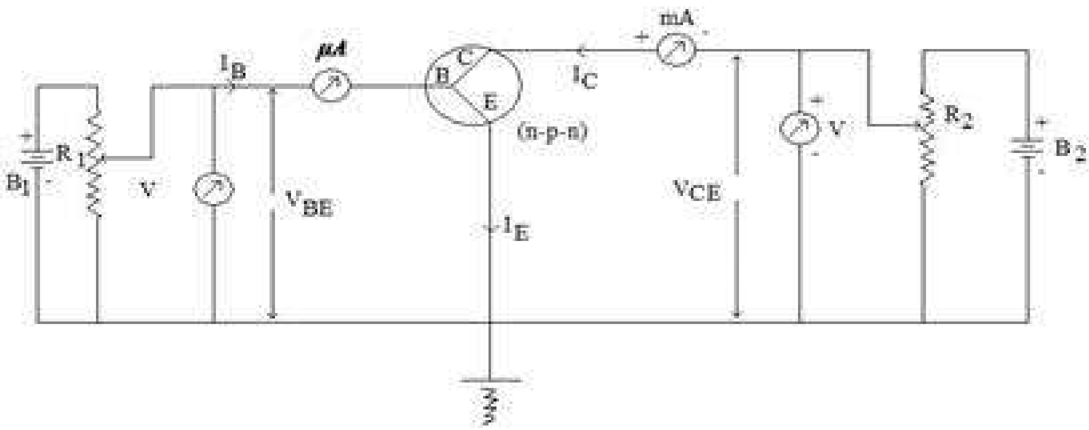
Using $I_E = I_B + I_C$

$I_E = 20 \times 10^{-6} + 2000\,\mu A$

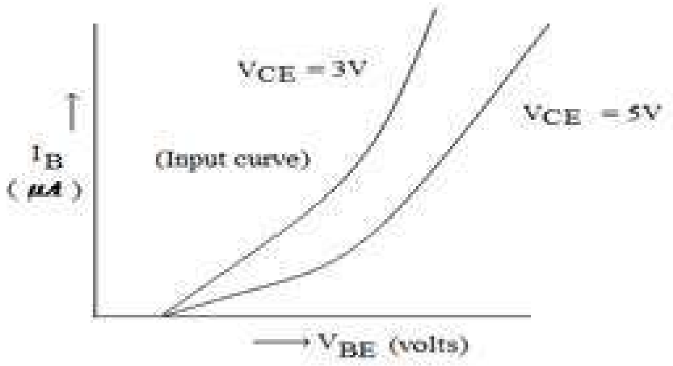
$I_E = 2020\,\mu A$

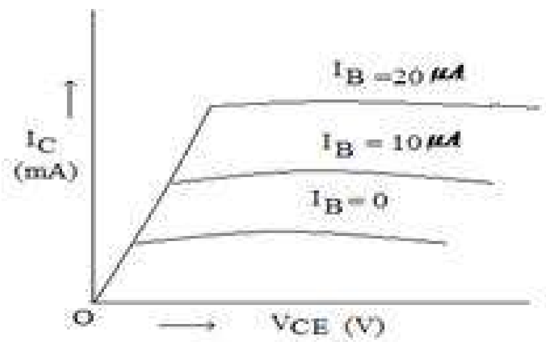
8. Draw the circuit diagram for common – emitter transistor characteristics using N-P-N transistor? Draw the input and output characteristic curve ?

Ans.



Input characteristic curve is the variation of base current I_B (Input) with base – emitter voltage (VEB) at constant collector emitter voltage (V_{CE}).





output characteristics is the variation of the collector current (I_C) with collector emitter voltage (V_{CE}) at constant base current (I_B) is called output characteristics.

---

9. For a CE-transistor amplifier, the audio signal voltage across the collected resistance of 2 $k\Omega$ is 2 V. Suppose the current amplification factor of the transistor is 100, find the input signal voltage and base current, if the base resistance is 1 $k\Omega$.

Ans. Collector resistance, $R_C = 2\ \mathrm{k\Omega} = 2000\Omega$

Audio signal voltage across the collector resistance, V = 2 V

Current amplification factor of the transistor, β = 100

Base resistance, $R_B = 1\ \mathrm{k\Omega} = 1000\ \Omega$

Input signal voltage = V_i

Base current = I_B

We have the amplification relation as:

Voltage amplification $= \frac{V}{V_1} = \beta \frac{R_c}{R_B}$

$$V_i = \frac{VR_B}{\beta R_C}$$

$$= \frac{2 \times 1000}{100 \times 2000} = 0.01V$$

Therefore, the input signal voltage of the amplifier is 0.01 V.

Base resistance is given by the relation:

$$R_B = \frac{V_i}{I_B}$$

$$= \frac{0.01}{1000} = 10 \times 10\mu^{-6}\text{A}$$

Therefore, the base current of the amplifier is 10 μA

---

10. Two amplifiers are connected one after the other in series (cascaded). The first amplifier has a voltage gain of 10 and the second has a voltage gain of 20. If the input signal is 0.01 volt, calculate the output ac signal.

Ans. Voltage gain of the first amplifier, V_1 = 10

Voltage gain of the second amplifier, V_2 = 20

Input signal voltage, V_i = 0.01 V

Output AC signal voltage = V_o

The total voltage gain of a two-stage cascaded amplifier is given by the product of voltage gains of both the stages, i.e.,

$V = V_1 \times V_2$

$= 10 \times 20$ = 200

We have the relation:

$$V = \frac{V_0}{V_1}$$

$V_0 = V \times V_i$

$= 200 \times 0.01$ = 2 V

Therefore, the output AC signal of the given amplifier is 2 V.

