S K Mondal's

GATE Mathematics

Chapter wise ALL GATE Questions of All Branch

Every effort has been made to see that there are no errors (typographical or otherwise) in the material presented. However, it is still possible that there are a few errors (serious or otherwise). I would be thankful to the readers if they are brought to my attention at the following e-mail address: swapan_mondal_01@yahoo.co.in

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1. Matrix Algebra

Previous Years GATE Questions

EC All GATE Questions

1. **Let, A =** $\begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ **and** A^{-1} **=** $\begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix}$ **Then (a + b) =** [EC: GATE-20005

(a) $\frac{7}{20}$ (b) $\frac{3}{20}$ (c) $\frac{19}{60}$ (d) $\frac{11}{20}$

1.(a)

We know $AA^{-1} = I_2$

$$\Rightarrow \begin{pmatrix} 2 & -0.1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & a \\ 0 & b \end{pmatrix} = \begin{pmatrix} 1 & 2a-0.1b \\ 0 & 3b \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow b = \frac{1}{3} \text{ and } a = \frac{1}{60}$$

$$\therefore a + b = \frac{7}{20}$$

2. **Given an orthogonal matrix A =** $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ **$[AA^T]^{-1}$ is** [EC: GATE-2005]

(a) $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$

2.(c).

We know

$AA^t = I_4$

$\left[AA^T\right]^{-1} = \left[I_4\right]^{-1} = I_4$

3. The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ **is** [EC: GATE-2006]

(a) 0 (b) 1
(c) 2 (d) 3

3. (c)

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow{R_3 - R_1} \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 - R_2} \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = A_1(\text{say}).$$

$\therefore \text{rank}(A) = 2.$

5. The eigen values of a skew-symmetric matrix are [EC: GATE-2010]

(a) Always zero (b) always pure imaginary
(c) Either zero or pure imaginary (d) always real

5. (c)

ME 20 Years GATE Questions

6. Rank of the matrix $\begin{bmatrix} 0 & 2 & 2 \\ 7 & 4 & 8 \\ -7 & 0 & -4 \end{bmatrix}$ is 3. [ME: GATE-1994]

6.Ans. False

$\text{As.} \det A = 0 \quad \text{so}, \text{rank}(A) < 3$

$$\text{But}\begin{vmatrix}0 & 2\\7 & 4\end{vmatrix} = -14 \neq 0$$

$$\therefore \text{rank(A)} = 2.$$

7. Rank of the matrix given below is: [ME: GATE-1999]

$$\begin{bmatrix}3 & 2 & -9\\-6 & -4 & 18\\12 & 8 & -36\end{bmatrix}$$

(a) 1 (b) 2 (c) 3 (d) $\sqrt{2}$

7. (a)

$$\begin{vmatrix}3 & 2 & -9\\-6 & -4 & 18\\12 & 8 & -36\end{vmatrix} \xrightarrow[R_2+2R_1]{R_3-4R_1} \begin{vmatrix}3 & 2 & -9\\0 & 0 & 0\\0 & 0 & 0\end{vmatrix}$$

$$\therefore \text{rank} = 1.$$

8. The rank of a 3×3 matrix C (=AB), found by multiplying a non-zero column matrix A of size 3×1 and a non-zero row matrix B of size 1×3, is [ME: GATE-2001]

(a) 0 (b) 1 (c) 2 (d) 3

8.(b)

$$\text{Let}A = \begin{vmatrix}a_1\\a_2\\a_3\end{vmatrix}, B = [b_1\ b_2\ b_3]$$

$$\text{Then } C = AB = \begin{bmatrix}a_1b_1 & a_1b_2 & a_1b_3\\a_2b_1 & a_2b_2 & a_2b_3\\a_3b_1 & a_3b_2 & a_3b_3\end{bmatrix}.\text{Then } \det(AB) = 0.$$

Then also every minor
of order 2 is also zero.

$$\therefore \text{rank(C)} = 1.$$

9. A is a 3 x 4 real matrix and A x = b is an inconsistent system of equations. The highest possible rank of A is **[ME: GATE-2005]**

(a) 1 (b) 2 (c) 3 (d) 4

9.(b). Highest possible rank of A= 2 ,as Ax = b is an inconsistent system.

10. Match the items in columns I and II. **[ME: GATE-2006]**

Column I	Column II
P. Singular matrix	**1. Determinant is not defined**
Q. Non-square matrix	**2. Determinant is always one**
R. Real symmetric	**3. Determinant is zero**
S. Orthogonal matrix	**4. Eigenvalues are always real**
	5. Eigenvalues are not defined

(a) P-3, Q-1, R-4, S-2
(b) P-2, Q-3, R-4, S-1
(c) P-3, Q-2, R-5, S-4
(d) P-3, Q-4, R-2, S-1

10.(a) (P) Singular matrix → Determinant is zero
(Q) Non-square matrix → Determinant is not defined
(R) Real symmetric → Eigen values are always real
(S) Orthogonal → Determinant is always one

CE 10 Years GATE Questions

Q1. [A] is a square matrix which is neither symmetric nor skew-symmetric and $[A]^T$ is its transpose. The sum and difference of these matrices are defined as $[S]=[A]+[A]^T$ and $[D]=[A]-[A]^T$, respectively. Which of the following statements is **TRUE? [CE-2011]**

(a) both [S] and [D] are symmetric
(b) both [S] and [D] are skew –symmetric
(c) [S] is skew-symmetric and [D] is symmetric
(d) [S] is symmetric and [D] is skew-symmetric.

Ans. **(d)**

Exp. Take any matrix and check.

11. **Given matrix [A] =** $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$, **the rank of the matrix is [CE: GATE – 2003]**

(a) 4 (b) 3 (c) 2 (d) 1

11.(c)

$$A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix} \xrightarrow[R_2-3R_3]{R_1-2R_3} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 4 \\ 2 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{R_2-4R_1} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$\therefore \text{Rank}(A) = 2$

12. **Real matrices** $[A]_{3\times1}, [B]_{3\times3}, [C]_{3\times5}, [D]_{5\times3}, [E]_{5\times5}$ **and** $[F]_{5\times1}$ **are given. Matrices [B] and [E] are symmetric. [CE: GATE – 2004]**
Following statements are made with respect to these matrices.
1. Matrix product $[F]^T [C]^T [B] [C] [F]$ **is a scalar.**
2. Matrix product $[D]^T [F] [D]$ **is always symmetric.**
With reference to above statements, which of the following applies?

(a) Statement 1 is true but 2 is false
(b) Statement 1 is false but 2 is true
(c) Both the statements are true
(d) Both the statements are false

12.(a)

$$\text{Let}[I] = [F]^T_{1\times5}[C]^T_{5\times3}[B]_{3\times3}[C]_{3\times5}[F]_{5\times1}$$
$$= [I]_{1\times1} = \text{scalar.}$$
$$\text{Let}[I'] = [D]^T_{3\times5}[F]_{5\times1}[D]_{5\times3} \text{ is not define.}$$

13. Consider the matrices X $_{(4\times3)}$, Y $_{(4\times3)}$ and P $_{(2\times3)}$. The order or P $(X^TY)^{-1}P^T]^T$ will be **[CE: GATE – 2005]**

(a) (2×2)
(b) (3×3)
(c) (4×3)
(d) (3×4)

13.(a)

$$\left[P_{2\times3}\left(X^T_{3\times4}Y_{4\times3}\right)^{-1}P^T_{3\times2}\right]^T$$
$$= \left[P_{2\times3}Z^{-1}_{3\times3}P^T_{3\times2}\right]^T \left[\text{Take } Z = XY,\right]$$
$$= \left[T_{2\times2}\right]^T = [T']_{2\times2} \begin{bmatrix} T = PZ^{-1}P^T \\ T' = T^T \end{bmatrix}$$

14. The inverse of the 2 × 2 matrix $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$ is, **[CE: GATE – 2007]**

(a) $\frac{1}{3}\begin{bmatrix} -7 & 2 \\ 5 & -1 \end{bmatrix}$
(b) $\frac{1}{3}\begin{bmatrix} 7 & 2 \\ 5 & 1 \end{bmatrix}$
(c) $\frac{1}{3}\begin{bmatrix} 7 & -2 \\ -5 & 1 \end{bmatrix}$
(d) $\frac{1}{3}\begin{bmatrix} -7 & -2 \\ -5 & -1 \end{bmatrix}$

14(a).

$$\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} -7 & 2 \\ 5 & -1 \end{bmatrix}$$

15. The product of matrices $(PQ)^{-1}$ P is **[CE: GATE – 2008]**

(a) P^{-1}
(b) Q^{-1}
(c) $P^{-1}Q^{-1}P$
(d) PQP^{-1}

15.(b)

$$(PQ)^{-1}P = Q^{-1}P^{-1}P = Q^{-1}$$

16. A square matrix B is skew-symmetric if **[CE: GATE – 2009]**

(a) $B^T = -B$
(b) $B^T = B$

(c) $B^{-1} = B$ (d) $B^{-1} = B^T$

16.(a)

$B^T = -B$

17. The inverse of the matrix $\begin{bmatrix} 3+2i & i \\ -i & 3-2i \end{bmatrix}$ **is** **[CE: GATE – 2010]**

(a) $\frac{1}{12}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$ (b) $\frac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

(c) $\frac{1}{14}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$ (d) $\frac{1}{14}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

17.(b)

$$\begin{pmatrix} 3+2i & i \\ -i & 3-2i \end{pmatrix}^{-1} = \frac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$$

IE All GATE Questions

18. For a given 2 × 2 matrix A, it is observed that **[IE: GATE-2006]**

$$\mathbf{A}\begin{bmatrix} \mathbf{1} \\ \mathbf{-1} \end{bmatrix} = -\begin{bmatrix} \mathbf{1} \\ \mathbf{-1} \end{bmatrix} \textbf{ and } \mathbf{A}\begin{bmatrix} \mathbf{1} \\ \mathbf{-2} \end{bmatrix} = -\mathbf{2}\begin{bmatrix} \mathbf{1} \\ \mathbf{-2} \end{bmatrix}$$

Then matrix A is

(a) $A = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

18.(c)

From these conditions eigen values are -1 and -2.

$$\text{Let } P = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$$

$$\Rightarrow P^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\therefore P^{-1} A P = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} = D(\text{say})$$

$$\Rightarrow A = PDP^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}$$

EE

Q27. The matrix $[\mathbf{A}] = \begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{4} & \mathbf{-1} \end{bmatrix}$ is decomposed into a product of a lower triangular matrix $[\mathbf{L}]$ and an upper triangular matrix $[\mathbf{U}]$. The properly decomposed $[\mathbf{L}]$ and $[\mathbf{U}]$ matrices respectively are

(a) $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{4} & \mathbf{-1} \end{bmatrix}$ and $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{4} & \mathbf{-1} \end{bmatrix}$

(b) $\begin{bmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{4} & \mathbf{-1} \end{bmatrix}$ and $\begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$

(c) $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{4} & \mathbf{1} \end{bmatrix}$ and $\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{0} & \mathbf{-1} \end{bmatrix}$

(d) $\begin{bmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{4} & \mathbf{-3} \end{bmatrix}$ and $\begin{bmatrix} \mathbf{1} & \mathbf{0.5} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$ [EE-2011]

Ans. **(d)**

2. Systems of Linear Equations

Previous Years GATE Question

EC All GATE Questions

1. The system of linear equations **[EC: GATE-2008]**

$$4x + 2y = 7$$
$$2x + y = 6$$

has

(a) A unique solution (b) no solution
(c) An infinite number of solutions (d) exactly two distinct solutions

1.(b)

This can be written as $AX = B$ Where $A = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$

Angemented matrix $\overline{A} = \begin{bmatrix} 4 & 2 & 7 \\ 2 & 1 & 6 \end{bmatrix}$

$$\overline{A} \xrightarrow{R_1 - 2R_2} = \begin{bmatrix} 0 & 0 & -5 \\ 2 & 1 & 6 \end{bmatrix}$$

$\text{rank}(A) \neq \text{rank}(\overline{A})$. The system is inconsistant .So system has no solution.

ME 20 Years GATE Questions

2. Using Cramer's rule, solve the following set of equations **[ME: GATE-1995]**

$$2x + 3y + z = 9$$
$$4x + y = 7$$
$$x - 3y - 7z = 6$$

2. Ans.

Given equations are

$$2x + 3y + 1z = 9$$
$$4x + 1y + 0z = 7$$
$$1x - 3y - 7z = 6$$

By Cramer's Rule

$$\frac{x}{\begin{vmatrix}9 & 3 & 1\\7 & 1 & 0\\6 & -3 & -7\end{vmatrix}}=\frac{y}{\begin{vmatrix}2 & 9 & 1\\4 & 7 & 0\\1 & 6 & -7\end{vmatrix}}=\frac{z}{\begin{vmatrix}2 & 3 & 9\\4 & 1 & 7\\1 & -3 & 6\end{vmatrix}}=\frac{1}{\begin{vmatrix}2 & 3 & 1\\4 & 1 & 0\\1 & -3 & -7\end{vmatrix}}$$

or $$\frac{x}{\begin{vmatrix}9 & 3 & 1\\7 & 1 & 0\\69 & 18 & -7\end{vmatrix}}=\frac{y}{\begin{vmatrix}2 & 9 & 1\\4 & -7 & 0\\15 & 69 & 0\end{vmatrix}}=\frac{z}{\begin{vmatrix}-10 & 0 & -12\\4 & 1 & 7\\13 & 0 & 27\end{vmatrix}}=\frac{1}{\begin{vmatrix}2 & 3 & 1\\4 & 1 & 0\\15 & 18 & 0\end{vmatrix}}$$

or $\frac{x}{57}=\frac{y}{171}=\frac{z}{-114}=\frac{1}{57}$ Hence x=1; y=3; z=-2

4. For the following set of simultaneous equations: [ME: GATE-1997]

$$1.5x - 0.5y = 2$$
$$4x + 2y + 3z = 9$$
$$7x + y + 5z = 10$$

(a) The solution is unique (b) Infinitely many solutions exist
(c) The equations are incompatible (d) Finite number of multiple solutions exist

4. (a)

$$\bar{A}=\begin{bmatrix}\frac{3}{2} & -\frac{1}{2} & 0 & 2\\ 4 & 2 & 3 & 9\\ 7 & 1 & 5 & 10\end{bmatrix}\xrightarrow[R_3-4R_1]{R_2-2R_1}\begin{bmatrix}3/2 & -1/2 & 0 & 2\\ 1 & 3 & 3 & 5\\ 1 & 3 & 5 & 2\end{bmatrix}$$

$$\xrightarrow{R_3-R_2}\begin{bmatrix}3/2 & -1/2 & 0 & 2\\ 1 & 3 & 3 & 5\\ 0 & 0 & 2 & -3\end{bmatrix}$$

$\therefore \text{rank of}(\bar{A}) = \text{rank of}(A) = 3$

$\therefore$ The system has unique solution.

5. Consider the system of equations given below: [ME: GATE-2001]

$$x + y = 2$$
$$2x + 2y = 5$$

This system has
(a) One solution (b) No solution (c) Infinite solution (d) Four solution

5. (b)

Same as Q.1

6. The following set of equations has [ME: GATE-2002]

3 x + 2 y + z = 4 x – y + z = 2 -2 x + 2 z = 5

(a) No solution (b) A unique solution (c) Multiple solution (d) An inconsistency

6.(b)

$$\overline{A} = \left[\begin{array}{ccc|c} 3 & 2 & 1 & 4 \\ 1 & -1 & 1 & 2 \\ -2 & 0 & 2 & 5 \end{array}\right] \xrightarrow[R_3+2R_2]{R_1-3R_2} \left[\begin{array}{ccc|c} 0 & 5 & -2 & -2 \\ 1 & -1 & 1 & 2 \\ 0 & -2 & 4 & 9 \end{array}\right]$$

$$\xrightarrow{\frac{-1}{2}R_3} \left[\begin{array}{ccc|c} 0 & 5 & -2 & -2 \\ 1 & -1 & 1 & 2 \\ 0 & -1 & -2 & \frac{-9}{2} \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 0 & 5 & -2 & -2 \\ 1 & 0 & -1 & -\frac{5}{2} \\ 0 & 1 & -2 & -\frac{9}{2} \end{array}\right]$$

$\therefore \text{rank}(A) = \text{rank}\left(\overline{A}\right) = 3$

$\therefore$ The system has unique solution

7. Consider the system of simultaneous equations [ME: GATE-2003]

$$x + 2y + z = 6$$
$$2x + y + 2z = 6$$
$$x + y + z = 5$$

This system has
(a) Unique solution (b) Infinite number of solutions
(c) No solution (d) Exactly two solution

7. (c)

$$\overline{A} = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 2 & 1 & 2 & 6 \\ 1 & 1 & 1 & 5 \end{array}\right] \xrightarrow[R_2-2R_3]{R_1-R_3} \left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & -4 \\ 1 & 1 & 1 & 5 \end{array}\right]$$

$$\xrightarrow[R_3-R_1]{R_2+R_1} \left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -3 \\ 1 & 0 & 1 & 4 \end{array}\right]$$

$\therefore \text{rank}(A) = 2 \neq 3 = \text{rank}\left(\overline{A}\right).$

$\therefore$ The system is inconsistent and has no solution.

8. Multiplication of matrices E and F is G. Matrices E and G are [ME: GATE-2006]

$E = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. What is the matrix F?

(a) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \cos\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8.(c)

Given $EF = G = I_3$

$\Rightarrow F = E^{-1}G = E^{-1}I_3 = E^{-1}$

9. For what value of a, if any, will the following system of equations in x, y and z have a solution? [ME: GATE-2008]

$$2x + 3y = 4$$
$$x+y+z = 4$$
$$x + 2y - z = a$$

(a) Any real number (b) 0
(c) 1 (d) There is no such value

9. (b)

$$\overline{A} = \left[\begin{array}{ccc|c} 2 & 3 & 0 & 4 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & -1 & a \end{array}\right] \xrightarrow[R_3 - R_2]{R_1 - 2R_2} \left[\begin{array}{ccc|c} 0 & 1 & -2 & -4 \\ 0 & -1 & 1 & 4 \\ 0 & 1 & -2 & a-4 \end{array}\right]$$

$$\xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|c} 0 & 1 & -2 & -4 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & a \end{array}\right]$$

If $a = 0$ then rank $(A) = \text{rank}(\overline{A}) = 2$. Therefore the system is consistant

$\therefore$ The system has sol^n.

CE 10 Years GATE Questions

33. Solution for the system defined by the set of equations 4y + 3z = 8; 2x – z = 2 and 3x + 2y = 5 is **[CE: GATE – 2006]**

(a) x = 0; y = 1; z = $\frac{4}{3}$ (b) x = 0; y = $\frac{1}{2}$; z = 2

(c) x = 1; y = $\frac{1}{2}$; z = 2 (d) non-existent

33. Ans.(d)

$$\textit{Consider the matrix } A = \begin{bmatrix} 0 & 4 & 3 \\ 2 & 0 & -1 \\ 3 & 2 & 0 \end{bmatrix}, \textit{Now } \det(A) = 0$$

So, by Cramer's Rule, the system has no solution.

10. Consider a non-homogeneous system of linear equations representing mathematically an over-determined system. Such a system will be **[CE: GATE – 2005]**

(a) consistent having a unique solution
(b) consistent having many solutions
(c) inconsistent having a unique solution
(d) Inconsistent having no solution

10. Ans.(b)

In an over determined system having more equations than variables, it is necessary to have consistent having many solutions .

11. For what values of α and β the following simultaneous equations have an infinite number of solutions? **[CE: GATE – 2007]**

x + y + z = 5; x + 3y + 3z = 9; x + 2y + αz = β

(a) 2, 7 (b) 3, 8 (c) 8, 3 (d) 7, 2

11.(d)

$$\overline{A} = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & 3 & 3 & 9 \\ 1 & 2 & \alpha & \beta \end{array}\right] \xrightarrow[R_2-R_1]{R_3-R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 1 & \alpha-1 & -5 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & \alpha-1 & \beta-5 \end{array}\right]$$

$$\xrightarrow[R_1-R_2]{R_3-R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & \alpha-2 & \beta-7 \end{array}\right]$$

For infinite solution of the system

$$\alpha - 2 = 0 \text{ and } \beta - 7 = 0$$

$$\Rightarrow \alpha = 2 \text{ and } \beta - 7.$$

12. The following system of equations **[CE: GATE – 2008]**

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + kz = 6$$

Will NOT have a unique solution for k equal to

(a) 0 (b) 5

(c) 6 (d) 7

12. (d)

$$\overline{A} = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & k & 6 \end{array}\right] \xrightarrow[R_2-R_1]{R_3-R_1} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & k-1 & 3 \end{bmatrix} \xrightarrow{R_3-3R_2} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & k-7 & 0 \end{bmatrix}$$

For not unique solution $k - 7 - 0$

$$\Rightarrow k = 7.$$

EE All GATE Questions

14. For the set of equations **[EE: GATE-2010]**

$$x_1 + 2x + x_3 + 4x_4 = 2$$

$$3x_1 + 6x_2 + 3x_3 + 12x_4 = 6$$

(a) Only the trivial solution $x_1 = x_2 = x_3 = x_4 = 0$ exists.

(b) There are no solutions.

(c) A unique non-trivial solution exists.

(d) Multiple non-trivial solutions exist

14.(d)

Because number of unknowns more them no. of equation.

IE All GATE Questions

15. Let A be a 3 × 3 matrix with rank 2. Then AX = 0 has **[IE: GATE-2005]**

(a) Only the trivial solution X = 0

(b) One independent solution

(c) Two independent solutions

(d) Three independent solutions

15. (b)

We know , rank (A) + Solution space X(A) = no. of unknowns.

$\Rightarrow 2 + X(A) = 3$. [Solution space X(A)= No. of linearly independent vectors]

$\Rightarrow X(A) = 1.$

17. Let $P \neq 0$ be a 3×3 real matrix. There exist linearly independent vectors x and y such that $Px = 0$ and $Py = 0$. The dimension of the range space of P is
[IE: GATE-2009]
(a) 0 (b) 1 (c) 2 (d) 3

17. (b)

3. Eigen Values and Eigen Vectors

EC All GATE Questions

1. Given the matrix $\begin{bmatrix} -4 & 2 \\ 4 & 3 \end{bmatrix}$, the eigenvector is [EC: GATE-2005]

(a) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

1. (c)

Characteristic equation

$|A - \lambda I_2| = 0$

$\Rightarrow \begin{vmatrix} -4-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda = -5, 4$

Take $\lambda = -5$, then $AX = \lambda X$ becomes

$\begin{bmatrix} -4 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -5x_1 \\ -5x_2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -4x_1 + 2x_2 \\ 4x_1 + 3x_2 \end{bmatrix} = \begin{bmatrix} -5x_1 \\ -5x_2 \end{bmatrix}$

$\therefore \left.\begin{array}{l} -4x_1 + 2x_2 = -5x_1 \\ -4x_1 + 3x_2 = -5x_2 \end{array}\right\} \Rightarrow x_1 = -2x_2$

$\therefore$ if $x_2 = -1$ then $x_1 = 2$

$\therefore \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is eigen vector corrosponding to $\lambda = -5.$

2. The eigen values and the corresponding eigen vectors of a 2 × 2 matrix are given by [EC: GATE-2006]

Eigenvalue	Eigenvector
$\lambda_1 = 8$	$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\lambda_2 = 4$	$v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The matrix is

(a) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

2. (a)

We know, sum of eigen values = trace (A). = Sum of diagonal element of A.
Therefore $\lambda_1 + \lambda_2 = 8 + 4 = 12$
Option (a) gives, $\text{trace(A)} = 6 + 6 = 12.$

3. For the matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$, the eigen value corresponding to the eigenvector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$ is

[EC: GATE-2006]

(a) 2 (b) 4
(c) 6 (d) 8

3. (c)

$$\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 101 \\ 101 \end{bmatrix} = \lambda \begin{bmatrix} 101 \\ 101 \end{bmatrix}$$

$$= \begin{bmatrix} 606 \\ 606 \end{bmatrix} = \begin{bmatrix} 101\lambda \\ 101\lambda \end{bmatrix} \Rightarrow 101\lambda = 606$$
$$\Rightarrow \lambda = 6$$

6. All the four entries of the 2 × 2 matrix P = $\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$ are nonzero, and one of its eigen values is zero. Which of the following statements is true? [EC: GATE-2008]

(a) $P_{11}P_{22} - P_{12}P_{21} = 1$ (b) $P_{11}P_{22} - P_{12}P_{21} = -1$
(c) $P_{11}P_{22} - P_{12}P_{21} = 0$ (d) $P_{11}P_{22} + {}_{12}P_{21} = 0$

6.(c) One eigen value is zero
$\Rightarrow \det P = 0$
$\Rightarrow P_{11}P_{22} - P_{12}P_{21} = 0$

7. The eigen values of the following matrix are [EC: GATE-2009]

$$\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

(a) 3, 3 + 5j, –6 – j (b) –6 + 5j, 3 + j, 3 – j

(c) 3+ j, 3 – j, 5 + j (d) 3, –1 + 3j, –1 – 3j

7. (d)

Let the matrix be A.

We know, Trace (A)=sum of eigen values.

ME 20 Years GATE Questions

8. Find the eigen value of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$ for any one of the eigen values, find out the corresponding eigenvector. [ME: GATE-1994]

8.

Same as Q.1

9. The eigen values of the matrix [ME: GATE-1999]

$$\begin{bmatrix} 5 & 3 \\ 3 & -3 \end{bmatrix}$$

(a) 6 (b) 5 (c) -3 (d) -4

9. (a), (d).

10. The three characteristic roots of the following matrix A [ME: GATE-2000]

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{vmatrix} \text{ are}$$

(a) 2,3 (b) 1,2,2 (c) 1,0,0 (d) 0,2,3

10.(b)

A is lower triangular matrix. So eigen values are only the diagonal elements.

11. For the matrix $\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ the eigen value are [ME: GATE-2003]

(a) 3 and -3 (b) –3 and -5 (c) 3 and 5 (d) 5 and 0

11. (c)

12. The sum of the eigen values of the matrix given below is [ME: GATE-2004]

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(a) 5 (b) 7 (c) 9 (d) 18

12.(b)

Sum of eigen values of A= trace (A)

13. For which value of x will the matrix given below become singular?

[ME:GATE-2004]

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

(a) 4 (b) 6 (c) 8 (d) 12

13. (a)

Let the given matrix be A.

A is singular.

$\Rightarrow \det A = 0$

$$\Rightarrow \begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix} = 0$$

$\Rightarrow x = 4.$

14. Which one of the following is an eigenvector of the matrix [ME: GATE-2005]

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$

14. (a)

Let the given matrix be A.
Eigen values of A are. 5, 5,
Take $\lambda = 5$, then $AX = \lambda X$ gives.

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5x_1 \\ 5x_2 \\ 5x_3 \\ 5x_4 \end{bmatrix}$$

$5x_1 = 5x_1$

$5x_2 + 5x_3 = 5x_2 \Rightarrow x_3 = 0$

$2x_3 + x_4 = 5x_3 \Rightarrow x_4 = 0 \quad [\therefore x_3 = 0$

$3x_3 + x_4 = 5x_4$

Thus the system of four equation has solution in the form $(K_1, K_2, 0, 0)$ where K_1, K_2 any real numbers. If we take $K_1 = K_2 = -2$ than (a) is ture.

15. Eigen values of a matrix $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1. What are the eigen values of the matrix S^2 = SS?

[ME: GATE-2006]

(a) 1 and 25 (b) 6 and 4 (c) 5 and 1 (d) 2 and 10

15. (a)

We know If λ be the eigen value of A
$\Rightarrow \lambda^2$ is an eigen value of A^2.

16. If a square matrix A is real and symmetric, then the eigenvaluesn [ME: GATE-2007]
(a) Are always real (b) Are always real and positive
(c) Are always real and non-negative (d) Occur in complex conjugate pairs

16. (a)

17. The number of linearly independent eigenvectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is [ME: GATE-2007]

(a) 0 (b) 1 (c) 2 (d) Infinite

17. (d)

Here $\lambda = 2, 2$

For $\lambda = 2$, $AX = \lambda X$ gives,

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 \\ 2x_2 \end{bmatrix}$$

$$\Rightarrow \left.\begin{matrix} 2x_1 + x_2 = 2x_1 \\ 2x_2 = 2x_2 \end{matrix}\right\} \Rightarrow x_2 = 0$$

$\therefore \begin{bmatrix} k \\ 0 \end{bmatrix}$ is the form of eigen vector corrosponding to λ=2. where $k \in R$.

18. The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one eigenvalue equal to 3. The sum of the other two eigenvalues is [ME: GATE-2008]

(a) p (b) p-1 (c) p-2 (d) p-3

18.(c) Let the given matrix be A.

we know we know $\sum \lambda i = \text{trace}(A)$.

Here $\lambda_1 = 3$ and $\text{trace}(A) = 1 + 0 + P = P + 1$

$\therefore \lambda_2 + \lambda_3 = P + 1 - 3 = P - 2$

19. The eigenvectors of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is a + b?

[ME: GATE-2008]

(a) 0 (b) ½ (c) 1 (d) 2

19.(b) Here $\lambda_1 = 1, \lambda_2 = 2$, Given $X_1 = \begin{bmatrix} 1 \\ a \end{bmatrix}$ and $X_2 = \begin{bmatrix} 1 \\ b \end{bmatrix}$

For $\lambda_1 = 1$, $AX_1 = \lambda_1 X_1$ gives

$$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ a \end{bmatrix} = \begin{bmatrix} 1 \\ a \end{bmatrix}$$

$$\Rightarrow \begin{matrix} 1+2a=1 \\ 2a=a \end{matrix} \Rightarrow a = 0$$

For $\lambda_2 = 2, \quad AX_2 = \lambda X_2$ gives

$$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 2b \end{bmatrix}$$

$$\Rightarrow \begin{matrix} 1+2b=2 \\ 2b=2b \end{matrix} \Rightarrow b = 1/2$$

$$\therefore a + b = 1/2$$

20. For a matrix $[M] = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$, the transpose of the matrix is equal to the inverse of the matrix, $[M]^T = [M]^{-1}$. The value of x is given by [ME: GATE-2009]

(a) $-\frac{4}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

20. (a)

Given $[M]^T = [M]^{-1}$

$\Rightarrow$ M is orthogonal matrix

$\Rightarrow MM^T = I_2$

$$\text{Now, } MM^T = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}\begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & \frac{3x}{5}+\frac{12}{25} \\ \frac{3x}{5}+\frac{12}{25} & x^2+\frac{9}{25} \end{bmatrix}$$

$\therefore MM^T = I_2$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3x}{5}+\frac{12}{25} \\ \frac{3x}{5}+\frac{12}{25} & x^2+\frac{9}{25} \end{bmatrix} = x = -\frac{12}{25}\times\frac{5}{3} = -\frac{4}{5}$$

21. One of the Eigen vectors of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ is [ME: GATE-2010]

(a) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ (b) $\begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$ (c) $\begin{Bmatrix} 4 \\ 1 \end{Bmatrix}$ (d) $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

21. (a)

The eigen vectors of A are given by $AX = \lambda X$
So we can check by multiplication.

$$\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} = 1\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is an eigen vactor of A. corrosponding to $\lambda = 1$

CE 10 Years GATE Questions

22. The eigen values of the matrix $\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$ **[CE: GATE – 2004]**

(a) are 1 and 4
(b) are –1 and 2
(c) are 0 and 5
(d) cannot be determined

22. (c)

23. Consider the system of equations $A_{(n \times n)}\, x_{(n \times t)} = \lambda_{(n \times l)}$ where, λ is a scalar. Let (λ_i, x_i) be an eigen-pair of an eigen value and its corresponding eigen vector for real matrix A. Let l be a (n × n) unit matrix. Which one of the following statement is NOT correct?

(a) For a homogeneous n × n system of linear equations, $(A - \lambda I)\, x = 0$ having a nontrivial solution, the rank of $(A - \lambda I)$ is less than n. **[CE: GATE – 2005]**

(b) For matrix A^m, m being a positive integer, (λ_i^m, x_i^m) will be the eigen-pair for all i.

(c) If $A^T = A^{-1}$, then $|\lambda_i| = 1$ for all i.

(d) If $A^T = A$, hen λ_i is real for all i.

23. (b)

If λ be the eigen value of A. then λ^m be the eigen value of $A^m.X^m$ is no the eigen vector of A^m

24. For a given matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigenvalues is 3. **[CE: GATE – 2006]**

The other two eigenvalues are

(a) 2, –5
(b) 3, –5
(c) 2, 5
(d) 3, 5

24(b).

we know $\lambda_1 + \lambda_2 + \lambda_3 = \text{trace(A)}.$

$$\Rightarrow 3 + \lambda_2 + \lambda_3 = 2 - 1 + 0 = 1$$

$$\Rightarrow \lambda_2 + \lambda_3 = -2$$

Only choice (b) is possible.

25. The minimum and the maximum eigen values of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are –2 and 6, respectively. What is the other eigen value? **[CE: GATE – 2007]**

(a) 5 (b) 3
(c) 1 (d) –1

25. (b)

We know $\lambda_1 + \lambda_2 + \lambda_3 = \text{trace(A)}$

by the condition, $-2 + 6 + \lambda_3 = 7$

$$\Rightarrow \lambda_3 = 3$$

26. The Eigen values of the matrix [P] = $\begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix}$ are **[CE: GATE – 2008]**

(a) – 7 and 8 (b) –6 and 5
(c) 3 and 4 (d) 1 and 2

26. (b).

EE All GATE Questions

29. The state variable description of a linear autonomous system is, X= AX,

Where X is the two dimensional state vector and A is the system matrix given by $A = \begin{bmatrix} \mathbf{0} & \mathbf{2} \\ \mathbf{2} & \mathbf{0} \end{bmatrix}$

The roots of the characteristic equation are **[EE: GATE-2004]**

(a) -2 and +2 (b)-j2 and +j2
(c)-2 and -2 (d) +2 and +2

29. (a)

30. In the matrix equation Px = q which of the following is a necessary condition for the existence of at least one solution for the unknown vector x: **[EE: GATE-2005]**
(a) Augmented matrix [Pq] must have the same rank as matrix P
(b) Vector q must have only non-zero elements
(c) Matrix P must be singular
(d) Matrix P must be square

30. (a).

31. For the matrix P= $\begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, *s* one of the eigen values is equal to -2. Which of the following is an eigen vector?

(a) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

31.(d).

$$AX = -2X$$

$$\Rightarrow \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} -2x_1 \\ -2x_2 \\ -2x_3 \end{bmatrix}$$

$$3x_1 - 2x_2 + 2x_3 = -2x_1 \quad -(i)$$

$$\Rightarrow \quad -2x_2 + x_3 = -2x_2 \quad -(ii)$$

$$x_3 = -2x_3 \quad -(iii)$$

From (ii)and (iii) we get

$x_2 = 0$ and $x_3 = 0$

From (i) $5x_1 = 2x_2 - 2x_3$ –(iv)

only choice (d) satisfies equation (iv).

32. If $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$, then top row of R^{-1} is **[EE: GATE-2005]**

(a) $\begin{bmatrix} 5 & 6 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & -3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -1 & 1/2 \end{bmatrix}$

32(b).

$$R^{-1} = \frac{1}{\det R}\text{adj } R$$

Now, det R = 1

$$\text{adj } R = \begin{bmatrix} 5 & -6 & 4 \\ -3 & 4 & -3 \\ 1 & -1 & 1 \end{bmatrix}^t = \begin{bmatrix} 5 & -3 & 1 \\ -6 & 4 & -1 \\ 4 & -3 & 1 \end{bmatrix}$$

$\therefore$ top row of $R^{-1} = \begin{bmatrix} 5 & -3 & 1 \end{bmatrix}$ as $\det R = 1.$

35. $x=[x_1 x_2 \ldots x_n]^T$ is an n-tuple nonzero vector. The n×n matrix $V=xx^T$ **[EE: GATE-2007]**
(a) has rank zero (b) has rank l
(c) is orthogonal (d) has rank n

35 (b).

As every minor of order 2 is zero.

Statement for Linked Answer Questions 37 & 38
Cayley - Hamiltion Theorem states that square matrix satisfies its own characteristic equation, Consider a matrix

$$\boldsymbol{A} = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$$

37. A satisfies the relation **[EE: GATE-2007]**
(a) $A + 3I + 2A^{-2} = 0$ (b) $A^2+2A+2I=0$
(c) $(A+I)(A+2I)=0$ (d) $\exp(A)=0$

37. (c)

Characteristic equation of A is

$|A - \lambda I_2| = 0$

$\Rightarrow \lambda^2 + 3\lambda + 2 = 0$

$\Rightarrow (\lambda + 3)(\lambda + 2) = 0$

By Cayley theorem $(A + 3I_2)(A + 2I_2) = 0$

38. A^9 equals **[EE: GATE-2007]**
(a) 511 A +510I (b) 309A +104I
(c) 154A +155I (d) exp (9A)

38.(a)

From Q.37. we get $A^2 + 3A + 2I = 0$

$\Rightarrow A^2 = -(3A + 2I)$. –(i)

$\therefore A^4 = A^2.A^2 = (3A - 2I).(3A - 2I)$

$= 9A^2 + 12A + 4I$

$= -15A - 14I$

Similarly, $A^8 = A^4.A^4 = -225A - 254I$ (by calculatoin)

and $A^9 = A.A^8 = 511A + 510I$

39. The characteristic equation of a (3×3) matrix P is defined as $\alpha(\lambda) = |\lambda I - P| = \lambda^3 + \lambda^2 + 2\lambda + 1 = 0$

If I denote identity matrix, then the inverse of matrix P will be **[EE: GATE-2008]**

(a) (P^2+P+2I)
(b) (P^2+P+I)
(c) $-(P^2+P+I)$
(c) $-(P^2+P+2I)$

39. (d)

Given ch. equ^n of A is

$\lambda^3 + \lambda^2 + 2\lambda + 1 = 0$

$\Rightarrow P^3 + P^2 + 2P + I = 0$ (By Cayley theorem).

$\Rightarrow P(P^2 + P + 2I) = -I$

$\Rightarrow P^{-1} = -(P^2 + P + 2I)$.

40. If the rank of a (5×6) matrix Q is 4, then which one of the following statements is correct? **[EE: GATE-2008]**

(a) Q will have four linearly independent rows and four linearly independent columns
(b) Q will have four lineally independent rows and five lineally independent columns
(c) QQ^T will be invertible
(d) Q^TQ will be invertible

40. (a).

Rank of a matrix is equal to the No. of linearly independent row or no. of linearly independent column vector.

42. Let P be a 2×2 real orthogonal matrix and $\vec{x}$ is a real vector $[x_1, x_2]^T$ with length $\|\vec{x}\| = \left(x_1^2 + x_2^2\right)^{\frac{1}{2}}$. Then which one of the following statements is correct?

[EE: GATE-2008]

(a) $\|P\vec{x}\| \le \|\vec{x}\|$ *where at least one vector statisfies* $\|P\vec{x}\| < \|\vec{x}\|$

(b) $\|P\vec{x}\| = \|\vec{x}\|$ *for all* vector $\vec{x}$

(c) $\|P\vec{x}\| \ge \|\vec{x}\|$ *where* at least one vector satisfies $\|P\vec{x}\| > \|\vec{x}\|$

(d) No relationship can be established between $\|\vec{x}\|$ *and* $\|P\vec{x}\|$

42. (b)

$$\text{Let } P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\therefore PP' = I$$

$$\text{Now, } P\overline{X} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1\cos\theta - x_2\sin\theta \\ x_1\sin\theta + x_2\cos\theta \end{bmatrix}$$

$$\therefore \left\|P\overline{X}\right\| = \sqrt{\left(x_1\cos\theta - x_2\sin\theta\right)^2 + \left(x_1\sin\theta + x_2\cos\theta\right)^2}$$

$$\therefore \left\|P\overline{X}\right\| = \sqrt{x_1^2 + x_2^2}$$

$$\therefore \left\|P\overline{X}\right\| = \left\|\overline{X}\right\| \text{ for all vector } \overline{x}.$$

43. The trace and determinate of a 2×2 matrix are known to be – 2 and – 35 respectively. Its eigenvalues are [EE: GATE-2009]

(a) -30 and – 5 (b) – 35 and – 1

(c) – 7 and 5 (d) 17.5 and - 2

43. (c)

Given $\lambda_1 + \lambda_2 = -2$ –(i)

$\lambda_1\lambda_2 = -35$

$\therefore \left(\lambda_1 - \lambda_2\right)^2 = \left(\lambda_1 + \lambda_2\right)^2 - 4\lambda_1\lambda_2 = 4 + 140 = 144$

$\Rightarrow \lambda_1 - \lambda_2 = \pm 12$

take $\lambda_1 - \lambda_2 = -12$ –(ii)

Solving (i) and (ii) we get

$\lambda_1 = -7$ and $\lambda_2 = 5$

44. An eigenvector of $P = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$ is

(a) $[-1\ 1\ 1]^T$ (b) $[1\ 2\ 1]^T$ (c) $[1\ -1\ 2]^T$ (d) $[2\ 1\ -1]^T$

44.(b)

Eigen values of P are 1,2,3

Take $\lambda = 3$

$$AX = \lambda X$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 \\ 3x_2 \\ 3x_3 \end{bmatrix}$$

$$\Rightarrow x_1 + x_2 = 3x_1$$

$$2x_1 + 2x_3 = 3x_2$$

$$3x_3 = 3x_3 \Rightarrow x_3 = 1$$

$$\therefore x_2 = 2 \text{ and } x_1 = 1$$

$$\therefore \text{For } \lambda = 3,\ X = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

IE All GATE Questions

16. Identify which one of the following is an eigenvector of the matrix

$\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{-2} \end{bmatrix}$ **[IE: GATE-2005]**

(a) $[-1\quad 1]^T$ (b) $[3\quad -1]^T$

(c) $[1\quad -1]^T$ (d) $[-2\quad 1]^T$

16. (b)

Eigen Value (λ) are $1, -2$.

