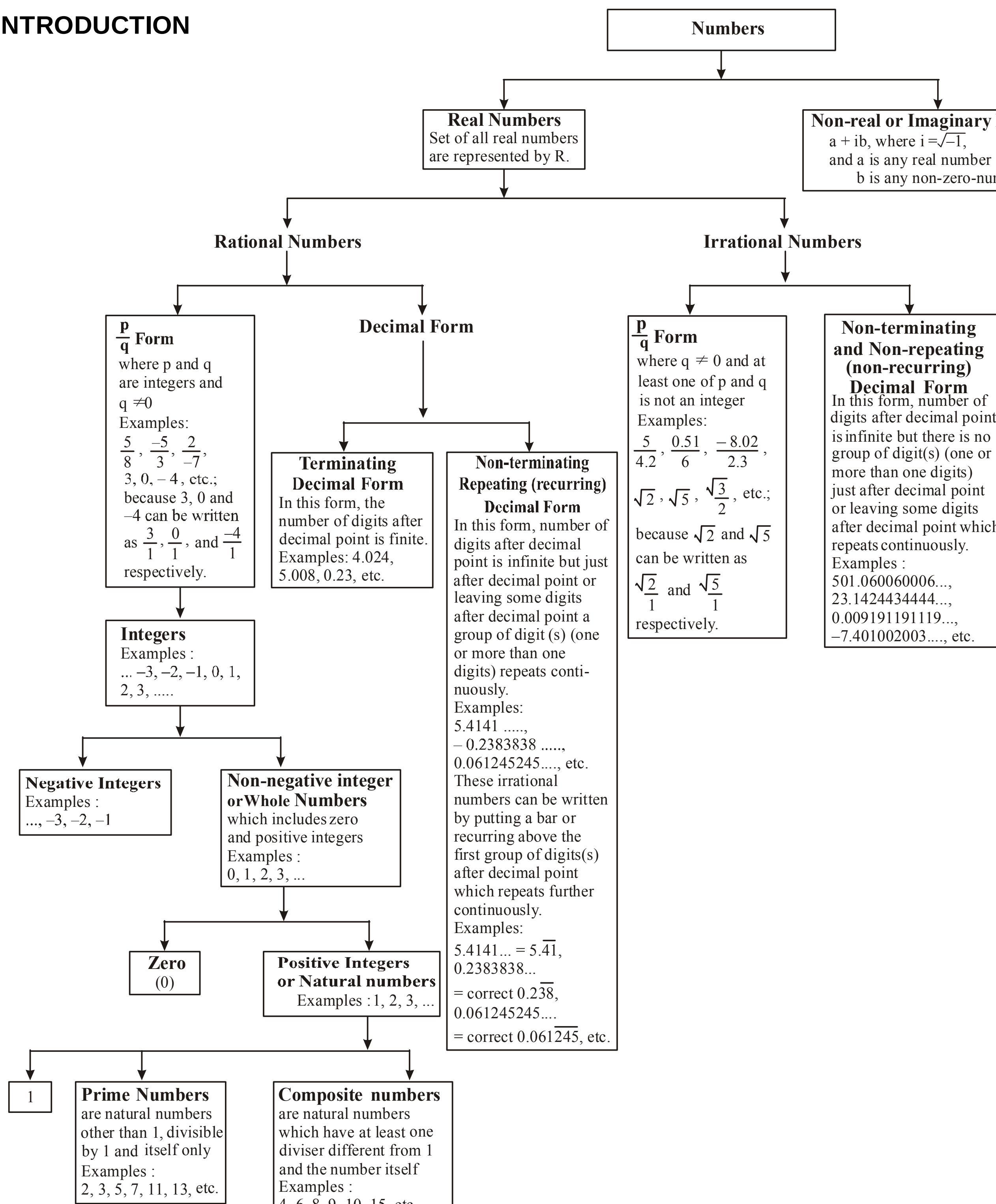
CHAPTER 1

Number System & Simplification

INTRODUCTION



REMEMBER

- ★ The ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called digits.
- ★ 1 is neither prime nor composite.
- ★ 1 is an odd integer.
- ★ 0 is neither positive nor negative.
- ★ 0 is an even integer.
- ★ 2 is prime & even both.
- ★ All prime numbers (except 2) are odd.

Natural Numbers :

These are the numbers (1, 2, 3, etc.) that are used for counting.
It is denoted by N.
There are infinite natural numbers and the smallest natural number is one (1).

Even numbers :

Natural numbers which are divisible by 2 are even numbers.
It is denoted by E.
E = 2, 4, 6, 8,...
Smallest even number is 2. There is no largest even number.

Odd numbers :

Natural numbers which are not divisible by 2 are odd numbers.
It is denoted by O.
O = 1, 3, 5, 7, ...
Smallest odd number is 1.
There is no largest odd number.

Based on divisibility, there could be two types of natural numbers : Prime and Composite.

Prime Numbers :

Natural numbers which have exactly two factors, i.e., 1 and the number itself are called prime numbers.
The lowest prime number is 2.
2 is also the only even prime number.

Composite Numbers :

It is a natural number that has atleast one divisor different from unity and itself.
Every composite number can be factorised into its prime factors.
For Example : $24 = 2 \times 2 \times 2 \times 3$. Hence, 24 is a composite number.
The smallest composite number is 4.

Twin-prime Numbers:

Pairs of such prime numbers whose difference is 2.
Example: 3 and 5, 11 and 13, 17 and 19.
How to check whether a given number is prime or not ?
Steps : (i) Find approximate square root of the given number.
(ii) Divide the given number by every prime number less than the approximate square root.
(iii) If the given number is exactly divisible by atleast one of the prime numbers, the number is a composite number otherwise a prime number.

Example : Is 401 a prime number?
Sol. Approximate square root of 401 is 20.
Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19
401 is not divisible by 2, 3, 5, 7, 11, 13, 17 or 19.
$\therefore$ 401 is a prime number.
(**Hint :** Next prime number after 19 and 23, which is greater than 20, so we need not check further.)
Co-prime Numbers : Co-prime numbers are those numbers which are prime to each other i.e., they don't have any common factor other than 1.
Since these numbers do not have any common factor, their HCF is 1 and their LCM is equal to product of the numbers.
Note : Co-prime numbers can be prime or composite numbers. Any two prime numbers are always co-prime numbers.
Example 1 : 3 and 5 : Both numbers are prime numbers.
Example 2 : 8 and 15 : Both numbers are composite numbers but they are prime to each other i.e., they don't have any common factor.

Face value and Place value :

Face Value is absolute value of a digit in a number.
Place Value (or Local Value) is value of a digit in relation to its position in the number.
Example : Face value and Place value of 9 in 14921 is 9 and 900 respectively.

Whole Numbers :

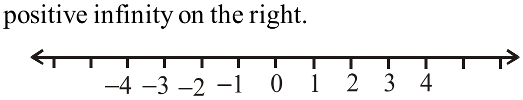
The natural numbers along with zero (0), form the system of whole numbers.
It is denoted by W.
There is no largest whole number and
The smallest whole number is 0.

Integers :

The number system consisting of natural numbers, their negative and zero is called integers.
It is denoted by Z or I.
The smallest and the largest integers cannot be determined.

The Number Line :

The number line is a straight line between negative infinity on the left to positive infinity on the right.



Rational numbers : Any number that can be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.

- It is denoted by Q.
- Every integer is a rational number.

- Zero (0) is also a rational number. The smallest and largest rational numbers cannot be determined. Every fraction (and decimal fraction) is a rational number.

$$Q = \frac{p \text{ (Numerator)}}{q \text{ (Denominator)}}$$

REMEMBER

★ If x and y are two rational numbers, then $\frac{x+y}{2}$ is also a rational number and its value lies between the given two rational numbers x and y.

★ An infinite number of rational numbers can be determined between any two rational numbers.

EXAMPLE 1. Find three rational numbers between 3 and 5.

Sol. 1st rational number = $\frac{3+5}{2} = \frac{8}{2} = 4$

2nd rational number (i.e., between 3 and 4)

$= \frac{3+4}{2} = \frac{7}{2}$

3rd rational number (i.e., between 4 and 5)

$= \frac{4+5}{2} = \frac{9}{2}$.

Irrational numbers : The numbers which are not rational or which cannot be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called irrational number.

It is denoted by Q' or Q^c.

$\sqrt{2}, \sqrt{3}, \sqrt{5}, 2+\sqrt{3}, 3-\sqrt{5}, 3\sqrt{3}$ are irrational numbers.

NOTE :

(i) Every positive irrational number has a negative irrational number corresponding to it.

(ii) $\sqrt{2}+\sqrt{3} \neq \sqrt{5}$

$\sqrt{5}-\sqrt{3} \neq \sqrt{2}$

$\sqrt{3} \times \sqrt{2} = \sqrt{3 \times 2} = \sqrt{6}$

$\sqrt{6} \div \sqrt{2} = \sqrt{\frac{6}{2}} = \sqrt{3}$

(iii) Some times, product of two irrational numbers is a rational number.

For example : $\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2} = 2$

$(2+\sqrt{3}) \times (2-\sqrt{3}) = (2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$

(iv) π is an irrational number. π : approximately equal to $\frac{22}{7}$ or 3.14.

Real Numbers :

All numbers that can be represented on the number line are called real numbers.

It is denoted by R.

R^+ : denotes the set of all positive real numbers and

R^- : denotes the set of negative real numbers.

Both rational and irrational numbers can be represented in number line.

Every real number is either rational or irrational.

EXAMPLE 2. Find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$.

Sol. We find by dividing, $\frac{1}{7} = 0.\overline{142857}$ and $\frac{2}{7} = 0.\overline{285714}$.

To find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$, we find a number which is non-terminating non-recurring lying between them.

So, 0.1501500150000... is an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$.

FRACTIONS

A fraction is a quantity which expresses a part of the whole.

$$\boxed{\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}}$$

EXAMPLE 3. Write a fraction whose numerator is $2^2 + 1$ and denominator is $3^2 - 1$.

Sol. Numerator $= 2^2 + 1 = 4 + 1 = 5$

Denominator $= 3^2 - 1 = 9 - 1 = 8$

$\therefore$ Fraction $= \frac{\text{Numerator}}{\text{Denominator}} = \frac{5}{8}$

TYPES OF FRACTIONS :

1. **Proper fraction :** If numerator is less than its denominator, then it is a proper fraction.

   For example : $\frac{2}{5}, \frac{6}{18}$

2. **Improper fraction :** If numerator is greater than or equal to its denominator, then it is a improper fraction.

For example : $\frac{5}{2}, \frac{18}{7}, \frac{13}{13}$

NOTE : *If in a fraction, its numerator and denominator are of equal value then fraction is equal to unity i.e. 1.*

3. **Mixed fraction :** It consists of an integer and a proper fraction.

For example : $1\frac{1}{2},\ 3\frac{2}{3},\ 7\frac{5}{9}$

NOTE : *Mixed fraction can always be changed into improper fraction and vice versa.*

For example : $7\frac{5}{9} = \frac{7\times9+5}{9} = \frac{63+5}{9} = \frac{68}{9}$

and $\frac{19}{2} = \frac{9\times2+1}{2} = 9+\frac{1}{2} = 9\frac{1}{2}$

4. **Equivalent fractions or Equal fractions :** Fractions with same value.

For example : $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}\left(=\frac{2}{3}\right)$.

NOTE : *Value of fraction is not changed by multiplying or dividing both the numerator or denominator by the same number.*

For example :

(i) $\frac{2}{5} = \frac{2\times5}{5\times5} = \frac{10}{25}$ So, $\frac{2}{5} = \frac{10}{25}$

(ii) $\frac{36}{16} = \frac{36\div4}{16\div4} = \frac{9}{4}$ So, $\frac{36}{16} = \frac{9}{4}$

5. **Like fractions:** Fractions with same denominators.

For example : $\frac{2}{7}, \frac{3}{7}, \frac{9}{7}, \frac{11}{7}$

6. **Unlike fractions :** Fractions with different denominators.

For example : $\frac{2}{5}, \frac{4}{7}, \frac{9}{8}, \frac{9}{2}$

NOTE : *Unlike fractions can be converted into like fractions.*

For example : $\frac{3}{5}$ and $\frac{4}{7}$

$\frac{3}{5}\times\frac{7}{7} = \frac{21}{35}$ and $\frac{4}{7}\times\frac{5}{5} = \frac{20}{35}$

7. **Simple fraction :** Numerator and denominator are integers.

For example : $\frac{3}{7}$ and $\frac{2}{5}$.

8. **Complex fraction :** Numerator or denominator or both are fractional numbers.

For example : $\frac{\frac{2}{5}}{7}, \frac{2\frac{1}{3}}{5\frac{2}{3}}, \frac{2+\frac{1+\frac{2}{7}}{3}}{2}$

9. **Decimal fraction :** Denominator with the powers of 10.

For example : $\frac{2}{10} = (0.2), \frac{9}{100} = (0.09)$

10. **Comparison of Fractions**

Comparison of two faction can be easily understand by the following example:

To compare two fraction $\frac{3}{5}$ and $\frac{7}{9}$, multiply each fraction by the LCM (45) of their denominators 5 and 9.

$\frac{3}{5}\times45 = 3\times9 = 27$

$\frac{7}{9}\times45 = 7\times5 = 35$

Since $27 < 35$

$\therefore\ \frac{3}{5} < \frac{7}{9}$

SHORT CUT METHOD

$\frac{3}{5} \times \frac{7}{9}$ [Write the each product on their numerator side]

$27 < 35$

$\therefore\ \frac{3}{5} < \frac{7}{9}$

EXAMPLE 4. Write 2.73 as a fraction.

Sol. $2.73 = \frac{273}{100}$

EXAMPLE 5. Express $\frac{2}{5}$ as a decimal fraction.

Sol. $\frac{2}{5} = \frac{2\times2}{5\times2} = \frac{4}{10}$

EXAMPLE 6. After doing 3/5 of the Biology homework on Monday night, Sanjay did 1/3 of the remaining homework on Tuesday night. What fraction of the original homework would Sanjay have to do on Wednesday night to complete the Biology assignment ?

(a) 1/15 **(b) 2/15**
(c) 4/15 **(d) 2/5**

Sol. (c) Remaining homework on Monday night

$$= 1 - \frac{3}{5} = \frac{2}{5}$$

Work done on Tuesday night

$$= \frac{1}{3} \text{ of } \frac{2}{5} = \frac{2}{15}$$

Remaining homework to complete the biology

$$\text{assignment} = \frac{2}{5} - \frac{2}{15} = \frac{6-2}{15} = \frac{4}{15}$$

ADDITION OF MIXED FRACTIONS

You can easily understand the addition of mixed fractions by the following example:

$$1\frac{3}{5} + 1\frac{8}{9} + 2\frac{4}{9} = \frac{8}{5} + \frac{17}{9} + \frac{14}{5}$$

$$= \frac{72+85+126}{45} = \frac{283}{45} = 6\frac{23}{45}$$

SHORT CUT METHOD

$$1\frac{3}{5} + 1\frac{8}{9} + 2\frac{4}{5} = (1+1+2) + \left(\frac{3}{5} + \frac{8}{9} + \frac{4}{5}\right)$$

$$= 4 + \frac{27+40+36}{45}$$

$$= 4 + \frac{103}{45} = 4 + 2\frac{13}{45} = 6\frac{13}{45}$$

Rounding off (Approximation) of Decimals :

There are some decimals in which numbers are found upto large number of decimal places.

For example : 3.4578, 21.358940789.

But many times we require decimal numbers upto a certain number of decimal places. Therefore,

If the digit of the decimal place is five or more than five, then the digit in the preceding decimal place is increased by one and if the digit in the last place is less than five, then the digit in the precedence place remains unchanged.

EXAMPLE 7.

(a) Write 21.3751 upto two places of decimal.

(b) Write 3.27645 upto three places of decimal.

Sol. (a) 21.3751 = 21.38

(b) 3.27645 = 3.276

CONVERSION OF RATIONAL NUMBER OF THE FORM NON-TERMINATING RECURRING DECIMAL INTO THE RATIONAL NUMBER OF THE FORM $\frac{p}{q}$

First write the non-terminating repeating decimal number in recurring form i.e., write

64.20132132132..... as $64.20\overline{132}$

Then using formula given below we find the required $\frac{p}{q}$ form of the given number.

Rational number in the form $\frac{p}{q}$

$$= \frac{\begin{bmatrix}\text{Complete number neglecting} \\ \text{the decimal and bar over} \\ \text{repeating digit (s)}\end{bmatrix} - \begin{bmatrix}\text{Non-recurring part of} \\ \text{the number neglecting} \\ \text{the decimal}\end{bmatrix}}{m \text{ times 9 followed by } n \text{ times 0}}$$

where m = number of recurring digits in decimal part

and n = number of non-recurring digits in decimals part

Thus, $\frac{p}{q}$ form of $64.20\overline{132} = \frac{6420132-6420}{99900}$

$$= \frac{6413712}{99900} = \frac{534476}{8325}$$

In short; $0.\overline{a} = \frac{a}{9}, 0.\overline{ab} = \frac{ab}{99}, 0.\overline{abc} = \frac{abc}{999},$ etc. and

$$0.a\overline{b} = \frac{ab-a}{90},\ 0.\overline{abc} = \frac{abc-a}{990},\ 0.ab\overline{c} = \frac{abc-ab}{900},$$

$$0.a\overline{bcd} = \frac{abcd-ab}{9900},\ ab.c\overline{de} = \frac{abcde-abc}{990}, \text{etc.}$$

EXAMPLE **8. Convert $2.\overline{45102}$ in the $\frac{p}{q}$ form of rational number.**

Sol. Required $\frac{p}{q}$ form $= \frac{246102-2}{99999} = \frac{246100}{99999}$

EXAMPLE **9. Convert $0.167\overline{3206}$ in the $\frac{p}{q}$ form of rational number.**

Sol. Required $\frac{p}{q}$ form $= \frac{1673206-167}{9999000} = \frac{1673039}{9999000}$

EXAMPLE **10. Convert 31.026415555 ... into $\frac{p}{q}$ form of rational number.**

Sol. First write 31.026415555... as $31.02641\overline{5}$

Now required $\frac{p}{q}$ form $= \frac{31026415-3102641}{900000} = \frac{27923774}{900000}$

$$= \frac{13961887}{450000}$$

PROPERTIES OF OPERATIONS :

The following properties of addition, subtraction and multiplication are valid for real numbers a , b and c.

(a) Commutative property of addition :
$a + b = b + a$

(b) Associative property of addition :
$(a + b) + c = a + (b + c)$

(c) Commutative property of multiplication:
$a \times b = b \times a$

(d) Associative property of multiplication :
$(a \times b) \times c = a \times (b \times c)$

(e) Distributive property of multiplication with respect to addition :
$(a + b) \times c = a \times c + b \times c$

DIVISIBILITY RULES

Divisibility by 2 :

A number is divisible by 2 if its unit's digit is even or 0.

Divisibility by 3 :

A number is divisible by 3 if the sum of its digits are divisible by 3.

Divisibility by 4 :

A number is divisible by 4 if the last 2 digits are divisible by 4, or if the last two digits are 0's.

Divisibility by 5 :

A number is divisible by 5 if its unit's digit is 5 or 0.

Divisibility by 6 :

A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

Divisiblity by 7 :

A number is divisible by 7 if unit's place digit is multiplied by 2 and subtracted from the remaining digits and the number obtained is divisible by 7.

For example,

$1680\boxed{7} = 1680 - 7 \times 2 = 1666$

It is difficult to decide whether 1666 is divisible by 7 or not. In such cases, we continue the process again and again till it become easy to decide whether the number is divisible by 7 or not.

$166\boxed{6} \longrightarrow 166 - 6 \times 2 = 154$

Again $15\boxed{4} \longrightarrow 15 - 4 \times 2 = 7$, divisible by7

Hence 16807 is divisible by 7.

Divisibility by 8 :

A number is divisible by 8 if the last 3 digits of the number are divisible by 8, or if the last three digits of a number are zeros.

Divisibility by 9 :

A number is divisible by 9 if the sum of its digits is divisible by 9.

Divisibility by 10 :

A number is divisible by 10 if its unit's digit is 0.

Divisibility by 11 :

A number is divisible by 11 if the sum of digits at odd and even places are equal or differ by a number divisible by 11.

Divisibility by 12 :

A number is divisible by 12 if the number is divisible by both 4 and 3.

Divisibility by 13 :

A number is divisible by 13 if its unit's place digit is multiplied by 4 and added to the remaining digits and the number obtained is divisible by 13.

For example,

$219\boxed{7} \longrightarrow 219 + 7 \times 4 = 247$

Again $24\boxed{7} \longrightarrow 24 + 7 \times 4 = 52$, divisible by 13.

Hence 2197 is divisible by 13.

Divisibility by 14 :

A number is divisible by 14 if the number is divisible by both 2 and 7.

Divisibility by 15 :

A number is divisible by 15 if the number is divisible by both 3 and 5.

Divisibility by 16 :

A number is divisible by 16 if its last 4 digits is divisible by 16 or if the last four digits are zeros.

Divisibility by 17 :

A number is divisible by 17 if its unit's place digit is multiplied by 5 and subtracted from the remaining digits and the number obtained is divisible by 17.

For example,

$491\boxed{3} \longrightarrow 491 - 3 \times 5 = 476$

Again, $47\boxed{6} \longrightarrow 47 - 6 \times 8 = 17$, divisible by 17.

Hence 4913 is divisible by 17.

Divisibility by 18 :

A number is divisible by 18 if the number is divisible by both 2 and 9.

Divisibility by 19 :

A number is divisible by 19 if its unit's place digit is multiplied by 2 and added to the remaining digits and the number obtained is divisible by 19.

For example,

$4873\boxed{7} \longrightarrow 4873 + 7 \times 2 = 4887$

$488\boxed{7} \longrightarrow 488 + 7 \times 2 = 502$

$50\boxed{2} \longrightarrow 50 + 2 \times 2 = 54$ not divisible by 19.

Hence 48737 is not divisible by 19.

Properties of Divisibility

(i) The product of 3 consecutive natural numbers is divisible by 6.

(ii) The product of 3 consecutive natural numbers, the first of which is even, is divisible by 24.

(iii) Difference between any number and the number obtained by writing the digits in reverse order is divisible by 9.

(iv) Any number written in the form $(10^n - 1)$ is divisible by 3 and 9.

(v) Any six-digits, twelve-digits, eighteen-digits or any such number with number of digits equal to multiple of 6, is divisible by each of 7, 11 and 13 if all of its digits are same.

For example 666666, 888888, 333333333333 are all divisible by 7, 11 and 13.

(vi) Any number in the form abcabc (a, b, c are three different digits) is divisible by 1001.

(vii) (a) $(a^n - b^n)$ is divisible both by $(a + b)$ and $(a - b)$, when n is even.

(b) $(a^n - b^n)$ is divisible only by $(a - b)$, when n is odd.

EXAMPLE 11. Without actual division, find which of the following numbers are divisible by 2, 3, 4, 5, 7, 9, 10, 11 :

(i) 36324 **(ii) 2211**

(iii) 87120 **(iv) 32625**

Sol. (i) 36324

It is divisible by 2 because 4 (unit's digit) is divisible by 2. It is divisible by 3 because 3 + 6 + 3 + 2 + 4 = 18 is divisible by 3. It is divisible by 4 becuase 24 is divisible by 4.

It is not divisible by 5.

It is not divisible by 7.

It is divisible by 9 because 3 + 6 + 3 + 2 + 4 = 18 is divisible by 9.

It is not divisible by 10.

It is not divisible by 11.

(ii) 2211

It is not divisible by 2.

It is divisible by 3 because 2 + 2 + 1 + 1 = 6 is divisible by 3.

It is not divisible by 4, 5, 7, 8, 10.

It is divisible by 11 because 2211 $\rightarrow (2+1)-(2+1) = 3-3 = 0$.

(iii) 87120

It is divisible by 2 because its unit's place digit is 0.

It is divisible by 3 because 8 + 7 + 1 + 2 + 0 = 18 is divisible by 3.

It is divisible by 4 because 20 is divisible by 4.

It is divisible by 5 because its unit's place digit is 0.

It is not divisible by 7.

It is divisible by 9 beacuse 8 + 7 + 1 + 2 + 0 = 18 is divisible by 9.

It is divisible by 10 because its unit's place digit is 0.

It is divisible by 11 beacuse 87120 $\rightarrow (8+1+0)-(7+2) = 9-9 = 0$.

EXAMPLE 12. Is 473312 divisible by 7?

Sol. $47331 - 2 \times 2 = 47327$

$4732 - 2 \times 7 = 4718$

$471 - 2 \times 8 = 455$

$45 - 2 \times 5 = 35$

35 is divisible by 7, therefore, 473312 is divisible by 7.

EXAMPLE 13. What is the value of M and N respectively if M39048458N is divisible by 8 and 11, where M and N are single digit integers?

(a) 7, 4 **(b) 8, 6**

(c) 6, 4 **(d) 3, 2**

Sol. (c) A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

i.e., 58N is divisible by 8.

Clearly, N = 4

Again, a number is divisible by 11 if the difference between the sum of digits at even places and sum of digits at the odd places is either 0 or is divisible by 11.

i.e. $(M+9+4+4+8)-(3+0+8+5+N)$

$= M + 25 - (16 + N)$

$= M - N + 9$ must be zero or it must be divisible by 11

i.e. $M - N = 2$

$\Rightarrow M = 2 + 4 = 6$

Hence, $M = 6, N = 4$

EXAMPLE 14. The highest power of 9 dividing 99! completely, is:

(a) 20 **(b) 24**

(c) 12 **(d) 11**

Sol. (c) $99! = 99 \times 98 \times 97 \times 96 \times 95 \times 94 ..... \times 1$

To find the highest power of 9 that divides this product, we have to find the sum of powers of all 9's in the expression.

In the nos. from 1 to 99, all the nos. divisible by 9 are 9, 18, 27, 36, 45, 54, 63, 72, 81 (9×9), 90, 99, i.e. 12 in no.

This clearly shows that 99! will be completely divisible by 9^{12}.

DIVISION ALGORITHM :

Dividend = (Divisor × Quotient) + Remainder

where, Dividend = The number which is being divided

Divisor = The number which performs the division process

Quotient = Greatest possible integer as a result of division

Remainder = Rest part of dividend which cannot be further divided by the divisor.

Complete remainder :

A complete remainder is the remainder obtained by a number by the method of successive division.

Complete remainder = [I divisor × II remainder] + I remainder

$$\boxed{\begin{array}{l} C.R. = d_1r_2 + r_1 \\ C.R. = d_1d_2r_3 + d_1r_2 + r_1 \end{array}}$$

Shortcut Approach

Two different numbers x and y when divided by a certain divisor D leave remainder r_1 and r_2 respectively. When the sum of them is divided by the same divisor, the remainder is r_3. Then,

$$\text{divisor } D = r_1 + r_2 - r_3$$

Method to find the number of different divisors (or factors) (including 1 and itself) of any composite number N :

STEP I : Express N as a product of prime numbers as $N = x^a \times y^b \times z^c$ ....

STEP II : Number of different divisors (including 1 and itself) $= (a + 1)(b + 1)(c + 1)$ ....

EXAMPLE 15. Find the number of different divisors of 50, besides unity and the number itself.

Sol. If you solve this problem without knowing the rule, you will take the numbers in succession and check the divisibility. In doing so, you may miss some numbers. It will also take more time.

Different divisors of 50 are : 1, 2, 5, 10, 25, 50

If we exclude 1 and 50, the number of divisors will be 4.

By rule : $50 = 2 \times 5 \times 5 = 2^1 \times 5^2$

$\therefore$ the number of total divisors $= (1 + 1) \times (2 + 1) = 2 \times 3 = 6$

or, the number of divisors excluding 1 and 50 = 6 – 2 = 4

EXAMPLE 16. A certain number when divided by 899 leaves the remainder 63. Find the remainder when the same number is divided by 29.

(a) 5 **(b) 4**

(c) 1 **(d) Cannot be determined**

Sol. (a) Number = 899Q + 63, where Q is quotient

$= 31 \times 29\,Q + (58 + 5) = 29\,[31Q + 2] + 5$

$\therefore$ Remainder = 5

HIGHEST COMMON FACTOR (HCF) OR GREATEST COMMON DIVISOR (GCD)

The highest (i.e. largest) number that divides two or more given numbers is called the highest common factor (HCF) of those numbers.

Methods to Find The HCF or GCD

There are two methods to find HCF of the given numbers

(i) Prime Factorization Method

When a number is written as the product of prime numbers, then it is called the prime factorization of that number. For example, 72 $= 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$. Here, $2 \times 2 \times 2 \times 3 \times 3$ or $2^3 \times 3^2$ is called prime factorization of 72.

To find the HCF of given numbers by this methods, we perform the prime factorization of all the numbers and then check for the common prime factors. For every prime factor common to all the numbers, we choose the least index of that prime factor among the given numbers. The HCF is the product of all such prime factors with their respective least indices.

EXAMPLE 17. Find the HCF of 72, 288 and 1080.

Sol: $72 = 2^3 \times 3^2$, $288 = 2^5 \times 3^2$, $1080 = 2^3 \times 3^3 \times 5$.

The prime factors common to all the given numbers are 2 and 3. The lowest indices of 2 and 3 in the given numbers are 3 and 2 respectively.

Hence, HCF $= 2^3 \times 3^2 = 72$.

(ii) Division Method

To find the HCF of two numbers by division method, we divide the larger number by the smaller number. Then we divide the smaller number by the first remainder, then first remainder by the second remainder.. and so on, till the remainder becomes 0. The last divisor is the required HCF.

EXAMPLE 18. Find the HCF of 288 and 1080 by the division method.

Sol.

$$\begin{array}{r|l|l} 288 & 1080 & 3 \\ & \underline{864} & \\ & 216 & 288 \mid 1 \\ & & \underline{216} \\ & & 72 \mid 216 \mid 3 \\ & & \quad\ \underline{216} \\ & & \quad\quad 0 \end{array}$$

The last divisor 72 is the HCF of 288 and 1080.

Shortcut Approach

To find the HCF of any number of given numbers, first find the difference between two nearest given numbers. Then find all factors (or divisors) of this difference. Highest factor which divides all the given numbers is the HCF.

EXAMPLE 19. Find the HCF of 12, 20 and 32.

Sol. Difference of nearest two numbers 12 and 20 = 20 – 12 = 8

All factors (or divisor) of 8 are 1, 2, 4 and 8.

1, 2 and 4 divides each of the three given numbers 12, 20 and 32. Out of 1, 2 and 4; 4 is the highest number. Hence, HCF = 4.

LEAST COMMON MULTIPLE (LCM)

The least common multiple (LCM) of two or more numbers is the lowest number which is divisible by all the given numbers.

Methods to Find The LCM

There are two methods to find the LCM.

(i) Prime Factorization Method

After performing the prime factorization of all the given numbers, we find the highest index of all the prime numbers among the given numbers. The LCM is the product of all these prime numbers with their respective highest indices because LCM must be divisible by all of the given numbers.

EXAMPLE **20. Find the LCM of 72, 288 and 1080.**

Sol. $72 = 2^3 \times 3^2$

$288 = 2^5 \times 3^2$

$1080 = 2^3 \times 3^3 \times 5$

Hence, $\text{LCM} = 2^5 \times 3^3 \times 5^1 = 4320$

(ii) Division Method

To find the LCM of 5, 72, 196 and 240, we use the division method in the following way:

Check whether any prime number that divides at least two of all the given numbers. If there is no such prime number, then the product of all these numbers is the required LCM, otherwise find the smallest prime number that divides at least two of the given numbers. Here, we see that smallest prime number that divides at least two given numbers is 2.

Divide those numbers out of the given numbers by 2 which are divisible by 2 and write the quotient below it. The given number(s) that are not divisible by 2 write as it is below it and repeat this step till you do not find at least two numbers that are not divisible by any prime number.

2	5,	72,	196,	240
2	5,	36,	98,	120
2	5,	18,	49,	60
3	5,	9,	49,	30
5	5,	3,	49,	10
	1,	3,	49,	2

After that find the product of all divisors and the quotient left at the end of the division. This product is the required LCM.

Hence, LCM of the given numbers = product of all divisors and the quotient left at the end.

$= 2 \times 2 \times 2 \times 3 \times 5 \times 3 \times 49 \times 2 = 35280$

Shortcut Approach

Using idea of co-prime, you can find the LCM by the following shortcut method:

LCM of 9, 10, 15 and 36 can be written directly as $9 \times 10 \times 2$.

The logical thinking that behind it is as follows:

Step 1: If you can see a set of 2 or more co-prime numbers in the set of numbers of which you are finding the LCM, write them down by multiply them.

In the above situation, since we see that 9 and 10 are co-prime to each other, we start off writing the LCM by writing 9×10 as the first step.

Step 2: For each of the other numbers, consider what prime factor(s) of it is/are not present in the LCM (if factorised into primes) taken in step 1. In case you see some prime factors of each of the other given numbers separately are not present in the LCM (if factorised into primes) taken in step 1, such prime factors will be multiplied in the LCM taken in step 1.

Prime factorisation of $9 \times 10 = 3 \times 3 \times 2 \times 5$

Prime factorisation of $15 = 3 \times 5$

Prime factorisation of $36 = 2 \times 2 \times 3 \times 3$

Here we see that both prime factors of 15 are present in the prime factorisation of 9×10 but one prime factor 2 of 36 is not present in the LCM taken in step 1. So to find the LCM of 9, 10, 15 and 36; we multiply the LCM taken in step 1 by 2.

Thus required $\text{LCM} = 9 \times 10 \times 2 = 180$

RULE FOR FINDING HCF AND LCM OF FRACTIONS

(I) HCF of two or more fractions

$$= \frac{\text{HCF of numerator of all fractions}}{\text{LCM of denominator of all fractions}}$$

(II) LCM of two or more fractions

$$= \frac{\text{LCM of numerator of all fractions}}{\text{HCF of denominator of all fractions}}$$

EXAMPLE **21. Find the HCF and LCM of .**

Sol. $\text{HCF} = \dfrac{\text{HCF of } 4, 6, 3}{\text{LCM of } 5, 11, 5} = \dfrac{1}{55}$

$\text{LCM} = \dfrac{\text{LCM of } 4, 6, 3}{\text{HCF of } 5, 11, 5} = \dfrac{12}{1} = 12$

SIMPLIFICATION

FUNDAMENTAL OPERATIONS :

1. Addition :

(a) Sum of two positive numbers is a positive number.
For example : $(+5) + (+2) = +7$

(b) Sum of two negative numbers is a negative number.
For example : $(-5)+(-3)=-8$

(c) Sum of a positive and a negative number is the difference between their magnitudes and give the sign of the number with greater magnitude.
For example : $(-3)+(+5)=2$ and $(-7)+(+2)=-5$

2. Subtractions :

Subtraction of two numbers is same as the sum of a positive and a negative number.

For Example :

$(+9)-(+2)=(+9)+(-2)=7$

$(-3)-(-5)=(-3)+5=+2.$

NOTE : *In subtraction of two negative numbers, sign of second number will change and become positive.*

3. Multiplication :

(a) Product of two positive numbers is positive.
(b) Product of two negative numbers is positive.
(c) Product of a positive number and a negative number is negative.
(d) Product of more than two numbers is positive or negative depending upon the presence of negative quantities.
If the number of negative numbers is even then product is positive and if the number of negative numbers is odd then product is negative.

For Example :

$(-3)\times(+2)=-6$

$(-5)\times(-7)=+35$

$(-2)\times(-3)\times(-5)=-30$

$(-2)\times(-3)\times(+5)=+30$

4. Division :

(a) If both the dividend and the divisor are of same sign, then quotient is always positive.
(b) If the dividend and the divisor are of different sign, then quotient is negative,

For Example :

$(-36)\div(+9)=-4$

$(-35)\div(-7)=+5$

Brackets :

Types of brackets are :
(i) Vinculum or bar –
(ii) Parenthesis or small or common brackets : ( )
(iii) Curly or middle brackets : { }
(iv) Square or big brackets : [ ]
The order for removal of brackets is (), {}, []

NOTE : *If there is a minus (–) sign before the bracket then while removing bracket, sign of each term will change.*

'BODMAS' RULE

Now a days it becomes 'VBODMAS' where,
'V' stands for "Vinculum"
'B' stands for "Bracket"
'O' stands for "Of"
'D' stands for "Division"
'M' stands for "Multiplication"
'A' stands for "Addition"
'S' stands for "Subtraction"
Same order of operations must be applied during simplification.

EXAMPLE 22. Simplify

$6+5-3\times2$ of $5-(15\div\overline{7-2})$
$=6+5-3\times2$ of $5-(15\div5)$ {Remove vinculum}
$=6+5-3\times2$ of $5-3$ {Remove common bracket}
$=6+5-3\times10-3$ {'Of' is done}
$=6+5-30-3$ {Multiplication is done}
$=11-33$ {Addition is done}
$=-22$ {Subtraction is done}.

Shortcut Approach

To simplify an expression, add all the positive numbers together and all the negative numbers separately and add or subtract the resulting numbers as the case will.

EXAMPLE 23. Simplify : $7-2+13-5-2+1$

Sol. $7-2+13-5-2+1$
$=7+13+1-2-5-2=21-9=12$
$[7+13+1=21$ and $-2-5-2=-9]$

EXAMPLE 24. $\dfrac{11\times11-21}{9\times6-(2)^2}=$

(a) 0 **(b)** $\frac{11}{52}$
(c) 2 **(d)** 40

Sol. (c) $\dfrac{11\times11-21}{9\times6-(2)^2}=\dfrac{121-21}{54-4}=\dfrac{100}{50}=2$

EXAMPLE 25. $\dfrac{1+1\times1-1\times1+1}{1+1\div1+(1+1)\times(1+1)}$

(a) 1/2 **(b)** 1/5
(c) 2/5 **(d)** 1/3

Sol. (d) $\dfrac{1+1-1+1}{1+1+2\times2}=\dfrac{3-1}{2+4}=\dfrac{2}{6}=\dfrac{1}{3}$

EXAMPLE 26. What is the missing figure in the expression given below?

$$\frac{16}{7}\times\frac{16}{7}-\frac{*}{7}\times\frac{9}{7}+\frac{9}{7}\times\frac{9}{7}=1$$

(a) 1 **(b)** 7
(c) 4.57 **(d)** 32

Sol. (d) Let the missing figure in the expression be x.

$$\frac{16}{7}\times\frac{16}{7}-\frac{x}{7}\times\frac{9}{7}+\frac{9}{7}\times\frac{9}{7}=1$$

$$\Rightarrow 16\times16-9x+9\times9=7\times7$$

$$\Rightarrow 9x=16\times16+9\times9-7\times7$$
$$=256+81-49=288$$

$$\Rightarrow x=\frac{228}{9}=32$$

POWERS OR EXPONENTS

When a number is multiplied by itself, it gives the square of the number. i.e., $a\times a=a^2$ (Example $5\times5=5^2$)

If the same number is multiplied by itself twice we get the cube of the number i.e., $a\times a\times a=a^3$ (Example $4\times4\times4=4^3$)

In the same way $a\times a\times a\times a\times a=a^5$

and $a\times a\times a\times \ldots$ upto n times $=a^n$

There are five basic rules of powers which you should know:

If a and b are any two real numbers and m and n are positive integers, then

(i) $a^m\times a^n=a^{m+n}$ (Example: $5^3\times5^4=5^{3+4}=5^7$)

(ii) $\frac{a^m}{a^n}=a^{m-n}$, if $m>n$ $\left(\text{Example}: \frac{6^5}{6^2}=6^{5-2}=6^3\right)$

$\frac{a^m}{a^n}=\frac{1}{a^{n-m}}$, if $m<n$ $\left(\text{Example}: \frac{4^3}{4^8}=\frac{1}{4^{8-3}}=\frac{1}{4^5}\right)$

and $\frac{a^m}{a^n}=a^0=1$, if $m=n$ $\left(\text{Example}: \frac{3^4}{3^4}=3^{4-4}=3^0=1\right)$

(iii) $(a^m)^n=a^{mn}=(a^n)^m$ (Example: $(6^2)^4=6^{2\times4}=6^8=(6^4)^2$

(iv) (a) $(ab)^n=a^n\,.\,b^n$ (Example: $(6\times4)^3=6^3\times4^3$)

(b) $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}, b\neq0$ $\left(\text{Example}: \left(\frac{5}{3}\right)^4=\frac{5^4}{3^4}\right)$

(v) $a^{-n}=\frac{1}{a^n}$ $\left(\text{Example}: 5^{-3}=\frac{1}{5^3}\right)$

(vi) For any real number a, $a^0=1$

ALGEBRIC IDENTITIES

Standard Identities

(i) $(a+b)^2=a^2+2ab+b^2$
(ii) $(a-b)^2=a^2-2ab+b^2$
(iii) $a^2-b^2=(a+b)(a-b)$
(iv) $(x+a)(x+b)=x^2+(a+b)x+ab$
(v) $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

Some More Identities

We have dealt with identities involving squares. Now we will see how to handle identities involving cubes.

(i) $(a+b)^3=a^3+b^3+3a^2b+3ab^2$
$\Rightarrow (a+b)^3=a^3+b^3+3ab(a+b)$
(ii) $(a-b)^3=a^3-b^3-3a^2b+3ab^2$
$\Rightarrow (a-b)^3=a^3-b^3-3ab(a-b)$
(iii) $a^3+b^3=(a+b)(a^2-ab+b^2)$
(iv) $a^3-b^3=(a-b)(a^2+ab+b^2)$
(v) $a^3+b^3+c^3-3abc$
$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
If $a+b+c=0$ then $a^3+b^3+c^3=3abc$

EXERCISE

Directions (Qs. 1 - 50): What will come in place of the question mark (?) in the following questions?

1. $16.02\times0.001=?$
   (a) 0.1602 (b) 0.001602
   (c) 1.6021 (d) 0.01602
   (e) None of these

2. $\frac{?}{50}=\frac{60.5}{?}$
   (a) 55 (b) 1512.5
   (c) 52.5 (d) 57.5
   (e) None of these

3. $5400\div9\div3=?$
   (a) 1800 (b) 900
   (c) 450 (d) 300
   (e) None of these

4. $10^{150}\div10^{146}=?$
   (a) 10^6 (b) 100000
   (c) 1000 (d) 10000
   (e) None of these

5. ?% of 360 = 129.6
   (a) 277 (b) 36
   (c) 64 (d) 72
   (e) None of these

6. $8265+2736+41320=?$
   (a) 51321 (b) 52231
   (c) 52321 (d) 52311
   (e) None of these

7. $\frac{(7\times?)^2}{49}=\sqrt{81}$
   (a) 9 (b) 2
   (c) 3 (d) 4
   (e) None of these

8. $\sqrt{625.04} \times 16.96 + 136.001 \div 17 = ?$
(a) 418 (b) 441
(c) 425 (d) 433
(e) 449

9. $48.25 \times 150 + 32 \times 16.5 - 125 \times 10.5 = ?$
(a) 6200 (b) 7500
(c) 6453 (d) 7100
(e) 6700

10. $36.0001 \div 5.9998 \times \sqrt{?} = 108.0005$
(a) 325 (b) 316
(c) 256 (d) 16
(e) 18

11. $138.009 + 341.981 - 146.305 = 123.6 + ?$
(a) 210.85 (b) 120.85
(c) 220.085 (d) 120.085
(e) None of these

12. $197 \times ? + 16^2 = 2620$
(a) 22 (b) 12
(c) 14 (d) 16
(e) None of these

13. $287.532 + 1894.029 - 657.48 = 743.095 + ?$
(a) 870 (b) 790
(c) 780 (d) 770
(e) 890

14. $27\frac{3}{11} + 118\frac{2}{5} - 32\frac{5}{22} = 11\frac{6}{11} + ?$
(a) $113\frac{9}{10}$ (b) $111\frac{9}{11}$
(c) $90\frac{9}{10}$ (d) $101\frac{9}{11}$
(e) None of these

15. $\frac{21}{25} \div \frac{9}{20} \times \frac{5}{12} \div \frac{10}{17} = ?$
(a) $7\frac{77}{125}$ (b) $11\frac{9}{10}$
(c) $\frac{119}{450}$ (d) $1\frac{29}{90}$
(e) None of these

16. $69012 - 20167 + (51246 \div 6) = ?$
(a) 57385 (b) 57286
(c) 57476 (d) 57368
(e) None of these

17. $98.98 \div 11.03 + 7.014 \times 15.99 = (?)^2$
(a) 131 (b) 144
(c) 12 (d) 121
(e) 11

18. $39.05 \times 14.95 - 27.99 \times 10.12 = (36 + ?) \times 5$
(a) 22 (b) 29
(c) 34 (d) 32
(e) 25

19. $2070.50 \div 15.004 + 39.001 \times (4.999)^2 = ?$
(a) 1005 (b) 997
(c) 1049 (d) 1213
(e) 1113

20. $\frac{45^2 \times 27^2}{135^2} = ?$
(a) 81 (b) 1
(c) 243 (d) 9
(e) None of these

21. $4\frac{1}{2} \times 4\frac{1}{3} - 8\frac{1}{3} \div 5\frac{2}{3} = ?$
(a) 8 (b) $18\frac{1}{34}$
(c) $1\frac{33}{34}$ (d) $\frac{7}{17}$
(e) None of these

22. $85.147 + 34.912 \times 6.2 + ? = 802.293$
(a) 400 (b) 450
(c) 550 (d) 600
(e) 500

23. $9548 + 7314 = 8362 + ?$
(a) 8230 (b) 8500
(c) 8410 (d) 8600
(e) None of these

24. $248.251 \div 12.62 \times 20.52 = ?$
(a) 400 (b) 450
(c) 600 (d) 350
(e) 375

25. $6.595 \times 1084 + 2568.34 - 1708.34 = ?$
(a) 6,000 (b) 12,000
(c) 10,000 (d) 8,000
(e) 9,000

26. $5679 + 1438 - 2015 = ?$
(a) 5192 (b) 5012
(c) 5102 (d) 5002
(e) None of these

27. $18\frac{2}{5}$ of $150.8 + ? = 8697.32 - 3058.16$
(a) 2764.44 (b) 2864.34
(c) 1864.44 (d) 2684.44
(e) None of these

28. $\frac{?}{24} = \frac{72}{\sqrt{?}}$
(a) 12 (b) 16
(c) 114 (d) 144
(e) None of these

29. $6\frac{5}{6}\times5\frac{1}{3}+17\frac{2}{3}\times4\frac{1}{2}=?$

(a) $112\frac{1}{3}$ (b) 663
(c) 240 (d) $116\frac{2}{3}$
(e) None of these

30. 35% of 1478 + 29% of 3214 = ?

(a) 1600 (b) 1250
(c) 1300 (d) 1450
(e) 1500

31. $\frac{5}{7}$ of 1596 + 3015 = ? − 2150

(a) 7200 (b) 48000
(c) 5300 (d) 58000
(e) 6300

32. 5798 − ? = 7385 − 4632

(a) 3225 (b) 2595
(c) 2775 (d) 3045
(e) None of these

33. $152\sqrt{?}+795=8226-3486$

(a) 425 (b) 985
(c) 1225 (d) 1025
(e) 675

34. $6.39\times15.266+115.8$ of $\frac{2}{5}=?$

(a) 145 (b) 165
(c) 180 (d) 130
(e) 135

35. 8597 − ? = 7429 − 4358

(a) 5706 (b) 5526
(c) 5426 (d) 5626
(e) None of these

36. 857 of 14% − 5.6 × 12.128 = ?

(a) 48 (b) 36
(c) 60 (d) 52
(e) 46

37. $5\frac{3}{5}\div3\frac{11}{15}+5\frac{1}{2}=?$

(a) 7 (b) $8\frac{1}{2}$
(c) $7\frac{1}{2}$ (d) $6\frac{1}{2}$
(e) None of these

38. 5978 + 6134 + 7014 = ?

(a) 19226 (b) 16226
(c) 19216 (d) 19126
(e) None of these

39. 9568 − 6548 − 1024 = ?

(a) 2086 (b) 1996
(c) 2293 (d) 1896
(e) None of these

40. 1.542 × 2408.69 + 1134.632 = ?

(a) 4600 (b) 4800
(c) 5200 (d) 6400
(e) 3600

41. 8.539 + 16.84 × 6.5 ÷ 4.2 = ?

(a) 25 (b) 42
(c) 44 (d) 35
(e) None of these

42. $17\frac{2}{3}$ of 180 + $\frac{1}{4}$ of 480 = ?

(a) 3180 (b) 3420
(c) 3200 (d) 3300
(e) None of these

43. $1325\sqrt{17}+508.24$ of 20% − 85.39 of $\frac{3}{4}=?$

(a) 5500 (b) 5200
(c) 5800 (d) 4900
(e) 5900

44. 45% of 1500 + 35% of 1700 = ?% of 3175.

(a) 50 (b) 45
(c) 30 (d) 35
(e) None of these

45. $3\frac{3}{5}$ of 157.85 + 39% of 1847 = ? − 447.30

(a) 1200 (b) 1500
(c) 1600 (d) 1800
(e) 2100

46. $47^{7.5}\div47^{3/2}\times47^{-3}=(\sqrt{47})^{?}$

(a) 3 (b) $2\frac{1}{2}$
(c) 6 (d) 3.5
(e) None of these

47. $33\frac{1}{3}$% of 768.9 + 25% of 161.2 − 68.12 = ?