---

11. A p-n photodiode is fabricated from a semiconductor with band gap of 2.8 eV. Can it detect a wavelength of 6000 nm?

Ans. Energy band gap of the given photodiode, *Eg* = 2.8 eV

Wavelength, λ = 6000 nm = $6000 \times 10^{-9}\ m$

The energy of a signal is given by the relation:

$E = \frac{hc}{\lambda}$

Where,

h = Planck's constant

= $6.626 \times 10^{-34}\ Js$

c = Speed of light

= $3 \times 10^{8}\ m/s$

$E = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{6000 \times 10^{-9}}$

$= 3.313 \times 10^{-20}\ J$

= $3.313 \times 10^{-20}\ J$

But $1.6 \times 10^{-19}\ J$ = 1 eV

$\therefore E = 3.313 \times 10^{-20}\ J$

$$= \frac{3.313 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.207 eV$$

The energy of a signal of wavelength 6000 nm is 0.207 eV, which is less than 2.8 eV - the energy band gap of a photodiode. Hence, the photodiode cannot detect the signal.

---

12. The number of silicon atoms per m^3 is 5×10^{28}. This is doped simultaneously with 5×10^{22} atoms per m^3 of Arsenic and 5×10^{20} per m^3 atoms of Indium. Calculate the number of electrons and holes. Given that $n_i = 1.5\times10^{16}\ \mathrm{m}^{-3}$. Is the material n-type or p-type?

Ans. Number of silicon atoms, $N = 5 \times 10^{28}\ atoms / m^3$

Number of arsenic atoms, $n_{As} = 5\times10^{22}\ atoms / m^3$

Number of indium atoms, $n_{In} = 5\times10^{20}\ atoms / m^3$

Number of thermally-generated electrons, $n_i = 1.5\times10^{16}\ electrons / m^3$

Number of electrons, $n_e = 5\times10^{22} - 1.5\times10^{16} \approx 4.99\times10^{22}$

Number of holes = n_h

In thermal equilibrium, the concentrations of electrons and holes in a semiconductor are related as:

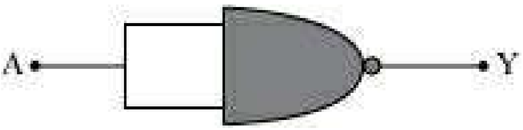
$$n_e n_h = n_i^2$$

$$\therefore n_h = \frac{n_i}{n_e}$$

$$= \frac{\left(1.5\times10^{16}\right)^2}{4.99\times10^{22}} \approx 4.51\times10^9$$

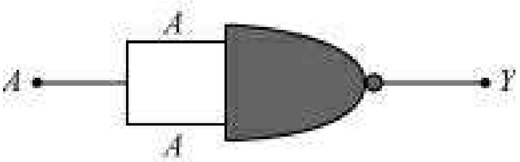
Therefore, the number of electrons is approximately 4.99×10^{22} and the number of holes is about 4.51×10^{9}. Since the number of electrons is more than the number of holes, the material is an n-type semiconductor.

13.Write the truth table for a NAND gate connected as given in Fig. 14.45.



Hence identify the exact logic operation carried out by this circuit.

Ans. *A* acts as the two inputs of the NAND gate and *Y* is the output, as shown in the following figure.



NAND Gate

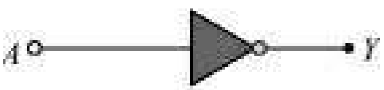
Hence, the output can be written as:

$Y = \overline{A \cdot B} = \overline{A} + \overline{B} = \overline{A}$ ....(i)

The truth table for equation (*i*) can be drawn as:

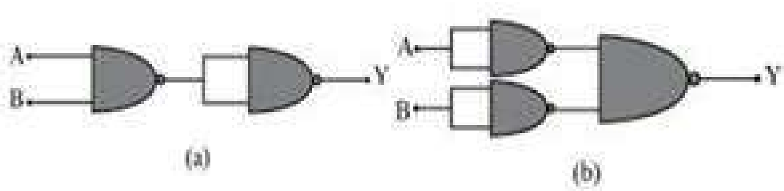
A	$Y\left(=\overline{A}\right)$
0	1
1	0

This circuit functions as a NOT gate. The symbol for this logic circuit is shown as:



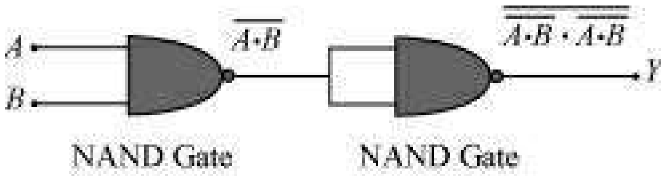
NOT Gate

14. You are given two circuits as shown in Fig. 14.46, which consist of NAND gates. Identify the logic operation carried out by the two circuits.



Ans. In both the given circuits, *A* and *B* are the inputs and *Y* is the output.

(a) The output of the left NAND gate will be $\overline{A \cdot B}$, as shown in the following figure.

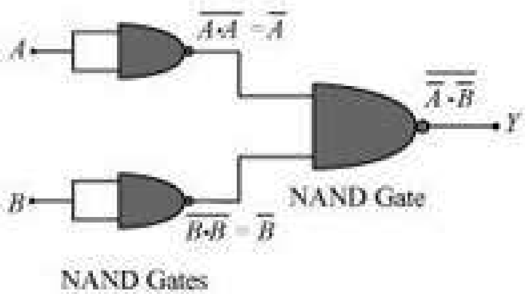


Hence, the output of the combination of the two NAND gates is given as:

$$Y = \overline{\left(\overline{A \cdot B}\right) . \left(\overline{A \cdot B}\right)} = \overline{\overline{AB}} + \overline{\overline{AB}} = AB$$

Hence, this circuit functions as an AND gate.

(b) $\overline{A}$ is the output of the upper left of the NAND gate and $\overline{B}$ is the output of the lower half of the NAND gate, as shown in the following figure.



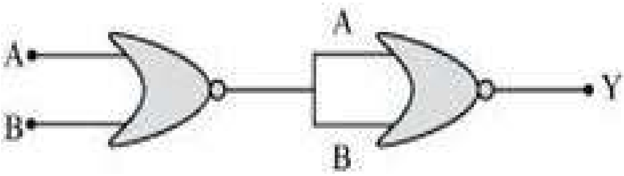
Hence, the output of the combination of the NAND gates will be given as:

$$Y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$$

Hence, this circuit functions as an OR gate.