Take $\lambda = 1$ and if $\begin{bmatrix} x \\ y \end{bmatrix}$ be the eigen veefor of A. Corresponding

To λ then.

$$\begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ -x-2y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow -x = 3y$$

when $y = -1$ then $x = 3$

$\therefore \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ be the eigen vector corrosponding to $\lambda = 1$

47. Let A be an n × n real matrix such that $A^2 = I$ and y = be an n – dimensional vector. Then the linear system of equations Ax = y has [IE: GATE-2007]

(a) No solution
(b) a unique solution
(c) More than one but finitely many independent solutions
(d) Infinitely many independent solutions

47. (b)

$$A^2 = I$$
$$\Rightarrow AA = I$$
$$\Rightarrow \det(AA) = 1$$
$$\Rightarrow \det A . \det A = 1$$
$$\Rightarrow \det A = \pm 1 \neq 0$$

By Cramer's rule $AX = y$ has unique solution.

48. Let A = $[a_{ij}]$, $1 \le i, j \le n$, with $n \ge 3$ and $a_{ij} = i.j$. Then the rank of A is [IE: GATE-2007]

(a) 0
(b) 1
(c) n – 1
(d) n

48.(b)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}, \text{ by the given condition}$$

$$\text{Now,} A \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank $(A) = 1$

51. A real n × n matrix A = $\{a_{ij}\}$ is defined as follows:

a_{ij} = i = 0, if

i = j, otherwise

The summation of all n eigen values of A is **[IE: GATE-2010]**

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$

(c) $\frac{n(n+1)(2n+1)}{6}$ (d) n^2

51.(a) It's a diagonal marix diagonal contain's n elements 1,2,----,n.

$\therefore 1+2+......+n = n\frac{(n+1)}{2}$

As diagonal elements are eigen valves.

$\therefore \sum \lambda i = \frac{n(n+1)}{2}$

CS All GATE Questions

Q40. Consider the matrix as given below.

$$\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{0} & \mathbf{4} & \mathbf{7} \\ \mathbf{0} & \mathbf{0} & \mathbf{3} \end{bmatrix}$$

What one sof the following options provides the **CORRECT** values of the eigenvalues of the matrix?

(a) 1, 4, 3 (b) 3, 7, 3 (c) 7, 3, 2 (d) 1, 2, 3 [CS-2011]

Ans. (a)

Exp. it's an upper triangular matrix.

52. F is an n × n real matrix. b is an n × 1 real vector. Suppose there are two n × 1 vectors, u and v such that u $\neq$ v, and Fu = b, Fv = b.

Which one of the following statements is false? [CS: GATE-2006]

(a) Determinant of F is zero

(b) There are an infinite number of solutions to Fx = b

(c) There is an $x \neq 0$ such that $Fx = 0$
(d) F must have two identical rows

52(d).
If F is non singular, then it has a unique inverse.
Now, $u = F^{-1} b$ and $v = F^{-1} b$
Since F^{-1} is unique u = v but it is given that u □ v. This is a contradiction. So F must be singular. This means that
(a) Determinate of F is zero is true. Also
(b) There are infinite number of solution to Fx = b is true since $|F| = 0$.
Given that Fu = b and Fv = b
(c) There is an $X \neq 0$ such that F X = 0 is also true, since X has infinite number of solutions, including the X = 0 solution.
(d) F must have 2 identical rows is false, since a determinant may become zero, even if two identical columns are present. It is not necessary that 2 identical rows must be present for $|F|$ to become zero

53. Consider the set of (column) vectors defined ty $X = \{x \in R^3 | x_1 + x_2 + x_3 = 0$, where $x^T = [x_1, x_2, x_3]^T\}$. which of the following is TRUE? [CS: GATE-2007]
(a) $\{[1, -1, 0]^T, [1, 0, -1]^T\}$ is a basis for the subspace X.
(b) $\{[1, -1, 0]^T, [1, 0, -1]^T\}$ is linearly independent set, but it does not span X and therefore is not a basis of X
(c) X is not a subspace for R^3
(d) None of the above

53.(b)

54. The following system of equations [CS: GATE-2008]
$x_1 + x_2 + 2x_3 = 1$
$x_1 + 2x_3 + 3x_3 = 2$
$x_1 + 4x_2 + ax_3 = 4$
has a unique solution. The only possible value(s) for a is/are
(a) 0 (b) either 0 or 1
(c) one of 0, 1 or –1 (d) any real number other than 5

54. (d)

$$\overline{A} = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 1 & 4 & a & 3 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & a-2 & 3 \end{array}\right]$$

$$\xrightarrow{R_3-3R_2}\begin{bmatrix}1 & 1 & 2 & 1\\0 & 1 & 1 & 1\\0 & 0 & a-5 & 0\end{bmatrix}$$

System has unique Sol^n if rank $(A) = \text{rank}\ (\overline{A}) = 3$. It is possible if a $\neq 5$.

55. How many of the following matrics have an eigenvalue 1? [CS: GATE-2008]

$$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\cdot\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\cdot\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix} \text{ and } \begin{bmatrix}-1 & 0\\1 & -1\end{bmatrix}$$

(a) One (b) two
(c) Three (d) four

55. (a)

Eigen valves of $\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$ are $1+i, 1-i$

Rest given matrix are triangular matrix. so diagonal elements are the eigen values.

$\therefore \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ has one eigen value 1.

56. Consider the following matrix. [CS: GATE-2010]

$$A = \begin{bmatrix}2 & 3\\x & y\end{bmatrix}$$

If the eigen values of A are 4 and 8, then
(a) x = 4, y = 10 (b) x = 5, y = 8
(c) x = –3, y = 9 (d) x = –4, y = 10

56.(d)

We know,

$\lambda_1 + \lambda_2 = 2 + y$ and $\lambda_1\lambda_2 = \det A = 2y - 3x$

$\Rightarrow 2 + y = 8 + 4 = 12$ $\Rightarrow 2y - 3x = 8.4 = 32$

$\Rightarrow y = 10$ $\Rightarrow x = \dfrac{2.10 - 32}{3} = -4$

57. Consider the following system of linear equations [CS: GATE-2003]

$$\begin{bmatrix}2 & 1 & -4\\4 & 3 & -12\\1 & 2 & -8\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}\alpha\\5\\7\end{bmatrix}$$

Notice that the second and the third columns of the coefficient matrix are linearly dependent. For how many values of α, does this system of equations have infinitely many solutions?
(a) 0 (b) 1
(c) 2 (d) infinitely many

57. (b)

$$\overline{A} = \left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 4 & 3 & -12 & 5 \\ 1 & 2 & -8 & 14 \end{array}\right] \xrightarrow[R_3 - \frac{1}{2}R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5-2\alpha \\ 0 & 3/2 & -6 & 7-\alpha/2 \end{array}\right]$$

$$\xrightarrow{2R_3} \left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5-2\alpha \\ 0 & 3 & -12 & 14-\alpha \end{array}\right] \xrightarrow{R_3 - 3R_2} \left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5-2\alpha \\ 0 & 0 & 0 & -1+5\alpha \end{array}\right]$$

The system has infinitely many solution

if $-1 + 5\alpha = 0 \Rightarrow \alpha = 1/5$.

$\therefore$ for only one value of α.

58. The number of different $n \times n$ symmetric matrices with each element being either 0 or 1 is: (Note : power (2, x) is same as 2^x) [CS: GATE-2004]

(a) Power (2, n)
(b) power (2, n^2)
(c) Power (2, $(n^2 + n)/2$
(d) power (2, $(n^2 - n)/2$)

58. Ans.(c)

In a symmetric matrix, the lower triangle must be the minor image of upper triangle using the diagonal as mirror. Diagonal elements may be anything. Therefore, when we are counting symmetric matrices we count how many ways are there to fill the upper triangle and diagonal elements. Since the first row has n elements, second (n – 1) elements, third row (n – 2) elements and so on upto last row, one element.

Total number of elements in diagonal + upper triangle

$$= n + (n-1) + (n-2) + \ldots + 1$$

$$= \frac{n(n+1)}{2}$$

Now, each one of these elements can be either 0 or 1. So that number of ways we can fill these elements is

$$2^{\frac{n(n+1)}{2}} = \text{power}\left(2, \frac{(n^2+n)}{2}\right)$$

Since there is no choice for lower triangle elements the answer is power $\left(2, \frac{(n^2+n)}{2}\right)$ which is choice (c).

59. Let A, B, C, D be $n \times n$ matrices, each with non-zero determinant, If ABCD = 1, then B^{-1} is [CS: GATE-2004]

(a) $D^{-1} C^{-1} A^{-1}$
(b) CDA
(c) ADC
(d) does not necessarily exist

59. (b).

ABCD = 1.

$\Rightarrow ABCD\,D^{-1}C^{-1} = D^{-1}C^{-1}$

$\Rightarrow AB = D^{-1}C^{-1}$

$\Rightarrow A^{-1}AB = A^{-1}D^{-1}C^{-1}$

$\Rightarrow B = (CDA)^{-1}$

$\Rightarrow B^{-1} = CDA.$

60. In an M × N matrix such that all non-zero entries are covered in a rows and b column. Then the maximum number of non-zero entries, such that no two are on the same row or column, is [CS: GATE-2004]

(a) ≤ a + b
(b) ≤ max (a, b)
(c) ≤ min[M–a, N–b]
(d) ≤ min {a, b}

60. (d)

61. How many solutions does the following system of linear equations have [CS: GATE-2004]

$$-x + 5y = -1$$
$$x - y = 2$$
$$x + 3y = 3$$

(a) Infinitely many
(b) two distinct solution
(c) Unique
(d) none

61. (c)

$$\overline{A} = \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & .3 & 3 \end{array}\right] \xrightarrow[R_3+R_1]{R_2+R_1} \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 0 & 4 & 1 \\ 0 & 8 & 2 \end{array}\right]$$

$$\xrightarrow{R_3-2R_2} \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 0 \end{array}\right] \therefore\ \text{rank}(A) = \text{rank}(\overline{A}) = 2$$

63. Consider the following system of equation in three real variables x_1, x_2 and x_3

$$2x_1 - x_2 + 3x_3 = 1$$
$$3x_1 - 2x_2 + 5x_3 = 2$$

$-x_1 - 4x_2 + x_3 = 3$

This system of equations has [CS: GATE-2005]

(a) No solution
(b) A unique solution
(c) More than one but a finite number of solutions
(d) An infinite number of solutions

63. Ans. (b)

$$\overline{A} = \left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 3 & -2 & 5 & 2 \\ -1 & -4 & 1 & 3 \end{array}\right] \xrightarrow[R_3+\frac{1}{2}R_1]{R_3-\frac{3}{2}R_1} \left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{-9}{2} & \frac{5}{2} & \frac{7}{2} \end{array}\right]$$

$$\xrightarrow{R_3-9R_2} \left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -2 & -1 \end{array}\right]$$

$\therefore$ Rank $(A) =$ Rank $(\overline{A}) = 3$

64. What are the eigen values of the following 2×2 matrix? [CS: GATE-2005]

$$\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$$

(a) –1 and 1
(b) 1 and 6
(c) 2 and 5
(d) 4 and –1

64. (b).

4. Determinants

Previous Years GATE Questions

EE All GATE Questions

1. The determinant of the matrix **[EE: GATE-2002]**

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 100 & 1 & 0 & 0 \\ 100 & 200 & 1 & 0 \\ 100 & 200 & 300 & 1 \end{bmatrix} \text{is}$$

(a) 100 (b) 200 (c)1 (d) 300

1.Ans(c)

5. Calculus

EC All GATE Questions

2. As x is increased from $-\infty$ to ∞, the function [EC: GATE-2006]

$$f(x) = \frac{e^x}{1+e^x}$$

(a) Monotonically increases
(b) Monotonically decreases
(c) Increases to a maximum value and then decreases
(d) Decreases to a minimum value and then increases

2. (a)

$$f'(x) = \frac{e^x}{(1+e^x)^2} > 0,\ \forall x \in (-\infty, \infty)$$

3. A function is given by f(t) = $\sin^2 t + \cos 2t$. Which of the following is true?
[EC: GATE-2009]

(a) f has frequency components at 0 and $\frac{1}{2\pi}$ Hz.

(b) f has frequency components at 0 and $\frac{1}{\pi}$ Hz.

(c) f has frequency components at $\frac{1}{2\pi}$ and $\frac{1}{\pi}$ Hz.

(d) f has frequency components at $\frac{0,1}{2\pi}$ and $\frac{1}{\pi}$ Hz.

3. Ans.(a)

$f(t) = \sin^2 t + \cos^2 t$

(i) $f(t) = \sin^2 t + 1 - 2\sin^2 t$

$= 1 - \sin 2t$

$= \cos^2 t$

Hence have $\frac{1}{2\pi}$ frequency components

(ii) $f(t) = \frac{1-\cos 2t}{2} + \cos 2t$

$= \frac{1+\cos 2t}{2}$

$= \cos^2 t$

4. $\lim_{\theta \to 0} \dfrac{\sin\left(\dfrac{\theta}{2}\right)}{\theta}$ is [EC: GATE-2007]

(a) 0.5 (b) 1 (c) 2 (d) not defined

4. (a)

$$\lim_{\theta \to 0} \frac{\sin(\theta/2)}{\theta} = \lim_{\theta \to 0} \frac{\sin(\theta/2)}{\theta/2} . \frac{1}{2} = \frac{1}{2}$$

ME 20 Years GATE Questions

5 Following are the values of a function y(x) : y(-1) = 5, y(0), y(1) = 8 $\dfrac{dy}{dx}$ at x = 0 as per Newton's central difference scheme is: [ME: GATE-1999]

(a) 0 (b) 1.5 (c) 2.0 (d) 3.0

5. Ans.(b)

$$\left(\frac{dy}{dx}\right)_{\text{at } x=0} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y(1) - y(-1)}{1 - (-1)} = \frac{8 - 5}{2} = 1.5$$

6. If, $y = x + \sqrt{x + \sqrt{x + \sqrt{x + \ ...\infty}}}$, then y (2)= [ME: GATE-2007]

(a) 4 or 1 (b) 4 only (c) 1 only (d) Undefined

6. Ans. (b)

Given $y = x + \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}}$ or, $(y-x) = \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}}$

Square both side, we get

$(y-x)^2 = x + y = x + \sqrt{x + \sqrt{x + ...\infty}}$ $\qquad (y-x)^2 = y$

$y^2 - 2x + 1)y + x^2 = 0$ $\qquad$ put $\qquad x = 2$

$\therefore\ y^2 - 5y + 4 = 0$ $\qquad (y-4)(y-1)=0$ $\qquad \therefore y=1$ or 4

But is always greater than x. Hence y = 4 only

7. The value of $\left[\lim_{x \to \infty} \dfrac{\sin x}{x}\right]$ is [ME: GATE-1994]

(a) ∞ (b) 2 (c) 1 (d) 0

7.(c)

$$\text{Put } x = \frac{1}{z}. \text{ Then} \lim_{x\to\infty}\frac{\sin x}{x} = \lim_{y\to 0}\frac{\sin 1/y}{1/y} = 1$$

8. The value of $\left[\lim_{x\to\infty}\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)\right]$ is [ME: GATE-1994]

(a) 0 (b) 2 (c) 1 (d) ∞

8.(d)

$$\lim_{x\to\infty}\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)$$

$$= \lim_{x\to\infty}\left(\frac{1-\cos x}{\sin x}\right) = \lim_{x\to\infty}\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \lim_{x\to\infty}\tan\frac{x}{2} = \infty$$

9. The function f(x) = |x+1| on the interval [-2, 0] [ME: GATE-1995]

(a) Continuous and differentiable

(b) Continuous on the integral but not differentiable at all points

(c) Neither continuous nor differentiable

(d) Differentiable but not continuous

9. (b)

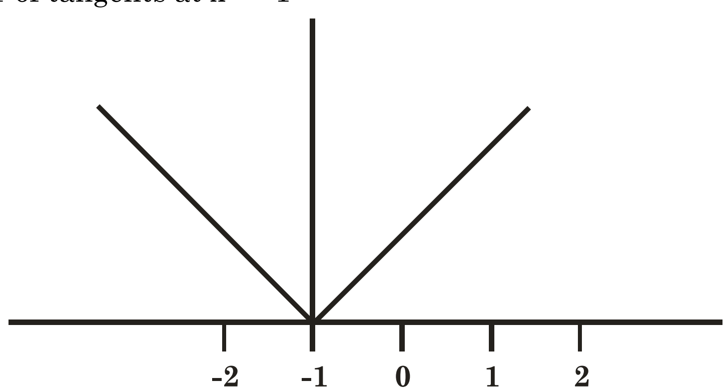
$f(x) = |x+1|$

f is continuous in $[-2,0]$

but not differentiable at

$x = -1$ because we can draw

infinite number of tangents at $x = -1$



10. $\lim_{x\to\infty}\frac{x^3 - \cos x}{x^2 + (\sin x)^2}$ equal [ME: GATE-1995]

(a) ∞ (b) 0 (c) 2 (d) Does not exist

10. Ans. (a)

cos x and sin x are finite whatever x may be

$$\therefore \lim_{x\to\infty} \frac{x^3 - \cos x}{x^2 + (\sin x)^2} \qquad = \lim_{x\to\infty} \frac{x^3}{x^2} = \infty.$$

11. If y=|x| for x<0 and y=x for $x \geq 0$, then [ME: GATE-1997]

(a) $\frac{dy}{dx}$ is discontinuous at x = 0 (b) y is discontinuous at x = 0

(c) y is not defend at x = 0 (d) Both y and $\frac{dy}{dx}$ are discontinuous at x = 0

11. (b)

12. $\underset{x\to1}{\text{Lt}}$ $(x^2-1)/(x-1)$ is [ME: GATE-2000]

(a) ∞ (b) 0 (c) 2 (d) 1

12. (c)

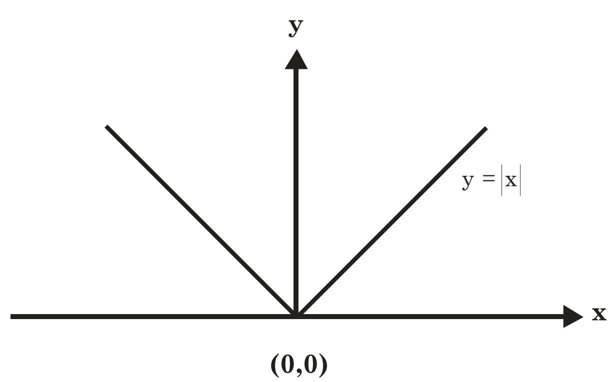
$$\lim_{x\to1}\frac{x^2-1}{x-1} = \lim_{x\to1}(x+1) = 2$$

13. What is the derivative of f(x) = |x| at x = 0? [ME: GATE-2001]

(a) 1 (b) -1 (c) 0 (d) Does not exist

13. (d)

$f(x) = |x|.$



At $x = 0$, we can draw infinitely many tangents at x=0.So limit does not exists.

14. Which of the following functions is not differentiable in the domain [-1,1]? [ME: GATE-2002]

(a) $f(x) = x^2$ (b) f(x) = x-1 (c) f(x) = 2 (d) f(x) = Maximum (x,-x)

14. Ans.(a)

15. $\underset{x \to 0}{\text{Lt}} \dfrac{\text{Sin}^2 x}{x}$ is equal to [ME: GATE-2003]

(a) 0 (b) ∞ (c) 1 (d) -1

15. (c)

$$\lim_{x \to 0} \frac{\sin^2 x}{x} = \lim_{x \to 0} \left(\frac{\sin x}{x} \right)^2 .x = \left(\lim_{x \to 0} \frac{\sin x}{x} \right)^2 . \lim_{x \to 0} x$$

$$= 1.0 = 1$$

16. If f(x)=$\dfrac{2x^2 - 7x + 3}{5x^2 - 12x - 9}$, then $\lim_{x \to 3}$ f(x) will be [ME: GATE-2006]

(a) – 1/3 (b) 5/18 (c) 0 (d) 2/5

16. (b)

$$\lim_{x \to 3} \frac{2x^2 - 7x + 3}{5x^2 - 12x - 9} \left[\frac{0}{0} \text{form} \right]$$

$$\lim_{x \to 3} \frac{4x - 7}{10x - 12} \left[\text{use L' Hospital rule} \right]$$

$$\frac{4.3 - 7}{10.3 - 12}$$

$$= \frac{5}{18}$$

17. $\lim_{x \to 0} \dfrac{e^x - \left(1 + x + \dfrac{x^2}{2}\right)}{x^3} =$ [ME: GATE-2007]

(a) 0 (b) 1/6 (c) 1/3 (d) 1

17. (b)

$$\lim_{x \to 0} \frac{e^x - \left(1 + x + \dfrac{x^2}{2}\right)}{x^3} = \lim_{x \to 0} \frac{1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \text{..........} - \left(1 + x + \dfrac{x^2}{2}\right)}{x^3}$$

$$= \lim_{x \to o} \frac{\dfrac{1}{3!} + \dfrac{x}{4!}}{1} \quad \textbf{(negelecting higher order term)}$$

$$= \frac{1}{6}$$

18. The Value of $\lim_{x \to 8} \dfrac{x^{1/3} - 2}{(x - 8)}$ [ME: GATE-2008]

(a) $\dfrac{1}{16}$ (b) $\dfrac{1}{12}$ (c) $\dfrac{1}{8}$ (d) $\dfrac{1}{4}$

18.(d) $\lim_{x\to 8}\frac{x^{\frac{1}{3}}-2}{x-8}=\lim_{x\to 8}\frac{x^{\frac{1}{3}}-2}{(x^{\frac{1}{3}}-2)(x^{2/3}+2x^{1/3}+4)}=\lim_{x\to 8}\frac{1}{x^{2/3}+2x^{1/3}+4}=\frac{1}{4}$

19. The function Y=| 2-3x | [ME: GATE-2010]

(a) is continuous $\forall x\in R$ and differentiable $\forall x\in R$

(b) is continuous $\forall x\in R$ and differentiable $\forall x\in R$ except at x = 3/2

(c) is continuous $\forall x\in R$ and differentiable $\forall x\in R$ except at x = 2/3

(d) is continuous $\forall x\in R$ except at x = 3 and differentiable $\forall x\in R$

19 (c)

same as 9.

CE 10 Years GATE Questions

Q27. What should be the value of λ such that the function defined below is continuous at x = π/22?

$$f(x)\begin{cases}\dfrac{\lambda\cos x}{\frac{\pi}{2}-x} & \text{if } x\neq \pi/2\\ 1 & \text{if } x=\pi/2\end{cases}$$

(a) 0 (b) $2/\pi$ (c) 1 (d) $\pi/2$ **[CE-2011]**

Ans. **(c)**

Exp. By the given condition

$$\lim_{x\to\pi/2} f(x)=f(\pi/2)$$

$$\Rightarrow \lim_{x\to\pi/2}\frac{\lambda\cos x}{\left(\pi/2-x\right)}=1 \qquad \dots (1)$$

Now, $\lim_{x\to\pi/2}\frac{\lambda\cos x}{\pi/2-x}\left[\frac{0}{0}\text{form}\right]$ … (2)

$=\lim_{x\to\pi/2}\frac{-\lambda\sin x}{-1}$ [use L'Hospital Rule]

$=\lambda$

From (1), $\lambda=1$

20. Given that one root of the equation $x^3-10x^2+31x-30=0$ is 5, the other two roots are

(a) 2 and 3 (b) 2 and 4

(c) 3 and 4 (d) – 2 and –3 **[CE: GATE – 2007]**

20. (a)

Given $x^3-10x^2+31x-30=0$......(i) and $x=5$ is one root of (i)

$\therefore (x-5)$ is a factor of (i)

$\therefore x^3 - 10x^2 + 31x - 30 = 0$

$\Rightarrow x^3 - 5x^2 - 5x^2 + 25x + 6x - 30 = 0$

$\Rightarrow x^2(x-5) - 5x(x-5) + 6(x-5) = 0$

$\Rightarrow (x-5)(x^2 - 5x + 6) = 0$

$\Rightarrow x = 5, 3, 2.$

21. The value of the function f(x) = $\lim_{x\to 0} \frac{x^3 + x^2}{2x^3 - 7x^2}$ is **[CE: GATE – 2004]**

(a) 0
(b) $-\frac{1}{7}$
(c) $\frac{1}{7}$
(d) ∞

21. (b)

$$\lim_{x\to 0} \frac{x^3 + x^2}{2x^3 - 7x^2} = \lim_{x\to 0} \frac{x+1}{2x-7} = -\frac{1}{7}$$

22. The $\lim_{x\to 0} \frac{\sin\left[\frac{2}{3}x\right]}{x}$ is **[CE: GATE – 2010]**

(a) $\frac{2}{3}$
(b) 1
(c) $\frac{3}{2}$
(d) ∞

22. (a)

$$\text{Hints:} - \lim_{x\to 0} \frac{\sin x}{x} = 1$$

IE All GATE Questions

24. If, $y = x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots\infty}}}$, then y (2)= [ME: GATE-2007]

(a) 4 or 1
(b) 4 only
(c) 1 only
(d) Undefined

24. Ans. (b)

Given $y = x + \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}}$ or, $(y-x) = \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}}$

Square both side, we get

$$(y-x)^2 = x + y = x + \sqrt{x + \sqrt{x + ...\infty}} \qquad (y-x)^2 = y$$

$y^2 - 2x + 1)y + x^2 = 0$ put $x = 2$

$\therefore\ y^2 - 5y + 4 = 0$ (y-4)(y-1)=0 $\therefore$ y=1 or 4

But is always greater than x. Hence y = 4 only

26. Consider the function f(x) = $|x|^3$, where x is real. Then the function f(x) at x = 0 is [IE: GATE-2007]

(a) Continuous but not differentiable
(b) Once differentiable but not twice
(c) Twice differentiable but not thrice
(d) Thrice differentiable

26. (a)

same as 13.

27. $\lim_{x \to 0} \frac{\sin x}{x}$ is [IE: GATE-2008]

(a) Indeterminate (b) 0 (c) 1 (d) 2

27. Ans. (c)

28. The expression $e^{-\ln x}$ for x > 0 is equal to [IE: GATE-2008]

(a) –x (b) x (c) x^{-1} (d) $-x^{-1}$

28. (c)

$$e^{-\text{In}x} = e^{\text{In}\frac{1}{x}} = \frac{1}{x}$$

29. At $t = 0$, the function $f(t) = \frac{\sin t}{t}$ has

(a) a minimum (b) a discontinuity
(c) a point of inflection (d) a maximum

29. (d)

$$\lim_{t \to 0} \frac{\sin t}{t} = 1$$

CS All GATE Questions

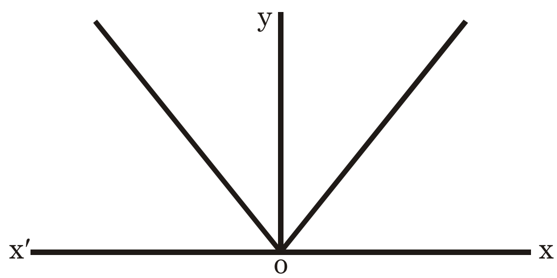
30. Consider the following two statements about the function f(x) = |x|
P: f(x) is continuous for all real values of x
Q: f(x) is differentiable for all real values of x
Which of the following is TRUE? [CS: GATE-2007]
(a) P is true and Q is false
(b) P is false and Q is true
(c) Both P and Q are true
(d) Both P and Q are false

30. Ans. (a)

$$f(x) = |x|$$

or $$f(x) = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$$

The graph of f(x) is



f(x) is continuous for all real values of x

$$\lim_{x \to 0^-} |x| = \lim_{x \to 0^+} |x| = 0$$

as can be seen from graph of |x|.

$$\lim_{x \to 0^-} f(x) = -1$$

and $\lim_{x \to 0^+} f(x) = +1$ as can be seen from graph of |x|

Left derivative $\neq$ Right derivative

So |x| is continuous but not differentiable at x = 0.

31. $\lim\limits_{x \to \infty} \dfrac{x - \sin x}{x + \cos x}$ equals [CS: GATE-2008]
(a) 1
(b) –1
(c) ∞
(d) $-\infty$

31(a).

$$\lim_{x\to\infty} \frac{x - \sin x}{x + \cos x} = \lim_{x\to\infty} \frac{1 - \frac{\sin x}{x}}{1 + \frac{\cos x}{x}} = \frac{1 - \lim\limits_{x\to\infty} \frac{\sin x}{x}}{1 + \lim\limits_{x\to\infty} \frac{\cos x}{x}}$$

put $x = \dfrac{1}{y}$ As $x \to \infty \Rightarrow y \to 0$

$$= \frac{1 - \lim_{x\to 0} y \sin\frac{1}{y}}{1 + \lim_{y\to 0} y \cos\frac{1}{y}} = \frac{1 - \lim_{y\to 0} y \sin\frac{1}{y}}{1 + \lim_{y\to 0} y \cos\frac{1}{y}} = \frac{1-0}{1+0} = 1$$

32. What is the value of $\lim_{n - \infty}\left(1 - \frac{1}{n}\right)^{2n}$? [CS: GATE-2010]

(a) 0 (b) e^{-2}
(c) $e^{-1/2}$ (d) 1

32. (b)

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n} = \lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)^{n}\right]^{2} = \left[\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n}\right]^{2}$$

$$= \left(e^{-1}\right)^{2} = e^{-2}$$

6. Mean Value Theorems

Previous Years GATE Questions

ME 20 Years GATE Questions

1. The value of ξ in the mean value of theorem of f(b) – f(a) = (b-a) f' (ξ) for $f(x) = Ax^2 + Bx + C$ in (a, b) is [ME: GATE-1994]

(a) b + a (b) b – a (c) $\frac{(b+a)}{2}$ (d) $\frac{(b-a)}{2}$

CE 10 Years GATE Questions

2. A rail engine accelerates from its stationary position for 8 seconds and travels a distance of 280 m. According to the Mean Value Theorem, the speedometer at a certain time during acceleration must read exactly **[CE: GATE – 2005]**

(a) 0 (b) 8 kmph
(c) 75 kmph (d) 126 kmph

Answer with Explanation

1. Ans. (c)

Exp. - Given $f(x) = Ax^2 + Bx + C$

$$f'(x) = 2Ax + B$$

and $\frac{f(b) - f(a)}{b - a} = f'(\xi)$, or $2A\xi + B = \frac{(Ab^2 + Bb + C) - (Aa^2 + Ba + C)}{b - a}$

$= \frac{A(b^2 + a^2) + B(b - a)}{b - a}$ $= A(b + a) + B$ Hence $\xi = \frac{b + a}{2}$

2. Ans. (d)

Since the position of rail engine S(t) is continuous and differentiable function, according to Lagranges mean value theorem

$\exists t$ Where $0 \le t \le 8$ such that

$$S'(t) = v(t) = \frac{S(8) - S(0)}{8 - 0}$$

$$= \frac{(280 - 0)}{(8 - 0)} \text{ m/sec}$$

$$= \frac{280}{8} \text{ m/sec}$$

$$= \frac{280}{8} \times \frac{3600}{1000}\ \text{kmph}$$
$$= 126\ \text{kmph}$$

Where v (t) is the velocity of the rail engine.

7. Theorems of Integral Calculus

EC All GATE Questions

1. The value of the integral I = $\frac{1}{2\pi}\int_0^\infty \exp\left(-\frac{x^2}{8}\right)$ dx is [EC: GATE-2005]

(a) 1 (b) π

(c) 2 (d) 2π

1.(a)

$$I = \frac{1}{\sqrt{2\pi}}\int_0^\infty \exp\left(\frac{-x^2}{8}\right)dx$$

$$\text{put } z = \frac{x^2}{8}$$

$$\Rightarrow dz = \frac{xdx}{4}$$

$$\Rightarrow dx = \frac{4dz}{\sqrt{8z}} = \frac{\sqrt{2}dz}{\sqrt{z}}$$

$$= \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-z}.\frac{\sqrt{2}}{\sqrt{z}}dz$$

$$= \frac{1}{\sqrt{\pi}}\int_0^\infty e^{-z}z^{-1/2}dz$$

$$= \frac{1}{\sqrt{\pi}}\int_0^\infty e^{-z}z^{-\frac{1}{2}-1}dz \quad \therefore\left[\Gamma(n) = \int_0^\infty e^{-z}z^{n-1}dz, n > 0\right]$$

$$= \frac{1}{\sqrt{\pi}}\Gamma(1/2) \quad \left[\therefore \Gamma(1/2) = \sqrt{\pi}\right]$$

$$= \frac{1}{\sqrt{\pi}}\sqrt{\pi} = 1$$

2. The integral $\int_0^\pi \sin^3\theta\, d\theta$ is given by [EC: GATE-2006]

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{4}{3}$ (d) $\frac{8}{3}$

2. (c)

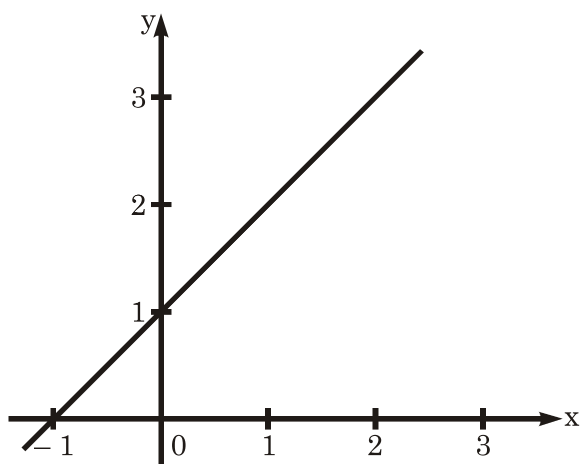
$$\int_0^{\pi} \sin 3\theta d\theta = \int_0^{\pi} (1-\cos 2\theta)\sin\theta d\theta. \qquad \text{put} z = \cos\theta$$

$$dz = -\sin\theta d\theta.$$

$$= -\int_1^{-1} (1-z^2) dz = \int_{-1}^{1} (1-z^2) dz$$

$$= 2\int_1^{1} (1-z^2) dz = 2\left[z - \frac{z3}{3}\right]_0^1 = 2(1-1/3) = \frac{3}{4}$$

3. The following plot shows a function y which varies linearly with x. The value of the integral I = $\int_1^2 y dx$ is [EC: GATE-2007]



(a) 1.0 (b) 2.5
(c) 4.0 (d) 5.0

3(b).

Here the points (0,1) and (-1,0) are on the time

$\therefore$ The equ^n of the line is

$$y - 1 = \frac{0-1}{-1-0}(x-0)$$

$$\Rightarrow y - 1 = x$$

$$\Rightarrow y = x + 1$$

$$\therefore \int_1^2 y dx = \int_1^2 (x+1) dx = \left[\frac{x2}{2} + x\right]_1^2 = 2.5$$

4. Which one of the following function is strictly bounded? [EC: GATE-2007]

(a) $\frac{1}{x^2}$ (b) e^x

(c) x^2 (d) e^{-x^2}

4. (d)

For a strictly bounded function f(x), limit should be finite

Here $\lim_{x\to\infty} e^{-x^2} \to = \text{(finite)}.$

ME 20 Years GATE Questions

6. The The value of $\int_0^\infty y^{\frac{1}{2}} e^{-y^3}\,dx$ is .... [ME: GATE-1994]

6. Ans.

$\int_0^\infty y^{1/2}.e^{-y^3}\,dy$ put $y^3 = z$

$$\Rightarrow 3y^2dy=dz$$

$$\Rightarrow dy=\frac{1}{3}y^{-2}dz$$

$$\Rightarrow dy = \frac{1}{3}z^{-\frac{2}{3}}dz$$

$$= \frac{1}{3}\int_0^\infty z^{\frac{1}{6}}.e^{-z}.z^{\frac{-2}{3}}dz$$

$$= \frac{1}{3}\int_0^\infty e^{-z}z^{-\frac{1}{2}}dz$$

$$= \frac{1}{3}\int_0^\infty e^{-z}z^{\frac{1}{2}-1}dz$$

$$= \frac{1}{3}\Gamma(\frac{1}{2})$$

$$= \frac{1}{3}.\sqrt{\pi}$$

$$= \frac{\sqrt{\pi}}{3}$$

8. $\int_{-a}^{a}\left(\sin^6 x+\sin^7 x\right)dx$ is equal to [ME: GATE-2005]

(a) $2\int_0^a \sin^6 x dx$ (b) $2\int_0^a \sin^7 x dx$ (c) $2\int_0^a (\sin^6 x+\sin^7 x)dx$ (d) Zero

8. (a)

$$\int_{-a}^{a}\left(\sin^6 x+\sin^7 x\right)dx$$

$$=2\int_0^a \sin^6 xdx.$$

$\sin x$ is odd function

$\Rightarrow \sin^6 x$ is even and $\sin^7 x$ is odd function.

$$\int_{-a}^{a}\sin^6 x = 2\int_0^a \sin xdx$$

and $\int_{-a}^{a}\sin^7 x = 0.$

9. The value of the integral $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$ is [ME: GATE-2010]

(a) $-\pi$ (b) $-\pi/2$ (c) $\pi/2$ (d) π

9. (d)

$$\int_{-\infty}^{\infty}\frac{dx}{1+x^2}=\left[\tan^{-1}x\right]_{-\infty}^{\infty}=\left[\pi/2-(\pi/2)\right]=\pi.$$

10. Which of the following integrals is unbounded? [ME: GATE-2008]

(a) $\int_0^{\pi/4}\tan x\, dx$ (b) $\int_0^{\infty}\frac{1}{x^2+1}dx$ (c) $\int_0^{\infty}xe^{-x}dx$ (d) $\int_0^1\frac{1}{1-x}dx$

10. (d)

At $x=1, \frac{1}{1-x}$ is unbounded.

21. The length of the curve $y=\frac{2}{3}x^{3/2}$ between x = 0 and x = 1 is [ME: GATE-2008]

(a) 0.27 (b) 0.67 (c) 1 (d) 1.22

21.(d)

$$\text{Length of the wire} = \int_0^1 \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\ dx$$
$$= \int_0^1 \sqrt{x+1}dx$$
$$= 1.22.$$

CE 10 Years GATE Questions

EE All GATE Questions

Q28. What is the value of the definite integral, $\int_0^a \frac{\sqrt{\mathbf{x}}}{\sqrt{\mathbf{x}} + \sqrt{\mathbf{a - x}}} \boldsymbol{dx}$?

(a) 0 (b) a/2 (c) a (d) 2a **[CE-2011]**

Ans. (b)

Exp. Let $\mathbf{f(x)} = \int_0^a \frac{\sqrt{\mathbf{x}}}{\sqrt{\mathbf{x}} + \sqrt{\mathbf{a - x}}} \boldsymbol{dx} = \mathbf{I_1}$ (say)

$\mathbf{f(a - x)} = \int_0^a \frac{\sqrt{\mathbf{a - x}}}{\sqrt{\mathbf{a - x}} + \sqrt{\mathbf{x}}} \boldsymbol{dx} = \mathbf{I_2}$ (say)

We know

$\int_0^a \mathbf{f(x)dx} = \int_0^a \mathbf{f(a - x)dx}$

$\Rightarrow \mathbf{I_1 = I_2 = I}$ (say)

$\therefore \mathbf{I_1 = I_2} = \int_0^a \frac{\sqrt{\mathbf{x}} + \sqrt{\mathbf{a - x}}}{\sqrt{\mathbf{a - x}} + \sqrt{\mathbf{x}}} \boldsymbol{dx} = \int_0^a \mathbf{dx = a}$

$\Rightarrow \mathbf{2I_1 = a}$

$\Rightarrow \mathbf{2I = a}$

$\Rightarrow \mathbf{I} = \frac{\mathbf{a}}{\mathbf{2}}$

11. If $S = \int_1^\infty x^{-3} dx$, then S has the value **[EE: GATE-2005]**

(a) $\frac{-1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

11. (c)

$$S = \int_1^\infty x^{-3}dx = \left[\frac{x^{-2}}{-2}\right]_1^\infty = \frac{1}{2}$$

16 The value of the quantity P, where $P = \int_0^1 xe^x dx$, is equal to **[EE: GATE-2010]**

(a) 0 (b) 1 (c) e (d) 1/e

16. (b)

$$P = \int_0^1 xe^x dx = \left[xe^x - e^x\right]_0^1 = 1$$

17. A continuous-time system is described by $y(t) = e^{-|x(t)|}$ where y (t) is the output and x (t) is the input. y(t) is bounded. **[EE: GATE-2006]**

(a) only when x(t) is bounded
(b) only when x(t) is non-negative
(c) only or $t \geq 0$ if x (t) is bounded for $t \geq 0$
(d) even when x(t) is not bounded

17. (d)

As $e^{-\infty} \to 0$(finite)

$\therefore$ y(t) is bounded even if x(t) is not bounded.

IE All GATE Questions

17. The value of the integral $\int_{-1}^{1} \frac{1}{x^2}dx$ is **[IE: GATE-2005]**

(a) 2 (b) does not exist (c) –2 (d) ∞

17. (b)

$\int_{-1}^{1} \frac{1}{x^2}dx$ does not exists because at $x = 0, \frac{1}{x^2}$ is not bounded.