(a) 230 (b) 225
(c) 235 (d) 220
(e) 240

48. $25^{7.5}\times5^{2.5}\div125^{1.5}=5^{?}$

(a) 16 (b) 17.5
(c) 8.5 (d) 13
(e) None of these

49. $16\sqrt{524}+1492-250.0521=?$

(a) 1600 (b) 1800
(c) 1900 (d) 2400
(e) 1400

50. $\sqrt{(0.798)^2+0.404\times0.798+(0.202)^2}+1=?$

(a) 0 (b) 2
(c) 1.596 (d) 10.404
(e) 3

Directions (Qs. 51-60) : Four of the following five parts (a), (b), (c), (d) and (e) are exactly equal. Which part is not equal to the other four parts ?

51. (a) $30 \times 14 \div 7 \times 5$ (b) $10^3 - 100 \times 7$
(c) $5 \times \sqrt{3600}$ (d) $450 \div 50 \times 50 - 5 \times 30$
(e) $10 \times 3 + 120 \times 2$

52. (a) $10.36 + 69.802 + 24.938$
(b) $2207.1 \div 21$ (c) $16\frac{2}{3}\%$ of 630.6
(d) 32.84375×3.2 (e) $\frac{1}{5}$ of $\frac{1}{9}$ of 4729.4

53. (a) $75 \times 8 \div 6$ (b) $98 \div 2.5 + 15.2 \times 4$
(c) $\sqrt{225} \times 2^3 - 5 \times 2^2$ (d) $76 \times 1.5 - 5.5 \times 2.6$
(e) $48 \times 1.2 + 127.2 \div 3$

54. (a) $115 \times 8 \div 10 + 8$ (b) $425 \div 17 \times 4$
(c) $36 \times 5 \div 6 + 17 \times 4$ (d) $2^6 + \sqrt{256} + 20$
(e) $35 \times 12 \div 14 + 14 \times 5$

55. (a) $45 \times 120 + 5^2 \times 10$
(b) $113 \times 25 \times 2$
(c) $27 \times 25 \times 8 + 15 \times 6 + 4 \times 40$
(d) $226 \times 5 + 113 \times 45$
(e) $50^2 \times 2 + 13 \times 50$

56. (a) $85 \div 17 \times 110$ (b) $45 \times 6 + 75 \times 4$
(c) $175 \div 25 \times 75 + 5^2$ (d) $36 \times 4 + 21^2 - 7 \times 5$
(e) $65 \times 12 - 46 \times 5$

57. (a) $120 \times 12 - 22 \times 20$ (b) 10% of $5000 + \frac{2}{5}$ of 1200
(c) $80 \times 40 - 20 \times 110$ (d) $8640 \div 60 + 53.5 \times 16$
(e) $5314 - 3029 - 1285$

58. (a) $16.80 \times 4.50 + 4.4$ (b) $1600 \div 40 + 16 \times 2.5$
(c) $5.5 \times 8.4 + 34.6$ (d) $1620 \div 20 - 1$
(e) $1856.95 - 1680.65 - 96.3$

59. (a) 40% of $160 + \frac{1}{3}$ of 240 (b) 120% of 1200
(c) $38 \times 12 - 39 \times 8$ (d) $1648 - 938 - 566$
(e) $6\frac{1}{2}$ of $140 - 2.5 \times 306.4$

60. (a) $732.534 + 412.256 - 544.29$
(b) $1256.214 - 355.514 - 300.2$
(c) $246.86 + 439.38 - 80.74$
(d) $1415.329 + 532.4 - 1347.229$
(e) $398.14 - 239.39 + 441.75$

61. Four of the following five parts lettered a, b, c, d and e are exactly equal. Which of the following is not equal to the other four?
(a) $24^2 - 12^2 + 112 \div 14$ (b) $17 \times 12 + 59 \times 4$
(c) $15 \times 28 + 20$ (d) $27 \times 16 + 56 \div 8$
(e) $185 \times 6 \div 2 - 23 \times 5$

62. Which of the following is the highest fraction?
(a) $\frac{5}{7}$ (b) $\frac{3}{4}$
(c) $\frac{2}{3}$ (d) $\frac{6}{7}$
(e) $\frac{7}{8}$

63. If $\frac{4}{9}$ of $\frac{3}{10}$ of $\frac{5}{8}$ of a number is 45, what is the number?
(a) 450 (b) 550
(c) 560 (d) 650
(e) None of these

64. Which of the following has fractions in ascending order?
(a) $\frac{2}{5}, \frac{3}{5}, \frac{1}{3}, \frac{4}{7}, \frac{5}{6}, \frac{6}{7}$ (b) $\frac{1}{3}, \frac{2}{5}, \frac{3}{5}, \frac{5}{6}, \frac{4}{7}, \frac{6}{7}$
(c) $\frac{1}{3}, \frac{2}{5}, \frac{3}{5}, \frac{4}{7}, \frac{5}{6}, \frac{6}{7}$ (d) $\frac{1}{3}, \frac{2}{5}, \frac{4}{7}, \frac{3}{5}, \frac{5}{6}, \frac{6}{7}$
(e) None of these

65. Find the value of $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right).....\left(1-\frac{1}{100}\right)$.
(a) $\frac{1}{5}$ (b) $\frac{1}{10}$
(c) $\frac{1}{50}$ (d) $\frac{2}{5}$
(e) $\frac{3}{5}$

66. If $\sqrt{15625} = 125$, then the value of $\sqrt{15625} + \sqrt{156.25} + \sqrt{1.5625}$ is
(a) 1.3875 (b) 13.875
(c) 138.75 (d) 156.25
(e) 162.235

67. When $0.\overline{47}$ is converted into a fraction the result is
(a) $\frac{46}{90}$ (b) $\frac{46}{99}$
(c) $\frac{47}{90}$ (d) $\frac{47}{99}$
(e) $\frac{99}{90}$

68. $(x^n - a^n)$ is completely divisible by $(x + a)$, when
(a) n is any natural number
(b) n is an even natural number
(c) n is an odd natural number
(d) n is prime
(e) None of these

Directions (Qs. 69-78): What approximate value will come in place of the question mark (?) in the following questions ? (You are not required to find the exact value).

69. $2371 \div 6 + (43 \times 4.35) = ?$
(a) 582 (b) 590
(c) 600 (d) 570
(e) 595

70. $\sqrt[3]{3380} + \sqrt{1300} = ?$
(a) 56 (b) 51
(c) 53 (d) 54
(e) 55

71. $(4.989)^2 + (21.012)^3 + \sqrt{1090} = ?$
(a) 9219 (b) 9391
(c) 9319 (d) 9129
(e) None of these

72. $7020 \div 2.99 \times \frac{13}{29} = ?$
(a) 1040 (b) 1100
(c) 1060 (d) 1050
(e) None of these

73. 24.99% of $5001 - 65.01\%$ of $2999 = ?$
(a) 840 (b) 500
(c) 700 (d) –500
(e) –700

74. $(81)^{-\frac{1}{2}} - (64)^{-\frac{2}{3}} = ?$
(a) $\frac{3}{19}$ (b) $\frac{1}{16}$
(c) $\frac{7}{144}$ (d) $\frac{1}{9}$
(e) None of these

75. $331.8 \div 23.7 + (-21)^2 - 94 = (?)^2$
(a) 15 (b) 16
(c) 18 (d) 19
(e) 17

76. 34% of $576 + 18\%$ of $842 = ?\%$ of $400 + 83.4$
(a) 75 (b) 72
(c) 62 (d) 65
(e) 66

77. $\frac{\sqrt{29241}}{\sqrt{361}} \times 5\frac{2}{9} = ?$
(a) 47 (b) 49
(c) 46 (d) 45
(e) 61

78. $3\frac{1}{4} + 6\frac{2}{7} + ? = 13\frac{3}{28}$
(a) $3\frac{2}{7}$ (b) $3\frac{4}{7}$
(c) $3\frac{3}{7}$ (d) $3\frac{5}{7}$
(e) $3\frac{6}{7}$

79. If the sum of two numbers is 55 and the H.C.F. and L.C.M. of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to:
(a) $\frac{55}{601}$ (b) $\frac{601}{55}$
(c) $\frac{11}{120}$ (d) $\frac{120}{11}$
(e) None of these

80. The least number which when divided by 48, 64, 90, 120 will leave the remainders 38, 54, 80, 110 respectively, is
(a) 2870 (b) 2860
(c) 2890 (d) 2880
(e) None of these

81. Product of two co-prime numbers is 117. Their L.C.M. should be:
(a) 1 (b) 117
(c) equal to their H.C.F. (d) cannot be calculated
(e) None of these

82. The number of prime factors in the expression $(6)^{10} \times (7)^{17} \times (11)^{27}$ is:
(a) 54 (b) 64
(c) 71 (d) 81
(e) None of these

83. The least number of five digits which is exactly divisible by 12, 15 and 18, is:
(a) 10010 (b) 10051
(c) 10020 (d) 10080
(e) None of these

84. The sum of two numbers is 462 and their highest common factor is 22. What is the maximum number of pairs that satisfy these conditions ?
(a) 1 (b) 3
(c) 5 (d) 6
(e) None of these

85. When n is divisible by 5 the remainder is 2. What is the remainder when n^2 is divided by 5.
(a) 2 (b) 3
(c) 1 (d) 4
(e) None of these

CHAPTER 2

Algebraic Expression & Inequalities

VARIABLE

An unknown quantity used in any equation may be constant known as variable. Variables are generally denoted by the last English alphabet x, y, z etc.
An equation is a statement of equality of two algebraic expressions, which involve one or more variables.

LINEAR EQUATION

An equation in which the highest power of variables is one, is called a linear equation. These equations are called linear because the graph of such equations on the x–y cartesian plane is a straight line.

Linear Equation in one variable

A linear equation which contains only one variable is called **linear equation in one variable**.
The general form of such equations is $ax + b = c$, where a, b and c are constants and $a \neq 0$.
All the values of x which satisfy this equation are called its solution(s).

NOTE : *An equation satisfied by all values of the variable is called an identity. For example : $2x + x = 3x$.*

EXAMPLE 1. ***Solve*** $2x - 5 = 1$

Sol. $2x - 5 = 1$

$\Rightarrow 2x = 1 + 5$

$\Rightarrow 2x = 6 \Rightarrow x = \frac{6}{2} = 3.$

EXAMPLE 2. ***Solve*** $7x - 5 = 4x + 11$

Sol. $7x - 5 = 4x + 11$

$\Rightarrow 7x - 4x = 11 + 5$ (Bringing like terms together)

$\Rightarrow 3x = 16 \Rightarrow x = \frac{16}{3} = 5\frac{1}{3}.$

EXAMPLE 3. ***Solve*** $\frac{4}{x} - \frac{3}{2x} = 5$

Sol. $\frac{4}{x} - \frac{3}{2x} = 5 \Rightarrow \frac{8-3}{2x} = 5$

$\Rightarrow \frac{5}{2x} = 5 \Rightarrow 10x = 5$

$\Rightarrow x = \frac{5}{10} = \frac{1}{2}$

Application of linear equations with one variables

EXAMPLE 4. The sum of the digits of a two digit number is 16. If the number formed by reversing the digits is less than the original number by 18. Find the original number.

Sol. Let unit digit be x.

Then tens digit $= 16 - x$

$\therefore$ Original number $= 10 \times (16 - x) + x$
$= 160 - 9x.$

On reversing the digits, we have x at the tens place and $(16 - x)$ at the unit place.

$\therefore$ New number $= 10x + (16 - x) = 9x + 16$

Original number – New number = 18

$(160 - 9x) - (9x + 16) = 18$

$160 - 18x - 16 = 18$

$-18x + 144 = 18$

$-18x = 18 - 144 \quad \Rightarrow 18x = 126$

$\Rightarrow x = 7$

$\therefore$ In the original number, we have unit digit = 7

Ten's digit $= (16 - 7) = 9$

Thus, original number = 97

EXAMPLE 5. The denominator of a rational number is greater than its numerator by 4. If 4 is subtracted from the numerator and 2 is added to its denominator, the new number becomes $\frac{1}{6}$. Find the original number.

Sol. Let the numerator be x.

Then, denominator $= x + 4$

$\therefore \frac{x-4}{x+4+2} = \frac{1}{6}$

$\Rightarrow \frac{x-4}{x+6} = \frac{1}{6}$

$\Rightarrow 6(x - 4) = x + 6$

$\Rightarrow 6x - 24 = x + 6 \Rightarrow 5x = 30$

$\therefore x = 6$

Thus, Numerator = 6, Denominator = 6 + 4 = 10.

Hence the original number $= \frac{6}{10}$.

EXAMPLE 6. A man covers a distance of 33 km in $3\frac{1}{2}$ hours; partly on foot at the rate of 4 km/hr and partly on bicycle at the rate of 10 km/hr. Find the distance covered on foot.

Sol. Let the distance covered on foot be x km.

$\therefore$ Distance covered on bicycle = (33 – x) km

$\therefore$ Time taken on foot $= \frac{\text{Distance}}{\text{Speed}} = \frac{x}{4}$ hr.

$\therefore$ Time taken on bicycle $= \frac{33-x}{10}$ hr.

The total time taken $= \frac{7}{2}$ hr.

$$\frac{x}{4}+\frac{33-x}{10}=\frac{7}{2}$$

$$\frac{5x+66-2x}{20}=\frac{7}{2}$$

$6x+132=140$

$6x=140-132$

$6x=8$

$x=\frac{8}{6}=1.33$ km.

$\therefore$ The distance covered on foot is 1.33 km.

Linear equation in two variables

General equation of a linear equation in two variables is $ax + by + c = 0$, where a, $b \neq 0$ and c is a constant, and x and y are the two variables.

The sets of values of x and y satisfying any equation are called its solution(s).

Consider the equation $2x + y = 4$. Now, if we substitute $x = -2$ in the equation, we obtain $2.(-2) + y = 4$ or $-4 + y = 4$ or $y = 8$. Hence $(-2, 8)$ is a solution. If we substitute $x = 3$ in the equation, we obtain $2.3 + y = 4$ or $6 + y = 4$ or $y = -2$

Hence $(3, -2)$ is a solution. The following table lists six possible values for x and the corresponding values for y, i.e. six solutions of the equation.

x	–2	–1	0	1	2	3
y	8	6	4	2	0	–2

Systems of Linear equation

Consistent System : A system (of 2 or 3 or more equations taken together) of linear equations is said to be consistent, if it has at least one solution.

Inconsistent System: A system of simultaneous linear equations is said to be inconsistent, if it has no solutions at all.

e.g. $X + Y = 9$; $3X + 3Y = 8$

Clearly there are no values of X & Y which simultaneously satisfy the given equations. So the system is inconsistent.

REMEMBER

★ The system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ has :

- a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.
- Infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
- No solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

★ The homogeneous system $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ has the only solution $x = y = 0$ when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

★ The homogeneous system $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ has a non-zero solution only when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, and in this case, the system has an infinite number of solutions.

EXAMPLE 7. Find k for which the system $3x - y = 4$, $kx + y = 3$ has a infinitely many solution.

Sol. The given system will have inifinite solution,

if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ i.e. $\frac{3}{k} = \frac{-1}{1}$ or $k = -3$.

EXAMPLE 8. Find k for which the system $6x - 2y = 3$, $kx - y = 2$ has a unique solution.

Sol. The given system will have a unique solution,

if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e. $\frac{6}{k} \neq \frac{-2}{-1}$ or $k \neq 3$.

EXAMPLE 9. What is the value of k for which the system $x + 2y = 3$, $5x + ky = -7$ is inconsistent?

Sol. The given system will be inconsistent if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e. if $\frac{1}{5} = \frac{2}{k} \neq \frac{3}{-7}$ or $k = 10$.

EXAMPLE 10. Find k such that the system $3x + 5y = 0$, $kx + 10y = 0$ has a non-zero solution.

Sol. The given system has a non zero solution,

if $\frac{3}{k} = \frac{5}{10}$ or $k = 6$

QUADRATIC EQUATION

An equation of the degree two of one variable is called quadratic equation.

General form : $ax^2 + bx + c = 0$...........(1) where a, b and c are all real number and $a \neq 0$.

For Example :

$2x^2 - 5x + 3 = 0$; $2x^2 - 5 = 0$; $x^2 + 3x = 0$

If $b^2 - 4ac \geq 0$, then the quadratic equation gives two and only two values (either same or different) of the unknown variable and both these values are called the roots of the equation.

The roots of the quadratic equation (1) can be evaluated using the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad ...(2)$$

The above formula provides both the roots of the quadratic equation, which are generally denoted by α and β,

say $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

The expression inside the square root $b^2 - 4ac$ is called the DISCRIMINANT of the quadratic equation and denoted by D. Thus, Discriminant (D) = $b^2 - 4ac$.

Shortcut Approach

Shortcut Approach to solve Quadratic equation $ax^2 + bx + c = 0$, if $b^2 - 4ac \geq 0$,

$\boxed{a}x^2 + bx + \boxed{c}$

ac ← [Product of coefficient a of x^2 and constant term c]

$p \quad q$ ← [Divide ac in two parts p and q such that their sum is equal to coefficient b of x and product equal to ac]

$\frac{p}{a} \quad \frac{q}{a}$ ← [Divide p and q by a]

$-\frac{p}{a} \quad -\frac{q}{a}$ ← [Change the sign]

Here $\frac{-p}{a}$ and $\frac{-q}{a}$ are two roots or solutions of quadratic equation $ax^2 + bx + c = 0$ i.e. $x = -\frac{p}{a}$ or $-\frac{q}{a}$.

EXAMPLE 11. Which of the following is a quadratic equation?

(a) $x^{\frac{1}{2}} + 2x + 3 = 0$

(b) $(x-1)(x+4) = x^2 + 1$

(c) $x^4 - 3x + 5 = 0$

(d) $(2x+1)(3x-4) = 2x^2 + 3$

Sol. (d) Equations in options (a) and (c) are not quadratic equations as in (a) max. power of x is fractional and in (c), it is not 2 in any of the terms.

For option (b), $(x-1)(x+4) = x^2 + 1$

or $x^2 + 4x - x - 4 = x^2 + 1$

or $3x - 5 = 0$

which is not a quadratic equations but a linear.

For option (d), $(2x+1)(3x-4) = 2x^2 + 3$

or $6x^2 - 8x + 3x - 4 = 2x^2 + 3$

or $4x^2 - 5x - 7 = 0$

which is clearly a quadratic equation.

EXAMPLE 12. *Solve* $2x^2 + 6 = 7x$

Sol. $2x^2 + 6 = 7x$

$\Rightarrow 2x^2 - 7x + 6 = 0$

$\boxed{2}x^2 + 7x + \boxed{6} = 0$

12 ← $[2 \times 6 = 12]$

$-4 \quad -3$ ← $[(-4) + (-3) = -7,\ (-4) \times (-3) = 12]$

$-4 \quad -\frac{3}{2}$ ← [Divide by coefficient 2 or x^2]

$2 \quad \frac{3}{2}$ ← [Change the sign]

∴ Solutions or roots of given quadratic equation $2x^2 + 6 = 7x$ are $\frac{3}{2}$ and 2.

EXAMPLE 13. *Solve* $x - \frac{1}{x} = 1\frac{1}{2}$

Sol. $x - \frac{1}{x} = 1\frac{1}{2} \Rightarrow \frac{x^2 - 1}{x} = \frac{3}{2}$

$\Rightarrow 2(x^2 - 1) = 3x$

$\Rightarrow 2x^2 - 2 = 3x$

$\Rightarrow 2x^2 - 3x - 2 = 0$

$\boxed{2}x^2 + (-3)x + \boxed{(-2)} = 0$

-4 ← $[2 \times (-2) = -4]$

$-4 \quad 1$ ← $[-4 + 1 = -3,\ -4 \times 1 = -4]$

$-2 \quad \frac{1}{2}$ ← [Divide by 2]

$2 \quad -\frac{1}{2}$ ← [Change the sign]

Either $2x + 1 = 0$ or $x - 2 = 0$

$\Rightarrow 2x = -1$ or $x = 2$

$\Rightarrow x = \frac{-1}{2}$ or $x = 2$

∴ $x = \frac{-1}{2}$, 2 are solutions.

Nature of Roots

The nature of roots of the equation depends upon the nature of its discriminant D.

1. If $D < 0$, then the roots are non-real complex, Such roots are always conjugate to one another. That is, if one root is $p + iq$ then other is $p - iq$, $q \neq 0$.

2. If D = 0, then the roots are real and equal. Each root of the equation becomes $-\frac{b}{2a}$. Equal roots are referred as repeated roots or double roots also.
3. If D > 0 then the roots are real and unequal.

Sign of Roots:

Let α, β are real roots of the quadratic equation $ax^2 + bx + c = 0$ that is $D = b^2 - 4ac \geq 0$. Then

1. Both the roots are positive if a and c have the same sign and the sign of b is opposite.
2. Both the roots are negative if a, b and c all have the same sign.
3. The Roots have opposite sign if sign of a and c are opposite.
4. The Roots are equal in magnitude and opposite in sign if $b = 0$ [that is its roots α and $-\alpha$]
5. The roots are reciprocal if a = c.
[that is the roots are α and $\frac{1}{\alpha}$]

EXAMPLE 14. The solutions of the equation $\sqrt{25 - x^2} = x - 1$ are :

(a) **$x = 3$ and $x = 4$** (b) **$x = 5$ and $x = 1$**
(c) **$x = -3$ and $x = 4$** (d) **$x = 4$ and $x = -3$**

Sol. (d) $\sqrt{25 - x^2} = x - 1$

or $25 - x^2 = (x-1)^2$ or $25 - x^2 = x^2 + 1 - 2x$

or $2x^2 - 2x - 24 = 0$ or $x^2 - x - 12 = 0$

or $(x-4)(x+3) = 0$ or $x = 4, x = -3$

EXAMPLE 15. Which of the following equations has real roots ?

(a) **$3x^2 + 4x + 5 = 0$** (b) **$x^2 + x + 4 = 0$**
(c) **$(x-1)(2x-5) = 0$** (d) **$2x^2 - 3x + 4 = 0$**

Sol. (c) Roots of a quadratic equation

$ax^2 + bx + c = 0$ are real if $b^2 - 4ac \geq 0$

Let us work with options as follows.

Option (a) : $3x^2 + 4x + 5 = 0$

$b^2 - 4ac = (4)^2 - 4(3)(5) = -44 < 0$.

Thus, roots are not real.

(b) : $x^2 + x + 4 = 0$

$b^2 - 4ac = (1)^2 - 4(1)(4) = 1 - 16 = -15 < 0$

Thus, roots are not real.

(c) : $(x-1)(2x-5) = 0 \Rightarrow 2x^2 - 7x + 5 = 0$

$b^2 - 4ac = (-7)^2 - 4 \times 2 \times 5 = 49 - 40 = 9 > 0$

Thus roots are real.

or $x = 1$ and $x = \frac{5}{2} > 0$; Thus, equation has real roots.

(d) : $2x^2 - 3x + 4 = 0$

$b^2 - 4ac = (-3)^2 - 4(2)(4) = 9 - 32 = -23 < 0$

Thus, roots are not real.

Hence, option (c) is correct.

EXAMPLE 16. If $2x^2 - 7xy + 3y^2 = 0$, then the value of x : y is :

(a) **3 : 2** (b) **2 : 3**
(c) **3 : 1 or 1 : 2** (d) **5 : 6**

Sol. (c) $2x^2 - 7xy + 3y^2 = 0$

$$2\left(\frac{x}{y}\right)^2 - 7\left(\frac{x}{y}\right) + 3 = 0$$

$$\frac{x}{y} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{49 - 24}}{2 \times 2} = \frac{7 \pm 5}{4} = 3, \frac{1}{2}$$

$$\Rightarrow \frac{x}{y} = \frac{3}{1} \text{ or } \frac{x}{y} = \frac{1}{2}$$

EXAMPLE 17. If $a + b + c = 0$ and a,b,c, are rational numbers then the roots of the equation
$(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are

(a) **rational** (b) **irrational**
(c) **non real** (d) **none of these.**

Sol. (a) The sum of coefficients

$= (b + c - a) + (c + a - b) + (a + b - c) = a + b + c = 0$ (given)

$\therefore x = 1$ is a root of the equation

$\therefore$ The other root is $\frac{a + b - c}{b + c - a}$, which is rational as a, b, c, are rational

Hence, both the roots are rational.

OTHER METHOD :

$D = (c + a - b)^2 - 4(b + c - a)(a + b - c)$

$= (-2b)^2 - 4(-2a)(-2c) = 4b^2 - 16ac$

$= 4(a + c)^2 - 16ac = 4[(a + c)^2 - 4ac] = [2(a - c)]^2$

D is a perfect square. Hence, the roots of the equation are rational.

EXAMPLE 18. Both the roots of the equation
$(x - b)(x - c) + (x - c)(x - a) + (x - a)(x - b) = 0$ are

(a) **dependent on a, b, c** (b) **always non real**
(c) **always real** (d) **rational**

Sol. (c) The equation is

$3x^2 - 2(a + b + c)x + (bc + ca + ab) = 0$

The discriminant

$D = 4(a + b + c)^2 - 4.3.(bc + ca + ab)$

$= 4[a^2 + b^2 + c^2 - ab - bc - ca]$

$= 2[(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)]$

$= 2[(a - b)^2 + (b - c)^2 + (c - a)^2] \geq 0$

$\therefore$ Roots are always real.

Symmetric Functions of Roots :

An expression in α, β is called a symmetric function of α, β if the function is not affected by interchanging α and β. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$ then,

Sum of roots : $\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

and Product of roots : $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Formation of quadratic Equation with Given Roots :

- An equation whose roots are α and β can be written as $(x - \alpha)(x - \beta) = 0$ or $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ or $x^2 -$ (sum of the roots) $x +$ product of the roots $= 0$.
- Further if α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then
  $ax^2 + bx + c = a(x - \alpha)(x - \beta)$ is an identity.

EXAMPLE 19. Of the following quadratic equations, which is the one whose roots are 2 and – 15 ?

(a) $x^2 - 2x + 15 = 0$ **(b)** $x^2 + 15x - 2 = 0$

(c) $x^2 + 13x - 30 = 0$ **(d)** $x^2 - 30 = 0$

Sol. (c) Sum of roots = 2 – 15 = – 13
Product of roots = 2 × (– 15) = – 30
Required equation

$= x^2 - x$ (sum of roots) + product of roots = 0

$\Rightarrow\ x^2 + 13x - 30 = 0$

EXAMPLE 20. If a and b are the roots of the equation $x^2 - 6x + 6 = 0$, then the value of $a^2 + b^2$ is :

(a) 36 **(b)** 24 **(c)** 17 **(d)** 6

Sol. (b) The sum of roots = a + b = 6
Product of roots = ab = 6

Now, $a^2 + b^2 = (a + b)^2 - 2ab = 36 - 12 = 24$

EXAMPLE 21. If a, b are the two roots of a quadratic equation such that a + b = 24 and a – b = 8, then the quadratic equation having a and b as its roots is :

(a) $x^2 + 2x + 8 = 0$ **(b)** $x^2 - 4x + 8 = 0$

(c) $x^2 - 24x + 128 = 0$ **(d)** $2x^2 + 8x + 9 = 0$

Sol. (c) a + b = 24 and a – b = 8
$\Rightarrow$ a = 16 and b = 8 $\Rightarrow$ ab = 16 × 8 = 128
A quadratic equation with roots a and b is

$x^2 - (a + b)x + ab = 0$ or $x^2 - 24x + 128 = 0$

INEQUATIONS :

A statement or equation which states that one thing is not equal to another, is called an inequation.

Symbols :

'<' means "is less than"

'>' means "is greater than"

'≤' means "is less than or equal to"

'≥' means "is greater than or equal to"

For example :

(a) $x < 3$ means x is less than 3.

(b) $y \geq 9$ means y is greater than or equal to 9.

Properties

1. Adding the same number to each side of an inequation does not effect the sign of inequality, i.e. if $x > y$ then, $x + a > y + a$.
2. Subtracting the same number to each side of an inequation does not effect the sign of inequaltiy, i.e., if $x < y$ then, $x - a < y - a$.
3. Multiplying each side of an inequality with same positive number does not effect the sign of inequality, i.e., if $x \leq y$ then $ax \leq ay$ (where, $a > 0$).
4. Multiplying each side of an inequality with a negative number reverse the sign of inequality i.e., if $x < y$ then $ax > ay$ (where $a < 0$).
5. Dividing each side of an inequation by a positive number does not effect the sign of inequality, i.e., if $x \leq y$ then $\frac{x}{a} \leq \frac{y}{a}$ (where $a > 0$).
6. Dividing each side of an inequation by a negative number reverses the sign of inequality, i.e., if $x > y$ then $\frac{x}{a} < \frac{y}{a}$ (where $a < 0$).

REMEMBER

★ If $a > b$ and a, b, n are positive, then $a^n > b^n$ but $a^{-n} < b^{-n}$. For example $5 > 4$; then $5^3 > 4^3$ or $125 > 64$, but

$5^{-3} < 4^{-3}$ or $\frac{1}{125} < \frac{1}{64}$.

★ If $a > b$ and $c > d$, then $(a + c) > (b + d)$.

★ If $a > b > 0$ and $c > d > 0$, then $ac > bd$.

★ If the signs of all the terms of an inequality are changed, then the sign of the inequality will also be reversed.

MODULUS :

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

1. If a is positive real number, x and y be the fixed real numbers, then

(i) $|x - y| < a \Leftrightarrow y - a < x < y + a$

(ii) $|x - y| \leq a \Leftrightarrow y - a \leq x \leq y + a$

(iii) $|x - y| > a \Leftrightarrow x > y + a$ or $x < y - a$

(iv) $|x - y| \geq a \Leftrightarrow x \geq y + a$ or $x \leq y - a$

2. Triangle inequality :

(i) $|x+y| \le |x| + |y|, \ \forall x, y \in R$

(ii) $|x-y| \ge |x| - |y|, \ \forall x, y \in R$

EXAMPLE 22. If a – 8 = b, then determine the value of $|a-b|-|b-a|$.

(a) 16 (b) 0 (c) 4 (d) 2

Sol. (b) $|a-b| = |8| = 8$

$\Rightarrow |b-a| = |-8| = 8$

$\Rightarrow |a-b| - |b-a| = 8 - 8 = 0$

EXAMPLE 23. *Solve :* $3x + 4 \le 19, \ x \in N$

Sol. $3x + 4 \le 19$

$3x + 4 - 4 \le 19 - 4$ [Subtracting 4 from both the sides]

$3x \le 15$

$\frac{3x}{3} \le \frac{15}{3}$ [Dividing both the sides by 3]

$x \le 5; \ x \in N$

$\therefore x = \{1, 2, 3, 4, 5\}$.

EXAMPLE 24. *Solve* $5 \le 2x - 1 \le 11$

Sol. $5 \le 2x - 1 \le 11$

$5 + 1 \le 2x - 1 + 1 \le 11 + 1$

[Adding 1 to each sides]

$6 \le 2x \le 12$

$\frac{6}{2} \le \frac{2x}{2} \le \frac{12}{2}$ [Dividing each side by 2]

$3 \le x \le 6$

$\Rightarrow x = \{3, 4, 5, 6\}$.

Applications Formulation of Equations/ Expressions :

A formula is an equation, which represents the relations between two or more quantities.

For example :

Area of parallelogram (A) is equal to the product of its base (b) and height (h), which is given by

$A = b \times h$

or $A = bh$.

Perimeter of triangle (P),

$P = a + b + c$, where a, b and c are length of three sides.

EXAMPLE 25. Form the expression for each of the following:

(a) 5 less than a number is 7.

(b) Monika's salary is 1500 less than thrice the salary of Surbhi.

Sol. (a) Expression is given by

$x - 5 = 7$, where x is any number

(b) Let the salary of Surbhi be Rs. x and salary of Monika be Rs. y.

Now, according to the question

$y = 3x - 1500$

More Applications of Equations :

Problems on Ages can be solved by linear equations in one variable, linear equations in two variables, and quadratic equations.

EXAMPLE 26. Kareem is three times as old as his son. After ten years, the sum of their ages will be 76 years. Find their present ages.

Sol. Let the present age of Kareem's son be x years.

Then, Kareem's age = 3x years

After 10 years, Kareem's age = 3x + 10 years

and Kareem's son's age = x + 10 years

$\therefore (3x + 10) + (x + 10) = 76$

$\Rightarrow 4x = 56 \Rightarrow x = 14$

$\therefore$ Kareem's present age = $3x = 3 \times 14 = 42$ years

Kareem's son's age = x = 14 years.

EXAMPLE 27. The present ages of Vikas and Vishal are in the ratio 15 : 8. After ten years, their ages will be in the ratio 5 : 3. Find their present ages.

Sol. Let the present ages of Vikas and Vishal be 15x years and 8x years.

After 10 years,

Vikas's age = 15x + 10 and

Vishal's age = 8x + 10

$\therefore \frac{15x + 10}{8x + 10} = \frac{5}{3}$

$\Rightarrow 3(15x + 10) = 5(8x + 10)$

$\Rightarrow 45x + 30 = 40x + 50$

$\Rightarrow 5x = 20 \Rightarrow x = \frac{20}{5} = 4$

$\therefore$ Present age of Vikas = $15x = 15 \times 4 = 60$ years

Present age of Vishal = $8x = 8 \times 4 = 32$ years.

SHORTCUT METHOD

(Ratio of present age of vikas and vishal) (After 10 years, ratio of age of vikash and vishal)

$\frac{15}{8} \times \frac{5}{3} \xrightarrow{\text{Difference}} 2 \xrightarrow{\text{Difference after 10 years}} 10 \times 2 = 20$

Difference = $15 \times 3 - 8 \times 5 = 5$

Common part = $\frac{20}{5} = 4$

Vikas Present age = $15 \times 4 = 60$ years

Vishal Present age = $8 \times 4 = 32$ years

EXAMPLE 28. Father's age is 4 less than five times the age of his son and the product of their ages is 288. Find the father's age.

Sol. Let the son's age be x years.

So, father's age = 5x – 4 years.

$\therefore x(5x - 4) = 288$

$\Rightarrow 5x^2 - 4x - 288 = 0 \Rightarrow 5x^2 - 40x + 36x - 288 = 0$

$\Rightarrow 5x(x - 8) + 36(x - 8) = 0$

$\Rightarrow (5x + 36)(x - 8) = 0$

Either $x - 8 = 0$ or $5x + 36 = 0 \Rightarrow x = 8$ or $x = \frac{-36}{5}$

x cannot be negative; therefore, x = 8 is the solution.

$\therefore$ Son's age = 8 years and Father's age = 5x – 4 = 36 years.

> **Shortcut Approach**
>
> If present age of the father is F times the age of his son. T years hence, the father's age become Z times the age of son then present age of his son is given by $\frac{(Z-1)T}{(F-Z)}$

EXAMPLE 29. Present age of the father is 9 times the age of his son. One year later, father's age become 7 times the age of his son. What are the present ages of the father and his son.

Sol. By the formula

$$\text{Son's age} = \frac{(7-1)}{(9-7)} \times 1 = \frac{6}{2} \times 1 = 3 \text{ years.}$$

So, father's age = 9 × son's age = 9 × 3 = 27 years.

SHORTCUT METHOD

$\begin{pmatrix}\text{Ratio of present age of father and son}\end{pmatrix}$ $\begin{pmatrix}\text{After one year ratio of father and son}\end{pmatrix}$

$\frac{9}{1} \times \frac{7}{1} \xrightarrow{\text{Difference}} 6 \xrightarrow[\text{after one year}]{\text{Difference}} 1 \times 6 = 6$

Difference = 9 × 1 – 1 × 7 = 2

Common part = $\frac{6}{2} = 3$

Present age of father = 9 × 3 = 27 years

Present age of son = 1 × 3 = 3 years

> **Shortcut Approach**
>
> If T_1 years earlier the age of the father was n times the age of his son, T_2 years hence, the age of the father becomes m times the age of his son then his son's age is given by
>
> $$\text{Son's age} = \frac{T_2(n-1) + T_1(m-1)}{n-m}$$

EXAMPLE 30. 10 years ago, Shakti's mother was 4 times older than her. After 10 years, the mother will be twice older than the daughter. What is the present age of Shakti?

Sol. By using formula,

$$\text{Shakti's age} = \frac{10(4-1) + 10(2-1)}{4-2} = 20 \text{ years.}$$

SHORTCUT METHOD

Comparisons of ages is given 10 years before the present time and 10 years after the present time.

Therefore time gap = 10 + 10 = 20 years.

$\begin{pmatrix}\text{10 years before ratio of age of mother and daughter}\end{pmatrix}$ $\begin{pmatrix}\text{10 years after, ratio of age of mother and daughter}\end{pmatrix}$

$\frac{4}{1} \times \frac{2}{1} \xrightarrow{\text{Difference}} 1 \xrightarrow[\text{after 20 years}]{\text{Difference}} 20 \times 1 = 20$

Difference = 4 × 1 – 1 × 2 = 2

Common part = $\frac{20}{2} = 10$

Present age of Shakti's mother = 4 × 10 + 10 = 50 years

Present age fo Shakti = 10 +10 = 20 years

> **Shortcut Approach**
>
> Present age of Father : Son = a : b
>
> After / Before T years = m : n
>
> Then son's age = $b \times \frac{T(m-n)}{an - bm}$
>
> and Father's age = $a \times \frac{T(m-n)}{an - bm}$

EXAMPLE 31. The ratio of the ages of the father and the son at present is 3 : 1. Four years earliar, the ratio was 4 : 1. What are the present ages of the son and the father?

Sol. Ratio of present age of Father and Son = 3 : 1

4 years before = 4 : 1

$$\text{Son's age} = 1 \times \frac{4(4-1)}{4 \times 1 - 3 \times 1} = 12 \text{ years.}$$

$$\text{Father's age} = 3 \times \frac{4(4-1)}{4 \times 1 - 3 \times 1} = 36 \text{ years.}$$

SHORTCUT METHOD

$\begin{pmatrix}\text{Ratio of present age of father and son}\end{pmatrix}$ $\begin{pmatrix}\text{4 years before, ratio of age of father and son}\end{pmatrix}$

$\frac{3}{1} \times \frac{4}{1} \xrightarrow{\text{Difference}} 3 \xrightarrow[\text{after 4 years}]{\text{Difference}} 4 \times 3 = 12$

Difference = 4 – 3 = 1

Common part = $\frac{12}{1} = 12$

Present age of father = 3 × 12 = 36 years

Present age of son = 1 × 12 = 12 years

EXERCISE

Directions (Qs. 1-5): In each question one/two equations are provided. On the basis of these you have to find out the relation between p and q.

Give answer (a) if $p = q$

Give answer (b) if $p > q$

Give answer (c) if $q > p$

Give answer (d) if $p \geq q$, and

Give answer (e) if $q \geq p$

1. I. $pq + 30 = 6p + 5q$
2. I. $2p^2 + 12p + 16 = 0$
   II. $2q^2 + 14q + 24 = 0$
3. I. $2p^2 + 48 = 20p$
   II. $2q^2 + 18 = 12q$
4. I. $q^2 + q = 2$
   II. $p^2 + 7p + 10 = 0$
5. I. $p^2 + 36 = 12p$
   II. $4q^2 + 144 = 48q$

Directions (Qs.6-10): For the two given equations I and II give answer

(a) if p is greater than q

(b) if p is smaller than q

(c) if p is equal to q.

(d) if p is either equal to or greater than q

(e) if p is either equal to or smaller than q.

6. I. $6p^2 + 5p + 1 = 0$
   II. $20q^2 + 9q = -1$
7. I. $3p^2 + 2p - 1 = 0$
   II. $2q^2 + 7q + 6 = 0$
8. I. $3p^2 + 15p = -18$
   II. $q^2 + 7q + 12 = 0$
9. I. $p = \frac{\sqrt{4}}{\sqrt{9}}$
   II. $9q^2 - 12q + 4 = 0$
10. I. $p^2 + 13p + 42 = 0$
   II. $q^2 = 36$

Directions (Qs. 11 - 14): In each of the following questions, one or two equation(s) is/are given. On their basis you have to determine the relation between x and y and then give answer

(a) if $x < y$ (b) if $x > y$

(c) if $x \leq y$ (d) if $x \geq y$

(e) if $x = y$

11. I $x^2 + 3x + 2 = 0$ II. $2y^2 = 5y$
12. I. $2x^2 + 5x + 2 = 0$ II. $4y^2 = 1$
13. I. $y^2 + 2y - 3 = 0$ II. $2x^2 - 7x + 6 = 0$
14. I. $x^2 - 5x + 6 = 0$ II. $y^2 + y - 6 = 0$

Directions (Qs. 15-18): In each of the following questions two equations are given. You have to solve them and Give answer

(a) if $p < q$ (b) if $p > q$

(c) if $p \leq q$ (d) if $p \geq q$

(e) if $p = q$

15. I. $p^2 - 7p = -12$
   II. $q^2 - 3q + 2 = 0$
16. I. $12p^2 - 7p = -1$
   II. $6q^2 - 7q + 2 = 0$
17. I. $p^2 - 8p + 15 = 0$
   II. $q^2 - 5q = -6$
18. I. $2p^2 + 20p + 50 = 0$
   II. $q^2 - 25$

Directions (Qs.19-23): In each of these questions two equations are given. You have to solve these equations and Give answer

(a) if $x < y$ (b) if $x > y$

(c) if $x = y$ (d) if $x \geq y$

(e) if $x \leq y$

19. I. $x^2 - 6x = 7$
   II. $2y^2 + 13y + 15 = 0$
20. I. $3x^2 - 7x + 2\ 0$
   II. $2y^2 - 11y + 15 = 0$
21. I. $10x^2 - 7x + 1 = 0$
   II. $35y^2 - 12y + 1 = 0$
22. I. $4x^2 = 25$
   II. $2y^2 - 13y + 21 = 0$
23. I. $3x^2 + 7x = 6$
   II. $6(2y^2 + 1) = 17y$

Directions (Qs. 24-26): In each question below one or more equation(s) is /are given. On the basis of these, you have to find out the relationship between p and q.

Give answer (a) if $p = q$

Give answer (b) if $p > q$

Give answer (c) if $p < q$

Give answer (d) if $p \leq q$

Give answer (e) if $p \geq q$

24. I. $2p^2 = 23p - 63$
   II. $2q\ (q^{-8}) = q^{-36}$
25. I. $p\ (p^{-1}) = (p^{-1})$
   II. $q^2 = 4q^{-1}$
26. I. $2p\ (p - 4) = 8\ (p + 5)$
   II. $q^2 + 12 + 7q$

Directions (Qs. 27-30): In each of the following questions two equations I and II are given. You have to solve both the equations and give answer

(a) if $a < b$ (b) if $a \leq b$

(c) if $a \geq b$ (d) if $a = b$

(e) if $a > b$

27. I. $a^2 - 5a + 6 = 0$
   II. $b^2 - 3b + 2 = 0$
28. I. $2a + 3b = 31$
   II. $3a = 2b + 1$
29. I. $2a^2 + 5a + 3 = 0$
   II. $2b^2 - 5b + 3 = 0$

30. I. $4a^2 = 1$
II. $4b^2 - 12b + 5 = 0$

Directions (Qs. 31-35) : In each of the following questions there are two equations. Solve them and choose the correct option

(a) If $P < Q$ (b) If $P > Q$
(c) If $P \leq Q$ (d) If $P \geq Q$
(e) If $P = Q$

31. **I.** $4P^2 - 8P + 3 = 0$ **II.** $2Q^2 - 13Q + 15 = 0$
32. **I.** $P^2 + 3P - 4 = 0$ **II.** $3Q^2 - 10Q + 8 = 0$
33. **I.** $3P^2 - 10P + 7 = 0$ **II.** $15Q^2 - 22Q + 8 = 0$
34. **I.** $20P^2 - 17P + 3 = 0$ **II.** $20Q^2 - 9Q + 1 = 0$
35. **I.** $20P^2 + 31P + 12 = 0$ **II.** $21Q^2 + 23Q + 6 = 0$

Diretions (Qs. 36-40) : For the two given equations I and II, give answer

(a) if a is greater than b
(b) if a is smaller than b
(c) if a is equal to b
(d) if a is either equal to or greater than b
(e) if a is either equal to or smaller than b

36. **I.** $\sqrt{2304} = a$
II. $b^2 = 2304$
37. **I.** $12a^2 - 7a + 1 = 0$
II. $15b^2 - 16b + 4 = 0$
38. **I.** $a^2 + 9a + 20 = 0$
II. $2b^2 - 10b + 12 = 0$
39. **I.** $3a + 2b = 14$
II. $a + 4b - 13 = 0$
40. **I.** $a^2 - 7a + 12 = 0$
II. $b^2 - 9b + 20 = 0$

Directions (Qs. 41-45) : In each question one or more equation(s) is (are) provided. On the basis of these you have

Give answer (a) if $p = q$
Give answer (b) if $p > q$
Give answer (c) if $q > p$
Give answer (d) if $p \geq q$ and
Give answer (e) if $q \geq p$

41. (i) $\frac{5}{28} \times \frac{9}{8} p = \frac{15}{14} \times \frac{13}{16} q$
42. (i) $p - 7 = 0$ (ii) $3q^2 - 10q + 7 = 0$
43. (i) $4p^2 = 16$ (ii) $q^2 - 10q + 25 = 0$
44. (i) $4p^2 - 5p + 1 = 0$ (ii) $q^2 - 2q + 1 = 0$
45. (i) $q^2 - 11q + 30 = 0$ (ii) $2p^2 - 7p + 6 = 0$

Directions (Qs. 46-48) : In each question below one or more equation(s) is/are provided. On the basis of these, you have to find out relation between p and q.

Give answer (a) if $p = q$, Give answer
(b) if $p > q$, Give answer
(c) if $q > p$, Give answer
(d) if $p \geq q$ and Give answers
(e) if $q \geq p$.

