

CHAPTER 7

INTEGRALS

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Evaluate the following integrals

1. $\int (\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}) dx.$

2. $\int_{-1}^1 e^{|x|} dx.$

3. $\int \frac{1}{1 - \sin^2 x} dx.$

4. $\int \left(8^x + x^8 + \frac{8}{x} + \frac{x}{8} \right) dx.$

5. $\int_{-1}^1 x^{99} \cos^4 x \, dx.$

6. $\int \frac{1}{x \log x \log(\log x)} dx.$

7. $\int_0^{\pi/2} \log \left(\frac{4 + 3 \sin x}{4 + 3 \cos x} \right) dx.$

8. $\int (e^{a \log x} + e^{x \log a}) dx.$

9. $\int \left(\frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} \right) dx.$

10. $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx.$

11. $\int (x^c + c^x) dx.$

12. $\frac{d}{dx} \left[\int f(x) dx \right].$

$$13. \int \frac{1}{\sin^2 x \cos^2 x} dx.$$

$$14. \int \frac{1}{\sqrt{x} + \sqrt{x-1}} dx.$$

$$15. \int e^{-\log e^x} dx.$$

$$16. \int \frac{e^x}{a^x} dx.$$

$$17. \int 2^x e^x dx.$$

$$18. \int \frac{x}{\sqrt{x+1}} dx.$$

$$19. \int \frac{x}{(x+1)^2} dx.$$

$$20. \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx.$$

$$21. \int \cos^2 \alpha dx.$$

$$22. \int \frac{1}{x \cos \alpha + 1} dx.$$

$$23. \int \sec x \cdot \log(\sec x + \tan x) dx.$$

$$24. \int \frac{1}{\cos \alpha + x \sin \alpha} dx.$$

$$25. \int \cot x \cdot \log \sin x dx.$$

$$26. \int \left(x - \frac{1}{2}\right)^3 dx.$$

$$27. \int \frac{1}{x(2+3 \log x)} dx.$$

$$28. \int \frac{1 - \sin x}{x + \cos x} dx.$$

$$29. \int \frac{1 - \cos x}{\sin x} dx.$$

$$30. \int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx.$$

$$31. \int \frac{(x+1)}{x} (x + \log x) dx.$$

$$32. \int \left(\sqrt{ax} - \frac{1}{\sqrt{ax}}\right)^2 dx.$$

$$33. \int_0^\pi |\cos x| dx.$$

$$34. \int_0^2 [x] dx \text{ where } [] \text{ is greatest integer function.}$$

35. $\int_0^{\sqrt{2}} [x^2] dx$ where $[]$ is greatest integer function.

36. $\int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx.$

37. $\int_{-2}^1 \frac{|x|}{x} dx.$

38. $\int_{-1}^1 x|x| dx.$

39. If $\int_0^a \frac{1}{1+x^2} = \frac{\pi}{4}$, then what is value of a .

40. $\int_a^b f(x) dx + \int_b^a f(x) dx.$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