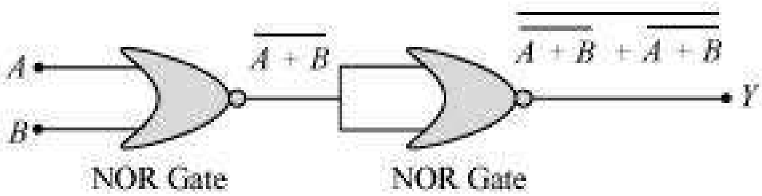
15.Write the truth table for circuit given in Fig. 14.47 below consisting of NOR gates and

identify the logic operation (OR, AND, NOT) which this circuit is performing.



(Hint: A = 0, B = 1 then A and B inputs of second NOR gate will be 0 and hence Y=1. Similarly work out the values of Y for other combinations of A and B. Compare with the truth table of OR, AND, NOT gates and find the correct one.)

Ans.A and B are the inputs of the given circuit. The output of the first NOR gate is $\overline{A+B}$. It can be observed from the following figure that the inputs of the second NOR gate become the out put of the first one.



Hence, the output of the combination is given as:

$$\overline{\overline{A+B}+\overline{A+B}} = \overline{\overline{A}\cdot\overline{B}+\overline{A}.\overline{B}}$$

$$= \overline{\overline{A}\cdot\overline{B}} = \overline{\overline{A}}+\overline{\overline{B}} = A+B$$

The truth table for this operation is given as:

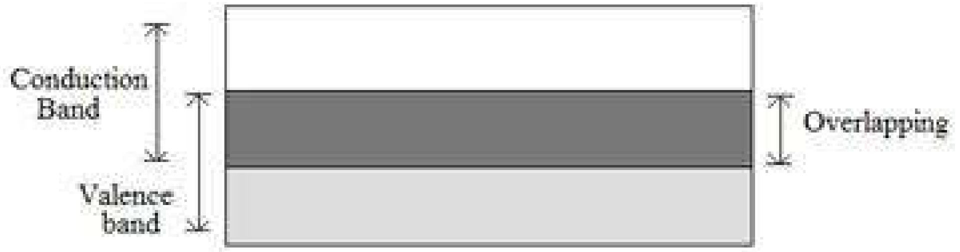
A	***B***	***Y*(=*A*+*B*)**
0	0	0
0	1	1
1	0	1
1	1	1

This is the truth table of an OR gate. Hence, this circuit functions as an OR gate.

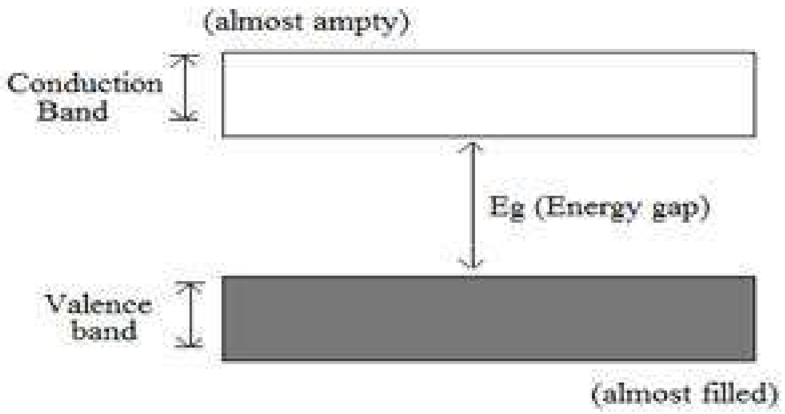
5 Mark Questions

1.Distinguish between conductors, insulators and semiconductors on the basis of energy band diagrams?

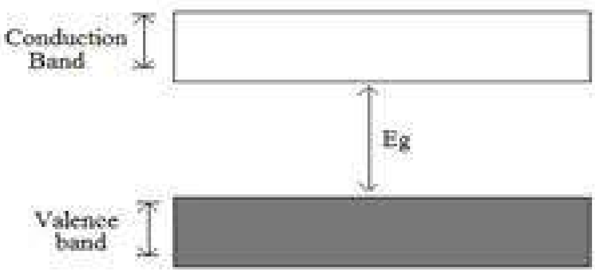
Ans.<u>Conductor</u> – Conduction band in a conductor is either partially filled or conduction and valence band overlaps each other. There is no energy gap in a conductor.



<u>Insulators</u> – conduction band and valence band of all insulator are widely separated by and energy gap of the order 6 to 9eV Also conduction band of an insulator is almost empty.



<u>Semiconductor</u> – In semiconductors the energy gap is very small i.e. about 1ev only.



2.The following truth table gives the output of a 2-input logic gate.

A B output

0 0 1

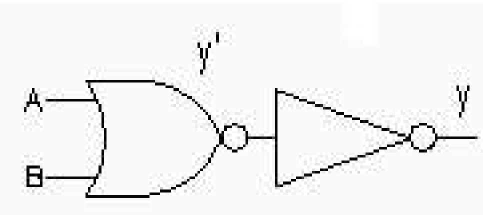
01 0

10 0

11 0

Identify the logic gate used and draw its logic symbol. If the output of this gate is fed as input to a NOT gate, name the new logic gate so formed?

Ans. The gate is NOR gate. If the output of NOR gate is connected to a NOT gate then the figure will be



New truth table is

A B Y

000

011

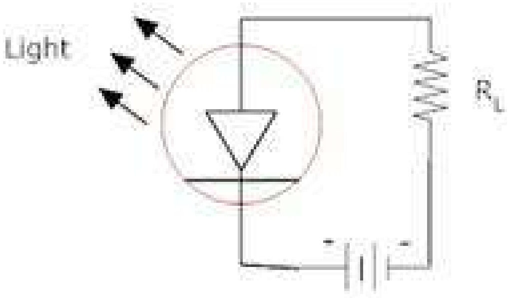
101

111

It is the truth table of OR gate

3.With the help of a diagram, show the biasing of a light emitting diode (LED). Give its two advantages over conventional incandescent lamps?