46. I. $4q^2 + 8q = 4q + 8$
II. $p^2 + 9p = 2p - 12$
47. I. $2p^2 + 40 = 18p$
II. $q^2 = 13q - 42$
48. I. $6q^2 + \frac{1}{2} = \frac{7}{2} q$
II. $12p^2 + 2 = 10p$

Directions (Qs. 49-53): In each of the following questions two equations are given. You have to solve them and give answer

(a) if $x > y$; (b) if $x < y$;
(c) if $x = y$; (d) if $x \geq y$;
(e) if $x \leq y$;

49. **I.** $y^2 - 6y + 9 = 0$ **II.** $x^2 + 2x - 3 = 0$
50. **I.** $x^2 - 5x + 6 = 0$ **II.** $2y^2 + 3y - 5 = 0$
51. **I.** $x = \sqrt{256}$ **II.** $y = (-4)^2$
52. **I.** $x^2 - 6x + 5 = 0$ **II.** $y^2 - 13y + 42 = 0$
53. **I.** $x^2 + 3x + 2 = 0$ **II.** $y^2 - 4y + 1 = 0$

54. If $3x - 5y = 5$ and $\frac{x}{x+y} = \frac{5}{7}$, then what is the value of $x - y$?
(a) 9 (b) 6
(c) 4 (d) 3
(e) None of these

55. $\frac{5}{7}$ of $\frac{4}{15}$ of a number is 8 more than $\frac{2}{5}$ of $\frac{4}{9}$ of the same number. What is half of that number?
(a) 630 (b) 315
(c) 210 (d) 105
(e) None of these

56. The difference between a two-digit number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number?
(a) 4 (b) 9
(c) 3 (d) Cannot be determined
(e) None of theses

57. By the how much is two-fifth of 200 greater than three - fifths of 125?
(a) 15 (b) 3
(c) 5 (d) 30
(e) None of these

58. If $\frac{x^2 - 1}{x + 1} = 2$, then, $x = ?$
(a) 1 (b) 0
(c) 2 (d) Can't be determined
(e) None of these

59. The difference between a number and its one-third is double of its one-third. What is the number?
(a) 60 (b) 18
(c) 30 (d) Cannot be determined
(e) None of these

60. Two pens and three pencils cost ₹ 86. Four pens and a pencil cost ₹ 112. What is the difference between the cost of a pen and that of a pencil?
(a) ₹ 25 (b) ₹ 13
(c) ₹ 19 (d) Cannot be determined
(e) None of these

61. The difference between a two-digit number and the number after interchanging the position of the two digits is 36. What is the difference between the two digits of the number?
(a) 4 (b) 6
(c) 3 (d) Cannot be determined
(e) None of these

62. If the digit in the unit's place of a two-digit number is halved and the digit in the ten's place is doubled, the number thus, obtained is equal to the number obtained by interchanging the digits. Which of the following is **definitely true**?
(a) Digits in the unit's place and the ten's place are equal.
(b) Sum of the digits is a two-digit number.
(c) Digit in the unit's place is half of the digit in the ten's place.
(d) Digit in the unit's place is twice the digit in the ten's place.
(e) None of these

63. If A and B are positive integers such that $9A^2 = 12A + 96$ and $B^2 = 2B + 3$, then which of the following is the value of $5A + 7B$?
(a) 31 (b) 41
(c) 36 (d) 43
(e) 27

64. On Children's Day, sweets were to be equally distributed among 175 children in a school. Actually on the Children's Day 35 children were absent and therefore, each child got 4 sweets extra. How many sweets were available in all for distribution?
(a) 2480 (b) 2680
(c) 2750 (d) 2400
(e) None of these

65. A two-digit number is seven times the sum of its digits. If each digit is increased by 2, the number thus obtained is 4 more than six times the sum of its digits. Find the number.
(a) 42 (b) 24
(c) 48 (d) Data inadequate
(e) None of these

66. One-third of Ramani's savings in National Savings Certificate is equal to one-half of his savings in Public Provident Fund. If he has ₹ 150000 as total savings, how much he saved in Public Provident Fund?
(a) ₹ 60000 (b) ₹ 50000
(c) ₹ 90000 (d) ₹ 30000
(e) None of these

67. $\frac{1}{5}$ of a number is equal to $\frac{5}{8}$ of the second number. If 35 is added to the first number then it becomes 4 times of second number. What is the value of the second number?
(a) 125 (b) 70
(c) 40 (d) 25
(e) None of these

68. In a two-digit number, the digit at unit place is 1 more than twice of the digit at tens place. If the digit at unit and tens place be interchanged, then the difference between the new number and original number is less than 1 to that of original number. What is the original number?
(a) 52 (b) 73
(c) 25 (d) 49
(e) 37

69. Free notebooks were distributed equally among children of a class. The number of notebooks each child got was one-eighth of the number of children. Had the number of children been half, each child would have got 16 notebooks. How many notebooks were distributed in all?
(a) 432 (b) 640
(c) 256 (d) 512
(e) None of these

70. Twenty times a positive integer is less than its square by 96. What is the integer?
(a) 24 (b) 20
(c) 30 (d) Cannot be determined
(e) None of these

71. The digit in the units place of a number is equal to the digit in the tens place of half of that number and the digit in the tens place of that number is less than the digit in units place of half of the number by 1. If the sum of the digits of the number is seven, then what is the number?
(a) 52 (b) 16
(c) 34 (d) Data inadequate
(e) None of these

72. The difference between a two-digit number and the number obtained by interchanging the digits is 9. What is the difference between the two digits of the number?
(a) 8 (b) 2
(c) 7 (d) Cannot be determined
(e) None of these

73. The difference between a number and its three-fifths is 50. What is the number?
(a) 75 (b) 100
(c) 125 (d) Cannot be determined
(e) None of these

74. If the numerator of a fraction is increased by 2 and the denominator is increased by 1, the fraction becomes $\frac{5}{8}$ and if the numerator of the same fraction is increased by 3 and the denominator is increased by I the fraction becomes $\frac{3}{4}$. What is the original fraction?
(a) Data inadequate (b) $\frac{2}{7}$
(c) $\frac{4}{7}$ (d) $\frac{3}{7}$
(e) None of these

75. If $2x + 3y = 26$; $2y + z = 19$ and $x + 2z = 29$, what is the value of $x + y + z$?
(a) 18 (b) 32
(c) 26 (d) 22
(e) None of these

76. If the sum of a number and its square is 182, what is the number?
(a) 15 (b) 26
(c) 28 (d) 91
(e) None of these

77. A certain number of tennis balls were purchased for ₹ 450. Five more balls could have been purchased for the same amount if each ball was cheaper by ₹ 15. Find the number of balls purchased.
(a) 15 (b) 20
(c) 10 (d) 25
(e) None of these

78. What will be the value of $n^4 - 10n^3 + 36n^2 - 49n + 24$,if $n = 1$?
(a) 21 (b) 2
(c) 1 (d) 22
(e) None of these

79. Out of total number of students in a college 12% are interested in sports. $\frac{3}{4}$th of the total number of students are interested in dancing. 10% of the total number of students are interested in singing and the remaining 15 students are not interested in any of the activities. What is the total number of students in the college?
(a) 450 (b) 500
(c) 600 (d) Cannot be determined
(e) None of these

80. The sum of four numbers is 64. If you add 3 to the first number, 3 is subtracted from the second number, the third is multiplied by 3 and the fourth is divided by 3, then all the results are equal. What is the difference between the largest and the smallest of the original numbers?
(a) 32 (b) 27
(c) 21 (d) Cannot be determined
(e) None of these

81. A classroom has equal number of boys and girls. Eight girls left to play Kho-kho, leaving twice as many boy as girls in the classroom. What was the total number of girls and boys present initially?
(a) Cannot be determined (b) 16
(c) 24 (d) 32
(e) None of these

82. The difference between the digits of a two-digit number is one-ninth of the difference between the original number and the number obtained by interchanging positions of the digits. What definitely is the sum of digits of that number?
(a) 5 (b) 14
(c) 12 (d) Data inadequate
(e) None of these

83. The denominator of a fraction is 2 more than thrice its numerator. If the numerator as well as denominator is increased by one, the fraction becomes 1/3. What was the original fraction?
(a) $\frac{4}{13}$ (b) $\frac{3}{11}$
(c) $\frac{5}{13}$ (d) $\frac{5}{11}$
(e) None of these

84. If $2x + y = 15$, $2y + z = 25$ and $2z + x = 26$, what is the value of z?
(a) 4 (b) 7
(c) 9 (d) 12
(e) None of these

85. Which of the following values of P satisfy the inequality $P(P-3) < 4P - 12$?
(a) $P > 4$ or $P \leq 3$ (b) $24 \leq P < 71$
(c) $P > 13; P < 51$ (d) $3 < P < 4$
(e) $P = 4, P = +3$

86. If the ages of P and R are added to twice the age of Q, the total becomes 59. If the ages of Q and R are added to thrice the age of P, the total becomes 68. And if the age of P is added to thrice the age of Q and thrice the age of R, the total becomes 108. What is the age of P?
(a) 15 years (b) 19 years
(c) 17 years (d) 12 years
(e) None of these

87. The product of the ages of Harish and Seema is 240. If twice the age of Seema is more than Harish's age by 4 years, what is Seema's age in years?
(a) 12 years (b) 20 years
(c) 10 years (d) 14 years
(e) Data inadequate

88. What would be the maximum value of Q in the following equation? $5P9 + 3R7 + 2Q8 = 1114$
(a) 8 (b) 7
(c) 5 (d) 4
(e) None of the above

89. Two-fifths of one-fourth of three-sevenths of a number is 15. What is half of that number?
(a) 96 (b) 196
(c) 94 (d) 188
(e) None of these

90. The sum of the digits of a two-digit number is 1 /11 of the sum of the number and the number obtained by interchanging the position of the'digits. What is the difference between the digits of that number?
(a) 3 (b) 2
(c) 6 (d) Data inadequate
(e) None of these

91. If a fraction's numerator is increased by 1 and the denominator is increased by 2 then the fraction becomes $\frac{2}{3}$. But when the numerator is increased by 5 and the denominator is increased by 1 then the fraction becomes $\frac{5}{4}$. What is the value of the original fraction?
(a) $\frac{3}{7}$ (b) $\frac{5}{8}$
(c) $\frac{5}{7}$ (d) $\frac{6}{7}$
(e) None of these

92. In a two-digit number the digit in the unit's place is more than the digit in the ten's place by 2. If the difference between the number and the number obtained by interchanging the digits is 18 what is the original number?
(a) 46 (b) 68
(c) 24 (d) Data inadequate
(e) None of these

93. If $2x + y = 17y$, $2z = 15$ and $x + z = 9$ then what is the value of $4x + 3y + z$?
(a) 41 (b) 43
(c) 55 (d) 45
(e) None of these

94. If the numerator of a fraction is increased by 2 and denominator is increased by 3, the fraction becomes 7/9; and if numerator as well as denominator are decreased by 1 the fraction becomes 4/5. What is the original fraction?
(a) $\frac{13}{16}$ (b) $\frac{9}{11}$
(c) $\frac{5}{6}$ (d) $\frac{17}{21}$
(e) None of these

95. The inequality $3n^2 - 18n + 24 > 0$ gets satisfied for which of the following values of n?
(a) $n < 2$ & $n > 4$ (b) $2 < n < 4$
(c) $n > 2$ (d) $n > 4$
(e) None of these

96. A sum is divided among Rakesh, Suresh and Mohan. If the difference between the shares of Rakesh and Mohan is ₹7000 and between those of Suresh and Mohan is ₹ 3000, what was the sum?
(a) ₹30,000 (b) ₹ 13,000
(c) ₹ 10,000 (d) Cannot be determined
(e) None of these

97. Three-fifths of a number is 30 more than 50 per cent of that number. What is 80 per cent of that number?
(a) 300 (b) 60
(c) 240 (d) Cannot be determined
(e) None of these

98. The difference between a two-digit number and the number obtained by interchanging the position of the digits of the number is 27. What is the difference between the digits of that number?
(a) 2 (b) 3
(c) 4 (d) Cannot be determined
(e) None of these

99. The sum of the ages of a father and his son is 4 times the age of the son. If the average age of the father and the son is 28 years, what is the son's age?
(a) 14 years (b) 16 years
(c) 12 years (d) Data inadequate
(e) None of these

100. Two-fifths of one-fourth of five-eighths of a number is 6. What is 50 per cent of that number?
(a) 96 (b) 32
(c) 24 (d) 48
(e) None of these

101. The sum of the digits of a two-digit number is $\frac{1}{5}$ of the difference between the number and the number obtained by interchanging the positions of the digits. What definitely is the difference between the digits of that number?
(a) 5 (b) 9
(c) 7 (d) Data inadequate
(e) None of these

102. Ashok gave 40 per cent of the amount he had to Jayant. Jayant in turn gave one-fourth of what he received from Ashok to Prakash. After paying ₹ 200 to the taxi-driver out of the amount he got from Jayant, Prakash now has ₹ 600 left with him. How much amount did Ashok have?
(a) ₹ 1,200 (b) ₹ 4,000
(c) ₹ 8,000 (d) Data inadequate
(e) None of these

103. What should be the maximum value of Q in the following equation?
$5P9 - 7Q2 + 9R6 = 823$
(a) 7 (b) 5
(c) 9 (d) 6
(e) None of these

104. The difference between a two-digit number and the number obtained by interchanging the position of the digits of that number is 54. What is the sum of the digits of that number?
(a) 6 (b) 9
(c) 15 (d) Data inadequate
(e) None of these

105. The product of two numbers is 192 and the sum of these two numbers is 28. What is the smaller of these two numbers?
(a) 16 (b) 14
(c) 12 (d) 18
(e) None of these

106. The age of Mr. Ramesh is four times the age of his son. After ten years the age of Mr. Ramesh will be only twice the age of his son. Find the present age of Mr. Ramesh's, son.
(a) 10 years (b) 11 years
(c) 12 years (d) Cannot be determined
(e) None of these

107. In an exercise room some discs of denominations 2 kg and 5 kg are kept for weightlifting. If the total number of discs is 21 and the weight of all the discs of 5 kg is equal to the weight of all the discs of 2 kg, find the weight of all the discs together.
(a) 80 kg (b) 90 kg
(c) 56 kg (d) Cannot be determined
(e) None of these

108. If the number of barrels of oil consumed doubles in a 10-year period and if B barrels were consumed in the year 1940, what multiple of B will be consumed in the year 2000?
(a) 64 (b) 60
(c) 12 (d) 32
(e) None of these

109. The sum of three consecutive even numbers is 14 less than one-fourth of 176. What is the middle number?
(a) 8 (b) 10
(c) 6 (d) Data inadequate
(e) None of these

110. The price of four tables and seven chairs is ₹ 12,090. **Approximately,** what will be the price of twelve tables and twenty-one chairs?
(a) ₹32,000 (b) ₹46,000
(c) ₹38,000 (d) ₹36,000
(e) ₹39,000

111. If the price of 253 pencils is ₹ 4263.05, what will be the **approximate** value of 39 such pencils'?
(a) ₹650 (b) ₹550
(c) ₹450 (d) ₹700
(e) ₹750

112. Sundari, Kusu and Jyoti took two tests each. Sundari secured $\frac{24}{60}$ marks in the first test and $\frac{32}{40}$ marks in the second test. Kusu secured $\frac{35}{70}$ marks in the first test and $\frac{54}{60}$ marks in the second test. Jyoti secured $\frac{27}{90}$ marks in the first test and $\frac{45}{50}$ marks in the second test. Who among them did register maximum progress?
(a) Only Sundari (b) Only Kusu
(c) Only Jyoti (d) Both Sundari and Kusu
(e) Both Kusu and Jyoti

CHAPTER 3

Average

AVERAGE

'Average' is a very simple but effective way of representing an entire group by a single value.

Average or Mean $= \dfrac{\text{Sum of given quantities}}{\text{Number of quantities}}$

To calculate the sum of quantities, they should be in the same unit.

Shortcut Approach

If X is the average of $x_1, x_2, x_3 \ldots x_n$ then

(a) The average of $x_1 + a, x_2 + a, x_3 + a, \ldots x_n + a$ is $X + a$.

(b) The average of $x_1 - a, x_2 - a, x_3 - a \ldots x_n - a$ is $X - a$

(c) The average of $ax_1, ax_2, \ldots ax_n$ is aX, provied $a \neq 0$

(d) The average of $\dfrac{x_1}{a}, \dfrac{x_2}{a}, \dfrac{x_3}{a}, \ldots \dfrac{x_n}{a}$ is $\dfrac{x}{a}$, provided $a \neq 0$

EXAMPLE 1. The average of the first nine prime numbers is:

(a) 9 **(b)** 11

(c) $11\frac{1}{9}$ **(d)** $11\frac{2}{9}$

Sol. (c) Average

$= \left(\dfrac{2+3+5+7+11+13+17+19+23}{9}\right) = \dfrac{100}{9} = 11\dfrac{1}{9}.$

EXAMPLE 2. In three numbers, the first is twice the second and thrice the third. If the average of these three numbers is 44, then the first number is :

Sol. (a) Let the three numbers be x, y and z

Therefore, x = 2y = 3z,

$y = \dfrac{x}{2}$ and $z = \dfrac{x}{3}$

Now, $\dfrac{x + \frac{x}{2} + \frac{x}{3}}{3} = 44$

or $\dfrac{11x}{18} = 44$ or $x = 72$

EXAMPLE 3. The average of five consecutive odd numbers is 61. What is the difference between the highest and lowest numbers?

(a) 2 **(b)** 5

(c) 8 **(d)** Cannot be determined

Sol. (c) Let the numbers be x, x + 2, x + 4, x + 6 and x + 8.

Then, $\dfrac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5} = 61$

or 5x + 20 = 305 or x = 57.

So, required difference = (57 + 8) – 57 = 8.

Average of a group consisting two different groups when their averages are known :

Let Group A contains m quantities and their average is a and Group B contains n quantities and their average is b, then **average of group C** containing m + n quantities

$= \dfrac{ma + mb}{m + n}$

EXAMPLE 4. There are 30 student in a class. The average age of the first 10 students is 12.5 years. The average age of the next 20 students is 13.1 years. The average age of the whole class is :

(a) 12.5 years **(b)** 12.7 years

(c) 12.8 years **(d)** 12.9 years

Sol. (d) Total age of 10 students = 12.5 × 10 = 125 years

Total age of 20 students = 13.1 × 20 = 262 years

Average age of 30 students $= \dfrac{125 + 262}{30} = 12.9$ years

EXAMPLE 5. The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of the number of boys to the number of girls in the class is

(a) 1 : 2 **(b)** 2 : 3

(c) 3 : 4 **(d)** 3 : 5

Sol. (b) Let the number of boys in a class be x.

Let the number of girls in a class be y.

$\therefore$ Sum of the ages of the boys = 16.4 x

Sum of the ages of the girls = 15.4 y

$\therefore$ 15.8 (x + y) = 16.4 x + 15.4 y

$\Rightarrow 0.6x = 0.4y \Rightarrow \dfrac{x}{y} = \dfrac{2}{3}$

$\therefore$ Required ratio = 2 : 3

WEIGHTED AVERAGE

If we have two or more groups of members whose individual averages are known, then combined average of all the members of all the groups is known as weighted average. Thus if there are k groups having member of number $n_1, n_2, n_3, \ldots\ldots n_k$ with averages $A_1, A_2, A_3 \ldots\ldots A_k$ respectively then weighted average.

$$A_w = \frac{n_1A_1 + n_2A_2 + n_3A_3 + \ldots + n_kA_k}{n_1 + n_2 + n_3 + \ldots + n_k}$$

EXAMPLE 6. The average monthly expenditure of a family was ₹2200 during the first 3 months; ₹2250 during the next 4 months and ₹3120 during the last 5 months of a year. If the total saving during the year were ₹1260, then the average monthly income was

Sol: Total annual income

$= 3 \times 2200 + 4 \times 2250 + 5 \times 3120 + 1260$

$= 6600 + 9000 + 15600 + 1260 = 32460$

$\therefore$ Average monthly income $= \dfrac{32460}{12} = ₹2705$

EXAMPLE 7. The average monthly expenditure of a family was ₹ 2200 during the first 3 months; ₹ 2250 during the next 4 months and ₹ 3120 during the last 5 months of a year. If the total saving during the year were ₹ 1260, then the average monthly income was

(a) ₹ 2605 **(b) ₹ 2805**

(c) ₹ 2705 **(d) ₹ 2905**

Sol. (c) Total annual income

$= 3 \times 2200 + 4 \times 2250 + 5 \times 3120 + 1260$

$= 6600 + 9000 + 15600 + 1260 = 32460$

$\therefore$ Average monthly income $= \dfrac{32460}{12} = ₹2705$

If the average of m observations is a and the average of n observations taken out of m is b, then

Average of rest of the observations $= \dfrac{ma - nb}{m - n}$

EXAMPLE 8. A man bought 20 cows in ₹ 200000. If the average cost of 12 cows is ₹ 12500, then what wil be the average cost of remaining cows?

Sol. Average cost of 20 cows $= \dfrac{200000}{20} = ₹\ 10000$

Here, m = 20, n = 12, a = 10000, b = 12500

Average cost of remaining (20 – 12) cows $= \dfrac{20 \times 10000 - 12 \times 12500}{20 - 12}$

$= \dfrac{200000 - 150000}{8} = \dfrac{50000}{8} = ₹\,6250$

EXAMPLE 9. The mean of the marks secured by 25 students of section A of class X is 47, that of 35 students of section B is 51 and that of 30 students of section C is 53. Find the combined mean of the marks of students of three sections of class X.

Sol. Mean of the marks of 25 students of XA = 47

$\therefore$ Sum of the marks of 25 students $= 25 \times 47 = 1175$ ....(i)

Mean of the marks of 35 students of XB = 51

$\therefore$ Sum of the marks of 35 students $= 35 \times 51 = 1785$ ....(ii)

Mean of the marks of 30 students of XC = 53

$\therefore$ Sum of the marks of 30 students $= 30 \times 53 = 1590$ ....(iii)

Adding (i), (ii) and (iii)

Sum of the marks of (25 + 35 + 30) i.e., 90 students

$= 1175 + 1785 + 1590 = 4550$

Thus the combined mean of the marks of students of three sections $= \dfrac{4550}{90} = 50.56$

☞ Shortcut Approach

If, in a group, one or more new quantities are added or excluded, then the new quantity or sum of added or excluded quantities = [Change in no. of quantities × original average] ± [change in average × final no. of quantities]

Take +ve sign if quantities added and take –ve sign if quantities removed.

EXAMPLE 10. The average weight of 29 students in a class is 48 kg. If the weight of the teacher is included, the average weight rises by 500 g. Find the weight of the teacher.

Sol. Here, weight of the teacher is added and final average of the group increases.

$\therefore$ Change in average is (+)ve, using the formula

Sum of the quantities added

$$= \left(\begin{matrix}\text{Change in no. of quantities} \\ \times \\ \text{Original average}\end{matrix}\right) + \left(\begin{matrix}\text{Change in average} \\ \times \\ \text{Final no. of quantities}\end{matrix}\right)$$

$\Rightarrow$ weight of teacher $= (1 \times 48) + (0.5 \times 30) = 63$ kg.

$\therefore$ weight of teacher is 63 kg.

EXAMPLE 11. The average age of 40 students in a class is 15 years. When 10 new students are admitted, the average is increased by 0.2 year. Find the average age of the new students.

Sol. Here, 10 new students are admitted.

$\therefore$ Change in average is +ve. Using the formula

Sum of the quantities added

$$= \left(\begin{matrix}\text{Change in no. of quantities} \\ \times \\ \text{Original average}\end{matrix}\right) + \left(\begin{matrix}\text{Change in average} \\ \times \\ \text{Final no. of quantities}\end{matrix}\right)$$

$\Rightarrow$ Sum of the weight of 10 new students admitted

$= (10 \times 15) + (0.2 \times 50) = 160$ kg.

$\therefore$ Average age of 10 new students $= \dfrac{s_a}{n_a} = \dfrac{160}{10} = 16$

$\therefore$ Average age of 10 new students is 16 years.

AVERAGE SPEED IF EQUAL DISTANCES ARE TRAVELLED BY TWO DIFFERENT SPEEDS

If a car travels at a speed S_1 from A to B and at a speed S_2 from B to A. Then

$$\boxed{\text{Average speed} = \frac{2S_1 . S_2}{S_1 + S_2}}$$

The above formula can be found out as follows:

If distance between A and B is d, then

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2d}{\frac{d}{S_1} + \frac{d}{S_2}} = \frac{2}{\frac{1}{S_1} + \frac{1}{S_2}} = \frac{2S_1 . S_2}{S_2 + S_1}$$

EXAMPLE 12. A motorist travels to a place 150 km away at an average speed of 50 km/hr and returns at 30 km/hr. His average speed for the whole journey in km/hr is :

(a) 35 (b) 37

(c) 37.5 (d) 40

Sol. (c) Average speed

$$= \frac{2xy}{x+y} \text{km/hr} = \left(\frac{2\times 50\times 30}{50+30}\right)\text{km/hr} = 37.5 \text{ km/hr.}$$

AVERAGE SPEED IF EQUAL DISTANCES ARE TRAVELLED BY THREE DIFFERENT SPEEDS

Average speed $\frac{3xyz}{xy+yz+zx}$

Where x, y and 2 are these different speeds.

EXAMPLE 13. A train covers the first 160 km at a speed of 120 km/h, another 160 km at 140 km/h and the last 160 km at 80 km/h. Find the average speed of the train for the entire journey.

Sol. Average speed $= \frac{3xyz}{xy+yz+zx}$

$$= \frac{3\times 120\times 140\times 80}{120\times 140+140\times 80+80\times 120}$$

$$= \frac{360\times 140\times 80}{16800+11200+9600} = \frac{4032000}{37600}$$

$$= 107\frac{11}{47} \text{ km/h}$$

REMEMBER

★ Average of first n natural numbers $= \frac{(n+1)}{2}$

★ Average of first n consecutive × 2 even numbers = (n + 1)

★ Average of first n consecutive × 2 odd numbers = n

★ Average of consecutive numbers $= \frac{\text{First number} + \text{Last number}}{2}$

★ Average of 1 to n odd numbers $= \frac{\text{Last odd number} + 1}{2}$

★ Average of 1 and n even numbers $= \frac{\text{Last even number} + 2}{2}$

★ Average of squares of first n natural numbers $= \frac{(n+1)(2n+1)}{6}$

★ Average of the cubes of first n natural numbers $= \frac{n(n+1)^2}{4}$

★ Average of n multiples of any number $= \frac{\text{Number}\times(n+1)}{2}$

★ If n is odd: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the middle number.

★ If n is even: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the average of the middle two numbers.

★ The average of squares of first n consecutive even number is $\frac{2(n+1)(2n+1)}{3}$.

★ The average of squares of consecutive even numbers till n is $\frac{(n+1)(n+2)}{3}$.

★ The average of square of consecutive odd numbers till n is $\frac{n(n+2)}{3}$.

★ If the average of n consecutive numbers is m, then the difference between the smallest and the largest number is $2(n-1)$.

EXAMPLE 14. Find the A.M. of the sequence 1, 2, 3, ..., 100.

Sol. We have sum of first n natural numbers $= \frac{n}{2}(n+1)$,

here n = 100

$$\Rightarrow \text{Sum} = \frac{100}{2}\times 101 = 101\times 50$$

$$\Rightarrow \text{AM} = \frac{\text{Sum}}{100} = \frac{101\times 50}{100} = 50.5$$

EXAMPLE 15. A sequence of seven consecutive integers is given. The average of the first five given integers is n. Find the average of all the seven integers.

Sol. Let the seven consecutive integers be

$x, x+1, x+2, ..., x+6$

The sum of the first five is

$x+x+1+x+2+x+3+x+4 = 5x+10$

The average of these five is $\frac{5x+10}{5} = x+2 = n$

The average of the seven will be

$$\frac{5x+10+x+5+x+6}{7} = \frac{7x+21}{7} = x+3$$

As $x+2 = n$, so $x+3 = x+2+1 = n+1$

EXAMPLE 16. The average of 11 results is 50. If the average of first six result is 49 and that of last six results is 52, find the sixth result.

Sol. Average of 11 results

1 2 3 4 5 6 7 8 9 10 11

Average of last 6 results = 52

Average of first 6 results = 49

It is quite obvious that the sixth result is included twice, once in the first six results and second in the last six results.

∴ Value of the sixth result = (Sum of first six results) + (Sum of last six results) – Sum of 11 results

$= 6\times 49 + 6\times 11\times 50 = 56$

EXAMPLE 17. Typist A can type a sheet in 5 minutes, typist B in 6 minutes and typist C in 8 minutes. The average number of sheets typed per hour per typist is .... sheets.

Sol. A types 12 sheets in 1 hour

B types 10 sheets in 1 hour

C types 7.5 sheets in 1 hour

Average number of sheets types per hour per typist

$$= \frac{12+10+7.5}{3} = \frac{29.5}{3} = 9.83$$

If a person or a motor car covers three equal distances at the speed of x km/h, y km/h and z km/h, respectively, then for the entire journey average speed of the person or motor car is $\left(\frac{3xzy}{xy + yz + zx}\right)$ km/h.

Shortcut Approach

- If average of n observations is a but the average becomes b when one observation is eliminated, then
  Value of eliminated observation = n (a – b) + b
- If average of n observations is a but the average becomes b when a new observation is added, then
  Value of added observation = n (b – a) + b
  We have n observations out of which some observations (a_1, a_2, a_3, ...) are replaced by some other new observations in this way, if the average increases or decreases by b, then
  Value of new observations = a ± nb
  where, $a = a_1 + a_2 + a_3 + ...$

NOTE : *In this formula, the signs of '+' and '–' depend upon the increment or decrement in the average.*

EXAMPLE 18. The average run scored by a batsman in 20 innings is 32. After 21st innings, the runs average becomes 34. How much runs does the batsman score in his 21st innings?

Sol. Runs scored in 20 innings = 20 × 32 = 640

Runs scored in 21 innings = 21 × 34 = 714

Runs scored in the 21st innings = 714 – 640 = 74

SHORTCUT METHOD

Here, n = 20; Initial average, a = 32; Last average, (b) = 34

∴ Runs scored in the 21st innings = n (b – a) + b

= 20 (34 – 32) + 34 = 20 × 2 + 34 = 74

EXAMPLE 19. The average weight of 3 women is increased by 4 kg, when one of them whose weight is 100 kg,is replaced by another woman. What is the weight of the new woman?

Sol. Total weight increased = 4 × 3 = 12 kg

∴ Weight of new woman = 100 + 12 = 112 kg

SHORTCUT METHOD

Here, n = 3, a = 100 kg, b = 4 kg

∴ Weight of new woman = a + nb = 100 + 3 × 4 = 112 kg

[here, '+' sign has been taken as average increases in this case.]

EXERCISE

1. The average of two numbers is XY. If one number is X, then the other number is
   (a) Y (b) $\frac{Y}{2}$
   (c) $2XY - X$ (d) $X(Y-1)$
   (e) None of these
2. The average of four consecutive odd numbers is 12.Which is the lowest odd number?
   (a) 9 (b) 3
   (c) 5 (d) Cannot be determined
   (e) None of these
3. The average marks scored by Ganesh in English, Science, Mathematics and History is less than 15 from that scored by him in English, History, Geography and Mathematics. What is the difference of marks in Science and Geography scored by him?
   (a) 40 (b) 50
   (c) 60 (d) Data inadequate
   (e) None of these
4. The average age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average reduces by one year. What is the age of the class teacher?
   (a) 50 years (b) 45 years
   (c) 40 years (d) Data inadequate
   (e) None of these
5. There are 50 boys in a class. Their average weight is 45 kg. When one boy leaves the class, the average reduces by 100 g. Find the weight of the boy who left the class.
   (a) 40.9 kg (b) 42.9 kg
   (c) 49.9 kg (d) 39.9 kg
   (e) None of these
6. The average age of 36 students in a group is 14 years. When teacher's age is included to it, the average increases by one. What is the teacher's age in years?
   (a) 31 (b) 36
   (c) 51 (d) cannot be determined
   (e) None of these
7. The average weight of 8 persons increases by 1.5 kg. If a person weighing 65 kg is replaced by a new person, what could be the weight of the new person?
   (a) 76 kg (b). 77 kg
   (c) 76.5 kg (d) Data inadequate
   (e) None of these
8. N number of persons decide to raise ₹ 3 lakhs by equal contributions from each. If they contributed ₹ 50 each extra, the contribution would be ₹ 3.25 lakhs. How many persons are there?
   (a) 600 (b) 400
   (c) 450 (d) Cannot be determined
   (e) None of these
9. The average of the first and the second of three numbers is 15 more than the average of the second and the third of these numbers. What is the difference between the first and the third of these three numbers?
   (a) 15 (b) 45
   (c) 60 (d) Data inadequate
   (e) None of these
10. The average of Suresh's marks in English and History is 55. His average of marks in English and Science is 65. What is the difference between the marks which he obtained in History and Science?
    (a) 40 (b) 60
    (c) 20 (d) Data inadequate
    (e) None of the above
11. The average of four consecutive even numbers is one-fourth of the sum of these numbers. What is the difference between the first and the last number?
    (a) 4 (b) 6
    (c) 2 (d) Cannot be determind
    (e) None of these
12. The average of three consecutive odd numbers is 14 more than one-third of the first of these numbers, what is the last of these numbers?
    (a) 17 (b) 19
    (c) 15 (d) Data inadequate
    (e) None of these
13. A mathematics teacher tabulated the marks secured by 35 students of 8th class. The average of their marks was 72. If the marks secured by Reema was written as 36 instead of 86 then find the correct average marks upto two decimal places.
    (a) 73.41 (b) 74.31
    (c) 72.43 (d) 73.43
    (e) Cannot be determined
14. The average of five consecutive odd numbers is 61. What is the difference between the highest and the lowest number?
    (a) 8 (b) 2
    (c) 5 (d) Cannot be determined
    (e) None of these
15. In a coconut grove, $(x + 2)$ trees yield 60 nuts per year, x trees yield 120 nuts per year and $(x - 2)$ trees yield 180 nuts per year. If the average yield per year per tree be 100, find x.
    (a) 3 (b) 4
    (c) 5 (d) 6
    (e) None of the above
16. 30 pens and 75 pencils were purchased for ₹ 510. If the average price of a pencil was ₹ 2.00, find the average price of a pen.
    (a) ₹ 10 (b) ₹ 11
    (c) ₹ 12 (d) cannot be determined
    (e) None of the above
17. A school has 4 section of Chemistry in Class X having 40, 35, 45 and 42 students. The mean marks obtained in Chemistry test are 50, 60, 55 and 45 respectively for the 4 sections. Determine the overall average of marks per student
    (a) 50.25 (b) 52.25
    (c) 51.25 (d) 53.25
    (e) None of the above

18. The average of 20 numbers is zero. Of them, at the most, how many may be greater than zero?
(a) 0 (b) 1
(c) 10 (d) 19
(e) None of the above

19. The average of six numbers is 3.95. The average of two of them is 3.4, while the average of the other two is 3.85. What is the average of the remaining two numbers?
(a) 4.5 (b) 4.6
(c) 4.7 (d) 4.8
(e) None of the above

20. Nine persons went to a hotel for taking their meals. Eight of them spent ₹ 12 each on their meals and the ninth spend ₹ 8 more than the average expenditure of all the nine. What was the total money spent by them?
(a) ₹ 115 (b) ₹ 117
(c) ₹ 119 (d) ₹ 122
(e) None of the above

21. The average age of A and B is 20 years. If C were to replace A, the average would be 19 and if C were to replace B, the averge would be 21. What are the age of A, B and C?
(a) 22, 18, 20 (b) 20, 20, 18
(c) 18, 22, 20 (d) 21,20,19
(e) None of the above

22. 3 years ago the average age of a family of 5 members was 17 years. With the birth of a new baby, the average age of six members remains the same even today. Find the age of the new baby.
(a) 1 year (b) 2 years
(c) $1\frac{1}{2}$ years (d) cannot be determined
(e) None of the above

23. The average age of a group of person going for picnic is 16 years. Twenty new persons with an average age of 15 years join the group on the spot due to which their average becomes 15.5 years. Find the number of persons initially going for picnic.
(a) 20 (b) 18
(c) 22 (d) 19
(e) None of the above

24. A batsman in his 12th innings makes a score of 65 and thereby increases his average by 2 runs. What is his average after the 12th innings if he had never been 'not out'?
(a) 42 (b) 43
(c) 44 (d) 45
(e) None of the above

25. A pupil's marks were wrongly entered as 83 instead of 63. Due to that the average marks for the class got increased by half. The number of pupils in the class is:
(a) 10 (b) 20
(c) 40 (d) 73
(e) None of the above

26. In the first 10 overs of a cricket game, the run rate was only 3.2. What should be the run rate in the remaining 40 overs to reach a target of 282 runs ?
(a) 6.25 (b) 6.50
(c) 6.75 (d) 7.00
(e) None of the above

27. The average number of printing error per page in a book of 512 pages is 4. If the total number of printing error in the first 302 pages is 1,208, the average number of printing errors per page in the remaining pages is
(a) 0 (b) 4
(c) 840 (d) 90
(e) None of the above

28. The average attendance in a school for the first 4 days of the week is 30 and for the first 5 days of the week is 32. The attendance on the fifth day is
(a) 32 (b) 40
(c) 38 (d) 36
(e) None of the above

29. The average expenditure of a labourer for 6 months was ₹ 85 and he fell into debt. In the next 4 months by reducing his monthly expenses to ₹ 60 he not only cleared off his debt but also saved ₹ 30. His monthly income is
(a) ₹ 70 (b) ₹ 72
(c) ₹ 75 (d) ₹ 78
(e) None of the above

30. The average of a batsman for 40 innings is 50 runs. His highest score exceeds his lowest score by 172 runs. If these two innings are excluded, his average drops by 2 runs. Find his highest score.
(a) 172 (b) 173
(c) 174 (d) 175
(e) None of the above

31. Last year, a Home Appliance Store sold an average (arithmetic mean) of 42 microwave ovens per month. In the first 10 months of this year, the store has sold an average (arithmetic mean) of only 20 microwave ovens per month. What was the average number of microwave ovens sold per month during the entire 22 months period ?
(a) 21 (b) 30
(c) 31 (d) 32
(e) None of the above

32. The captain of a cricket team of 11 players is 25 years old and the wicket-keeper is 3 years older. If the age of these two players are replaced by that of another two players, the average of the cricket team drops by 2 years. Find the average age of these two players.
(a) 15 years (b) 15.5 years
(c) 17 years (d) 16.5 years
(e) None of the above

33. A batsman makes a score of 87 runs in the 17th inning and thus increases his average by 3. Find his average after 17th inning.
(a) 36 (b) 39
(c) 42 (d) 45
(e) None of the above

34. Nine men went to a hotel. 8 of them spent ₹ 3 each over their meals and the ninth spent ₹ 2 more than the average expenditure of all the nine. The total money spent by all of them was
(a) ₹ 26 (b) ₹ 40
(c) ₹ 29.25 (d) ₹ 27
(e) None of the above

35. The mean of 30 values was 150. It was detected on rechecking that one value 165 was wrongly copied as 135 for the computation of the mean. Find the correct mean.
(a) 151 (b) 149
(c) 152 (d) 148
(e) None of the above

36. A cricketer whose bowling average is 12.4 runs per wicket takes 5 wickets for 26 runs and thereby decreases his average by 0.4. The number of wickets taken by him till the last match was:
(a) 64 (b) 72
(c) 80 (d) 85
(e) None of the above

37. In an examination, a pupil's average marks were 63 per paper. If he had obtained 20 more marks for his Geography paper and 2 more marks for his History paper, his average per paper would have been 65. How many papers were there in the examination?
(a) 8 (b) 9
(c) 10 (d) 11
(e) None of the above

38. A car owner buys petrol at ₹ 7.50, ₹ 8.00 and ₹ 8.50 per litre for three successive years. What approximately is his average cost per litre of petrol if he spends ₹ 4000 each year?
(a) ₹ 8 (b) ₹ 9
(c) ₹ 7.98 (d) ₹ 8.50
(e) None of the above

39 A batsman has scored an average of 46 runs for a certain number of innings played in England. When he came back to India, he played another two test matches of two innings each and scored at an average of 55 runs. For the innings in England and in India taken together, he has improved his average by 2 runs over the matches played in England. Find the number of innings played in England.
(a) 12 (b) 13
(c) 14 (d) 15
(e) None of these

40 There were 35 students in a hostel. Due to the admission of 7 new students, the expenses of mess were increased by ₹ 42 per day while the average expenditure per head diminished by ₹ 1. What was the original expenditure of the mess?
(a) ₹ 400 (b) ₹ 420
(c) ₹ 445 (d) ₹ 465
(e) None of the above

41. A family consists of grandparents, parents and three grandchildren. The average age of the grandparents is 67 years, that of the parents is 35 years and that of the grandchildren is 6 years. What is the average age of the family?
(a) $28\frac{4}{7}$ years (b) $31\frac{5}{7}$ years
(c) $32\frac{1}{7}$ years (d) $27\frac{1}{2}$ years
(e) None of the above

42. In Arun's opinion, his weight is greater than 65 kg but less than 72 kg. His brother does not agree with Arun and he thinks that Arun's weight is greater than 60 kg but less than 70 kg. His mother's view is that his weight cannot be greater than 68 kg. If all of them are correct in their estimation, what is the average of different probable weights of Arun?
(a) 67 kg (b) 68 kg
(c) 69 kg (d) 66.5 kg
(e) None of the above

43. The average age of a board of 8 functional directors in a company is the same as it was 3 years ago, a younger man having been substituted for one of the directors. How much younger was the new man than the director whose place he took.
(a) 24 years (b) 26 years
(c) 28 years (d) 27 years
(e) None of the above

44. A batsman makes a scores of 98 runs in his 19th inning and thus increases his average by 4. What is his average after 19th inning ?
(a) 22 (b) 24
(c) 28 (d) 26
(e) None of the above

45. The average weight of 45 students in a class is 52 kg. 5 of them whose average weight is 48 kg leave the class and other 5 students whose average weight is 54 kg join the class. What is the new average weight (in kg) of the class ?
(a) $51\frac{1}{3}$ (b) $52\frac{2}{3}$
(c) $52\frac{1}{3}$ (d) 43.42
(e) None of the above

46. The average of 10 numbers is 40.2. Later it is found that two numbers have been wrongly copied. The first is 18 greater than the actual number and the second number added is 13 instead of 31. Find the correct average.
(a) 40.2 (b) 40.4
(c) 40.6 (d) 40.8
(e) None of the above

CHAPTER 4

Percentage

PER CENT

The word "per cent" is derived from the latin words "per centum", which means "per hundred".

A **percentage** is a fraction with denominator hundred.

It is denoted by the symbol %.

Numerator of the fraction is called the **rate per cent**.

VALUE OF PERCENTAGE :

Value of percentage always depends on the quantity to which it refers.

Consider the statement :

"65% of the students in this class are boys". From the context, it is understood that boys form 65% of the total number of students in the class. To know the value of 65% of the total number of students in the class, the value of the total number of boys student should be known.

If the total number of students is 200, then, the number of boys

$= \frac{200 \times 65}{100} = 130$; It can also be written as $(200) \times (0.65) = 130$.

If the total number of students is 500, then the number of boys

$$= \frac{500 \times 65}{100} = 325$$

NOTE *that the expressions 6%, 63%, 72%, 155% etc. do not have any value to themselves. Their values depend on the quantities to which they refer.*

Some Quick Results:

5% of a number $= \frac{\text{Number}}{20}$, $\quad$ 10% of a number $= \frac{\text{Number}}{10}$

$12\frac{1}{2}\%$ of a number $= \frac{\text{Number}}{8}$, 20% of a number $= \frac{\text{Number}}{5}$

25% of a number $= \frac{\text{Number}}{4}$, $\quad$ 50% of a number $= \frac{\text{Number}}{2}$

EXAMPLE 1. 20% of 300 = ?

Sol. According to the formula,

$$20\% \text{ of } 300 = 300 \times \frac{20}{100} = 3 \times 20 = 60$$

SHORTCUTS METHOD

Here, number = 300

$$20\% \text{ of number} = \frac{\text{Number}}{5} = \frac{300}{5} = 60$$

To express the fraction equivalent to % :

Express the fraction with the denominator 100, then the numerator is the answer.

EXAMPLE 2. Express the fraction $\frac{11}{12}$ into the per cent.

Sol. $\frac{11}{12} = \frac{\frac{11}{12} \times 100}{100} = \frac{91\frac{2}{3}}{100} = 91\frac{2}{3}\%$

To express % equivalent to fraction :

$$\mathbf{a\% = \frac{a}{100}}$$

EXAMPLE 3. Express $45\frac{5}{6}\%$ into fraction.

Sol. $45\frac{5}{6}\% = \frac{45\frac{5}{6}}{100} = \frac{275}{6 \times 100} = \frac{11}{24}$.

Fractional Equivalents of % (Percentage)	
$1\% = \frac{1}{100}$	$33\frac{1}{3}\% = \frac{1}{3}$
$2\% = \frac{1}{50}$	$40\% = \frac{2}{5}$
$4\% = \frac{1}{25}$	$50\% = \frac{1}{2}$
$5\% = \frac{1}{20}$	$66\frac{2}{3}\% = \frac{2}{3}$
$6\frac{1}{4}\% = \frac{1}{16}$	$60\% = \frac{3}{5}$
$10\% = \frac{1}{10}$	$75\% = \frac{3}{4}$
$11\frac{1}{3}\% = \frac{17}{150}$	$80\% = \frac{4}{5}$

$12\frac{1}{2}\% = \frac{1}{8}$	$96\% = \frac{24}{25}$
$16\% = \frac{4}{25}$	$100\% = 1$
$16\frac{2}{3}\% = \frac{1}{6}$	$115\% = \frac{23}{20}$
$20\% = \frac{1}{5}$	$133\frac{1}{3}\% = \frac{4}{3}$
$25\% = \frac{1}{4}$	

EXPRESSING ONE QUANTITY AS A PER CENT WITH RESPECT TO OTHER

To express a quantity as a per cent with respect to other quantity following formula is used.

$$\frac{\text{The quantity to be expressed in per cent}}{\text{2nd quantity (in respect of which the per cent has to be obtained)}} \times 100\%$$

Note : *To apply this formula, both the quantities must be in same unit.*

EXAMPLE 4. 60 kg is what per cent of 240 kg?

Sol. According to the formula,

Required percentage

$$= \frac{\text{The quantity to be expressed in per cent}}{\text{2nd quantity (in respect of which the per cent has to be obtained)}} \times 100\%$$

$$= \frac{60}{240} \times 100\% = \frac{100}{4}\% = 25\%$$

PERCENTAGE INCREASE OR DECREASE OF A VALUE

$$\boxed{\text{Increase } \% = \frac{\text{Increase value}}{\text{Original value}} \times 100\%}$$

$$\boxed{\text{Decrease } \% = \frac{\text{Decrease value}}{\text{Original value}} \times 100\%}$$

EXAMPLE 5. Rent of the house is increased from ₹ 7000 to ₹ 7700. Express the increase in price as a percentage of the original rent.

Sol. Increase value = ₹ 7700 – ₹ 7000 = ₹ 700

$$\text{Increase } \% = \frac{\text{Increase value}}{\text{Original value}} \times 100\% = \frac{700}{7000} \times 100\%$$

$$= 10\%$$

$\therefore$ Percentage rise = 10 %.

EXAMPLE 6. The cost of a bike last year was ₹19000. Its cost this year is ₹ 17000. Find the per cent decrease in its cost.

Sol. $\%\text{ decrease} = \frac{19000 - 17000}{19000} \times 100\%$

$$= \frac{2000}{19000} \times 100\% = 10.5\,\%.$$

$\therefore$ Per cent decrease = 10.5 %.

Shortcut Approach

When a number x is increased or decreased by y%, then the new number will be $\frac{100 \pm y}{100} \times x$.

NOTE : 1. '+' *sign is used in case of increase.*
2. '–' *sign is used in case of decrease.*

EXAMPLE 7. The monthly income of a person is ₹ 8000. If his income is increased by 20%, then what wil be his new monthly income?

Sol. Monthly income of a person = ₹ 8000

$$\text{Increment in income} = 8000 \times \frac{20}{100} = 1600$$

New income = 8000 + 1600 = ₹ 9600

SHORTCUT METHOD

Here, x = ₹ 8000 and y = 20%

According to the formula,

$$\text{New income} = \frac{100 + 20}{100} \times 8000$$

['+' sign is used for increase in income]

$$= \frac{120}{100} \times 8000 = ₹\ 9600$$

Shortcut Approach

- If x is a% more than y, then y is $\left(\frac{a}{100 + a} \times 100\right)\%$ less than x.
- If x is a% less than y, then y is $\left(\frac{a}{100 - a} \times 100\right)\%$ more than x.

EXAMPLE 8. If income of Ravi is 20% more than that of Ram, then income of Ram is how much per cent less than that of Ravi?

Sol. Let Ram's income be 100.

Then, Ravi's income = 120

$$\therefore \text{Required percentage} = \frac{120 - 100}{120} \times 100\% = \frac{20}{120} \times 100\%$$

$$= 16\frac{2}{3}\%$$

SHORTCUT METHOD

Here, a = 20%

According to the formula,

$$\text{Required percentage} = \left(\frac{a}{100+a}\times 100\right)\%$$

$$= \left(\frac{20}{100+20}\times 100\right)\% = \frac{50}{3}\% = 16\frac{2}{3}\%$$

Shortcut Approach

- If A's income is r % more than that of B, then B's income is less than that of A by

$$\left(\frac{r}{100+r}\times 100\right)\%$$

- If A's income is r % less than that of B, then B's income is more than that of A by

$$\left(\frac{r}{100-r}\times 100\right)\%$$

SHORTCUT METHOD

Tara		Ravi		Meena
100		25		100
		40		
Now, Tara	:	Ravi	:	Meena
500	:	200	:	800

Meena weight 98 compare to Tara's weight

$$= \frac{800}{500}\times 100 = 160\%$$

Shortcut Approach

If A is x % of C and B is y % of C, then A is $\frac{x}{y}\times 100$ % of B.

EXAMPLE 9. A positive number is divided by 5 instead of being multiplied by 5. By what per cent is the result of the required correct value ?

Sol. Let the number be 1, then the correct answer = 5

The incorrect answer that was obtained = $\frac{1}{5}$.

$\therefore$ The required % = $\frac{1}{5\times 5}\times 100 = 4\%$.

Shortcut Approach

- x % of a quantity is taken by the first, y % of the remaining is taken by the second and z % of the remaining is taken by third person. Now, if A is left in the fund, then the initial amount

$$= \frac{A\times 100\times 100\times 100}{(100-x)(100-y)(100-z)} \text{ in the begining.}$$

- x % of a quantity is added. Again, y % of the increased quantity is added. Again z % of the increased quantity is added. Now it becomes A, then the initial amount

$$= \frac{A\times 100\times 100\times 100}{(100+x)(100+y)(100+z)}$$

EXAMPLE 10. 3.5 % income is taken as tax and 12.5 % of the remaining is saved. This leaves ₹ 4,053 to spend. What is the income ?

Sol. By direct method,

$$\text{Income} = \frac{4053\times 100\times 100}{(100-3.5)(100-12.5)} = ₹\,4800.$$

Shortcut Approach

If the value of a number is first increased by a% and later decreased by a%, then the net effect is always a decrease which is equal to a% of a and is written as $\frac{a^2}{100}\%$ or $\left(\frac{a^2}{10}\right)\%$.

EXAMPLE 11. The salary of a worker is first increased by 5% and then it is decreased by 5%. What is the change in his salary?

Sol. Let the initial salary of the worker be ₹ 100.
Firstly, the salary of worker is increased by 5%.

So, increased salary = 105% of 100 = $\frac{105\times 100}{100}$ = ₹ 105

Now, the salary is reduced by 5% after the increase.

$\therefore$ Reduced salary = 95% of 105 = $\frac{95\times 105}{100} = 99.75$

$\therefore$ Required change is a decrease

i.e., $100 - 99.75 = 0.25$

So, required percentage decrease in salary = $\frac{0.25\times 100}{100}\%$

$= 0.25\%$

SHORTCUT METHOD

Here, a = 5%

We know that, change in the salary of worker is a decrease.

According to the formula,

$$\text{Decrease percentage} = \frac{a^2}{100}\% = \frac{5^2}{100}\% = \frac{25}{100}\% = 0.25\%$$

Shortcut Approach

If the price of a commodity increases by r %, then reduction in consumption, so as not to increase the expenditure is $\left(\frac{r}{100+r}\times 100\right)\%$.

If the price of a commodity decreases by r %, then the increase in consumption so as not to decrease the expenditure is $\left(\frac{r}{100-r}\times 100\right)\%$

EXAMPLE 12. If the price of coal be raised by 20%, then find by how much a householder must reduce his consumption of this commodity so as not to increase his expenditure ?

Sol. Reduction in consumption $=\left(\frac{20}{100+20}\times 100\right)\%$

$$=\left(\frac{20}{120}\times 100\right)\% = 16.67\,\%$$

Shortcut Approach

If due to r% decrease in the price of an item, a person can buy A kg more in ₹ x, then

Actual price of that item = ₹ $\frac{r\,x}{(100-r)\,A}$ per kg

EXAMPLE 13. If due to 10% decrease in the price of sugar, Ram can buy 5 kg more sugar in ₹ 100, then find the find actual price of sugar.