41. (i) $\int \frac{x \operatorname{cosec}(\tan^{-1} x^2)}{1+x^4} dx.$

(ii) $\int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx.$

(iii) $\int \frac{1}{\sin(x-a) \sin(x-b)} dx.$

(iv) $\int \frac{\cos(x+a)}{\cos(x-a)} dx.$

(v) $\int \cos x \cos 2x \cos 3x dx.$

(vi) $\int \cos^5 x dx.$

(vii) $\int \sin^2 x \cos^4 x dx.$

(viii) $\int \cot^3 x \operatorname{cosec}^4 x dx.$

(ix) $\int \frac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} dx.$

(x) $\int \frac{1}{\sqrt{\cos^3 x \cos(x+a)}} dx.$

(xi) $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx.$

(xii) $\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx.$

42. Evaluate :

$$(i) \int \frac{x}{x^4 + x^2 + 1} dx.$$

$$^{*}(ii) \int \frac{1}{x [6 (\log x)^2 + 7 \log x + 2]} dx.$$

$$(iii) \int \frac{dx}{1 + x - x^2}.$$

$$(iv) \int \frac{1}{\sqrt{9 + 8x - x^2}} dx.$$

$$(v) \int \frac{1}{\sqrt{(x - a)(x - b)}} dx.$$

$$(vi) \int \sqrt{\frac{\sin(x - \alpha)}{\sin(x + \alpha)}} dx.$$

$$(vii) \int \frac{5x - 2}{3x^2 + 2x + 1} dx.$$

$$(viii) \int \frac{x^2}{x^2 + 6x + 12} dx.$$

$$(ix) \int \frac{x + 2}{\sqrt{4x - x^2}} dx.$$

$$(x) \int x \sqrt{1 + x - x^2} dx.$$

$$(xi) \int (3x - 2) \sqrt{x^2 + x + 1} dx.$$

$$(xii) \int \sqrt{\sec x + 1} dx.$$

43. Evaluate :

$$(i) \int \frac{dx}{x(x^7 + 1)}.$$

$$(ii) \int \frac{\sin x}{(1 + \cos x)(2 + 3 \cos x)} dx.$$

$$(iii) \int \frac{\sin \theta \cos \theta}{\cos^2 \theta - \cos \theta - 2} d\theta.$$

$$(iv) \int \frac{x-1}{(x+1)(x-2)(x+3)} dx.$$

$$(v) \int \frac{x^2 + x + 2}{(x-2)(x-1)} dx.$$

$$(vi) \int \frac{(x^2+1)(x^2+2)}{(x^3+3)(x^2+4)} dx.$$

$$(vii) \int \frac{dx}{(2x+1)(x^2+4)}.$$

$$(viii) \int \frac{dx}{\sin x (1-2 \cos x)}.$$

$$(ix) \int \frac{\sin x}{\sin 4x} dx.$$

$$(x) \int \frac{x^2-1}{x^4+x^2+1} dx.$$

$$(xi) \int \sqrt{\tan x} dx.$$

$$(xii) \int \frac{x^2+9}{x^4+81} dx.$$

44. Evaluate :

$$(i) \int x^5 \sin x^3 dx.$$

$$(ii) \int \sec^3 x dx.$$

$$(iii) \int e^{ax} \cos (bx+c) dx.$$

$$(iv) \int \sin^{-1} \frac{6x}{1+9x^2} dx.$$

$$(v) \int \cos \sqrt{x} dx.$$

$$(vi) \int x^3 \tan^{-1} x dx.$$

$$(vii) \int e^{2x} \left(\frac{1+\sin 2x}{1+\cos 2x} \right) dx.$$

$$(viii) \int e^x \left(\frac{x-1}{2x^2} \right) dx.$$

$$(ix) \int \sqrt{2ax-x^2} dx.$$

$$(x) \int e^x \frac{(x^2+1)}{(x+1)^2} dx.$$

$$(xi) \int e^x \frac{(2+\sin 2x)}{(1+\cos 2x)} dx.$$

$$(xii) \int \left\{ \log (\log x) + \frac{1}{(\log x)^2} \right\} dx.$$

$$(xiii) \int (6x + 5) \sqrt{6 + x - x^2} dx.$$

$$(xiv) \int (x - 2) \sqrt{\frac{x+3}{x-3}} dx.$$

$$(xv) \int (2x - 5) \sqrt{x^2 - 4x + 3} dx.$$

$$(xvi) \int \sqrt{x^2 - 4x + 8} dx.$$

45. Evaluate the following definite integrals :

$$(i) \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx.$$

$$(ii) \int_0^{\frac{\pi}{2}} \cos 2x \log \sin x dx.$$

$$(iii) \int_0^1 x \sqrt{\frac{1-x^2}{1+x^2}} dx.$$

$$(iv) \int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx.$$

$$(v) \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx.$$

$$(vi) \int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx.$$

$$(vii) \int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx.$$

46. Evaluate :

$$(i) \int_1^3 \{|x-1| + |x-2| + |x-3|\} dx. \quad (ii) \int_0^{\pi} \frac{x}{1 + \sin x} dx.$$