Ans. Light emitting diode is forward biased i.e. energy is released at the junction.



<u>Advantages of LED</u>

(1) They are used in numerical displays as compact in size.

(2) It works at low voltage and has longer life than incandescent bulbs.

4.The input resistance of a silicon transistor is 665 Ω. Its base current is changed by 15 μ A, which results in the change in collector current by 2mA. This transistor is used as a common emitter amplifier with a load resistance of 5k Ω. Calculate current gain (βac).

Ans.(1) Trans conductance (gm) (2) voltage gain (Av) of the amplifier.

Here $\Delta I_B = 15\mu A = 15\times10^{-6} A$

$\Delta I_C = 2mA = 2\times10^{-3} A$

$Rin = 665\Omega, R_2 = 5k\Omega = 5\times10^3 \Omega$

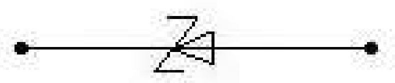
$$\beta ac = \frac{\Delta I_C}{\Delta I_B} = \frac{2\times10^{-3}}{15\times10^{-6}} = 133.3$$

(1) Trans conductance, gm = $\frac{\beta ac}{Rin} = \frac{133.3}{665} = 0.2\Omega^{-1}$

(2) Voltage gain (Av) = $gmR_L = 0.2 \times 5 \times 10^3 = 1000$

5.Draw the symbol for zener diode? Zener diodes have higher dopant densities as compared to ordinary p-n junction diodes. How dos it affect the (i) width of the depletion layer (i) junction field?

Ans. Symbol for zener diode



N P

(i) Width of the depletion layer of zener diode becomes very small due to heavy doping of p and n-regions

(ii) Junction field will be high.

6.A P-N-P transistor is used in common – emitter mode in an amplifier circuit. A change of $40\mu A$ in the base current brings a change of 2mA in collector current and 0.04V in base – emitter voltage. Find (i) input resistance (ii) current amplification factor (β) . If a load resistance of 6kΩ is used, then find voltage gain?

Ans. $\Delta I_B = 40\mu A = 40 \times 10^{-6} A$

$\Delta I_C = 2mA = 2 \times 10^{-3} A$

$\Delta V_{BE} = 0.04V$

$R_L = 6k\Omega = 6 \times 10^3 \Omega$

$$Rin = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{0.04}{40\times10^{-6}} = 1\times10^3\,\Omega = 1k\Omega$$

$$\beta = \frac{\Delta \upsilon_C}{\Delta I_B} = \frac{2\times10^{-3}}{40\times10^{-6}} = 50$$

Voltage gain $= \beta^{RL}/_{Ri} = \dfrac{50\times6\times10^3}{1\times10^3} = 300$

7.A semiconductor has equal electron and whole concentration of $6\times10^8/m^3$ **.**

On doping with certain impurity, electron concentration increases to $8\times10^{12}/m^3$ **.**

(i) Identify the new semiconductor

(ii) Calculate the new whole concentration.

(iii) How does the energy gap vary with doping?

Ans.(i) New semiconductor obtained is N-type because

(ii) $nenh = ne^2$

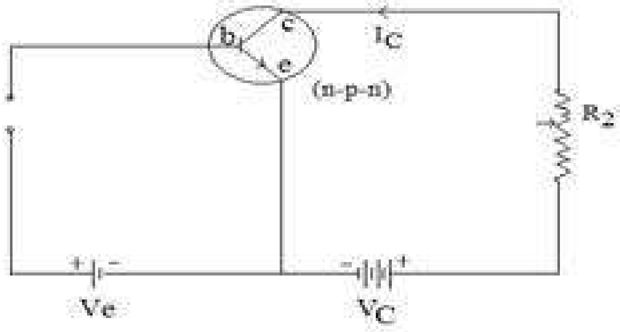
$$nh = \frac{ni^2}{ne} = \frac{36\times10^{16}}{8\times10^{12}}$$

$$nh = 4.5\times10^4/m^3$$

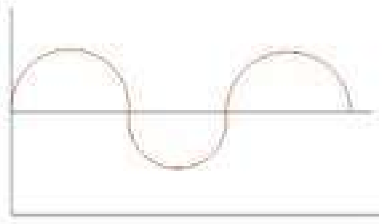
(iii) Energy gap decreases due to creation of donor level in between the valence band and the conduction band.

8.Draw a labeled circuit diagram of a common emitter transistor amplifier. Draw the input and the output wave forms and also state the relation between input and output signal?

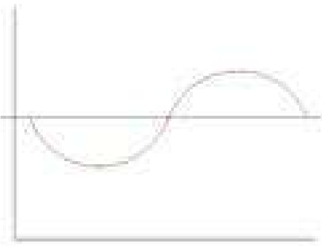
Ans.



Input wave from



Output wave form



Relation – output waveform has 180^o phase reversal as compared to input and also the output is being amplified.

9.In an intrinsic semiconductor the energy gap E_g is 1.2 eV. Its hole mobility is much smaller than electron mobility and independent of temperature. What is the ratio between conductivity at 600K and that at 300K? Assume that the temperature dependence of intrinsic carrier concentration n_i is given by

$$n_i = n_o \exp\left[-\frac{Eg}{2k_B T}\right]$$

where n_0 is a constant.