CS All GATE Questions

20. $\int_0^{\frac{\pi}{4}} \frac{(1-\tan x)}{(1+\tan x)} dx$ evaluates to [CS: GATE-2009]

(a) 0 (b) 1 (c) ln 2 (d) $\frac{1}{2}\ln 2$

20. Ans.(d)

Since

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$\therefore \quad 1 = \int_0^{\frac{\pi}{4}} \frac{1-\tan x}{1+\tan x} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{1-\tan\left(\frac{\pi}{4}-x\right)dx}{1+\tan\left(\frac{\pi}{4}-x\right)}$$

Since $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

$$\therefore \quad I = \int_0^{\frac{\pi}{4}} \frac{\left[\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]}{\left[\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{1-\left[\frac{1-\tan x}{1+\tan x}\right]}{1+\left[\frac{1-\tan x}{1+\tan x}\right]} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{2\tan x}{2} dx$$

$$= \int_0^{\frac{\pi}{4}} \tan x\, dx$$

$$= [\log(\sec x)]_0^{\frac{\pi}{4}}$$

$$= \ln\left(\sec\frac{\pi}{4}\right) - \ln(\sec 0)$$

$$= \ln(\sqrt{2}) - \ln(1)$$

$$= \ln(2^{1/2}) - 0 = \frac{1}{2}\ln 2$$

8. Partial Derivatives

EC All GATE Questions

1. Consider the function f(x) = $x^2 - x - 2$. The maximum value of f(x) in the closed interval [–4, 4] is [EC: GATE-2007]

(a) 18 (b) 10
(c) –2.25 (d) indeterminate

1.(a)

$$f(x) = x^2 - x - 2$$
$$\therefore f'(x) = 2x - 1$$
$$f^1(x) = 0 \Rightarrow x = \frac{1}{2} \in [-4, 4]$$
$$\text{Now} f''(x) = 2 > 0$$
$$\therefore f(x) \text{has minimum at } x = 1/2$$

It Shows that a maximum value that will be at x = 4 or x = - 4

$$\text{At} \quad x = 4,\ f(x) = 10$$
$$\therefore \text{At} \quad x = -4,\ f(x) = 18$$
$$\therefore \text{At} \quad x = -4,\ f(x) \text{ has a maximum.}$$

2. For real values of x, the minimum value of the function f(x) = exp (x) + exp (–x) is [EC: GATE-2008]

(a) 2 (b) 1
(c) 0.5 (d) 0

2. (a)

$$f(x) = e^x + e^{-x}$$

For extrema,

$$f'(x) = 0 \Rightarrow e^x - e^{-x} = 0$$
$$\Rightarrow x = 0.$$
$$f''(x) = e^x - e^{-x}$$
$$f''(x)\Big|_{x=0} = 2 > 0$$
$$\text{Heve minimum at } x = 0,\ f10) = 2.$$

3. If $e^y = x^{\frac{1}{x}}$ then y has a [EC: GATE-2010]

(a) Maximum at $x = e$ (b) minimum at $x = e$
(c) Maximum at $x = e^{-1}$ (d) minimum at $x = e^{-1}$

3. (a)

$$e^y = x\frac{1}{x}$$

Take log both side

$$y = \frac{1}{x}\log x$$

For extrema,

$$\frac{dy}{dx} = 0 \Rightarrow \frac{1}{x^2} - \frac{1}{x^2}\log x = 0$$

Now

$$\frac{d^2y}{dx^2}\Big|_{x=e} = \frac{1}{e^3} < 0$$

$\therefore$ Max at $x = e$.

ME 20 Years GATE Questions

5. Let f =y^x. What is $\frac{\partial^2 f}{\partial x \partial y}$ at x = 2, y = 1? [ME: GATE-2008]

(a) 0 (b) In 2 (c) 1 (d) 1/In 2

5(c).

$$f = y^x$$

Take log both side

$$\log f = x\log y$$

Differentiate

$$\frac{1}{f}\frac{\partial f}{\partial y} = \frac{x}{y} \Rightarrow \frac{\partial f}{\partial y} = y^x\left(\frac{x}{y}\right) = y^{x-1}.x$$

$$\Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}(y^{x-1}.x) = xy^{x-1}\ln y + y^{x-1}$$

$$\therefore \frac{\partial^2 f}{\partial x dy}\Big|_{(2,1)} = 1$$

6. If II (x,y) is a homogeneous function of degree n, then $x\frac{\partial N}{\partial x} + y\frac{\partial N}{\partial y} = nH.$

[ME: GATE-1994]

6. Euler's Theorem for homogeneous function

7. If $\phi(x)=\int_0^{x^2}\sqrt{t}\,dt$, then $\frac{d\phi}{dx}$ is [ME: GATE-1998]

(a) $2x^2$ (b) $\sqrt{x}$ (c) 0 (d) 1

.

7. (a)

$$\Phi(x) = \int_0^{x^2} \sqrt{t}\, dt = \frac{2}{3}x^3$$

$$\therefore \frac{d\Phi}{dx} = \frac{2}{3}3x^2 = 2x^2$$

8. If z = f(x,y), dz is equal to [ME: GATE-2000]

(a) $(\partial f/\partial x)dx + (\partial f/\partial y)dy$ (b) $(\partial f/\partial y)dx + (\partial f/\partial x)dy$

(c) $(\partial f/\partial x)dx - (\partial f/\partial y)dy$ (b) $(\partial f/\partial y)dx - (\partial f/\partial x)dy$

8. (a)

9. The function $f(x) = x^3 - 6x^2 + 9x + 25$ has [ME: GATE-1995]

(a) A maxima at x = 1 and a minima at x = 3
(b) A maxima at x = 3 and a minima at x = 1
(c) No maxima, but a minima at x = 3
(d) A maxima at x = 1, but not minima

9.(a)

$$f(x) = x^3 - 6x^2 + 9x + 25$$

For extrema, $f^1(x) = 0 \Rightarrow 3x^2 - 12x + 9 = 0 \Rightarrow x = 1, 3.$

For extrema, $f^1(x) = 0 \Rightarrow 3x^2 - 12x + 9 = 0 \Rightarrow x = 1, 3.$

Now, $f''(x) = 6x - 12$

$\therefore f''(x)\big|_{x=1} = -6 < 0.$ $f(x)$ has mix. value at $x = 1$

$f''(x)\big|_{x=3} = 6 > 0.$ $f(x)$ has min. value at $x = 3$

10. The minimum point of the function $f(x) = (x^2/3) - x$ is at [ME: GATE-2001]

(a) x = 1 (b) x = -1 (c) x = 0 (d) $x = \frac{1}{\sqrt{3}}$

10. (a)

For extrema, $f^1(x) = 0$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow x = \pm 1$

$$f''(x) = 2x$$
$$f''(1) = 2 > 0 \quad \text{and}\, f''(-1) = -2 < 0$$
$$\Rightarrow f \text{ has min value at } x = 1$$

11. The function f(x,y) = $2x^2 + 2xy - y^3$ has [ME: GATE-2002]
(a) Only one stationary point at (0,0)
(b) Two stationary points at (0,0) and (1/6, -1/3)
(c) Two stationary points at (0,0) and (1,-1)
(d) No stationary point

11. Ans.(b)
See theory.

12. If $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$, then dy/dx will be equal [ME: GATE-2004]
(a) $\sin\left(\frac{\theta}{2}\right)$ (b) $\cos\left(\frac{\theta}{2}\right)$ (c) $\tan\left(\frac{\theta}{2}\right)$ (d) $\cot\left(\frac{\theta}{2}\right)$

12. (c)

$$\frac{dx}{dy} = a(1+\cos\theta)\frac{dy}{d\theta} = a\sin\theta$$
$$\therefore \frac{dy}{dx} = \frac{2\sin\theta/2\,\cos\theta/2}{2\cos^2\theta/2} = \tan\theta/2$$

13. Equation of the line normal to function f(x) = $(x-8)^{2/3}+1$ at P(0,5) is [ME: GATE-2006]
(a) y = 3x -5 (b) y = 3x +5 (c) 3y = x+15 (d) 3y = x -15

13.(b)

$$m = \left.\frac{dy}{dx}\right|_{(0,5)} = -\frac{1}{3} \quad \therefore mm_1 = -1$$
$$\Rightarrow m_1 = 3, \text{ where } m_1 = \text{slope of the normal.}$$
$$\therefore \text{ Equation of normal at (0,5) is}$$
$$y - 5 = 3(x-1)$$
$$\Rightarrow y = 3x + 5$$

14. The minimum value of function y = x^2 in the interval [1, 5] is [ME: GATE-2007]
(a) 0 (b) 1 (c) 25 (d) Undefined

14. (b)

$y = x^2$ is strictly increasing function on [1,5]

$\therefore y = x^2$ has minimum value at $x = 1$ is 1.

23. The distance between the origin and the point nearest to it on the surface $z^2 = 1 + xy$ is [ME: GATE=2009]

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) -2

23. Ans(a)

CE 10 Years GATE Questions

15. The function $f(x) = 2x^3 - 3x^2 - 36x + 2$ has its maxima at **[CE: GATE – 2004]**

(a) $x = -2$ only (b) $x = 0$ only

(c) $x = 3$ only (d) both $x = -2$ and $x = 3$

15. (a)

$$f(x) = 2x^3 - 3x^2 - 36x + 2$$

$$f''(x) = 6x^2 - 6x - 36$$

For extrema, $f^1(x) = 0$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow x = 3, -2$$

$$f''(x) = 12x - 6$$

$$\left|f''(x)\right|_{x=3} = 30 > 0 \quad \Rightarrow f \text{ has minimum at } x = 3$$

$$\left|f''(x)\right|_{x=-2} = -30 < 0 \quad \Rightarrow f \text{ has maximum at } x = -2$$

16. Given a function **[CE: GATE – 2010]**

$f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$

The optimal value of f(x, y)

(a) is a minimum equal to $\frac{10}{3}$

(b) is a maximum equal to $\frac{10}{3}$

(c) is a minimum equal to $\frac{8}{3}$

(a) is a maximum equal to $\frac{8}{3}$

16. (a)

$$f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$$

$$\frac{\partial f}{\partial x} = 8x - 8. \qquad \frac{\partial f}{\partial y} = 12y - 4.$$

$\frac{\partial f}{\partial x} = 0$ gives $x = 1$ and only $\frac{\partial f}{\partial y} =$ gives $y = 1/3$

$\therefore (1, 1/3)$ is only stationary point.

Now $r = \left[\frac{\partial^2 f}{\partial x^2}\right]_{(1,1/3)} = 8 > 0$

$t = \left[\frac{\partial^2 f}{\partial y^2}\right]_{(1,1/3)} = 12 > 0$

and $s = \left[\frac{\partial^2 f}{\partial x dy}\right]_{(1,1/3)} = 0$

$\therefore \quad rt - s^2 = 96 > 6.$

$\therefore \quad (1, 1/3)$ is a pointof minima.

$\therefore \quad f(1, 1/3) = 4 \times 1^2 + 6 \times \frac{1}{3^2} - 8.1 - 4.1/3 + 8$

$= \frac{10}{3}.$

EE All GATE Questions

Q27. What should be the value of λ such that the function defined below is continuous at x = $\pi/22$?

$$f(x)\begin{cases} \dfrac{\lambda \cos x}{\frac{\pi}{2} - x} & \text{if } x \neq \pi/2 \\ 1 & \text{if } x = \pi/2 \end{cases}$$

(a) 0 (b) $2/\pi$ (c) 1 (d) $\pi/2$ **[CE-2011]**

Ans. **(c)**

Exp. By the given condition

$$\lim_{x \to \pi/2} f(x) = f(\pi/2)$$

$$\Rightarrow \lim_{x \to \pi/2} \frac{\lambda \cos x}{(\pi/2 - x)} = 1 \qquad \ldots (1)$$

$$\lim_{x \to \pi/2} \frac{\lambda \cos x}{\pi/2 - x}\left[\frac{0}{0} \text{form}\right] \qquad \ldots (2)$$

$\lim_{x \to \pi/2} \frac{-\lambda \sin x}{-1}$ [use L'Hospital Rule]

$= \lambda$

From (1), $\lambda = 1$

17. For the function $f(x) = x^2e^{-x}$, the maximum occurs when x is equal to **[EE: GATE-2005]**
(a) 2 (b) 1 (c) 0 (d) -1

17. (a)

$$f'(x) = 2xe^{-2} - x^2e^{-x}$$
$$\text{For extrema } f'(x) = 0$$
$$\Rightarrow 2xe^{-x} - x^2e^{-x} = 0$$
$$\Rightarrow x = 0, 2$$
Now
$$f''(x) = 2e^{-x} - 2xe^{-x} - 2xe^{-x} + x^2e^{-x}$$
$$= 2e^{-x} - 4xe^{-x} + x^2e^{-x}$$
$$\left[f''(x)\right]_{x=0} = 2 > 0 \quad \text{and} \left[f''(x)\right]_{x=2} = -2e^{-2} < 0$$
$$\therefore \text{ at } x = 2, f(x) \text{ has a maximum value.}$$

18. Consider function $f(x) = (x^2-4)^2$ where x is a real number. Then the function has **[EE: GATE-2008]**
(a) Only one minimum
(b) Only two minima
(c) Three minima
(d) Three maxima

18.(b)

$$f(x) = (x^2 - 4)^2$$
$$f'(x) = 2(x^2 - 4).2x = 4x(x^2 - 4).$$
$$\text{For extrema}, f'(x) = 0$$
$$\Rightarrow x = 0, -2, 2.$$
$$f''(x) = 4(x^2 - 4) + 8x^2$$
$$= 12x^2 - 16$$
$$\left[f''(x)\right]_{x=0} = -16 < 0$$
$$\left[f''(x)\right]_{x=-2} = 32 > 0$$
$$\text{and}\left[f''(x)\right]_{x=2} = 32 > 0$$
$$\therefore \text{At } x = 0, f(x) \text{ has maxima.}$$
$$\text{At } x = -2, 2, f(x) \text{ has minima.}$$

19. A cubic polynomial with real coefficients [EE: GATE-2009]
(a) Can possibly no extrema and no zero crossings
(b) May have up to three extrema and up to 2 zero crossings
(c) Cannot have more than two extrema and more than three zero crossings
(d) Will always have an equal number of extrema and zero crossings

19. Ans. (c)

$$F(x) = Ax^3 + Bx^2 + Cx + D$$

$\therefore \quad F(x) = 3Ax^2 + 2Bx + C$

First max: $F'(x) = 6Ax + 2B$

Second max: $F''(x) = 6A$

$F'''(x) = 0$

So maximum two extrema and three zero crossing

IE All GATE Questions

20. If f = $a_0x^n + a_1x^{n-1}y + ... + a_{n-1}xy^{n-1} + a_ny^n$, where a_i (i = 0 to n) are constants, then $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}$ is **[IE: GATE-2005]**

(a) $\dfrac{f}{n}$ (b) $\dfrac{n}{f}$

(c) nf (d) $n\sqrt{f}$

20. (e)

$x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = xf$ – Euler's theorem for homogeneous function

21. Given y = x^2 + 2x + 10, the value of $\left.\dfrac{dy}{dx}\right|_{x=1}$ is equal to **[IE: GATE-2008]**

(a) 0 (b) 4

(c) 12 (d) 13

21. (b)

Given, $y = x^2 + 2x + 10$

$\therefore \quad \dfrac{dy}{dx} = 2x + 2$

$\Rightarrow \quad \left.\dfrac{dy}{dx}\right|_{x=1} = 4$

22. For real x, the maximum value of $\dfrac{e^{\sin x}}{e^{\cos x}}$ is **[IE: GATE-2007]**

(a) 1 (b) e

(c) $e^{\sqrt{2}}$ (d) ∞

22(c).

$y = e^{\sin x - \cos x}$

Take log both side

$$\log y = \cos x - \sin x$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \cos x + \sin x$$

$$\Rightarrow \frac{dy}{dx} = y(\cos x + \sin x) = e^{\sin x - \cos x}(\cos x + \sin x)$$

For extrema $\frac{dy}{dx} = 0$ gives.

$$\tan x = -1 = \tan\left(\pi - \pi/4\right) = \tan\frac{3\pi}{4}$$

$$\Rightarrow x = \frac{3\pi}{4}$$

$$\frac{d^2y}{dx^2} = e^{(\sin x - \cos x)}.(\cos x + \sin x)^2 + e^{(\sin x - \cos x)}(-\sin x + \cos x)$$

$$\left[\frac{d2y}{dx2}\right]_{x=\frac{3\pi}{4}} = -\sqrt{2}e^{\sqrt{2}} < 0.$$

so ,y has max at $x = \frac{3\pi}{4}$

At that point , $y = e^{\sqrt{2}}$

23. Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 is [IE: GATE-2008]

(a) 1 (b) 3

(c) 4 (d) 9

23. (b)

$y' = 0$ gives $2x - 6 = 0$

$\Rightarrow x = 3$

$y''(x) = 2$

CS All GATE Questions

24. A point on a curve is said to be an extremum if it is a local minimum or a local maximum. The number of distinct exterma for the curve $3x^4 - 16x^3 - 24x^2 + 37$ is [CS: GATE-2008]

(a) 0 (b) 1

(c) 2 (d) 3

24.(d)

Let $f(x) = 3x^4 - 16x^3 - 24x^2 + 37$

For extrema, $f'(x) = 0$ gives

$12x^3 - 48x^2 - 48x = 0$

$\Rightarrow x(x^2 - 4x - 4) = 0$

$\Rightarrow x = 0, 2 \pm 2\sqrt{2}$

$\therefore f(x)$ has three extrema points.

9. Gradient

EC All GATE Questions

1. $\nabla \times \nabla \times P$, where P is a vector, is equal to [EC: GATE-2006]

(a) $P \times \nabla \times P - \nabla^2 P$
(b) $\nabla^2 P + \nabla(\nabla \bullet P)$
(c) $\nabla^2 P + \nabla \times P$
(d) $\nabla(\nabla \bullet P) - \nabla^2 P$

1. (d) (formula)

2. $\iint (\nabla \times P) \bullet ds$, where P is a vector, is equal to [EC: GATE-2006]

(a) $\oint P \bullet dl$
(b) $\oint \nabla \times \nabla \times P \bullet dl$
(c) $\oint \nabla \times P \bullet dl$
(d) $\iiint \nabla \bullet P\, dv$

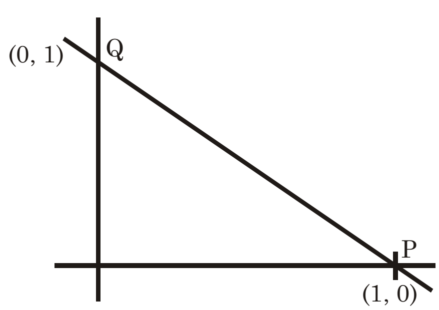
2. (a) Hints (Stokes Theorem).

1. Consider points P and Q in the x-plane, with P= (1, 0) and Q = (0, 1). The line integral $2\int_P^Q (xdx + ydy)$ along the semicircle with the line segment PQ as its diameter [EC: GATE-2008]

(a) Is –1
(b) is 0
(c) Is 1
d) depends on the direction (clockwise or anti-clockwise) of the semicircle

1.Ans. (b)

The straight line equation is x + y = 1



Then,
$$I = 2\int_0^1 (1-y)\,.\,(-dy) + 2\int_0^1 y\,dy$$
$$= 2\left[\frac{y^2}{2} - y\int_0^1\right] + 2\left[\frac{y}{2}\int_0^1\right] = 0$$

5. The value of the integral of the function $g(x, y) = 4x^3 + 10y^4$ along the straight line segment from the point (0, 0) to the point (1, 2) in the x-y plane is [EC: GATE-2008]

(a) 33 (b) 35
(c) 40 (d) 56

5(a).

The equation of the line passing through (0,0) and (1,2)
is y = 2x

Given $y(x,y) = 4x^3 + 10y^4 = 4x^3 + 10(2x)^4 = 4x^3 + 160xy$

$$\therefore I = \int_0^1 \left(4x^3 + 160x^4\right)dx = 33.$$

ME 20 Years GATE Questions

3. The magnitude of the gradient of the function $f = xyz^3$ at (1,0,2) is [ME:GATE-1998]

(a) 0 (b) 3 (c) 8 (d) ∞

3. (c)

$$\overline{\nabla}.f = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).xyz^3$$

$$= yz^3\hat{i} + xz^3\hat{j} + 3xyz^2\hat{k}$$

$$\overline{\nabla}.f\Big|_{(1,0,2)} = 8\hat{j}$$

$$\therefore \left|\overline{\nabla}.f\right|_{(1,0,2)} = \left|8\hat{j}\right| = 8.$$

4. If $\vec{V}$ is a differentiable vector function and f is a sufficient differentiable scalar function, then curl $((f\vec{V})$ is equal to [ME: GATE-1995]

(a) (grad f)×($\vec{v}$)+(f curl $\vec{v}$) (b) $\vec{O}$ (c) f curl ($\vec{v}$) (d) (grade f)×($\vec{v}$)

4.(a)

$$\overline{\nabla} \times \left(f\overline{v}\right) = \left(\overline{\nabla}f\right) \times \overline{v} + f(\nabla \times \overline{V})$$

$$= \left(\text{gradf}\right) \times \overline{v} + f(\text{curl}\,\overline{v})$$

5. The expression curl (grad f), where f is a scalar function, is [ME: GATE-1996]

(a) Equal to $\nabla^2 f$ (b) Equal to div (grad f)
(c) A scalar of zero magnitude (d) A vector of zero magnitude

5. (d)

$$\overline{\nabla}\times\left(\overline{\nabla}.f\right)=\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z}\end{bmatrix}$$

$$=\left(\frac{\partial f}{\partial y\partial z}-\frac{\partial^2 f}{\partial y\partial z}\right)\hat{i}-\hat{j}\left(\frac{\partial^2 f}{\partial x\partial z}-\frac{\partial^2 f}{\partial x\partial z}\right)\hat{j}+\hat{k}\left(\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial^2 f}{\partial x\partial y}\right)$$

$$=\overline{0}$$

6. The temperature field in a body varies according to the equation T(x,y) = x^3+4xy. The direction of fastest variation in temperature at the point (1,0) is given by

(a) $3\hat{i} + 8\hat{j}$ (b) $\hat{i}$ (c) $0.6\hat{i} + 0.8\hat{j}$ (d) $0.5\hat{i} + 0.866\hat{j}$ [ME: GATE-1997]

6. Ans. (c)

Given $T=x^3+4xy$

$$\frac{\partial T}{\partial x}=3x^2+4y$$

$$\left(\frac{\partial T}{\partial x}\right)_{(1,0)}=3$$

$$\frac{\partial T}{\partial y}=4x$$

$$\left(\frac{\partial T}{\partial y}\right)_{(1,0)}=4$$

$\therefore$ Direction of fastest variation in temperature at (1,0) is given by

$(3\hat{i}+4\hat{j})$ or $0.6\hat{i}+0.8\hat{j}$

7. If the velocity vector in a two – dimensional flow field is given by $\vec{v}=2xy\vec{i}+(2y^2-x^2)\vec{j},$ the vorticity vector, curl $\vec{v}$ will be [ME: GATE-1999]

(a) $2y^2\vec{j}$ (b) $6y\vec{j}$ (c) zero (d) $-4x\vec{k}$

7. (d)

$$\overline{\nabla}\times\overline{V}=\begin{pmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0\\ 2xy & 2y^2-x^2 & 0\end{pmatrix}=\left(-2x-4x\right)\hat{k}$$

$$=-4x\hat{k}$$

8. The divergence of vector $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ is [ME: GATE-2001]

(a) $\vec{i}+\vec{j}+\vec{k}$ (b) 3 (c) 0 (d) 1

8. (b)

$$\overline{\nabla}\times\overline{F}=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\hat{k}\right).\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$$
$$=1+1+1=3$$

9. The cector field $\vec{F}=x\hat{i}-y\hat{j}$ (where $\hat{i}$ and $\hat{j}$ are unit vector) is [ME: GATE-2003]

(a) Divergence free, but not irrotational is
(b) Irrotational, but not divergence free
(c) Divergence free and irrotational
(d) Neither divergence free nor irrotational

9. (c).

$$\overline{\nabla}\times\overline{F}=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\bar{j}+\frac{\partial}{\partial z}\hat{k}\right).\left(x\hat{i}-y\hat{j}\right)$$
$$=1-1=0 \qquad \Rightarrow \overline{F} \text{ is devergence free}$$
$$\overline{\nabla}\times\overline{F}=\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & 0 \end{vmatrix}=\vec{0}$$
$\Rightarrow$ F is irrotational vector.

10. The divergence of the vector $(x-y)\hat{i}+(y-x)\hat{j}+(x+y+z)\hat{k}$ is
(a) 0 (b) 1 (c) 2 (d) 3

10.(d)

11. The divergence of the vector field $3xz\hat{i}+2xy\hat{j}-yz^2\hat{k}$ at a point (1 ,1,1) is equal to
(a) 7 (b) 4 (c) 3 (d) 0 [ME: GATE-2009]

11. (c)

$$\overline{\nabla}\times\overline{F}=3z+2x-2yz$$
$$\left.\overline{\nabla}\times\overline{F}\right]_{(1,1,1)} = 3.1+2.1-2.1.1$$
$$=3.$$

12. Velocity vector of a flow field is given as $\vec{V}=2xy\hat{i}-x^2z\hat{j}$. The vorticity vector at (1, 1, 1) is [ME: GATE-2010]

(a) $4\hat{i}-\hat{j}$ (b) $4\hat{i}-\hat{k}$ (c) $\hat{i}-4\hat{j}$ (d) $\hat{i}-4\hat{k}$

12. (d)

curl $\overline{V}$ is called vorticity vector.

$$\text{Now, curl } \overline{V} = \overline{\nabla}\times\overline{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & -2x^2z & 0 \end{vmatrix} = x^2\hat{i}+0+\hat{k}(-2xz-2x)$$

0

$$\therefore \left[\text{curl}\overline{V}\right]_{(1,1,1)} = \hat{i}+\hat{k}(-2-2) = \hat{i}-4\hat{k}$$

13. Among the following, the pair of vectors orthogonal to each other is [ME: GATE-1995]
(a) [3,4,7], [3,4,7] (b) [0,0,0], [1,1,0] (c) [1,0,2], [0,5,0] (d) [1,1,1], [-1,-1,-1]

13. (c)

Let $\vec{a},\vec{b}$ be two vector st $-\vec{a}.\vec{b}=0.$ Then we say that they are orthogonal.
Choice (c) is correct.

14. The angle between two unit-magnitude co-planar vectors P (0.866, 0.500, 0) and Q (0.259, 0.966, 0) will be [ME: GATE-2004]
(a) 0^0 (b) 30^0 (c) 45^0 (d) 60^0

14. (c)

$$\cos\theta = \frac{\overline{P}.\overline{Q}}{|\overline{P}||\overline{Q}|} = \frac{(0.866\times0.259)+(0.5\times0.966)+0}{\sqrt{(.866)^2+(.5)^2+0^2}\sqrt{(.259)^2+(.966)^2+0^2}}$$

= 0.707.

$\Rightarrow \theta = 45^o$

15.The area of a triangle formed by the tips of vectors $\overline{a},\ \overline{b},$ and $\overline{c}$ is [ME: GATE-2007]

(a) $\frac{1}{2}(\overline{a}-\overline{b}).(\overline{a}-\overline{c})$ (b) $\frac{1}{2}\left|(\overline{a}-\overline{b})\times(\overline{a}-\overline{c})\right|$

(c) $\frac{1}{2}\left|\overline{a}\times\overline{b}\times\overline{c})\right|$ (d) $\frac{1}{2}(\overline{a}\times\overline{b}).\overline{c}$

15.(b)

16. In a flow field in x, y-plane, the variation of velocity with time t is given by v $v=(x^2+yt)\vec{i}$
$\vec{v}=(x^2+y^2)\vec{i}$ [ME: GATE-1999]

The acceleration of the particle in this field, occupying point (1,1) at time t = 1 will be

(a) $\vec{i}$ (b) $2\vec{i}$ (c) $3\vec{i}$ (d) $5\vec{i}$

16. Ans.(d)

$\vec{v} = (x^2 + yt)i$

$v = x^2 + yt,$

at t=1, $v_{(1,1)} = 1 + 1 \times 1 = 2$

$\frac{\partial u}{\partial x} = 2x = 2 \times 1 = 2,$

$\frac{\partial u}{\partial t} = y = 1, \quad \frac{\partial u}{\partial y} = t = 1$

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \quad = (2\times2+0+0+1)i = 5i$

17. The maximum value of the directional derivative of the function $\phi = 2x^2 + 3y^2 + 5z^2$ at a point (1,1,-1) is [ME:GATE-2000]

(a) 10 (b) -4 (c) $\sqrt{152}$ (d) 152

17. (c)

$\vec{\nabla}\Phi = 4x\hat{i} + 6y\hat{j} + 10z\hat{k}$

$\vec{\nabla}\Phi\Big]_{(1,1,-1)} = 4\hat{i} + 6\hat{j} - 10\hat{k}$

$\therefore \left|\vec{\nabla}\Phi\right|_{(1,1-1)} = \sqrt{4^2 + 6^2 + (-10)^2} = \sqrt{152}$

18. The directional derivative of the scalar function f(x, y, z) = $x^2 + 2y^2 + z$ at the point P = (1,1, 2) in the direction of the vector $\vec{a} = 3\hat{i} - 4\hat{j}$ is [ME: GATE-2008]

(a) – 4 (b) -2 (c) -1 (d) 1

18.(b)

Required directional derivatives at P(1,1,-1)

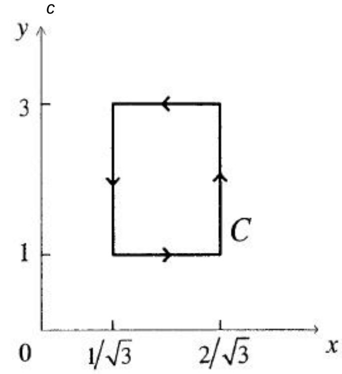
$\therefore \left|\vec{\nabla}\Phi.\hat{n}\right|_{(1,1,2)}$, where $\hat{n}$ is the unit vector in the direction of $\vec{a}$

$= \left|\left(2\hat{i} + 4\hat{j} + \hat{k}\right).\frac{1}{5}\left(3\hat{i} - 4\hat{j}\right)\right| = \frac{\vec{a}}{|\vec{a}|} = \frac{3\vec{i} - 4\vec{j}}{\sqrt{3^2 + (-4)^2}} = \frac{1}{5}\left(3\vec{i} - 4\vec{j}\right).$

$= \frac{1}{5}(6 - 16)$

$= -2.$

19. If $\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$ then $\oint_c \vec{A} \bullet d\vec{l}$ over the path shown in the figure is



(a) 0 (b) $\frac{2}{\sqrt{3}}$ (c) 1 (d) $2\sqrt{3}$

19. (a)

$\oint \overline{A}.d\overline{l} = 0$ is the curve is closed.

20. The line integral $\int \vec{V}.d\vec{r}$ of the vector $\vec{V}.(\vec{r}) = 2xyz\hat{i} + x^2z\hat{j} + x^2y\hat{k}$ from the origin to the point P (1, 1, 1) [ME: GATE-2005]
(a) Is 1 (b) Is zero
(c) Is -1 (d) Cannot be determined without specifying the path

20.Ans(a)

$$\int \vec{V}.d\vec{r} = \int [2xyz\hat{i} + x^2z\hat{j} + x^2y\hat{k}].(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int (2xyzdx + x^2zdy + x^2ydz)$$

Along the line joining $(0,0,0)$ *to the po*int $(1,1,1)$ *is given*
by the parametric form by
$x = t, y = t, z = t$

$$Then \int \vec{V}.d\vec{r} = \int_0^1 (2t.t.tdt + t^2.tdt + t^2.tdt)$$

$$= \int_0^1 4t^3dt = 4.\frac{1}{4} = 1$$

24. The area enclosed between the curves $y^2 = 4x$ and $X^2 = 4y$ is [ME: GATE-2009]
(a) $\frac{16}{3}$ (b) 8 (c) $\frac{32}{3}$ (d) 16

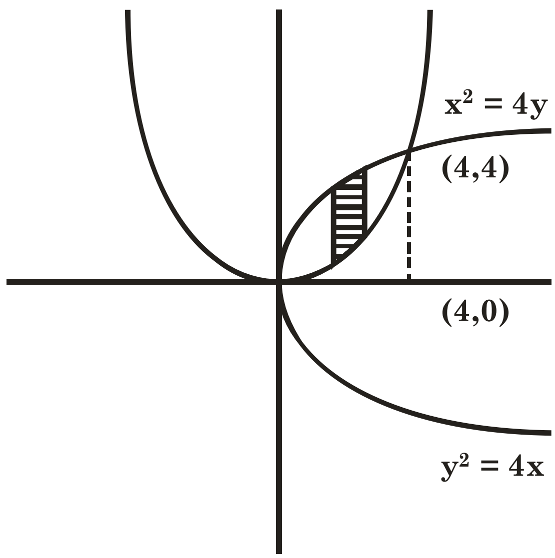
24. (a)

Let $y^2 = 4x$ be curve (i) $= y_1$(say)

$x^2 = 4y$ be curve (ii) $= y_2$(say)

$\therefore$ Area $= \int_0^4 (y_1 - y_2)\,dx$

$= \int_0^4 \left(\sqrt{4x} - \dfrac{x^2}{4}\right)dx = \dfrac{16}{3}$



25. Stokes theorem connects [ME: GATE-2005]
(a) A line integral and a surface integral
(b) A surface integral and a volume integral
(c) A line integral and a volume integral
(d) Gradient of a function and its surface integral

25. (a)

27. The Gauss divergence theorem relates certain [ME: GATE-2001]
(a) Surface integrals to volume integrals
(b) Surface integrals to line integrals
(c) Vector quantities to other vector quantities
(d) Line integrals to volume integrals

27. (a)

4. For the function $\phi = ax^2y - y^3$ to represent the velocity potential of an ideal fluid $\nabla^2\phi$ should be equal to zero. In that case, the value of 'a' has to be: [ME: GATE-1999]
(a) -1 (b) 1 (c) -3 (d) 3

4. (d)

$$\Phi = ax^2y - y^3$$
$$\frac{\partial^2\Phi}{\partial x^2} = 2ay \text{ and } \frac{\partial^2\Phi}{\partial y^2} = -6y$$
$$\therefore \nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 2ay - 6y$$
$$\therefore \nabla^2\Phi = 0 \Rightarrow a = 3.$$

CE 10 Years GATE Questions

Q29. If a and b are two arbitrary vectors with magnitudes a and b, respectively $\left|\vec{\mathbf{a}} \times \vec{\mathbf{b}}\right|^2$ will be equal to
(a) $\mathbf{a^2b^2} - \left(\vec{\mathbf{a}}.\vec{\mathbf{b}}\right)^2$ (b) $\mathbf{ab} - \vec{\mathbf{a}}.\vec{\mathbf{b}}$ (c) $\mathbf{a^2b^2} + \left(\vec{\mathbf{a}}.\vec{\mathbf{b}}\right)^2$ (d) $\mathbf{ab} + \vec{\mathbf{a}}.\vec{\mathbf{b}}$ **[CE-2011]**

Ans. (a)

Exp. $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \left|\vec{\mathbf{a}}\right|\left|\vec{\mathbf{b}}\right| \mathbf{\sin\theta}\, \hat{\mathbf{n}}$

$= \mathbf{ab\ \sin\theta}\, \hat{\mathbf{n}}$ [Taking $\left|\vec{\mathbf{p}}\right| = \mathbf{P}$]

$\therefore \left|\vec{\mathbf{a}} \times \vec{\mathbf{b}}\right|^2 = \left(\vec{\mathbf{a}} \times \vec{\mathbf{b}}\right)^2$

$= \mathbf{a^2b^2\ \sin\theta} \left|\hat{\mathbf{n}}\right|^2$

$= \mathbf{a^2b^2} \left(\mathbf{1 - \cos^2\theta}\right)\left[\because \left|\hat{\mathbf{n}}\right| = \mathbf{1}\right]$

$= \mathbf{a^2b^2 - a^2b^2\ \cos^2\theta}$

$= \mathbf{a^2b^2} - \left(\mathbf{ab\cos\theta}\right)^2$

$= \mathbf{a^2b^2} - \left(\left|\vec{\mathbf{a}}\right|\left|\vec{\mathbf{b}}\right|\mathbf{\cos\theta}\right)^2$

$= \mathbf{a^2b^2} - \left(\vec{\mathbf{a}}.\vec{\mathbf{b}}\right)^2$

28. For a scalar function f(x, y, z) = $x^2 + 3y^2 + 2z^2$, the gradient at the point P (1, 2, –1) is
(a) $2\vec{i} + 6\vec{j} + 4\vec{k}$ (b) $2\vec{i} + 12\vec{j} - 4\vec{k}$ **[CE: GATE – 2009]**
(c) $2\vec{i} + 12\vec{j} + 4\vec{k}$ (d) $\sqrt{56}$

28. (b)

$$\vec{\nabla}f\Big|_{(1,2,-1)} = 2\hat{i} + 12\hat{j} - 4\hat{k}$$

29. The inner (dot) product of two vectors $\vec{P}$ and $\vec{Q}$ is zero. The angle (degrees) between the two vectors is **[CE: GATE – 2008]**

(a) 0 (b) 30
(c) 90 (d) 120

29. (c)

30. If P, Q and R are three points having coordinates (3, –2, –1), (1, 3, 4), (2, 1, –2) in XYZ space, then the distance from point P to plane OQR (O being the origin of the coordinate system) is given by **[CE: GATE – 2003]**

(a) 3 (b) 5
(c) 7 (d) 9

30. (a)

The equation of the plane OQR is (O being origin).

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 0 & y - 0 & z - 0 \\ 1 & 3 & 4 \\ 2 & 1 & -2 \end{vmatrix} = 0$$

$$\Rightarrow 2x - 2y + z = 0 \qquad -(i)$$

Now 1 distance from P to plane (1) is

$$\left| \frac{2.3 - 2.(-2) + (-1)}{\sqrt{2^2 + (-2)^2 + (1)^2}} \right| = 3.$$

31. For a scalar function f(x, y, z) = $x^2 + 3y^2 + 2z^2$, the directional derivative at the point P(1, 2, –1) in the direction of a vector $\vec{i} - \vec{j} + 2\vec{k}$ is **[CE: GATE – 2009]**

(a) –18 (b) $-3\sqrt{6}$
(c) $3\sqrt{6}$ (d) 18

31. (b)

Same as Q.18.

32. Value of the integral $\oint_c (xydy - y^2dx)$, where, c is the square cut from the first quadrant by the lines x = 1 and y = 1 will be (Use Green's theorem to change the line integral into double integral) **[CE: GATE – 2005]**

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) $\frac{5}{3}$

32. (c) Green's theorem say,

$$\oint(Mdx + Ndy) = \iint_{xy}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy$$

Here $\oint(xydy - y^2dx)$

$$= \oint(-y^2dx + xydy) = \iint_{xy}\left(\frac{\partial(xy)}{\partial x} - \frac{\partial(-y^2)}{\partial y}\right)dxdy$$

$$= \iint_{x=0\ y=0} (y + 2y)dxdy$$

$$= \int_0^1 dx\int_0^1 3ydy$$

$$= \frac{3}{2}$$

EE All GATE Questions

34. For the scalar field u=$\frac{x^2}{2} + \frac{y^2}{3}$, magnitude of the gradient at the point(1,3) is **[EE: GATE-2005]**

(a) $\sqrt{\frac{13}{9}}$ (b) $\sqrt{\frac{9}{2}}$

(c) $\sqrt{5}$ (d) $-\frac{9}{2}$

34. (c)

$$\overline{\nabla}u = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}\right)u$$

$$= x\hat{i} + \frac{2}{3}y\hat{j} \qquad \therefore \overline{\nabla}u\Big|_{(1,3)} = \hat{i} + 2\hat{j}$$

$$\therefore \left|\overline{\nabla}u\right|_{(1,3)} = \sqrt{1^2 + 2^2}$$
$$= \sqrt{5}$$

35. Let x and y be two vectors in a 3 dimensional space and <x, y> denote their dot product. Then the determinant det $\begin{bmatrix} <x,x> & <x,y> \\ <y,x> & <y,y> \end{bmatrix}$ **[EE: GATE-2007]**
(a) is zero when x and y are linearly independent
(b) is positive when x and y are linearly independent
(c) is non-zero for all non-zero x and y
(d) is zero only when either x or y is zero

35. Ans (d)

$$\det\begin{bmatrix} x.x & x.y \\ y.x & y.y \end{bmatrix} = \begin{vmatrix} x.x & x.y \\ y.x & y.y \end{vmatrix}$$

is zero only when either x or y is zero.

46. A sphere of unit radius is centered at the origin. The unit normal at a point (x, y, z) on the surface of the sphere is the vector **[IE: GATE-2009]**

(a) (x, y, z)
(b) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$
(b) $\left(\frac{x}{\sqrt{3}}, \frac{y}{\sqrt{3}}, \frac{z}{\sqrt{3}}\right)$
(d) $\left(\frac{x}{\sqrt{2}}, \frac{y}{\sqrt{2}}, \frac{z}{\sqrt{2}}\right)$

46. (b)

47. If a vector $\vec{R}(t)$ has a constant magnitude, then **[IE: GATE-2005]**

(a) $\vec{R} \bullet \frac{d\vec{R}}{dt} = 0$
(b) $\vec{R} \times \frac{d\vec{R}}{dt} = 0$
(c) $\vec{R} \bullet \vec{R} - \frac{d\vec{R}}{dt}$
(d) $\vec{R} \times \vec{R} = \frac{d\vec{R}}{dt}$

47. (a)

Let $\vec{R}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

$|\vec{R}(t)| = k$ (constant)

i.e., $x^2(t) + y^2(t) + z^2(t) = \text{constant}.$

On analysing the given (a) option, we find that $\vec{R}(t).\frac{d\vec{R}(t)}{dt}$ will give constant magnitude, so first differentiation of the integration will be zero.

15. $F(x,y) = (x^2 + xy)a_x + (y^2 + xy)a_y$. It's line integral over the straight line from (x,y) = (0,2) to (x, y) = (2,0) evaluates to **[EE: GATE-2009]**

(a) – 8 (b) 4 (c) 8 (d) 0

15. (d)

The equation of the line passing through (0,2) and (2,0) is x + y = 2

$$\therefore \int F(x,y)dxdy = \int_0^2 (x^2 + xy)dx + \int_2^0 [y^2 + y(2-y)]dy$$

$$\therefore \int_0^2 [x^2 + x(2-x)]dx + \int_2^0 [y^2 + y(2-y)]dy$$

= 0.