$$(iii) \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx.$$

$$(iv) \int_0^{\frac{\pi}{2}} \log \sin x dx.$$

$$(v) \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx.$$

$$(vi) \int_{-2}^2 f(x) dx \text{ where } f(x) = \begin{cases} 2x - x^3 & \text{when } -2 \leq x < -1 \\ x^3 - 3x + 2 & \text{when } -1 \leq x < 1 \\ 3x - 2 & \text{when } 1 \leq x < 2. \end{cases}$$

$$(vii) \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx.$$

$$(viii) \int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx.$$

47. Evaluate the following integrals

$$(i) \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$$

$$(ii) \int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx.$$

$$(iii) \int_{-1}^1 \log \left(\frac{1 + \sin x}{1 - \sin x} \right) dx.$$

$$(iv) \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx.$$

$$(v) \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx.$$

$$(iv) \int_{-a}^a \sqrt{\frac{a-x}{a+x}} dx.$$

48. $\int_0^1 [2x] dx$ where $[\]$ is greatest integer function.

49. $\int e^{\log x + \log \sin x} dx.$

50. $\int e^{\log(x+1) - \log x} dx.$

51. $\int \frac{\sin x}{\sin 2x} dx.$

52. $\int \sin x \sin 2x dx.$

53. $\int_{-\pi/4}^{\pi/4} |\sin x| dx.$

54. $\int_a^b f(x) dx + \int_b^a f(a+b-x) dx.$

55. $\int \frac{1}{\sec x + \tan x} dx.$

56. $\int \frac{\sin^2 x}{1 + \cos x} dx.$

57. $\int \frac{1 - \tan x}{1 + \tan x} dx.$

58. $\int \frac{a^x + b^x}{c^x} dx.$

59. Evaluate

(i) $\int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx, x \in [0, 1]$

(ii) $\int \sqrt{\frac{1 - \sqrt{x}}{1 + \sqrt{x}}} dx$

(iii) $\int \frac{\sqrt{x^2 + 1} [\log(x^2 + 1) - 2 \log x]}{x^4} dx$

(iv) $\int \frac{x^2}{(x \sin x + \cos x)^2} dx$

(v) $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$

(vi) $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

$$(vii) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin |x| - \cos |x|) dx$$

$$(viii) \int_1^2 [x^2] dx, \text{ where } [x] \text{ is greatest integer function}$$

$$(ix) \int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx.$$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