Ans. Energy gap of the given intrinsic semiconductor, E_g = 1.2 eV

The temperature dependence of the intrinsic carrier-concentration is written as:

$$n_i = n_o \exp\left[-\frac{Eg}{2k_B T}\right]$$

Where k_B = Boltzmann constant = $8.62 \times 10^{-5} eV/K$

T = Temperature

n_0 = Constant

Initial temperature, T_1 = 300 K

The intrinsic carrier-concentration at this temperature can be written as:

$$n_{i1} = n_0 \exp\left[-\frac{E_g}{-2k_B \times 300}\right] \ldots (1)$$

Final temperature, T_2 = 600 K

The intrinsic carrier-concentration at this temperature can be written as:

$$n_{i2} = n_0 \exp\left[-\frac{E_g}{-2k_B \times 600}\right] \ldots (2)$$

The ratio between the conductivities at 600 K and at 300 K is equal to the ratio between the respective intrinsic carrier-concentrations at these temperatures.

$$\frac{n_{i2}}{n_{i1}} = \frac{n_o \exp\left[-\frac{E_g}{2k_B 600}\right]}{n_o \exp\left[-\frac{E_g}{2k_B 300}\right]}$$

$$= \exp \frac{E_g}{2k_B}\left[\frac{1}{300}-\frac{1}{600}\right]$$

$$= \exp\left[\frac{1.2}{2\times 8.62\times 10^{-5}}\times\frac{2-1}{600}\right]$$

$$= \exp[11.6] = 1.09\times 10^5$$

Therefore, the ratio between the conductivities is 1.09×10^5.

10.In a p-n junction diode, the current I can be expressed as

$$I = I_0 \exp\left(\frac{eV}{2k_BT}-1\right)$$

where *I*0 is called the reverse saturation current, *V* is the voltage across the diode and is positive for forward bias and negative for reverse bias, and *I* is the current through the diode, *k* Bis the Boltzmann constant (8.6×10^{-5} eV/K) and T is the absolute temperature. If for a given diode $I_0 = 5\times 10^{-12}$ A and T = 300 K, then

(a) What will be the forward current at a forward voltage of 0.6 V?

(b) What will be the increase in the current if the voltage across the diode is increased to 0.7 V?

(c) What is the dynamic resistance?

(d) What will be the current if reverse bias voltage changes from 1 V to 2 V?

Ans. In a p-n junction diode, the expression for current is given as:

$$I = I_0 \exp\left(\frac{eV}{2k_BT}-1\right)$$

Where,

I_0 = Reverse saturation current = $5 \times 10^{-12}\ A$

T = Absolute temperature = 300 K

k_B = Boltzmann constant = $8.6\times10^{-5}eV/K = 1.376\times10^{-23}JK^{-1}$

V = Voltage across the diode

(a) Forward voltage, V = 0.6 V

$\therefore$Current, $I = 5\times10^{-12}\left[\exp\left(\dfrac{1.6\times10^{-19}\times0.6}{1.376\times10^{-23}\times300}\right)-1\right]$

$$=5\times10^{-12}\times\exp[22.36]=0.0256\text{A}$$

Therefore, the forward current is about 0.0256 A.

(b) For forward voltage, V' = 0.7 V, we can write:

$$=5\times10^{-12}\left[\exp\left(\frac{1.6\times10^{-19}\times0.7}{1.376\times10^{-23}\times300}\right)-1\right]$$

$$=5\times10^{-12}\times\exp[26.25]=1.257\text{A}$$

Hence, the increase in current, $\Delta I = I' - I$

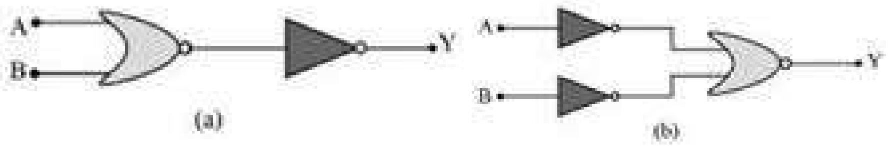
= 1.257 - 0.0256 = 1.23 A

(c) Dynamic resistance $=\dfrac{Change\ in\ voltage}{Change\ in\ current}$

$$=\frac{0.7-0.6}{1.23}=\frac{0.1}{1.23}=0.081\Omega$$

(d) If the reverse bias voltage changes from 1 V to 2 V, then the current (I) will almost remain equal to I_0 in both cases. Therefore, the dynamic resistance in the reverse bias will be infinite.

11. You are given the two circuits as shown in Fig. 14.44. Show that circuit (a) acts as OR gate while the circuit (b) acts as AND gate.



Ans.(a) *A* and *B* are the inputs and *Y* is the output of the given circuit. The left half of the given figure acts as the NOR Gate, while the right half acts as the NOT Gate. This is shown in the following figure.

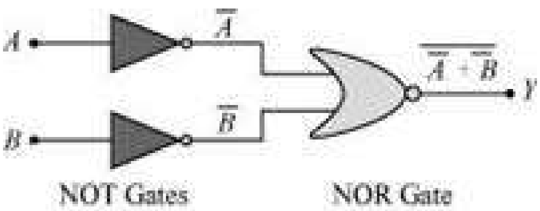
Hence, the output of the NOR Gate = $\overline{A+B}$

This will be the input for the NOT Gate. Its output will be $\overline{\overline{A+B}}$ = *A* + *B*

$\therefore Y = A + B$

Hence, this circuit functions as an OR Gate.

(b) *A* and *B* are the inputs and *Y* is the output of the given circuit. It can be observed from the following figure that the inputs of the right half NOR Gate are the outputs of the two NOT Gates.



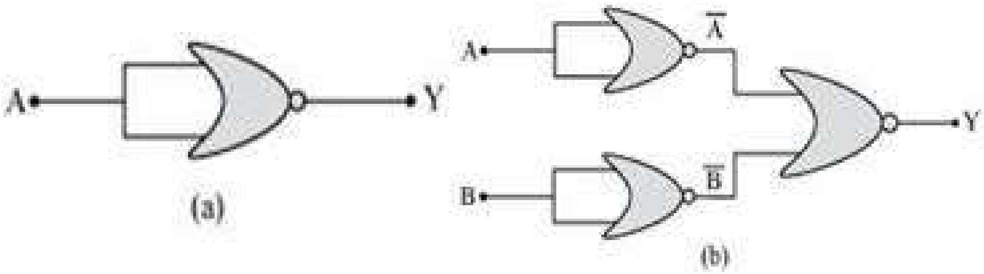
Hence, the output of the given circuit can be written as:

$$Y = \overline{\overline{A}+\overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}} = A \cdot B$$

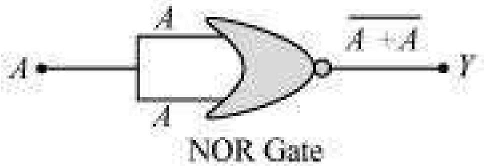
Hence, this circuit functions as an AND Gate.