10. Multiple Integrals

EC All GATE Questions

ME 20 Years GATE Questions

2. A triangle ABC consists of vertex points A (0,0) B(1,0) and C(0,1). The value of the integral $\iint 2x\ dxdy$ over the triangle is

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{8}$ (d) $\frac{1}{9}$ [ME: GATE-1997]

2. (b)

The equation of the line AB is

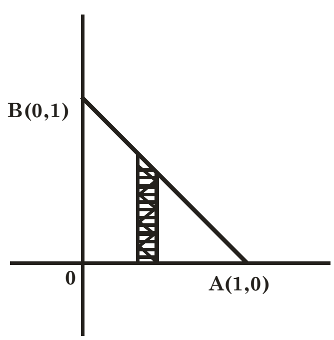
$$y - 0 = \frac{1-0}{0-1}(x-1).$$

$$\Rightarrow y + x = 1$$

$$\therefore \iint 2x dx dy = 2\int_{x=0}^{1} \left\{\int_{y=0}^{1-x} x dy\right\} dx$$

$$= 2\int_{x=0}^{1} x.(1-x)dx = 2\int_{0}^{1}\left(x - x^2\right)dx$$

$$= 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3}$$



3. $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y)\,dx\,dy$ is [ME: GATE-2000]

(a) 0 (b) π (c) $\pi/2$ (d) 2

3. (d)

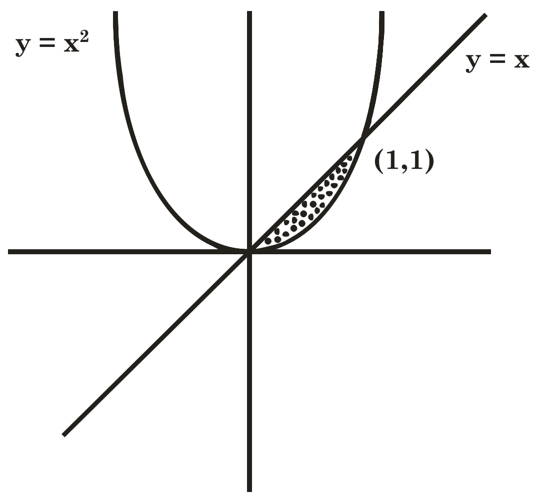
$$\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y)dxdy$$
$$= \int_0^{\pi/2} \sin x dx . \int_0^{\pi/2} \cos y dy + \int_0^{\pi/2} \cos x dx \int_0^{\pi/2} \sin y dy$$
$$= \left[-\cos x\right]_0^{\pi/2} \left[\sin y\right]_0^{\pi/2} + \left[\sin x\right]_0^{\pi/2} \left[-\cos y\right]_0^{\pi/2}$$
$$= 1.1 + 1.1 = 2$$

4. The area enclosed between the parabala $y = x^2$ and the straight line $y = x$ is [ME: GATE-2003]

(a) 1/8 (b) 1/6 (c) 1/3 (d) ½

4. (b)

$$\therefore \text{Area} = \left|\int_0^1 \left(x^2 - x\right) dx\right|$$
$$= \left|\frac{1}{3} - \frac{1}{2}\right| = \frac{1}{6} \text{units.}$$



5. The volume of an object expressed in spherical co-ordinates is given by

$V = \int_0^{2\pi} \int_0^{\pi/3} \int_0^1 r^2 \sin\phi \, dr \, d\phi \, d\theta$ The value of the integral is

[ME: GATE-2004]

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{4}$ **5. (a)**

$$V = \int_0^{2\pi} \int_0^{\pi/3} \int_0^1 r^2 \sin\phi dr d\phi . d\theta.$$

$$=\int_0^1 r^2 d\theta \int_0^{2\pi} d\theta \int_0^{\pi/3} \sin\phi d\phi$$
$$=\frac{1}{3}.2\pi\left[-\cos\theta\right]_0^{\pi/3}$$
$$=\frac{1}{3}.2\pi.\tfrac{1}{2}$$
$$=\pi/3.$$

5. Ans. (a)

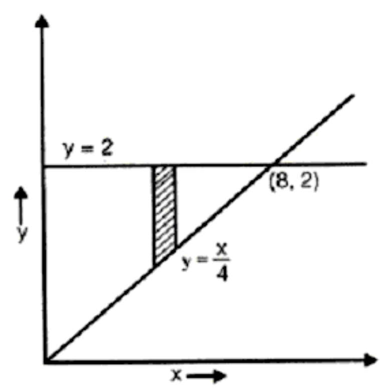
$$V=\int_0^{2\pi}\int_0^{\pi/3}\int_0^1 r^2\sin\phi.dr.d\phi.d\theta, \quad =\int_0^{2\pi}\int_0^{\pi/3}\int_2^1 \sin\phi.d\phi.d\theta,$$
$$=\frac{1}{2}\int_0^{2\pi}[1-\cos\phi]_0^{\pi/3}d\theta, \quad =\frac{1}{3}\int_0^{2\pi}\frac{1}{2}d\theta=\frac{1}{6}\times 2\pi=\frac{\pi}{3}$$

6. Changing the order of the integration in the double integral [ME: GATE-2005]
$I=\int_0^8\int_{x/4}^2 f(x,y)dydx$ leads to $I=\int_r^s\int_{xp}^q f(x,y)dxdy$. What is q?
(a) 4 y (b) 16 y^2 (c) x (d) 8

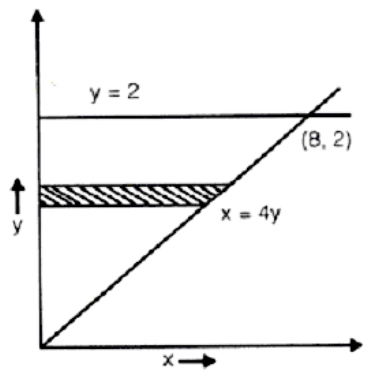
6. Ans. (a)

When $I=\int_0^8\int_{x/4}^2 f(x.y)dx\,dy$

i.e.



Now,



Figure

$$I=\int_0^2\int_0^{4Y} f(x.y)dx\, dy$$

7. By a change of variable x (u, y) = uv, y (u, v) = v/u is double integral, the integrand f(x, y) changes to f(uv, v/u) ϕ (u,v). Then, ϕ (u, v) is [ME: GATE-2005]

(a) 2 u/v (b) 2 uv (c) v^2 (d) 1

7. Ans. (a)

$$\frac{\partial x}{\partial u}=v$$

$$\frac{\partial x}{\partial v}=u$$

and $\dfrac{\partial y}{\partial u}=-\dfrac{v}{u^2}$ $\qquad \dfrac{\partial y}{\partial v}=\dfrac{1}{u}$

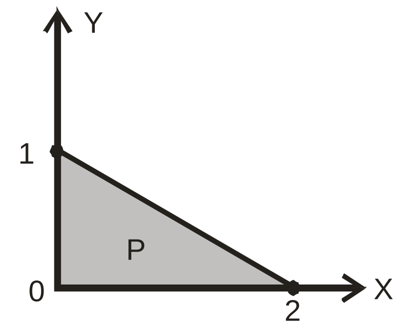
and $\phi(u,v)=\begin{vmatrix}\dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v}\\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v}\end{vmatrix} = \begin{vmatrix} v & u\\ -\dfrac{v}{u^2} & \dfrac{1}{u}\end{vmatrix} = \dfrac{v}{u}+\dfrac{v}{u}=\dfrac{2v}{u}$

8. The right circular cone of largest volume that can be enclosed by a sphere of 1 m radius has a height of [ME: GATE-2005]

(a) 1/3 m (b) 2/3 m (c) $\dfrac{2\sqrt{2}}{3}$ m (d) 4/3 m

8. Ans. (c)

9. Consider the shaded triangular region P shown in the figure. What is $\iint_P xydxdy$?



Figure

(a) $\frac{1}{6}$ (b) $\frac{2}{9}$ (c) $\frac{7}{16}$ (d) 1 [ME: GATE-2008]

9. (a)

The equation of the line AB is

$$\frac{x}{2}+\frac{y}{1}=1$$

$$\Rightarrow x+2y=2 \quad \infty$$

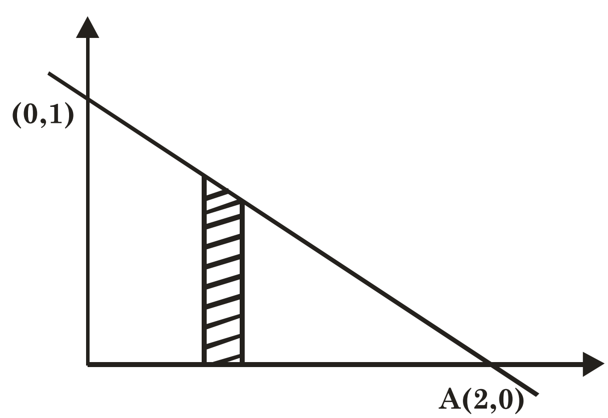
$$\therefore \text{Area}=\int_{x=0}^{2}\left\{\int_{y=0}^{\frac{2-x}{2}} xydy\right\}dx$$

$$\int_{x=0}^{2} x\left[\frac{y^2}{2}\right]_0^{\frac{2-x}{2}} dx=\frac{1}{8}\int_0^2 x\left(4-4x+x^2\right)dx$$

$$=\frac{1}{8}\left[\frac{x^4}{4}-\frac{4}{3}x^3+2x^2\right]_0^2$$

$$=\frac{1}{8}\left[4-\frac{4}{3}.8+8\right]$$

$$=\frac{1}{6}$$



11. **the parabolic arc $y=\sqrt{x}, 1\le x\le 2$ is revolved around the x-axis. The volume of**

[ME: GATE-2010]

(a) $\pi/4$ (b) $\pi/2$ (c) $3\pi/4$ (d) $3\pi/2$

11. Ans. (d)

Differential volume

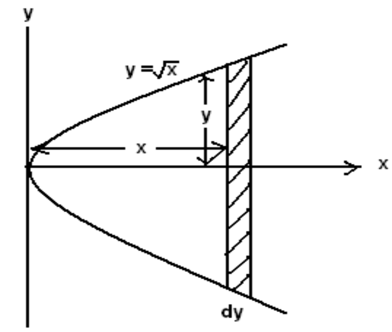
$dv = \pi y^2 dy$

Volume from x = 1 to x = 2

$v = {}^{2}\int \pi y^2 dy$

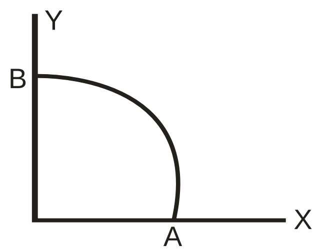
$y^1 = \sqrt{x}, \quad y^2 = x$

$v = \pi \int_1^2 x dx = \pi \left(\frac{x^2}{2} \right)_1^2 = \frac{3\pi}{2}$



22. A path AB in the form of one quarter of a circle of unit radius is shown in the figure. Integration of $(x + y)^2$ on path AB traversed in a counterclockwise sense is

[ME: GATE-2009]



Figure

(a) $\frac{\pi}{2}-1$ (b) $\frac{\pi}{2}+1$ (c) $\frac{\pi}{2}$ (d) 1

22. (b)

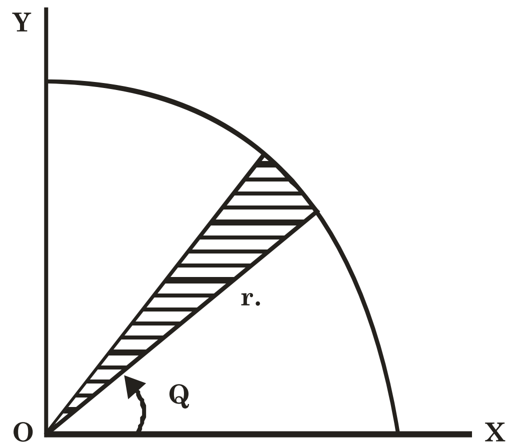
$\iint (x+y)^2 \, dxdy$

Path AB

$= \int_0^{\frac{\pi}{2}} (r\cos\theta + r\sin\theta)^2 . rd\theta \quad$,here $r = 1$

$= \int_0^{\pi/2} (1 + \sin 2\theta) d\theta$

$= \pi/2 + 1.$



7. Using definite integrals find the area of the region bounded by the curves

$y = x^2 + 2$ [ME: GATE-1995]

$y = x$

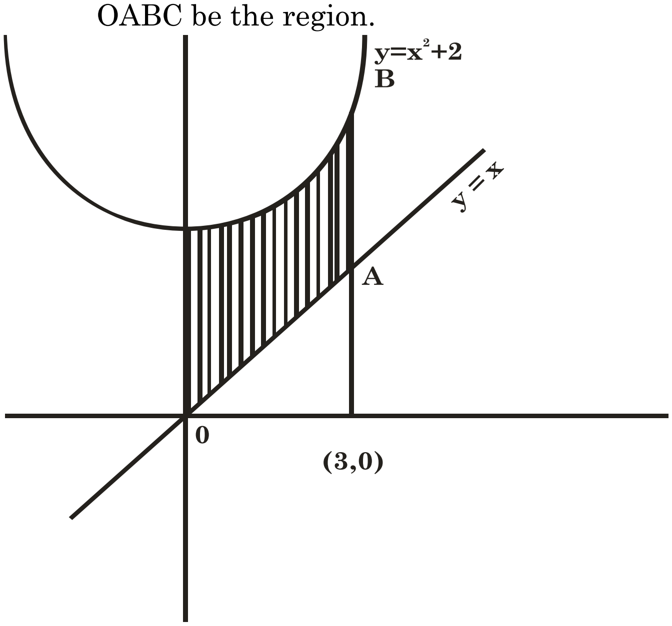
$x = 0$

and $x = 3$

Also sketch the region bounded by these curves.

7.

OABC be the region.



CE 10 Years GATE Questions

12. What is the area common to the circles r = a and r = 2a cos θ?

(a) $0.524\ a^2$ (b) $0.614\ a^2$ **[CE: GATE – 2006]**

(c) $1.047\ a^2$ (d) $1.228\ a^2$

12. Ans. (d)

Area common to circles r = a

And $r = 2a \cos \theta$ is $1.228\ a^2$

13. The value of $\int_0^3 \int_0^x (6 - x - y)\, dx\, dy = 0$ **[CE: GATE – 2008]**

(a) 13.5 (b) 27.0

(c) 40.5 (d) 54.0

13.(a)

$$\int_0^3 \int_0^x (6 - x - y)\, dxdy$$

$$= \int_{x=0}^3 \left\{ \int_{y=0}^x (6 - x - y)\, dy \right\} dx$$

$$= \int_0^3 \left(6y - xy - \frac{y^2}{2} \right) dx$$

$$= \int_0^3 \left(6x - \frac{3x^2}{2} \right) dx$$

$$= \left[3x^2 - \frac{x^3}{2} \right]_0^3 = 27 - \frac{27}{2} = \frac{27}{2} = 13.5$$

14. A parabolic cable is held between two supports at the same level. The horizontal span between the supports is L. The sag at the mid-span is h. The equation of the parabola is $y = 4h\frac{x^2}{L^2}$, where x is the horizontal coordinate and y is the vertical coordinate with the origin at the centre of the cable. The expression for the total length of the cable is

(a) $\int_0^L \sqrt{1 + 64\frac{h^2x^2}{L^4}}\, dx$ (b) $2\int_0^{\frac{L}{2}} \sqrt{1 + 64\frac{h^3x^2}{L^4}}\, dx$

[CE: GATE – 2010]

(c) $\int_0^{\frac{L}{2}} \sqrt{1 + 64\frac{h^2x^2}{L^4}}\, dx$ (d) $2\int_0^{\frac{L}{2}} \sqrt{1 + 64\frac{h^2x^2}{L^4}}\, dx$

14(d). We know length of the curve f(x) between x = a and x = b given by

$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Given $\lambda = 4h\frac{x^2}{L^2}$

$$\frac{dy}{dx} = 8h\frac{x}{L^2}$$

Since, $y = 0$. at $x = 0$

and $y = h$ at $x = \frac{L}{2}$

$\therefore \left(\text{Length of cable}\right) = \int_0^{\frac{L}{2}} \sqrt{1 + \left(\frac{8hx}{L^2}\right)^2}\, dx$

Length of cable

$= 2\int_o^{\frac{L}{2}} \sqrt{1 + 64\frac{h^2x^2}{L^4}} dx$

EE All GATE Questions

15. The expression $V = \int_0^H \pi R^2 (1 - h/H)^2 dh$ for the volume of a cone is equal to **[EE: GATE-2006]**

(a) $\int_0^R \pi R^2 (1 - h/H)^2 dr$ (b) $\int_0^R \pi R^2 (1 - h/H)^2 dh$

(c) $\int_0^H 2\pi r\ H(1 - r/R) dh$ (d) $\int_0^R 2\pi r\ H\left(1 - \frac{r}{R}\right)^2 dr$

15. Ans. (d)

Choices (a) and (d) *can* be correct becouse

variable is r in these two.

By integrating (d), *we* get

$\frac{1}{3}\pi r^2 H$, which is volume of cone.

16. A surface S(x,y)=2x+5y-3 is integrated once over a path consisting of the points that satisfy $(x+1)2 + (y-1)2 = \sqrt{2}$. The integral evaluates to **[EE: GATE-2006]**

(a) $17\sqrt{2}$ (b) $17/\sqrt{2}$

(c) $\sqrt{2}/17$ (d) 0

16. Ans. (d)

18. $f(x,y)$ **is a continuous defined over** $(x,y) \in [0,1] \times [0,1]$**. Given two constrains,** $x > y^2$ **and** $y > x^2$**, the volume under** $f(x,y)$ **is** **[EE: GATE-2009]**

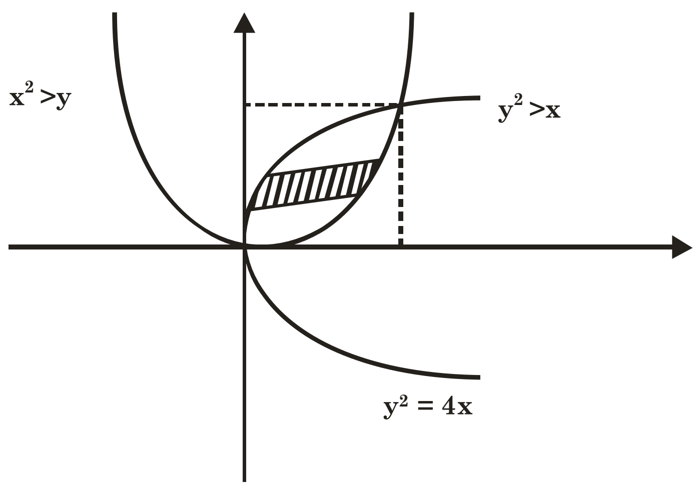
(a) $\int\limits_{y=0}^{y=1}\int\limits_{x=y^2}^{x=\sqrt{y}} f(x,y)\,dxdy$ (b) $\int\limits_{y=x^2}^{y=1}\int\limits_{x=y^2}^{x=1} f(x,y)\,dxdy$

(c) $\int\limits_{y=0}^{y=1}\int\limits_{x=0}^{x=1} f(x,y)\,dxdy$ (d) $\int\limits_{y=0}^{y=\sqrt{x}}\int\limits_{x=0}^{x=\sqrt{y}} f(x,y)\,dxdy$

18. (a)

$\therefore$ volume

$$\int\limits_{y=0}^{1}\int_{x=y^2}^{\sqrt{y}} f(x,y)dxdy$$



IE All GATE Questions

19. **The value of integral $\int\limits_0^\infty\int\limits_0^\infty e^{-x^2}e^{-y^2}$ dx dy is** **[IE: GATE-2007]**

(a) $\sqrt{\frac{\pi}{2}}$ (b) $\sqrt{\pi}$

(c) π (d) $\frac{\pi}{4}$

19. (d)

$$I=\int\limits_0^\infty\int\limits_0^\infty e^{-x^2}e^{-y^2}dxdy=\int\limits_0^\infty e^{-x^2}dx\int\limits_0^\infty e^{-y^2}dy$$

put $z=x^2$, $dz=2xdx$

$$\int_0^\infty e^{-x^2}dx=\frac{1}{2}\int_0^\infty e^{-z}z^{-1/2}dz=\frac{1}{2}\int_0^\infty e^{-z}z^{1/2-1}dz=\frac{1}{2}\Gamma(\frac{1}{2})=\frac{\sqrt{\pi}}{2}$$

$$\therefore \int_0^\infty e^{-y^2}\,dy = \frac{\sqrt{\pi}}{2}$$

$$\therefore I = \frac{\pi}{4}.$$

11. Fourier Series

EC All GATE Questions

1. Choose the function f(t); $-\infty < t < \infty$, for which a Fourier series cannot be defined. [EC: GATE-2005]

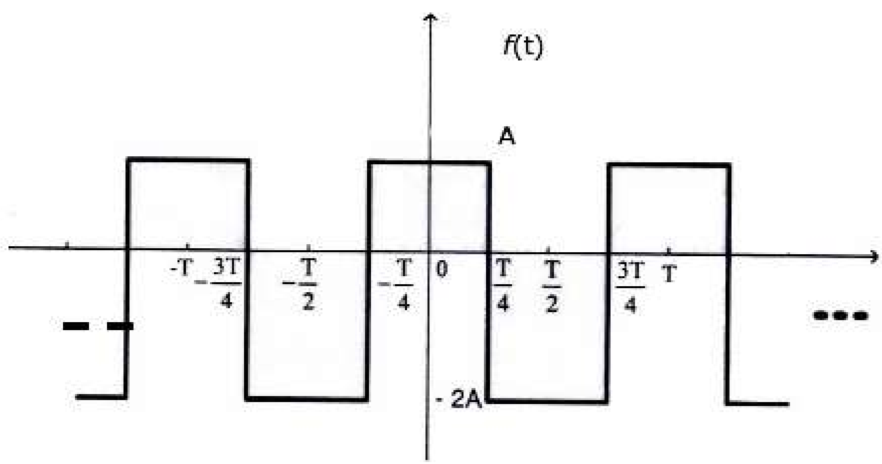
(a) 3 sin (25 t) (b) 4 cos (20 t + 3) + 2 sin (710 t)
(c) exp (–|t|) sin (25 t) (d) 1

1.(c)

2. The Fourier series of a real periodic function has only [EC: GATE-2009]
P. cosine terms if it is even
Q. sine terms if it is even
R. cosine terms if it is odd
S. sine terms if it is odd
Which of the above statements are correct?
(a) P and S (b) P and R
(c) Q and S (d) Q and R

2. (a) Because sine function is odd and cosine is even function.

3. The trigonometric Fourier series for the waveform $f(t)$ shown below contains [EC: GATE-2010]



(a) Only cosine terms and zero value for the dc component
(b) Only cosine terms and a positive value for the dc component
(c) Only cosine terms and a negative value for the dc component
(d) Only sine terms and a negative for the dc component

3. (c) From figure it's an even function. so only cosine terms are present in the series and for DC value,

$$\text{So} = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt$$

$$= \frac{1}{T}\left[\int_{-T/2}^{-T/4} -'2Adt + \int_{-T/4}^{T/4} Adt + \int_{T/4}^{T/2} -2Adt\right]$$

$$= \frac{1}{T}\left[-2A\left(\frac{T}{4}+\frac{T}{2}\right) + A\left(\frac{T}{4}+\frac{T}{4}\right) - 2A\left(\frac{T}{2}-\frac{T}{4}\right)\right]$$

$$= \frac{1}{T}\left[-2A.\frac{T}{4} + \frac{AT}{2} - \frac{2AT}{4}\right]$$

$$= \frac{1}{T}\left[\frac{-AT}{2}\right]$$

$$= -\left(\frac{A}{2}\right)$$

So DC take negative value.

5. For the function e^{-x}, the linear approximation around x = 2 is [EC: GATE-2007]

(a) $(3 - x)\, e^{-2}$ (b) $1 - x$

(c) $[3 + 2\sqrt{2} - (1 + \sqrt{2})\, x]e^{-2}$ (d) e^{-2}

5. Ans.(a)

$$f(x) = f(x_0) + \frac{(x - x_0)\, f'(x_0)}{\underline{|1}} + \frac{(x - x_0)^2\, f''(x_0)}{\underline{|2}} + \ldots\ldots.$$

$$= e^{-2} + (x - 2)\,(-e^{-2}) + \frac{(x - 2)^2}{\underline{|2}}(+e^{-2})\ldots\ldots..$$

$$= e^{-2} + \left(2 - x + \frac{(x - 2)^2}{\underline{|2}}\right)e^{-2} + \ldots\ldots..$$

$$= (3 - x)\,e^{-2}$$

(neglecting higher power of x)

6. Which of the following functions would have only odd powers of x in its Taylor series expansion about the point x = 0? [EC: GATE-2008]

(a) $\sin(x^3)$ (b) $\sin(x^2)$

(c) $\cos(x^3)$ (d) $\cos(x^2)$

6. (a)

$$\text{We know, } \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots.$$

$$\therefore \sin x^3 = x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + ...$$

7. In the Taylor series expansion of exp(x) + sin(x) about the point x = π, the coefficient of $(x - \pi)^2$ is [EC: GATE-2008]

(a) exp (π) (b) 0.5 exp(π)
(c) exp (π) + 1 (d) exp(π) – 1

7. (b)

Let $f(x) = e^x + \sin x$

Taylor's series is

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a)$$

where $a = \pi$

$$f(x) = f(\pi) + (x-\pi)f'(\pi) + \frac{(x-\pi)^2}{2!}f''(\pi).$$

$$\therefore \text{cofficient of } (x-\pi)^2 \text{ is } \frac{f''(\pi)}{2}$$

$$\text{Now, } f''(\pi) = e^x - \sin x\big|_{x=\pi} = e^\pi$$

$$\therefore \text{cofficient of } \underline{\underline{(x-\pi)^2}} = 0.5\exp(\pi).$$

8. The Taylor series expansion of $\dfrac{\sin x}{x - \pi}$ at x = π is given by [EC: GATE-2009]

(a) $1 + \frac{(x-\pi)^2}{3!} + ...$ (b) $-1 - \frac{(x-\pi)^2}{3!} + ...$

(c) $1 - \frac{(x-\pi)^2}{3!} + ...$ (d) $-1 + \frac{(x-\pi)^2}{3!} + ...$

8. (d)

We know.

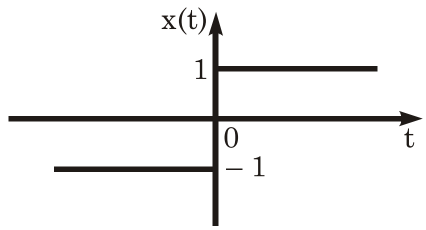
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

$$\therefore \sin(x-\pi) = (x-\pi) - \frac{(x-\pi)^3}{3!} + \frac{(x-\pi)^5}{5!} - \frac{(x-\pi)^7}{7!} + ...$$

$$\Rightarrow \frac{-\sin x}{x-\pi} = 1 - \frac{(x-\pi)^2}{3!} + \frac{(x-\pi)^4}{5!} - \frac{(x-\pi)^6}{7!} + ...$$

$$\Rightarrow \frac{\sin x}{x-\pi} = -1 + \frac{(x-\pi)^2}{3!} - \frac{(x-\pi)^4}{5!} + \frac{(x-\pi)^6}{7!} - \ldots\ldots..$$

1. The function x(t) is shown in the figure. Even and odd parts of a unit-step function u(t) are respectively, [EC: GATE-2005]



(a) $\frac{1}{2}, \frac{1}{2}x(t)$ (b) $-\frac{1}{2}, \frac{1}{2}x(t)$ (c) $\frac{1}{2}, -\frac{1}{2}x(t)$ (d) $-\frac{1}{2}, -\frac{1}{2}x(t)$

1. Ans.(a)

Even part = $\frac{u(t)+u(-t)}{2}$

Now $u(t) = 0; \quad t < 0$

$= 1, \quad t \geq 0$

$\therefore \quad u(-t) = 0, \quad -t < 0$

$= 1, \quad -t \geq 0$

i.e., $u(-t) = 1, \quad t \leq 0$

$= 0, \quad t > 0$

$\therefore \quad \frac{u(t)+u(-t)}{2} = \frac{1}{2}; \quad t \leq 0$

$= \frac{1}{2} \quad t > 0$

$\therefore \quad \text{Even}[u(t)] = \frac{1}{2}$

$$\text{Odd}(u(t)) = \frac{u(t)+u(-t)}{2}\begin{bmatrix} -\frac{1}{2}, & t \leq 0 \\ \frac{1}{2}, & t > 0 \end{bmatrix}$$

$= \frac{x(t)}{2}$ from given figure

ME 20 Years GATE Questions

9. For $x = \frac{\pi}{6}$, the sum of the series $\sum_{1}^{\infty}(\cos x)^{2n} = \cos^2 x + \cos^4 x + ...$is

(a) π (b) 3 (c) ∞ (d) 1 [ME: GATE-1998]

9. Ans. (b)

$$\sum_{n=0}^{\infty} (\cos x)^{2n} = \cos^2 x + \cos^4 x + ...$$

$$\text{At } x = \frac{\pi}{6}, \sum_{n=0}^{\infty} \left(\cos x/6\right)^{2n} = \cos^2 \frac{\pi}{6} + \cos^4 \frac{\pi}{6} + \cos^6 \frac{\pi}{6} + ...$$

$$= \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + ...$$

$$= \frac{\frac{3}{4}}{1 - \frac{3}{4}} = \frac{3}{4} \times \frac{4}{1} = 3.$$

11. In the Taylor series expansion of e^x about x = 2, the coefficient of $(x - 2)^4$ is

(a) 1/4! (b) $2^4/4!$ (c) $e^2/4!$ (d) $e^4/4!$ [ME: GATE-2008]

11. (d)

Taylor series of
f(x) in the neighborhood of a,

$$f(x) = \sum_{n=0}^{\infty} b_n (x - a)^n \text{ ,here } a = 2.$$

$$\text{where } b_n = \frac{f^n(a)}{n!} \qquad \therefore b_4 = \frac{f^4(2)}{4!} = \frac{e^4}{4!}$$

12. The sum of the infinite series, $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...$ is [ME: GATE-1997]

(a) π (b) infinity (c) 4 (d) $\frac{\pi^2}{4}$

12. Ans. ()

CE 10 Years GATE Questions

14. The Fourier series expansion of a symmetric and even function, f(x) where

$$f(x) = 1 + \left(\frac{2x}{\pi}\right), \qquad -\pi \le x \le 0$$ [CE: GATE – 2003]

And $$= 1 - \left(\frac{2x}{\pi}\right), \qquad 0 \le x \le \pi$$

Will be

(a) $\sum_{n=1}^{\infty} \frac{4}{\pi^2 n^2}(1 + \cos n\pi)$ (b) $\sum_{n=1}^{\infty} \frac{4}{\pi^2 n^2}(1 - \cos n\pi)$

(c) $\sum_{n=1}^{\infty} \frac{4}{\pi^2 n^2}(1 - \sin n\pi)$ (d) $\sum_{n=1}^{\infty} \frac{4}{\pi^2 n^2}(1 + \sin n\pi)$

14. (b)

f(x) is symmetric and even, it's Fourier series contain only cosine term.
Now.

$$an = \frac{2}{\pi}\int_0^{\pi}\left(1 - \frac{2x}{\pi}\right)\cos nx dx$$

$$= \frac{2}{\pi}\left[\frac{\sin nx}{n}\right]_0^{\pi} - \frac{4}{\pi^2}\left\{\left[\frac{.x \sin nx}{n}\right]_0^{\pi} - \left[-\frac{\cos nx}{n^2}\right]_0^{\pi}\right\}$$

$$= -\frac{4}{\pi^2 n^2}(\cos n\pi - 1)$$

$$= \frac{4}{\pi^2 n^2}\left[1 - \cos n\pi\right]$$

15. The summation of series $S = 2 + \frac{5}{2} + \frac{8}{2^2} + \frac{11}{2^3} + \ldots\ldots\ldots \infty$ is [CE: GATE – 2004]

(a) 4.50 (b) 6.0
(c) 6.75 (d) 10.0

15. (d)

Let $S = 2 + x.$

were $x = \frac{5}{2} + \frac{8}{2^2} + \frac{11}{2^3} + \ldots\ldots.\infty$ –(i)

$\Rightarrow \frac{1}{2}x = \frac{5}{2^2} + \frac{8}{2^3} + \frac{11}{2^4} + \ldots\ldots.\infty$ –(ii)

Apply (i) – (ii) we get.

$$x - \frac{1}{2}x = \frac{5}{2} + \frac{8-5}{2^2} + \frac{11-8}{2^3} + \frac{14-11}{2^4} + \ldots..$$

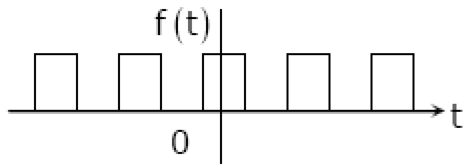
$$\Rightarrow \frac{x}{2} = \frac{5}{2} + \left(\frac{3}{2^2} + \frac{3}{2^3} + \frac{3}{2^4} + \ldots.\right)$$

$$\Rightarrow \frac{x}{2} = \frac{5}{2} + \left[\frac{\frac{2}{2^2}}{1 - \frac{1}{2}} \right] = \frac{5}{2} + \frac{3}{2} = 4$$

$\therefore x = 8 \qquad \therefore S = 2 + 8 = 10.$

EE All GATE Questions

Q16. The Fourier series expansion $\mathbf{f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t}$ of the periodic signal shown below will contain the following nonzero terms



(a) $\mathbf{a_0}$ and $\mathbf{b_n, n = 1,3,5...\infty}$
(b) $\mathbf{a_0}$, **and** $\mathbf{a_n, n = 1,2,3...\infty}$
(c) $\mathbf{a_0}$ and $\mathbf{a_n, n = 1,2,3...\infty}$
(d) $\mathbf{a_0}$ and $\mathbf{a_n, n = 1,3,5...\infty}$ [EE-2011]

Ans. **(b)**
Exp. from the figure, we can say that
f(t) is an symmetric and even function of t.
as cost is even function so choice (b) is correct.

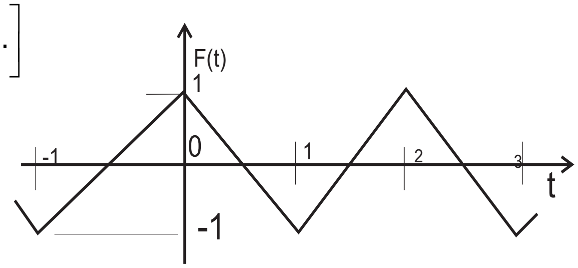
16. Fourier series for the waveform, f (t) shown in fig. is **[EE: GATE-2002]**

(a) $\frac{8}{\pi^2}\left[\sin(\pi t) + \frac{1}{9}\sin(3\pi t) + \frac{1}{25}\sin(5\pi t) + ...\right]$

(b) $\frac{8}{\pi^2}\left[\sin(\pi t) - \frac{1}{9}\cos(3\pi t) + \frac{1}{25}\sin(5\pi t) + ...\right]$

(c) $\frac{8}{\pi^2}\left[\cos(\pi t) - \frac{1}{9}\cos(3\pi t) + \frac{1}{25}\cos(5\pi t) + ...\right]$

(d) $\frac{8}{\pi^2}\left[\cos(\pi t) - \frac{1}{9}\sin(3\pi t) + \frac{1}{25}\sin(5\pi t) + ...\right]$



16.(c)
From the figure, we say f (x) is even functions. so choice (c) is correct.

17. The Fourier series for the function f(x)=$\sin^2 x$ is [EE: GATE-2005]
(a) sinx+sin2x
(b) 1-cos2x

(c) sin2x+cos2x
(d) 0.5-0.5cos2x

17. (d)

Here $f(x)=\sin^2 x$ is even function, hence $f(x)$ has no sine term.

Now, $a_0=\frac{2}{\pi}\int_0^{\pi}\sin^2 x dx.=\int_0^{\pi}(1-\cos 2x)dx=1.$ we know

$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx.$

$=0.5+\text{term contain cosine}$

18. X(t) is a real valued function of a real variable with period T. Its trigonometric Fourier Series expansion contains no terms of frequency $\omega=2\pi(2k)/T; k=1, 2,....$ Also, no sine terms are present. Then x(t) satisfies the equation [EE: GATE-2006]

(a) $x(t)=-x(t-T)$
(b) $x(t)=-x(T-t)=-x(-t)$ (c) $x(t)=x(T-t)=-x(t-T/2)$
(d) $x(t)=x(t-T)=x(t-T/2)$

18. (d)

No sine terms are present.
$\therefore$x(t) is even function.

19. The Fourier Series coefficients, of a periodic signal x (t), expressed as $x(t)=\sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$ are given by

$a_{-2}=-j1; a_{-1}=0.5+j0.2; a_0=j2; a_1=0.5-j0.2; a_2=2+j1; and\ a_k=0; for\ \backslash k\backslash>2.$

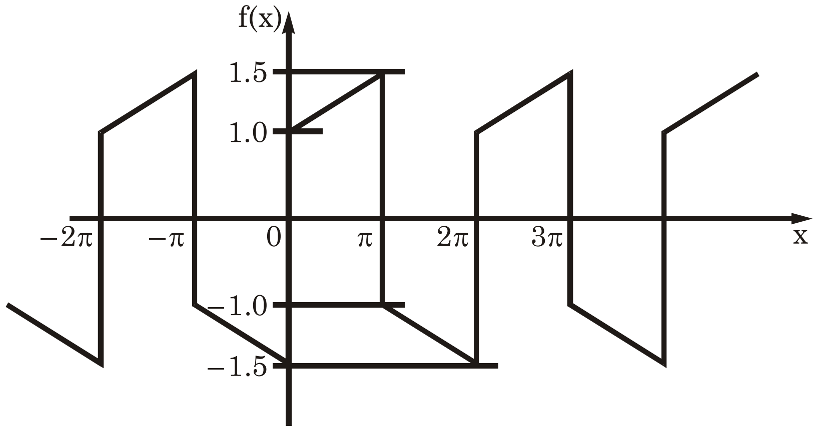
Witch of the following is true? [EE: GATE-2009]

(a) x(t) has finite energy because only finitely many coefficients are non- zero
(b) x(t) has zero average value because it is periodic
(c) The imaginary part of x(t) is constant
(d) The real part of x(t) is even

19. (a)

IE All GATE Questions

20. f(x), shown in the figure is represented by $f(x) = \mathbf{a_0 + \sum_{n=1}^{\infty}\{a_n \cos(nx) + b_n \sin(nx)\}}$. The value of $\mathbf{a_0}$ is [IE: GATE-2010]



(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

20. (a).

From the figure, we say that , f(x) is odd function.

$$\therefore a_0 = \frac{2}{T_0}\int_0^T f(x)dx = 0.$$

21. Given the discrete-time sequence $x[n] = \mathbf{[2, 0, \underset{\uparrow}{-1}, -3, 4, 1, -1]}$, X $(e^{j\pi})$ is

(a) 8 (b) 6π (c) 8π (d) 6**[IE: GATE-2005]**

21. Ans.(c)

25. The infinite series $f(x) = \mathbf{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} .... \infty}$ converges to [IE: GATE-2010]

(a) cos (x) (b) sin (x)
(c) sin h (x) (d) e^x

25. (b).

We know Taylor series at

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{iv}(0) + ...\infty$$

For $f(x) = \sin x.\ f'(x) = \cos x,\ f''(x) = -\sin x,\ f'''(x) = -\cos x,........$

$\therefore f(0) = 0,\ f'(0) = 1,\ f''(0) = 0\ f'''(0) = -1,..........$

$$\therefore f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - - - - — = \sin x$$

12. Differential Equations

Previous Years GATE Questions

EC All GATE Questions

1. The following differential equation has [EC: GATE-2005]

$$3\frac{d^2y}{dt^2} + 4\left(\frac{dy}{dt}\right)^3 + y^2 + 2 = x$$

(a) degree = 2, order = 1
(b) degree = 3, order = 2
(c) degree = 4, order = 3
(d) degree = 2, order = 3

2. The order of the differential equation [EC: GATE-2009]

$$\frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^3 + y^4 = e^{-t} \text{ is}$$

(a) 1 (b) 2 (c) 3 (d) 4

3. A solution of the following differential equation is given by [EC: GATE-2005]

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

(a) $y = e^{2x} + e^{-3x}$
(b) $y = e^{2x} + e^{3x}$
(c) $y = e^{-2x} + e^{3x}$
(d) $y = e^{-2x} + e^{-3x}$

4. For the differential equation $\frac{d^2y}{dx^2} + k^2y = 0$, the boundary conditions are

(i) $y = 0$ for $x = 0$, and
(ii) $y = 0$ for $x = a$ **[EC: GATE-2006]**
The form of non-zero solutions of y (where m varies over all integers) are

(a) $y = \sum_m A_m \sin^{\frac{m\pi x}{a}}$
(b) $y = \sum_m A_m \cos^{\frac{m\pi x}{a}}$
(c) $y = \sum_m A_m x^{\frac{m\pi}{a}}$
(d) $y = \sum_m A_m e^{-\frac{m\pi x}{a}}$

5. The solution of the differential equation $k^2\frac{d^2y}{dx^2} = y - y_2$ under the boundary conditions **[EC: GATE-2007]**

(i) $y = y_1$ At $x = 0$ and
(ii) $y = y_2$ At $x = \infty$,

Where k, y_1 and y_2 are constants, is

(a) $y = (y_1 - y_2)\exp\left(\frac{-x}{k^2}\right) + y_2$

(b) $y = (y_2 - y_1)\exp\left(\frac{-x}{k}\right) + y_1$

(c) $y = (y_1 - y_2)\sin h\left(\frac{x}{k}\right) + y_1$

Ans(d)

(d) $y = (y_1 - y_2)\exp\left(\frac{-x}{k}\right) + y_2$

6. Which of the following is a solution to the differential equation $\frac{dx(t)}{dt} + 3x(t) = 0$?

[EC: GATE-2008]

(a) $x(t) = 3e^{-1}$ (b) $x(t) = 2e^{-3t}$

(c) $x(t) = -\frac{3}{2}t^2$ (d) $x(t) = 3t^2$

7. A function $n(x)$ satisfies the differential equation $\frac{d^2n(x)}{dx^2} - \frac{n(x)}{L^2} = 0$ where L is a constant. The boundary conditions are: $n(0)$=K and $n(\infty) = 0$. The solution to this equation is

[EC: GATE-2010]

(a) $n(x) = K\exp(x/L)$ (b) $n(x) = K\exp(x/\sqrt{L})$

(c) $n(x) = K^2\exp(-x/L)$ (d) $n(x) = K\exp(-x/L)$

ME 20 Years GATE Questions

8. For the differential equation $\frac{dy}{dt} + 5y = 0$ with y(0)=1, the general solution is

(a) e^{5t} (b) e^{-5t} (c) $5e^{-5t}$ (d) $e^{\sqrt{-5t}}$

[ME: GATE-1994]

9. A differential equation of form $\frac{dy}{dx} = y(x,y)$ is homogeneous if the function f(x,y) depends only on the ratio $\frac{y}{x}$ or $\frac{x}{y}$.