Sol. Let the actual price of sugar be ₹ y.

Amount of sugar bought in ₹ 100 $=\frac{100}{y}$ ...(i)

Price of sugar after 10% decrease = 90% of $y=\frac{9}{10}y$

Now, amount of sugar bought in ₹ 100 $=\frac{1000}{9y}$ ...(ii)

According to the question,

$$\frac{1000}{9y}-\frac{100}{y}=5$$

$$\Rightarrow \quad \frac{1000-900}{9y}=5$$

$$\Rightarrow \quad y=\frac{100}{9\times 5}$$

$$\Rightarrow \quad y=\frac{20}{9}=₹\,2\frac{2}{9}$$

SHORTCUT METHOD

Here, r = 10%, x = ₹ 100 and A = 5 kg

$\therefore$ Actual price of sugar $=\frac{r\times x}{(100-r)\,A}$

$$=\frac{10\times 100}{(100-10)\times 5}=\frac{1000}{450}=₹\,2\frac{2}{9}$$

Shortcut Approach

Population Formula

If the original population of a town is P, and the annual increase is r %, then the population after n years is $P\left(1+\frac{r}{100}\right)^n$ and population before n years $=\frac{P}{\left(1+\frac{r}{100}\right)^n}$

If the annual decrease be r %, then the population after n years is $P\left(1-\frac{r}{100}\right)^n$ and

Population before n years $=\frac{P}{\left(1-\frac{r}{100}\right)^n}$

EXAMPLE 14. The population of a certain town increased at a certain rate per cent per annum. Now it is 456976. Four years ago, it was 390625. What will it be 2 years hence ?

Sol. Suppose the population increases at r% per annum. Then,

$$390625\left(1+\frac{r}{100}\right)^4=456976$$

$$\therefore \quad \left(1+\frac{r}{100}\right)^2=\sqrt{\frac{456976}{390625}}=\frac{676}{625}$$

Population 2 years hence $=456976\left(1+\frac{r}{100}\right)^2$

$$=456976\times\frac{676}{625}=494265 \text{ approximately.}$$

EXAMPLE 15. The population of a city increases at the rate of 4% per annum. There is an additional annual increase of 1 % in the population due to the influx of job seekers. Therefore, the % increase in the population after 2 years will be :

(a) 10 **(b)** 10.25

(c) 10.55 **(d)** 10.75

Sol. (b) The net annual increase = 5%.

Let the initial population be 100.

Then, population after 2 years = 100 × 1.05 × 1.05

= 110.25

Therefore, % increase in population

= (110.25 – 100) = 10.25%

Shortcut Approach

First Increase and then decrease :

If the value is first increased by x % and then decreased by y % then there is $\left(x - y - \frac{xy}{100}\right)$% increase or decrease, according to the + ve or – ve sign obtained respectively.

EXAMPLE 16. A number is increased by 10%. and then it is decreased by 10%. Find the net increase or decrease per cent.

Sol. % change $= 10 - 10 - \frac{10 \times 10}{100} = -1\%$

i.e 1% decrease.

Shortcut Approach

Average percentage rate of change over a period.

$= \frac{(\text{New Value} - \text{Old Value})}{\text{Old Value}} \times \frac{100}{n}$% where n = period.

The percentage error $= \frac{\text{The Error}}{\text{True Value}} \times 100\%$

Successive increase or decrease

If the value is increased **successively by x % and y %** then the final **increase** is given by

$$\left(x + y + \frac{xy}{100}\right)\%$$

If the value is decreased **successively by x % and y %** then the final **decrease** is given by

$$\left(-x - y + \frac{xy}{100}\right)\%$$

EXAMPLE 17. The price of a car is decreased by 10 % and 20 % in two successive years. What per cent of price of a car is decreased after two years ?

Sol. Put x = – 10 and y = – 20, then

$$-10 - 20 + \frac{(-10)(-20)}{100} = -28\%$$

∴ The price of the car decreases by 28%.

Shortcut Approach

Student and Marks

The percentage of passing marks in an examination is x%. If a candidate who scores y marks fails by z marks, then the maximum marks $M = \frac{100(y+z)}{x}$

A candidate scoring x % in an examination fails by 'a' marks, while another candidate who scores y% marks gets 'b' marks more than the minimum required passing marks.

Then the maximum marks $M = \frac{100(a+b)}{y-x}$.

EXAMPLE 18. Vishal requires 40% to pass. If he gets 185 marks, falls short by 15 marks, what was the maximum he could have got ?

Sol. If Vishal has 15 marks more, he could have scored 40% marks.

Now, 15 marks more than 185 is 185 + 15 = 200

Let the maximum marks be x, then

40% of x = 200

$$\Rightarrow \frac{40}{100} \times x = 200$$

$$\Rightarrow x = \frac{200 \times 100}{40} = 500$$

Thus, maximum marks = 500

SHORTCUT METHOD

Maximum marks $= \frac{100(185+15)}{40} = \frac{100 \times 200}{40} = 500$.

EXAMPLE 19. A candidate scores 15% and fails by 30 marks, while another candidate who scores 40% marks, gets 20 marks more than the minimum required marks to pass the examination. Find the maximum marks of the examination.

Sol. Quicker method :

Maximum marks $= \frac{100(30+20)}{40-15} = 200$

Shortcut Approach

If two candidates contested in an election and one candidate got x% of total votes and still lose by y votes, then

Total number of votes casted $= \frac{100 \times y}{100 - 2x}$

EXAMPLE 20. In an election contested by two candidates, one candidate got 30% of total votes and still lost by 500 votes, then find the total number of votes casted.

Sol. Let total number of votes casted be x.

Number of votes got by first candidate $= \frac{30}{100}x.$

Number of votes got by second candidate $= \frac{70}{100}x.$

According to the question,

Difference in votes = 500

$$\Rightarrow \frac{70}{100}x - \frac{30}{100}x = 500$$

$$\Rightarrow \frac{40}{100}x = 500$$

$$\therefore \quad x = \frac{500 \times 100}{40} = 1250$$

SHORTCUT METHOD

Here, x = 30 and y = 500

$$\therefore \text{ Total number of votes} = \frac{100 \times y}{100 - 2x}$$

$$= \frac{500 \times 100}{100 - 2 \times 30}$$

$$= \frac{500 \times 100}{40} = 1250$$

Shortcut Approach

2–dimensional figure and area

If the sides of a triangle, square, rhombus or radius of a circle are increased by a%, its area is increased by $\frac{a(a+200)}{100}\%$

EXAMPLE 21. If the radius of a circle is increased by 10 %, what is the percentage increase in its area ?

Sol. Radius is increased by 10%.

So, Area is increased by $\frac{10(10+200)}{100} = 21\ \%.$

EXAMPLE 22. If the length and width of a rectangular garden were each increased by 20%, then what would be the per cent increase in the area of the graden ?

(a) 20% (b) 24%

(c) 36% (d) 44%

Sol. (d) Let the original length and width of the garden be x and y units, respectively.

Then, the original area = x × y = xy square units

New area = 1.2 x × 1.2y = 1.44xy square units

$$\% \text{ increase in area} = \frac{(1.44xy - xy)}{xy} \times 100 = 44\%$$

EXAMPLE 23. If A's salary is 50 % more than B's, then by what percent B's salary is less than A's salary ?

Sol. By direct method,

B's salary is less than A's salary by

$$\left(\frac{50}{100+50} \times 100\right)\%$$

$$= \frac{50}{150} \times 100\ \% = 33.33\ \%.$$

EXAMPLE 24. Ravi's weight is 25% that of Meena's and 40% that of Tara's. What percentage of Tara's weight is Meena's weight.

Sol. Let Meena's weight be x kg and Tara's weight be y kg.

Then Ravi's weight = 25% of Meena's weight

$$= \frac{25}{100} \times x \quad \text{.... (i)}$$

Also, Ravi's weight = 40% of Tara's weight

$$= \frac{40}{100} \times y \quad \text{.... (ii)}$$

From (i) and (ii), we get

$$\frac{25}{100} \times x = \frac{40}{100} \times y$$

$$\Rightarrow 25x = 40y$$

$$\Rightarrow 5x = 8y \Rightarrow x = \frac{8}{5}y$$

Meena's weight as the percentage of Tara's weight

$$= \frac{x}{y} \times 100 = \frac{\frac{8}{5}y}{y} \times 100 = \frac{8}{5} \times 100 = 160$$

Hence, Meena's weight is 160% of Tara's weight.

EXERCISE

1. The income of a company increases 20% per annum. If its income is ₹ 2664000 in the year 1999 what was its income in the year 1997?
   (a) ₹ 2220000 (b) ₹ 2850000
   (c) ₹ 2121000 (d) ₹ 1855000
   (e) None of these
2. If the growth in production of company *A* from 1994 to 1995 was 25% and that from 1995 to 1996 was 60%, then what percentage growth took place from 1994 to 1996?
   (a) 85% (b) 75%
   (c) 200% (d) 100%
   (e) None of these
3. A shopkeeper employed a servant at a monthly salary of ₹ 1500. In addition to it, he agreed to pay him a commission of 15% on the monthly sale. How much sale in Rupees should the servant do if he wants his monthly income as ₹ 6000?
   (a) ₹ 30000 (b) ₹ 415000
   (c) ₹ 31500 (d) ₹ 50000
   (e) None of these
4. Mr Yadav spends 60% of his monthly salary on consumable items and 50% of the remaining on clothes and transport. He saves the remaining amount. If his savings at the end of the year were ₹ 48456, how much amount per month would he have spent on clothes and transport?
   (a) ₹ 4038 (b) ₹ 8076
   (c) ₹ 9691.20 (d) ₹4845.60
   (e) None of these
5. In a class of 60 children, 30% children can speak only English, 20 % Hindi and English both and the rest of the children can speak only Hindi. How many children can speak Hindi?
   (a) 42 (b) 36
   (c) 30 (d) 48
   (e) None of these
6. Somesh bought a microwave oven and paid 10% less than its original price. He sold it at 30% profit on the price he had paid. What percentage of profit did Somesh earn on the original price?
   (a) 32% (b) 11%
   (c) 20% (d) 17%
   (e) None of these
7. The price of commodity X increases by 40 paise every year, while the price of commodity Y increases by 15 paise every year. If in 1988, the price of commodity X was ₹ 4.20 and that of Y was ₹ 6.30, in which year will commodity X cost 40 paise more than commodity Y?
   (a) 1997 (b) 1998
   (c) 1999 (d) 2000
   (e) None of these
8. Mr X, a businessman, had income in the year 1995 such that he earned a profit of 20% on his investment in the business. In the year 1996 his investment was less by ₹ 5000 but still had the same income (Income = Investment + Profit) as that in 1995. Thus the per cent profit earned in 1996 increased by 6%. What was his investment in 1995?
   (a) ₹ 100000 (b) ₹ 100500
   (c) ₹ 105000 (d) Data inadequate
   (e) None of these
9. The strength of a school increases and decreases every alternate year. It starts with increase by 10% and there-after the percentage of increase/decrease is the same. Which of the following is **definitely true** about the strength of the school in 2001as compared to that in 1996?
   (a) Increase approximately by 2%
   (b) Decrease approximately by 2%
   (c) Increase approximately by 20%
   (d) Decrease approximately by 20%
   (e) None of these
10. Milk contains 5% water. What quantity of pure milk should be added to 10 litres of milk to reduce this to 2% ?
   (a) 5 litres (b) 7 litres
   (c) 15 litres (d) Cannot be determined
   (e) None of these
11. $A = 10\%$ of x, $B = 10\%$ of y, $C = 10\%$ of $x + 10\%$ of y. On the basis of the above equalities, what is true in the following?
   (a) *A* is equal to *B*
   (b) *A* is greater than *B*
   (c) *B* is greater than *A*
   (d) Relation cannot be established between *A* and *B*
   (e) None of these
12. If inflation increases at a rate of 8 p.c.p.a. what will a ₹ 20 article cost at the end of two years?
   (a) Between ₹ 20 and ₹ 21
   (b) Between ₹ 21 and ₹ 22
   (c) Between ₹ 22 and ₹ 23
   (d) Between ₹ 23 and ₹ 24
   (e) None of these
13. In a recent survey 40% houses contained two or more people. Of those houses containing only one person 25% were having only a male. What is the percentage of all houses which contain exactly one female and no males?
   (a) 75 (b) 40
   (c) 15 (d) Cannot be determined
   (e) None of these
14. Sumitra has an average of 56% on her first 7 examinations. How much should she make on her eighth exami nation to obtain an average of 60% on 8 examinations?
   (a) 88% (b) 78%
   (c) 98% (d) Cannot be determined
   (e) None of these

15. If $3x + 7 = x^2 + M = 7x + 5$, what is the value of 120% of M?
(a) 8.90 (b) 9.90
(c) 9.98 (d) Cannot be determined
(e) None of these

16. Four-fifths of three-eighths of a number is 24. What is 250 per cent of that number?
(a) 100 (b) 160
(c) 120 (d) 200
(e) None of these

17. What **approximate** value should come in place of the question mark (?) in the following equation?
159% of 6531.8 + 5.5 × 1015.2 = ? + 5964.9
(a) 10,000 (b) 10,900
(c) 11,000 (d) 10,600
(e) 12,000

18. When 35 per cent of a number is added to another number, the second number increases by its 20 per cent. What is the ratio between the second number and the first number?
(a) 4 : 7 (b) 7 : 4
(c) 8 : 5 (d) Data inadequate
(e) None of these

19. Two-fifths of thirty per cent of one-fourth of a number is 15. What is 20 per cent of that number'?
(a) 90 (b) 150
(c) 100 (d) 120
(e) None of these

20. Jeevan bought an article with 30 per cent discount on the labelled price. He sold the article with 12 per cent profit on the labelled price. What was his per cent profit on the price he bought?
(a) 40 (b) 50
(c) 60 (d) Data inadequate
(e) None of these

21. Naresh's monthly income is 30% more than that of Raghu. Raghu's monthly income is 20% less than that of Vishal. If the difference between the monthly incomes of Naresh and Vishal is ₹ 800, what is the monthly income of Raghu?
(a) ₹ 16,000 (b) ₹ 20,000
(c) ₹ 12,000 (d) Data inadequate
(e) None of these

22. In a certain year, the population of a certain town was 9000. If in the next year the population of males increases by 5% and that of the females by 8% and the total population increases to 9600, then what was the ratio of population of males and females in that given year?
(a) 4:5 (b) 5:4
(c) 2:3 (d) Data inadequate
(e) None of these

23. A petrol pump owner mixed leaded and unleaded petrol in such a way that the mixture contains 10% unleaded petrol. What quantity of leaded petrol should be added to 1 ltr mixture so that the percentage of unleaded petrol becomes 5%?
(a) 1000 ml (b) 900 ml
(c) 1900 ml (d) 1800 ml
(e) None of these

24. Out of a total 85 children playing badminton or table tennis or both, total number of girls in the group is 70% of the total number of boys in the group. The number of boys playing only badminton is 50% of the number of boys and the total number of boys playing badminton is 60% of the total number of boys. The number of children playing only table tennis is 40% of the total number of children and a total of 12 children play badminton and table tennis both. What is the number of girls playing only badminton?
(a) 16 (b) 14
(c) 17 (d) Data inadequate
(e) None of these

25. When the price of a product was increased by 15%, the number sold was decreased by 20%. What was the net effect?
(a) 8% gain (b) 5% loss
(c) 8% loss (d) Cannot be determined
(e) None of these

26. If $\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of a number is 12 then 30 per cent of the number will be
(a) 48 (b) 64
(c) 54 (d) 42
(e) None of these

27. Venkat purchased twenty dozens of toys at the rate of ₹ 375 per dozen. He sold each one of them at the rate of ₹ 33. What was his percentage profit?
(a) 6.5 (b) 5.6
(c) 3.5 (d) 4.5
(e) None of these

28. A speaks truth in 75% and B in 80% cases. In what percentage of cases are they likely to contradict each other when narrating the same incident?
(a) 35 (b) 30
(c) 25 (d) 20
(e) None of these

29. Pradip spends 40 per cent of his monthly income on food items, and 50 per cent of the remaining on clothes and conveyance. He saves one-third of the remaining amount after spending on food, clothes and conveyance. If he saves ₹ 19,200 every year, what is his monthly income?
(a) ₹ 24,000 (b) ₹ 12,000
(c) ₹ 16,000 (d) ₹ 20,000
(e) None of these

30. When 30 per cent of a number is added to another number the second number increases by its 20 per cent. What is the ratio between the first and the second number?
(a) 3 : 2 (b) 2 : 3
(c) 2 : 5 (d) Data inadequate
(e) None of these

31. Rajesh solved 80 per cent of the questions in an examination correctly. If out of 41 questions solved by Rajesh 37 questions are correct and of the remaining questions out of 8 questions 5 questions have been solved by Rajesh correctly then find the total number of questions asked in the examination.
(a) 75 (b) 65
(c) 60 (d) Cannot be determined
(e) None of these

32. 10% of the inhabitants of a village having died of cholera, a panic set in, during which 25% of the remaining inhabitants left the village. The population is then reduced to 4050. Find the number of original inhabitants.
(a) 5000 (b) 6000
(c) 7000 (d) 8000
(e) None of these

33. Chunilal invests 65% in machinery 20% in raw material and still has ₹ 1,305 cash with him. Find his total investment.
(a) ₹ 6,500 (b) ₹ 7, 225
(c) ₹ 8,500 (d) ₹ 7, 395
(e) None of these

34. When the price of a pressure cooker was increased by 15%, the sale of pressure cookers decreased by 15%. What was the net effect on the sales?
(a) 15% decrease (b) no effect
(c) 2.25% increase (d) 2.25% decrease
(e) None of these

35. When the price of a radio was reduced by 20%, its sale increased by 80%. What was the net effect on the sale?
(a) 44% increase (b) 44% decrease
(c) 66% increase (d) 75% increase
(e) None of these

36. When the price of sugar was increased by 32%, a family reduced its consumption in such a way that the expenditure on sugar was only 10% more than before. If 30 kg were consumed per month before, find the new monthly consumption.
(a) 20 kg (b) 25 kg
(c) 30 kg (d) 35 kg
(e) None of these

37. If 10 % of an electricity bill is deducted, ₹ 45 is still to be paid. How much was the bill?
(a) ₹ 50 (b) ₹ 60
(c) ₹ 55 (d) ₹ 70
(e) None of these

38. The ratio of salary of a worker in July to that in June was $2\frac{1}{2} : 2\frac{1}{4}$, by what % the salary of July more than salary of June. Also find by what %, salary of June was less than that of July.
(a) $11\frac{1}{9}\%$ and 10% (b) 10% and $11\frac{1}{9}\%$
(c) Both 10% (d) Both $11\frac{1}{9}\%$
(e) None of these

39. 405 sweets were distributed equally among children in such a way that the number of sweets received by each child is 20% of the total number of children. How many sweets did each child receive?
(a) 7 (b) 9
(c) 18 (d) 45
(e) None of these

40. There is an increase of 30% in the production of milk chocolates in Amul Dairy in one month. If now it is 9,100 milk chcolates per month, what was it one month ago?
(a) 10,000 chocolates (b) 9000 chocolates
(c) 8000 chocolates (d) 7000 chocolates
(e) None of these

41. In a college election between two rivals, a candidate who got 40% of the total votes polled, was defeated by his rival by 160 votes. The total number of votes polled was
(a) 900 (b) 800
(c) 700 (d) 600
(e) None of these

42. A scooter costs ₹ 25, 000 when it is brand new. At the end of each year, its value is only 80% of what it was at the beginning of the year. What is the value of the scooter at the end of 3 years?
(a) ₹ 10,000 (b) ₹ 12,500
(c) ₹ 12,800 (d) ₹ 12,000
(e) None of these

43. A number is increased by 11% and then reduced by 10%. After these operations, the number :
(a) does not change (b) decreases by 1%
(c) increases by 1% (d) increases by 0.1%
(e) None of these

CHAPTER 5

Ratio & Proportion

RATIO

Ratio is strictly a mathematical term to compare two similar quantities expressed in the same units.

The ratio of two terms 'x' and 'y' is denoted by x : y.

In general, the ratio of a number x to a number y is defined as the quotient of the numbers x and y.

COMPARISON OF TWO OR MORE RATIOS

Two or more ratios may be compared by reducing the equivalent fractions to a common denominator and then comparing the magnitudes of their numerator. Thus, suppose 2 : 5, 4 : 3 and 4 : 5 are three ratios to be compared then the fractions $\frac{2}{5}, \frac{4}{3}$ and $\frac{4}{5}$ are reduced to equivalent fractions with a common denominator. For this, the denominator of each is changed to 15 equal to the L.C.M. their denominators Hence the given ratios are expressed $\frac{6}{15}, \frac{20}{15}$ and $\frac{12}{15}$ or 2 : 5, 4 : 3, 4 : 5 according to magnitude.

EXAMPLE 1. Which of the ratios 2 : 3 and 5 : 9 is greater ?

Sol. In the form of fractions, the given ratios are $\frac{2}{3}$ and $\frac{5}{9}$,

Reducing them to fractions with a common denominator they are written as $\frac{6}{9}$ and $\frac{5}{9}$.

Hence the greater ratio is $\frac{6}{9}$ or 2 : 3.

EXAMPLE 2. Are the ratios 3 to 4 and 6:8 equal ?

Sol. The ratios are equal if 3/4 = 6/8.

These are equal if their cross products are equal; that is, if 3 × 8 = 4 × 6. Since both of these products equal 24, the answer is yes, the ratios are equal.

Remember to be careful! Order matters!

A ratio of 1 : 7 is not the same as a ratio of 7:1.

REMEMBER

★ In the ratio of two quantities the two quantities must be of the same kind and in same unit.

★ The ratio is a pure number, i.e., without any unit of measurement.

★ The ratio would stay unaltered even if both the numerator and the denominator are multiplied or divided by the same number.

COMPOUND RATIO

Ratios are compounded by multiplying together the numerators for a new denominator and the denominator for a new denominator.

The compound ratio of a : b and c : d is $\frac{a \times c}{b \times d}$, i.e., ac : bd.

EXAMPLE 3. Find the compound ratio of the four ratios : 4 : 5, 15 : 13, 26 : 3 and 6 : 17

Sol. The required ratio $= \frac{4 \times 15 \times 26 \times 6}{5 \times 13 \times 3 \times 17} = \frac{48}{17}$

or 48 : 17

Shortcut Approach

The duplicate ratio of x : y is $x^2 : y^2$.

The triplicate ratio of x : y is $x^3 : y^3$.

The subduplicate ratio of x : y is $\sqrt{x} : \sqrt{y}$.

The subtriplicate ratio of x : y is $\sqrt[3]{x} : \sqrt[3]{y}$.

Reciprocal ratio of a : b is $\frac{1}{a} : \frac{1}{b}$ or b : a

Inverse ratio

Inverse ratio of x : y is y : x.

PROPERTIES OF RATIOS

1. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$, i.e., the inverse ratios of two equal ratios are equal. The property is called **Invertendo**.
2. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$, i.e., the ratio of antecedents and consequents of two equal ratios are equal. This property is called **Alternendo**.
3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$. This property is called **Componendo**.
4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$. This property is called **Dividendo**.
5. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. This property is called **Componendo and Dividendo.**

6. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=$ .... Then,

Each ratio $=\frac{\text{sum of Numerators}}{\text{sum of Denominators}}$

i.e. $\frac{a}{b}=\frac{c}{d}=\frac{a+c+e+...}{b+d+f+...}$

7. If we have two equations containing three unknowns as
$a_1x + b_1y + c_1z = 0$ and ... (i)
$a_2x + b_2y + c_2z = 0$ ... (ii)
then, the values of x, y and z cannot be resolved without having a third equation.
However, in the absence of a third equation, we can find the ratio x : y : z.
This will be given by
$b_1c_2 - b_2c_1 : c_1a_2 - c_2a_1 : a_1b_2 - a_2b_1$.

Shortcut Approach

To divide a given quantity into a given ratio.
Suppose any given quantity a, is to be divided in the ratio m : n.
Let one part of the given quantity be x then the other part will be a – x.

$\therefore \frac{x}{a-x}=\frac{m}{n}$ or $nx = ma - mx$ or $(m+n)x = ma$

$\therefore$ one part is $\frac{ma}{m+n}$ and the other part will be

$a-\frac{ma}{m+n}=\frac{na}{m+n}$

EXAMPLE 4. Divide 70 in the ratio 3 : 7.

Sol. Let one part be x
then the other part = 70 – x

$\therefore \frac{x}{70-x}=\frac{3}{7}$ or $7x = 210 - 3x$

or $x = 21$ and $70 - x = 49$
Hence the two required parts of 70 are 21 and 49.

EXAMPLE 5. What is the least integer which when subtracted from both the numerator and denominator of $\frac{60}{70}$ will give a ratio equal to $\frac{16}{21}$?

Sol. Let x be the required integer. Then,

$\frac{60-x}{70-x}=\frac{16}{21}$

$\Rightarrow 1260 - 21x = 1120 - 16x$

$\Rightarrow 5x = 140 \Rightarrow x = 28.$

EXAMPLE 6. If $\frac{x}{y}=\frac{4}{5}$, find the value of $\frac{3x+4y}{4x+3y}$.

Sol. $\frac{3x+4y}{4x+3y}=\frac{\frac{3x}{y}+4}{\frac{4x}{y}+3}=\frac{3\times\frac{4}{5}+4}{4\times\frac{4}{5}+3}=\frac{32}{31}$.

EXAMPLE 7. Find the value of $\frac{x+a}{x-a}+\frac{x+b}{x-b}$, if $x=\frac{2ab}{a+b}$.

Sol. $x=\frac{2ab}{a+b}\Rightarrow\frac{x}{a}=\frac{2b}{a+b}$

By componendo – dividendo,

$\frac{x+a}{x-a}=\frac{3b+a}{b-a}$

Similarly, $\frac{x}{b}=\frac{2a}{a+b}$

$\Rightarrow \frac{x+b}{x-b}=\frac{3a+b}{a-b}$

$\therefore \frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$

$=\frac{-(3b+a)}{a-b}+\frac{3a+b}{a-b}=\frac{2a-2b}{a-b}=2$.

EXAMPLE 8. Divide ₹ 581 among A, B and C such that four time A's share is equal to 5 times B's share which is equal to seven times C's share.

Sol. 4 times A's share = 5 times B's share
= 7 times C's share.

$\frac{\text{A's share}}{35}=\frac{\text{B's share}}{28}=\frac{\text{C's share}}{20}$

[Dividing by L.C.M. of 4, 5 and 7 i.e. 140]

$\therefore$ A : B : C = 35 : 28 : 20

$\therefore$ Share of A $=\frac{35}{35+28+20}\times 581$ = ₹ 245.

Share of B $=\frac{28}{83}\times 581$ = ₹ 196.

Share of C $=\frac{20}{83}\times 581$ = ₹ 140.

Shortcut Approach

(i) If A : B = a : b and B : C = m : n, then A : B : C = am : mb : nb and A : C = am : bn
(ii) If A : B = a : b, B : C = c : d and C : D = e : f, then A : B : C : D = ace : bce : bde : bdf

EXAMPLE 9. The ratio of A : B = 1 : 3, B : C = 2 : 5 and C : D = 2 : 3. Find the value of A : B : C : D.

Sol. A : B = 1 : 3, B : C = 2 : 5, C : D = 2 : 3
A : B : C : D = (1 × 2 × 2) : (3 × 2 × 2) : (3 × 5 × 2) : (3 × 5 × 3)
= 4 : 12 : 30 : 45

Shortcut Approach

Two numbers are in ratio a : b and x is subtracted from the numbers, then the ratio becomes c : d. The two numbers will be $\frac{xa(d-c)}{ad-bc}$ and $\frac{xb(d-c)}{ad-bc}$, respectively.

EXAMPLE 10. Two numbers are in the ratio 3 : 5. If 9 is subtracted from each, the ratio becomes 12 : 23. Find the greater number.

Sol. Here a = 3, b = 5, c = 12, d = 23 and x = 9

Then, 1st number $= \frac{xb(d-c)}{ad-bc} = \frac{9\times3(23-12)}{3\times23-5\times12}$

$= \frac{27\times11}{69-60} = \frac{297}{9} = 33$

2nd number $= \frac{xb(d-c)}{ad-bc} = \frac{9\times5(23-12)}{3\times23-5\times12}$

$= \frac{45\times11}{69-60} = \frac{45\times11}{9} = 55$

Shortcut Approach

In any 2-dimensional figures, if the corresponding sides are in the ratio x : y, then their areas are in the ratio $x^2 : y^2$.

EXAMPLE 11. The ratio of the radius of two circles is 2 : 5. Find the ratio of their areas.

Sol. Ratio of their areas = $2^2 : 5^2 = 4 : 25$

Shortcut Approach

- In any two 3-dimensional similar figures, if the corresponding sides are in the ratio x : y, then their volumes are in the ratio $x^3 : y^3$.
- If the ratio between two numbers is a : b and if each number is increased by x, the ratio becomes c : d. Then, two numbers are given as $\frac{xa(c-d)}{ad-bc}$ and $\frac{xb(c-d)}{ad-bc}$

EXAMPLE 12. The ratio between two numbers is 3 : 4. If each number be increased by 2, the ratio becomes 7 : 9. Find the number.

Sol. Numbers are $\frac{2\times3\ (7-9)}{3\times9-4\times7}$ and $\frac{2\times4\ (7-9)}{3\times9-4\times7}$ or 12 and 16.

Shortcut Approach

If the sum of two numbers is A and their difference is a, then the ratio of numbers is given by A + a : A – a.

EXAMPLE 13. The sum of two numbers is 60 and their difference is 6. What is the ratio of the two numbers ?

Sol. The required ratio of the numbers

$= \frac{60+6}{60-6} = \frac{66}{54} = \frac{11}{9}$ or 11 : 9

EXAMPLE 14. Three persons A, B, C whose salaries together amount to ₹14400, spend 80, 85 and 75 per cent of their salaries respectively. If their savings are in the ratio 8 : 9 : 20, find their respective salaries.

Sol. A, B and C spend 80 %, 85 % and 75 % respectively of their salaries.

$\Rightarrow$ A, B and C save 20 %, 15 % and 25 % respectively of their salaries.

So, 20 % of A's salary : 15 % of B's salary : 25 % of C's salary = 8 : 9 : 20

$\Rightarrow \frac{1}{5}$ of A's salary : $\frac{3}{20}$ of B's salary :

$\frac{1}{4}$ of C's salary = 8 : 9 : 20 ....(i)

Now $\frac{\frac{1}{5} \text{ of A's salary}}{\frac{3}{20} \text{ of B's salary}} = \frac{8}{9}$

$\Rightarrow \frac{\text{A's salary}}{\text{B's salary}} = \frac{3}{20}\times8\times\frac{5}{9} = \frac{2}{3}$

$\therefore$ A's salary : B's salary = 2 : 3 ...(ii)

Similarly, B's salary : C's salary = 3 : 4 ...(iii)

From (ii) and (iii)

A's salary : B's salary : C's salary = 2 : 3 : 4.

$\therefore$ A's salary $= \frac{2}{2+3+4}\times14400$ = ₹ 3200

B's salary $= \frac{3}{2+3+4}\times14400$ = ₹ 4800

C's salary $= \frac{4}{2+3+4}\times14400$ = ₹ 6400.

Removal and Replacement

Shortcut Approach

(i) Let a vessel contains Q unit of mixture of ingredients A and B. From this, R unit of mixture is taken out and replaced by an equal amount of ingredient B only.

If this process is repeated n times, then after n operations

$$\frac{\text{Quantity of A left}}{\text{Quantity of A originally present}} = \left(1-\frac{R}{Q}\right)^n$$

and Quantity of B left = Q – Quantity of A Left

EXAMPLE 15. A container contains 40 litres of milk. From this container, 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container ?

Sol.

	Milk	Water
To start with	40 litres	
After 1st operation	36 litres	4 litres
After 2nd operation	$36-\frac{4}{40}\times36$	$4-\frac{4}{40}\times4+4$
	= 32.4 litres	= 4 – 0.4 + 4
		= 7.6 litres

After 3rd operation $32.4-\frac{4}{40}\times 32.4$ $\quad 7.6-\frac{4}{40}\times 7.6+4$

$=32.4-3.24 \quad =7.6-0.76+4$

$=29.16 \quad =10.84$

$\therefore$ The quantity of milk in the container is 29.16 litres.

SHORTCUT METHOD

Quantity of milk in container : $40\left(1-\frac{4}{40}\right)^3 = 29.16$ litres

☞ Shortcut Approach

In a container, milk and water are present in the ratio a : b. If x L of water is added to this mixture, the ratio becomes a : c. Then,

Quantity of milk in original mixture $= \frac{ax}{c-b}$ L

and quantity of water in original mixture $= \frac{bx}{c-b}$ L

EXAMPLE 16. In a container, milk and water are present in the ratio 7 : 5. If 15 L water is added to this mixture, the ratio of milk and water becomes 7 : 8. Find the quantity of water in the new mixture.

Sol. Let the quantity of milk and water in initial mixture be 7x and 5x L.

Then, according to the question,

$$\frac{7x}{5x+15}=\frac{7}{8} \Rightarrow 7x\times 8 = 7(5x+15)$$

$$\Rightarrow 56x = 35x+105 \Rightarrow 56x-35x = 105$$

$$\Rightarrow 21x = 105 \Rightarrow x = \frac{105}{21} = 5$$

$\therefore$ Quantity of water in initial mixture $= 5\times 5 = 25$ L

and quantity of water in new mixture $= 25+15 = 40$ L

SHORTCUT METHOD

Here, a = 7, b = 5, c = 8 and x = 15 L

According to the formula,

Quantity of water in original mixture $= \frac{bx}{c-b} = \frac{5\times 15}{8-5} = 25$L

$\therefore$ Quantity of water in new mixture $= 25+15 = 40$ L

☞ Shortcut Approach

A container has milk and water in the ratio a : b, a second container has milk and water in the ratio c : d. If both the mixtures are emptied into a third container, then the ratio of milk of water in third container is given by

$$\left(\frac{a}{a+b}+\frac{c}{c+d}\right):\left(\frac{b}{a+b}+\frac{d}{c+d}\right)$$

EXAMPLE 17. 2 containers have milk and water in the ratio 2 : 1 and 3 : 1, respectively. If both containers are emptied into a bigger container, then find the ratio of milk of water in bigger container?

Sol. Given, ratio of milk and water in 1st container = 2 : 1

$\therefore$ Quantity of milk in 1st container $= \frac{2}{3}$

and quantity of water in 1st container $= \frac{1}{3}$

Similarly, ratio of milk and water in 2nd container = 3 :1

$\therefore$ Quantity of milk in 2nd container $= \frac{3}{4}$

and quantity of water in second container $= \frac{1}{4}$

Now, after pouring both mixture in one container

Quantity of milk $= \frac{2}{3}+\frac{3}{4} = \frac{17}{12}$

and quantity of water $= \frac{1}{4}+\frac{1}{3} = \frac{7}{12}$

Hence, required ratio = 17 : 7

SHORTCUT METHOD

Here, a = 2, b = 1, c = 3, d = 1

Ratio of milk of water in bigger container

$$=\left(\frac{a}{a+b}+\frac{c}{c+d}\right):\left(\frac{b}{a+b}+\frac{d}{c+d}\right)$$

$$=\left(\frac{2}{2+1}+\frac{3}{3+1}\right):\left(\frac{1}{2+1}+\frac{1}{3+1}\right) = \frac{17}{12}:\frac{7}{12} = 17:7$$

PROPORTION

When two ratios are equal, the four quantities composing them are said to be in proportion.

If $\frac{a}{b}=\frac{c}{d}$, then a, b, c, d are in proportions.

This is expressed by saying that 'a' is to 'b' as 'c' is to 'd' and the proportion is written as

a : b :: c : d or a : b = c : d

The terms a and d are called the extremes while the terms b and c are called the means.

REMEMBER

★ If four quantities are in proportion, the product of the extremes is equal to the product of the means.
Let a, b, c, d be in proportion, then
$\frac{a}{b}=\frac{c}{d} \Rightarrow ad = bc.$

★ If three quantities a, b and c are in continued proportion, then a : b = b : c
$\therefore$ $ac = b^2$
b is called mean proportional.

EXAMPLE 18. Find the mean proportional between 3 and 75.

Sol. Let x be the required mean proportional. Then,

$3 : x :: x : 75$

$\therefore \quad x = \sqrt{3 \times 75} = 15$

EXAMPLE 19. What must be added to each of the four numbers 10, 18, 22, 38 so that they become in proportion ?

Sol. Let the number to be added to each of the four numbers be x

By the given condition, we get

$(10+x) : (18+x) :: (22+x) : (38+x)$

$\Rightarrow (10+x)(38+x) = (18+x)(22+x)$

$\Rightarrow 380 + 48x + x^2 = 396 + 40x + x^2$

Cancelling x^2 from both sides, we get

$380 + 48x = 396 + 40x$

$\Rightarrow 48x - 40x = 396 - 380$

$\Rightarrow 8x = 16 \Rightarrow x = \frac{16}{8} = 2$

Therefore, 2 should be added to each of the four given numbe₹

EXAMPLE 20. Find the fourth proportional to $p^2 - pq + q^2, p^3 + q^3, p - q$

Sol. Let x be the fourth proportional

$\therefore \ (p^2 - pq + q^2) : (p^3 + q^3) = (p - q) : x$

$\Rightarrow (p^2 - pq + q^2) \times x = (p^3 + q^3)(p - q)$

$\therefore \quad x = \frac{(p^3 + q^3)(p - q)}{(p^2 - pq - q^2)}$

$\Rightarrow \quad x = \frac{(p + q)(p^2 - pq + q^2)(p - q)}{(p^2 - pq - q^2)}$

$\Rightarrow \quad x = (p + q)(p - q) = p^2 - q^2$

$\therefore$ The required fourth proportional is $p^2 - q^2$

EXAMPLE 21. Find third proportional to $a^2 - b^2$ and $a + b$.

Sol. Let x be the required third proportional

Then $a^2 - b^2 : a + b = a + b : x$

$\therefore \ (a^2 - b^2)x = (a + b)(a + b)$

$\therefore \quad x = \frac{(a + b)(a + b)}{a^2 - b^2} = \frac{a + b}{a - b}$

DIRECT PROPORTION

If on the increase of one quantity, the other quantity increases to the same extent or on the decrease of one, the other decreases to the same extent, then we say that the given two quantities are directly proportional. If A and B are directly proportional then we denote it by $A \propto B$.

Some Examples :

1. Work done $\propto$ number of men
2. Cost $\propto$ number of Articles
3. Work $\propto$ wages
4. Working hour of a machine $\propto$ fuel consumed
5. Speed $\propto$ distance to be covered

INDIRECT PROPORTION (OR INVERSE PROPORTION)

If on the increase of one quantity, the other quantity decreases to the same extent or vice versa, then we say that the given two quantities are indirectly proportional. If A and B are indirectly proportional then we denote it by $A \propto \frac{1}{B}$.

Also, $A = \frac{k}{B}$ (k is a constant)

$\Rightarrow \quad AB = k$

If b_1, b_2 are the values of B corresponding to the values a_1, a_2 of A respectively, then

$a_1 b_1 = a_2 b_2$

Some Examples :

1. More men, less time
2. Less men, more time
3. More speed, less taken time to be covered distance

EXAMPLE 22. A garrison of 3300 men had provision for 32 days, when given at the rate of 850 gm per head. At the end of 7 days, a reinforcement arrived and it was found that the provision would last 17 days more, when given at the rate of 825 gm per head. What was the strength of the reinforcement ?

(a) 1500 **(b)** 1700

(c) 1800 **(d)** 1600

Sol. (b) There is a provision for 2805 × 32 kg for 3300 men for 32 days @ 850 gm per head per day.

In 7 days, 3300 men consumed

$\frac{2805 \times 32}{32} \times 7 = 2805 \times 7$ kg.

Let the strength of the reinforcement arrived after 7 days be x.

$\therefore$ (3300 + x) men had provision of 2805 × 25 kg for 17 days @ 825 gm per head per day, i.e.

$\therefore \ \frac{(3300 + x) \times 825 \times 17}{1000} = 2805 \times 25$

$\Rightarrow (3300 + x) = \frac{1000 \times 2805 \times 25}{825 \times 17} = 5000$

$\Rightarrow x = 1700$

$\therefore$ Strength of the reinforcement arrived after 7 days = 1700.

RULE OF THREE

In a problem on simple proportion, usually three terms are given and we have to find the fourth term, which we can solve by using Rule of three. In such problems, two of given terms are of same kind and the third term is of same kind as the required fourth term. First of all we have to find whether given problem is a case of direct proportion or indirect proportion.

For this, write the given quantities under their respective headings and then mark the arrow in increasing direction. If both arrows are in same direction then the relation between them is direct otherwise it is indirect or inverse proportion. Proportion will be made by either head to tail or tail to head.

The complete procedure can be understand by the examples.

EXAMPLE 23. A man completes 5/8 of a job in 10 days. At this rate, how many more days will it take him to finish the job ?

(a) 5 **(b)** 6

(c) 7 **(d)** $7\frac{1}{2}$

Sol. (b) Work done $= \frac{5}{8}$. Balance work $= \left(1-\frac{5}{8}\right)=\frac{3}{8}$.

Less work, Less days (Direct Proportion)

Let the required number of days be x. Then,

Work	days
↑ 5/8	10 ↑
3/8	x

Then, $\frac{5}{8}:\frac{3}{8}::10:x \Rightarrow \frac{5}{8}\times x=\frac{3}{8}\times 10$

$$\Rightarrow x=\left(\frac{3}{8}\times 10\times\frac{8}{5}\right)=6\cdot$$

EXAMPLE 24. A fort had provision of food for 150 men for 45 days. After 10 days, 25 men left the fort. The number of days for which the remaining food will last, is :

(a) $29\frac{1}{5}$ **(b)** $37\frac{1}{4}$

(c) 42 **(d)** 54

Sol. (c) After 10 days : 150 men had food for 35 days. Suppose 125 men had food for x days. Now,

Less men, More days (Indirect Proportion)

Then,

men	days
↑ 150	35 ↓
125	x

$\therefore 125:150::35:x \Rightarrow 125\times x=150\times 35$

$$\Rightarrow x=\frac{150\times 35}{125}\Rightarrow x=42.$$

Hence, the remaining food will last for 42 days.

EXAMPLE 25. If the cost of printing a book of 320 leaves with 21 lines on each page and on an average 11 words in each line is ₹ 19, find the cost of printing a book with 297 leaves, 28 lines on each page and 10 words in each line.

(a) ₹ $22\frac{3}{8}$ **(b)** ₹ $20\frac{3}{8}$

(c) ₹ $21\frac{3}{8}$ **(d)** ₹ $21\frac{3}{4}$

Sol. (c) **Less leaves, less cost (Direct Proportion)**
More lines, more cost (Direct Proportion)
Less words, less cost (Direct Proportion)

$$\left.\begin{array}{ll}\text{leaves} & 320:297\\ \text{lines} & 21:28\\ \text{words} & 11:10\end{array}\right\}::19:x$$

$\therefore 320\times 21\times 11\times x=297\times 28\times 10\times 19$

$$\Rightarrow x=\frac{171}{8}=21\frac{3}{8}$$

PARTNERSHIP

A partnership is an association of two or more persons who invest their money in order to carry on a certain business.

A partner who manages the business is called the **working partner** and the one who simply invests the money is called the **sleeping partner.**

Partnership is of two kinds :

(i) Simple (ii) Compound.

Simple partnership :

If the capitals is of the partners are invested for the same period, the partnership is called simple.

Compound partnership :

If the capitals of the partners are invested for different lengths of time, the partnership is called compound.

Shortcut Approach

If the period of investment is the same for each partner, then the profit or loss is divided in the ratio of their investments.

If A and B are partners in a business, then

$$\frac{\text{Investment of A}}{\text{Investment of B}}=\frac{\text{Profit of A}}{\text{Profit of B}}=\frac{\text{Loss of A}}{\text{Loss of B}}$$

If A, B and C are partners in a business, then

Investment of A : Investment of B : Investment of C
= Profit of A : Profit of B : Profit of C, or
= Loss of A : Loss of B : Loss of C

EXAMPLE 26. Three partner Rahul, Puneet and Chandan invest ₹ 1600, ₹ 1800 and ₹ 2300 respectively in a business. How should they divide a profit of ₹ 399 ?

Sol. Profit is to be divided in the ratio 16 : 18 : 23

$$\text{Rahul's share of profit}=\frac{16}{16+18+23}\times 399$$

$$=\frac{16}{57}\times 399=₹\ 112$$

$$\text{Puneet's share of profit}=\frac{18}{57}\times 399=₹\ 126$$

$$\text{Chandan's share of profit}=\frac{23}{57}\times 399=₹\ 161$$

EXAMPLE 27. A and B invested in the ratio 3 : 2 in a business. If 5% of the total profit goes to charity and A's share is ₹ 855, find the total profit.

Sol. Let the total profit be ₹ 100.

Then, ₹ 5 goes to charity.

Now, ₹ 95 is divided in the ratio 3 : 2.

$$\therefore \quad \text{A's share}=\frac{95}{3+2}\times 3=₹\ 57$$

But A's actual share is ₹ 855.

$\therefore$ Actual total profit $= 855\left(\frac{100}{57}\right) =$ ₹ 1500

Shortcut Approach

When the amount of capital invested by different partners is same (say ₹ x) for differend time periods, $t_1, t_2, t_3, \ldots$, then Ratio of profit/loss = Ratio of time period for which the capital is invested

$$P_1 : P_2 : P_3 : \ldots = t_1 : t_2 : t_3 : \ldots$$

EXAMPLE 28. A, B and C start a business with investment of ₹ 50000 each. A remians in partnershi for 9 months, B for 6 months and C for 12 months. Then, find the ratio of their profits.

Sol. Ratio of profit of A, B and C will be in the ratio of time period of investment

So, A's profit : B's Profit : C's profit = 9 : 6 : 12 = 3 : 2 : 4

MONTHLY EQUIVALENT INVESTMENT

It is the product of the capital invested and the period for which it is invested.

If the period of investment is different, then the profit or loss is divided in the ratio of their Monthly Equivalent Investment.

$$\frac{\text{Monthly Equivalent Investment of A}}{\text{Monthly Equivalent Investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}} \text{ or } \frac{\text{Loss of A}}{\text{Loss of B}}$$

i.e., $$\frac{\text{Investment of A} \times \text{Period of Investment of A}}{\text{Investment of B} \times \text{Period of Investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}} \text{ or } \frac{\text{Loss of A}}{\text{Loss of B}}$$

Shortcut Approach

If A, B and C are partners in a business, then

Monthly Equivalent Investment of A : Monthly Equivalent Investment of B : Monthly Equivalent Investment of C

= Profit of A : Profit of B : Profit of C.

= Loss of A : Loss of B : Loss of C.

EXAMPLE 29. A and B start a business. A invests ₹ 600 more than B for 4 months and B for 5 months. A's share is ₹ 48 more than that of B, out of a total profit of ₹ 528. Find the capital contributed by each.

Sol. B's profit $= \frac{528 - 48}{2} =$ ₹240

A's profit = 528–240 = ₹ 288

$$\frac{\text{A's capital} \times 4}{\text{B's capital} \times 5} = \frac{288}{240} = \frac{6}{5}$$

$$\therefore \frac{\text{A's capital}}{\text{B's capital}} = \frac{6}{5} \times \frac{5}{4} = \frac{3}{2}$$

$$\Rightarrow \frac{\text{B's capital} + 600}{\text{B's capital}} = \frac{3}{2}$$

$\Rightarrow$ B's capital = ₹ 1200 and A's capital = ₹ 1800

EXAMPLE 30. Three persons A, B, C rent the grazing of a park for ₹ 570. A puts in 126 oxen in the park for 3 months, B puts in 162 oxen for 5 months and C puts in 216 oxen for 4 months. What part of the rent should each person pay ?

Sol. Monthly equivalent rent of A = 126 × 3 = 378

Monthly equivalent rent of B = 162 × 5 = 810

Monthly equivalent rent of C = 216 × 4 = 864

$\therefore$ Rent is to be divided in the ratio

378 : 810 : 864, ie. 7 : 15 : 16

$\therefore$ A would have to pay $\frac{7}{7+15+16}$ of the rent

$= \frac{7}{38}$ of the rent $= \frac{7}{38} \times 570 =$ ₹ 105

$\therefore$ B would have to pay $\frac{15}{38}$ of the rent $= \frac{15}{38} \times 570 =$ ₹225

and C would have to pay $\frac{16}{38}$, i.e. $\frac{8}{19}$ of the rent

$= \frac{8}{19} \times 570 =$ ₹ 240

Shortcut Approach

When capital invested by the partners is givne as $X_1, X_2, X_3, \ldots$ for different time period $t_1, t_2, t_3, \ldots$ in a business, then

Ratio of their profits $P_1 : P_2 : P_3 : \ldots = X_1t_1 : X_2t_2 : X_3t_3 : \ldots$

EXAMPLE 31. A starts a business with ₹ 4000 and B joins the business 4 months later with an investment of ₹ 5000. After 1 yr. they earn a profit of ₹ 22000. Find the share of A and B.

