S K Mondal's

Industrial Engineering

Contents

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Note

"Asked Objective Questions" is the total collection of questions from:-

20 yrs IES (2010-1992) [Engineering Service Examination]

21 yrs. GATE (2011-1992)

and 14 yrs. IAS (Prelim.) [Civil Service Preliminary]

Every effort has been made to see that there are no errors (typographical or otherwise) in the material presented. However, it is still possible that there are a few errors (serious or otherwise). I would be thankful to the readers if they are brought to my attention at the following e-mail address: swapan_mondal_01@yahoo.co.in

S K Mondal

1. Forecasting

Theory at a Glance (For IES, GATE, PSU)

Forecasting means estimation of type, quantity and quality of future works e.g. sales etc. It is a calculated economic analysis.

1. Basic elements of forecasting:

1. Trends
2. Cycles
3. Seasonal Variations
4. Irregular Variations

2. Sales forecasting techniques:

a. Historic estimation
b. Sales force estimation
c. Trend line (or Time-series analysis) technique
d. Market survey
e. Delphi Method
f. Judge mental techniques
g. Prior knowledge
h. Forecasting by past average
i. Forecasting from last period's sales
j. Forecasting by Moving average
k. Forecasting by weighted moving average
l. Forecasting by Exponential smoothing
m. Correlation Analysis
n. Linear Regression Analysis.

I. Average method:

Forecast sales for next period = Average sales for previous period

Example:	Period No	Sales
	1	7
	2	5
	3	9
	4	8
	5	5
	6	8

Forecast sales for Period No 7 $= \dfrac{7+5+9+8+5+8}{6} = 7$

II. Forecast by Moving Average:

In this method the forecast is neither influenced by very old data nor does it solely reflect the figures of the previous period.

Example:	Year	Period	Sales	Four-period average forecasting
	1987	1	50	
		2	60	
		3	50	
		4	40	
	1988	1	50	
		2	55	

Forecast for 1988 period 1 $= \frac{50 + 60 + 50 + 40}{4} = 50$

Forecast for 1988 period 2 $= \frac{60 + 50 + 40 + 50}{4} = 50$

III. Weighted Moving Average:

A weighted moving Average allows any weights to be placed on each element, providing of course, that the sum of all weights equals one.

Example:	Period	Sales
	Month-1	100
	Month-2	90
	Month-3	105
	Month-4	95
	Month-5	110

Forecast (weights 40%, 30%, 20%, 10% of most recent month)

Forecast for month-5 would be:

$$F_5 = 0.4 \times 95 + 0.3 \times 105 + 0.2 \times 90 + 0.1 \times 100 = 97.5$$

Forecast for month-6 would be:

$$F_6 = 0.4 \times 110 + 0.3 \times 95 + 0.2 \times 105 + 0.1 \times 90 = 102.5$$

IV. Exponential Smoothing:

New forecast = α (latest sales figure) + $(1 - \alpha)$ (old forecast) [VIMP]

Where: α is known as the smoothing constant.

The size of α should be chosen in the light of the stability or variability of actual sales, and is normally from 0.1 to 0.3.

The smoothing constant, α, that gives the equivalent of an N-period moving average can be calculated as follows, $\alpha = \frac{2}{N + 1}$.

For e.g. if we wish to adopt an exponential smoothing technique equivalent to a nine-period moving average then, $\alpha = \frac{2}{9 + 1} = 0.2$

Basically, exponential smoothing is an average method and is useful for forecasting one period ahead. **In this approach, the most recent past period demand is weighted most heavily.** In a continuing manner the weights assigned to successively past period demands decrease according to exponential law.

Generalized equation:

$$F_t = \alpha.(1-\alpha)^0 d_{t-1} + \alpha.(1-\alpha)^1 d_{t-2} + \alpha.(1-\alpha)^2 d_{t-3} + \ldots\ldots\ldots + \alpha(1-\alpha)^{k-1} d_{t-k} + (1-\alpha)^k F_{t-k}$$

[Where k is the number of past periods]

It can be seen from above equation that the weights associated with each demand of equation are not equal but rather the successively older demand weights decrease by factor $(1-\alpha)$. In other words, the successive terms $\alpha(1-\alpha)^0, \alpha(1-\alpha)^1, \alpha(1-\alpha)^2, \alpha(1-\alpha)^3$ decreases exponentially.

This means that the more recent demands are more heavily weighted than the remote demands.

Exponential smoothing method of Demand Forecasting: (ESE-06)

(i) Demand for the most recent data is given more weightage.
(ii) This method requires only the current demand and forecast demand.
(iii) This method assigns weight to all the previous data.

V. Regression Analysis:

Regression analysis is also known as method of curve fitting. On this method the data on the past sales is plotted against time, and the best curve called the 'Trend line' or 'Regression line' or 'Trend curve'. The forecast is obtained by extrapolating this trend line or curve.

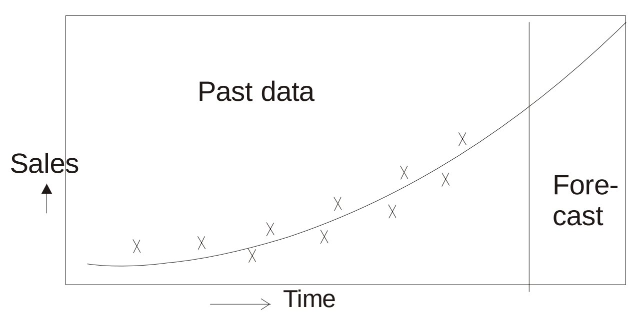
For linear regression

$$y = a + bx$$

$$a = \frac{\Sigma y - b\Sigma x}{n}$$

$$b = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}$$

$$\text{Standard error} = \sqrt{\frac{\Sigma(y-y_1)^2}{(n-2)}}$$



OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. Which one of the following forecasting techniques is not suited for making forecasts for planning production schedules in the short range? [GATE-1998]

(a) Moving average (b) Exponential moving average
(c) Regression analysis (d) Delphi

GATE-2. A moving average system is used for forecasting weekly demand. $F_1(t)$ and $F_2(t)$ are sequences of forecasts with parameters m_1 and m_2, respectively, where m_1 and m_2 ($m_1 > m_2$) denote the numbers of weeks over which the moving averages are taken. The actual demand shows a step increase from d_1 to d_2 at a certain time. Subsequently, [GATE-2008]

(a) Neither $F_1(t)$ nor $F_2(t)$ will catch up with the value d_2
(b) Both sequences $F_1(t)$ and $F_2(t)$ will reach d_2 in the same period
(c) $F_1(t)$ will attain the value d_2 before $F_2(t)$
(d) $F_2(t)$ will attain the value d_2 before $F_1(t)$

GATE-3. When using a simple moving average to forecast demand, one would

(a) Give equal weight to all demand data **[GATE-2001]**
(b) Assign more weight to the recent demand data
(c) Include new demand data in the average without discarding the earlier data
(d) Include new demand data in the average after discarding some of the earlier demand data

GATE-4. Which of the following forecasting methods takes a fraction of forecast error into account for the next period forecast? [GATE-2009]

(a) Simple average method
(b) Moving average method
(c) Weighted moving average method
(d) Exponential smoothening method

GATE-5. The demand and forecast for February are 12000 and 10275, respectively. Using single exponential smoothening method (smoothening coefficient = 0.25), forecast for the month of March is: [GATE-2010]

(a) 431 (b) 9587 (c) 10706 (d) 11000

GATE-6. The sales of a product during the last four years were 860, 880, 870 and 890 units. The forecast for the fourth year was 876 units. If the forecast for the fifth year, using simple exponential smoothing, is equal to the forecast using a three period moving average, the value of the exponential smoothing constant a is: [GATE-2005]

$(a)\frac{1}{7}$ $(b)\frac{1}{5}$ $(c)\frac{2}{7}$ $(d)\frac{2}{5}$

GATE-7. **For a product, the forecast and the actual sales for December 2002 were 25 and 20 respectively. If the exponential smoothing constant (α) is taken as 0.2, then forecast sales for January, 2003 would be: [GATE-2004]**
(a) 21 (b) 23 (c) 24 (d) 27

GATE-8. **The sales of cycles in a shop in four consecutive months are given as 70, 68, 82, and 95. Exponentially smoothing average method with a smoothing factor of 0.4 is used in forecasting. The expected number of sales in the next month is: [GATE-2003]**
(a) 59 (b) 72 (c) 86 (d) 136

GATE-9. **In a forecasting model, at the end of period 13, the forecasted value for period 14 is 75. Actual value in the periods 14 to 16 are constant at 100. If the assumed simple exponential smoothing parameter is 0.5, then the MSE at the end of period 16 is: [GATE-1997]**
(a) 820.31 (b) 273.44 (c) 43.75 (d) 14.58

GATE-10. **The most commonly used criteria for measuring forecast error is:**
(a) Mean absolute deviation (b) Mean absolute percentage error
(c) Mean standard error (d) Mean square error **[GATE-1997]**

GATE-11. **In a time series forecasting model, the demand for five time periods was 10, 13, 15, 18 and 22. A linear regression fit resulted in an equation F = 6.9 + 2.9 t where F is the forecast for period t. The sum of absolute deviations for the five data is: [GATE-2000]**
(a) 2.2 (b) 0.2 (c) –1.2 (d) 24.3

Previous 20-Years IES Questions

IES-1. **Which one of the following is not a purpose of long-term forecasting? [IES 2007]**
(a) To plan for the new unit of production
(b) To plan the long-term financial requirement.
(c) To make the proper arrangement for training the personnel.
(d) To decide the purchase programme.

IES-2. **Which one of the following is not a technique of Long Range Forecasting? [IES-2008]**
(a) Market Research and Market Survey (b) Delphi
(c) Collective Opinion (d) Correlation and Regression

IES-3. **Assertion (A): Time series analysis technique of sales-forecasting can be applied to only medium and short-range forecasting.**
Reason (R): Qualitative information about the market is necessary for long-range forecasting. [IES-2001]
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-4. Which one of the following forecasting techniques is most suitable for making long range forecasts? [IES-2005]

(a) Time series analysis (b) Regression analysis
(c) Exponential smoothing (d) Market Surveys

IES-5. Which one of the following methods can be used for forecasting when a demand pattern is consistently increasing or decreasing?

(a) Regression analysis (b) Moving average **[IES-2005]**
(c) Variance analysis (d) Weighted moving average

IES-6. Which one of the following statements is correct? [IES-2003]

(a) Time series analysis technique of forecasting is used for very long range forecasting
(b) Qualitative techniques are used for long range forecasting and quantitative techniques for short and medium range forecasting
(c) Coefficient of correlation is calculated in case of time series technique
(d) Market survey and Delphi techniques are used for short range forecasting

IES-7. Given *T* = Underlying trend, *C* = Cyclic variations within the trend, *S* = Seasonal variation within the trend and *R* = Residual, remaining or random variation, as per the time series analysis of sales forecasting, the demand will be a function of: [IES-1997]

(a) T and C (b) R and S
(c) T, C and S (d) T, C, S and R

IES-8. Which one of the following methods can be used for forecasting the sales potential of a new product? [IES-1995]

(a) Time series analysis
(b) Jury of executive opinion method
(c) Sales force composite method
(d) Direct survey method

IES-9. Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-2001]

List-I	**List-II**
A. Decision making under complete certainty	1. Delphi approach
B. Decision making under risk	2. Maximax criterion
C. Decision making under complete uncertainly	3 Transportation mode
D. Decision making based on expert opinion	4. Decision tree

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	4	1	2	(b)	4	3	2	1
(c)	3	4	2	1	(d)	4	3	1	2

IES-10. Assertion (A): Moving average method of forecasting demand gives an account of the trends in fluctuations and suppresses day-to-day insignificant fluctuations. [IES-2009]

Reason (R): Working out moving averages of the demand data smoothens the random day-to-day fluctuations and represents only significant variations.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-11. Which one of the following is a qualitative technique of demand forecasting? [IES-2006]
(a) Correlation and regression analysis (b) Moving average method
(c) Delphi technique (d) Exponential smoothing

IES-12. Match List-I (Methods) with List-II (Problems) and select the correct answer using the codes given below the lists: [IES-1998]

List-I	**List-II**
A. Moving average	1. Assembly
B. Line balancing	2. Purchase
C. Economic batch size	3. Forecasting
D. Johnson algorithm	4. Sequencing

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	1	3	2	4	(b)	1	3	4	2
(c)	3	1	4	2	(d)	3	1	2	4

IES-13. Using the exponential smoothing method of forecasting, what will be the forecast for the fourth week if the actual and forecasted demand for the third week is 480 and 500 respectively and $\alpha = 0{\cdot}2$? [IES-2008]
(a) 400 (b) 496 (c) 500 (d) 504

IES-14. The demand for a product in the month of March turned out to be 20 units against an earlier made forecast of 20 units. The actual demand for April and May turned to be 25 and 26 units respectively. What will be the forecast for the month of June, using exponential smoothing method and taking smoothing constant α as 0.2? [IES-2004]
(a) 20 units (b) 22 units (c) 26 units (d) 28 units

IES-15. A company intends to use exponential smoothing technique for making a forecast for one of its products. The previous year's forecast has been 78 units and the actual demand for the corresponding period turned out to be 73 units. If the value of the smoothening constant α is 0.2, the forecast for the next period will be: [IES-1999]
(a) 73 units (b) 75 units (c) 77 units (d) 78 units

IES-16. It is given that the actual demand is 59 units, a previous forecast 64 units and smoothening factor 0.3. What will be the forecast for next period, using exponential smoothing? [IES-2004]
(a) 36.9 units (b) 57.5 units (c) 60.5 units (d) 62.5 units

IES-17. Consider the following statements: [IES 2007]
Exponential smoothing
1. **Is a modification of moving average method**
2. **Is a weighted average of past observations**

3. Assigns the highest weight age to the most recent observation
Which of the statements given above are correct?
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 and 3 only

IES-18. In a forecasting situation, exponential smoothing with a smoothing constant $\alpha = 0.2$ is to be used. If the demand for n^{th} period is 500 and the actual demand for the corresponding period turned out to be 450, what is the forecast for the $(n + 1)^{th}$ period? [IES-2009]
(a) 450 (b) 470 (c) 490 (d) 500

IES-19. Consider the following statement relating to forecasting: [IES 2007]
1. The time horizon to forecast depends upon where the product currently lies its life cycle.
2. Opinion and judgmental forecasting methods sometimes incorporate statistical analysis.
3. In exponential smoothing, low values of smoothing constant, alpha result in more smoothing than higher values of alpha.
Which of the statements given above are correct?
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 1 and 3 only (d) 2 and 3 only

IES-20. Which one of the following statements is not correct for the exponential smoothing method of demand forecasting? [IES-2006]
(a) Demand for the most recent data is given more weightage
(b) This method requires only the current demand and forecast demand
(c) This method assigns weight to all the previous data
(d) This method gives equal weightage to all the periods

IES-21. Match List-I (Activity) with List-II (Technique) and select the correct answer using the code given below the lists: [IES-2005]

List-I	**List-II**
A. Line Balancing	1. Value analysis
B. Product Development	2. Exponential smoothing
C. Forecasting	3. Control chart
D. Quality Control	4. Selective control
	5. Rank position matrix

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	1	4	3	(b)	5	3	2	1
(c)	2	3	4	1	(d)	5	1	2	3

IES-22. For a product, the forecast for the month of January was 500 units. The actual demand turned out to be 450 units. What is the forecast for the month of February using exponential smoothing method with a smoothing coefficient = 0.1? [IES-2005]
(a) 455 (b) 495 (c) 500 (d) 545

IES-23. Which of the following is the measure of forecast error? [IES-2009]
(a) Mean absolute deviation (b) Trend value
(c) Moving average (d) Price fluctuation

Previous 20-Years IAS Questions

IAS-1. For sales forecasting, pooling of expert opinions is made use of in **[IAS-1996]**

(a) Statistical correlation (b) Delphi technique
(c) Moving average method (d) Exponential smoothing

IAS-2. To meet short range changes in demand of a product, which of the following strategies can be considered? **[IAS-2004]**

1. Overtime 2. Subcontracting
3. Building up inventory 4. New investments

Select the correct answer from the codes given below:

(a) 1, 2 and 3 (b) 1, 3 and 4
(c) 2 and 3 (d) 1 and 2

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (d) Moving, average, Exponential moving average is used for short range.
Regression is used for short and medium range.
Delphi is used for long range forecasting.

GATE-2. Ans. (d)

GATE-3. Ans. (d)

GATE-4. Ans. (d)

GATE-5. Ans. (d) $d_{n-1} = 12000, \quad F_{n-1} = 10275, \quad F_n = ?$

According to single exponential smoothing method

$$F_n = \alpha d_{n-1} + (1-\alpha) F_{n-1} = 0.25 \times 12000 + 0.75 \times 10275 = 10706.25$$

GATE-6. Ans. (c) Using simple exponential smoothing, new forecast = Old forecast + α (Actual – old forecast) and forecast using a three period moving average = (880 + 870 + 890)/3 and equate.

GATE-7. Ans. (c) Use new forecast = old forecast + α (actual demand – old forecast)

GATE-8. Ans. (b) Let expected number of sales in the next month = u_t

$$\therefore \quad u_t = \alpha s_t + \alpha(1-\alpha)s_{t-1} + \alpha(1-\alpha)^2 s_{t-2} + \alpha(1-\alpha)^3 s_{t-3}$$

where s_t = sales for the t period and so on.

$$\Rightarrow \quad u_t = 0.4 \times 95 + 0.4 \times 0.6 \times 82 + 0.4 \times (0.6)^2 68 + 0.4 \times (0.6)^3 70 = 73.52$$

GATE-9 Ans. (b)

Period	**14.0**	**15.00**	**16.000**
X_t	100.0	100.00	100.000
F_t	75.0	87.50	93.750
$(X_t - F_t)$	25.0	12.50	6.250
$\alpha(X_t - F_t)$	12.5	6.25	3.125
F_{t+1}	87.5	93.75	96.875
$(X_t - F_t)^2$	625	156.25	39.0625
$\Sigma(X_t - F_t)^2$		820.31	

Mean squared error, MSE = $\dfrac{820.31}{3}$ = 273.44

GATE-10. Ans. (d)

GATE-11. Ans. (a) Sum of absolute deviation
= $(D_1 - F_1) + (D_2 - F_2) + (D_3 - F_3) + (D_4 - F_4) + (D_5 - F_5)$
= (10 – 6.9 – 2.9x1) + (13 – 6.9 – 2.9x2) + (15 – 6.9 – 2.9x3) + …………….

Previous 20-Years IES Answers

IES-1. Ans. (c)

IES-2. Ans. (d) Correlation and Regression method is used for short and medium range forecasting.

IES-3. Ans. (b)

IES-4. Ans. (d)

IES-5. Ans. (a)

IES-6. Ans. (b)

IES-7. Ans. (c) Sale forecasting should not be influenced by the random variations in demand.
IES-8. Ans. (d)
IES-9. Ans. (c)
IES-10. Ans. (a)
IES-11. Ans. (c)
IES-12. Ans. (d)
IES-13. Ans. (b) $F_4 = \alpha d_3 + (1-\alpha) F_3 = (0.2)(480) + (0.8)500 = 96 + 400 = 496$

IES-14. Ans. (b) $\alpha = 0.2, \quad D_{\text{March}} = 20 \text{ units} \quad D_{\text{April}} = 25 \quad D_{\text{May}} = 26$

$$F_{\text{Mar}} = 20 \text{ units} \quad F_{\text{April}} = 20 \quad F_{\text{May}} = 21 \quad F_{\text{Jun}} = ?$$

$$F_{\text{April}} = \alpha \times D_{\text{Mar}} + (1-\alpha) \; F_{\text{Mar}}$$

$$= 0.2 \times 20 + 0.8 \times 20$$

$$F_{\text{May}} = \alpha \times D_{\text{April}} + (1-\alpha) \; F_{\text{April}} = 0.2 \times 25 + 0.8 \times 20 = 21$$

$$F_{\text{June}} = \alpha \times D_{\text{May}} + (1-\alpha) \; F_{\text{May}} = 0.2 \times 26 + 0.8 \times 21 = 22 \text{ units}$$

IES-15. Ans. (c) New forecast = Old forecast + α(actual – old forecast)
= 78 + 0.2 (73 – 78) = 77
IES-16. Ans. (d) $D = 59$ units, $F = 64$ units, $\alpha = 0.3$

$$\text{New forecast} = \alpha \times (\text{latest sales figure}) + (1-\alpha)(\text{old forecast})$$

$$= 0.3 \times 59 + (1-0.3) \times 64 = 62.5$$

IES-17. Ans. (c) 1 is false: Exponential smoothing is a modification of weightage moving average method.
IES-18. Ans. (c) $F_{n+1} = \alpha d_n + (1-\alpha) F_n = (0.2)(450) + (1-0.2)500 = 90 + 400 = 490$

Forecast for $(n+1)^{th}$ period = 490

IES-19. Ans. (b) Higer the value of α-is more responsive & lower is most stable.
IES-20. Ans. (d)
IES-21. Ans. (d)
IES-22. Ans. (b) $F_n = \alpha D_{n-1} + (1-\alpha) F_{n-1} = 0.1 \times 450 + (1-0.1) \times 500 = 495$ units
IES-23. Ans. (a)

Previous 20-Years IAS Answers

IAS-1. Ans. (b)
IAS-2. Ans. (b)

Conventional Questions with Answer

Conventional Question [ESE-2010]

Question: What are moving average and exponential smoothing models for forecasting? A dealership for Honda city cars sells a particular model of the car in various months of the year. Using the moving average method, find the exponential smoothing forecast for the month of October 2010. Take exponential smoothing constant as 0.2:

Jan.	2010	80	cars
Feb.	2010	65	cars
March	2010	90	cars
April	2010	70	cars
May	2010	80	cars
June	2010	100	cars
July	2010	85	cars
Aug.	2010	65	cars
Sept.	2010	75	cars

[15 Marks]

Answer:

(i) **Moving average model for forecasting:** Refer theory part of this book**.**

(ii) **Exponential smoothing model for forecasting:** Refer theory part of this book

Months	Sells cars	Forecast demand (n = 3)
Jan.	80	
Feb.	65	
March	90	
April	70	(80+65+90)/3=78.33
May	80	(65+90+70)/3=75
June	100	(90+70+80)/3=80
July	85	(70+80+100)/3=83.33
Aug.	60	(80+100+85)/3=88.33
Sep.	75	(100+85+60)/3=81.67

Forecast of oct. by exponential smoothing method

$$F_{oct} = F_{sep} + \propto (D_{sep.} - F_{sep.})$$

$$\propto = 0.2 \quad F_{sep} = 73.33 \quad D_{spt.} = 75$$

$$F_{oct} = 81.67 + 0.2\ (75 - 81.67)$$

$$F_{oct} = 80.33$$

$$\simeq 81$$

Forecast for the month of October using moving average

$$F_{oct} = \frac{D_{July} + D_{Aug} + D_{Sep}}{3}$$

$$= \frac{80 + 60 + 75}{3}$$

$$= 71.67$$

Conventional Question [ESE-2006]

Explain the need for sales forecasting. How are forecasting methods classified?

The past data about the load on a machine centre is as given below:

Month	Load, Machine-Hours
1	585
2	611
3	656
4	748
5	863
6	914
7	964

(i) If a five month moving average is used to forecast the next month's demand, compute the forecast of the load on the centre in the 8th month.
(ii) Compute a weighted three moving average for the 8th month, where the weights are 0.5 for the latest month, 0.3 and 0.2 for the other months, respectively. [10-Marks]

Solution: Most organisations are not in a position to wait unit orders are received before they begin to determine what production facilities, process, equipment, manpower, or materials are required and in what quantities. Most successful organizsation nticipate the future and for their products and translate that information into factor inputs required to satisfy expected demand. Forecasting provides a blue print for managerial planning. Forecasting is the estimation of the future on the basis of the past.

In many organizations, sales forecasts are used to establish production levels, facilitate scheduling, set inventory levels, determine man power loading, make purchasing decisions, establish sales conditions (pricing and advertising) and aid financial planning (cash budgeting and capital budgeting).

A good forecast should have the following attributes. It should be accurate, simple, easy, economical, quick and upto date. Following are the basic steps involved in a systematic demand forecast.

(i) State objectives
(ii) Select method
(iii) Identify variables
(iv) Arrange data
(v) Develop relationship
(vi) Prepare forecast and interpret
(vii) Forecast in specific units.

(i) Forecast for 8th month on the basis of five month moving average
$= (964 + 914 + 863 + 748 + 656)/5 = 829$

(ii) Forecast for 8th month on the basis of weighted average
$= 0.5 \times 964 + 0.3 \times 914 + 0.2 \times 863 = 928.8$

Conventional Question [ESE-2009]

(i) List common time-series forecasting models. Explain simple exponential smoothing method of forecasting demand. What are its limitations?
(ii) The monthly forecast and demand values of a firm are given below:

Month	Forecast units	Demand units
Jan	100	97
Feb	100	93
Mar	100	110
Apr	100	98
May	102	130
Jun	104	133
Jul	106	129

Aug	108	138
Sep	110	136
Oct	112	124
Nov	114	139
Dec	116	125

Calculate Tracking Signal for each month. Comment on the forecast model.

[10-Marks]

Solution: **(i)** Component of time series models

(1) Trend (T)
(2) Cyclic variation (C)
(3) Seasonal variation (S)
(4) Random variation (R)

Exponential Smoothing

This is similar to the weighted average method. The recent data is given more weightage and the weightages for the earlier periods are successfully being reduced. Let $\mathbf{x_1}$ is the actual (historical) data of demand during the period t. Let α is the weightage given for the period t and $\mathbf{F_1}$ is the forecast for the time t then forecast for the time (t + 1) will be given as

$$\mathbf{F_{t+1} = F_t + \alpha(x_t - F_t)}$$

$$\mathbf{F_{(t+1)} = \alpha\ x_t + (1-\alpha)F_t}$$

(ii) Tracking signal

$$= \frac{\textbf{Cumulative deviation}}{\textbf{MAD}}$$

$$= \frac{\sum(\mathbf{x_t - F_t})}{\textbf{MAD}}$$

Where,

MAD = Mean Absolute deviation

$$= \frac{\textbf{Sum of absolute deviations}}{\textbf{Total number of datas}}$$

$$== \frac{\sum(\mathbf{x_t - F_t})}{\mathbf{n}}$$

Month	**Forecast Unit**	**Demand Unit (x_t)**	$(x_t - F_t)$	**MAD**	$\sum(x_t - F_t)$	$\mathbf{T.S} = \frac{\sum(x_t - F_t)}{\mathbf{MAD}}$
January	100	97	- 3	3	-3	-1
February	100	93	- 7	5	-10	-2
March	100	110	10	6.67	0	0
April	100	98	- 2	5.5	-2	-0.3636
May	102	130	28	10	26	2.6
June	104	133	29	13.167	55	4.177
July	106	129	23	14.571	78	5.353
August	108	138	30	16.5	108	6.545
September	110	136	26	17.55	134	7.635
October	112	124	12	17	146	8.588
November	114	139	25	17.727	171	9.646
December	116	125	9	17	180	10.588

Mean square error (MSE) $= \dfrac{\sum\left(\mathbf{x_t - F_t}\right)^2}{\mathbf{n}} = \dfrac{\mathbf{4742}}{\mathbf{12}} = \mathbf{395.167}$

Upper limit $= \mathbf{3} \times \sqrt{\mathbf{MSE}} = \mathbf{3} \times \sqrt{\mathbf{395.167}} = \mathbf{59.636}$

Since upper limit of $\mathbf{T.S} < \mathbf{59.636}$ hence modal should not be revised.

Conventional Question **[ESE-2001]**

Demand for a certain item has been as shown below:

The forecast for April was 100 units with a smoothing constant of 0.20 and using first order exponential smoothing what is the July forecast? What do you think about a 0.20 smoothing constant?

Time	Actual Demand	
April	**200**	
May	**50**	
June	**150**	**[10]**

Solution: Using exponential smoothing average:

$$F_{\text{May}} = \alpha \times D_{\text{April}} + (1-\alpha)\ F_{\text{April}} = 0.2 \times 200 + (1-0.2) \times 100 = 120$$

$$F_{\text{June}} = \alpha \times D_{\text{May}} + (1-\alpha) F_{\text{May}} = 0.2 \times 50 + (1-.2) \times 120 = 106$$

$$F_{\text{July}} = \alpha \times D_{\text{June}} + (1-\alpha) \times F_{\text{June}} = 0.2 \times 150 + 0.8 \times 106 = 114.8 \simeq 115$$

Conventional Question **[GATE-2000]**

In a time series forecasting model, the demand for five time periods was 10, 13, 15 18 and 22. A linear regression fit results in as equation $F = 6.9 + 2.9\ t$ where F is the forecast for period t. The sum of absolute deviation for the five data is?

Solution: Sum of absolute deviation

$= (D_1 - F_1) + (D_1 - F_2) + (D_3 - F_3) + (D_4 - F_4) + (D_5 - F_5)$

$= (10 - 6.9 - 2.91) + (13 - 6.9 - 2.92) + (15 - 6.9 - 2.93)$

$\quad + (18 - 6.9 - 2.9 - 2.94) + (22 - 6.9 - 2.95)$

$= 0.2 + 0.3 + 0.6 + 0.5 + 0.6 = 2.2$

2. Routing & Scheduling

Theory at a Glance (For IES, GATE, PSU)

Routing

Routing includes the planning of: what work shall be done on the material to produce the product or part, where and by whom the work shall be done. It also includes the determination of path that the work shall follow and the necessary sequence of operations which must be done on the material to make the product.

Routing procedure consist of the following steps:

The finished product is analysed thoroughly from the manufacturing stand point, including the determination of components if it is an assembly product. Such an analysis must include:

(i) Material or parts needed.

(ii) Whether the parts are to be manufactured, are to be found in stores (either as raw materials or worked materials), or whether they are-to be purchased.

(iii) Quantity of materials needed for each part and for the entire order.

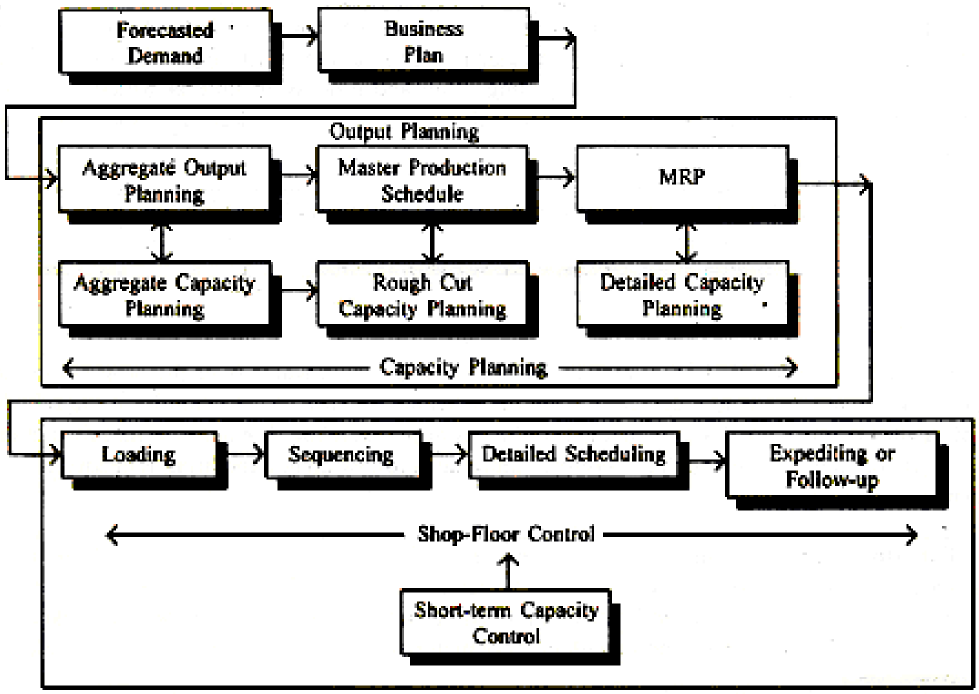
The following activities are to be performed in a particular sequence for routing a product

1. Analysis of the product and breaking it down into components.
2. Taking makes or buys decisions.
3. Determination of operations and processing time requirement.
4. Determination of the lot size.

Scheduling

Introduction

Scheduling is used to allocate resources over time to accomplish specific tasks. It should take account of technical requirement of task, available capacity and forecasted demand. Forecasted demand determines plan for the output, which tells us when products are needed. The output-plan should be translated into operations, timing and schedule on the shop-floor. This involves loading, sequencing, detailed scheduling, expediting and input/output control.



The Planning and Scheduling Function

Loading

The customer order for each job has certain job contents, which need to be performed on various work centers or facilities. During each planning period, jobs orders are assigned on facilities. This ultimately determines the work-load or jobs to be performed in a planned period.

The assignment of specific jobs to each operational facility during a planning period is known as loading.

Sequencing

When number of jobs are waiting in queue before an operational facility (such as, a milling machine), there is a need to decide the sequence of processing all the waiting jobs. Sequencing is basically an order in which the jobs, waiting before an operational facility, are processed. For this, priority rule, processing time, etc., are needed.

The decision regarding order in which jobs-in-waiting are processed on an operational facility or work-centre is called as *sequencing*.

Detailed Scheduling

Once the priority rule of job sequencing is known, we can sequence the jobs in a particular order. This order would determine which job is done first, then which the next one is and so on. However, sequencing does not tell us the day and time at which a particular job is to be done. This aspect is covered in detailed scheduling. In this, estimates are prepared regarding setup and processing time at which a job is due to start and finish. Detailed

Detailed scheduling encompasses the formation of starting and finishing time of all jobs at each operational facility.

Expediting

Once the detailed schedule is operationalized, we need to keep a watch over the progress in the shop-floor. This is necessary to avoid a deviation from the schedule. In case of deviation from the schedule, the causes of deviation are immediately attended to. For example, machine breakdown, non-availability of a tool, etc., cause disruption in schedule. Therefore, continuous follow up or expediting is needed to overcome the deviations from schedule.

Expediting or follow-up involves continuous tracking of the job's progress and taking specific action if there is a deviation from the detailed schedule. The objective of expediting is to complete the jobs as per the detailed schedule and overcome any special case causing delay, failure, break-down, non-availability of material and disruption of detailed schedule.

Short-term Capacity (Input-output) Control

Schedules are made so that jobs are completed at a specific time on every facility. For this, each facility has certain capacity to perform. In real situation, the utilization of the capacity of each facility may be different from the planned one. This difference should be monitored carefully because under-utilization of capacity means waste resource and over-utilization may cause disruption, failure, delays, or even breakdown. Therefore, in case of discrepancy in input and output of the capacities, some adjustments in schedule are needed.

Short-term capacity control involves monitoring of deviation between actual and planned utilization of the capacity of an operational facility.

There are two types of schedules used: Master Schedules and Shop or Production Schedule.

1. **Master schedule:** The first step in scheduling is to prepare the Master Schedule. ***A master schedule specifies the product to be manufactured, the quality to be produced and the delivery date to the customer.*** It also indicates the relative importance or manufacturing orders. The scheduling periods used in the master schedule are usually months. Whenever a new order is received, it is scheduled on the master schedule taking into account the production capacity of the plant. Based on the master schedule, individual components and sub-assemblies that make up each product are planned:
   (i) Orders are placed for purchasing raw materials to manufacture the various components.
   (ii) Orders are placed for purchasing components from outside vendors.
   (iii) Shop or production schedules are prepared for parts to be manufactured within the plant.

The objectives of master schedule are:

1. It helps in keeping a running total of the production requirements.
2. With its help, the production manager can plan in advance for any necessity of shifting from one product to another or for a possible overall increase or decrease in production requirements.
3. It provides the necessary data for calculating the back log of work or load ahead of each major machine.
4. After an order is placed in the master schedule, the customer can be supplied with probable or definite date of delivery.

2. Shop or production schedule: After preparing the master schedule, the next step is to prepare shop or production schedule. This includes the department machine and labour-load schedules, and the start dates and finish dates for the various components to be manufactured within the plant.

A scheduling clerk does this job so that all processing and shipping requirements are relatively met. For this, the following are the major considerations to be taken case of:

(i) Due date of the order.
(ii) Whether and where the machine and labour capacity are available.
(iii) Relative urgency of the order with respect to the other orders.

Objectives of Production Schedule:

1. It meets the output goals of the master schedule and fulfils delivery promises.
2. It keeps a constant supply of work ahead of each machine.
3. It puts manufacturing orders in the shortest possible time consistent with economy.

The Scheduling Problem

List Scheduling Algorithms

This class of algorithms arranges jobs on a list according to some rule. The next job on the list is then assigned to the first available machine.

Random List

This list is made according to a random permutation.

Longest Processing Time (LPT)

The longest processing time rule orders the jobs in the order of decreasing processing times. Whenever a machine is free, the largest job ready at the time will begin processing. This algorithm is a heuristic used for finding the minimum make span of a schedule. It schedules the longest jobs first so that no one large job will "stick out" at the end of the schedule and dramatically lengthen the completion time of the last job.

Shortest Processing Time (SPT)

The shortest processing time rule orders the jobs in the order of increasing processing times. Whenever a machine is free, the shortest job ready at the time will begin processing. This algorithm is optimal for finding the minimum total completion time and weighted completion time. In the single machine environment with ready time at 0 for all jobs, this algorithm is optimal in minimizing the mean flow time, minimizing the mean number of jobs in the system, minimizing the mean waiting time of the jobs from the time of arrival to the start of processing, minimizing the maximum waiting time and the mean lateness.

Weighted Shortest Processing Time (WSPT)

The weighted shortest processing time rule is a variation of the SPT rule. Let t[i] and w[i] denote the processing time and the weight associated with the job to be done in the sequence ordered by the WSPT rule. WSPT sequences jobs such that the following inequality holds,

$$t[1]/w[1] \Leftarrow t[2]/w[2] \Leftarrow \ldots \Leftarrow t[n]/w[n]$$

In the single machine environment with ready time set at 0 for all jobs, the WSPT minimizes the weighted mean flow time.

Earliest Due Date (EDD)

In the single machine environment with ready time set at 0 for all jobs, the earliest due date rule orders the sequence of jobs to be done from the job with the earliest due date to the job with the latest due date. Let d[i] denote the due date of the ith job in the ordered sequence . EDD sequences jobs such that the following inequality holds,

$$d[1] \Leftarrow d[2] \Leftarrow \ldots d[n]$$

EDD, in the above setting, finds the optimal schedule when one wants to minimize the maximum lateness, or to minimize the maximum tardiness.

Minimum Slack Time (MST)

The minimum slack time rule measures the "urgency" of a job by its slack time. Let $d[i]$ and $t[i]$ denote the due date and the processing time associated with the i^{th} job to be done in the ordered sequence. MST sequences jobs such that the following inequality holds,

$$d[1] - t[1] \Leftarrow d[2] - t[2] \Leftarrow \ldots \Leftarrow d[n] - t[n]$$

In the single machine environment with ready time set at 0, MST maximizes the minimum lateness.

Other Algorithms

Hodgson's Algorithm

Hodgson's Algorithm minimizes the number of tardy jobs in the single machine environment with ready time equal to zero.

Let E denote the set of early jobs and L denote the set of late jobs. Initially, all jobs are in set E and set L is empty.

Step 1: Order all jobs in the set E using EDD rule.

Step 2: If no jobs in E are late, stop; E must be optimal. Otherwise, find the first late job in E. Let this first late job be the k^{th} job in set E, job $[k]$.

Step 3: Out of the first k jobs, find the longest job. Remove this job from E and put it in L. Return to step 2.

Scheduling of *n* Jobs on One Machine (*n*/1 Scheduling)

There are five jobs in waiting for getting processed on a machine. Their sequence of arrival, processing time and due-date are given in the table below. Schedule the jobs using FCFS, SPT, D Date, LCFS, Random, and STR rules. Compare the results.

Job (In Sequence of Arrival)	Processing Time (Days)	Due Date (i.e., Days From Now)
J1	4	6
J2	5	7
J3	3	8
J4	7	10
J5	2	3

***Solution*:**

(i) FCFS (First-come-first-serve) Rule

In this, the job, which arrives first, is scheduled first. Then the next arrived job is scheduled, and so on.

Job Arrival (In Sequence) (i)	Processing Time (Days) (p_i)	Due Date (Days From Today) (d_i)	Flow Time (Days) $F_i = (F_{i-1}+p_i)$	Lateness of Job = (F_i-d_i); if $F_i > d_i$ Otherwise Zero
J1	4	6	0 + 4 = 4	0
J2	5	7	4 + 5 = 9	2
J3	3	8	9 + 3 = 12	4
J4	7	10	12 + 7 = 19	9
J5	2	3	19 + 2 = 21	18

Total flow time = 4 + 9 + 12 + 19 + 21 = 65 days

Mean flow time = $\dfrac{\text{Total flow time}}{\text{Number of jobs}} = \dfrac{65}{5}$ = 13 days

Total lateness of job = 0 + 2 + 4 + 9 + 18 = 33 days

Average lateness of job = $\dfrac{33}{5}$ = 6.6 days.

(ii) SPT (Shortest Processing Time) Rule or SOT (Shortest Operation Time) Rule

This rule gives highest priority to that job, which has shortest processing time. This approach gives following sequence of jobs for the given problem:

Job Sequence (i)	Processing Time (Days) (p_i)	Due-date (Days From Hence) (d_i)	Flow Time (Days) $F_i = (F_{i-1} + p_i)$	Lateness of Job $L_i = (F_i-d_i)$ if $F_i > d_i$ Otherwise Zero
J5	2	3	0 + 2 = 2	0
J3	3	8	2 + 3 = 5	0
J1	4	6	5 + 4 = 9	9 – 6 = 3
J2	5	7	9 + 5 = 14	14 – 7 = 7
J4	7	10	14 + 7 = 21	21 – 10 = 11

Total flow time = 2 + 5 + 9 + 14 + 21 = 51 days

Mean flow time = $\dfrac{51}{5}$ = 10.2 days

Total lateness of jobs = 3 + 7 + 11 = 21 days

Average lateness of job = $\dfrac{21}{5}$ = 4.2 days.

This rule gives highest priority to the job having earliest due-date:

Job Sequence (i)	Processing Time (Days) p_i	Due-date (Days From Hence) d_i	Flow Time (Days) $F_i = (F_{i-1} + p_i)$	Lateness of Job $L_i = (F_i - d_i)$ if $F_i > d_i$ Otherwise Equal to Zero
J5	2	3	0 + 2 = 2	0
J1	4	6	2 + 4 = 6	0
J2	5	7	6 + 5 = 11	11 – 7 = 4
J3	3	8	11 + 3 = 14	14 – 8 = 6
J4	7	10	14 + 7 = 21	21 – 10 = 11

Total flow time = 2 + 6 + 11 + 14 + 21 = 54 days

Mean flow time = $\frac{54}{5}$ = 10.8 days

Total lateness of job = 0 + 0 + 4 + 6 + 11 = 21 days

Average lateness of job = $\frac{21}{5}$ = 4.2 days.

(iv) LCFS (Last-come-first-serve) Rule

This rule gives highest priority to that job, which has arrived most recently. Most recent job is the last arrived job. The scheduling of jobs on this rule is explained through the earlier example.

Job Sequence (i)	Processing Time (Days) p_i	Due-date (Days From Hence) d_i	Flow Time (Days) $F_i = (F_{i-1} + p_i)$	Lateness of Job $L_i = (F_i - d_i)$ if $F_i > d_i$ Otherwise Equal to Zero
J5	2	3	0 + 2 = 2	0
J4	7	10	2 + 7 = 9	0
J3	3	8	9 + 3 = 12	12 – 8 = 4
J2	5	7	12 + 5 = 17	17 – 7 = 10
J1	4	6	17 + 4 = 21	21 – 6 = 15

Total flow time = 2 + 9 + 12 + 17 + 21 = 61 days

Mean flow time = $\frac{61}{5}$ = 12.2 days

Total lateness of job = 4 + 10 + 15 = 29 days

Average lateness of job = $\frac{29}{5}$ = 5.8 days.

(v) Random Schedule Rule

Take any job randomly. The rule gives priority of jobs in a random order. Let the random selection of job be: J4 $\rightarrow$ J3 $\rightarrow$ J1 $\rightarrow$ J5 $\rightarrow$ J2.

Job Sequence (i)	Processing Time (Days) (p_i)	Due-date (Days From Hence) (d_i)	Flow Time (Days) $F_i = (F_{i-1} + p_i)$	Lateness of Job $L_i = (F_i - d_i)$ if $F_i > d_i$ Otherwise Equal to Zero
J4	7	10	0 + 7 = 7	0
J3	3	8	7 + 3 = 10	10 − 8 = 2
J1	4	6	10 + 4 = 14	14 − 6 = 8
J5	2	3	14 + 2 = 16	16 − 3 = 13
J2	5	7	16 + 5 = 21	21− 7 = 14

Total flow time = 7 + 10 + 14 + 16 + 21 = 68 days

Mean flow time = $\frac{68}{5}$ = 13.6 days

Total lateness of job = 2 + 8 + 13 + 14 = 37 days

Average lateness of job = $\frac{37}{5}$ = 7.4 days.

(vi) STR (Slack Time Remaining) Rule

STR is calculated as the difference between the times remaining before the due-date minus remaining processing time.

Job Sequence (i)	Processing Time (Days) p_i	Due-date (Days From Hence) d_i	Flow Time (Days) $F_i = (F_{i-1} + p_i)$	Lateness of Job $L_i = (F_i - d_i)$ if $F_i > d_i$ Otherwise Equal to Zero
J5	2	3	0 + 2 = 2	0
J1	4	6	2 + 4 = 6	0
J2	5	7	6 + 5 = 11	11 − 7 = 4
J4	7	10	11 + 7 = 18	18 − 10 = 8
J3	3	8	18 + 3 = 21	21 − 8 = 13

Total flow time = 2 + 6 + 11 + 18 + 21 = 58 days

Mean flow time = $\frac{58}{5}$ = 11.6 days

Total lateness of job = 4 + 8 + 13 = 25 days

Average lateness of job = $\frac{25}{5}$ = 5 days.

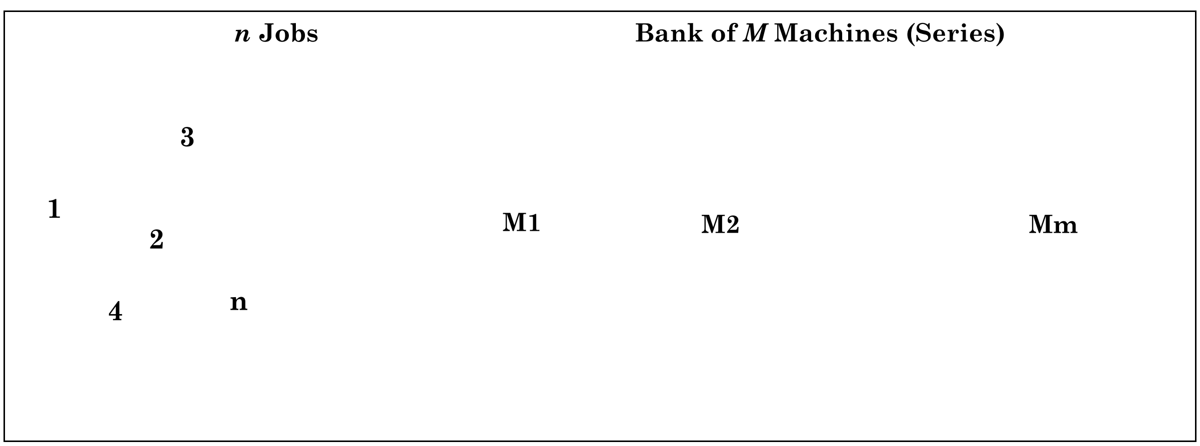
Comparison of Sequencing Rules (for the given problem)

Rule	Total Flow Time to Complete Jobs (Days)	Average Time to Complete Jobs (Days)	Average Lateness
FCFS	65	13	6.6
SPT	51	10.2	4.2
DDate	54	10.8	4.2
LCFS	61	12.2	5.8
Random	68	13.6	7.4
STR	58	11.6	5

It is observed that SPT sequencing rule (for single machine and many jobs) performs better than other rules in minimizing total flow time, average flow time, and average lateness of jobs. It may be noted that this observation is valid for any **"*n* job- one machine"** ***(n/1)*** **scheduling problem.**

Johnson's Rule

Flow Shop Scheduling
(*n* jobs, *m* machines)



Flow shop with two machines in series with unlimited storage in between the two machines.

There are n jobs and the processing time of job j on machine 1 is p_{1j} and the processing time on machine 2 is p_{2j} the rule that minimizes the make span is commonly referred to as Johnson's rule.

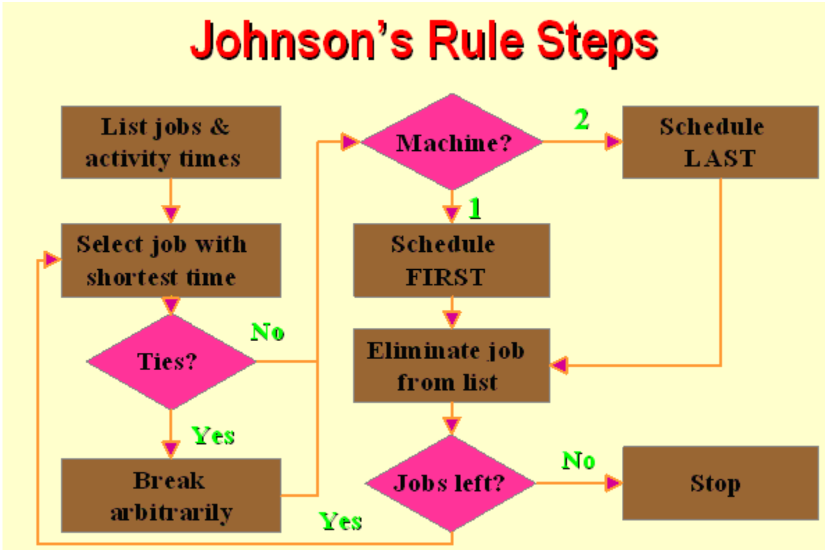
Algorithm of Johnson's Rule

1. Identify the job with the **smallest** processing time (on either machine).
2. If the smallest processing time involves:

   Machine 1, schedule the job at the beginning of the schedule.

Machine 2, schedule the job toward the end of the schedule.

3. If there is some unscheduled job, go to 1 otherwise stop.



Johnson's Algorithm for 3 Machines

At first we have to convert it equivalent two-machine problem.
Solving an equivalent two-machine problem with processing times:

$$p'_{1j} = p_{1j} + p_{2j} \quad \text{and} \quad p'_{2j} = p_{2j} + p_{3j}$$

Then apply the above rules to p'_1 and p'_2

***Example*:**

JOB	1	2	3	4	5
p1j	4	4	10	6	2
p2j	5	1	4	10	3

Apply Algorithm of Johnson's rule easily find the sequence

Ans. 5 – 1 – 4 – 3 – 2.

Analysis of Result

The present sequence is analyzed for time on machines as follows:

Jobs	Machine 1		Machine 2	
	Time in	Time out	Time in	Time out
J6	0	1	1*	11
J1	1	5	11	16
J2	5	11	16	24
J5	11	20	24	30
J4	20	28	30	33

* Processing time for J6 on M_2 is 1 min and its processing on M_1 is over only after 1 min. Therefore, only after 1 min, next job J1 will start on M_1 and J6 will go on M_2.

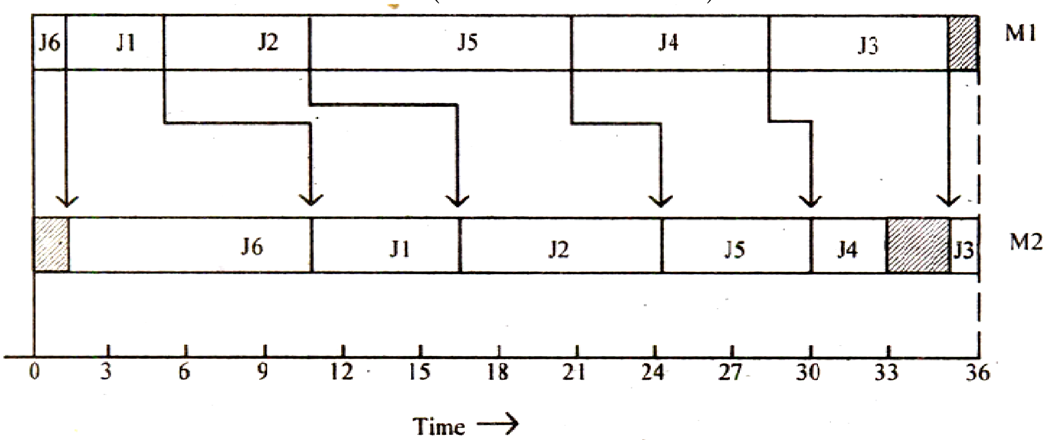
* * Job3 will start on M_2 only after 35 min as it's out-time on M_1 is 35 min. In all other cases, the jobs are waiting to be loaded on M_2 (except J6 and J3).

(a) Idle time for machine 1 = (Total elapsed time) – (Total busy time for machine 1)

$$= T - \sum_{i=1}^{6} t_{i1} = 36 - 35 = 1 \text{ min.}$$

(b) Idle time for machine 2 = $T - \sum_{i=1}^{6} t_{i2}$

$$= 36 - (5 + 8 + 1 + 3 + 6 + 10) = 36 - 33 = 3 \text{ min.}$$



Scheduling of Six Jobs on 2 Machine

Process *n*-Jobs on 3 Machines (*n*/3 Problem) and Jackson Algorithm

For a special *n* jobs and 3 machines problem, Jackson provided an extension of Johnson's algorithm. For this, let t_{ij} be the processing time of job *i* on machine *j*. Here, $i = 1, 2, \ldots n,$ and $j = 1, 2, 3$.

At lease one of the following conditions must be satisfied before we can use this algorithm:

(i) Minimum $\{t_{i1}\} \geq$ Maximum $\{t_{i2}\}$
(ii) Minimum $\{t_{i3}\} \geq$ Maximum $\{t_{i2}\}$

Step 1

Take two hypothetical machines R and S. The processing time on R and S is calculated as follows:

$$t_{iR} = t_{i1} + t_{i2}$$
$$t_{iS} = t_{i2} + t_{i3}$$

Step 2

Use Johnson's algorithm to schedule jobs on machines *R* and *S* with t_{iR} and t_{iS}.

Example:

Six jobs are to be processed on three machines. The processing time is as follows

	J2	J3	J4	J5	J6
	3	5	4	2	1
	4	6	3	1	2
	6	7	9	7	7

Solution:

Check for necessary conditions:

$$\text{Min } \{t_{i1}\} = 1$$
$$\text{Max } \{t_{i2}\} = 6$$
$$\text{Min } \{t_{i3}\} = 6$$

Now, since Min $\{t_{i3}\} \geq$ Max $\{t_{i2}\}$; and, Min $\{t_{i1}\} \geq$ Max $\{t_{i2}\}$ are satisfied, **the Jackson's algorithm may be used.**

Now, let us frame two hypothetical machines R and S on which the processing times are:

	J1	J2	J3	J4	J5	J6
	12	7	11	7	3	3
	10	10	11	12	8	9

Using Johnson's algorithm the optimum sequence for two machines R and S and six jobs is:

J5	J6	J2	J4	J3	J1

The time calculations are as follows:

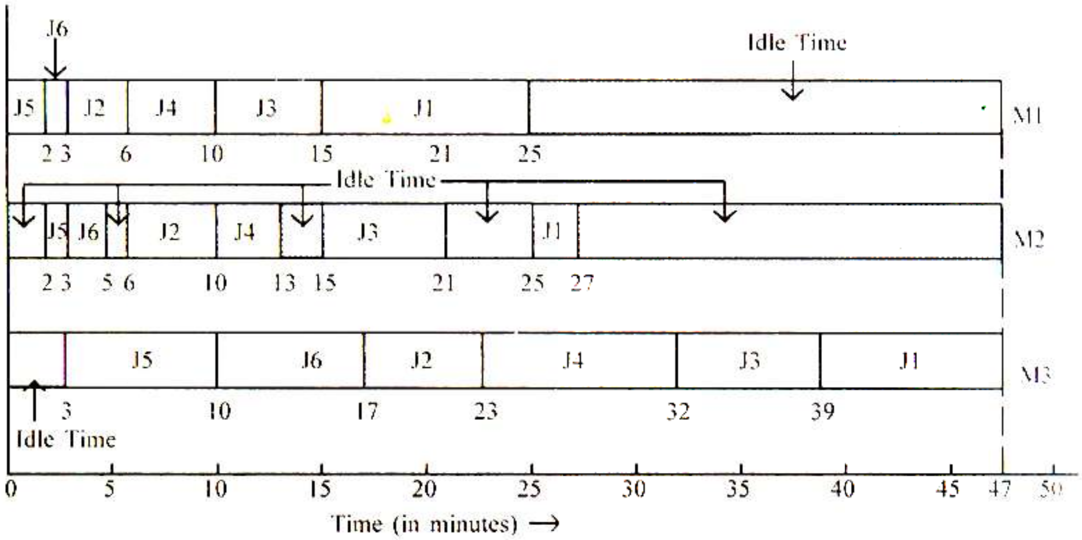
Job	[illegible] $_1$		M_2		M_3	
	Time in	Time out	Time in	Time out	Time in	Time out
J5	0	2	2	3	[illegible]	[illegible]
J6	[illegible]	3	3	5	[illegible]	[illegible]
J2	3	6	6	10	17	[illegible]
J4	6	10	10	[illegible]	23	32
[illegible]	10	15	15	[illegible]	32	39
[illegible]	15	25	25	[illegible]	39	[illegible]

Calculation of Machine Idle Time:

Idle time for machine 2(M_2) = (2 – 0) + (6 – 5) + (15 – 13) + (25 – 21) + (47 – 27)

= 2 + 1 + 2 + 4 + 20 = 29 min.

Idle time for machine 3(M_3) = (3 – 0) = 3 min.



Gantt Chart for *n/3* Problems

Processing of 2 Jobs on *m* Machine (2/*m*) Problem

Let there be two jobs: J1 and J2. Each job is to be processed on *m* machines: M_1, M_2 ... M_m. There are two different sequences, one each for each job. It is not permissible to have alternative sequences. Only one job can be performed at a time on the two machines. The processing time is known and is deterministic. The problem is to find the sequence of processing so as to minimize the total elapsed time in the system.

Technique: Graphical method is used to solve this problem. It can be illustrated with an example.

***Example*:**

Two jobs *J*1 and *J*2 are to be processed on five machines M_1, M_2, ... M_5. The processing time and job sequences are as follows:

Job 1

Machine Sequence	M_1	M_2	M_3	M_4	M_5
Processing Time	2	5	6	6	7

Job 2

Machine Sequence	M_3	M_1	M_4	M_5	M_2
Processing Time	5	6	4	3	7

Find the total minimum elapsed time using graphical approach.

***Solution*:**

Step 1: On a graph paper, represent processing times of jobs *J*1 and *J*2 on *X* and *Y*-axes,

Step 3: Shade the common area for each machines (Fig. below).

Step 4: Start from origin. Draw a line in phases of diagonally (at 45°), horizontally and vertically. The only condition to be avoided is to cross a shaded area by the diagonal line.

The line moving horizontally (i.e, along job l) means that J1 is processed and J2 is idle; while line moving vertically means that J2 is processed and J1 is idle. A diagonal line means that both J1 and J2 are processed.

The shaded portion is avoided to be crossed by diagonal line, because at any time both J1 and J2 cannot be processed on the same machine.

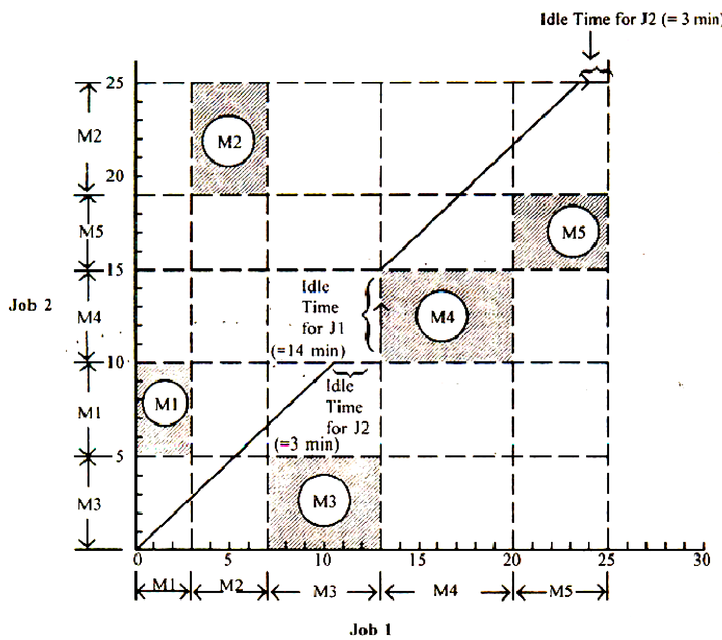
Step 5: Note the idle time for each job from graph.

Calculation of Elapsed Time

Elapsed time = Processing time + Idle Time

For Job 1: Elapsed time = (2 + 5 + 6 + 6 + 7) + (5)
= 26 + 5 = 31 min.

For Job 2: Elapsed time = (5 + 6 + 4 + 3 + 7) + (3 + 3)
= 25 + 6 = 31 min.



Graphical Solution of (2/5) Problem

***Example*:**

Use Hodgson's algorithm to schedule five jobs for which the processing time *(t_i)* and due-date *(d_i)* are as follows:

Task *(i)*	1	2	3	4	5
t_i	5	3	1	6	2
d_i	5	8	10	12	15

***Solution*:**

Using EDD Rule

Task (i)	1	2	3	4	5	
t_i (Given)	5	3	1	6	2	Processing time of *ith* job
C_i	0+5)=5	(5+3)=8	(8+1)=9	(9+6)=15	(15+2)=17	Completion time of *ith* job
d_i (Given)	5	8	10	12	15	Due date of *ith* job
$L_i = (C_i - d_i)$, if positive, or = 0; if negative	0	0	0 (as negative is unacceptable	(15–12)=3	(17–15)=2	Lateness of *ith* job

Steps: Higden Algorithm *(step explained below).*

Step 1: Identify first job which is late = 4*th* job.

Step 2: Form a string of jobs into first late job.

1	2	3	4	← String of jobs
5	3	1	6	Processing time

Step 3: Identify in this string the job of maximum processing time = Job 4 with maximum Processing time = job 4.

Step 4: Remove this job from string of jobs and put in the new late job in the string and repeat Steps 1 to 4.

i	1	2	3	5
t_i (Given)	5	3	1	2
C_i	5	8	9	11
d_i (Given)	5	8	10	15
$L_i = (C_i - d_i)$, if positive, or = 0; if negative	0	0	0	0

Step 5: Since at this stage there is no late job, we will stop.

Hence, solution is:

Task (i)	1	2	3	5	4
t_i	5	3	1	2	6
C_i	5	8	9	11	17
d_i	5	8	10	15	12
L_i	0	0	0	0	(17–12)=5

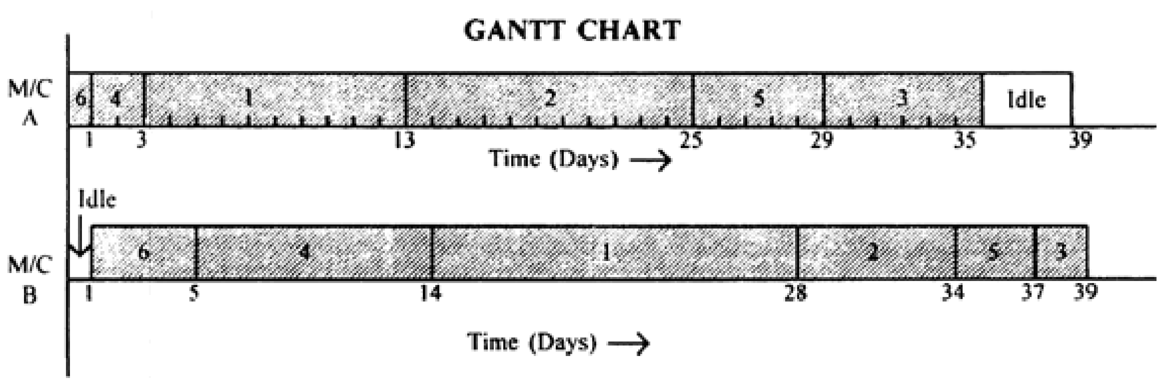
***Example*: Use Johnson's algorithm to schedule six jobs and two machines:**

Task	Time on Machine A	Time on Machine B
1	10	14
2	12	6
3	6	2
4	2	9
5	4	3
6	1	4

***Solution*: Using Johnson's rule:**

1. Select task with least processing time in the string of the given jobs. If it is on machine A, place at the left-end otherwise on right-end.
2. Remove that task from string and apply rule again.
3. Repeat steps 1 – 2 till all jobs are over. Sequencing is as follows as per Johnson's rule:

# Job/Task →	6	4	1	2	5	3



***Example*: Use *Jackson's Extension of Johnson's Rule* to schedule five jobs on three machines.**

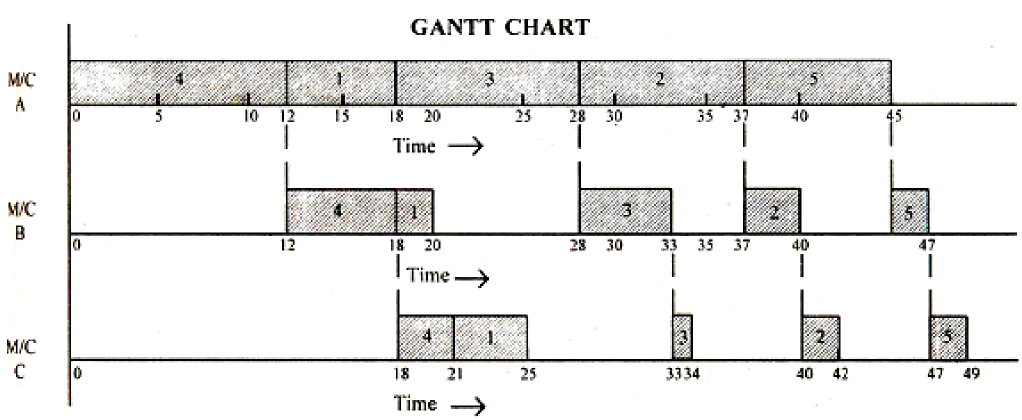
Job	Processing Time		
	M/C A	M/C B	M/C A
1	6	2	4
2	9	3	2
3	10	5	1
4	12	6	3
5	8	2	2

***Solution*:**
Since machine B is dominated by machine A: as maximum processing time of machine B (=6) is less than or equal to the minimum processing time on machine A (= 6). Hence, above problem is converted to fit into 2 machine *n* job as follows:

Job	Processing Time	
	M/C I (A+B)	M/C II (B+C)
1	8	6
2	12	5
3	15	6
4	18	9
5	10	4

Using Johnson's rule the optimal sequence is:

# Job →	4	1	3	2	5

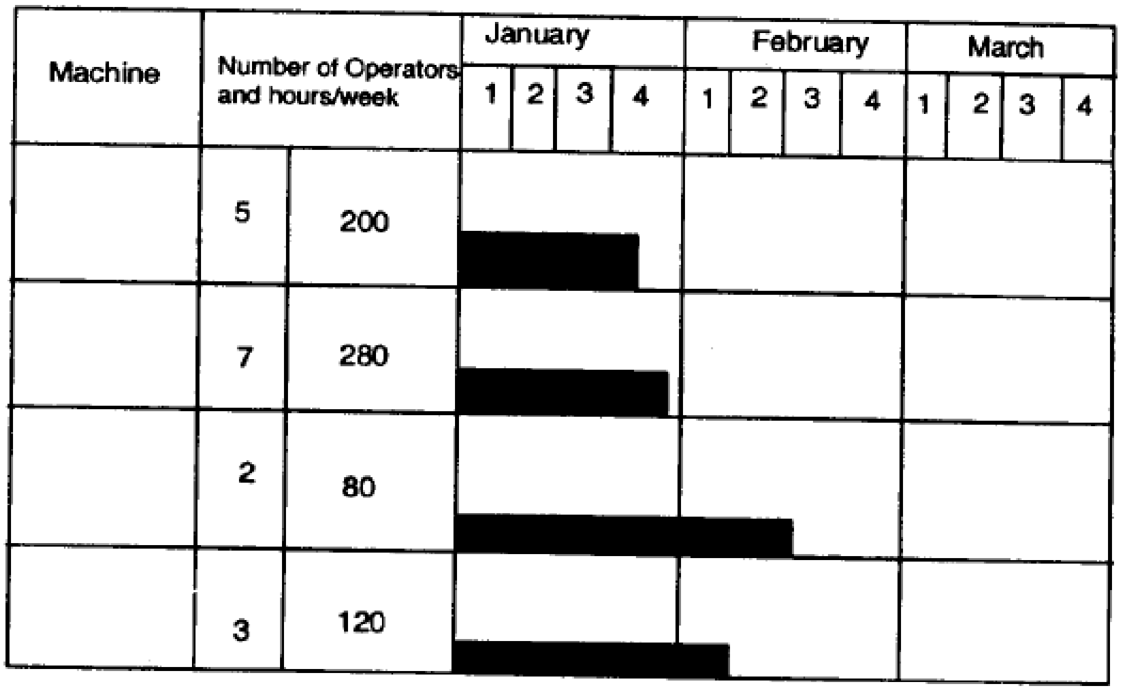


Make Span in 49 unit of Time

Machine Loading

Allocating the **job to work** centres is referred to as "Machine Loading", while allocation of jobs to the entire shop is called "Shop Loading". The production planners can safeguard the and equipment. The load capacity of a machine may be expressed in terms of pieces for a given length of time or in time for a given number of pieces. In either case, the capacity may be determined very readily from the standard time values of the operations performed by the machine.

A machine load chart is a chart for showing the work ahead for various machines and processes. A typical machine load chart is shown in figure. Here, the load is expressed in terms of the number of hours for a given number of pieces. Such a chart is known as 'Bar Chart' or 'Gantt Chart'. A bar represents a task. It is shown along the horizontal axis which indicates time scale.



A Typical Machine Load Chart

Despatching

After the schedule has been completed, the production planning and control department makes a master manufacturing order with complete information including routing, the desired completion dates within each department or on each machine and the engineering drawings. From this master manufacturing order, departmental manufacturing orders can be made up giving only the information necessary for each individual foreman.

These include inspection tickets and authorization to move the work from one department to the next when each department's work is completed. When a foreman of a particular department receives the manufacturing order, he is authorized to begin production in his department. The despatching of these orders and instructions at the proper time to the proper people is usually done by a person know as **"Despatcher".**

So, "Despatcher" function consists of issuing the orders and instruction which sets production in "motion in accordance with production schedules and routings. This function is purely a clerical function and requires voluminous paper work.

Duties of a Despatcher

1. Initiate the work by issuing the current work order instructions and drawings to the different production departments, work stations, machine operators or foremen. The various documents despatched include: detailed machine schedules, route sheets, operations sheets, materials requisition forms, machine loading cards, move or material ticket and inspection ticket plus work order.

   [**Note:** It is not raw material it is material from store]
2. Release materials from stores.
3. Release production tooling, that is, all tools, jigs, fixtures and gauges for each operation before operation is started.
4. Keep a record of the starting and completion date of each operation.
5. Getting reports back from the men when they finish the jobs.

Works order documents. The usual formats of various works order documents used by the despatcher are the route sheet (card), operation sheet and machine loading chart.

1. Work order
2. Machine load chart
3. Material requisition form [but not raw material]
4. Move ticket
5. Inspection ticket

The term "despatching" is not much heard. in decentralized control where the foreman passes out the jobs. It is used mainly with centralized control where the production control's branch office (despatch office) in each department tells men what jobs to work on. In this system, one shop order copy, known as "traveller", circulates through the shop with the parts.

Product Development

What is Product Development?

Product development is the process of designing, creating, and marketing an idea or product. The product can either be one that is new to the market place or one that is new to your particular company, or, an existing product that has been improved. In many instances a product will be labelled new and improved when substantial changes have been made.

The Product Development Process

All product development goes through a similar planning process. Although the process is a continuous one, it is crucial that companies stand back after each step and evaluate whether the new product is worth the investment to continue. That evaluation should be based on a specific set of objective criteria, not someone's gut feeling. Even if the product is wonderful, if no one buys it the company will not make a profit.

Brainstorming and developing a concept is the first step in product development. Once an idea is generated, it is important to determine whether there is a market for the product, what the target market is, and whether the idea will be profitable, as well as whether it is feasible from an engineering and financial standpoint. Once the product is determined to be feasible, the idea or concept is tested on a small sample of customers within the target market to see what their reactions are.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Scheduling

GATE-1. Production flow analysis (PFA) is a method of identifying part families that uses data from [GATE-2001]

(a) Engineering drawings (b) Production schedule
(c) Bill of materials (d) Route sheets

The Scheduling Problem and Johnson's Rule

GATE-2. A manufacturing shop processes sheet metal jobs, wherein each job must pass through two machines (M_1 and M_2, in that order). The processing time (in hours) for these jobs is:

Machine	Jobs					
	P	Q	R	S	T	U
M1	15	32	8	27	11	16
M2	6	19	13	20	14	7

The optimal make-span (in hours) of the shop is: [GATE-2006]

(a) 120 (b) 115 (c) 109 (d) 79

Common Data Q3 and Q4: [GATE-2010]

Four jobs are to be processed on a machine as per data listed in the table.