[ME: GATE-1995]

10. The solution of the differential equation $\frac{dy}{dx} + y^2 = 0$ is [ME: GATE-2003]

(a) $y = \frac{1}{x+c}$ (b) $y = \frac{-x^2}{3} + c$

(c) ce^x (d) Unsolvable as equation is non-linear

11. If $x^2 \frac{dy}{dx} + 2xy = \frac{2\ell nx}{x}$, and y(1)=0, then what is y(e)? [ME: GATE-2005]

(a) e (b) 1 (c) 1/e (d) $1/e^2$

12. The solution of the differential equation $\frac{dy}{dx} + 2xy = e^{-x^2}$ with y(0) = 1 is

(a) $(1+x)e^{+x^2}$ (b) $(1+x)e^{-x^2}$ (c) $(1-x)e^{+x^2}$ (d) $(1-x)e^{-x^2}$ [ME: GATE-2006]

13. The solution of $dy/dx = y^2$ with initial value y (0) = 1 bounded in the interval

(a) $-\infty \le x \le \infty$ (b) $-\infty \le x \le 1$ [ME: GATE-2007]

(c) $x < 1$, x>1 (d) $-2 \le x \le 2$

14. The solution of $x\frac{dy}{dx} + y = x^4$ with the condition y(1)= $\frac{6}{5}$ is [ME: GATE-2009]

(a) y= $\frac{x^4}{5} + \frac{1}{x}$ (b) y= $\frac{4x^4}{5} + \frac{4}{5x}$ (c) y= $\frac{x^4}{5}$ +1 (d) y= $\frac{x^5}{5}$ +1

15. Solve for y, if $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$; with y(0) = 1 and yω(0)= - 2 [ME:GATE-1994]

16. The solution to the differential equation f''(x)+4f'(x)+4f(x)=0 is [ME: GATE-1995]

(a) $f_1(x) = e^{-2x}$ (b) $f_1(x) = e^{2x}$, $f_2(x) = e^{-2x}$
(c) $f_1(x) = e^{-2x}$, $f_2(x) = xe^{-2x}$ (d) $f_1(x) = e^{-2x}$, $f_2(x) = e^{-x}$

17. The general solution of the differential equation $x^2 = \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ is

(a) $Ax + Bx^2$ (A, B are constants) [ME: GATE-1998]
(b) Ax + B log x (A, B are constants)
(c) $Ax + Bx^2 \log x$ (A, B are constants)
(d) Ax + Bx log x (A, B are constants)

18. $\frac{d^2y}{dx^2} + (x^2 + 4x)\frac{dy}{dx} + y = x^8 - 8$ [ME: GATE-1999]

The above equation is a
(a) Partial differential equation (b) Nonlinear differential equation
(c) Non-homogeneous differential equation (d) Ordinary differential equation

19. The solution of the differential equation $d^2y/dx^2 + dy/dx + y = 0$ is [ME: GATE-2000]

Statement for Linked Answer Questions 20 & 21:

20. The complete solution of the ordinary differential equation

$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = 0$ is y = $c_1e^{-x} + c_2e^{-x}$ **[ME: GATE-2005]**

Then, p and q are
(a) p = 3, q = 3 (b) p = 3, q = 4 (c) p=4,q=3 (d) p = 4, q = 4

21. Which of the following is a solution of the differential equation
$\frac{d^2y}{dx^2}+p\frac{dy}{dx}+(q+1)=0$? [ME: GATE-2005]
(a) e^{-3x} (b) $x\,e^{-x}$ (c) $x\,e^{-2x}$ (d) x^2e^{-2x}

22. For $\frac{d^2}{dx^2}+4\frac{dy}{dy}+3y=3e^{2x}$, the particular integrals is [ME: GATE-2006]
(a) $\frac{1}{15}e^{2x}$ (b) $\frac{1}{5}e^{2x}$ (c) $3e^{2x}$ (d) $C_1e^{-x}+C_2e^{-3x}$

23. Given that $\ddot{x}+3x=0,$ and x(0)=1, $\dot{x}(0)=0$ what is x(1)? [ME: GATE-2008]
(a) -0.99 (b) -0.16 (c) 0.16 (d) 0.99

24. It is given that y" + 2y' + y = 0, y(0) = 0, y(1)=0. What is y (0.5)? [ME: GATE-2008]
(a) 0 (b) 0.37 (c) 0.62 (d) 1.13

25. The Blasius equation, $\frac{d^3f}{d\eta^3}+\frac{f}{2}\frac{d^2f}{d\eta^2}=0$**, is a** [ME: GATE-2010]

(a) Second order nonlinear ordinary differential equation
(b) Third order nonlinear ordinary differential equation
(c) Third order linear ordinary differential equation
(d) Mixed order nonlinear ordinary differential equation

26. The partial differential equation [ME: GATE-2007]
$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\left(\frac{\partial\phi}{\partial x}\right)+\left(\frac{\partial\phi}{\partial y}\right)=0$ has
(a) Degree 1 order 2 (b) Degree 1 order 1
(c) Degree 2 order 1 (d) Degree 2 order 2

CE 10 Years GATE Questions

27. The degree of the differential equation $\frac{d^2x}{dt^2}+2x^3=0$ is **[CE: GATE –2007]**
(a) 0 (b) 1 (c) 2 (d) 3

29. The order and degree of the differential equation **[CE: GATE – 2010]**
$\frac{d^3y}{dx^3}+4\sqrt{\left(\frac{dy}{dx}\right)^3+y^2}=0$ are respectively
(a) 3 and 2 (b) 2 and 3 (c) 3 and 3 (d) 3 and 1

30. The solution for the differential equation $\frac{dy}{dx} = x^2y$ with the condition that y = 1 at x = 0 is

(a) $y = e^{\frac{1}{2x}}$ (b) $\ln(y) = \frac{x^3}{3} + 4$ **[CE: GATE – 2007]**

(c) $\ln(y) = \frac{x^2}{2}$ (d) $y = e^{\frac{x^3}{3}}$

31. Biotransformation of an organic compound having concentration (x) can be modelled using an ordinary differential equation $\frac{dx}{dt} + kx^2 = 0,$ where k is the reaction rate constant. If x = a at t = 0, the solution of the equation is **[CE: GATE – 2004]**

(a) $x = ae^{-kt}$ (b) $\frac{1}{x} = \frac{1}{a} + kt$ (c) $x = a(1 - e^{-kt})$ (d) $x = a + kt$

32. The solution of the differential equation, $x^2\frac{dy}{dx} + 2xy - x + 1 = 0$, given that at x = 1, y = 0 is

[CE: GATE – 2006]

(a) $\frac{1}{2} - \frac{1}{x} + \frac{1}{2x^2}$ (b) $\frac{1}{2} - \frac{1}{x} - \frac{1}{2x^2}$

(c) $\frac{1}{2} + \frac{1}{x} + \frac{1}{2x^2}$ (d) $-\frac{1}{2} + \frac{1}{x} + \frac{1}{2x^2}$

33. Transformation to linear form by substituting $v = y^{1-n}$ of the equation $\frac{dy}{dt} + p(t)y = q(t)y^n$; n > 0 will be **[CE: GATE – 2005]**

(a) $\frac{dv}{dt} + (1-n)pv = (1-n)q$ (b) $\frac{dv}{dt} + (1-n)pv = (1+n)q$

(c) $\frac{dv}{dt} + (1+n)pv = (1-n)q$ (d) $\frac{dv}{dt} + (1+n)pv = (1+n)q$

34. A spherical naphthalene ball exposed to the atmosphere loses volume at a rate proportional to its instantaneous surface area due to evaporation. If the initial diameter of the ball is 2 cm and the diameter reduces to 1 cm after 3 months, the ball completely evaporates in

[CE: GATE – 2006]

(a) 6 months (b) 9 months

(c) 12 months (d) infinite time

35. A body originally at 60°C cools down to 40°C in 15 minutes when kept in air at a temperature of 25°C. What will be the temperature of the body at the end of 30 minutes?

[CE: GATE – 2007]

(a) 35.2° C (b) 31.5° C

(c) 28.7° C (d) 15° C

36. Solution of $\frac{dy}{dx} = -\frac{x}{y}$ at x = 1 and y = $\sqrt{3}$ is **[CE: GATE – 2008]**

(a) $x - y^2 = -2$ (b) $x + y^2 = 4$

(c) $x^2 - y^2 = -2$ (d) $x^2 + y^2 = 4$

37. Solution of the differential equation $3y\frac{dy}{dx} + 2x = 0$ represents a family of

(a) Ellipses (b) circles **[CE: GATE – 2009]**
(c) Parabolas (d) hyperbolas

39. Match each differential equation in Group I to its family of solution curves from Group II. **[CE: GATE-2009]**

Group I	**Group II**
P. $\frac{dy}{dx} = \frac{y}{x}$	1. Circles
Q. $\frac{dy}{dx} = -\frac{y}{x}$	2. Straight lines
R. $\frac{dy}{dx} = \frac{x}{y}$	3. Hyperbolas
S. $\frac{dy}{dx} = -\frac{x}{y}$	

Codes:

	P	**Q**	**R**	**S**		**P**	**Q**	**R**	**S**
(a)	2	3	3	1	(b)	1	3	2	1
(c)	2	1	3	3	(d)	3	2	1	2

40. The solution of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 0$; y (0) = 1, $\frac{dy}{dx}\left(\frac{x}{4}\right) = 0$ in the range $0 < x < \frac{\pi}{4}$ is given by **[CE: GATE – 2005]**

(a) $e^{-x}\left(\cos 4x + \frac{1}{4}\sin 4x\right)$ (b) $e^{x}\left(\cos 4x - \frac{1}{4}\sin 4x\right)$
(c) $e^{-4x}\left(\cos x - \frac{1}{4}\sin x\right)$ (d) $e^{-4x}\left(\cos 4x - \frac{1}{4}\sin 4x\right)$

41. The general solution of $\frac{d^2y}{dx^2} + y = 0$ is **[CE: GATE – 2008]**

(a) y = P cos x + Q sin x (b) y = P cos x
(c) y = P sin x (d) y = P $\sin^2$ x

42. The equation $k_x\frac{\partial^2 h}{\partial x^2} + k_z\frac{\partial^2 h}{\partial z^2} = 0$ can be transformed to $\frac{\partial^2 h}{\partial x_1^2} + \frac{\partial^2 h}{\partial z^2} = 0$ by substituting **[CE: GATE – 2008]**

(a) $x_t = x\frac{k_z}{k_x}$ (b) $x_t = x\frac{k_x}{k_z}$

(c) $x_t = x\sqrt{\dfrac{k_x}{k_z}}$ (d) $x_t = x\sqrt{\dfrac{k_z}{k_x}}$

43. The solution to the ordinary differential equation $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 6y = 0$ is

[CE: GATE – 2010]

(a) $y = c_1e^{3x} + c_2e^{-2x}$ (b) $y = c_1e^{3x} + c_2e^{2x}$
(c) $y = c_1e^{-3x} + c_2e^{2x}$ (d) $y = c_1e^{-3x} + c_2e^{-2x}$

45. The partial differential equation that can be formed from z = ax + by + ab has the form $\left(\text{with } p = \dfrac{\partial z}{\partial x} \text{ and } q = \dfrac{\partial z}{\partial y}\right)$ **[CE: GATE – 2010]**

(a) z = px + qy (b) z = px + pq
(c) z = px + qy + pq (d) z = qy + pq

Q30. The solution of the differential equation $\dfrac{dy}{dx} + \dfrac{y}{x} = x,$ with the condition that y = 1 at x = 1, is

(a) $y = \dfrac{2}{3x^2} + \dfrac{x}{3}$ (b) $y = \dfrac{2}{2} + \dfrac{1}{2x}$ (c) $y = \dfrac{2}{3} + \dfrac{x}{3}$ (d) $y = \dfrac{2}{3x} + \dfrac{x^2}{3}$ **[CE-2011]**

Ans. **(d)**

Exp, $\dfrac{dy}{dx} + \dfrac{y}{x} = x$

It's a linear differential equation

$\therefore \mathbf{I.E.} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

Solution is $xy = \int x^2 dx + C$

$\Rightarrow xy = \dfrac{x^3}{3} + C$ ... (1)

Given y (1) = 1,

$\therefore$ from (1): $c = \dfrac{2}{3}$

$\therefore xy = \dfrac{x^3}{3} + \dfrac{2}{3}$

$\Rightarrow y = \dfrac{2}{3x} + \dfrac{x^3}{3}$

EE All GATE Questions

46. The solution of the first order differential equation $x(t) = -3x(t), x(0) = x_0$ is

[EE: GATE-2005]

(a) $X(t) = x_0 e^{-3t}$ (b) $X(t) = x_0 e^{-3}$ (c) $X(t) = x_0 e^{-1/3}$
(d) $X(t) = x_0 e^{-1}$

48. For the differential equation $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 0$ with initial conditions x (0) = 1 and $\left.\frac{dx}{dt}\right|_{t=0} = 0$, the solution is **[EE: GATE-2010]**

(a) x (t) = $2e^{-6t} - e^{-2t}$ (b) x (t) = $2e^{-2t} - e^{-4t}$
(c) x (t) = $-e^{-6t} + 2e^{-4t}$ (d) x (t) = $e^{--2t} - e^{-4t}$

Q13. With K as constant, the possible solution for the first order differential equation $\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{e^{-3x}}$ is

(a) $-\frac{1}{3}e^{-3x} + K$ (b) $-\frac{1}{3}e^{3x} + K$ (c) $-3e^{-3x} + K$ (d) $-3e^{-x} + K$ [EE-2011]

Ans. **(a)**

Exp. $\frac{dy}{dx} = e^{-3x}$

$\Rightarrow dy = e^{-3x}dx$

$y = \frac{e^{-3x}}{-3} + K$

IE All GATE Questions

51. Consider the differential equation $\frac{dy}{dx}$ = 1 + y^2. Which one of the following can be a particular solution of this differential equation? **[IE: GATE-2008]**

(a) y = tan (x + 3) (b) y = tan x + 3
(c) x = tan (y + 3) (d) x = tan y + 3

22. The boundary-value problem $y^n + \lambda y = 0$, $y(0) = y(\lambda) = 0$ will have non-zero solutions if and only if the values of λ are [IE: GATE-2007]

(a) 0, ±1, ±2, ... (b) 1, 2, 3, ...

(c) 1, 4, 9, ... (d) 1, 9, 25, ...

22 Ans. (c)

Differential Equation

1. Ans. (b) $3\dfrac{d^2y}{dt^2} + 4\left(\dfrac{dy}{dt}\right)^3 + y^2 + 2 = x$

Order of highest derivative = 2

Hence, most appropriate answer is (b)

2. Ans. (b) The order of a differential equation is the order of the highest derivative involving in equation, so answer is 2.

3. (b)

Let y = mx be the trial sol^n of the given differental equation

$\therefore$ The corrosponding auxiliary equation is

$m^2 - 5m + 6 = 0$

$\Rightarrow m = 2, 3$

$\therefore \quad y = c_1 e^{2x} + c_2 e^{3x}$

4. (a)

Here $y = c_1 \cos kx + c_2 \sin kx$ .... (1) be the solution of the given differential equation.

Now use boundary conditions

For $x = 0, y = 0$ gives $c_1 = 0$. Equation – (1) becomes

$y = c_2 \sin kx \quad ....(2)$

For $\quad x = a, y = 0$ given, $\quad c_2 \sin ka = 0.$ If $c_2 = 0$ then

(2) becomes $y = 0,$ so it gives trival solution.

So take $\sin ka = 0$

$\Rightarrow \sin ka = \sin n\pi, \qquad \eta = 0,1,2,.......$

$\Rightarrow ka = n\pi$

$\Rightarrow k = \dfrac{n\pi}{a}$

$\therefore \quad y = c_2 \quad \sin\dfrac{n\pi x}{a}$ be the solution, $n = 0,1,2,3.......$

6. (b)

Hints : $m + 3 = 0 \qquad \Rightarrow m = -3$

$\therefore x(t) = c_1 e^{-3t}$

7.(d)

Hints :

$m^2 - \dfrac{1}{L^2} = 0 \qquad \Rightarrow m = \pm\dfrac{1}{L}$

$\therefore \quad n(x) = c_1\, e^{\frac{x}{L}} + c_2\, e^{-\frac{x}{L}} \qquad -(1)$

Use boundary condition

i) $n(o) = k.$ This implies

$K = c_1 + c_2 \qquad -(ii)$

(ii) $n(\infty) = 0$ gives $\quad 0 = c_1\, e^{\infty} + c_2.0.$ For finite solution $c_1 = 0$

$\therefore$ From (ii) $\quad K = c_2$

$\therefore \quad y = Ke^{-\frac{x}{L}}$

8. (b)

Hints :

$m = -5. \Rightarrow y = c_1 e^{-5t}$

Given $y(0) = 1$

$\therefore C_1 = 1$

Hence $y = e^{-5t}$

9. Ans. True

10. (a)

Given differential equation is

$\dfrac{dy}{dx} + y^2 = 0 \Rightarrow \dfrac{dy}{y^2} = -dx$

Integra ling we get

$$-\frac{1}{y} = -x + c$$

$$\Rightarrow y = \frac{1}{x-c} = \frac{1}{x+c_1}$$

11. (d)

$$x^2\frac{dy}{dx} + 2xy = \frac{2\ln x}{x} \quad -(i)$$

$$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = \frac{2\ln x}{x^3}.$$

It is linear differential equation.

$$\therefore \quad \text{I.F.} = e^{\int \frac{2}{x}dx} = x^2$$

Multiplying I.F both side of (i) then we get

$$d(yx^2) = \frac{2\ln x}{x}$$

Integrating we get

$$yx^2 = 2\int \frac{\ln x}{x}dx + c = 2\frac{(\ln x)^2}{2} + c$$

Using boundary condition y(1) = 0 we get

C = 0

$$\therefore \quad y = \frac{(\ln x)^2}{x^2}$$

$$\therefore \quad y(e) = \frac{1}{e^2} \quad [\therefore \ln e = 1]$$

12 (b)

It is a linear diff. equation

$$\text{I.F} = e^{\int 2x dx} = e^{x^2}$$

$$\therefore \quad \text{Solution is} \qquad ye^{x^2} = \int e^{-x^2}e^{x^2}dx + c = x + c$$

$$\text{At } x = 0, \quad y = 1, \quad \text{gives } c = 1$$

$$\therefore \quad y = (1+x)e^{-x^2}$$

13 (c)

$$\text{Given } \frac{dy}{dx} = y^2$$

Integrating,

$$\Rightarrow \int \frac{dy}{y^2} = \int dx$$

$$\Rightarrow -\frac{1}{y} = x + c$$

$\Rightarrow y = -\frac{1}{x+c}$ ....(1)

At $y(0) = 1$

Equation(1) gives, $1 = -\frac{1}{e} \Rightarrow c = -1$

$\therefore \quad y = -\frac{1}{x-1}, x-1 \neq 0 \Rightarrow x \neq 1$

$\Rightarrow x < 1 \text{ and } x > 1$

14.(d)

Given $x\frac{dy}{dx} + y = x^4$

$\Rightarrow \frac{dy}{dx} + \left(\frac{y}{x}\right) = x^3$

Which is 1st order linear differential equation.

$I.F = e^{\int \frac{1}{x}dx} = x$

$\therefore \quad \text{solution } xy = \int x^4 dx + c = \frac{x^5}{5} + c$

Given condition $y(1) = \frac{6}{5}$

$\therefore \quad \frac{6}{5} = \frac{1}{5} + c$

$\Rightarrow \quad c = 1$

$\therefore \quad xy = \frac{x^5}{5} + 1$

$\Rightarrow \quad y = \frac{x^4}{5} + \frac{1}{x}$

15.

Let $y = e^{mx} (m \neq 0)$ be the trial sol^n of the given equation.

$\therefore$ Auxiliary equation is $m^2 + 2m + 1 = 0 \Rightarrow (m+1)^2 = 0 \Rightarrow m = -1, -1.$

$\therefore \quad y = (A + Bt)e^{-t} \qquad \text{and } \frac{dy}{dt} = -(A + Bt)e^{-t} + Be^{-t}$

Boundary condition $y(0) = 1$ and $\frac{dy}{dt}(0) = -2$

$\therefore \quad 1 = A \quad \text{and } -2 = -A + B$

$\Rightarrow \quad A = 1 \quad \text{and} -1 + B = -2$

$\Rightarrow \quad B = -1$

$\therefore \quad y = (1-t)e^{-t}$

16.(c)

Let $y(x) = e^{mx} (m \neq 0)$ be the trial sol^n.

Auxiliary equation. $m^2 + 4m + 4 = 0 \qquad \Rightarrow (m+2)^2 = 0$

$\Rightarrow m = -2, -2$

$f(x) = (A + Bx)e^{-2x}$

In particular, when $A = 1, B = 1, \text{then } f(x) = (1+x)e^{-2x}$

$= e^{-2x} + xe^{-2x}$

17. (d)

The given homogeneous differential equation reduces to

$D(D-1) - D + 1 = 0, \quad \text{Where } D = \dfrac{d}{dz}$

$\Rightarrow \ D = 1,1$

$\therefore \ \ y = (c_1 + c_2 z)e^z = (c_1 + c_2 \log x)x. = c_1 x + c_2 (x \log x)$

18.(c)

19.

$$y = e^{\frac{-1}{2}x}\left(c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right)$$

20. (c)

Given $y = c_1 e^{-x} + c_2 e^{-3x}$ is the solution of $\dfrac{d^2y}{dx^2} + p\dfrac{dy}{dx} + qy = 0 - (i)$

Let $y = e^{mx}\,(m \neq 0)$ be the trial solution of (i). Therefore $m = -1, -3.$

Then $m^2 + pm + q = (m+1)(m+3)$

$\Rightarrow \ m^2 + pm + q = m^2 + 4m + 3$

$\Rightarrow \ p = 4 \ \text{ and } \ q = 3$

21. (c)

Here $p = 4 \text{ and } q = 3.$ The given equation becomes

$\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 4y = 0 - (i)$

Now solution of (i) is $y = (c_1 + c_2 x)e^{-2x}$

$\therefore$ solutions are e^{-2x} and xe^{-2x}

22. (b)

$P.I. = \dfrac{1}{D^2 + 4D + 3}.3e^{2x}, \ D = \dfrac{d}{dx}$

$= 3e^{2x}\dfrac{1}{2^2 + 4.2 + 3}$

$= \dfrac{3e^{2x}}{15} = \dfrac{e^{2x}}{5}$

23. (d)

Auxiliary equ^n of $x^{11} + 3x = 0$ is $m^2 + 3 = 0$

$\Rightarrow \; m = \pm i\sqrt{3}.$

solution is $x(t) = A\cos\sqrt{3}t + B\sin\sqrt{3}t$

At $t = 0,\ 1 = A$ and $0 = B.$

$\therefore \; x(t) = \cos\sqrt{3}t$

$x(1) = \cos\sqrt{3} = 0.99\,(\text{degree})$

24. (a)

Auxiliary equation is $m^2 + 2m + 1 = 0 \Rightarrow m = -1, -1$

$\therefore$ solution $y = (c_1 + c_2x)e^{-x}$

Using boundary condition $y(0) = 0$ and $y(1) = 0$

we get $y = 0$

25. (b) f is non linear.

26. (a)

27. (b)

29. (a)

30. (d)

$$\frac{dy}{dx} = x^2y \Rightarrow \frac{dy}{y} = x^2dx$$

Integrating we get

$$\log y = \frac{x^3}{3} + c$$

Given $y(0) = 1$ then $c = 0$

$\therefore$ solution is $y = e^{\frac{x^3}{3}}$

31.(b)

$$\frac{dx}{dt} + Kx^2 = 0$$

$$\Rightarrow \frac{dx}{x^2} = -Kdt$$

Integrating, we get

$$-\frac{1}{x} = -Kt + c$$

At $t = 0, x = a,\quad c = -\frac{1}{a}$

$\therefore$ solution is $\frac{1}{x} = \frac{1}{a} + Kt$

32. (a)

Given $x^2\frac{dy}{dx}+2xy-x+1=0$

$\Rightarrow \frac{dy}{dx}+\frac{2}{x}y=\frac{x-1}{x^2}$

$\text{I.F.}=e^{\int\frac{2}{x}dx}=x^2$

$\therefore\ yx^2=\int\frac{(x-1)}{x^2}x^2dx+c=\int(x-1)dx+c=\frac{x^2}{2}-x+c$

At $x=1, y=0$ gives $c=\frac{1}{2}$

$\therefore\ yx^2=\frac{x^2}{2}-x+\frac{1}{2}$

$\Rightarrow\ y=\frac{1}{2}-\frac{1}{x}+\frac{1}{2x^2}$

33.(a)

Given, $\frac{dy}{dt}+p(t)\,y=q(t)\,y^n;\ n>0$

Putting $v=y^{1-n}$

$\frac{dv}{dt}=(1-n)\,y^{-n}\frac{dy}{dt}$

$\frac{dy}{dt}=\frac{1}{(1-n)\,y^{-n}}\frac{dv}{dt}$

Substituting in the given differential equation, we get,

$\frac{1}{(1-n)\,y^{-n}}\frac{dv}{dt}+p(t)\,y=q(t)y^n$

Multiplying by $(1-n)\,y^{-n}$, we get

$\frac{dv}{dt}+p(t)\,(1-n)\,y^{1-n}=q(t)\,(1-n)$

Now since $y^{1-n}=v$, we get

$\frac{dv}{dt}+(1-n)\,pv=(1-n)\,q$

Where p is p (t) and q is q (t)

34. (a)

By the given condition,

$\frac{dV}{dt}=KA, K=\text{constant}.-(i).$

where $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$

$\therefore \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\therefore$ (i) becomes

$\frac{dr}{dt} = -K$

$\Rightarrow \int dr = -\int K dt$

$\Rightarrow r = -Kt + c$

At $t = 0, r = 1$ cm

$\therefore \quad c = 1$

$\therefore \quad r = -Kt + 1 \qquad -(ii)$

Now, at $t = 3$ months, then $r = 0.5$cm

(ii) gives $K = \frac{0.5}{3}$

$\therefore \quad r = \frac{-0.5}{3}t + 1 \quad -(iii)$

Now, put $r = 0$ in (iii)

we get $t = 6$ months

35(b).

$\frac{d\theta}{dt} = -k(\theta - \theta_0)$ (Newton's law of cooling)

$\Rightarrow \quad \frac{d\theta}{\theta - \theta_0} = -kdt$

$\Rightarrow \quad \int \frac{d\theta}{\theta - \theta_0} = \int -kdt$

$\Rightarrow \quad \ln(\theta - \theta_0) = -kt + C_1$

$\Rightarrow \quad \theta - \theta_0 = C.e^{-kt}$

$\theta = \theta_0 + C.e^{-kt}$

Given, $\theta_0 = 25^\circ C$

Now at $t = 0, \theta = 60^\circ$

$60 = 25 + C.e^0$

$\Rightarrow \quad C = 35$

$\therefore \quad \theta = 25 + 35\, e^{-kt}$

at $t = 15$ minutes $\theta = 40^\circ C$

$\therefore \quad 40 = 25 + 35\, e^{(-k \times 15)}$

$\Rightarrow \quad e^{-15k} = \frac{3}{7}$

Now at $t = 30$ minutes

$\theta = 25 + 35\, e^{-30k}$

$= 25 + 35\, (e^{-15k})^2$

$$= 25 + 35 \times \left(\frac{3}{7}\right)^2 \qquad \left(\text{since } e^{-15k} = \frac{3}{7}\right)$$
$$= 31.428^{\circ}C$$
$$\approx 31.5^{\circ}C$$

36. (d)

$$\frac{dy}{dx} = -\frac{x}{y}$$
$$\Rightarrow \quad ydy = -xdx$$
Integrating
$$\frac{y^2}{2} = -\frac{x^2}{2} + c \quad -(i)$$
At $x = 1,\ y = \sqrt{3}$ gives
$c = 2$
$\therefore$ (i) becomes $x^2 + y^2 = 4$

37.(a)

$$3y\frac{dy}{dx} + 2x = 0$$
$$\Rightarrow 3ydy + 2xdx = 0$$
Integrating
$$\frac{3y^2}{2} + x^2 = c$$
$$\Rightarrow \frac{x^2}{c} + \frac{y^2}{\frac{2c}{3}} = 1$$
- an ellipse.

39. (a)

1. $\dfrac{dy}{dx} = \dfrac{y}{x}$
$$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$$
$$\log y = \log x + \log c$$
$$\Rightarrow \quad y = xc$$
- straight line

2. $\dfrac{dy}{dx} = -\dfrac{y}{x}$
$$\Rightarrow \frac{dy}{y} + \frac{dx}{x} = 0$$
Integrating we get

$\log y + \log x = \log c$
$\Rightarrow xy = c$
–hyperbola

3. $\frac{dy}{dx} = \frac{y}{x}$
$\Rightarrow \ ydy = xdx$
Integrating
$y^2 - x^2 = c$
–hyperbola

4. $\frac{dy}{dx} = -\frac{x}{y}$
$\Rightarrow \ ydy = -xdx$
Integrating
$y^2 = -x^2 + c$
$\Rightarrow \ x^2 + y^2 = c$ –circle

40. (a)

Let $y = e^{mx}(m \neq 0)$ be the trial solution.
Auxiliary equation is $m^2 + 2m + 17 = 0$

$\Rightarrow \ m = \frac{-2 \pm \sqrt{4 - 4.17.1}}{2.1}$

$= \frac{-2 \pm 8i}{2}$

$= -1 \pm 4i$

$\therefore \ y = e^{-x}\left(A\cos 4x + B\sin 4x\right)$

now, $\frac{dy}{dx} = -e^{-x}\left(A\cos 4x + \sin 4x\right) + e^{-x}\left(-4A\sin 4x + 4B\cos 4x\right)$

At $x = 0, y = 1$ gives
A=1.
At x= $\frac{\pi}{4}$, y=0 gives,

$0 = e^{-\frac{\pi}{4}}\left(-1\right) + e^{-\frac{\pi}{4}}.4\left(-B\right)$

$\Rightarrow \ 4B = 1$

$\Rightarrow \ B = \frac{1}{4}$

$\therefore \ y = e^{-x}\left(\cos 4x + \frac{1}{4}\sin 4x\right)$

41. (a)

Let $y = e^{mx}\left(m \neq 0\right)$ be the trial solution.

Auxiliary equation is $m^2 + 1 = 0$

$\Rightarrow \; m = \pm i$

$\therefore \; y = P\cos x + Q\sin x$

42(d).

Put $\quad x_t = x\sqrt{\dfrac{k_z}{k_x}}$

$$\frac{\partial x_t}{\partial x} = \sqrt{\frac{k_z}{k_x}}$$

$$\Rightarrow \quad \frac{\partial x}{\partial x_t} = \sqrt{\frac{k_z}{k_x}} \qquad \ldots \text{(i)}$$

Now given equation is

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \qquad \ldots \text{(ii)}$$

$$\frac{\partial^2 h}{\partial x_t^2} = \frac{\partial}{\partial x_t}\left(\frac{\partial h}{\partial x_t}\right) = \frac{\partial}{\partial x_t}\left(\frac{\partial h}{\partial x} \times \frac{\partial x}{\partial x_t}\right)$$

$$= \frac{\partial}{\partial x_t}\left(\frac{\partial h}{\partial x} \times \sqrt{\frac{k_x}{k_z}}\right) \qquad \text{[from eqn. (i)]}$$

$$= \sqrt{\frac{k_x}{k_z}} \times \frac{\partial}{\partial x_t}\left(\frac{\partial h}{\partial x}\right)$$

$$= \sqrt{\frac{k_x}{k_z}} \times \left(\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) \times \frac{\partial h}{\partial x}\right)$$

$$= \sqrt{\frac{k_x}{k_z}} \times \left(\frac{\partial^2 h}{\partial x^2} \times \sqrt{\frac{k_x}{k_z}}\right)$$

$$= \frac{k_x}{k_z} \times \frac{\partial^2 h}{\partial x^2}$$

$$\therefore \frac{\partial^2 h}{\partial x^2} = \frac{k_z}{k_x} \times \frac{\partial^2 h}{\partial x_t^2}$$

Now substitute in equation (ii) we get

$$k_x \times \frac{k_z}{k_x} \times \frac{\partial^2 h}{\partial x_t^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

$$\Rightarrow \quad k_z \times \frac{\partial^2 h}{\partial x_t^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

$$\Rightarrow \quad \frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

This is the desired form

$\therefore \quad x_t = x\sqrt{\frac{k_z}{k_x}}$ is the correct transformation

43. (c)

Let $y = e^{mx}\left(m \neq 0\right)$ be the trial solution.

Auxiliary equation is $m^2 + m - 6 = 0$

$\Rightarrow \quad m = -3, 2.$

$\therefore \quad y = c_1 e^{-3x} + c_2 e^{2x}$

44. Ans. (c) $\frac{dy}{dx} = 0.25\,y^2$ (y = 1 at x = 0) h = 1

Iterative equation for backward (implicit) Euler methods for above equation would be

$$\frac{y_{k+1} - y_k}{h} = 0.25\,y_{k+1}^2$$

$$\Rightarrow \quad y_{k+1} - y_k = 0.25\,h\,y_{k+1}^2$$

$$\Rightarrow \quad 0.25\,h\,y_{k+1}^2 - y_{k+1} + y_k = 0$$

Putting k = 0 in above equation

$$0.25\,h\,y_1^2 - y_1 + y_0 = 0$$

Since $y_0 = 1$ and h = 1

$$0.25\,y_1^2 - y_1 + 1 = 0$$

$$y_1 = 2$$

45. (c)

46. (a)

Let $x = e^{mt} (m \neq 0)$ be trial solution

Auxiliary equation is $m + 3 = 0$

$\Rightarrow \quad m = -3$

$\therefore x(t) = c_1 e^{-3t}$

$x(0) = x_0$ gives. $c_1 = x_0$

$\therefore \quad x(t) = x_0 e^{-3t}$

48. (b)

Let$= e^{mt}\left(m \neq 0\right)$ be trial solution.

Auxiliary equation is $m^2 + 6m + 8 = 0$

$\Rightarrow \quad m = -2, -4$

$\therefore \quad x(t) = c_1 e^{-2t} + c_2 e^{-4t} \qquad -(i)$

and $\dfrac{dx}{dt} = -2c_1 e^{-2t} - 4c_2 e^{-4t}$

At t=0, x=1 gives, $c_1 + c_2 = 1$ ....(ii)

At $t = 0, \dfrac{dx}{dt} = 0$ gives

$-2c_1 - 4c_2 = 0$

$\Rightarrow c_1 + 2c_2 = 0 \quad -(iii)$

Solving (ii) & (iii) we get, $c_1 = 2, c_2 = -1$

$x(t) = 2e^{-2t} - e^{-4t}$

51. (a)

$\dfrac{dy}{dx} = 1 + y^2$

$\Rightarrow \dfrac{dy}{1+y^2} = dx$

Integrating

$\tan^{-1} y = x + c$

$\Rightarrow y = \tan(x + c)$

13. Complex Variables

QUESTION AND ANSWERE

Complex Analysis:

1. The residue of the function f(z) = $\frac{1}{(z+2)^2\,(z-2)^2}$ at z = 2 is [EC: GATE-2008]

(a) $-\frac{1}{32}$ (b) $-\frac{1}{16}$ (c) $\frac{1}{16}$ (d) $\frac{1}{32}$

1. (a)

$$\text{Residue at } z=2 \text{ is } \lim_{z\to 2}\frac{d}{dz}\left[(z-2)^2 f(z)\right]$$

$$=\lim_{z\to 2}\frac{d}{dz}\left[\left(\frac{1}{z+2}\right)^2\right]$$

$$=\lim_{z\to 2}\frac{-2}{(z+2)^3}$$

$$=\lim_{z\to 2}\frac{-2}{(z+2)^3}=-\frac{1}{32}$$

2. If f(z) = $c_2 + c_1 z^{-1}$, than $\oint_{\substack{\text{unit}\\\text{circle}}} \frac{1+f(z)}{z}\,dz$ is given by [EC: GATE-2009]

(a) $2\pi c_1$ (b) $2\pi(1+c_0)$ (c) $2\pi jc_1$ (d) $2\pi j\,(1+c_0)$

2. (d)

$$\text{Let}\quad g(z)=\frac{1+f(z)}{z}=\frac{(1+c_0)z+c_1}{z^2}$$

$\therefore$ g(z) has a pole of order two at $z=0$

$$\therefore\quad \operatorname{Res}(g,o)=\lim_{z\to 0}\frac{d}{dz}\left[(z-0)^2 g(z)\right]$$

$$=\lim_{z\to 0}(1+c_0)$$

$$=\quad 1+c_0$$

$$\therefore\quad \int_{|z|=1} g(z)dz=2\pi i(1+c_0)$$

3. The residues of a complex function $X(z)=\dfrac{1-2z}{z(z-1)(z-2)}$ at its poles are [EC: GATE-2010]

(a) $\frac{1}{2},-\frac{1}{2}$ and 1 (b) $\frac{1}{2},\frac{1}{2}$ and -1 (c) $\frac{1}{2},1$ and $-\frac{3}{2}$ (d) $\frac{1}{2},-1$ and $\frac{3}{2}$

3. (c)

$x(z)$ has simple poles at $z = 0,1,2.$

$$\therefore \quad \operatorname{Res}(x,0)=\lim_{z\to 0}\left[(z-0)x(z)\right]=\lim_{z\to 0}\frac{1-2z}{(z-1)(z-2)}=\frac{1}{2}$$

$$\operatorname{Res}(x,1)=\lim_{z\to 1}\left[(z-1)x(z)\right]=\lim_{z\to 1}\frac{1-2z}{z(z-2)}=1$$

$$\operatorname{Res}(x,2)=\lim_{z\to 2}\left[(z-2)x(z)\right]=\lim_{z\to 2}\frac{1-2z}{z(z-1)}=\frac{-3}{2}$$

4. For the function of a complex variable W = ln Z (where, W = u + jv and Z = x + jy), the u = constant lines get mapped in Z-plane as [EC: GATE-2006]

(a) set of radial straight lines
(b) set of concentric circles
(c) set of confocal hyperbolas
(d) set of confocal ellipses

4. Ans. (b)

Given, $W = \log_e z$

$$\Rightarrow \quad u + iv = \log_e(x+iy) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{y}{x}\right)$$

Since, u is constant, therefore

$$\frac{1}{2}\log(x^2+y^2) = c$$

$$\Rightarrow \quad x^2 + y^2 = c$$

Which is represented set of concentric circles.

5. The value of the contour integral $\displaystyle\oint_{|z-j|=2}\frac{1}{z^2+4}\,dz$ in positive sense is [EC: GATE-2006]

(a) $\frac{j\pi}{2}$ (b) $-\frac{\pi}{2}$

(c) $-\frac{j\pi}{2}$ (d) $\frac{\pi}{2}$

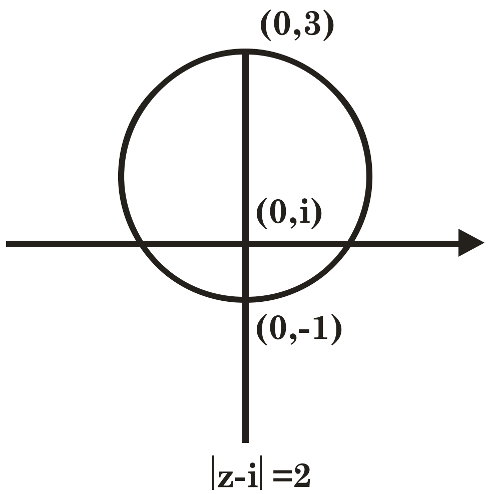
5. (d)

$$\text{Let } f(z) = \frac{1}{z^2+4} = \frac{1}{(z+2i)(z-2i)}$$

In $|z-i| = 2$, $z = 2i$ is a pole of order 1.

$$\therefore \quad \operatorname{Res}(f, 2i) = \lim_{z \to 2i}\left[(z-2i)f(z)\right] = \frac{1}{4i}$$

$$\therefore \quad \int_{|z-i|=2} f(z)dz = 2\pi i \times \frac{1}{4i} = \frac{\pi}{2}$$



ME 20 Years GATE Questions

7. i^i, where $i=\sqrt{-1}$, is given by [ME: GATE-1996]

(a) 0 (b) $e^{-\pi/2}$ (c) $\frac{\pi}{2}$ (d) 1

7. (b)

$i^i = e^{i\log i}$.

Now, $\log i = \log|i| + \left(2k\pi + \frac{\pi}{2}\right)i, \qquad k = 0,1,2,.........$

$= \frac{\pi i}{2}, \qquad \text{for } k = 0.$

$\therefore \quad i^i = e^{i.\frac{\pi i}{2}} = e^{-\frac{\pi}{2}}$

8. The integral $\oint f(z)dz$ evaluated around the unit circle on the complex plane for $f(z)\frac{\cos\ z}{z}$ is [ME: GATE-2008]

(a) $2\pi i$ (b) $4\pi i$ (c) $-2\pi i$ (d) 0

8. (a)

$f(z) = \frac{\cos z}{z}$,

$\therefore$ f(z) has a simple pole at $z = 0$

$\therefore \quad \operatorname{Res}(f,0) = \lim_{z\to 0}\left[(z-0)f(z)\right] = 1$

$\therefore \quad \oint f(z)dz = 2\pi i \times 1 = 2\pi i$

9. Assuming $i = \sqrt{-2}$ and t is a real number $\int_0^{\pi/3} e^{it}dt$ is [ME: GATE-2006]

(a) $\frac{\sqrt{3}}{2} + i\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2} - i\frac{1}{2}$ (c) $\frac{1}{2} + i\left(\frac{\sqrt{3}}{2}\right)$ (d) $\frac{1}{2} + i\left(1 + \frac{\sqrt{3}}{2}\right)$

9.(a)

$$\int_0^{\pi/3} e^{it}dt - \left[\frac{e^{it}}{i}\right]_0^{\pi/3} = \frac{1}{i}\left(e^{i\pi/3} - 1\right) = -i\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} - 1\right)$$

$$= -i\left(\frac{1}{2} + i\frac{\sqrt{3}}{2} - 1\right) = \frac{\sqrt{3}}{2} + i.\frac{1}{2}$$

10. If ϕ(x,y) and ψ(x,y) are functions with continuous second derivatives, then ϕ(x,y) + iψ(x,y) can be expressed as an analytic function of x+ iψ(i=$\sqrt{-1}$), when [ME: GATE-2007]

(a) $\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial x}, \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial y}$ (b) $\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}, \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$

(c) $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 1$ (d) $\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = 0$

10. (b)

$\Phi(x,y) + i\Psi(x,y)$ is analylic, so it satisfies Cauchy-Riemann equation

$$\frac{\partial \Phi}{\partial x} = \frac{\partial \Psi}{\partial y}, \qquad \frac{\partial \Phi}{\partial y} = -\frac{\partial \Psi}{\partial x}$$

11. An analytic function of a complex variable z = x + iy is expressed as f(z) = u (x, y) + i v(x, y) where $i = \sqrt{-1}$. If u = xy, the expression for v should be [ME: GATE-2009]

(a) $\frac{(x+y)^2}{2}$+k (b) $\frac{x^2-y^2}{2}$+k (c) $\frac{y^2-x^2}{2}$+k (d) $\frac{(x-y)^2}{2}$+k

11. (c)

Here u and v are analytic as f(z) is analytic.