60. Evaluate the following integrals :

$$(i) \int \frac{x^5 + 4}{x^5 - x} dx.$$

$$(ii) \int \frac{dx}{(x-1)(x^2+4)} dx$$

$$(iii) \int \frac{2x^3}{(x+1)(x-3)^2} dx$$

$$(iv) \int \frac{x^4}{x^4 - 16} dx$$

$$(v) \int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx.$$

$$(vi) \int \frac{1}{x^4 + 1} dx.$$

$$(vii) \int_0^{\infty} \frac{x \tan^{-1} x}{(1+x^2)^2} dx.$$

61. Evaluate the following integrals as limit of sums :

$$(i) \int_2^4 (2x + 1) dx.$$

$$(ii) \int_0^2 (x^2 + 3) dx.$$

$$(iii) \int_1^3 (3x^2 - 2x + 4) dx.$$

$$(iv) \int_0^4 (3x^2 + e^{2x}) dx.$$

$$(v) \int_2^5 (x^2 + 3x) dx.$$

62. Evaluate

$$(i) \int_0^1 \cot^{-1}(1 - x + x^2) dx$$

$$(ii) \int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)}$$

$$(iii) \int_0^1 \frac{\log(1+x)}{1+x^2} dx$$

$$(iv) \int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx.$$

$$63. \int \frac{1}{\sin x + \sin 2x} dx.$$

$$64. \int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta.$$

$$65. \int \sec^3 x dx.$$

$$66. \int e^{2x} \cos 3x dx.$$

ANSWERS

$$1. \frac{\pi}{2} x + c.$$

$$2. 2e - 2$$

$$3. \tan x + c.$$

$$4. \frac{8^x}{\log 8} + \frac{x^9}{9} + 8 \log |x| + \frac{x^2}{16} + c.$$

$$5. 0$$

$$6. \log | \log (\log x) | + c$$

7. 0
8. $\frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$
9. $\tan x + c$
10. 0
11. $\frac{x^{c+1}}{c+1} + \frac{c^x}{\log c} + c$
12. $f(x) + c$
13. $\tan x - \cot x + c$
14. $\frac{2}{3}x^{3/2} - \frac{2}{3}(x-1)^{3/2} + c$
15. $\log |x| + c$
16. $\left(\frac{e}{a}\right)^x / \log(e/a) + c$
17. $\frac{2^x e^x}{\log(2e)} + c$
18. $\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + c.$
19. $\log|x+1| + \frac{1}{x+1} + c.$
20. $2e^{\sqrt{x}} + c$
21. $x \cos^2 \alpha + c$
22. $\frac{\log |x \cos \alpha + 1|}{\cos \alpha} + c.$
23. $\frac{(\log |\sec x + \tan x|)^2}{2} + c$
24. $\frac{\log |\cos \alpha + x \sin \alpha|}{\sin \alpha} + c$
25. $\frac{(\log \sin x)^2}{2} + c$
26. $\frac{x^4}{4} + \frac{1}{2x^2} - \frac{3x^2}{2} + 3|\log x| + c.$
27. $\frac{1}{3} \log |2 + 3 \log x| + c.$
28. $\log |x + \cos x| + c$
29. $2 \log |\sec x/2| + c.$
30. $\frac{1}{e} \log |x^e + e^x| + c.$

$$31. \frac{(x + \log x)^2}{2} + c$$

$$32. a \frac{x^2}{2} + \frac{\log|ax|}{a} - 2x + c.$$

$$33. 0$$

$$34. 1$$

$$35. (\sqrt{2} - 1)$$

$$36. \frac{b-a}{2}$$

$$37. -1$$

$$38. 0$$

$$39. 1$$

$$40. 0$$

$$41. (i) \frac{1}{2} \log \left[\operatorname{cosec}(\tan^{-1} x^2) - \frac{1}{x^2} \right] + c.$$

$$(ii) \frac{1}{2} (x^2 - x\sqrt{x^2 - 1}) + \frac{1}{2} \log |x + \sqrt{x^2 - 1}| + c.$$

$$(iii) \frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$$

$$(iv) x \cos 2a - \sin 2a \log |\sec(x-a)| + c.$$

$$(v) \frac{1}{48} [12x + 6 \sin 2x + 3 \sin 4x + 2 \sin 6x] + c.$$

$$(vi) \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c.$$

$$(vii) \frac{1}{32} \left[2x + \frac{1}{2} \sin 2x - \frac{1}{2} \sin 4x - \frac{1}{6} \sin 6x \right] + c.$$

$$(viii) - \left(\frac{\cot^6 x}{6} + \frac{\cot^4 x}{4} \right) + c.$$

$$(ix) \frac{1}{(a^2 - b^2) \sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} + c.$$

[Hint : put $a^2 \sin^2 x + b^2 \cos^2 x = t$]

$$(x) \quad -2 \operatorname{cosec} a \sqrt{\cos a - \tan x \cdot \sin a} + c.$$

[Hint. : Take $\sec^2 x$ as numerator]

$$(xi) \quad \tan x - \cot x - 3x + c.$$

$$(xii) \quad \sin^{-1} (\sin x - \cos x) + c.$$

$$42. \quad (i) \quad \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 + 1}{\sqrt{3}} \right) + c. \quad [\text{Hint : put } x^2 = t]$$

$$(ii) \quad \log \left| \frac{2 \log x + 1}{3 \log x + 2} \right| + C \quad [\text{Hint : put } \log x = t]$$

$$(iii) \quad \frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5} - 1 + 2x}{\sqrt{5} + 1 - 2x} \right| + c$$

$$(iv) \quad \sin^{-1} \left(\frac{x - 4}{5} \right) + c.$$

$$(v) \quad 2 \log |\sqrt{x - a} + \sqrt{x - b}| + c$$

$$(vi)$$

$$- \cos \alpha \sin^{-1} \left(\frac{\cos x}{\cos \alpha} \right) - \sin \alpha \cdot \log |\sin x + \sqrt{\sin^2 x - \sin^2 \alpha}| + c$$