Job	Processing time (in days)	Due date
1	4	6
2	7	9
3	2	19
4	8	17

GATE-3. If the Earliest Due-date (EDD) rule is used to sequence the jobs, the number of jobs delayed is: [GATE-2010]

(a) 1 (b) 2 (c) 3 (d) 4

GATE-4. Using the Shortest Processing Time (SPT) rule, total tardiness is:

(a) 0 (b) 2 (c) 6 (d) 8

[GATE-2010]

Previous 20-Years IES Questions

Routing

IES-1. Routing in production planning and control refers to the
(a) Balancing of load on machines **[IES-2000]**
(b) Authorization of work to be performed
(c) Progress of work performed
(d) Sequence of operations to be performed

IES-2. The routing function in a production system design is concerned with. [IES-1996]
(a) Manpower utilization
(b) Machine utilization
(c) Quality assurance of the product
(d) Optimizing material flow through the plant

Scheduling

IES-3. Consider the following statements: [IES 2007]
Scheduling
1. Is a general timetable of manufacturing
2. Is the time phase of loading
3. Is loading all the work in process on
4. Machines according to their capacity
Which of the statements given above are correct?
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 and 3 only

IES-4. Consider the following statements: [IES-2004]
1. Preparation of master production schedule is an iterative process
2. Schedule charts are made with respect to jobs while load charts are made with respect to machines
3. MRP is done before master production scheduling
Which of the statements given above are correct?
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-5. Which of the following factors are to be considered for production scheduling? [IES-1995]
1. Sales forecast 2. Component design
3. Route sheet 4. Time standards
Select the correct answer using the codes given below:
Codes: (a) 1, 2 and 3 (b) 1, 2 and 4 (c) 1, 3 and 4 (d) 2, 3 and 4

IES-6. Assertion (A): Planning and scheduling of job order manufacturing differ from planning and scheduling of mass production manufacturing. [IES-1994]
Reason (R): In mass production manufacturing, a large variety of products are manufactured in large quantity.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A

(c) A is true but R is false
(d) A is false but R is true

IES-7. Production scheduling is simpler, and high volume of output and high labour efficiency are achieved in the case of: [IES-1993]
(a) Fixed position layout (b) Process layout
(c) Product layout (d) A combination of line and process layout

IES-8. A manufacturer's master product schedule of a product is given below: [IES-1999]

Period Planned:	**Week-l**	**Week-2**	**Week-3**
Planned Production:	**50**	**100**	**100**
	Week-4	**Week-5**	**Week-6**
	100	**150**	**50**

Each product requires a purchased component A in its sub-assembly. Before the start of week-1, there are 400 components of type A in stock. The lead time to procure this component is 2 weeks and the order quantity is 400. Number of components A per product is only one. The manufacturer should place the order for
(a) 400 components in week-l
(b) 400 components in week-3
(c) 200 components in week-l and 200 components in week-3
(d) 400 components in week-5

Machine Loading

IES-9. Which one of the following charts gives simultaneously, information about the progress of work and machine loading? [IAS-1995]
(a) Process chart (b) Machine load chart
(c) Man-machine chart (d) Gantt chart

IES-10. Which one of the following is required for the preparation of the load chart machine? [IAS-1998]
(a) Process chart (b) Sequencing of jobs on the machine
(c) Route sheet of jobs (d) Schedule of jobs for the machine

Despatching

IES-11. Despatching function of production planning and control refers to:
(a) A dispatch of finished goods on order **[IES-2001; IAS-1997, 1999]**
(b) Movement of in-process material from shop to shop
(c) Authorizing a production work order to be launched
(d) Dispatch of bills and invoices to the customer

IES-12. Which one of the following statements is not correct? [IES-2008]
(a) Schedule chart shows the processing of a job on various work centres against time
(b) Load chart shows the processing of various jobs on a work centre against time
(c) Dispatching is the activity related with dispatching of goods to the customers
(d) Routing is the activity related with the operations and their sequence to be performed on the job.

IES-13. Which one of the following statements is correct in relation to production, planning and control? [IES-1999]

(a) Expediting initiates the execution of production plans, whereas dispatching maintains them and sees them through to their successful completion
(b) Dispatching initiates the execution of production plans, whereas expediting maintains them and sees them through to their successful completion
(c) Both dispatching and expediting initiate the execution of production plans
(d) Both dispatching and expediting maintain the production plans and see them through to their successful completion

IES-14. Consider the following statement [IES-1998]
Dispatching
1. **Is the action of operations planning and control**
2. **Releases work to the operating divisions.**
3. **Conveys instructions to the shop floor.**

Of these statements:

(a) 1, 2 and 3 are correct (b) 1 and 2 are correct
(c) 2 and 3 are correct (d) 1 and 3 are correct

IES-15. Which one of the following statements correctly defines the term 'despatching'? [IES-2003]

(a) Maintaining the record of time of starting and completion of each operation
(b) The appraisal and evaluation of human work in terms of time
(c) Taking all such steps which are meant to affect and implement the programme of production according to plans
(d) Moving the work after completion to the next process or machine on the route

IES-16. In production, planning and control, the document which authorizes the start of an operation on the shop floor is the [IES-2001]

(a) Dispatch order (b) Route plan (c) Loading chart (d) Schedule

Product Development

IES-17. The value engineering technique in which experts of the same rank assemble for product development is called [IES-1993]

(a) Delphi (b) Brain storming
(c) Morphological analysis (d) Direct expert comparison

IES-18. Which one of the following is the preferred logical sequence in the development of a new product? [IES-2002]

(a) Technical feasibility, social acceptability and economic viability
(b) Social acceptability, economic viability and technical feasibility
(c) Economic viability, social acceptability and technical feasibility
(d) Technical feasibility, economic viability and social acceptability

IES-19. Consider the following aspects: [IES-2009]

1. Functional 2. Operational 3. Aesthetic
Which of the above aspects is/are to be analyzed in connection with the product development?
(a) 1, 2 and 3 (b) 1 and 2 only (c) 2 and 3 only (d) 3 only

IES-20. Consider the following statements: [IES-2009]
The immediate objective of a product is:
1. **To simulate sales function**
2. **To utilize the existing equipment and power**
3. **To monopolize the market**

Which of the above statements is/are correct?
(a) 1, 2 and 3 (b) 1 and 2 only (c) 2 and 3 only (d) 3 only

Previous 20-Years IAS Questions

Routing

IAS-1. The following activities are to be performed in a particular sequence for routing a product [IAS-1994]
1. **Analysis of the product and breaking it down into components**
2. **Determination of the lot size**
3. **Determination of operations and processing time requirement**
4. **Taking makes or buys decisions**

The correct sequence of these activities is
(a) 1, 2, 3, 4 (b) 3, 1, 2, 4 (c) 3, 1, 4, 2 (d) 1, 4, 3, 2

Scheduling

IAS-2. Which of the following are the objectives of scheduling? [IAS-2007]
1. **Reducing average waiting time of a batch**
2. **To meet the deadline of order fulfillment**
3. **To improve quality of products**
4. **To increase facility utilization**

Select the correct answer using the code given below:
(a) 1, 2 and 4 (b) 2, 3 and 4 (c) 1, 2 and 3 (d) 1 and 3 only

IAS-3. Which one of the following is the correct definition of critical ratio in scheduling? [IAS-2004]
(a) Demand time/supply lead time
(b) Supply lead time/demand time
(c) Demand time/manufacturing lead time
(d) Manufacturing lead time/demand time

IAS-4. Consider the following advantages: [IAS-2000]
1. **Very flexible**
2. **Simple to understand**
3. **Detailed operation can be visualized**

Which of these are the advantages of master scheduling?
(a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) 1, 2 and 3

IAS-5. **Activities involved in production planning and control system are listed below:** **[IAS-1997]**

1. Planning	**2. Loading**
3. Scheduling	**4. Despatching**
5. Routing	**6. Follow up**

The correct sequence of these activities in production planning and control system is:

(a) 1, 3, 5, 4, 2, 6 (b) 1, 5, 3, 4, 2, 6 (c) 1, 5, 3, 2, 4, 6 (d) 1, 3, 5, 2, 4, 6

IAS-6. **Assertion (A): Conventional production planning techniques cannot be used for managing service operations.** **[IAS-2002]**
Reason (R): Service operations cannot be inventoried.

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-7. **Which of the following pairs are correctly matched?** **[IAS-1996]**

1. Project scheduling — Critical path analysis
2. Batch production — Line of balance scheduling
3. Despatching — Job order
4. Routing — Gantchart

Select the correct answer using the codes given below:
Codes: (a) 1,3 and 4 (b) 1,2 and 4 (c) 2 and 3 (d) 1, 2 and 3

IAS-8. **Consider the following advantages** **[IAS-1994]**

1. Lower in-process inventory
2. Higher flexibility in rescheduling in case of machine breakdown
3. Lower cost in material handling equipment

When compared to process layout, the advantages of product layout would include

(a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) 1, 2, and 3

IAS-9. **Match List-I (Charts) with List-II (Applications) and select the correct answer using the codes given below the lists:** **[IAS-2003]**

List-I	**List-II**
A. Operation process chart	**1.** Scheduling project operations
B. Flow process chart	**2.** To study backtracking and traffic congestion
C. Flow diagram	**3.** To analyze indirect costs such as material handling cost
D. PERT chart	**4.** To study relations between operations

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	1	4	3	(b)	4	3	2	1
(c)	2	3	4	1	(d)	4	1	2	3

IAS-10. **In which one of the following types of industrial activities, the problem of loading and scheduling becomes more difficult?**

(a) Single-product continuous (b) Multi-product continuous **[IAS-2001]**
(c) Batch production (d) Continuous or process production

IAS-11. Which of the following factors necessitate a change in schedule?
1. Change in Board of Directors 2. Capacity modification
3. Lack of capital 4. Change in priority
5. Unexpected rush orders **[IAS-2004]**
Select the correct answer using the codes given below:
(a) 2, 3 and 4 (b) 1, 2 and 5 (c) 2, 4 and 5 (d) 1, 3 and 4

The Scheduling Problem and Johnson's Rule

IAS-12. Johnson's rule is applicable for planning a job shop for **[IAS-2002]**
(a) n machines and 2 jobs (b) 2 machines and n jobs
(c) n machines and n jobs (d) 1 machine and n jobs

Machine Loading

IAS-13. Which one of the following charts gives simultaneously, information about the progress of work and machine loading? **[IAS-1995]**
(a) Process chart (b) Machine load chart
(c) Man-machine chart (d) Gantt chart

IAS-14. Which one of the following is required for the preparation of the load chart machine? **[IAS-1998]**
(a) Process chart (b) Sequencing of jobs on the machine
(c) Route sheet of jobs (d) Schedule of jobs for the machine

Despatching

IAS-15. Dispatching function of production planning and control refers to:
(a) A dispatch of finished goods on order **[IAS-1997, 1999; IES-2001]**
(b) Movement of in-process material from shop to shop
(c) Authorizing a production work order to be launched
(d) Dispatch of bills and invoices to the customer

IAS-16. In a low volume production, the dispatching function is not concerned with issuing of which one of the following? **[IAS-2007]**
(a) Work tickets
(b) Requisition of raw materials, parts and components
(c) Route sheets to production supervisor
(d) Requisition of tools and facilities

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (b, c)
GATE-2. Ans. (b) Using Johnson's algorithm

	M1		M2	
	IN	OUT	IN	OUT
R	0	8	8	21
T	8	19	21	35
S	19	46	46	66
Q	46	78	78	97
U	78	94	97	104
P	94	109	109	115

GATE-3. Ans. (c) Alternating According to EDD

J of	Processing time (in days)	Due date	Flow time	Lateness
1	4	6	4	0
2	7	9	11	2
4	8	17	19	2
3	2	19	21	2

Lateness = flow time- due date
It is take it zero
So, we can see there are 3 joins are determine

GATE-4. Ans. (b) Arranging According to S P T

J of	Processing time (in days)	Due date	Flow time	Lateness
3	2	19	2	0
1	4	6	6	0
2	7	9	13	4
4	8	17	21	4

So total tardiness = 4 + 4 = 8/4 = 2

Previous 20-Years IES Answers

IES-1. Ans. (d)
IES-2. Ans. (d)
IES-3. Ans. (a)
IES-4. Ans. (b)
IES-5. Ans. (d)
IES-6. Ans. (c) A is true and R is false.
IES-7. Ans. (c)
IES-8. Ans. (b)
IES-9. Ans. (b)
IES-10. Ans. (d)
IES-11. Ans. (c)

to the operating facility through the release of orders and instruction in accordance with a previously developed plan of activity (time and sequence) establish by scheduling section of the production planning and control department.

IES-13. Ans. (b)
IES-14. Ans. (a)
IES-15. Ans. (c)
IES-16. Ans. (a)
IES-17. Ans. (b) Value engineering technique in which experts of the same rank assemble for product development is called brain storming.
IES-18. Ans. (b)
IES-19. Ans. (a)
IES-20. Ans. (a)

Previous 20-Years IAS Answers

IAS-1. Ans. (d)
IAS-2. Ans. (a)
IAS-3. Ans. (a)
IAS-4. Ans. (a)
IAS-5. Ans. (c)
IAS-6. Ans. (a)
IAS-7. Ans. (d)
IAS-8. Ans. (b)
IAS-9. Ans. (b)
IAS-10. Ans. (c)
IAS-11. Ans. (c)
IAS-12. Ans. (b)
IAS-13. Ans. (b)
IAS-14. Ans. (d)
IAS-15. Ans. (c)
IAS-16. Ans. (b)

Conventional Questions with Answer

Conventional Question **[ESE-2001]**

Consider the following jobs and their processing times at corresponding machines:

	Duration (hr)		
Job	**Machine I**	**Machine II**	**Machine III**
A	**13**	**5**	**9**
B	**5**	**3**	**7**
C	**6**	**4**	**5**
D	**7**	**2**	**6**

Using Johnson's rule, find the optimal sequence.

Solution: At first we convert it equivalent two-machine problem.

Job	$J_1 = M_1 + M_2$	$J_2 = M_2 + M_3$
A	18	14
B	8	10
C	40	9
D	9	8

Apply Johnson's Algorithm for 2 machine we can easily find the sequence

Ans. B – A – C – D

***Question*:** **Processing time (in minute) of six jobs on two machines are given below. Use Johnson's rule to schedule these jobs.**

Job	J1	J2	J3	J4	J5	J6
Machine M_1	4	6	7	8	9	1
Machine M_2	5	8	1	3	6	10

***Solution*:**

Minimum processing time of 1 min is for J3 on M_2 and J6 on M_1.

Place J6 at the first and J3 at the end of the sequence.

J6					J3

Now, remove J3 and J6 from the consideration. We have the following jobs:

Job	J1	J2	J4	J5
Machine M_1	4	6	8	9
Machine M_2	5	8	3	6

Out of all the remaining processing times, J4 on M_2 is least and equal to 3 minutes. So, place it at the last of the sequence. It is in the last because of being least processing time on M_2 and not on M_1.

After eliminating J4 from the above list, we have J1, J2 and J5. Out of all remaining processing times, J1 on M_1 is least and is equal to 4 min. Therefore, place this job at the beginning of the list. After placing J4 at the end and J1 in the beginning we have the following sequence:

J6	J1			J4	J3

Now, the remaining jobs are J2 and J5. Looking at their processing times, it is observed that the least time is 6 min. for J2 on M_1 and J5 on M_2. Therefore, place J2 at the beginning of left-most slot of sequence and J5 at the right-most slot of the sequence. The optimal sequence is J6, J1, J2, J5, J4 and J3:

J6	J1	J2	J5	J4	J3

3. Line Balancing

Theory at a Glance (For IES, GATE, PSU)

Assembly Line Balancing

Introduction

An assembly line is a flow-oriented production system where the productive units performing the operations, referred to as stations, are aligned in a serial manner. The workpiece visit stations successively as they are moved along the line usually by some kind of transportation system, e.g. a conveyor belt.

Objective in Line Balancing Problem

In an assembly line, the problem is to design the work station. Each work station is designed to complete few processing and assembly tasks. The objective in the design is to assign processes and tasks to individual stations so that the total time required at each work station is approximately same and nearer to the desired cycle time or production rate.

In case, all the work elements which can be grouped at any station have same station time, then this is a case of perfect line balancing. Production flow would be smooth in this case. However, it is difficult to achieve this in reality. When perfect line balancing is not achieved, the station time of slowest station would determine the production rate or cycle time.

Example: Let us consider a five-station assembly system in which the station times are 12, 16, 13, 11 and 15 minutes respectively. The slowest station is station 2, which takes 16 min., while station 4 is fastest with 11 min. of station time. Work carrier enters at station: and leaves at station 5. Now a work carrier

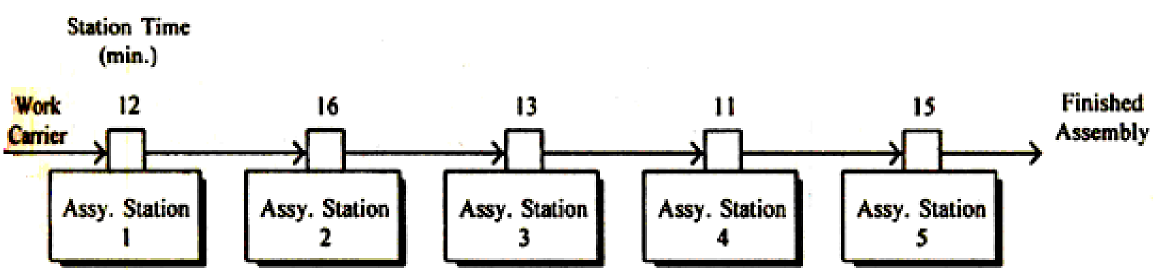


Fig. A

At station 1 cannot leave station 1 after 12 minutes as station 2 is not free after 12 minutes of work on a previously arrived work carrier. Only after 16 minutes it is free to pull work carrier from station 1. Therefore, station 1 will remain idle for (16 – 12) = 4 min. Similarly, in each cycle, station 3,4 and 5 would be idle for 3, 2 and 4 min.

Since, idle time at any station is the un-utilized resource, the objective of line balancing is to minimise this.

An assembly line consists of (work) stations k = 1, ..., m usually arranged along a conveyor belt or a similar mechanical material handling equipment. The workpieces (jobs) are consecutively launched down the line and are moved from station to station. At each station, certain operations are repeatedly performed regarding the cycle time (maximum or average time available for each work cycle).

Manufacturing a product on an assembly line requires partitioning the total amount of work into a set $V = \{1, ..., n\}$ of elementary operations named tasks. Performing a task j takes a task time t_j and requires certain equipment of machines and/or skills of workers. The total work load necessary for assembling a workpiece is measured by the sum of task times t_{sum}. Due to technological and organizational conditions precedence constraints between the tasks have to be observed.

These elements can be summarized and visualized by a precedence graph. It contains a node for each task, node weights for the task times, arcs for the direct and paths for the indirect precedence constraints. Figure 1 shows a precedence graph with n = 9 tasks having task times between 2 and 9 (time units).

Any type of ALBP consists in finding a feasible line balance, i.e., an assignment of each task to a station such that the precedence constraints (Figure 1) and further restrictions are fulfilled.

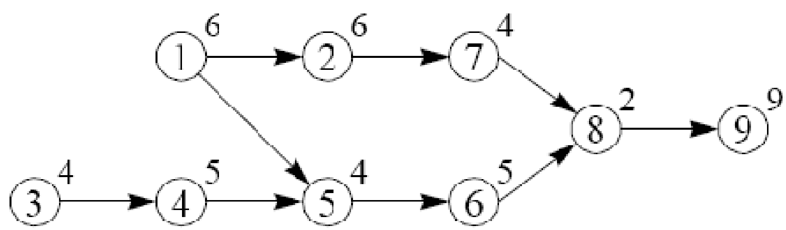


Figure 1. Precedence graph

The set S_k of task assigned to a station k (= 1, ..., m) constitutes its station load or work content, the cumulated task time $t(s_x) = \sum_{j \in s_k} t_j$ is called station time.

When a fixed common cycle time c is given (paced line), a line balance is feasible only if the station time of neither station exceeds c. In case of $t(S_k) < c$, the station k has an idle time of $c - t(S_k)$ time units in each cycle. For the example of Figure 1, a feasible line balance with cycle time c = 11 and m = 5 stations is given by the station loads $S_1=\{1, 3\}$, $S_2 = \{2, 4\}$, $S_3 = \{5, 6\}$, $S_4 = \{7, 8\}$, $S_5 = \{9\}$.

Because of the long-term effect of balancing decisions, the used objectives have to be carefully chosen considering the strategic goals of the enterprise. From an economic point of view cost and profit related objectives should be considered. However, measuring and predicting the cost of operating a line over months or years and the profits achieved by selling the products assembled is rather complicated and error-prone. A usual surrogate objective consists in maximizing the line utilization which is measured by the line efficiency E as the productive fraction of the line's total operating time and directly depends on the cycle time c and the number of stations m. In the most simple case, the line efficiency is defined as follows: $\boldsymbol{E = t_{sum} / (m.c)}$.

- The classic example is **Henry Ford's** auto chassis line.
  - Before the "moving assembly line" was introduced in 1913, each chassis was assembled by one worker and required 12.5 hours.
  - Once the new technology was installed, this time was reduced to 93 minutes.

- Favorable Conditions
  - Volume adequate for reasonable equipment utilization.
  - Reasonably stable product demand.
  - Product standardization.
  - Part interchange-ability.
  - Continuous supply of material.

- Not all of the above must be met in every case.

❖ Minimum rational work element
- Smallest feasible division of work.

❖ Flow time = time to complete all stations

❖ Cycle time
- Maximum time spent at any one workstation.
- Largest workstation time.
- How often a product is completed.
- Inverse of the desired hourly output rate = the amount of time available at each work station to complete all assigned work.

$$\boxed{1} \longrightarrow \boxed{2} \longrightarrow \boxed{3}$$

4 min 5 min 4 min

Flow time = 4 + 5 + 4 = 13
Cycle time = max (4, 5, 4) = 5

❖ Total work content: Sum of the task times for all the assembly tasks for the product.

❖ Precedence diagram: network showing order of tasks and restrictions on their performance.

❖ Measure of efficiency.

$$\text{Efficiency} = \frac{\text{Sum of task times } (T)}{\text{Actual number of workstations } (N_a) \times \text{Cycle time } (C)}$$

Constraints in Line Balancing Problem

The operations in any line follow same precedence relation. For example, operation of super-finishing cannot start unless earlier operations of turning, etc., are over. While designing the line balancing problem, one has to satisfy the ***precedence constraint.*** This is also referred as *technological constraint,* which is due to sequencing requirement in the entire job.

Another constraint in the balancing problem is z*oning constraint*. If may be either *positive zoning constraint* or *negative zoning constraint*. Positive zoning constraint compels the designer to accommodate specified work-elements to be grouped together at one station. For example, in an automobile assembly line, workers are doing work at both sides of automobile. Therefore, at any station, few operations have to be combined. Many times, operation and inspection are grouped together due to *positive zoning constraint*.

In a *negative zoning constraint* few operations are separated from each other. For example, any work station, which performs spray painting, may be separate from a station, which performs welding, due to safety considerations.

Therefore, following constraints must be following in a line balancing problem:

1. Precedence relationship.
2. Zoning constraints (if any).
3. Restriction on number of work stations (n), which should lie between one and total number of work elements (N). Thus:

$$\mathbf{1 \leq \boldsymbol{n} \leq \boldsymbol{N}}$$

4. Station time (T_{si}) must lie between cycle time and maximum of all work element time (Max $\{T_{iN}\}$):

$$\boldsymbol{Max\ \{T_{iN}\} \leq T_{si} \leq T_c}$$

Definition and Terminology in Assembly Line

1. Work Element (*i*)

The job is divided into its component tasks so that the work may be spread along the line. Work element is a part of the total job content in the line. Let N be the maximum number of work element, which is obtained by dividing the total work element into minimum rational work element. Minimum rational work element is the smallest practical divisible task into which a work can be divided. Thus, the work element number (i) is

$$1 \le i \le N$$

The time in a work element, i say (T_{iN}), is assumed as constant. Also, all T_{iN} are additive in nature. This means that we assume that if work elements, 4 and 5, are done at any one station, the station time would be ($T_{4N} + T_{5N}$). Where N is total number of work elements?

2. Work Stations (*w*)

It is a location on the assembly line where a combination of few work elements is performed. Since minimum number of work stations (w) cannot be less than 1, we have

$$w \ \ge\ 1$$

3. Total Work Content (T_{wc})

This is the algebric sum of time of all the work elements on the line. Thus;

$$T_{wc} = \sum_{i=1}^{N} T_{iN}$$

4. Station Time (T_{si})

It is the sum of all the work elements (i) on work station (s). Thus, if there are n_1 to n_2 work elements assigned at station s, then

$$T_{si} = \sum_{n_1}^{n_2} T_{iN}$$

5. Cycle Time (T_c)

Cycle time is the rate of production. This is the time between two successive assemblies coming out of a line. Cycle time can be greater than or equal to the maximum of all times, taken at any station. Necessary clarification is already given in the previous example.

$$T_c \ge Max\ \{T_{si}\}$$

If, T_c = max $\{T_{si}\}$, then there will be ideal time at all stations having station time less than the cycle time.

6. Delay or Idle Time at Station (T_{ds})

This is the difference between the cycle time of the line and station time.

$$T_{ds} = T_c - T_{si}$$

7. Precedence Diagram

This is a diagram in which the work elements are shown as per their sequence relations. Any job cannot be performed unless its predecessor is completed. A graphical representation, containing arrows from predecessor to successor work element, is shown in the precedence diagram. Every node in the diagram represents a work element.

8. Balance Delay or Balancing Less (*d*)

This is a measure of line-inefficiency. Therefore, the effort is done to minimise the balance delay. Due to imperfect allocation of work elements along various stations, there is idle time at station. Therefore, balance delay:

$$d = \frac{nT_c - T_{wc}}{nT_c} = \frac{nT_c - \sum_{i=1}^{N} T_{iN}}{nT_c}$$

Where:

T_c = Total cycle time;

T_{wc} = Total work content;

n = Total number of stations.

9. Line Efficiency (*LE*)

It is expressed as the ratio of the total station time to the cycle time, multiplied by the number of work stations (n):

$$LE = \frac{\sum_{i=1}^{n} T_{si}}{(n)(T_c)} \times 100\%$$

Where

T_{si} = Station time at station i

n = Total number of stations

T_c = Total cycle time

10. Smoothness Index (*SI*)

Smoothness index is a measure of relative smoothness of a line:

$$SI = \sqrt{\sum_{i=1}^{n} \left[(T_{si})_{max} - T_{si} \right]^2}$$

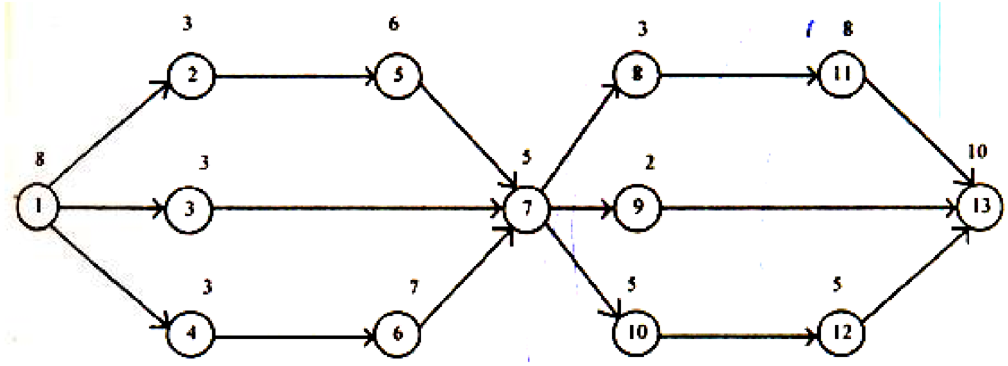
Where,

$(T_{si})_{max}$ = Maximum station time.

Methods of Line Balancing

It is not possible (to date) to have an approach, which may guarantee an optimal solution for a line balancing problem. Many heuristics exist in literature for this problem. The heuristic provides satisfactory solution but does not guarantee the optimal one (or the best solution). We would discuss some of the heuristics on a sample problem of line balancing, as given below:

Problem: Let us consider the precedence diagram of 13 work elements shown below. The time for each work element is at the top of each node:



Precedence diagram

In a tabular form, this precedence diagram is represented as follows:

Work Element Element	Duration (min)	Immediate Precedence
1	8	–
2	3	1
3	3	1
4	3	1
5	6	2
6	7	4
7	5	3,5,6
8	3	7
9	2	7
10	5	7
11	8	8
12	5	10
13	10	9,11,12
Total	**68**	

Heuristic: Largest Candidate Rule

Step 1: List all work elements (i) in descending order of their work elements (T_{iN}) value.

Step 2: Decide cycle time (T_c).

Step 3: Assign work element to the station. Start from the top of the list of unassigned elements. Select only feasible elements as per the precedence and zoning constraints. Select till the station does not exceed cycle time.

Step 4: Continue step 3 for next station.

Step 5: Till all work elements are over, repeat steps 3, 4.

Problem 1: Refer the problem shown in figure below. Decide cycle time.
Total work content = 68 min.
Largest work element time = 10 min.
Thus, cycle time (T_c) must satisfy : $T_c \geq 10$ min.

For minimum cycle time of 10 min., number of stations would be $\frac{68}{10} = 6.8$. Therefore, we must take stations lesser than this. Let us select 5 stations design. For 5 stations, the station time should be nearly equal to $\frac{68}{5} = 13.6$ min. List work elements in descending order of their work element.

Work element	T_{iN}	Immediate Precedence
13	10	9,11,12

6	7	4
5	6	2
7	5	3,5,6
10	5	7
12	5	10
2	3	1
3	3	1
4	3	1
8	3	7
9	2	7

Step 3:

Station	Element	T_{iN}	Σ T_{iN} at Station
I	1	8	
	2	3	
	3	3	14
II	4	3	
	6	7	
	5	6	16
III	7	5	
	10	5	
	8	3	13
IV	11	8	
	12	5	
	9	2	15
V	13	10	10

Here, final cycle time is maximum station time which is 16 min.

Balance delay $= \dfrac{nT_c - \sum T_{iN}}{\sum nT_c}$

$$= \frac{5\times16-68}{5\times16}\times100\% = 15\%$$

Let us consider a 4-station design:

Approximate cycle time $= \dfrac{\sum T_{iN}}{\text{No. of Stations}}$

$$= \frac{68}{4} = 17\,\text{min}.$$

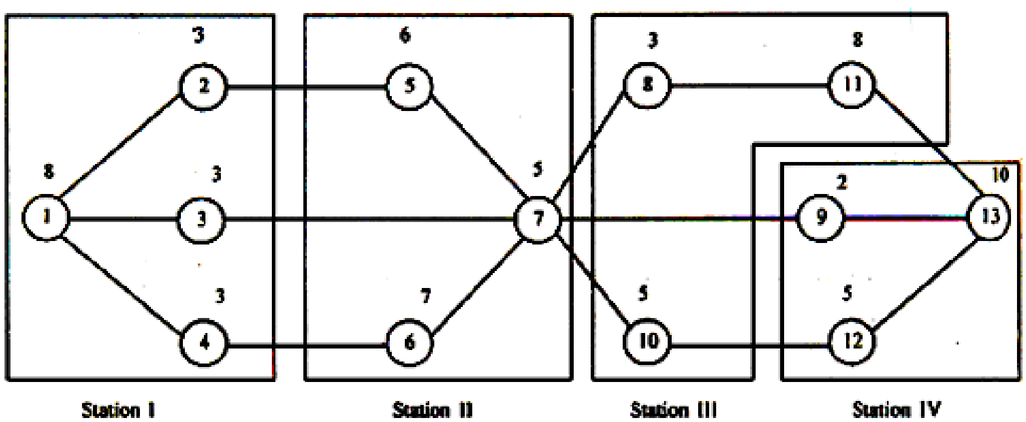
Station	Element	T_{iN}	Σ T_{iN} at Station
I	1	8	
	2	3	
	3	3	
	4	3	17
II	6	7	

	5	6	
	7	5	18
III	10	5	
	8	3	
	11	8	16
IV	12	5	
	9	2	
	13	10	17

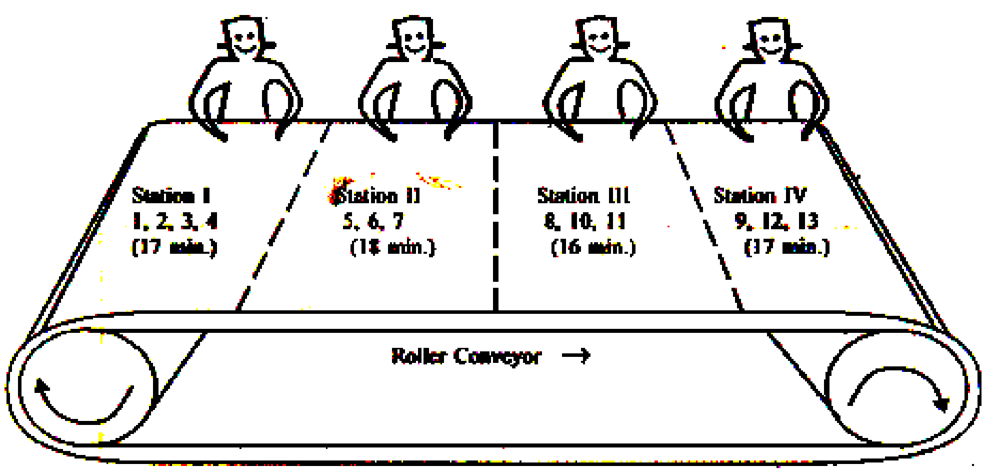
Since maximum station time is 18 min. (for station II), the cycle time would also be 18 min.

Here, **Balance delay** $= \dfrac{4 \times 18 - 68}{4 \times 18} \times 100\% = 5.55\%$.

As the balance delay is quite less in 4-station design, we may select 4-station designs provided the capacity of station II is at least 18 min.



Station line design for Problem I



Physical layout of 4-station design

Kilbridge-Wester Heuristic for Line Balancing

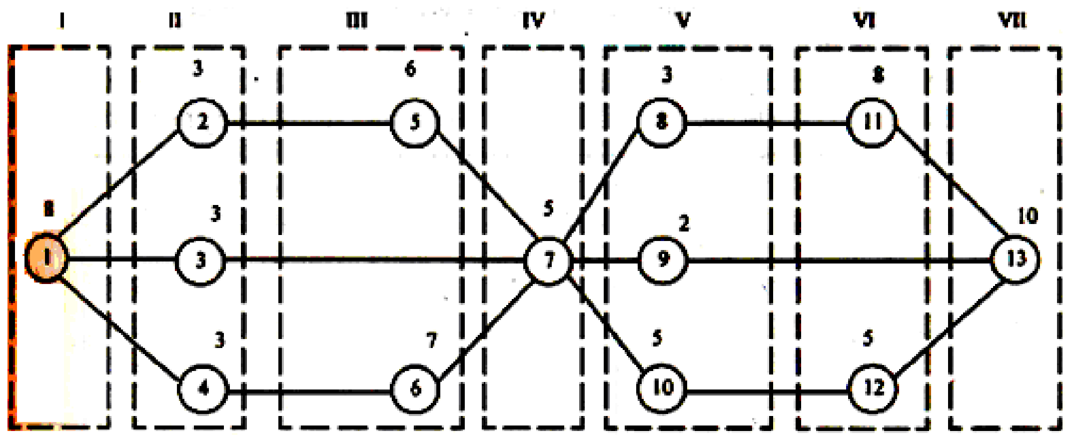
Step 1: Construct precedence diagram. Make a column I, in which include all work elements, which do not have a precedence work element. Make column II in which list all elements, which follow elements in column I. Continue till all work elements are exhausted.

Step 2: Determine cycle time (T_c) by finding all combinations of the primes of $\sum_{i=1}^{N} T_{iN}$ which is the total elemental time. A feasible cycle time is selected. Number of stations would be:

$$n = \frac{\sum_{i=1}^{N} T_{iN}}{T_c}$$

Step 3: Assign the work elements in the work station so that total station time is equal to or slightly less than the cycle time.

Step 4: Repeat step 4 for unassigned work elements.



Seven column initial assignment

Now, selecting cycle time as equal to 18 seconds we follow these steps:

Column	Work Element, (*i*)	T_{iN}	Column Sum	Cumulative Sum
I	1	8	8	8
II	2	3		
	3	3		
	4	3	9	17
III	5	6		
	6	7	13	30
IV	7	5	5	35
V	8	3		
	9	2		
	10	5	10	45
VI	11	8		
	12	5	13	58
VII	13	10 (t_{max})	10	68

Total elemental time is 68 minutes which is 2 × 2 × 17. The cycle time must lie between 68 (for one station) to 10 min. (which is max of all T_{iN}):

$$10 \le T_c \le 68$$

The possible combinations of primes (17, 2 and 2) of work content time (68 min) are as follows:

Feasible Cycle Time	Infeasible Cycle Time
17 $17 \times 2 = 34$ $17 \times 2 \times 2 = 68$	2 $2 \times 2 = 4$

Let us arbitrarily select 17 as the cycle time. Now, regroup elements in columns I and II till we get 17 min. of station time. Thus, elements 1, 2, 3, 4 are selected at station I. We proceed in the same way for remaining elements:

Station	Element (i)	T_{iN}	Station Sum (T_{si})	($T_c - T_{si}$) for T_c = 17 min.
I	1	8		
	2	3		
	3	3		
	4	3	17	0
II	5	6		
	6	7	13	4
III	7	5		
	8	3		
	9	2		
IV	10	5	15	2
	11	8		
	12	5	13	4
V	13	10	10	7

Line Efficiency $= \dfrac{68}{5 \times 17} \times 100 = 80\%$

Smoothness Index $= \sqrt{0^2 + 4^2 + 2^2 + 4^2 + 7^2} = \sqrt{85} = 9.22$

Balance Delay $= \dfrac{5 \times 17 - 68}{5 \times 17} \times 100 = 20\%$

Now, locking at the previous table, little readjustment in work element is possible if the cycle time is extended to 18 min. This is apparent when we consider the following grouping:

Column	Work Element (i)	T_{iN}	Station Sum (T_{si})	(T_c - T_{si}) for T_c = 18 min.
I	1	8		
	2	3		
	3	3		
II	4	3	17	1
	5	6		
	6	7		
	7	5	18	0
III	8	3		
	9	2		
	10	5		
	11	8	18	0
IV	12	5		
	13	10	15	3

Line Efficiency = $\dfrac{68}{4 \times 18} \times 100 = 94.44\%$

Smoothness Index = $\sqrt{1^2 + 3^2} = \sqrt{10} = 3.16$

Balance Delay = $\dfrac{4 \times 18 - 68}{4 \times 18} \times 100 = 5.56\%$

Heuristic: Helgeson-Birnie (Ranked Positional Weight) Method

Following steps are followed:

Step 1: Draw the precedence diagram.

Step 2: For each work element, determine the positional weight. It is the total time on the longest path from the beginning of the operation to the last operation of the network.

Step 3: Rank the work elements in descending order of ranked positional weight (R.P.W.).

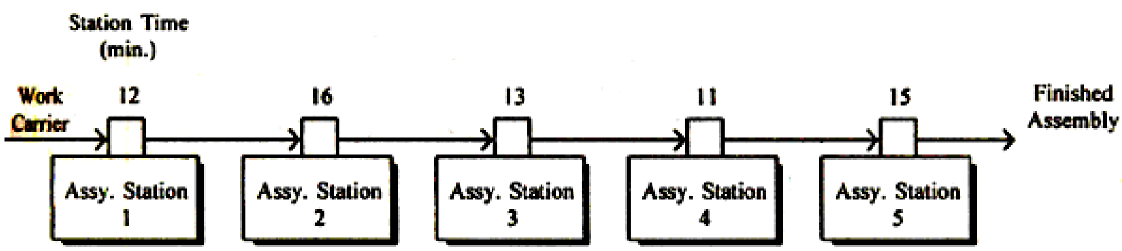
Calculation of RPW would be explained in the example to follow.

Step 4: Assign the work element to a station. Choose the highest RPW element. Then, select the next one. Continue till cycle time is not violated. Follow the precedence constraints also.

Step 5: Repeat step 4 till all operations are allotted to one station.

Example: Let us consider the previous example. The precedence diagram is shown in figure above. Assume cycle time is 18 min.

Solution:



Refer figure above RPW of any work element (i) is the sum of the time of work elements on the longest path, starting from i^{th} work element to the last work element. Therefore, for all activities, first find the longest path, starting from that element to the last work element. This is given in last column of table below:

The ranked positional weight (RPW) of all work elements, *i*, is shown below:

Work Element, *i*	Rank	RPW	Longest Path
1.	1	44	1–4–6–7–8–11–13
2.	3	35	2–5–7–8–11–13
3.	6	29	3–7–8–11–13
4.	2	36	4–6–7–8–11–13
5.	5	32	5–7–8–11–13
6.	4	33	6–7–8–11–13
7.	7	26	7–8–11–13
8.	8	21	8–11–13
9.	12	12	9–13

12.	11	15	12–13
13.	13	10	13

Assignment of work station is done as follows:

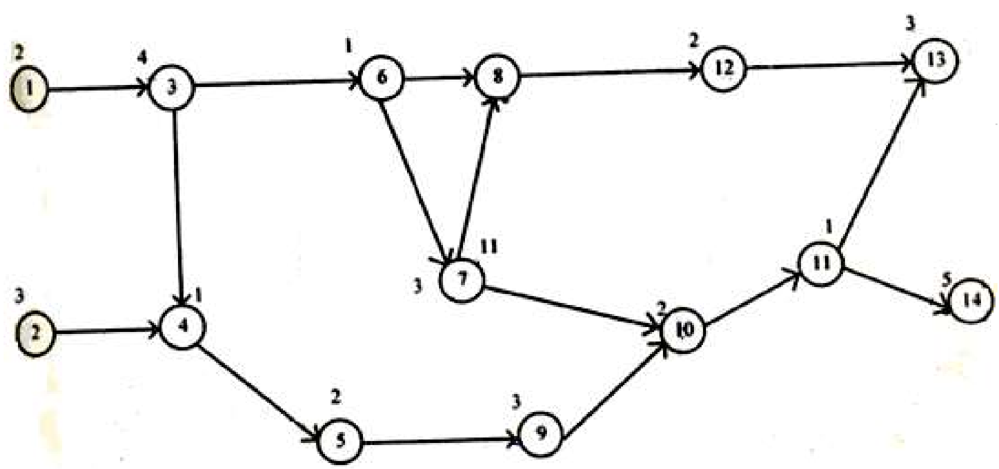
Station	Element, (*i*)	Element Time, (T_{iN})	Station Time, (T_{iN})	$T_c - T_{in}$
I	1	8		
	4	3		
	6	7	18	0
II	2	3		
	3	3		
	5	6		
III	7	5	17	1
	8	3		
	9	2		
	10	5		
	11	8	18	0
IV	12	5		
	13	10	15	3

Line Efficiency $= \dfrac{68}{18 \times 4} \times 100 = 94.44\%$

Smoothness Index $= \sqrt{0^2 + 1^2 + 0^2 + 3^2} = 3.16$

Balance Delay $= \dfrac{4 \times 18 - 68}{4 \times 18} \times 100 = 5.56\%$

Problem: Design the work stations for an assembly line shown below. Use RPW method. Desired cycle time is 10 minutes.



Solution:

$$T_{wc} = \sum_{i=1}^{N} T_{iN} = \text{Total work content} = 2 + 4 + 1 + 2 + 2 + 3 + 3 + 2 + 1 + 5 + 3 + 2 + 1 + 3 = 34$$

Range of cycle time:

$$Max\,(T_{iN}) \le T_c \le \sum_{i=1}^{N} T_{iN} \text{ or } 5 \le T_c \le 34$$

Desired cycle time C = 10 min.

Minimum number of work stations $= \dfrac{\sum T_{iN}}{T_c} = \dfrac{34}{10} = 3.4 \simeq 4$

Using Rank Position Weight (RPW) method:

Task	RPW
1	(20)
3	(18)
2	(17)
4	(14)
5	(13)
9	(11)
6	(12)
7	(11)
10	(8)
8	(7)
11	(6)
12	(5)
14	(5)
13	(3)

Now, grouping on the basis of weight:

Work Station	→	Work Station 1	Work Station 2	Work Station 3	Work Station 4
Job	→	1, 3, 2, 4	5, 6, 9, 7	10, 8, 11, 12, 13	14
Work Station Time	→	(10)	(9)	(10)	(5)

(a) Balance Delay $= \left[1 - \dfrac{\sum_{i=1}^{N} T_{iN}}{nT_c}\right] = \dfrac{\text{Total Ideal Time}}{\text{Cycle Time} \times \text{No. of Stations}}$

$$= \left[1 - \frac{34}{4*10}\right]*100 = 15\%$$

(b) Line Efficiency = [1 – Balance delay] * 100

= [1 – 0.15] * 100
= 85%

(c) Smoothness Index $= \sqrt{\sum_{i=1}^{n}\left[(T_s)_{\max} - T_{si}\right]^2}$

$$= \sqrt{(10-10)^2 + (10-9)^2 + (10-10)^2 + (10-5)^2} = \sqrt{0+1+0+25} = \sqrt{26} = 5.1$$

Selective Assembly

Selective assembly in manufacturing is a technique of assembly in which parts are not assembled with any of the parts in the corresponding category of the mating component. This allows for greater variability in the production of the individual components, but this benefit is at least partially negated by the introduction of part sorting.

Selective assembly is a method of obtaining high-precision assemblies from relatively low-precision components. In selective assembly, the mating parts are manufactured with wide tolerances. The mating part population is partitioned to form selective groups, and corresponding selective groups are then assembled interchangeably. If the mating parts are manufactured in different processes and in different machines, their standard deviations will be different. It is impossible that the number of parts in the selective group will be the same. A large number of surplus parts are expected according to the difference in the standard deviations of the mating parts. A method is proposed to find the selective groups to minimize the assembly variation and surplus parts when the parts are assembled linearly. A genetic algorithm is used to find the best combination of the selective groups to minimize the assembly variation. Selective assembly is successfully applied using a genetic algorithm to achieve high-precision assemblies without sacrificing the benefit of wider tolerance in manufacturing.

Capacity Planning

Capacity planning

Capacity planning forms the second principal step in the production system, the Product and Service design step being the first. The term "Capacity" of a plant is used to denote the maximum rate of production that the plant can achieve under given set of assumed operating conditions, for instance, number of shifts and number of plant operating days etc.

Capacity planning is concerned with determining labour and equipment capacity requirements to meet the current master production schedule and long term future needs of the plant.

Short term capacity planning involves decisions on the following factors:

(a) Employment levels
(b) Number of work shifts
(c) Labour overtime hours
(d) Inventory stock piling
(e) Order back logs
(f) Subcontracting jobs to other plants/shops in busy periods.

Long term capacity planning involves decisions on the following factors

(i) Investment in new machines/equipments
(ii) New plant construction
(iii) Purchase of existing plants
(iv) Closing down/selling obsolete facilities.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. The table given details of an assembly line. [GATE-2006]

Work station	I	II	III	IV	V	VI
Total task time at the workstation (in minutes)	7	9	7	10	9	6

What is the line efficiency of the assembly line?
(a) 70% (b) 75% (c) 80% (d) 85%

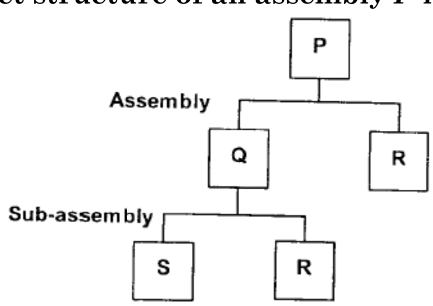
GATE-2. In an assembly line for assembling toys, five workers are assigned tasks which take times of 10, 8, 6, 9 and 10 minutes respectively. The balance delay for the line is: [GATE-1996]
(a) 43.5% (b) 14.8% (c) 14.0% (d) 16.3%

GATE-3. An electronic equipment manufacturer has decided to add a component sub-assembly operation that can produce 80 units during a regular 8-hour shift. This operation consists of three activities as below: [GATE-2004]

Activity	Standard time (min.)
M. Mechanical assembly	12
E. Electric wiring	16
T. Test	03

For line balancing the number of work stations required for the activities M, E and T would respectively be
(a) 2, 3, 1 (b) 3, 2, 1 (c) 2, 4, 2 (d) 2, 1, 3

GATE-4. The product structure of an assembly *P* is shown in the figure.



Estimated demand for end product P is as follows: [GATE-2008]

Week	**1**	**2**	**3**	**4**	**5**	**6**
Demand	**1000**	**1000**	**1000**	**1000**	**1200**	**1200**

Ignore lead times for assembly and sub-assembly. Production capacity (per week) for component R is the bottleneck operation. Starting with zero inventory, the smallest capacity that will ensure a feasible production plan up to week 6 is:
(a) 1000 (b) 1200 (c) 2200 (d) 2400

Previous 20-Years IES Questions

Reason (R): Assembly line balancing reduces in-process inventory.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false **[IES-2009]**
(d) A is false but R is true

IES-2. Consider the following characteristics of assembly line balancing:
1. shareout of sequential work activities into work stations
2. High utilization of equipment
3. Minimization of idle time
Which of the statements given above are correct? [IES-2008]
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 and 3 only

IES-3. Which of the following are the benefits of assembly line balancing?
1. It minimises the in-process inventory [IES-1998]
2. It reduces the work content.
3. It smoothens the production flow
4. It maintains the required rate of output.
Select the correct answer using the codes given below:
Codes: (a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 4 (d) 1, 2 and 4

IES-4. In an assembly line, what is the balance delay? [IES 2007]
(a) Line efficiency × 100 (b) 100 – Line efficiency (in percentage)
(c) $\dfrac{\text{Line efficiency}}{100}$ (d) None of the above

IES-5. In an assembly line, when the workstation times are unequal, the overall production rate of an assembly line is determined by the:
(a) Fastest station time **[IES-2006]**
(b) Slowest station time
(c) Average of all station times
(d) Average of slowest and fastest station times

IES-6. Which one of the following is true in respect of production control for continuous or assembly line production? [IES-2002]
(a) Control is achieved by PERT network
(b) Johnson algorithm is used for sequencing
(c) Control is on one work centre only
(d) Control is on flow of identical components through several operations

IES-7. Manufacturing a product requires processing on four machines A, B, C, D in the order A – B – C – D. The capacities of four machines are A = 100, B = 110, C = 120 and D = 130 units per shift. If the expected output is 90% of the system capacity, then what is the expected output? [IES-2006]
(a) 90 units (b) 99 units (c) 108 units (d) 117 units

IES-8. Match List-l (Parameter) with List-II (Definition) and select the correct answer using the code given below the lists: [IES-2006]

List-I	**List-II**
A. Total work content	**1.** Aggregate of all the work elements to be done on line
B. Workstation process time	**2.** Line inefficiency which results from the idle time due to imperfect allocation of work among stations

C. Cycle time					**3.** Time interval between parts coming off the line				
D. Balance delay					**4.** Sum of the times of the work elements done at a station				
Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	1	4	3	2	(b)	1	2	3	4
(c)	3	4	1	2	(d)	3	2	1	4

IES-9. **A production line is said to be balanced when** **[IES-1997]**
(a) There are equal number of machines at each work station
(b) There are equal number of operators at each work station
(c) The waiting time for service at each station is the same
(d) The operation time at each station is the same.

IES-10. **A production system has a product type of layout in which there are four machines laid in series. Each machine does a separate operation. Every product needs all the four operations to be carried out. The designed capacity of each of the four machines is 200, 175, 160 and 210 products per day. The system capacity would be:**
(a) 210 products per day (b) 200 products per day **[IES-2000]**
(c) 175 products per day (d) 160 products per day

IES-11. **A new facility has to be designed to do all the welding for 3 products: A, B and C, Per unit welding time for each product is 20 s, 40 s and 50 s respectively. Daily demand forecast for product A is 450, for B is 360 and for C is 240. A welding line can operate efficiently for 220 minutes a day. Number of welding lines required is:** **[IES-2001]**
(a) 5 (b) 4 (c) 3 (d) 2

IES-12. **A company has four work centres A, B, C and D, with per day capacities of 450 units, 390 units, 360 units and 400 units respectively. The machines are laid down in order A, B, C, and D and product has to be operated on all these machines for getting converted into finished product. The actual output turns to be 306 units per day. What is the system efficiency?** **[IES-2004]**
(a) 68% (b) 78% (c) 80% (d) 85%

IES-13. **An operations consultant for an automatic car wash wishes to plan for enough capacity of stalls to handle 60 cars per hour. Each car will have a wash time of 3 minutes, but there is to be a 20% allowance for set-up time, delays and payment transactions. How many car wash stalls should be installed?** **[IES-1999; 2003]**
(a) 3 (b) 4 (c) 5 (d) 6

IES-14. **An operations consultant for an automatic car wash wishes to plan for enough capacity to handle 60 cars per hour. Each car will have a wash time of 4 minutes, but there is to be a 25% allowance for setup time, delays and payment transactions. How many car wash stalls should be installed?** **[IES-2009]**
(a) 3 (b) 4 (c) 5 (d) 6

IES-15. **Consider the following sets of tasks to complete the assembly of an engineering component:** **[IES-1997]**

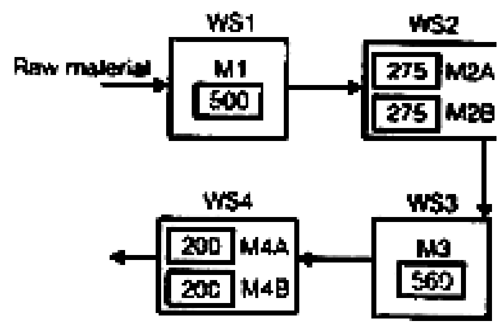
Task Time (in seconds) Precedence

Task	Time (in seconds)	Precedence
A	10	–
B	20	–
C	15	A
D	5	B
E	30	C
F	15	E
G	5	D

The expected production rate is 3000 units per shift of 8 hour duration. The minimal number of workstations that are needed to achieve this production level is:

(a) 4 (b) 8 (c) 10 (d) 11

IES-16. **A product is manufactured by processing on the four work-station (WS). The capacity of each machine on these work-stations is given in the diagram as shown above. In the diagram M1, M2A, M2B, M3, M4A and M4B are the machines and 500, 275, 275, 560, 200 and 200 are their capacities in number of products made per shift. If the products made in this system are 5%, then what will be the output from this system?**



[IES-2009]

(a) 380 (b) 475 (c) 522 (d) 532

Capacity Planning

IES-17. **Match List-I (PPC functions) with List-II (Activity) and select the correct answer using the codes given below the lists:** **[IES-2004]**

List-I	List-II
A. Capacity planning	**1.** Listing products to be assembled and when to be delivered
B. Shop floor control	**2.** Rescheduling orders based on production priorities
C. Master production schedule	**3.** Closure tolerances
D. Material requirement planning	**4.** Monitor progress of orders and report their status
	5. Planning of labour and equipment

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	1	4	3	2	(b)	5	2	1	4
(c)	1	2	3	4	(d)	5	4	1	2

1. Capacity planning **[IES-2008]**
2. Material requirement planning
3. Purchasing
4. Design decisions

Which one of the following is the correct sequence of the above steps in operations management?

(a) 2 – 3 – 4 – 1 (b) 2 – 4 – 3 – 1
(c) 4 – 1 – 2 – 3 (d) 1 – 2 – 4 – 3

Previous 20-Years IAS Questions

IAS-1. A work shift is for 8 hours duration; 30 minutes lunch break and two 15 minutes (each) tea breaks are allowed per shift. If products are to go out after assembly at the rate of 60 per shift, and total assembly time content for a product is 42 minutes, then minimum number of work stations needed is: [IAS-2002]

(a) 8 (b) 12 (c) 6 (d) 5

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (c) Line efficiency $= \dfrac{\text{Total time used}}{\text{Number of work stations} \times \text{Cycle time}} \times 100$

$$= \frac{48}{6 \times 10} \times 100 = 80\%$$

GATE-2. Ans. (c) Balance delay = 100 – Line efficiency (in percentage).

GATE-3. Ans. (a)

GATE-4. Ans. (c)

Previous 20-Years IES Answers

IES-1. Ans. (b)

IES-2. Ans. (a) All the three statements are correct with respect to assembly line balancing:

1. Apportionment of sequential work activities into work stations
2. High utilization of equipment
3. Minimization of idle time

IES-3. Ans. (c)

IES-4. Ans. (b)

IES-5. Ans. (b)

IES-6. Ans. (b)

IES-7. Ans. (a) In the sequence of A – B – C – D only minimum output have to be calculated. Other machines will be on empty position.

$\therefore$ $\text{Output} = \eta \times 100 = 0.9 \times 100 = 90\,\text{units}$

IES-8. Ans. (a)

IES-9. Ans. (d)

IES-10. Ans. (d)

IES-11. Ans. (c) $N = \dfrac{450 \times 20 + 360 \times 40 + 240 \times 50}{220 \times 60} = 2.68$ above whole number is 3.

IES-12. Ans. (d) Maximum possible output 360 units per day

Actual output is 306 per day

$\therefore$ $\eta = \dfrac{306}{360} \times 100\% = 85\%$

IES-13. Ans. (b) One stall can wash $\dfrac{60}{3 \times 1.20} \approx 16\,\text{car/hr}$

No. of wash stall = 4

IES-14. Ans. (d) Wash time for each car = 4 × 1.25 = 5 min.

Number of cars washed in one hour in one stall = $\dfrac{60}{5} = 12$

Number of car wash stalls to be installed = $\dfrac{60}{12} = 5$

IES-15. Ans. (d) Precedence is such that there is no waiting time. Total time for one assembly is 100 sec.

$\therefore$ No. of stations $= \dfrac{3000 \times 100}{8 \times 60 \times 60} \simeq 11$

IES-16. Ans. (a) Given that products made in the system are 5% defective. Therefore percentage of items without any defects = 100 – 5 = 95.

Among all the four station WS4 station has minimum number of raw material = 400

$\therefore$ Output from this system = (200 + 200) × 0.95 = 380

IES-17. Ans. (b) PPC: Production planning and control.

IES-18. Ans. (c) Design decision → Capacity planning → Material requirement planning → Purchasing.

Previous 20-Years IAS Answers

IAS-1. Ans. (c) Effective work 7 hr/shift. Effective work hour needed to produce 60 per shift = 42 hr/shift.

$\therefore$ Work station needed = 42/7 = 6.

Conventional Questions with Answer

Assembly Line Balancing

Conventional Question **[CSE-1990]**

The following elemental data pertains to the assembly of a new model toy:

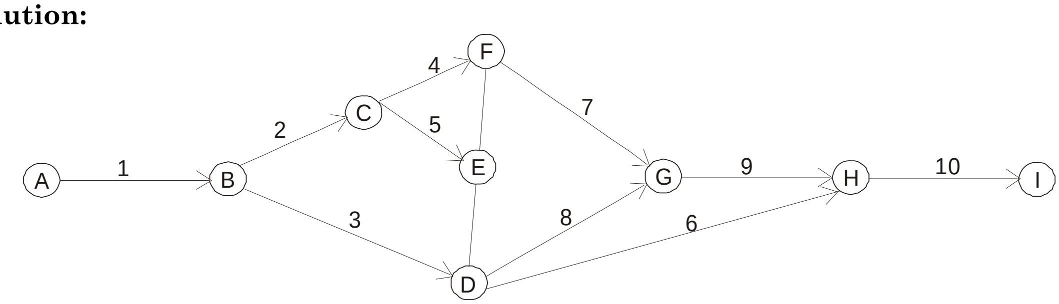
Element No.	Elemental Time (Min)	Immediate Predecessors
1	0.5	—
2	0.3	1
3	0.8	1
4	0.2	2
5	0.1	2
6	0.6	3
7	0.4	4, 5
8	0.5	3, 5
9	0.3	7, 8
10	0.6	6, 9

The desired value of the cycle times 1.0 min, compute.

(i) The balance delay and

(ii) The theoretically minimum number of stations required so as to minimize the balance delay. Also mention some of the steps you would recommended to improve the line balance.

Solution:



(i) $E = \dfrac{4.3}{10\times 1} = 43\%;\quad \text{Therefore Delay} = 100 - 43 = 57\%$

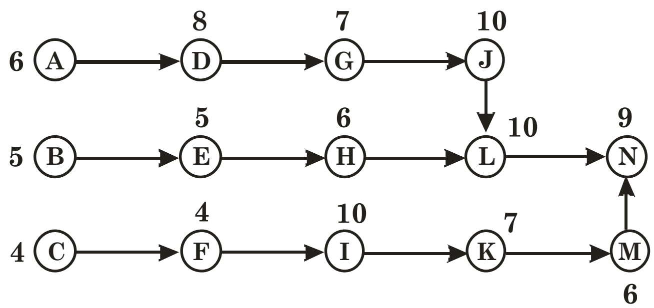
Conventional Question **[ESE-2007]**

An Assembly of an equipment involves the tasks A to N, the precedence tasks and processing times of these tasks in minutes are given in the following table. Considering cycle time of 20 minutes at any work station, balance the assembly line and find the optimum number of work stations. Also find the idle time at each work station.

Task	Predecessor	Duration
A	-	6
B	-	5
C	-	4
D	A	8
E	B	5
F	C	4
G	D	7
H	E	6
I	F	10

J	G	10	
K	I	7	
L	J, H	10	
M	K	6	
N	L, M	9	**[10-Marks]**

Solution:



Precedence diagram

The table below shows some possibilities of grouping the tasks in to work stations.
Balancing the work station has been shown in the below table.
The optimum number of work station is 6.

Work station number	Grouping (First)	Work station time in minutes	Grouping (Second)	Work station time in minutes	Grouping (Third)	Work station time in minutes	Idle time in minutes
1	A, B, C	15	A, D, B	19	A, B, D	19	1
2	D, E, F	17	C, F, I	18	C, E, F	13	7
3	G, J	17	G, H, E	18	G, K	17	3
4	H, L	16	J, L	20	H, L	16	4
5	I, K	17	K, M	13	I, K	17	3
6	M, N	15	N	9	M, N	15	5
		97		97		97	

Also the idle time has been shown in the below the table against grouping no 3.
Also the balance delay

$$= \frac{\textbf{Total idle time for the assembly line}}{\textbf{Total time taken by a product from the first work station to the last work station}}$$

$$= \frac{100\left(nC - \sum t_i\right)}{nC}$$

Where, n = total number of work stations
C = cycle time
t_i = time for the i^{th} elemental task

$$= \frac{100(6 \times 20 - 97)}{6 \times 20}$$

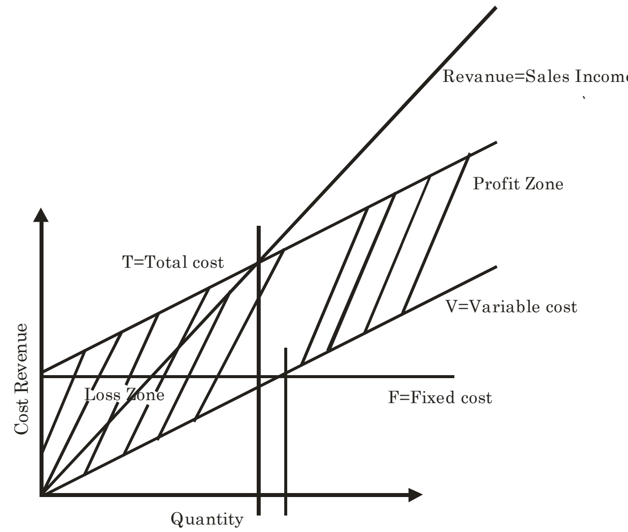
= 19.2%

4. Break Even Analysis

Theory at a Glance (For IES, GATE, PSU)

A. It usually refers to the number of pieces for which a business neither makes a profit nor incurs a loss.

In other words, the selling price of the product is the total cost of production of the component.



(i) No. Profit no loss
Fixed cost + variable cost × Quantity = Selling price × Quantity

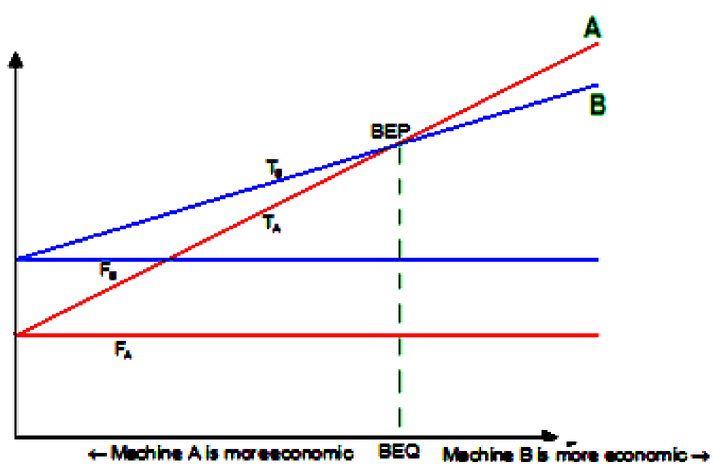
$$\boxed{F + VQ = SQ}$$

(ii) Fixed profit '*P*'

$$\boxed{F + VQ + P = SQ}$$

B. Break-even point analysis is also used to make a choice between two machine tools to produce a given component.

The intersection of Total cost line of Machine A and Machine B is BEP.



At break even point
Total cost of machine A = Total cost of machine B

$$\boxed{F_A + QV_A = F_B + QV_B}$$

$$\therefore\ Q = \frac{F_B - F_A}{V_A - V_B}$$

Here note if $F_A > F_B$ and $V_A < V_B$ or $F_A < F_B$ and $V_A > V_B$ only then Q will be positive.

But if Q comes out negative then, if

(i) $F_A = F_B$ but $V_A \neq V_B$: Whose Variable cost is less that one is economical.

(ii) $V_A = V_B$ but $F_A \neq F_B$: Whose Fixed cost is less that one is economical.

(iii) $F_A \neq F_B$ and $V_A \neq V_B$: Whose both Fixed and Variable cost is less that one is economical.

❖ The same type of analysis can also be used to decide whether an item should be manufactured or purchased and what capacity manufacturing the item would be more economical then purchasing it.

Contribution: Contribution is the measure of economic value that tells how much the sale of one unit of the product will contribute to cover fixed cost, with the remainder going to profit.

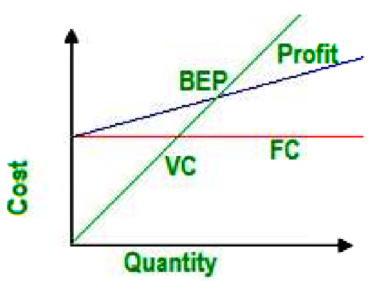
Contribution = Sales – total variable cost (Q.V.)

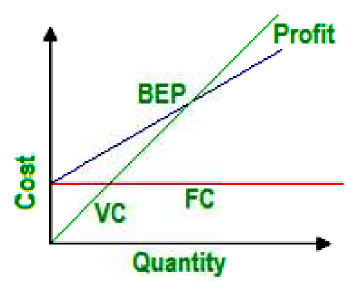
As Sales = $F + QV + P$

Therefore contribution = $F + P$

Since both sales and variable cost vary with output, contribution also vary with output.

At BEP, contribution = F





(A) (i) Capital-intensive industry
(ii) High contribution
(iii) High FC, Low VC

B) (i) Labour-intensive industry
(ii) Low contribution
(iii) Low FC; High VC

Case (A): Requires a large volume of output to reach break even, but once it has attained its profitability increases rapidly.

Case (B): Profitability after BEP increases slowly.

Case (A): When fixed costs are a large portion of total cost, small changes in volume or prices can result in significant changes in profit.

Case (B): When variable costs are high a reduction is variable cost may be more effective in generating profits than changes in the total volume or per-unit prices.

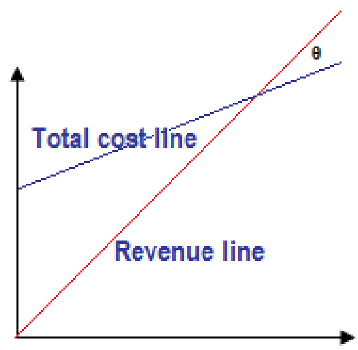
Margin of safety ratio (M/S) ratio

$$\left(M/S\right)\ \text{ratio} = \frac{\text{Margine of safety}}{\text{Present sale}}$$

Higher is the ratio, more sound of the economics of the firm. At BEP (M/S) = 0

Angle of incidence: θ

This is the angle between the lines of total cost and total revenue. Higher is the angle of incidence faster will be the attainment of considerable profit for given increase in production over BEP. Thus the higher value of **θ** make system more sensitive to changes near BEP.