Sol. A's share : B's share

= 4000 × 12 : 5000 × (12 – 4) = 4 × 12 : 5 × 8 = 6 : 5

Now, let the share of A = 6x, and the share of B = 5x

According to the question, 6x + 5x = 22000 ⇒ 11x = 22000

$\therefore$ x = ₹ 2000

Share of A = 6x = 6 × 2000 = ₹ 12000,

and share of B = 5x = 5 × 2000 = ₹ 10000

SHORTCUT METHOD

A's share : B's share = 4000 × 12 : 5000 × 8 = 6 : 5

Now, A's share $= \frac{6}{6+5} \times 22000 =$ ₹ 12000

and B's share $= \frac{5}{6+5} \times 22000 =$ ₹ 10000

Shortcut Approach

If $P_1 : P_2 : P_3 : \ldots$ is the ratio of profit and $t_1 : t_2 : t_3 : \ldots$ is the ratio of time periods, then ratio of invesements is given by

$$\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} : \ldots$$

EXAMPLE 32. A, B and C each does certain investments for time periods in the ratio of 5 : 6 : 8. At the end of the business terms, they received the profit in the ratio of 5 : 3 : 12. Find the ratio of investments of A, B and C.

Sol. Here, $t_1 : t_2 : t_3 = 5 : 6 : 8$ and $P_1 : P_2 : P_3 = 5 : 3 : 12$

$$\text{Required ratio} = \frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} = \frac{5}{5} : \frac{3}{6} : \frac{12}{8} = 1 : \frac{1}{2} : \frac{3}{2} = 2 : 1 : 3$$

MIXTURE

Simple Mixture : When two different ingredients are mixed together, it is known as a simple mixture.

Compound Mixture : When two or more simple mixtures are mixed together to form another mixture, it is known as a compound mixture.

Alligation : Alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together.

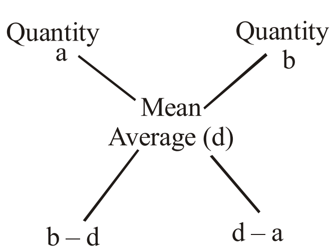
The word 'Alligation' literally means 'linking'.

ALLIGATION RULE

It states that when different quantities of the same or different ingredients of different costs are mixed together to produce a mixture of a mean cost, the ratio of their quantities is inversely proportional to the difference in their cost from the mean cost.

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Price of Dearer} - \text{Mean Price}}{\text{Mean Price} - \text{Price of Cheaper}}$$

Graphical representation of Alligation Rule :



$$\frac{\text{Quantity of a}}{\text{Quantity of b}} = \frac{b-d}{d-a}$$

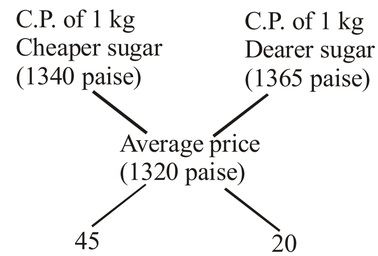
Applications of Alligation Rule :

(i) To find the mean value of a mixture when the prices of two or more ingredients, which are mixed together and the proportion in which they are mixed are given.

(ii) To find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price.

EXAMPLE 33. In what proportion must sugar at ₹ 13.40 per kg be mixed with sugar at ₹ 13.65 per kg, so that the mixture be worth ₹ 13.20 a kg ?

Sol.

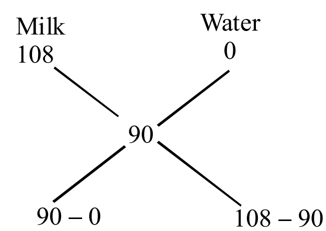


$$\frac{\text{Quantity of cheaper sugar}}{\text{Quantity of dearer sugar}} = \frac{45}{20} = \frac{9}{4}$$

$\therefore$ They must be mixed in the ratio 9 : 4.

EXAMPLE 34. A mixture of a certain quantity of milk with 16 litres of water is worth 90 P per litre. If pure milk be worth ₹ 1.08 per litre, how much milk is there in the mixture ?

Sol. The mean value is 90P and the price of water is 0 P.



By the Alligation Rule, milk and water are in the ratio of 5 : 1.

$\therefore$ Quantity of milk in the mixture = $5 \times 16 = 80$ litres.

Shortcut Approach

Price of the Mixture :

When quantities Q_i of ingredients M_i's with the cost C_i's are mixed then cost of the mixture C_m is given by

$$C_m = \frac{\Sigma C_i Q_i}{\Sigma Q_i}$$

EXAMPLE 35. 5 kg of rice of ₹ 6 per kg is mixed with 4 kg of rice to get a mixture costing ₹ 7 ker kg. Find the price of the costlier rice.

Sol. Let the price of the costlier rice be ₹ x.

By direct formula,

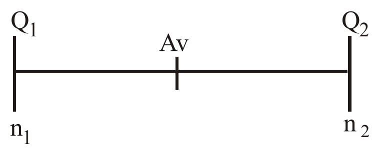
$$7 = \frac{6 \times 5 + 4 \times x}{9}$$

$$\Rightarrow \quad 63 - 30 = 4x \Rightarrow 4x = 33$$

$$\Rightarrow \quad x = \frac{33}{4} = 8.25$$

STRAIGHT LINE APPROACH OF ALLIGATION

Let Q_1 and Q_2 be the two quantities, and n_1 and n_2 are the number of elements present in the two quantities respectively,



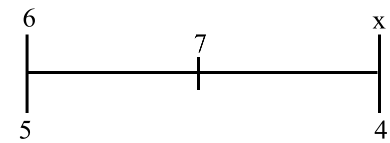
where Av is the average of the new group formed then

n_1 corresponds to Q_2 – Av, n_2 corresponds to Av – Q_1 and $(n_1 + n_2)$ corresponds to $Q_2 - Q_1$.

Let us consider the previous example.

EXAMPLE 36. 5 kg of rice at ₹ 6 per kg is mixed with 4 kg of rice to get a mixture costing ₹ 7 per kg. Find the price of the costlier rice.

Sol. Using straight line method,



4 corresponds to 7 – 6 and 5 corresponds to x – 7.

i.e. $4 \to 1$

$5 \to 1.25$

Hence, $x - 7 = 1.25$

$\Rightarrow \quad x = 8.25$

EXERCISE

1. Salaries of A, B and C were in the ratio of 3 : 5 : 7 respectively. If their salaries were increased by 50%, 60% and 50% respectively, what will be the new ratio of their respective salaries?
(a) 3 : 6 : 7 (b) 4 : 5 : 7
(c) 4 : 5 : 8 (d) Data inadequate
(e) None of these
2. If 40% of a number is equal to two-thirds of another number, what is the ratio of the first number to the second?
(a) 7 : 3 (b) 3 : 7
(c) 2 : 5 (d) 5 : 3
(e) None of these
3. Radha started a business, investing ₹ 75,000. After 3 months, Sunidhi joined her with an amount of ₹ 1,25,000 and after another six months Neha joined them with an amount of ₹ 1,50,000. Profit earned at the end of three years from when Radha started the business should be distributed in what ratio among Radha, Sunidhi and Neha respectively?
(a) 36 : 55 : 54 (b) 18 : 28 : 27
(c) 35 : 54 : 55 (d) Cannot be determined
(e) None of these
4. What should come in place of the question mark(?) in the following equation? $\frac{28}{?}=\frac{?}{112}$
(a) 70 (b) 56
(c) 48 (d) 64
(e) None of these
5. An empty fuel tank to a car was filled with A type of petrol. When the tank was half empty, it was filled with B type of petrol. Again when the tank was half empty, it was filled with A type of petrol. When the tank was half empty again, it was filled with B type of petrol. At this time, what was the percentage of A type of petrol in the tank?
(a) 50% (b) 40%
(c) 33.5% (d) 37.5%
(e) None of these
6. The ratio of A's and B's salary is 9 : 4. If A's salary is increased by 15%, then his total salary becomes ₹ 5175. What is the salary of B?
(a) ₹ 2,000 (b) ₹ 4,000
(c) ₹ 4,500 (d) ₹ 2,500
(e) None of these
7. Three friends *A, B* and *C* started a business by investing a sum of money in the ratio of 5 : 7 : 6. After 6 months *C* withdraws half of his capital. If the sum invested by '*A*' is ₹ 40,000, out of a total annual profit of ₹ 33,000, *C*'s share will be
(a) ₹ 9,000 (b) ₹ 12,000
(c) ₹ 11,000 (d) ₹ 10,000
(e) None of these
8. Seats for Maths, Physics and Biology are in the ratio of 5 : 7 : 8 respectively. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the respective ratio of increased seats?
(a) 2 : 3 : 4 (b) 6 : 7 : 8
(c) 6 : 8 : 9 (d) Cannot be determined
(e) None of these
9. Mr Sharad started a business investing ₹ 50000. Four months later Mr Praveen joined the business by investing ₹ 90000. If the profit in the business at the end of the year was ₹ 22000 how much amount would Mr Praveen have received as the profit?
(a) ₹ 16000 (b) ₹ 14000
(c) ₹ 12000 (d) ₹ 11000
(e) None of these
10. The ratio of Gomati's and Rashmi's ages is 3 : 5 respectively. After ten years this ratio will become 2 : 3. What is Rashmi's age in years?
(a) 50 (b) 40
(c) 60 (d) Cannot be determined
(e) None of these
11. Salaries of Rajesh and Sunil are in the ratio of 2 :3. If the salary of each one is increased by ₹ 4000 the new ratio becomes 40 : 57. What is Sunil's present salary?
(a) ₹ 17000 (b) ₹ 20000
(c) ₹ 25500 (d) Csannot be determined
(e) None of these
12. The numbers of students speaking English and Hindi are in the ratio of 4:5. If the number of students speaking English increased by 35% and that speaking Hindi increased by 20%, what would be the new respective ratio?
(a) 19 : 20 (b) 7 : 8
(c) 8 : 9 (d) Cannot be determined
(e) None of these
13. Abhijit started a business investing ₹ 70000. Anuja joined him after six months with an amount of ₹ 105000 and Sunil joined them with ₹ 1.4 lakhs after another six months. The amount of profit earned should be distributed in what ratio among Abhijit, Anuja and Sunil respectively, three years after Abhijit started the business?
(a) 42 : 45 : 56 (b) 7 : 6 : 10
(c) 12 : 15 : 16 (d) Cannot be determined
(e) None of these
14. The ratio of males and females in a city is 7 : 8 and the percentage of children among males and females is 25% and 20% respectively. If the number of adult females in the city is 156800 what is the total population?
(a) 245000 (b) 367500
(c) 196000 (d) 171500
(e) None of these
15. Hariprasad and Madhusudan started a business, investing sums in the ratio of 2 : 3. If Hariprasad had invested an additional amount of ₹ 10,000 the ratio of Hariprasad's investment to Madhusudan's investment would have been 3 : 2. What was the amount invested by Hariprasad?
(a) ₹ 8000 (b) ₹ 12000
(c) ₹ 9000 (c) Data inadequate
(e) None of these

16. The ratio of the present ages of a son and his father is 1 : 5 and that of his mother and father is 4 : 5. After 2 years the ratio of the age of the son to that of his mother becomes 3 : 10. What is the present age of the father?
(a) 30 years (b) 28 years
(c) 37 years (d) Data inadequate
(e) None of these

17. The ratio of the number of students appearing for examination in the year 1998 in the states *A, B* and *C* was 3 : 5 : 6. Next year if the number of students in these states increases by 20%, 10% and 20% respectively, the ratio in states *A* and *C* would be 1 : 2. What was the number of students who appeared for the examination in the state *A* in 1998?
(a) 7200 (b) 6000
(c) 7500 (d) Data inadequate
(e) None of these

18. A man spends ₹ 1810 for buying bedsheets at ₹ 200 each and pillows at ₹ 70 each. What will be the ratio of bedsheets to pillows when maximum number of bedsheets are bought?
(a) 3:8 (b) 8:3
(c) 9:1 (d) 1:9
(e) None of these

19. Mr Shivkumar started a business, investing ₹ 25000 in 1996. In 1997 he invested an additional amount of ₹ 10000 and Mr Rakesh joined him with an amount of ₹ 35000. In 1998, Mr Shivkumar invested another additional amount of ₹ 10000 and Mr Suresh joined them with an amount of ₹ 35000. What will be Rakesh's share in the profit of ₹ 150000 earned at the end of three years from the start of the business in 1996?
(a) ₹ 70000 (b) ₹ 50000
(c) ₹ 45000 (d) ₹ 75000
(e) None of these

20. Incomes of two companies A and B are in the ratio of 5 : 8. Had the income of company A been more by ₹ 25 lakh, the ratio of their incomes would have been 5 : 4. What is the income of company B?
(a) ₹ 80 lakh (b) ₹ 50 lakh
(c) ₹ 40 lakh (d) ₹ 60 lakh
(e) None of these

21. The ratio of number of students studying Arts, Commerce and Science in a College is 3 : 5 : 8. What is the new ratio of the number of students studying Arts, Commerce and Science respectively if there is an increase of 20%, 40% and 25% in the number of students studying Arts, Commerce and Science?
(a) 18:35:50 (b) 3:10:10
(c) 4:8:5 (d) 32:35:25
(e) None of these

22. Abhishek started a business investing ₹ 50,000. After one year he invested another ₹ 30,000 and Sudin also joined him with a capital of ₹ 70,000. If the profit earned in three years from the starting of business was ₹ 87,500, then find the share of Sudin in the profit.
(a) ₹ 37,500 (b) ₹ 32,500
(c) ₹ 38,281 (d) ₹ 52,500
(e) None of these

23. Weights of two friends Ram and Shyam are in the ratio of 4 : 5. Ram's weight increases by 10% and the total weight of Ram and Shyam together becomes 82.8 kg, with an increase of 15%. By what per cent did the weight of Shyam increase?
(a) 12.5% (b) 17.5%
(c) 19% (d) 21%
(e) None of these

24. When 50% of one number is added to a second number, the second number increases to its four-thirds. What is the ratio between the first number and the second number?
(a) 3 : 2 (b) 3 : 4
(c) 2 : 3 (d) Data inadequate
(e) None of these

25. The ratio of present ages of Nisha and Shilpa is 7:8 respectively. Four years hence this ratio becomes 9:10 respectively. What is Nisha's present age in years?
(a) 18 (b) 14
(c) 17 (d) Data inadequate
(e) None of these

26. When a number is added to another number the total becomes $33\frac{1}{3}$ per cent of the second number. What is the ratio between the first and the second number?
(a) 3 : 7 (b) 7 : 4
(c) 7 : 3 (d) Data inadequate
(e) None of these

27. The ratio between the present ages of *P* and *Q* is 5 : 8. After four years, the ratio between their ages will be 2 : 3. What is *Q*'s age at present?
(a) 36 years (b) 20 years
(c) 24 years (d) Data inadequate
(e) None of these

28. Jaydeep purchased 25 kg of rice at the rate of ₹ 16.50 per kg and 35 kg of rice at the rate of ₹ 24.50 per kg. He mixed the two and sold the mixture. Approximately, at what price per kg did he sell the mixture to make 25 per cent profit?
(a) ₹ 26.50 (b) ₹ 27.50
(c) ₹ 28.50 (d) ₹ 30.00
(e) ₹ 29.00

29. In 1 kg mixture of sand and iron, 20% is iron. How much sand should be added so that the proportion of iron becomes 10%?
(a) 1 kg (b) 200 gms
(c) 800 gms (d) 1.8 kg
(e) None of these

30. The ratio of *P*'s and *Q*'s ages is 5 : 7. If the difference between the present age of *Q* and the age of *P* six years hence is 2 then what is the total of present ages of *P* and *Q*?
(a) 52 years (b) 48 years
(c) 56 years (d) Data inadequate
(e) None of these

31. There is a ratio of 5 :4 between two numbers. If forty percent of the first number is 12 then what would be the 50 percent of the second number?
(a) 12 (b) 24
(c) 18 (d) Data inadequate
(e) None of the above

32. An amount of money is to be distributed among P, Q and R in the ratio of 5 : 8 : 12 respectively. If the total share of Q and R is four times that of P, what is definitely P's share?

(a) ₹ 3,000 (b) ₹ 5,000
(c) ₹ 8,000 (d) Data inadequate
(e) None of these

33. When 30 per cent of a number is added to another number the second number increases to its 140 per cent. What is the ratio between the first and the second number?
(a) 3 : 4 (b) 4 : 3
(c) 3 : 2 (d) Data inadequate
(e) None of these

34. If 25% of a number is subtracted from a second number the second number reduces to its five-sixths. What is the ratio between the first number and the second number?
(a) 2 : 3 (b) 3 : 2
(c) 1 : 3 (d) Data inadequate
(e) None of these

35. Two friends *P* & *Q* started a business investing amounts in the ratio of 5 : 6. R joined them after six months investing an amount equal to that of *Q*'s amount. At the end of the year 20% profit was earned which was equal to ₹ 98,000. What was the amount invested by R?
(a) ₹ 2,10,000 (b) ₹ 1,05,000
(c) ₹ 1,75,000 (d) Data inadequate
(e) None of these

36. One year ago the ratio of Yamini's and Gamini's ages was 6 : 7 respectively. Four years hence this ratio would become 7 : 8. How old is Gamini?
(a) 35 years (b) 30 years
(c) 31 years (d) Cannot be determined
(e) None of these

37. Ratio of present age of P and Q is 7 : 3. After four years their ages are in the ratio of 2 : 1. What is the present age of P ?
(a) 24 years (b) 28 years
(c) 32 years (d) Data inadequate
(e) None of these

38. If 40 per cent of a number is added to an other number then it becomes 125 per cent of itself. What will be the ratio of first and second numbers?
(a) 8 : 5 (b) 5 : 7
(c) 5 : 8 (d) Data inadequate
(e) None of these

39. An amount of money is to be divided among *P, Q* and *R* in the ratio of 4 : 9 : 16. If *R* gets 4 times more than *P*, what is *Q*'s share in it?
(a) ₹ 1,800 (b) ₹ 2,700
(c) ₹ 3,600 (d) Data inadequate
(e) None of these

40. Jagtap purchases 30 kg of wheat at the rate of ₹ 11.50 per kg and 20 kg of wheat at the rate of ₹ 14.25 per kg. He mixed the two and sold the mixture. **Approximately** at what price per kg should he sell the mixture to make 30 per cent profit?
(a) ₹ 16.30 (b) ₹ 18.20
(c) ₹ 15.60 (d) ₹ 14.80
(e) ₹ 15.40

41. Mr. Gangadhar, Mr. Ramesh and Mr. Shridhar together earned ₹ 19800. The ratio of earnings between Mr. Gangadhar and Mr. Ramesh is 2 : 1 while that between Mr. Ramesh and Mr. Shridhar is 3 : 2. How much did Mr. Ramesh earn?
(a) ₹ 3600 (b) ₹ 5400
(c) ₹ 1800 (d) ₹ 6300
(e) None of these

42. Mr. Kutty has only hens and sheep. If the total number of their heads is 38 and the total number of legs is 100 then what is the ratio between the numbers of hens and sheep?
(a) 2 : 1 (b) 1 : 2
(c) 6 : 13 (d) 13 : 6
(e) None of these

43. If A : B : C = 2 : 3 : 4, then $\frac{A}{B}:\frac{B}{C}:\frac{C}{A}$ is equal to :
(a) 4 : 9 : 16 (b) 8 : 9 : 12
(c) 8 : 9 : 16 (d) 8 : 9 : 24
(e) None of these

44. A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets ₹ 1000 more than D, what is B's share?
(a) ₹ 500 (b) ₹ 1500
(c) ₹ 2000 (d) ₹ 1400
(e) None of these

45. The sum of three numbers is 98. If the ratio of the first to the second is 2 : 3 and that of the second to the third is 5 : 8, then the second number is :
(a) 20 (b) 30
(c) 38 (d) 48
(e) None of these

46. The ratio of number of ladies to gents at a party was 1 : 2, but when 2 ladies and 2 gents left, the ratio became 1 : 3. How many people were originally present at the party?
(a) 6 (b) 9
(c) 12 (d) 10
(e) None of these

47. A man divides his property so that his son's share to his wife's and the wife's share to his daughter are both in the ratio 3 : 1. If the daughter gets ₹ 10,000 less than the son, find the total worth of the property.
(a) ₹ 16,200 (b) ₹ 16,250
(c) ₹ 16,500 (d) ₹ 15,300
(e) None of these

48. A bag contains an equal number of one rupee, 50 paise and 25 paise coins respectively. If the total value is ₹ 35, how many coins of each type are there?
(a) 20 coins (b) 30 coins
(c) 28 coins (d) 25 coins
(e) None of these

49. The salaries of A,B,C are in the ratio 2 : 3 : 5. If the increments of 15%, 10% and 20% are allowed respectively in their salaries, then what will be the new ratio of their salaries?
(a) 3 : 3 : 10 (b) 10 : 11 : 20
(c) 23 : 33 : 60 (d) Cannot be determined
(e) None of these

50. In an express train, the passengers travelling in A.C. sleeper class, First class and Sleeper class are in the ratio 1:2:7, and rate for each class is in the ratio 5 : 4 : 2. If the total income from this train is ₹ 54, 000, find the income of Indian Railways from A.C. sleeper class.
(a) ₹ 12,000 (b) ₹ 20,000
(c) ₹ 22,000 (d) ₹ 10,000
(e) None of these

51. What is the ratio whose terms differ by 40 and the measure of which is $\frac{2}{7}$?
(a) 16 : 56 (b) 14 : 56
(c) 15 : 56 (d) 16 : 72
(e) None of these

52. The average age of three boys is 25 years and their ages are in the proportion 3: 5 : 7. The age of the youngest boy is:
(a) 21 years (b) 18 years
(c) 15 years (d) 9 years
(e) None of these

53. A photograph measuring $2\frac{1"}{2}\times1\frac{7"}{8}$ is to be enlarged so that the length will be 4". How many inches will the enlarged breadth be?
(a) $1\frac{1}{2}$ (b) $2\frac{1}{8}$
(c) 3 (d) $3\frac{3}{8}$
(e) None of these

54. In a partnership, A invests $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time, B invests $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time and C, the rest of the capital for whole time. Find A's share of the total profit of ₹ 2,300.
(a) ₹ 100 (b) ₹ 200
(c) ₹ 300 (d) ₹ 400
(e) None of these

55. A, B and C start a business each investing ₹ 20,000. After 5 months A withdrew ₹ 5000, B withdrew ₹ 4000 and C invests ₹ 6000 more. At the end of the year, a total profit of ₹ 69,900 was recorded. Find the share of B.
(a) ₹ 20,000 (b) ₹ 21,200
(c) ₹ 28,200 (d) ₹ 20,500
(e) None of these

56. A is a working partner and B is a sleeping partner in a business. A puts in ₹ 50,000 and B ₹ 60,000. A gets 12.5% of the profit for managing the business, and the rest is divided in proportion to their capitals. Find the share of A in profit of ₹ 8800.
(a) ₹ 3500 (b) ₹ 4600
(c) ₹ 5400 (d) ₹ 4800
(e) None of these

57. A began business with ₹ 12500 and is joined afterwards by B with ₹ 37500. When did B join, if the profits at the end of the year are divided equally?
(a) 8 months (b) 9 months
(c) 10 months (d) 7 months
(e) None of these

58. A began business with ₹ 45,000 and was later joined by B with ₹ 54,000. When did B join if the profit at the end of the year were divided in the ratio 2 : 1?
(a) 5 months after (b) 10 months after
(c) 7 months after (d) 12 months after
(e) None of these

59. A and B enter into partnership with capitals in the ratio 3 : 4. At the end of 10 months A withdraws, and the profits now are divided in the ratio of 5 : 6. Find how long B remained in the business?
(a) 9 months (b) 8 months
(c) 6 months (d) 7 months
(e) None of these

60. A and B invest ₹ 3,000 and ₹ 4,000 in a business. A receives ₹ 10 per month out of the profit as a remuneration for running the business and the rest of profit is divided in proportion to the investments. If in a year 'A' totally receives ₹ 390, what does B receive?
(a) ₹ 375 (b) ₹ 360
(c) ₹ 350 (d) ₹ 260
(e) None of these

61. A started a business with ₹ 4500 and another person B joined after some period with ₹ 3000. Determine this period after B joined the business if the profit at the end of the year is divided in the ratio 2 : 1
(a) After 3 months (b) After 4 months
(c) After 6 months (d) After $2\frac{1}{2}$ months
(e) None of these

62. A and B entered into a partnership with capitals in the ratio of 4 : 5. After 3 months, A withdrew $\frac{1}{4}$ of his capital and B withdrew $\frac{1}{5}$ of his capital. The gain at the end of 10 months was ₹ 760. Find the profit of B.
(a) ₹ 450 (b) ₹ 430
(c) ₹ 410 (d) ₹ 340
(e) None of these

63. A and B rent a pasture for 10 months; A puts in 80 cows for 7 months. How many can B put in for the remaining 3 months, if he pays half as much again as A?
(a) 120 (b) 180
(c) 200 (d) 280
(e) None of these

64. In a partnership between X and Y, X's capital is $\frac{2}{5}$ of total and is invested for $\frac{2}{3}$ year. If his share of the profit is $\frac{4}{7}$ of the total, for how long is Y's capital in the business?
(a) 1 year (b) $\frac{1}{8}$ years
(c) $\frac{1}{3}$ years (d) $\frac{1}{4}$ years
(e) None of these

65. X and Y put in ₹ 3,000 and ₹ 4,000 respectively into a business. X reinvests into the business his share of the first year's profit of ₹ 2,100 whereas Y does not reinvest. In what ratio should they share the second year's profit?
(a) 39 : 40 (b) 3 : 4
(c) 3 : 7 (d) 40 : 79
(e) None of these

66. Gold is 19 times as heavy as water and copper is 9 times heavy. In what ratio must these metals be mixed so that the mixture may be 15 times as heavy as water?
(a) 2 : 3 (b) 3 : 2
(c) 1 : 3 (d) 2 : 1
(e) None of these

67. Six litres of a 20% solution of alcohol in water are mixed with 4 litres of a 60% solution of alcohol in water. The % alcoholic strength of the mixture is
(a) 80 (b) 40
(c) 36 (d) 48
(e) None of these

68. A merchant lent out ₹ 1,000 in two parts, one at 8% and the other at 10% interest. The yearly average comes out to be 9.2%. Find the amount lent in two parts.
(a) ₹ 400, ₹ 600 (b) ₹ 500, ₹ 500
(c) ₹ 300, ₹ 700 (d) cannot be determined
(e) None of these

69. One litre of water was mixed to 3 litres of sugar solution containing 4% of sugar. What is the percentage of sugar in the solution?
(a) 3 (b) 4
(c) 6 (d) Insuffficient data
(e) None of these

70. How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for ₹ 20 so as to have a mixture worth ₹ $10\frac{2}{3}$ a litre?
(a) 10 litres (b) 12 litres
(c) 15 litres (d) 18 litres
(e) None of these

71. How many kg of salt at 42 paise per kg must a man mix with 25 kg of salt at 24 paise per kg so that he may, on selling the mixture at 40 paise per kg gain 25% on the outlay?
(a) 15 kg (b) 18 kg
(c) 20 kg (d) 24 kg
(e) None of these

72. A trader mixes 80 kg of tea at ₹15 per kg with 20 kg of tea at cost price of ₹ 20 per kg. In order to earn a profit of 25%, what should be the sale price of the mixed tea?
(a) ₹ 23.75 (b) ₹ 22
(c) ₹ 20 (d) ₹ 19.20
(e) None of these

73. A company blends two varieties of tea from two different tea gardens, one variety costing ₹ 20 per kg and other ₹ 25 per kg, in the ratio 5 : 4. He sells the blended tea at ₹ 23 per kg. Find his profit percent :
(a) 5% profit (b) 3.5% loss
(c) 3.5% profit (d) No profit, no loss
(e) None of these

74. Alcohol cost ₹ 3.50 per litre and kerosene oil cost ₹ 2.50 per litre. In what proportion these should be mixed so that the resulting mixture may be ₹ 2.75 per litre?
(a) 2 : 5 (b) 1 : 3
(c) 4 : 7 (d) 2 : 3
(e) None of these

75. Pure milk costs ₹ 3.60 per litre. A milkman adds water to 25 litres of pure milk and sells the mixture at ₹ 3 per litre. How many litres of water does he add?
(a) 2 litres (b) 5 litres
(c) 7 litres (d) 11 litres
(e) None of these

CHAPTER 6

Profit and Loss

INTRODUCTION

Cost Price

The amount paid to purchase an article or the price at which an article is made, is known as its cost price.

The cost price is abbreviated as C.P.

Selling Price

The price at which an article is sold, is known as its selling price.

The selling price is abbreviated as S.P.

Profit

If the selling price (S.P.) of an article is greater than the cost price (C.P.), then the difference between the selling price and cost price is called profit.

Thus, If S.P. > C.P., then

Profit = S.P. – C.P.

$\Rightarrow$ S.P. = C.P. + Profit

$\Rightarrow$ C.P. = S.P. – Profit.

Loss

If the selling price (S.P.) of an article is less than the cost price (C.P.), then the difference between the cost price (C.P.) and the selling price (S.P.) is called loss.

Thus, if S.P. < C.P., then

Loss = C.P. – S.P.

$\Rightarrow$ C.P. = S.P. + Loss

$\Rightarrow$ S.P. = C.P. – Loss

EXAMPLE 1. An article was bought for ₹ 2000 and sold for ₹ 2200. Find the gain or loss.

Sol. C.P. of the article = ₹ 2000

S.P. of the article = ₹ 2200

Since S.P. > C.P. So there is gain.

Gain (profit) = S.P. – C.P.

= ₹ 2200 – ₹ 2000 = ₹ 200

Profit and Loss percentage

The profit per cent is the profit that would be obtained for a C.P. of ₹ 100.

Similarly, the loss per cent is the loss that would be made for a C.P. of ₹ 100.

$$\text{Profit per cent} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$\text{Loss per cent} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

REMEMBER

★ $\text{Profit} = \dfrac{\text{C.P.} \times \text{Profit \%}}{100}$

★ $\text{Loss} = \dfrac{\text{C.P.} \times \text{Loss \%}}{100}$

★ $\text{S.P.} = \left(\dfrac{100 + \text{Profit\%}}{100}\right) \times \text{C.P.}$

★ $\text{S.P.} = \left(\dfrac{100 - \text{Loss\%}}{100}\right) \times \text{C.P.}$

★ $\text{C.P.} = \dfrac{100 \times \text{S.P.}}{100 + \text{Profit \%}}$

★ $\text{C.P.} = \dfrac{100 \times \text{S.P.}}{100 - \text{Loss \%}}$

NOTE

(i) *If an article is sold at a certain gain (say 45%), then SP = 145% of CP*

(ii) *If an article is sold at certain loss (say 25%), then SP = 75% of CP.*

EXAMPLE 2. A cycle was purchased for ₹ 1600 and sold for ₹ 1400. Find the loss and loss %.

Sol. C.P. of the cycle = ₹ 1600

S.P. of the cycle = ₹ 1400

Since S.P < C.P, so there is a loss.

Loss = C.P. – S.P.

= ₹ 1600 – ₹ 1400 = ₹ 200.

$$\text{Loss \%} = \frac{\text{Loss}}{\text{C.P.}} \times 100 = \frac{200}{1600} \times 100 = 12\frac{1}{2}\%$$

EXAMPLE 3. By selling a table for ₹ 330, a trader gains 10%. Find the cost price of the table.

Sol. S.P. = ₹ 330, Gain = 10%

$$\therefore \quad \text{C.P.} = \left(\frac{100}{100 + \text{Gain }\%}\right) \times \text{S.P}.$$

$$= ₹\ \frac{100}{100+10} \times 330 = \frac{100}{110} \times 330 = ₹\ 300.$$

EXAMPLE 4. A sells a bicycle to B at a profit of 20% and B sells it to C at a profit of 25%. If C pays ₹ 225 for it, what did A pay for it.

Sol. $\text{C.P. of A} = 225 \times \frac{100}{100+20} \times \frac{100}{100+25}$

$$= 225 \times \frac{100}{120} \times \frac{100}{125} = ₹\ 150.$$

EXAMPLE 5. A mobile phone is sold for ₹ 5060 at a gain of 10%. What would have been the gain or loss per cent if it had been sold for ₹ 4370 ?

Sol. S.P. = ₹ 5060, gain = 10%

$$\therefore \text{C.P.} = \frac{5060 \times 100}{100+10} = ₹\ 4600.$$

2nd S.P. = ₹ 4370

Since, S.P. < C.P., so there is loss.

$$\therefore \text{Loss }\% = \frac{(4600-4370)\times 100}{4600} = 5\%$$

Shortcut Approach

Dishonest dealing

$$\text{Gain }\% = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

$$\frac{\text{True Scale}}{\text{False Scale}} = \frac{100 + \text{gain}\%}{100 - \text{loss}\%}$$

EXAMPLE 6. A cloth merchant says that due to slump in the market, he sells the cloth at 10% loss, but he uses a false metre-scale and actually gain 15%. Find the actual length of the scale.

(a) 78 cm **(b) 78.25 cm**
(c) 78.5 cm **(d) 78.75 cm**

Sol. (b) $\frac{\text{True scale}}{\text{False scale}} = \frac{100+\text{gain}\%}{100-\text{loss}\%}$

$$\frac{100}{\text{False scale}} = \frac{100+15}{100-10}$$

$$\Rightarrow \text{False scale} = \frac{100 \times 90}{115} = 78.26\,\text{cm}$$

Shortcut Approach

Real Profit/Loss percentage :

If the profit or loss is calculated on S.P., then it is not actual profit or loss.

Real profit (loss)% is the profit (loss)% on C.P.

$$\text{Real Profit }\% = \frac{\%\text{ profit on S.P.}}{100 - \%\text{ profit on S.P.}} \times 100$$

EXAMPLE 7. A dishonest dealer professes to sell his goods at cost price, but he uses a weight of 960 g for the kg weight. Find his gain per cent.

Sol. Error = 1 kg – 960 g

= 1000 g – 960 g = 40 g.

$$\therefore \quad \text{Gain }\% = \frac{40}{1000-40} \times 100$$

$$= \frac{40}{960} \times 100 = 4\frac{1}{6}\%$$

Shortcut Approach

Goods passing through successive hands

When there are two successive profits of a% and b%, then the resultant profit per cent is given by

$$\left(a + b + \frac{ab}{100}\right)\%$$

When there are two successive loss a% and b%, then the resultant loss per cent is given by $\left(-a - b + \frac{ab}{100}\right)\%$

When there is a profit of a% and loss by b% in a transaction, then the resultant profit or loss per cent is given by $\left(a - b - \frac{ab}{100}\right)\%$, according to the +ve or –ve sign respectively.

When cost price and selling price are reduced by the same amount (A) and profit increases then cost price (C.P.)

$$= \frac{[\text{Initial profit }\% + \text{Increase in profit }\%] \times A}{\text{Increase in profit }\%}$$

EXAMPLE 8. A table is sold at a profit of 20%. If the cost price and selling price are ₹ 200 less, the profit would be 8% more. Find the cost price.

Sol. $\text{C.P.} = ₹\frac{(20+8)\times 200}{8} = ₹\ 28 \times 25 = ₹\ 700.$

☞ Shortcut Approach

If cost price of x articles is equal to the selling price of y articles, then profit/loss percentage = $\frac{x-y}{y}\times 100\%$, according to +ve or –ve sign respectively.

EXAMPLE 9. If the C.P. of 15 tables be equal to the S.P. of 20 tables, find the loss per cent.

Sol. Profit/Loss% = $\frac{-5}{20}\times 100$ = 25% loss, since it is –ve.

EXAMPLE 10. If the cost price of 20 articles is equal to the selling price of 18 articles, then find the profit per cent.

Sol. Here, x = 20 and y = 18

According to the formula,

$$\text{Profit}\,\% = \left(\frac{x-y}{y}\times 100\right)\% = \left(\frac{20-18}{18}\times 100\right)\%$$

$$= \frac{100}{9}\% = 11\frac{1}{9}\%$$

EXAMPLE 11. If the C.P. of 6 articles is equal to the S.P. of 4 articles. Find the gain per cent.

Sol. Let C.P. of an article be ₹ x; then,

C.P. of 6 articles = ₹ 6x

C.P. of 4 articles = ₹ 4x

But S.P. of 4 articles = C.P. of 6 articles

∴ S.P. of 4 articles = 6x

Thus, gain = S.P – C.P. = ₹ (6x – 4x) = ₹ 2x

$$\therefore \text{Gain}\,\% = \frac{2x}{4x}\times 100 = 50$$

Thus, gain in the transaction = 50%

EXAMPLE 12. By selling 33 metres of cloth, a man gains the sale price of 11 metres. The gain % is

(a) 50% **(b)** 25%

(c) $33\frac{1}{3}$% **(d)** 20%

Sol. (a) Gain = S.P. of 33 metres – C.P. of 33 metres

= S.P. of 11 metres

⇒ S.P. of 22 metres = C.P. of 33 metres

$$\therefore \%\,\text{gain} = \frac{\text{gain}}{\text{C.P. of metres}}\times 100$$

$$= \frac{\text{S.P. of 11 metres}}{\text{C.P. of 33 metres}}\times 100$$

$$= \frac{\text{S.P. of 11 metres}}{\text{S.P. of 22 metres}}\times 100 = \frac{11}{22}\times 100 = 50\%$$

SHORTCUT METHOD

If on selling 'x' articles a man gains equal to the S.P. of y articles. Then,

$$\%\,\text{gain} = \frac{y}{x-y}\times 100 = \frac{11}{33-11}\times 100 = \frac{11}{22}\times 100 = 50\%$$

☞ Shortcut Approach

- A man purchases a certain number of articles at x a rupee and the same number at y a rupee. He mixes them together and sells them at z a rupee. Then his gain or loss %

  $$= \left[\frac{2xy}{z(x+y)} - 1\right]\times 100$$ according as the sign is +ve or –ve.

- If two items are sold, each at ₹. x, one at a gain of p% and the other at a loss of p%, there is an overall loss given by $\frac{p^2}{100}$%. The absolute value of the loss is given by $\frac{2p^2x}{100^2 - p^2}$.

- If CP of two items is the same and % Loss and % Gain on the two items are equal, then net loss or net profit is zero.

EXAMPLE 13. A shopkeeper sold two radio sets for ₹ 792 each, gaining 10% on one, and losing 10% on the other. Then he

(a) neither gains nor loses **(b)** gains 1%

(c) loses 1% **(d)** gains 5%

Sol. (c) When selling price of two articles is same and

% gain = % loss

then there will be always loss.

and overall % loss = $\frac{(10)^2}{100}\% = 1\%$

EXAMPLE 14. A man bought two housing apartments for ₹ 2 lakhs each. He sold one at 20% loss and the other at 20% gain. Find his gain or loss.

(a) 4% loss **(b)** 4 % gain

(c) No loss, no gain **(d)** 10 % loss

Sol. (c) When C.P. of two articles is same and

% gain = % loss

Then, on net, there is no loss, no gain

when two different articles sold at same S.P. and x_1 and x_2 are % gain (or loss) on them. Then, overall % gain or loss

$$= \left[\frac{100 - 2(100 \pm x_1)(100 \pm x_2)}{(100 \pm x_1) + (100 \pm x_2)}\right]\%$$

(Taking + or – according to gain or loss)

EXAMPLE 15. A man sold two watches for ₹ 1000 each. On one he gains 25% and on the other 20% loss. Find how much % does he gain or lose in the whole transaction?

(a) $\frac{100}{41}$**% loss** **(b)** $\frac{100}{41}$**% gain**

(c) No gain, no loss **(d) Cannot be determined**

Sol. (b) When $S_1 = S_2$, then

overall % gain or % loss

$$= \left[100 - \frac{2(100 + x_1)(100 - x_2)}{(100 + x_1) + (100 - x_2)}\right]\%$$

$$= \left(100 - \frac{2(125)(80)}{(125) + (80)}\right)\% = \left(100 - \frac{2 \times 125 \times 80}{205}\right)\%$$

$$= \frac{100}{41}\% \text{ gain } (\because \text{ it is +ve})$$

Shortcut Approach

A businessman sells his items at a profit/loss of a%. If he had sold it for ₹ R more, he would have gained/lost b%. Then,

$$\text{CP of items} = \frac{R}{b \pm a} \times 100$$

'–' = When both are either profit or loss
'+' = When one is profit and other is loss

EXAMPLE 16. A person sold a table at a profit of $6\frac{1}{2}$%. If he had sold it for ₹ 1250 more, he would have gained 19%. Find the CP of the table.

Sol. Here, $a = 6\frac{1}{2}\% = \frac{13}{2}\%$

$b = 19\%$ and $R = ₹\,1250$

According to the formula,

$$\text{CP of table} = \frac{R}{b - a} \times 100$$

$$= \frac{1250}{19 - \frac{13}{2}} \times 100 = \frac{1250 \times 2}{25} \times 100$$

$$= ₹\,10000$$

Shortcut Approach

If A sold an article to B at a profit (loss) of r_1 % and B sold this article to C at a profit (loss) of r_2 %, then cost price of article for C is given by (cost price for A) $\times \left(1 \pm \frac{r_1}{100}\right)\left(1 \pm \frac{r_2}{100}\right)$.

EXAMPLE 17. Nikunj sold a machine to Sonia at a profit of 30%. Sonia sold this machine to Anu at a loss of 20%. If Nikunj paid ₹ 5000 for this machine, then find the cost price of machine for Anu.

Sol. Here $r_1 = 30\%$ and $r_2 = 20\%$

CP of a machine for Nikunj = ₹ 5000

∴ CP of machine for Anu = CP of machine for Nikunj

$$\left(1 + \frac{r_1}{100}\right)\left(1 - \frac{r_2}{100}\right)$$

$$= 5000\left(1 + \frac{30}{100}\right)\left(1 - \frac{20}{100}\right) = 5000 \times \frac{130}{100} \times \frac{80}{100} = ₹\,5200$$

Shortcut Approach

If a man purchases m items for ₹ x and sells n items for ₹ y, then

$$\text{Profit or loss per cent is given by } \frac{my - nx}{nx} \times 100\%$$

[Positive result means profit and negative result means loss].

EXAMPLE 18. If Karan purchases 10 or anges for ₹ 25 and sells 9 oranges for ₹ 25, then find the gain percentage.

Sol. Here, m = 10, x = 25, n = 9 and y = 25

$$\therefore \text{ Profit per cent} = \frac{my - nx}{nx} \times 100\%$$

$$= \frac{25 \times 10 - 9 \times 25}{9 \times 25} \times 100\%$$

$$= \frac{250 - 225}{225} \times 100\%$$

$$= \frac{25}{255} \times 100\% = \frac{100}{9}\% = 11\frac{1}{9}\%$$

Marked Price

The price on the lable is called the marked price or list price. The marked price is abbreviated as M.P.

Discount

The reduction made on the 'marked price' of an article is called the discount.

NOTE : *When no discount is given, 'selling price' is the same as 'marked price'.*

- Discount = Marked price × Rate of discount.
- S.P. = M.P. – Discount.
- Discount % = $\dfrac{\text{Discount}}{\text{M.P.}} \times 100$.
- Buy x get y free i.e., if x + y articles are sold at cost price of x articles, then the percentage discount = $\dfrac{y}{x+y} \times 100$.

EXAMPLE 19. How much % must be added to the cost price of goods so that a profit of 20% must be made after throwing off a discount of 10% from the marked price?

(a) **20%** **(b)** **30%**

(c) **$33\frac{1}{3}$ %** **(d)** **25%**

Sol. (c) Let C.P. = ₹ 100, then S.P. = ₹ 120

Also, Let marked price be ₹ x. Then

90% of x = 120

$$\Rightarrow x = \frac{120 \times 100}{90} = 133\frac{1}{3}$$

$\therefore$ M.P. should be ₹ $133\frac{1}{3}$

or M.P. = $33\frac{1}{3}$% above C.P.

EXAMPLE 20. At a clearance sale, all goods are on sale at 45% discount. If I buy a skirt marked ₹ 600, how much would I need to pay?

Sol. M.P. = ₹ 600, Discount = 45%

$$\text{Discount} = \frac{\text{M.P.} \times \text{Discount\%}}{100} = \frac{600 \times 45}{100} = ₹\,270.$$

$\therefore$ S.P. = M.P. – Discount

= ₹ 600 – ₹ 270 = ₹ 330.

Hence, the amount I need to pay is ₹ 330.

EXAMPLE 21. After allowing a discount of 12% on the marked price of an article, it is sold for ₹ 880. Find its marked price.

Sol. S.P. = ₹ 880 and Discount % = 12

Let M.P. = x

$$\text{Discount} = \frac{\text{M.P.} \times \text{Discount\%}}{100} = \frac{x \times 12}{100} = \frac{3}{25}x$$

Now, M.P. = S.P. + Discount

$$x = 880 + \frac{3}{25}x$$

$$\Rightarrow x - \frac{3}{25}x = 880 \Rightarrow \frac{22x}{25} = 880$$

$$\Rightarrow x = \frac{880 \times 25}{22} = 40 \times 25 = ₹\,1000$$

$\therefore$ Marked price of the article is ₹ 1000.

EXAMPLE 22. A shopkeeper offers his customers 10% discount and still makes a profit of 26%. What is the actual cost to him of an article marked ₹ 280?

Sol. M.P. = ₹ 280 and Discount % = 10

$$\text{Discount} = \frac{\text{M.P.} \times \text{Discount \%}}{100} = \frac{280 \times 10}{100} = ₹\,28$$

S.P. = M.P. – Discount = ₹ 280 – ₹ 28 = ₹. 252

Now, S.P. = ₹. 252 and profit = 26%

$$\therefore \text{C.P.} = \frac{100}{100 + \text{gain \%}} \times \text{S.P.}$$

$$= \frac{100}{100+26} \times 252 = ₹\,200$$

Hence, the actual cost of the article is ₹ 200.

REMEMBER

★ In **successive discounts**, first discount is subtracted from the marked price to get net price after the first discount. Taking this price as the new marked price, the second discount is calculated and it is subtracted from it to get net price after the second discount. Continuing in this manner, we finally obtain the final selling price.

In case of successive discounts a% and b%, the effective discount is $\left(a + b - \dfrac{ab}{100}\right)\%$

EXAMPLE 23. Find the single discount equivalent to successive discounts of 15% and 20%.

Sol. By direct formula,

$$\text{Single discount} = \left(a + b - \frac{ab}{100}\right)\%$$

$$= \left(15 + 20 - \frac{15 \times 20}{100}\right)\% = 32\,\%$$

NOTE : *If the list price of an item is given and discounts d_1 and d_2 are given succesively on it then,*

$$\textit{Final price} = \textit{list price}\left(1 - \frac{d_1}{100}\right)\left(1 - \frac{d_2}{100}\right)$$

EXAMPLE 24. An article is listed at ₹ 65. A customer bought this article for ₹ 56.16 and got two successive discounts of which the first one is 10%. The other rate of discount of this scheme that was allowed by the shopkeeper was :

(a) 3 % **(b)** 4 %

(c) 6 % **(d)** 2 %

Sol. (b) Price of the article after first discount

$65 - 6.5 = ₹\ 58.5$

Therefore, the second discount

$$= \frac{58.5 - 56.16}{58.5} \times 100 = 4\%$$

EXAMPLE 25. A shopkeeper offers 5% discount on all his goods to all his customers. He offers a further discount of 2% on the reduced price to those customers who pay cash. What will you actually have to pay for an article in cash if its M.P. is ₹ 4800?

Sol. M.P. = ₹ 4800

First discount = 5% of M.P.

$$= \frac{5}{100} \times 4800 = ₹\ 240$$

Net price after discount = ₹ 4800 – ₹ 240

= ₹ 4560

Second discount = 2% of ₹ 4560

$$= \frac{2}{100} \times 4560 = ₹\ 91.20$$

Net price after discount = ₹ 4560 – ₹ 91.20

= ₹ 4468.80

SHORTCUT METHOD

$$\text{S.P.} = 4800\left(1 - \frac{5}{100}\right)\left(1 - \frac{2}{100}\right) = ₹\ 4468.80$$

SALES TAX

To meet government's expenditures like construction of roads, railway, hospitals, schools etc. the government imposes different types of taxes. Sales tax (S.T.) is one of these tax.
Sales tax is calculated on selling price (S.P.)

NOTE : *If discount is given, selling price is calculated first and then sales tax is calculated on the selling price of the article.*

EXAMPLE 26. Sonika bought a V.C.R. at the list price of ₹ 18,500. If the rate of sales tax was 8%, find the amount she had to pay for purchasing the V.C.R.

Sol. List price of V.C.R. = ₹ 18,500

Rate of sales tax = 8%

∴ Sales tax = 8% of ₹ 18,500

$$= \frac{8}{100} \times 18500 = ₹\ 1480$$

So, total amount which Sonika had to pay for purchasing the V.C.R. = ₹ 18,500 + ₹ 1480

= ₹ 19,980.

EXAMPLE 27. The sale price of an article including the sales tax is ₹ 616. The rate of sales tax is 10%. If the shopkeeper has made a profit of 12%, then the cost price of the article is :

(a) ₹ 500 **(b)** ₹ 515

(c) ₹ 550 **(d)** ₹ 600

Sol. (a) Let the CP of the article be ₹ x

Then, SP = $x \times 1.12 \times 1.1$

Now, $x \times 1.12 \times 1.1 = 616$

$$\Rightarrow x = \frac{616}{1.232} = ₹\ 500$$

Shortcut Approach

If 'a'th part of some items is sold at x% loss, then required gain per cent in selling rest of the items in order that there is neither gain nor loss in whole transaction, is $\frac{ax}{1-a}\%$

EXAMPLE 28. A medical store owner purchased medicines worth ₹ 6000 from a company. He sold 1/3 part of the medicine at 30% loss. On which gain he should sell his rest of the medicines, so that he has neither gain nor loss?