12.Write the truth table for the circuits given in Fig. 14.48 consisting of NOR gates only.

Identify the logic operations (OR, AND, NOT) performed by the two circuits.



Ans.(a)A acts as the two inputs of the NOR gate and Y is the output, as shown in the following figure. Hence, the output of the circuit is $\overline{A+A}$.



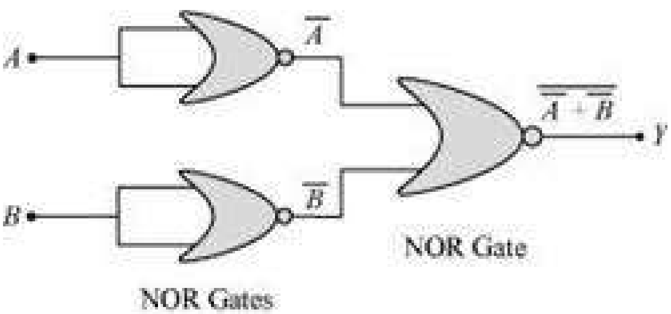
Output, $Y=\overline{A+A}=\overline{A}$

The truth table for the same is given as:

A	***Y*** $\left(=\overline{A}\right)$
0	1
1	0

This is the truth table of a NOT gate. Hence, this circuit functions as a NOT gate.

(b) A and B are the inputs and Y is the output of the given circuit. By using the result obtained in solution **(a)**, we can infer that the outputs of the first two NOR gates are $\overline{A}\ and\ \overline{B}$,as shown in the following figure.



$\overline{A}\ and\ \overline{B}$ are the inputs for the last NOR gate. Hence, the output for the circuit can be written as:

$$Y = \overline{\overline{A}+\overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}} = A \cdot B$$

The truth table for the same can be written as:

A	***B***	***Y***($= \mathrm{A} - B$)
0	0	0
0	1	0
1	0	0
1	1	1

This is the truth table of an AND gate. Hence, this circuit functions as an AND gate.

CBSE Class 12 physics
Important Questions
Chapter 15
Communication Systems

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1 Mark Questions

1.Name the type of the communication system in which the signal is discrete and binary coded version of message or information?
Ans. Digital communication.

2.What is the purpose of modulating a signal in transmission?
Ans. Modulation is done because low frequency signal cannot be transmitted to a longer distance so in order to increase the range of transmission modulation is done.

3.What is the requirement of transmitting microwaves form one position to another on the earth?
Ans. The transmitting and receiving antenna must be in the line of sight.

4.A T.V. tower has a height of 300m. What is the maximum distance up to which the T.V transmission can be received?
Ans. $d = \sqrt{2Rh} = \sqrt{2 \times 6400 \times 1000 \times 300}$

$d = 62km.$

5.Why ground wave propagation is not suitable for high frequencies?
Ans. Ground waves are not suitable for propagation high frequencies because signals having frequency more than 1500 KHz are greatly absorbed by the surface of the earth and cannot be transmitted.

6.What type of modulation is used for commercial broadcast of voice signal?
Ans. Amplitude Modulation.

2 Mark Questions

1. A signal jumps from one level to another instantaneously. What will be its frequency?

Ans. It means that signal jumps form one level to another in no time so its frequency will be infinite.

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2. Sky has no limit but sky wave propagation has its limit. Explain why?

Ans. Sky wave propagation is due to the reflection of radio waves by the ionosphere but high frequency waves gets absorbed by the ionosphere and cannot be reflected by the ionosphere.

---

3. A transmitting antenna has a height of 50m. If radius of the earth is taken as 6250 km. find the area covered by it?

Ans. $d = \sqrt{2rh}$

$d = \sqrt{2 \times 6250 \times 50 \times 10^3}$

$d = 2.5 \times 10^4 m$

Area covered $= \pi d^2 = 3.14 \times (2.5 \times 10^4)^2$

Area covered $= 1963 km^2$

---

4. What is the role of F_2 layer in communication?

Ans. F_2 is the topmost layer of ionosphere. Its height is upto 400km and is called as a reflecting layer for high frequency radio wave.

5.A carrier wave of peak voltage 12V is used to transmit a message signal. What should be the peak voltage of the modulating signal in order to have a modulation index of 75%?

Ans. $\mu = \frac{A_m}{A_c}$

$A_m = \mu \times A_c$

$A_m = \frac{75}{100} \times 12$

$A_m = 9V$

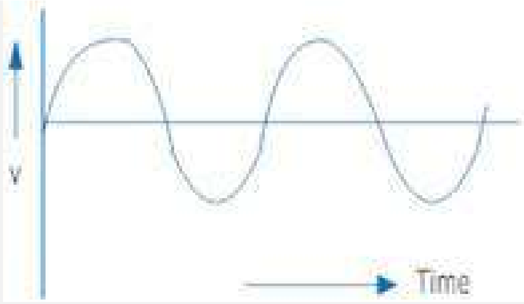
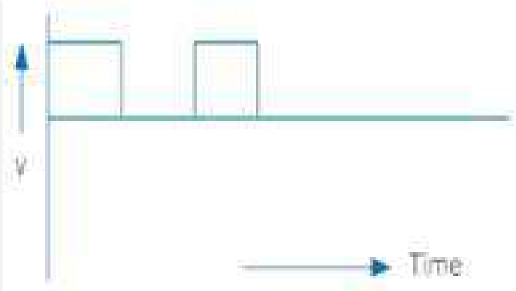
6.Give the set up of a basic communication system?

Ans. Basic communication system consist of an information source, a transmitter, a link and a receiver.

7.Distinguish between analog and digital communication?

Ans.

Analog communication	**Digital communication**
(1) In analog communication analog signal is used which continuous varies with time. (2)	(1) In digital communication digital signal is used which has only two levels i.e. high and low. (2)

8.Which of the following frequencies will be suitable for beyond-the-horizon communication using sky waves?