Profit volume ratio:

$$\frac{\text{Sale} - \text{Variable cost}}{\text{Sale}}$$

Higher is the profit volume ratio, greater will be angle of incidence and vice-versa.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. **A standard machine tool and an automatic machine tool are being compared for the production of a component. Following data refers to the two machines. [GATE-2004]**

	Standard Machine Tool	Automatic Machine Tool
Setup time	30 min.	2 hours
Machining time per piece	22 min.	5 min
Machine rate	Rs.200 per hour	Rs.800 per hour

The breakeven production batch size above which the automatic machine tool will be economical to use, will be
(a) 4 (b) 5 (c) 24 (d) 225

GATE-2. **A company produces two types of toys: *P* and *Q*. Production time of *Q* is twice that of *P* and the company has a maximum of 2000 time units per day. The supply of raw material is just sufficient to produce 1500 toys (of any type) per day. Toy type *Q* requires an electric switch which is available @ 600 pieces per day only. The company makes a profit of Rs. 3 and Rs. 5 on type *P* and *Q* respectively. For maximization of profits, the daily production quantities of *P* and *Q* toys should respectively be: [GATE-2004]**
(a) 100, 500 (b) 500, 1000 (c) 800, 600 (d) 1000, 1000

GATE-3. **A component can be produced by any of the four processes I, II, III and IV. Process I has a fixed cost of Rs. 20 and variable cost of Rs. 3 per piece. Process II has a fixed cost Rs. 50 and variable cost of Re. 1 per piece. Process III has a fixed cost of Rs. 40 and variable cost of Rs. 2 per piece. Process IV has a fixed cost of Rs. 10 and variable cost of Rs. 4 per piece. If the company wishes to produce 100 pieces of the component, from economic point of view it should choose [GATE-2005]**
(a) Process I (b) Process II (c) Process III (d) Process IV

GATE-4. **Two machines of the same production rate are available for use. On machine 1, the fixed cost is Rs. 100 and the variable cost is Rs. 2 per piece produced. The corresponding numbers for the machine 2 are Rs. 200 and Re. 1 respectively. For certain strategic reasons both the machines are to be used concurrently. The sale price of the first 300 units is Rs. 3.50 per unit and subsequently it is only Rs. 3.00. The breakeven production rate for each machine is: [GATE-2003]**
(a) 75 (b) 100 (c) 150 (d) 600

Previous 20-Years IES Questions

IES-1. Last year, a manufacturer produced 15000 products which were sold for Rs. 300 each. At that volume, the fixed costs were Rs. 15.2 lacs and total variable costs were Rs. 21 lacs. The break even quantity of product would be: [IES-2000]
(a) 4000 (b) 7800 (c) 8400 (d) 9500

IES-2. Assertion (A): It is possible to have more than one break-even point in break even charts. [IES-1999]
Reason (R): All variable costs are directly variable with production.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-3. On a lathe, the actual machining time required per work piece is 30 minutes. Two types of carbide tools are available, both having a tool life of 60 minutes. [IES-1998]
Type I : Brazed type of original cost Rs. 50/-.
Type II : Throwaway tip (square) of original cost Rs. 70/-
If the overall cost of grinding the cutting edge is Rs. 10/-, assuming all the costs are the same for both the types, for break even costs, the appropriate batch size would be:
(a) 2 pieces (b) 4 pieces (c) 6 pieces (d) 8 pieces

IES-4. Two alternative methods can produce a product first method has a fixed cost of Rs. 2000/- and variable cost of Rs. 20/- per piece. The second method has a fixed cost of Rs. 1500/- and a variable cost of Rs. 30/-. The break even quantity between the two alternatives is: [IES-1996]
(a) 25 (b) 50 (c) 75 (d) 100

IES-5. For a small scale industry, the fixed cost per month is Rs. 5000/-. The variable cost per product is Rs. 20/- and sales price is Rs. 30/- per piece. The break-even production per month will be: [IES-1995]
(a) 300 (b) 460 (c) 500 (d) 10000

IES-6. In the production of a product the fixed costs are Rs. 6,000/- and the variable cost is Rs. 10/- per product. If the sale price of the product is Rs. 12/-, the break even volume of products to be made will be: [IES-2008]
(a) 2000 (b) 3000 (c) 4000 (d) 6000

IES-7. Process I requires 20 units of fixed cost and 3 units of variable cost per piece, while Process II required 50 units of fixed cost and 1 unit of variable cost per piece. For a company producing 10 piece per day [IES-1997]
(a) Process I should be chosen (b) Process II should be chosen
(c) Either of the two processes could be chosen
(d) A combination of process I and process II should be chosen

IES-8. Match List-I (Methods) with List-II (Applications) and select the correct answer using the codes given below the lists: [IES-1998]
List-I **List-II**

A. Break even analysis	**1.** To provide different facility at different locations
B. Transportation problem	**2.** To take action from among the paths with uncertainty
C. Assignment problem	**3.** To choose between different methods of manufacture
D. Decision tree	4. To determine the location of the additional plant

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	4	3	1	2	(b)	3	4	1	2
(c)	3	4	2	1	(d)	4	3	2	1

IES-9. **M/s. ABC & Co. is planning to use the most competitive manufacturing process to produce an ultramodern sports shoe. They can use a fully automatic robot-controlled plant with an investment of Rs. 100 million; alternately they can go in for a cellular manufacturing that has a fixed cost of Rs. 80 million. There is yet another choice of traditional manufacture that needs in investment of Rs. 75 million only. The fully automatic plant can turn out a shoe at a unit variable cost of Rs. 25 per unit, whereas the cellular and the job shop layout would lead to a variable cost of Rs. 40 and Rs. 50 respectively. The break even analysis shows that the break even quantities using automatic plant *vs* traditional plant are in the ratio of 1: 2. The per unit revenue used in the break even calculation is: [IES-1997]**

(a) Rs. 75 (b) Rs. 87 (c) Rs. 57 (d) Rs. 55

IES-10. **Process X has fixed cost of Rs. 40,000 and variable cost of Rs. 9 per unit whereas process Y has fixed cost of Rs.16, 000 and variable cost of Rs. 24 per unit. At what production quantity, the total cost of X and Yare equal? [IES-2004]**

(a) 1200 units (b) 1600 units (c) 2000 units (d) 2400 units

IES-11. **Which one of the following information combinations has lowest break-even point? [IES-2004]**

Fixed cost (in Rs.)	**Variable cost / unit (in Rs.)**	**Revenue/units (in Rs.)**
(a) 30,000	10	40
(b) 40,000	15	40
(c) 50,000	20	40
(d) 60,000	30	40

IES-12. **The indirect cost of a plant is Rs 4,00,000 per year. The direct cost is Rs 20 per product. If the average revenue per product is Rs 60, the break-even point is: [IES-2003]**

(a) 10000 products (b) 20000 products
(c) 40000 products (d) 60000 products

IES-13. **If the fixed cost of the assets for a given period doubles, then how much will the break-even quantity become? [IES-2007]**

(a) Half the original value (b) Same as the original value
(c) Twice the original value (d) Four times the original value

IES-14. **Process X has a fixed cost of Rs 40,000 per month and a variable cost of Rs 9 per unit. Process Y has a fixed cost of Rs 16,000 per month and a variable cost of Rs 24 per unit. At which value, total costs of processes X and Y will be equal? [IES-2009]**

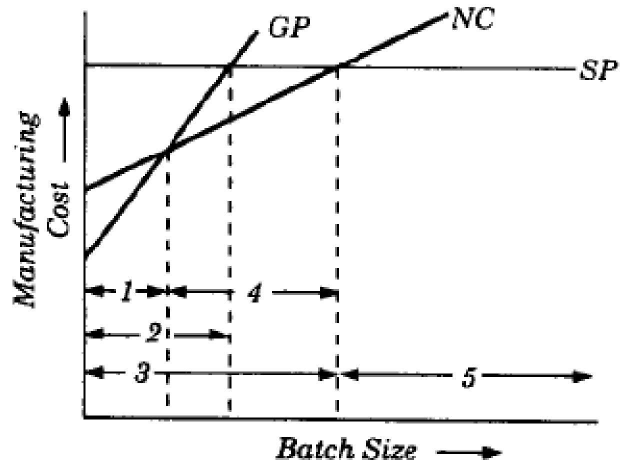
(a) 800 (b) 1200 (c) 1600 (d) 2000

IES-15. Consider the following statements: [IES-2009]
The break-even point increases
1. **If the fixed cost per unit increases**
2. **If the variable cost per unit decreases**
3. **If the selling price per unit decreases**

Which of the above statements is/are correct?
(a) 1 only (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-16. If the total investment is Rs. 5,00,000 for a target production, the income for the current year is Rs. 3,00,000 and total operating cost is Rs. 1,00,000; what is the economic yield? [IES-2006]
(a) 10% (b) 30% (c) 20% (d) 40%

IES-17. Based on the given graph, the economic range of batch sizes to be preferred for general purpose machine (OP), NC machine (NC) and special purpose machine (SP) will be:



Codes:

	GP	NC	SP
(a)	2	5	4
(b)	1	4	5
(c)	3	2	4
(d)	1	4	2

[IES-1997]

IES-18. Assertion (A): A larger margin of safety in break-even analysis is helpful for management decision. [IES-1997]
Reason (R): If the margin of safety is large, it would indicate that there will be profit even when there is a serious drop in production.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-19. Match List-I (Element of cost) with List-II (Nature of cost) and select the correct answer using the codes given below the lists: [IES-1994]

List-I	**List-II**
A. Interest on capital	**1.** Variable
B. Direct labour	**2.** Semi-variable
C. Water and electricity	**3.** Fixed

Codes:

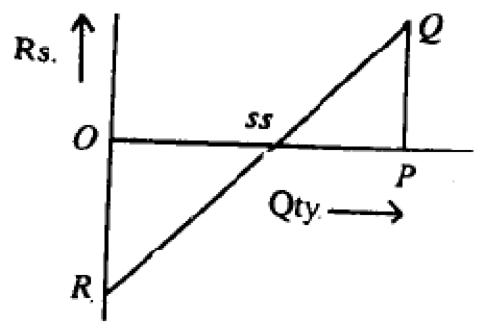
	A	B	C		A	B	C
(a)	3	1	2	(b)	2	1	3
(c)	3	2	1	(d)	2	3	1

Previous 20-Years IAS Questions

IAS-1. Fixed investments for manufacturing a product in a particular year is Rs. 80,000/- The estimated sales for this period is 2, 00,000/-. The variable cost per unit for this product is Rs. 4/-. If each unit is sold at Rs.20/-, then the break even point would be: [IAS-1994]
(a) 4,000 (b) 5,000 (c) 10,000 (d) 20,000

IAS-2. The fixed costs for a year is Rs. 8 lakhs, variable cost per unit is Rs. 40/- and the selling price of each unit is Rs. 200/-. If the annual estimated sales is Rs. 20,00,000/-, then the break-even volume is: [IAS-1997]
(a) 2000 (b) 3000 (c) 3333 (d) 5000

IAS-3. Match List-I (Symbols) with List-II (Meaning) and select the correct answer using the codes given below the Lists; related to P/V chart on Break-Even Analysis as shown in the above figure:



List-I	List-II [IAS-2002]
A. OR	**1.** Profit
B. PQ	**2.** Break-Even Point
C. SS	**3.** Profit/Volume Ratio
D. RQ	**4.** Cost for new design
	5. Fixed cost

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	5	4	2	3	(b)	2	1	3	5
(c)	5	1	2	3	(d)	2	4	3	5

IAS-4. If Break-even point = Total fixed cost $\div \left(1 - \dfrac{\textbf{Variable cost per unit}}{X}\right)$,
then *X* is the [IAS-2000]
(a) Overheads (b) Price per unit
(c) Direct cost (d) Materials cost

IAS-5. A company sells 14,000 units of its product. It has a variable cost of Rs. 15 per unit. [IAS-1999]
Fixed cost is Rs. 47,000 and the required profit is Rs. 23,000
Per unit product price (in Rs.) will be:
(a) 60 (b) 40 (c) 30 (d) 20

IAS-6. Two jigs are under consideration for a drilling operation to make a particular part. Jig A costs Rs. 800 and has operating cost of Rs. 0.10 per part. Jig B costs Rs. 1200 and has operating cost of Rs. 0.08 per part. The quantity of parts to be manufactured at which either jig will prove equally costly is: [IAS-1998]
(a) 8000 (b) 15000 (c) 20000 (d) 23000

IAS-7. **Assertion (A): Marginal cost in linear break-even analysis provides the management with useful information for price fixing. [IAS-1996]**
Reason (R): The marginal cost is the maximum value at which the product selling price must be fixed to recover all the costs.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-8. **The variable cost per unit associated with automated assembly line (V_A), cellular manufacturing (V_B), and job shop production (V_C) will be such that [IAS-1995]**
(a) $V_A > V_B > V_C$ (b) $V_B > V_A > V_C$
(c) $V_C > V_B > V_A$ (d) $V_C > V_A > V_B$

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (d) Given data

	Standard Machine tool	Automatic machine tool
Set up time	30 min	2 hours
Machining time per piece	22 min	5 min
Machine rate	200 per hour	Rs. 800 per hour

Total cost of z_1 component by using standard machine tool,

$$(T_C)_1 = \left[\frac{30}{60} + \frac{22 \times z_1}{60}\right] \times 200 = 100 + \frac{2200}{30} z_1$$

Total cost of z_2 component by using Automatic Machine tool,

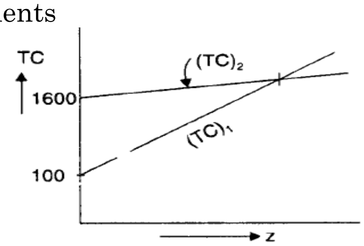
$$(T_C)_2 = \left[2 + \frac{5}{60} \times z_2\right] \times 800 = 1600 + \frac{2000}{30} z_2$$

Let break even point be z number of components

$$\therefore \quad 100 + \frac{2200}{30} z = 1600 + \frac{2000}{30} z$$

or $\quad \frac{200}{30} z = 1500$

or $\quad z = \frac{1500 \times 30}{200} = 225$



Alternately

Let N be the Break even number

At Break even point

$$\left(\frac{1}{2} + \frac{22}{60} \cdot N\right) 200 = \left(2 + \frac{5N}{60}\right) \times 800 \quad \Rightarrow \frac{1}{2} + \frac{22}{60} N = 8 + \frac{20N}{60}$$

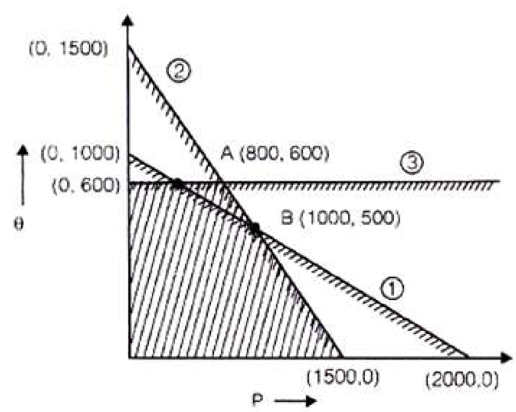
$$\Rightarrow \frac{22N}{60} - \frac{20N}{60} = -\frac{1}{2} \quad \Rightarrow \left(\frac{22-20}{60}\right) N = \frac{16-1}{2} \quad \Rightarrow N = \frac{15 \times 60}{2 \times 2} = 225.$$

GATE-2. Ans. (c) Clearly,

$P + 2Q \leq 2000$

$P \leq 1500$

$Q \leq 600$



GATE-3. Ans. (b) Total cost = fixed cost + (number of piece × variable cost)

GATE-4. Ans. (a) Let both machine produce 'Q' unit, so total production $2Q$

or $Q = 75$

Previous 20-Years IES Answers

IES-1. Ans. (d)B.E.Q.= $\dfrac{\text{fixed cost}}{\text{selling price - variable cost price}}$

IES-2. Ans. (d)

IES-3. Ans. (a)

IES-4. Ans. (b) $2000 + 20n = 1500 + 30n$, $10n = 500$ and $n = 50$.

IES-5. Ans. (c) Break even production per month is 500.

IES-6. Ans. (b) $(S-V).x = F \Rightarrow x = \dfrac{F}{S-V} = \dfrac{6000}{(12-10)} = 3000$

IES-7. Ans. (a) For 10 pieces, it is economical to use process I.

IES-8. Ans. (b)

IES-9. Ans. (a)

IES-10. Ans. (b) Total cost of X = Total cost of Y or $F_x + Q.V_x = F_y + Q.V_y$

or $Q = \dfrac{F_x - F_y}{V_y - V_x} = \dfrac{40000-16000}{24-9} = 1600$ units

IES-11. Ans. (a) Without any calculation we observe that Revenue of each unit is same for all cases. And Fixed cost and variable cost both are minimum in case of (a). So, it will give us minimum BFQ.

Alternatively $F + Q.V = Q.R$ or $Q = \left(\dfrac{F}{R-V}\right)$

(a) 1000 units (b) 1600 units (c) 2500 units (d) 6000 units

IES-12. Ans. (a) Sales cost = Fixed cost + variable cost [where, N = Number of variable]
or, $60 \times N = 4{,}00{,}000 + 20 \times N$ or, $40N = 4{,}00{,}000$
or, N = 10000 Products

IES-13. Ans. (c) $F + V.Q = S.Q$ or $F = Q.(V - S)$
If $F\uparrow$ 2 times Q also $\uparrow$ 2 times

IES-14. Ans. (c) $C_{F_1} + C_{V_1}x = C_{F_2} + C_{V_2}x \Rightarrow 40000 + 9x = 16000 + 29x$

$\Rightarrow 24000 = 15x \qquad \Rightarrow x = \dfrac{24000}{15} = 1600$

IES-15. Ans. (d) $C_F + C_V x = C_S x; \qquad \therefore x = \dfrac{C_F}{C_S - C_V}$

Therefore if the Fixed Cost/Unit i.e. C_F increases the value of x increased i.e. B.E.P. increases.
If the variable cost/unit, i.e. C_V decreases × decreases i.e. B.E.P. decreases.
If the selling price i.e. C_S decreases the value of × increases i.e. B.E.P. increases. Therefore statements **(1) and (3)** are correct.

IES-16. Ans. (d) Economic yield = $\dfrac{\text{Profit}}{\text{Investment}} \times 100\% = \dfrac{(300000-100000)}{500000} \times 100\% = 40\%$

IES-17. Ans. (b)

IES-18. Ans. (a)

IES-19. Ans. (c)

Previous 20-Years IAS Answers

IAS-1. Ans. (b) For break even point,

Fixed cost (F) + Variable cost (V) × Quantity (Q)

= Selling price (S) × Quantity (Q)

or, $Q = \frac{F}{S - V} = \frac{80000}{20 - 4} = 5000$

IAS-2. Ans. (d) $F + Q.V = Q.S$ or, $700000 + Q \times 40 = Q \times 200$

$\therefore \quad Q = 4375$ nearest as 5000.

IAS-3. Ans. (c)

IAS-4. Ans. (b) $F + VQ = SQ$ [S is selling cost per unit]

IAS-5. Ans. (d) $F + Q.V + P = Q.S$

or $4700 + 14000 \times 15 + 23000 = 14000 \times S$ or $S = 20$ per unit.

IAS-6. Ans. (c) $F_A + Q.V_A = F_B + Q.V_B$ or $Q = \frac{F_B - F_A}{V_A - V_B} = \frac{1200 - 800}{0.10 - 0.08} = 20{,}000$ units

IAS-7. Ans. (a)

IAS-8. Ans. (c) Variable cost per unit in least with automated assembly line, and maximum with job shop production. Thus $V_C < V_B < V_A$.

Conventional Questions with Answer

Conventional Question [ESE-2009]

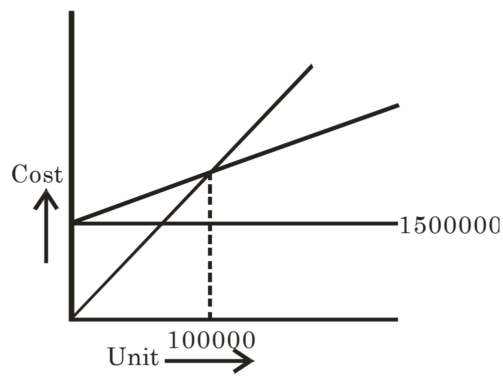
A company is faced with a situation where it can either produce some item by adding additional infrastructure which will cost them Rs. 15,00,000/- but unit cost of production will be Rs. 5/- each. Alternatively it can buy the same item from a vendor at a rate of Rs. 20/- each. When should the company add to its capacity in terms of demand of items per annum? Draw the diagram to show the BEP. **[2-Marks]**

Answer: Let the capacity is x when company will meet its demand, so

1500000 + 5x = 20x

15x = 1500000

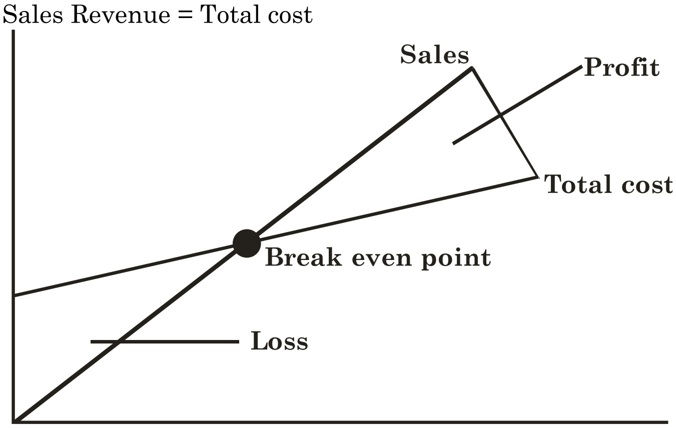
$\Rightarrow$ x = 100000



Conventional Question [ESE-2008]

What is meant by break-even point? Draw a figure to illustrate your answer. **[2 Marks]**

Solution: Break even point: Break even point is the point at which cost or expenses and revenue are equal. i.e. there is no loss or gain.
At B.E.P Sales Revenue = Total cost



Conventional Question [ESE]

***Question*: The following data refers to a manufacturing unit**

Fixed cost = Rs. 100000/-

Variable cost = 100/- per unit

Selling price = Rs. 200/- per unit

(i) Calculate the BEP

(ii) Calculate the number of component needed to be product to get a profit of Rs. 20000/-

***Solution*:**

(i) At break even point

$$F + Q.V = S \times Q$$

$$\therefore \quad Q = \frac{F}{S - V} = \frac{100000}{200 - 100} = 1000 \text{ pieces}$$

(ii) For fixed profit Rs. 20000/-

$$F + Q.V + P = S \times Q$$

$$\therefore \quad Q = \frac{F + P}{S - V} = \frac{100000 + 20000}{200 - 100} = 1200 \text{ units}$$

Conventional Question [ESE-2006]

What is break-even analysis? How is it useful to the manager?
For a particular product, the following information is given:
Selling price per unit : Rs. 100
Variable cost per unit : Rs. 60
Fixed costs : Rs. 10,00,000
Due to inflation the variable costs have increased by 10% while fixed costs have increased by 5%. If the break-even quantity is to remain constant by what percentage should the Sales price be raised? [IES-2006, 15-Marks]

Solution: The break-even point means the level of output or sales at which no profit or loss is made. It represents the position at which marginal profit or contribution is just sufficient to cover fixed over heads. When production exceeds the break-even the business, makes a profit and when production is below the volume of production at break-even point the business makes a loss.

The break-even analysis helps the manager/management in solving the following problems.

(i) The total profit of business is ascertained at various levels of activity and different patterns of production and sales.
(ii) Reporting the top management the effect on net profits of introducing a, new line or discontinuing the existing line.
(iii) Where severe competition is being met and it is desired to reduce the selling price, the effect of any reduction on profits can be easily ascertained.
(iv) Where reduction in selling price is intended to increase sales, the increase necessary to allow to earn the previous profit can be calculated.
(v) The controllability and postponement of expenditure can be worked out from the break even point.
(vi) It helps in planning and managerial control.
(vii) Break-even point can be helpful in detecting the effect of gradual changes that may have crept into the operation of budget planning and evaluating new proposals and alternative courses of action.

Thus, the utility of break-even analysis to the management/manager, lies in the fact that it represents a cross-sectional view of the profit structure. Also it highlights the areas of economic strength and weaknesses in the firm.

Solution to the problem:

$$\mathbf{S_1 = Rs.100 / unit, \quad V_1 = Rs.60 / unit}$$

We know $N_1 = S = F + N_1 V$

$$N_1 = \frac{F}{S-V} = \frac{1000000}{100-60} = 25000$$

$S_2 = ?$

$V_2 = 60 + 10\% \text{ of } 60 = Rs.66$

$F_2 = 1000000 + 5\% \text{ of } 1000000$

$= Rs.1050000$

$$N_2 = \frac{1050000}{S_2 - V_2}$$

$$S_2 - V_2 = \frac{1050000}{25000} = 42$$

$S_2 = V_2 + 42$

$S_2 = 66 + 42 = 108 \text{ Rs}$

$\therefore \%$ change in $S_2 = 8\%$

5. PERT and CPM

Theory at a Glance (For IES, GATE, PSU)

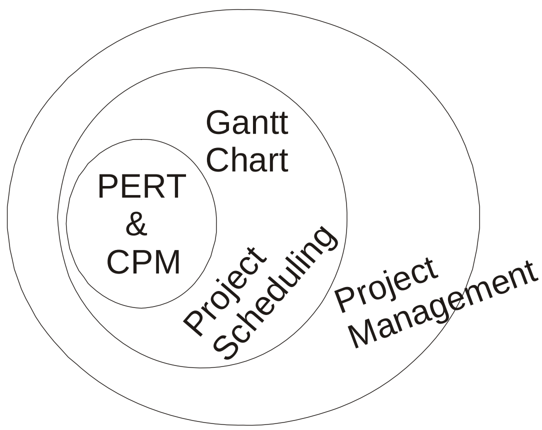
PROJECT MANAGEMENT

⇓

(Project planning and scheduling) — Gantt Chart

⇓

(Special Scheduling Techniques: PERT and CPM



Gantt chart: Is one of the first scientific techniques for project planning and scheduling.
CPM: Critical Path Method.
PERT: Program Evaluation and Review Technique.
The principal feature of PERT is that its activity time estimates are probabilistic. The activity time in CPM applications were relatively less uncertain and were, thus, of deterministic nature.

With the passage of time, PERT and CPM applications started overlapping and now they are used almost as a single techniques and difference between the two is only of the historical or academic interest.

Difference between PERT and CPM

Main difference: In PERT activity time is **probabilistic**. In CPM activity time is **deterministic.**

The other difference: PERT is **Event – Oriented**. While the CPM is **Activity – Oriented** (in CPM we actually know the Activity time).

Time in PERT & CPM

In CPM all time estimates are assumed to be **deterministic** for every activity of the project.

In PERT all activity time is **probabilistic.**

(i) For PERT. Employs **Beta-distribution** for the time – expectation for activity.
(ii) **Optimistic time (t_o):** If everything in the project goes well.
(iii) **Most Likely Time (t_m):** It is the time for completing an activity that is best.
(iv) **Pessimistic Time (t_p):** If everything in the project goes wrong.

Expected time

$$t_e = \left[\frac{t_o + 4t_m + t_p}{6}\right]$$

In PERT, The completion time for the project has a **normal distribution** about the expected completion time.

Critical Path: Critical path is the on the network of project activities which takes **longest** time from start to finish. **[Definition: ESE-2003]**

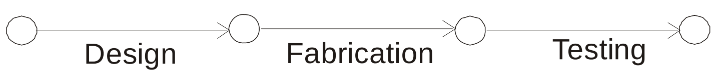
- The critical path in the network is that sequence of activities and events where there is **no "Slack".**
- If any activity on the critical path gets delayed by t_x time, then the total project will be delayed by t_x.
- Same is not true for activities, not lying on critical path.
- Critical path determines the focal activities for which no tolerance in terms of delay is desirable.

Work Breakdown Structure (W.B.S.)

A project is a combination of interrelated activities which must be performed in a certain order for its completion. The process of dividing the project into these activities is called the Work-Break-Down structure (W.B.S.). The activity or a unit of work, also called work content is a clearly identifiable and manageable work unit. Let us consider a very simple situation to illustrate the W.B.S. A group of students is given the project of designing, fabricating and testing a small centrifugal pump. The project can be broken down into the following sub-parts.

(i) Design, (ii) Fabrication, (iii) Testing

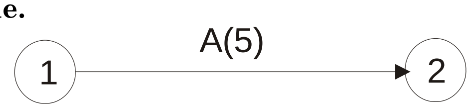
The Network at this level of detail will look as shown in figure.



Terminology

Activity: It is a time consuming effort that is required to perform part of a work.

Example: Drilling a hole.



Activity 1-2 is A and required 5 unit time

An activity with zero slack is known as **critical activity.**

Event: It is the beginning, completion point, or mile stone accomplishment within the project. An activity beginning and ends with events. An event triggers on activity of the project.

An event is a point in time within the project which has significance to the management. No expenditure of manpower or resources may be associated with an event.

Dummy Activity: An activity that **consumes no time** but shows precedence among activities. It is useful for proper representation in the network. **[Definition: ESE-1995]**

Crashing & Crash Cost: The process of reducing on activity time by adding fresh resource and hence usually increasing cost. Crashing is needed for finishing the task before estimated time. Cost associated to crash is crash cost.

Float and Slack: That the float of an activity has the same significance as the slack of the events. Slack corresponds to events and hence to PERT while Float corresponds to activities and hence to CPM.

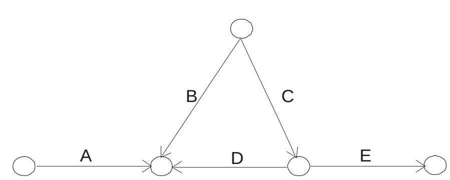
Negative Float and Negative slack: The latest allowable occurrence time (T_L) for the end event in a CPM network is usually assumed to be equal to the earliest expected time (T_E) for that event. But in a PERT network, there is specified a date by which the project is expected to be complete. This is called the scheduled completion time T_S and for the backward pass computation, T_L for the end event is taken equal to T_s. Now there may be three cases: $T_s > T_E$, $T_S = T_E$ and $T_s < T_E$ When $T_S > T_E$, a positive float results and the events have positive slacks.

$T_S = T_E$, a zero float results and critical events have zero slacks.

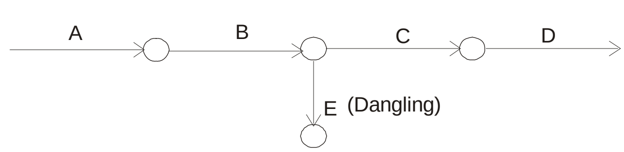
So, when $T_S < T_E$, the critical activity will not have zero float. In such cases the critical path is the path of least float.

Network Construction
(i) What activities must be completed before a particular activity starts?
(ii) What activities follow this?
(iii) What activities must be performed concurrently with this?

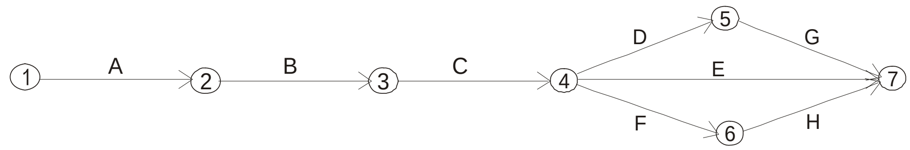
Faulty Network
(i) Looping



(ii) Dangling



Numbering the Events (Fulkersons's Rule)



Fulkerson's Rule

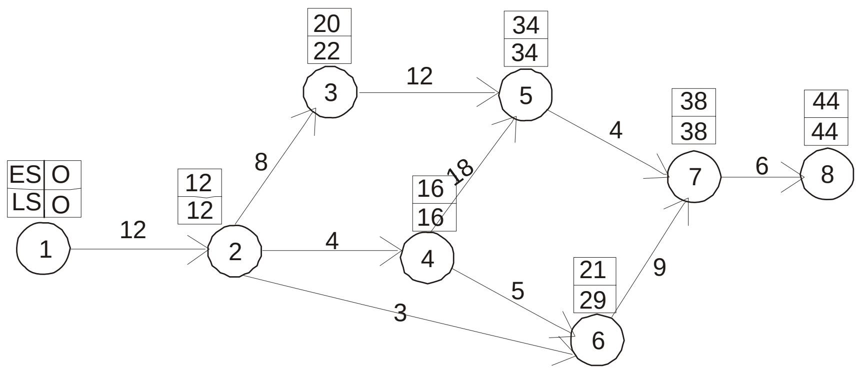
Time (4)

ES = The earliest start time for an activity. The assumption is that all predecessor activities are started at their earliest start time. **[Definition ESE-2003]**

EF = The earliest finish time for an activity. The assumption is that the activity starts on its ES and takes the expected time '*t*'. Therefore **EF = ES + t**

LF = The latest finish time for an activity, without Delaying the project. The assumption is that successive activities take their expected time.

LS = The latest start time for an activity, without delaying the project. **LS = LF – t**



To Calculate ES

Forward Pass: Start from first event and go upto last end.

$ES_1 = 0$

$ES_2 = ES_1 + t = 0+12 = 12$

$ES_3 = ES_2 + t = 12+8 = 20$

$ES_4 = ES_2 + t = 12+4 = 16$

$ES_5 = \max \left\{ \left(ES_4 + t \right) \; ; \; \left(ES_3 + t \right) \right\} = \max$

$\left(34, 32\right) = 34$

$ES_6 = \max \left\{ \left(ES_2 + 3 \right) \; ; \; \left(ES_4 + 5 \right) \right\} = \max$

$\left(15, 21\right) = 21$

$ES_7 = \max \left\{ \left(ES_6 + 9 \right) \; ; \; \left(ES_5 + 4 \right) \right\} = \max$

$\left(30, 38\right) = 38$

$ES_8 = ES_7 + 6 = 44$

To Calculate LS

Backward Pass: Start from last event and come upto first.

(i) $(LS)_8 = ES_8 = 44$

(ii) $(LS)_7 = LS_8 - 6 = 38$

(iii) $(LS)_5 = LS_7 - 4 = 34$

(iv) $(LS)_6 = (LS)\ 7 - 9 = 29$

(v) $(LS)_4 = \min \{((LS)_6 - 5)\ ;\ (LS)_5 - 18\}$

$= \min (24, 16) = 16$

(vi) $(LS)_3 = (LS)\ 5 - 12 = 22$

(vii) $(LS)_2 = \min \{((LS)_3 - 8)\ ;\ (LS)_4 - 4\ ;$

$(LS_6 - 3)\} = \min (14, 12, 26) = 12$

EF = ES + t
LF = LS + t

Activity	Time	ES	LS	EF	LF	Stack
1-2	12	0	0	12	12	0
2-3	8	12	14	20	22	2
2-4	4	12	12	16	16	0
2-6	3	12	26	15	29	14
3-5	12	20	22	32	34	2
4-5	18	16	16	34	34	0
4-6	5	16	24	21	29	8
5-7	4	34	34	38	38	0
6-7	9	21	29	30	38	8
7-8	6	38	38	44	44	0

Critical path: Slack = 0
1 – 2 – 4 – 5 – 7 – 8

Same
Same
or, from diagram if ES = LS

Float or slack: It is defined as the amount of time on activity can be delayed without affecting the duration of the project.

Total Float: It is the maximum time, which is available to complete an activity minus the actual time which the activity takes.

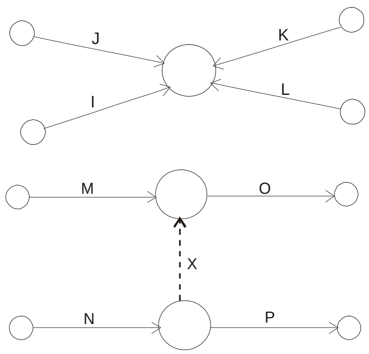
$$\therefore \text{Total float} = (LS)_{same(i)} - (ES)_{same(i)}$$
$$= [(LF)_{next(i)} - (ES)_{previous(i)}] - t_{ij}$$

Free Slack: It is used to denote the amount of time an activity can be delayed without delaying the earliest start of any succeeding activity.

$$= [(EF)_{next\ (j)} - (ES)_{\text{previous(i)}}] - t_{ij}$$

Independent Float: It is important when the network of the project runs on earliest time. If an activity reaches next stage at the latest time, independent float will indicate if the considered activity will reach at the next stage so as to allow the following activity to begin at the earliest time.

Independent Float = $(EF)_j - (LS)_j - t_{ij}$

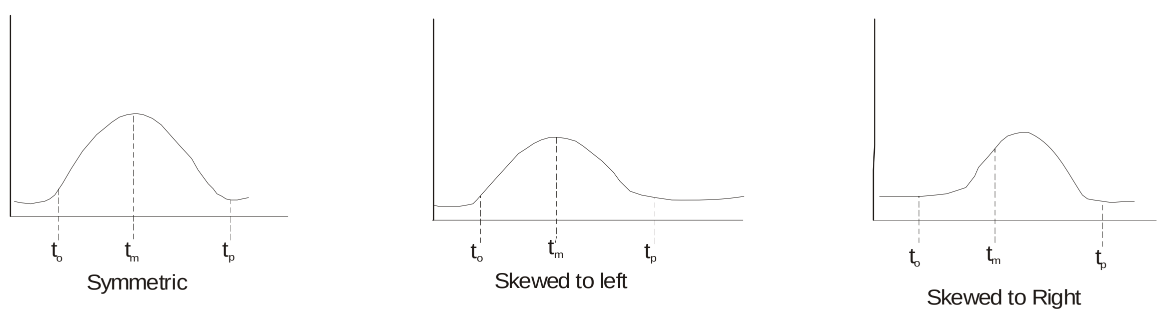


1. K or L will not start until both I and J finished.
2. I or J may or may not end in same time.
3. K and L may or may not start same time

1. Both activity M & N must be finished before O can start.
2. Activity P depends only on N not on activity M, so when N finish P may start but don't need to know about M

Frequency Distribution Curve for PERT

It is assumed to be a β - distribution curve with a unimodal point occurring at t_m and its end points occurring at t_o and t_p. The most likely time need not be the midpoint of t_0 and t_p and hence the frequency distribution curve may be skewed to the left, skewed to the right or symmetric.



β - Distribution curve

Though the β - distribution curve is not fully described by the mean (μ) and the standard deviation (σ), yet in PERT the following relations are approximated for μ and σ :

PERT	Expected time i.e. mean (μ)	$t_e = \left[\dfrac{t_o + 4t_m + t_p}{6}\right]$	i.e. mean if β - distribution
	Standard deviation	$(\sigma) = \dfrac{t_p - t_o}{6}$	
	Variance (V)	$\sigma^2 = \left(\dfrac{t_p - t_o}{6}\right)^2$	it is the variance of an activity.

(i) Variance of the expected time of the project, $(\sigma_{cp})^2$ is obtained by adding the variance of the expected time of all activities along the critical path.

$$\sigma_{cp}^2 = \sum (\sigma_i)^2$$

(ii) The expected time of the project is the sum of the expected time of all activities lying on the critical path.

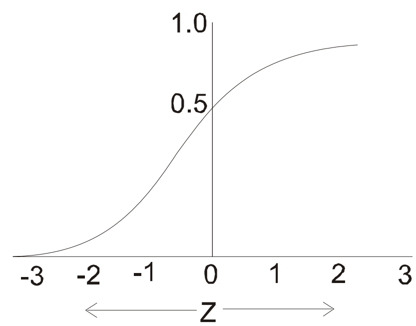
$$t_{cp} = \sum t_e$$

(iii) Probability that the project will be completed in a given time. (T)
a > the expected completion time (t_{cp})
b > standard deviation (σ_{cp})
Calculate $(Z) = \left(\frac{T - t_{cp}}{\sigma_{cp}}\right)$

Probability, $P = \phi(Z)$ assuming that the completion time for the project has a Normal Distribution about the expected completion time.

Where $\phi(Z) =$ cumulative distribution function after the variable Z corresponding to a standardize normal distribution.

If $Z = 0$ i.e. $T = t_{cp}$ there is a 50% probability that the project completing on the scheduled time.

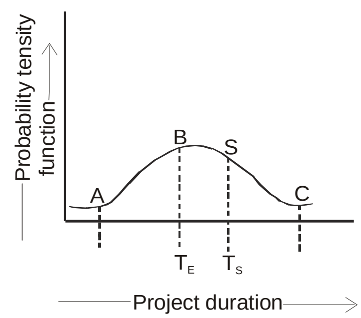


Cumulative distribution function

What is the probability that the activity will be completed in this expected time?
Variance is the measure of this uncertainty. Greater the value of variance, the larger will be the uncertainty.

Probability of Meeting the Scheduled Dates
The standard normal distribution curve

Note:
(i) It has an area equal to unity.
(ii) Its standard deviation is one.
(iii) It is symmetrical about the mean value.



T_E = project expected time, i.e. critical path time (or Scheduled completion time)

T_s = Contractual obligation time, (or Schedule completion time)

Therefore, probability of completing a project in time T_S is given by

$$P(T_s) = \frac{\text{Area under ABS}}{\text{Area under ABC}}$$

Standard deviation for network

$$\sigma = \sqrt{\text{Sum of the varience along critical path}}$$

$= \sqrt{\sum \sigma_{ij}^2}$ Where varience for an activity, $V = \left(\frac{t_p - t_o}{6}\right)^2$

Since the standard deviation for a normal curve is 1, the σ calculated above is used as a scale factor for calculating the normal deviate.

Normal deviation, $Z = \frac{T_S - T_E}{\sigma}$

The values of probability for a normal distribution curve, corresponding to the different value of normal deviate are given in a simplified manner.

For a normal deviate of +1, the corresponding probability is 84.1% and for $Z = -1$ corresponding $P = 15.9$ %.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. In PERT analysis a critical activity has [GATE-2004]
(a) Maximum Float (b) Zero Float
(c) Maximum Cost (d) Minimum Cost

GATE-2. A project consists of three parallel paths with durations and variances of (10, 4), (12, 4) and (12, 9) respectively. According to the standard PERT assumptions, the distribution of the project duration is: [GATE-2002]
(a) Beta with mean 10 and standard deviation 2
(b) Beta with mean 12 and standard deviation 2
(c) Normal with mean 10 and standard deviation 3
(d) Normal with mean 12 and standard deviation 3

GATE-3. A dummy activity is used in PERT network to describe [GATE-1997]
(a) Precedence relationship (b) Necessary time delay
(c) Resource restriction (d) Resource idleness

GATE-4. In PERT, the distribution of activity times is assumed to be: [GATE-1995; IES-2002]
(a) Normal (b) Gamma (c) Beta (d) Exponential

GATE-5. The expected time (t_e) of a PERT activity in terms of optimistic time (t_o), pessimistic time (t_p) and most likely time (t_l) is given by: [GATE-2009]
(a) $t_e = \frac{t_o + 4t_l + t_p}{6}$ (b) $t_e = \frac{t_o + 4t_p + t_l}{6}$
(c) $t_e = \frac{t_o + 4t_l + t_p}{3}$ (d) $t_e = \frac{t_o + 4t_p + t_l}{3}$

Statement for Linked Answer Questions Q6 & Q7:
Consider a PERT network for a project involving six tasks (a to f)

Task	Predecessor	Expected task time (in days)	Variance of the task time (in days^2)
a	-	30	25
b	a	40	64
c	a	60	81
d	b	25	9
e	b, c	45	36
f	d, e	20	9

GATE-6. The expected completion time of the project is: [GATE-2006]
(a) 238 days (b) 224 days (c) 171 days (d) 155 days

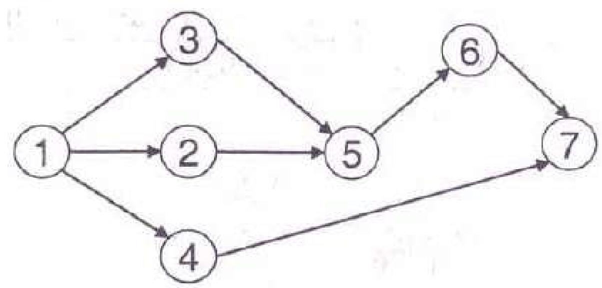
GATE-7. The standard deviation of the critical path of the project is:

[GATE-2006]

(a) $\sqrt{151}$ days (b) $\sqrt{155}$ days (c) $\sqrt{200}$ days (d) $\sqrt{238}$ days

Common Data for Questions Q8 and Q9:

Consider the following PERT network:



The optimistic time, most likely time and pessimistic time of all the activities are given in the table below:

Activity	Optimistic time (Days)	Most Likely time (Days)	Pessimistic time (days)
1 – 2	1	2	3
1 – 3	5	6	7
1 – 4	3	5	7
2 – 5	5	7	9
3 – 5	2	4	6
5 – 6	4	5	6
4 – 7	4	6	8
6 – 7	2	3	4

GATE-8. The critical path duration of the network (in days) is: [GATE-2009]

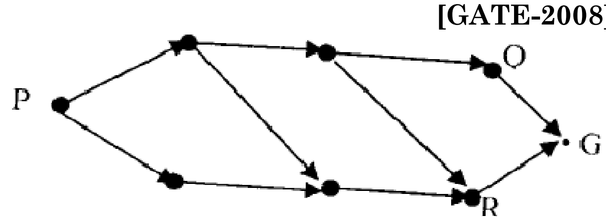
(a) 11 (b) 14 (c) 17 (d) 18

GATE-9. The standard deviation of the critical path is: [GATE-2009]

(a) 0.33 (b) 0.55 (c) 0.88 (d) 1.66

GATE-10. For the network below, the objective is to find the length of the shortest path from node P to node G. Let d_{ij} be the length of directed arc from node *i* to node *j*. [GATE-2008]

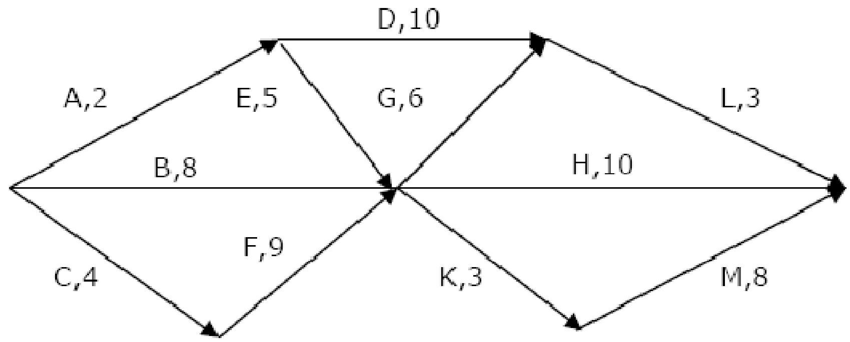
Let s_j be the length of the shortest path from P to node j. Which of the following equations can be used to find s_G?



(a) $s_G = \text{Min}\{s_Q, s_R\}$ (b) $s_G = \text{Min}\{s_Q - D_{QG}, S_R - d_{RG}\}$

(c) $s_G = \text{Min}\{s_Q + d_{QG}, S_R + d_{RC}\}$ (d) $s_G = \text{Min}\{d_{QG}, d_{RG}\}$

GATE-11. A Project consists of activities A to M shown in the net in the following figure with the duration of the activities marked in days



The project can be completed: **[GATE-2003]**
(a) Between 18, 19 days (b) Between 20,22 days
(c) Between 24, 26 days (d) Between 60, 70 days

GATE-12. **The project activities, precedence relationships and durations are described in the table. The critical path of the project is:** **[GATE-2010]**

Activity	Precedence	Duration (in days)
P	–	**3**
Q	–	**4**
R	**P**	**5**
S	**Q**	**5**
T	**R, S**	**7**
U	**R, S**	**5**
V	**T**	**2**
W	**U**	**10**

(a) P-R-T-V (b) Q-S-T-V (c) P-R-U-W (d) Q-S-U-W

CPM

GATE-13. **A project has six activities (A to F) with respective activity durations 7, 5, 6, 6, 8, 4 days. The network has three paths A-B, C-D and E-F. All the activities can be crashed with the same crash cost per day. The number of activities that need to be crashed to reduce the project duration by 1 day is:** **[GATE-2005]**
(a) 1 (b) 2 (c) 3 (d) 6

Previous 20-Years IES Questions

IES-1. **Consider the following statements:** **[IES-2007]**
PERT considers the following time estimates
1. Optimistic time **2. Pessimistic time** **3. Most likely time**
Which of the statements given above are correct?
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 3 only (d) 1 and 3 only

IES-2. **Consider the following statements with respect to PERT** **[IES-2004]**
1. **It consists of activities with uncertain time phases**
2. **This is evolved from Gantt chart**

3. **Total slack along the critical path is not zero**
4. **There can be more than one critical path in PERT network**
5. **It is similar to electrical network**

Which of the statements given above are correct?

(a) 1, 2 and 5 (b) 1, 3 and 5 (c) 2, 4 and 5 (d) 1, 2 and 4

IES-3. **Dummy activities are used in a network to: IES-1992, 2000]**

(a) Facilitate computation of slacks
(b) Satisfy precedence requirements
(c) Determine project completion time
(d) Avoid use of resources

IES-4. **A PERT activity has an optimistic time estimate of 3 days, a pessimistic time estimate of 8 days, and a most likely time estimate of 10 days. What is the expected time of this activity? [IES-2008]**

(a) 5·0 days (b) 7·5 days (c) 8·0 days (d) 8.5 days

IES-5. **Which one of the following statements is not correct? [IES-2008]**

(a) PERT is activity oriented and CPM is event oriented
(b) In PERT, three time estimates are made, whereas in CPM only one time estimate is made
(c) In PERT slack is calculated whereas in CPM floats are calculated
(d) Both PERT and CPM are used for project situations

IES-6. **If the earliest starting time for an activity is 8 weeks, the latest finish time is 37 weeks and the duration time of the activity is 11 weeks, then the total float is equal to: [IES-2000]**

(a) 18 weeks (b) 14 weeks (c) 56 weeks (d) 40 weeks

IES-7. **The earliest occurrence time for event '1' is 8 weeks and the latest occurrence time for event' I' is 26 weeks. The earliest occurrence time for event '2' is 32 weeks and the latest occurrence time for event '2' is 37 weeks. If the activity time is 11 weeks, then the total float will be: [IES-1998]**

(a) 11 (b) 13 (c) 18 (d) 24

IES-8. **Which of the following are the guidelines for the construction of a network diagram? [IES-1996]**

1. **Each activity is represented by one and only one arrow in the network.**
2. **Two activities can be identified by the same beginning and end events.**
3. **Dangling must be avoided in a network diagram.**
4. **Dummy activity consumes no time or resource.**

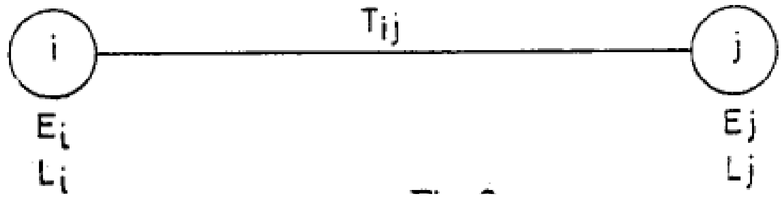
Select the correct answer using the codes given below:
Codes:

(a) 1, 2 and 3 (b) l, 3 and 4 (c) 1, 2 and 4 (d) 2, 3 and 4

IES-9. **Earliest finish time can be regarded as [IES-1993]**

(a) EST + duration of activity (b) EST – duration of activity
(c) LFT + duration of activity (d) LFT – duration of activity

IES-10. **Consider an activity having a duration time of T_{ij}. E is the earliest occurrence time and L the latest occurrence time (see figure given).**



Consider the following statements in this regard: **[IES-1993]**

1. **Total float = L_j - E_i - T_{ij}**
2. **Free float = E_j - E_i - T_{ij}**
3. **Slack of the tail event = L_j- E_i**

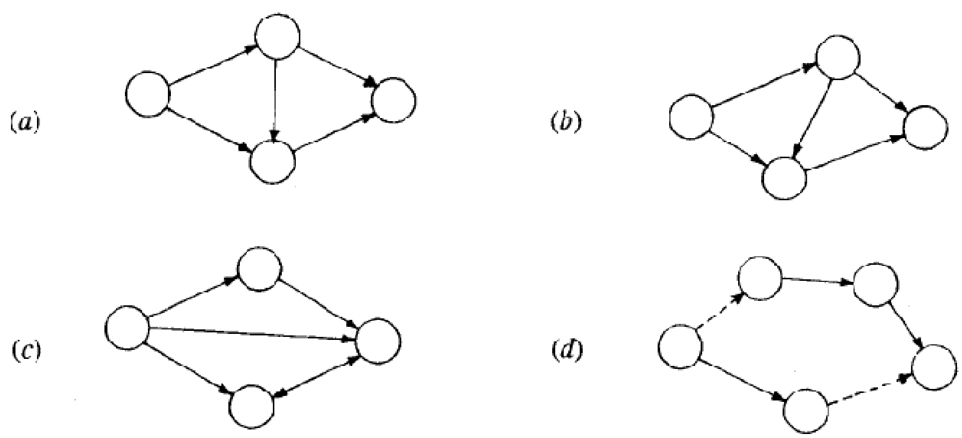
Of these statements:

(a) 1, 2 and 3 are correct (b) 1 and 2 are correct
(c) 1 and 3 are correct (d) 2 and 3 are correct

IES-11. **What is the additional time available for the performance of an activity in PERT and CPM calculated on the basis that all activities will start at their earliest start time, called?** **[IES-2008]**

(a) Slack (b) Total float (c) Free float (d) Independent float

IES-12. Which one of the following networks is correctly drawn? **[IES-1993]**



IES-13. **The essential condition for the decompression of an activity is:**

(a) The project time should change due to decompression **[IES-1992]**
(b) After decompression the time of an activity invariably exceeds its normal time.
(c) An activity could be decompressed to the maximum extent of its normal time
(d) None of the above.

IES-14. **A PERT network has three activities on critical path with mean time 3, 8 and 6, and standard deviation1, 2 and 3 respectively. The probability that the project will be completed in 20 days is:[IES-1993]**

(a) 0.50 (b) 0.66 (c) 0.84 (d) 0.95

IES-15. **Time estimates of an activity in a PERT network are:** **[IES-1999]**

Optimistic time t_o = 9 days; pessimistic time t_p = 21 days and most likely time t_e = 15 days. The approximates probability of completion of this activity in 13 days is:
(a) 16% (b) 34% (c) 50% (d) 84%

IES-16. In a PERT network, expected project duration is found to be 36 days from the start of the project. The variance is four days. The probability that the project will be completed in 36 days is: [IES-1997]
(a) Zero (b) 34% (c) 50% (d) 84%

IES-17. In a small engineering project, for an activity, the optimistic time is 2 minutes, the most likely time is 5 minutes and the pessimistic time is 8 minutes. What is the expected time of the activity? [IES-2005]
(a) 1 minutes (b) 5 minutes (c) 8 minutes (d) 18 minutes

IES-18. Assertion (A): Generally PERT is preferred over CPM for the purpose of project evaluation. [IES-1996]
Reason (R): PERT is based on the approach of multiple time estimates for each activity.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-19. Which one of the following statements is not correct? [IES 2007]
(a) PERT is probabilistic and CPM is
(b) In PERT, events are used deterministic and in CPM activities are used
(c) In CPM, the probability to complete
(d) In CPM crashing is carried the project in a given time-duration is out calculated

IES-20. Consider the following statements in respect of PERT and CPM:
1. PERT is event-oriented while CPM is activity-oriented.
2. PERT is probabilistic while CPM is deterministic.
3. Levelling and smoothing are the techniques related to resource scheduling in CPM.
Which of the statements given above are correct? [IES-2006]
(a) 1, 2 and 3 (b) Only 1 and 2 (c) Only 2 and 3 (d) Only 1 and 3

IES-21. Match List-I with List-II and select the correct answer using the code given below the lists: [IES-2005]

	List-I		**List-II**
A.	Transportation Problem	**1.**	Critical Path
B.	Assignment Problem	**2.**	Stage Coach
C.	Dynamic Problem	**3.**	Vogel's Approximate Method
D.	PERT	**4.**	Hungarian Method

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	1	3	4	(b)	3	4	2	1
(c)	2	4	3	1	(d)	3	1	2	4

IES-22. **Match List-I (Term) with List-II (Characteristics) and select the correct answer using the code given below the lists: [IES-2007]**

List-I	**List-II**
A. Dummy activity	**1.** Follows β distribution
B. Critical path	**2.** It is built on activity oriented diagram
C. PERT activity	**3.** Constructed only to establish sequence
D. Critical path method	**4.** Has zero total slack

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	4	1	2	(b)	4	2	3	1
(c)	3	4	2	1	(d)	4	2	1	3

IES-23. **Match List-I (Techniques/Methods) with List-II (Models) and select the correct answer using the codes given below the lists: [IES-2004]**

List-I	**List-II**
A. Vogel's approximation method	**1.** Assignment model
B. Floods technique	**2.** Transportation model
C. Two phase method	**3.** PERT and CPM
D. Crashing	**4.** Linear programming

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	4	1	2	(b)	2	1	4	3
(c)	3	1	4	2	(d)	2	4	1	3

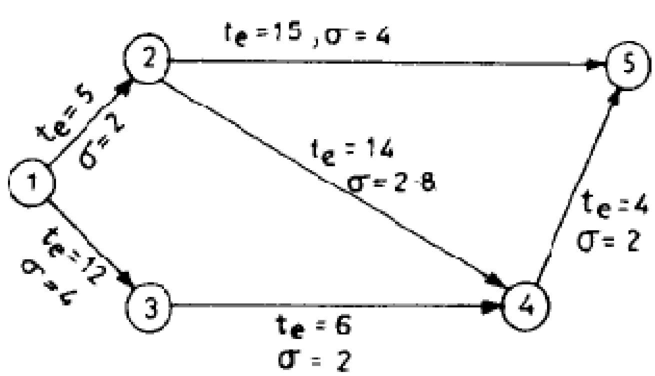
IES-24. **Estimated time T_e and variance of the activities 'V' on the critical path in a PERT new work are given in the following table:**

Activity	**T_e (days)**	**V (days)^2**
a	**17**	**4**
b	**15**	**4**
c	**8**	**1**

The probability of completing the project in 43 days is: [IES-1998]

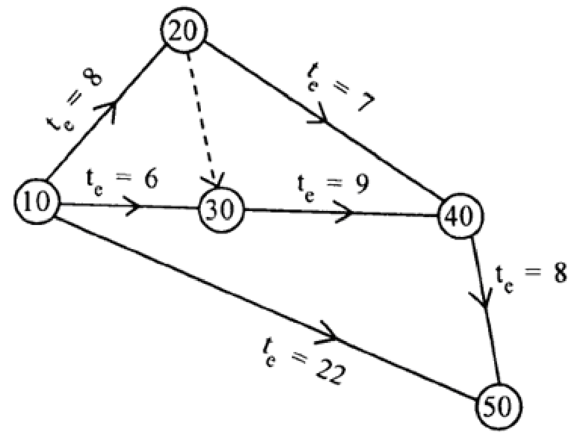
(a) 15.6% (b) 50.0% (c) 81.4% (d) 90.0%

IES-25. **For the PERT network shown in the given figure, the probability of completing the project in 27 days is: [IES-1994]**



IES-26. **If critical path of a project is 20 months with a standard deviation 4 months, what is the probability that the project will be completed in 24 months? [IES-2008]**
(a) 15·85% (b) 68·3% (c) 84·2% (d) 95·50%

IES-27. **Consider the network. Activity times are given in number of days. The earliest expected occurrence time (TE) for event 50 is:**
(a) 22
(b) 23
(c) 24
(d) 25



[IES-2008]

IES-28. **The three time estimates of a PERT activity are: optimistic time = 8 min, most likely time = 10 min and pessimistic time = 14 min. The expected time of the activity would be: [IES-2002]**
(a) 10.00 min (b) 10.33 min (c) 10.66 min (d) 11.00 min

IES-29. **Assertion (A): The change in critical path required rescheduling in a PERT network. [IES-2002]**
Reason (R): Some of the activities cannot be completed in time due to unexpected breakdown of equipments or non-availability of raw materials.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-30. **Match List-I (OR-technique) with List-II (Model) and select the correct answer using the codes given below the lists: [IES-2001]**

List-I		List-II	
A.	Branch and Bound technique	**1.**	PERT and CPM
B.	Expected value approach	**2.**	Integer programming
C.	Smoothing and Leveling	**3.**	Queuing theory
D.	Exponential distribution	**4.**	Decision theory

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	1	4	3	(b)	2	4	1	3
(c)	3	4	1	2	(d)	3	1	4	2

IES-31. **Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-2000]**
List-I **List-II**

A. Control charts for variables	**1.** Binomial distribution
B. Control chart for number of non-conformities	**2.** Beta distribution
C. Control chart for fraction rejected	**3.** Normal distribution
D. Activity time distribution in PERT	**4.** Poisson distribution
	5. Exponential distribution

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	4	1	5	(b)	5	4	3	1
(c)	4	3	1	2	(d)	3	4	1	2

CPM

IES-32. Latest start time of an activity in CPM is the **[IES-2001]**

(a) Latest occurrence time of the successor event minus the duration of the activity
(b) Earliest occurrence time for the predecessor event plus the duration of the activity
(c) Latest occurrence time of the successor event
(d) Earliest occurrence time for the predecessor event

IES-33. In CPM, the cost slope is determined by: **[IES-1994]**

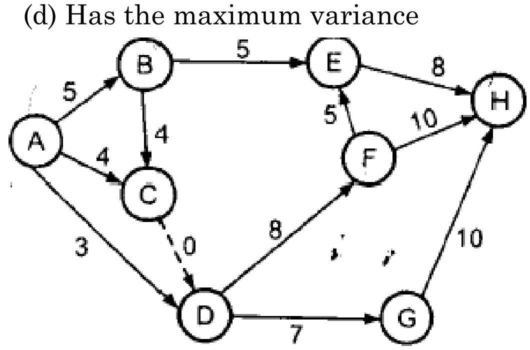
(a) $\dfrac{\text{Crash cost}}{\text{Normal cost}}$ (b) $\dfrac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$

(c) $\dfrac{\text{Normal cost}}{\text{Crash cost}}$ (d) $\dfrac{\text{Normal cost} - \text{Crash cost}}{\text{Normal time} - \text{Crash time}}$

IES-34. The critical path of a network is the path that: **[IES-2005]**

(a) Takes the shortest time (b) Takes the longest time
(c) Has the minimum variance (d) Has the maximum variance

IES-35. For the network shown in the given figure, the earliest expected completion time of the project is:



(a) 26 days
(b) 27 days
(c) 30 days
(d) Indeterminable

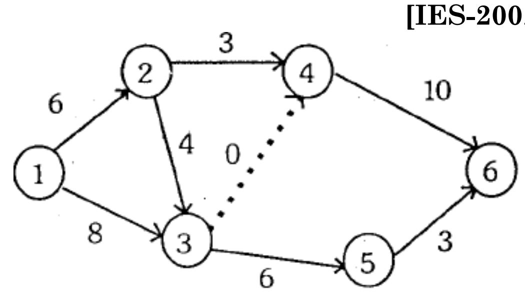
[IES-2001]

IES-36. In a network, what is total float equal to? **[IES-2006]**

(a) $LFT_j - EST_i + t_{i-j}$ (b) $EST_j - LFT_i + t_{i-j}$
(c) $EST_j - LFT_i - t_{i-j}$ (d) $LFT_j - EST_i - t_{i-j}$

Where, LFT = latest finish time of an activity; EST = earliest start time of an activity; t_{i-j} = time of activity i-j)

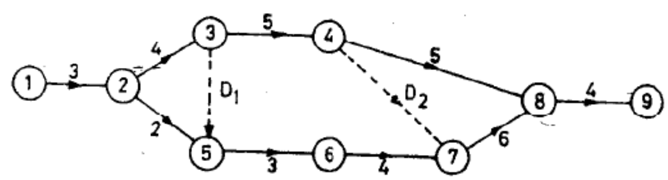
IES-37. **For the network shown in the figure, the variance along the critical path is 4. [IES-2002]**
The probability of completion of the project in 24 days is:



(a) 68.2%
(b) 84.1 %
(c) 95.4%
(d) 97.7%

IES-38. **The variance (V_1) for critical path [IES-1997]**
a $\rightarrow$b = 4 time units, b$\rightarrow$c = 16 time units, c $\rightarrow$ d = 4 time units, d $\rightarrow$ e = 1 time unit.
The standard deviation d the critical path a $\rightarrow$e is:
(a) 3 (b) 4 (c) 5 (d) 6

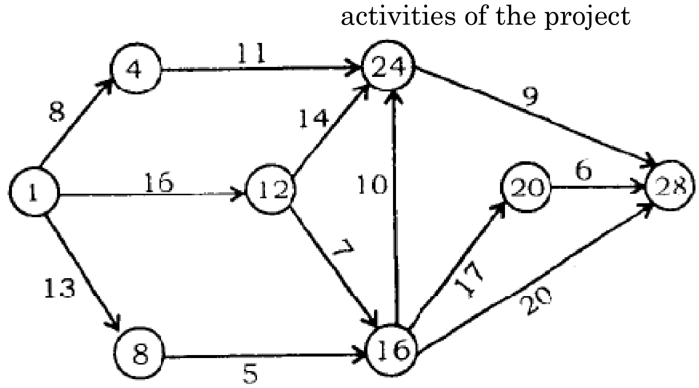
IES-39. **In the network shown below. The critical path is along**



(a) 1-2-3-4-8-9
(b) 1-2-3-5-6-7-8-9
(c) 1-2-3-4-7-8-9
(d) 1-2-5-6-7-8-9

IES-40. **The variance of the completion time for a project is the sum of variances of: [IES-2003]**
(a) All activity times
(b) Non-critical activity times
(c) Critical activity times
(d) Activity times of first and last activities of the project

IES-41.



The earliest time of the completion of the last event in the above network in weeks is: [IES-2003]
(a) 41 (b) 42 (c) 43 (d) 46

IES-42. **Consider the following statements regarding updating of the network: [IES-2002]**
1. **For short duration project, updating is done frequently**

2. For large duration project, frequency of updating is decreased as the project is nearing completion
3. Updating is caused by overestimated or underestimated times of activities
4. The outbreak of natural calamity necessitates updating

Which of the above statements are correct?

(a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 4 (d) 1, 2 and 4

Previous 20-Years IAS Questions

CPM

IAS-1. In CPM network critical path denotes the [IAS-2002]

(a) Path where maximum resources are used
(b) Path where minimum resources are used
(c) Path where delay of one activity prolongs the duration of completion of project
(d) Path that gets monitored automatically

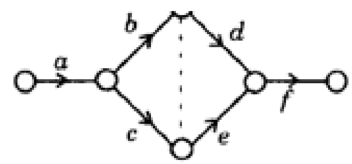
IAS-2. Time estimates of a project activity are: [IAS-2002]
t_{op} optimistic time = 10 days.
t_{ml}, most likely time = 15 days.
t_{pcs}, pessimistic time =22 days.
Variance in days for this activity as per BETA distribution is:

(a) 12 (b) 7 (c) 5 (d) 4

Answers with Explanation (Objective)

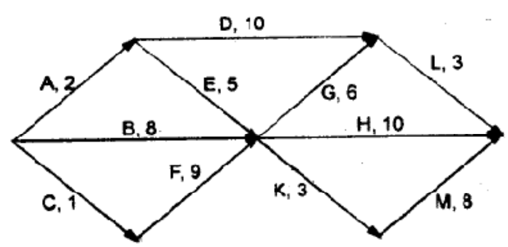
Previous 20-Years GATE Answers

GATE-1. Ans. (b)
GATE-2. Ans. (d) Since PERT is a Beta distribution, therefore Beta with mean 12 and standard deviation is correct.
GATE-3. Ans. (a)
GATE-4. Ans. (c)
GATE-5. Ans. (a)
GATE-6. Ans. (d) Critical path = a – c – e – f
= 30 + 60 + 45 + 20
= 155 days



Standard deviation,
$$\sigma = \sqrt{25+81+36+9} \text{ days}$$
$$= \sqrt{151}$$

GATE-7. Ans. (a)
GATE-8. Ans. (d)
GATE-9. Ans. (c)
GATE-10. Ans. (c)
GATE-11. Ans. (c) Project completed
= Activity C + Activity F
+ Activity K + Activity M
= 4 + 9 + 3 + 8 = 24



GATE-12. Ans. (d) Q – S – V – W is haring maximum duration = 24 days so it is the critical path.
GATE-13. Ans. (c)

Previous 20-Years IES Answers

IES-1. Ans. (a)
IES-2. Ans. (d)
IES-3. Ans. (b)
IES-4. Ans. (d) t_e = Expected time $= \dfrac{t_o + 4t_m + t_p}{6} = \dfrac{3+(4\times 10)+8}{6}$

$$= \frac{3+40+8}{6} = 8.5 \text{ days.}$$

IES-5. Ans. (a) (a) PERT → Event oriented
CPM → Activity oriented
(b) PERT → 3 time estimates are made

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

t_o = optimistic time

t_m = most likely time

t_p = pessimistic time

CPM → only one time estimate

(c) In PERT slack is calculated, CPM floats calculated

(d) Both PERT and CPM are used for project situation.

IES-6. Ans. (a) $\text{T.F.} = LS_i - ES_{ij} - t_{ij} = 37 - 8 - 11 = 18$

IES-7. Ans. (c) Total float = 37 – 8 – 11 = 18 days.

IES-8. Ans. (b)

IES-9. Ans. (a)

IES-10. Ans. (a)

IES-11. Ans. (c)

IES-12. Ans. (a) Diagram (a) is correct as in (b) & (c) diagrams backward arrows are seen which is not correct. In (d) both activity is dummy it is also not correct.

IES-13. Ans. (c)

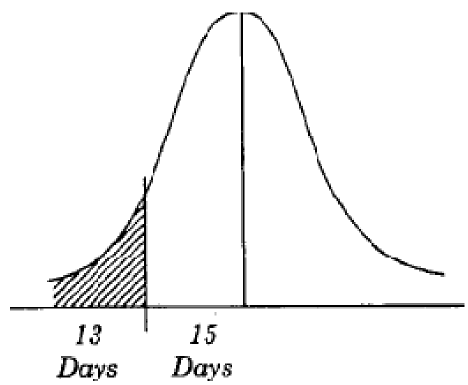
IES-14. Ans. (b) $z = \frac{T_s - T_{cp}}{\sigma_{cp}} = \frac{20 - 17}{6} = 0.5$

IES-15. Ans. (a) Expected time = $\frac{t_o + 4t_e + t_p}{6}$

$= \frac{9 + 4 \times 15 + 21}{6} = 15$ dats and

$\sigma = \frac{t_p - t_o}{6} = \frac{21 - 9}{6} = 2$

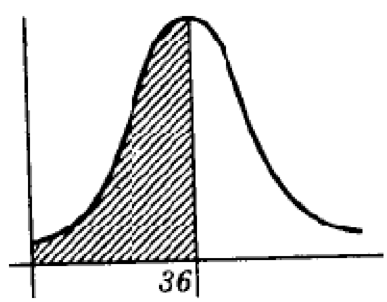
Probability of completing in 13 days is shaded area = 50% – Area for 1σ = 50 – 34 = 16%.



IES-16. Ans. (c) Variance = 4 days,

Std. dev. = $\sqrt{2}$ = 2 days

Probability in this case is shaded area in given figure, which is 50%.



IES-17. Ans. (b) Expected time $(t_e) = \frac{t_0 + 4t_m + t_p}{6} = \frac{2 + 4x9 + 8}{6} = 5\,\text{min}$

IES-18. Ans. (a)

IES-19. Ans. (c) In PERT, the probability to complete the project in a given time-duration is calculated but in CPM we know the activity time definitely so no question of probability.

IES-20. Ans. (b)

IES-21. Ans. (b)

IES-22. Ans. (a)

IES-23. Ans. (b)

IES-24. Ans. (c) Expected project time = 17 + 15 + 8 = 40 days and variance V

= 4 + 4 + 1= 9, ($\sigma = \sqrt{V}$ = 3 days

Project is to be completed in 43 days.

∴ Probability ± Shaded area = 50 + 34 = 84%.

IES-25. Ans. (a) Critical path is 1 – 2 – 4 – 5

t_e = expected project time = 5 + 14 + 4 = 23 days and $\sigma = \sqrt{2^2 + 2.8^2 + 2^2} = 4$

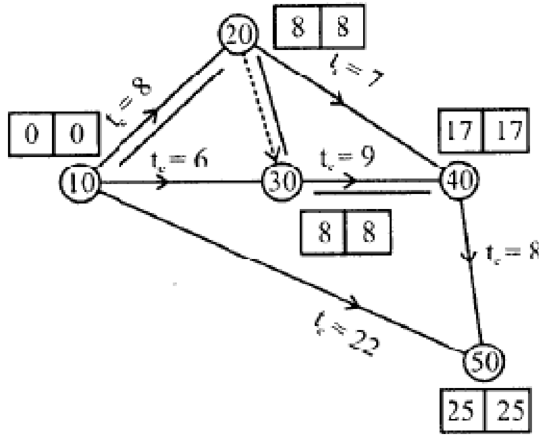
$Z = \frac{27-23}{4} = 1$ Area for Z = I is 0.341.

Therefore Probability = 0.5 + 0.341 = 0.841

IES-26. Ans. (c) $z = \frac{x - \bar{x}}{\sigma} = \frac{24-20}{4} = 1;$ $P(1) = 0.842 = 84.2\%$

IES-27. Ans. (d) Critical path is given by 10 – 20 – 30 – 40 – 50

∴ The earliest expected occurrence time (TE) for the event is 25.



IES-28. Ans. (b)

IES-29. Ans. (b)

IES-30. Ans. (a)

IES-31. Ans. (d)

IES-32. Ans. (a)

IES-33. Ans. (b)

IES-34. Ans. (b)

IES-35. Ans. (c)

IES-36. Ans. (d)

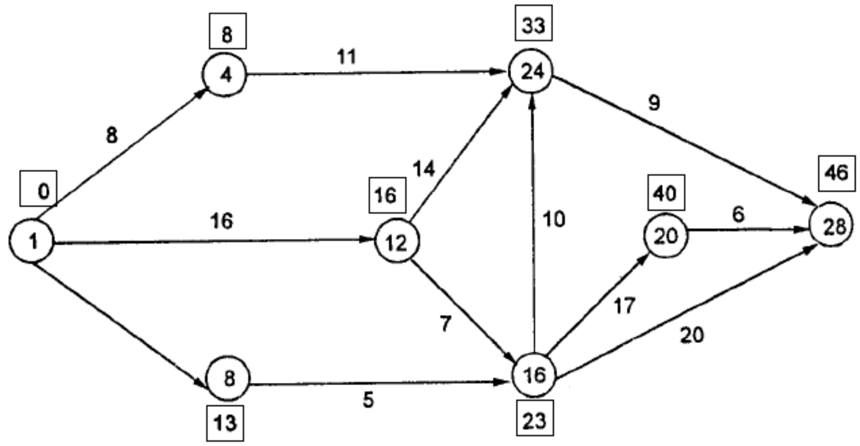
IES-37. Ans. (d)

IES-38. Ans. (c) Standard Deviation = $\sqrt{4+16+4+1} = 5$

IES-39. Ans. (b)

IES-40. Ans. (c)

IES-41. Ans. (d)



Earliest time, $T_E = 46$

IES-42. Ans. (a)

Previous 20-Years IAS Answers

IAS-1. Ans. (c) Total float in critical path is zero so delay in any activity is delayed project.

IAS-2. Ans. (d) Variance (s^2) = $\left(\frac{T_p - T_0}{6}\right)^2 = \left(\frac{22-10}{6}\right)^2 = 4$

Conventional Questions with Answer

Conventional Question [ESE-2010]

What is float or slack and when does a sub critical path becomes critical?

[2 Marks]

Ans. **Float or slack:** It is time by which completion of an activity can be delayed without delaying the project: It is two type of float or slack

(i) Event float (ii) Activity float

Critical path: The critical activities of network that constitute an unit erupted path which spans the entire network from start finish is known as critical path.

Conventional Question [ESE-2006]

What is a critical path? Why is the critical path of such importance in large project scheduling and control? Can a critical path change during the course of a project? **[2 Marks]**

Solution: The 'Critical Path' connects those events for which the earliest and the latest times are the same, i.e. these events have zero slack time. The activities connecting these nodes are called critical activities. For these nodes the two time estimates are the same, which means that as soon as the proceeding activity is over the succeeding activity has to begin with no slack if the project is to be completed on schedule.

If the path i.e. critical path is affected, the total completion of the project will be affected. In such cases, larger project needs constant supervision and revision. Critical path can also change during the course of a project if the variables affecting the project completion time or indirect cost, changes.

Conventional Question [ESE-2007]

A typical activity i-j in CPM network has activity duration $\left(t_{ij}\right)$ of 2.5 time units. The earlier expected time $\left(T^i{}_E\right)$ and latest allowable occurrence time $\left(T^i{}_L\right)$ of event i are compound as 8 and 11 units respectively. The corresponding times of event j, i.e., $T^j{}_E$ and $T^j{}_L$ are respectively 13.5 and 13.5 units. Find the three floats of the activity i-j. **[2 Marks]**

Solution: Total float is the spare time available when all preceding activities occur at the earliest possible times and all succeeding activities occur at the latest possible times.

Total Float = latest start – Earliest Start

So, the three floats of the activity i-j are 2.5, 3, 0

Conventional Question [ESE-2000]

What is the standard deviation of the project completion time along the critical path? If the standard deviation of the corresponding activity are s_1, s_2 and s_3,

Solution:

Corresponding activity variance = $s^2{}_1$, $s^2{}_2$ and $s^2{}_3$,

Total Variance along critical path ($\sigma\ ^2{}_{cp}$) = $s^2{}_1 + s^2{}_2 + s^2{}_3$

Standard deviation along critical path = $= \sqrt{\text{Variance}} = \sqrt{s_1^2 + s_2^2 + s_3^2}$

Conventional Question [ESE-1996]

In a PERT analysis the critical path of a project is of 120 days with a variance of 16 (day)^2 determine the 95% confidence limit of project completion time.

Solution:

Z = 1. 605;

Z = 1. 605 for P = 0.9505

$$P = \varphi\left(\frac{T - T_{cp}}{\sigma}\right) \quad \Rightarrow 1.605 = \left(\frac{T - 120}{\sqrt{16}}\right) \quad \Rightarrow T = 120 + 6.42 = 126.42 \text{ days}$$

Conventional Question [ESE-2008]

A project consists of 7 jobs. Jobs A and F can be started and completed independently. Jobs B and C can start only after job A has been completed. Jobs D, E and G can start only after jobs B, (C and D) and (E and F) are completed, respectively. Time estimates of all the jobs are given in the following table:

Job	Time Estimates (Days)		
	Optimistic	Pessimistic	Most Likely
A	3	7	5
B	7	11	9
C	4	18	14
D	4	12	8
E	4	8	6
F	5	19	12
G	2	6	4

Draw the network and determine the critical path, and its expected duration (T_e). What is the probability of completing the project in T_e days? Also, determine the total and free slacks of all the jobs.

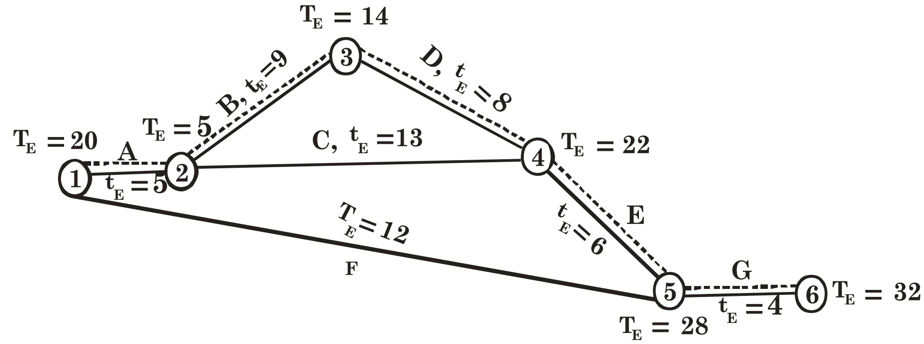
[15-Marks]

Solution:

Time Estimate (Days)						
Job	t_o	t_p	T_m	$t_E = \frac{t_o + 4t_m + t_p}{6}$	$S.D(\sigma) = \left(\frac{t_p - t_o}{6}\right)$	Variance $V = \sigma^2$
A	3	7	5	$\frac{3+4\times5+7}{6} = 5$	$= \frac{4}{6} = \frac{2}{3}$	$\frac{4}{9}$
B	7	11	9	$\frac{7+4\times9+11}{6} = \frac{54}{6} = 9$	$= \frac{4}{6} = \frac{2}{3}$	$\frac{4}{9}$
C	4	18	14	$\frac{4+4\times14+18}{6} = \frac{78}{6} = 13$	$= \frac{14}{6} = \frac{7}{3}$	$\frac{49}{9}$
D	4	12	8	$\frac{4+4\times8+12}{6} = \frac{48}{6} = 8$	$= \frac{8}{6} = \frac{4}{3}$	$\frac{16}{9}$
E	4	8	6	$\frac{4+4\times6+8}{6} = \frac{36}{6} = 6$	$= \frac{4}{6} = \frac{2}{3}$	$\frac{4}{9}$
F	5	19	12			
G	2	6	4			

				$\frac{5+4\times12+19}{6}=\frac{72}{6}=12$ $\frac{2+4\times4+6}{6}=\frac{24}{6}=4$	$=\frac{14}{6}=\frac{7}{3}$ $=\frac{4}{6}=\frac{2}{3}$	$\frac{49}{9}$ $\frac{4}{9}$

The network is drawn below:



$\therefore$ Schedule duration, $\mathbf{T_E = 32}$ days

The following paths for 1st event to last event.

