CHAPTER 9

DIFFERENTIAL EQUATIONS

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the order and degree of the following differential equations.

(i) $\frac{dy}{dx} + \cos y = 0.$

(ii) $\left(\frac{dy}{dx}\right)^2 + 3\frac{d^2y}{dx^2} = 4.$

(iii) $\frac{d^4y}{dx^4} + \sin x = \left(\frac{d^2y}{dx^2}\right)^5.$

(iv) $\frac{d^5y}{dx^5} + \log\left(\frac{dy}{dx}\right) = 0.$

(v) $\sqrt{1 + \frac{dy}{dx}} = \left(\frac{d^2y}{dx^2}\right)^{1/3}.$

(vi) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = k\frac{d^2y}{dx^2}.$

(vii) $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 = \sin x.$

(viii) $\frac{dy}{dx} + \tan\left(\frac{dy}{dx}\right) = 0$

2. Write the general solution of following differential equations.

(i) $\dfrac{dy}{dx} = x^5 + x^2 - \dfrac{2}{x}$.

(ii) $(e^x + e^{-x})\ dy = (e^x - e^{-x})dx$

(iii) $\dfrac{dy}{dx} = x^3 + e^x + x^e$.

(iv) $\dfrac{dy}{dx} = 5^{x+y}$.

(v) $\dfrac{dy}{dx} = \dfrac{1 - \cos 2x}{1 + \cos 2y}$.

(vi) $\dfrac{dy}{dx} = \dfrac{1 - 2y}{3x + 1}$.

3. Write integrating factor of the following differential equations

(i) $\dfrac{dy}{dx} + y \cos x = \sin x$

(ii) $\dfrac{dy}{dx} + y \sec^2 x = \sec x + \tan x$

(iii) $x^2 \dfrac{dy}{dx} + y = x^4$.

(iv) $x \dfrac{dy}{dx} + y \log x = x + y$

(v) $x \dfrac{dy}{dx} - 3y = x^3$

(vi) $\dfrac{dy}{dx} + y \tan x = \sec x$

(vii) $\dfrac{dy}{dx} + \dfrac{1}{1 + x^2} y = \sin x$

4. Write order of the differential equation of the family of following curves

(i) $y = Ae^x + Be^{x + c}$

(ii) $Ay = Bx^2$

(iii) $(x - a)^2 + (y - b)^2 = 9$

(iv) $Ax + By^2 = Bx^2 - Ay$

(v) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 0$.

(vi) $y = a \cos (x + b)$

(vii) $y = a + be^{x+c}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

5. (i) Show that $y = e^{m \sin^{-1} x}$ is a solution of

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0.$$

(ii) Show that $y = \sin(\sin x)$ is a solution of differential equation

$$\frac{d^2y}{dx^2} + (\tan x)\frac{dy}{dx} + y\cos^2 x = 0.$$

(iii) Show that $y = Ax + \frac{B}{x}$ is a solution of $\frac{x^2 d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$

(iv) Show that $y = a\cos(\log x) + b\sin(\log x)$ is a solution of

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

(v) Verify that $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ satisfies the differential equation :

$$\left(a^2 + x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$$

(vi) Find the differential equation of the family of curves

$y = e^x (A\cos x + B\sin x)$, where A and B are arbitrary constants.

(vii) Find the differential equation of an ellipse with major and minor axes $2a$ and $2b$ respectively.

(viii) Form the differential equation representing the family of curves $(y - b)^2 = 4(x - a)$.

6. Solve the following differential equations.

(i) $\frac{dy}{dx} + y\cot x = \sin 2x.$ (ii) $x\frac{dy}{dx} + 2y = x^2 \log x.$

(iii) $\dfrac{dx}{dy} + \dfrac{1}{x} . y = \cos x + \dfrac{\sin x}{x}, \quad x > 0.$