Sol. Given, $a = \frac{1}{3}$ and $x = 30\%$

According to the formula,

$$\text{Required gain \%} = \frac{ax}{1-a}\% = \frac{\frac{1}{3} \times 30}{1 - \frac{1}{3}}\% = \frac{10 \times 3}{2}\% = 15\%$$

EXERCISE

1. If by selling twelve note-books, the seller earns profit equal to the selling price of two note-books, what is his percentage profit?
(a) 20% (b) 25%
(c) $16\frac{2}{3}\%$ (d) Data inadequate
(e) None of these
2. A grocer purchased 20 kg of rice at the rate of ₹ 15 per kg and 30 kg of rice at the rate of ₹ 13 per kg. At what price per kg should he sell the mixture to earn $33\frac{1}{3}\%$ profit on the cost price?
(a) ₹ 28.00 (b) ₹ 20.00
(c) ₹ 18.40 (d) ₹ 17.40
(e) None of these
3. By selling an article for ₹ 96, double profit is obtained than the profit that would have been obtained by selling it for ₹ 84. What is the cost price of the article?
(a) ₹ 72.00 (b) ₹ 75.00
(c) ₹ 70.00 (d) ₹ 68.00
(e) None of these
4. A shopkeeper sold a TV set for ₹ 17940, at a discount of 8% and gained 19.6%. If no discount is allowed, what will be his gain per cent?
(a) 25% (b) 36.4%
(c) 24.8% (d) Can't be determined
(e) None of these
5. Deepa bought a calculator at 30% discount on the listed price. Had she not got the discount, she would have paid ₹ 82.50 extra. At what price did she buy the calculator ?
(a) ₹ 192.50 (b) ₹ 275
(c) ₹ 117.85 (d) Cannot be determined
(e) None of these
6. A shopkeeper sells a TV set for ₹ 16560 at 10% discount on its marked price and earns 15% profit. If no discount is offered, then what will be his present per cent profit?
(a) $27\frac{7}{9}$ (b) $22\frac{7}{9}$
(c) $25\frac{7}{9}$ (d) Data inadequate
(e) None of these
7. A builder purchased a plot of land for ₹ 80 lakh and constructed a five-storey building inclusive of ground floor on it. How much should he charge for each flat to make 25% profit on his investment on land, if there are five flats on each storey?
(a) ₹ 50000 (b) ₹ 100000
(c) ₹ 500000 (d) ₹ 2000000
(e) None of these
8. A trader purchased on old bicycle for ₹ 480. He spent 20% of the cost on its repair. If he wants to earn ₹ 144 as net profit on it, how much percentage should he add to the purchase price of the bicycle?
(a) 50% (b) 48%
(c) 96% (d) 100%
(e) None of these
9. The price of 2 sarees and 4 shirts is ₹ 16000. With the same money one can buy 1 saree and 6 shirts. If one wants to buy 12 shirts, how much shall one have to pay?
(a) ₹ 2,400 (b) ₹ 4,800
(c) ₹ 1,200 (d) Cannot be determined
(e) None of these
10. A shopkeeper bought 150 calculators at the rate of ₹ 250 per calculator. He spent ₹ 2500 on transportation and packing. If the marked price of calculator is ₹ 320 per calculator and the shopkeeper gives a discount of 5% on the marked price then what will be the percentage profit gained by the shopkeeper?
(a) 20% (b) 14%
(c) 15% (d) 16%
(e) None of these
11. A garment company declared 15% discount for whole sale buyers. Mr Sachdev bought garments from the company for ₹ 25,000 after getting discount. He fixed up the selling price of garments in such a way that he earned a profit of 8% on original company price. What is the **approximate** total selling price?
(a) ₹ 28,000 (b) ₹ 29,000
(c) ₹ 32,000 (d) ₹ 28,500
(e) ₹ 29,500
12. A shopkeeper sold an article for ₹ 720 after giving 10% discount on the labelled price and made 20% profit on the cost price. What would have been the percentage profit, had he not given the discount'?
(a) 25% (b) 30%
(c) 23% (d) 28%
(e) None of these
13. The difference between a discount of 35% and two successive discounts of 20% and 20% on a certain bill was ₹ 22. Find the amount of the bill.
(a) ₹ 1,100 (b) ₹ 200
(c) ₹ 2,200 (d) Data inadequate
(e) None of these
14. A shopkeeper labels the price of articles 20% above the cost price. If he allows ₹ 31.20 off on a bill of ₹ 312, find his profit per cent on the article?
(a) 8 (b) $12\frac{1}{3}$
(c) $11\frac{2}{3}$ (d) $8\frac{1}{3}$
(e) None of these

15. A shopkeeper sold an article offering a discount of 5% and earned a profit of 23.5%. What would have been the percentage of profit earned if no discount had been offered?
(a) 28.5 (b) 27.675
(c) 30 (d) Data inadequate
(e) None of these

16. A shopkeeper sold sarees at ₹ 266 each after giving 5% discount on labelled price. Had he not given the discount, he would have earned a profit of 12% on the cost price. What was the cost price of each saree?
(a) ₹ 280 (b) ₹ 260
(c) ₹ 38 mph (d) Data inadequate
(e) None of these

17. The profit earned by selling an article for ₹ 832 is equal to the loss incurred when the same article is sold for ₹ 448. What should be the sale price of the article for making 50 per cent profit?
(a) ₹ 960 (b) ₹ 1060
(c) ₹ 1,200 (d) ₹ 920
(e) None of these

18. Prabhu purchased 30 kg of rice at the rate of ₹ 17.50 per kg and another 30 kg rice at a certain rate. He mixed the two and sold the entire quantity at the rate of ₹ 18.60 per kg and made 20 per cent overall profit. At what price per kg did he purchase the lot of another 30 kg rice?
(a) ₹ 14.50 (b) ₹ 12.50
(c) ₹ 15.50 (d) ₹ 13.50
(e) None of these

19. An article when sold for ₹ 200 fetches 25 per cent profit. What would be the percentage profit/loss if 6 such articles are sold for ₹ 1,056?
(a) 10 pre cent loss (b) 10 per cent profit
(c) 5 per cent loss (d) 5 per cent profit
(e) None of these

20. A shopkeeper gave an additional 20 per cent concession on the reduced price after giving 30 per cent standard concession on an article. If Arun bought that article for ₹ 1,120, what was the original price?
(a) ₹ 3,000 (b) ₹ 4,000
(c) ₹ 2,400 (d) ₹ 2,000
(e) None of these

21. What per cent of selling price would be 34% of cost price if gross profit is 26% of the selling price?
(a) 17.16 (b) 74.00
(c) 25.16 (d) 88.40
(e) None of these

22. A man sold 10 eggs for 5 rupees and gained 20%.How many eggs did he buy for 5 rupees?
(a) 10 eggs (b) 12 eggs
(c) 14 eggs (d) 16 eggs
(e) None of these

23. A person sells 36 oranges per rupee and suffers a loss of 4%. Find how many oranges per rupee to be sold to have a gain of 8%?
(a) 30 (b) 31
(c) 32 (d) 33
(e) None of these

24. Coconuts were purchased at ₹ 150 per hundred and sold at ₹ 2 per coconut. If 2000 coconuts were sold, what was the total profit made?
(a) ₹ 500 (b) ₹ 1000
(c) ₹ 1500 (d) ₹ 2000
(e) None of these

25. A shopkeeper's price is 50% above the cost price. If he allows his customer a discount of 30% what profit does he make?
(a) 5% (b) 10%
(c) 15% (d) 20%
(e) None of these

26. A shopkeeper purchases 10 kg of rice at ₹ 600 and sells at a loss as much the selling price of 2 kg of rice . Find the sale rate of rice/ kg.
(a) ₹ 60 per kg (b) ₹ 50 per kg
(c) ₹ 80 per kg (d) ₹ 70 per kg
(e) None of these

27 If 15 oranges are bought for a rupee, how many must be sold for a rupee to gain 25%?
(a) 12 (b) 10
(c) 20 (d) 18
(e) None of these

28. A man buys milk at ₹ 6 per litre and adds one third of water to it and sells mixture at ₹ 7.20 per litre. The gain is
(a) 40% (b) 80%
(c) 60% (d) 25%
(e) None of these

29. A milk man makes a profit of 20% on the sale of milk. If he were to add 10% water to the milk, by what %would his profit increase?
(a) 30 (b) $\frac{40}{3}$
(c) 22 (d) 10
(e) None of these

30. A grocer purchased 80 kg of sugar at ₹ 13.50 per kg and mixed it with 120 kg sugar at ₹ 16 per kg. At what rate should he sell the mixture to gain 16%?
(a) ₹ 17 per kg (b) ₹ 17.40 per kg
(c) ₹ 16.5 per kg (d) ₹ 16 per kg
(e) None of these

31. A dishonest fruit seller professes to sell his goods at the cost price but weighs 800 grams for a kg weight. Find his gain percent.
(a) 100% (b) 150%
(c) 50% (d) 200%
(e) None of these

32. A shopkeeper purchased 150 identical pieces of calculators at the rate of ₹ 250 each. He spent an amount of ₹ 2500 on transport and packing. He fixed the labelled price of each calculator at ₹ 320. However, he decided to give a discount of 5% on the labelled price. What is the percentage profit earned by him ?
(a) 14% (b) 15%
(c) 16% (d) 20%
(e) None of these

33. A dishonest dealer sells his goods at the cost price but still earns a profit of 25% by underweighing. What weight does he use for a kg?
(a) 750 g (b) 800 g
(c) 825 g (d) 850 g
(e) None of these

34. A shopkeeper marks up his goods to gain 35%. But he allows 10% discount for cash payment. His profit on the cash transaction therefore, in percentage, is
(a) $13\frac{1}{2}$ (b) 25
(c) $21\frac{1}{2}$ (d) $31\frac{1}{2}$
(e) None of these

35. A man sold two steel chairs for ₹ 500 each. On one he gains 20% and on other, he loses 12%. How much does he gain or lose in the whole transaction?
(a) 1.5% gain (b) 2% gain
(c) 1.5% loss (d) 2% loss
(e) None of these

36. A firm of readymade garments makes both men's and women's shirts. Its average profit is 6% of the sales. Its profit in men's shirts average 8% of the sales and women's shirts comprise 60% of the output. The average profit per sale rupee in women shirts is
(a) 0.0466 (b) 0.0666
(c) 0.0166 (d) 0.0366
(e) None of these

37. A man purchases two watches at ₹ 560. He sells one at 15% profit and other at 10% loss. Then he neither gains nor lose. Find the cost price of each watch.
(a) ₹ 224, ₹ 300 (b) ₹ 200, ₹ 300
(c) ₹ 224, ₹ 336 (d) ₹ 200, ₹ 336
(e) None of these

38. A man bought a horse and a carriage for ₹ 3000. He sold the horse at a gain of 20% and the carriage at a loss 10%, thereby gaining 2% on the whole. Find the cost of the horse.
(a) ₹ 1000 (b) ₹ 1200
(c) ₹ 1500 (d) ₹ 1700
(e) None of these

39. Two electronic musical instruments were purchased for ₹ 8000. The first was sold at a profit of 40% and the second at loss of 40%. If the sale price was the same in both the cases, what was the cost price of two electronic musical instruments?
(a) ₹ 2000, ₹ 5000 (b) ₹ 2200, ₹ 5500
(c) ₹ 2400, ₹ 5000 (d) ₹ 2400, ₹ 5600
(e) None of these

40. A man sells an article at a gain 15%. If he had bought it at 10% less and sold it for ₹ 4 less, he would have gained 25%. Find the cost price of the article.
(a) ₹ 150 (b) ₹ 160
(c) ₹ 170 (d) ₹ 180
(e) None of these

41. A man sells an article at 5% profit. If he had bought it at 5% less and sold it for ₹ 1 less, he would have gained 10%. The cost price of the article is :
(a) ₹ 200 (b) ₹ 150
(c) ₹ 240 (d) ₹ 280
(e) None of these

42. Five kg of butter was bought by a shopkeeper for ₹ 300. One kg becomes unsaleable. He sells the remaining in such a way that on the whole he incurs a loss of 10%. At what price per kg was the butter sold?
(a) ₹ 67.50 (b) ₹ 52.50
(c) ₹ 60 (d) ₹ 72.50
(e) None of these

43. A fruitseller sells 8 oranges at a cost price of 9. The profit per cent is
(a) $12\frac{1}{2}$ (b) $11\frac{1}{9}$
(c) $5\frac{15}{17}$ (d) $8\frac{2}{3}$
(e) None of these

44. The cost price of 20 articles is equal to the selling price of 25 articles. The loss percent in the transaction is
(a) 5 (b) 20
(c) 25 (d) 30
(e) None of these

45. By selling 66 metres of cloth a person gains the cost price of 22 metres. Find the gain per cent.
(a) 22% (b) $22\frac{1}{2}\%$
(c) 33% (d) $33\frac{1}{3}\%$
(e) None of these

46. By selling 66 metres of cloth a man loses the selling price of 22 metres. Find the loss per cent.
(a) 20% (b) 25%
(c) 30% (d) 35%
(e) None of these

47. A single discount equal to a discount series of 10% and 20% is
(a) 25% (b) 28%
(c) 30% (d) 35%
(e) None of these

48. The list price of a watch is ₹ 160. A retailer bought the same watch ₹ 122.40. He got two successive discounts one at 10% and the other at a rate which was not legible. What is the second discount rate?
(a) 12% (b) 14%
(c) 15% (d) 18%
(e) None of these

49. A tradesman is marketing his goods 20% above the cost price of the goods. He gives 10% discount on cash payment, find his gain percent.
(a) 12% (b) 8%
(c) 15% (d) 18%
(e) None of these

50. For a certain article, if discount is 25%, the profit is 25%. If the discount is 10%, then the profit is
(a) 10% (b) 20%
(c) 35% (d) 50%
(e) None of these

51. A trader marks his goods at such a price that he can deduct 15% for cash and yet make 20% profit. Find the marked price of an item which costs him ₹ 90 :
(a) ₹ $135\frac{11}{13}$ (b) ₹ $105\frac{3}{21}$
(c) ₹ $127\frac{1}{17}$ (d) ₹ $95\frac{1}{21}$
(e) None of these

52. A trader wants 10% profit on the selling price of a product whereas his expenses amount to 15% on sales. What should be his rate of mark up on an article costing ₹ 9?
(a) 20% (b) $66\frac{2}{3}\%$
(c) 30% (d) $\frac{100}{3}\%$
(e) None of these

CHAPTER 7

Simple & Compound Interest

INTEREST

Interest is the fixed amount paid on borrowed money.
The sum lent is called the **Principal**.
The sum of the principal and interest is called the **Amount**.
Interest is of two kinds :

(i) Simple interest
(ii) Compound interest

(I) SIMPLE INTEREST

When interest is calculated on the original principal for any length of time, it is called simple interest.

REMEMBER

★ Simple interest $= \dfrac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$

i.e. $\text{S.I.} = \dfrac{P \times R \times T}{100}$

★ Principal (P) $= \dfrac{100 \times \text{S.I.}}{R \times T}$

★ Rate (R) $= \dfrac{100 \times \text{S.I.}}{T \times P}$

★ Time (T) $= \dfrac{100 \times \text{S.I.}}{P \times R}$

★ If rate of simple interest differs from year to year, then

$$\text{S.I.} = P \times \frac{(R_1 + R_2 + R_3 + ...)}{100}$$

★ Amount = Principal + Interest

i.e. $A = P + I = P + \dfrac{PRT}{100} = P\left(1 + \dfrac{RT}{100}\right)$

EXAMPLE 1. Find the interest to be paid on a loan of ₹ 6000 at 5% per year for 5 years

Sol. P = ₹ 6000, R = 5% and T = 5 years

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{6000 \times 5 \times 5}{100} = ₹\,1500$$

EXAMPLE 2. Find the amount to be paid back on a loan of ₹ 18,000 at 5.5% per annum for 3 years

Sol. P = ₹ 18000, R = 5.5%, T = 3 years

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{18000 \times 5.5 \times 3}{100} = ₹\,2970$$

Amount = P + I = 18000 + 2970 = ₹ 20970

EXAMPLE 3. In how many years will a sum of money triple itself, at 25% per annum simple interest.

Sol. Let the sum of money be ₹ P. So, A = 3P

and S.I. = A – P = 3P – P = 2P

R = 25%

$$\therefore \quad T = \frac{100 \times \text{S.I.}}{P \times R} = \frac{100 \times 2P}{P \times 25} = 8 \text{ years}$$

EXAMPLE 4. What rate per cent per annum will produce ₹ 250 as simple interest on ₹ 6000 in 2.5 years

Sol. P = ₹ 6000; Time (T) = 2.5 years; S.I. = ₹ 250

$$\therefore \quad \text{Rate} = \frac{\text{S.I.} \times 100}{P \times T} = \frac{250 \times 100}{6000 \times 2.5} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}\%$$

EXAMPLE 5. To buy furniture for a new apartment, Sylvia Chang borrowed ₹5000 at 11 % simple interest for 11 months. How much interest will she pay?

Sol. From the formula, I = Prt, with P = 5000, r = 0.11, and t = 11/12 (in years). The total interest she will pay is

I = 5000 (0.11) (11/12) = 504.17

or ₹ 504.17

Shortcut Approach

If $\frac{1}{x}$ part of a certain sum P is lent out at R_1% SI, $\frac{1}{y}$ part is lent out at R_2 % SI and the remaining $\frac{1}{z}$ part at R_3 % SI and this way the interest received by I, then $P = \dfrac{I \times 100}{\dfrac{R_1}{x} + \dfrac{R_2}{y} + \dfrac{R_3}{z}}$

EXAMPLE 6. Alok lent out a certain sum. He lent 1/3 part of his sum at 7% SI, 1/4 part at 8% SI and remaining part at 10% SI. If ₹ 510 is his total interest, then find the money lent out.

Sol. Here, $R_1 = 7\%$, $R_2 = 8\%$, $R_3 = 10\%$

and $\dfrac{1}{z} = \dfrac{1}{3}, \dfrac{1}{y} = \dfrac{1}{4}$, I = ₹ 510

$$\therefore \quad \frac{1}{x} = \left[1 - \left(\frac{1}{3} + \frac{1}{4}\right)\right] = \frac{5}{12}$$

According to the formula,

$$P = \frac{I \times 100}{\frac{R_1}{x} + \frac{R_2}{y} + \frac{R_3}{z}} = \frac{510 \times 100}{\frac{7}{3} + \frac{8}{4} + \frac{50}{12}}$$

$$= \frac{51000}{\frac{7}{3} + 2 + \frac{25}{6}} = \frac{51000}{51} \times 6 = ₹\ 6000$$

Shortcut Approach

If a sum of money becomes n times in T yr at simple interest, then formula for calculating rate of interest will be given as

$$R = \frac{100(n-1)}{T}\%$$

EXAMPLE 7. A sum of money becomes four times in 20 yr at SI. Find the rate of interest.

Sol. Here, T = 20 yr, n = 4

$$\therefore R = \frac{100(n-1)}{T} = \frac{100(4-1)}{20} = \frac{100 \times 3}{20} = 15\%$$

Shortcut Approach

If a sum of money at a certain rate of interest becomes n times in T_1 yr and m times in T_2 yr, then formula for T_2 will be given as

$$T_2 = \left(\frac{m-1}{n-1}\right) \times T_1$$

EXAMPLE 8. A sum becomes two times in 5 yr at a certain rate of interest. Find the time in which the same amount will be 8 times at the same rate of interest.

Sol. Here, n = 2, m = 8, $T_1 = 5$, $T_2 = ?$

$$\therefore \quad T_2 = \left(\frac{m-1}{n-1}\right) \times T_1 = \left(\frac{8-1}{2-1}\right) \times 5 = 35 \text{ yr}$$

(II) COMPOUND INTEREST

Money is said to be lent at compound interest when at the end of a year or other fixed period, the interest that has become due is not paid to the lender, but is added to the sum lent, and the amount thus obtained becomes the principal in the next year or period. The process is repeated until the amount for the last period has been found. Hence,

When the interest charged after a certain specified time period is added to form new principal for the next time period, the interest is said to be compounded and the total interest accrued is compound interest.

REMEMBER

★ $C.I. = P\left[\left(1 + \frac{r}{100}\right)^n - 1\right];$

★ $\text{Amount (A)} = P\left(1 + \frac{r}{100}\right)^n$

★ If rate of compound interest differs from year to year, then

$$\text{Amount} = P\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)\left(1 + \frac{r_3}{100}\right) \ldots\ldots$$

Compound interest – when interest is compounded annually but time is in fraction

If time = $t\frac{p}{q}$ years, then

$$\boxed{A = P\left(1 + \frac{r}{100}\right)^t \left(1 + \frac{\frac{p}{q}r}{100}\right)}$$

Compound interest – when interest is calculated half-yearly

Since r is calculated half-yearly therefore the rate per cent will become half and the time period will become twice, i.e.,

Rate per cent when interest is paid half-yearly $= \frac{r}{2}\%$

and time = 2 × time given in years

Hence,

$$\boxed{A = P\left(1 + \frac{r}{2 \times 100}\right)^{2n}}$$

Compound interest – when interest is calculated quarterly

Since 1 year has 4 quarters, therefore rate of interest will become $\frac{1}{4}$th of the rate of interest per annum, and the time period will be 4 times the time given in years

Hence, for quarterly interest

$$\boxed{A = P\left(1 + \frac{r/4}{100}\right)^{4 \times n} = P\left(1 + \frac{r}{400}\right)^{4n}}$$

EXAMPLE 9. Find the compound interest on ₹ 70000 for 4 years at the rate of 14% per annum compounded annually.

Sol. P = ₹ 70000, n = 4, r = 14%

$$A = P\left(1 + \frac{r}{100}\right)^n = 70000\left(1 + \frac{14}{100}\right)^4 = ₹\ 118227.20$$

C.I. = A – P = 118227.20 – 70000 = ₹ 48227.20

EXAMPLE 10. If ₹ 60000 amounts to ₹ 68694 in 2 years then find the rate of interest.

Sol. Given : A = ₹68694

P = ₹ 60000

n = 2 years

r = ?

$$\because \ A = P\left(1 + \frac{r}{100}\right)^n$$

$$\therefore \ 68694 = 60000\left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \ \frac{68694}{60000} = \left(1 + \frac{r}{100}\right)^2 \Rightarrow \frac{11449}{10000} = \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \quad 1+\frac{r}{100}=\sqrt{\frac{11449}{10000}}=\sqrt{1.1449}$$

$$\Rightarrow \quad 1+\frac{r}{100}=1.07$$

$$\Rightarrow \quad \frac{r}{100}=1.07-1=0.07$$

$\therefore \quad r=0.07\times 100=7\%$

EXAMPLE 11. In how many years, the sum of ₹ 10000 will become ₹ 10920.25 if the rate of compound interest is 4.5% per annum?

Sol. A = ₹ 10920.25
P = ₹ 10000
Rate of interest = 4.5%
Time (n) = ?

$$\because \ A=P\left(1+\frac{r}{100}\right)^n$$

$$\therefore \ 10920.25=10000\left(1+\frac{4.5}{100}\right)^n$$

$$\frac{10920.25}{10000}=\left(1+\frac{0.9}{20}\right)^n=\left(\frac{20.9}{20}\right)^n$$

$$\Rightarrow \frac{436.81}{400}=\left(\frac{20.9}{20}\right)^n \Rightarrow \left(\frac{20.9}{20}\right)^2=\left(\frac{20.9}{20}\right)^n$$

Hence ₹ 10000 will become ₹10920.25 in 2 years at 4.5%.

EXAMPLE 12. Suppose ₹1000 is deposited for 6 years in an account paying 8.31% per year compounded annually.

(a) Find the compound amount.
In the formula above, P = 1000, i = .0831, and n = 6.
The compound amount is
$A = P(1+i)^n$
$A = 1000(1.0831)^6$
A = ₹ 1614.40.

(b) Find the amount of interest earned.
Subtract the initial deposit from the compound amount.
Amount of interest = ₹1614.40 – ₹ 1000 = ₹ 614.40.

EXAMPLE 13. Find the compound interest on ₹ 8000 at 15% per annum for 2 years 4 months, compound annually.

Sol. Time = 2 years 4 months = $2\frac{4}{12}$ years = $2\frac{1}{3}$ years

$$\text{Amount} = ₹\left[8000\left\{\left(1+\frac{15}{100}\right)\right\}^2\left(1+\frac{\frac{1}{3}\times 15}{100}\right)\right]$$

$$= ₹\left(8000\times\frac{23}{20}\times\frac{23}{20}\times\frac{21}{20}\right)= ₹\ 11109$$

$\therefore$ C.I. = ₹ (11109 – 8000) = ₹ 3109.

EXAMPLE 14. What will be the compound interest on ₹ 4000 in 4 years at 8 per cent annum. If the interest is calculated half-yearly.

Sol. Given : P = ₹ 4000, r = 8%, n = 4 years
Since interest is calculated half-yearly, therefore,

$r=\frac{8}{2}\%=4\%$ and $n=4\times 2=8$ half years

$$\therefore \ A=4000\left(1+\frac{4}{100}\right)^8=4000\times\left(\frac{26}{25}\right)^8$$

$$=4000\times 1.3685=5474.2762$$

Amount = ₹ 5474.28

$\therefore$ Interest = Amount – Principal
= ₹ 5474.28 – ₹ 4000 = ₹ 1474.28

EXAMPLE 15. Find the compound interest on ₹ 25625 for 12 months at 16% per annum, compounded quarterly.

Sol. Principal (P) = ₹ 25625

Rate (r) = 16% = $\frac{16}{4}\%=4\%$

Time = 12 months = 4 quarters

$$A=25625\left(1+\frac{4}{100}\right)^4=25625\left(\frac{26}{25}\right)^4$$

$$25625\times\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}= ₹29977.62$$

C.I. = A – P = 29977.62 – 25625 = ₹ 4352.62

Shortcut Approach

Difference between Compound Interest and Simple Interest

When T = 2

(i) $\text{C.I.}-\text{S.I.}=P\left(\frac{R}{100}\right)^2$

(ii) $\text{C.I.}-\text{S.I.}=\frac{R\times \text{S.I.}}{2\times 100}$

When T = 3

(i) $\text{C.I.}-\text{S.I.}=\frac{PR^2}{10^4}\left(\frac{300+R}{100}\right)$

(ii) $\text{C.I.}-\text{S.I.}=\frac{\text{S.I.}}{3}\left[\left(\frac{R}{100}\right)^2+3\left(\frac{R}{100}\right)\right]$

NOTE: SI and CI for one year on the same sum and at same rate are equal.

EXAMPLE 16. The difference between compound interest and simple interest on a certain amount of money at 5% per annum for 2 years is ₹ 15. Find the sum :

(a) ₹ 4500 **(b) ₹ 7500**
(c) ₹ 5000 **(d) ₹ 6000**

Sol. (d) Let the sum be ₹ 100.

Therefore, $SI = \frac{100 \times 5 \times 2}{100} = ₹\ 10$

and $CI = 100\left(1+\frac{5}{100}\right)^2 - 100$

$= 100 \times \frac{21 \times 21}{20 \times 20} - 100 = ₹\ \frac{41}{4}$

Difference of CI and SI $= \frac{41}{4} - 10 = \frac{1}{4}$

If the difference is $\frac{1}{4}$, the sum = 100

⇒ If the difference is ₹ 15, the sum
= 400 × 15 = ₹ 6000

EXAMPLE 17. The difference between the simple interest and the compound interest compounded annually at the rate of 12% per annum on ₹ 5000 for two years will be :

(a) ₹ 47.50 (b) ₹ 63
(c) ₹ 45 (d) ₹ 72

Sol. (d) Required difference

$= \left[5000\left(1+\frac{12}{100}\right)^2 - 5000\right] - \frac{5000 \times 12 \times 2}{100}$

$= 5000\left(\frac{28}{25} \times \frac{28}{25} - 1\right) - 1200$

$= 5000\left(\frac{784-625}{625}\right) - 1200 = ₹72$

EXAMPLE 18. The difference between CI and SI for 3 yr at the rate of 20% pa is ₹ 152. What is the principal lent ?

Sol. Difference between CI and SI for 3 yr = 152

$\therefore \quad P\left(\frac{R}{100}\right)^2\left(\frac{R}{100}+3\right) = 152$

$\Rightarrow \quad P\left(\frac{20}{100}\right)^2\left(\frac{20}{100}+3\right) = 152$

$\Rightarrow \quad P\left(\frac{1}{25}\right)\left(\frac{16}{5}\right) = 152 \Rightarrow P = \frac{152 \times 25 \times 5}{16}$

$\Rightarrow \quad P = 9.5 \times 25 \times 5 \Rightarrow P = 1187.5$

EXAMPLE 19. Subash purchased a refrigerator on the terms that he is required to pay ₹1,500 cash down payment followed by ₹ 1,020 at the end of first year, ₹ 1,003 at the end of second year and ₹ 990 at the end of third year. Interest is charged at the rate of 10% per annum. Calculate the cash price :

(a) ₹ 3,000 (b) ₹ 2,000
(c) ₹ 4,000 (d) ₹ 5,000

Sol. (c) Cash down payment = ₹ 1500
Let ₹ x becomes ₹ 1020 at the end of first year.

Then, $1020 = x\left(1+\frac{10}{100}\right)$

or $x = \frac{1020 \times 100}{110} = ₹\,927.27$

Similarly, $1003 = y\left(1+\frac{10}{100}\right)^2$

or $y = \frac{1003 \times 20 \times 20}{22 \times 22} = ₹\ 828.92$

and $z = \frac{990 \times 20 \times 20 \times 20}{22 \times 22 \times 22} = ₹\ 743.80$

Hence, CP = 1500 + 927.27 + 828.92 + 743.80
= 3999.99 or ₹ 4000.

EXAMPLE 20. The difference between the interest received from two different banks on ₹ 500 for 2 yrs is ₹ 2.5. Find the difference between their rates.

Sol. $I_1 = \frac{500 \times 2 \times r_1}{100} = 10 r_1$

$I_2 = \frac{500 \times 2 \times r_2}{100} = 10 r_2$

$I_1 - I_2 = 10r_1 - 10r_2 = 2.5$

Or, $I_1 - I_2 = \frac{2.5}{10} = 0.25\%$

SHORTCUT METHOD

When $t_1 = t_2$,

$(r_1 - r_2) = \frac{I_d \times 100}{\text{sum} \times t} = \frac{2.5 \times 100}{500 \times 2} = 0.25\%$

☞ ***Shortcut Approach***

If a certain sum at compound interest becomes x times in n_1 yr and y times in n_2 yr, then $x^{\frac{1}{n_1}} = y^{\frac{1}{n_2}}$

EXAMPLE 21. If a certain sum at compound interest becomes double in 5 yr, then in how many years, it will be 16 times at the same rate of interest ?

Sol. Here, $n_1 = 5$yr, $x = 2$, $y = 16$ and $n_2 = ?$
According to the formula,

$$x^{\frac{1}{n_1}} = y^{\frac{1}{n_2}} \Rightarrow 2^{\frac{1}{5}} = 16^{\frac{1}{n_2}}$$

$$\Rightarrow \quad 2^{\frac{1}{5}} = (2)^{4 \times \frac{1}{n_2}}$$

$$\Rightarrow \quad \frac{1}{5} = \frac{4}{n_2} \quad \text{[on comparing both sides]}$$

$$\therefore \quad n_2 = 5 \times 4 = 20 \text{ yr}$$

Alternative method:

If sum = x, then

x becomes 2x in 5 yr.

2x becomes 4x in 10 yr.

4x becomes 8x in 15 yr.

8x becomes 16x in 20 yr.

EXAMPLE 22. At what rate per cent compound interest does a sum of money becomes nine - fold in 2 years?

Sol. Let the sum be ₹ x and the of compound interest be r% per annum; then

$$9x = x\left(1+\frac{r}{100}\right)^2 \text{ or, } 9 = \left(1+\frac{r}{100}\right)^2$$

$$\text{or, } 3 = 1+\frac{r}{100}; \text{ or, } \frac{r}{100} = 2$$

$$\therefore r = 200\%$$

SHORTCUT METHOD

The general formula of compound interest can be changed to the following form :

If a certain sum becomes 'm' times in 't' years, the rate of compound interest r is equal to $100\left[(m)^{1/t}-1\right]$

In this case, $r = 100\left[(9)^{1/t}-1\right] = 100\,(3-1) = 200\%$

EXAMPLE 23. The simple interest on a certain sum of money at 4% per annum for 4 years is ₹ 80 more than the interest on the same sum for 3 years at 5% per annum. Find the sum.

Sol. Let the sum be ₹ x, then at 4% rate for 4 years the simple

$$\text{Interest} = \frac{x \times 4 \times 4}{100} = ₹\frac{4x}{25}$$

$$\text{At 5\%rate for 3 yrs the simple interest} = \frac{x \times 5 \times 3}{100} = ₹\frac{3x}{20}$$

$$\text{Now, we have, } \frac{4x}{25} - \frac{3x}{20} = 80$$

$$\text{or } \frac{16x-15x}{100} = 80 \qquad \therefore x = ₹8000$$

SHORTCUT METHOD

For this type of question

$$\text{Sum} = \frac{\text{Difference} \times 100}{[r_2t_1 - r_2t_2]} = \frac{80 \times 100}{4 \times 4 - 3 \times 5} = ₹8000$$

EXAMPLE 24. Some amount out of ₹ 7000 was lent at 6 % per annum and the remaining at 4 % per annum. If the total simple interest from both the fractions in 5 years was ₹ 1600, find the sum lent at 6 % per annum.

Sol. Suppose ₹ x was lent at 6 % per annum.

$$\text{Thus, } \frac{x \times 6 \times 5}{100} + \frac{(7000-x) \times 4 \times 5}{100} = 1600$$

$$\text{or, } \frac{3x}{10} + \frac{7000-x}{5} = 1600$$

$$\text{or, } \frac{3x+14,000-2x}{10} = 16000$$

$$\therefore x = 16000 - 14000 = ₹2000$$

SHORTCUT METHOD

$$\text{Overall rate of interest} = \frac{1600 \times 100}{5 \times 7000} = \frac{32}{7}\%$$

∴ ratio of two amounts = 2 : 5

6 %　　　　4 %

$\frac{32}{7}\%$

$\frac{4}{7}\%$　　　　$\frac{10}{7}\%$

$$\therefore \text{Amount lent at 6 \%} = \frac{7000}{7} \times 2 = ₹2000$$

EXAMPLE 25. As n amount of money grows upto ₹ 4840 in 2 years and upto ₹ 5324 in 3 years on compound interest. Find the rate percent

Sol. We have,

P + CI of 3 yrs = ₹ 5324.......(1)

P + CI of 2yrs = ₹ 4840........(2)

Subtracting (2) from (1), we get

CI of 3rd year = 5324 – 4840 = ₹ 484.

Thus, the CI calculated in the third year which is ₹ 484 is basically the amount of interest on the amount generated after 2 years which is ₹ 4840.

$$\therefore r = \frac{484 \times 100}{4840 \times 1} = 10\%$$

SHORTCUT METHOD

$$\frac{\text{Difference of amount after n years and (n+1) years} \times 100}{\text{Amount after 2 years}}$$

In this, n = 2.

$$\therefore \text{rate} = \frac{\text{Difference of amount after 2 years and 3 years} \times 100}{\text{Amount after 2 years}}$$

$$= \frac{(5324-4840)}{4840} \times 100 = \frac{484 \times 100}{4840} = 10\%$$

EXAMPLE 26. A certain amount of money at compound interest grows upto ₹ 51168 in 15 yrs and upto ₹ 51701 in 16 years. Find the rate per cent per annum.

$$\textbf{Sol. } \text{Rate} = \frac{(51701-51168) \times 100}{51168} = \frac{533 \times 100}{51168}$$

$$= \frac{100}{96} = \frac{25}{24} = 1\frac{1}{24}\%$$

EXAMPLE 27. Find the compound interest on ₹ 18,750 in 2 years the rate of interest being 4% for the first year and 8% for the second year.

Sol. After first year the amount

$$= 18750\left(1+\frac{4}{100}\right) = 18750\left(\frac{104}{100}\right)$$

After 2nd year the amount $= 18750\left(\frac{104}{100}\right)\left(\frac{108}{100}\right)$

$$= 18750\left(\frac{26}{25}\right)\left(\frac{27}{25}\right) = 21060$$

$\therefore$ CI = 21060 – 18,750 = ₹ 2310.

> **Shortcut Approach**
>
> If the population of a city is P and it increases with the rate of R% per annum, then
>
> (i) Population after n yr $= P\left(1+\frac{R}{100}\right)^n$
>
> (ii) Population n yr ago $= \dfrac{P}{\left(1+\frac{R}{100}\right)^n}$

Note: • If population decreases with the rate of R%, then (–) sign will be used in place of (+) in the above mentioned formula

• If the rate of growth per year is $R_1\%, R_2\%, R_3\%, \ldots\ldots, R_n\%$, then
Population after n yr

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)\ldots\ldots\left(1+\frac{R_n}{100}\right)$$

(This formula can also be used, if there is increase/ decrease in the price of an article.)

EXAMPLE 28. The population of a particular area A of a city is 5000. It increases by 10% in 1st yr. It decreases by 20% in the 2nd yr because of some reason. In the 3rd yr, the population increases by 30%. What will be the population of area A at the end of 3 yr?

Sol. Given that, P = 5000, $R_1 = 10\%$, $R_2 = -20\%$ (decrease) and $R_3 = 30\%$

$\therefore$ Population at the end of 3rd year

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

$$= 5000\left(1+\frac{10}{100}\right)\left(1-\frac{20}{100}\right)\left(1+\frac{30}{100}\right)$$

$$= 5000\times\frac{11}{10}\times\frac{4}{5}\times\frac{13}{5} = 10\times11\times4\times13 = 5720$$

A computer gives the following results for various values of n.

Interest is compounded	n	$\left(1+\frac{1}{n}\right)^n$
Annually	1	$\left(1+\frac{1}{1}\right)^1 = 2$
Semiannually	2	$\left(1+\frac{1}{2}\right)^2 = 2.25$
Quarterly	4	$\left(1+\frac{1}{4}\right)^4 \approx 2.4414$
Monthly	12	$\left(1+\frac{1}{12}\right)^{12} \approx 2.6130$

EXERCISE

1. Arun borrowed a sum of money from Jayant at the rate of 8% per annum simple interest for the first four years, 10% per annum for the next six years and 12% per annum for the period beyond ten years. If he pays a total of ₹ 12,160 as interest only at the end of 15 years, how much money did he borrow?
   (a) ₹8000 (b) ₹10,000
   (c) ₹12,000 (d) ₹9,000
   (e) None of these
2. A sum fetched total simple interest of ₹4016.25 at the rate of 9 p.c.p.a. in 5 years. What is the sum?
   (a) ₹8925 (b) ₹8032.50
   (c) ₹4462.50 (d) ₹8900
   (e) None of these
3. At a simple interest ₹800 becomes ₹956 in three years. If the interest rate, is increased by 3%, how much would ₹800 become in three years?
   (a) ₹1020.80 (b) ₹1004
   (c) ₹1028 (d) Data inadequate
   (e) None of these
4. On ₹ 3,000 invested at a simple interest rate 6 p.c.p.a, ₹ 900 is obtained as interest in certain years. In order to earn ₹ 1,600 as interest on ₹ 4,000 in the same number of years, what should be the rate of simple interest?
   (a) 7 p.c.p.a. (b) 8 p.c.p.a.
   (c) 9 p.c.p.a. (d) Data inadequate
   (e) None of these
5. The difference between CI and SI on a certain sum of money at 10% per annum for 3 years is ₹620. Find the principal if it is known that the interest is compounded annually.
   (a) ₹200, 000 (b) ₹20,000
   (c) ₹10,000 (d) ₹100, 000
   (e) None of these
6. A certain amount earns simple interest of ₹1750 after 7 years. Had the interest been 2% more, how much more interest would it have earned?
   (a) ₹35 (b) ₹350
   (c) ₹245 (d) Cannot be determined
   (e) None of these
7. What will be the difference in simple and compound interest on ₹ 2000 after three years at the rate of 10 percent per annum?
   (a) ₹160 (b) ₹42
   (c) ₹62 (d) ₹20
   (e) None of these
8. Nikhilesh invested certain amount in three different schemes *A, B* and *C* with the rate of interest 10 p.c.p.a., 12 p.c.p.a. and 15 p.c.p.a. respectively. If the total interest accrued in one year was ₹3200 and the amount invested in scheme *C* was 150% of the amount invested in scheme *A* and 240% of the amount invested in scheme *B,* what was the amount invested in scheme *B*?
   (a) ₹8000 (b) ₹5000
   (c) ₹6500 (d) Cannot be determined
   (e) None of these
9. Aniket deposited two parts of a sum of ₹25000 in different banks at the rates of 15% per annum and 18% per annum respectively. In one year he got ₹4050 as the total interest. What was the amount deposited at the rate of 18% per annum?
   (a) ₹9000 (b) ₹18000
   (c) ₹15000 (d) Data inadequate
   (e) None of these
10. Mr *X* invested an amount for 2 years @ 15 p.c.p.a at simple interest. Had the interest been compounded, he would have earned ₹ 450/- more as interest. What was the amount invested?
    (a) ₹22000 (b) ₹24000
    (c) ₹25000 (d) Data inadequate
    (e) None of these
11. Difference between the compound interest and the simple interest accrued on an amount of ₹18000, in two years was ₹405. What was the rate of interest p.c.p.a?
    (a) 16 (b) 12
    (c) 15 (d) Cannot be determined
    (e) None of these
12. Anish borrowed ₹ 15000 at the rate of 12% and an other amount at the rate of 15% for two years. The total interest paid by him was ₹9000. How much did he borrow?
    (a) ₹32,000 (b) ₹33,000
    (c) ₹30,000 (d) ₹35,000
    (e) None of these
13. The compound interest on any sum at the rate of 5% for two years is ₹512.50. Find the sum.
    (a) ₹5200 (b) ₹4800
    (c) ₹5000 (d) ₹5500
    (e) None of these
14. Mr Amin borrowed some money from Mr Vishwas. The rate of interest for first two years is 8% p.a., for the next three years is 11 % p.a. and for the period beyond 5 years 14% p.a. Mr Vishwas got an amount of ₹10920 as an interest at the end of eight years. Then what amount was borrowed by Mr Amin'?
    (a) ₹12000 (b) ₹15000
    (c) ₹1400 (d) Data inadequate
    (e) None of these

15. The C.I. on a certain sum of money for the 4th year at 8% p.a. is ₹ 486. What was the compound interest for the third year on the same sum at the same rate?
(a) ₹ 450 (b) ₹ 475
(c) ₹ 456 (d) ₹ 480
(e) None of these

16. Seema invested an amount of ₹ 16000 for two years at compound interest and received an amount of ₹ 17640 on maturity. What is the rate of interest?
(a) 8 pcpa (b) 5 pcpa
(c) 4 pcpa (d) Data inadequate
(e) None of these

17. Amit Kumar invested an amount of ₹ 15,000 at compound interest rate of 10 pcpa for a period of two years. What amount will he receive at the end of two years?
(a) ₹ 18,000 (b) ₹ 18,500
(c) ₹ 17,000 (d) ₹ 17,500
(e) None of these

18. In a business *A* and *C* invested amounts in the ratio 2:1. Whereas the ratio between amounts invested by *A* and *B* was 3:2. If ₹ 1,57,300 was their profit, how much amount did B receive?
(a) ₹ 72,600 (b) ₹ 48,400
(c) ₹ 36,300 (d) ₹ 24,200
(e) None of these

19. Mr. Sane invested a total amount of ₹ 16,500 for two years in two schemes A and B with rate of simple interest 10 p.c.p.a. and 12 p.c.p.a. respectively. If the total amount of interest earned was ₹ 3,620, what was the amount invested in scheme B?
(a) ₹ 8,000 (b) ₹ 8,600
(c) ₹ 8,150 (d) Data inadequate
(e) None of these

20. The difference between the simple and the compound interest
compounded every six months at the rate of 10% p.a. at the end of two years is ₹ 124.05. What is the sum?
(a) ₹ 10,000 (b) ₹ 6,000
(c) ₹ 12,000 (d) ₹ 8,000
(e) None of these

21. Parameshwaran invested an amount of ₹ 12,000 at the simple interest rate of 10 pcpa and another amount at the simple interest rate of 20 pcpa. The total interest earned at the end of one year on the total amount invested became 14 pcpa. Find the total amount invested.
(a) ₹ 22,000 (b) ₹ 25,000
(c) ₹ 20,000 (d) ₹ 24,000
(e) None of these

22. Raviraj invested an amount of ₹ 10,000 at compound interest rate of 10 pcpa for a period of three years. How much amount will Raviraj get after three years?
(a) ₹ 12,310 (b) ₹ 13,210
(c) ₹ 13,320 (d) ₹ 13,120
(e) None of these

23. Nelson borrowed some money at the rate of 6 p.c.p.a. for the first three years, 9 p.c.p.a. for the next five years and 13 p.c.p.a. for the period beyond eight years. If the total interest paid by him at the end of eleven years is ₹ 8,160, how much money did he borrow?
(a) ₹ 12,000 (b) ₹ 10,000
(c) ₹ 8,000 (d) Data inadequate
(e) None of these

24. Amal borrowed a sum of money with simple interest as per the following rate structure:
(1) 6 p.c. p.a. for the first three years
(2) 8 p.c. p.a. for the next five years
(3) 12 p.c. p.a. for the next eight years
If he paid a total of ₹ 5,040 as interest at the end of twelve years, how much money did he borrow?
(a) ₹ 8,000 (b) ₹ 10,000
(c) ₹ 12,000 (d) ₹ 6,000
(e) None of these

25. The simple interest in 14 months on a certain sum at the rate of 6 per cent per annum is ₹ 250 more than the interest on the same sum at the rate of 8 per cent in 8 months. How much amount was borrowed?
(a) ₹ 15000 (b) ₹ 25000
(c) ₹ 7500 (d) ₹ 14500
(e) None of these

26. On retirement, a person gets 1.53 lakhs of his provident fund which he invests in a scheme at 20% p.a. His monthly income from this scheme will be
(a) ₹ 2, 450 (b) ₹ 2,500
(c) ₹ 2, 550 (d) ₹ 2, 600
(e) None of these

27. A sum was put at simple interest at a certain rate for 4 years Had it been put at 2% higher rate, it would have fetched ₹ 56 more. Find the sum.
(a) ₹ 500 (b) ₹ 600
(c) ₹ 700 (d) ₹ 800
(e) None of these

28. Simple interest on a certain sum is 16 over 25 of the sum. Find the rate per cent and time, if both are equal.
(a) 8% and 8 years (b) 6% and 6 years
(c) 10% and 10 years (d) 12 % and 12 years
(e) None of these

29. The simple interest on ₹ 200 for 7 months at 5 paise per rupee per month is
(a) ₹ 70 (b) ₹ 7
(c) ₹ 35 (d) ₹ 30.50
(e) None of these

30. A tree increases annually by $\frac{1}{8}$th of its height. By how much will it increase after $2\frac{1}{2}$ yearly, if it stands today 10ft high?
(a) 3 ft (b) 3.27 ft
(c) 3.44 ft (d) 3.62 ft
(e) None of these

31. If there are three sum of money P,Q and R so that P is the simple interest on Q and Q is the simple interest of R, rate % and time are same in each case, then the relation of P, Q and R is given by
(a) $P^2 = QR$ (b) $Q^2 = PR$
(c) $R^2 = PQ$ (d) $PQR = 100$
(e) None of these

32. In how many minimum number of complete years, the interest on ₹ 212.50 P at 3% per annum will be in exact number of rupees?
(a) 6 (b) 8
(c) 9 (d) 7
(e) None of these

33. A milk man borrowed ₹ 2,500 from two money lenders. For one loan, he paid 5% p.a. and for the other, he paid 7% p.a. The total interest paid for two years was ₹ 275. How much did he borrow at 7% rate?
(a) ₹ 600 (b) ₹ 625
(c) ₹ 650 (d) ₹ 675
(e) None of these

34. What annual instalment will discharge a debt of ₹4,200 due in 5 years at 10% simple interest?
(a) ₹ 500 per year (b) ₹ 600 per year
(c) ₹ 700 per year (d) ₹ 800 per year
(e) None of these

35. Adam borrowed some money at the rate of 6% p.a. for the first two years, at the rate of 9% p.a. for the next three years, and at the rate of 14% p.a. for the period beyond five years. If he pays a total interest of ₹ 11,400 at the end of nine years, how much money did he borrow?
(a) ₹ 10,000 (b) ₹ 12,000
(c) ₹ 14,000 (d) ₹ 16,000
(e) None of these

36. A person borrows ₹ 5000 for 2 years at 4% p.a. simple interest. He immediately lends it to another person at $6\frac{1}{4}\%$ p.a. for 2 years. Find his gain in the transaction per year.
(a) ₹ 112.50 (b) ₹ 125
(c) ₹ 150 (d) ₹ 167.50
(e) None of these

37. A certain amount earns simple interest of ₹ 1750 after 7 years Had the interest been 2% more, how much more interest would it have earned?
(a) ₹ 35 (b) ₹ 245
(c) ₹ 350 (d) Cannot be determined
(e) None of these

38. What will be the ratio of simple interest earned by certain amount at the same rate of interest for 6 years and that for 9 years?
(a) 1 : 3 (b) 1 : 4
(c) 2 : 3 (d) Data inadequate
(e) None of these

39. Two equal sums of money were invested, one at 4% and the other at 4.5%. At the end of 7 years, the simple interest received from the latter exceeded to that received from the former by ₹ 31.50. Each sum was :
(a) ₹ 1,200 (b) ₹ 600
(c) ₹ 750 (d) ₹ 900
(e) None of these

40. The simple interest on a sum of money is $\frac{1}{16}$th of the principal and the number of years is equal to the rate per cent per annum. The rate per cent annum is ________ .
(a) $6\frac{1}{4}\%$ (b) $6\frac{1}{3}\%$
(c) $6\frac{1}{5}\%$ (d) $4\frac{1}{5}\%$
(e) None of these

41. An automobile financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest of 10%, the effective rate of interest becomes :
(a) 10% (b) 10.25%
(c) 10.5% (d) None of these
(e) None of these

42. A lent ₹ 5000 to B for 2 years and ₹ 3000 to C for 4 years on simple interest at the same rate of interest and received ₹ 2200 in all from both of them as interest. The rate of interest per annum is:
(a) 5% (b) 7%
(c) $7\frac{1}{8}\%$ (d) 10%
(e) None of these

43. A sum of ₹ 725 is lent in the beginning of a year at a certain rate of interest. After 8 months, a sum of ₹ 362.50 more is lent but at the rate twice the former. At the end of the year, ₹ 33.50 is earned as interest from both the loans. What was the original rate of interest?
(a) 3.6% (b) 4.5%
(c) 5% (d) 3.46%
(e) None of these

44. The difference between the simple interest received from two different sources on ₹ 1500 for 3 years is ₹ 13.50. The difference between their rates of interest is:
(a) 0.1% (b) 0.2%
(c) 0.3% (d) 0.4%
(e) None of these

45. The rates of simple interest in two banks A and B are in the ratio 5 : 4. A person wants to deposit his total savings in two banks in such a way that he received equal half-yearly interest from both. He should deposit the savings in banks A and B in the ratio.
(a) 2 : 5 (b) 4 : 5

(c) 5 : 2 (d) 5 : 4
(e) None of these

46. The price of a T.V. set worth ₹ 20,000 is to paid in 20 instalments of ₹ 1000 each. If the rate of interest be 6% per annum, and the first instalment be paid at the time of purchase, then the value of the last instalment covering the interest as well will be :
(a) ₹ 1050 (b) ₹ 2050
(c) ₹ 3000 (d) None of these
(e) ₹ 2020

47. A man buys a music system valued at ₹ 8000. He pays ₹ 3500 at once and the rest 18 months later, on which he is charged simple interest at the rate of 8% per annum. Find the total amount he pays for the music system.
(a) ₹ 9260 (b) ₹ 8540
(c) ₹ 8720 (d) ₹ 9410
(e) None of these

48. An amount of ` 1,00,000 is invested in two types of shares. The first yields an interest of 9% p.a. and the second, 11% p.a. If the total interest at the end of one year is $9\frac{3}{4}\%$, then the amount invested in each share was:
(a) ₹ 52,500; ₹ 47,500
(b) ₹ 62, 500; ₹ 37,500
(c) ₹ 72,500: ₹ 27,500
(d) ₹ 82, 500; ₹ 17,500
(e) None of these

49. Find the compound interest on ₹ 12450 for 9 months at 12% per annum compounded quarterly.
(a) ₹ 1154.45 (b) ₹ 1125.18
(c) ₹ 1198.72 (d) 1164.32
(e) None of these

50. A person invested in all ₹ 2600 at 4%, 6% and 8% per annum simple interest. At the end of the year, he got the same interest in all the three cases. The money invested at 4% is:
(a) ₹ 200 (b) ₹ 600
(c) ₹ 800 (d) ₹ 1200
(e) None of these

51. Divide ₹ 2379 into 3 parts so that their amounts after 2, 3 and 4 years respectively may be equal, the rate of interest being 5% per annum at simple interest. The first part is:
(a) 759 (b) 792
(c) 818 (d) 828
(e) None of these

CHAPTER 8

Time & Work

TIME AND WORK

In most of the problems on time and work, either of the following basic parameters are to be calculated :

Shortcut Approach

- If A can do a piece of work in X days, then A's one day's work = $\frac{1}{X}$th part of whole work.
- If A's one day's work = $\frac{1}{X}$th part of whole work, then A can finish the work in X days.
- If A can do a piece of work in X days and B can do it in Y days then A and B working together will do the same work in $\frac{XY}{X+Y}$ days.
- If A, B and C can do a work in X, Y and Z days respectively then all of them working together can finish the work in

$$\frac{XYZ}{XY+YZ+XZ}\text{ days.}$$

- If (A + B) can do a piece of work in X days, (B + C) can do a place of work in Y days and (C + A) can do a piece of work in Z days.
Then,
(A + B + C) can do a piece of work in $\frac{2XYZ}{XY+YZ+ZX}$ days

EXAMPLE 1. A can do a piece of work in 5 days, and B can do it in 6 days. How long will they take if both work together?