(i) 1 – 2 – 3 – 4 – 5 – 6
(ii) 1 – 2 – 4 – 5 – 6
(iii) 1 – 5 – 6

Sum of $\mathbf{T_E}$ is for all path

For, (i) 5 + 9 + 8 + 6 + 4 = 32
(ii) 5 + 13 + 6 + 4 = 28
(iii) 12 + 4 =16

Hence critical path is.
1 – 2 – 3 – 4 – 5 – 6 (A – B – D – E – G)

$\therefore$ Expected duration, $\mathbf{T_E = 32}$ days

$$\sigma_{\text{critical path}} = \sqrt{\frac{4}{9}+\frac{4}{9}+\frac{16}{9}+\frac{4}{9}+\frac{4}{9}} = 1.8856$$

$$Z = \left(\frac{T - t_{cp}}{\sigma_{cp}}\right)$$

At $\mathbf{T = T_e = t_{cp} \therefore Z = 0}$

Hence there is 50% chance to complete project on excepted time.

Conventional Question **[ESE-1993]**

A building project consists of 10 activities; their estimated duration is given below.

Activity	**Duration**
1 – 2	**5**
2 – 3	**2**
2 – 4	**6**
3 – 5	**4**
3 – 6	**4**
4 – 5	**2**
4 – 7	**3**
5 – 8	**7**
6 – 8	**8**

7 – 8	2

Draw the network and compute

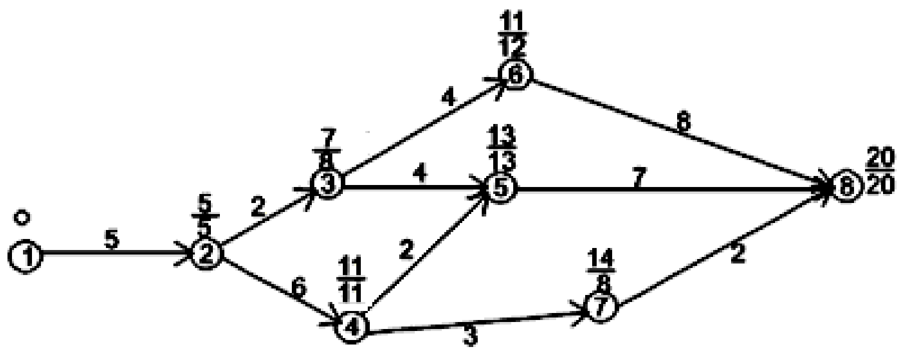
(i) Event times

(ii) Activity time

(iii) Total float and determine

(iv) Critical path

Solution:



(i) Critical path 1 – 2 – 4 – 5 – 8

(ii)

Activity	Duration	ES	LS	Total Float
1-2	5	0	0	0
2-3	2	5	6	1
2-4	6	5	5	0
3-5	4	7	9	2
3-6	4	7	8	1
4-5	2	11	11	0
4-7	3	11	15	4
5-8	7	13	13	0
6-8	8	11	12	1
7-8	2	14	18	4

Conventional Question **[ESE-1991]**

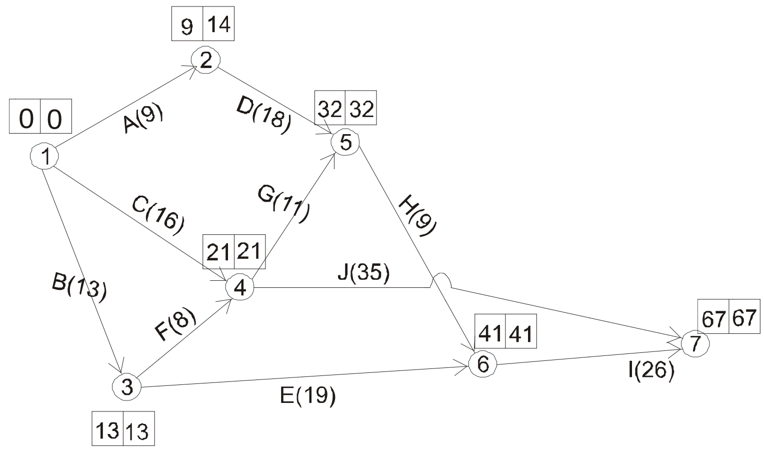
A small plant layout job consists of 10 steps their precedence relationship and activity times are identified as follows.

Step	Predecessor	Time (Hours)
A	None	9
B	None	13
C	None	16
D	A	18
E	B	19
F	B	8
G	C, F	11
H	D, G	9
I	E, H	26

J	**C, F**	**35**

Draw the network complete the forward and backward passes what activities make up the critical path? Which activity has the most slack?

Solution:



Critical path B – F – G – H – I
Slack of A – 5
Slack of C – 5
Slack of J – 32
Slack of E – 28
Max Slack is in activity J.

Conventional Question **[ESE-1990]**

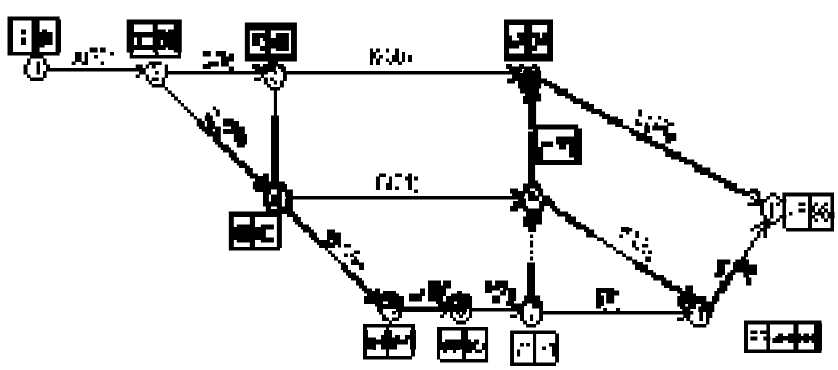
Table 1 gives the different activities associated with a project consisting of 12 tasks (A, B, ..., L) in which the following precedence relationships must hold (XLY Means X must be completed before Y can start):
A < C; A < B; B < D; B < G; B < K; C < D; C < G; D < E; E < F; F < H; F < I; F < L; G < I; G < L; H < J; I < J and K < L

Table-1:

Task	A	B	C	D	E	F	G	H	I	J	K	L
Time (Days)	30	7	10	14	10	7	21	7	12	15	30	15

Draw the network diagram and determine the critical path. Also determine the critical path time.

Solution:



Critical path A – C – D – E – F – I – J
Critical time = 98 (days).

6. Inventory Control

Theory at a Glance (For IES, GATE, PSU)

A fundamental objective of a good system of operation control of Inventories is to be able to place an order at the right time,

from the right source

to acquire the right quantity

at right price.

and right quality.

"Inventory is the life blood of a production system."

Categories:

1. Production inventories → go to final product
2. MRO (Maintenance, Repair and operating supplies) e.g. spare parts, oils grease.
3. In-process inventories (semi-finish products at various production stages)
4. Finished goods inventories
5. Miscellaneous inventory

Another way of classifying industrial inventories are

(i) Transition inventory

(ii) Speculative inventory

(iii) Precautionary inventory

Selective Inventory Control

Different type of inventory analysis?

(i) ABC analysis (class A, class B, class C)

(ii) VED Analysis (vital, Essential, Desirable)

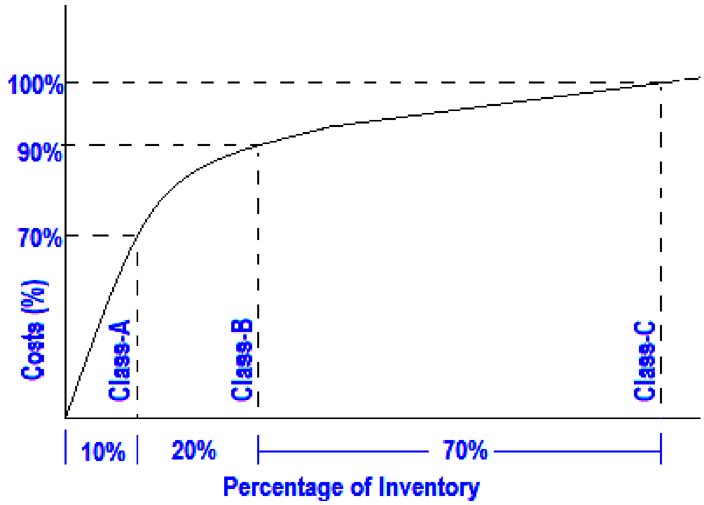
(iii) SDE Analysis (Scarce, Difficult, Easily Available)

(iv) HML Analysis (High, Medium, Low Cost)

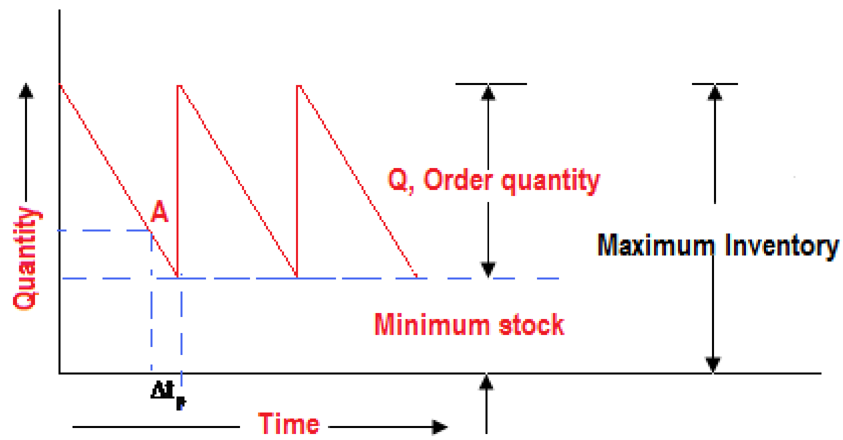
(v) FSN Analysis (Fast, Slow, Non-moving items)

ABC Analysis: The common and important of the selective inventory control of ABC analysis. ABC Analysis is done for items on stock and the basis of analysis is the annual consumption in terms of money value.

Control of A - item: 10 % of the item accounts 70% costs.
Control of B - item: 20% of the item accounts 20% costs.
Control of C - item: 70% of the item accounts 10% costs.



Inventory Management System



a. Minimum inventory or buffer stock
b. Reorder point (A)
c. Procurement lead time (Δt_p)
d. Recorder quantity (Q)

Costs:

1. Unit cost of inventory

(i) Costs paid to the supplies for procuring one unit.
(ii) House manufactured product → direct Manufacturing cost.

Note: For discount model cost of inventory is considered.

2. Ordering costs: Total cost to procure 1 time.

Includes: (i) Originating, placing and paying for an order
(ii) Salary of purchase department
(iii) Telephones, postage, stationary etc

Note: For batch production it is Set-up costs.

3. Carrying costs or holding costs.

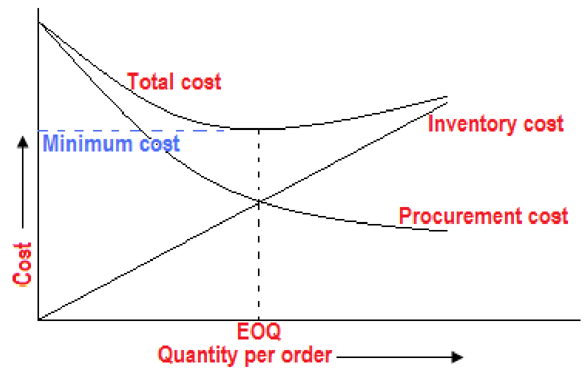
Includes: (i) Interest
(ii) Cost of storage
(iii) Handling and transfer

(iv) Insurance
(v) Personal property tax
(vi) Risk of obsoleteness
(vii) Depreciation
(viii) Salaries and wages to the store personnel
(ix) Pilferage/ theft of material

Generally carrying cost is expressed as a percentage of the inventory value.

4. Shortage or stock-out costs.
(i) Due to shortage how many products does not sold directly.
(ii) Good-will loss i.e. customer reduction.

EOQ, Economic Order Quantity

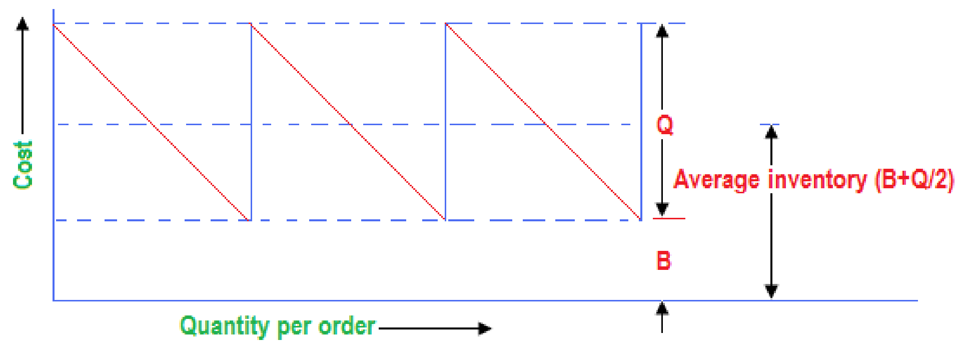


Let,

Q = Economic Order Quantity
C = Unit cost of Part
I_c = Inventory carrying costs per unit
U = Annual Usage i.e. Annual Demand.
R = Ordering, set up, procurement cost per order.
T = Total cost

Model-I (Deterministic Demand)

Uniform demand Rate, Infinite production Rate.



Total Cost (T) = (Ordering Cost) × (Number of order placed in a year)

+ (Carrying cost per unit) × (Average inventory level during year)

Number of order placed = $\dfrac{U}{Q}$

Average inventory carried during the year = (B + Q/2)

$$\therefore\ T = R\times\frac{U}{Q} + I_c\times\left(B+\frac{Q}{2}\right)$$

$$\therefore\ \frac{dT}{dQ} = -\frac{RU}{Q^2} + \frac{I_c}{2}$$

$$\therefore\ Q = \sqrt{\frac{2RU}{I_c}}$$

[VIMP]

This is Wilson's formula for Economic Order Quantity.

If Buffer stock is zero then, Ordering cost = carrying cost **[VIMP for MCQ]**

Minimum Total cost (T_{min}) = $\dfrac{R\times U}{\sqrt{\dfrac{2RU}{I_c}}} + I_c\left\{B+\dfrac{1}{2}\sqrt{\dfrac{2RU}{I_c}}\right\}$

$= [\sqrt{2RUI_C} + BI_C]$

If Buffer stock, B = 0 then

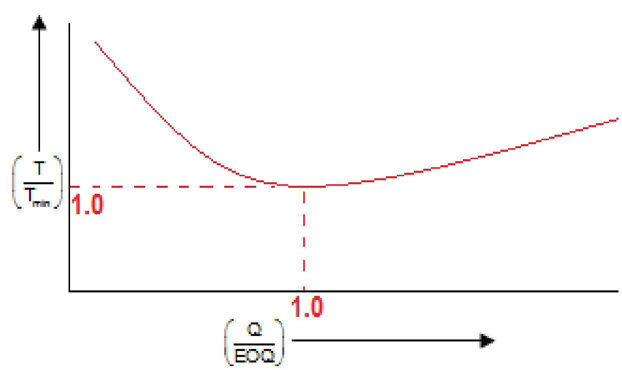
$$T_{\min} = \sqrt{2URI_C}$$

Sensitivity of EOQ Model

$$\frac{T}{T_{\min}} = \frac{\dfrac{RU}{Q} + I_c\left\{B+\dfrac{Q}{2}\right\}}{\left(\sqrt{2RUI} + BI_c\right)}$$

if B=0 then

$$\text{Sensitivity}\ \left(\frac{T}{T_{\min}}\right) = \frac{1}{2}\left\{\frac{EOQ}{Q} + \frac{Q}{EOQ}\right\}$$

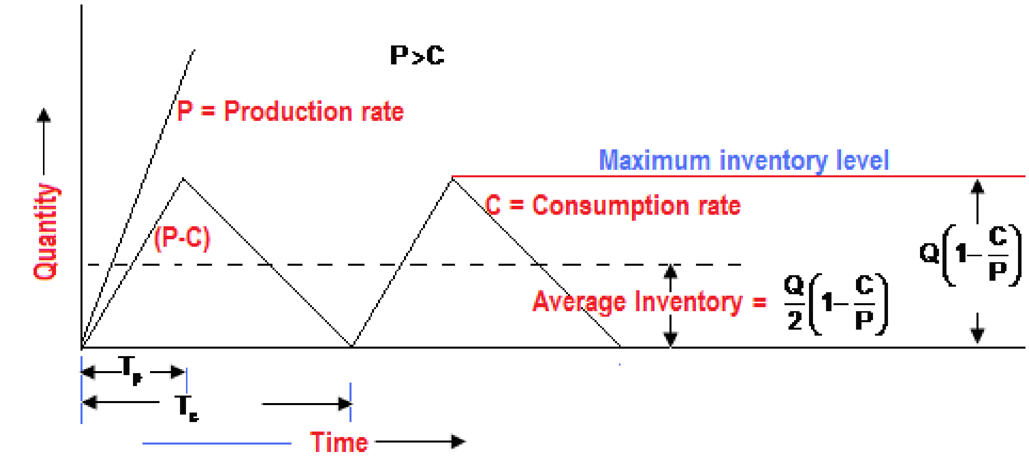


Where Q = Any amount of order
EOQ = Economic order quantity

Note: As EOQ $\infty\sqrt{U}$ and $T\infty\sqrt{U}$

Total cost and number of order per year is proportional to square root of demand. We therefore conclude that ***unless*** the demand is ***highly uncertain*** the EOQ model gives fairly satisfactory decision values. That so why EOQ model is very useful.

Model-II (Gradual Replacement Model)



Let, Q = Economic Batch quantity

P = Production Rate per day

C = consumption Rate per day

T_p= Production Time: U = Annual Demand

T_c= Consumption Time: R = Set up cost

$$\therefore\ T_p = \frac{Q}{P} \quad \text{and} \quad T_p \times P = T_c \times C = Q$$

Accumulation rate = (P – C); (in time T_p)

Maximum inventory = (P – C) × T_p = (P – C) × $\frac{Q}{P}$ = $Q\left(1-\frac{C}{P}\right)$

$$\therefore \text{ Average inventory} = \frac{Q}{2}\left(1-\frac{C}{P}\right)$$

$\therefore$ Total cost (T) = Total orderig cost + Total carrying cost

$$= \frac{U}{Q}\times R + \frac{Q}{2}\left(1-\frac{C}{P}\right)\times I_c$$

$$\therefore \frac{dT}{dQ} = -\frac{UR}{Q^2} + \frac{1}{2}\left(1-\frac{C}{P}\right)\times I_c$$

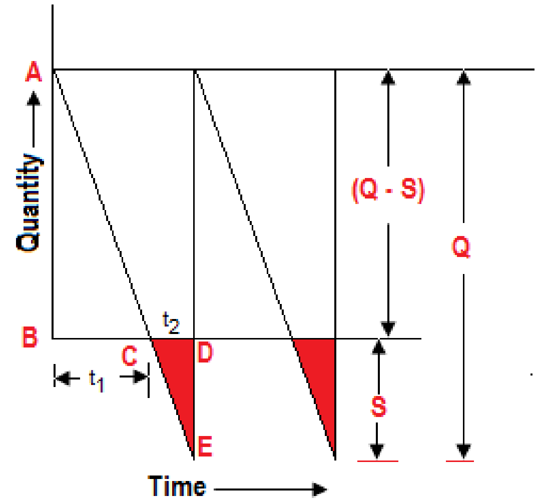
$$\therefore EBQ = \sqrt{\frac{2UR}{\left(1-\frac{C}{P}\right)I_c}}$$

In gradual replacement model if Buffer stock 'B' then Same EOQ formula

And Total cost (T) = $\frac{U}{Q}\times R + \frac{Q}{2}\left(1-\frac{C}{P}\right)I_c + B\times I_c$

Model-III

Inventory control for deterministic demand lead time zero, reordering allowed and shortages allowed



Let,
Q = Economic order quantity
S = Shortage
(Q – S) = Inventory remaining after backlog is satisfied
R = Cost of ordering
I_c = Annual cost of calming one unit for one year
I_p = Penalty for the shortage of one unit per year
t_1 = Stock replenishment time for zero inventory
t_2 = Backlog time
U = annual Demand

Number of cycle per year = $\dfrac{U}{Q}$

Time for 1 cycle ($t_1 + t_2$) = $\dfrac{Q}{U}$

Δ ABC & Δ CDE similar traingle

$$\frac{t_1}{Q-S} = \frac{t_2}{S} = \frac{t_1+t_2}{Q} = \frac{\frac{Q}{U}}{Q} = \frac{1}{U}$$

$$\therefore \quad t_1 = \frac{Q-S}{U} \quad \text{and} \therefore \quad t_2 = \frac{S}{U}$$

(i) Carrying cost per cycle

Average inventory = $\left(\dfrac{Q-S}{2}\right)$; Time = t_1 = $\left(\dfrac{Q-S}{U}\right)$

Cost = $\dfrac{Q-S}{2} \times t_1 \times I_c = \dfrac{Q-S}{2} \times \dfrac{Q-S}{U} \times I_c$ --------- (i)

(ii) Penalty per cycle

Average shortage = $\dfrac{S}{2}$, Time = t_2 = $\dfrac{S}{U}$

Penalty = $\dfrac{S}{2} \times t_2 \times I_p = \dfrac{S}{2} \times \dfrac{S}{U} \times I_p$ ----------- (ii)

Total cost per cycle = {(i) + (ii)} + Ordering cost per cycle

$$= \frac{(Q-S)^2}{2U} I_c + \frac{S^2}{2U} I_p + R$$

Annual cost (T) (Inventory) = Cost per cycle ×Number of cycle per year

$$= \left[\frac{(Q-S)^2}{2U} I_c + \frac{S^2}{2U} I_p + R \right] \times \frac{U}{Q}$$

$$= \frac{(Q-S)^2}{2Q} I_c + \frac{S^2}{2Q} I_p + \frac{RU}{Q}$$

$\frac{dT}{dQ} = 0$ gives

$$\frac{d}{dQ}\left[QI_c - 2SI_c + \frac{S^2}{Q} I_c + \frac{S^2}{Q} I_p + \frac{2RU}{Q} \right] = 0$$

$$\text{or} \quad I_c - 0 - \frac{S^2}{Q^2}\left(I_c + I_p\right) - \frac{2RU}{Q^2} = 0$$

$$\text{or,} \quad Q^2 = \frac{S^2\left(I_c + I_p\right) + 2RU}{I_c} \qquad (iii)$$

$\frac{dT}{dS} = 0$ gives

$$\frac{d}{dS}\left[QI_c - 2SI_c + \frac{S^2}{Q}\left(I_c + I_p\right) + \frac{2RU}{Q} \right]$$

$$\text{or} \quad 0 - 2I_c + \frac{2S}{Q}\left(I_c + I_p\right) = 0$$

$$\text{or,} \quad S = Q\left(\frac{I_c}{I_c + I_p}\right) \qquad (iv)$$

From (iii) and (iv), we get

$$Q^2 = Q^2\left(\frac{I_c}{I_c + I_p}\right)^2 \left(\frac{I_c + I_p}{I_c}\right) + \frac{2RU}{I_c}$$

$$or \; Q^2\left(1 - \frac{I_c}{I_c + I_p}\right) = \frac{2RU}{I_c}$$

$$\text{or } Q = \left(\sqrt{\frac{2RU}{I_c}}\right) \times \left(\sqrt{\frac{I_c + I_p}{I_p}}\right) = (Wilson's \text{ EOQ}) \times \sqrt{\frac{I_c + I_p}{I_p}}$$

$$= \text{Max}^{m} \text{ Inventry} = (Q-S)$$

First calculate Q and then calculate S and find (Q-S)

Total Optimal Cost (T_{opt})

$$= \sqrt{2URI_c \times \frac{I_p}{I_p + I_c}}$$

[VIMP Formula]

$$Q_o = \sqrt{\frac{I_c + I_p}{I_p}} \times \sqrt{\frac{P}{P-C}} \times \sqrt{\frac{2RU}{I_c}}$$

Units/run or units/procurement

Case-I: Infinite production rate (P = ∞) and Shortage allowed

$$Q = \sqrt{\frac{I_c + I_p}{I_p}} \times \sqrt{\frac{2RU}{I_c}}$$

Case-II: If shortage are not allowed ($I_p = \infty$)

$$Q = \sqrt{\frac{P}{P-C}} \times \sqrt{\frac{2RU}{I_c}}$$

Case-III: If P = α and $I_p = \alpha$

$$Q = \sqrt{\frac{2RU}{I_c}}$$

Model-IV (Inventory Model with Single Discount)

Order Quantity	Unit price
$1 \le Q \le M$	C
$Q \ge M$	$(1 - d) \times C = C'$

Method:

Step-I: Determine (EOQ)' with C'

Step-II: Cheek (EOQ)' >, = or < M
if (EOQ)' ≥M accept discount

ELSE go to next step

Step III:

Calculate

(i) $$T_{optimum} = UC + \frac{RU}{(EOQ)_{with\ C}} + I_c\left(\frac{EOQ}{2}\right)_{with\ C}$$

(ii) $$T_M = UC' + R\times\frac{U}{M} + I_c'\left(\frac{M}{2}\right)$$

[I_c = x% of C and I_c' = x% of C']

If $T_M < T_{optimum}$ then accept discount

Objective Questions (GATE, IES, IAS)

Previous 20-Years GATE Questions

EOQ Model

GATE-1. Setup costs do not include **[GATE-1997]**

(a) Labour cost of setting up machines
(b) Ordering cost of raw material
(c) Maintenance cost of the machines
(d) Cost of processing the work piece

GATE-2. There are two products P and Q with the following characteristics

Product	Demand (Units)	Order Cost (Rs/order)	Holding Cost (Rs./unit/year)
P	100	50	4
Q	400	50	1

The economic order quantity (EOQ) of products P and Q will be in the ratio **[GATE-2004]**

(a) 1: 1 (b) 1: 2 (c) 1: 4 (d) 1: 8

GATE-3. In inventory planning, extra inventory is unnecessarily carried to the end of the planning period when, using one of the following lot size decision policies: **[GATE-1998]**

(a) Lot-for-lot production
(b) Economic Order Quantity (EOQ) lot size
(c) Period Order Quantity (POQ) lot size
(d) Part Period total cost balancing

GATE-4. Market demand for springs is 8,00,000 per annum. A company purchases these springs in lots and sells them. The cost of making a purchase order is Rs.1,200. The cost of storage of springs is Rs.120 per stored piece per annum. The economic order quantity is: **[GATE-2003]**

(a) 400 (b) 2,828 (c) 4,000 (d) 8,000

GATE-5. An item can be purchased for Rs 100. The ordering cost is Rs. 200 and the inventory carrying cost is 10% of the item cost annum. If the annual demand is 4000 units, then economic order quantity (in units) is: **[GATE-2002]**

(a) 50 (b) 100 (c) 200 (d) 400

GATE-6. If the demand for an item is doubled and the ordering cost halved, the economic order quantity **[GATE-1995]**

(a) Remains unchanged (b) Increases by a factor of $\sqrt{2}$
(c) Is doubled (d) Is halved

GATE-7. **Annual demand for window frames is 10000. Each frame costs Rs. 200 and ordering cost is Rs. 300 per order. Inventory holding cost is Rs. 40 per frame per year. The supplier is willing to offer 2% discount if the order quantity is 1000 or more, and 4% if order quantity is 2000 or more. If the total cost is to be minimized, the retailer should** **[GATE-2010]**
(a) Order 200 frames every time (b) Accept 2% discount
(c) Accept 4% discount (d) Order Economic Order Quantity

GATE-8. **A company has an annual demand of 1000 units, ordering cost of Rs. 100/ order and carrying cost of Rs. 100/unit-year. If the stock-out costs are estimated to be nearly Rs. 400 each time the company runs out-of-stock, then safety stock justified by the carrying cost will be:** **[GATE-2004]**
(a) 4 (b) 20 (c) 40 (d) 100

GATE-9. **The maximum level of inventory of an item is 100 and it is 100 and it is achieved with infinite replenishment rate. The inventory becomes zero over one and half month due to consumption at a uniform rate. This cycle continues throughout the year. Ordering cost is Rs. 100 per order and inventory carrying cost is Rs. 10 per item per month. Annual cost (in Rs.) of the plan, neglecting material cost, is:** **[GATE-2007]**
(a) 800 (b) 2800 (c) 4800 (d) 6800

GATE-10. **In a machine shop, pins of 15 mm diameter are produced at a rate of 1000 per month and the same is consumed at a rate of 500 per month. The production and consumption continue simultaneously till the maximum inventory is reached. Then inventory is allowed to reduce to zero due to consumption. The lot size of production is 1000. If backlog is not allowed, the maximum inventory level is:** **[GATE-2007]**
(a) 400 (b) 500 (c) 600 (d) 700

GATE-11. **Consider the following data for an item.**
Annual demand: 2500 units per year Ordering cost: Rs. 100 per order Inventory holding rate: 25% of unit price.

Price quoted by a supplier

Order quantity (units)	Unit price (Rs.)
< 500	10
≥ 500	9

[GATE-2006]

The optimum order quantity ?
(a) 447 (b) 471 (c) 500 (d) $\geq$600

GATE-12. **In computing Wilson's economic lot size for an item, by mistake the demand rate estimate used was 40% higher than the tree demand rate. Due to this error in the lot size computation, the total cost of setups plus inventory holding per unit time. Would rise above the true optimum by approximately** **[GATE-1999]**
(a) 1.4 % (b) 6.3% (c) 18.3% (d) 8.7%

GATE-13. **One of the following statements about PRS (Periodic Reordering System) is not true. Identify** **[GATE-1998]**

(a) PRS requires continuous monitoring of inventory levels
(b) PRS is useful in control of perishable items
(c) PRS provides basis for adjustments to account for variations in demand
(d) In PRS, inventory holding costs are higher than in Fixed Recorder Quantity System

GATE-14. **The net requirements of an item over 5 consecutive weeks are 50-015-20-20. The inventory carrying costs are Re. 1 per item per week and Rs. 100 per order respectively. Starting inventory is zero. Use "Least Unit Const Technique" for developing the plan. The const of the plan (in Rs.) is: [GATE-2007]**
(a) 200 (b) 250 (c) 255 (d) 260

Previous 20-Years IES Questions

IES-1. **Which of the following are the benefits of inventory control?**
1. Improvement in customers relationship. [IES-2007]
2. Economy in purchasing.
3. Elimination of the possibility of duplicate ordering.
Select the correct answer using the code given below:
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 and 3 only

ABC Analysis

IES-2. **In ABC analysis, A items require:**
[IES-2005]
(a) No safety stock (b) Low safety stock
(c) Moderate safety stock (d) High safety stock

IES-3. **Classifying items in A, B and C categories for selective control in inventory management is done by arranging items in the decreasing order of: [IES-1995]**
(a) Total inventory costs (b) Item value
(c) Annual usage value (d) Item demand

IES-4. **Assertion (A): Selective control manages time more effectively.**
Reason (R): ABC analysis is based on Pareto distribution. [IES-2005]
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-5. **ABC analysis in materials management is a method of classifying the inventories based on [IES-2003]**
(a) The value of annual usage of the items
(b) Economic order quantity
(c) Volume of material consumption
(d) Quantity of materials used

IES-6. **Consider the following statements: [IES-1995]**
1. ABC analysis is based on Pareto's principle

2. **FIFO and LIFO policies can be used for material valuation in materials management.**
3. **Simulation can be l1sed for inventory control.**
4. **EOQ (Economic Order Quantity) formula ignores variations in demand pattern.**

Of these statements:

(a) 1 alone is correct (b) 1 and 3 are correct
(c) 2, 3 and 4 are correct (d) 1, 2, 3 and 4 are correct

IES-7. Out of the following item listed below, which two items you would consider under category (c) under ABC analysis: [IES-1992]

Annual Usage of items		
Items No.	**Annual usage × 1000**	**Unit cost Rs.**
A	**30**	**0.10**
B	**300**	**0.15**
C	**2**	**200.00**
D	**60**	**0.10**
E	**5**	**0.30**
F	**300**	**0.10**
G	**10**	**0.05**
H	**7**	**0.10**
I	**20**	**0.10**
J	**5**	**0.20**

(a) B and F (b) C and E (c) E and J (d) G and H

IES-8. In the ABC method of inventory control, Group A constitutes costly items. What is the usual percentage of such items of the total items? [IES-2006]

(a) 10 to 20% (b) 20 to 30% (c) 30 to 40 % (d) 40 to 50 %

IES-9. Which one of the following is correct? [IES-2008]
In the basic EOQ model, if lead time increases from 5 to 10 days, the EOQ will:

(a) Double (b) Decrease by a factor of two
(c) Remain the same (d) The data is insufficient to find EOQ

EOQ Model

IES-10. In the EOQ model, if the unit ordering cost is doubled, the EOQ

(a) Is halved (b) Is doubled **[IES-2007]**
(c) Increases 1.414 times (d) Decreases 1.414 times

IES-11. Economic Order Quantity is the quantity at which the cost of carrying is: [IES-2002]

(a) Minimum (b) Equal to the cost of ordering
(c) Less than the cost or ordering (d) Cost of over-stocking

IES-12. In the basic EOQ model, if demand is 60 per month, ordering cost is Rs. 12 per order, holding cost is Rs. 10 per unit per month, what is the EOQ? [IES-2008]

(a) 12 (b) 144 (c) 24 (d) 28

IES-13. **If the annual demand of an item becomes half, ordering cost double, holding cost one-fourth and the unit cost twice, then what is the ratio of the new EOQ and the earlier EOQ? [IES-2006]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 2

IES-14. **If demand is doubled and ordering cost, unit cost and inventory carrying cost are halved, then what will be the EOQ? [IES-2009]**

(a) Half (b) Same (c) Twice (d) Four times

IES-15. **Which one of the following is an inventory system that keeps a running record of the amount in storage and replenishes the stock when it drops to a certain level by ordering a fixed quantity? [IES-2006]**

(a) EOQ (b) Periodic (c) Peripheral (d) ABC

IES-16. **Match List-I with List-II and select the correct answer using the code given below the Lists: [IES-2007]**

List-I	List-II
A. Procurement cost	**1.** Cost of holding materials
B. Carrying cost	**2.** Cost of receiving order
C. Economic order quantity	**3.** Procurement lead time
D. Reorder point	**4.** Break-even analysis

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	1	4	2	(b)	3	4	1	2
(c)	2	1	4	3	(d)	2	4	1	3

IES-17. **There are two products A and B with the following characteristics product demand (in units), order cost (in Rs./order), holding cost (in Rs./unit/years) [IES-1994]**

A.	**100**	**100**	**4**
B.	**400**	**100**	**1**

The economic order quantities (EOQ) of product A and B will be in the ratio of:

(a) 1: 1 (b) 1: 2 (c) 1: 4 (d) 1 : 8

IES-18. **A shop owner with an annual constant demand of 'A' units has ordering costs of Rs. 'P' per order and carrying costs Rs. '1' per unit per year. The economic order quantity for a purchasing model having no shortage may be determined from [IES-2002]**

(a) $\sqrt{24P/AI}$ (b) $\sqrt{24AP/I}$ (c) $\sqrt{2AP/I}$ (d) $\sqrt{2AI/P}$

IES-19. **In inventory control theory, the economic order quantity (E.O.Q.) is:**

(a) Average level of inventory **[IES-1995]**
(b) Optimum lot size.
(c) Lot size corresponding to break-even analysis
(d) Capacity of a warehouse.

IES-20. **Consider the following costs: [IES-1999]**

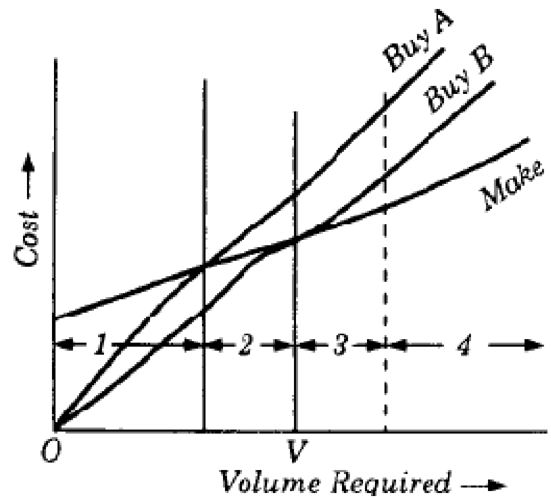
1. Cost of inspection and return of goods
2. Cost of obsolescence

3. Cost of scrap
4. Cost of insurance
5. Cost of negotiation with suppliers

Which of these costs are related to inventory carrying cost?

(a) 1,2 and 3 (b) 1, 3 and 4 (c) 2, 3 and 4 (d) 2, 4 and 5

IES-21. **Details of cost for make or buy decision are shown in the given graph. A discount is offered for volume of purchase above 'V'. Which one of the following ranges would lead to the economic decision?**



Buy A, B	Make
(a) 1 and 2	3 and 4
(b) 1 and 3	2 and 4
(c) 2 and 4	1 and 3
(d) 1 and 4	2 and 3

[IES-1998]

IES-22. **Which of the following cost elements are considered while determining the Economic Lot Size for purchase?** **[IES-1998]**

1. Inventory carrying cost 2. Procurement cost
3. Set up cost

Select the correct answer using the codes given below:

Codes: (a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-23. **Annual demand for a product costing Rs. 100 per piece is Rs. 900. Ordering cost per order is Rs. 100 and inventory holding cost is Rs. 2 per unit per year. The economic lot size is:** **[IES-1997]**

(a) 200 (b) 300 (c) 400 (d) 500

IES-24. **A furniture company is maintaining a constant work force which can produce 3000 tables per quarter. The annual demand is 12000 units and is distributed seasonally in accordance with the quarterly indexes Q_1 = 0.80, Q_2 = 1.40, Q_3 = 1.00 and Q_4 = 0.80. Inventories are accumulated when demand is less than the capacity and are used up during periods of strong demand to supply the total demand. To take into account any seasonal demand the inventories on hand at the beginning of the first quarter should be at least** **[IES-2003]**

(a) 0 (b) 600 (c) 1200 (d) 2400

IES-25. **Consider the data given in the following table:** **[IES-1997]**

Period	Demand	Production plan		
		Regular production	Overtime production	Others

1	500	500	–	–
2	650	650	–	
3	800	650	150	–
4	900	650	150	?

Give the fact that production in regular and overtime is limited to 650 and 150 respectively, the balance demand of 100 units in the 4th period can be met by

(a) Using overtime in period 2
(b) Using regular production in period 1
(c) Subcontracting
(d) Using any of the steps indicated in (a), (b) and (c)

Previous 20-Years IAS Questions

IAS-1. **Which one of the following correctly represents the average inventory turnover ratio for raw materials? [IAS-2003]**

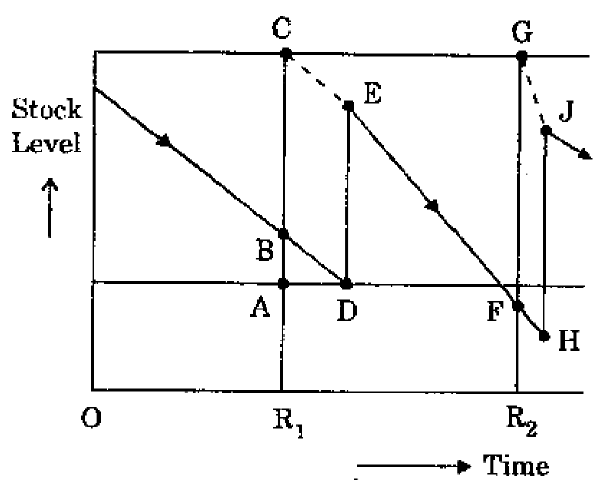
(a) Annual sales/annual inventory
(b) Average working process volume/total production volume
(c) Annual consumption / annual inventory
(d) Volume of spare parts/total annual sale

IAS-2. **The inventory carrying cost includes [IAS-1999]**

(a) Expenditure incurred for payment of bills
(b) Placing an order
(c) Receiving and inspecting
(d) Obsolescence and depreciation

EOQ Model

IAS-3. **The given figure shows the details of stock-level in the periodic review inventory control system. Match List-I (Characteristic) with List-II (Line) and select the correct answer using the codes given below the lists:**



List-I **List-II** **[IAS-2003]**

A. Lead time **1.** DE

B. Ordered quantity **2.** FH
C. Safety stock **3.** CG
D. Review period **4.** R1A
5. AD

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	4	2	5	(b)	5	1	4	3
(c)	3	1	4	5	(d)	5	4	2	3

IAS-4. **If orders are placed once a month to meet an annual demand of 6,000 units, then the average inventory would be: [IAS-1994]**
(a) 200 (b) 250 (c) 300 (d) 500

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1 Ans. (c)

GATE-2. Ans. (c) $(EOQ)_P = \sqrt{\frac{2AD}{H}} = \sqrt{\frac{2\times 50\times 100}{4}} = 50$

$$(EOQ)_Q = \sqrt{\frac{2AD}{H}} = \sqrt{\frac{2\times 50\times 400}{1}} = 200$$

$$\therefore \quad \frac{(EOQ)_P}{(EOQ)_Q =} = \frac{50}{200} = \frac{1}{4}$$

GATE-3. Ans. (b)

GATE-4. Ans. (c) $EOQ = \sqrt{\frac{2NA}{CI}}$

Where, $N = 8,00,000;$ $A = 1200$ Rsm; $CI = 120$ Rs/stored piece/annum

$$\therefore \; EOQ = \sqrt{\frac{2\times 8\times 10^5 \times 1200}{120}} = \sqrt{16\times 10^6} = 4000$$

GATE-5. Ans. (d) $q = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2\times 4000\times 200}{10}} = 400$ units

GATE-6. Ans. (a) $EOQ = \sqrt{\frac{2AD}{H}}$

where, A = Ordering cost; D = Demand; h = Unit holding cost

GATE-7. Ans. (c) U = 10000/per year; C = 200 Rs/ frame

R = 300 Rs/ per order; I_C = Rs 40 per year/ per item

$$\text{EOQ}' = \sqrt{\frac{2\text{RU}}{\text{I}_\text{C}}} = \sqrt{\frac{2\times 300\times 10000}{40}} = 387$$

Total cost with EOQ

Without discount

$$T = U.C + \frac{U}{EOQ}\times R + \frac{EOQ}{2}\times I_C$$

$$= 10000\times 200 + \frac{10000}{387}\times 300 + \frac{387}{2}\times 40 = 2,015,492$$

2 % Discount

$$T = 10000\times(200\times 0.98) + \frac{10000}{1000}\times 300 + \frac{1000}{2}\times 40 = 1,983,000/-$$

4 % Discount

$$T = 10000\times 200\times 0.96 + \frac{10000}{2000}\times 300 + \frac{1000}{2}\times 40 = 1,983,000/-$$

50 Accept 4%

GATE-8. Ans. (c) Given: D = 1000; Ordering cost, A = Rs. 100/order

Holding cost, H = Rs. 100/unit-year; Stock out cost, S = Rs. 400

$\therefore$ Optimum level of stock out $= \sqrt{\frac{2AD}{H}} \times \sqrt{\frac{S}{H+S}}$

$$= \sqrt{\frac{2 \times 100 \times 1000}{100}} \times \sqrt{\frac{400}{400+100}} = 40$$

GATE-9. Ans. (d)

GATE-10. Ans. (b) MAXIMUM INVENTORY $= (P - C)T_P = Q\left(1 - \frac{C}{P}\right)$

GATE-11. Ans. (c) Case I: Let EOQ is less than 500

$$\therefore \quad EOQ = \sqrt{\frac{2Q \times C_0}{C_c}} = 447.21$$

Case II: Let EOQ is greater than 500

$\therefore\ EOQ = \sqrt{\frac{2Q \times C_0}{C_c}} = 471.40$ which is against the assumption.

$\therefore\ EOQ = 447.21$

GATE-12. Ans. (c) $\therefore \quad EOQ = \sqrt{\frac{2RC_0}{C_c}}$

Inventory cost = $\sqrt{2RC_0C_c}$; Cost rise = $\sqrt{2 \times 1.4RC_0C_c} = 1.183\ \sqrt{2RC_0C_c}$

$\therefore$ Percentage increase = $\frac{1.183 - 1}{1} \times 100 = 18.3\%$ increase.

GATE-13. Ans. (a)
GATE-14. Ans. (b)

Previous 20-Years IES Answers

IES-1. Ans. (a)
IES-2. Ans. (b)
IES-3. Ans. (c)
IES-4. Ans. (b)
IES-5. Ans. (a)
IES-6. Ans. (d)
IES-7. Ans. (d)
IES-8. Ans. (a)
IES-9. Ans. (c)

IES-10. Ans. (c) EOQ = $\sqrt{\frac{2RU}{I_c}}$ if R$\uparrow$ 2 times EOQ will$\uparrow$ $\sqrt{2}$ times

IES-11. Ans. (b)
IES-12. Ans. (a)

IES-13. Ans. (d) $EOQ = \sqrt{\frac{2UR}{I_c}} = \sqrt{\frac{2UR}{I.C}}$

$$\Rightarrow \quad \frac{EOQ_2}{EOQ_1} = \sqrt{\left(\frac{U_2}{U_1}\right) \times \left(\frac{R_2}{R_1}\right) \times \left(\frac{T_1}{T_2}\right) \times \left(\frac{C_1}{C_2}\right)} = \sqrt{\left(\frac{1}{2}\right) \times 2 \times \left(\frac{4}{1}\right) \times \left(\frac{1}{2}\right)} = 2$$

IES-14. Ans. (c) $EOQ = \sqrt{\frac{2UR}{I_c}}$ if $U_2 = 2U;\ R_2 = \frac{R}{2}$

Here we have to think twice as carrying cost is halved therefore % of cost is halved. Again the unit cost is also halved therefore

$$I_{c2} = \frac{\%}{2} \times \frac{\text{Unit cost}}{2} = \frac{I_c}{4}; \qquad (EOQ)_2 = \sqrt{\frac{2(2U)\frac{R}{2}}{\frac{I_c}{4}}} = 2 \times EOQ$$

IES-15. Ans. (a)
IES-16. Ans. (c)
IES-17. Ans. (c) EOQ of A and B is in ratio of 1: 4 being

$$\sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \times \text{Order cost} \times \text{Demand}}{\text{Holding cost}}}$$

IES-18. Ans. (c)
IES-19. Ans. (b) In inventory control theory the economic order quantity is optimum lot size.
IES-20. Ans. (c)
IES-21. Ans. (a)
IES-22. Ans. (b)
IES-23. Ans. (b)
IES-24. Ans. (b)
IES-25. Ans. (b)

Previous 20-Years IAS Answers

IAS-1. Ans. (b)
IAS-2. Ans. (d)
IAS-3. Ans. (b)
IAS-4. Ans. (b)

Conventional Questions with Answer

Model-I (Deterministic Demand)

Conventional Question [ESE-2009]

What is the effect on order quantity when the demand increases by four-fold in basic order point inventory system and other factors remain unchanged? Explain.

[2-Marks]

Solutions:

Annual usage, U in Units per year

Carrying cost (I_c) in Rs./Unit

Ordering cost (R) in Rs.

Order quantity, EOQ = $\sqrt{\dfrac{2UR}{I_c}}$

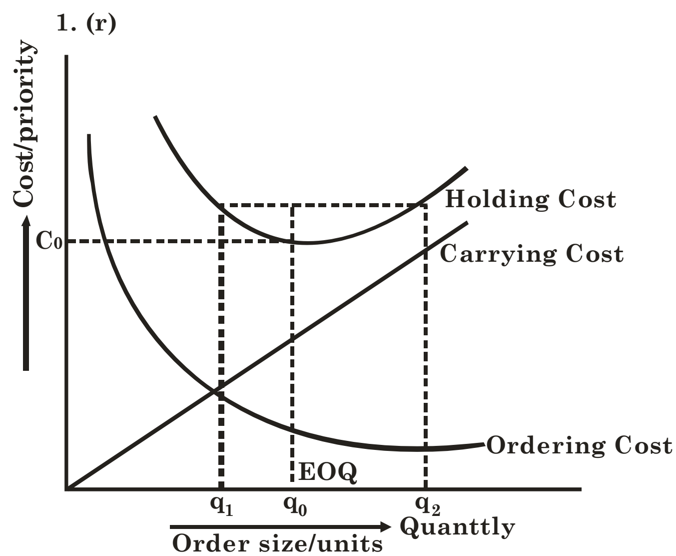
Thus, when U increased by four times, the EOQ (order quantity) will increase two times.

Conventional Question [ESE-2006]

With the help of quantity-cost curve, explain the significance of Economic Order Quantity (EOQ). What are the limitations of using EOQ formula?

[2 Marks]

Solution:



If the ordered quantity is $\mathbf{q_o}$, then the quantity is minimum i.e. $\mathbf{C_o}$. If the ordered quantity is less than EOQ or more than EOQ, then the W total cost will rise. The limitation of using EOQ formula lies in the assumption mode is continuous supply or one time supply etc.

Conventional Question [ESE]

The annual demand for an item is 3200 parts. The unit cost is Rs. 6 and the inventory carrying charges are estimated as 25% per annum. If the cost of one procurement is Rs. 150 find

(i) EOQ

(ii) Number of order per year

(iii) Time between two consecutive order

(iv) The optimal cost

Solution:

Given Annual usage, U = 3200 Units per year

Carrying cost (I_c) = 25% of units cost = 0.25 × 6 = 1.5 Rs./Unit

Ordering cost (R) = Rs. 150

(i) EOQ = $\sqrt{\frac{2UR}{I_c}} = \sqrt{\frac{2 \times 3200 \times 150}{1.5}}$ = 800 Units/order

(ii) Number of order per year = $\frac{3200}{800}$ = 4

(iii) Time between two consecutive order = ¼ year = 3 months.

(iv) Minimum inventory cost = $\sqrt{2URI_c} = \sqrt{2 \times 3200 \times 150 \times 1.5} = \text{Rs.}1200/-$

and Product cost = U × C = 3200 × 6 = Rs. 19200/–

Optimum cost = Rs. 20400/–

Conventional Question **[ESE-2002]**

The demand for a component is 10000 pieces per year. The cost per item is Rs. 50 and the interest cost is @ 1% per month. The cost associated with placing the order is Rs 240/-. What is the EOQ?

Solution:

Given U = 10,000

C = Rs. 50/- Per item

I_c = I × C = 1 × 12 × 50/100 = Rs. 6/ year / item

R = Rs. 240/–

$$(EOQ)_{th} = \sqrt{\frac{2UR}{I_c}} = \sqrt{\frac{2 \times 10000 \times 240}{6}} \simeq 895$$

∴ No of order placed = $\frac{10000}{895} \simeq 11.17$ times/year.

[**Note:** No of order placed may or may not be whole number]

Conventional Question **[ESE-1999]**

A company uses a certain component x at the rate of 5000/year. The cost/item is Rs.20/- and it costs Rs. 200/- to place an order. The annual carrying cost of inventory is 10% of the price of the item. Storage cost is negligible. Assuming zero safety stock calculate EOQ.

Solution:

U = 5000/year

C = Rs. 20/- per item

I_c = I × C = 0.1 × 20 = Rs. 2/– per item per year.

R = Rs. 200/– per procurement

∴ $(EOQ) = \sqrt{\frac{2RU}{I_C}} = \sqrt{\frac{2 \times 5000 \times 200}{2}} = 1000$ per procurement.

Conventional Question **[ESE-1995]**

A store sells 5200 cases of cold drinks per year. The supplies charge Rs100/- per each delivery regardless of how many cases have been ordered. Delivery always occurs the day after ordering and the average carrying cost Rs 10.40/- per/item/year. Find the number of cases per order.

Solution:

Given U = 5200/ Year
R = Rs. 100/ Year
I_c = 10.40 / item/ year

$$\therefore \qquad EOQ = \sqrt{\frac{2UR}{I_C}} = \sqrt{\frac{2\times 5200\times 100}{10.40}} \approx 317$$

Conventional Question [CSE-2002]

A material manager had recently attended a short training program on material management he thought of applying some of the optimization concept that he had learnt. He picked on one item BV1960, which was essentially a brass valve. From the current record he found that the average annual demand was 10000 valves. The accounting information system revealed that the carrying cost Rs. 0.40 per valve per year whereas the ordering cost was Rs. 5.50/- per order. The current policy adopted in the company was to order for 400 valves at a time. Is this an optimal policy? What would be the annual savings if the EOQ concept was applied?

Solution:

Given U = 10,000/ Year
R = Rs. 5.5/– per year
I_c = 0.40 / item/ year

$$\therefore \qquad EOQ = \sqrt{\frac{2UR}{I_C}} = \sqrt{\frac{2\times 5.50\times 10000}{0.40}} \approx 525 \text{ per order.}$$

So the current system is not EOQ i.e. it is not optimal.
For current system total inventory cost to the company (T)

$$\frac{U}{Q}\times R + \text{Average item } x \text{ inventory cost per year per item}$$

$$= \frac{10000}{400}\times 5.5 + \frac{400}{2}\times 0.4 = \text{Rs.}\,217.5 \text{ per year}$$

For EOQ Modal

$$T = \frac{10000}{525}\times 5.5 + \frac{525}{2}\times 0.4 = Rs.\,209.76 \text{ Per year.}$$

Savings per year = Rs. (217.5 – 209.76) = Rs. 7.74 per year.

Conventional Question [CSE-2001]

Explain the salient feature of the following inventory models

(i) Deterministic models
(ii) Probabilistic models
(iii) Model under uncertainty

In a deterministic model the ordering cost is Rs.4500/- per order. The cost of each item is Rs. 2500/- and carrying cost is 10% per year. If the annual demand is 10000 units determine EOQ. If the inventory carrying cost decrease by 10% and ordering cost increased by 10% then determine the charge of % in EOQ. What do you infer?

Solution:

Given Case-I:

U = 10000
R = 4500/per year
I_c = I × C = 0.1 × 2500 = 250 per unit /year

$$EOQ = \sqrt{\frac{2UR}{I_C}} = \sqrt{\frac{2\times10000\times4500}{250}}$$ = 600 per order.

Case-II:

U = 10000

R = 4500 × (1.1) = Rs. 4950 per year

I_c =250 × (1 – 0.1) Rs. 225 per unit /year

$$EOQ = \sqrt{\frac{2UR}{I_C}} = \sqrt{\frac{2\times10000\times4950}{225}} = 663$$

% increased

$$= \frac{(EOQ)_{IIth} - (EOQ)_{Ith}}{(EOQ)_{Ith}} \times 100\% = 10.55\%$$

* If inventory cost is decreased but same percent cost of procurement increased then also economic order quantity increased.

Q2. A company has to manufacture 150,000 brackets in a year. It orders raw material for the brackets in lots of 40000 units from a supplies. It cost Rs 40 to place an order and estimated inventory carrying cost, which is Rs 0.15. Calculate the variation in percentage in their order quantity from optimal, and what this variation cost. [20]

Solution:

Given U = 150,000/ year

R = Rs 40/- per procurement

I_c = Rs 0.15 per part per year

C = 0.15/0.2 = Rs. 0.75 per part

$$EOQ = \sqrt{\frac{2UR}{I_C}} = \sqrt{\frac{2\times150,000\times40}{0.15}} = 8945$$

%Variation from optimal cost = $\frac{Q_{company} - EOQ}{EOQ} \times 100 = 347.21\%$

Now, Company cost = $\frac{150000}{40000}\times40 + \frac{40000}{2}\times0.15 + 150000\times0.75 = \text{Rs.}115650/-$

Optimal cost = $\frac{150000}{8945}\times40 + \frac{8945}{2}\times0.15 + 150000\times0.75 = \text{Rs.}113842/-$

% Variation from optimal cost = $\frac{115650-113842}{113842}\times100 = 1.588\%$

Model-II (Gradual Replacement Model)

Conventional Question **[ESE-1994]**

In kelvinator produces refrigerator in batches. How many units in a batch should they produce? In each batch once the production starts they can make 80 units per day. The demand during the production period is 60 units per day. Estimated demand for the year is 10000 units. Set-up cost of the manufacturing process is 3000 per setup. Carrying cost is Rs. 15 unit per year.

Solution:

Given: U = 10000 units/year

R = Rs. 3000/- per setup

I_c = Rs. 15 per unit per year

P = 80 unit per day

C = 60 unit per day

$$\therefore \quad EBQ = \sqrt{\frac{2 \times 10000 \times 3000}{\left(1 - \frac{60}{80}\right) \times 15}} = 4000 \text{ per lot.}$$

Model-III

Inventory control for deterministic demand lead time zero, reordering allowed and shortages allowed

Conventional Question **[ESE-2000]**

ABC Company has to supply 30000 switches per year to its consumer. This demand is fixed and known. The customer uses its item in assembly operation and has no storage space. A shortage cost is Rs10/- is incurred if the company fails to deliver the required units. The set-up cost per Run is Rs 3500/-

Determine

(i) The optimum Run size Q

(ii) The optimum level of inventory at the beginning of any period?

(iii) The optimum scheduling period

(iv) The minimum total expected annual cost

[*Note: If Penalty cost is not given then we will assume that I_p = 3 to 5 times of I_c.*]

Solution:

[In this problem I_p is given Rs. 10/- so let us assume I_c= 2.5 per unit per year]

U = 30000 per year

I_c = Rs 2.5/ assume per unit

R = Rs 3500/-

(i) Optimum Run size (Q) $= \sqrt{\frac{2UR}{I_c} \times \left(\frac{I_c + I_p}{I_p}\right)}$

$$= \sqrt{\frac{2 \times 30000 \times 3500}{2.5} \times \frac{(2.5 + 10)}{10}} = 10247 \text{ per lot}$$

(ii) Shortage (S)

$$Q \times \frac{I_c}{I_c + I_p} = 2050 \text{ units}$$

Therefore Optimum level of inventory at the beginning of any period=(Q-S) = 8198 Units

(iii) The optimum Scheduled period = $\frac{Q}{U}$ year $= \frac{10247}{30000} \approx 125 \text{ days}$

(iv) Optimum cost (Inventory cost) $= \sqrt{2RUI_c \times \frac{I_p}{I_p + I_c}}$

$$= \sqrt{2 \times 3500 \times 2.5 \times \frac{10}{10 + 2.5}} = \text{Rs. } 20499/-$$

Conventional Question **[CSE-1998]**

The demand for an item in a company is 18000 units per year. The company can produce this item @ 3000 units per months. The cost of one setup is Rs 50/- The

holding a is Rs 0.15 per unit per month. The shortage cost of one units is Rs 20/- per year determine

(i) Economic production Quantity
(ii) No of shortage permited
(iii) The manufacturing time
(iv) Time between set-up and maximum inventory level.

Solution:

U = 18000 unit/year;
P = 3000 unit/ month
R = 50 per setup;
I_c = Rs. 0.15 × 12 = Rs. 1.8; and I_p = Rs. 20/-

(i) $$Q_o = \sqrt{\frac{1.8+20}{20}} \times \sqrt{\frac{3000\times 12}{3000\times 12 - 18000}} \times \sqrt{\frac{2\times 50\times 18000}{1.8}} \approx 1477$$

Model-IV (Inventory Model with Single Discount)

Conventional Question [ESE-2008]

Name the three costs involved in inventory control. A store procures and sells certain items. Information about an item is as follows:
Expected annual sales = 8000 units
Ordering cost = Rs. 1,800 per order
Holding cost = 10% of average inventory value
The items can be purchased according to the following schedule:

Lot size	**Unit price (Rs)**
1 - 999	220
1000 - 1499	200
1500 - 1999	190
2000 and above	185

Determine the best order size. **[10-Marks]**

Solution: Three costs involved in inventory control are:

(i) **Ordering Cost (C_o):** This represents the expenses involved in placing an order with the outside supplier. This occurs whenever inventory is replenished. It is expressed as cost in rupee per order.

(ii) **Carrying Cost (C_c)**: This represents the cost of holding and storage of inventory. It is proportional to the amount of inventory and time over which it is held. It consists of cost involved in:

- Storage and handling
- Interest on funds tied up in inventory
- Insurance
- Obsolescence and deterioration
- Stock and record keeping

Carrying cost is expressed as cost per unit time. This is also expressed as a % of average annual investment in inventory.

$\therefore \mathbf{C_c = C_u I}$

Where $\mathbf{C_u}$ = unit cost

(iii) **Unit cost(C_u)**: It refers to the nominal cost of the inventory item per unit. It is the purchase price of the item. If it is bough from outside. It is bought from outside. It is the production if the item is produced within the organisation. It is expressed as Rupees per unit.

R = annual sales = 8000 unit

C_o = Ordering cost = Rs. 1800/order

C_c = Carrying cost or holding cost = 10% of average inventory value

Lot size	unit price (Rs)	Type
1 – 999	220	I
1000 – 1499	200	II
1500 – 1999	190	III
2000 and above	185	IV

Case I

Let Q = economic order quantity

$$= \sqrt{\frac{2RC_o}{C_c}} = \sqrt{\frac{2 \times 8000 \times 1800}{0.1 \times 220}} = 1144$$

Which is more than 999

Hence Q = 999

Ordering cost $= \frac{8000}{999} \times 1800 = \text{Rs.}14414.414$

Average inventory cost $= \frac{999}{2} \times 0.1 \times 220 = \text{Rs.}10989$

Total inventory cost $= 14414.414 + 10989$

$= \text{Rs.}25403.414$

$(\text{T.C})_\text{I} = 25403.414 + 220 \times 8000$

$\therefore (\text{T.C})_\text{I} = \text{Rs.}1785403.414$ and $(\text{EOQ})_\text{I} = 999$

Case II

$$Q = \sqrt{\frac{2 \times 8000 \times 1800}{0.1 \times 200}} = 1200$$

Which lies on range

Total inventory cost $= \frac{8000}{1200} \times 1800 + \frac{1200}{2} \times 0.1 \times 200 = \text{Rs.}24000$

$(\text{T.C})_\text{II} = 24000 + 200 \times 8000 = \text{Rs.}1624000$

$\therefore (\text{T.C})_\text{II} = \text{Rs.}1624000$ and $(\text{EOQ})_\text{II} = 1200$

Case III

$$Q = \sqrt{\frac{2 \times 8000 \times 1800}{0.1 \times 190}} = 1231$$

Which does not lies within range

$\therefore (\text{EOQ})_\text{III} = 1500$

Total inventory cost $= \frac{8000}{1500} \times 1800 + \frac{1500}{2} \times 0.1 \times 190$

$= \text{Rs.}23850$

$(\text{T.C})_\text{III} = 23850 + 190 \times 8000 = \text{Rs.}1543850$

$\therefore (\mathbf{T.C})_{\mathbf{III}} = \mathbf{Rs.1543850}$ and $(\mathbf{EOQ})_{\mathbf{III}} = \mathbf{1500}$

Case IV

$$\mathbf{Q} = \sqrt{\frac{\mathbf{2 \times 8000 \times 1800}}{\mathbf{0.1 \times 185}}} = \mathbf{1248}$$

Does not lies within range

$\therefore (\mathbf{EOQ})_{\mathbf{IV}} = \mathbf{2000}$

Total inventory cost $= \dfrac{\mathbf{8000}}{\mathbf{2000}} \times \mathbf{1800} + \dfrac{\mathbf{2000}}{\mathbf{2}} \times \mathbf{0.1} \times \mathbf{185}$

= Rs. 25700

$(\mathbf{T.C})_{\mathbf{IV}} = \mathbf{25700 + 185 \times 8000 = Rs.1505700}$

$\therefore (\mathbf{T.C})_{\mathbf{IV}} = \mathbf{Rs.1505700}$ and $(\mathbf{EOQ})_{\mathbf{IV}} = \mathbf{2000}$

Hence best lot size is of 2000

Conventional Question [ESE-2004]

For XYZ Company, the annual requirement of an item is 2400 units. Each item cost the company Rs 6/-. The supplies offer a discount of 5% if 500 or more quantity is purchased. The ordering cost is Rs. 32/- per order and the average inventory cost is 16%. Is it advisable to accept the discount? Comment on the result.

Solution:

Order Quantity	**Unit price**
$Q < 500$	C = Rs. 6
$Q \geq 500$	Rs. 6 × (1 – 5/100) = Rs. 5.70/–

R = Rs. 32/–; U = 2400; $I_c = 0.16 \times 6 = 0.96$; $I_c = 0.16 \times 5.7 = 0.912$

Step-I: $EOQ' = \sqrt{\dfrac{2RU}{I_c'}} = \sqrt{\dfrac{2 \times 32 \times 2400}{0.912}} = 410; E.O.Q. = \sqrt{\dfrac{2 \times 32 \times 2400}{0.96}} = 400$

Step-II: $T_{optimum} = 2400 \times 6 + 32 \times \dfrac{2400}{400} + \dfrac{400}{2} \times 0.96 = Rs.14784/-$

Step-II: $T_{500} = 2400 \times 5.7 + 32 \times \dfrac{2400}{500} + \dfrac{500}{2} \times 0.912 = Rs.14061.6/-$

So, I advised to *XYZ* Company to accept the offer. Because it will save Rs. 722.4 per year.

Conventional Question [GATE-2000]

A Company places order for supply of two items A and B. The order cost for each of the items is Rs. 300/- per order. The inventory carrying cost is 18% of the unit price per year per unit. The unit prices of the items are Rs. 40 and Rs. 50 respectively. The annual demand are 10000 and 20000 respectively (a) Find the EOQ. (b) Supplier is willing to give the 1% discounts if both the item is ordered from him and if the order quantities for each item is ordered from him and if the order quantities for each item or 1000 unit or more. Is it profitable to avail the discount?

Solution:

A		**B**	
U = 10000;	R = 300	U = 20000;	R = 300
C = 40;	$I_c = 0.18 \times 40 = 7.2$	C = 50;	$I_c = 0.18 \times 50 = 9$

$$EOQ = \sqrt{\frac{2RU}{I_c}} = \sqrt{\frac{2 \times 10000 \times 300}{7.2}} = 913 \qquad EOQ = \sqrt{\frac{2RU}{I_c}} = \sqrt{\frac{2 \times 20000 \times 300}{9}} = 1155$$

$$T = 10000 \times 40 + \frac{10000}{913} \times 300 + \frac{913}{2} \times 7.2 = 406573 \qquad T = 20000 \times 50 + \frac{20000}{1155} \times 300 + \frac{1155}{2} \times 9 = 1010392$$

Total cots = Rs. 1,416,965/–

If the discount is accepted then

A	B
$C_A' = 0.99C = 39.6$	$C_B' = 0.99 \times 50 = 49.5$
$I_{CA}' = 0.18 \times C_A' = 7.128$	$I_{CB}' = 0.18 \times C_B' = 8.91$
$(EOQ')_A = \sqrt{\frac{2 \times 10000 \times 300}{7.182}} = 914$	$(EOQ')_B = \sqrt{\frac{2 \times 20000 \times 300}{8.91}} = 1161$

If discount accepted then $Q_A = 1000$ & $Q_B = 1161$ per order

$$T_M = U_A C'_A + \frac{U_A}{Q_A} \times R_A + \frac{M}{2} \times I'_{CA} + U_B C_B' + \frac{U_B}{(EOQ')} \times R_B + \frac{EOQ'}{2} \times I'_{CB}$$

$$= 10000 \times 39.6 + \frac{10000}{1000} \times 300 + 500 \times 7.128 + 2000 \times 49.5 + \frac{20000 \times 300}{1161} + \frac{1161}{2} \times 891$$

$$= 1402904 / -$$

If discount accepted saving per year = $T - T_M$ = 1416,965 – 1402904 = 14,061/–

7. MRP

Theory at a Glance (For IES, GATE, PSU)

MRP System

An MRP system has three major input components:

- **Master Production Schedule (MPS):** MPS is designed to meet the market demand (both the firm orders and forecasted demand) in future in the taken planning horizon. MPS mainly depicts the detailed delivery schedule of the end products. However, orders for replacement components can also be included in it to make it more comprehensive.
- **Bill of Materials (BOM):** BOM represents the product structure. It encompasses information about all sub components needed, their quantity, and their sequence of buildup in the end product. Information about the work centers performing buildup operations is also included in it.
- **Inventory Status File:** Inventory status file keeps an up-to-date record of each item in the inventory. Information such as, item identification number, quantity on hand, safety stock level, quantity already allocated and the procurement lead time of each item is recorded in this file.

After getting input from these sources, MRP logic processes the available information and gives information about the following:

- **Planned Orders Receipts:** This is the order quantity of an item that is planned to be ordered so that it is received at the beginning of the period under consideration to meet the net requirements of that period. This order has not yet been placed and will be placed in future.
- **Planned Order Release:** This is the order quantity of an item that is planned to be ordered and the planned time period for this order that will ensure that the item is received when needed. Planned order release is determined by offsetting the planned order receipt by procurement lead time of that item.
- **Order Rescheduling:** This highlight the need of any expediting, de-expediting, and cancellation of open orders etc. in case of unexpected situations.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Materials Requirement Planning

GATE-1. In an MRP system, component demand is: [GATE-2006]
(a) Forecasted
(b) Established by the master production schedule
(c) Calculated by the MRP system from the master production schedule
(d) Ignored

GATE-2. For planning the procurement or production of dependent demand items, the technique most suitable is................. (MRP/EOQ) [GATE-1995]

Previous 20-Years IES Questions

Materials Requirement Planning

IES-1. The proper sequence of activities for material requirement planning is: [IES-2002]
(a) Master production schedule, capacity planning, MRP and order release
(b) Order release, master production schedule, MRP and capacity planning
(c) Master production schedule, order release, capacity planning and MRP
(d) Capacity planning, master production schedule, MRP and order release

IES-2. Match List-I (Files in MRP) with List-II (Inputs required) and select the correct answer: [IES-2002]

List-I	List-II
A. Master production schedule	**1.** Scheduled receipts
B. Bills of materials	**2.** Units costs and discounts
C. Inventory records	**3.** Production capacity
	4. Product structure

Codes:	**A**	**B**	**C**		**A**	**B**	**C**
(a)	4	1	3	(b)	3	4	2
(c)	3	4	1	(d)	4	3	1

IES-3. Which one of the following is not a necessary information input to Material Requirements Planning? [IES-2006]
(a) Inventory on hand
(b) Bill of materials
(c) Sequence of operations on a job
(d) Master production schedule (MPS)

IES-4. Assertion (A): Master production schedule drives the whole of production and inventory control system in a manufacturing organization. [IES-2000]

Reason (R): Master production schedule is a list of daily and weekly work released by PPC to production.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-5. Which of the following input data are needed for MRP? [IES-1998]
1. Master production schedule 2. Inventory position
3. Machine capacity 4. Bill of materials
Select the correct answer using the codes given below:
Codes: (a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 2 and 4 (d) 1, 3 and 4

IES-6. Consider the following: [IES-2009]
1. A master production schedule
2. An inventory status file
3. Bill of material
Which of the above are the inputs to MRP systems?
(a) 1, 2 and 3 (b) 1 and 2 only (c) 2 and 3 only (d) 1 and 3 only

IES-7. Match List-I (Production control function) with List-II (Explanation) and select the correct answer using the code given below the lists: [IES-2005]

List I	List II
A. Bill of Materials (BOM)	**1.** A technique for determining the quantity and timing of dependent demand items
B. Capacity Resource Planning (CRP)	**2.** A technique for determining personnel and equipment capacities needed to meet the production objective
C. Material Requirement Planning (MRP)	**3.** Specifies what end items are to be produced and when
D. Master Production Schedule (MPS)	**4.** The part numbers & quantity required per assembly

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	4	1	2	3	(b)	3	2	1	4
(c)	4	2	1	3	(d)	3	1	2	4

IES-8. Which of the following are needed as the input data for materials requirement planning? [IES-2005]
1. Weekly production schedule 2. Bill of material
3. Supplier lead time 4. Market forecast
Select the correct answer using the code given below:
(a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1 and 4 (d) 1, 2, 3 and 4

Job Design

IES-9. Assertion (A): In job design, instead of each job consisting of a single task, a large group of tasks are clustered for a job holder.

Reason (R): A single job should encompass not only production tasks but also the set up, scheduling and control tasks related to the operation. [IES-1998]
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-10. A process of discovering and identifying the pertinent information relating to the nature of a specific job is called [IES-1999]
(a) Job identification (b) Job description
(c) Job analysis (d) Job classification

Job Standards

IES-11. Assertion (A): Job enrichment increases the job satisfaction of the employee. [IES-2002]
Reason (R): The jobs of wireman and lineman doing indoor and outdoor works respectively can be integrated for better results.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-12. Procedure of modifying work content to give more meaning and enjoyment to the job by involving employees in planning, organization and control of their work, is termed as [IES-1996]
(a) Job enlargement (b) Job enrichment
(c) Job rotation (d) Job evaluation

Previous 20-Years IAS Questions

Job Standards

IAS-1. A systematic job improvement sequence will consist of: [IAS-1994]
(i) Motion Study (ii) Time Study
(iii) Job Enrichment (iv) Job Enlargement
An optimal sequence would consist of:
(a) i, ii, iii and iv (b) ii, i, iii and iv
(c) iii, i, ii and iv (d) iii, iv, i and ii

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (c) In a MRP system, component demand is calculated by the MRP system from the Master Production Schedule.

GATE-2. Ans. MRP

Previous 20-Years IES Answers

IES-1. Ans. (a)
IES-2. Ans. (c)
IES-3. Ans. (c)
IES-4. Ans. (c)
IES-5. Ans. (c)
IES-6. Ans. (a)
IES-7. Ans. (d)
IES-8. Ans. (d)
IES-9. Ans. (b)
IES-10. Ans. (b)
IES-11. Ans. (c)
IES-12. Ans. (b)

Previous 20-Years IAS Answers

IAS-1. Ans. (a)

Conventional Questions with Answer

Conventional Question - IES 2010

Question: Distinguish between material requirements planning and manufacturing resource planning. **[2 Marks]**

Answer:

Material Requirement Planning	Manufacturing Resource Planning
1. It uses information about end product demands, product structure and component requirement. **2.** Purchase lead's time and current inventory product & purchasing schedule.	Manufacturing resource planning evolved from material requirement planning to integrate other functions in planning process. These functions may include engineering, marketing, purchasing, production scheduling business planning and finance.

Conventional Question - IES 2008

Question: Is Material Requirement planning a material planning system, a production planning system or both? Explain. **[2 Marks]**

Answer: Material requirement planning (MRP) is used for material requirement and production planning. Material requirement planning (MRP) is a computational technique that converts the master schedule for end products into a detailed schedule for the raw materials and components used in the end products. The detailed schedule identifies the quantities of each raw material and component item. It also indicates when each item must be ordered and delivered so as to meet the master schedule for final products.

MRP is often though of as a method of inventory control. While it is an effective tool for minimizing unnecessary inventory investment. MRP is also useful in production scheduling and purchasing of materials.

The concept of MRP is relatively straight forward. What complicates the application of the technique is the sheer magnitude of the data to be processed. The master schedule provides the overall production plan for the final products in terms of month-by-month deliveries. Each of the products may contain hundreds of individual components. These components are produced from raw materials, some of which are common among the components. For example, several components may be made out of the same sheet steel. The components are assembled into simple subassemblies and these subassemblies are put together into more complex subassemblies and so on until the final products are assembled. Each step in the manufacturing and assembly sequence takes time. All of these factors must be incorporated into the MRP calculations. Although each calculation is uncomplicated, the magnitude of the data is so large that the application of MRP is virtually impossible unless carried out on a digital computer.