(a) 10 kHz

(b) 10 MHz

(c) 1 GHz

(d) 1000 GHz

Ans.(b) 10 MHz

Forbeyond-the-horizon communication, it is necessary for the signal waves to travel a large distance. 10 KHz signals cannot be radiated efficiently because of the antenna size. The high energy signal waves (1GHz - 1000 GHz) penetrate the ionosphere. 10 MHz frequencies get reflected easily from the ionosphere. Hence, signal waves of such frequencies are suitable for beyond-the-horizon communication.

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9. Frequencies in the UHF range normally propagate by means of:

(a) Ground waves.

(b) Sky waves.

(c) Surface waves.

(d) Space waves.

Ans.(d) Space waves

Owing to its high frequency, an ultra high frequency (UHF)wave can neither travel along the trajectory of the ground nor can it get reflected by the ionosphere. The signals having UHF are propagated through line-of-sight communication, which is nothing but space wave propagation.

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10. Digital signals

(i) Do not provide a continuous set of values,

(ii) Represent values as discrete steps,

(iii) Can utilize binary system, and

(iv) Can utilize decimal as well as binary systems.

Which of the above statements are true?

(a) (i) and (ii) only

(b) (ii) and (iii) only

(c) (i), (ii) and (iii) but not (iv)

(d) All of (i), (ii), (iii) and (iv).

Ans. (c) A digital signal uses the binary (0 and 1) system for transferring message signals. Such a system cannot utilise the decimal system (which corresponds to analogue signals). Digital signals represent discontinuous values.

---

11. Is it necessary for a transmitting antenna to be at the same heightas that of the receiving antenna for line-of-sight communication? A TV transmitting antenna is 81m tall. How much service area can it cover if the receiving antenna is at the ground level?

Ans. Line-of-sight communication means that there is no physical obstruction between the transmitter and the receiver. In such communications it is not necessary for the transmitting and receiving antennas to be at the same height.

Height of the given antenna, h = 81 m

Radius of earth, R = 6.4×10^6 m

For range, $d = (2Rh)\frac{1}{2}$, the service area of the antenna is given by the relation:

$A = \pi d^2$

$= \pi\ (2Rh)$

$= 3.14 \times 2 \times 6.4 \times 10^6 \times 81$

$= 3255.55 \times 10^6\ m^2$ = 3255.55

$\sim 3256 m^2$

---

12. A carrier wave of peak voltage 12 V is used to transmit a message signal. What should be the peak voltage of the modulating signal in order to have a modulation index of 75%?

Ans. Amplitude of the carrier wave, A_c = 12 V

Modulation index, m = 75% = 0.75

Amplitude of the modulating wave = A_m

Using the relation for modulation index:

$$m = \frac{A_m}{A_c}$$

$$A_m = mA_c$$

$$= 0.75 \times 12 = 9V$$

---

13. For an amplitude modulated wave, the maximum amplitude is found to be 10 V while the minimum amplitude is found to be 2 V. Determine the modulation index μ . What would be the value of μ if the minimum amplitude is zero volt?

Ans. Maximum amplitude, A_{max} = 10 V

Minimum amplitude, $A_{\min}$ = 2 V

Modulation index μ, is given by the relation:

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

$$= \frac{10-2}{10+2} = \frac{8}{12} = 0.67$$

If $\mathbf{A_{min}} = 0$

Then $\mu = \frac{A_{\max}}{A_{\max}} = \frac{10}{10} = 1$

3 Mark Questions

1. Define the term modulation index for A.M. wave. What would be the modulation index for an A.M. wave for which the maximum amplitude is 'a' write the minimum amplitude is 'b'?

Ans. Modulation index is the ratio of amplitude E_m of caries wave to the amplitude E_c of carries (original) wave.

i.e. $\mu = \frac{E_m}{E_c}$

Here Maximum Amplitude $a = E_c + E_m$

Minimum Amplitude $b = E_c - E_m$

$$\Rightarrow E_c = \frac{a+b}{2} \text{ and } E_m = \frac{a-b}{2}$$

$$\Rightarrow \mu = \frac{a-b}{a+b}$$

2. A T.V. tower has a height of 80m. By how much the height of tower be increased to triple its coverage?

Ans. Here $h_1 = 80m$

$$d_1 = \sqrt{2h_1R} = \sqrt{2 \times 80 \times R} = \sqrt{160R}$$

$$d_2 = \sqrt{2h_2R} = 3d_1$$

$$\sqrt{2h_2R} = 3\sqrt{160R}$$

$$\Rightarrow h_2 = 720m$$

---

3. An audio signal of amplitude one half of the carries amplitude is employed in amplitude modulation. What is the modulation index? Hence define amplitude modulation?

Ans. $E_m = 0.5\ E_c$

$$E_{max} = E_m + 0.5\ E_c = 1.5\ E_c$$

$$E_{min} = E_m - 0.5\ E_c = 0.5\ E_c$$

$$\mu = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}} = \frac{1.5E_c - 0.5E_c}{1.5E_c + 0.5E_c}$$

$$\mu = 0.5$$

---

4. An audio signal of 32kHz modulates a carrier of frequency 84MHz and produces a frequency deviation of 96kHz.

Find (a) frequency modulation index

(b) frequency range of the frequency modulated wave?

Ans. $fm = 3.2KHz$

$fc = 34MHz$

$\delta = 96KHz$

(a) Frequency modulated index

$$mf = \frac{\delta}{fm} = \frac{96}{3.2} = 30$$

(b) Frequency range of the modulated wave

$= fc \pm fm$

$= 84 \times 10^3 \pm 3.2KHz$

$=83.997MHz$ to $84.003MHz$

---

5. Define the following terms

(a) Ground wave propagation

(b) Space wave propagation

(c) Sky wave propagation

Ans.(a) **Ground wave propagation** – Radio travel along the surface of the earth and are called ground waves and the propagation along the surface of the earth is called ground wave propagation. It is limited to a frequency below 1.5 MHz

(b) **Space wave propagation** – The radio waves which are reflected back to the earth by ionosphere are known as sky waves and this mode of propagation of sky waves is known as sky wave propagation.