Sol. A's 1 day's work = $\frac{1}{5}$th part of whole work and

B's 1 day's work = $\frac{1}{6}$th part of whole work

$\therefore$ (A + B)'s one day's work = $\frac{1}{5}+\frac{1}{6}=\frac{11}{30}$th part of whole work. So, both together will finish the work in $\frac{30}{11}$ days $=2\frac{8}{11}$ days.

By Direct Formula :

A + B can do the work in $\frac{5\times6}{5+6}$ days $=\frac{30}{11}=2\frac{8}{11}$ days.

EXAMPLE 2. Two men, Vikas and Vishal, working separately can mow a field in 8 and 12 hours respectively. If they work in stretches of one hour alternately, Vikas beginning at 8 a.m, when will the mowing be finished?

Sol. In the first hour, Vikas mows $\frac{1}{8}$ of the field.

In the second hour, Vishal mows $\frac{1}{12}$ of the field.

$\therefore$ In the first 2 hours, $\left(\frac{1}{8}+\frac{1}{12}=\frac{5}{24}\right)$ of the field is mown.

$\therefore$ In 8 hours, $\frac{5}{24}\times4=\frac{5}{6}$ of the field is mown.

Now, $\left(1-\frac{5}{6}\right)=\frac{1}{6}$ of the field remains to be mown.

In the 9th hour, Vikas mows $\frac{1}{8}$ of the field.

Remaining work $=\frac{1}{6}-\frac{1}{8}=\frac{1}{24}$

$\therefore$ Vishal will finish the remaining work in $\left(\frac{1}{24}\div\frac{1}{12}\right)$

or $\frac{1}{2}$ of an hour.

$\therefore$ The total time required is $\left(8+1+\frac{1}{2}\right)$ or $9\frac{1}{2}$ hours.

Thus, the work will be finished at $8+9\frac{1}{2}=17\frac{1}{2}$ or 5.30 pm.

EXAMPLE 3. A can do a piece of work in 36 days, B in 54 days and C in 72 days. All the three began the work together on the Dec. 15, 2014, but A left 8 days and B 12 days before the completion of the work. If C took the rest for a week then in how many days, the work was finished from the day it started ?

Sol. Let the total time taken be x days.

According to the given condition

$$\Rightarrow \frac{x-8}{36}+\frac{x-12}{54}+\frac{x}{72}=1$$

$$\Rightarrow \frac{6(x-8)+4(x-12)+3x}{216}=1$$

$$\Rightarrow \frac{6x-48+4x-48+3x}{216}=1 \Rightarrow \frac{13x-96}{216}=1$$

$$\Rightarrow 13x-96=216 \Rightarrow 13x=216+96=312$$

$$\Rightarrow x=\frac{312}{13}=24$$

Since, C takes the rest for a week, so the number of days in which the work was finished from one day it started = 31 i.e. on 14.01.2015.

EXAMPLE 4. A and B can do a certain piece of work in 18 days, B and C can do it in 12 days and C and A can do it in 24 days. How long would each take separately to do it ?

Sol. (A + B)'s one days's work = 1/18,
(A + C)'s one days's work = 1/24,
(B + C)'s one days's work = 1/12,
Now add up all three equations :

$$2(A+B+C)\text{'s one days's work} = \frac{1}{18}+\frac{1}{24}+\frac{1}{12}=\frac{13}{72}$$

$$(A+B+C)\text{'s one days's work} = \frac{13}{144}$$

A's one days's work = (A + B + C)'s one days's work

$$-(B+C)\text{'s one days's work} = \frac{13}{144}-\frac{1}{12}=\frac{1}{144}$$

Since A completes of the work in 1 day, he will complete 1 work in $\frac{144}{1}=144$ days

By similar logic we can find that B needs $\frac{144}{7}$ days and C will require $\frac{144}{5}$ days.

Shortcut Approach

- If A and B together can do a piece of work in X days and A alone can do it in Y days, then B alone can do the work in $\frac{XY}{Y-X}$ days.
- If (A + B + C) can do a piece of work in X days and (B + C) can do a piece of work in Y days then

  A can do a piece of work $\frac{XY}{Y-X}$ days

EXAMPLE 5. A and B together can do a piece of work in 6 days and A alone can do it in 9 days. In how many days can B alone do it?

Sol. $(A+B)$'s 1 day's work $=\frac{1}{6}$th part of the whole work.

A's 1 day's work $=\frac{1}{9}$th part of the whole work.

$\therefore$ B's 1 day's work $=\frac{1}{6}-\frac{1}{9}=\frac{3-2}{18}=\frac{1}{18}$th part of the whole work.

$\therefore$ B alone can do the work in 18 days.

SHORTCUT METHOD

B alone can do the whole work in

$$\frac{6\times 9}{9-6}=\frac{54}{3}=18 \text{ days}$$

Shortcut Approach

- A and B can do a work in 'X' and 'Y' days respectively. They started the work together but A left 'a' days before completion of the work. Then, time taken to finish the work is $\frac{Y(X+a)}{X+Y}$
- If 'A' is 'a' times efficient than B and A can finish a work in X days, then working together, they can finish the work in $\frac{aX}{a+1}$ days.
- If A is 'a' times efficient than B and working together they finish a work in Z days then, time taken by A = $\frac{Z(a+1)}{a}$ days. and time taken by B = Z(a + 1) days.
- If A working alone takes 'x' days more than A and B together, and B working along takes 'y' days more than A and B together then the number of days taken by A and B working together is given by $[\sqrt{xy}]$ days.

EXAMPLE 6. A and B can do alone a job in 6 days and 12 days. They began the work together but 3 days before the completion of job, A leaves off. In how many days will the work be completed?

(a) 6 days **(b)** 4 days
(c) 5 days **(d)** 7 days

Sol. (a) Let work will be completed in x days. Then, work done by A in (x – 3) days + work done by B in x days = 1

$$\text{i.e. } \frac{x-3}{6}+\frac{x}{12}=1$$

$$\Rightarrow \frac{3x-6}{12}=1 \Rightarrow x=6 \text{ days}$$

SHORTCUT METHOD

$$\text{Required time} = \frac{12(6+3)}{12+6}=6 \text{ days}$$

EXAMPLE 7. A is half good a workman as B and together they finish a job in 14 days. In how many days working alone will B finish the job.

(a) 20 days (b) 21 days
(c) 22 days (d) None of these

Sol. (b) Let B can do the work in x days and A can do the work in 2x days

$$\text{Then, } \frac{1}{x}+\frac{1}{2x}=\frac{1}{14} \quad \text{(given)}$$

$$\Rightarrow x=\frac{3}{2}\times 14=21 \text{ days}$$

SHORTCUT METHOD

$$\text{Time taken by B}=14\left(\frac{1}{2}+1\right)=21 \text{ days}$$

☞ Shortcut Approach

If a_1 men and b_1 boys can complete a work in x days, while a_2 men and b_2 boys can complete the same work in y days, then

$$\frac{\text{One day work of 1 man}}{\text{One day work of 1 boy}}=\frac{(yb_2-xb_1)}{(xa_1-ya_2)}$$

EXAMPLE 8. If 12 men and 16 boys can finish a work in 5 days, while 13 men and 24 boys can finish the same work in 4 days. Compare the one day work of 1 man and 1 boy.

Sol. Here, $a_1=12, b_1=16, x=5, a_2=13, b_2=24$ and $y=4$

$$\frac{\text{One day work of 1 man}}{\text{One day work of 1 boy}}=\frac{(yb_2-xb_1)}{(xa_1-ya_2)}$$

$$=\frac{4\times 24-5\times 16}{5\times 12-4\times 13}$$

$$=\frac{96-80}{60-52}=\frac{16}{8}=\frac{12}{1}$$

☞ Shortcut Approach

If n men or m women can do a piece of work in X days, then N men and M women together can finish the work in $\frac{nmX}{nM+mN}$ days.

EXAMPLE 9. 10 men can finish a piece of work in 10 days, where as it takes 12 women to finish it in 10 days. If 15 men and 6 women undertake to complete the work, how many days they will take to complete it?

(a) 7 days (b) 5 days
(c) 4 days (d) 6 days

Sol. (b) It is clear that 10 men = 12 women or 5 men = 6 women

$\Rightarrow$ 15 men + 6 women = (18 + 6) i.e. 24 women

Now 12 women can complete the work in 10 days

$\therefore$ 24 women will do it in 5 days.

SHORTCUT METHOD

$$\text{Required time}=\frac{10\times 12\times 10}{10\times 6+12\times 15}=5 \text{ days}$$

EXAMPLE 10. If 3 men or 4 women can reap a field in 43 days, how long will 7 men and 5 women take to reap it ?

Sol. First Method : 3 men reap $\frac{1}{43}$ of the field in 1 day.

$\therefore$ 1 men reaps $\frac{1}{43\times 3}$ of the field in 1 day.

4 women reap $\frac{1}{43}$ of the field in 1 day.

$\therefore$ 1 woman reaps $\frac{1}{43\times 4}$ of the field in 1 day.

$\therefore$ 7 men and 5 women reap $\left(\frac{7}{43\times 3}+\frac{5}{43\times 4}\right)=\frac{1}{12}$ of the field in 1 day.

$\therefore$ 7 men and 5 women will reap the whole field in 12 days.

Second Method : 3 men = 4 women

$\therefore$ 1 man $=\frac{4}{3}$ women $\quad \therefore$ 7 men $=\frac{28}{3}$ women

$\therefore$ 7 men + 5 women $=\frac{28}{3}+5=\frac{43}{3}$ women now, the question becomes :

If 4 women can reap a field in 43 days, how long will $\frac{43}{3}$ women take to reap it ?

The basic-formula gives

$$4\times 43=\frac{43}{3}\times D_2 \quad \text{or,} \quad D_2=\frac{4\times 43\times 3}{43}=12 \text{ days.}$$

SHORTCUT METHOD

$$\text{Required number of days}=\frac{1}{\left[\frac{7}{43\times 3}+\frac{5}{43\times 4}\right]}$$

$$=\frac{43\times 3\times 4}{7\times 4+5\times 3}=12 \text{ days.}$$

EXAMPLE 11. If 12 men and 16 boys can do a piece of work in 5 days and 13 men and 24 boys can do it in 4 days, how long will 7 men and 10 boys take to do it ?

Sol. 12 men and 16 boys can do the work in 5 days .... (1)

13 men and 24 boys can do the work in 4 days .... (2)

Now it is easy to see that if the no. of workers be multiplied by any number, the time must be divided by the same number (derived from : more worker less time).

Hence multiplying the no. of workers in (1) and (2) by 5 and 4 respectively, we get 5 (12 men + 16 boys) can do the work in 5/5 = 1 day

4 (13 men + 24 boys) can do the work in $\frac{4}{4} = 1$ day

or, $5(12m + 16b) = 4(13m + 24b)$

or, $60\,m + 80\,b = 52\,m + 96\,b$

or, $60\,m - 52\,m = 96\,b - 80\,b$

or, $8\,m = 16\,b$

$\therefore$ 1 men = 2 boys.

Thus, 12 men + 16 boys = 24 boys + 16 boys = 40 boys

and 7 men + 10 boys = 14 boys + 10 boys = 24 boys

The question now becomes :

"If 40 boys can do a piece of work in 6 days how long will 24 boys take to do it ?"

Using basic formula, we have

$40 \times 5 = 24 \times D_2$

or, $D_2 = \frac{40 \times 5}{24} = 8\frac{1}{3}$ days

EXAMPLE 12. Two men and 7 boys can do a piece of work in 14 days. 3 men and 8 boys can do it in 11 days. In how many days can 8 men and 6 boys do a work 3 times as big as the first ?

Sol. 2 men + 7 boys in 14 days $\Rightarrow$ 28 men + 98 boys in 1 day

3 men + 8 boys in 11 days $\Rightarrow$ 33 men + 88 boys in 1 day

$\therefore$ 28 men + 98 boys = 33 men + 88 boys

$\therefore$ 2 boys $\equiv$ 1 man

Now, 2 men + 7 boys = 11 boys; 8 men + 6 boys = 22 boys

More boys, fewer days; more work, more days

Boys	Days	Work
11 ↓	14 ↑	1 ↑
22	x	3

$\therefore \frac{x}{14} = \frac{11}{22} \times \frac{3}{1}$ $\therefore$ Number of days = 21 days.

EXAMPLE 13. Kaberi takes twice as much time as Kanti and thrice as much as Kalpana to finish a place of work. They together finish the work in one day. Find the time taken by each of them to finish the work.

Sol. Here, the alone time of kaberi is related to the alone times of other two persons, so assume the alone time of kaberia = x,

Then, alone time of kanti = $\frac{x}{2}$ and of kalpana = $\frac{x}{3}$

Kaberi's 1 day work + kanti's 1 day work + kalpana's 1 day work = combined 1 days work

$$\Rightarrow \frac{1}{x} + \frac{1}{x/2} + \frac{1}{x/3} = \frac{1}{1} \Rightarrow \quad x = 6$$

$\therefore$ Alone time for kaberi = 6 days, for kanti = 6/2 = 3 days, kalpana = 6/3 = 2 days,

EXAMPLE 14. 1 man or 2 women or 3 boys can do a work in 44 days. Then in how many days will 1 man, 1 woman and 1 boy do the work?

Sol. Number of required days

$$= \frac{1}{\frac{1}{44 \times 1} + \frac{1}{44 \times 2} + \frac{1}{44 \times 3}} = \frac{44 \times 1 \times 2 \times 3}{6 + 3 + 2} = 24 \text{ days}$$

Shortcut Approach

A and B do a piece of work in a and b days, respectively. Both begin together but after some days, A leaves off and the remaining work is completed by B in x days. Then, the time after which A left, is given by

$$T = \frac{(b - x)a}{a + b}$$

EXAMPLE 15. A and B can do a piece of work in 40 days and 50 days, respectively. Both begin together but after a certain time, A leaves off. In this case B finishes the remaining work in 20 days. After how many days did A leave?

Sol. Here, a = 40 days, b = 50 days, x = 20 and T = ?

$$\therefore \text{Required time} = \frac{(b - x)a}{a + b} = \frac{(50 - 20) \times 40}{(40 + 50)}$$

$$= \frac{30 \times 40}{90} = \frac{40}{3} = 13\frac{1}{3} \text{ days}$$

Shortcut Approach

If 'M_1' persons can do 'W_1' works in 'D_1' days and 'M_2' persons can do 'W_2' works in 'D_2' days then

$M_1 D_1 W_2 = M_2 D_2 W_1$

If T_1 and T_2 are the working hours for the two groups then

$M_1 D_1 W_2 T_1 = M_2 D_2 W_1 T_2$

Similarly,
$M_1D_1W_2T_1E_1 = M_2D_2W_1T_2E_2$, where E_1 and E_2 are the efficiencies of the two groups.

- If the number of men to do a job is changed in the ratio a : b, then the time required to do the work will be in the ratio b : a, assuming the amount of work done by each of them in the given time is the same, or they are identical.
- A is K times as good a worker as B and takes X days less than B to finish the work. Then the amount of time required by A and B working together is $\frac{K \times X}{K^2 - 1}$ days.
- If A is n times as efficient as B, i.e. A has n times as much capacity to do work as B, then A will take $\frac{1}{n}$ of the time taken by B to do the same amount of work.

EXAMPLE **16. 5 men prepare 10 toys in 6 days working 6 hrs a day. Then in how many days can 12 men prepare 16 toys working 8 hrs a day ?**

Sol. $M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$
Here, $5 \times 6 \times 6 \times 16 = 12 \times D_2 \times 8 \times 10$

$\therefore D_2 = \frac{5 \times 6 \times 6 \times 16}{12 \times 8 \times 10} = 3$ days.

EXAMPLE **17. A and B can do a work in 45 days and 40 days respectively. They began the work together, but A left after some time and B finished the remaining work in 23 days. After how many days did A leave ?**

Sol. B works alone for 23 days.

$\therefore$ Work done by B in 23 days $= \frac{23}{40}$ work

$\therefore$ Work done by A + B together $= 1 - \frac{23}{40} = \frac{17}{40}$ work

Now, A + B do 1 work in $\frac{40 \times 45}{40 + 45} = \frac{40 \times 45}{85}$ days

$\therefore$ A + B do $\frac{17}{40}$ work in $\frac{40 \times 45}{85} \times \frac{17}{40} = 9$ days.

SHORTCUT METHOD

If we ignore the intermediate steps, we can write a direct formula as : $\frac{40 \times 45}{40 + 45}\left(\frac{40 - 23}{40}\right) = 9$ days.

EXAMPLE **18. Two friends take a piece of work for ₹ 960. One alone could do it in 12 days, the other in 16 days with the assistance of an expert they finish it in 4 days. How much remuneration the expert should get ?**

Sol. First friend's 4 day's work $= \frac{4}{12} = \frac{1}{3}$ (Since, the work is finished in 4 days, when expert assists)

Second friends's 4 day's work $= \frac{4}{16} = \frac{1}{4}$

The expert's 4 day's work $= 1 - \left(\frac{1}{3} + \frac{1}{4}\right) = \frac{5}{12}$

Now, total wages of ₹ 960 is to be distributed among two friends and the expert in proportion to the amount of work done by each of them.
So, 960 is to be divided in the proportion of

$\frac{1}{3} : \frac{1}{4} : \frac{5}{12}$ or 4 : 3 : 5

$\therefore$ Share of expert $= \frac{5}{12} \times 960 =$ ₹ 400

Hence, the expert should get ₹ 400.

EXAMPLE **19. A certain number of men can do a work in 60 days. If there were 8 men more it could be finished in 10 days less. How many men are there ?**

Sol. Let there be x men originally.
(x + 8) men can finish the work in (60 – 10) = 50 days.
Now, 8 men can do in 50 days what x men do in 10 days, then by basic formula we have

$\therefore x = \frac{8 \times 50}{10} = 40$ men.

SHORTCUT METHOD (1) :

We have :
x men to the work in 60 days and (x + 8) men do th work in (60 – 10 =) 50 days.
Then by "basic formula", 60x = 50(x + 8)

$\therefore x = \frac{50 \times 8}{10} = 40$ men.

SHORTCUT METHOD (2) :

There exists a relationship :
Original number of workers

$= \frac{\text{No. of more wor ker} \times \text{Number of days taken by the sec ond group}}{\text{No. of less days}}$

$= \frac{8 \times (60 - 10)}{10} = \frac{8 \times 50}{10} = 40$ man

EXAMPLE **20. Two coal loading machines each working 12 hours per day for 8 days handles 9,000 tonnes of coal with an efficiency of 90%. While 3 other coal loading machines at an efficiency of 80 % set to handle 12,000 tonnes of coal in 6 days. Find how many hours per day each should work.**

Sol. Here $\frac{N_1 \times D_1 \times R_1 \times E_1}{W_1} = \frac{N_2 \times D_2 \times R_2 \times E_2}{W_2}$

$N_1 = R_1 = 12h/\text{ day} : N_2 = 3R_2 = ?$

$E_1 = \frac{90}{100}$ $W_1 = 9{,}000$;

$E_2 = \frac{80}{100}$ $W_2 = 12{,}000$

$\Rightarrow \frac{2 \times 8 \times 12 \times 90}{9{,}000 \times 100} \frac{3 \times 6 \times R_2\, 80}{12{,}000 \times 100}$

$\Rightarrow R_2 = 16$ h / day.

$\therefore$ Each machine should work 16 h/ day.

WORK AND WAGES

Wages are distributed in proportion to the work done and in indirect proportion to the time taken by the individual.

EXAMPLE 21. A, B and C can do a work in 6, 8 and 12 days respectively. Doing that work together they get an amount of ₹ 1350. What is the share of B in that amount?

Sol. A's one day's work $= \frac{1}{6}$

B's one day's work $= \frac{1}{8}$

C's one day's work $= \frac{1}{12}$

A's share : B's share : C's share

$= \frac{1}{6} : \frac{1}{8} : \frac{1}{12}$

Multiplying each ratio by the L.C.M. of their denominators, the ratios become 4 : 3 : 2

$\therefore$ B's share $= \frac{1350 \times 3}{9} = ₹450$

EXAMPLE 22. If 6 men working 8 hours a day earn ₹1680 per week, then how much will 9 men working 6 hours a day earn per week ?

Sol. 6m 8 hours ₹ 1680

9m 6 hours ?

$1680 \times \frac{6}{8} \times \frac{9}{6} = ₹\, 1890$

SHORTCUT METHOD

As earnings are proportional to the work done, we have

$$\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2} \Rightarrow \frac{6 \times 8}{1680} = \frac{9 \times 6}{W_2} \Rightarrow W_2 = ₹\, 1890$$

EXAMPLE 23. A can do a piece of work in 15 days and B in 20 days. They finished the work with the assistance of C in 5 days and got ₹ 45 as their wages, find the share for each in the wages.

Sol. A did in 5 days 1/3 of the work,

B did in 5 days 1/4 of the work.

C did in 5 days $1 - \frac{1}{3} + \frac{1}{4} = \frac{5}{12}$ of the work

Since A, B, C did in 5 days 1/3, 1/4, 5/12 of the work respectively.

A's share $= ₹\ 45 \times \frac{1}{3} = ₹\, 15$

B's share $= ₹\ 45 \times \frac{1}{4} = ₹\, 11.25$

C's share $= ₹\, 45 \times \frac{5}{12} = ₹\, 18.75$

EXAMPLE 24. If 8 men, working 9 hours per day can build a wall 18 meter long, 2 meters wide and 12 meters high in 10 days, how many men will be required to build a wall 32 meters long, 3 meters wide and 9 meters high by working 6 hours a day in 8 days ?

Sol. This method is a substitute for the conventional method and can be safely employed for most of the problems.

Step 1 : Assume the thing to be found as 'X'

Step 2 : In the first place look for X's counterpart.

e.g. in the above example, X = no. of men

So X's counterpart = No. of men, given = 8.

So write X = 8x.......

Now see the direct and indirect variation or simply see by which operation more men will be required & by which fewer:

We have $X = 8 \times \frac{32}{18} \times \frac{3}{2} \times \frac{9}{12} \times \frac{10}{8} \times \frac{9}{6} = 30$ men

SHORTCUT METHOD

Given that

$M_1 = 8$ men	$M_2 = x$ (Let)
$t_1 = 9$ hour	$t_2 = 6$ hour
$D_1 = 10$ days	$D_2 = 8$ days
$W_1 = 18 \times 2 \times 12\ m^3$	$W_2 = 32 \times 3 \times 9\ m^3$

Now, $\frac{M_1 t_1 d_1}{w} = \frac{M_2 d_2 t_2}{w^2}$

$$= \frac{8 \times 9 \times 10}{18 \times 2 \times 12} = \frac{x \times 6 \times 8}{32 \times 3 \times 9} = \boxed{x = 30 \text{ men}}$$

EXAMPLE 25. If 5 engines consume 6 tonnes of coal when each runs 9 hours per day, how much coal will be needed for 8 engines, each running 10 hours. per day, it being given that 3 engines of the former type consume as much as 4 engines of latter type ?

Sol. We have $X = 6 \times \frac{8}{5} \times \frac{10}{9} \times \frac{3}{4} = 8$ tons

Explanation : (1) More engines more coal (> 1)

(2) More time, more coal (> 1)

(3) Latter consumes less coal than former (< 1).

In case of men working we have more time, less men (< 1) but here we have more time, more coal (> 1).

Here let W = 6 tonnes ≡ 5 × 9 × 4/3 engine hours

and let X ≡ 8 × 10 × 1 engine hours.

or $X = 6 \text{ tons} \times \frac{8 \times 10 \times 1}{5 \times 9 \times (4/3)} = 8$ tons

SHORTCUT METHOD

Given that

$M_1 = 5$ engines	$M_2 = 8$ engines
$t_1 = 9$ hour	$t_2 = 10$ hour
$w_1 = 6$ tones	$n_2 =$ (efficiency)
n_1 (efficiency) $= 4$	$w_2 = x$ (let)

Now, $\frac{m_1 t_1 n_1}{w_1} = \frac{m_2 t_2 n_2}{w_2}$

$\therefore \quad \frac{5 \times 9 \times 4}{6} = \frac{8 \times 10 \times 3}{x}$

$x = 8$ tonnes

EXAMPLE 26. A garrison of 1500 men is provisioned for 60 days. After 25 days the garrison is reinforced by 500 men, how long will the remaining provisions last ?

Sol. Since the garrison is reinforced by 500 men therefore then are $(1500 + 500)$ or 2000 men now,
since $60 - 25 = 35$ days.
$\Rightarrow$ The provisions left would last 1500 men 35 days
$\Rightarrow$ Provisions left would last 1 man 35×1500 days
$\Rightarrow$ Provisions left would last 2000 men

$$35 \times \frac{1500}{2000} = 26.25 \text{ days}$$

By work equivalence method
$1500 \times 60 = (1500 \times 25) + (2000 \times X)$
Solve to get $X = 26.25$ days.

EXAMPLE 27. 40 men can cut 60 trees is 8 hours. If 8 men leaves the job how many trees will be cut in 12 hours ?

Sol. 40 men – working 8 hours – cut 60 trees

or, 1 man – working 1 hour – cuts $\frac{60}{40 \times 8}$ trees

Thus, 32 men– working 12 hours – cuts $\frac{60 \times 32 \times 12}{40 \times 8} = 73$ trees.

Using basic concepts :
$M_1 = 40$, $D_1 = 8$ (As days and hrs both denote time)
$W_1 = 60$ (cutting of trees is taken as work)
$M_2 = 40 - 8 = 32$, $D_2 = 12$, $W_2 = ?$
Putting the values in the formula
$M_1 D_1 W_2 = M_2 D_2 W_1$
We have , $40 \times 8 \times W_2 = 32 \times 12 \times 60$

or, $W_2 = \frac{32 \times 12 \times 60}{40 \times 8} = 72$ trees.

EXAMPLE 28. I can finish a work in 15 days at 8 hours a day. You can finish it in $6\frac{2}{3}$ days at 9 hrs a day. Find in how many days we can finish it working together 10 hrs a day.

Sol. First suppose each of us works for only one hr a day.
Then I can finish the work in $15 \times 8 = 120$ days and you can finish the work in $\frac{20}{3} \times 9 = 60$ days.
But here we are given that we do the work 10 hrs a day. Then clearly we can finish the work in 4 days.

EXAMPLE 29. A can do a work in 6 days. B takes 8 days to complete it. C takes as long as A and B would take working together. How long will it take B and C to complete the work together ?

Sol. $(A + B)$ can do the work in $\frac{6 \times 8}{6 + 8} = \frac{24}{7}$ days.

$\therefore$ C takes $\frac{24}{7}$ days to complete the work.

$\therefore$ $(B + C)$ takes $\frac{\frac{24}{7} \times 8}{\frac{24}{7} + 8} = \frac{24 \times 8}{24 + 56} = 2\frac{2}{5}$ days.

EXAMPLE 30. A group of 20 cows can graze a field 3 acres in size in 10 days. How many cows can graze a field twice as large in 8 days ?

Sol. Here, first of all, let us see how work can be defined. It is obvious that work can be measured as "acres grazed".
In the first case, there were 20 cows in the group.
They had to work for 10 days to do the work which we call W (which = 3)
$\Rightarrow \; 20 \times 10 = 3$ .... (1)
Do not be worried about the numerical values on either side. The point is that logically this equation is consistent as the LHS indicates "Cowdays" and the RHS indicates "Acres", both of which are correct ways of measuring work done.
Now the field is twice as large. Hence the new equation is
$\Rightarrow \; C \times 8 = 6$ .... (2)
Just divide (2) by (1) to get the answer.

$$\frac{8C}{200} = \frac{6}{3}$$

$\Rightarrow \; 8C = 2 \times 200 \Rightarrow \; C = \frac{400}{8} = 50$ cows.

Hence, there were 50 cows in the second group.

SHORTCUT METHOD

No. of cows $(M_1) = 20$	No. of cows $(m_2) = 2$
No. of dogs $(D_1) = 10$	No. of dogs $(d_2) = 8$
Field graze $(w_1) = 3$	Field graze $= 3 \times 2 = 6$

Now, By formula

$$\frac{M_1 d_1}{W_1} = \frac{M_2 d_2}{W_2}$$

$$\Rightarrow \quad \frac{20 \times 10}{3} = \frac{n_2 \times 8}{6}$$

$n_2 = 50$ cows

EXAMPLE 31. In how many days can the work be completed by A and B together ?

I. A alone can complete the work in 8 days.
II. If A alone works for 5 days and B alone works for 6 days, the work gets completed.
III. B alone can complete the work in 16 days.

(a) I and II only **(b) II and III only**
(c) Any two of the three **(d) II and either I or III**

Sol. (c) I. A can complete the job in 8 days. So, A's 1 days' work $= \frac{1}{8}$

II. A works for 5 days, B works for 6 days and the work is completed.

III. B can complete the job in 16 days. So B's 1 days's work $= \frac{1}{16}$

I and III : $(A + B)$'s 1 days' work $= \left(\frac{1}{8} + \frac{1}{16}\right) = \frac{3}{16}$

$\therefore$ Both can finish the work in $\frac{16}{3}$ days.

II and III : Suppose A takes x days to finish the work

Then, $\frac{5}{x} + \frac{6}{16} = 1 \Rightarrow \frac{5}{x} = \left(1 - \frac{3}{8}\right) = \frac{5}{8} \Rightarrow x = 8$

$\therefore$ $(A + B)$'s 1 days' work $= \left(\frac{1}{8} + \frac{1}{16}\right) = \frac{3}{16}$

$\therefore$ Both can finish it in $\frac{16}{3}$ days.

I and II : A' 1 day's work $= \frac{1}{8}$.

Suppose B takes x days to finish the work

Then from II, $\left(5 \times \frac{1}{8} + 6 \times \frac{1}{x} = 1\right)$

$\Rightarrow \frac{6}{x} = \left(1 - \frac{5}{8}\right) = \frac{3}{8} \Rightarrow x = \left(\frac{8 \times 6}{3}\right) = 16$

$\therefore$ $(A + B)$'s 1 days' work $= \left(\frac{1}{8} + \frac{1}{16}\right) = \frac{3}{16}$

$\therefore$ Both can finish it in $\frac{16}{3}$ days.

EXAMPLE 32. In how many days can the work be done by 9 men and 15 women ?

I. 6 men and 5 women can complete the work in 6 days
II. 3 men and 4 women can complete the work in 10 days
III. 18 men and 15 women can complete the work in 2 days.

(a) III only **(b) All I, II and III**
(c) Any two of the three **(d) Any of the three**

Sol. (c) Clearly, any two of the three will give two equations in x and y, which can be solved simultaneously.
Foir example I and II together give

$$\left(6x + 5y = \frac{1}{6}, 3x + 4y = \frac{1}{10}\right)$$

EXAMPLE 33. 8 men and 14 women are working together in a field. After working for 3 days, 5 men and 8 women leave the work. How many more days will be required to complete the work ?

I. 19 men and 12 women together can complete the work in 18 days.
II. 16 men complete two-third of the work in 16 days
III. In a day, the work done by three men is equal to the work done by four women.

(a) I only **(b) II only**
(c) III only **(d) I or II or III**

Sol. (d) Clearly, I only gives the answer
Similarly, II only gives the answer
And, III only gives the answer

PIPES AND CISTERNS

The same principle of Time and Work is employed to solve the problems on Pipes and Cisterns. The only difference is that in this case, the work done is in terms of filling or emptying a cistern (tank) and the time taken is the time taken by a pipe or a leak (crack) to fill or empty a cistern respectively.

Inlet : A pipe connected with a tank (or a cistern or a reservoir) is called an inlet, if it fills it.

Outlet : A pipe connected with a tank is called an outlet, if it empties it.

Shortcut Approach

- If a pipe can fill a tank in x hours, then the part filled in 1 hour $= \frac{1}{x}$
- If a pipe can empty a tank in y hours, then the part of the full tank emptied in 1 hour $= \frac{1}{y}$.
- If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then the net part filled in 1 hour, when both the pipes are opened $= \left(\frac{1}{x} - \frac{1}{y}\right)$.

  $\therefore$ Time taken to fill the tank, when both the pipes are opened $= \frac{xy}{y - x}$.
- If a pipe can fills or empties tank in x hours and another can fill or empties the same tank in y hours, then time taken to fill or empty the tank $= \frac{xy}{y + x}$, when both the pipes are opened
- If a pipe fills a tank in x hours and another fills the same tank is y hours, but a third one empties the full tank in z hours, and all of them are opened together, then net part filled in 1 hr $= \left[\frac{1}{x} + \frac{1}{y} - \frac{1}{z}\right]$

  $\therefore$ Time taken to fill the tank $= \frac{xyz}{yz + xz - xy}$ hours.
- A pipe can fill a tank in x hrs. Due to a leak in the bottom it is filled in y hrs. If the tank is full, the time taken by the leak to empty the tank $= \frac{xy}{y - x}$ hrs.

- A cistern has a leak which can empty it in X hours. A pipe which admits Y litres of water per hour into the cistern is turned on and now the cistern is emptied in Z hours. Then the capacity of the cistern is $\frac{X+Y+Z}{Z-X}$ litres.
- A cistern is filled by three pipes whose diameters are X cm., Y cm. and Z cm. respectively (where X < Y < Z). Three pipes are running together. If the largest pipe alone will fill it in P minutes and the amount of water flowing in by each pipe is proportional to the square of its diameter, then the time in which the cistern will be filled by the three pipes is $\left[\frac{PZ^2}{X^2+Y^2+Z^2}\right]$ minutes.
- If one filling pipe A is n times faster and takes X minutes less time than the other filling pipe B, then the time they will take to fill a cistern, if both the pipes are opened together, is $\left[\frac{nX}{(n^2-1)}\right]$ minutes. A will fill the cistern in $\left(\frac{X}{n-1}\right)$ minutes and B will take to fill the cistern $\left(\frac{nX}{n-1}\right)$ minutes.

  Here, A is the faster filling pipe and B is the slower one.
- Two filling pipes A and B opened together can fill a cistern in t minutes. If the first filling pipe A alone takes X minutes more or less than t and the second fill pipe B along takes Y minutes more or less than t minutes, then t is given by $[t=\sqrt{xy}]$ minutes.

EXAMPLE 34. A pipe can fill a cistern in 6 hours. Due to a leak in its bottom, it is filled in 7 hours. When the cistern is full, in how much time will it be emptied by the leak?

(a) 42 hours (b) 40 hours

(c) 43 hours (d) 45 hours

Sol. (a) Part of the capacity of the cistern emptied by the leak in one hour $=\left(\frac{1}{6}-\frac{1}{7}\right)=\frac{1}{42}$ of the cistern.

The whole cistern will be emptied in 42 hours.

EXAMPLE 35. Three pipes A, B and C can fill a cistern in 6 hrs. After working together for 2 hrs, C is closed and A and B fill the cistern in 8 hrs. Then find the time in which the cistern can be filled by pipe C.

Sol. $A+B+C$ can fill in 1 hr $=\frac{1}{6}$ of cistern.

$A+B+C$ can fill in 2 hrs $=\frac{2}{6}=\frac{1}{3}$ of cistern.

Remaining part $=\left(1-\frac{1}{3}\right)=\frac{2}{3}$ is filled by $A+B$ in 8 hrs.

$\therefore$ $(A+B)$ can fill the cistern in $\frac{8\times 3}{2}=12$ hrs.

Since $(A+B+C)$ can fill the cistern in 6 hrs.

$\therefore$ $C=(A+B+C)-(A+B)$ can fill the cistern in $\frac{12\times 6}{12-6}$ hours = 12 hours.

EXAMPLE 36. Pipe A can fill a tank in 20 hours while pipe B alone can fill it in 30 hours and pipe C can empty the full tank in 40 hours. If all the pipes are opened together, how much time will be needed to make the tank full?

Sol. By direct formula,

The tank will be fill in $=\frac{20\times 30\times 40}{30\times 40+20\times 40-20\times 30}$

$=\frac{120}{7}=17\frac{1}{7}$ hrs.

EXAMPLE 37. Three pipes A, B and C can fill a tank in 6 minutes, 8 minutes and 12 minutes, respectively. The pipe C is closed 6 minutes before the tank is filled. In what time will the tank be full ?

(a) 4 min (b) 6 min

(c) 5 min (d) Data inadequate

Sol. (a) Let it takes t minutes to completely fill the tank.

Now, $\frac{t}{6}+\frac{t}{8}+\frac{t-6}{12}=1$

or $\frac{4t+3t+2t-12}{24}=1$

or $9t-12=24$

or $9t=36 \Rightarrow t=4$ min.

EXAMPLE 38. If three taps are opened together, a tank is filled in 12 hrs. One of the taps can fill it in 10 hrs and another in 15 hrs. How does the third tap work ?

Sol. We have to find the nature of the third tap, whether it is a filler or a waste pipe.

Let it be a filler pipe which fills in x hrs.

Then, $\frac{10\times 15\times x}{10\times 15+10x+15x}=12$

or, $150x=150\times 12+25x\times 12$

or $-150x=1800$ $\therefore$ $x=-12$

–ve sign shows that the third pipe is a waste pipe which vacates the tank in 12 hrs.

EXAMPLE 39. 4 pipes can fill a reservoir in 15, 20, 30 and 60 hours respectively. The first was opened at 6 am, second at 7 am third at 8 am and fourth at 9 am. When will the reservoir be full?

Sol. (1) Let the time be t hours after 6 am.

$\therefore$ $\frac{1}{15}\times t+\frac{(t-1)}{20}+\frac{(t-2)}{30}+\frac{(t-3)}{60}=1$

$\therefore$ $4t+3(t-1)+2(t-2)+(t-3)=60$

$\therefore$ $t=7$ hours $\therefore$ It is filled at 1 pm

EXAMPLE 40. A and B can fill a cistern in 7.5 minutes and 5 minutes respectively and C can carry off 14 litres per minute. If the cistern is already full and all the three pipes are opened, then it is emptied in 1 hour. How many litres can it hold ?

Sol. If the capacity is L litres, water filled in 1 hour = Water removed in 1 hour.

$$L+\frac{L}{7\frac{1}{2}}\times 60+\frac{L}{5}\times 60=14\times 60$$

$$\therefore\quad L+\frac{2L}{15}\times 60+12L=14\times 60\Rightarrow L+8L+12L=14\times 60$$

$\Rightarrow$ $21L=14\times 60$ or $L=40$ litres.

So the capacity of the cistern is 40 litres.

EXAMPLE 41. A cistern can be filled by two taps A and B in 25 minutes and 30 minutes respectively can be emptied by a third in 15 minutes. If all the taps are turned on at the same moment, what part of the cistern will remain unfilled at the end of 100 minutes ?

Sol. We have $\frac{1}{25}+\frac{1}{30}-\frac{1}{15}=\frac{1}{150}$ part filled in 1 minute

Hence, $1-100\left(\frac{1}{150}\right)$ = 1/3rd of the tank is unfilled after 100 minutes.

EXAMPLE 42. A barrel full of beer has 2 taps one midway, which draw a litre in 6 minutes and the other at the bottom, which draws a litre in 4 minutes. The lower tap is lower normally used after the level of beer in the barrel is lower than midway. The capacity of the barrel is 36 litres. A new assistant opens the lower tap when the barrel is full and draws out some beer. As a result the lower tap has been used 24 minutes before the usual time. For how long was the beer drawn out by the new assistant ?

Sol. The top tab is operational till 18 litres is drawn out.

$\therefore$ Time after which the lower tap is usually open $=18\times 6=108$ minutes

$\therefore$ Time after which it is open now $=108-24=84$ minutes

$\therefore$ Litres drawn $=84/6=14$ litres

$\therefore$ $18-14=4$ litres were drawn by the new assistant.

$\therefore$ Time $=4\times 4=16$ minutes

EXAMPLE 43. A cistern can be filled by two pipes filling separately in 12 and 16 min. respectively. Both pipes are opened together for a certain time but being clogged, only 7/8 of the full quantity of water flows through the former and only 5/6 through the latter pipe. The obstructions, however, being suddenly removed, the cistern is filled in 3 min. from that moment. How long was it before the full flow began?

(a) 2.5 min **(b) 4.5 min**

(c) 3.5 min **(d) 5.5 min**

Sol. (b) Both the pipes A and B can fill $\frac{1}{12}+\frac{1}{16}=\frac{7}{48}$ of the cistern in one minute, when their is no obstruction.

With obstruction, both the pipes can fill

$\frac{1}{12}\times\frac{7}{8}+\frac{1}{16}\times\frac{5}{6}=\frac{7}{96}+\frac{5}{96}=\frac{1}{8}$ of the cistern in one minute.

Let the obstructions were suddenly removed after x minutes.

$\therefore$ With obstruction, $\frac{x}{8}$ of the cistern could be filled in x minutes and so the remaining $1-\frac{x}{8}=\frac{8-x}{8}$ of the cistern was filled without obstruction in 3 minutes, i.e.

In one minute, $\frac{8-x}{24}$ of the cistern was filled.

$$\Rightarrow\frac{8-x}{24}=\frac{7}{48}\Rightarrow 16-2x=7\ \Rightarrow\ x=4.5$$

EXERCISE

1. 12 men take 18 days to complete a job whereas 12 women in 18 days can complete $\frac{3}{4}$ of the same job. How many days will 10 men and 8 women together take to complete the same job?
   (a) 6 (b) $13\frac{1}{2}$
   (c) 12 (d) Data inadequate
   (e) None of these
2. Seven men and four boys can complete a work in 6 days. A man completes double the work than a boy. In how many days will 5 men and 4 boys complete the work?
   (a) 5 (b) 4
   (c) 6 (d) Cannot be determined
   (e) None of these
3. The work done by a woman in 8 hours is equal to the work done by a man in 6 hours and by a boy in 12 hours. If working 6 hours per day, 9 men can complete a work in 6 days then in how many days can 12 men, 12 women and 12 boys together finish the same work by working 8 hours per day?
   (a) $1\frac{1}{3}$ days (b) $3\frac{2}{3}$ days
   (c) 3 days (d) $1\frac{1}{2}$ days
   (e) None of these
4. Tap '*A*' can fill a water tank in 25 minutes, tap '*B*' can fill the same tank in 40 minutes and tap '*C*' can empty that tank in 30 minutes. If all the three taps are opened together, in how many minutes will the tank be completely filled up or emptied?
   (a) $3\frac{2}{13}$ (b) $15\frac{5}{13}$
   (c) $8\frac{2}{13}$ (d) $31\frac{11}{19}$
   (e) None of these
5. Machine A can print one lakh books in 8 hours. Machine B can do the same job in 10 hours. Machine C can do the same job in 12 hours. All the three machines start job at 9.00 am. A breaks down at 11.00 am and the other two machines finish the job. **Approximately** at what time will the job be finished?
   (a) 12.00 noon (b) 1.30 pm
   (c) 1.00 pm (d) 11.30 am
   (e) None of these
6. Suresh can complete a job in 15 hours. Ashutosh alone can complete the same job in 10 hours. Suresh works for 9 hours and then the remaining job is completed by Ashutosh. How many hours will it take Ashutosh to complete the remaining job alone?
   (a) 4 (b) 5
   (c) 6 (d) 12
   (e) None of these
7. 10 men and 15 women finish a work in 6 days. One man alone finishes that work in 100 days. In how many days will a woman finish the work?
   (a) 125 days (b) 150 days
   (c) 90 days (d) 225 days
   (e) None of these
8. A tank is filled in 5 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank?
   (a) 35 hours (b) 25 hours
   (c) 20 hours (d) Cannot be determined
   (e) None of these
9. 24 men working 8 hours a day can finish a work in 10 days. Working at the rate of 10 hours a day, the number of men required to finish the same work in 6 days is :
   (a) 30 (b) 32
   (c) 34 (d) 36
   (e) None of these
10. A water tank is $\frac{2}{5}$ th full. Pipe A can fill the tank in 10 minutes and the pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?
    (a) 6 minutes to empty (b) 6 minutes to fill
    (c) 9 minutes to empty (d) 9 minutes to fill
    (e) None of these
11. A water tank has three taps A, B and C. Tap A, when opened, can fill the water tank alone in 4 hours. Tap B, when opened, can fill the water tank alone in 6 hours and tap C, when opened, can empty the water tank alone in 3 hours. If taps A, B and C are opened simultaneously, how long will it take to fill the tank completely?
    (a) 10 hours (b) 8 hours
    (c) 18 hours (d) 12 hours
    (e) None of these
12. Twenty-four men can complete a work in sixteen days. Thirty-two women can complete the same work in twenty-four days. Sixteen men and sixteen women started working and worked for twelve days. How many more men are to be added to complete the remaining work in 2 days?
    (a) 48 (b) 24
    (c) 36 (d) 16
    (e) None ofthe above
13. The total monthly income of four men and two women is ₹ 46,000. If every woman earns ₹ 500 more than a man then what is the monthly income of a woman?
    (a) ₹ 7,500 (b) ₹ 8,000
    (c) ₹ 9,000 (d) ₹ 6,500
    (e) None of these

14. 10 men can complete a piece of work in 15 days and 15 women can complete the same work in 12 days. If all the 10 men and 15 women work together, in how many days will the work get completed?