8. Work Study and Work Measurement

Theory at a Glance (For IES, GATE, PSU)

Definition: Work study may be defined as the analysis of a job for the purpose of finding the preferred method of doing it and also determining the standard time to perform it by two areas of study, method study **(motion study)** and time study **(work measurement).**

Role of Work Study in Improving Productivity

In order to understand the role of work study, we need to understand the role of method study and that of time study.

Method study (also sometimes called Work Method Design) is mostly used to improve existing method of doing work although it is equally well applicable to new jobs. When applied to existing jobs, method study aims to find better methods of doing the jobs that are economical and safe, require less human effort, and need shorter make-ready / put-away time. The better method involves the optimum use of best materials and appropriate manpower so that work is performed in well, organized manner leading to utilization, better quality and lower costs.

We can therefore say that through method study we have a systematic way of developing human resource effectiveness, providing high machine and equipment utilization, and making economical use of materials.

Time study, on the other hand, provides the standard time, that is the time needed by worker to complete a job by the specified method. Therefore for any job, the method of doing it is first improved by method study, the new method is implemented as a standard practice and for that job to be done by the new method, and standard time is established by the use of time study. Standard times are essential for any organization, as they are needed for proper estimation of:

- Manpower, machinery and equipment requirements.
- Daily, weekly or monthly requirement of materials.
- Production cost per unit as an input to selling price determination.
- Labur budgets.
- Worker's efficiency and make incentive wage payments.

By the application of method study and time study in any organization, we can thus achieve greater output at less cost and of better quality, and hence achieve higher productivity.

Motion Study and Motion Economy

Motion study is a technique of analyzing the body motions employed in doing a task in order to eliminate or reduce ineffective movements and facilitates effective movements. By using motion study and the principles of motion economy the task is redesigned to be more effective and less time consuming.

The **Gilbreths** pioneered the study of manual motions and developed basic laws of motion economy that are still relevant today. They were also responsible for the development of detailed motion picture studies, termed as *Micro Motion Studies*, which are extremely useful for analyzing *highly repetitive manual* operations. With the improvement in technology, of course, video camera has replaced the traditional motion picture film camera.

In a broad sense, motion study encompasses micro motion study and both have the same objective: job simplification so that it is less fatiguing and less time consuming while motion study involves a simple visual analysis, micro motion study uses more expensive equipment. The two types of studies may be compared to viewing a task under a magnifying glass versus viewing the same under a microscope. The added detail revealed by the microscope may be needed in exceptional cases when even a minute improvement in motions matters, i.e. on extremely short repetitive tasks.

Taking the cine films @ 16 to 20 frames per second with motion picture camera, developing the film and analyzing the film for micro motion study had always been considered a costly affair. To save on the cost of developing the film and the cost of film itself, a technique was used in which camera took only 5 to 10 frames per minute. This saved on the time of film analysis too. In applications where infrequent shots of camera could provide almost same information, the technique proved fruitful and acquired the name *Memo Motion Study*.

Traditionally, the data from micro motion studies are recorded on a ***Simultaneous Motion (simo) Chart*** while that from motion studies are recorded on a ***Right Hand - Left Hand Process Chart.***

Therbligs

As result of several motion studies conducted Gilbreths concluded that any work can be done by using a combination of 17 basic motions, called Therbligs (Gilbreth spelled backward). These can be classified as effective therbligs and ineffective therbligs. Effective therbligs take the work progress towards completion. Attempts can be made to shorten them but they cannot be eliminated. Ineffective therbligs do not advance the progress of work and therefore attempts should be made to eliminate them by applying the ***Principles of Motion Economy.***

SIMO Chart

It is a graphic representation of the sequence of the therbligs or group of therbligs performed by body members of operator. It is drawn on a common time scale. In other words, it is a two-hand process chart drawn in terms of therbligs and with a time scale. A video film or a motion picture film is shot of the operation. The film is analyzed frame by frame. For the left hand, the sequence of therbligs (or group of therbligs) with their time values are recorded on the column corresponding to the left hand. The symbols are added against the length of column representing the duration of the group of therbligs. The procedure is repeated for the right and other body members (if any) involved in carrying out the operation.

It is generally not possible to time individual therbligs. A certain number of therbligs may be grouped into an element large enough to be measured.

Uses of SIMO Chart

From the motion analysis shown about the motions of the two hands (or other body members) involved in doing an operation, inefficient motion pattern can be identified and any violation of the principle of motion economy can be easily noticed. The chart, therefore, helps in improving the method of doing the operation so that balanced two-handed actions with coordinated foot and eye motions can be achieved and ineffective motion can be either reduced or eliminated. The result is a smoother, more rhythmic work cycle that keeps both delays and operator fatigue to the minimum extent.

Cycle Graph and Chrono Cycle Graph

These techniques of analyzing the paths of motion made by an operator were developed by the Gilbreths. To make a cycle graph, a small electric bulb is attached to the finger, hand, or any other part of the body whose motion is to be recorded. By using still Photography, the path of light of bulb (in other words, that of the body member) as it moves through space for one complete cycle is photographed by keeping the working area relatively less illuminated. More than one camera may be used in different planes to get more details. The resulting picture (cycle graph) shows a permanent record of the motion pattern employed in the form of a closed loop of white continuous line with the working area in the background. A cycle graph does not indicate the direction or speed of motion.

It can be used for

- Improving the motion pattern and
- Training purposes in that two cycle graphs may be shown with one indicating a better motion pattern than the other.

The Chrono cycle graph is similar to the cycle graph, but the power supply to the bulb is interrupted regularly by using an electric circuit. The bulb is thus made to flash. The procedure for taking photograph remains the same. The resulting picture (Chrono cycle graph), instead of showing continuous line of motion pattern, shows short dashes of line spaced in proportion to the speed of the body member photographed. Wide spacing would represent fast moves while close spacing would represent slow moves. The jumbling of dots at one point would indicate fumbling or hesitation of the body member. A chrono cycle graph can thus be used to study the motion pattern as well as to compute velocity, acceleration and retardation experienced by the body member at different locations.

The world of sports has used this analysis tool, updated to video, for extensively the purpose of training in the development of from and skill.

Work Measurement (Time study)

Work measurement refer to the estimation of standard time, that is the time allowed for completing one piece of job using the given method. This is the time taken by an average experienced worker for the job with provisions for delays beyond the workers control.

Definition: Time study is a technique to estimate the time to be allowed to a qualified and well-trained worker working at a normal pace to complete a specified task.
There are several techniques used for estimation of standard time in industry. These include time study, work sampling, standard data, and predetermined time systems.

Application

Standard times for different operations in industry are useful for several applications like

- Estimating material machinery and equipment requirements.

- Estimating the production cost per unit as an input to
    - ❖ Preparation of budgets
    - ❖ Determination of selling price
    - ❖ Make or buy decision
- Estimating manpower requirements.
- Estimating delivery schedules and planning the work
- Balancing the work of operators working in a group.
- Estimating performance of workers and use as basis for incentive payment to those direct and in director labur who show greater productivity.

Time Study Procedure

The procedure for time study can best be described step-wise, which are self explanatory.

Step 1: **Define** objective of the study. This involves statement of the use of the result, the precision desired, and the required level of confidence in the estimated time standards.

Step 2: **Analyse** the operation to determine whether standard method and conditions exist and whether the operator is properly trained. If need is felt for method study or further training of operator, the same may be completed before starting the time study.

Step 3: **Select** Operator to be studied if there is more than one operator doing the same task.

Step 4: **Record** information about the standard method, operation, operator, product, equipment, quality and conditions.

Step 5: **Divide** the operation into reasonably small elements.

Step 6: **Time** the operator for each of the elements. Record the data for a few number of cycles. Use the data to estimate the total numbers of observations to be taken.

Step 7: Collect and record the data of required number of cycles by timing and rating the operator.

Step 8: For each element **calculate** the representative watch time. Multiply it by the rating factory to get normal time.

Normal time = Observed time * Rating factor

Add the normal time of various elements to obtain the normal time for the whole operation.

Step 9: Determine allowances for various delays from the company's policy book or by conducting an independent study.

Step 10: Determine standard time by adding allowances to the normal time of operation.

Standard time = Normal time + allowances

Time Study Equipment

The following equipment is needed for time study work.

- Timing device
- Time study observation sheet
- Time study observation board
- Other equipment

Timing Device

The stop watch and the electronic timer are the most widely used timing devices used for time study. The two perform the same function with the difference that electronics timer can measure time to the second or third decimal of a second and can keep a large volume of time data in memory.

Time Study Observation Sheet

It is a printed form with space provided for nothing down the necessary information about the operation being studied like name of operation, drawing number, name of the operator, name of time study person, and the date and place of study. Space are provided in the form for writing detailed description of the process (element-wise), recording stop-watch readings for each element of the process, performance rating(s) of the operator, and computation Figure 2 Shows a typical time study observation sheet.

Time Study Board

It is a light -weight board used for holding the observation sheet and stopwatch in position. It is of size slightly larger than that of observation sheet used. Generally, the watch is mounted at the center of the top edge or as shown in Figure 3 near the upper right-hand corner of the board. The board has a clamp to hold the observation sheet. During the time study, the board is held against the body and the upper left arm by the time study person in such a way that the watch could be operated by the thumb/index finger of the left hand. Watch readings are recorded on the observation sheet by the right hand.

Other Equipment

This includes pencil, eraser and device like tachometer for checking the speed, etc.

Normal Performance

There is no universal concept of Normal Performance. However, it is generally defined as the working rate of an average qualified worker working under capable supervision but not under any incentive wage payment scheme. This rate of working is characterized by the fairly steady exertion of reasonable effort, and can be maintained day after day without undue physical or mental fatigue.

The level of normal performance differs considerably from one company to another. What company a calls 100 percent performance, company B may call 80 percent, and company C may call 125 percent and so on. It is important to understand that the level that a company selects for normal performance is not critical but maintaining that level uniform among time study person and constant with the passage of time within the company is extremely important.

There are, of course, some universally accepted benchmark examples of normal performance, like dealing 52 cards in four piles in 0.5 minute, and walking at 3 miles per hour (4.83 km/hr). In order to make use of these benchmarks, it is important that a complete description about these be fully understood, like in the case of card dealing, what is the distance of each pile with respect to the dealer, technique of grasping, moving and disposal of the cards.

Some companies make use of video films or motion pictures for establishing what they consider as normal speed or normal rate of movement of body members. Such films are made of typical factory jobs with the operator working at the desired normal pace. These films are reported to be useful in demonstrating the level of performance expected from the operators and also for training of time study staff.

Performance Rating

During the time study, time study engineer carefully observes the performance of the operator. This performance seldom conforms to the exact definition of normal or standard.

Therefore, it becomes necessary to apply some 'adjustment' to the mean observed time to arrive at the time that the normal operator would have needed to do that job when working at an average pace. This 'adjustment' is called **Performance Rating.**

Determination of performance rating is an important step in the work measurement procedures. It is based entirely on the experience, training, and judgment of the work-study engineer. It is the step most subjective and therefore is subject to criticism.

It is the procedure in which the time study engineer compares the performance of operator(s) under observation to the Normal Performance and determines a factor called **Rating Factor.**

$$\text{Rating factor} = \frac{\text{Observed performance}}{\text{Normal performance}}$$

System of Rating

There are several systems of rating, the performance of operator on the job. These are

1. Pace Rating
2. Westinghouse System of Rating
3. Objective Rating
4. Synthetic Rating

A brief description of each rating method follows.

Pace Rating

Under this system, performance is evaluated by considering the rate of accomplishment of the work per unit time. The study person measures the effectiveness of the operator against the concept of normal performance and then assigns a percentage to indicate the ratio of the observed performance to normal or standard performance.

In this method, which is also called the speed rating method, the time study person judges the operators speed of movements, i.e. the rate at which he is applying himself, or in other words "how fast" the operator the motions involved.

Westinghouse System of Rating

This method considers four factors in evaluating the performance of the operator : Skill, effort, conditions and Consistency.
Skill may be defined as proficiency at following a given method. It is demonstrated by co ordination of mind and hands. A person's skill in given operation increases with his experience on the job, because increased familiarity with work bring speed, smoothness of motions and freedom from hesitations.

The Westinghouse system lists six classes of skill as poor fair, average, good, excellent in a Table1. The time study person evaluates the skill displayed by the operator and puts it in one of the six classes. As equipment % value of each class of skill is provided in the table, the rating is translated into its equivalent percentage value, which ranges from +15 % (for super skill) to – 22 % (for poor skill).

In a similar fashion, the ratings for effort, conditions, and consistency are given using Table2 for each of the factors. By algebraically combining the ratings with respect to each of the four factors, the final performance-rating factor is estimated.

Objective Rating

In this system, speed of movements and job difficulty are rated separately and the two estimates are combined into a single value. Rating of speed or pace is done as described earlier, and the rating of job difficulty is done by selecting adjustment factors corresponding to characteristics of operation with respect to (i) amount of body used, (ii) foot pedals, (iii) bimanual ness, (iv) eye-hand co ordination, (v) handling requirements and (vi) weight handled or resistance encountered Mundel and Danner have given Table of % values (adjustment factor) for the effects of various difficulties in the operation performed.

For an operation under study, the numerical value for each of the six factors is assigned, and the algebraic sum of the numerical values called job difficulty adjustment factor is estimated.

The rating factor R can be expressed as

$$R = P \times D$$

Where: P = Pace rating factor
D = Job difficulty adjustment factor.

Synthetic Rating

This method of rating has two main advantages over other methods that (i) it does not rely on the judgment of the time study person and (ii) it give consistent results.

The time study is made as usual. Some manually controlled elements of the work cycle are selected. Using a PMT system (Pre-determined motion time system), the times for these elements are determined. The times of these elements are the performance factor is determined for each of the selected elements.

Performance or Rating Factor, $R = P / A$

Where P = Predetermined motion time of the element,
A = Average actual Observed time of the element.

The overall rating factor is the mean of rating factors determined for the selected elements, which is applied uniformly to all the manually controlled elements of the work cycle.

Example: A work cycle has been divided into 8 elements and time study has been conducted. The average observed times for the elements are as:

Element No.	1	2	3	4	5	6	7	8
Element Type	M	M	P	M	M	M	M	M
Average actual time (minutes)	0.14	0.16	0.30	0.52	0.26	0.45	0.34	0.15

M = Manually Controlled, P = Power Controlled
Total observed time of work cycle = **2.32 min**.

Suppose we select elements number 2, 5 and 8 (These must be manually controlled elements). By using some PMT system, suppose we determine the times of these elements as

Elements No.	2	5	8
PMT System times (mins)	0.145	0.255	0.140

Rating factor for element 2 = 0.145 / 0.16 = **90.06%**
Rating factor for element 5 = 0.255 / 0.26 = **98.08%**
Rating factor for element8 = 0.140 / 0.15 = **96.66%**

The mean of the rating factors of selected elements = **94.93%** or say **95%** is the rating factor that will be used for all the manual elements of the work cycle.
The normal time of the cycle is calculated as given in the following table.

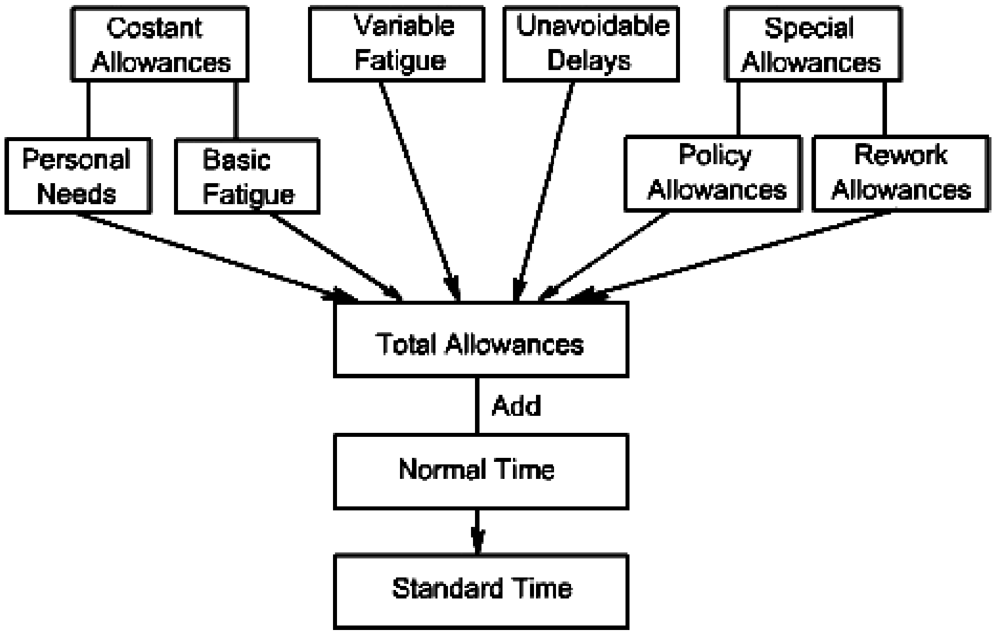
Element No.	1	2	3	4	5	6	7	8
Element Type	M	M	P	M	M	M	M	M
Average actual time(min)	0.14	0.16	0.30	0.52	0.26	0.45	0.34	0.15
PMT system time(min)		0.145			0.255			0.14
Performance Rating Factor	95	95	100	95	95	95	95	95

Normal Cycle Time = 0.95(0.14 + 0.16 + 0.52 + 0.26 + 0.45 + 0.34 + 0.15) + 1.00(0.30)
= 1.92 + 0.30 = **2.22 minutes**

Allowances

The readings of any time study are taken over a relatively short period of time. The normal time arrived at, therefore does not include unavoidable delay and other legitimate lost time, for example, in waiting for materials, tools or equipment; periodic inspection of parts; interruptions due to legitimate personal need, etc. It is necessary and important that the time study person applies some adjustment, or allowances to compensate for such losses, so that fair time standard is established for the given job.

Allowances are generally applied to total cycle time as some percentage of it, but sometimes these are given separately as some % for machine time and some other % for manual effort time. However no allowance are given for interruptions which maybe due to factor which are within the operator's control or which are avoidable.



Most companies allow the following allowances to their employees.

- Delay Allowance
- Fatigue Allowance
- Personal Allowance
- Special Allowance

Delay Allowance

This time allowance is given to an operator for the numerous interruptions that he experiences every day during the course of his work. These interruptions include interruptions from the supervisor, inspector, planners, expediters, fellow workers, production personnel and others. This allowance also covers interruptions due to material irregularities, difficulty in maintaining specifications and tolerances, and interference delays where the operator has to attend to more than one machine.

Fatigue Allowance

This allowance can be divided into two parts: (i) basic fatigue allowance and (ii) variable fatigue allowance. The basic fatigue allowance is given to the operator to compensate for the energy expended for carrying out the work and to alleviate monotony. For an operator who is doing light work while seated, under good working conditions and under normal demands on the sensory or motor system, a 4% of normal time is considered adequate. This can be treated as a constant allowance.

The magnitude of variable fatigue allowance given to the operator depends upon the severity of the factor or conditions, which cause extra (more than normal) fatigue to him. As we know, fatigue is not homogeneous, it range from strictly physical to purely psychological and includes combinations of the two. on some people it has a marked effect while on others, it has apparently little or no effect. Whatever may be the kind of fatigue-physical or mental, the result is same-it reduces the work output of operator. The major factors that cause more than just the basic fatigue includes sever working conditions, especially with respect to noise, illumination, heat and humidity; the nature of work, especially with respect to posture, muscular exertion and tediousness and like that.

It is true that in modern industry, heavy manual work, and thus muscular fatigue is reducing day by day but mechanization is promoting other fatigue components like monotony and mental stress. Because fatigue in totality cannot be eliminated, proper allowance has to be given for adverse working conditions and repetitiveness of the work.

Personal Allowance

This is allowed to compensate for the time spent by worker in meeting the physical needs. A normal person requires a periodic break in the production routine. The amount of personal time required by operator varies with the individual more than with the kind of work, though it is seen that workers need more personal time when the work is heavy and done under unfavorable conditions.

The amount of this allowance can be determined by making all-day time study or work sampling. Mostly, a 5 % allowance for personal time (nearly 24 minutes in 8 hours) is considered appropriate.

Special Allowance

These allowances are given under certain special circumstances. Some of allowances and the conditions under which they are given are:

Small Lot Allowance: This allowance is given when the actual production period is too short to allow the worker to come out of the initial learning period. When an operator completes several small-lot jobs on different setups during the day, an allowance as high as 15 percent may be given to allow the operator to make normal earnings.

Training Allowance: This allowance is provided when work is done by trainee to allow him to maker reasonable earnings. It may be a sliding allowance, which progressively decreases to zero over certain length of time. If the effect of learning on the job is known, the rate of decrease of the training allowance can be set accordingly.

Rework Allowance: This allowance is provided on certain operation when it is known that some present of parts made are spoiled due to factors beyond the operator's control. The time in which these spoiled parts may be reworked is converted into allowance.

Different organizations have decided upon the amount of allowances to be given to different operators by taking help from the specialists/consultants in the field and through negotiations between the management and the trade unions. ILO has given its recommendations about the magnitude of various allowances, Table 4.

Example: In making a time study of a laboratory technician performing an analysis of processed food in a canning factory, the following times were noted for a particular operation.

Run	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Operation time(sec.)	21	21	16	19	20	16	20	19	19	20	40	19
Run	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Operation time(sec.)	21	18	23	19	15	18	18	19	21	20	20	19

If the technician's performance has been rated at 120 percent, and the company policy for allowance (personal, fatigue, etc.) stipulates 13 percent,

(i) Determine the normal time

(ii) Determine the standard time

Watch readings falling 50% above and 25% below the average may be considered as abnormal.

Answer: $T_{av} = \frac{\sum \text{Cycle time}}{\text{No. of cycles}} = \frac{481}{24} = 20.04$ sec.

$\Rightarrow 1.5T_{av} = 30$ sec. $\Rightarrow 0.75T_{av} = 15$ sec.

Discarding the time values which are greater than .75 T_{av} or less than 1.5 T_{av}, the average observed cycle time = $\frac{441}{23} = 19.2$ sec.

Normal time = $19.2 \times \frac{120}{100} = 23.04$ sec.

Standard time = Normal time + Allowances = $23.04 \times \frac{100}{100-13} = 26.5$ sec.

Predetermined Motion Time System

A predetermined motion time system (PMTS) may be defined as a procedure that analyzes any manual activity in terms of basic or fundamental motions required to performing it. Each of these motions is assigned a previously established standard time value in such a way that the timings for the individual motions can be synthesized to obtain the total time for the performance of the activity.

The main use of PMTS lies in the estimation of time for the performance of a task before it is performed. The procedure is particularly useful to some organizations because it does not require troublesome rating with each study.

Applications of PMTS are for:

(i) Determination of job time standards.
(ii) Comparing the times for alternative proposed methods so as to find the economics of the proposals prior to production run.
(iii) Estimation of manpower, equipment and space requirements prior to setting up the facilities and start of production.
(iv) Developing tentative work layouts for assembly line prior to their working.
(v) Checking direct time study results.

A number of PMTS are in use, some of which have been developed by individual organizations for their own use, while other organizations have publicized for universal applications.

The following are commonly used PMT systems

- Work factor (1938)
- Method Time Measurement (1948)
- Basic Motion Time (1951)
- Dimension Motion Time (1954)

Some important factors which be considered while selecting a PMT system for application to particular industry are:

1. **Cost of Installation**. This consists mainly of the cost of getting expert for applying the system under consideration.

2. **Application Cost**. This is determined by the length of time needed to set a time standard by the system under consideration.
3. **Performance Level of the System**. The level of performance embodied in the system under consideration may be different from the normal performance established in the industry where the system is to be used. However, this problem can be overcome by 'calibration' which is nothing but multiplying the times given in the Tables by some constant or by the application of an adjustment allowance.
4. **Consistency of Standards**. Consistency of standards set by a system on various jobs is a vital factor to consider. For this, the system can be applied on a trial basis on a set of operations in the plant and examined for consistency among them.
5. **Nature of Operation**. Best results are likely to be achieved if the type and nature of operations in the plant are similar to the nature and type of operations studied during the development of the system under consideration.

Advantages and limitations of using PMT systems

Advantage

Compared to other work measurement techniques, all PMT system claim the following advantages:

1. There is no need to actually observe the operation running. This means the estimation of time to perform a job can be made from the drawings even before the job is actually done. This feature is very useful in production planning, forecasting, equipment selection etc.
2. The use of PMT eliminates the need of troublesome and controversial performance rating. For the sole reason of avoiding performance rating, some companies have been using this technique.
3. The use of PM times forces the analyst to study the method in detail. This sometimes helps to further improve the method.
4. A bye-product of the use of PM time is a detailed record of the method of operation. This is advantageous for installation of method, for instructional purposes, and for detection and verification of any change that might occur in the method in future.
5. The PM times can be usefully employed to establish elemental standard data for setting time standards on jobs done on various types of machines and equipment.
6. The basic times determined with the use of PMT system are relatively more consistent.

Limitations

There are two main limitations to the use of PMT system for establishing time standards. These are:

(i) Its application to only manual contents of job and
(ii) The need of trained personnel. Although PMT system eliminates the use of rating, quite a bit of judgment is still necessarily exercised at different stages.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Motion Study and Motion Economy

GATE-1. The principles of motion economy are mostly used while conducting **[GATE-2002]**
(a) A method study on an operation
(b) A time study on an operation
(c) A financial appraisal of an operation
(d) A feasibility study of the proposed manufacturing plant.

Work Measurement (Time Study)

GATE-2. The individual human variability in time studies to determine the production standards is taken care of by: **[GATE-1996]**
(a) Personal allowances (b) Work allowances
(c) Rating factor (d) None of the above

GATE-3. A welding operation is time-studied during which an operator was pace-rated as 120%. The operator took, on an average, 8 minutes for producing the weld-joint. If a total of 10% allowances are allowed for this operation, the expected standard production rate of the weld-joint (in units per 8 hour day) is: **[GATE-2005]**
(a) 45 (b) 50 (c) 55 (d) 60

GATE-4. The standard time of an operation while conducting a time study is: **[GATE-2002]**
(a) Mean observed time + Allowances
(b) Normal time + Allowances
(c) Mean observed time × Rating factor + Allowances
(d) Normal time × Rating factor + Allowances

GATE-5. Fifty observations of a production operation revealed a mean cycle time of 10 min. The worker was evaluated to be performing at 90% efficiency. Assuming the allowances to be 10% of, the normal time, the standard time (in seconds) for the job is: **[GATE-2001]**
(a) 0.198 (b) 7.3 (c) 9.0 (d) 9.9

GATE-6. In a time study exercise, the time observed for an activity was 54 seconds. The operator had a performance rating of 120. A personal time allowance of 10% is given. The standard time for the activity, in seconds, is: **[GATE-2000]**
(a) 54 (b) 60.8 (c) 72 (d) 58.32

GATE-7. A stop watch time study on an operator with a performance rating of 120 yielded a time of 2 minute. If allowances of 10% of the total available time are to be given, the standard time of the operation is: **[GATE-1995]**
(a) 2 minutes (b) 2.4 minutes (c) 2.64 minutes (d) 2.67 minutes

GATE-8. The actual observed time for an operation was 1 minute per piece. If the performance rating of the operator was 120 and a 5 per cent

personal time is to be provided, the standard time in minute per piece is: **[GATE-1993]**
(a) 1.000 (b) 1.200 (c) 1.250 (d) 1.263

GATE-9. A soldering operation was work-sampled over two days (16 hours) during which an employee soldered 108 joints. Actual working time was 90% of the total time and the performance rating was estimated to be 120 percent. If the contract provides allowance of 20 percent of the total time available, the standard time for the operation would be: [GATE-2004]
(a) 8 min. (b) 8.9 min. (c) 10 min. (d) 12 min.

Previous 20-Years IES Questions

Motion Study and Motion Economy

IES-1. Work study is mainly aimed at [IES-1995]
(a) Determining the most efficient method of performing a job
(b) Establishing the minimum time of completion of job
(c) Developing the standard method and standard time of a job
(d) Economizing the motions involved on the part of the worker while performing a job

IES-2. Which of the following hand-motion belongs to 'Therblig' in motion study') [IES-2001]
1. Unavoidable delay **2. Pre-position**
3. Select **4. Reach**
Select the correct answer from the codes given below:
Codes: (a) 1 and 4 (b) 1 and 2 (c) 1, 2 and 3 (d) 2, 3 and 4

IES-3. Match List-I (Charts) with List-II (Details) and select the correct answer using the codes given below the lists: [IES-1998]

List-I	List-II
A. Multiple activity chart	**1.** Work factor system
B. SIMO chart	**2.** Movement of material
C. String diagram	**3.** Motion analysis
D. MTM	**4.** Working an idle time of two or more men/machines

Codes:	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	3	4	2	1
(c)	4	3	1	2	(d)	3	4	1	2

IES-4. Assertion (A): SIMO chart reveals the deficiencies in the motion pattern of the process chart. [IES-1992]
Reason (R): SIMO chart and operator processes chart yield the same results.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-5. Match List-I (Study) with List-II (Related factors) and select the correct answer using the codes given below the lists: [IES-2004]

List-I	List-II
A. Job enrichment	**1.** Gilbert's principles
B. Job evaluation	**2.** Movement of limbs by work factor system
C. Method study	**3.** Herzberg motivators
D. Time study	**4.** Jacques time span of discretion

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	1	4	3	(b)	3	4	1	2
(c)	2	4	1	3	(d)	3	1	4	2

IES-6. Repetitive fast speed activities can be effectively analyzed by taking photograph at [IES-2002]
(a) Low speed and screening at low speed
(b) High speed and screening at high speed
(c) High speed and screening at low speed
(d) Low speed and screening at high speed

Work Measurement (Time Study)

IES-7. Which one of the following statements is correct? [IES-2006]
Standard time is obtained from normal time by adding the policy allowance and
(a) Personal allowances only
(b) Fatigue allowances only
(c) Delay allowances only
(d) Personal, fatigue and delay allowances

IES-8. A time standard for a data entry clerk is to be set. A job is rated at 120 percent, it takes 30 seconds to enter each record and the allowances are 15%. What is the normal time? [IES-2008]
(a) 25 seconds (b) 30 seconds (c) 36 seconds (d) 40 seconds

IES-9. If in a time study, the observed time is 0.75 min, rating factor = 110% and allowances are 20% of normal time, then what is the standard time? [IES-2009]
(a) 0.82 min (b) 0.975 min (c) 0·99 min (d) 1·03

IES-10. Determination of standard time in complex job system is best done through [IES-1996]
(a) Stop watch time study
(b) Analysis of micro motions
(c) Group timing techniques
(d) Analysis of standard data system

IES-11. Standard time is: [IES-2003]
(a) Normal time + Allowances
(b) (Normal time × Rating) + Allowances
(c) $\left(\dfrac{\text{Normal time}}{\text{Rating}}\right)$ + Allowances
(d) Normal time + (Allowances × Rating)

IES-12. The standard time of an operation has been calculated as 10 min. The worker was rated at 80%. If the relaxation and other allowances were 25%, then the observed time would be: [IES-1999]
(a) 12.5 min (b) 10 min (c) 8 min (d) 6.5 min

IES-13. In time study, the rating factor is applied to determine [IES-1995]
(a) Standard time of a job
(b) Merit rating of the worker

(c) Fixation of incentive rate (d) Normal time of a job

IES-14. Match List-I (Charts) with List-II (Operations/Information) using the codes given below the lists: [IES-2001]

List-I	List-II
A. Standard process sheet	**1.** Operations involving assembly and inspection without machine
B. Multiple activity chart	**2.** Operations involving the combination of men and machines
C. Right and left hand operation chart	**3.** Work measurement
D. SIMO chart	**4.** Basic information for routing
	5. Therblig

Codes:	A	B	C	D		A	B	C	D
(a)	4	3	1	2	(b)	1	2	4	5
(c)	1	3	4	2	(d)	4	2	1	5

IES-15. MTM is a work measurement technique by: [IES-1999]
(a) Stopwatch study (b) Work sampling study
(c) Pre-determined motion time systems (d) Past data comparison

IES-16. Consider the following objectives: [IES-1994]
1. To train the individual regarding motion economy.
2. To assist in research projects in the field of work study.
3. To help in the collection of Motion Time data.
The objectives of Micromotion Study would include
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-17. Which one of the following is NOT a work measurement technique?
(a) Time study (b) Work sampling **[IES-2009]**
(c) Motion time data (d) Micromotion study

Predetermined Motion Time System

IES-18. Consider the following steps: [IES-1994]
1. Method time measurement 2. Work sampling
3. Work factor system.
PMTS (Predetermined motion time systems) in work study would include:
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-19. Which one of the following is not a technique under Predetermined Motion Time System (PMTS)? [IES-2000, 2003, 2006]
(a) Work factor (b) Synthetic data
(c) Stopwatch time study (d) MTM

IES-20. Which one of the following statements is not correct? [IES-2008]
(a) Work sampling is a technique of work measurement
(b) Method study is a technique aimed at evolving improved methods
(c) Synthetic data is not a technique covered under pre-determined motion time systems
(d) 'Select' is the first step of method study

Previous 20-Years IAS Questions

Motion Study and Motion Economy

IAS-1. Work study involves **[IAS 1994]**

(a) Only method study (b) Only work measurement
(c) Method study and work measurement (d) Only motion study

IAS-2. Match List-I with List-II and select the correct answer using the codes given below the lists: **[IAS-1996]**

List-I	**List-II**
A. Time study	**1.** Stop watch
B. Work factor system	**2.** Clinograph
C. Micromotion study	**3.** Body member
D. Cycle graph	**4.** High speed film
	5. Small electric bulb

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	4	3	2	1	(b)	5	3	4	2
(c)	1	4	3	5	(d)	1	3	4	5

IAS-3. A Left Hand – Right Hand activity chart is given below: **[IAS-1999]**

Left hand Activity	**Symbol**	**Time in Min.**	**Symbol**	**Right Hand Activity**
Lift the work piece		**0.1**		
		0.1		**Open the vice**
Clamp the work piece		**0.2**		**Clamp the work piece**
		1.0		**Take the file**
Do hand Filing		**1.5**		**Do hand filing**
		0.1		**Take the micrometer**
Check the dimension		**0.4**		**Check the dimension**
		0.1		**Open the vice**
Remove the work piece		**0.1**		

The cycle time for the operation is:

(a) 2.3 min. (b) 2.5 min. (c) 2.7 min. (d) 2.2 min.

IAS-4. Motions of limbs are through **[IAS-2003]**

1. Elbow **2. Finger** **3. Hip**
4. Shoulder **5. Wrist**

What is the correct sequence in descending order of motion in terms of time of fatigue involved?

(a) 3-4-1-5-2 (b) 2-5-1-4-3 (c) 5-2-3-1-4 (d) 4-3-2-1-5

IAS-5. Match List-I (Type of Chart) with List-II (Definition) and select the correct answer using the codes given below the lists: **[IAS-2004]**

List-I **List-II**

A.	Outline process chart	**1.**	It is used to record the activities of one subject in relation to one or more others
B.	Multiple Activity chart	**2.**	It is a chart in which activities of the machine or machines are recorded in relation to that of the operator
C.	SIMO chart	**3.**	It records the main activities of the process through the symbols of operation and inspection
D.	Non-machining chart	**4.**	It is used to record the activities of the hands of an operator
		5.	It makes use of Therblig for charting minute elements of an operation

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	2	5	1	(b)	5	1	4	2
(c)	3	1	5	2	(d)	5	2	4	1

IAS-6. **A SIMO chart should be used with** **[IAS-2001]**
(a) Process charts (b) Flow diagrams
(c) Man-machine operation charts (d) Therbligs

IAS-7. **A graphical representation of the coordinated activities (i.e., fundamental motions) of an operator's body members, on a common time scale is known as** **[IAS-1998]**
(a) SIMO chart (b) Man-Machine chart
(c) Two-handed process chart (d) Gantt chart

IAS-8. **Consider the following charts:** **[IAS-1997]**
1. Man-machine chart **2. SIMO chart** **3. Load chart**
Those used in method study would include:
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

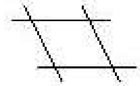
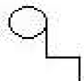
IAS-9. **Consider the following statements:** **[IAS-2007]**
Method study is carried out to achieve
1. **The most effective use of plant and equipment.**
2. **The most effective use of human efforts.**
3. **Evaluation of human work.**

Which of the statements given above are correct?
(a) 1 and 3 only (b) 2 and 3 only (c) 1 and 2 only (d) 1, 2 and 3

IAS-10. **Consider the following activities:** **[IAS-2000]**
1. **Body movement**
2. **Work capability of individuals**
3. **Movement of materials**

Which of these activities is observed in method study?
(a) 1, 2 and 3 (b) 2 and 3 (c) 1 and 3 (d) 1 and 2

IAS-11. **Match List-I (Therbligs symbols) with List-II (Motions) and select the correct answer using the codes given below the lists:** **[IAS-1999]**

	List I	List II
A		1. Find
B		2. Rest
C		3. Transport Loaded
D		4. Assemble

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	1	2	3	4	(b)	4	3	2	1
(c)	3	4	1	2	(d)	4	3	1	2

IAS-12. **Therbligs are essential in** **[IAS-1997]**
(a) Flow process chart (b) Motion economy
(c) Method-analysis chart (d) Suggestion system

IAS-13. **Simo charts are used for** **[IAS-1996]**
(a) Inspection of parts (b) Operator movements
(c) Work done at one place (d) Simulation of models

IAS-14. **In which of the following operations, micro motion study technique is used?** **[IAS-2004]**
1. Short cycle operations lasting two minutes or less
2. Long cycle operations lasting more than five minutes
3. Medium cycle operations lasting between two and five minutes
Select the correct answer using the codes given below:
(a) 1 only (b) 1 and 2 (c) 2 and 3 (d) 1, 2 and 3

Work Measurement (Time Study)

IAS-15. **In industrial engineering, the standard time is equal to:** **[IAS-2001]**
(a) Normal time + Allowance (b) Observed time + Allowance
(c) Observed time × Rating factor (d) Normal time × Rating factor

IAS-16. **Standard time is:** **[IAS-2002]**
(a) Average time + Allowances (b) Average time – Allowances
(c) Normal time – Allowances (d) Normal time + Allowances

IAS-17. **Standard times (ST) and labour rates are as in the table. Labour overheads are 20% of labour cost.** **[IAS-2002]**

Activity	ST, min	Labour rate Rs/hr.
Cutting	**2**	**550**
Inspection	**0·5**	**400**
Packaging	**0·5**	**400**

If the material cost is Rs. 25/unit, what will be the total cost of production in Rs/unit?
(a) 25 (b) 55 (c) 45 (d) 35

IAS-18. **For a manual operation to study the total processing standard time using a chronometer, following observations were made: [IAS-2007]**

Processing time	**:**	**16 minutes**
Rating of the worker	**:**	**120%**
Personal allowance	**:**	**0.6 minutes**
Basic fatigue allowance	**:**	**1.92 minutes**
Unavoidable delay allowance	**:**	**1.08 minutes**

What is the standard operating time for the operation?
(a) 16 minutes (b) 17.92 minutes
(c) 21.52 minutes (d) None of the above

IAS-19. **In a stop-watch time study, the observed time was 0.16 minute; the performance rating factor was 125on the 100 normal (percentage scale). What is the standard time in minutes if 10% allowances are permitted? [IAS-2004]**
(a) 0.180 (b) 0.200 (c) 0.220 (d) 0.240

IAS-20. **There is 8 hours duty and a job should take 30 minutes to complete it. But after 8 hours, an operator is able to complete only 14 such jobs. The operator's performance is: [IAS-1997]**
(a) 77.5% (b) 78.5% (c) 87.5% (d) 97.5%

IAS-21. **Assertion (A): Split hand type stopwatch is preferred in time study by stopwatch method. [IAS-1997]**
Reason (R): Split hand type stopwatch eliminates possibilities of delay in nothing the reading.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-22. **Match List-I (Techniques of work measurement) with List-II (Usages in Indian industry) and select the correct answer using the codes given below the lists: [IAS-2001]**

List-I	**List-II**
A. Stop-watch study	**1.** 5%
B. Standard based study	**2.** 10%
C. Work sampling	**3.** 80%
D. Historical data based	

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	1	1	2	(b)	2	1	3	1
(c)	3	2	1	2	(d)	2	3	1	1

IAS-23. **Match List-I with List-II and select the correct answer using the codes given below the lists: [IAS-1995]**

List-I	**List-II**
A. Statistic	**1.** Performance Rating
B. MTM	**2.** Motion Study
C. Stop Watch	**3.** Work Measurement
D. Man Machine Chart	**4.** Work Sampling
E. Standard Time	

Codes:

	A	B	C	D	E		A	B	C	D	E
(a)	4	3	3	2	1	(b)	4	2	3	2	1
(c)	2	3	4	1	2	(d)	3	1	4	2	1

IAS-24. **Which one of the following techniques is used for determining allowances in time study?**
(a) Acceptance sampling (b) Linear regression (c) Performance rating (d) work sampling

IAS-25. **Consider the following statements:** **[IAS-2001]**
The principle of motion economy related to the use of human body is that the two hands should
1. **Begin as well as complete their motions at the same time**
2. **Not be idle at the same time except during rest**
3. **Be relieved of all work that can be done more advantageously by a jig, fixture or foot-operated device**

Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IAS-26. **Consider the following principles:** **[IAS-1997]**
1. **Trying to avoid the use of hands for holding.**
2. **Relieving the hands of work whenever possible.**
3. **Keeping the work in the normal work area.**
4. **Avoiding having both hands doing the same thing at the same time.**

Principles of motion economy would include:
(a) 1, 2 and 3 (b) 1, 3 and 4 (c) 1, 3, and 4 (d) 1, 2 and 3

IAS-27. **From the point of motion economy it is preferable to move**
(a) Both hands in the same direction **[IAS-1995]**
(b) Right hand first and then left hand
(c) Only one hand at a time
(d) Both hands in opposite directions.

IAS-28. **The number of cycles to be timed in a stopwatch time study depends upon the** **[IAS-1996]**
(a) Discretion of the time study engineer
(b) Time of each cycle and the accuracy of results desired
(c) Time available to the time study engineer
(d) Nature of the operation and as well as the operator

IAS-29. **In respect of time study, match List-I (Situations) with List-II (Allowance) and select the correct answer using the codes given below the lists:** **[IAS-2003]**

	List-I		List-II
A.	To allow for personal needs	**1.**	Contingency allowance
B.	To meet legitimate delay in work	**2.**	Policy allowance
C.	Offered under special circumstances	**3.**	Injury allowance

to add to the earnings — 4. Relaxation allowance

Codes:	A	B	C		A	B	C
(a)	4	1	2	(b)	3	2	4
(c)	4	2	3	(d)	3	1	2

IAS-30. Which one of the following statements regarding time study is correct? [IAS-2000]

(a) While conducting time study, it is important to get historical records of preceding times
(b) It is based on a procedure originally proposed by Frederick W. Taylor
(c) It is necessary to multiply the measured time by the reciprocal of learning curve
(d) One should be careful to collect data under just-in-time conditions

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (a)

GATE-2. Ans. (c)

GATE-3. Ans. (a) Normal time = 1.2 × 8 = 9.6 minute

Standard time = $9.6 + 9.6 \times \frac{10}{100} = 10.56$ minute/piece

Total time available = 8 × 60 minute

So, production rate = $\frac{480}{10.56} = 45.45 \approx 45$ weld joint

GATE-4. Ans. (c)

GATE-5. Ans. (d)

GATE-6. Ans. (c) Normal time = Observed time × $\frac{\text{Performance rating}}{100} = 54 \times \frac{120}{100}$

Standard time = Normal time × $\frac{100}{100 - \%\text{ allowance}}$

$$= 54 \times \frac{120}{100} \times \frac{100}{(100-10)} = 72 \text{ sec.}$$

GATE-7. Ans. (c) Standard time of the operation = Basic time + Allowances

Basic time = $\frac{\text{Observed time} \times \text{Rating factor}}{100} = \frac{2 \times 120}{100} = 2.4$ minutes

Allowances = 10% of total available time = $2.4 \times \frac{10}{100} = 2.4$ minutes

Standard time of the operation = 2.4 + 0.24 = 2.64 minutes.

GATE-8. Ans. (d) Normal performance = $\frac{\text{Observed performance}}{\text{Performance rating}} = \frac{100}{120} = \frac{1}{1.2}$

Normal time = $\frac{\text{Observed time}}{\text{Normal performance}} = 1 \times 1.2 = 1.2$ minutes

Standard time = Normal time × Allowance = 1.2 × 1.05 = 1.260 minutes

GATE-9. Ans. (d) Actual time = $\frac{90}{100} \times 16 \times 60 = 864$ minutes

Normal time = Actual time × $\frac{120}{100} = 864 \times 1.2 = 1036.8$

Standard time = 1036.8(1.2) = 1244.16 minutes

Standard time per joint = $\frac{1244.16}{108} \simeq 12$ minutes

Previous 20-Years IES Answers

IES-1. Ans. (c)
IES-2. Ans. (c)
IES-3. Ans. (a)
IES-4. Ans. (c) A is true but R is false.
IES-5. Ans. (b)
IES-6. Ans. (c)
IES-7. Ans. (d) Standard time = Normal time + PDA Allowances
IES-8. Ans. (c)
IES-9. Ans. (c) Normal time = Observed Time × Rating Factor = 0.75 ×1.1 = 0.825 min
Standard time = Normal Time × (1 + % age allowance)
= 0.825 × (1 + 0.20) = 0.99 minutes
IES-10. Ans. (d)
IES-11. Ans. (a) Normal time = Actual time × Rating factor
Standard time = Normal time + Allowances
IES-12. Ans. (b) Observed time = (Standard time + Allowances) × Rating of worker
= 10 × (1 + 0.25) × 0.8 = 10
IES-13. Ans. (b) In time study, the rating factor is applied to determine merit rating of the worker.

IES-14. Ans. (d)
IES-15. Ans. (c) MTM (Methods-time measurement) is based on use of standard time for work elements that have been predetermined from long periods of observation and analysis.
IES-16. Ans. (d) Objectives 1 and 3 are true for Micromotion study.
IES-17. Ans. (d)
IES-18. Ans. (d)
IES-19. Ans. (c)
IES-20. Ans. (c)

Previous 20-Years IAS Answers

IAS-1. Ans. (c)
IAS-2. Ans. (d)
IAS-3. Ans. (c) Just add all the times.
IAS-4. Ans. (a)
IAS-5. Ans. (a)
IAS-6. Ans. (d)
IAS-7. Ans. (a)
IAS-8. Ans. (b)
IAS-9. Ans. (d)
IAS-10. Ans. (a)
IAS-11. Ans. (d)
IAS-12. Ans. (b)

IAS-13. Ans. (b)

IAS-14. Ans. (b)

IAS-15. Ans. (a)

IAS-16. Ans. (d)

IAS-17. Ans. (b) Total production cost = Material cost + Labour cost

$$= 25 + \left[\frac{550}{60}\times 2+\frac{400}{60}\times 0.5+\frac{400}{60}\times 0.5\right]\times 1.2$$

$$= 25 + 30 = 55$$

IAS-18. Ans. (d) Standard time = Normal time + PDF allowance

= Observed time $\times$ Performance rating + PDF allowance

= 16×1.2 + (0.6 + 1.92 + 1.08) min = 22.8 min

IAS-19. Ans. (c) Standard time = Observed time $\times$ Performance rating

= $0.16\times 1.25\times (1.10)$ = 0.220 min

IAS-20. Ans. (c) Operator's performance = $\dfrac{\text{Observed performance}}{\text{Normal performance}} = \dfrac{14}{8\times 2}\times 100\% = 87.5\%$

IAS-21. Ans. (a)

IAS-22. Ans. (d)

IAS-23. Ans. (a)

IAS-24. Ans. (d)

IAS-25. Ans. (a)

IAS-26. Ans. (b)

IAS-27. Ans. (d)

IAS-28. Ans. (b)

IAS-29. Ans. (a)

IAS-30. Ans. (b)

Conventional Questions with Answer

Conventional Question - IES 2010

Give the symbols, activity names used in method study for charting of the processes. [2 Marks]

Ans. Activity name and symbols used in work study for charting of the process.

Activity Name	Symbol
Operation	O
Inspection	
Transportation	⇒
Delay	D
Storage	▽

Conventional Question - IES 2009

Define the term 'standard time' and list the common allowances given in work standard. [2-Marks]

Solution: Standard time
It is defined as the sum of base time and the allowances i.e,
Standard time = Base or normal time + Allowances

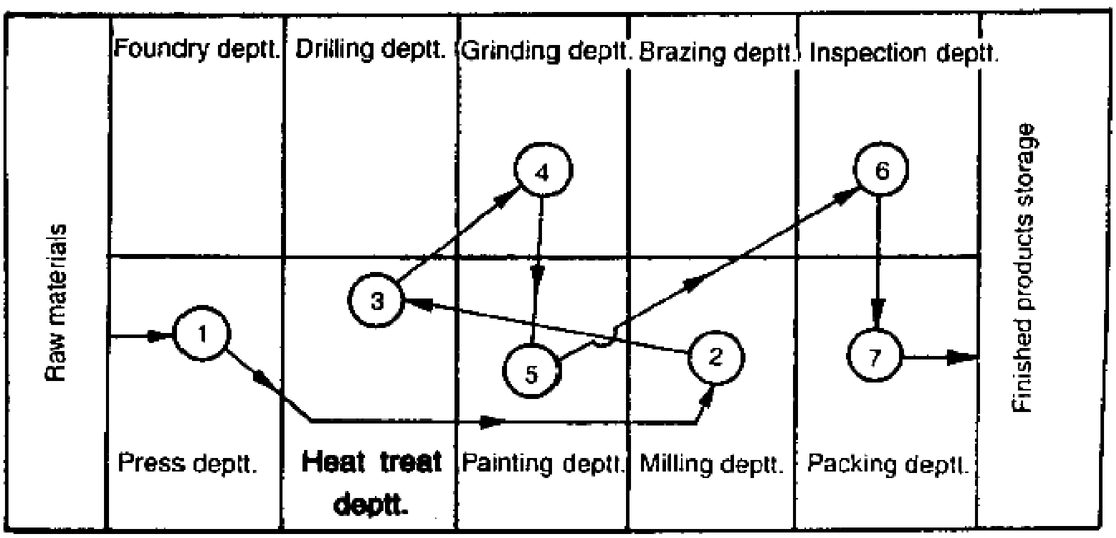
Common allowances
(1) Delays allowances
(2) Fatigue allowances
(3) Interruption allowances
(4) Adverse allowances
(5) Extreme job conditions rework allowances
(6) Personal needs allowances
(7) Rest period allowances
(8) For circumstances peculiar to the operation.

9. Layout Plant

Theory at a Glance (For IES, GATE, PSU)

Functional Layout

Here, machines performing similar operations are grouped together, and are not arranged according to any particular sequence of operations. The work is brought to a machine from a machine on which the previous operation was carried out; this machine may be in another department or even a building. This results in lot of back tracking or cross-movements of the work. Such a layout is suitable for low volume production (Batch production or Jobbing production), and where the product is not standardized.



Functional Layout

Advantages of Functional Layout:

1. Greater flexibility of production. Change in product design can be easily accommodated.
2. Lower initial investment in machinery because of less duplication of equipment.
3. Break down of one machine will not shut-down the production as the work of that machine can be transferred to another machine or worker.

Disadvantages:

1. Generally, more floor space is required.
2. More handling costs because of back-tracking and cross-movements of work, resulting in chaotic material movement.

Objective Questions (GATE, IES, IAS)

Previous 20-Years GATE Questions

Plant Layout

GATE-1. The manufacturing area of a plat is divided into four quadrants. Four machines have to be located, one in each quadrant. The total number of possible layouts is: [GATE-1995]
(a) 4 (b) 8 (c) 16 (d) 24

GATE-2. The layout with a higher material handling effort is a........ layout. (product/process) [GATE-1995]

GATE-3. Vehicle manufacturing assembly line is an example of: [GATE-2010]
(a) Product layout (b) Process layout
(c) Manual layout (d) Fixed layout

Work Flow Diagram

GATE-4. The symbol used for Transport in work study is: [GATE-2003]
(a) ⇒ (b) T (c) ▭ (d) ▽

GATE-5. An assembly activity is represented on an Operation Process Chart by the symbol [GATE-2005]
(a) AA (b) A (c) D (d) O

Previous 20-Years IES Questions

Plant Layout

IES-1. Consider the following statements regarding plant location and plant layout: [IES-2000]
1. **Qualitative factor analysis is a method of evaluating a potential-location without applying quantitative values to the decision criteria.**
2. **The three determinants of the type of layout are type of product, type of process and the volume of production.**
3. **An appliance manufacturing plant where products are made on assembly lines would be classified as job shop type of layout.**

Which of these statements is/are correct?
(a) 1, 2, and 3 (b) 1 and 2 (c) 2 alone (d) 3 alone

IES-2. Match List-I (Object) with List-II (Tool) and select the correct answer: [IES-1996]

	List-I		List-II
A.	Improving utilization of supervisory staff	**1.**	Micromotion study
B.	Improving plant layout	**2.**	Work sampling

C. Improving work place layout — **3.** Flow process chamber
D. Improving highly repetitive hand movements — **4.** Chronocyclegraph

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	3	1	4	(b)	3	2	1	4
(c)	2	3	4	1	(d)	3	2	4	1

IES-3. **Which of the following charts are used for plant layout design?** **[IES-1995]**
1. Operation process chart **2. Man machine chart**
3. Correlation chart **4. Travel chart**
Select the correct answer using the codes given below:
Codes: (a) 1, 2, 3 and 4 (b) 1, 2 and 4 (c) 1, 3 and 4 (d) 2 and 3

IES-4. **Which one of the following is correct?** **[IES-2008]**
Production planning and control functions are extremely complex in:
(a) Job-production shop producing small number of pieces only once
(b) Job-production shop producing small number of pieces intermittently
(c) Batch production shop producing a batch only once
(d) Batch production shop producing a batch at irregular intervals

IES-5. **Which one of the following combinations is valid for product layout?**
(a) General purpose machine and skilled labour **[IES-2001]**
(b) General purpose machine and unskilled labour
(c) Special purpose machine and semi-skilled labour
(d) Special purpose machine and skilled labour

IES-6. **The type of layout suitable for use of the concept, principles and approaches of 'group technology' is:** **[IES-1999]**
(a) Product layout (b) Job-shop layout
(c) Fixed position layout (d) Cellular layout

IES-7. **Match List-I (Type of products) with List-II (Type of layout) and select the correct answer.** **[IES-1996]**

List-I	**List-II**
A. Ball bearings	**1.** Process layout
B. Tools and gauges	**2.** Product layout
C. Large boilers	**3.** Combination of product and process layout
D. Motor cycle assembly	**4.** Fixed position layout

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	1	3	4	2	(b)	3	1	4	2
(c)	1	2	4	3	(d)	3	1	2	4

IES-8. **Assertion (A): Product layout is more amenable to automation than process layout.** **[IES-1995]**
Reason (R): The work to be performed on the product is the determining factor in the positioning of the manufacturing equipment in product layout.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false

(d) A is false but R is true

IES-9. Consider the following situations that would warrant a study of the layout: [IES-1994]
1. Change in the work force 2. Change in production volume
3. Change in product design 4. Competition in the market
The situation(s) that would lead to a change in the layout would include:
(a) 1, 2, 3 and 4 (b) 1, 3 and 4 (c) 3 alone (d) 2 alone

IES-10. Assertion (A): In centralized inspection, material handling is less.
Reason (R): Less number of gauges and instruments are required as inspection is carried out in one location. [IES-2008]
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Fixed Position Layout

IES-11. Which one of the following types of layout is used for the manufacture of huge aircrafts? [IES-2003]
(a) Product layout (b) Process layout
(c) Fixed position layout (d) Combination layout

IES-12. Air cargo movements fall under: [IES-1993]
(a) Fixed path system (b) Continuous system
(c) Intermittent system (d) Variable path system

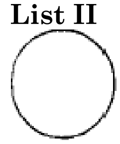
IES-13. Consider the following features/characteristics: [IES-1993]
1. Need for greater variety of skills in labour
2. Intermittent flow of materials and parts
3. Preference for flexible layout
The characteristics of job order layout would include
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

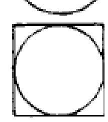
Work Flow Diagram

IES-14. A diagram showing the path followed by men and materials while performing a task is known as [IES-1993]
(a) String Diagram (b) Flow Process Chart
(c) Travel Chart (d) Flow Diagram

IES-15. Match List-I (Activity) with List-II (Symbol) and select the correct answer using the codes given below the lists: [IES-1993]

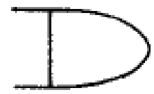
List-I / **List II**

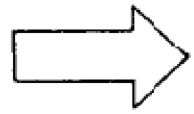
A. A man is doing some productive work — **1.** 

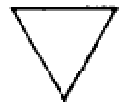
B. A load is moving from one place to another — **2.** 

C. A hand is not accomplishing any thing and is waiting

D. A hand is holding an object

3. 

4. 

5. 

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	1	4	3	5	(b)	1	3	4	5
(c)	3	2	1	4	(d)	3	4	5	2

Flow Process Chart

IES-16. Flow process chart contains [IES-2001]
(a) Inspection and operation
(b) Inspection, operation and transportation
(c) Inspection, operation, transportation and delay
(d) Inspection, operation, transportation, delay and storage

Computerized Techniques for Plant layout:

CORELAP, CRAFT, ALDEP, PLANET, COFAD, CAN-Q

IES-17. Which one of the following software packages is used for plant layout? [IES-1995]
(a) SIMSCRIPT (b) DYNAMO (c) CRAFT (d) MRP

IES-18. Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-1994]

List-I	**List-II**
A. Memory	**1.** Assembler
B. Software for layout	**2.** Buffer
C. Compiler	**3.** GPSS
D. Simulation	**4.** Hardware
	5. CRAFT

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	3	1	4	(b)	3	2	4	5
(c)	4	5	1	3	(d)	2	5	4	3

Previous 20-Years IAS Questions

Plant Layout

IAS-1. Which of the following are the characteristics of job order production? [IAS-2003]
1. **High degree of production control is required**
2. **Division of labour is effective**
3. **Detailed schedule is needed for each component**

4. A flexible layout is preferred
Select the correct answer using the codes given below:
Codes: (a) 1, 3 and 4 (b) 2 and 4 (c) 1 and 3 (d) 3 and 4

IAS-2. Which of the following can be considered to be the advantages of product layout [IAS-1997]
1. Reduced material handling
2. Greater flexibility
3. Lower capital investment
4. Better utilization of men and machines
Codes: (a) 1 and 3 (b) 2 and 3 (c) 1 and 4 (d) 2 and 4

IAS-3. Match List-I (Plant layout) with List-II (Characteristic/Use) and select the correct answer using the code given below the lists: [IAS-2007]

	List-I		**List-II**
A.	Fixed position	**1.**	Avoids back tracking
B.	Functional layout	**2.**	Ship building
C.	Product layout	**3.**	Machines performing similar operations are grouped together
D.	Process layout	**4.**	Helps in reducing total production time

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	4	3	1	(b)	1	3	4	2
(c)	1	4	3	2	(d)	2	3	4	1

IAS-4. Consider the following statements: [IAS-2004]
In designing a plant layout, a "Product Layout" should be preferred if:
1. The variety of the products is low.
2. The variety of the products is very large
3. The quantity of production is very small in each variety
4. The quantity of production is very large in each variety
5. The in-process inspection is maximum
6. The in-process inspection is minimum
Which of the statements given above are correct?
(a) 1, 3 and 6 (b) 1, 4 and 5 (c) 2, 3 and 5 (d) 1, 4 and 6

IAS-5. Which of the following are true of product type layout? [IAS-1996]
1. No flexibility **2. Reduced material handling**
3. Most suitable for low-volume production
Select the correct answer using the codes given below:
Codes: (a) 1, 2 and 3 (b) 1 and 2 (c) 1 and 3 (d) 2 and 3

IAS-6. Assertion (A): A product layout is preferred when the flexibility in sequence of operations is required. [IAS-2002]
Reason (R): Product layout reduces inventories as well as labour cost.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A

(c) A is true but R is false
(d) A is false but R is true

IAS-7. A set of requirement for a product-layout in an industry is: **[IAS-2001]**
(a) General purpose machine and skilled labour
(b) Special purpose machine and skilled labour
(c) General purpose machine and unskilled labour
(d) Special purpose machine and semi-skilled labour

Process Layout

IAS-8. Consider the following limitations: **[IAS-2002]**
1. Movement of machines and equipments for production centre is costly.
2. Long flow lines lead to expensive handling.
3. Breakdown in one machine leads to stoppage of production.
4. Large work-in-process during production.
5. Higher grade skills are required.
Process layout has which of the above limitations?
(a) 1, 2 and 4 (b) 2, 4 and 5 (c) 2 and 3 (d) 1, 4 and 5

IAS-9. Assertion (A): Process layout is preferred for processing non-standard products. **[IAS-2002]**
Reason (R): Process layout has well defined mathematical flow which facilitates mechanized material handling.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-10. To avoid excessive multiplication of facilities, the layout preferred is: **[IAS-1995]**
(a) Product layout (b) Group layout
(c) Static layout (d) Process layout

IAS-11. Consider the following statements: **[IAS-1999]**
A process layout
1. Has machines of same functions arranged in a place.
2. Is suitable for batch production.
3. Has machines of different functions arranged according to processing sequence.
4. Is suitable for mass production.
Of these statements:
(a) 1 and 2 are correct (b) 3 and 4 are correct
(c) 2 and 3 are correct (d) 1 and 4 are correct

IAS-12. The layout suitable for the low demand and high variety product is:
(a) Group layout (b) Process layout **[IAS-1999]**
(c) Product layout (d) Static layout

Fixed Position Layout

IAS-13. The layout of ship-building industry should be: [IAS-2003]
(a) Process layout (b) Group layout
(c) Fixed location layout (d) Product layout

IAS-14. Assertion (A): Fixed position layout is used in manufacturing huge aircrafts, ships, vessels etc. [IAS-1998]
Reason (R): Capital investment is minimum in fixed position layout.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-15. Match List-I (Types of layout) with List-II (Uses) and select the correct answer using the codes given below the lists: [IAS-2001]

List-I	List-II
A. Product layout	**1.** Where a large quantity of products is to be produced
B. Process layout	**2.** Where a large variety of products is manufactured
C. Combined layout	**3.** Where item is being made in different types of sizes
D. Fixed position layout	**4.** Where too heavy or huge item is used as raw material

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	1	2	3	4	(b)	2	1	4	3
(c)	1	2	4	3	(d)	2	1	3	4

IAS-16. Which one of the following pairs is NOT correctly matched?
(a) Job production …. Process layout **[IAS-1999]**
(b) Mass production …. Product layout
(c) Job production …. Special purpose machine
(d) Job production …. Production on order

IAS-17. Match List-I (Type of layout of facilities) with List-II (Application) and select the correct answer using the codes given below the Lists: [IAS-1997]

List-I	List-II
A. Flow-line layout	**1.** Flammable, explosive products
B. Process layout	**2.** Automobiles
C. Fixed position layout	**3.** Aeroplanes
D. Hybrid layout	**4.** Jobshop production

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	4	1	3	2	(b)	2	3	4	1
(c)	2	4	3	1	(d)	2	3	1	4

IAS-18. Match List-I (Type of job) with List-II (Appropriate method-study technique) and select the correct answer using the code given below the lists: [IAS-1995, 2007]

List-I	List-II
A. Complete sequence of operations	**1.** Flow diagram

B. Factory layout: movement of materials — **2.** Flow process chart
C. Factory layout: movement of workers — **3.** Multiple activity chart
D. Gang work — **4.** String diagram

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	4	3	2	1	(b)	2	1	4	3
(c)	4	1	2	3	(d)	2	3	4	1

IAS-19. **Consider the following: [IAS-2004]**

1. **Simplified production planning and control systems**
2. **Reduced material handling**
3. **Flexibility of equipment and personnel**

The advantages of flow-line layout in a manufacturing operation are

(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

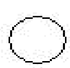
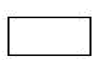
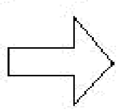
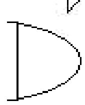
IAS-20. **Match List-I (Equipment) with List-II (Typical situations) and select the correct answer using the codes given below the lists: [IAS-2002]**

List-I	**List II**
A. Conveyor	**1.** Driverless vehicle with varying path
B. Cranes	**2.** Vertical movement of materials
C. Industrial trucks	**3.** Varying paths of movement
D. Lifts	**4.** Movement of intermittent load
	5. Fixed route movement

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	1	3	2	4	(b)	5	4	3	2
(c)	1	4	3	2	(d)	5	3	2	4

Work Flow Diagram

IAS-21. **Match List-I (Symbol) with List-II (Name) and select the correct answer: [IAS-2000]**

List-I	**List-II**
A. 	1. Delay
B. 	2. Transportation
C. 	3. Operation
D. 	4. Inspection

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	4	1	2	(b)	3	4	2	1
(c)	4	3	2	1	(d)	4	3	1	2

IAS-22. A cutter breaks while cutting gears and is removed by the operator. Which of the following represents this activity on the flow process chart? [IAS-2004]

(a) Delay-D (b) Operation-O
(c) Operation cum transportation-$\Rightarrow$ (d) inspection- $\square$

IAS-23. What is a chart that shows the step-by-step procedure used, and the interrelationship between anyone or more persons, forms or products when the procedure involves movement from place to place, called? [IAS-2007]

(a) Multi activity chart
(b) Simo chart
(d) Flow process chart – man analysis
(d) Process chart – combined analysis

Computerized Techniques for Plant layout:

CORELAP, CRAFT, ALDEP, PLANET, COFAD, CAN-Q

IAS-24. Consider the following computer packages: [IAS-2001]

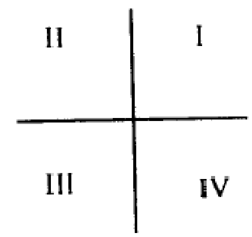
1. CORELAP 2. CRAFT
3. DYNAMO 4. SIMSCRIPT

Which of these packages can be employed for layout analysis of facilities?

(a) 1 and 4 (b) 1 and 2 (c) 2 and 3 (d) 3 and 4

Answers with Explanation (Objective)

Previous 20-Years GATE Answers



GATE-1. Ans. (d) In quadrant I, we can locate any one of the four machines (i.e.) we can allocate quadrant I in 4 ways. Thereafter quadrant II in 3 ways, thereafter quadrant III in 2 ways. No further choice for quadrant IV.

$\therefore$ Total number of possible layouts

$= 4 \times 3 \times 2 = 24$

GATE-2. Ans. Process

GATE-3. Ans. (a)

GATE-4. Ans. (a)

GATE-5. Ans. (d)

Previous 20-Years IES Answers

IES-1. Ans. (c)

IES-2. Ans. (c)

IES-3. Ans. (b)

IES-4. Ans. (d) Production, planning and control function are extremely complex in batch production shop producing a batch at irregular intervals.

IES-5. Ans. (c)

IES-6. Ans. (d)

IES-7. Ans. (a)

IES-8. Ans. (a)

IES-9. Ans. (c)

IES-10. Ans. (b)

IES-11. Ans. (c)

IES-12. Ans. (a) Air cargo movements fall under fixed path system using conveyors.

IES-13. Ans. (a)

IES-14. Ans. (b) A diagram showing the path followed by men and materials while performing a task is known as flow process chart.

IES-15. Ans. (a)

IES-16. Ans. (d)

IES-17. Ans. (c)

IES-18. Ans. (d)

Previous 20-Years IAS Answers

IAS-1. Ans. (b)

IAS-2. Ans. (c)

IAS-3. Ans. (d)

IAS-4. Ans. (d)

IAS-5. Ans. (b)

IAS-6. Ans. (d) A is false. Product layout has lowest flexibility. For flexibility in sequence of operation use process layout.

IAS-7. Ans. (d)

IAS-8. Ans. (a)

IAS-9. Ans. (c) *R is false.* Process layout does not have defined mathematical flow. That so why material handling is costly and lot of back-tracking and cross-movements of work.

IAS-10. Ans. (d)

IAS-11. Ans. (a)

IAS-12. Ans. (b)

IAS-13. Ans. (b)

IAS-14. Ans. (b)

IAS-15. Ans. (a)

IAS-16. Ans. (c)

IAS-17. Ans. (c)

IAS-18. Ans. (a)

IAS-19. Ans. (b)

IAS-20. Ans. (b)

IAS-21. Ans. (b)

IAS-22. Ans. (b)

IAS-23. Ans. (d)

IAS-24. Ans. (b)

10. Control Chart

Theory at a Glance (For IES, GATE, PSU)

Control charts are statistical tool, showing whether a process is in control or not. It is a graphical tool for monitoring the activities of an ongoing process also referred as Shewhart control charts. Control charts are used for process monitoring and variability reduction.

Before discussing and calculating the limits etc. of control charts, it is necessary to understand causes of variations present in the system. Variability is an inherent feature of every process. Production data always have some variability.

Causes of Variations

Two types of causes are present in the production system

- **Special causes:** Variation due to identifiable factors in the production process. Examples of special causes include: wrong tool, wrong production method, improper raw material, operator's skill, wrong die etc. Control of process is achieved through the elimination of special causes. According to Deming, only 15% of the problems are due to the special causes. Special causes or also sometimes referred as **Assignable causes**
- **Common causes:** Variation inherent in the process. Improvement of process is accomplished through the reduction of common causes and improving the system. According to Deming, 85% of the problems are due to the common causes.

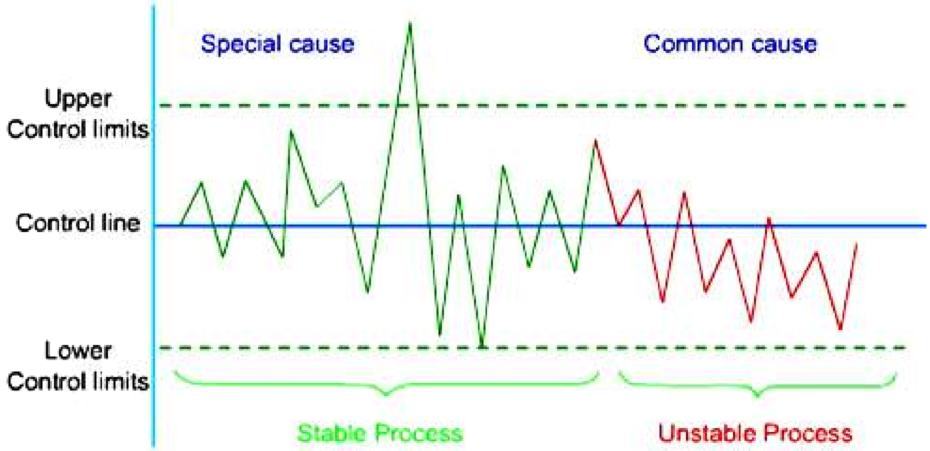
Assignable causes are controlled by the use of statistical process charts.

Steps in constructing a control chart

- Decide what to measure or count
- Collect the sample data
- Plot the samples on a control chart
- Calculate and plot the control limits on the control chart
- Determine if the data is in control
- If non-random variation is present, discard the data (fix the problem) and recalculate the control limits
- When data are with in the control limits we leave the process assuming there are only chance causes present

A process is in control IF

- No sample points outside control limits
- Most points near process average or center line
- About equal number of points above and below the center line
- Sample point are distributed randomly



Control Chart Representing Limits, Special Causes, Common Causes

Types of Process Data

Two types of process data:

1. **Variable:** Continuous data. Things we can measure. Example includes length, weight, time, temperature, diameter, etc.
2. **Attribute:** Discrete data. Things we count. Examples include number or percent defective items in a lot, number of defects per item etc.

Types of Control Charts

The classification of control charts depend upon the type of data.

1. **Variable charts:** are meant for variable type of data. X bar and R Chart, X bar and sigma chart for the individual units
2. **Attribute chats:** are meant for attribute type of data. p chart, np chart, c chart, u chart, U chart. For remember [CPU]

$\bar{X}$ – Chart and R – Chart

Control charts for the variable type of data (*X* bar and *R* charts)

In the x bar chart the sample means are plotted in order to control the mean value of a variable. In R chart, the sample ranges are plotted in order to control the variability of a variable.