(c) **Space wave propagation** – High frequency waves which cannot be reflected back to the earth by transmitting antenna to receiving antenna by the mode called line of sight communication. It is also called as space wave propagation.

6. Which two communication methods make use of space wave propagation method? If the sum of the heights of transmitting and receiving antenna is line of sight communication is fixed at h, show that the range is maximum when the two antenna have a height $\frac{h}{2}$ each?

Ans. Satellite communication and line of sight (LOS) communication make use of space waves.

Now $d_1 = \sqrt{2Rh_1}$

$d_2 = \sqrt{2Rh_2}$

For maximum range

$d_m = \sqrt{2Rh_1} + \sqrt{2Rh_2}$

$d_m = d_1 + d_2 = d$

Given $h_1 + h_2 = h$

Let $h_1 = x$ then $h_2 = h - x$

$d_m = \sqrt{2Rx} + \sqrt{2R(h-x)}$

Differentiating wr.t x

$$\frac{dd_m}{d_x} - \sqrt{\frac{R}{2x}} = \sqrt{\frac{R}{2(h-x)}} = 0 i.e. \frac{1}{2x} = \frac{1}{2(h-x)}$$

$$\Rightarrow x = \frac{h}{2} \Rightarrow h_1 = h_2 = \frac{h}{2}$$

7. A frequency modulated wave is represented by an equation.

Find (1) carrier frequency

(2) modulating signal frequency

(3) Power dissipated if load resistor is of 100 Ω ?

Ans. Given e = 10 sin $\left(5\times10^{8}++6\sin1000t\right)$

Compare if with general equation

e = E sin (w_ct + m_f sin w_mt)

Carries frequency $v_c = \frac{w_c}{2\pi} = \frac{5\times10^{8}}{2\times3.14}$

$v_c = 79.62MHz$

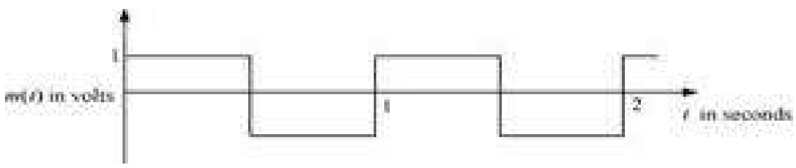
Modulating signal frequency

$$v_m = \frac{Wm}{2\pi} = \frac{100}{2\times3.14} = 15.92Hz$$

Power dissipated $P = \frac{(Erms)^2}{R} = \frac{\left(\frac{10}{\sqrt{2}}\right)^2}{100} = 0.5\ Watts$

---

8. A modulating signal is a square wave, as shown in Fig. 15.14.



The carrier wave is given by $c(t) = 2\sin(8\Omega t)volts.$

(i) Sketch the amplitude modulated waveform

(ii) What is the modulation index?

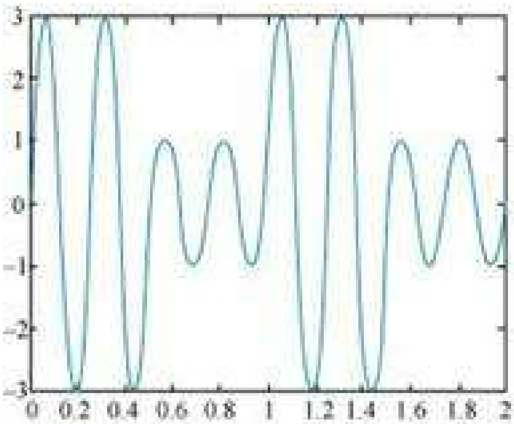
Ans. It can be observed from the given modulating signal that the amplitude of the modulating signal, Am= 1 V

It is given that the carrier wave $c(t) = 2\ sin\ (8nt)$

∴Amplitude of the carrier wave, Ac= 2 V

Time period of the modulating signal Tm= 1 s

The angular frequency of the modulating signal is calculated as:



$$(t)_m = \frac{2\pi}{T_m}$$

$= 2\pi rads^{-1}$ ....(i)

The angular frequency of the carrier signal is calculated as:

$(t)_c = 2\pi$ rad s^{-1} ....(ii)

From equations (i) and (ii), we get:

$(t)_c = 4(t)_m$

The amplitude modulated waveform of the modulating signal is shown in the following figure.

(ii) Modulation index, $m = \frac{A_m}{A_c} = \frac{1}{2} = 0.5$

9.Due to economic reasons, only the upper sideband of an AM wave is transmitted, but at the receiving station, there is a facility for generating the carrier. Show that if a device is available which can multiply two signals, then it is possible to recover the modulating signal at the receiver station.

Ans. Let ω_c and ω_s be the respective frequencies of the carrier and signal waves.

Signal received at the receiving station, $V = V_1 \cos\left(\omega_c + \omega_s\right)t$

Instantaneous Voltage of the carrier wave, $V_{in} = V_c \cos \omega_c t$

$$\therefore VV_{in} = V_1 \cos\left(\omega_c + \omega_s\right)t.\left(V_c \cos \omega_c t\right)$$

$$V_1 V_c = \left[\cos\left(\omega_c + \omega_s\right)t.\cos \omega_c t\right]$$

$$\frac{V_1 V_c}{2} = \left[2\cos\left(\omega_c + \omega_s\right)t.\cos \omega_c t\right]$$

$$\frac{V_1 V_c}{2} = \left[\cos\left\{\left(\omega_c + \omega_s\right)t + \cos \omega_c t\right\} + \cos\left\{\left(\omega_c + \omega_s\right)t - \omega_c t\right\}\right]$$

$$\frac{V_1 V_c}{2} = \left[\cos\left\{\left(2\omega_c + \omega_s\right)t + \cos \omega_s t\right\}\right]$$

At the receiving station, the low-pass filter allows only high frequency signals to pass through it. Obstructs the low frequency signal ω_s . Thus, at the receiving station, one can record the modulating signal $\frac{V_1 V_c}{2} \cos \omega_s t$, which is the signal frequency.