(iv) $\cos^3 x \dfrac{dy}{dx} + \cos x = \sin x.$

(v) $ydx + \left(x - y^3\right) dy = 0$

(vi) $ye^y dx = \left(y^3 + 2xe^y\right) dy$

7. Solve each of the following differential equations :

(i) $y - x\dfrac{dy}{dx} = 2\left(y^2 + \dfrac{dy}{dx}\right).$

(ii) $\cos y \, dx + (1 + 2e^{-x}) \sin y \, dy = 0.$

(iii) $x\sqrt{1 - y^2} dy + y\sqrt{1 - x^2} dx = 0.$

(iv) $\sqrt{\left(1 - x^2\right)\left(1 - y^2\right)} \, dy + xy \, dx = 0.$

(v) $(xy^2 + x) \, dx + (yx^2 + y) \, dy = 0; \; y(0) = 1.$

(vi) $\dfrac{dy}{dx} = y \sin^3 x \cos^3 x + xy \, e^x.$

(vii) $\tan x \; \tan y \, dx + \sec^2 x \sec^2 y \, dy = 0$

8. Solve the following differential equations :

(i) $x^2 \, y \, dx - (x^3 + y^3) \, dy = 0.$

(ii) $x^2 \dfrac{dy}{dx} = x^2 + xy + y^2.$

(iii) $\left(x^2 - y^2\right) dx + 2xy \, dy = 0, \quad y(1) = 1.$

(iv) $\left(y\sin\frac{x}{y}\right)dx = \left(x\sin\frac{x}{y} - y\right)dy.$ (v) $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right).$

(vi) $\frac{dy}{dx} = \frac{2xy}{x^2 + y^2}$ (vii) $\frac{dy}{dx} = e^{x+y} + x^2 e^y.$

(viii) $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}.$

(ix) $\left(3xy + y^2\right)dx + \left(x^2 + xy\right)dy = 0$

9. (i) Form the differential equation of the family of circles touching y-axis at (0, 0).

(ii) Form the differential equation of family of parabolas having vertex at (0, 0) and axis along the (i) positive y-axis (ii) positive x-axis.

(iii) Form differential equation of family of circles passing through origin and whose centre lie on x-axis.

(iv) Form the differential equation of the family of circles in the first quadrant and touching the coordinate axes.

10. Show that the differential equation $\frac{dy}{dx} = \frac{x+2y}{x-2y}$ is homogeneous and solve it.

11. Show that the differential equation :

$(x^2 + 2xy - y^2)\,dx + (y^2 + 2xy - x^2)\,dy = 0$ is homogeneous and solve it.

12. Solve the following differential equations :

(i) $\frac{dy}{dx} - 2y = \cos 3x.$

(ii) $\sin x\frac{dy}{dx} + y\cos x = 2\sin^2 x\cos x$ if $y\left(\frac{\pi}{2}\right) = 1$

(iii) $3e^x \tan y\,dx + \left(1 - e^x\right)\sec^2 y\,dy = 0$

13. Solve the following differential equations :

(i) $(x^3 + y^3)\ dx = (x^2y + xy^2)dy.$

(ii) $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx.$

(iii) $y\left\{x\cos\left(\dfrac{y}{x}\right) + y\sin\left(\dfrac{y}{x}\right)\right\}dx$

$$- x\left\{y\sin\left(\dfrac{y}{x}\right) - x\cos\left(\dfrac{y}{x}\right)\right\}dy = 0.$$

(iv) $x^2dy + y(x + y)\ dx = 0$ given that $y = 1$ when $x = 1$.

(v) $xe^{\frac{y}{x}} - y + x\dfrac{dy}{dx} = 0$ if $y(e) = 0$

(vi) $(x^3 - 3xy^2)\ dx = (y^3 - 3x^2y)dy.$

(vii) $\dfrac{dy}{dx} - \dfrac{y}{x} + \operatorname{cosec}\left(\dfrac{y}{x}\right) = 0$ given that $y = 0$ when $x = 1$

16. Solve the following differential equations :

(i) $\cos^2 x\dfrac{dy}{dx} = \tan x - y.$

(ii) $x\cos x\dfrac{dy}{dx} + y\,(x\sin x + \cos x) = 1.$

(iii) $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \dfrac{x}{y}\right)dy = 0.$

(iv) $(y - \sin x)\ dx + \tan x\ dy = 0,\ y(0) = 0.$

LONG ANSWER TYPE QUESTIONS (6 MARKS EACH)

17. Solve the following differential equations :

(i) $(x\,dy - y\,dx)\,y \sin\left(\frac{y}{x}\right) = (y\,dx + x\,dy)\,x \cos\left(\frac{y}{x}\right)$

(ii) $3e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$ given that $y = \frac{\pi}{4}$, when $x = 1$.

(iii) $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$ given that $y(0) = 0$.

ANSWERS

1.(i) order = 1, degree = 1 (ii) order = 2, degree = 1

(iii) order = 4, degree = 1 (iv) order = 5, degree is not defined.