$$\left[\text{Hint : } \sqrt{\frac{\sin(x - \alpha)}{\sin(x + \alpha)}} = \frac{\sin(x - \alpha)}{\sin^2 x - \sin^2 \alpha} \right]$$

$$(vii) \quad \frac{5}{6} \log |3x^2 + 2x + 1| + \frac{(-11)}{3\sqrt{2}} \tan^{-1} \left(\frac{3x + 1}{\sqrt{2}} \right) + c$$

$$(viii) \quad x - 3 \log |x^2 + 6x + 12| + 2\sqrt{3} \tan^{-1} \left(\frac{x + 3}{\sqrt{3}} \right) + c$$

$$(ix) \quad -\sqrt{4x - x^2} + 4 \sin^{-1} \left(\frac{x - 2}{2} \right) + c$$

$$(x) \quad \frac{-1}{3} (1 + x - x^2)^{\frac{3}{2}} + \frac{1}{8} (2x - 1) \sqrt{1 + x - x^2} + \frac{5}{16} \sin^{-1} \left(\frac{2x - 1}{\sqrt{5}} \right) + c$$

$$(xi) \quad (x^2 + x + 1)^{\frac{3}{2}} - \frac{7}{2} \left[\left(x + \frac{1}{2} \right) \sqrt{x^2 + x + 1} + \frac{3}{8} \log \left| x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right| \right] + c$$

$$(xii) \quad -\log \left| \cos x + \frac{1}{2} + \sqrt{\cos^2 x + \cos x} \right| + c$$

[Hint : Multiply and divide by $\sqrt{\sec x + 1}$]

$$43. \quad (i) \quad \frac{1}{7} \log \left| \frac{x^7}{x^7 + 1} \right| + c$$

$$(ii) \quad \log \left| \frac{1 + \cos x}{2 + 3 \cos x} \right| + c$$

$$(iii) \quad \frac{-2}{3} \log |\cos \theta - 2| - \frac{1}{3} \log |1 + \cos \theta| + c.$$

$$(iv) \quad \frac{9}{10} \log |x + 3| + \frac{4}{15} \log |x - 2| - \frac{1}{6} |x + 1| + c$$

$$(v) \quad x + 4 \log \left| \frac{(x - 2)^2}{x - 1} \right| + c$$

$$(vi) \quad x + \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) - 3 \tan^{-1} \left(\frac{x}{2} \right) + c$$

[Hint : put $x^2 = t$]

$$(vii) \quad \frac{2}{17} \log |2x + 1| - \frac{1}{17} \log |x^2 + 4| + \frac{1}{34} \tan^{-1} \frac{x}{2} + c$$

$$(viii) \quad -\frac{1}{2} \log |1 - \cos x| - \frac{1}{6} \log |1 + \cos x| + \frac{2}{3} \log |1 - 2 \cos x| + c$$

[Hint : Multiply N^r and D^r by $\sin x$ and put $\cos x = t$]

$$(ix) \quad \frac{-1}{8} \log \left| \frac{1 + \sin x}{1 - \sin x} \right| + \frac{1}{4\sqrt{2}} \log \left| \frac{1 + \sqrt{2} \sin x}{1 - \sqrt{2} \sin x} \right| + c$$

$$(x) \quad \frac{1}{2} \log \left| \frac{x^2 - x + 1}{x^2 + x + 1} \right| + c$$

$$(xi) \quad \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan x - 1}{\sqrt{2} \tan x} \right) + \frac{1}{2\sqrt{2}} \log \left| \frac{\tan x - \sqrt{2} \tan x + 1}{\tan x + \sqrt{2} \tan x + 1} \right| + c$$

$$(xii) \quad \frac{1}{3\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 9}{3\sqrt{2}} \right) + c$$

$$44. \quad (i) \quad \frac{1}{3} [-x^3 \cos x^3 + \sin x^3] + c$$

$$(ii) \quad \frac{1}{2} [\sec x \tan x + \log |\sec x + \tan x|] + c$$

[Hint : Write $\sec^3 x = \sec x \cdot \sec^2 x$ and take $\sec x$ as first function]