Centre line, upper control limit, lower control limit for x bar and R charts are calculated. The formulae used are as following:

$$\overline{X_i} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$\overline{X_i}$ = mean of the i^{th} sample; n = sample size; X_i = i^{th} data

$$R_i = X_{\max(i)} - X_{\min(i)}$$

R_i = range of i^{th} sample
$X_{max(i)}$ = maximum value of the data in i^{th} sample
$X_{min(i)}$ = minimum value of the data in i^{th} sample

$$\overline{R} = \frac{\sum_{i=1}^{g} R_i}{g}$$

$\overline{R}$ = mean of g samples

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{g} \overline{X_i}}{g} = CL_{\overline{X}}$$ **(Centre line for X bar chart)**

$\overline{\overline{X}}$ = mean of mean of g samples
g = number of samples

$$UCL_{\overline{X}} = \overline{\overline{X}} + 3\sigma_x$$

σ_x = standard deviation of samples

$$\sigma_x = \frac{\hat{\sigma}}{\sqrt{n}}$$

$\hat{\sigma} = \frac{R}{d_2}$ = estimate of standard deviation of population
d_2 = parameter depends on sample size n

$$UCL_{\overline{X}} = \overline{\overline{X}} + \frac{3\hat{\sigma}}{\sqrt{n}} = \overline{\overline{X}} + \frac{3\overline{R}}{\sqrt{n}\, d_2}$$

$VCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R}$ **(Upper control limit for X bar chart)**

$A_2 = \frac{3}{d_2\sqrt{n}} =$ Parameter depends on sample size
value of A_2 can be directly obtained from the standard tables
$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$ **(Lower control limit for X bar chart)**

$$UCL_R = \overline{R} + 3\hat{\sigma}R,\ \hat{\sigma}R = d_3\hat{\sigma} = d_3\frac{\overline{R}}{d_2}$$

$UCL_R = D_4\overline{R}$ where $D_4 = \overline{R}\left(1 + \frac{3d_3}{d_2}\right)$ **(Upper control limit for R chart)**

$UCL_R = D_3\overline{R}$ where $D_3 = \overline{R}\left(1 - \frac{3d_3}{d_2}\right)$ **(Lower control limit for R chart)**

Example: Mean values and ranges of data from 20 samples (sample size = 4) are shown in the table below:

S. No.	Mean of Sample	Range	S. No.	Mean of Sample	Range	S. No.	Mean of Sample	Range	S. No.	Mean of Sample	Range	S. No.	Mean of Sample	Range
1	10	4	5	9	5	9	10	4	13	12	4	17	12	4
2	15	4	6	11	6	10	11	6	14	12	3	18	15	3
3	12	5	7	11	4	11	12	5	15	11	3	19	11	3
4	11	4	8	9	4	12	13	4	16	15	4	20	10	4

Sum of mean of 20 samples = $\sum_{i=1}^{20} \overline{X} = 232$

Average of mean values of 20 samples = $\dfrac{\sum_{i=1}^{20} \overline{X}}{20} = 11.6$ (Center Line of X bar Chart)

Average of Ranges of 20 samples = $\dfrac{\sum_{i=1}^{20} R}{20} = 4.15$ (Center Line of R Chart)

Upper Control Limit of X bar chart = 11.6 + A_2 4.15 (A_2 = 0.729 for sample size 4)

= 14.63

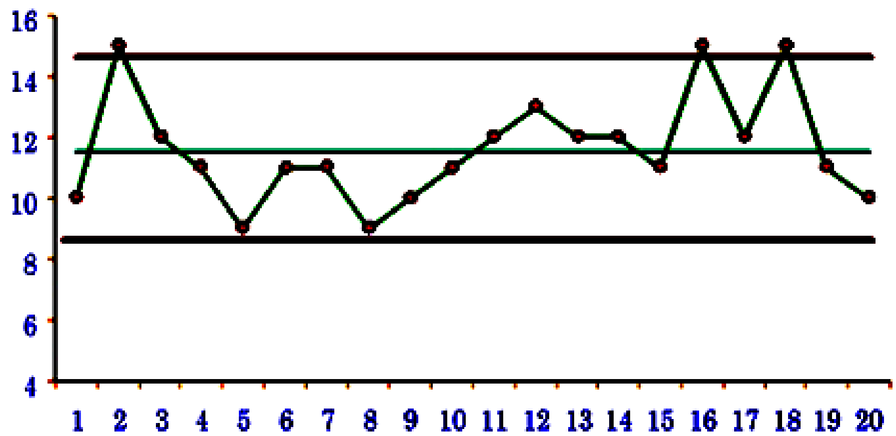
Lower Control Limit of X bar chart = 11.6 – A_2 4.15 (A_2 = 0.729 for sample size 4)

= 8.57

Upper Control Limit of R chart = D_3 4.15 (D_3 = 2.282 for sample size 4)

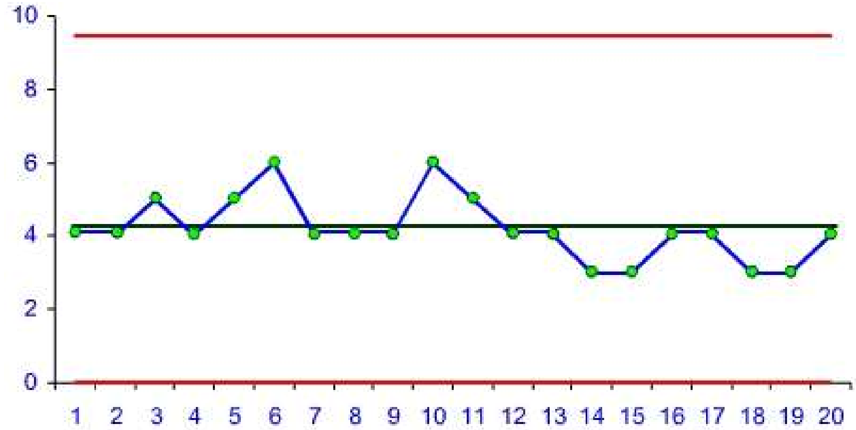
= 9.47 ≈ 9.5

Lower Control Limit of R chart = D_4 4.15 (D_4 = 0 for sample size 4)



X-Bar Chart

Sample data at S.N 2, 16, and 18 are slightly above the UCL. Efforts must be made to find the special causes and revised limits are advised to calculate after deleting these data.



R Chart

All the data are within the LCL and UCL in R Chart. Hence variability of the process data is not an issue to worry.

C – Chart and P – Chart

Control charts for Attribute type data (p, c, u charts)

p-charts calculates the percent defective in sample. p-charts are used when observations can be placed in two categories such as yes or no, good or bad, pass or fail etc.

c-charts counts the number of defects in an item. c-charts are used only when the number of occurrence per unit of measure can be counted such as number of scratches, cracks etc.

u-chart counts the number of defect per sample. The u chart is used when it is not possible to have a sample size of a fixed size.

For attribute control charts, the estimate of the variability of the process is a function of the process average.

Centre line, upper control limit, lower control limit for c, p, and u charts are calculated. The formulae used are as following:

p-chart formulae

$$\overline{p} = \frac{\text{Sum of defectivess peice in all samples}}{\text{Total number of items in all samples}}$$ **= centre line of p chart**

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \quad \text{and} \quad LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

Where n is the sample size. Sample size in p chart must be ≥ 50
Sometimes LCL in p chart becomes negative, in such cases LCL should be taken as 0

c-chart formulae

$$\overline{c} = \frac{\text{Sum of defects in all samples}}{\text{Total number of samples}} = \text{Centre line of } c \text{ chart}$$

$$UCL = \overline{c} + 3\sqrt{\overline{c}} \quad \text{and} \quad LCL = \overline{c} - 3\sqrt{\overline{c}}$$

u-chart formulae

$$\bar{u} = \frac{\text{Sum of defects in all samples}}{\text{Total number of items in all samples}} = \frac{\sum_{i=1}^{k} c_i}{\sum_{i=1}^{k} n_i}$$

c_i = Number of defects in i^{th} sample
k = Number of samples
n_i = Size of i^{th} samples

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}} \quad \text{and} \quad LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$$

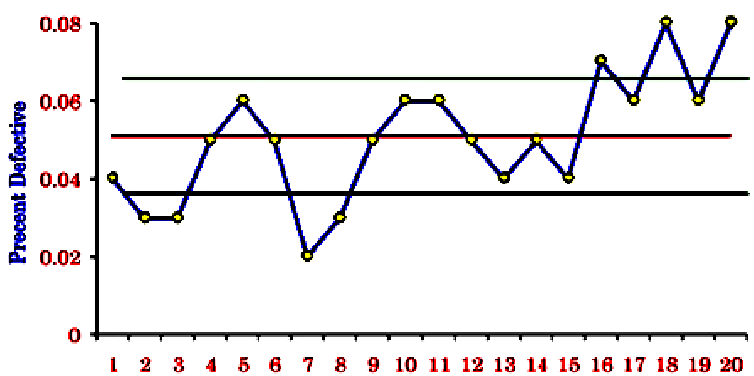
Example: p-chart
Data for defective CDs from 20 samples (sample size = 100) are shown in the table below:

Sample No.	No. of Defective CDs = *x*	Proportion Defective = *x*/sample size	Sample No.	No. of Defective CDs = *x*	Proportion Defective = *x*/sample size
1	4	0.04	11	6	0.06
2	3	0.03	12	5	0.05
3	3	0.03	13	4	0.04
4	5	0.05	14	5	0.05
5	6	0.06	15	4	0.04
6	5	0.05	16	7	0.07
7	2	0.02	17	6	0.06
8	3	0.03	18	8	0.08
9	5	0.05	19	6	0.06
10	6	0.06	20	8	0.08

$$CL = \frac{\text{Sum of defectives}}{\text{Sum of all samples}} = \frac{101}{2000} = 0.051$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.051 + 3\sqrt{\frac{0.051(1-0.051)}{100}} = 0.066$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{N}} = 0.051 - 3\sqrt{\frac{0.051(1-0.051)}{100}} = 0.036$$



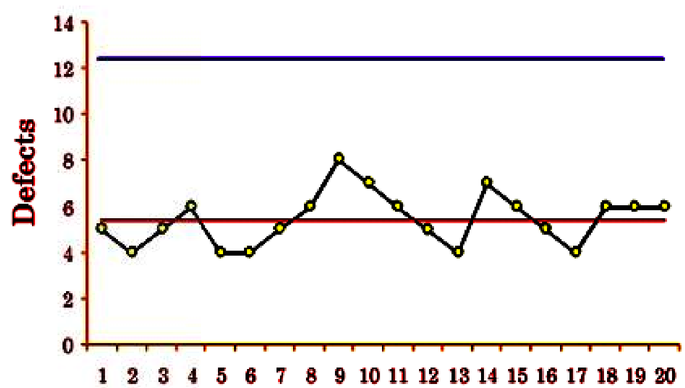
p-Chart

Sample data at S.N 16, 18, and 20 are above the UCL. Efforts must be made to find the special causes and revised limits are advised to calculate after deleting these data. There is important observation that is clearly visible from the data points that there is an increasing trend in the average proportion defectives beyond sample number15 also, data show cyclic pattern. Process appears to be out of control and also there is a strong evidence that data are not from independent source.

Example: c-chart
Data for defects on TV set from 20 samples (sample size = 10) are shown in the table below:

Sample No.	No. of Defects	Sample No.	No. of Defects	Sample No.	No. of Defects	Sample No.	No. of Defects
1	5	6	4	11	6	16	5
2	4	7	5	12	5	17	4
3	5	8	6	13	4	18	6
4	6	9	8	14	7	19	6
5	4	10	7	15	6	20	6

$$CL = \frac{\text{Sum of defects}}{\text{Number of samples}}$$
$$= \frac{109}{20} = 5.45$$
$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 5.45 + 3\sqrt{5.45}$$
$$= 12.45$$
$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 5.45 - 3\sqrt{5.45}$$
$$= -1.55 = 0$$



C-Chart

None of the sample is out of the LCL and UCL. But the chart shows cyclic trend.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Quality Analysis and Control

GATE-1. Statistical quantity control was developed by: [GATE-1995]

(a) Frederick Taylor (b) Water shewhart
(c) George Dantzig (d) W.E. Deming

GATE-2. Match the following quantity control objective functions with the appropriate statistical tools: [GATE-1992]

Objective functions		**Statistical Tools**	
A.	A casting process is to be controlled with respect to hot tearing tendency	**P.**	X-chart
B.	A casting process is to be controlled with respect to the number of blow holes, of any, produced per unit casting	**Q.**	c-chart
		R.	Random sampling
C.	A machining process is to be controlled with respect to the diameter of shaft machined	**S.**	p-chart
D.	The process variability in a milling operation is to be controlled with respect to the surface finish of components	**T.**	Hypothesis testing
		U.	R-charts

GATE-3. In a weaving operation, the parameter to be controlled is the number of defects per 10 square yards of material. Control chart appropriate for, his task is: [GATE-1998]

(a) P-chart (b) C-chart (c) R-chart (d) X-chart

Previous 20-Years IES Questions

Quality Analysis and Control

IES-1. Match List-I (Quality control concepts) with List-II (Quality control techniques) and select the correct answer using the codes given below the lists: [IES-2004]

List-I		**List-II**	
A.	Tightened and reduced inspection	**1.**	Dodge Romig tables
B.	Lot tolerance percent defective	**2.**	Control chart for variables
C.	Poisson distribution	**3.**	MIL standards
D.	Normal distribution	**4.**	Control chart for number of nonconformities

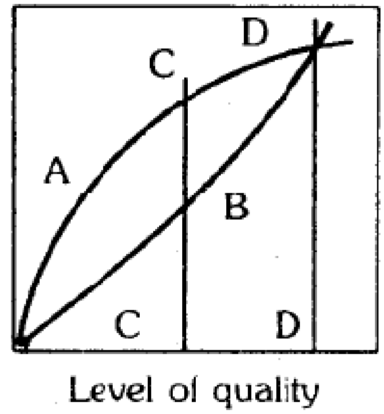
Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	1	4	3	(b)	3	4	1	2
(c)	3	1	4	2	(d)	2	4	1	3

IES-2. Quality control chart for averages was maintained for a dimension of the product. After the control was established, it was found that the standard deviation (σ) of the process was 1.00 mm The

dimension of the part is 70 ± 2.5 mm. Parts above 72.5 mm can be reworked but parts below 67.5 mm have to be scrapped. What should be the setting of the process to ensure production of no scrap and to minimize the rework? [IES-2004]

(a) 68.5 mm (b) 70 mm (c) 70.5 mm (d) 72.5 mm

IES-3. **The graph shows the results of various quality levels for a component**



Consider the following statements:

1. **Curve A shows the variation of value of component**
2. **Curve B shows the variation of cost of the component**
3. **Graph is called as fish bone diagram**
4. **The preferred level of quality is given by line CC**
5. **The preferred level of quality is given by line DD**

[IES-2002]

Which of the above statements are correct?

(a) 1, 2 and 5 (b) 1, 3 and 4 (c) 2, 3 and 4 (d) 1, 2 and 4

IES-4. **Match List-I (Scientist) with List-II (Research work) and select the correct answer using the codes given below the lists: [IES-2000]**

List-I	List-II
A. Schewart	**1.** Less function in quality
B. Taguchi	**2.** Queuing model
C. Erlang	**3.** Zero defects
	4. Control charts

Codes:	A	B	C		A	B	C
(a)	3	1	2	(b)	4	3	1
(c)	4	1	2	(d)	3	4	1

IES-5. **Assertion (A): In case of Control Chart for fraction rejected (p-chart), binomial distribution is used. [IES-2008]**
Reason (R): In binomial distribution probability of the event varies with each draw.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-6. **Assertion (A): In case of Control Charts for variables, the averages of sub-groups of readings are plotted instead of plotting individual readings. [IES-2008]**
Reason (R): It has been proved through experiments that averages will form normal distribution

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-7. **Assertion (A): In case of control charts for variables, the average of readings of a subgroup of four and more is plotted rather than the individual readings.** **[IES-2000, 2003]**
Reason (R): Plotting of individual readings needs a lot of time and effort.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-8. **The span of control refers to the** **[IES-2003]**
(a) Total amount of control which can be exercised by the supervisor
(b) Total number of persons which report to any- one supervisor
(c) Delegation of authority by the supervisor to his subordinates
(d) Delegation of responsibility by the supervisor to his subordinates

IES-9. **Consider the following statements:** **[IES-2001]**
Control chart of variables provides the
1. Basic variability of the quality characteristic.
2. Consistency of performance.
3. Number of products falling outside the tolerance limits.
Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-10. **Which one of the following steps would lead to interchangeability?**
(a) Quality control (b) Process planning **[IES-1994]**
(c) Operator training (d) Product design

IES-11. **Match List-I (Trend/Defect) with List-II (Chart) and select the correct answer using the codes given below the lists:** **[IES-2003]**

List-I	**List II**
A. Trend	**1.** R-Chart
B. Dispersion	**2.** C-Chart
C. Number of defects	**3.** $\overline{X}$-Chart
D. Number of defectives	**4.** np-Chart
	5. u-Chart

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	5	3	2	4	(b)	3	1	4	2
(c)	3	1	2	4	(d)	3	4	5	2

IES-12. **Assertion (A): In case of control charts for variables, if some points fall outside the control limits, it is concluded that the process is not under control.** **[IES-1999]**
Reason (R): It was experimentally proved by Shewhart that averages of four or more consecutive readings from a universe (population) or from a process, when plotted, will form a normal distribution curve.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-13. **Consider the following statements with respect to control charts for attributes:** **[IES-2004]**

1. The lower control limit is non-negative
2. Normal distribution is the order for this data
3. The lower control limit is not significant
4. These charts give the average quality characteristics

Which of the statements given above are correct?

(a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 4 (d) 1, 2 and 4

IES-14. **If in a process on the shop floor, the specifications are not met, but the charts for variables show control, then which of the following actions should be taken?** **[IES-2009]**

(a) Changes the process
(b) Change the method of measurement
(c) Change the worker or provide him training
(d) Change the specifications or upgrade the process

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (b) Dr. Waher Shewhart an American scientist during World War-II.
GATE-2. Ans. A – S, B – Q, C – P, D – U
GATE-3. Ans. (b)

Previous 20-Years IES Answers

IES-1. Ans. (c)
IES-2. Ans. (c) Standard deviation from mean = 0.5 mm
Therefore for no scrap D = 70 + 0.5 = 70.5 mm
IES-3. Ans. (c)
IES-4. Ans. (c)
IES-5. Ans. (c)
IES-6. Ans. (a)
IES-7. Ans. (b)
IES-8. Ans. (d)
IES-9. Ans. (a)
IES-10. Ans. (a) Quality control leads to interchangeability.
IES-11. Ans. (c)
IES-12. Ans. (b)
IES-13. Ans. (c)
IES-14. Ans. (c)

11. Sampling, JIT, TQM, etc.

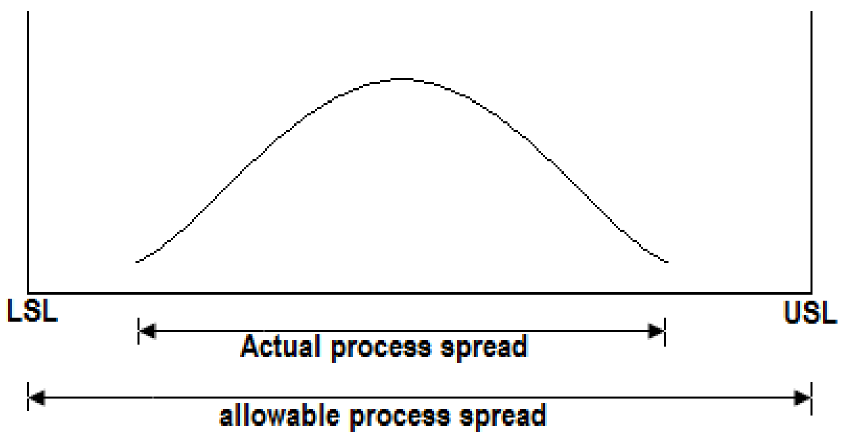
Theory at a Glance (For IES, GATE, PSU)

Process Capability

Process capability compares the output of an in-control process to the specification limits by using capability indices.

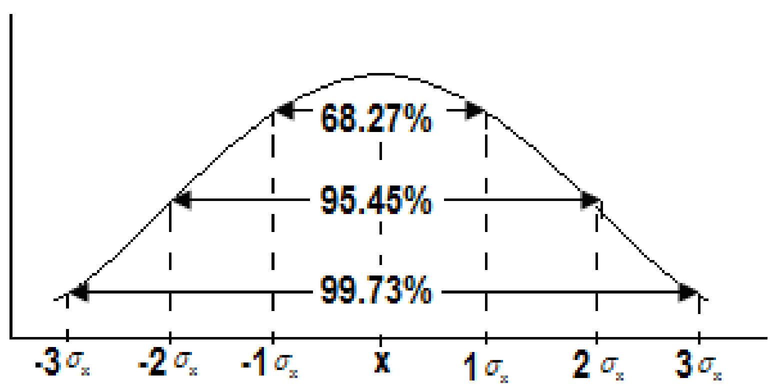
Capability Indices: A process capability index uses both the process variability and the process specifications to determine whether the process is 'capable'.

Capable Process: A capable process is one where almost all the measurements fall inside the specification limits. This can be represented pictorially by the plot below.



Work Sampling

Curve of normal distribution



To make things easier we speak 95% confidence level than 95.45%.

Remember:

95% confidence level or 95% of the area under the curve = 1.96

99% confidence level or 99% of the area under the curve = 2.58

99.9% confidence level or 99.9% of the area under the curve = 3.3

Standard error of proportion (σ_p) = $\sqrt{\dfrac{pq}{n}}$

Where, p = percentage of idle time

q = percentage of working time = $(1 - p)$

n = number of observation

So you may use this equation as (σ_p) = $\sqrt{\dfrac{p(1-p)}{n}}$ also.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Sampling Plan (Single, Double, Sequential Sampling Plan)

GATE-1. **In carrying out a work sampling study in a machine shop, it was found that a particular lathe was down for 20% of the time. What would be the 95% confidence interval of this estimate if 100 observations were made? [GATE-2002]**
(a) 0.16, 0.24 (b) 0.12, 0.28
(c) 0.08, 0.32 (d) None of these

GATE-2. **Preliminary work sampling studies show that machine was idle 25% of the time based on a sample of 100 observations. The number of observations needed for a confidence level of 95% and an accuracy of ± 5% is: [GATE-1996]**
(a) 400 (b) 1200 (c) 3600 (d) 4800

Just in Time (JIT)

GATE-3. [GATE-1995]

List-I (Problem areas)	**List-II (Techniques)**
A. JIT	**1.** CRAFT
B. Computer assisted layout	**2.** PERT
C. Scheduling	**3.** Johnson's rule
D. Simulation	**4.** Kanbans
	5. EOQ rule
	6. Monte Carlo

Previous 20-Years IES Questions

Process Capability

IES-1. **Which one of the following correctly explains process capability? [IES-1998]**
(a) Maximum capacity of the machine
(b) Mean value of the measured variable
(c) Lead time of the process
(d) Maximum deviation of the measured variables of the components

IES-2. **Process capability of a machine is defined as the capability of the machine to: [IES-1993]**
(a) Produce a definite volume of work per minute
(b) Perform definite number of operations
(c) Produce job at a definite spectrum of speed
(d) Hold a definite spectrum of tolerances and surface finish

Operation Characteristic Curve (OC Curve)

IES-3. Match List-I with List-II and select the connect answer using the codes given below the lists: [IES-2001]

List-I		**List-II**	
A.	OC Curve	**1.**	Acceptance sampling
B.	AOQL	**2.**	Dodge Roming table
C.	Binomial distribution	**3.**	p-charts
D.	Normal curve	**4.**	Control charts for variables

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	1	2	3	4	(b)	1	3	2	4
(c)	4	2	3	1	(d)	4	3	2	1

IES-4. The curve representing the level of achievement with reference to time is known as [IES-2002]
(a) Performance curve (b) Operating characteristic curve
(c) S-curve (d) Learning curve

IES-5. An operating characteristic curve (OC curve) is a plot between
(a) Consumers' risk and producers' risk **[IES-2009]**
(b) Probability of acceptance and probability of rejection
(c) Percentage of defective and probability of acceptance
(d) Average outgoing quality and probability of acceptance

Sampling Plan (Single, Double, Sequential Sampling Plan)

IES-6. Assertion (A): Double sampling is preferred over single sampling when the quality or incoming lots is expected to be either very good or very bad. [IES-2000]
Reason (R): With double sampling, the amount of inspection required will be lesser than that in the case of single sampling.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-7. Which one of the following statements is not correct? [IES-2008]
(a) The operating characteristic curve of an acceptance sampling plan shows the ability of the plan to distinguish between good and bad lots.
(b) No sampling plan can give complete protection against the acceptance of defective products.
(c) C-chart has straight line limits and U chart has zig-zag limits.
(d) Double sampling results in more inspection than single sampling if the incoming quality is very bad.

IES-8. Which one of the following is not the characteristic of acceptance sampling? [IES-2007]
(a) This is widely suitable in mass production
(b) It causes less fatigue to inspectors
(c) This is much economical
(d) It gives definite assurance for the conformation of the specifications for all the pieces

IES-9. Assertion (A): Sampling plans with acceptance number greater than zero are generally better than sampling plans with acceptance number equal to zero. [IES-2006]

Reason (R): Sampling plans with acceptance number greater than zero have a larger sample size as compared to similar sampling plans with acceptance number equal to zero.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-10. Assertion (A): In attribute control of quality by sampling, the sample size has to be larger than variable control. [IES-2005]
Reason (R): Variables are generally continuous, and attributes have few discrete levels.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-11. Consider the following statements in respect of double sampling plan: [IES-2003]
1. **Average number of pieces inspected is double that of single sampling**
2. **Average number of pieces inspected is less than that for single sampling**
3. **Decision to accept or reject the lot is taken only after the inspection of both samples**
4. **Decision to accept or reject the lot is reached sometimes after one sample and sometimes after two samples**

Which of these statements are correct?
(a) 1, 2 and 3 (b) 2 and 4 (c) 1 and 4 (d) 2 and 3

IES-12. Assertion (A): In dodge romig sampling tables, the screening inspection of rejected lots is also included. [IES-2001]
Reason (R): Dodge romig plans are indexed at an LTPD of 10%
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-13. The product is assembled from parts A and B. The probability of defective parts A and B are 0.1 and 0.2 respectively. Then the probability of the assembly of A and B to be non defective is: [IES-1992]
(a) 0.7 (b) 0.72 (c) 0.8 (d) 0.85

IES-14. A control chart is established with limits of ± 2 standard errors for use in monitoring samples of size n = 20. Assume the process to be in control. What is the likelihood of a sample mean falling outside the control limits? [IES-2005]
(a) 97.7% (b) 95.5% (c) 4.5% (d) 2.3%

IES-15. In a study to estimate the idle time of a machine, out of 100 random observations the machine was found idle on 40 observations. The

total random observations required for 95% confidence level and ± 5% accuracy is: [IES-2001]
(a) 384 (b) 600 (c) 2400 (d) 9600

IES-16. **The management is interested to know the percentage of idle time of equipment. The trial study showed that percentage of idle time would be 20%. The number of random observations necessary for 95% level of confidence and ± 5% accuracy is: [IES-2000]**
(a) 6400 (b) 1600 (c) 640 (d) 160

IES-17. **If one state occurred four times in hundred observations while using the work-sampling technique, then the precision of the study using a 95% confidence level will be: [IES-1997]**
(a) 90% (b) 92% (c) 95% (d) 98%

IES-18. **For a confidence level of 95% and accuracy ± 5%, the number of cycles to be timed in a time study is equal to: [IES-2009]**

$$\left[k\sqrt{\frac{N\sum X^2 - (\sum X)^2}{\sum X}} \right]^2$$

Where, N = Number of observations taken; $X = X_1, X_2, \ldots, X_N$ are individual observations. What is the value of K?
(a) 10 (b) 20 (c) 30 (d) 40

Previous 20-Years IAS Questions

Just in Time (JIT)

IAS-1. **Match List-I with List-II and select the correct answer: [IAS-2000]**

	List-I		List-II
A.	Just-in-time inventory and procurement techniques	**1.**	Intermittent production
B.	Work-in-progress likely to be low compared to output	**2.**	Repetitive production
C.	Scheduling typically most complex processes	**3.**	Continuous production
D.	Flexible manufacturing cell	**4.**	Job production

Codes:	A	B	C	D		A	B	C	D
(a)	2	3	4	1	(b)	3	2	1	4
(c)	2	3	1	4	(d)	3	2	4	1

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (b)

GATE-2. Ans. (d) $S \times P = K\sqrt{\dfrac{P(1-P)}{N}}$

Where, K = 2 for 95% confidence level
S = 0.05 (accuracy)
P = 0.25 (idle time)

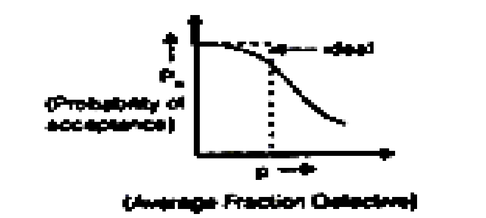
Number of observations needed for 95% confidence level and ± 5% accuracy is:

$$N = \frac{K^2(P)\,(1-P)}{S^2P^2} = \frac{(2)^2\,(0.25)(1-0.25)}{(0.05)^2\;(0.25)^2} = 4800$$

GATE-3. Ans. (a) – 4, (B) – 1, (C) – 3, (D) – 6

Previous 20-Years IES Answers

IES-1. Ans. (d)
IES-2. Ans. (d) Process capability of a machine is defined as the capability of the machine to hold a definite spectrum of tolerances and surface finish.
IES-3. Ans. (a)
IES-4. Ans. (a)
IES-5. Ans. (c) OC Curve (Operating Characteristic Curve)
∴ OC CURVE is a plot between percentage of defective and probability of acceptance.



OC CURVE (OPERATING CHARACTERISTIC CURVE)

IES-6. Ans. (d)
IES-7. Ans. (d)
IES-8. Ans. (d)
IES-9. Ans. (d)
IES-10. Ans. (a)
IES-11. Ans. (b)
IES-12. Ans. (b)
IES-13. Ans. (b) It is a case of mutually independent events. Probability that the part A is non-defective = 1 – 0.1 = 0.9

Probability that Part-B is non-defective, = 1 – 0.2 = 0.8

Hence, probability that both Part-A and Part-B are non-defective = 0.9 × 0.8 = 0.72.

IES-14. Ans. (c)

IES-15. Ans. (c) $1.96\sigma_p = 5$ or, $\sigma_p = 2.5 = \sqrt{\dfrac{40 \times 60}{n}}$ or, $n \approx 384$

IES-16. Ans. (a) $1.96\sigma_p = 5 \quad \text{or}, \sigma_p = 2.5 = \sqrt{\dfrac{20 \times 80}{n}} \quad \text{or}, n \approx 160$

IES-17. Ans. (d) Accuracy = $\sqrt{\dfrac{p(1-p)}{N}}$ for 95% confidence level = $\sqrt{\dfrac{0.05 \times 0.95}{100}} \approx 0.02 = 2\%$

$\therefore$ Precision = 98%

IES-18. Ans. (d) For 95% confidence level

$K = 2$

S = Accuracy = 0.05

$K = \dfrac{K}{S} = \dfrac{2}{0.05} = 40$

Previous 20-Years IAS Answers

IAS-1. Ans. (a)

Conventional Questions with Answer

Conventional Question **[ESE-2007]**

From the data of a pilot study, the percentage of occurrence of an activity is 60%. Find the number of observations for 95% confidence level and an accuracy of ± 2%. **[2 Marks]**

Solution: The formulae for determining the number of observations is

$$\mathbf{S_p = K\sqrt{\frac{p(1-p)}{N}}}$$

or $\mathbf{N = K^2 \dfrac{p(1-p)}{S_p^2}}$ where

S = the desired relative accuracy

$\mathbf{(\pm 2\% = \pm 0.02)}$

$\mathbf{S_p}$ = the desired absolute accuracy

P = percentage accuracy of an activity of interest or a classification being measured. (60% = 0.6)

N = total number of random observations.

$$\mathbf{\therefore N = \frac{2^2 \times 0.6(1-0.6)}{(0 \pm 0.02 \times 0.6)^2}}$$

$$\mathbf{= \frac{4 \times 0.6 \times 0.4}{(\pm 0.012)^2}}$$

$$\mathbf{= \frac{0.96}{+0.000144} \simeq 6667}$$

12. Graphical Method

Theory at a Glance (For IES, GATE, PSU)

What is LPP (Q-ESE)

Linear programming is a technique which allocates scare available resources under conditions of certainty in an optimum manner, (i.e. maximum or minimum) to achieve the company objectives which may be maximum overall profit or minimum overall cost.

Linear programming deals with the optimization (maximization or minimization) of linear functions subjects to linear constraints.

One L.P.P

Maximize $(z) = 3x_1 + 4x_2$ (i)

Subject to $4x_1 + 2x_2 \geq 80$ (ii)

$2x_1 + 5x_2 \leq 180$ (iii)

$x_1, x_2 \geq 0$ (iv)

1. The variables that enter into the problem are called ***decision variables.*** e.g., x_1, x_2.
2. The expression showing the relationship between the manufacture's goal and the decision variables is called the ***objective function.*** e.g. $z = 3x_1 + 4x_2$ (maximize).
3. The inequalities (ii); (iii); (iv) are called ***constraints*** being all linear, it is a linear programming problem (L.P.P).This is an example of a real situation from industry.

Graphical Method

Working Procedure:

Step-1: Formulate the given problem as a linear programming problem.

Step-2: Plot the given constraints as equalities on $x_1.x_2$ co-ordinate plane and determine the convex region formed by them.

[A region or a set of points is said to be convex if the line joining any two of its points lies completely in the region (or the set)]

Step-3: Determine the vertices of the convex region and find the value of the objective function and find the value of the objective function at each vertex. The vertex which gives the optimal value of the objective function gives the desired optimal solution the problem.

Otherwise:

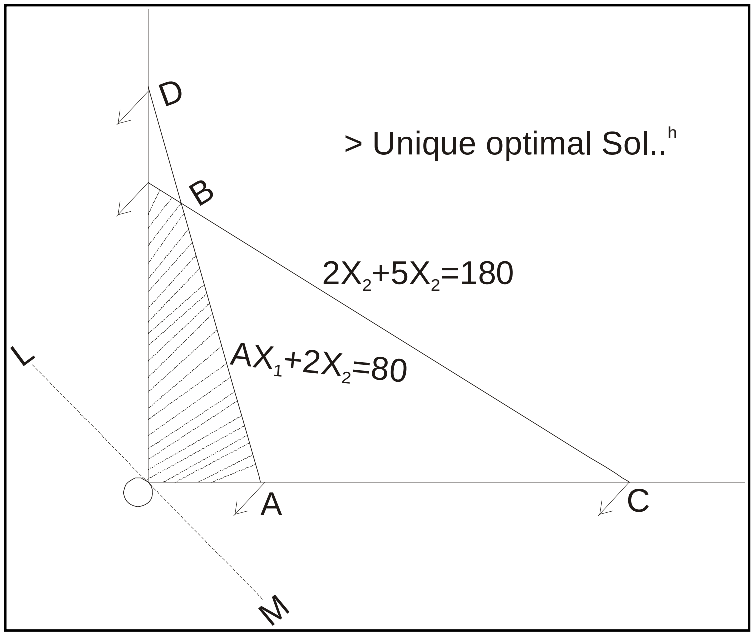
Draw a dotted line through the origin representing the objective function with $z = 0$. As z is increased from zero, this line moves to the right remaining parallel to itself. We go on sliding this line (parallel to itself), till it is farthest away from the origin and passes through only one vertex on the convex region. This is the vertex where the maximum value of z is attained.

When it is required to minimize z_n value z is increased till the dotted line passes through the nearest vertex of the convex region.

Example: Maximize $z = 3x_1 + 4x_2$

Subject to $4x_1 + 2x_2 \geq 80$
$2x_1 + 5x_2 \leq 180$
$x_1, x_2 \geq 0$

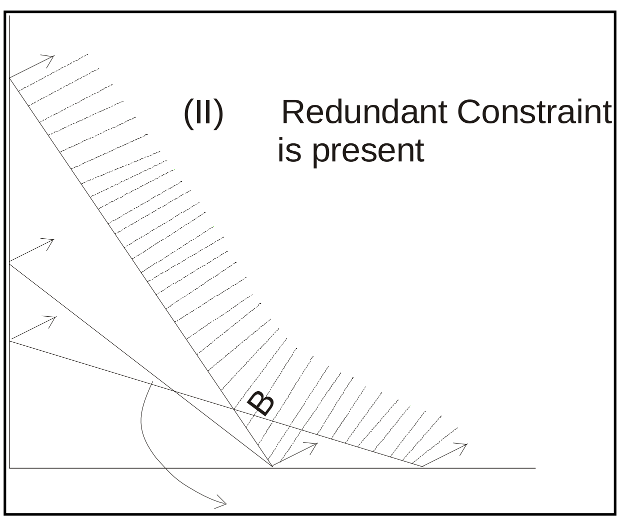
1.



Here farthest point B(2.5, 35) therefore it is the solution
$x_1 = 2.5$, $x_2 = 35$ and $z_{max} = 147.5$

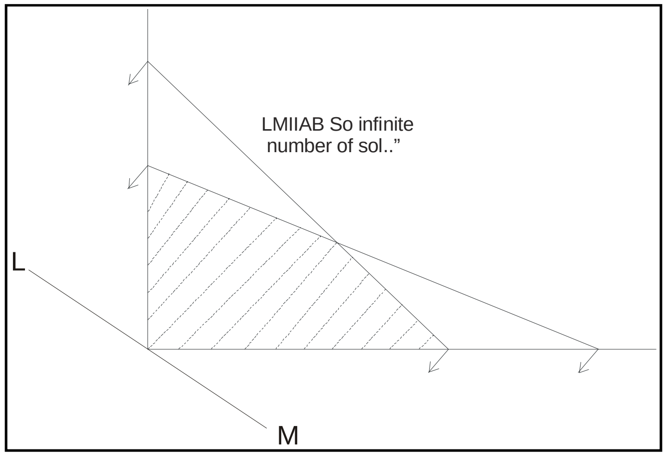
2. **Note:** dotted line parallel to the line LM is called the **iso-cost line** since it represents all possible combinations of x_1, x_2 which produce the same total cost.

3. A minimization problem

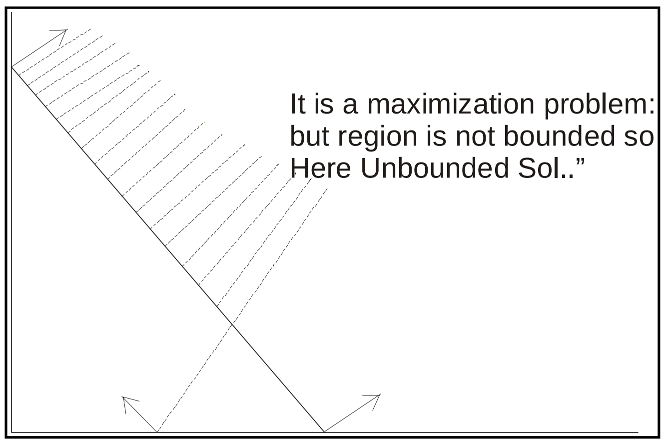


B is solution but this is redundant Constraint as it dominated by another two constraints.

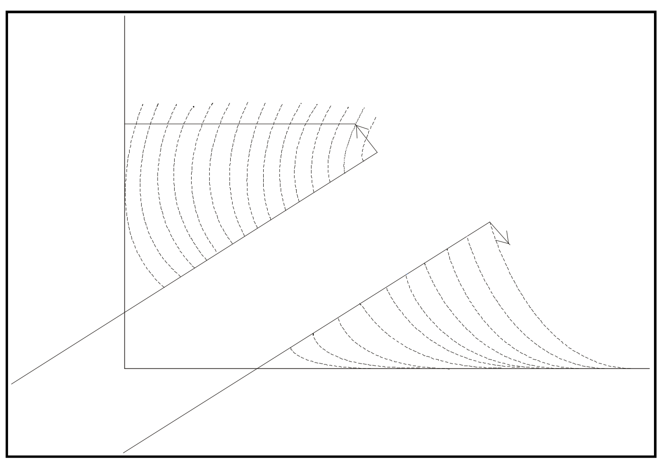
4.



5.



6.



Here the constraints were incompatible. So, there is no solution.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. The first algorithm for Linear Programming was given by: **[GATE-1999]**

(a) Bellman (b) Dantzig (c) Kulm (d) Van Neumann

GATE-2. If at the optimum in a linear programming problem, a dual variable corresponding to a particular primal constraint is zero, then it means that **[GATE-1996]**

(a) Right hand side of the primal constraint can be altered without affecting the optimum solution
(b) Changing the right hand side of the primal constraint will disturb the optimum solution
(c) The objective function is unbounded
(d) The problem is degenerate

GATE-3. Consider the following Linear Programming Problem (LPP):

Maximize $z = 3x_1 + 2x_2$ **[GATE -2009]**

Subject to $x_1 \le 4$

$x_2 \le 6$

$3x_1 + x_2 \le 18$

$x_2 \ge 0, x_2 \ge 0$

(a) The LPP has a unique optimal solution
(b) The LPP is infeasible
(c) The LPP is unbounded
(d) The LPP has multiple optimal solutions.

Graphical Method

GATE-4. A manufacturer produces two types of products, 1 and 2, at production levels of x_1 and x_2 respectively. The profit is given is $2x_1 + 5x_2$. The production constraints are: **[GATE-2003]**

$x_1 + 3x_2 \le 40$ $3x_1 + x_2 \le 24$

$x_1 + x_2 \le 10$ $x_1 > 0,\ x_2 > 0$

The maximum profit which can meet the constraints is:

(a) 29 (b) 38 (c) 44 (d) 75

Statement for Linked Answer Questions Q5 and Q6:

Consider a linear programming problem with two variables and two constraints. The objective function is: Maximize $X_1 + X_2$. The corner points of the feasible region are (0, 0), (0, 2) (2, 0) and (4/3, 4/3) **[GATE-2005]**

GATE-5. If an additional constraint $X_1 + X_2 \le 5$ is added, the optimal solution is:

(a) $\left(\frac{5}{3}, \frac{5}{3}\right)$ (b) $\left(\frac{4}{3}, \frac{4}{3}\right)$ (c) $\left(\frac{5}{2}, \frac{5}{2}\right)$ (d) (5, 0)

GATE-6. **Let Y_1 and Y_2 be the decision variables of the dual and v_1 and v_2 be the slack variables of the dual of the given linear programming problem. The optimum dual variables are:**
(a) Y_1 and Y_2 (b) Y_1 and v_1 (c) Y_1 and v_2 (d) v_1 and v_2

Previous 20-Years IES Questions

IES-1. **Which one of the following is the correct statement? [IES-2007]**
In the standard form of a linear programming problem, all constraints are:
(a) Of less than or equal to, type.
(b) Of greater or equal to, type.
(c) In the form of equations.
(d) Some constraints are of less than equal to, type and some of greater than equal to, type.

IES-2. **Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-1995]**

List-I	**List-II**
A. Linear programming	**1.** Ritchie
B. Dynamic programming	**2.** Dantzig
C. 'C' programming	**3.** Bell
D. Integer programming	**4.** Gomory

Codes:

	A	B	C	D		A	B	C	D
(a)	2	1	4	3	(b)	1	2	3	4
(c)	2	3	1	4	(d)	2	3	4	1

IES-3. **A feasible solution to the linear programming problem should**
(a) Satisfy the problem constraints **[IES-1994]**
(b) Optimize the objective function
(c) Satisfy the problem constraints and non-negativity restrictions
(d) Satisfy the non-negativity restrictions

IES-4. **Consider the following statements: [IES-1993]**
Linear programming model can be applied to:
1. Line balancing problem **2. Transportation problem**
3. Project management
Of these statements:
(a) 1, 2 and 3 are correct (b) 1 and 2 are correct
(c) 2 and 3 are correct (d) 1 and 3 are correct.

IES-5. **Solution to $Z = 4x_1 + 6x_2$ [IES-1992]**
$x_1 + x_2 \le 4;$ $3x_1 + x_2 \le 12;$ $x_1, x_2 \ge 0$ **is:**
(a) Unique (b) Unbounded (c) Degenerate (d) Infinite

IES-6. **The primal of a LP problem is maximization of objective function with 6 variables and 2 constraints. [IES-2002]**
Which of the following correspond to the dual of the problem stated?

1. It has 2 variables and 6 constraints
2. It has 6 variables and 2 constraints
3. Maximization of objective function
4. Minimization of objective function
Select the correct answer using the codes given below:
(a) 1 and 3 (b) 1 and 4 (c) 2 and 3 (d) 2 and 4

Graphical Method

IES-7. In a linear programming problem, which one of the following is correct for graphical method? [IES-2009]
(a) A point in the feasible region is not a solution to the problem
(b) One of the corner points of the feasible region is not the optimum solution
(c) Any point in the positive quadrant does not satisfy the non-negativity constraint
(d) The lines corresponding to different values of objective functions are parallel

IES-8. In case of solution of linear programming problem using graphical method, if the constraint line of one of the non-redundant constraints is parallel to the objective function line, then it indicates [IES-2004, 2006]
(a) An infeasible solution (b) A degenerate solution
(c) An unbound solution (d) A multiple number of optimal solutions

IES-9. Which of the following are correct in respect of graphically solved linear programming problems? [IES-2005]
1. The region of feasible solution has concavity property.
2. The boundaries of the region are lines or planes.
3. There are corners or extreme points on the boundary
Select the correct answer using the code given below:
(a) 1 and 2 (b) 2 and 3 (c) 1 and 3 (d) 1, 2 and 3

IES-10. Which one of the following statements is NOT correct? [IES-2000]
(a) Assignment model is a special ease of a linear programming problem
(b) In queuing models, Poisson arrivals and exponential services are assumed
(c) In transportation problems, the non-square matrix is made square by adding a dummy row or a dummy column
(d) In linear programming problems, dual of a dual is a primal

IES-11. Consider the following statements regarding the characteristics of the standard form of a linear programming problem: [IES-1999]
1. All the constraints are expressed in the form of equations.
2. The right-hand side of each constraint equation is non-negative.
3. All the decision variables are non-negative.
Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-12. **A variable which has no physical meaning, but is used to obtain an initial basic feasible solution to the linear programming problem is referred to as: [IES-1998]**

(a) Basic variable (b) Non-basic variable
(c) Artificial variable (d) Basis

IES-13. **Which of the following conditions are necessary of applying linear programming? [IES-1992]**

1. There must be a well defined objective function
2. The decision variables should be interrelated and non-negative
3. The resources must be in limited supply

(a) 1 and 2 only (b) 1 and 3 only (c) 2 and 3 only (d) 1, 2 and 3

IES-14. **If m is the number of constraints in a linear programming with two variables x and y and non-negativity constraints $x \geq 0$, $y \geq 0$; the feasible region in the graphical solution will be surrounded by how many lines? [IES-2007]**

(a) m (b) $m + 1$ (c) $m + 2$ (d) $m + 4$

IES-15. **Consider the following linear programming problem: [IES-1997]**
Max. $Z = 2A + 3B$, subject to $A + B < 10$, $4A + 6B < 30$, $2A + B < 17$, $A, B \geq 0$.
What can one say about the solution?

(a) It may contain alternative optima
(b) The solution will be unbounded
(c) The solution will be degenerate
(d) It cannot be solved by simplex method

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (b)

GATE-2. Ans. (c)

GATE-3. Ans. (a)

GATE-4. Ans. (a) Rearranging the above equations $\frac{x_1}{40}+\frac{x_2}{\left(\frac{40}{3}\right)}\le 1$, $\frac{x_1}{8}+\frac{x_2}{24}\le 1$ and $x_1+x_2\le 1.$ Draw the lines and get solution.

GATE-5. Ans. (b)

GATE-6. Ans. (d)

Previous 20-Years IES Answers

IES-1. Ans. (c)

IES-2. Ans. (c)

IES-3. Ans. (c) A feasible solution to the linear programming problem should satisfy the problem constraints.

IES-4. Ans. (b) Linear programming model can be applied to line balancing problem and transportation problem but not to project management.

IES-5. Ans. (a)

$$\begin{array}{r} X_1 + X_2 = 4 \\ 3X_1 + X_2 = 12 \\ \hline -2X_1 \quad\quad = -8 \end{array} \qquad \Rightarrow X_1 = 4 \quad \text{and} \quad X_2 = 0$$

IES-6. Ans. (b)

IES-7. Ans. (a)

IES-8. Ans. (d) All points on the line is a solution. So there are infinite no of optimal solutions.

IES-9. Ans. (b)

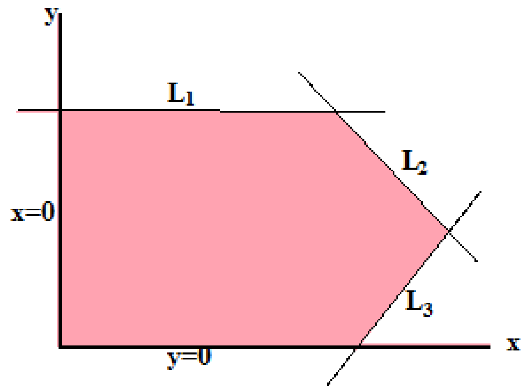
IES-10. Ans. (c)

IES-11. Ans. (a)

IES-12. Ans. (c)

IES-13. Ans. (d)

IES-14. Ans. (c) Constraints = 3 the feasible region is surrounded by more two lines *x*-axis and *y*-axis.



IES-15. Ans. (a)

Conventional Questions with Answer

Conventional Question **[ESE-2007]**

Two product A and B are to be machined on three machine tools, P, Q and R. Product A takes 10 hrs on machine P, 6 hrs on machine Q and 4 hrs on machine R. The product B takes 7.5 hrs on machine P, 9 hrs on machine Q and 13 hrs on machine R. The machining time available on these machine tools, P, Q, R are respectively 75 hrs, 54 hrs and 65 hrs per week. The producer contemplates profit of Rs. 60 per product A, and Rs. 70 per product B. Formulate LP model for the above problem and show the feasible solutions to the above problem? Estimate graphically/ geometrically the optimum product mix for miximizing the profit. Explain why one of the vertics of the feasible region becomes the optimum solution point.

(Note: Graph sheet need not be used) **[15-Marks]**

Solution: The given data are tabulated as follows:

Machine No.	Time taken for operation on A	Time taken for operation on B	Available hours per week
P	10	7.5	75
Q	6	9	54
R	5	13	65

Let $\mathbf{x_1}$ = number of A to be machined

$\mathbf{x_2}$ = number of B to be machined, for profit maximization.

Problem formulation:

(i) Restriction on availability of machines for operation.

(a) If only A were operated on machine P

Then, $\mathbf{x_1 \le 75/10}$

$\mathbf{10x_1 \le 75}$

(b) If only B were operated on machine P then, $\mathbf{7.5x_2 \le 75}$

Since, both A and B are operated on machine P

$\therefore \mathbf{10x_1 + 7.5x_2 \le 75}$ … (i)

Similarly, $\mathbf{6x_1 + 9x_2 \le 54}$ … (ii)

And $\mathbf{5x_1 + 13x_2 \le 65}$ … (iii)

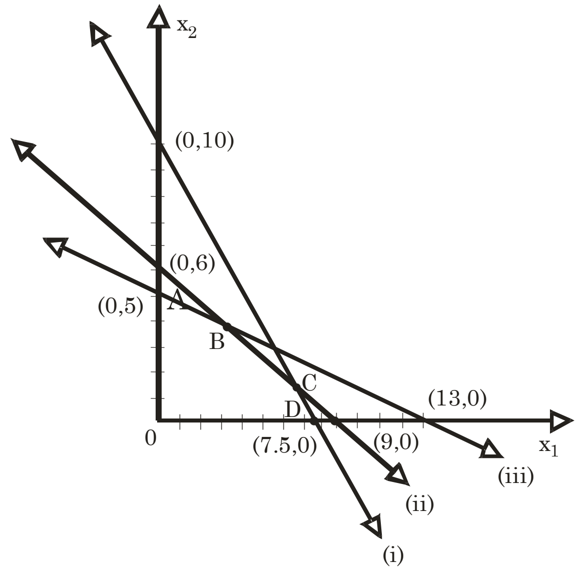
Now, profit equation:

Profit fper product A = Rs.60 and profit per product B = Rs. 70

$\therefore$Total profit is $\mathbf{60x_1 + 70x_2}$

$\therefore$Objective function which is to be maximized is

$$\mathbf{Z = 60x_1 + 70x_2}$$



OABCD in the graph is feasible reagion

On solving equation (ii) and (iii) we get co-ordinates of B

$\therefore \mathbf{B \equiv (3.5454, 3.6363)}$

On solving equation (i) and (ii) we get co-ordinates of C

$\therefore \mathbf{C \equiv (6, 2)}$

And $\mathbf{A \equiv (0, 5)}$; $\mathbf{D \equiv (7.5, 0)}$

Now,

$\mathbf{Z(A) = 60 \times 0 + 70 \times 5 = 350}$

$\mathbf{Z(B) = 60 \times 3.5454 + 70 \times 3.6363 = 467.265}$

$\mathbf{Z(C) = 60 \times 6 + 70 \times 2 = 500}$

$\mathbf{Z(D) = 60 \times 7.5 + 70 \times 0 = 450}$

Hence optimum solution is A = 6 and B = 2

13. Simplex Method

Theory at a Glance (For IES, GATE, PSU)

General Linear Programming Problem

Optimize {Minimize or maximize}

$$Z = c_1x_1 + c_2x_2 + c_3x_3 + ... + c_1x_1$$

Subjected to the constraints

$a_{11}x_1 + a_{12}x_2 + ... + a_{1x}x_n \le b_1$

$a_{21}x_1 + a_{22}x_2 + ... + a_{2x}x_n \le b_2$

$am_1x_1 + am_2x_2 + ... + ax_{mn}x_n \le b_m$

and meet the non-negativity restrictions $x_1, x_2, x_3, ... x_n \ge 0$

1. A set of values $x_1, x_2, x_3, ... x_n$ which constrains of the L.P.P. is called its solutions.
2. Any solution to a L.P.P. which satisfies the non-negativity restrictions of the problem is called its ***feasible Solution.***
3. Any feasible solution which maximizes (or minimizes) the objective function of the L.P.P. is called its ***optimal solution.***
4. A constraints

   $$\sum_{j=1}^{n} a_{ij}x_i \le b_i, (i = 1, 2, ...m)$$

   $$\sum_{j=1}^{n} a_{ij}x_i + s_i = b_i, (i = 1, 2, ...m)$$

   Then the s_i is called ***slack variables.***
5. A constraints

   $$\sum_{j=1}^{n} a_{ij}x_i \ge b_i, (i = 1, 2, ...m)$$

   $$\sum_{j=1}^{n} a_{ij}x_i - s_i = b_i, (i = 1, 2, ...m)$$

   Then the Si is called ***surplus variables.***
6. ***Canonical forms of L.P.P.***

   Maximize $z = c_1x_1 + c_2x_2 + ... + c_nx_n$

   Subject to the Constraints

   $a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n \le b_i$; $[i = 1, 2, ... m]$

   $x_1, x_2, ..., x_n \ge 0$
7. ***Standard from of L.P.P***

   Maximize z=c_1x_1+ c2x2+....+c_nx_n

   Subject to the constraint

   $a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n \le b_i$ $[i = 1, 2, .. m]$

   $x_1, x_2, ..., x_n \ge 0$
8. Convert the following L.PP. to the standard form

   Maximize $z = 3x_1 + 5x_2 + 7x_3$

Subject to $6x_1 - 4x_2 \leq 5$

$3x_1 + 2x_2 + 5x_3 \geq 11$

$4x_1 + 3x_3 \leq 2$

$x_1, x_2 ..... \geq 0$

As x_3 is unrestricted let $x_3 = x_3' - x_3''$

Where x_3', x_3''

And Introducing the slack/surplus variables, the problem in standard form becomes:

Maximize $z_3 x_1 + 5x_2 + 7x_3' - 7x_3''$

Subject to $6x_1 - 4x_2 + 5x_3 = 5$

$3x_1 + 2x_2 + 5x_3' - 5x_3'' - s_2 = 11$

$4x_1 + 3x_3' - 3x_3'' - s_3 = 2$

$x_1, x_2, x_3', x_3''; s_1, s_2, s_3 \geq 0$

Big-M Method

In the simplex method was discussed with required transformation of objective function and constraints. However, all the constraints were of inequality type withless-than-equal-to' (δ) sign. However, 'greater-than-equal-to' (ε) and 'equality' (=) constraints are also possible. In such cases, a modified approach is followed, which will be discussed in this chapter. Different types of LPP solutions in the context of Simplex method will also be discussed. Finally, a discussion on minimization *vs* maximization will be presented.

Simplex method with 'greater-than-equal-to' (ε) and equality (=) constraints

The LP problem, with 'greater-than-equal-to' (ε) and equality (=) constraints, is transformed to its standard form in the following way :

1. One 'artificial variable' is added to each of the 'greater-than-equal-to' (ε) and equality (=) constraints to ensure an initial basic feasible solution.
2. Artificial variables are 'penalized' in the objective function by introducing a large negative (positive) coefficient M for maximization (minimization) problem.
3. Cost coefficients, which are supposed to be placed in the Z-row in the initial simplex tableau, are transformed by 'pivotal operation' considering the column of artificial variable as 'pivotal column' and the row of the artificial variable as 'pivotal row'.
4. If there are more than one artificial variable, step 3 is repeated for all the artificial variables one by one.

Let us consider the following LP problem

Maximize $Z = 3x_1 + 5x_2$

Subject to $x_1 + x_2 \geq 2$

$x_2 \leq 6$

$3x_1 + 2x_2 = 18$

$x_1, x_2 \geq 0$

After incorporating the artificial variables, the above LP problem becomes as follows:

Maximize $Z = 3x_1 + 5x_2 - Ma_1 - Ma_2$

Subject to $x_1 + x_2 - x_3 + a_1 = 2$

$$x_2 + x_4 = 6$$
$$3x_1 + 2x_2 + a_2 = 18$$
$$x_1,\ x_2 \geq 0$$

Where x_3 is surplus variable, x_4 is slack variable and a_1 and a_2 are the artificial variables. Cost coefficients in the objective function are modified considering the first constraint as follows:

$$Z - 3x_1 - 5x_2 + Ma_1 + Ma_2 = 0 \qquad (E_1)$$
$$x_1 + x_2 - x_3 + a_1 = 2 \qquad (E_2) \leftarrow \text{Pivotal Row}$$

Pivotal Column

Thus, pivotal operation is $E_1 - M \times E_2$, which modifies the cost coefficients as follows:

$$Z - (3 + M)x_1 - (5 + M)x_2 + Mx_3 + 0\,a_1 + Ma_2 = -2M$$

Next, the revised objective function is considered with third constraint as follows:

$$Z - (3 + M)x_1 - (5 + M)\,x_2 + Mx_3 + 0\,a_1 + Ma_2 = -2M \qquad (E_3)$$
$$3\,x_1 + 2\,x_2 + a_2 = 18 \qquad (E_4) \leftarrow \text{Pivotal Row}$$

Pivotal Column

Obviously pivotal operation is $E_3 - M \times E_4$, which further modifies the cost coefficients as follows:

$$Z - (3 + 4M)x_1 - (5 + 3M)x_2 + Mx_3 + 0a_1 + 0a_2 = -20\ \mathrm{M}$$

The modified cost coefficients are to be used in the Z-row of the first simplex tableau. Next, let us move to the construction of simplex tableau. Pivotal column, pivotal row and pivotal element are marked (same as used in the last class) for the ease of understanding.

Iteration	Basis	Z	Variables						b_r	$\frac{b_r}{c_{rs}}$
			x_1	x_2	x_3	x_4	a_1	a_2		
1	Z	1	$-3-4M$	$-5-3M$	M	0	0	0	$-20M$	--
	a_1	0	1	1	-1	0	1	0	2	2
	x_4	0	0	1	0	1	0	0	6	--
	a_2	0	3	2	0	0	0	1	18	6

Note: That while comparing $(-3 - 4M)$ and $(-5 - 3M)$, it is decided that $(-3 - 4M) < (-5 - 3M)$ as M is any arbitrarily large number.

Successive iterations are shown as follows:

Iteration	Basis	Z	x_1	x_2	x_3	x_4	a_1	a_2	b_r	$\frac{b_r}{c_{rs}}$
2	Z	1	0	$-2+M$	$-3-3M$	0	$3+4M$	0	$6-12M$	--
	x_1	0	1	1	-1	0	1	0	2	--
	x_4	0	0	1	0	1	0	0	6	--
	a_2	0	0	-1	3	0	-3	1	12	4

Iteration	Basis	Z	x_1	x_2	x_3	x_4	a_1	a_2	b_r	$\frac{b_r}{c_{rs}}$
3	Z	1	0	-3	0	0	M	$1+M$	18	--
	x_1	0	1	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$	6	9
	x_4	0	0	1	0	1	0	0	6	6
	x_3	0	0	$-\frac{1}{3}$	1	0	-1	$\frac{1}{3}$	4	--
4	Z	1	0	0	0	3	M	$1+M$	36	--
	x_1	0	1	0	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	2	--
	x_2	0	0	1	0	1	0	0	6	--
	x_3	0	0	0	1	$\frac{1}{3}$	-1	$\frac{1}{3}$	6	--

It is found that, at iteration 4, optimality has reached. Optimal solution is $Z = 36$ with $x_1 = 2$ and $x_2 = 6$. The methodology explained above is known as *Big-M* method. Hope, reader has already understood the meaning of the terminology!

'Unbounded', 'Multiple' and 'Infeasible' solutions in the context of Simplex Method

As already discussed in lecture notes 2, a linear programming problem may have different type of solutions corresponding to different situations. Visual demonstration of these different types of situations was also discussed in the context of graphical method. Here, the same will be discussed in the context of Simplex method.

Unbounded Solution

If at any iteration no departing variable can be found corresponding to entering variable, the value of the objective function can be increased indefinitely, i.e., the solution is unbounded.

sMultiple (Infinite) Solutions

If in the final tableau, one of the non-basic variables has a coefficient 0 in the Z-row, it indicates that an alternative solution exists. This non-basic variable can be incorporated in the basis to obtain another optimal solution. Once two such optimal solutions are obtained, infinite number of optimal solutions can be obtained by taking a weighted sum of the two optimal solutions.

Consider the slightly revised above problem,

Maximize $Z = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \geq 2$

$x_2 \leq 6$

$3x_1 + 2x_2 = 18$

$x_1,\ x_2 \geq 0$

Curious readers may find that the only modification is that the coefficient of x_2 is changed from 5 to 2 in the objective function. Thus the slope of the objective function and that of third constraint are now same. It may be recalled from lecture notes 2, that if the *Z line* is parallel to any side of the feasible region (i.e., one of the constraints) all the points lying on that side constitute optimal solutions (refer Fig. 3 in lecture notes 2). So, reader should be able to imagine graphically that the LPP is having infinite solutions. However, for this particular set of constraints, if the objective function is made parallel (with equal slope) to either the first constraint or the second constraint, it will not lead to multiple solutions. The reason is very simple and left for the reader to find out. As a hint, plot all the constraints and the objective function on an arithmetic paper.

Now, let us see how it can be found in the simplex tableau. Coming back to our problem, final, tableau is shown as follows. Full problem is left to the reader as practice.

Final tableau:

Iteration	Basis	Z	Variables						b_r	$\frac{b_r}{c_{rs}}$
			x_1	x_2	x_3	x_4	a_1	a_2		
3	Z	1	0	0	0	0	M	$1+M$	18	--
	x_1	0	1	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$	6	9
	x_4	0	0	1	0	1	0	0	6	6
	x_3	0	0	$-\frac{1}{3}$	1	0	-1	$\frac{1}{3}$	4	--

Coefficient of non-basic variable x_2 is zero

As there is no negative coefficient in the Z-row the optimal is reached. The solution is Z = 18, with x_1 = 6 and x_2 = 0. However, the coefficient of non-basic variable x_2 is zero as shown in the final simplex tableau. So, another solution is possible by incorporating x_2 in the basis. Based on the b_r / c_{rs}, 4x will be the exiting variable. The next tableau will be as follows:

Iteration	Basis	Z	Variables						b_r	$\frac{b_r}{c_{rs}}$
			x_1	x_2	x_3	x_4	a_1	a_2		
4	Z	1	0	0	0	0	M	$1+M$	18	--
	x_1	0	1	0	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	2	--
	x_2	0	0	1	0	1	0	0	6	6
	x_3	0	0	0	1	$\frac{1}{3}$	-1	$\frac{1}{3}$	6	18

Coefficient of non-basic variable x_4 is zero

Thus, another solution is obtained, which is Z = 18 with x_1 = 2 and x_2 = 6. Again, it may be noted that, the coefficient of non-basic variable x_4 is zero as shown in the tableau. If one more similar step is performed, same simplex tableau at iteration 3 will be obtained.

Thus, we have two sets of solutions $\begin{Bmatrix}6\\0\end{Bmatrix}$ and $\begin{Bmatrix}2\\6\end{Bmatrix}$. Other optimal solutions will be obtained as $\beta\begin{Bmatrix}6\\0\end{Bmatrix} + (1-\beta)\begin{Bmatrix}2\\6\end{Bmatrix}$ where, $\beta \in [0, 1]$. For example, let $\beta = 0.4$, corresponding solution is $\begin{Bmatrix}3.6\\3.6\end{Bmatrix}$, i.e., x_1 = 3.6 and x_2 = 3.6.

Note: That values of the objective function are not changed for different sets of solution; for all the cases Z = 18.

Infeasible Solution

If in the final tableau, at least one of the artificial variables still exists in the basis, the solution is indefinite.

Reader may check this situation both graphically and in the context of Simplex method by considering following problem:

Maximize $Z = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \le 2$

$3x_1 + 2x_2 \ge 18$

$x_1, x_2 \ge 0$

Minimization Versus Maximization Problems

As discussed earlier, standard form of LP problems consist of a maximizing objective function. Simplex method is described based on the standard form of LP problems, i.e., objective function is of maximization type. However, if the objective function is of minimization type, simplex method may still be applied with a small modification. The required modification can be done in either of following two ways:

1. The objective function is multiplied by –1 so as to keep the problem identical and 'minimization' problem becomes 'maximization'. This is because of the fact that minimizing a function is equivalent to the maximization of its negative.
2. While selecting the entering non-basic variable, the variable having the maximum coefficient among all the cost coefficients is to be entered. In such cases, optimal solution would be determined from the tableau having all the cost coefficients as non-positive (≤ 0).

Still one difficulty remains in the minimization problem. Generally the minimization problems consist of constraints with 'greater-than-equal-to' (≥) sign. For example, minimize the price (to compete in the market); however, the profit should cross a minimum threshold. Whenever the goal is to minimize some objective, lower bounded requirements play the leading role.

Constraints with 'greater-than-equal-to' (≥) sign are obvious in practical situations.

To deal with the constraints with 'greater-than-equal-to' (≥) and = sign, *Big-M* method is to be followed as explained earlier.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. Simplex method of solving linear programming problem uses **[GATE-2010]**
(a) All the points in the feasible region
(b) Only the comer points of the feasible region
(c) Intermediate points within the infeasible region
(d) Only the interior points in the feasible region

Common data for Question Q2 and Q3 **[GATE-2008]**

Consider the Linear Programme (LP)

Maximize $4x + 6y$

subject to $3x + 2y \le 6$

$2x + 3y \le 6$

$x, y \ge 0$

GATE-2. After introducing slack variables s and t, the initial basic feasible solution is represented by the tableau below (basic variables are $s = 6$ and $t = 6$, and the objective function value is 0).

	–4	–6	0	0	0
s	3	2	1	0	6
t	2	3	0	1	6
	x	*y*	*s*	*T*	*RHS*

After some simplex iteration, the following tableau is obtained

	0	0	0	2	12
s	5/3	0	1	–1/3	2
y	2/3	1	0	1/3	2
	x	*y*	*s*	*T*	*RHS*

From this, one can conclude that
(a) The LP has a unique optimal solution
(b) The LP has an optimal solution that is not unique
(c) The LP is infeasible
(d) The LP is unbounded

GATE-3. The dual for the LP in Q 2 is:
(a) Min $6u + 6v$ subject to $3u + 2v \ge 4$; $2u + 3v \ge 6$ u; and $v \ge 0$
(b) Max $6u + 6u$ subject to $3u + 2v \le 4$; $2u + 3v \le 6$; and $u, v \ge 0$
(c) Max $4u + 6v$ subject to $3u + 2v \ge 6$; $2u + 3v \ge 6$; and $u, v \ge 0$
(d) Min $4u + 6u$ subject to $3u + 2v \le 6$; $2u + 3v \le 6$; and $u, v \ge 0$

Previous 20-Years IES Questions

IES-1. Which one of the following is true in case of simplex method of linear programming? [IES-2009]
(a) The constants of constraints equation may be positive or negative
(b) Inequalities are not converted into equations
(c) It cannot be used for two-variable problems
(d) The simplex algorithm is an iterative procedure

IES-2. Which one of the following subroutines does a computer implementation of the simplex routine require? [IES 2007]
(a) Finding a root of a polynomial
(b) Solving a system of linear equations
(c) Finding the determinant of a matrix
(d) Finding the eigenvalue of a matrix

IES-3. A tie for leaving variables in simplex procedure implies: [IES-2005]
(a) Optimality (b) Cycling (c) No solution (d) Degeneracy

IES-4. In the solution of linear programming problems by Simplex method, for deciding the leaving variable [IES-2003]
(a) The maximum negative coefficient in the objective function row is selected
(b) The minimum positive ratio of the right-hand side to the first decision variable is selected
(c) The maximum positive ratio of the right-hand side to the coefficients in the key column is selected
(d) The minimum positive ratio of the right-hand side to the coefficient in the key column is selected

IES-5. Match List-I (Persons with whom the models are associated) with List-II (Models) and select the correct answer: [IES-2002]

List-I	**List-II**
A. J. Von Newmann	**1.** Waiting lines
B. G. Dantzig	**2.** Simulation
C. A.K. Erlang	**3.** Dynamic programming
D. Richard Bellman	**4.** Competitive strategies
	5. Allocation by simplex method

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	1	5	4	(b)	4	5	1	3
(c)	2	5	1	4	(d)	4	1	5	3

IES-6. Consider the following statements regarding linear programming:
1. Dual of a dual is the primal. [IES-2001]
2. When two minimum ratios of the right-hand side to the coefficient in the key column are equal, degeneracy may take place.
3. When an artificial variable leaves the basis, its column can be deleted from the subsequent Simplex tables.
Select the correct answer from the codes given below:

Codes: (a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-7. In the solution of a linear programming problem by Simplex method, if during iteration, all ratios of right-hand side b_i to the coefficients of entering variable a are found to be negative, it implies that the problem has [IES-1999]

(a) Infinite number of solutions (b) Infeasible solution
(c) Degeneracy (d) Unbound solution

IES-8. A simplex table for a linear programming problem is given below:

	5 X_1	2 X_2	3 X_3	0 X_4	0 X_5	0 X_6	Z
X_4	1	2	2	1	0	0	8
X_5	3	4	1	0	1	0	7
X_6	2	3	4	0	0	1	10

Which one of the following correctly indicates the combination of entering and leaving variables? [IES-1994]

(a) X_1 and X_4 (b) X_2 and X_6 (c) X_2 and X_5 (d) X_3 and X_4

IES-9. While solving a linear programming problem by simplex method, if all ratios of the right-hand side (b_i) to the coefficient, in the key row (a_{ij}) become negative, then the problem has which of the following types of solution? [IES-2009]

(a) An unbound solution (b) Multiple solutions
(c) A unique solution (d) No solution

Big-M Method

IES-10. Consider the following statements: [IES-2000]

1. **A linear programming problem with three variables and two constraints can he solved by graphical method.**
2. **For solutions of a linear programming problem with mixed constraints. Big-M-method can be employed.**
3. **In the solution process of a linear programming problem using Big-M-method, when an artificial variable leaves the basis, the column of the artificial variable can be removed from all subsequent tables.**

Which one these statements are correct?

(a) 1, 2 and 3 (b) 1 and 2 (c) 1 and 3 (d) 2 and 3

IES-11. A linear programming problem with mixed constraints (some constraints of ≤ type and some of ≥ type) can be solved by which of the following methods? [IES-2009]

(a) Big-M method (b) Hungarian method
(c) Branch and bound technique (d) Least cost method

IES-12. When solving the problem by Big-M method, if the objective functions row (evaluation row) shows optimality but one or more artificial variables are still in the basis, what type of solution does it show? [IES-2009]

(a) Optimal solution (b) Pseudooptimal solution

(c) Degenerate solution (d) Infeasible solution

IES-13. Which one of the following statements is not correct? [IES-2008]

(a) A linear programming problem with 2 variables and 3 constraints can be solved by Graphical Method.
(b) In Big-M method if the artificial variable can not be driven out it depicts an optimal solution.
(c) Dual of a dual is the primal problem.
(d) For mixed constraints either Big-M method or two phase method can be employed.

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (b) Simplex method of solving linear programming problem uses only the corner points of the feasible region.

GATE-2. Ans. (b) As C_j = 0, 0 for x and y respectively therefore It is an optimal solution but not unique.

GATE-3. Ans. (a) Duplex method:

Step-I: Convert the problem to maximization form so Choice may be (b) or (c).

Step-II: Convert (≥) type constraints if any to (≤) type by multiplying such constraints by (–1) so our choice is (b).

Previous 20-Years IES Answers

IES-1. Ans. (d)
IES-2. Ans. (b)
IES-3. Ans. (d)
IES-4. Ans. (b)
IES-5. Ans. (b)
IES-6. Ans. (a)
IES-7. Ans. (d)

IES-8. Ans. (a) The combination of entering and leaving variables corresponds to Z being minimum and maximum value of row in table.

IES-9. Ans. (a) While solving a linear programming problem by simplex method, if all the ratios of the right hand side (b_i) to the coefficient in the key row (a_{ij}) become negative, it means problem is having unbounded solution.

IES-10. Ans. (d)

IES-11. Ans. (a) A linear programming problem with mixed constraints (some constraints of ≤ type and some of ≥ type) can be solved by Big M-method which involves.

(i) Objective function should be changed to maximization function.

(ii) If the constraint is ≥ type, along with a slack variable an artificial variable is also used.

IES-12. Ans. (d) When solving the problem by Big-M method if the objective functions row (evaluation row) shows optimality but one or more artificial variables are still in the basis, this shows infeasible solution.

IES-13. Ans. (b)

14. Transportation Model

Theory at a Glance (For IES, GATE, PSU)

Transportation problem: This is a special class of L.P.P. in which the objective is to transport a single commodity from various origins to different destinations at a minimum cost. The problem can be solved by simplex method. But the number of variables being large, there will be too many calculations.

Formulation of Transportation problem: There are m plant locations (origins) and n distribution centres (destinations). The production capacity of the i^{th} plant is a_i and the number of units required at the j^{th} destination b_j. The Transportation cost of one unit from the i^{th} plant to the j^{th} destination c_{ij}.Our objective is to determine the number of units to be transported from the i^{th} plant to j^{th} destination so that the total transportation cost is minimum.

Let x_{ij} be the number of units shipped from i^{th} plant to j^{th} destination, then the general transportation problem is:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \; x_{ij}$$

Subjected to

$$x_{i1} + x_{i2} + - - - + x_{in} = a_i \left(\text{for } i^{th} \text{ origin } i = 1, 2, ..., m\right)$$

$$x_{1j} + x_{2j} + - - - + x_{mj} = b_j \left(\text{for } j^{th} \text{ origin } i = 1, 2, ..., n\right)$$

$$x_{ij} \geq 0$$

The two sets of constraints will be consistent if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, which is the condition for a transportation problem to have a feasible solution? Problems satisfying this condition are called balanced transportation problem.

Degenerate or non-Degenerate: A feasible solution to a transportation problem is said to be a basic feasible solution if it contains at the most (m + n – 1) strictly positive allocations, otherwise the solution will 'degenerate'. If the total number of positive (non-zero) allocations is exactly (m + n – 1), then the basic feasible solution is said to be non-degenerate, if '(m + n – 1)' $\rightarrow$ no. of allocated cell. Then put an $\varepsilon \rightarrow 0$ at a location so that all u_i, v_j can be solved and after optimality check put $\varepsilon = 0$.

Optimal solution: The feasible solution which minimizes the transportation cost is called an optimal solution.

Working Procedure for transportation problems:

Step-1: Construct transportation table: if the supply and demand are equal, the problem is balanced. If supply and demand is not same then add dummy cell to balance it.