(v) order = 2, degree = 2 (vi) order = 2, degree = 2

(vii) order = 3, degree = 2 (viii) order = 1, degree is not defined

2.(i) $y = \frac{x^6}{6} + \frac{x^3}{6} - 2\log|x| + c$ (ii) $y = \log_e\left|e^x + e^{-x}\right| + c$

(iii) $y = \frac{x^4}{4} + e^x + \frac{x^{e+1}}{e+1} + c.$ (iv) $5^x + 5^{-y} = c$

(v) $2(y - x) + \sin 2y + \sin 2x = c.$ (vi) $2\log|3x + 1| + 3\log|1 - 2y| = c.$

3.(i) $e^{\sin x}$ (ii) $e^{\tan x}$

(iii) $e^{-1/x}$ (iv) $e^{\frac{(\log x)^2}{2}}$

(v) $\frac{1}{x^3}$ (vi) $\sec x$

(vii) $e^{\tan^{-1} x}$

4.(i) 1 (ii) 1

(iii) 2 (iv) 1

(v) 1 (vi) 1

(vii) 2

5.(vi) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ (vii) $x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} = y\frac{dy}{dx}$

(viii) $2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$

6.(i) $y \sin x = \frac{2 \sin^3 x}{3} + c$ (ii) $y = \frac{x^2 \left(4 \log_e x - 1\right)}{16} + \frac{c}{x^2}$

(iii) $y = \sin x + \frac{c}{x},\ x > 0$ (iv) $y = \tan x - 1 + ce^{-\tan x}$

(v) $xy = \frac{y^4}{4} + c$ (vi) $x = -y^2e^{-y} + cy^2$

7.(i) $cy = (x + 2)(1 - 2y)$ (ii) $\left(e^x + 2\right) \sec y = c$

(iii) $\sqrt{1 - x^2} + \sqrt{1 - y^2} = c$

(iv) $\frac{1}{2}\log\left|\frac{\sqrt{1 - y^2} - 1}{\sqrt{1 - y^2} + 1}\right| = \sqrt{1 - x^2} - \sqrt{1 - y^2} + c$

(v) $\left(x^2 + 1\right)\left(y^2 + 1\right) = 2$

(vi) $\log y = -\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + xe^x - e^x + c$

$$= \frac{1}{16}\left[\frac{\cos^3 2x}{3} - \cos 2x\right] + (x-1)e^x + c$$

(vii) $\log|\tan y| - \frac{\cos 2x}{y} = c$

8.(i) $\frac{-x^3}{3y^3} + \log|y| = c$

(ii) $\tan^{-1}\left(\frac{y}{x}\right) = \log|x| + c$

(iii) $x^2 + y^2 = 2x$

(iv) $y = ce^{\cos(x/y)}$ [**Hint :** Put $\frac{1}{x} = v$]

(v) $\sin\left(\frac{y}{x}\right) = cx$

(vi) $c\left(x^2 - y^2\right) = y$

(vii) $-e^{-y} = e^x + \frac{x^3}{3} + c$

(viii) $\sin^{-1} y = \sin^{-1} x + c$

(ix) $x\log\left(x^3 y\right) + y = cx$

9.(i) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

(ii) $2y = x\frac{dy}{dx}$, $y = 2x\frac{dy}{dx}$

(iii) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

(iv) $(x-y)^2(1+y')^2 = (x+yy')^2$

10. $\log\left|x^2 + xy + y^2\right| = 2\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3x}}\right) + c$

11. $\frac{x^3}{x^2+y^2} = \frac{c}{x}(x+y)$

12.(i) $y = \frac{3\sin 3x}{13} - \frac{2\cos 3x}{13} + ce^{2x}$ (ii) $y = \frac{2}{3}\sin^2 x + \frac{1}{3}\operatorname{cosec} x$

(iii) $\tan y = k\left(1 - e^{x}\right)^3$

13.(i) $-y = x\log\left\{c\left(x - y\right)\right\}$ (ii) $cx^2 = y + \sqrt{x^2 + y^2}$

(iii) $xy\cos\left(\frac{y}{x}\right) = c$ (iv) $3x^2y = y + 2x$

(v) $y = -x\log\left(\log|x|\right),\ x \neq 0$ (vi) $c\left(x^2 + y^2\right) = \sqrt{x^2 - y^2}$.

(vii) $\cos\frac{y}{x} = \log|x| + 1$

16. (i) $y = \tan x - 1 + ce^{\tan^{-1} x}$ (ii) $y = \frac{\sin x}{x} + c\frac{\cos x}{x}$

(iii) $x + ye^{\frac{x}{y}} = c$ (iv) $2y = \sin x$

17. (i) $c\,xy = \sec\left(\frac{y}{x}\right)$

(ii) $(1 - e)^3 \tan y = (1 - e^x)^3$

(iii) $y = x^2$.