$$(iii) \quad \frac{e^{ax}}{a^2 + b^2} [a \cos (bx + c) + b \sin (bx + c)] + c_1$$

$$(iv) \quad 2x \tan^{-1} 3x - \frac{1}{3} \log |1 + 9x^2| + c \quad [\text{Hint : put } 3x = \tan \theta]$$

$$(v) \quad 2 [\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$$

$$(vi) \quad \left(\frac{x^4 - 1}{4} \right) \tan^{-1} x - \frac{x^3}{12} + \frac{x}{4} + c.$$

$$(vii) \quad \frac{1}{2} e^{2x} \tan x + c.$$

$$(viii) \quad \frac{e^x}{2x} + c.$$

$$(ix) \quad \frac{x-a}{2} \sqrt{2ax - x^2} - \frac{a^2}{2} \sin^{-1} \left(\frac{x-a}{a} \right) + c$$

$$(x) \quad e^x \left(\frac{x-1}{x+1} \right) + c.$$

$$(xi) \quad e^x \tan x + c.$$

$$(xii) \quad x \log |\log x| - \frac{x}{\log x} + c. [\text{Hint : put } \log x = t \Rightarrow x = e^t]$$

$$(xiii) \quad -2(6+x-x^2)^{3/2} + 8 \left[\frac{2x-1}{4} \sqrt{6+x-x^2} + \frac{25}{8} \sin^{-1} \left(\frac{2x-1}{5} \right) \right] + c$$

$$(xiv) \quad \frac{1}{2} (x+2) \sqrt{x^2-9} - \frac{3}{2} \log |x + \sqrt{x^2-9}| + c$$

$$(xv) \quad \frac{2}{3} (x^2 - 4x + 3)^{3/2} - \left(\frac{x-2}{2} \right) \sqrt{x^2 - 4x + 3} + \frac{1}{2} \log |x - 2 + \sqrt{x^2 - 4x + 3}| + c$$

$$(xvi) \quad \left(\frac{x-2}{2} \right) \sqrt{x^2 - 4x + 8} + 2 \log |(x-2) + \sqrt{x^2 - 4x + 8}| + c$$

$$45. \quad (i) \quad \frac{1}{20} \log 3.$$

$$(ii) \quad -\frac{\pi}{4}$$

$$(iii) \quad \frac{\pi}{4} - \frac{1}{2}. \quad [\text{Hint : put } x^2 = t] \quad (iv) \quad \frac{\pi}{4} - \frac{1}{2} \log 2.$$

$$(v) \quad \frac{\pi}{2}.$$

$$(vi) \quad 5 - 10 \log \frac{15}{8} + \frac{25}{2} \log \left(\frac{6}{5} \right).$$

$$(vii) \quad \pi/2. \quad \left[\text{Hint : } \left(\frac{x}{1 + \cos x} + \frac{\sin x}{1 + \cos x} \right) dx. \right]$$

$$46. \quad (i) \quad 8. \quad (ii) \quad \pi.$$

$$(iii) \quad \frac{\pi}{8} \log 2. \quad (iv) \quad \frac{-\pi}{2} \log 2.$$

$$(v) \quad \frac{1}{4} \pi^2.$$

$$(vi) \quad 95/12.$$

$$\left[\text{Hint : } \int_{-2}^2 f(x) dx = \int_{-2}^{-1} f(x) dx + \int_{-1}^1 f(x) dx + \int_1^2 f(x) dx \right]$$

$$(vii) \quad \frac{\pi^2}{16}.$$

$$(viii) \quad \frac{\pi^2}{2ab}. \quad \left[\text{Hint : Use } \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$47. \quad (i) \quad \frac{\pi}{12}. \quad (ii) \quad \frac{\pi}{2} - \log 2.$$