Step-2: Find the initial basic feasible solution. For this use ***Vogel's approximation Method (VAM).*** The VAM takes into account not only the least cost c_{ij} but also the costs that just exceed the least cost c_{ij} and therefore yields a better initial solution than obtained from other methods. As such we shall confine our selves to VAM only which consists of the following steps:

Steps in VAM solution:

1. Determine the difference between the two lowest distribution costs for each row and each column.
2. Select the row or column with the greatest difference. If greatest difference not unique, make an arbitrary choice.
3. Assign the largest possible allocations within the restrictions to the lowest cost square in the row and column selected.
4. Omit any row or column that has been completely satisfied by the assignment just made.
5. Reallocate the differences as in step1, except for rows and columns that have been omitted.
6. Repeat steps 2 to 5, until all assignment have been made.

Step-3: ***Apply Optimality Check***

In the above solution, the ***number of allocations must be 'm+n-1' otherwise the basic solution degenerates.*** Now to test optimality, we apply the **Modified Distribution (MODI) Method** and examine each unoccupied cell to determine whether making an allocation in it reduces the total transportations cost and then repeat this procedure until lowest possible transportation cost is obtained. This method consists of the following steps:

1. Note the numbers u_i along the left and v_j along the top of the cost matrix such that their sums equals to original costs of occupied cells i.e. solve the equations [$u_i+v_j=c_{ij}$] starting initially with some $u_i=0$ (For allocated cell) (u or v –ive)
2. Compute the net evaluations $w_{ij}=u_i+v_j-c_{ij}$ for all the empty cells and enter them in upper right hand corners of the corresponding cells.
3. Examine the sign of each w_{ij}. If all $w_{ij} \le 0$, then the current basic feasible solution is optimal. If even one $w_{ij} > 0$, this solution is not optimal and we proceed further.

Step-4: ***Iterate towards optimal solution:***

1. Choose the unoccupied cell with the largest w_{ij} and mark θ in it.
2. Draw a closed path consisting of horizontal and vertical lines beginning and ending at θ-cell and having its other corners at the allocated cells.
3. Add and subtract θ alternately to an from the transition cells of the loop subjected to rim requirements. Assign a maximum value to θ so that one basic variable becomes zero and the other basic variables remain non-negative. Now the basic cell whose allocation has been reduced to zero leaves the basis.

Step-5: Return to step 3 and repeat the process until an optimal basic feasible solution is obtained.

Example: 1

Destination

S \ D	A	B	C	D	
I	21	16	25	13	11
II	17	18	14	23	13
III	32	41	18	41	19
Requirement	6	10	12	15	43 \ 43

Source (rows I, II, III); Availability (last column)

Solution:

Step-1: Above it is a balanced type transportation problem

Step-2: VAM

Table-1

21	16	25	13	11-11=0(3)
17	18	14	23	13 (3)
32	27	18	41	19 (9)
6	10	12	15-11=4	
(4)	(2)	(4)	(10)	

Table-2

17	18	14	23	13-4 = 9(3)
32	27	18	41	19 (9)
6	10	12	4-4=0	
(15)	(9)	(4)	(18)	

Table-3

17	18	14	9-6=3 (3)
32	27	18	19 (9)
6-6=0	10	12	
(15)	(9)	(4)	

Table- 4,5

3 |18	|14	3= (4)
7 |27	12 |18	19-12=7 (9)
10	12	
(9)	(4)	

Table-6

[4]	[16]	[25]	[13] 11
[17] 6	[18] 3	[14]	[23] 4
[22]	[27] 7	[18] 12	[41]

Step-3: Apply optimality check

As number of allocation = $m + n - 1$ i.e. '6', we can apply MODI method

	$V_1=17$	$V_2=18$	$V_3=9$	$V_4=23$
$U_1^{-10}=0$	[21] (-)	[16] (-)	[25] (-)	[13] 11
$U_2=0$	[17] 6	[18] 3	[14] (-)	[23] 4
$U_3=9$	[32] (-)	[27] 7	[18] 12	[41] (-)

Since all the net evaluation is (*–ive*) then current solution is optimal solution

$\therefore \quad T_{opt} = 6\times17 + 3\times18 + 7\times27 + 12\times18 + 11\times13 + 4\times23 = 796$

To remove degeneracy

$m + n - 1 = 4 + 6 - 1 = 9$

Allocated cell = 8 degeneracy

$\therefore$ Required one ε but where?

For that put u_1, u_2, u_3, u_4, and v_1, v_2, v_3, v_4, v_5, v_6.

At first assume $u_3=0$

$\therefore$ $u_3 + v_1 = 6$ or $v_1 = 6$

And $u_4 + v_1 = 6$ or $u_4 = 0$

And $u_3 + v_3 = 9$ or $v_3 = 9$

$u_1 + v_3 = 9$ or $u_1 = 0$

$u_4 + v_4 = 2$ or $v_4 = 2$

$u_4 + v_5 = 2$ or $v_5 = 2$

but $u_2 + v_2 = 3$

$u_2 + v_6 = 5$

Table (by VAM Method)

	$V_1=6$	V_2	$V_3=9$	$V_4=2$	$V_5=2$	V_6
$U_1=0$	[9]	[12]	(5) [9]	[6]	[9]	[10]
U_2	* [7]	(4) [3]	* ε [7]	* [7]	* [5]	(2) [5]
$U_3=0$	(1) [6]	[5]	(1) [9]	[11]	[3]	[11]
$U_4=0$	(3) [6]	[8]	[11]	(2) [2]	(4) [2]	[10]
	4^0	4^2	6^2	2^4	4^1	5

Here the problem so put one ε so that u_2 is found out for that we can put ε on any of '*' cell but allocated cell column (3) has maximum cost is '9' so we will put ε on cell (2 – 3) i.e.,

$u_2 + v_3 = 7$

$u_2 = 7 - 9 = -2$

then $v_2 = 3 + 2 = 5$

$v_6 = 5 + 2 = 7$

Then check optimality of the above problem.

Example: 2 In the next table there is some allocated cell and we only check optimality.

Step-3: Optimality check

$m + n - 1 = 3 + 4 - 1 = 6$

Allocated cell = 6

$\therefore$ We can check optimality

1. Let $u_1 = 0$ then according to all allocated cell find out u_2, u_3 ---- v_1, v_2, ----

Using $[u_i + v_j = c_{ij}]$

	$V_1=2$	$V_2=3$	$V_3=12$	$V_4=6$
$U_1=0$	(1) 2	(5) 3	11	7
$U_2=-5$	1	0	6	(1) 1
$U_3=3$	(6) 5	8	(3) 15	(1) 9

2.

Empty cell Row-column	Calculation $w_{ij} = u_i + v_j - c_{ij}$	Improvement
2 – 1	$-5 + 2 - 1 = -4$	No
2 – 2	$-5 + 3 - 0 = -2$	No
3 – 2	$3 + 3 - 8 = 0 - 2$	No
1 – 3	$0 + 12 - 11 = 1$	Yes
2 – 3	$-5 + 12 - 11 = 1$	Yes
1 – 4	$0 + 6 - 7 = -1$	No

Since the net evaluations in two cells are positive a better solution can be found

Step-4: ***Iteration towards optimal solution.***

A. First iteration

(a) Next basic feasible solution

(i) Chose the unoccupied cell with the maximum w_{ij}. In case of a tie, select the one with lower original cost. In Table 1, cells (1 – 3) and (2, 3) each have $w_{ij} = 1$ and out of these cell (2, 3) has lower original cost 6, therefore we take this as the next basic cell and note θ in it.

(ii) Draw a closed path beginning and ending at θ-cell. Add and subtract θ, alternately to and from the transition cells of the loop subject the rim requirement s. Assign a maximum value to θ so that one basic variable becomes zero and the other basic variables remain ≥ 0. Now the basic cell whose allocation has been reduced to zero leaves the basis. This gives the second basic solution (Table 2).

Table - 2

(1) 2	(5) 3	11	7
1	0	+Q 6	(1) -Q 1
(6) 5	8	(3) -Q 15	(1) +Q 9

Put θ=1 so that one cell can empty

(1) 2	(5) 3	11	7
1	0	(1) 6	1
(6) 5	8	(2) 15	(2) 9

Here total transportation cost $= \text{Rs.}\left[1\times2+5\times3+1\times6+6\times5+2\times15+2\times9\right]\times100$
$= \text{Rs. } 10{,}100$

B. Optimality check

	V_1=2	V_2=3	V_3=12	V_4=6
U_1=0	(1) 2	(5) 3	11	7
U_2=-6	1	0	(1) 6	1
U_3=3	(6) 5	8	(2) 15	(2) 9

Empty cell	**Calculation** $\left[\boldsymbol{W_{ij} = U_i + V_j - C_{ij}}\right]$	**Improvement**
2 – 1	– 6 + 2 – 1 = – 5	No
2 – 2	– 6 + 3 – 0 = – 3	No
3 – 2	3 + 3 – 8 = – 2	No
1 – 3	0 + 12 – 11 = 1	Yes
1 – 4	0 + 6 – 7 = – 1	No
2 – 4	– 6 + 6 – 1 = – 1	No

Second Iteration

(a) Next basic feasible solution:

(1) -Q 2	(5) 3	+Q 11	7
1	0	(1) 6	1
(6) +Q 5	8	(2) -Q 15	(2) 9

Put θ = 1

(b) Optimality Check

	$V_1=1$	$V_2=3$	$V_3=11$	$V_4=5$
$U_1=0$	2	(5) 3	(1) 11	7
$U_2=-9$	1	0	(1) 6	1
$U_3=4$	(7) 5	8	(1) 15	(2) 9

Empty cell	**Calculation** $\left[W_{ij} = U_i + V_j - C_{ij}\right]$	**Improvement**
1 – 1	0 + 1 – 2 = – 1	No
2 – 1	– 5 + 3 – 1 = – 5	No
2 – 2	– 5 + 3 – 0 = – 2	No
3 – 2	4 + 3 – 8 = – 1	No
1 – 4	0 + 5 – 7 = – 2	No
2 – 4	– 5 + 5 – 1 = – 1	No

So this basic feasible solution is optimal

Optimal Transportation cost $= \text{Rs.}\left[5\times 3 + 1\times 11 + 1\times 6 + 7\times 5 + 1\times 15 + 2\times 9\right]\times 100$

$= \text{Rs. } 10{,}000/-$

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. **For the standard transportation linear programme with *m* sources and *n* destinations and total supply equaling total demand, an optimal solution (lowest cost) with the smallest number of non-zero x_{ij} values (amounts from source *i* to destinationj) is desired. The best upper bound for this number is: [GATE-2008]**

(a) mn (b) $2(m + n)$ (c) $m + n$ (d) $m + n - 1$

GATE-2. **A company has two factories *S*1, *S*2 and two warehouses *D*1, *D*2. The supplies from *S*1 and *S*2 are 50 and 40 units respectively. Warehouse *D*1 requires a minimum of 20 units and a maximum of 40 units. Warehouse *D*2 requires a minimum of 20 units and, over and above, it can take as much as can be supplied. A balanced transportation problem is to be formulated for the above situation. The number of supply points, the number of demand points, and the total supply (or total demand) in the balanced transportation problem respectively are: [GATE-2005]**

(a) 2, 4, 90 (b) 2, 4, 110 (c) 3, 4, 90 (d) 3, 4, 110

GATE-3. **The supply at three sources is 50, 40 and 60 units respectively whilst the demand at the four destinations is 20, 30, 10 and 50 units. In solving this transportation problem [GATE-2002]**

(a) A dummy source of capacity 40 units is needed
(b) A dummy destination of capacity 40 units is needed
(c) No solution exists as the problem is infeasible
(d) None solution exists as the problem is degenerate

GATE-4. **A firm is required to procure three items (*P*, *Q*, and *R*). The prices quoted for these items (in Rs.) by suppliers *S*1, *S*2 and *S*3 are given in table. The management policy requires that each item has to be supplied by only one supplier and one supplier supply only one item. The minimum total cost (in Rs.) of procurement to the firm is: [GATE-2006]**

Item	Suppliers		
	S1	S2	S3
P	110	120	130
Q	115	140	140
R	125	145	165

(a) 350 (b) 360 (c) 385 (d) 395

Previous 20-Years IES Questions

IES-1. Consider the following statements: [IES-2007]
The assignment problem is seen to be the special case of the transportation problem in which
1. $m = n$ 2. All $a_i = 1$ 3. $x_{ij} = 1$
(The symbols have usual meaning)

Which of the statements given above are correct?
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 and 3 only

IES-2. In order for a transportation matrix which has six rows and four columns, not to be degenerate, how much must be the number of allocated cells in the matrix? [IES-2007]
(a) 6 (b) 9 (c) 15 (d) 24

IES-3. Which one of the following is not the solution method of transportation problems? [IES-2006]
(a) Hungarian method (b) Northwest corner method
(c) Least cost method (d) Vogel's approximation method

IES-4. Which one of the following conditions should be satisfied for the application of optimality test on an initial solution of transportation model? [IES-2004]
(a) Number of allocations should be less than $m + n - 1$
(b) Number of allocations should be equal to $m + n - 1$
(c) Number of allocations should be equal to $m + n$
(d) Number of allocations should be more than $m + n$

IES-5. In a connected network of '*n*' arcs (roads) joining '*m*' vertices (towns), a selection of roads is taken up for resurfacing based on a minimum spanning tree of network as being the least cost solution. This spanning tree will contain [IES-1994]
(a) m arcs (b) $(m + 1)$ arcs
(c) $(m - 1)$ arcs (d) $(m + n - 1)$ arcs

IES-6. Consider the following statements on transportation problem:
1. In Vogel's approximation method, priority allotment is made in the cell with lowest cost in the column or row with least penalty
2. The North-West corner method ensures faster optimal solution
3. If the total demand is higher than the supply, transportation problem cannot be solved
4. A feasible solution may not be an optimal solution.
Which of these statements are correct? [IES-2003]
(a) 1 and 4 (b) 1 and 3 (c) 2 and 3 (d) 2 and 4

IES-7. Assertion (A): In the solution of transportation problem, for application of optimality test, the number of allocations required is $m + n - 1$ and these should be in independent positions. [IES-2003]
Reason (R): If the number of allocations is not $m + n - 1$, values of all oddments, i.e., u_i and v_j cannot be found.
(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-8. Assertion (A): Vogel's approximation method yields the best initial basic feasible solution of a transportation problem. [IES-2000]
Reason (R): Vogel's method gives allocations to the lowest cost elements of the whole matrix.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-9. In a transportation problem North-West corner rule would yield
(a) An optimum solution (b) An initial feasible solution **[IES-1999]**
(c) A Vogel's approximate solution (d) A minimum cost solution

IES-10. In a transportation problem, the materials are transported from 3 plants to 5 warehouses. The basis feasible solution must contain exactly, which one of the following allocated cells? [IES-1998]
(a) 3 (b) 5 (c) 7 (d) 8

IES-11. When there are 'm' rows and 'n' columns in a transportation problem, degeneracy is said to occur when the number of allocations is: [IES-1997]
(a) Less than $(m + n - 1)$ (b) Greater than $(m + n - 1)$
(c) Equal to $(m - n - 1)$ (d) Less than $(m - n - 1)$

IES-12. In order for a transportation matrix which has six rows and four columns not to degenerate, what is the number of occupied cells in the matrix? [IES-2008]
(a) 6 (b) 9 (c) 15 (d) 24

IES-13. Assertion (A): Transportation problem can be solved by VAM heuristic much faster as compared to the solution through linear programming method. [IES-1996]
Reason (R): VAM heuristic gives an approximate solution. It is checked for optimality test. If it is optimal, the algorithm stops there. If it is not an optimal solution, then improved solutions are found out through very little iteration till optimality is reached.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-14. A solution is not a basic feasible solution in a transportation problem if after allocations. [IES-1996]
(a) There is no closed loop
(b) There is a closed loop

(c) Total number of allocations is one less than the sum of numbers of sources and destinations
(d) There is degeneracy

IES-15. **Match List-I (O.R. Techniques) with List-II (Application) and select the correct answer using the codes given below the lists: [IES-1995]**

List-I	List-II
A. Linear programming	**1.** Warehouse location decision
B. Transportation	**2.** Machine allocation decision
C. Assignment	**3.** Product mix decision
D. Queuing theory	**4.** Project management decision
	5. Number of servers decision

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	1	2	3	5	(b)	3	1	2	5
(c)	1	3	4	5	(d)	3	2	1	4

IES-16. **The solution in a transportation model (of dimension $m \times n$) is said to be degenerate if it has [IES-1995]**
(a) Exactly $(m + n - 1)$ allocations (b) Fewer than $(m + n - 1)$ allocations
(c) More than $(m + n - 1)$ allocations (d) $(m \times n)$ allocations

IES-17. **Assertion (A): In distribution problem, unit cost of production as well as transportation cost is considered. [IES-1994]**
Reason (R): The Vogel approximation method can reduce the number of iterations required to move from the initial assignment to the optimal solution.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-18. **Consider the following statements: [IES-1993]**
In a transportation problem, North-West corner method would yield
1. An optimum solution **2. An initial feasible solution**
3. Vogel's approximate solution
Of these statements:
(a) I alone is correct (b) 2 alone is correct
(c) 3 alone is correct (d) 2 and 3 are correct

IES-19. **In a 6 × 6 transportation problem, degeneracy would arise, if the number of filled slots were: [IES-1993]**
(a) Equal to thirty six (b) More than twelve
(c) Equal to twelve (d) Less than eleven

Previous 20-Years IAS Questions

IAS-1. **Consider the location of a warehouse to distribute books for the cities of Bombay, Bangalore and Calcutta. The estimated volume of distribution to Bombay, Bangalore and Calcutta are 55,000, 20,000**

and 25,000 units respectively. Using some appropriate origin, the (*x*, *y*) co-ordinates of Bombay, Bangalore and Calcutta can be approximated as (10, 20), (20, 10) and (30, 30) respectively. The (*x*, *y*) co-ordinates or the optimal location would be: [IAS-1998]

(a) (20, 20) (b) (l0, 20) (c) (30, 20) (d) (20, 30)

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

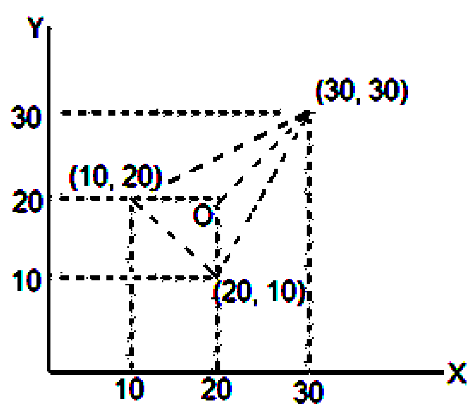
GATE-1. Ans. (d)
GATE-2. Ans. (c)
GATE-3. Ans. (b)
GATE-4. Ans. (c)

Previous 20-Years IES Answers

IES-1. Ans. (b) 3 – is wrong, here $\sum x_{ij} = 1$ if allotted, and $\sum x_{ij} = 0$ if not allotted
IES-2. Ans. (b) m + n – 1 = 4 + 6 – 1 = 9
IES-3. Ans. (a) Hungarian method is used in assignment problem (but remember assignment problem is a special case of transportation problem).
IES-4. Ans. (b)
IES-5. Ans. (d) Spanning tree will have $m + n - 1$ acrs
IES-6. Ans. (d)
IES-7. Ans. (a)
IES-8. Ans. (c)
IES-9. Ans. (b)
IES-10. Ans. (c)
IES-11. Ans. (a)
IES-12. Ans. (b) $m + n - 1$
IES-13. Ans. (a)
IES-14. Ans. (b)
IES-15. Ans. (b)
IES-16. Ans. (b)
IES-17. Ans. (b) Both A and R are true and R is not correct explanation for A.
IES-18. Ans. (b) Statement 2 alone is correct.
IES-19. Ans. (d)

Previous 20-Years IAS Answers

IAS-1. Ans. (a)



Conventional Questions with Answer

Conventional Question **[ESE-2000]**

A company has three plants at A, B, C which supply to warehouses located at D, E, F, G and H. Weekly plant capacities are 200,125 and 225 tons respectively. Weekly warehouses requirements are 75, 105, 130, 155 and 85 tons respectively. Unit transportation cost matrix is given below:

To / FR	D	E	F	G	H
A	50	82	65	60	35
B	45	70	70	65	50
C	80	45	75	60	40

Determine the optimum cost distribution pattern and also the minimum total cost.

Solution:

Step-1:

To / FR	D	E	F	G	H	
A	50	82	65	60	35	200
B	45	70	70	65	50	125
C	80	45	75	60	40	225
	75	105	130	155	85	550 / 550

It is a balanced type transportation problem.

Step-2: VAM

To / FR	D	E	F	G	H			Penalties
A	50	82	65 (80)	60 (120)	35	200	-120	15, 15, 10, 5, 5
B	45 (75)	70	70 (50)	65	50	125	-75 =50	5, 5, 20, ← 5, 5
C	80	45 (105)	75	60 (35)	40 (85)	225		5, 23, ← 12, 12 ←
	75	105	130	155	85	550 / 550		

D	E	F	G	H
5	25 ↑	5	0	5
5		5	0	5
5		5	0	
		5	0	
		5	5	

Step-3: $m + n - 1 = 3 + 5 - 1 = 7$

And number of allocated cell = 7

$\therefore$ It is not degenerate so we can check optimality.

Step-4: Modi Method

First Iteration

	$V_1=40$	$V_2=45$	$V_3=65$	$V_4=60$	$V_5=40$
$U_1=0$	50	82	(80) 65	(120) -Q 60	+Q 35
$U_2=5$	(75) 45	70	(50) 70	65	50
$U_3=0$	80	(105) 45	75	(35) +Q 60	(85) -Q 40

Put Q = 85

Empty Cell	Calculation $\left[W_{ij} = U_i + V_j - C_{ij}\right]$	Improvement
1 – 1	0 + 40 – 50 = – 10	No
1 – 2	0 + 45 – 82 = – 37	No
1 – 5	0 + 40 – 35 = 5	Yes
2 – 2	5 + 45 – 70 = – 20	No
2 – 4	5 + 60 – 65 = 0	No
2 – 5	5 + 40 – 50 = – 5	No
3 – 1	0 + 40 – 80 = – 40	No
3 – 3	0 + 65 – 75 = – 10	No

Second Iteration

	$V_1=40$	$V_2=95$	$V_3=65$	$V_4=60$	$V_5=35$
$U_1=0$	(-) 50	(-) 82	(80) 65	(35) 60	(85) 35
$U_2=5$	(75) 45	(-) 70	(50) 70	(-) 65	(-) 50
$U_3=0$	(-) 80	(105) 45	(-) 75	(120) 60	(-) 40

Empty Cell Calculation Improvement
(Do yourself and will find no improvement)

So this basic feasible solution is optimal

Minimum cost: $75 \times 45 + 105 \times 45 + 80 \times 65 + 50 \times 70 + 35 \times 60 + 120 \times 60 + 85 \times 35 = \text{Rs.}29,075$

Conventional Question **[UPSC-1998]**

In the state of Bihar, in a particular region there are 5 coalmines which produce the following output at the indicated production costs:

Mine	Output m.tons/day	Production Cost units of 100 Rs. Per metric ton
1	120	25
2	150	29
3	80	34
4	160	26
5	140	28

Before the coal can be sold to the steel making units, it must be cleaned and graded at one of the 3 coal preparation plants. The capacities and operating cost of the 3 plants areas follows.

Plants	**Capacity metric tons/day**	**Opterating cost units of 100 Rs. Per metric ton**
A	300	2
B	200	3
C	200	3

All coal is transported by rail at a cost Rs. 50 per metric ton kilometre and the distance in kilometre from each mine in the preparation plants indicated below:

Preparation Plants	Distance kilometres to Mines				
	1	2	3	4	5
A	22	44	26	52	24
B	18	16	24	42	48
C	44	32	16	16	22

(i) Using a transportation model, determine how the output of each mine should be allocated to the three preparation plants to optimise cost.

(ii) Are alternative approaches possible? If so, what is the logic of 1st allocation in these alternatives?

(iii) What is degeneration and when can it happen? [20]

Solution: ***Using Transportation Model***

Step-1: Cost matrix per metric tonne is given by [Rs.]

Coal mines	**A**	**B**	**C**	**Supply**
1	2500+200+22*50 3800	2500+300+18*150 =3700	2500+300+44*50 =5000	120
2	2900+200+2200 5300	2900+300+16*50 =4000	2900+300+32*50 =4500	150
3	3400+200+1300 =4900	3900+300+24*50 =4900	3400+300+16*50 4500	80
4	2600+200+2600 =5400	2600+300+42+50 5000	2600+300+16*50 =3700	160
5	2800+200+1200 4200	2800+300+48+50 =5500	2800+300+22*50 4200	140
Demand	300	200	200	

Here Demand > supply, hence add dummy source with infinite cost.

Step-2: Using VAM allocate

plant / From	A	B	C	Supply	
1	(70) 38	(50) 37	50	120	1, 1, 1, 12 ←
2	53	(150) 40	48	150	8, 8 ←
3	(40) 49	49	(40) 45	80	0, 0, 0, 4, 4, 4
4	54	50	(160) 37	160	13 ←
5	(140) 42	55	42	140	0, 0, 0, 0
Dummy	(50) ∞	∞	∞	50	0, 0, 0, 0, 0
Demand	300	200	200	700 / 700	
	4 4 4,4, 7, ∞	3 3 12	5 3 3 ,3,3, ∞		

Step-3: $m + n - 1 = 8$

And number of allocated cell = 8

$\therefore$ It is not degenerate so we can check optimality.

Step-4: Modi Method (Optimality Check)

	$V_1=38$	$V_2=37$	$V_3=34$
$U_1=0$	(70) 38	(50) 37	(-) 50
$U_2=3$	(-) 53	(150) 40	(-) 48
$U_3=11$	(40) 49	(-) 49	(40) 45
$U_4=3$	(-) 54	(-) 50	(160) 37
$U_5=4$	(140) 42	(-) 55	(-) 42
$U_6=a$	(50) a	(-) a	(-) a

Optimum cost = (70*38 + 50*37 + 150 $\times$ 40 + 40 $\times$ 49 + 40*45 + 160*37 + 140*42)*100

=Rs. 2,607,000

15. Assignment Model

Theory at a Glance (For IES, GATE, PSU)

An assignment problem is a special type of transportation problem in which the objective is to assign a number of origins to an equal number of destinations at a minimum cost (or maximum profit).

Formulation of an assignment problem: There are n new machines m_i ($I = 1, 2, \ldots\ldots, n$) which are to be installed in a machine shop. There are n vacant spaces s_j ($j = 1, 2, \ldots\ldots, n$) available. The cost of installing the machine m_i at space S_j is C_{ij} rupees.

Let us formulate the problem of assigning machines to spaces so as to minimize the overall cost.

Let x_{ij} be the assignment of machine m_i to space s_j i.e. Let x_{ij} be a variable such that

$$x_{ij} = \begin{cases} 1, \text{ if } i^{th} \text{ machine is installed at } j^{th} \text{ space} \\ \qquad\qquad 0, \text{ otherwise} \end{cases}$$

Since one machine can only be installed at each place, we have

$x_{i1} + x_{i2} + \ldots\ldots\ x_{in} = 1$ for m_i ($I = 1, 2, 3, \ldots, n$)

$x_{1i} + x_{2i} + \ldots. + x_{ij} = 1$ for s_j ($j = 1, 2, 3, \ldots, n$)

Also the total installation cost is $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$

Thus assignment problem can be stated as follows:

Determine $x_{ij} \geq 0$ ($j = 1, 2, 3, \ldots.., n$) so as to minimize$(z) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1,\ 2, \ldots.., n \qquad \text{and} \qquad \sum_{j=1}^{n} x_{ij} = 1,\ i = 1, 2, 3, \ldots., n$$

This problem is explicitly represented by the following $n \times n$ cost matrix:

$$\begin{array}{c|cccccc} & \multicolumn{6}{c}{\text{Spaces}} \\ & S_1 & S_2 & S_3 & \cdots & S_n \\ \hline M_1 & C_{11} & C_{12} & C_{13} & \cdots & C_{1n} \\ M_2 & C_{21} & C_{22} & C_{23} & \cdots & C_{2n} \\ M_3 & C_{31} & C_{32} & C_{33} & \cdots & C_{3n} \\ \text{Machines} & \vdots & \vdots & \vdots & & \vdots \\ M_n & C_{n1} & C_{n2} & C_{n3} & \cdots & C_{nn} \end{array}$$

This assignment problem constitutes $n!$ possible ways of installing n machines at n spaces. If we enumerate all these $n!$ alternatives and evaluate the cost of each one of them and select the one with the minimum cost, the problem would be solved. But this method would be very slow and time consuming, even for small value of *n* and hence it is not at all suitable. However, a much more efficient method of solving such problems in available this is the **Hungarian method** for solution of assignment problems which we describe below:

1. Given: ***Original Cost Matrix***
2. Subtract the lowest entry of each column and get ***Job- opportunity matrix***
3. Subtract the lowest entry from each row of Job- opportunity matrix and get ***total opportunity cost matrix.***

Example:

	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

It is a minimization problem

Original Cost Matrix

Subtract lowest entry of each column

10	14	12	7	4
18	0	0	0	0
19	3	5	9	1
0	14	13	15	0
7	9	12	14	3

Job-opportunity matrix

Subtract lowest entry of each row

$6-_{2}$	$10-_{2}$	$8-_{2}$	$3-_{2}$	O
18	O	O	O	O_{+z}
18^{-2}	2_{-2}	4^{-2}	8_{-2}	O
O	14	13	15	O_{+z}
4_{-2}	6_{-2}	9_{-2}	11=2	O

Subtract lowest element of undotted matrix from all uncut element and add the same to corners of cut line element.

4	8	6_{-1}	1^{-1}	0
18_{+1}	0^{+1}	0	0	2+1
16	0	2^{-1}	6^{-1}	0
0	14	13^{-1}	15_{-1}	2
2	4	7^{-1}	9^{-1}	0

Iteration-2

	A	B	C	D	E
1	4	8	5	0	0
2	19	1	0	0	3
3	16	0	1	5	0
4	0	14	12	14	2
5	2	4	6	8	0

Assign
A is 4; B is 3; C is 2; D is 1; E is 5

VIMP: For maximization by assignment we have to firstly changed original cost matrix by putting (–*ve*) at all cost element.

Example:

Maximize

	A	B	C
1	32	38	40
2	40	24	28
3	41	27	33

Then first change all element (–*ve*), then original cost matrix is:

Minimize:

A	B	C
-32	-38	-40
-40	-24	-28
-41	-27	-33

Subtract Column

9	0	0
1	14	12
0	11	7

Subtract Row

9	0	0
1	13	11
0	11	7

A	B	C
16	0	0
0	6	5
0	5	0

B-1
C-3
A-2

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years IES Questions

IES-1. Consider the following statements in respect of assignment method of optimization: [IES-2006]

1. **The matrix format of the method must be a square matrix.**
2. **Some type of rating has to be given to the performance of each pairing.**

Which of the statements given above is/are correct?

(a) Only 1 (b) Only 2 (c) Both 1 and 2 (d) Neither 1 nor 2

IES-2. In an assignment problem having *n* facilities and *n* jobs, what is the number of possible ways of making assignments? [IES-2005]

(a) n^l (b) n^2 (c) $2n$ (d) 2^n

IES-3. The assignment algorithm is applicable to which of the following combined situations for the purpose of improving productivity?

1. **Identification of sales force-market [IES-1998]**
2. **Scheduling of operator-machine**
3. **Fixing machine-location.**

Select the correct answer using the codes given below:

Codes: (a) 1, 2 and 3 (b) 1 and 3 (c) 2 and 3 (d) 1 and 2

Answers with Explanation (Objective)

Previous 20-Years IES Answers

IES-1. Ans. (a) There is nothing about performance.

IES-2. Ans. (a)

IES-3. Ans. (c)

Conventional Questions with Answer

Conventional Question **[ESE-2010]**

Question: Transportation costs from manufacturing plants to warehouses are given in table. They are in euros. Solve this problem to minimize the cost of transportation by stating the steps used in the algorithm. [15 Marks]

Warehouse	PLANT			
	A	B	C	D
1	10	8	10	8
2	10	7	9	10
3	11	9	8	7
4	12	14	13	10

Answer:

We have to use Hungarian method for this assignment problem. It is a special case of transportation problem.

To minimize the cost of Transportation:

Warehouse	PLANT			
	A	B	C	D
1	10	8	10	8
2	10	7	9	10
3	11	9	8	7
4	12	14	13	10

Step 1.

Select minimum entry in column. Subtract from each element.

Warehouse	A	B	C	D
1.	0	1	2	1
2.	0	0	1	3
3.	1	2	0	0
4.	2	7	5	3

Step 2.

Select minimum entry in each row and subtract it from each element and draw matrix and draw minimum no. of lines either horizontal and vertical covering all zeros.

Warehouse	A	B	C	D
1	0	1	2	1
2	0	0	1	3
3	1	2	0	0
4	0	5	3	1

Step.3.

The lines are less than number of rows and column. Therefore select minimum entry from rest element and subtract it from all non zero rows and column element and add same entry at the intersection points of lines.

Warehouse	A	B	C	D
1	0	1	2 $^{-1}$	1 $^{-1}$
2	0	0	1 $^{-1}$	3 $^{-1}$
3	1 $^{+1}$	2 $^{+1}$	0	0
4	0	5	3 $^{-1}$	1 $^{-1}$

Repeat step 2

Warehouse	A	B	C	D
1	0	1	1	0
2	0	0	0	2
3	2	3	0	0
4	0	5	2	0

No. of lines is equal is to no. of rows or columns hence solution is optimal

Step for assignment – select rows and columns that has minimum of zeros

Warehouse	A	B	C	D
1	0	1	1	0
2	0	0	0	2
3	2	3	0	0
4	0	5	2	0

Select: - Select that row or column that has minimum number of zero.

			Cost
2	-	B	7
3	-	C	8
1	-	A	10
4	-	D	10

Total cost = 35 euros

Conventional Question [ESE-2009]

Four technicians are required to do four different jobs. Estimates of time to complete every job as provided by the technicians are as below:

Technician	Hours to Complete Job			
	Job 1	Job 2	Job 3	Job 4
A	20	36	31	27
B	24	34	45	22
C	22	45	38	18
D	37	40	35	28

Assign the jobs to technicians to minimize the total work-time. State the steps taken in the algorithm used. [15-Marks]

Solution:. Step (1): Subtracting the minimum value in the row from each element in that row.

	1	2	3	4
A	20	36	31	37
B	24	34	45	22
C	22	45	38	18
D	37	40	35	28

Step (2): From the resulting matrix, in each column, subtract the minimum value in the column from each element in that column.

0	16	11	17
2	12	23	0
4	27	20	0
9	12	7	0

Step (3): Checking whether a feasible assignment could be obtained by assigning at the elements contain zeros.

0	4	4	17
2	0	16	0
4	15	13	0
9	0	0	0

0	4	4	17
2	0	16	0
4	15	13	0
9	0	0	0

Hence we find feasible solution, the optimum assignments is
$A \rightarrow 1$, $B \rightarrow 2$, $C \rightarrow 4$, $D \rightarrow 3$
Total work time, T
= 20 + 34 + 18 + 35
= 107

Conventional Question **[ESE-2008]**

What are the differences between transportation and assignment problems, in relation to

(i) Structure of the problem;

(ii) Procedure for solving? **[2 Marks]**

Solution: The transportation problem is one of the sub-classes of L.P.Ps. in which the objective is to transport various quantities of a single homogeneous commodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum. To achieve this objective we must know the amount and location of available supplies and the quantities demanded. In addition, we must know the costs that result from transporting one unit of commodity from various origins to various destinations.

(ii) An assignment problem is a particular case of transportation problem in which a number of operations are to be assigned to an equal number of operators, where each operator performs only one operation. The objective is to maximize overall profit or minimize overall cost for a given assignment schedule.

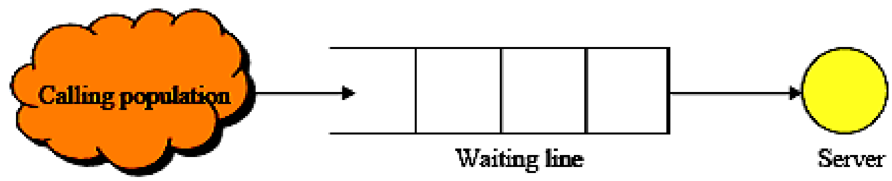
An assignment problem is a completely degenerate form of a transportation problem. The units available at each origin and units demanded at each destination are all equal to one. That means exactly one occupied cell in each row and each column of the transportation table i.e., only n occupied cells in place of the required n + n – 1 (= 2 n -1)

16. Queuing Model

Theory at a Glance (For IES, GATE, PSU)

Queuing Theory or Waiting Line

A simple but typical queuing model



Typical measures of system performance are server utilization, length of waiting lines, and delays of customers.

Key elements of queuing systems

- **Customer:** refers to anything that arrives at a facility and requires service, e.g., people, machines, trucks, emails.
- **Server:** refers to any resource that provides the requested service, e.g., receptionist, repairpersons, retrieval machines, runways at airport.

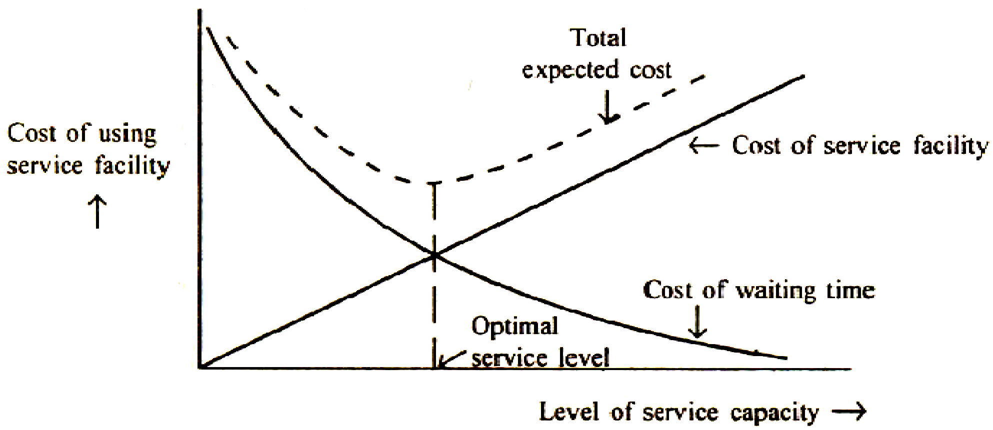
(i) **Calling population:** *The population of potential customers may be assumed to be finite or infinite.*

- **Finite population model:** If arrival rate depends on the number of customers being served and waiting, e.g., model of one corporate jet, if it is being repaired, the repair arrival rate becomes zero.
- ♦ **Infinite population model:** If arrival rate is not affected by the number of customers being served and waiting, e.g., systems with large population of potential customers.

Table: Some Applications of Waiting Line Problem

S. No.	Application Area	Arrival	Waiting Line	Service Facility
1.	Factory	Material/tools	In-process inventory (WIP)	Work stations
2.	Assembly line	Sub-assemblies	WIP	Employees currently processing the WIP
3.	Machine maintenance	Repair tools & equipment	Machines needing repair	Maintenance crew
4.	Airport	Plane	Planes ready to fly	Runway
5.	Bank	Customer	Deposit/withdrawal	Bank employed & computer
6.	Walk-in	Job seekers	Applicants	Interviewers

	interview			
7.	Phone exchange	Dialed number	Caller	Switchboard
8.	Govt. office	Files	Backlog files	Clerks
9.	Post office	Letters	Mailbox	Postal employees
10.	Executive note	Dictation note	Letters to be typed	Secretary
11.	Grocery shop	Customers	Customer on the counter	Checkout clerks and bag packers
12.	Traffic light crossing	Vehicles	Vehicles in line	Traffic signals
13.	Car service station	Cars	Unserviced cars	Service facilities
14.	Railways	Passengers	Waiting passengers on platform/waiting room	Trains
15.	Tool crib	Mechanics	Waiting mechanics	Store keeper
16.	Hospital	Patients	Sick people	Doctor & operation facility



Waiting line cost and service levels

(ii) Arrival Pattern at the System: The probability distribution of the inter-arrival times, which is the time between two consecutive arrivals, may also be governed by a probability distribution.

For a given arrival rate (λ), a discrete Poisson distribution is given by:

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \text{For } x = 0, 1, 2, 3, \ldots\ldots$$

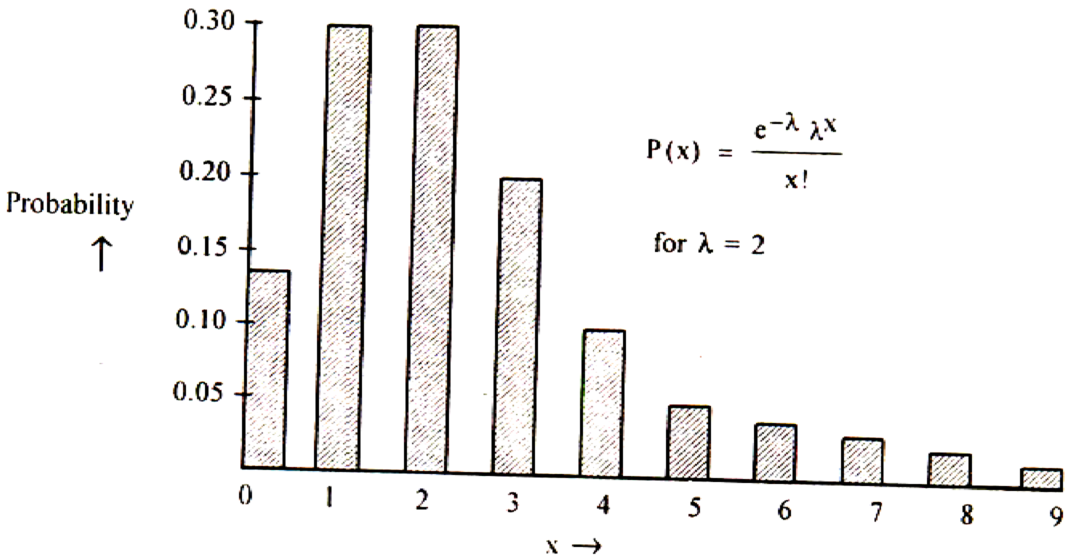
where:

$P(x)$: Probability of x arrival.
x : Number arrivals per unit time.
λ : Average arrival rate.
$1/\lambda$: Mean time between arrivals or inter-arrival time.

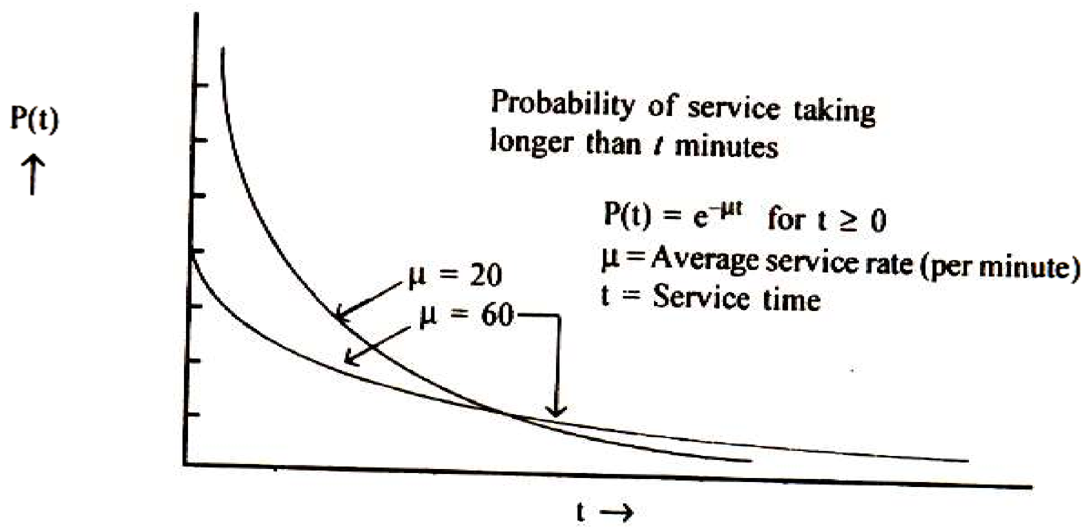
It can be shown mathematically that the probability distribution of inter-arrival time is governed by the exponential distribution when the probability distribution of number of arrivals is Poisson distribution.

The corresponding exponential distribution for inter-arrival time is given by:

$$P(t) = \lambda e^{-\lambda t}$$



Poisson distribution for arrival time



Exponential distribution for service times

Queue Characteristics

The queue may be considered to be limited when its length cannot exceed a certain number. It may be unlimited or infinite otherwise.

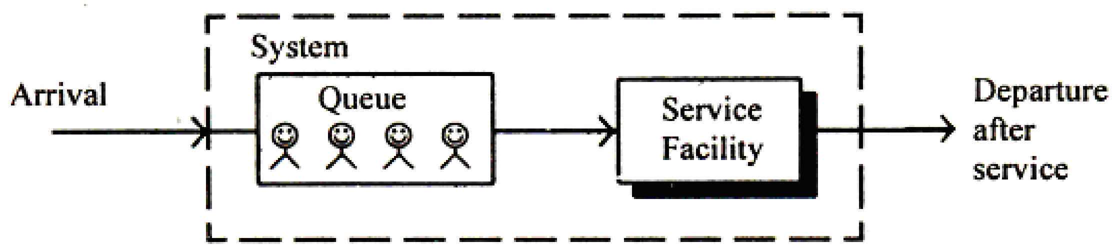
Another characteristic of a queue is its discipline. Queue discipline is the rule by which customers waiting in queue would receive service. These rules may be:

FIFO : First – In – First – Out
LIFO : Last – In – First – Out
SIRO : Service – In – Random – Order, etc.

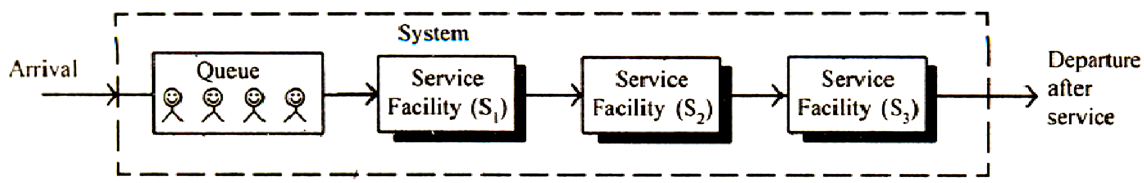
For example, in a railway reservation counter, the customer, who enters first in queue, will receive service prior to other customers joining this queue later. This is a FIFO system.

Service Characteristics

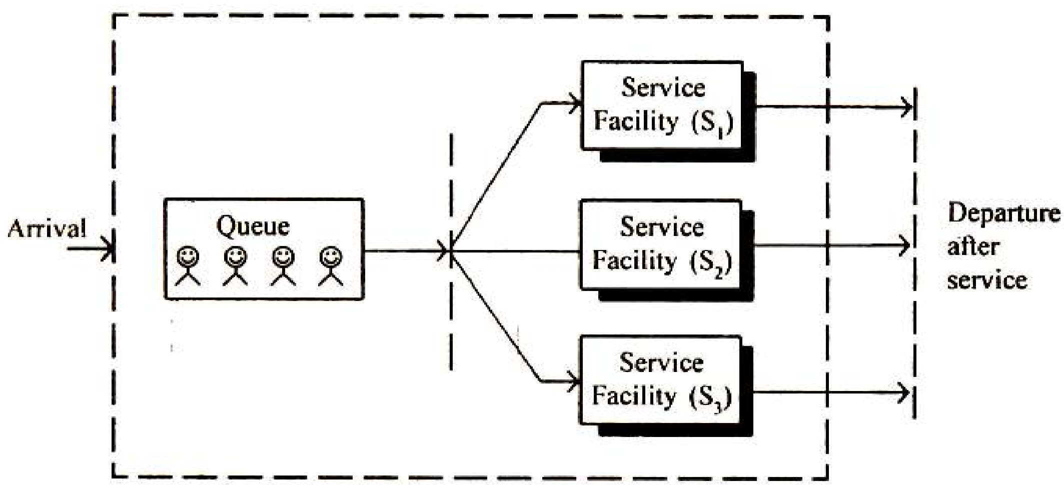
Service system may vary depending upon the number of service channels, number of servers, number of phases, etc. A single channel server has one server. A three-phase service means that once an arrival enters the service, it is served at three stations (or phases).



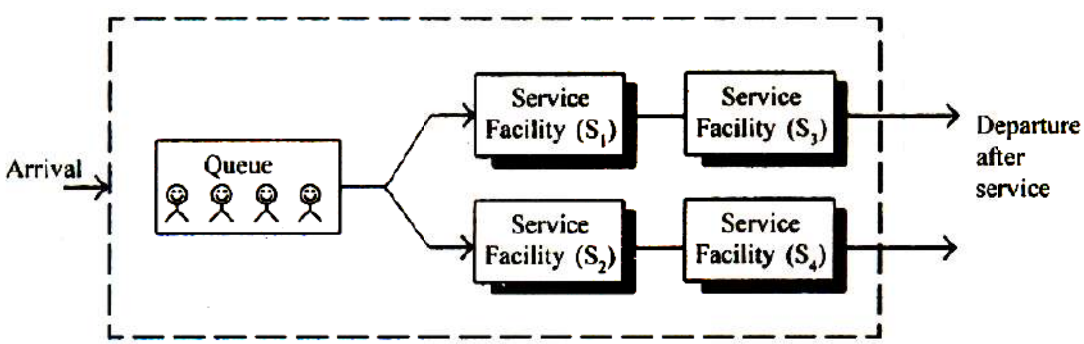
Single channel, single phase system



Single channel, multiphase (3-phase) system



Multi channel, single phase system



Multi Channel, Multiphase System

Single-Line-Single-Server Model

Queuing models may be formulated on the basis of some fundamental assumptions related to following five features:

- Arrival process
- Queue configuration
- Queue discipline
- Service discipline, and
- Service facility.

Let us understand the M/M/1 model first. Following set of assumptions is needed:

1. **Arrival Process:** The arrival is through infinite population with no control or restriction. Arrivals are random, independent and follow Poisson distribution. The arrival process is stationary and in single unit (rather than batches).
2. **Queue Configuration:** The queue length is unrestricted and there is a single queue.
3. **Queue Discipline:** Customers are patient.
4. **Service Discipline:** First-Come-First-Serve (FCFS)
5. **Service Facility:** There is one server, whose service times are distributed as per exponential distribution. Service is continuously provided without any prejudice or breakdown, and all service parameters are state independent.

Relevance of this Model

Despite being simple, this model provides the basis for many other complicated situations. It provides insight and helps in planning process. Waiting line for ticket window for a movie, line near the tool crib for checking out tools, railway reservation window, etc., are some direct applications of this model.

Operating Characteristics

It is the measure of performance of a waiting line application. How well the model performs, may be known by evaluating the operating characteristics of the queue. We analyze the steady state of the queue, when the queue has stabilized after initial transient stage. Similarly, we do not consider the last or shutting down stage of the service.

There are two major parameters in waiting line: **arrival rate (λ)** and service rate (μ). They follow Poisson and exponential probability distribution, respectively. When arrival rate (λ) is less than service rate (μ), i.e., traffic density $\left(\rho \equiv \frac{\lambda}{\mu}\right)$ is less than one, we may have a real waiting line situation, because otherwise there would be an infinitely long queue and steady state would never be achieved.

Following are the lists of parameters:

$\lambda \equiv$ Mean arrival rate in units per period

$\mu \equiv$ Mean service rate in units per period

$\rho \equiv \frac{\lambda}{\mu} \equiv$ Traffic int ensity

$n \equiv$ Number of units in the system

$w \equiv$ Random variate for time spent in the system.

Following are the lists of operating characteristics, which may be derived for steady state situation and for $\rho < 1$:

Queue Related Operating Characteristics

1. **Average line length or expected number of units in queue,**

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho\lambda}{(\mu-\lambda)}$$

2. **Average waiting time or expected time in queue,**

$$W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu-\lambda}$$

System Related Operating Characteristics

3. **Average line length or expected number of units in the system,**

$$L_s = L_q + \text{units being served} = \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} = \frac{\rho\lambda}{\mu-\lambda}$$

4. **Average waiting time or expected time in the system**

$$W_s = \frac{L_s}{\lambda} = \frac{1}{\mu-\lambda}$$

5. **Utilization of service facility,**

$$U = \frac{\lambda}{\mu} = \rho$$

6. **Expected number of units in queue for busy system,**

$$L_b = \frac{\lambda}{\mu-\lambda}$$

7. **Expected time in queue for busy system,**

$$W_b = \frac{1}{\mu-\lambda}$$

Probabilities Related Operating Characteristics

8. **Probability of no unit in the system (i.e., system is idle),**

$$P_o = 1-\rho$$

9. **Probability of system being occupied or busy,**

$$P(n>0) = 1 - P_o = \rho$$

10. **Probability of *n* units in the system,**

$$P_n = P_o\rho^n \qquad \text{(Geometric distribution)}$$

11. **Probability density function for time spent in the system,**

$$f(w) = (\mu-\lambda)e^{-(\mu-\lambda)w} \qquad w \ge 0$$

12. Variance of number of units in the system,

$$V_{ls} = \frac{\lambda\mu}{(\mu-\lambda)^2}$$

13. Variance of time in the system,

$$V_{ws} = \frac{1}{(\mu-\lambda)^2}$$

Model-I (M/M/1:N/ FIFO)

In this model, the capacity of the queue is limited to N rather than infinity as in earlier model. For this model,

$$P_o = \begin{cases} \dfrac{1-\rho}{1-\rho^{N+1}} & \text{for } \lambda \neq \mu \\ \dfrac{1}{N+1} & \text{for } \lambda = \mu \end{cases}$$

$$P(n>0) = 1 - P_o$$

$$P_n = P_o\rho^n \qquad \text{for } n \le N$$

$$L_s = \begin{cases} \dfrac{\rho}{1-\rho} - \dfrac{(N+1)\rho^{N+1}}{1-\rho^{N+1}} & \text{for } \lambda \neq \mu \\ \dfrac{N}{2} & \text{for } \lambda = \mu \end{cases}$$

$$L_q = L_s - (1-P_o); \qquad L_b = \frac{L_q}{1-P_o}$$

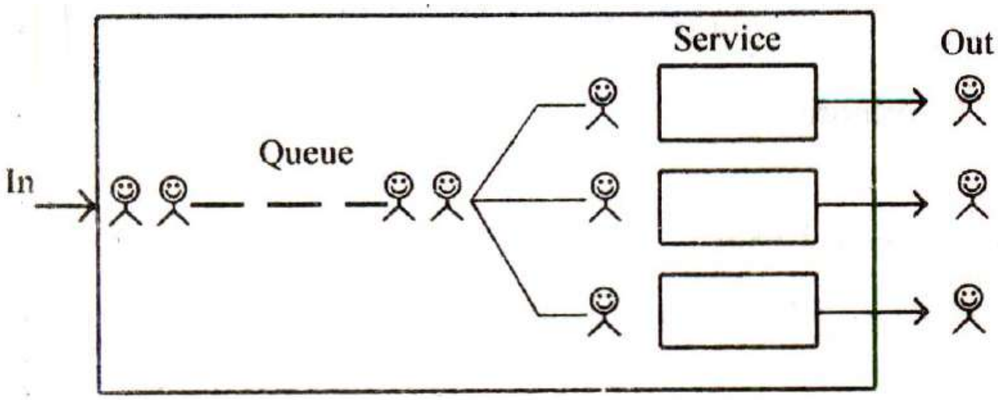
$$Ws = \frac{L_q}{\lambda(1-P_N)} + \frac{1}{\mu}; \qquad W_q = W_s - \frac{1}{\mu}; \qquad W_b = \frac{W_q}{1-P_o}$$

When N is ∞, i.e., the queue length may be infinite, the simplified relations are given in the earlier model.

Model-II (M/M/C: ∞/FIFO)

Multiple Channels Queuing Model

In this model, more than one server is assumed to provide service. Each service station is assumed to provide same type of service and is equipped with similar facility for service. The waiting line breaks into shorter lines, one each for each service station.



Multiple channel queue

There may be two situations:

(a) Number of parallel service stations (C) is greater than or equal to number of customers in the system (n): i.e. $C \geq n$

For this situation, there will be no queue and thus the mean service rate will be equal to (n μ).

(b) Case when $C < n$, queue will be formed. For this situation, the utilization factor is given by the probability that a service channel is being used (ρ_C). This is the ratio of average arrival rate (λ) and maximum service rate of all the C channels, which is C times μ. Thus,

(i) $\rho_C = \dfrac{\lambda}{\mu C}$

(ii) Probability of *n* units in the multi channel system (for n < C),

$$P_n = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_o \qquad 0 \leq n \leq (C-1)$$

Probability of *n* units in the multi channel system (for n > C),

$$P_n = \frac{1}{C!C^{(n-C)}}\left(\frac{\lambda}{\mu}\right)^n P_o$$

(iii) Probability that a service station is idle or waiting for customer = Probability that atleast C customers are present in the system,

$$P(n \leq C) = \frac{\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(\mu C-\lambda)} \times P_o$$

(iv) Probability of no customer in the system,

$$P_o = \frac{1}{\left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \left[\frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{\mu C}{\mu C-\lambda}\right]}$$

(v) $$L_s = \left\{ \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(\mu C-\lambda)^2} P_o \right\} + \frac{\lambda}{\mu}$$

(vi) $$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(\mu C-\lambda)^2} P_o$$

(vii) $$W_s = \frac{L_s}{\lambda}$$

(viii) $$W_q = \frac{L_q}{\lambda}$$

Little's Law

In queuing theory, Little's result, theorem, or law are three names for the same law.

The average waiting time and the average number of items waiting for a service in a service system are important measurements for a manager. Little's law relates these two metrics via the average rate of arrivals to the system. This fundamental law has found numerous uses in operations management and managerial decision making.

Highlights

(i) Distribution of arrival is Poisson's distribution
(ii) Distribution of Service Time follows Exponential law

First calculate:

Mean arrival rate (λ) = $\dfrac{1}{\text{Time of one arrival}}$, arrivals/min

Mean service rate $(\mu) = \dfrac{1}{\text{Time of one service}}$, service/min

1. Probability that one has to wait, $(P) = (1 - P_o) = \dfrac{\lambda}{\mu}$
2. Probability that one arrival does not have to wait, $(P_o) = 1 - \dfrac{\lambda}{\mu}$
3. Average waiting time of an arrival who waits, $(W) = \dfrac{1}{\mu - \lambda}$
4. Average number of units in system, $(L) = \dfrac{\lambda}{\mu - \lambda}$
5. Mean waiting time of an arrival, $(W_m) = \dfrac{\lambda}{\mu(\mu - \lambda)}$
6. Average queue length, $(L_q) = \dfrac{\lambda^2}{\mu(\mu - \lambda)}$
7. Average length of non-empty queues (L_{neq}) = $(L) = \dfrac{\mu}{\mu - \lambda}$
8. In an *M/M/1* queuing system, the number of arrivals in an interval of length T is a Poisson random variable (i.e. the probability of there being n arrivals in an interval of length T is $\dfrac{e^{-\lambda T}(\lambda T)^n}{n!}$). And the probability density function $f(t)$ of the inter-arrival time is given by $\lambda e^{-\lambda t}$.
9. Consider a single server queuing model with Poisson arrivals (λ) and exponential service (μ). The number in the system is restricted to a maximum of N. The probability that a person who comes in leaves without joining the queue is $\dfrac{1}{N+1}$.
10. For a $M/M/1: \infty / FCFS$ queue, the probability of the queue size being greater than N is given by $\left(\dfrac{\lambda}{\mu}\right)^N$.
11. If the arrival rate of units is λ. and the service rate is μ for a waiting line system having 'm' number of service stations, then the probability of a services unit being turned out in the time interval (t; $t + \Delta t$) (given that there are 'n' units in the system at time 't' and 'n' being less than 'm' is equal to $\mu.\Delta t$.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. In an *M/M/1* queuing system, the number of arrivals in an interval of length *T* is a Poisson random variable (i.e. the probability of there being *n* arrivals in an interval of length *T* is $\frac{e^{-\lambda T}(\lambda T)^n}{n!}$). The probability density function *f*(*t*) of the inter-arrival time is given by: [GATE-2008]

(a) $\lambda^2(e^{-\lambda^2 t})$ (b) $\frac{e^{-\lambda t}}{\lambda^2}$ (c) $\lambda e^{-\lambda t}$ (d) $\frac{e^{-\lambda t}}{\lambda}$

GATE-2. Consider a single server queuing model with Poisson arrivals (λ = 4/ hour) and exponential service (λ = 4/hour). The number in the system is restricted to a maximum of 10. The probability that a person who comes in leaves without joining the queue is: [GATE-2005]

$(a)\frac{1}{11}$ $(b)\frac{1}{10}$ $(c)\frac{1}{9}$ $(d)\ \frac{1}{2}$

GATE-3. In a single serve infinite population queuing model, arrivals follow a Poisson distribution with mean λ = 4 per hour. The service times are exponential with mean service time equal to 12 minutes. The expected length of the queue will be: [GATE-2000]

(a) 4 (b) 3.2 (c) 1.25 (d) 5

GATE-4. The cost of providing service in a queuing system increases with [GATE-1997]

(a) Increased mean time in the queue
(b) Increased arrival rate
(c) Decreased mean time in the queue
(d) Decreased arrival rate

GATE-5. At a production machine, parts arrive according to a Poisson process at the rate of 0.35 parts per minute. Processing time for parts have exponential distribution with mean of 2 minutes. What is the probability that a random part arrival finds that there are already 8 parts in the system (in machine! in queue)?[GATE-1999]

(a) 0.0247 (b) 0.0576 (c) 0.0173 (d) 0.082

GATE-6. The number of customers arriving at a railway reservation counter is Poisson distributed with an arrival rate of eight customers per hour. The reservation clerk at this counter takes six minutes per customer on an average with an exponentially distributed service time. The average number of the customers in the queue will be: [GATE-2006]

(a) 3 (b) 3.2 (c) 4 (d) 4.2

GATE-7. **A maintenance service facility has Poisson arrival rates, negative exponential service time and operates on a 'first come first served' queue discipline. Breakdowns occur on an average of 3 per day with a range of zero to eight. The maintenance crew can service an average of 6 machines per day with a range of zero to seven. The mean waiting time for an item to be serviced would be: [GATE-2004]**

$(a)\frac{1}{6}$ day (b) $\frac{1}{3}$ day (c) 1 day (d) 3 day

GATE-8. **Arrivals at a telephone booth are considered to be Poisson, with an average time of 10 minutes between successive arrivals. The length of a phone call is distributed exponentially with mean 3 min. The probability that an arrival does not have to wait before service is: [GATE-2002]**

(a) 0.3 (b) 0.5 (c) 0.7 (d) 0.9

GATE-9. **Little's law is a relationship between: [GATE-2010]**

(a) Stock level and lead time in an inventory system
(b) Waiting time and length of the queue in a queuing system
(c) Number of machines and job due dates in a scheduling problem
(d) Uncertainty in the activity time and project completion time

Previous 20-Years IES Questions

IES-1. **Which of the following distributions is followed by the number of arrivals in a given time in a single-server queueing model?**

(a) Negative exponential distribution (b) Poisson distribution
(c) Normal distribution (d) Beta distribution **[IES-2009]**

IES-2. **In single server queuing model if arrival rate is λ and service rate is μ, then what is the probability of the system being idle? [IES-2005]**

(a) $\frac{\lambda}{\mu}$ (b) $\frac{\mu}{\lambda}$ (c) $1-\frac{\lambda}{\mu}$ (d) $\left(\frac{1-\lambda}{\mu}\right)$

IES-3. **Which one of the following statements is correct? [IES-2004]**
Queuing theory is applied best in situations where

(a) Arrival rate of customers equal to service rate
(b) Average service time is greater than average arrival time
(c) There is only one channel of arrival at random and the service time is constant
(d) The arrival and service rate cannot be analyzed through any standard statistical distribution

IES-4. **Consider two queuing disciplines in a single server queue. Case 1 has a first come first served discipline and case 2 has a last come first served discipline. If the average waiting times in the two cases are W_1 and W_2 respectively, then which one of the following inferences would be true? [IES-1997]**

$(a) W_1 > W_2$ $(b) W_1 < W_2$ $(c) W_1 = W_2$
(d) Data insufficient to draw any tangible inference

IES-5. **In a single server queue customers are served at a rate of μ· If W and W_q represent the mean waiting time in the system and mean waiting time in the queue respectively, then W will be equal to:** **[IES-1997]**

(a) $(a)\ W_q - \mu$ $(b) W_q - \mu$ $(c) W_q + 1/\mu$ $(d) W_q - 1/\mu$

IES-6. **Assertion (A): In a queuing model, the assumption of exponential distribution with only one parameter family for service times is found to be unduly restrictive.** **[IES-1995]**
Reason (R): This is partly because the exponential distribution has the property that smaller service times are inherently less probable than larger service times.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-7. **Match List-I with List-II and select the correct answer using the codes given below the lists:** **[IES-1994]**

List-I	**List-II**
A. Linear programming problem	**1.** Travelling salesman
B. Queuing problem	**2.** Saddle point
C. Dynamic problem	**3.** Product mix
D. Game theory problem	**4.** Normal distribution

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	4	1	2	(b)	4	3	1	2
(c)	3	4	2	1	(d)	4	3	2	1

IES-8. **If the arrivals at a service facility are distributed as per the Poisson distribution with a mean rate of 10 per hour and the services are exponentially distributed with a mean service time of 4 minutes, what is the probability that a customer may have to wait to be served?** **[IES-2009]**

(a) 0.40 (b) 0.50 (c) 0.67 (d) 1.00

IES-9. **The inter-arrival times at a tool crib are exponential with an average time of 10 minutes and the length of the service time is assumed to be exponential with mean 6 minutes. The probability that a person arriving at the booth will have to wait is equal to:** **[IES-2008]**

(a) 0·15 (b) 0·40 (c) 0·42 (d) 0·6

IES-10. **A single-bay car wash with a Poisson arrival rate and exponential service time has cars arriving at an average rate of 10 minutes apart and an average service time of 4 minutes. What is the system utilization?** **[IES-2009]**

(a) 1·00 (b) 0·67 (c) 0.40 (d) 0·24

IES-11. **In a single server queuing system with arrival rate of 'λ' and mean service time of 'μ' the expected number of customers in the system is**

$\frac{\lambda}{(\mu-\lambda)}$. **What is the expected waiting time per customer in the system?** **[IES-2008]**

(a) $\frac{\lambda^2}{(\mu-\lambda)}$ (b) $\mu-\lambda$ (c) $\frac{1}{(\mu-\lambda)}$ (d) $\frac{\mu-\lambda}{\lambda}$

IES-12. If the arrival takes place every 10 minutes with a service times of 4 minutes per unit, then the mean arrival rate, mean service rate and the probability that one would have to wait will be respectively. **[IES-1994]**

(a) 10, 4 and 0.25 (b) 0.1, 0.25 and 0.4
(c) 10, 0.4 and 0.25 (d) 0.1, 0.25 and 0.1

IES-13. In a M/M/I queuing system, the expected waiting time of a unit that actually waits is given by: **[IES-1994]**

(a) $\frac{\lambda}{\mu(\mu-\lambda)}$ (b) $\frac{\lambda}{\mu-\lambda}$ (c) $\frac{1}{\mu-\lambda}$ (d) $\frac{\lambda}{\mu^2(\mu-\lambda)}$

IES-14. Service time in queuing theory is usually assumed to follow: **[IES-1992]**

(a) Normal distribution (b) Poisson's distribution
(c) Erlang distribution (d) Exponential law

IES-15. If the arrivals are completely random, then what is the probability distribution of number of arrivals in a given time? **[IES-2005]**

(a) Negative exponential (b) Binomial
(c) Normal (d) Poisson

IES-16. If the number of arrivals in a queue follows the Poisson distribution, then the inner arrival time obeys which one of the following distributions? **[IES-2007]**

(a) Poisson's distribution (b) Negative exponential law
(c) Normal distribution (d) Binomial

IES-17. Weekly production requirements of a product are 1000 items. The cycle time of producing one product on a machine is 10 minutes. The factory works on two shift basis in which total available time is 16 hours. Out of the available time about 25% is expected to be wasted on break downs, material unavailability and quality related problems. The factory works for 5 days in a week. How many machines are required to fulfil the production requirements? **[IES-2008]**

(a) 2 (b) 3 (c) 4 (d) 6

IES-18. If average arrival rate in a queue is 6/hr and the average service rate is 10/hr, which one of the following is the average number of customers in the line, including the customer being served? **[IES-2007]**

(a) 0.3 (b) 0.6 (c) 1.2 (d) 1.5

IES-19. For a *M/M/*1: ∞ /*FCFS* queue, the mean arrival rate is equal to 10 per hour and the mean service rate is 15 per hour: The expected queue length is: [IES-1998]

(a) 1.33 (b) 1.53 (c) 2.75 (d) 3.20

IES-20. For a *M/M/*1: ∞ /*FCFS* queue, the probability of the queue size being greater than *N* is given; [IES-1993]

(λ = mean arrival rate and μ = mean service rate)

(a) $\left(\frac{\lambda}{\mu}\right)^N$ (b) $\left(\frac{\mu}{\lambda}\right)^N$ (c) $(\lambda\mu)^N$ (d) $\lambda\mu^N$

IES-21. Assertion (A): In waiting line model, it is assumed that arrival rate is described by a Poisson probability distribution. [IES-1993]

Reason (R): The arrival rate is a probabilistic variable and queue discipline is first come first served.

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-22. Customers arrive at a counter randomly at the rate of 6 customers per hour. The service is provided at the counter by a server. The mean time of the service is 4 minutes per customer. The services are exponentially distributed. What is the probability that a newly arrived customer has to wait? [IES-2004]

(a) 0.4 (b) 0.6 (c) 0.66 (d) 0.8

IES-23. At a self-service store, a cashier can serve 10 customers in 5 minutes. On an average 15 customers arrive every 10 minutes. If the arrivals are as per Poisson distribution and services as per exponential distribution, the probability that the cashier would be idle is: [IES-2003]

(a) 0.5 (b) 0.75 (c) 0.25 (d) 0

IES-24. Arrivals at a telephone booth are considered to be according to Poisson distribution, with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to he distributed exponentially with mean of 3 minutes.

The probability that a person arriving at the booth will have to wait, is: [IES-2000, 2001]

(a) $\frac{3}{10}$ (b) $\frac{7}{10}$ (c) $\frac{7}{30}$ (d) $\frac{13}{30}$

IES-25. The average time between two arrivals of customers at a counter in a readymade garment store is 4 min. The average time of the counter clerk to serve the customer is 3 min. The arrivals are

distributed as per Poisson distribution and the services are as per the exponential distribution. The probability that a customer arriving at the counter will have to wait, is: [IES-1999]

(a) Zero (b) 0.25 (c) 0.50 (d) 0.75

IES-26. **Assertion (A): The number of vehicles arriving per unit time at a vehicle maintenance shop at any instant of time can be assessed if the probability of that number and the average rate of arrivals is known. [IES-1998]**

Reason (R): The vehicles, '*k*' arriving at a vehicle maintenance shop with probability *p*(*k*) is given by $p(k) = \dfrac{\lambda^k e^{-\lambda}}{k!}$.

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-27. **If the arrival rate of units is λ. and the service rate is μ for a waiting line system having 'm' number of service stations, then the probability of a services unit being turned out in the time interval (t; t + Δ t) (given that there are 'n' units in the system at time 't' and 'n' being less than 'm' is equal to: [IES-1996]**

(a) Zero (b) $\mu.\Delta t$ (c) $m.\mu.\Delta t$ (d) $n.\mu.\Delta t$

IES-28. **In a queuing problem, if the arrivals are completely random, then the probability distribution of number of arrivals in a given time follows: [IES-2006]**

(a) Poisson distribution (b) Normal distribution
(c) Binomial distribution (d) Exponential distribution

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (c)

GATE-2. Ans. (a) If $\rho_n = \frac{1}{N+1}$; when, $\rho = 1$

GATE-3. Ans. (b) Average queue length, $(L_q) = \frac{\lambda^2}{\mu(\mu - \lambda)}$

GATE-4. Ans. (c)

GATE-5. Ans. (c) $\left(\frac{\lambda}{\mu}\right)^N = (0.7)^8$

GATE-6. Ans. (b) Average number of customer in queue = $\frac{\lambda^2}{\mu(\mu - \lambda)}$

$$= \frac{8 \times 8}{10 \cdot (10 - 8)} = 3.2$$

GATE-7. Ans. (a) Given: λ = 3 per day (arrival rate)

μ = 6 per day (service rate)

Mean waiting time for an item to serviced = $\frac{\lambda}{\mu(\mu - \lambda)} = \frac{3}{6 \cdot (6 - 3)}$

$$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \text{ day.}$$

GATE-8. Ans. (c) The probability that an arrival does not have towait = $1 - \frac{\lambda}{\mu}$

GATE-9. Ans. (b)

Previous 20-Years IES Answers

IES-1. Ans. (b)
IES-2. Ans. (c)
IES-3. Ans. (a)
IES-4. Ans. (c)
IES-5. Ans. (c)
IES-6. Ans. (c)
IES-7. Ans. (a)
IES-8. Ans. (c) Arrivals at a rate of 10/hour (λ = 10)

Service is at the rate of 4 minutes interval (μ = 15)

$$\rho = \frac{\lambda}{\mu} = \frac{10}{15} = 0.67$$

$\lambda = \frac{10}{60}$ per minutes

$\rho = \frac{0.1666}{1/4}$; $\mu = 1/4$ per minutes

IES-9. Ans. (d) Probability that person has to wait $= \rho = \frac{\lambda}{\mu}$

$$\lambda = \frac{60}{10} = 6 \text{ persons/hour} \quad \text{and} \quad \mu = \frac{60}{6} = 10 \text{ persons/hour}$$

Therefore probability that person has to wait $= \frac{\lambda}{\mu} = \frac{6}{10} = 0.6$

IES-10. Ans. (c)

IES-11. Ans. (c)

IES-12. Ans. (b) $\lambda = \frac{1}{10} = 0.1,\ \mu = \frac{1}{4} = 0.25 \text{ and } P = \frac{\lambda}{\mu} = \frac{0.1}{0.25} = 0.4$

IES-13. Ans. (a)

IES-14. Ans. (d)

IES-15. Ans. (d)

IES-16. Ans. (b)

IES-17. Ans. (b) Expected waiting time per customer in the system $= w_s = \frac{L_s}{\lambda}$

$$= \left(\frac{\lambda}{\mu - \lambda}\right)\left(\frac{1}{\lambda}\right) = \frac{1}{\mu - \lambda}$$

IES-18. Ans. (d) $\frac{\lambda}{\mu - \lambda} = \frac{6}{10 - 6} = 1.5$

IES-19. Ans. (a) Expected length of queue $= \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{10^2}{15(15 - 10)} = 1.33$

IES-20. Ans. (a)

IES-21. Ans. (a) Both A and R are true. Also R gives satisfactory explanation for A.