$\therefore$ u, v satisfy Cauchy-Riemann equation.

$u_x = v_y$ – (i) and $u_y = -v_x$ – (ii)

Given $u = xy$

$\therefore \quad u_x = y$

$\Rightarrow \quad v_y = y \qquad$ [by (i)]

Integrating

$v = \frac{y^2}{2} + c(x) \qquad$ –(iii)

Again

$v_x = c'(x)$

$\Rightarrow \quad -u_y = c'(x) \qquad$ [by (ii)]

$\Rightarrow \quad -x = c'(x)$

Integreting,

$c(x) = \frac{-x^2}{2} + k$

From (iii) we get

$v = \frac{y^2 - x^2}{2} + k$

12. The modulus of the complex number $\left(\frac{3+4i}{1-2i}\right)$ is. [ME: GATE-2010]

$(a)5 \qquad (b)\sqrt{5} \qquad (c)1/\sqrt{5} \qquad (d)1/5$

12. (b)

$$\frac{3+4i}{1-2i}=\frac{(3+4i)(1+2i)}{(1-2i)(1+2i)}=\frac{-5+10i}{5}=-1+2i$$

$$\therefore \quad \left|\frac{3+4i}{1-2i}\right|=\left|-1+2i\right|=\sqrt{5}$$

CE 10 Years GATE Questions

Q26. For an analytic function, $f(x+iy)=u(x,y)+iv(x,y), u$ is given by $u=3x^2-3y^2$. The expression for v, considering K to be constant is

(a) $3y^2-3x^2+K$ (b) $6x-6y+K$ (c) $6y-6x+K$ (d) $6xy+K$ **[CE-2011]**

Ans. (d)

Exp. Cauchy Riemann equations for

$\mathbf{f(z)=u+iv}$

$\mathbf{u_x=v_y}$ and $\mathbf{u_y=-v_x}$ … (ii)

Given $\mathbf{u=3x^2-3y^2}$

$\Rightarrow \mathbf{u_x=6x \Rightarrow u_y=6x}$ [using (i)]

$\Rightarrow \mathbf{v=6xy+\phi(x)}$ … (iii)

Differentiating w.r.t.x

$\mathbf{v_x=6y+\phi'(x)}$

$\Rightarrow \mathbf{-u_y=6y+\phi'(x)}$

[using (ii)]

$\Rightarrow \mathbf{6y=6y+\phi'(x)}$

$\Rightarrow \mathbf{\phi'(x)=0}$

$\Rightarrow \mathbf{\phi(x)=K}$ (Constant)

$\therefore$ from (iii) we get

$\mathbf{v=6xy+K}$

13. The analytic function f(z) = $\frac{z-1}{z^2+1}$ has singularities at **[CE: GATE – 2009]**

(a) 1 and –1 (b) 1 and i

(c) 1 and –i (d) i and –i

13. (d)

$$f(z)=\frac{z-1}{z^2+1}=\frac{z-1}{(z+i)(z-i)}$$

$\therefore \quad f(z)$ has singularities at $z=i,-i$

14. Using Cauchy's integral theorem, the value of the integral (integration being taken in counter clockwise direction) $\oint_c \frac{z^3-6}{3z-1}dz$ is **[CE: GATE – 2006]**

(a) $\frac{2\pi}{81} - 4\pi i$ (b) $\frac{\pi}{8} - 6\pi i$ (c) $\frac{4\pi}{81} - 6\pi i$ (d) 1

14. (a)

Let $f(z) = \frac{z^3 - 6}{3z - i}$. Here $f(z)$ has a singularities at $z = i/3$

$\therefore$ $\text{Res}(f, \frac{1}{3}) = \lim_{z \to i/3}\left[(z - i/3) \times \frac{z^3 - 6}{3z - i}\right] = \frac{1}{3}.\left(\frac{i^3}{27} - 6\right)$

$\therefore$ $\int_c f(z)dx = 2\pi i \times \frac{1}{3}\left(\frac{i^3}{27} - 6\right) = \frac{2\pi}{81} i^4 - 4\pi i = \frac{2\pi}{81} - 4\pi i$

15. Consider likely applicability of Cauchy's Integral Theorem to evaluate the following integral counter clockwise around the unit circle c. **[CE: GATE – 2005]**

$I = \oint_c \sec z dz,$

z being a complex variable. The value of I will be

(a) I = 0: singularities set = ϕ

(a) $I = 0$: singularities set = $\left\{\pm \frac{2n+1}{2}\pi ; n = 0, 1, 2........\right\}$

(c) $I = \frac{\pi}{2}$: singularities set = $\{\pm n\pi;\ n = 0, 1, 2 ....\}$

(d) None of above

15. Ans. (a)

$$\int \sec z\, dz = \int \frac{1}{\cos z} dz$$

The poles are at $z_0 = \left(n + \frac{1}{2}\right)\pi$

$$= .... \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{+3\pi}{2} ....$$

None of these poles lie inside the unit circle $|z| = 1$

Hence, sum of residues at poles = 0

$\therefore$ Singularities set = ϕ and

I = $2\pi i$ [sum of residues of f(z) at the poles]

= $2\pi i \times 0 = 0$

16. The value of the integral $\int_C \frac{\cos(2\pi z)}{(2z - 1)(z - 3)} dz$ (where C is a closed curve given by $|z| = 1$) is

[CE: GATE – 2009]

(a) $-\pi i$ (b) $\frac{\pi i}{5}$ (c) $\frac{2\pi i}{5}$ (d) πi

16. (c)

Let $f(z) = \dfrac{\cos(2\pi z)}{(2z-1)(z-3)}$. $f(z)$ has singularity at $z = 1/2$

in C $(1z1 = 1)$.

$\therefore \quad \text{Res}(f, 1/2) = \lim\limits_{z\to 1/2}\left[(z-1/2)f(z)\right]$

$= \lim\limits_{z\to 1/2}\left[\dfrac{1}{2}.\dfrac{\cos(2\pi z)}{z-3}\right]$

$= \dfrac{1}{5}$

$\therefore \quad \int\limits_c f(z)dz = \dfrac{2\pi i}{5}$

17. Which one of the following is NOT true for complex number Z_1 and Z_2?

(a) $\dfrac{Z_1}{Z_2} = \dfrac{Z_1\overline{Z}_2}{|Z_2|^2}$ (b) $|Z_1 + Z_2| \le |Z_1| + |Z_2|$ **[CE: GATE – 2005]**

(c) $|Z_1 - Z_2| \le |Z_1| - |Z_2|$ (d) $|Z_1 + Z_2|^2 + |Z_1 - Z_2|^2 = 2|Z_1|^2 + 2|Z_2|^2$

17. Ans. (d)

(a) is true since

$$\frac{Z_1}{Z_2} = \frac{Z_1\overline{Z}_2}{Z_2\overline{Z}_2} = \frac{Z_1\overline{Z}_2}{|Z_2|^2}$$

(b) is true by triangle inequality of complex number.

(c) is not true since $|Z_1 - Z_2| \ge |Z_1| - |Z_2|$

(d) is true since

$$\begin{aligned} |Z_1 + Z_2|^2 &= (Z_1 + Z_2)\,(\overline{Z_1 + Z_2}) \\ &= (Z_1 + Z_2)\,(\overline{Z}_1 + \overline{Z}_2) \\ &= Z_1\overline{Z}_1 + Z_2\overline{Z}_2 + Z_2\overline{Z}_1 + Z_1\overline{Z}_2 \qquad \ldots \text{(i)} \end{aligned}$$

And

$$\begin{aligned} |Z_1 - Z_2|^2 &= (Z_1 + Z_2)\,(\overline{Z_1 - Z_2}) \\ &= (Z_1 - Z_2)\,(\overline{Z}_1 - \overline{Z}_2) \\ &= Z_1\overline{Z}_1 + Z_2\overline{Z}_2 - Z_2\overline{Z}_1 - Z_1\overline{Z}_2 \qquad \ldots \text{(ii)} \end{aligned}$$

Adding (i) and (ii) we get

$$\begin{aligned} |Z_1 + Z_2|^2 + |Z_1 - Z_2|^2 &= 2Z_1\overline{Z}_1 + 2Z_2\overline{Z}_2 \\ &= 2|Z_1|^2 + 2|Z_2|^2 \end{aligned}$$

EE All GATE Questions

Q14. Roots of the algebraic equation $\mathbf{x^3 + x^2 + x + 1 = 0}$ are

(a) $(+1,+j,-j)$ (b) $(+1,-1,+1)$ (c) $(0,0,0)$ (d) $(-1,+j,-j)$ [EE-2011]

Ans. **(d)**

Exp, $\mathbf{x^3 + x^2 + x + 1 = 0}$ ... (1)

Now $\mathbf{f}(-1) = 0$

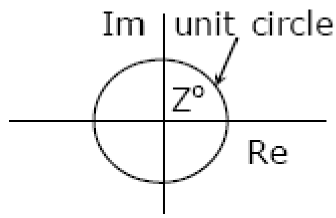
So, $(\mathbf{x+1})$ is a factor of (1)

$\therefore \mathbf{x^3 + x^2 + x + 1 = 0}$

$\Rightarrow \mathbf{x^2(x+1) + 0(x+1) + 1(x+1) = 0}$

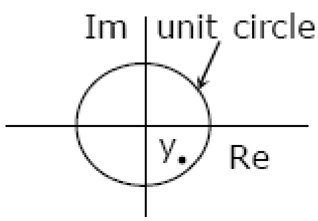
$\Rightarrow \mathbf{(x+1)(x^2+1) = 0 \Rightarrow x = 1, -j, +j}$

Q1. A point z has been plotted in the complex plane, as shown in figure below.

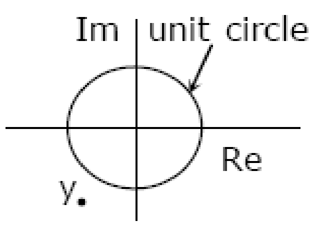


The plot of the complex number $\mathbf{y = \frac{1}{z}}$ [EE-2011]

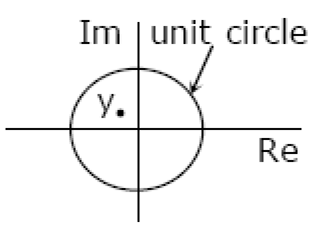
(a)



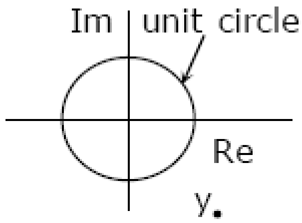
(b)



(c)



(d)



y.

Ans. (d)

18. Given X(z)= $\dfrac{z}{(z-a)^2}$ with $|z| > a$, the residue of X(z) z^{n-1} at z=a for $n \geq 0$ will be

[EE: GATE-2008]

(a) a^{n-1} (b) a^n (c) na^n (d) na^{n-1}

18. Ans. (d)

$$X(z) = \frac{z}{(z-a)^2} \text{ with } |z| > a$$

$$Let\ f(z) = X(z)z^{n-1} = \frac{z^n}{(z-a)^2}$$

$\therefore$ Re*sidue* of F(z) *at* z=a

$$= \frac{1}{1!}\lim_{dz \to a}\frac{d}{dz}[(z-a)^2 F(z)]$$

$$= \lim_{dz \to a}\frac{d}{dz}(z^n)$$

$$F(a) = na^{n-1}$$

19. For the equation, $s^3 - 4s^2 + s + 6 = 0$
The number of roots in the left half of s-plane will be **[EE: GATE-2004]**
(a) zero (b) one (c) two (d) three

19. Ans. (c)
Constructing Routh-array

S^3	1	1	0
S^2	-4	6	0
S^1	2	0	
S^0	6		

Number of sign changes in the first column is two, therefore the number of roots in the left half s-plane is 2

20. The algebraic equation **[EE: GATE-2006]**
$F(s) = s^5 - 3s^4 + 5s^3 - 7s^2 + 4s + 20$ is given $F(s) = 0$ has
(a) a single complex root with the remaining roots being real
(b) one positive real root and four complex roots, all with positive real parts

(c) one negative real root, two imaginary roots, and two roots with positive real parts
(d) once positive real root, two imaginary roots, and two roots with negative real parts

20. Ans. (c)

$F(s)=s^5-3s^2-7s^2+4s=20$

we can solve it by making Routh Hurwitz array.

$$\begin{array}{cccc} s^5 & 1 & 5 & 4 \\ s^4 & -3 & -7 & 20 \\ s^3 & 8/3 & 20/3 & 0 \\ s^2 & 5 & 20 & 0 \\ s^1 & 0 & 0 & 0 \\ s^0 & 20 & 0 & 0 \end{array}$$

We can replace 1st element of s^1 by 10.
If we observe the 1st column, sign is changing two times.
So we have two poles on right half side of imaginary
Axis and $5s^2+20=0$
So, $s = \pm 2j$ and1 pole on left side of imaginary axis .

36. The value of $\oint_C \frac{dz}{(1+z^2)}$ where C is the contour $|z - i/2| = 1$ is **[EE: GATE-2007]**

(a) $2\pi i$
(b) π
(c) $\tan^{-1} z$
(d) $\pi i \tan^{-1} z$

36. Ans (b)

IE All GATE Questions

21. Consider the circle |z – 5 – 5i| = 2 in the complex plane (x, y) with z = x + iy. The minimum distance from the origin to the circle is [IE: GATE-2005]

(a) $5\sqrt{2}-2$ (b) $\sqrt{54}$

(c) $\sqrt{34}$ (d) $5\sqrt{2}$

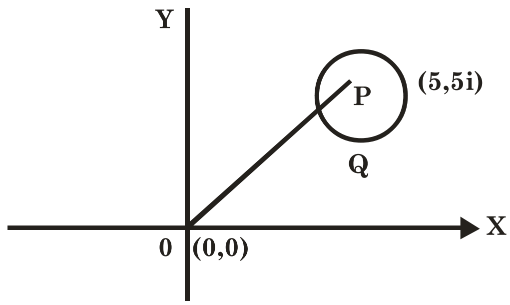
21. (a)

$|z-5-5i|=2$

$\Rightarrow |z-(5+5i)|=2$ represents a circle of radius 2. and center (5,5)

From figure,

$OP=\sqrt{5^2+5^2}=2\sqrt{5}$



OQ is minimum distance from the origin.

$OQ=2\sqrt{5-2}$ as $PQ=\text{radius}=2.$

22. Let $z^3=\bar{z}$, where z is a complex number not equal to zero. Then z is a solution of
[IE: GATE-2005]

(a) $z^2 = 1$ (b) $z^3 = 1$
(c) $z^4 = 1$ (d) $z^9 = 1$

22. Ans. (c)

Given, $z^3 = \bar{z}$

$\Rightarrow$ $z^4 = |z|^2$ (on multiplying z both side)

Now by hit and trial method we see the solution being

$Z^4 = 1$

23. The value of the integral of the complex function **[IE: GATE-2006]**

$$\mathbf{f(s)=\frac{3s+4}{(s+1)\,(s+2)}}$$

Along the path $|s| = 3$ is

(a) $2\pi j$ (b) $4\pi j$
(c) $6\pi j$ (d) $8\pi j$

23. (c)

Given $f(s)=\dfrac{3s+4}{(s+1)(s+2)}$

$f(s)$ has singularities at $s=-1,-2$ which are inside the given circle

$$|s| = 3$$

$$\therefore \quad \text{Re}\,s(f,-1) = \lim_{s\to -1}\left[(s+1)f(s)\right] = 1.$$

$$\text{Re}\,s(f,-2) = \lim_{s\to -2}\left[(s+2)f(s)\right] = 2.$$

$$\int_{|s|=3} f(s)ds = 2\pi j \times (1+2) = 6\pi j$$

24. Let j = $\sqrt{-1}$. Then one value of j^j is **[IE: GATE-2007]**

(a) $\sqrt{j}$ (b) -1 (c) $\frac{\pi}{2}$ (d) $e^{-\frac{\pi}{2}}$

24. (d) same as Q.7

25. The polynomial $p(x) = x^5 + x + 2$ has **[IE: GATE-2007]**

(a) all real roots (b) 3 real and 2 complex roots
(c) 1 real and 4 complex roots (d) all complex roots

25. (c)

Given $f(x) = x^5 + x + 2.$

$P(+x) = +++$ (Taking only sign of each term)

$\Rightarrow P(x)$ has no +ve real roots.

P(-x)= - - + (Taking only sign of each term)

$\Rightarrow$ P(x) has one $-$ve real root

As, P(x) of degree 5 .So other four roots are complex.

26. For the function $\frac{\sin z}{z^3}$ of a complex variable z, the point z = 0 is **[IE: GATE-2007]**

(a) a pole of order 3 (b) a pole of order 2
(c) a pole of order 1 (d) not a singularity

26. (b)

As. $\lim_{z\to 0}\frac{\sin z}{z} = 1.$

Therefore the function has z = 0 is a pole of order 2.

27. It is known that two roots of the nonlinear equation $x^3 - 6x^2 + 11x - 6 = 0$ are 1 and 3. The third root will be **[IE: GATE-2008]**

(a) j (b) –j (c) 2 (d) 4

27. (c)

Let third root be α. of $x^3 - 6x^2 + 11x - 6 = 0$

Then $1 + 3 + \alpha = 6 \Rightarrow \alpha = 2$

28. If z = x + jy, where x and y are real, the value of $|e^{jz}|$ is **[IE: GATE-2009]**

(a) 1 (b) $e\sqrt{x^2 + y^2}$ (c) e^y (d) e^{-y}

28. (d)

$\left|e^{jz}\right| = \left|e^{j(x+jy)}\right| = \left|e^{-y+jx}\right| = \left|e^{-y}e^{jx}\right| = \left|e^{-y}\right|\left|e^{jx}\right|$

$= e^{-y}\left[\because e^{-y} > 0, \text{for all } \ y \in \Re \text{ and} \left|e^{ix}\right| = 1\ \right]$

29. One of the roots of the equation $x^3 = j$, where j is the positive square root of –1, is **[IE: GATE-2009]**

(a) j (b) $\dfrac{\sqrt{3}}{2} + j\dfrac{1}{2}$ (c) $\dfrac{\sqrt{3}}{2} - j\dfrac{1}{2}$ (d) $-\dfrac{\sqrt{3}}{2} - j\dfrac{1}{2}$

29. (b)

$x^3 = j$

$\Rightarrow x^3 = e^{\frac{j\pi}{2}}$

$\Rightarrow x = e^{\frac{j\pi}{6}} = \cos\dfrac{\pi}{6} + j\sin\dfrac{\pi}{6}$

$\Rightarrow \dfrac{\sqrt{3}}{2} + j.\dfrac{1}{2}$

30. The root mean squared value of x(t) = 3 + 2 sin (t) cos (2t) is **[IE: GATE-2008]**

(a) $\sqrt{3}$ (b) $\sqrt{8}$ (c) $\sqrt{10}$ (d) $\sqrt{11}$

30. Ans. (d)

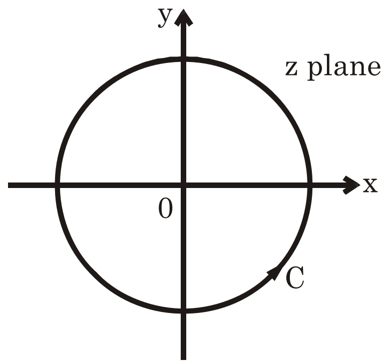
x(t) = 3 + 2 sin t cos 2t

x(t) = 3 + sin 3t – sin t

$\therefore$ Root mean square value = $\sqrt{3^2 + 1^2 + 1^2} = \sqrt{11}$

31. Contour C in the adjoining figure is described by $x^2 + y^2 = 16$.

The value of $\oint_C \dfrac{z^2 + 8}{0.5z - 1.5j}\,dz$ **(Note : j = $\sqrt{-1}$)** **[IE: GATE-2010]**



(a) $-2\pi j$ (b) $2\pi j$ (c) $4\pi j$ (d) $-4\pi j$

31. (d)

Let $f(z) = \dfrac{z^2+8}{0.5z-1.5j} = \dfrac{2(z^2+8)}{z-3j}$

f(z) has a singularity at z=3j which is inside the given circle $x^2+y^2=16$.

$\therefore \ \mathrm{Res}(f,3j) = \lim_{z\to 3j}\left[(z-3j)f(z)\right] = -2$

$\therefore \ \oint f(z)ds = 2\pi j \times (-2) = -4\pi j$

14. Probability and Statistics

Probability & Statistics

1. A fair dice is rolled twice. The probability that an odd number will follow an even number is [EC: GATE-2005]

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

1. (d) Here the sample space S = 6

Therefore $P(\text{odd number}) = \frac{3}{6} = \frac{1}{2}$

and $P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$

since events are independent,

therefore, $P(\text{odd / even}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

2. A probability density function is of the form [EC: GATE-2006]

$p(x) = Ke^{-\alpha|x|}, x \in (-\infty, \infty)$

The value of K is

(a) 0.5 (b) 1 (c) 0.5α (d) α

2. (c)

As (x) is a probability density function

$$\therefore \int_{-\infty}^{\infty} p(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} ke^{-\alpha|x|}dx = 1$$

$$\Rightarrow \int_{-\infty}^{0} ke^{\alpha x}dx + \int_{0}^{\infty} ke^{-\alpha x}dx = 1 \quad \begin{bmatrix} \because |x| = x, \text{for } x > 0 \\ = -x, \text{for } x < 0 \end{bmatrix}$$

$$\Rightarrow k = 0.5\alpha$$

3. Three companies X, Y and Z supply computers to a university. The percentage of computers supplied by them and the probability of those being defective are tabulated below

[EC: GATE-2006]

Company	% of computers supplied	Probability of being defective
X	60%	0.01
Y	30%	0.02
Z	10%	0.03

Given that a computer is defective, the probability that it was supplied by Y is
(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

3. Ans. (d)

4. If E denotes expectation, the variance of a random variable X is given by [EC: GATE-2007]
(a) $E[X^2] - E^2[X]$ (b) $E[X^2] + E^2[X]$
(c) $E[X^2]$ (d) $E^2[X]$

4. ans (a)

$$\text{Varience of } X = E\left\{(X-m)^2\right\}, m = \text{mean of the distribution}$$

$$\therefore \text{Var}(X) = E\left\{\left(X^2 - 2mX + m^2\right)\right\}$$

$$= E\left(X^2\right) - 2mE(X) + m^2$$

$$= E\left(X^2\right) - 2E^2(X) + E^2(X) \quad \left[\because m = E(X), \text{by defination of mean}\right]$$

$$= \mathbf{E(X^2) - E^2(X)}$$

5. An examination consists of two papers, Paper 1 and Paper 2. The probability of failing in Paper 1 is 0.3 and that in Paper 2 is 0.2. Given that a student has failed in Paper 2, the probability of failing in Paper 1 is 0.6. The probability of a student failing in both the papers is [EC: GATE-2007]

(a) 0.5 (b) 0.18
(c) 0.12 (d) 0.06

5.Ans(c).

Let A be the event that 'failed in paper 1'.
B be the event that 'failed in paper 2'.
Given P(A) = 0.3, P(B) = 0.2.

And also given $P\left(\frac{A}{B}\right) = 0.6$

$$\text{we know } P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \quad P(A \cap B) = 0.6 \times 0.2 = 0.12$$

6. $P_x(x) = M\exp(-2|x|) - N\exp(-3|x|)$ is the probability density function for the real random variable X, over the entire x axis. M and N are both positive real numbers. The equation relating M and N is [EC: GATE-2008]

(a) $M - \frac{2}{3}N = 1$ (b) $2M + \frac{1}{3}N = 1$

(c) M + N = 1 (d) M + N = 3

6. Ans.(a)

Given $P_x(x)$ is the probability density function for the random variable X.

$$\Rightarrow \int_{-\infty}^{\infty}\left(Me^{-2|x|}-Ne^{-3|x|}\right)dx=1$$

$$\Rightarrow \int_{-\infty}^{0}\left(Me^{2x}-Ne^{3x}\right)dx+\int_{0}^{\infty}\left(Me^{-2x}+Ne^{-3x}\right)dx=1$$

$$\Rightarrow \left(\frac{M}{2}-\frac{N}{3}\right)+\left(\frac{M}{2}-\frac{N}{3}\right)=1$$

$$\Rightarrow M-\frac{2}{3}N=1$$

7. A fair coin is tossed 10 times. What is the probability that ONLY the first two tosses will yield heads? [EC: GATE-2009]

(a) $\left(\frac{1}{2}\right)^2$ (b) ${}^{10}C_2\left(\frac{1}{2}\right)^3$ (c) $\left(\frac{1}{2}\right)^{10}$ (d) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$

7. (c)

Let A be the event that first toss is head

And B be the event that second toss is head.

$\therefore \quad P(A)=\frac{1}{2}, \qquad P(B)=\frac{1}{2}$

By the given condition rest all 8 tosses should be tail

$\therefore$ The probability of getting head in first two cases

$= \quad \left(\frac{1}{2}\right)^2\cdot\left(\frac{1}{2}\right)^8=\left(\frac{1}{2}\right)^{10}.$

8. A fair coin is tossed independently four times. The probability of the event "the number of time heads shown up is more than the number of times tails shown up" is [EC: GATE-2010]

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{5}{16}$

8. Ans (d)

Here we have to find

$P(H,H,H,T)+P(H,H,H,H)$

$=4c_3\left(\frac{1}{2}\right)^3\cdot\left(\frac{1}{2}\right)+4c_4\left(\frac{1}{2}\right)^4\cdot\left(\frac{1}{2}\right)^0$

$=4.\left(\frac{1}{2}\right)^4+\left(\frac{1}{2}\right)^4=\frac{5}{16}$

ME 20 Years GATE Questions

9 In a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total of 900 bolts are respectively

[ME: GATE-2000]

(a) 90 and 9 (b) 9 and 90

9. Ans (a)

It's a poission distribution. Here $n = 900, p = 0.1$

$\therefore$ $\text{mean}(m) = np = 900 \times 0.1 = 90$

Standard deviation $(\sigma) = \sqrt{npq} = \sqrt{90 \times .9}, \text{Here}\, q = 1 - p.$

$= \sqrt{81} = 9 \quad (\because \sigma > 0).,$

10. Consider the continuous random variable with probability density function

$f(t) = 1 + t$ for $-1 \le t \le 0$

$= 1 - t$ for $0 \le t \le 1$

[ME: GATE-2006]

The standard deviation of the random variables is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{6}}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

10. Ans. (b)

$\text{Var}(T) = \sigma_t^2 = \int_{-\infty}^{\infty} t^2 f(t)dt,$ T being the random variable of f(t).

$= \int_{-1}^{0} t^2(1+t)dt + \int_{0}^{1} t^2(1-t)dt$

$= \frac{1}{6}$

$\therefore \quad \sigma_t \frac{1}{\sqrt{6}} [\because \sigma_t > 0]$

11. The standard deviation of a uniformly distributed random variable between 0 and 1 is

[ME: GATE-2009]

(a) $\frac{1}{\sqrt{12}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{5}{\sqrt{12}}$ (d) $\frac{7}{\sqrt{12}}$

11. (a)

$\text{Here p.d.f. is}\, f(x) = \frac{1}{1-0} = 1, \quad 0 < x < 1.$

$\therefore \quad \text{mean}(m) = E(x) = \int_0^1 x f(x)dx = \int_0^1 x dx = \frac{1}{2}$

$\therefore \quad \text{Var}(x) = \sigma^2 = \int_0^1 \left(\frac{x-1}{2}\right)^2 .1.dx = \int_0^1 \left(x^2 - x + \frac{1}{4}\right)dx = \frac{1}{3} - \frac{1}{2} + \frac{1}{4} = \frac{1}{12}$

$\therefore \sigma = \frac{1}{\sqrt{12}} [\because \sigma > 0]$

12 The probability that two friends share the same birth-month is [ME: GATE-1998]

(a) $\frac{1}{6}$ (b) $\frac{1}{12}$ (c) $\frac{1}{144}$ (d) $\frac{1}{24}$

12. (b)

Let A = the event that the birth month of first friend
And B= that of second friend.

$\therefore \quad P(A) = 1,$ as 1st friend can born in any month

and $P(B) = \frac{1}{12}$, by the condition.

$\therefore$ Probability of two friends share same birth-month

is $1 \times \frac{1}{12} = \frac{1}{12}$

13. The probability of a defective piece being produced in a manufacturing process is 0.01. The probability that out of 5 successive pieces, only one is defective, is

(a) $(0.99)^2 (0.01)$ (b) $(0.99)(0.01)^4$ [ME: GATE-1996]
(c) 5×(0.99)(0.01)4 (d) $5\times(0.99)^4(0.01)$

13. (d)

The required probability $= {}^5c_1(.01)^1 \times (.99)^4 = 5 \times (0.99)^4 \times (.01).$

14. A box contains 5 block balls and 3 red balls. A total of three balls are picked from the box one after another, without replacing them back. The probability of getting two black balls and one red ball is [ME: GATE-1997]

(a) 3/8 (b) 2/15 (c) 15/28 (d) ½

14. (c)

Here the possible combination of picking up three balls without replacement is
BBR, BRB, RBB.
(B = Black ball, R = Red balls)

$\therefore \quad P(BBR) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{5}{28}$

$P(BRB) = \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} = \frac{5}{28}$

$P(RBB) = \frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} = \frac{5}{28}$

$\therefore$ Probability of getting two black balls and one red ball is $\frac{15}{28}$.

15. An unbiased coin is tossed three times. The probability that the head turns up in exactly two cases is [ME: GATE-2001]

(a) 1/9 (b) 1/8 (c) 2/3 (d) 3/8

15. (d)

Required probability $= {}^{3}c_{2} \times \left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{2}\right) = \frac{3}{8}$

16. Two dice are thrown. What is the probability that is the sum of the numbers on the two dice is eight? [ME: GATE-2002]

(a) 5/36 (b) 5/18 (c) ¼ (d) 1/3

16. (a)

Here sample space = 6 × 6 = 36

Here, there are five such points whose sum is 8. They are (2,6), (3,5), (4,4), (5,3), (6,2).

$\therefore$ Requireprobability $= \frac{5}{36}$

17. Manish has to travel from A to D changing buses at stops B and C enroute. The maximum waiting time at either stop can be 8 minutes each, but any time of waiting up to 8 minutes is equally likely at both places. He can afford up to 13 minutes of total waiting time if he is to arrive at D on time. What is the probability that Manish will arrive late at D? [ME: GATE-2002]

(a) 8/13 (b) 13/64 (c) 119/128 (d) 9/128

17.Ans(a)

18. Arrivals at a telephone booth are considered to be poison, with an average time of 10 minutes between successive arrivals. The length of a phone call is distributes exponentially with mean 3 minutes. The probability that an arrival does not have to wait before service is [ME: GATE-2002]

(a) 0.3 (b) 0.5 (c) 0.7 (d) 0.9

18.Ans(a)

19. A box contains 5 black and 5 red balls. Two balls are randomly picked one after another from the box, without replacement. The probability for both balls being red is [ME: GATE-2003]

(a) $\frac{1}{90}$ (b) $\frac{1}{2}$ (c) $\frac{19}{90}$ (d) $\frac{2}{9}$

19. (d)

The probability of drawing two red balls without replacement

$= \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$

20. From a pack of regular from a playing cards, two cards are drawn at random. What is the probability that both cards will be Kings, if first card in NOT replaced

(a) $\frac{1}{26}$ (b) $\frac{1}{52}$ (c) $\frac{1}{169}$ (d) $\frac{1}{221}$ [ME: GATE-2004]

20. (d)

Here sample space S = 52

$\therefore$ The probability of drawing both cards are king without replacement

$$= \frac{^4c_1}{52} \times \frac{^3c_1}{51} = \frac{1}{221}$$

21. A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is [ME: GATE-2005]

(a) 0.0036 (b) 0.1937 (c) 0.2234 (d) 0.3874

21.(b)

Let A be the event that items are defective and B be the event that items are non- defective.

$\therefore P(A) = 0.1$ and $P(B) = 0.9$

$\therefore$ Probability that exactly two of those items are defective

$= \ ^{10}c_2 (.1)^2 (.9)^8 = 0.1937$

22. A single die is thrown twice. What is the probability that the sum is neither 8 nor 9?

[ME: GATE-2005]

(a) 1/9 (b) 5/36 (c) 1/4 (d) 3/4

22. (d)

Here sample space = 36

Total No. of way in which sum is either 8 or 9 are

(2,6), (3,5),(3,6), (4,4), (4,5), (5,3), (5,4), (6,2), (6,3)

So probability of getting sum 8 or 9 = $\frac{9}{36} = \frac{1}{4}$

So the probability of not getting sum 8 or 9 $= 1 - \frac{1}{4} = \frac{3}{4}$.

24. A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective?

[ME: GATE-2006]

(a) 1/5 (b) 1/25 (c) 20/99 (d) 11/495

24(d)

The probability of defective items $= \frac{20}{100}$.

Therefore the probability of first two defective items without replacement

$$= \frac{20}{100} \times \frac{19}{99} = \frac{19}{495}.$$

25. A coin is tossed 4 times. What is the probability of getting heads exactly 3 times?

(a) $\frac{1}{4}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ [ME: GATE-2008

25. (a)

Probability of getting exactly three heads

$$= {}^4c_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right) = 4 \times \frac{1}{2^4} = \frac{1}{4}$$

26. If three coins are tossed simultaneously, the probability of getting at least one head is [ME: GATE-2009]

(a) 1/8 (b) 3/8 (c) 1/2 (d) 7/8

26. (d)

Here the sample space $S = 2^3 = 8$.
No. of ways to get all tails =1.

$\therefore$ probability to get all tails $= \frac{1}{8}$

$\therefore$ Probability to get at least one head is $=1-\frac{1}{8} = \frac{7}{8}$

27. A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability **of** drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is [ME: GATE-2010]

(a) 2/315 (b) 1/630 (c) 1/1260 (d) 1/2520

27. (c)

Here sample space = 9
The required probability of drawing 2 washers, 3 nuts and 4 bolts respectively without replacement

$$= \frac{2}{9} \times \frac{1}{8} \times \frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4} \times \frac{3}{3} \times \frac{2}{2} \times \frac{1}{1}$$

$$= \frac{1}{1260}$$

28. If 20 per cent managers are technocrats, the probability that a random committee of 5 managers consists of exactly 2 technocrats is [ME: GATE-1993]

(a) 0.2048 (b) 0.4000 (c) 0.4096 (d) 0.9421

28. (a)

The probability of technocrats manager $= \frac{20}{100} = \frac{1}{5}$

$\therefore$ Probability of non technocrats manager $= \frac{4}{5}$

Now the require probability $= {}^5c_2 \times \left(\frac{1}{5}\right)^2 \times \left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)^3 = 0.2048$

29. Analysis of variance is concerned with: [ME: GATE-1999]
(a) Determining change in a dependent variable per unit change in an independent variable
(b) Determining whether a qualitative factor affects the mean of an output variable
(c) Determining whether significant correlation exists between an output variable and an input variable.
(d) Determining whether variance in two or more populations are significantly different.

29. Ans.(d)
Analysis of variance is used in comparing two or more populations, e.g. Different types of manures for yelding a single crop.

30. Four arbitrary point (x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4), are given in the x, y – plane Using the method of least squares, if, regressing y upon x gives the fitted line y = ax + b; and regressing y upon x given the fitted line y = ax + b; and regressing x upon y gives the fitted line x = cy + d then [ME: GATE-1999]
(a) The two fitted lines must coincide (b) The two fitted lines need not coincide
(c) It is possible that ac = 0 (d) A must be 1/c

30. (d)

$y = ax + b - (i)$ and $x = cy + d - (ii)$

From (ii) we get $x - d = cy \Rightarrow y = \frac{1}{c}x - \frac{d}{c} - (iii)$

comparing (i) and (ii), $a = \frac{1}{c}$ and $b = \frac{-d}{c}$

31. A regression model is used to express a variable Y as a function of another variable X. This implies that [ME: GATE-2002]
(a) There is a causal relationship between Y and X
(b) A value of X may be used to estimate a value of Y
(c) Values of X exactly determine values of Y
(d) There is no causal relationship between Y and X

31. (b)

32. Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is FALSE? [ME: GATE-2007]
(a) E (XY) = E (X) E (Y) (b) Cov (X, Y) = 0
(c) Var (X + Y) = Var (X) + Var (Y) (d) $E(X^2 y^2) = (E(X))^2 (E(y))^2$

32. (b).

CE 10 Years GATE Questions

33. A class of first year B. Tech. Students is composed of four batches A, B, C and D, each consisting of 30 students. It is found that the sessional marks of students in Engineering Drawing in batch C have a mean of 6.6 and standard deviation of 2.3. The mean and standard deviation of the marks for the entire class are 5.5 and 4.2, respectively. It is decided by the course instructor to normalize the marks of the students of all batches to have the same mean and standard deviation as that of the entire class. Due to this, the marks of a student in batch C are changed from 8.5 to

(a) 6.0 (b) 7.0 **[CE: GATE – 2006]**
(c) 8.0 (d) 9.0

33.Ans(d). Let mean and stander deviation of batch C be μ_c and σ_c respectively and mean and standard deviation of entire class of 1st year students be μ and σ respectively.

Given $\mu_c = 6.6$ and $\sigma_c = 2.3$

and $\mu = 5.5$ and $\sigma = 4.2$

In order to normalize batch C to entire class, the normalized score must be equated

Since $Z = \dfrac{x-\mu}{\sigma}$

$$Z_c = \frac{x_c - \mu_c}{\sigma_c} = \frac{8.5 - 6.6}{2.3}$$

Now $Z = \dfrac{x-\mu}{\sigma} = \dfrac{x-5.5}{4.2}$

$$\therefore Z = Z_c \Rightarrow \frac{x-5.5}{4.2} = \frac{8.5-6.6}{2.3}$$

$$\Rightarrow x = 8.969 \cong 9.0$$

34. Three values of x and y are to be fitted in a straight line in the form y = a + bx by the method of least squares. Given $\Sigma x = 6$, $\Sigma y = 21$, $\Sigma x^2 = 14$ and $\Sigma xy = 46$, the values of a and b are respectively. **[CE: GATE – 2008]**

(a) 2 and 3 (b) 1 and 2
(c) 2 and 1 (d) 3 and 2

34.Ans(d)

$y = a + bx$

Given

$n = 3, \Sigma x = 6, \Sigma y = 21, \Sigma x^2 = 14$

And $\Sigma xy = 46$

$$b = \frac{n\Sigma xy - \Sigma x\,\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$a = \bar{y} - b\bar{x}$$

$$= \frac{\Sigma y}{n} - b\frac{\Sigma x}{n}$$

Substituting, we get

$$b = \frac{(3\times 46)-(6\times 21)}{(3\times 14)-(6)^2} = 2$$

$$a = \frac{21}{3} - 2\times\left(\frac{6}{3}\right) = 3$$

$\therefore$ $a = 3$ and $b = 2$

35. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be

(a) 100% (b) 50% **[CE: GATE – 2003]**
(c) 49% (d) None of these

35. (d)

Non defective screws =7

$\therefore$ Probability of the two screws are non defective

$$= \frac{{}^3c_0 \times {}^7c_2}{{}^{10}c_2} \times 100\%$$

$$= \frac{7}{15} \times 100\% = 46.6 \simeq 47\%$$

36. A hydraulic structure has four gates which operate independently. The probability of failure off each gate is 0.2. Given that gate 1 has failed, the probability that both gates 2 and 3 will fail is **[CE: GATE – 2004]**

(a) 0.240 (b) 0.200
(c) 0.040 (d) 0.008

36.(c)

P(gate to and gate 3/gate 1 failed)

$$\begin{aligned} &= P(\text{gate}\,2 \text{ and gate } 3) \\ &= P(\text{gate}\,2) \times P(\text{gate } 3) \\ &= 0.2\times 0.2 = 0.04 \end{aligned} \qquad \left[\begin{aligned} &\therefore \text{ all three gates are} \\ &\text{independent corrosponding} \\ &\text{to each other} \end{aligned}\right]$$

37. Which one of the following statements is NOT true?

(a) The measure of skewness is dependent upon the amount of dispersion
(b) In a symmetric distribution, the values of mean, mode and median are the same
(c) In a positively skewed distribution; mean > median > mode
(d) In a negatively skewed distribution; mode > mean > median **[CE: GATE – 2005**

37. (d)

(d) is not true since in a negatively skewed distribution, **mode > median > mean**

38. There are 25 calculators in a box. Two of them are defective. Suppose 5 calculators are randomly picked for inspection (i.e., each has the same chance of being selected), what is the probability that only one of the defective calculators will be included in the inspection?