$$(iii) \quad 0. \quad (iv) \quad \pi/2.$$

$$(v) \quad \frac{\pi^2}{4}$$

$$(vi) \quad a\pi.$$

$$48. \quad \frac{1}{2}$$

$$49. \quad -x \cos x + \sin x + c.$$

$$50. \quad x + \log x + c.$$

$$51. \quad \frac{1}{2} \log |\sec x + \tan x| + c.$$

$$52. \quad -\frac{1}{2} \left(\frac{\sin 3x}{3} - \sin x \right)$$

$$53. \quad 2 - \sqrt{2}$$

$$54. \quad 0$$

$$55. \quad \log |1 + \sin x| + c$$

$$56. \quad x - \sin x + c$$

$$57. \quad \log |\cos x + \sin x| + c$$

$$58. \quad \frac{(a/c)^x}{\log(a/c)} + \frac{(b/c)^x}{\log(b/c)} + C.$$

$$59. \quad (i) \quad \frac{2(2x-1)}{\pi} \sin^{-1} \sqrt{x} + \frac{2\sqrt{x-x^2}}{\pi} - x + c$$

$$(ii) \quad -2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x-x^2} + c$$

$$(iii) \quad -\frac{1}{3} \left(1 + \frac{1}{x^2} \right)^{3/2} \left[\log \left(1 + \frac{1}{x^2} \right) - \frac{2}{3} \right] + c$$

$$(iv) \frac{\sin x - x \cos x}{x \sin x + \cos x} + c$$

$$(v) (x + a) \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{ax} + c$$

$$(vi) 2 \sin^{-1} \frac{\sqrt{3} - 1}{2}$$

$$(vii) 0$$

$$(viii) -\sqrt{2} - \sqrt{3} + 5$$

$$(ix) \frac{3}{\pi} + \frac{1}{\pi^2}.$$

$$60. \quad (i) \quad x - 4 \log |x| + \frac{5}{4} \log |x - 1| + \frac{3}{4} \log |x + 1| \\ + \log |x^2 + 1| - \frac{1}{2} \tan^{-1} x + c.$$

$$x + \frac{1}{2} \log \left| \frac{x - 1}{x + 1} \right| - \frac{1}{2} \tan^{-1} x + \log \left| \frac{x^2 - 1}{x^4 + 1} \right| + c.$$

$$(ii) \quad \frac{1}{5} \log |x - 1| - \frac{1}{10} \log |x^2 + 4| - \frac{1}{10} \tan^{-1} \left(\frac{x}{2} \right) + c.$$

$$(iii) \quad 2x - \frac{1}{8} \log |x + 1| + \frac{81}{8} \log |x - 3| - \frac{27}{2(x - 3)} + c.$$

$$(iv) \quad x + \frac{1}{2} \log \left| \frac{x - 2}{x + 2} \right| - \tan^{-1} \left(\frac{x}{2} \right) + c.$$

$$(v) \quad \pi/\sqrt{2}.$$

$$(vi) \quad \frac{1}{2\sqrt{2}} \tan^{-1} \frac{(x^2 - 1)}{\sqrt{2x}} - \frac{1}{4\sqrt{2}} \log \left| \frac{x^2 - \sqrt{2x} + 1}{x^2 + \sqrt{2x} + 1} \right| + c$$

(vii) $\pi/8$.

61. (i) 14. (ii) $\frac{26}{3}$.

(iii) 26.

(iv) $\frac{1}{2}(127 + e^8)$.

(v) $\frac{141}{2}$.

62. (i) $\frac{\pi}{2} - \log 2$

(ii) $-\frac{1}{5} \log \left| \frac{\tan x - x}{2 \tan x + 1} \right| + c$

(iii) $\frac{\pi}{8} \log 2$.

(iv) $\frac{\pi}{2} \log \left(\frac{1}{2} \right)$.

63. $\frac{1}{6} \log |1 - \cos x| + \frac{1}{2} \log (1 + \cos x) - \frac{2}{3} \log |1 + 2 \cos x| + c$.

64. $3 \log |(2 - \sin \theta)| + \frac{4}{2 - \sin \theta} + c$.

65. $\frac{1}{2} \sec x + \tan x + \frac{1}{2} \log |\sec x + \tan x| + c$.

66. $\frac{e^{2x}}{13} (2 \cos 3x + 3 \sin 3x) + c$.