IES-22. Ans. (a) Mean arrival rate $(\lambda) = \frac{1}{\text{Time of one arrival}} = 6$ customes arrivals/hr

Mean Service rate $(\mu) = \frac{1}{\text{Time of one service}} = 15$ custome service/hr

Probability that one has to wait $(P) = \frac{\lambda}{\mu} = \frac{6}{15} = 0.4$

IES-23. Ans. (c) Mean arrival rate, $\lambda = \frac{15}{10} = 1.5$ customer / min

Mean service rate, $\mu = \frac{10}{5} = 2$ customer / min.

Probability of idle $= \left(1 - \frac{\lambda}{\mu}\right) = 0.25$

IES-24. Ans. (a) Probability = $\frac{\lambda}{\mu} = \frac{3}{10}$

IES-25. Ans. (d) Utilization parameter $= \frac{\lambda}{\mu} = \frac{15}{20} = 0.75$

Probability that system is idle $= 1 - \frac{\lambda}{\mu} = 0.25$

Probabilitythat a customer has to wait $= 0.75$

IES-26. Ans. (a)

IES-27. Ans. (b)

IES-28. Ans. (a)

Conventional Questions with Answer

Conventional Question **[ESE-2004]**

In a machine shop certain type of machine breakdown are in accurate with Poisson process. The estimated cost of Idle machine is Rs. 15/-per hour. Two repair man A & B with different skill are being consider being haired as repairmen. Repairman a taken 6 min on an average to repair a machine and his wages are Rs. 8 per hour. When is repairman B taken 5 min to repair to and the wages are Rs. 10/- per hour. Which repairman should be used and why? Assume the work shift taken 8 hrs.

Solution:

Consider A: Mean arrival rate (M) = $\frac{6}{60}$ breakdown/min = 0.1 breakdown/min

Mean service time (μ) = $\frac{1}{6}$ machine/min.

Mean waiting time of an arrival $= \dfrac{1/10}{\frac{1}{6}\left(\frac{1}{6}-\frac{1}{10}\right)} = 9$ min

In a day machine loss due to idle = $\text{Rs. } \dfrac{6\times 8\times 9}{60}\times 15 = \text{Rs. } 108/-$

Workman charge = Rs. 8 × 8 = Rs. 64/–

Total Cost = Rs. 172/–

Consider B: Mean arrival rate (M) = $\frac{6}{60}$ breakdown/min = 0.1 breakdown/min

Mean service time (μ) = $\frac{1}{5}$ repaire/min.

Mean waiting time of an arrival $= \dfrac{M}{\mu(\mu - M)} = \dfrac{10^{-1}}{5^{-1}\left(5^{-1}-10^{-1}\right)} = 5\,\text{min}$

M/c Idle loss = $\dfrac{6\times 8\times 5}{60}\times 15 = \text{Rs. } 60/-$

Man B charge = Rs. 8 × 10 = Rs. 80/–

Total Cost = Rs. 140/–

Use B workman.

Example 1: Arrival of machinists at a tool crib are considered to be distributed as Poisson distribution with an average rate of 7 per hour. The service time at the tool crib is exponentially distributed with mean of 4 minutes.

(a) What is the probability that a machinist arriving at the tool crib will have to wait?
(b) What is the average number of machinists at the tool crib?
(c) The company made a policy decision that it will install a second crib if a machinist has to wait atleast five minutes before being served. What should be additional flow of machinist to the tool crib to justify a second tool crib?

Solution:

Give λ = 7 per hour = $\frac{7}{60}$ = 0.117 machinist per minute

$\mu = \frac{1}{4}$ per min = 0.25 machinist per minute

$\rho = \frac{\lambda}{\mu} = \frac{0.117}{0.25} = 0.467$

(a) Probability of no machinists in the queue, $P_o = 1 - \frac{\lambda}{\mu} = 0.533$

Hence, probability of atleast one machinist in queue = $1 - P_o$
$= 1 - 0.533 = 0.467.$

The probability that a machinist has to wait would be the case when there is atleast one machinist already present in the queue, which is 0.467.

(b) $L_s = \frac{\lambda}{\mu - \lambda} = \frac{0.117}{0.25 - 0.117} = 0.875$ machinists.

(c) Let the new arrival rate is λ^1 when the average waiting time is 5 minutes. Since

$W_q = \frac{\lambda^1}{\mu(\mu - \lambda^1)} \quad \Rightarrow 5 = \frac{\lambda^1}{0.25(0.25 - \lambda^1)} \quad \Rightarrow 0.3125 - 1.25\lambda^1 = \lambda^1$

$\Rightarrow \lambda^1 = \frac{0.3125}{2.25} = 0.1389$ per min. $= 0.1389 \times 60 = 8.33$ machinists per hour.

Example 2: At a telephone booth, arrivals are assumed to follow Poisson distribution with average time of 10 minutes between two calls. The average length of a telephone call is 4 minutes and it is assumed to be exponentially distributed. Find:

(a) Average number of calls (customers) in the system.
(b) Average number of calls waiting to be served.
(c) Average time a call spends in the system.
(d) Average waiting time of a call before being served.
(e) Fraction of time during which booth is empty.
(f) Probability of atleast one customer in the booth.
(g) Probability of more than three calls in the system.

***Solution*:**

Given; $\lambda = \frac{1}{10} = 0.1$ and $\mu = \frac{1}{4} = 0.25$

Traffic intensity, $\rho = \frac{\lambda}{\mu} = \frac{4}{10} = 0.4$

$(a)\quad L_s = \frac{\lambda}{\mu - \lambda} = \frac{0.1}{0.25 - 0.1} = 0.667$

$(b)\quad L_w = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(0.1)^2}{0.25(0.25 - 0.1)} = 0.267$

$(c)\quad W_s = \frac{1}{\mu - \lambda} = \frac{1}{0.25 - 0.1} = 6.67$

$(d)\quad W_q = \dfrac{\rho}{\mu-\lambda} = \dfrac{0.4}{0.25-0.1} = 2.67$

$(e)\quad P_o = 1-\rho = 1-0.4 = 0.6$

$(f)\quad P_n(n>0) = 1-P_o = 1-0.6 = 0.4$

$(g)\quad P_n(n>3) = 1-(P_o+P_1+P_2) = 1-\left(P_o+P_o\rho+P_o\rho^2\right) = 1-P_o\left(1+\rho+\rho^2\right)$

$$= 1-0.6(1+0.4+0.16) = 0.064$$

Example 3: A repair shop is manned by a single worker. Customers arrive at the rate of 30 per hour. Time required to provide service is exponentially distributed with mean of 100 seconds. Find the mean waiting time of a customer, needing repair facility in the queue.

Solution

Given; $\lambda = 30$ per hour

$\mu = \dfrac{60\times 60}{100} = 36$ Customer per hour.

Mean waiting time of a customer in the queue,

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{30}{36(36-30)} = 0.139 \text{ hour} = 8.33 \text{ minutes.}$$

Example 4: A commercial bank has three tellers counter for its customers. The services at these tellers are exponentially distributed with mean of 5 minutes per customer. The arrival of customers is Poisson distributed with mean arrival rate of 36 per hour. Analyses the system

Solution

Given; $\lambda = 36$ per hour

$\mu = \dfrac{60}{5} = 12$ per hour

$\dfrac{\lambda}{\mu} = \dfrac{36}{12} = 3$

$C = 4$ tellers.

(i) $$P_o = \frac{1}{\left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\left[\frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C\frac{\mu C}{\mu C-\lambda}\right]}$$

$$= \frac{1}{\left[1+\left(\frac{\lambda}{\mu}\right)+\frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2+\frac{1}{6}\left(\frac{\lambda}{\mu}\right)^3\right]+\left[\frac{1}{4!}\left(\frac{\lambda}{\mu}\right)^4\left(\frac{\mu C}{\mu C-\lambda}\right)\right]}$$

$$= \frac{1}{\left[1+3+\frac{1}{2}(3)^2+\frac{1}{6}(3)^3\right]+\left[\frac{1}{24}(3)^4\left(\frac{12\times 4}{12\times 4-36}\right)\right]}$$

$$= \frac{1}{1+3+4.5+4.5+4.5+13.5} = 0.0377$$

(ii) Average number of customers in the queue,

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(\mu C-\lambda)^2} P_o = \frac{(36)(12)\left(\frac{36}{12}\right)^4}{(4-1)!\left[(12)4-36\right]^2} \times 0.0377$$

(iii) Average number of customers in the system,

$$L_s = L_q + \frac{\lambda}{\mu} = 1.530 + 3 = 4.53$$

(iv) Average time, which a customer waits in queue,

$$W_q = \frac{\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(\mu C-\lambda)^2} P_o = \frac{12\times(3)^4}{(4-1)!(12\times 4-36)^2} \times 0.0377$$

$= 0.0424$ hour $= 2.54$ minutes.

(v) Average time a customer spends in system,

$$W_s = W_q + \frac{1}{\mu} = 0.0424 + \frac{1}{12} = 0.1257 \text{ hour} = 7.54 \text{ minutes}$$

(vi) Number of hours the tellers are busy during the 6-day week,

Utilization factor, $\rho c = \dfrac{\lambda}{\mu C} = \dfrac{36}{12\times 4} = 0.75$

Hence, if the bank works for 6 days on 6 hours daily basis, the teller is busy for 75% of time, i.e., $0.75 \times 6 \times 6 = 27$ hours per week.

(vii) Expected number of tellers idle at any point of time.

For this, let us find the probability of no customer (P_0), probability of 1 customer (P_1), probability of two customers (P_2) and probability of three customers (P_3):

$P_0 = 0.0377$ (already found earlier)

as $\quad P_n = \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0$

$$P_1 = \frac{1}{1!}\left(\frac{36}{12}\right)0.0377 = 0.1131$$

$$P_2 = \frac{1}{2!}\left(\frac{36}{12}\right)^2 0.0377 = 0.1696$$

$$P_3 = \frac{1}{3!}\left(\frac{36}{12}\right)^3 0.0377 = 0.1696$$

Now, when there is no customer, all the four tellers are idle. When there is one customer, one teller is occupied while three are idle. Similarly, for two customers, two tellers are idle and for three customers, one teller is idle.

Thus, expected number of idle tellers

$= P_0\,(4) + P_1\,(3) + P_2\,(2) + P_3\,(1)$

$= 0.0377 \times 4 + 0.1131 \times 3 + 0.1696 \times 2 + 0.1696 = 0.9989.$

Thus, on the average 0.9989 or one teller will remain idle at any point of time.

Conventional Question [ESE- 2006]

What are the six categories of queueing models as per kendall notation?

[2 Marks]

Solution: Refer Theory part of this book

17. Value Analysis for Cost/Value

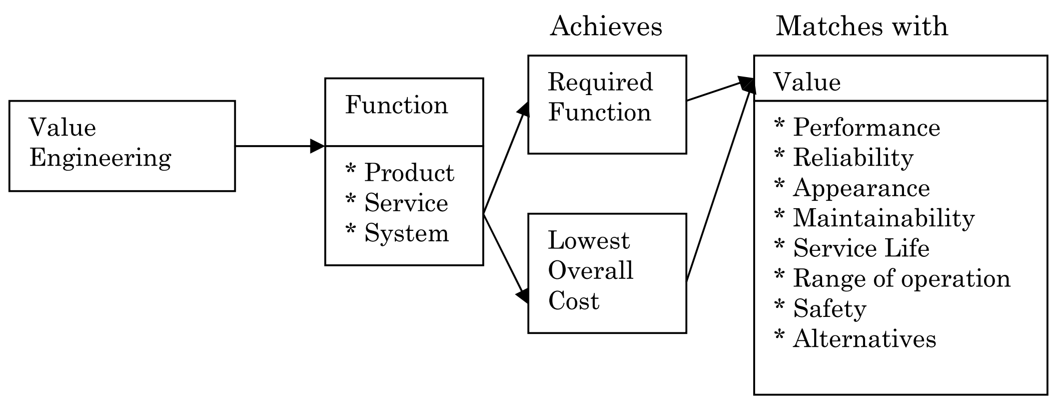
Theory at a Glance (For IES, GATE, PSU)

Value Engineering

Value engineering some meaning

Value analysis
Value management
Value performance
Value control etc.

1. **Define value Engineering** **[IAS-1996; 2004]**
   What is value analysis? **[IAS-2003]**
   "A systematic inter-disciplinary examination of design and other factors affecting the cost of a product or service in order to devise means of achieving the specified purpose most economically, at the required standard of quality and reliability".



What is Value Engineering

2. **What is the objective of value Engineering?** **[IAS-1984]**
   The systematic application of recognized technique
   - To identify the function of a product or service.
   - Establish norms for the function.
   - And provide the necessary function with lowest cost.

3. **What is value?**
   Value is the required or needed performance at minimum cost.
   Value in general is the ratio of function and cost.

$$\text{Value} = \frac{\text{Function}}{\text{Cost}}$$

4. **Define the terms**
   (i) Use value
   (ii) Esteem value

(iii) **Cost value**
(iv) **Exchange value**
as related to value engineering. **[IAS-1994]**

Use Value: It is defined as the qualities and prosperities needed to accomplish a service, product or work.

Esteem Value: It provides properties, features and attractiveness to a service, product or work which make the ownership desirable.

Scrap Value: It is the money which can be recovered when the item is not needed. It is the scrap value.

Cost Value: It is the total cost of material, labour, overhead and services to produce an item.

Exchange Value: It is the property and qualities which enables to exchange a product for something else, which is needed by the exchange.

5. How can the value be increased? **[IAS-2003]**

- To secure the best combination of ideas, processes materials methods and approach to problems involving the least expenditure of resources, time and money.
- Identification and removal of unnecessary costs and hidden costs which provide neither quality, nor use, nor life, nor appearance, nor customer features, are the just rewards for value studies.

$$\text{Value} = \frac{\text{Function}}{\text{Cost}}$$

So to increase value we many increase Function taking cost constant and may decrease cost without affecting function.

6. List down the question which needs to be answered when carrying out value analysis for a product (or) what are the value tests for developing better value alternatives? **[IAS-1996]**

1. Does it use contribute value?
2. Is its cost proportionate to its usefulness?
3. Does it need all its features?
4. Is there anything better for the intended uses?
5. Can a usable part be made by lower cost method?
6. Can a standard product be found which will be usable?
7. Is it made on proper tooling considering quantities used?
8. Do materials reasonable?
9. Will another dependable supplier provide it for less?
10. Is anyone buying it for less?

7. In value engineering approach, functional approach a key feature. Discuss three techniques of functional approach. **[IAS-1984]**

In functional approach

(i) Functions of the items are identified
(ii) Each function is evaluated and compared
(iii) Alternative strategy for cost reduction or function improvement is adopted.

(A) FAST Diagram (Function Analysis system Techniques)
(B) FIRST (Functional Ideas Regarding System Techniques)
(C) FACTS (Functional Analysis of Components of Total System)
(D) PROFIT (Product Return Opportunities by Function Investigated Technique)

8. Other approaches in value engineering

(i) MISS: Modify, Substitute or subdivide or exchange/eliminate to help change.

M	od
I	fy
S	ubstitute
S	ub divide

(ii) DARSIRI: Data colleting, analysis, record ideas, speculate, innovate, review and implement.

D	ata collection
A	nalysis
R	ecod
S	peculate
I	nnovate
R	eview
I	mplement

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years IES Questions

IES-1. **Consider the following:** **[IES-2009]**

1. A system-oriented approach
2. A multidiscipline team approach
3. A function-oriented approach

Which of the above refer to value engineering?

(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 and 3 only

Codes:

(a) Both A and R are individually true and R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

IES-2. **Consider the following steps which are involved in constructing a function analysis system technique of value engineering:** **[IES-2003]**

1. Find the critical path **2. Prepare a function worksheet**
3. Listing of functions **4. Diagram layout**

Which of the following gives the correct sequence of steps?

(a) 1-3-2-4 (b) 2-1-3-4 (c) 4-2-3-1 (d) 3-2-4-1

IES-3. **Assertion (A): Value Engineering is concerned with increasing the quality of the product even at enhanced cost to fulfil the customer.**
Reason (R): Customer requirements are changing very rapidly and survival of a manufacturing industry is linked with fulfilling the customer's requirements. **[IES-2008]**

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-4. **Consider the following phases:** **[IES-1994]**

1. Information phase **2. Evaluation phase**
3. Creative phase **4. Investigation phase**

The correct sequence of these phases in value engineering is:

(a) 1, 3, 4, 2 (b) 1, 3, 2, 4 (c) 3, 1, 4, 2 (d) 3, 1, 2, 4

IES-5. **Consider the following statements:** **[IES-2006]**

The objective of value analysis is:

1. To reduce the cost **2. To increase the profit**
3. To improve quality

Which of the statements given above are correct?

(a) Only 1 and 2 (b) Only 1 and 3 (c) Only 2 and 3 (d) 1, 2 and 3

IES-6. **In value engineering important consideration is given to:** **[IES-1995]**

(a) Cost reduction (b) Profit maximization
(c) Function concept (d) Customer satisfaction.

IES-7. Assertion(A): Value engineering is a technique applied to compete in the market only at the time of introduction of a product or service. [IES-2007]
Reason(R): Increasing the functional worth of a product appreciably, keeping the cost almost constant, is the real objective of value engineering.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-8. In value engineering the term 'value' refers to: [IES-1992]
(a) Market value
(b) Relation between cost and efficiency
(c) Relation between function and cost
(d) Relation between predictability and cost

IES-9. Value analysis is particularly of interest when [IES-1992]
(a) Jobbing work economics are involved
(b) Production is one large scale
(c) Only few components are involved
(d) Costly equipment is used.

IES-10. Assertion (A): Value analysis is superior to other conventional cost reduction techniques. [IES-2005]
Reason (R): In conventional cost reduction techniques, value is increased by widening tolerance bands.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-11. Consider the following basic steps involved in value analysis:
1. Create 2. Blast 3. Refine
The correct sequence of these steps is: [IES-1997]
(a) 1, 2, 3 (b) 3, 1, 2 (c) 1, 3, 2 (d) 2, 1, 3

IES-12. Value is usually considered as a relationship between IES-1996]
(a) Utility and cost (b) Profit and cost
(c) Psychology and reliability (d) Appearance and utility

IES-13. Aluminium tie pin and gold tie pin, both, serve the purpose of keeping the tie in position. But the gold pin has significance due to: [IES-1996]
(a) Exchange value (b) Use value
(c) Esteem value (d) Cost value

Answers with Explanation (Objective)

Previous 20-Years IES Answers

IES-1. Ans. (a)

IES-2. Ans. (d)

IES-3. Ans. (d) Value: It differs from both price and cost in sense, that it is cost proportionate to function i.e.

$$\text{Value} = \frac{\text{Function}\left(\text{or utility}\right)}{\text{Cost}}$$

It can be therefore seen that value of a product can be increased either by increasing its utility with the same cost or decreasing its cost for the same function. Function specifies the purpose of the product or what the product does, what is its utility etc. Thus Assertion is wrong reason is correct answer is (d). A large increase in utility with a small increase in cost.

IES-4. Ans. (b)

IES-5. Ans. (b) Profit is not direct objective of value analysis but it indirectly increases profitability.

IES-6. Ans. (d) In value engineering important consideration is given to customer satisfaction.

IES-7. Ans. (d) A is false. $\text{Value} = \frac{\text{Function}}{\text{Cost}}$, it is used every time; not only at the time of introduction of a product or service.

IES-8. Ans. (c)

IES-9. Ans. (a)

IES-10. Ans. (b)

IES-11. Ans. (d)

IES-12. Ans. (a)

IES-13. Ans. (c)

18. Miscellaneous

Theory at a Glance (For IES, GATE, PSU)

Wages Plan

- Wage incentives include all the plans that provide extra pay for extra performance. In addition to regular wages for the job.
- It implies monetary inducements offered to employees to perform beyond acceptance standards.

Individual Incentive Plans:

Under a system of individual incentives, all or a portion of an individual's pay.

Incentive plan must be:

- Customized
- Flexible
- Up-to-date
- Simple
- Minimum wages should be guaranteed

Advantages

a. Raise productivity, lower production costs
b. Increase earnings
c. Less direct supervision required
d. Objective evaluations

Disadvantages

a. Quality vs. quantity issues
b. Encounter resistance of introduction of new technology
c. New production methods/improvments may be resisted
d. Lack of cooperation in on-the-job-training
e. Elevated levels of mistrust between management & workers

Types of Incentives

Time Based	Output Based
• Halsey plan	• Taylor's differential piece rate
• Rowan plan	• Merrick differential piece rate
• Barth plan	• Gantt task system
• Bedaux plan	• Emerson Efficiency plan

1. **Straight piece rate:** In the straight piece rate system, a worker is paid straight for the number of pieces which he produces per day. In this plan, quality may suffer.

2. **Straight piece rate with a guaranteed base wage:** A worker is paid straight for output set by management even if worker produces less then the target level output. If worker exceeds this target output, he is given wage in direct proportion to the number of pieces produced by him at the straight piece rate.

3. **Halsey Plan:** $W = R.T + (P/100)(S - T).R$ where W: wage of worker, R: wage rate, T: actual time taken to complete job, P: percentage of profit shared with worker, S: std. time allowed. Output standards are based upon previous production records available. Here management also shares a percentage of bonuses.

4. **Rowan Plan:** $W = R.T + [(S - T)/S].R.T$ Unlike Halsey Plan gives bonus on $(S - T)/S$, thus it can be employed even if the output standard is not very accurate.

5. **Emerson Efficiency plan:**
   - Upto 67% of Emerson the worker is determined by dividing the time taken by the standard time rate.
   - Upto 100% Emerson 20% bonus is paid to the worker.
   - An additional bonus of 1% is added each additional 1% efficiency.

Depreciation

Depreciation expense is calculated utilizing either a straight line depreciation method or an accelerated depreciation method. The straight line method calculates depreciation by spreading the cost evenly over the life of the fixed asset. Accelerated depreciation methods such as declining balance and sum of years digits calculate depreciation by expensing a large part of the cost at the beginning of the life of the fixed asset.

The required variables for calculating depreciation are the cost and the expected life of the fixed asset. Salvage value may also be considered.

What is Depreciation?

Depreciation is the way to avail tax benefits for a tangible property or intangible property, which loses the value due to passage of time. Tangible property is such property, which can be seen or touched, e.g., office car, furniture and machine. Intangible property is the property, which cannot be touched or seen, e.g., copyright, licence, franchise, patent, etc. Generally, the value of a property becomes lesser as time passes. This is because of following reasons:

1. Obsolescence or becoming out-of-date
2. Depletion
3. Wear and tear
4. Rusting and corrosion
5. Improper repair
6. Frequent breakdown and accidents
7. Insufficient capacity to cope up with the changed demand situation
8. High maintenance.

Notations Used

As we charge depreciation, the value of asset decreases by the amount of depreciation. This remained value of the asset (which is the difference between purchase price and total depreciation charged till that period) is also called as *value* of the asset.

Let

$(BV)_n$	=	Book value at the end of n^{th} year
d_n	=	Depreciation charged in the n^{th} year
P	=	Purchase price (or the original cost) of the asset
N	=	Service life of the asset
S	=	Scrap value of the asset at the end of its life
D_n	=	Total depreciation charged till n^{th} year

Thus; as defined earlier:

$$\boxed{(BV)_n = P - \sum_{n=1}^{n} d_n}$$

$$\boxed{S = P - \sum_{n=1}^{N} d_n = (BV)_{n=N}}$$

and, **total depreciation charged till the end of n^{th} year:**

$$\boxed{D_N = \sum_{n=1}^{n} d_n}$$

and,

$$\boxed{(BV)_n = (BV)_{n-1} - d_n}$$

We would like to clarify two common misconceptions in depreciation:

(1) Depreciation is not charged for the purpose of procuring another asset as its life ends. Rather it is for the purpose of accounting of the capital expenditure and tax calculation.

(2) Value is not to be confused with the resale price of the asset. In fact, resale price and book value are generally different in all cases.

Accounting Concept of Depreciation

Suppose an asset is purchased for Rs. 10,000. This cost is viewed as the pre-paid operating expense, which will occur during the use of the asset. Therefore, it should be charged against profit during its life time. The scheme of charging this expense provides the real significance of depreciation.

Value Concept of Depreciation

The physical capital such as machine, building, etc., in real sense, does not get spent. But the accounting considers this as an expense, spread over its life-time. Original price minus the retained value of the asset (i.e., value) is the total depreciation of the asset.

Classification of Depreciation

Depreciation is classified as:

(1) Physical depreciation
(2) Functional depreciation
(3) Accident

Depreciation	Description	Example
Physical	❖ Physical impairment of an asset. ❖ Wear and tear. ❖ Deterioration in the item ❖ More vibration, shock, abrasion, impact, noise	(1) Wearing of tire (2) Rushing of pipes (3) Corrosion in metal (4)Chemical decomposition (5) Old car, etc.
Functional	❖ Due to change in demand, the service of the asset becomes inadequate ❖ More efficient model of asset is available (or obsolescence of existing one)	(1) An office has one good manual type-writer. Still, it is profitable and desirable to dispose the manual type-writer and purchase a computer-printer for DTP job. (2) A 386-computer with mono-screen is in good condition, yet it may be inadequate for current use, and a Pentium-II computer with multimedia kit and colour monitor is needed.
Accident	❖ Accidental failure or partial damage	❖ Due to sudden voltage fluctuation, the TV burns out

Methods to Charge Depreciation

Straight Line Method (SLM)

In this, the value of the asset decreases uniformly thought the life of the asset. Thus;

$$d_1, = d_2 = d_3, \ldots = d_N = d \text{ (say)}$$

Since depreciation is charged for ***N*** years during the life-time and total loss in value is the difference between **purchase price (*P*)** and **scrap value (*S*);**

$$d_n = d = \frac{P - S}{N}$$

$$D_n = \sum_{n=1}^{n} d_n = \sum_{n=1}^{n} d = nd$$

or**, Total depreciation charged upto *n* years:**

$$D_n = nd = n\left[\frac{P-S}{N}\right]$$

Therefore,

$$(BV)_n = P - D_n = P - n\left[\frac{P-S}{N}\right]$$

Example: Let purchase price of an asset is Rs. 20,000 and scrap value is Rs. 2,000. The life of asset is 10 years. Then total depreciation, which should be charged in the life-time, is:

$D_N = P - S = 20{,}000 - 2000 = 18000$
$N = 10$ years
$d = 18000/10 = 1800$

End of Year (n)	Depreciation Charged during Year (n) = d_n	Book Value at the end of Year = $(BV)_n = P - \Sigma d_n$
0	—	20,000 (= P)
1	1800	18,200
2	1800	16,400
3	1800	14,600
4	1800	12,800
5	1800	11,000
6	1800	9,200
7	1800	7,400
8	1800	5,600
9	1800	3,800
10	1800	2,000 (= S)

Declining Balance Method (DBM)

Here, we assume that the asset loses its value faster in early life period. A fixed percentage of book value at the beginning of any year is taken as the depreciation charge for that year. Therefore, every year the book value of the asset decreases by a fixed percentage. One of the weaknesses of DBM is that asset never depreciates to zero. If the value declines by α per cent every year, then:

$$d_t = \alpha(BV)_{t-1}$$

$$(BV)_t = (BV)_{t-1} - d_{\hat{t}} = (BV)_{t-1} - \alpha(BV)_{t-1} = (1-\alpha)(BV)_{t-1}$$

Example: For an asset worth 20,000, we will have the following depreciation charges at 20% declining balance method.

End of Year (t)	Depreciation Charged during the Year (d_t) = 0.2 $(BV)_{t-1}$	Book Value at the end of the Year = (BVt) = $(BV)_{t-1} - d_t$
0	—	Rs. 20,000
1	(0.2) (20,000) = 4000	20,000 – 4000 = 16000
2	(0.2) (16,000) = 3200	16,000 – 3200 = 12800
3	(0.2) (12,800) = 2560	12,800 – 2560 = 10240
4	(0.2) (10,240) = 2048	10,240 – 2048 = 8192
5	(0.2) (8192) = 1638	8192 – 1628 = 6554

Double Declining Balance Method (DDBM)

It is noticed that in straight, line method (SLM) of depreciation, the rate of depreciation is $\frac{1}{N}$ where N is the asset useful life. This is because in SLM, $d_t = \frac{(P-V)}{N}$ and is constant. Now, if one takes double of this rate, i.e., $\left(\frac{2}{N}\right)$ and applies the declining balance method, the method to charge depreciation is called as double declining balance method. Therefore, DDBM is a special case of DBM where percentage α is taken as $\frac{2}{N}$.

Hence, in the previous example, if we use DDBM, the value for α would be $\frac{2}{5}$ = 0.4 when the asset life is 5 years. Method to calculate depreciation and book value remains same.

Example: Show that for the declining balance method the rate of depreciation:

$$\alpha = 1-\left(\frac{S}{P}\right)^{\frac{1}{N}};$$

Where, S and P are salvaging values and purchase prices, and N is the asset life.

Solution:

Depreciation during first year, $d_1 = \alpha P$

Value at the end of first year, $(BY)_1 = P - d_1 = P(1-\alpha)$

Similarly, $d_2 = \alpha (BV)_1 = \alpha(1-\alpha)P$

$(BV)_2 = P(1-\alpha) - \alpha(1-\alpha)P = P(1-\alpha)(1-\alpha) = P(1-\alpha)^2$

$d_3 = \alpha(BV)_2 = \alpha(1-\alpha)^2 P$

$(BV)_3 = (BV)_2 - d_3 = (1-\alpha)^2 P - \alpha(1-\alpha)^2 P = (1-\alpha)^2(1-\alpha)P = (1-\alpha)^3 P$

Therefore, at the end of asset life after N years:

$$(\mathrm{BV})_{\mathrm{N}} = (1-\alpha)^{\mathrm{N}}\,\mathrm{P}$$

The value at the end of asset life is also the salvage value. Therefore,

$$S = (1-\alpha)^N P$$

or, $$(1-\alpha)^N = \frac{S}{P}$$

or, $$1-\alpha = \left(\frac{S}{P}\right)^{\frac{1}{N}}$$

or, $$\boxed{\alpha = 1 - \left(\frac{S}{P}\right)^{\frac{1}{N}}}$$

Sum of Year Digits Method (SYD)

In this method, we assume that the value of the asset decreases with a decreasing rate as it becomes older. Let us understand this through an example.

Example: Let *P* = Rs. 20,000, *S* = Rs. 2000, *N* = 5 years. In sum of year digits method, the first step is to sum all digits starting from 1 to *N*. We call it sum of year digit. Thus, for *N* = 5, this sum is 1 + 2 + 3 + 4 + 5 = 15.

Next step is to calculate depreciation and book value. The depreciation in the first year would be $\frac{5}{15}$ time purchase price minus salvage value. Here, numerator indicates the last year digit, i.e., 5. The denominator indicates sum of year digit which we have calculated in step 1. The book value at the end of first year would be purchase price minus depreciation charged. In the second year, the depreciation charge would be $\frac{4}{15}$ times, purchase price minus salvage value. Similarly, the depreciation charged in third year is $\frac{3}{15}$ times book value at the end of second year.

Calculations: Now, the value of asset which would depreciate in 5 years = $P - S$
= 20,000 – 2000 = Rs. 18,000

End of Year (t)	Year in Reverse order	Depreciation Charged during the Year, d_t	Book Value at the End of Year, $(BV)_t$
0	—		20,000
1	5	(5/15) (18,000) = 6000	20,000 – 6000 = 14000
2	4	(4/15) (18,000) = 4800	14,000 – 4800 = 9200
3	3	(3/15) (18,000) = 3600	9200 – 3600 = 5600
4	2	(2/15) (18,000) = 2400	5600 – 2400 = 3200
5	1	(1/15) (18,000) = 1200	3200 – 1200 = 2000(=S)
	Sum = 15		

Example: Show that for SYD method, depreciation during n^{th} year is $(P-S)\frac{2(N-n+1)}{N(N+1)}$

Solution

Let N = year of life for the asset.

Sum of digit from 1 to $N = 1 + 2 + 3 + ... + N = \frac{N(N+1)}{2}$

For n^{th} year the reverse number of year is = $(N - n + 1)$

Hence, depreciation factor for the n^{th} year = $\frac{N-n+1}{\frac{N(N+1)}{2}} = \frac{2(N-n+1)}{N(N+1)}$

Hence, depreciation in the n^{th} year

$$d_n = (P-S)\frac{2(N-n+1)}{N(N+1)}$$

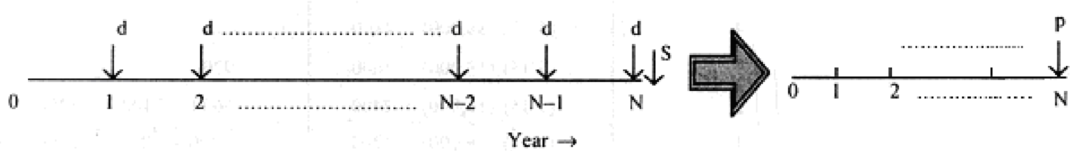
Hence, proved.

SYD method has following features:

1. It gives rapid depreciation in early years.
2. The asset depreciates to the salvage value at the end of life. This is not the case in DBM or DDB method.

Sinking Fund Method (SFM)

We assume that a sinking fund is established and accumulated in this method. By sinking fund we mean that each year depreciation is so charged that the future worth of all depreciation and salvage value becomes equal to purchase price of the asset. Thus, if rate of interest is *i,* the cash flow diagram is:



$$P = S + \sum_{n=1}^{N} \text{(Future worth of depreciation till } N^{th} \text{ year)}$$

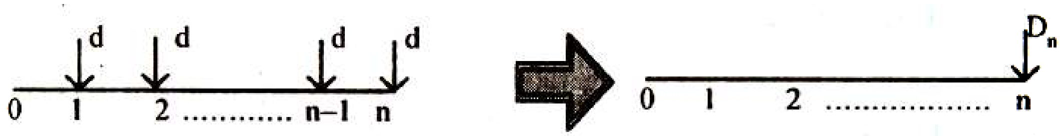
$$= S + d + d(1+i) + d(1+i)^2 + \dots + d(1+i)^{N-1}$$

$$= S + d\left[1 + (1+i) + (1+i)^2 + \dots + (1+i)^{N-1}\right]$$

$$= S + d\left[\frac{(1+i)^N - 1}{(1+i)-1}\right] = S + d\left[\frac{(1+i)^N - 1}{i}\right]$$

or, $$\boxed{d = (P-S)\left[\frac{i}{(1+i)^N - 1}\right]}$$

Total depreciation till n^{th} year is shown as D_n



Here;

$$D_n = d + d(1+i) + \dots + d(1+i)^{n-1} = d\left[\frac{(1+i)^n - 1}{i}\right]$$

Putting the value of d from earlier derivation:

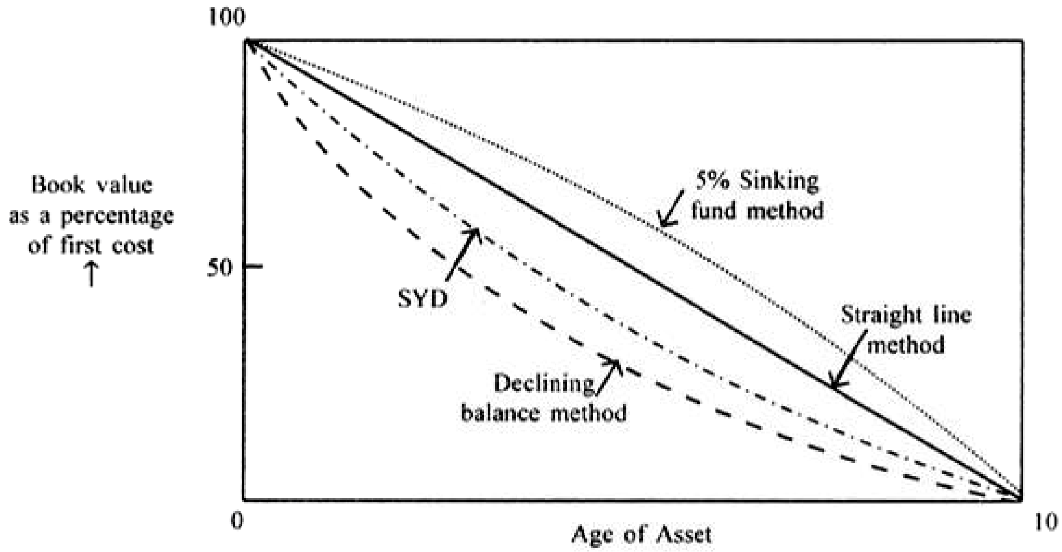
$$= (P-S)\left[\frac{i}{(1+i)^N - 1}\right]\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\boxed{D_n = (P-S)\left[\frac{(1+i)^n - 1}{(1+i)^N - 1}\right]}$$

Also,

$$\mathbf{(BV)_n = P - D_n}$$

Sinking fund method is generally not very common for accounting purpose. It gives a low depreciation in early year.



Comparison of different methods of depreciation

Example: A machine is purchased for Rs. 75,000 with an estimated age of 10 years. Its scrap value is Rs. 5000 at the 10^{th} year. What will be the depreciation for 6^{th} year and book value at the end of 6^{th} year? Assume an interest rate of 5%.

Solution: Given; P = Rs. 75,000, S = Rs. 5000, N = 10 years, n = 6 year

Straight line method

$$d = \frac{(P-S)}{N} = \frac{75,000-5000}{10} = \text{Rs. } 7000$$

In 6^{th} year depreciation charged is Rs. 7000

Total depreciation charged till 6^{th} year = 6 × 7000

$$D_6 = 42,000$$

Book value at the end of 6^{th} year = $P - D_6$

$$= 75,000 - 42,000 = \text{Rs. } 33,000$$

Declining balance method

$$\alpha = 1 - \left(\frac{S}{P}\right)^{\frac{1}{N}}$$

$$\alpha = 1 - 10\sqrt{\frac{5000}{75,000}} = 0.237$$

$$(BV)_N = (1-\alpha)_N P$$

$$(BV)_5 = 75,000\left(\frac{5000}{75,000}\right)^{\frac{5}{10}} = 19365$$

$$(BV)_6 = 75,000\left(\frac{5000}{75,000}\right)^{\frac{6}{10}} = 14771$$

Therefore, $d_6 = (BV)_6 - (BV)_5 = 19365 - 14771 = \text{Rs. } 4594$

Double declining method

$$\alpha = \frac{2}{N} = \frac{2}{10} = 0.2$$

$$(BV)_5 = 75,000(1-0.2)^5 = 24576$$

$$(BV)_6 = 75,000(1-0.2)^6 = 9661$$

$$d_6 = 24576 - 19661 = \text{Rs. } 4915$$

Sum of years digit method (SYD)

Sum of year = 1 + 2 + 3 + . . . + 10 = 55

Depreciation factor for 6th year = $\frac{5}{55}$

$$d_6 = (75,000 - 5000)\frac{5}{55} = \text{Rs. } 6363.6$$

$$(BV)_6 = 75,000 - \sum_1^6 d_n$$

$$= 75,000 - (75,000 - 5000)\left[\frac{10}{55} + \frac{9}{55} + \frac{8}{55} + \frac{7}{55} + \frac{6}{55} + \frac{5}{55}\right]$$

$$= 75,000 - 70,000 \times 0.818 = \text{Rs. } 17727.27$$

Sinking fund method (SFM)

$$d = (75,000 - 5000)\left[\frac{i}{(1+i)^N - 1}\right] = 70,000\left[\frac{0.05}{(1+0.05)^{10} - 1}\right] = \text{Rs. } 5565.32$$

$$d_6 = 5565.32\left[\frac{(1+i)^5 - 1}{i}\right] = 5565.32\left[\frac{(1.05)^5 - 1}{0.05}\right] = 30751.90$$

$$(BV)_6 = 75,000 - (75,000 - 5000)\left[\frac{(1+i)^6 - 1}{(1+i)^{10} - 1}\right]$$

$$= 75,000 - 70,000\frac{0.34}{0.63} = 75,000 - 70,000 \times 0.54 = \text{Rs. } 37145$$

Method of depreciation →	SLM	DBM	DDB	SYD	SFM
Depreciation in 6*th* year (Rs.)	7000	4594	4915	6363.64	30751.90
Book value at the end of 6*th* year (Rs.)	33000	14771	19661	17727.27	37145.00

Service Life of Asset

The estimate of service life has important bearing on the calculation of depreciation. It depends upon the experience of the industry. Some guidelines are as follows:

Asset	Range for the life of asset in calculating depreciation
Furniture	8 – 12
Aircraft	5 – 7
Computer	5 – 7
Electric equipment	9 – 15
Motor Car	9 – 15
Construction	4 – 6

Gantt Chart

The Gantt chart is a very useful graphical tool for representing a production schedule. A common production schedule involves a large number of jobs that have to be processed on a number of production facilities, such as machine testing, etc. Gantt chart contains time on its one axis. The status and scheduling of jobs on a time scale is schematically represented. This gives a clear pictorial representation of relationship among different production-related activities of a firm on a time horizon. Gantt chart is equally suited for any other non-production activity, where the work is similar to a project involving many activities.

How to prepare a Gantt chart

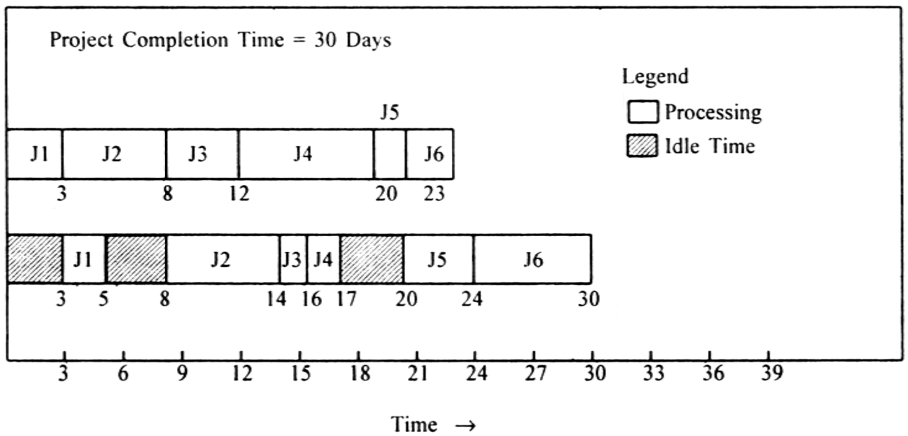
The Gantt chart consists of two axes. On X-axis, generally, time is represented. This may be in the units of years, months, weeks, days, hours or minutes. on Y-axis, various activities or tasks, machine-centers or facilities are represented.

Example: Let us consider two machines and six jobs. The processing time (in hour) for each job is given below:

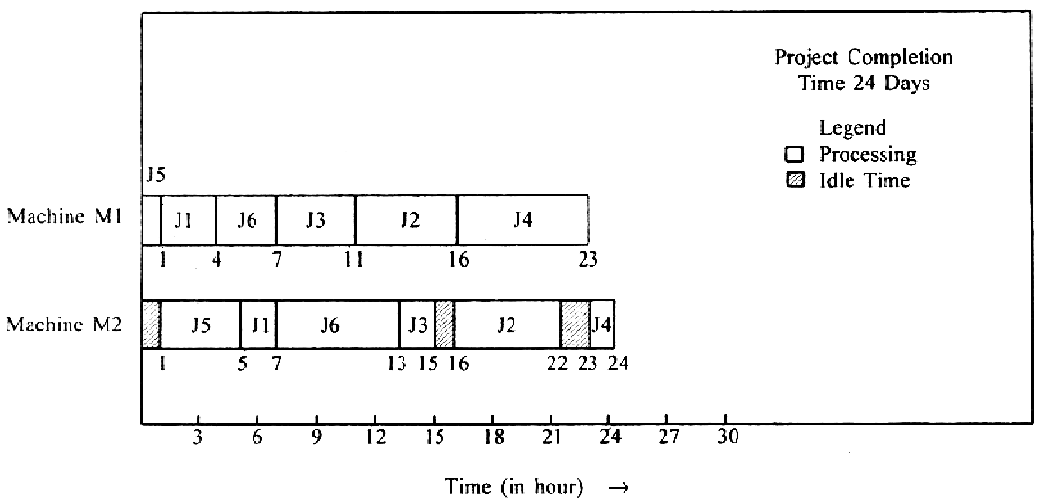
Assume that process-sequence such that machine M_1 is need before M_2. We have to draw a Gantt Charted:

Jobs	J1	J2	J3	J4	J5	J6
Machine M_1	3	5	4	7	1	3
Machine M_2	2	6	2	1	4	6

The Gantt chart is as follows:



Gantt chart on *First-come-first served* Basis



Gantt chart for *Shortest Processing Time* (SPT) Sequence on Machine M_1

The project, consisting of six jobs on two machines, is scheduled in such a way that processing on the first machine should be over before processing on the second machine is undertaken. This is due to sequence-of-operations requirements. For the sequencing rule of FCFS, the Gantt chart is shown in Figure Above. For the SPT rule, the Gantt chart is shown in Fig. above. The methodology is quite simple and already explained.

In Fig. above the sequencing rule is *first-cum-first-served*. Therefore, the jobs are sequenced as $J1 \to J2 \to J3 \to J4 \to J5 \to J6$. Till job $J1$ is processed on machine $M1$, the other machine $M2$ is idle. It cannot process other jobs in this period due to *sequence-of-operation* constraint. As soon as $J1$ is released from $M1$. it goes on $M2$. Now, processing of $J1$ on $M2$ is over at 5^{th} hour. But, $J2$ will be free from machine $M1$ after processing at 8^{th}

hour. Hence, machine $M2$ will remain idle from the time when $J1$ is over on $M2$ (i.e., 5^{th} hour) to the earliest possible loading time (i.e., 8^{th} hour) on $M2$. Similarly, all other jobs are scheduled on Gantt chart.

In Fig. above, the sequencing rule is SPT. Therefore, the jobs are sequenced as: $J5 \rightarrow J1 \rightarrow J6 \rightarrow J3 \rightarrow J2 \rightarrow J4$. Idle and processing time on 1 and 2 are shown on the Gantt Charts.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Depreciation

GATE-1. A machine is purchased for Rs. 32,000, and its assumed life is 20 years. The scrap value at the end of its life is Rs. 8,000. If the depreciation Is charged by the diminishing balance method, then the percentage reduction in its value, at the end of the first year is: [GATE-1997]

(a) 6.7% (b)7.1% (c) 7.2% (d)7.6%

Others

GATE-2. Six jobs arrived in a sequence as given below: [GATE-2009]

Jobs	Processing Time (days)
I	4
II	9
III	5
IV	10
V	6
VI	8

Average flow time (in days) for the above jobs using Shortest Processing Time rule is:

(a) 20.83 (b) 23.16 (c) 125.00 (d) 139.00

GATE-3. A set of 5 jobs is to be processed on a single machine. The processing time (in days) is given in the table below. The holding cost for each job is Rs. *K* per day. [GATE-2008]

Job	Processing time
P	5
Q	2
R	3
S	2
T	1

(a) T-S-Q-R-P (b) P-R-S-Q-T (c) T-R-S-Q-P (d) P-Q-R-S-T

GATE-4. Capacities of production of a item over 3 consecutive months in regular time are 100, 100 and 80 and in overtime are 20, 20 and 40. The demands over those 3 months are 90, 130 and 110. The cost of production in regular time and overtime are respectively Rs. 20 per item and Rs. 24 per item. Inventory carrying cost is Rs. 2 per item per month. The levels of starting and final inventory are nil. Back-order is not permitted. For minimum cost of plan, the level of planned production in overtime in the third month is: [GATE-2007]

(a) 40 (b) 30 (c) 20 (d) 0

GATE-5. A stockist wishes to optimize the number of perishable items he needs to stock in any month in his store. The demand distribution for this perishable item is: [GATE-2006]

Demand (in units)	2	3	4	5
Probability	0.10	0.35	0.35	0.20

The stockist pays Rs. 70 for each item and he sells each at Rs. 90. If the stock is left unsold in any month, he can sell the item at Rs. 50 each. There is no penalty for unfulfilled demand. To maximize the expected profit, the optimal stock level is:

(a) 5 units (b) 4 units (c) 3 units (d) 2 units

GATE-6. A company uses 2555 units of an item annually. Delivery lead time is 8 days. The reorder point (in number of units) to achieve optimum inventory is: [GATE-2009]

(a) 7 (b) 8 (c) 56 (d) 60

GATE-7. The distribution of lead time demand for an item is as follows:

Lead time demand	Probability
80	0.20
100	0.25
120	0.30
140	0.25

The reorder level is 1.25 times the expected value of the lead time demand. The service level is: [GATE-2005]

(a) 25% (b) 50% (c) 75% (d) 100%

GATE-8. A residential school stipulates the study hours as 8.00 pm to 10.30 pm. Warden makes random checks on a certain student 11 occasions a day during the study hours over a period of 10 days and observes that he is studying on 71 occasions. Using 95% confidence interval, the estimated minimum hours of his study during that 10 day period is: [GATE-2003]

(a) 8.5 hours (b) 13.9 hours (c) 16.1 hours (d) 18.4 hours

GATE-9. Cellular manufacturing is suitable for [GATE-2000]

(a) A single product in large volumes
(b) One-off production of several varieties
(c) Products with similar features made in batches
(d) Huge variety of products in large volumes

GATE-10. In a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total of 900 bolts are respectively [GATE-2000]

(a) 90 and 9 (b) 9 and 90 (c) 81 and 9 (d) 9 and 81

GATE-11. Analysis of variance is concerned with [GATE-1999]

(a) Determining change in a dependent variable per unit change in an independent variable
(b) Determining whether a qualitative factor affects the mean of an output variable

(c) Determining whether significant correlation exists between an output variable and an input variable
(d) Determining whether variances in two or more populations are significantly different.

GATE-12. A 750 hours life test is performed on ten components. If one component fails after 350 hours of operation and all others survive the test, then the failure per hour is: [GATE-1997]
(a) 0.000141 (b) 0.000133 (c) 0.00141 (d) 0.00133

GATE-13. The probability of a defective piece being produced in a manufacturing process is 0.01. The probability that out of 5 successive pieces, only one is defective, is: [GATE-1996]
(a) $(0.99)^4 (0.01)$
(b) $(0.99) (0.01)^4$
(c) $5 \times (0.99) (0.01)^4$
(d) $5 \times (0.99)^4 (0.01)$

Previous 20-Years IES Questions

Wages Plan

IES-1. Consider the following conditions: [IES-1996]
1. Minimum wages should be guaranteed
2. Providing incentive to group efficiency performance
3. A differential price rate should exist
4. All standards should be based on optimum standard of production.
Those essential for an incentive plan include
(a) 1 and 4 (b) 1 and 2 (c) 3 and 4 (d) 2 and 3

IES-2. Consider the following factors: [IES-1995]
1. Adequate incentive
2. Ease of administration
3. Flexibility
4. Guaranteed basic pay
5. Higher wages
6. Simplicity
Among these, the factors which are to be considered while developing a good wage incentive plan would include
(a) 1, 2, 3 and 5
(b) 2, 3, 4 and 5
(c) 1, 2, 4 and 6
(d) 1, 2, 5 and 6.

IES-3. A sum of Rs. 500/- Paid as wages for erecting a machine should be debited to: [IES-1992]
(a) Machinery account
(b) Suspense account
(c) Wages account
(d) Repair account

IES-4. Match List-I (Wage payment plans) with List-II (Method of payment) and select the correct answer: [IES-2002]

List-I	List-II
A. Time based	**1.** Stock distribution
B. Price rate	**2.** 100% bonus
C. Gain sharing	**3.** Taylor differential piece rate
D. Indirect payments	**4.** Straight salary

Codes; A B C D A B C D

(a)	4	3	2	1	(b)	2	1	4	3
(c)	4	1	2	3	(d)	2	3	4	1

IES-5. **Given that E = Earnings, R = Rate per hour, T = Time worked in hours, S = Standard time on the basis of data in hours. [IES-1996]**
Rowan wage incentive plan is:
(a) $E = RT + \left(\frac{S-T}{S}\right)R$ (b) $E = RT + (S-T)R$
(c) $E = RT + 0.4(S-T)R$ (d) $E = RT + \left(\frac{S-T}{S}\right)RT$

IES-6. **Earning in Rowan system =** $R \times T_s + \frac{T_s - T_a}{T_s} T_a R$ **[IES-1994]**
Earning in 50% Halsey plan = $R \times T_s + p(T_s - T_a)R$
Where R = hourly rate, T_a **= actual completion time of task**
T_s **= standard time for the task, p = percentage allowed.**
Both Rowan system and 50% Halsey plan will provide the same earning when the actual time is:
(a) Equal to standard time (b) Half the standard time
(c) One-quarter of standard time (d) Twice the standard time.

IES-7. **Which of the following plans guarantees minimum wage to a workers and bonus based on fixed percentage of time saved?**
(a) Gantt plan (b) Halsey plan **[IES-1992]**
(c) Rowan plan (d) Bedaux plan

IES-8. **Match List-I (Topic) with List-II (Method of solving) and select the correct answer using the codes given below the lists: [IES-1997]**

List-I	List-II
A. Forecasting	**1.** North-West corner method
B. Linear programming	**2.** Rowan plan
C. Wage incentive	**3.** Method of penalty
D. Work measurement	**4.** Time series analysis
	5. Work factor system

Codes:	A	B	C	D		A	B	C	D
(a)	4	3	1	5	(b)	4	1	5	3
(c)	4	3	2	5	(d)	3	1	2	4

IES-9. **Rowan incentive plan is given by (R = Hourly rate,** T_a**= Actual time taken for job,** T_s **= Standard time for job and E = Earnings)**

$$E = R \times T_a + \frac{(T_s - T_a)}{T_s} \times T_a \times R$$

The shape of the curves between bonus earned and percentage time saved is a [IES-2000]
(a) Straight line (b) Parabola
(c) Horizontal line (d) Vertical line

IES-10. **Given that: [IES-1999]**
E = Earning in time T_a

Ta = Actual time of work
T_s = Standard time set to complete the task
R = Rate per unit time
If $E = R.T_a + \frac{R}{2}(T_s - T_a)$, then the graph between bonus earned and time saved is a
(a) Straight line (b) Convex curve
(c) Concave curve (d) Parabola

IES-11. The following data pertain to a worker: [IES-1993]
Base rate = Rs. 20 per hour
Time taken for completing the job = 2 hours.
Standard time = 3 hours.
Under Halsey plan, the total earning of the worker is:
(a) Rs. 36.67 (b) 40.67 (c) Rs.46.67 (d) Rs. 56.67

IES-12. For the maintenance section of an industry, the most suitable incentive plan would be: [IES-1993]
(a) Piece rate system (b) Group incentive plan
(c) Bonus plan (d) Profit sharing plan

IES-13. In Bedaux skill and effort rating, a normal worker is rated at [IES-1992]
(a) B (b) 60 B (c) 100 B (d) 144 B

Depreciation

IES-14. A piece of equipment has been purchased by the city for Rs. 10,000 with an anticipated salvage at Rs. 500 after 8 years. Which of the depreciation models will yield the greatest reduction in the book value of the equipment? [IES-1992]
(a) Straight line depreciation
(b) Sum-of-years digits depreciation
(c) Declining balance depreciation
(d) Declining balance depreciation is switched to straight line.

IES-15. An equipment has been purchased for Rs. 120 and is estimated to have 10 years life and a scrap value of Rs. 20/- at the end of life. The book value of the equipment at the end of sixth year then the interest rate is 5% (using declining balance methods) will be: [IES-1992]
(a) Rs. 40.95 (b) Rs. 51.25 (c) 55.00 (d) Rs. 59.25

IES-16. A machine costs Rs. 2000 installed. The cost of repairs increases Rs. 40 annually. The scrap value of the machine is Rs. 200. The economic repair life of the machine is nearly [IES-1992]
(a) $5\frac{1}{2}$ years (b) $7\frac{1}{2}$ years (c) $9\frac{1}{2}$ years (d) $11\frac{1}{2}$ years

Others

IES-17. The reason for diversification is to [IES-2002]

(a) Reduce production cost
(b) Balance low demand high capacity situation
(c) Satisfy more customers
(d) Improve capacity utilization

IES-18. Match List-I (Limits in normal distribution) with List-II (Population covered) and select the correct answer: [IES-2002]

List-I	List-II
A. $\pm 3\,\sigma$	**1.** 0.3413
B. $\pm 2\,\sigma$	**2.** 0.6826
C. $\pm 1\sigma$	**3.** 0.9973
	4. 0.9545

Codes:	**A**	**B**	**C**		**A**	**B**	**C**
(a)	3	4	2	(b)	3	2	4
(c)	4	2	3	(d)	4	3	2

IES-19. The characteristic life-cycle of a product consists of four periods. The rate of consumption increases rapidly at the beginning of the
(a) Incubation period (b) Growth period **[IES-1998]**
(c) Maturity period (d) Decline period

IES-20. Which of the following are the principles of material handling?
1. Keep all handling to the minimum [IES-1996]
2. Move as few pieces as possible in one unit.
3. Move the heaviest weight to the least distance
4. Select only efficient handling equipment
Select the correct answer using the codes given below:
Codes: (a) 1, 2, 3, and 4 (b) 1, 3, and 4 (c) 1, 2 and 4 (d) 2 and 4

IES-21. Production cost refers to prime cost plus [IES-1995]
(a) Factory overheads
(b) Factory and administration overheads
(c) Factory, administration and sales overheads
(d) Factory, administration and sales overheads and profit.

IES-22. Match List-I (Equipment) with List-II (Application) and select the correct answer using the codes given below the lists: [IES-1993]

List-I	List-II
A. Hoist	**1.** For moving over a fixed route
B. Conveyor	**2.** For transporting material over a varying path
C. Fork truck	**3.** For vertically raising or lowering material in a fixed location
D. Elevators	**4.** For overhead lifting of loads in a fixed area

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	4	2	3	5	(b)	2	3	4	5
(c)	4	1	2	3	(d)	2	1	4	3

IES-23. Goodwill of an enterprise is termed as [IES-1992]
(a) Liquid asset (b) Volatile
(c) Fictitious asset (d) Liability

IES-24. The minimum attractive rate return is different for various types of projects. All of the following factors will affect the minimum attractive rate of return for a new project except [IES-1992]
(a) The amount of capital that is available in the retained earnings
(b) The interest paid on capital, which is obtained through debt financing
(c) The rates used by the lending institutions or other firms
(d) None of the above

IES-25. A dealer sells a radio set at Rs. 900 and makes 80% profit on his investment. If he can sell it at Rs. 200 more, his profit as percentage of investment will be: [IES-1999]
(a) 160 (b) 180 (c) 100 (d) 120

Previous 20-Years IAS Questions

Wages Plan

IAS-1. Money required for the purchase of stores, payment of wages etc. is known as [IAS-1994]
(a) Block capital (b) Reserved capital
(c) Authorised capital (d) Working capital

IAS-2. Under the Emerson Efficiency Plan, the worker gets a normal wage at the efficiency of: [IAS-1996]
(a) 50% (b) 66.2/3% (c) 75% (d) 85%

Depreciation

IAS-3. An equipment costs P and its service in number of years is N. If the annual depreciation charge is $\frac{P-L}{N}$ then L is the [IAS-2000]
(a) Maintenance cost (b) Salvage value
(c) Production cost (d) Idle cost

Others

IAS-4. In trouble-shooting brain-storming session, views of different people can be best represented and integrated by which one of the following? [IAS-2007]
(a) Deming's Circle Diagram (b) Crosby's
(c) Ishikava Diagram (d) Juran Diagram

IAS-5. In case of mutually exclusive projects, which one is the most suitable criterion for evaluation of projects? [IAS-2007]
(a) Net Present Value (b) Payback Period
(c) Internal Rate of Return (d) Rate of return

IAS-6. Which of the following can be solved by the Brown and Gibson procedure? [IAS-2004]
(a) Transportation problem (b) CPM network
(c) Site location problem (d) Product-mix problem

IAS-7. **Several sequencing rules can be used to sequence jobs. The performance of these rules can be studied using several performance measures. Consider the following sequencing rules:**
SPT (Shortest Processing Time) **[IAS-2004]**
EDD (Expected Due Date)
And the following performance measures:
MFT (Mean Flow Time)
ML (Mean Lateness)
Which one of the following is not correct?
(a) SPT minimizes MFT (b) EDD minimizes MFT
(c) EDD minimizes ML (d) SPT minimizes ML

IAS-8. **Assertion (A): Shortest processing time algorithm reduces waiting time of a batch while processing.** **[IAS-2002]**
Reason (R): SPT rule always meets the due date requirements.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-9. **Which one of the following pairs is not correctly matched?**
(a) SPT Rule -----------fairness to all jobs **[IAS-1995]**
(b) EDD rule---lateness minimization
(c) Johnsons Rule-- two machine sequencing
(d) FIFO Rule----fairness to all jobs

IAS-10. **For economic manufacture, the total annual cost is given as 12 M $\left(P+\dfrac{D}{Q}\right)$. Where, M is number of parts made per month P is processing cost per part.** **[IAS-2004]**
Which one of the following is represented by D/Q in the above expression?
(a) Setting up cost (b) Profit per part
(c) Bonus (d) Setting up cost/part

IAS-11. **Profit volume chart techniques is an effective tool of application for analyse is when the Company is dealing with** **[IAS-2004]**
(a) One product only (b) A loss situation
(c) Only turn-key assignments (d) More than one product

IAS-12. **Standard material cost of a product is Rs. 20/- @ Rs. 10 per kg. In one batch, on an average the consumption of material was 1.8 kg and rate of material was Rs.12 per kg. What is the material usage variance?** **[IAS-2004]**
(a) Rs. 2/– adverse (b) Rs. 2/– favourable
(c) Rs. 2.40 favourable (d) Rs. 1.60 adverse

IAS-13. **Which one of the following does not form a part of the direct cost of a component?** **[IAS-2003]**
(a) Cost of special tooling used

(b) Cost of material used
(c) Cost of material wasted
(d) Wages of the labour actually involved

IAS-14. Match List-I (Cost Element) with List II (Type of Cost) and select the correct answer using the codes given below the lists: [IAS-2003]

List-I		**List-II**	
A.	Discount	**1.**	Ordering cost
B.	Preparation of the machine for a product	**2.**	Material cost
C.	Negotiations with vendors	**3.**	Set-up cost
D.	Rent for the warehouse	**4.**	Carrying cost

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	1	3	4	(b)	4	3	1	2
(c)	2	3	1	4	(d)	4	1	3	2

IAS-15. Which of the following is the expression for the market price?
(a) Selling price + discount to distributor **[IAS-2003]**
(b) Selling price – discount to distributor
(c) Total cost + discount to distributor
(d) Office cost + selling and distribution expenses

IAS-16. Which one of the following pairs is correctly matched? [IAS-2000]
(a) Work space design : Ergonomics
(b) Motion economy : Terotechnology
(c) Method study : Milestone chart
(d) Time study : Climograph

IAS-17. Assertion (A): Chronocycle graph is useful in getting the direction as well as speed of the movement of the human body elements.
Reason (R): A record of path of movement is affected by a continuous source of light. [IAS-2003]
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-18. The type of industry that provides the lowest capital output ratio is:
(a) Steel industry (b) Small scale industry **[IAS-2002]**
(c) Watch industry (d) Machine tool industry

IAS-19. The cost of producing one more unit is referred to as [IAS-2001]
(a) Capacity planning (b) Extra cost
(c) Opportunity cost (d) Marginal cost

IAS-20. Which one of the following conditions warrants that a business should be closed? [IAS-2001]
(a) Flat variable cost line (b) Vertical variable cost line

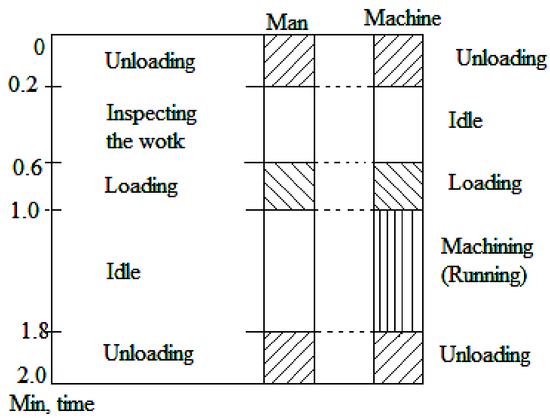
(c) Steep revenue line
(d) Capacity utilization can never reach 100%

IAS-21. For a product, the direct labour cost is Rs. 5 and direct materials cost is Rs. 10. The annual cost of direct materials is expected to be Rs. 2, 00,000 and the annual overhead to be absorbed is Rs. 1, 20,000. The total cost of the product will be: [IAS-2000]
(a) Rs. 16 (b) Rs.18 (c) Rs.19 (d) Rs.21

IAS-22. Assertion (A): Japanese mass production methods use robots.
Reason (R): Japanese production philosophy is to use pull system of manufacture. [IAS-1999]
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

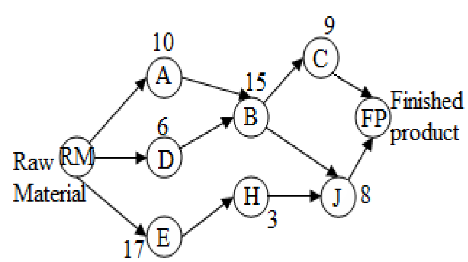
IAS-23. Standardization of products is done to: [IAS-1998]
(a) Eliminate unnecessary varieties in design
(b) Simplify manufacturing process '
(c) Make interchangeable manufacture possible
(d) Reduce material cost

IAS-24. Man-Machine chart for a production activity is represented in the figure given below. The percentage utility of man and machine in the production activity are respectively Man Machine
(a) 50% and 7%
(b) 55.5% and 77.7%
(c) 80% and 60%
(d) 50% and 90%



[IAS-1998]

IAS-25. The raw material can be routed through different machines for manufacturing a specific product. Figure shown below indicates the time taken in hours for each machine viz. A, B, C, D, E Hand J. Which one of the shown routes is to be allowed for maximum productivity? [IAS-1998]



(a) RM – A – B – C – FP
(b) RM – D – B – J – FP

(c) RM – D – B – C – FP (d) RM – E – H – J – FP

IAS-26. Balance sheet refers to the financial position of the Company
(a) For a particular year (b) For a particular month **[IAS-1996]**
(c) In a particular shop (d) On a particular date

IAS-27. Production cost per unit can be reduced by: [IAS-1995]
(a) Producing more with increased inputs
(b) Producing more with the same inputs
(c) Eliminating idle time
(d) Minimizing resource waste

IAS-28. Which of the following characteristics are more important in the equipment selected for mass production shops? [IAS-1995]
(a) Fast output (b) Low tooling cost
(c) Low labour cost (d) Versatility

IAS-29. Assertion (A): Job shop production leads to large work-in-process inventory. [IAS-1994]
Reason (R): Jobbing production is used to manufacture medium variety production.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false (d) A is false but R is true

IAS-30. Assertion (A): FIFO rules for sequencing are accepted easily by all as it appears fair to all. [IAS-1994]
Reason (R): FIFO rule is optimum for most scheduling situations.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-31. Assertion (A): Gang process chart is an aid in studying the activities of a group of people working together. [IAS-1994]
Reason (R): Gang process chart analyses the cycle or routine followed by each member of the gang.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-32. Consider the following constituent steps of capital budgeting:
1. Short range capital budgeting [IAS-2003]
2. Long range capital budgeting
3. Search for opportunities and sources
4. Measurement of worth and selection
The correct sequence of these steps from the commencement is:
(a) 3-2-1-4 (b) 2-3-4-1 (c) 3-1-2-4 (d) 2-4-3-1

IAS-33. Which of the following are the elements of disbursements in capital budgeting? [IAS-2003]

1. Dividend **2. Profits retained**
3. Loan to other companies **4. Depreciation**
5. New investments
Select the correct answer using the codes given below:
Codes: (a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 5 (d) 2, 4 and 5

IAS-34. In the Capital Budget, which one of the following project-expenditures CANNOT be called capital spending? [IAS-2001]
(a) Building new dams
(b) Building new roads
(c) Expenditure on disaster management
(d) Purchases of aircraft for defense

IAS-35. Assertion (A): Companies investing in countries with high inflation rates use payback period method for capital budgeting. [IAS-2003]
Reason (R): The operating cash flows in such investments are precisely and easily determined.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is **not** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Answers with Explanation (Objective)

Previous 20-Years GATE Answers

GATE-1. Ans. (a) Book value after k^{th} year, $B_k = P\left(\frac{S}{P}\right)^{k/n}$

Where P = Purchase value
S = Scrape value
n = Life period in years

Given P = Rs. 32,000; S = Rs. 8000; n = 20 years

$\therefore \quad B_k = 32000\left(\frac{8000}{32000}\right)^{1/20} = 32000(0.933)$

Reduction in value = 1 – 0.9333 = 0.067
% reduction in value = 6.7%

GATE-2. Ans. (a) mean flow time time $= \frac{125}{6} = 20.83$

GATE-3. Ans. (a)
GATE-4. Ans. (b)
GATE-5. Ans. (b)
GATE-6. Ans. (c)
GATE-7. Ans. (d)
GATE-8. Ans. (c) Number of total observations in 10 days = 11 × 10 = 110
Number of observation when studying = 71

$\therefore$ p = probability of studying $= \frac{71}{110} = 0.6455$

Total studying hour in 10 days = (2.5 hours) × 10 = 25 hours
Hence, minimum number of hours of studying in 10 days = (25 hours) × p
= 25 × 0.6455
= 16.13 hours

GATE-9. Ans. (c)
GATE-10. Ans. (a) Mean, μ = nρ = 900 × 0.1 = 90
Standard deviation, σ = $n\rho^2$ = 900 × 0.1 × 0.1 = 9
GATE-11. Ans. (d) Analysis of variance is used in comparing two or more populations, e.g. different types of manures for yielding a single crop.
GATE-12. Ans. (a) During the test, 10 components are tested for 750 hours.
$\therefore$ Total time, T = 10 × 750 = 7500 unit hours.
Total operating time = 7500 – 350 = 7150 hours.

Number of failures/hour $= \frac{1}{7150} = 0.000141$

GATE-13. Ans. (d) Probability $= {}^5C_1\left(0.01\right)^1\left(0.99\right)^4 = 5\times\left(0.99\right)^4\left(0.01\right)$

Previous 20-Years IES Answers

IES-1. Ans. (a)
IES-2. Ans. (c)
IES-3. Ans. (a)
IES-4. Ans. (a)
IES-5. Ans. (d)
IES-6. Ans. (b)
IES-7. Ans. (b)
IES-8. Ans. (c)
IES-9. Ans. (b)
IES-10. Ans. (a)
IES-11. Ans. (c) Assuming 50 – 50; Under Halsey Plan

$$\text{Wage (W)} = R.T + \frac{P}{100}\left(S - T\right).R = 20 \times 2 + \frac{\left(3-2\right) \times 20}{2} = \text{Rs.}50/\text{-}$$

R = base rate = Rs. 20/- hrs.
T = actual time = 2 hrs.
S = standard time = 3 hrs.

IES-12. Ans. (c) For the maintenance section, it is desirable that worker does the job assigned fast and is rewarded suitably. Thus bonus plan is best suited.
IES-13. Ans. (b)
IES-14. Ans. (d)
IES-15. Ans. (c) Rate of depreciation $= 1 - \left(\frac{S}{C}\right)^{1/T} = 1 - \left(\frac{20}{120}\right)^{1/10} = 0.164$

Book value at the end of six years $= C(1-p)t$

$$= 120(1 - 0.164)6 = \text{Rs. } 40.95$$

IES-16. Ans. (c) Economic repair life, $x = \sqrt{\frac{2(C-s)}{b}}$

Where, C = Initial cost of the machine = Rs. 2000
s = Scrap value of machine = Rs. 200

$\therefore \quad x = \sqrt{\frac{2(2000-200)}{40}} = \sqrt{90} = 9\frac{1}{2}$ years.

IES-17. Ans. (c)
IES-18. Ans. (a)
IES-19. Ans. (b)
IES-20. Ans. (b) Handle product in as large a unit as practical.
IES-21. Ans. (b) Production cost refers to prime cost plus factory and administrative overheads.
IES-22. Ans. (c)
IES-23. Ans. (c)
IES-24. Ans. (d) MARR: Minimum attractive rate of return or minimum acceptable rate of return, is the minimum return on a project a manager is willing to accept before starting a project. The MARR generally increases with increased risk.
IES-25. Ans. (d)

Previous 20-Years IAS Answers

IAS-1. Ans. (d)
IAS-2. Ans. (b)
IAS-3. Ans. (b) In straight line depreciation method. Annual depreciation is constant during the production life of the product. Thus the annual depreciation change is given by $\frac{P-L}{N}$, where L is the salvage value.
IAS-4. Ans. (c)
IAS-5. Ans. (d)
IAS-6. Ans. (d)
IAS-7. Ans. (b) SPT minimize MFT & ML and EDD minimize only ML
IAS-8. Ans. (a)
IAS-9. Ans. (a)
IAS-10. Ans. (d)
IAS-11. Ans. (d)
IAS-12. Ans. (d) Actual material cost = Rs. 1.8×12=Rs. 21.6
Standard material cost = Rs. 20
Variance = Rs. 1.60 adverse
IAS-13. Ans. (c)
IAS-14. Ans. (c)
IAS-15. Ans. (d)
IAS-16. Ans. (a) Terotechnology : Economic management of asset
Climograph: Graphical depiction of a monthly prediction and temperature of a place.
IAS-17. Ans. (c)
IAS-18. Ans. (d)
IAS-19. Ans. (d)
IAS-20. Ans. (b)
IAS-21. Ans. (d) No of product per year = $\frac{200000}{10}=20000$ units.
Therefore overhead cost per unit = $\frac{120000}{20000}=6$ per unit.
Total cost = Labour cost + Material cost + Overhead cost = 5 + 10 + 6 = 21
IAS-22. Ans. (b)
IAS-23. Ans. (c)
IAS-24. Ans. (c)
IAS-25. Ans. (d)
IAS-26. Ans. (a)
IAS-27. Ans. (b)
IAS-28. Ans. (a) Fast output and low labour cost are more important characteristics for mass production shop.
IAS-29. Ans. (b)
IAS-30. Ans. (c)
IAS-31. Ans. (a)

IAS-32. Ans. (b)

IAS-33. Ans. (a)

IAS-34. Ans. (c)

IAS-35. Ans. (c) ***Job enlargement*** involves adding new tasks to a job in order to make it less boring and more challenging.

- It is especially useful for assembly- line jobs that are repetitive and monotonous and do not involve the worker's mental processes.

Job rotation among management trainees has been practiced for many years to give them an overall view of the firm's operations and to prepare them for promotion.

Job enrichment is the process of redesigning jobs to satisfy higher- level needs and organizational needs by improving worker satisfaction and task efficiency.

- It gives workers more responsibility, authority, and autonomy in planning and doing their work.