[CE: GATE – 2006]

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

38. (b)

Probability of only one is defective out of 5 calculators

$$= \frac{{}^{2}c_1 \times {}^{23}c_4}{{}^{25}c_5} = \frac{1}{3}$$

39. If the standard deviation of the spot speed of vehicles in a highway is 8.8 kmph and the mean speed of the vehicles is 33 kmph, the coefficient of variation in speed is

[CE: GATE – 2007]

(a) 0.1517 (b) 0.1867 (c) 0.2666 (d)0.3645

39. (c)

$$C\nu = \frac{\sigma}{\mu} = \frac{8.8}{33} = 0.2666$$

40. If probability density functions of a random variable X is

$f(x) = x^2$ for $-1 \le x \le 1$, and **[CE: GATE – 2008]**

$= 0$ for any other value of x

Then, the percentage probability $P\left(-\frac{1}{3} \le x \le \frac{1}{3}\right)$ is

(a) 0.247 (b) 2.47 (c) 24.7 (d) 247

40. (b)

$$P\left(\frac{-1}{3} \le x \le \frac{1}{3}\right) = \int_{-\frac{1}{3}}^{\frac{1}{3}} x^2 dx \qquad = \left[\frac{x^3}{3}\right]_{-\frac{1}{3}}^{\frac{1}{3}} = \frac{2}{81}$$

$$\therefore \quad \text{Percentage probability} = \frac{2}{81} \times 100 \simeq 2.47\%$$

41. A person on a trip has a choice between private car and public transport. The probability of using a private car is 0.45. While using the public transport, further choices available are bus and metro, out of which the probability of commuting by a bus is 0.55. In such a situation, the probability (rounded up to two decimals) of using a car, bus and metro, respectively would be

[CE: GATE – 2008]

(a) 0.45, 0.30 and 0.25 (b) 0.45, 0.25 and 0.30

(c) 0.45, 0.55 and 0.00 (d) 0.45, 0.35 and

41. (a)

Given

p(private car) = 0.45
p(bus / public transport) = 0.55

Since a person has a choice between private car and public transport

$$p\,(\text{public transport}) = 1 - p(\text{private car}) = 1 - 0.45 = 0.55$$

$$p\,(\text{bus}) = p(\text{bus} \cap \text{public transport}) = p(\text{bus} \mid \text{public transport}) \times p(\text{public transport}) = 0.55 \times 0.55 = 0.3025 \simeq 0.30$$

Now $p\,(\text{metro}) = 1 - [p(\text{private car}) + p(\text{bus})] = 1 - (0.45 + 0.30) = 0.25$

$\therefore$ p (private car) = 0.45
p (bus) = 0.30
and p(metro) = 0.25

42. The standard normal probability function can be approximated as

$$F(x_N) = \frac{1}{1 + \exp(-1.7255\, x_n\, |x_n|^{0.12})}$$ **[CE: GATE – 2009]**

Where x_N = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be between 90 cm and 102 cm is

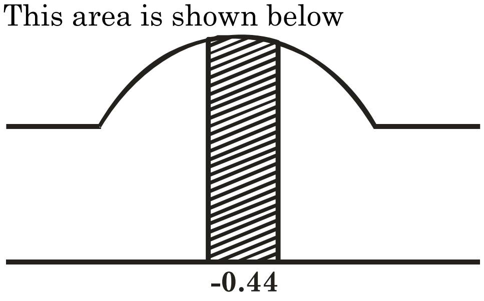
(a) 66.7% (b) 50.0% (c) 33.3% (d) 16.7%

42. (d)

Here $\mu = 102\text{cm}$ and $\sigma = 27\text{cm}$

$$P(90 \le x \le 102) = P\left(\frac{90-102}{27} \le x \le \frac{102-102}{27}\right) = P(-0.44 \le x \le 0)$$

This area is shown below



The shaded area in above figure is given by $F(0) - F(-0.44)$

$$= \frac{1}{1+\exp(0)} - \frac{1}{1+\exp(-1.7255 \times (-0.44) \times (0.44)^{0.12})}$$
$$= 0.5 - 0.3345$$
$$= 0.1655 \simeq 16.55\%$$

43. Two coins are simultaneously tossed. The probability of two heads simultaneously appearing is **[CE: GATE – 2010]**

(a) $\frac{1}{8}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

43.(c)

Probability of two head $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

Q3. There are two containers, with one containing 4 Red and 3 Green balls and the other containing Blue and 4 Green balls. One bal is drawn at random form each container. The probability that one of the ball is Red and the other is Blue will be

(a) 1/7 (b) 9/49 (c) 12/49 (d) 3/7 **[CE-2011]**

Ans. (c)

EE All GATE Questions

45. A fair coin is tossed three times in succession. If the first toss produces a head, then the probability of getting exactly two heads in three tosses is **[EE: GATE-2005]**

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

45.(d)

After first head in first toss, probability of tails in 2^{nd} and 3^{rd} toss $=\frac{1}{2}.\frac{1}{2}=\frac{1}{4}$

$\therefore$ Probability of exactly two heads $=1-\frac{1}{4}=\frac{3}{4}$

46. Two fair dice are rolled and the sum r of the numbers turned up is considered

[EE: GATE-2006]

(a) $\Pr(r>6)=\frac{1}{6}$ (b) Pr(r/3 is an integer)= $\frac{5}{6}$

(c) Pr(r=8 | r/4 is an integer) $=\frac{5}{9}$ (d) Pr(r=6 | r/5 is an integer)= $\frac{1}{18}$

46. (c)

47. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that the first removed ball is white, the probability that the second removed ball is red is **[EE: GATE-2010]**

(a) 1/3 (a) 3/7
(a) 1/2 (a) 4/7

47. (c)

After first ball is drawn white then sample space has 4 + 3 – 1 = 6 balls.
Probability of second ball is red without replacement

$$= \frac{{}^3c_0 \times {}^3c_1}{6} = \frac{1}{2}$$

14. X is a uniformly distributed random variable that takes values between 0 and 1. The value of E{X^3} will be **[EE: GATE-2008]**

(a) 0 (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

14. Ans. (c)

$$f_x(x) = \begin{cases} 1, 0 < x < 1 \\ 0, other \text{ wise} \end{cases}$$

$$E(X^3) = \int_{-\infty}^{\infty} x^3 f_x(x)dx = \int_0^1 x^3 dx = \left.\frac{x^4}{4}\right|_0^1$$

$$= \frac{1}{4} - 0 = \frac{1}{4}$$

IE All GATE Questions

48. Consider a Gaussian distributed radom variable with zero mean and standard deviation σ. The value of its cumulative distribution function at the origin will be **[IE: GATE-2008]**

(a) 0 (b) 0.5 (c) 1 (d) 10σ

48 Ans. (b)

49. A random variable is uniformly distributed over the interval 2 to 10. Its variance will be **[IE: GATE-2008]**

(a) $\frac{16}{3}$ (b) 6 (c) $\frac{256}{9}$ (d) 36

49. (a)

The p.d.f $f(x) = \frac{1}{10-2} = \frac{1}{8}$, $x \in (2,10)$

mean of $x = E(x) = \int_2^{10} \frac{1}{8} x\, dx = \frac{1}{8}\left[\frac{x^2}{2}\right]_2^{10} = \frac{1}{16}.96 = 6.$

Varience of x $= (\sigma_x^2) = E\left[(x-6)^2\right]$

$= \int_2^{10} (x-6)^2 \frac{1}{8} dx = \frac{1}{8}\left[\frac{x^3}{3} - \frac{12x^2}{2} + 36x\right]$

$= \frac{16}{3}$

50. The probability that there are 53 Sundays in a randomly chosen leap year is

(a) $\frac{1}{7}$ (b) $\frac{1}{14}$ (c) $\frac{1}{28}$ (d) $\frac{2}{7}$

[IE: GATE-2005]

50. (d)

No. of days in a leap year are 366 days. In which there are 52 complete weeks and 2 days extra.

This 2 days may be of following combination.

1. Sunday & Monday
2. Monday & Tuesday
3. Tuesday & Wednesday
4. Wednesday & Thursday
5. Thursday & Friday
6. Friday & Saturday
7. Saturday & Sunday

There are two combination of Sunday in (1.) and (7).

$\therefore$ Required probability

$= \frac{2}{7}$

51. You have gone to a cyber-café with a friend. You found that the cyber-café has only three terminals. All terminals are unoccupied. You and your friend have to make a random choice of selecting a terminal. What is the probability that both of you will NOT select the same terminal? [IE: GATE-2006]

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1

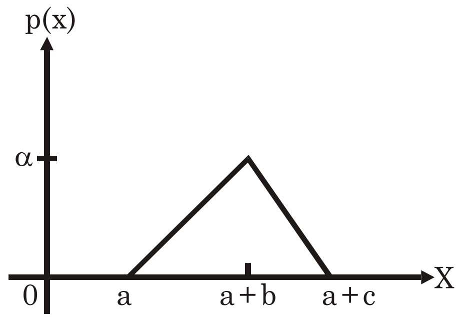
51.(c)

Out of three terminals probability of selecting terminals of two friends is $= \frac{1}{3}$

$\therefore$ Probability of not selecting same terminal $= 1 - \frac{1}{3} = \frac{2}{3}$

52. Probability density function p(x) of a random variable x is as shown below. The value of α is **[IE: GATE-2006]**

(a) $\frac{2}{c}$ (b) $\frac{1}{c}$ (c) $\frac{2}{(b+c)}$ (d) $\frac{1}{(b+c)}$



52.(a) p(x) is p.d.f. $\Rightarrow \int_{-\infty}^{\infty} p(x)dx = 1$

From figure, area of traiangle $= \frac{1}{2}.c.\alpha = \frac{\alpha c}{2}$

$\therefore \quad \frac{\alpha c}{2} = 1 \quad \Rightarrow \alpha = \frac{2}{c}$

53. Two dices are rolled simultaneously. The probability that the sum of digits on the top surface of the two dices is even, is **[IE: GATE-2006]**

(a) 0.5 (b) 0.25 (c) 0.167 (d) 0.125

53. (a)

Here sample space S= 6 × 6 = 36

Total no. of way in which sum of digits on the top surface of the two dice is is even is 18.

$\therefore$ The require probability $= \frac{18}{36} = 0.5$.

55. Poisson's ratio for a metal is 0.35. Neglecting piezo-resistance effect, the gage factor of a strain gage made of this metal is **[IE: GATE-2010]**

(a) 0.65 (b) 1 (c) 1.35 (d) 1.70

55. (d)

Poission's ratio $\sigma = 0.36$

Gage factor, $Gr = 1 + 2\sigma = 1 + 2 \times 0.35 = 1.70$

56. Assume that the duration in minutes of a telephone conversation follows the exponential distribution f(x) = $\frac{1}{5}e^{-\frac{x}{5}}, x \geq 0$. The probability that the conversation will exceed five minutes is **[IE: GATE-2007]**

(a) $\frac{1}{e}$ (b) $1-\frac{1}{e}$ (c) $\frac{1}{e^2}$ (d) $1-\frac{1}{e^2}$

56. (a)

Required probability $= \int_5^{\infty} \frac{1}{5}e^{-\frac{x}{5}}dx = \frac{1}{e}$

22. Using the given data points tabulated below, a straight line passing through the origin is fitted using least squares method. The slope of the line is

x	1.0	2.0	3.0
y	1.5	2.2	2.7

[IE: GATE-2005]

(a) 0.9 (b) 1.0
(c) 1.1 (d) 1.5

22. Ans.(c)

Suppose the line being, $y = mx$

Since, it has been fit by least square method, therefore

$$\sum y = \mu \sum x, \text{ and} \sum xy = \mu \sum x^2$$

$\therefore$ $m = 1.1$

23. The function y = sin ϕ, **(ϕ > 0)** is approximated as y = ϕ, where ϕ is in radian. The maximum value of ϕ for which the error due to the approximation is with in **±2% is**

[IE: GATE-2006]

(a) 0.1 rad (b) 0.2 rad
(c) 0.3 rad (d) 0.4 rad

23. Ans.(c)

CS All GATE Questions

Q3. If two fair coins are flipped and at least one of the outcome is know to be a head, what is the probability that both outcomes are heads?

(a) 1/3 (b) ¼ (c) ½ (d) 2/3 [CS-2011]

Ans. (c)

Q18. If the difference between the expectation of the square of a random variable $\left(E[X]\right)^2$ is denoted by R, then

R = 0 (b) R< 0 (c) **R ≥ 0** (d) R > 0 [CS-2011]

(a)

Ans. (c)

Exp. We know,

The second control momnt,

$$\mu_2 = E\left\{(X-m)\right\}^2 \left[\mathbf{m = mean\ of\ the\ distribution\ of\ X}\right]$$

$$= E\left(X^2\right) - 2m \times E(X) + m^2$$

$$= E\left(X^2\right) - 2\left[E(X)\right]^2 + E(X) \qquad \left[\because m = E(X)\right]$$

$$E\left(X^2\right) - \left[E(X)\right]^2$$

$$\mu_2 \geq 0$$

$$\therefore E\left(X^2\right) - \left[E(X)\right]^2 \geq 0$$

Q34. A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card?

(a) 1/5 (b) 4/25 (c) ¼ (d) 2/5 [CS-2011]

Ans. *

57. For each element is a set of size 2n, an unbiased coin is tossed. The 2n coin tossed are independent. An element is chosen if the corresponding coin toss were head. The probability that exactly n elements are chosen is [CS: GATE-2006]

(a) $\dfrac{\binom{2n}{n}}{4^n}$ (b) $\dfrac{\binom{2n}{n}}{2^n}$ (c) $\dfrac{1}{\binom{2n}{n}}$ (d) $\dfrac{1}{2}$

57.(a)

The probability that exactly n elements are chosen

= the probability of getting n heads out of 2n tosses

$$= {}^{2n}c_n \left(\frac{1}{2}\right)^n \times \left(\frac{1}{2}\right)^{2n-n}$$

$$= \frac{{}^{2n}c_n}{2^{2n}}$$

$$= \frac{{}^{2n}c_n}{4^n}$$

59. Suppose we uniformly and randomly select a permutation from the 20! permutations of 1, 2, 3 ….., 20. What is the probability that 2 appears at an earlier position that any other even number in the selected permutation? [CS: GATE-2007]

(a) $\frac{1}{2}$ (b) $\frac{1}{10}$ (c) $\frac{9!}{20!}$ (d) None of these

59. (d)

Number of permutations with '2' in the first position = 19!

Number of permutations with '2' in the second position = 10 × 18!

(fill the first space with any of the 10 odd numbers and the 18 spaces after the 2 with 18 of the remaining numbers in 18! ways)

Number of permutations with '2' in 3^{rd} position = 10 × 9 × 17!

(fill the first 2 places with 2 of the 10 odd numbers and then the remaining 17 places with remaining 17 numbers)

and so on until '2' is in 11^{th} place. After that it is not possible to satisfy the given condition, since there are only 10 odd numbers available to fill before the '2'. So the desired number of permutations which satisfies the given condition is

19! + 10 × 18! + 10 × 9 × 17! + 10 × 9 × 8 × 16! + … + 10! × 9!

Now the probability of this happening is given by

$$\frac{19!+10\times 18!+10\times 9\times 17!\ldots+10!\times 9!}{20!}$$

Which is clearly not choices (a), (b) or (c)

Thus, Answer is (d) none of these.

60. Aishwarya studies either computer science or mathematics everyday. if the studies computer science on a day, then the probability that she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability that she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probability that she studies computer science on Wednesday? [CS: GATE-2008]

(a) 0.24 (b) 0.36 (c) 0.4 (d) 0.6

60. (c)

Let C denote computes science study and M denotes maths study.

P(C on monday and C on wednesday)

= p(C on monday, M on tuesday and C on wednesday)

+ p(C on monday, C on tuesday and C on wednesday)

= 1 × 0.6 × 0.4 + 1 × 0.4 × 0.4

= 0.24 + 0.16

= 0.40

61. Let X be a randon variable following normal distribution with mean +1 and variance 4. Let Y be another normal variable with mean –1 and variance unknown. If $P(X \le -1) = P(Y \ge 2)$ the standard deviation of Y is [CS: GATE-2008]

(a) 3 (b) 2 (c) $\sqrt{2}$ (d) 1

61. Ans. (a) Given $\Psi_x = 1,\ \sigma_x^2 = 4 \Rightarrow \sigma_x = 2$ and $\mu_Y = -1,\ \sigma_Y$ is unknown

given, $p(X \le -1) = p(Y \ge 2)$

Converting into standard normal variates,

$$p\left(z \le \frac{-1-\mu_x}{\sigma_x}\right) = p\left(z \ge \frac{2-\mu_y}{\sigma_y}\right)$$

$$p\left(z \le \frac{-1-1}{2}\right) = p\left(z \ge \frac{2-(-1)}{\sigma_y}\right)$$

$$P(z \le -1) = p\left(z \ge \frac{3}{\sigma_y}\right) \quad \ldots \text{(i)}$$

Now since us know that in standard normal distribution,

$$P(z \le -1) = p(z \ge 1) \quad \ldots \text{(ii)}$$

Comparing (i) and (ii) we can say that

$$\frac{3}{\sigma_y} = 1 \Rightarrow \sigma_y = 3$$

62. An unbalanced dice (with 6 faces, numbered from 1 to 6) is thrown. The probability that the face value is odd is 90% of the probability that the face value is even. The probability of getting any even numbered face is the same.

If the probability that the face is even given that it is greater than 3 is 0.75, which one of the following options is closed to the probability that the face value exceeds 3? [CS: GATE-2009]

(a) 0.453 (b) 0.468 (c) 0.485 (d) 0.492

62. (b)

It is given that

$P(\text{odd}) = 0.9\ p(\text{even})$

Now since $\Sigma p(x) = 1$

$\therefore\quad p(\text{odd}) + p\ (\text{even}) = 1$

$\Rightarrow\quad 0.9\ p(\text{even}) + p\ (\text{even}) = 1$

$\Rightarrow\quad p(\text{even}) = \frac{1}{1.9} = 0.5263$

Now, it is given that p (any even face) is same

i.e $p(2) = p(4) = p(6)$

Now since,

$$p(\text{even}) = p(2) \text{ or } p(4) \text{ or } p(6)$$
$$= p(2) + p(4) + p(6)$$

$$\therefore\quad p(2) = p(4) = p(6) = \frac{1}{3}p(\text{even})$$
$$= \frac{1}{3}(0.5263)$$

$$= 0.1754$$

It is given that

$$p(\text{even} \mid \text{face} > 3) = 0.75$$

$$\therefore \quad \frac{p(\text{even} \cap \text{face} > 3)}{p(\text{face} > 3)} = 0.75$$

$$\Rightarrow \quad \frac{p(\text{face} = 4, 6)}{p(\text{face} > 3)} = 0.75$$

$$\Rightarrow \quad p(\text{face} > 3) = \frac{p(\text{face} = 4, 6)}{0.75} = \frac{p(4) + p(6)}{0.75}$$

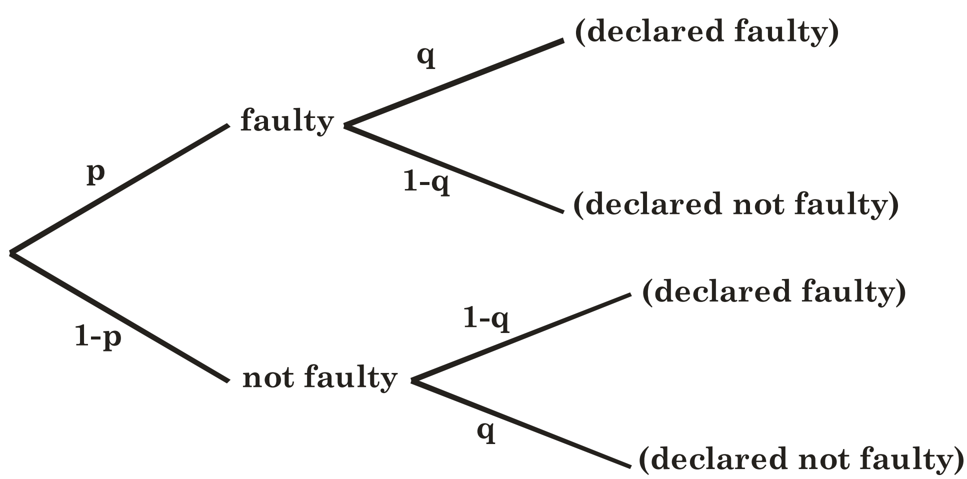
$$= \frac{0.1754 + 0.1754}{0.75}$$

$$= 0.4677 \simeq 0.468$$

63. Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q.
What is the probability of a computer being declared faulty? [CS: GATE-2010]

(a) $pq + (1 - p)(1 - q)$ (b) $(1 - q)p$ (c) $(1 - p)q$ (d) pq

63.(a)



From the diagram,

$$P(\text{declared faulty}) = pq + (1 - p)(1 - q)$$

64. What is the probability that a divisor of 10^{99} is a multiple of 10^{96}? [CS: GATE-2010]

(a) $\frac{1}{625}$ (b) $\frac{4}{625}$ (c) $\frac{12}{625}$ (d) $\frac{16}{625}$

64. Ans. (a)

p(multiple of 10% | divisor of 10^{99})

$$= \frac{n(\text{multiple of } 10^{96} \text{and divisor of } 10^{99)}}{n(\text{divisor of } 10^{99})}$$

Since $10 = 2.5$

$10^{99} = 2^{99} \, . \, 5^{99}$

Any divisor of 10^{99} is of the form $2^a \, . \, 5^b$ where $0 \le a \le 99$ and $0 \le b \le 99$.
The number of such possibilities is combination of 100 values of a and 100 values of b = 100 × 100 each of which is a divisor of 10^{99}.
So, no. of divisors of $10^{99} = 100 \times 100$.
Any number which is a multiple of 10^{96} as well as divisor of 10^{99} is of the form $2^a \, . \, 5^b$ where $96 \le a \le 99$ and $96 \le b \le 99$. The number of such combinations of 4 values of a and 4 values of b is 4 × 4 combinations, each of which will be a multiple of 10^{96} as well as a divisor of 10^{99}.
$\therefore$ p(multiple of 10^{96} | divisor of 10^{99})

$$= \frac{4 \times 4}{100 \times 100} = \frac{1}{625}$$

65. Let P(E) denote the probability of the even E. Given P(A) = 1, P(B) = $\frac{1}{2}$, the values of $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$ respectively are [CS: GATE-2003]

(a) $\frac{1}{4}, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{1}{4}$ (c) $\frac{1}{2}, 1$ (d) $1, \frac{1}{2}$

65.(d)

Here, $P(A) = 1, P(B) = \frac{1}{2}$

Since A, B are independent events,

$\therefore \quad P(AB) = P(A)P(B)$

$$P\left(\frac{A}{B}\right) = \frac{P(AB)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A) = 1$$

$$P\left(\frac{B}{A}\right) = \frac{P(A)P(B)}{P(A)} = P(B) = \frac{1}{2}$$

66. A program consists of two modules executed sequentially. Let f_1 (t) and f_2 (t) respectively denote the probability density functions of time taken to execute the two modules. The probability density function of the overall time taken to execute the program is given by [CS: GATE-2003]

(a) $f_1(t) + f_2(t)$ (b) $\int_0^t f_1(x) f_2(x) dx$

(c) $\int_0^t f_1(x) f_2(t - x) dx$ (d) $\max\{f_1(t), f_2(t)\}$

66.(c)

Let the time taken for first and second modules be represented by x and y and total time = t.

and y and total time = t.

$\therefore t = x + y$ is a random variable

Now the joint density function

$$g(t) = \int_0^t f(x, y)\, dx$$

$$= \int_0^t f(x, t - x)\, dx$$

$$= \int_0^t f_1(x)\, f_2(t - x)\, dx$$

which is also called as convolution of f_1 and f_2, abbreviated as $f_1 * f_2$.

Correct answer is therefore, choice (c).

67. If a fair coin is tossed four times. What is the probability that two heads and two tails will result? [CS: GATE-2004]

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{5}{8}$ (d) $\frac{3}{4}$

67. (a)

Here $P(H) = P(T) = \frac{1}{2}$

It's a Bernoulli's trials.

$\therefore$ Required probability

$$= {}^4c_2 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^2$$

$$= \frac{{}^4c_2}{2^4} = \frac{3}{8}$$

68. An examination paper has 150 multiple-choice questions of one mark each, with each question having four choices. Each incorrect answer fetches – 0.25 mark. Suppose 1000 students choose all their answers randomly with uniform probability. The sum total of the expected marks obtained all these students is [CS: GATE-2004]

(a) 0 (b) 2550 (c) 7525 (d) 9375

68. (d)

Let the marks obtained per question be a random variable X. It's probability distribution table is given below:

X	1	-0.25
P (X)	$\frac{1}{4}$	$\frac{3}{4}$

Expected mark per question $= E(x) = \sum x\,p(x)$

$$= 1 \times \frac{1}{4} + (-0.25) \times \frac{3}{4} = \frac{1}{16} \text{ marks}$$

Total marks expected for 150 questions

$$= \frac{1}{16} \times 150 = \frac{75}{8} \text{ marks per student.}$$

Total expected marks of 1000 students

$$= \frac{75}{8} \times 1000 = 9375 \text{ marks}.$$

69. Two n bit binary strings, S_1 and S_2 are chosen randomly with uniform probability. The probability that the Hamming distance between these strings (the number of bit positions where the two strings differ) is equal to d is [CS: GATE-2004]

(a) $\frac{{}^nC_d}{2^n}$ (b) $\frac{{}^nC_d}{2^d}$ (c) $\frac{d}{2^n}$ (d) $\frac{1}{2^d}$

69.(a)

It's a binomial distribution

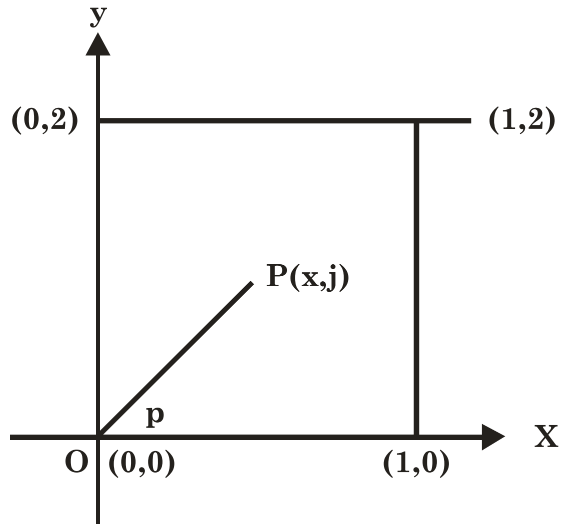
$$P(x = d) = {}^nc_d \left(\frac{1}{2}\right)^d \left(\frac{1}{2}\right)^{n-d}$$

$$= \frac{{}^nc_d}{2^n}$$

70. A point is randomly selected with uniform probability in the X-Y. plane within the rectangle with corners at (0, 0), (1, 0), (1, 2) and (0, 2). If p is the length of the position vector of the point, the expected value of p^2 is [CS: GATE-2004]

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

70. (d)



$\therefore \quad p = \sqrt{x^2 + y^2} \quad \Rightarrow p^2 = x^2 + y^2$

$\therefore E(p^2) = E(x^2 + y^2) = E(x^2) + E(y^2)$

Since x and y are uniformly distributed in the interval $0 \le x \le 1$ and $0 \le y \le 2$ respectively.

$\therefore$ Probability density function of x,

$$p(x) = \frac{1}{1-0} = 1$$

and probability density function of y,

$$p(y) = \frac{1}{2-0} = \frac{1}{2}$$

$$\therefore E(x^2) = \int_0^1 x^2 p(x)dx = \int_0^1 x^2\, dx = \frac{1}{3}$$

$$\text{And } E(y^2) = \int_0^2 y^2 p(y)dy = \int_0^2 \frac{y^2}{2} dy = \frac{4}{3}$$

$$\therefore E(p^2) = E(x^2) + E(y^2) = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$

71. Let f(x) be the continuous probability density function of a random variable X. The probability that $a < X \le b$, is [CS: GATE-2005]

(a) f (b –a) (b) f(b) – f(a) (c) $\int_a^b f(x)dx$ (d) $\int_a^b xf(x)dx$

71.(c)

For continuous cases,

$$P(a < x \le b) = \int_a^b f(x)dx$$

15. Numerical Methods

1. The equation $x^3 - x^2 + 4x - 4 = 0$ is to be solved using the Newton-Raphson method. If x = 2 is taken as the initial approximation of the solution, then the next approximation using this method will be [EC: GATE-2007]

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) 1 (d) $\frac{3}{2}$

1.(a)

Newton-Raphson iteration scheme is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n = 0,1,2......$$

Here $x_0 = 2,$

$$\therefore x_1 = 2 - \frac{f(2)}{f'(2)} = \frac{8}{12} = \frac{2}{3}.$$

2. The recursion relation to solve $x = e^{-x}$ using Newton-Raphson method is [EC: GATE-2008]

(a) $X_{n+1} = e^{-x_n}$ (b) $X_{n+1} = X_n - e^{-x_n}$

(c) $X_{n+1} = (1 + X_n)\frac{e^{-x_n}}{1 + e^{-x_n}}$ (d) $X_{n+1} = \frac{X_n^2 - e^{-x_n}(1 + x_n) - 1}{x_n - e^{-x_n}}$

2. (c)

Newton-Raphson iteration scheme is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, f(x) = x - e^{-x}.$$

$$= x_n - \frac{x_n - e^{x_n}}{1 + e^{-x_n}}$$

$$= \frac{\left(1 + e^{-x_n}\right)x_n - \left(x_n - e^{-x_n}\right)}{1 + e^{-x_n}}$$

$$= \frac{\left(1 + x_n\right)e^{-x_n}}{1 + e^{-x_n}}$$

4. We wish to solve $x^2 - 2 = 0$ by Netwon Raphson technique. Let the initial guess b $x_0 = 1.0$ Subsequent estimate of x(i.e.x_1) will be: [ME: GATE-1999]

(a) 1.414 (b) 1.5 (c) 2.0 (d) None of the above

4.(b).

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}, \; here \; f(x) = x^2 - 2$$
$$= 1 - \frac{-1}{2}$$
$$= \frac{3}{2}$$
$$= 1.5$$

5. The values of a function f(x) are tabulated below [ME: GATE-2004]

x	f(x)
0	1
1	2
2	1
3	10

Using Newton's forward difference formula, the cubic polynomial that can be fitted to the above data, is

(a) $2x^3+7x^2-6x+2$ (b) $2x^3-7x^2+6x-2$

(c) $x^3-7x^2-6x^2+1$ (d) $2x^3-7x^2+6x+1$

5. Ans. (d)

x	f(x)	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^2 f(x)$
0 1 2 3	1 2 1 10	 1 -1 9	 2 10	 1 2

Using Newton's forward interpolation formula we get

$$f(x) = f(0) + \frac{x}{1}\nabla f(0) + \frac{x(x-1)}{1.2}\nabla^2 f(0) + \frac{x(x-1)(x-2)}{1.2.3}\nabla^3 f(0)$$
$$= 1 + x(1) + \frac{x(x-1)}{2}(-2) + \frac{x(x-1)(x-2)}{6}(12), \quad =1+x+(x-x^2)+2x(x^2-3x+2)$$
$$=1+x+x-x^2+2x^3-6x^2+4x, \quad =2x^3-7x^2+6x+1$$

6. Starting from $X_0 = 1$, one step of Newton-Raphson method in solving the equation $x^3 + 3x-7 = 0$ gives the next value (x_1) as [ME: GATE-2005]

(a) $x_1= 0.5$ (b) $x_1= 1.406$ (c) $x_1= 1.5$ (d) $x_1 = 2$

6. (c)

Newton-Raphson iteration scheme is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n = 0,1,2......$$

Given $x_0 = 1$

$$\therefore x_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-3}{6} = \frac{3}{2} = 1.5$$

7. The order of error is the Simpson's rule for numerical integration with a step size h is [ME: GATE-1997]

(a) h (b) h^2 (c) h^3 (d) h^4

7. Ans. (b)

8. The accuracy of Simpson's rule quadrature for a step size h is [ME: GATE-2003]

(a) $O(h^2)$ (b) $O(h^3)$ (c) $O(h^4)$ (d) $O(h^2)$

8. Ans. (d)

9. With a 1 unit change in b, what is the change in x in the solution of the system of equations x + y = 2, 1.01 x + 0.99 y = b? [ME: GATE-2005]

(a) Zero (b) 2 units (c) 50 units (d) 100 units

9.. Ans. (c)

Given $x + y = 2$ .... (i)

$1.01 x + 0.99 y = b$ .... (ii)

Multiply 0.99 is equation (i), and subtract from equation (ii), we get

$(1.01-0.99)\, x = b - 2\times 0.99$ $\quad 0.02x = b-1.98$ $\quad \therefore\ 0.02\,\Delta x=\Delta b$ $\quad \therefore\ \Delta x=\frac{1}{0.02}=50$ unit

10. Match the items in columns I and II. [ME: GATE-2006]

Column I	Column II
P. Gauss-Seidel method	1. Interpolation
Q. Forward Newton-Gauss method	2. Non-linear differential equations
R. Runge-Kutta method	3. Numerical integration
S. Trapezoidal Rule	4. Linear algebraic equations

(a) P-1, Q-4, R-3, S-2 (b) P-1, Q-4, R-2, S-3
(c) P-1, Q-3, R-2, S-4 (d) P-4, Q-1, R-2, S-3

10. (d)

(P) Gauss – Seidal method → Linear algebraic equation
(Q) Forward Newton – Gauss method → Interpolation
(R) Runge – Kutta method → Non-linear differential equations
(S) Trapezoidal Rule → Numerical integration

11. A calculator has accuracy up to 8 digits after decimal place. The value of $\int_0^{2\pi} \sin x\, dx$ when evaluated using this calculator by trapezoidal method with 8 equal intervals, to 5 significant digits is [ME: GATE-2007]

(a) 0.00000 (b) 1.0000 (c) 0.00500 (d) 0.00025

11. Ans. (a)

$$h = \frac{2\pi - 0}{8} = \frac{\pi}{4}$$
$$y_0 = \sin(0) = 0$$
$$y_1 = \sin\left(\frac{\pi}{4}\right) = 0.70710$$
$$y_2 = \sin\left(\frac{\pi}{2}\right) = 1$$
$$y_3 = \sin\left(\frac{3\pi}{4}\right) = 0.7010$$
$$y_4 = \sin(\pi) = 0$$
$$y_5 = \sin\left(\frac{5\pi}{4}\right) = -0.70710$$
$$y_6 = \sin\left(\frac{6\pi}{4}\right) = -1$$
$$y_7 = \sin\left(\frac{7\pi}{4}\right) = -0.70710$$
$$y_8 = \sin\left(\frac{8\pi}{4}\right) = 0$$

Trapezoidal rule

$$\int_{x_0}^{x_0+nh} f(x).dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})]$$
$$\int_0^{2\pi} \sin x.dx = \frac{h}{8}$$
$$[(0+0) + 2(0.70710 + 1 + 0.70710 + 0 - 0.70710 - 0.70710)] \quad = 0.00000$$

13. In the solution of the following set of linear equations by Gauss elimination using partial pivoting 5x + y + 2z = 34; 4y – 3z = 12; and 10x – 2y + z = –4; the pivots for elimination of x and y are **[CE: GATE – 2009]**

(a) 10 and 4
(b) 10 and 2
(c) 5 and 4
(d) 5 and –4

13. Ans.(a)

The equations are

$$5x + y + 2z = 34$$
$$0x + 4y - 3z = 12$$
and $$10x - 2y + z = -4$$

The augmented matrix for gauss-elimination is

$$\left[\begin{array}{ccc|c} 5 & 1 & 2 & 34 \\ 0 & 4 & -3 & 12 \\ 10 & -2 & 1 & -4 \end{array}\right]$$

Since in the first column maximum element in absolute value is 10, we need to exchange row 1 with row 3.

$$\begin{bmatrix} 5 & 1 & 2 & | & 34 \\ 0 & 4 & -3 & | & 12 \\ 10 & -2 & 1 & | & -4 \end{bmatrix} \xrightarrow{R(1,3)} \begin{bmatrix} 10 & -2 & 1 & | & -4 \\ 0 & 4 & -3 & | & 12 \\ 5 & 1 & 2 & | & 34 \end{bmatrix}$$

So the pivot for eliminating x is $a_{11} = 10$

Now to eliminate y, we need to compass the eliminate in second column at and below the diagonal.

Since $a_{22} = 4$ is already larger in absolute value compares to $a_{32} = 1$

$\therefore$ The pivot element for eliminating y is $a_{22} = 4$ itself.

$\therefore$ The pivots for eliminating x and y are respectively 10 and 4.

CE 10 Years GATE Questions

Q2. The square root of a number N is to be obtained by applying the Newton Raphson iterations to the equation $\mathbf{x^2 - N = 0}$. If *i* denotes the iteration index, the correct iterative scheme will be

(a) $\mathbf{x_{i+1} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)}$

(b) $\mathbf{x_{i+1} = \frac{1}{2}\left(x^2{}_i + \frac{N}{x^2{}_i}\right)}$

(b) (c) $\mathbf{x_{i+1} = \frac{1}{2}\left(x_i + \frac{N^2}{x_i}\right)}$

(d) $\mathbf{x_{i+1} = \frac{1}{2}\left(x_i - \frac{N}{x_i}\right)}$ **[CE-2011]**

Ans. **(a)**

Exp. $\mathbf{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0, 1, 2...}$

$$\mathbf{= x_i - \frac{x^2{}_i - N}{2x_i}\left[f(x) = x^{\vee} - N\right]}$$

$$\mathbf{= \frac{1}{2}\left[\frac{2x^2{}_i - x^2{}_i + N}{x_i}\right]}$$

$$\mathbf{= \frac{1}{2}\left[\frac{x^2{}_i + N}{x_i}\right]}$$

$$\mathbf{= \frac{1}{2}\left[x_i + \frac{N}{x_i}\right]}$$

Statement for Linked Answer Questions 12 and 13:

Give a > 0, we wish to calculate its reciprocal value $\frac{1}{a}$ by using Newton Raphson Method for f(x) = 0.

12. The Newton Raphson algorithm for the function will be **[CE: GATE – 2005]**

(a) $x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right)$

(b) $x_{k+1} = \left(x_k + \frac{a}{2}x_k^2\right)$

(c) $x_{k+1} = 2x_k - ax_k^2$

(d) $x_{k+1} = x_k - \frac{a}{2}x_k^2$

12. (c)

$$x = \frac{1}{a} \Rightarrow \frac{1}{x} - a = 0$$

Let $f(x) = \frac{1}{x} - a$

Newton Rapshon iteration scheme

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{\frac{1}{x_n} - a}{-\frac{1}{{x_n}^2}}$$

$$= x_n + {x_n}^2\left(\frac{1}{x_n} - a\right)$$

$$= 2x_n - a{x_n}^2$$

13. For a = 7 and starting with $X_0 = 0.2$, the first two iterations will be

(a) 0.11, 0.1299
(b) 0.12, 0.1392
(c) 0.12, 0.1416
(d) 0.13, 0.1428

13.(b)

$$\begin{aligned} x_1 &= 2x_0 - a{x_0}^2 \\ &= 2 \times 0.2 - 7 \times 0.04 \\ &= 0.12 \\ x_2 &= 2x_1 - ax_1^2 \\ &= 2 \times .12 - 7 \times 0.0144 \\ &= 0.24 - 0.1008 \\ &= 0.1392 \end{aligned}$$

14. The following equation needs to be numerically solved using the Newton-Raphson method.

$$x^3 + 4x - 9 = 0$$

The iterative equation for this purpose is (k indicates the iteration level)

[CE: GATE – 2007]

(a) $x_{k+1} = \dfrac{2x_k^3 + 9}{3x_k^2 + 4}$
(b) $x_{k+1} = \dfrac{3x_k^2 + 4}{2x_k^2 + 9}$
(c) $x_{k+1} = x_k - 3x_k^2 + 4$
(d) $x_{k+1} = \dfrac{4x_k^2 + 3}{9x_k^2 + 2}$

14.(a)

Newton –Rapshon iteration scheme is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^3 + 4x_n - 9}{3x_n^2 + 4}$$

$$= \frac{2x_n^3 + 9}{3x_n^2 + 4}$$

15. A 2nd degree polynomial, f(x) has values of 1, 4 and 15 at x = 0, 1 and 2, respectively. The integral $\int_0^2 f(x)\,dx$ is to be estimated by applying the trapezoidal rule to this data. What is the error (defined as "true value – approximate value") in the estimate?

(a) $-\frac{4}{3}$ (b) $-\frac{2}{3}$ **[CE: GATE – 2006]**

(c) 0 (d) $\frac{2}{3}$

15. (a)

Given

(x)	0	1	2
f(x)	1	4	15

$$\therefore f(x) = \frac{(x-1)(x-2)}{(0-1)(0-2)} f(0) + \frac{(x-0)(x-2)}{(1-0)(1-2)} f(1) + \frac{(x-0)(x-1)}{(2-0)(2-1)}.f(2)$$

$$= \frac{x^2 - 3x + 2}{2}.1 + \frac{x^2 - 2x}{-1}4 + \frac{x^2 - x}{2} : 15$$

$$= 4x^2 - x + 1$$

$$\text{Error} = \int_0^2 f(x)\,dx - \frac{b}{2}\left[y_0 + y_2 + 2y_1\right]$$

$$= \int_0^2 \left(4x^2 - x + 1\right) dx - \frac{1}{2}\left[1 + 15 + 2.4\right]$$

$$= \frac{32}{3} - 12$$

$$= -\frac{4}{3}$$

16. The table below gives values of a function F(x) obtained for values of x at intervals of 0.25.

x	0	0.25	0.5	0.75	1.0
F(x)	1	0.9412	0.8	0.64	0.50

[CE: GATE – 2010]

The value of the integral of the function between the limits 0 to 1 using Simpson's rule is

(a) 0.7854 (b) 2.3562

(c) 3.1416 (d) 7.5000

16. (a)

$$\int_0^1 f(x)\,dx = \frac{h}{3}\left[\left(y_0 + y_4\right) + 4\left(y_1 + y_3\right) + 2y_2\right]$$

$$= \frac{0.25}{3}\left[1 + 0.5 + 4\left(0.9412 + 0.64\right) + 2 \times 0.8\right]$$

$= 0.7854$

EE All GATE Questions

17. Equation e^x-1=0 is required to be solved using Newton's method with an initial guess x_0=-1 Then after one step of Newton's method, estimate x_1 of the solution will be given by
(a) 0.71828 (b) 0.36784 (c) 0.20587 (d) 0.00000
[EE: GATE-2008]

17. (a)

$f(x) = e^x - 1$

Newton iteration scheme

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{e^{x_n} - 1}{e^{x_n}}$$

$$\therefore x_1 = x_0 - \frac{e^{x_0} - 1}{e^{x_0}} = -1. - \frac{\frac{1}{e} - 1}{\frac{1}{e}}. = -1 - (1 - e)$$

$= e - 2$

$= .71828$

18. (a)

Let $f(x) = x^2 - 117$

Newton iteration scheme is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^2 - 117}{2x_n}$$

$$= \frac{x_n^2 + 117}{2x_n}$$

$$= \frac{1}{2}\left(x_n + \frac{117}{x_n}\right)$$

19. A differential equation $\frac{dx}{dt} = e^{-2t}u(t)$ has to be solved using trapezoidal rule of integration with a step size h=0.01s. Function u(t) indicates a unit step function. If x(0-)=0, then value of x at t=0.01s will be given by
[EE: GATE-2008]
(a) 0.00099 (b) 0.00495 (c) 0.0099 (d) 0.0198

19. Ans. (c)

49. The differential equation $\frac{dx}{dt} = \frac{1-x}{\tau}$ is discretised using Euler's numerical integration method with a time step $\Delta T > 0$. What is the maximum permissible value of ΔT to ensure stability of the solution of the corresponding discrete time equation?

[EE: GATE-2007]

(a) 1 (b) $\tau / 2$ (c) τ (d) 2τ

49. Ans. (d)

IE All GATE Questions

20. For k = 0, 1, 2, …… the steps of Newton-Raphson method for solving a non-linear equation is given as **[IE: GATE-2006]**

$$x_{k+1} = \frac{2}{3}x_k + \frac{5}{3}x_k^{-2}$$

Starting from a suitable initial choice as k tends to ∞, the iterate x_k tends to

(a) 1.7099 (b) 2.2361

(c) 3.1251 (d) 5.0000

20. (a)

$$x_{k+1} = \frac{2}{3}x_k + \frac{5}{3}x_k^{-2}$$

$$= x_k - \frac{1}{3}x_k + \frac{5}{3}x_k^{-2}$$

$$\Rightarrow x_{k+1} - x_k = -\frac{1}{3}x_k + \frac{5}{3}x_k^{-2}$$

$$\Rightarrow \frac{f(x_k)}{f'(x_k)} = \frac{1}{3}x_k - \frac{5}{3}x_k^{-2} = \frac{x_k^3 - 5}{3x_k^2}$$

$\therefore f(x) = x^3 - 5$ (by newton-Rapshon medhod)

$f(x) = 0$

$\Rightarrow x^3 = 5$

$\Rightarrow x = 1.7099$

21. Identify the Newton-Raphson iteration scheme for finding the square root of 2.

(a) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$ (b) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{2}{x_n}\right)$

(c) $x_{n+1} = \frac{2}{x_n}$ (d) $x_{n+1} = \sqrt{2 + x_n}$ **[IE: GATE-2007]**

21.(a)

$x = \sqrt{2}$

$\therefore f(x) = x^2 - 2$

N − R scheme is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n = \frac{x_n^2 - 2}{2x_n}$$

$$= \frac{x_n^2 + 2}{2x_n}$$

$$= \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$$

CS All GATE Questions

23. Consider the series $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}$, $x_0 = 0.5$ obtained from the Newton-Raphson method. The series converges to [CS: GATE-2007]

(a) 1.5 (b) $\sqrt{2}$ (c) 1.6 (d) 1.4

23. (a)

$$x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}; \qquad x_0 = 0.5$$

The series converges when $x_{n+1} = x_n = \alpha$

$$\therefore \alpha = \frac{\alpha}{2} + \frac{9}{8\alpha} = \frac{4\alpha^2 + 9}{8\alpha}$$

$$\Rightarrow 4\alpha^2 = 9$$

$$\Rightarrow \alpha = \frac{3}{2} = 1.5$$

24. The Newton-Raphson iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n^{-}}\right)$ can be used to compute the [CS: GATE-2008]

(a) square of R (b) reciprocal of R
(c) square root of R (d) logarithm of R

24.(c)

25. Newton-Raphson method is used to compute a root of the equation $x^2 - 13 = 0$ with 3.5 as the initial value. The approximation after one iteration is [CS: GATE-2010]

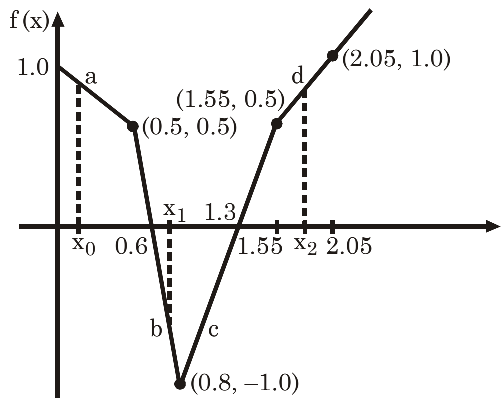
(a) 3.575 (b) 3.677 (c) 3.667 (d) 3.607

25. (d) N-R iteration scheme is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3.5 - \frac{(3.5)^2 - 13}{2 \times 3.5}$$
$$= 3.607.$$

26. A piecewise linear function f(x) is plotted using thick solid lines in the figure below (the plot is drawn to scale). [CS: GATE-2003]



If we use the Newton-Raphson method to find the roots of f(x) = 0 using x0, x1 and x2 respectively as initial guesses, the roots obtained would be

(a) 1.3, 0.6 and 0.6 respectively
(b) 0.6, 0.6 and 1.3 respectively
(c) 1.3, 1.3 and 0.6 respectively
(d) 1.3, 0.6 and 1.3 respectively

26. Ans. (d)

Starting from x_0,

slope of line a = $\frac{1 - 0.5}{0 - 0.5} = -1$

y-intercept = 1

Eqn, of a is y = mx + c = –1x + 1

This line will cut x axis (i.e., y = 0), at x = 1

Since x = 1 is > than x = 0.8, a perpendicular at x = 1 will cut the line c and not line b.

$\therefore$ root will be 1.3

Starting from x_1,

the perpendicular at x_1 is cutting line b and root will be 0.6.

Starting from x_2,

Slope of line d = $\frac{1 - 0.5}{2.05 - 1.55} = 1$

Equation of d is y – 0.5 = 1(x – 1.55)

i.e. y = x – 1.05

This line will cut x axis at x = 1.05

Since x = 1.05 is > than x = 0.8, the perpendicular at x = 1.05 will cut the line c and not line b. the root will be therefore equal to 1.3. So starting from x_0, x_1 and x_2 the roots will be respectively 1.3, 0.6 and 1.3.

27. The minimum Number of equal lenth subintervals needed to approximater $\int_1^2 xe^x\,dx$ to an accuracy of at least $\frac{1}{3}\times 10^{-6}$ using the trapezoidal rule is [CS: GATE-2008]

(a) 1000e (b) 1000 (c) 100e (d) 100

27 Ans. (a)

Here, the function being integrated is

$$f(x) = xe^x$$
$$f(x) = xe^x + e^x = e^x(x+1)$$
$$f'(x) = xe^x + e^x + e^x = e^x(x+2)$$

Truncation Error for trapezoidal rule

$$= \text{TE (bound)}$$
$$= \frac{h^3}{12}\max|f''(\xi)|.N_i$$

Where N_i is number of subintervals

$$N_i = \frac{b-a}{h}$$

$$\Rightarrow \qquad T_E = \frac{h^3}{12}\max|f''(\xi)|.\frac{b-a}{h}$$
$$= \frac{h^2}{12}(b-a)\max|f''(\xi)|\,1\le\xi\le 2$$
$$= \frac{h^2}{12}(2-1)\,[e^2(2+2)]$$
$$= \frac{h^2}{3}e^2 = \frac{1}{3}\times 10^{-6}$$
$$\Rightarrow \qquad h^2 = \frac{10^{-6}}{e^2}$$
$$\Rightarrow \qquad h = \frac{10^{-3}}{e}$$
$$N_i = \frac{b-a}{h}$$
$$= \frac{2-1}{\left(\frac{10^{-3}}{e}\right)} = 1000\ e$$

16. Transform Theory

Previous years GATE Questions

EC All GATE Questions

1. Let x(n) = $\left(\frac{1}{2}\right)^n u(n)$, y(n) = x^2(n) and Y(e^{jw}) be the Fourier transform of y(n). Then Y(e^{j0}) is

[EC: GATE-2005]

(a) $\frac{1}{4}$ (b) 2 (c) 4 (d) $\frac{4}{3}$

1. Ans. (a)

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$\therefore \quad y(n) = \left(\frac{1}{2}\right)^{2n} u^2(n)$$

$$y(n) = \left(\frac{1}{2}\right)^{2n} u(n) = \left(\left(\frac{1}{2}\right)^2\right)^n u(n)$$

$$\therefore \quad y(n) = \left(\frac{1}{4}\right)^n u(n)$$

$$\therefore \quad y(e^{10}) = \frac{1}{4}$$

2. The signal $\dot{x}(t)$ is described by [EC: GATE-2008]

$$x(:) = \begin{cases} 1 & \text{for} - \le t \le +1 \\ 0 & \text{otherwise} \end{cases}$$

Two of the angular frequencies at which its Fourier transform becomes zero are

(a) π , 2π (b) 0.5 π, 1.5 π
(c) 0, π (d) 2π, 2.5 π

2. Ans. (a)

Given : $x(t) = \begin{bmatrix} 1 & -1 < + < 1 \\ 0 & \text{otherwise} \end{bmatrix}$

Fourier transform is

$$\int_{-1}^{1} e^{-i\,st}\,dt = -\frac{1}{i^s}[e^{-i\,st}]_{-1}^{1}$$

$$= \frac{1}{i^s}[e^{i\,st} - e^{-i\,st}] = \frac{2}{s}[\sin st]$$
$$= 0 \text{ for } s = \pi \text{ and } 2\pi$$

3. Consider the function f(t) having Laplace transform

$$F(s) = \frac{\omega_0}{s^2 + \omega_0^2} \ \text{Re}[s] > 0$$

The final value of f(t) would be [EC: GATE-2006]

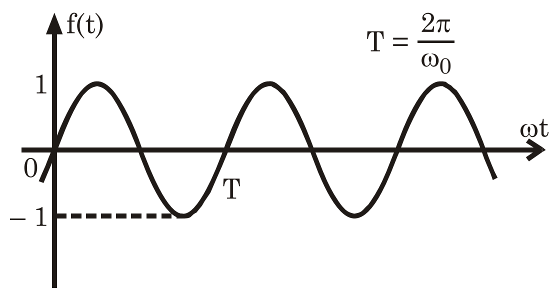
(a) 0 (b) 1
(c) $-1 \le f(\infty) \le 1$ (d) ∞

3. Ans. (c)

$$f(t) = L^{-1} f(x)$$
$$= \sin w_0 t$$

As $-1 \le \sin\theta \le 1$

Thus, $-1 \le f(\infty) \le 1$



4. Given that F(s) is the one-sided Laplace transform of f(t), the Laplace transform of $\int_0^t f(\tau)\,d\tau$ is [EC: GATE-2009]

(a) $sF(s) - f(0)$ (b) $\frac{1}{2}F(s)$ (c) $\int_0^s F(\tau)\,d\tau$ (d) $\frac{1}{2}[F(s) - f(0)]$

4. Ans. (b)

$$\int_0^4 f(\tau)\,d\tau = \frac{1}{s} f(s) \qquad \text{……(Lapalace formule)}$$

5. Given $f(t) = L^{-1}\left[\frac{3s+1}{s^3 + 4s^2 + (K-3)s}\right]$. If $\lim_{t\to\infty} f(t) = 1$, then the value of K is [EC: GATE-2010]

(a) 1 (b) 2 (c) 3 (d) 4

5. Ans. (d)

$$f(t) = L^{-1}\left\{\frac{3s+1}{s^3+4s^2+(K-3)s}\right\}$$

$$F(s) = L\left[f(t)\right]$$

$$= \frac{(3s+1)}{s^3+4s^2+(K-3)s}$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} SF(s) = 1$$

$$\Rightarrow \lim_{s\to 0}\frac{(3s+1)}{s^2+4s+(K-3)s} = 1$$

$$\Rightarrow \quad K-3 = 1 \quad \Rightarrow \quad K = 4$$

(a) $|z| < \frac{5}{6}$ (b) $|z| > \frac{6}{5}$ (c) $\frac{5}{6} < |z| < \frac{6}{5}$ (d) $\frac{6}{5} < |z| < \infty$

6. The region of convergence of Z-transform of the sequence

$$\left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1) \text{ must be}$$

6. Ans. (c)

$$f(n) = \left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$$

Now, $\left(\frac{6}{5}\right)^n = \left(\frac{5}{6}\right)^{-n}$

$\therefore \quad \frac{5}{6}\left(\frac{6}{5}\right)^n = \frac{5}{6}.\left(\frac{5}{6}\right)^{-n}$

or $\left(\frac{6}{5}\right)^n = \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)^{-n-1}$

$\therefore$ f(n) =

$$\left(\frac{5}{6}\right)^n u(n) - \frac{5}{6}\left(\frac{5}{6}\right)^{-n-1} u(-n-1)$$

$\therefore$ F(z) =

$$\left(1-\frac{5}{6}z^{-1}\right) - \frac{5}{6}\left(1-\frac{5}{6}z^{-1}\right)^{-1}.z^{-1}$$

Hence, region of convergence, $|z| < \frac{6}{5}$ and $|z| < \frac{5}{6}$.

For two terms $\frac{5}{6} < |z| < \frac{6}{5}$

[EC: GATE-2005]

7. Consider the z-transform $X(z) = 5z^2 + 4z^{-1} + 3$; $0 < |z| < \infty$. The inverse z-transform x[n] is

[EC: GATE-2010]

(a) $5\,\delta[n+2] + 3\,\delta[n] + 4\,\delta[n-1]$ (b) $5\,\delta[n-2] + 3\,\delta[n] + 4\,\delta[n+1]$

(c) 5 u[n + 2] + 3 u[n] + 4 u[n – 1] (d) 5 u[n – 2] + 3 u[n] + 4 u[n + 1]

7. Ans. (a)

$x(z) = 5z^2 + 4z^{-1} + 3$

$0<|Z| < \infty$

$x[n] = 5\,\delta[n + 2] + 4\,\delta[n - 1] + 3\,\delta[n]$

ME 20 Years GATE Questions

8. If f(t) is a finite and continuous function for t, the Laplace transformation is given by $F = \int_0^\infty e^{-st}\, f(t)\, dt$. For f(t) = cos h mt, the Laplace transformation is.....[ME: GATE-1994]

8. Ans. $\dfrac{s}{s^2 - m^2}$

9. The Laplace transform of $\cos\ \omega t$ is $\dfrac{w}{\delta^2 + \omega^2}$. [ME: GATE-1995]

(a) True (b)False

9. Ans. (b)False

Laplace transform of $\cos\ \omega t$ is $\dfrac{w}{\delta^2 - \omega^2}$.

10. $(s+1)^{-2}$ is the Laplace transform of [ME: GATE-1998]

(a) t^2 (b) t^3 (c) e^{-2t} (d) te^{-t}

10. Ans.(d)

$$L(t) = \frac{1}{s^2}$$

By first shifting theorem

$$L(e^{-t}.t) = \frac{1}{(s+2)^2}$$

11. Laplace transform of $(a + bt)^2$ where 'a' and 'b' are constants is given by: [ME: GATE-1999]

(a) $(a+bs)^2$ (b) $\dfrac{1}{(a+bs)^2}$ (c) $\dfrac{a^2}{s} + \dfrac{2ab}{s^2} + \dfrac{2b^2}{s^3}$ (d) $\dfrac{a^2}{s} + \dfrac{2ab}{s^2} + \dfrac{b^2}{s^3}$

11. Ans.(c)

$(a+bt)^2 = a^2 + b^2t^2 + 2abt.$

Laplace transform of 1 $= \dfrac{1}{s}$ Laplace transform of $t^n = \dfrac{Ln}{s^n + 1}$

$$\therefore\ L\ (a+bt)^2 = \frac{a^2}{s} + \frac{2b^2}{s^3} + \frac{2ab}{s^2}$$

12. The Laplace transform of the function $\sin^2 2t$ is [ME: GATE-2000]

(a) $(1/2s)-s/[2(s^2+16)]$ (b) $s/(s^2+16)$

(c) $(1/s)-s/(s^2+4)$ (d) $s/(s^2+4)$

12. Ans.(a)

$$\sin^2 2t=\frac{1-\cos 4t}{2}$$

$$\therefore\ L.\left\{\frac{1-\cos 4t}{2}\right\}=\frac{1}{2}\left\{\frac{1}{s}-\frac{s}{s^2+16}\right\}=\left\{\frac{1}{2s}-\frac{s}{(s^2+16)}\right\}$$

13. Laplace transform of the function $\sin\ \omega t$ [ME: GATE-2003]

(a) $\frac{s}{s^2+\omega^2}$ (b) $\frac{\omega}{s^2+\omega^2}$ (c) $\frac{s}{s^2-\omega^2}$ (d) $\frac{\omega}{s^2-\omega^2}$

13. Ans. (b)

$$L[\sin\omega t]=\frac{\omega}{s^2+\omega^2}$$

14. A delayed unit step function is defined as $u(t\text{-}a)=\begin{cases}0, & \text{for } t<a\\ 1, & \text{for } t\geq a\end{cases}$. Its Laplace transform is

(a) $a.e^{-as}$ (b) $\frac{e^{-es}}{s}$ (c) $\frac{e^{es}}{s}$ (d) $\frac{e^{as}}{s}$

14. Ans. (d)

$$L[U(t-a)]=\int_0^\infty e^{-st}\ U(t-a)dt,\quad =\int_0^a e^{-st}.0.dt+\int_0^\infty e^{-st}.1.dt,\ =0.\int_a^\infty e^{-st}dt,\ =\left[\frac{e^{-st}}{-s}\right]_a^\infty=\frac{e^{-as}}{s}$$

15. If F(s) is the Laplace transform of function f (t), then Laplace transform of $\int_0^t f(\tau)d\tau$ is [ME: GATE-2007]

(a) $\frac{1}{s}F(s)$ (b) $\frac{1}{s}F(s)$ - f(0) (c) $sF(s)-f(0)$ (d) $\int F(s)$ ds

15. Ans. (a)

$$L\left[\int_0^t f(t)\ dt\right]=\frac{1}{s}F(s)$$

16. The Inverse Laplace transform of $\frac{1}{(S^2+S)}$ is [ME: GATE-2009]

(a) $1 + e^t$ (b) $1\text{-}e^t$ (b) $1\text{-}\ e^{-t}$ (d) $1 + e^{-t}$

16. Ans. (c)

$$L^{-1}\left(\frac{1}{s^2+s}\right)=?$$

$$\frac{1}{s^2+s}=\frac{1}{s(s+1)}=\frac{1}{s}-\frac{1}{s+1}$$

$$L^{-1}\left(\frac{1}{s^2+s}\right) \quad =L^{-1}\left(\frac{1}{s}\right)--L^{-1}\left(\frac{1}{s+1}\right) \quad =1\text{-}e^{-t}$$

[Using standard formulae] Standard formula:

$$L^{-1}\left(\frac{1}{s}\right)=1 \quad \Rightarrow \quad L^{-1}\left(\frac{1}{s+a}\right)=e^{-at} \quad \Rightarrow \quad L^{-1}\left(\frac{1}{s-a}\right)=e^{at}$$

17. The Laplace transform of a function f(t) is $\frac{1}{s^2(s+1)}$. The function f(t) is

(a) $t-1+e^{-t}$ (b) $t+1+e^{-t}$ (c) $-1+e^{-t}$ (d) $2t+e^{t}$

[ME: GATE-2010]

17. **Ans. (a)**

$$L[f(t)]=\frac{1}{S^2(S+1)}$$

$$f(t)=L^{-1}\left\{\frac{1}{S^2(S+1)}\right\}$$

$$L^{-1}\left\{\frac{1}{S+1}\right\}=e^{-t}$$

$$L^{-1}\left\{\frac{1}{S(S+1)}\right\}=\int_0^t e^{-t}\,dt=1-e^{-t}$$

$$L^{-1}\left\{\frac{1}{S^2(S+1)}\right\}=\int_0^t 1-e^{-t}=t-1+e^{-t}$$

CE 10 Years GATE Questions

18. If L defines the Laplace Transform of a function, L [sin (at)] will be equal to

(a) $\frac{a}{s^2-a^2}$ (b) $\frac{a}{s^2+a^2}$ **[CE: GATE – 2003]**

(c) $\frac{s}{s^2+a^2}$ (d) $\frac{s}{s^2-a^2}$

18. Ans. (b)

$$L\,[f\,(t)] \quad =\int_0^\infty e^{-st}\,f(t)\,dt$$

$$\Rightarrow \quad L\,[\sin\,(at)] \quad =\int_0^\infty e^{-st}\,\sin(at)\,dt$$

$$=\frac{a}{s^2+a^2}$$

19. Laplace transform for the function f(x) = cosh (ax) is **[CE: GATE – 2009]**

(a) $\frac{a}{s^2 - a^2}$ (b) $\frac{s}{s^2 - a^2}$

(c) $\frac{a}{s^2 + a^2}$ (d) $\frac{s}{s^2 + a^2}$

19. Ans. (b)

It is a standard result that

L (cosh at) = $\frac{s}{s^2 - a^2}$.

Q3. There are two containers, with one containing 4 Red and 3 Green balls and the other containing Blue and 4 Green balls. One bal is drawn at random form each container. The probability that one of the ball is Red and the other is Blue will be

(a) 1/7 (b) 9/49 (c) 12/49 (d) 3/7 **[CE-2011]**

Ans. (c)

EE All GATE Questions

Statement for Linked Answer Question (20) and (21)

A state variable system X(t)= $\begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$, with the initial condition X(0)[-1 3]^T and the unit step input u(t) has

20. The state transition equation **[EE: GATE-2005]**

(a) $\begin{bmatrix} 1 & \frac{1}{3}(1 - e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \frac{1}{3}(e^{-t} - e^{-3t}) \\ 0 & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 1 & \frac{1}{3}(e^{-t} - e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}$ (d) $\begin{bmatrix} 1 & (1 - e^{-t}) \\ 0 & e^{-t} \end{bmatrix}$

20. Ans. (a)

$$(sI-A)^{-1}=\begin{bmatrix} s & 1 \\ 0 & s+3 \end{bmatrix}^{-1}=\frac{Adj(sI-A)}{|sI-A|}$$

$$=\frac{\begin{bmatrix} s+3 & 0 \\ -1 & s \end{bmatrix}}{\begin{bmatrix} s & 1 \\ 0 & s+3 \end{bmatrix}}=\frac{\begin{bmatrix} s+3 & -1 \\ 0 & s \end{bmatrix}}{s(s+3)}.$$

$$=\begin{bmatrix} \frac{1}{s} & \frac{-1}{s(s+3)} \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$\therefore \phi(t)=L^{-1}(sI-A)^{-1}$$

$$=\begin{bmatrix} 1 & \frac{1}{3}(1-e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}$$

21. The state transition equation **[EE: GATE-2005]**

(a) $X(t)=\begin{bmatrix} t-e^{-t} \\ e^{-t} \end{bmatrix}$ (b) $X(t)=\begin{bmatrix} t-e^{-t} \\ 3e^{-3t} \end{bmatrix}$

(c) $X(t)=\begin{bmatrix} t-e^{-3t} \\ 3e^{-3t} \end{bmatrix}$ (d) $X(t)=\begin{bmatrix} t-e^{-3t} \\ e^{-t} \end{bmatrix}$

21. Ans. (c)

zero state response $=L^{-1}\phi(s)BU(S)$

$$=L^{-1}\begin{bmatrix} \frac{1}{s} & \frac{-1}{s(s+3)} \\ 0 & \frac{1}{s+3} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\frac{1}{s}$$

$$=L^{-1}\begin{bmatrix} \frac{1}{s^2} \\ 0 \end{bmatrix}=\begin{bmatrix} t \\ 0 \end{bmatrix}$$

State transition equation

=zero input response+zero state response.

$$\therefore X(t)=\phi(X)X(0)+t$$

$$=\begin{bmatrix} -1+1-e^{-3t} \\ 0+3e^{-3t} \end{bmatrix}+\begin{bmatrix} t \\ 0 \end{bmatrix}=\begin{bmatrix} t-e^{-3t} \\ 3e^{-3t} \end{bmatrix}$$

22. Let x(t)= rect $\left(t-\frac{1}{2}\right)$(where rect (x) =1 for $-\frac{1}{2}\le x\le\frac{1}{2}$ and zero otherwise). Then if since (x)= $\frac{\sin(\pi x)}{\pi x}$, the Fourier Transform of x(t)+x(-t) will be given by **[EE: GATE-2008]**

(a) sinc$\left(\frac{\omega}{2\pi}\right)$ (b) 2sinc$\left(\frac{\omega}{2\pi}\right)$

(c) $2\text{sinc}\left(\frac{\omega}{2\pi}\right)\cos\left(\frac{\omega}{2}\right)$ (d) $\text{sinc}\left(\frac{\omega}{2\pi}\right)\sin\left(\frac{\omega}{2}\right)$

22. Ans. (c)

$rect(x) = 1$ for $\frac{-1}{2} \leq x \leq \frac{1}{2}$

Given $x(t) = rect\left(t - \frac{1}{2}\right)$

Simpliying $\mathrm{x}(\mathrm{t})$ *with* the help of equation (1).

$\therefore \quad x(t) = 1, 0 \le t \le 1$

$=0$, therewise

Now, $F\left[x(t)\right] = \int_{-\omega}^{\omega} x(t) e^{-j\omega t} at$

$= \int_{0}^{1} 1.e^{-j\omega t} at$

$= \frac{1}{-j\omega}(e^{-j\omega t})_0^1 = \frac{1}{j\omega}(1 - e^{-j\omega})$

$= \frac{1}{j\omega}\left[\frac{e^{\frac{j\omega}{2}} - e^{\frac{-j\omega}{2}}}{e^{\frac{j\omega}{2}}}\right]$

$= \frac{2}{\omega}\left[\frac{e^{\frac{j\omega}{2}} - e^{\frac{-j\omega}{2}}}{2j}\right].e^{\frac{-j\omega}{2}}$

$\therefore F\left[x(t)\right] = \frac{\sin\frac{\omega}{2}}{\omega/2} e^{\frac{-j\omega}{2}}$

$x(-t) = t, -1 \le t \le 0$

$= 0, otherwise$

$F\left[x(t)\right] = \int_{-\infty}^{\infty} x(-t) e^{-j\omega t} at$

$= \int_{-1}^{0} 1.\ e^{-j\omega t} at$

$= \frac{1}{-j\omega}\left(e^{-j\omega t}\right)_{-1}^{0}$

$= \frac{1}{j\omega}\left(1 - e^{j\omega} - 1\right)$

$= \frac{1}{j\omega}\left[e^{\frac{j\omega}{2}} - e^{\frac{-j\omega}{2}}\right] e^{\frac{j\omega}{2}}$

$= \frac{2}{\omega}\left[\frac{e^{\frac{j\omega}{2}} - e^{\frac{-j\omega}{2}}}{2j}\right] e^{\frac{j\omega}{2}}$

$F\left[x(-t)\right] = \frac{\sin\frac{\omega}{2}}{\omega/2} e^{\frac{j\omega}{2}}$

$\therefore F\left[x(t) + x(-t)\right] = \frac{\sin\frac{\omega}{2}}{\omega/2}\left[e^{\frac{-j\omega}{2}} + e^{\frac{j\omega}{2}}\right]$

$= \frac{\sin\frac{\omega}{2}}{\omega/2}\left(2\cos\frac{\omega}{2}\right)$

$= 2\sin\left(\frac{\omega}{2\pi}\right)\cos\left(\frac{\omega}{2}\right)$

23. Let s(t) be the step response of a linear system with zero initial conditions; then the response of this system to an input u(t) is [EE: GATE-2002]

(a) $\int_0^t s(t-\tau)u(\tau)d\tau$ (b) $\frac{d}{dt}\left[\int_0^t s(t-\tau)u(\tau)d\tau\right]$

(c) $\int_0^t s(t-\tau)\left[\int_0^t u(\tau_1)d\tau_1\right]d\tau$ (d) $\int_0^t s(t-\tau)^2 u(\tau)d\tau$

23. Ans. (b)

24. Let Y(s) be the Laplace transformation of the function y (t), then final value of the function is **[EE: GATE-2002]**

(a) $\lim_{s\to 0} Y(s)$ (b) $\lim_{s\to\infty} Y(s)$

(c) $\lim_{s\to 0} sY(s)$ (d) $\lim_{s\to\infty} sY(s)$

24. Ans. (c)

25. Consider the function, F(s) = $\frac{5}{s(s^2+3s+2)}$ where F(s) is the Laplace transform of the function f(t).The initial value of f(t) is equal to **[EE: GATE-2004]**

(a) 5 (b) $\frac{5}{2}$ (c) $\frac{5}{3}$ (d) **0**

25. Ans. (d)

Initial value= $\lim_{S\to\infty} F(s) = \lim_{S\to\infty} \frac{5}{s^2+3s+2} = 0$

26. The Laplace transform of a function f(t) is F(s)= $\frac{5s^2+23s+6}{s(s^2+2s+2)}$. *As* $t\to\infty$, f(t) approaches

(a) 3 (b) 5 (c) $\frac{17}{2}$ (d) ∞ **[EE: GATE-2005]**

26. Ans. (a) $\underset{t\to\infty}{Lt} f(t) = \underset{t\to\infty}{Lt}\, sF(s) = \underset{t\to\infty}{Lt} \frac{5s^2+23s+6}{s^2+2s+2} = 3$

27. If u(t), r(t) denote the unit step an unit ramp functions respectively and u(t)* r(t) their convolution, then the function u(t+1)* r(t-2) is given by **[EE: GATE-2007]**

(a) (1/2)(t-1) (t-2)
(b) (1/2)(t-1)(t-2)
(c) (1/2)$(t-1)^2$u(t-1)
(d) None of these

27. Ans. (c)

$$L\left[u(t+1) = \frac{1}{s}(e^s)\right]$$

$$L[r(t-2)] = \frac{1}{s^2}e^{-2se}$$

$$\therefore L^{-1}\left[\frac{1}{s}e^s \frac{1}{s^2}e^{-2s}\right] = L^{-1}\left[\frac{e^{-s}}{s^3}\right]$$

$$= \frac{1}{2}(t-1)^2 u(t-1)$$

28. A function y(t) satisfies the following differential equation $\frac{dy(t)}{dt} + y(t) = \delta(t)$

Where $\delta(t)$ is the delta function. Assuming zero initial condition, and denoting the unit step function by $u(t), y(t)$ can be of the form **[EE: GATE-2008]**

(a) e^{t} (b) e^{-t} (c) $e^{t}u(t)$ (d) $e^{-t}u(t)$

28. Ans. (d)

$$\frac{dy(t)}{dt}+y(t)=\delta t$$

Taking Laplace transfrom of both sides, we have

sy(s) -y(o)+y(0)=1

$\Rightarrow$ $(s+1)y(s)-0=1$

$\Rightarrow$ $y(s)=\dfrac{1}{s+1}$

Taking inverse Laplace transform, we get

$y(t)=e^{-t}u(t)$

29. The Laplace transform of g(t) is **[EE: GATE-2010]**

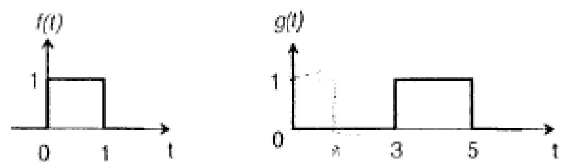
(a) $\dfrac{1}{s}(e^{3s}-e^{5s})$ (b) $\dfrac{1}{s}(e^{-5s}-e^{-3s})$

(c) $\dfrac{e^{-3s}}{s}(1-e^{-2s})$ (d) $\dfrac{1}{s}(e^{5s}-e^{3s})$

29. Ans. (c)

Common Data for Questions 30 and 31:

Given *f(t) and g(t) as* shown below:



30. g (t) can be expressed as **[EE: GATE-2010]**

(a) $g(t)=f(2t-3)$ (b) $g(t)=f\left(\dfrac{t}{2}-3\right)$

(c) $g(t)=f\left(2t-\dfrac{3}{2}\right)$ (d) $g(t)=f\left(\dfrac{t}{2}-\dfrac{3}{2}\right)$

30. Ans. (d)

31. The Laplace transform of g(t) is **[EE: GATE-2010]**

(a) $\dfrac{1}{s}(e^{3s}-e^{5s})$ (b) $\dfrac{1}{s}(e^{-5s}-e^{-3s})$

(c) $\dfrac{e^{-3s}}{s}(1-e^{-2s})$ (d) $\dfrac{1}{s}(e^{5s}-e^{3s})$

31. Ans. (c)

32. If u(t) is the unit step and δ (t) is the unit impulse function, the inverse z-transform of F(z)= $\dfrac{1}{z+1}$ for k>0 is **[EE: GATE-2005]**

(a) $(-1)^k \delta(k)$ (b) $\delta(k)-(-1)^k$

(c) $(-1)^k u(k)$ (d) $u(k)-(-1)^k$

32. Ans. (b)

$$F(z)=\frac{1}{z+1}=\frac{z+1-z}{z+1}$$

$$=1-\frac{z}{z-(-1)}.$$

$$\therefore z^{-1}\left[F(z)\right]=8(t)-\left(-1^n\right)$$

$$\left[\because z^{-1}\left(\frac{z}{z-a}\right)=a^n\right]$$

12. The running integrator, given by **[EE: GATE-2006]**

$$y(t)\int_{-\infty}^{t} x(t')dt'$$

(a) has no finite singularities in its double sided Laplace Transfrom Y(s)
(b) Produces a bounded output for every causal bounded input
(c) Produces a bounded output for every anticausal bounded input
(d) has no finite zeroes in its double sided Laplace Transfrom Y(s)

12. Ans. (b)

27. The state transition matrix for the system $\dot{X}=AX$ with initial state X(0) is **[EE: GATE-2002]**

(a) $(sI-A)^{-1}$
(b) $e^{At}X(0)$
(c) Laplace inverse of $[(s\,I-A)^{-1}]$
(d) Laplace inverse of $[(sI-A)^{-1}X\,(0)]$

27. Ans. (c)

4. Consider the matrix P = $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$. **The value of e^p is** [EC: GATE-2008]

(a) $\begin{bmatrix} 2e^{-2}-3e^{-1} & e^{-1}-e^{-2} \\ 2e^{-2}-2e^{-1} & 5e^{-2}-e^{-1} \end{bmatrix}$ (b) $\begin{bmatrix} e^{-1}+e^{-2} & 2e^{-2}-e^{-1} \\ 2e^{-1}-4e^{-2} & 3e^{-1}+2e^{-2} \end{bmatrix}$

(c) $\begin{bmatrix} 5e^{-2}+e^{-1} & 3e^{-1}-e^{-2} \\ 2e^{-2}-6e^{-1} & 4e^{-2}+e^{-1} \end{bmatrix}$ (d) $\begin{bmatrix} 2e^{-1}+e^{-2} & e^{-1}-e^{-2} \\ -2e^{-1}-2e^{-2} & -e^{-1}+2e^{-2} \end{bmatrix}$

4. Ans. (d) $e^P = L^{-1}[(sI-P)^{-1}]$

and $P=\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

where $(sI-P)^{-1}=\begin{bmatrix} s & 1 \\ 2 & s+3 \end{bmatrix}^{-1}$

$$= \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

$$\therefore \qquad e^{P} = L^{-1}\left\{\begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{2}{s+2} - \frac{1}{s+1} \end{bmatrix}\right\}$$

$$= \begin{bmatrix} 2e^{-1} + e^{-2} & e^{-1} - e^{-2} \\ -2e^{-1} + 2e^{-2} & 2e^{-2} - e^{-1} \end{bmatrix}$$

Q40. Let the Laplace transform of a function $f(t)$ which exists for $t > 0$ be $F_1(s)$ and the Laplace transform of its delayed version $f(t-\tau)$ be $F_2(s)$. Let $F^*_1(s)$ be the complex conjugate $F_1(s)$ with the Laplace variable set as $s = \sigma + j\omega$. If $G(s) = \frac{F_2(s).F^*_2(s)}{|F_1(s)|^2}$, then the inverse Laplace transform of $G(s)$ is

(a) An ideal impulse $\delta(t)$
(b) an ideal delayed impulse $\delta(t-\tau)$
(c) An ideal step function $u(t)$
(d) an ideal delayed step function $u(t-\tau)$

[EE-2011]

Ans. (b)

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33. If the Fourier transform of x[n] is $X(e^{j\omega})$, then the Fourier transform of $(-1)^n x[n]$ is

[IE: GATE-2004]

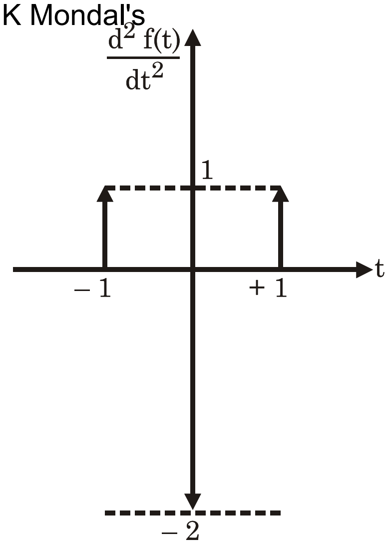
(a) $(-j)^{\omega} X(e^{j\omega})$
(b) $(-1)^{\omega} X(e^{j\omega})$
(c) $X(e^{j(\omega-\pi)})$
(d) $\frac{d}{d\omega}(X(e^{j\omega}))$

33. Ans. (c)

34. If the waveform, shown in the following figure, corresponds to the second derivative of a given function f (t), then the Fourier transform of f (t) is

(a) $1 + \sin\omega$
(b) $1 + \cos\omega$
(c) $\frac{2(1-\cos\omega)}{\omega^2}$
(d) $\frac{2(1+\cos\omega)}{\omega^2}$

[IE: GATE-2006]



34. Ans. (c)

$$\frac{d^2f(t)}{dt^2} = \delta\,(t-1) + \delta\,(t+1) - 2\delta\,(t)$$

Taking Laplace transform of both sides, we get

$$s^2\,F(s) = e^{-s} + e^{s} - 2$$

$$\Rightarrow\quad (j\omega)^2\,F\,(j\omega) = e^{-j\omega} + e^{j\omega} - 2$$

$$\Rightarrow\qquad F\,(j\omega) = \frac{2(1-\cos\omega)}{\omega^2}$$

35. The Fourier transform of a function g (t) is given as

$$\mathbf{G(\omega) = \frac{\omega^2 + 21}{\omega^2 + 9}}$$

Then the function g (t) is given as **[IE: GATE-2006]**

(a) $\delta\,(t) + 2\exp\,(-3\,|\,t\,|)$
(b) $\cos 3\omega t + 21\exp\,(-3t)$
(c) $\sin 3\omega t + 7\cos\omega t$
(d) $\sin 3\omega t + 21\exp\,(3t)$

35. Ans. (a)

$$g(t) = \square(t) + 2\,\exp.\,(-3\,|\,t\,|)$$

Taking Laplace transform both sides,

$$G\,(\omega) \qquad = 1 + 2\int_{-\infty}^{0}\exp\,(3\,t)\,\exp\,(-\,j\,\omega\,t)\,.\,dt + 2\int_{0}^{\infty}\exp\,(-3\,t)\,.\,\exp\,(-\,j\,\omega\,t)\,.\,dt$$

$$= 1 + 2\int_{-\infty}^{0}\exp\,(3 - j\,\omega)\,t\,.\,dt + 2\int_{0}^{\infty}\exp\,(-3 - j\,\omega)\,t\,.\,dt$$

$$= \ 1 + \frac{2}{3 - j\,\omega} + \frac{2}{3 + j\,\omega}$$

$$G\,(\omega) = \frac{\omega^2 + 21}{\omega^2 + 9}$$

36. The Fourier transform of $x(t) = e^{-at}\,u(-t)$, where u(t) is the unit step function,
[IE: GATE-2008]

(a) Exists for any real value of a
(b) Does not exist for any real value of a
(c) Exists if the real value of a is strictly negative
(d) Exists if the real value of a is strictly positive

36. Ans. (d)

37. The fundamental period of $x(t) = 2 \sin \pi t + 3 \sin 3\pi t$, with t expressed in seconds, is

[IE: GATE-2009]

(a) 1 s (b) 0.67 s
(c) 2 s (d) 3s

37. Ans. (d)

H.C.F. of 2π and 3π is 6π.

Then, fundamental frequency = 6π

$\therefore$ Period, $T = \dfrac{6\pi}{2\pi} = 3$ sec

38. u(t) represents the unit step function. The Laplace transform of $u(t - \tau)$ is

[IE: GATE-2010]

(a) $\dfrac{1}{s\tau}$ (b) $\dfrac{1}{s-\tau}$

(c) $\dfrac{e^{-s\tau}}{s}$ (d) $e^{-s}\tau$

38. Ans. (c)

$f(t) = u(t-\tau)$

$L\{f(t)\} = L\{u(t-\tau)\}$

$F(s) = \dfrac{e^{-s\tau}}{s}$

39. A measurement system with input x(t) and output y(t) is described by the different equation $\mathbf{3\dfrac{dy}{dt} + 5y} = 8x$. The static sensitivity of the system is [IE: GATE-2010]

(a) 0.60 (b) 1.60 (c) 1.67 (d) 2.67

39. Ans. (d)

$$\frac{3\,dy}{dt} + 5y = 8x$$

Taking Laplace transform, we have

$$3sy(s) + 5y(s) = 8X(s)$$

$$y(s)\,[3s + 5] = 8X(s)$$

$$\frac{y(s)}{x(s)} = \frac{8}{3s+5}$$

For static sensitivity, $s \to 0$

$$\therefore \quad \frac{Y(s)}{X(s)} = \frac{8}{3} \times \frac{1}{0 + \frac{5}{3}} = \frac{8}{5} = 1.6$$

40. The fundamental period of the discrete-time signal $x[n] = e^{j\left(\frac{5\pi}{6}\right)n}$ is

[IE: GATE-2008]

(a) $\dfrac{6}{5\pi}$ (b) $\dfrac{12}{5}$ (c) 6 (d) 12

40. Ans. (b)

$$\omega = \frac{5\pi}{6}$$

or $$\frac{2\pi}{T} = \frac{5\pi}{6}$$

or $$T = \frac{12}{5}$$

41. A plant with a transfer function $\frac{\mathbf{2}}{\mathbf{s(s+3)}}$ is controlled by a PI controller with $K_p = 1$ and $\mathbf{K_i} \geq 0$ in a unity feedback configuration. The lowest value of K_i that ensures zero steady state error for a step change in the reference input is

[IE: GATE-2009]

(a) 0 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

41. Ans. (b)

$$G'(s) = \left[k_p + \frac{k_i}{s}\right]\left[\frac{2}{s(s+3)}\right]$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G'(s)} \qquad \left[R(s) = \frac{1}{s}\right]$$

$$= \lim_{s\to 0} \frac{1}{1+\left[k_p + \frac{k_i}{s}\right]\left[\frac{2}{s(s+3)}\right]}$$

$$= \lim_{s\to 0} \frac{s(s+3)}{s(s+3)\,(k_p s + k_i)^2}$$

Lowest value of $k_i = \frac{1}{3}$ for G_s to be zero.