(a) 6 (b) $7\frac{2}{3}$

(c) $6\frac{2}{3}$ (d) $6\frac{1}{3}$

(e) None of these

15. 'A' completes a work in 12 days. *'B'* completes the same work in 15 days. *'A'* started working alone and after 3 days B joined him. How many days will they now take together to complete the remaining work?

(a) 5 (b) 8

(c) 6 (d) 4

(e) None of these

16. Rajani has to read a book of 445 pages. She has already read the first 157 pages of the book and if she reads 24 pages of the book everyday then how long will she take now to complete the book?

(a) 25 days (b) 20 days

(c) 46 days (d) 21 days

(e) None of these

17. 10 horses and 15 cows eat grass of 5 acres in a certain time. How many acres will feed 15 horses and 10 cows for the same time, supposing a horse eats as much as 2 cows ?

(a) 40/7 acres (b) 39/8 acres

(c) 40/11 acres (d) 25/9 acres

(e) None of these

18. X and Y can do job in 25 days and 30 days respectively. They work together for 5 days and then X leaves. Y will finish the rest of the work in how many days?

(a) 18 days (b) 19days

(c) 20days (d) 21 days

(e) None of these

19. A and B can do a job is 16 days and 12 days respectively. 4 days before finishing the job, A joins B. B has started the work alone. Find how many days B has worked alone?

(a) 6 days (b) 4 days

(c) 5 days (d) 7 days

(e) None of these

20. A contractor undertakes to built a walls in 50 days. He employs 50 peoples for the same. However after 25 days he finds that only 40% of the work is complete. How many more man need to be employed to complete the work in time?

(a) 25 (b) 30

(c) 35 (d) 20

(e) None of these

21. A is 30% more efficient than B. How much time will they, working together, take to complete a job which A along could have done in 23 days?

(a) 11 days (b) 13 days

(c) $20\frac{3}{17}$ days (d) 12 days

(e) None of these

22. A and B can finish a work in 10 days while B and C can do it in 18 days. A started the work, worked for 5 days, then B worked for 10 days and the remaining work was finished by C in 15 days. In how many days could C alone have finished the whole work ?

(a) 30 days (b) 15 days

(c) 45 days (d) 24 days

(e) None of these

23. 12 men complete a work in 18 days. Six days after they had started working, 4 men joined them. How many days will all of them take to complete the remaining work ?

(a) 10 days (b) 12 days

(c) 15 days (d) 9 days

(e) None of these

24. A tyre has two punctures. The first puncture along would have made the tyre flat in 9 minutes and the second alone would have done it in 6 minutes. If air leaks out at a constant rate, how long does it take both the punctures together to make it flat?

(a) $1\frac{1}{2}$ minutes (b) $3\frac{1}{2}$ minutes

(c) $3\frac{3}{5}$ minutes (d) $4\frac{1}{4}$ minutes

(e) None of these

25. A man, a woman or a boy can do a job in 20 days, 30 days or 60 days respectively. How many boys must assist 2 men and 8 women to do the work in 2 days?

(a) 15 boys (b) 8 boys

(c) 10 boys (d) 11 boys

(e) None of these

26. A can do 50% more work as B can do in the same time. B alone can do a piece of work in 20 hours. A, with help of B, can finish the same work in how many hours ?

(a) 12 (b) 8

(c) $13\frac{1}{3}$ (d) $5\frac{1}{2}$

(e) None of these

27. If 15 women or 10 men can complete a project in 55 days, in how many days will 5 women and 4 men working together complete the same project ?

(a) 75 (b) 8

(c) 9 (d) 85

(e) None of these

28. 12 buckets of water fill a tank when the capacity of each tank is 13.5 litres. How many buckets will be needed to fill the same tank, if the capacity of each bucket is 9 litres ?

(a) 8 (b) 15

(c) 16 (d) 18

(e) None of these

29. Water flows at 3 metres per sec through a pipe of radius 4 cm. How many hours will it take to fill a tank 40 metres long, 30 metres broad and 8 metres deep, if the pipe remains full?
(a) 176.6 hours (b) 120 hour
(c) 135.5 hours (d) None of these
(e) None of these

30. A can do a piece of work in 10 days, while B alone can do it in 15 days. They work together for 5 days and the rest of the work is done by C in 2 days. If they get ₹ 450 for the whole work, how should they divide the money ?
(a) ₹ 225, ₹ 150, ₹ 75 (b) ₹ 250, ₹ 100, ₹ 100
(c) ₹ 200, ₹ 150, ₹ 100 (d) ₹ 175, ₹ 175, ₹ 100
(e) None of these

31. A alone would take 8 days more to complete the job than if both A and B would together. If B worked alone, he took $4\frac{1}{2}$ days more to complete the job than A and B worked together. What time would they take if both A and B worked together?
(a) 7 days (b) 5 days
(c) 4 days (d) 6 days
(e) None of these

32. A alone can complete work in 15 days and B alone in 20 days. Starting with A, the work on alternate days. The total work will be completed in
(a) 17 days (b) 16 days
(c) 14 days (d) 13 days
(e) None of these

33. A contractor undertook to do a piece of work in 9 days. He employed certain number of laboures but 6 of them were absent from the very first day and the rest could finish the work in only 15 days. Find the number of men originally employed .
(a) 15 (b) 6
(c) 13 (d) 9
(e) None of these

34. After working for 8 days, Anil finds that only $\frac{1}{3}$ of the work has been done. He employs Rakesh who is 60 % efficient as Anil. How many more days will Anil take to complete the job?
(a) 15 days (b) 12 days
(c) 10 days (d) 8 days
(e) None of these

35. A sum of ₹ 25 was paid for a work which A can do in 32 days, B in 20 days, B and C in 12 days and D in 24 days. How much did C receive if all the four work together ?
(a) ₹ $\frac{14}{3}$ (b) ₹ $\frac{16}{3}$
(c) ₹ $\frac{15}{3}$ (d) ₹ $\frac{17}{3}$
(e) None of these

36. A and B can do a job in 15 days and 10 days, respectively. They began the work together but A leaves after some days and B finished the remaining job in 5 days. After how many days did A leave?
(a) 2 days (b) 3 days
(c) 1 day (d) 4 days
(e) None of these

37. Mr. Suresh is on tour and he has ₹ 360 for his expenses. If he exceeds his tour by 4 days he must cut down daily expenses by ₹ 3. The number of days of Mr. Suresh's tour programme is :
(a) 20 days (b) 24 days
(c) 40 days (d) 42 days
(e) None of these

38. A can knit a pair of socks in 3 days. B can knit the same thing in 6 days. If they are knitting together, in how many days will they knit two pairs of socks?
(a) 4 days (b) 2 days
(c) $4\frac{1}{2}$ days (d) 3 days
(e) None of these

39. A can do a job in 3 days less time than B. A works at it alone for 4 days and then B takes over and completes it. If altogether 14 days were required to finish the job, how many days would each of them take alone to finish it?
(a) 17 days, 20 days (b) 12 days, 15 days
(c) 13 days, 16 days (d) 14 days, 11 days
(e) None of these

40. Two workers A and B working together completed a job in 5 days. If A worked twice as efficiently as he actually did and B worked $\frac{1}{3}$ as efficiently as he actually did, the work would have completed in 3 days. Find the time for A to complete the job alone.
(a) $6\frac{1}{4}$ days (b) $5\frac{3}{4}$ days
(c) 5 days (d) 3 days
(e) None of these

41. X can do a piece of work in 15 days. If he is joined by Y who is 50% more efficient, in what time will X and Y together finish the work?
(a) 10 days (b) 6 days
(c) 18 days (d) Data insufficient
(e) None of these

42. A can build up a wall in 8 days while B can break it in 3 days. A has worked for 4 days and then B joined to work with A for another 2 days only. In how many days will A alone build up the remaining part of wall?
(a) $13\frac{1}{3}$ days (b) $7\frac{1}{3}$ days
(c) $6\frac{1}{3}$ days (d) 7 days
(e) None of these

43. Sakshi can do a piece of work in 20 days. Tanya is 25% more efficient than Sakshi. The number of days taken by Tanya to do the same piece of work is:
(a) 15 (b) 16
(c) 18 (d) 25
(e) None of these

44. Two taps can fill a tank in 12 and 18 minutes respectively. Both are kept open for 2 minutes and the first is turned off. In how many minutes more will the tank be filled ?
(a) 15 min. (b) 20 min.
(c) 11 min. (d) 13 min.
(e) None of these

45. A cistern normally takes 6 hours to be filled by a tap but because of a leak, 2 hours more. In how many hours will the leak empty a full cistern ?
(a) 20 hours (b) 24 hours
(c) 26 hours (d) 18 hours
(e) None of these

46. One fill pipe A is 3 times faster than second fill pipe B and takes 10 minutes less time to fill a cistern than B takes. Find when the cistern will be full if fill pipe B is only opened.
(a) 20 min (b) 18 min
(c) 15 min (d) 10 min
(e) None of these

47. Two pipes can fill a cistern in 14 and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom, 32 minutes extra are taken for the cistern to be filled up. If the cistern is full, in what time would the leak empty it ?
(a) 110 hours (b) 112 hours
(c) 115 hours (d) 100 hours
(e) None of these

48. Two pipes A and B can fill a cistern in 10 and 15 minutes respectively. Both fill pipes are opened together, but at the end of 3 minutes, 'B' is turned off. How much time will the cistern take to fill ?
(a) 6 min (b) 8 min
(c) 10 min (d) 12 min
(e) None of these

49. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern ?
(a) 10 min (b) 12 min
(c) 15 min (d) 9 min
(e) None of these

50. Two taps A and B can fill a cistern in 12 minutes and 18 minutes respectively. They are turned on at the same time. If the tap A is turned off after 4 minutes, how long will tap B take to fill the rest of the cistern ?
(a) 8 min. (b) 9 min.
(c) 10 min. (d) 7 min.
(e) None of these

51. A pipe can fill a tank in 15 minutes and another one in 10 minutes. A third pipe can empty the tank in 5 minutes. The first two pipes are kept open for 4 minutes in the beginning and then the third pipe is also opened. In what time will the tank be empited ?
(a) 35 min (b) 15 min
(c) 20 min (d) Cannot be emptied
(e) None of these

52. Two fill pipes A and B can fill a cistern in 12 and 16 minutes respectively. Both fill pipes are opened together, but 4 minutes before the cistern is full, one pipe A is closed. How much time will the cistern take to fill ?
(a) $9\frac{1}{7}$ min. (b) $3\frac{1}{3}$ min.
(c) 5 min. (d) 3 min.
(e) None of these

53. Two fill taps A and B can separately fill a cistern in 45 and 40 minutes respectively. They started to fill a cistern together but tap A is turned off after few minutes and tap B fills the rest part of cistern in 23 minutes. After how many minutes, was tap A turned-off ?
(a) 9 min (b) 10 min
(c) 12 min (d) 7 min
(e) None of these

54. Three fill pipes A, B and C can fill separately a cistern in 3, 4 and 6 minutes respectively. A was opened first. After 1 minute, B was opened and after 2 minutes from the start of A, C was also opened. Find the time when the cistern will be full ?
(a) $2\frac{1}{9}$ min (b) $4\frac{1}{2}$ min
(c) $3\frac{3}{4}$ min (d) 3 min
(e) None of these

55. A tap can fill a tank in 16 minutes and another can empty it in 8 minutes. If the tank is already 1/2 full and both the taps are opened together, will the tank be filled or emptied? How long will it take before the tank is either filled or emptied completely as the case may be ?
(a) Emptied; 16 min (b) Filled; 8 min
(c) Emptied; 8 min (d) Filled; 12 min
(e) None of these

CHAPTER 9

Time, Speed and Distance

TIME, SPEED AND DISTANCE

Speed

The rate at which any moving body covers a particular distance is called its speed.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}; \ \text{Time} = \frac{\text{Distance}}{\text{Speed}};$$

$$\text{Distance} = \text{Speed} \times \text{time}$$

Unit :

SI unit of speed is metre per second (mps). It is also measured in kilometers per hour (kph) or miles per hour (mph).

Basic Conversions :

(i)
- 1 hour = 60 minutes = 60 × 60 seconds.
- 1 km = 1000 m
- 1 km = 0.6214 mile
- 1 mile = 1.609 km i.e. 8 km = 5 miles
- 1 yard = 3 feet
- 1 foot = 12 inches
- 1 km/h $= \frac{5}{18}$ m/sec,
- 1 m/sec $= \frac{18}{5}$ km/h
- 1 miles/hr $= \frac{22}{15}$ ft/sec

Shortcut Approach

- Average speed $= \frac{\text{Total Distance}}{\text{Total time}}$
- While travelling a certain distance (d), if a man changes his speed in the ratio m : n, then the ratio of time taken becomes n : m.
- If a certain distance (d), say from A to B, is covered at 'a' km/hr and the same distance is covered again say from B to A in 'b' km/hr, then the average speed during the whole journey is given by :

  Average speed $= \left(\frac{2ab}{a+b}\right)$ km/hr

  Also, if t_1 and t_2 is time taken to travel from A to B and B to A respectively, the distance 'd' from A to B is given by :

  $$d = (t_1 + t_2)\left(\frac{ab}{a+b}\right)$$

  $$d = (t_1 - t_2)\left(\frac{ab}{b-a}\right)$$

  $$d = (b-a)\left(\frac{t_1 t_2}{t_1 - t_2}\right)$$

- If first part of the distance is covered at the rate of v_1 in time t_1 and the second part of the distance is covered at the rate of v_2 in time t_2, then the average speed is $\left(\frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}\right)$

Relative Speed

When two bodies are moving in same direction with speeds S_1 and S_2 respectively, their relative speed is the difference of their speeds.

i.e., Relative Speed $= S_1 - S_2$, If $S_1 > S_2$

$= S_2 - S_1$, if $S_2 > S_1$

When two bodies are moving in opposite direction with speeds S_1 and S_2 respectively, then their relative speed is the sum of their speeds.

i.e., Relative Speed $= S_1 + S_2$

EXAMPLE 1. The driver of a maruti car driving at the speed of 68 km/h locates a bus 40 metres ahead of him travelling in the same direction. After 10 seconds, the bus is 60 metres behind. The speed of the bus is.

Sol. Let speed of Bus $= S_B$ km/h.

Now, in 10 sec., car covers the relative distance

$= (60 + 40)$ m $= 100$ m

$\therefore$ Relative speed of car $= \frac{100}{10} = 10$ m/s

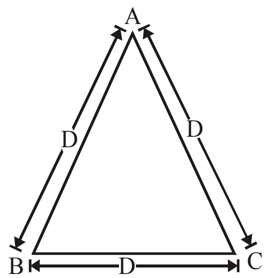
$$= 10 \times \frac{18}{5} = 36 \text{ km/h}$$

$$\therefore \ 68 - S_B = 36$$

$$\Rightarrow S_B = 32 \text{ km/h}$$

EXAMPLE 2. If a person goes around an equilateral triangle shaped field at speed of 10, 20 and 40 kmph on the first, second and third side respectively and reaches back to the starting point, then find his average speed during the journey.

Sol. Let the measure of each side of triangle is D km. The person travelled the distance from A to B with 10 kmph, B to C with 20 kmph and C to A with 40 kmph.



If T_{AB} = Time taken by the person to travel from A to B,
T_{BC} = Time taken by the person to travel from B to C and
T_{CA} = Time taken by the person to travel from C to A.
Then total time $= T_{AB} + T_{BC} + T_{CA}$

$$= \frac{D}{10} + \frac{D}{20} + \frac{D}{40} = D\left(\frac{8+4+2}{80}\right) = \frac{7D}{40}$$

Total distance travelled $= D + D + D = 3D$
Hence, average speed

$$= \frac{3D}{\frac{7D}{40}} = \frac{120}{7} = 17\frac{1}{7} \text{ kmph.}$$

EXAMPLE 3. Two guns were fired from the same place at an interval of 15 min, but a person in a bus approaching the place hears the second report 14 min 30 sec after the first. Find the speed of the bus, supposing that sound travels 330 m per sec.

Sol. Distance travelled by the bus in 14 min 30 sec could be travelled by sound in (15 min – 14 min 30 sec) = 30 second.

$\therefore$ Bus travels 330×30m in $14\frac{1}{2}$ min.

$\therefore$ Speed of the bus per hour

$$= \frac{330 \times 30 \times 2 \times 60}{29 \times 1000} = \frac{99 \times 12}{29} = \frac{1188}{29} = 40\frac{28}{29} \text{km / hr}$$

EXAMPLE 4. A hare sees a dog 100 m away from her and scuds off in the opposite direction at a speed of 12 km/h. A minute later the dog perceives her and gives chase at a speed of 16 km/h. How soon will the dog overtake the hare and at what distance from the spot where the hare took flight?

Sol. Suppose the hare at H sees the dog at D.

$\therefore$ DH = 100 m
Let K be the position of the hare where the dog sees her.
$\therefore$ HK = the distance gone by the hare in 1 min

$$= \frac{12 \times 1000}{60} \times 1\text{m} = 200 \text{ m}$$

$\therefore$ DK = 100 + 200 = 300 m
The hare thus has a start of 300 m.
Now the dog gains (16 – 12) or 4 km/h.

$\therefore$ The dog will gain 300 m in $\frac{60 \times 300}{4 \times 1000}$ min or $4\frac{1}{2}$ min.

Again, the distance gone by the hare in $4\frac{1}{2}$ min

$$= \frac{12 \times 1000}{60} \times 4\frac{1}{2} = 900\text{m}$$

$\therefore$ Distance of the place where the hare is caught from the spot H where the hare took flight = 200 + 900 = 1100 m

Shortcut Approach

If two persons (or vehicles or trains) start at the same time in opposite directions from two points A and B, and after crossing each other they take x and y hours respectively to complete the journey, then

$$\frac{\text{Speed of first}}{\text{Speed of second}} = \sqrt{\frac{y}{x}}$$

EXAMPLE 5. A train starts from A to B and another from B to A at the same time. After crossing each other they complete their journey in $3\frac{1}{2}$ and $2\frac{4}{7}$ hours respectively. If the speed of the first is 60 km/h, then find the speed of the second train.

Sol.

$$\frac{\text{1st train's speed}}{\text{2nd train's speed}} = \sqrt{\frac{y}{x}} = \sqrt{\frac{2\frac{4}{7}}{3\frac{1}{2}}} = \sqrt{\frac{18}{7} \times \frac{2}{7}} = \frac{6}{7}$$

$$\therefore \quad \frac{60}{\text{2nd train's speed}} = \frac{6}{7}$$

$\Rightarrow$ 2nd train's speed = 70 km/h.

Shortcut Approach

Usual speed : If a man changes his speed to $\frac{a}{b}$ of his usual speed, reachss his destination late/earlier by t minutes then,

$$\text{Usual time} = \frac{\text{Change in time}}{\left(\frac{b}{a} - 1\right)}$$

EXAMPLE 6. A boy walking at $\frac{3}{5}$ of his usual speed, reaches his school 14 min late. Find his usual time to reach the school.

Sol. Usual time $= \frac{14}{\frac{5}{3} - 1} = \frac{14 \times 3}{2} = 21$ min

EXAMPLE 7. A train after travelling 50 km, meets with an accident and then proceeds at $\frac{4}{5}$ of its former rate and arrives at the terminal 45 minutes late. Had the accident happened 20 km

further on, it would have arrived 12 minutes sooner. Find the speed of the train and the distance.

Sol. Let A be the starting place, B the terminal, C and D the places where the accidents to be placed.

By travelling at $\frac{4}{5}$ of its original rate the train would take $\frac{5}{4}$ of its usual time, i.e., $\frac{1}{4}$ of its original time more.

$\therefore$ $\frac{1}{4}$ of the usual time taken to travel the distance

CB = 45 min. ...(i)

and $\frac{1}{4}$ of the usual time taken to travel the distance

DB = (45 – 12) min ...(ii)

Subtracting (ii) from (i),

$\frac{1}{4}$ of the usual time taken to travel the distance

CD = 12 min.

$\therefore$ Usual time taken on travel 20 km = 48 min.

$\therefore$ Speed of the train per hour $= \frac{20}{48} \times 60$ or 25 km/h.

From (i), we have

Time taken to travel CB = 45 × 4 min = 3 hrs.

$\therefore$ The distance CB = 25 × 3 or 75 km.

Hence the distance AB = the distance (AC + CB)

= 50 + 75 or 125 km.

Shortcut Approach

A man covers a certain distance D. If he moves S_1 speed faster, he would have taken t time less and if he moves S_2 speed slower, he would have taken t time more. The original speed is given by

$$\frac{2 \times (S_1 \times S_2)}{S_2 \sim S_1}$$

EXAMPLE 8. A man covers a certain distance on scooter. Had he moved 3 km/h faster, he would have taken 20 min less. If he had moved 2 km/h slower, he would have taken 20 min more. Find the original speed.

Sol. Speed $= \frac{2 \times (3 \times 2)}{3 - 2} = 12 \text{km/hr.}$

Shortcut Approach

If a person with two different speeds U & V cover the same distance, then required distance

$$= \frac{U \times V}{U \sim V} \times \text{Difference between arrival time}$$

Also, required distance $= \text{Total time taken} \times \frac{U \times V}{U + V}$

EXAMPLE 9. A boy walking at a speed of 10 km/h reaches his school 12 min late. Next time at a speed of 15 km/h reaches his school 7 min late. Find the distance of his school from his house?

Sol. Difference between the time = 12 – 7 = 5 min $= \frac{5}{60} = \frac{1}{12}$ hr

Required distance $= \frac{15 \times 10}{15 - 10} \times \frac{1}{12} = \frac{150}{5} \times \frac{1}{12} = 2.5$ km

Shortcut Approach

A policemen sees a thief at a distance of d. He starts chasing the thief who is running at a speed of 'a' and policeman is chasing with a speed of 'b' (b > a). In this case, the distance covered by the thief when he is caught by the policeman, is given by $d\left(\frac{a}{b-a}\right)$.

Shortcut Approach

A man leaves a point A at t_1 and reaches the point B at t_2. Another man leaves the point B at t_3 and reaches the point A at t_4, then they will meet at

$$t_1 + \frac{(t_2 - t_1)(t_4 - t_1)}{(t_2 - t_1) + (t_4 - t_3)}$$

EXAMPLE 10. A bus leaves Ludhiana at 5 am and reaches Delhi at 12 noon. Another bus leaves Delhi at 8 am and reaches Ludhiana at 3 pm. At what time do the buses meet?

Sol. Converting all the times into 24 hour clock time, we get 5 am = 500, 12 noon = 1200, 8 am = 800 and 3 pm = 1500

Required time $= 500 + \frac{(1200 - 500)(1500 - 500)}{(1200 - 500) + (1500 - 800)}$

$= 500 + \frac{700 \times 1000}{700 + 700} = 1000 = 10$ am.

Shortcut Approach

Relation between time taken with two different modes of transport : $t_{2x} + t_{2y} = 2(t_x + t_y)$

where,

t_x = time when mode of transport x is used single way.

t_y = time when mode of transport y is used single way.

t_{2x} = time when mode of transport x is used both ways.

t_{2y} = time when mode of transport y is used both ways.

EXAMPLE 11. A man takes 6 hours 30 min. in going by a cycle and coming back by scooter. He would have lost 2 hours 10

min by going on cycle both ways. How long would it take him to go by scooter both ways?

Sol. Clearly, time taken by him to go by scooter both way

$$= 6h.30m - 2h.10m = 4h.20m = 4\frac{1}{3}h$$

EXAMPLE 12. A man travels 120 km by ship, 450 km by rail and 60 km by horse taking altogether 13 hrs 30 min. The speed of the train is 3 times that of the horse and 1½ times that of the ship. Find the speed of the train.

Sol. If the speed of the horse is x km/hr; that of the train is 3x and that of the ship is $\frac{3x}{1\frac{1}{2}} = 2x$ km/hr

$$\therefore \quad \frac{120}{2x} + \frac{450}{3x} + \frac{60}{x} = \frac{27}{2}$$

$$\therefore \quad \frac{60}{x} + \frac{150}{x} + \frac{60}{x} = \frac{27}{2} \qquad \therefore \qquad \frac{270}{x} = \frac{27}{2}$$

$\therefore$ x = 20 $\quad \therefore$ Speed of the train = 60 km/hr.

EXAMPLE 13. Rajesh travelled from the city A to city B covering as much distance in the second part as he did in the first part of his journey. His speed during the second part was twice his speed during the first part of the journey. What is his average speed of journey during the entire travel ?

(1) His average speed is the harmonic mean of the individual speed for the two parts.

(2) His average speed is the arithmetic mean of the individual speed for the two parts.

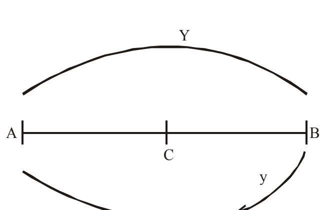
(3) His average speed is the geometric mean of the individual speeds for the two parts.

(4) Cannot be determined.

Sol. (1) The first part is ½ of the total distance & the second part is ½ of the total distance. Suppose, he travels at a km/hr speed during the first half & b km/hr speed during the second half. When distance travelled is the same in both parts of the journey, the average speed is gives by the formula $\frac{2ab}{a+b}$ i.e. the harmonic mean of the two speeds.

EXAMPLE 14. Two friends X and Y walk from A to B at a distance of 39 km, at 3 km an hour and $3\frac{1}{2}$ km an hour respectively. Y reaches B, returns immediately and meet x at C. Find the distance from A to C .

Sol. When Y meets X at C, Y has walked the distance AB + BC and X has walked the distance AC.



So, both X and Y have walked together a distance

$$= 2 \times AB = 2 \times 39 = 78 \text{ km}.$$

The ratio of the speeds of X and Y is 3 : $3\frac{1}{2}$ i.e, $\frac{6}{7}$

Hence , the distance travelled by X = AC

$$= \frac{6}{6+7} \times 78 = 36 \text{ km}$$

EXAMPLE 15. A man rides one-third of the distance from A to B at the rate of 'a' kmph and the remaining at the rate of '2b' kmph. If he had travelled at the uniform rate of 3c kmph, he could have rode from A to B and back again in the same time. Find a relationship between a, b and c.

Sol. Let the distance between A and B is X km and T_1 and T_2 be the time taken, then

$$T_1 = \frac{X}{3a}, \quad T_2 = \frac{2X}{6b} = \frac{X}{3b}, \quad T_1 + T_2 = \frac{X}{3}\left[\frac{a+b}{ab}\right]$$

Let T_3 be the time taken in third case, then $T_3 = \frac{2X}{3c}$

$$\Rightarrow \frac{2X}{3c} = \frac{X}{3ab}(a+b) \Rightarrow \frac{2}{c} = \frac{a+b}{ab} \Rightarrow c = \frac{2ab}{a+b}$$

EXAMPLE 16. Two cyclists start from the same place to ride in the same direction. A starts at noon at 8 kmph and B at 1.30 pm at 10kmph. How far will A have ridden before he is overtaken by B ? Find also at what times A and B will be 5 km apart.

Sol. If A rides for X hours before he is overtaken, then B rides for (X – 15) hrs.

$\Rightarrow$ 8X = 10 (X – 1.5) $\Rightarrow$ X = 7.5

$\Rightarrow$ A will have ridden 8 × 7.5 km or 60 km.

For the second part, if Y = the required number of hours after noon, then

8X = 10 (X – 1.5) ± 5

$\Rightarrow$ X = 10 or 5 according as B is ahead or behind A.

$\Rightarrow$ The required times are 5 p.m. and 10 p.m.

EXAMPLE 17. Two men A and B start from a place P walking at 3 kmph and 3½ kmph respectively. How many km apart will they be at the end of 2½ hours ?

(i) If they walk in opposite directions ?

(ii) If they walk in the same direction ?

(iii) What time will they take to be 16 km apart if.

(a) they walk in opposite directions ?

(b) in the same direction ?

Sol. (i) When they walk in opposite directions, they will be

$$\left(3 + 3\frac{1}{2}\right) = 6\frac{1}{2} \text{ km apart in 1 hour.}$$

$$\therefore \text{ In } 2\frac{1}{2} \text{ hours they will be } 6\frac{1}{2} \times \frac{5}{2} = 16\frac{1}{4} \text{ km apart.}$$

(ii) If they walk in the same direction, they will be

$3\frac{1}{2} - 3 = \frac{1}{2}$ km apart in 1 hour.

$\Rightarrow$ In $2\frac{1}{2}$ hours they will be $\frac{1}{2} \times \frac{5}{2} = 1\frac{1}{4}$ km apart.

(iii) Time to be 16 km apart while walking in opposite directions $= \dfrac{16}{3 + 3\frac{1}{2}} = 2\frac{6}{13}$ hours.

But if they walk in the same direction,

time $= \dfrac{16}{3\frac{1}{3} - 3} = 32$ hours

TRAINS

A train is said to have crossed an object (stationary or moving) only when the last coach of the train crosses the object completely. It implies that the total length of the train has crossed the total length of the object.

Shortcut Approach

- Time taken by a train to cross a pole/a standing man

$$= \frac{\text{Length of train}}{\text{Speed of train}}.$$

- Time taken by a train to cross platform/bridge etc. (i.e. a stationary object with some length)

$$= \frac{\text{length of train + length of platform/bridge etc.}}{\text{speed of train}}.$$

- When two trains with lengths L_1 and L_2 and with speeds S_1 and S_2 respectively, then
(a) When they are moving in the same direction, time taken by the faster train to cross the slower train

$$= \frac{L_1 + L_2}{\text{difference of their speeds}}.$$

(b) When they are moving in the opposite direction, time taken by the trains to cross each other

$$= \frac{L_1 + L_2}{\text{sum of their speeds}}.$$

- Suppose two trains or two bodies are moving in the same direction at u km/hr and v km/hr respectively such that u > v, then
their relative speed = (u – v) km/hr.
If their lengths be x km and y km respectively, then time taken by the faster train to cross the slower train (moving in the same direction) $= \left(\frac{x+y}{u-v}\right)$ hrs.

- Suppose two trains or two bodies are moving in opposite directions at u km/hr and v km/hr, then their relative speed = (u + v) km/hr.
If their lengths be x km & y km, then :
time taken to cross each other $= \left(\frac{x+y}{u+v}\right)$ hrs.

- If a man is running at a speed of u m/sec in the same direction in which a train of length L meters is running at a speed v m/sec, then (v – u) m/sec is called the speed of the train relative to man. Then the time taken by the train to cross the man $= \frac{1}{v-u}$ seconds

- If a man is running at a speed of u m/sec in a direction opposite to that in which a train of length L meters is running with a speed v m/sec, then (u + v) is called the speed of the train relative to man.
Then the time taken by the train to cross the man

$$= \frac{1}{v+u} \text{ seconds.}$$

- If two trains start at the same time from two points A and B towards each other and after crossing, they take (a) and (b) hours in reaching B and A respectively. Then,
A's speed : B's speed $= \left(\sqrt{b} : \sqrt{a}\right)$.

EXAMPLE 18. How long does a train 90 m long running at the rate of 54 km/h take to cross –

(a) a Mahatma Gandhi's statue?
(b) a platform 120 m long?
(c) another train 150 m long, standing on another parallel track?
(d) another train 160 m long running at 36 km/h in same direction?
(e) another train 160 m long running at 36 km/h in opposite direction?
(f) a man running at 6 km/h in same direction?
(g) a man running at 6 km/h in opposite direction?

Sol. (a) The statue is a stationary object, so time taken by train is same as time taken by train to cover a distance equal to its own length.

Now, 54 km/h $= 54 \times \frac{5}{18} = 15 \text{m/s}$

$\therefore$ Required time $= \frac{90}{15} = 6$ sec.

(b) The platform is stationary of length = 120 m.
Length to be covered
= Length of the train + Length of the platform
= 90 + 120 = 210 m

$\therefore$ Required time $= \frac{210}{15} = 14$ sec.

(c) Length to be covered
= Length of the train + length of the other train
= 90 + 150 = 240 m.

$\therefore$ Required time $= \dfrac{240}{15} = 16$ sec.

(d) Another train is moving in same direction.
Length to be covered
= Length of the train + length of the other train
$= 90 + 160 = 250$ m
Relative speed $= 54 - 36 = 18$ kmph.

$\therefore$ Required time $= \dfrac{250}{18 \times \frac{5}{18}} = 50$ sec.

(e) Another train is moving in opposite direction.
Length to be covered
= Length of the train + length of the other train
$= 90 + 160 = 250$ m
Relative speed $= 54 + 36 = 90$ kmph

$\therefore$ Required speed $= \dfrac{250}{\frac{5}{18} \times 90} = 10$ sec .

(f) The man is moving in same direction,
so Length to be covered = Length of the train,
and relative speed = speed of train – speed of man

$\therefore$ Required time $= \dfrac{90}{(54-6) \times \frac{5}{18}}$

$= \dfrac{90}{40} \times 3 = \dfrac{27}{4} = 6\dfrac{3}{4}$ sec.

(g) The man is moving in opposite direction, so
Length to be covered = Length of the train, and
relative speed = speed of train + speed of man

$\therefore$ Required time $= \dfrac{90}{(54+6) \times \frac{5}{18}} = \dfrac{27}{5} = 5\dfrac{2}{5}$ sec.

EXAMPLE 19. Two trains of equal lengths are running on parallel tracks in the same direction at 46 km/h and 36 km/h, respectively. The faster train passes the slower train in 36 sec. The length of each train is :

(a) 50 m (b) 80 m
(c) 72 m (d) 82 m
(e) None of these

Sol. (a) Let the length of each train be x metres.
Then, the total distance covered $= (x + x) = 2x$ m

Relative speed $= (46 - 36) = 10$ km/h $= \dfrac{10 \times 5}{18}$ m/s

Now, $36 = \dfrac{2x \times 18}{50}$ or $x = 50$ m

EXAMPLE 20. A train 110 m in length travels at 60 km/h. How much time does the train take in passing a man walking at 6 km/h against the train ?

(a) 6 s (b) 12 s
(c) 10 s (d) 18 s
(e) None of these

Sol. (a) Relative speeds of the train and the man

$= (60 + 6) = 66$ km/h $= \dfrac{66 \times 5}{18}$ m/s

Distance = 110 m
Therefore, time taken in passing the men

$= \dfrac{110 \times 18}{66 \times 5} = 6$s

EXAMPLE 21. Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmph. In what time will they be clear of each other from the moment they meet?

(a) 10 sec (b) 12 sec
(c) 14 sec (d) cannot be determined
(e) None of these

Sol. (b) Relative speed of the trains

$= (42 + 48)$ kmph $= 90$ kmph

$= \left(90 \times \dfrac{5}{18}\right)$ m/sec $= 25$ m/sec.

Time taken by the trains to pass each other
= Time taken to cover $(137 + 163)$ m at 25 m/sec

$= \left(\dfrac{300}{25}\right)$ sec = 12 seconds.

Shortcut Approach

If a train of length L m passes a platform of x m in t_1s, then time taken t_2s by the same train to pass a platform of length y m is given as

$$t_2 = \left(\frac{L+y}{L+x}\right) t_1$$

EXAMPLE 22. A train of length 250m, passes a platform of 350 m length in 50s. What time will this train take to pass the platform of 230m length.

Sol. Here, $L = 250$ m, $x = 350$ m, $t_1 = 50$s,
$y = 230$ m and $t_2 = ?$

$\therefore t_2 = \left(\dfrac{L+y}{L+x}\right) t_1 = \left(\dfrac{250+230}{250+350}\right) \times 50$

$= \dfrac{480}{600} \times 50 = 40$s

Shortcut Approach

From stations P and Q, two trains start moving towards each other with the speeds a and b, respectively. When they meet each other, it is found that one train covers distance d more than that of another train. In such cases, distance between stations P and Q is given as $\left(\frac{a+b}{a-b}\right)\times d$.

EXAMPLE 23. From stations A and B, two trains start moving towards each other with the speeds of 150 km/h and 130 km/h, respectively. When the two trains meet each other, it is found that one train covers 20 km more than that of another train. Find the distance between stations A and B.

Sol. Here, a = 150km/h, b = 130 km/h and d = 20 km

According to the formula,

Distance between stations A and B$=\left(\frac{a+b}{a-b}\right)\times d$

$$=\left(\frac{150+130}{150-130}\right)\times 20=\frac{280}{20}\times 20=280\,\text{km}$$

Shortcut Approach

- The distance between P and Q is (d) km. A train with (a) km/h starts from station P towards Q and after a difference of (t) hr another train with (b) km/h starts from Q towards station P, then both the trains will meet at a certain point after time T. Then,

$$T=\left(\frac{d\pm tb}{a+b}\right)$$

- If second train starts after the first train, then t is taken as positive.
  If second train starts before the first train, then t is taken as negative.

EXAMPLE 24. The distance between two stations P and Q is 110km. A train with speed of 20km/h leaves station P at 7:00 am towards station Q. Another train with speed of 25 km/h leaves station Q at 8:00 am towards station P. Then, at what time both trains meet?

Sol. Here, d = 110 km, t = 8 : 00 – 7 : 00 = 1 h

a = 20 km/h and b = 25 km/h

Time taken by trains to meet, $T=\left(\frac{d+tb}{a+b}\right)$

$$\Rightarrow \quad T=\frac{110+(1)(25)}{20+25}=\frac{135}{45}$$

$\Rightarrow \quad t=3$ h

$\therefore$ They will meet at = 7 : 00 am + 3 h = 10 : 00 am.

Shortcut Approach

The distance between two stations P and Q is d km. A train starts from P towards Q and another train starts from Q towards P at the same time and they meet at a certain point after t h. If train starting from P travels with a speed of x km/h slower or faster than another train, then

(i) Speed of faster train $=\left(\frac{d+tx}{2t}\right)$km/h

(ii) Speed of slower train $=\left(\frac{d-tx}{2t}\right)$km/h

EXAMPLE 25. The distance between two stations A and B is 138 km. A train starts from A towards B and another from B to A at the same time and they meet after 6 h. The train travelling from A to B is slower by 7 km/h compared to other train from B to A, then find the speed of the slower train?

Sol. Here, d = 138 km, t = 6 h and x = 7 km/h

$$\therefore \text{ Speed of slower train}=\frac{d-tx}{2t}=\frac{138-(6)(7)}{2(6)}$$

$$=\frac{138-42}{12}=\frac{96}{12}=8\,\text{km/h}$$

Shortcut Approach

A train covers distance d between two stations P and Q in t_1 h. If the speed of train is reduced by (a) km/h, then the same distance will be covered in t_2 h.

(i) Distance between P and Q is

$$d=a\left(\frac{t_1t_2}{t_2-t_1}\right)\text{ km}$$

(ii) Speed of the train $=\left(\frac{at_2}{t_2-t_1}\right)$km/h

EXAMPLE 26. A train covers distance between two stations A and B in 2h. If the speed of train is reduced by 6 km/h, then it travels the same distance in 3 h. Calculate the distance between two stations and speed of the train.

Sol. Here, $t_1 = 2$h, $t_2 = 3$h, a = 6 km/h and d = ?

(i) Distance between A and B is

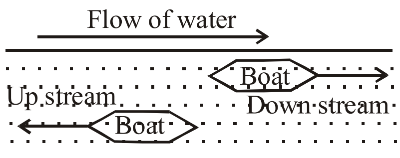
$$d=a\left(\frac{t_1\,t_2}{t_2-t_1}\right)\text{ km}$$

$$\Rightarrow \quad d=6\left(\frac{2\times 3}{3-2}\right)\Rightarrow d=36\,\text{km}$$

(ii) Speed of the train $=\frac{a\,t_2}{t_2-t_1}=\frac{6\times 3}{3-2}=18$ km/h

BOATS AND STREAMS

Stream : It implies that the water in the river is moving or flowing.
Upstream : Going against the flow of the river.
Downstream : Going with the flow of the river.
Still water : It implies that the speed of water is zero (generally, in a lake).



Let the speed of a boat (or man) in still water be X m/sec and the speed of the stream (or current) be Y m/sec. Then,

Shortcut Approach

- Speed of boat with the stream (or downstream or D/S) $= (X + Y)$ m/sec.
- Speed of boat against the stream (or upstream or U/S) $= (X - Y)$ m/sec.
- Speed of boat in still water is
$$X = \frac{(X+Y)+(X-Y)}{2} = \frac{\text{Upstream} + \text{Downstream}}{2}$$
- Speed of the stream or current is $Y = \frac{(X+Y)-(X-Y)}{2}$
$$= \frac{\text{Downstream} - \text{Upstream}}{2}$$

EXAMPLE 27. A boat is rowed down a river 28 km in 4 hours and up a river 12 km in 6 hours. Find the speed of the boat and the river.

Sol. Downstream speed is $\frac{28}{4} = 7$ kmph

Upstream speed is $\frac{12}{6} = 2$ kmph

Speed of Boat $= \frac{1}{2}$ (Downstream + Upstream Speed)

$$= \frac{1}{2}[7+2] = 4.5 \text{ kmph}$$

Speed of current $= \frac{1}{2}$ (Downstream–Upstream speed)

$$= \frac{1}{2}(7-2) = 2.5 \text{ kmph}$$

EXAMPLE 28. P, Q, and R are the three towns on a river which flows uniformly. Q is equidistant from P and R. I row from P to Q and back in 10 hours and I can row from P to R in 4 hours. Compare the speed of my boat in still water with that of the river.

(a) 4 : 3 **(b)** 5 : 3
(c) 6 : 5 **(d)** 7 : 3
(e) None of these

Sol. (c) Let the speed of the boat be v_1 and the speed of the current be v_2 and d be the distance between the cities.

$$\text{Now, } \frac{d}{v_1 + v_2} = 4 \text{ and } \frac{d}{v_1 - v_2} = 6$$

$$\Rightarrow \frac{v_1 + v_2}{v_1 - v_2} = \frac{6}{4}$$

$$\text{or } \frac{2v_1}{2v_2} = \frac{10}{2} \text{ or } \frac{v_1}{v_2} = 5:1$$

Required ratio $= (5 + 1) : 5 = 6 : 5$

Shortcut Approach

A man can row X km/h in still water. If in a stream which is flowing of Y km/h, it takes him Z hours to row to a place and back, the distance between the two places is

$$\frac{Z(X^2 - Y^2)}{2X}$$

EXAMPLE 29. A man can row 6 km/h in still water. When the river is running at 1.2 km/h, it takes him 1 hour to row to a place and back. How far is the place?

Sol. Man's rate downstream $= (6 + 1.2) = 7.2$ km/h.
Man's rate upstream $= (6 – 1.2)$ km/h $= 4.8$ km/h.
Let the required distance be x km.

$$\text{Then } \frac{x}{7.2} + \frac{x}{4.8} = 1 \text{ or } 4.8x + 7.2x = 7.2 \times 4.8$$

$$\Rightarrow x = \frac{7.2 \times 4.8}{12} = 2.88\text{km}$$

SHORTCUT METHOD

$$\text{Required distance } = \frac{1 \times \left(6^2 - (1.2)^2\right)}{2 \times 6}$$

$$= \frac{36 - 1.44}{12} = \frac{34.56}{12} = 2.88 \text{ km}$$

Shortcut Approach

- A man rows a certain distance downstream in X hours and returns the same distance in Y hours. If the stream flows at the rate of Z km/h, then the speed of the man in still water is given by
$$\frac{Z(X+Y)}{Y-X} \text{km/hr}$$
- And if speed of man in still water is Z km/h then the speed of stream is given by
$$\frac{Z(Y-X)}{X+Y} \text{km/hr}$$

Shortcut Approach

If speed of stream is a and a boat (swimmer) takes n times as long to row up as to row down the river, then

$$\text{Speed of boat (swimmer) in still water} = \frac{a(n+1)}{(n-1)}$$

Note: *This formula is applicable for equal distances.*

EXAMPLE 30. Rajnish can row 12 km/h in still water. It takes him twice as long to row up as to row down the river. Find the rate of stream.

Sol. Here, speed of Rajnish in still water = 12 km/h

n = 2; Speed of stream (a) = ?

According to the formula,

$$\text{Speed in still water} = \frac{a(n+1)}{(n-1)}$$

$$\Rightarrow 12 = \frac{a(2+1)}{(2-1)}$$

$$\Rightarrow 3a = 12$$

$$\therefore a = \frac{12}{3} = 4 \text{ km/h}$$

EXAMPLE 31. Vikas can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of 3 km/h, find the speed of Vikas in still water.

Sol. By the formula,

$$\text{Vikas's speed in still water} = \frac{3(9+6)}{9-6} = 15 \text{ km/h}$$

Shortcut Approach

If a man capable of rowing at the speed (u) m/sec in still water, rows the same distance up and down a stream flowing at a rate of (v) m/sec, then his average speed through the journey is

$$= \frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}} = \frac{(u-v)(u+v)}{u}$$

EXAMPLE 32. Two ferries start at the same time from opposite sides of a river, travelling across the water on routes at right angles to the shores. Each boat travels at a constant speed though their speeds are different. They pass each other at a point 720m from the nearer shore. Both boats remain at their sides for 10 minutes before starting back. On the return trip they meet at 400m from the other shore. Find the width of the river.

(a) 1760m **(b) 1840m**

(c) 2000m **(d) Cannot be found**

(e) None of these

Sol. (a)

Let the width of the river be x.

Let a, b be the speeds of the ferries.

$$\frac{720}{a} = \frac{(x-720)}{b} \quad \text{......... (i)}$$

$$\frac{(x-720)}{a} + 10 + \frac{400}{a} = \frac{720}{b} + 10 + \frac{(x-400)}{b} \quad \text{......... (ii)}$$

(Time for ferry 1 to reach other shore + 10 minute wait + time to cover 400m)

= Time for freely 2 to cover 720m to other shore + 10 minute wait + Time to cover (x – 400m))

$$\text{Using (i), we get} \quad \frac{a}{b} = \frac{720}{(x-720)}$$

$$\text{Using (ii),} \quad \frac{(x-320)}{a} = \frac{(x+320)}{b} \Rightarrow \frac{a}{b} = \frac{(x-320)}{(x+320)}$$

On, solving we get, x = 1760m

EXAMPLE 33. A man rows 27km with the stream and 15km against the stream taking 4 hours each time. Find this rate per hour in still water and the rate at which the stream flows.

Sol. Speed with the stream $= \frac{27}{4} = 6\frac{3}{4}$ kmph

$\therefore$ Speed against the stream $= \frac{15}{4} = 3\frac{3}{4}$ kmph.

$\therefore$ Speed of the man in still water

$$= \frac{1}{2}\left(6\frac{3}{4} + 3\frac{3}{4}\right) = 5\frac{1}{4} \text{ kmph}$$

$$\therefore \text{Speed of the stream} = \frac{1}{2}\left(6\frac{3}{4} - 3\frac{3}{4}\right) = 1.5 \text{ kmph}$$

EXAMPLE 34. On a river, B is between A and C and is also equidistant from A and C. A boat goes from A to B and back in 5 hours 15 minutes and from A to C and back in 7 hours. How long will it take to go from C to A if the river flows from A to C?

Sol. If the speed in still water is x kmph and speed of the river is y kmph, speed down the river = x + y and speed up the river = x – y.

$$\therefore \frac{d}{x+y} + \frac{d}{x-y} = 5\frac{1}{4} \quad \text{............. (1)}$$

$$\frac{2d}{x+y} = 7 \quad \text{............. (2)}$$

$$\text{Multiplying (1) by 2, we get} \quad \frac{2d}{x+y} + \frac{2d}{x-y} = 10\frac{1}{2}$$

$$\Rightarrow 7 + \frac{2d}{x-y} = \frac{21}{2} \quad \left[\because \frac{2d}{x-y} = 7\right]$$

$$\Rightarrow \frac{2d}{x-y} = 3\frac{1}{2} \text{ hours} = \text{Time taken to travel from C to A.}$$

Shortcut Approach

If boat's (swimmer's) speed in still water is a km/h and river is flowing with a speed of b km/h, then average speed in going to a certain place and coming back to starting point is given by $\frac{(a+b)(a-b)}{a}$ km/h.

EXAMPLE 35. Ramesh rows in still water with speed of 4.5 km/h to go to a certain place and to come back. Find his average speed for the whole journey, if the river is flowing with a speed of 1.5 km/h.

Sol. Here, a = 4.5 km/h, b = 1.5 km/h

$$\text{Average speed} = \frac{(a+b)(a-b)}{a}$$

$$= \frac{(4.5+1.5)(4.5-1.5)}{4.5} = \frac{6 \times 3}{4.5} = \frac{18}{4.5} = 4 \text{ km/h}$$

EXERCISE

1. A car finishes a journey in ten hours at the speed of 80 km/hr. If the same distance is to be covered in eight hours how much more speed does the car have to gain?
   (a) 8 krn/hr (b) 10 km/hr
   (c) 12 km/hr (d) 16 krn/hr
   (e) None of these
2. Two cars *A* and *B* are running towards each other from different places 88 km apart. If the ratio of the speeds of the cars *A* and *B* is 5 : 6 and the speed of the car *B* is 90 km per hour then after how long will the two meet each other?
   (a) $26\frac{2}{3}$ minutes (b) 24 minutes
   (c) 32 minutes (d) 36 minutes
   (e) None of these
3. Train '*A*' leaves Mumbai Central for Lucknow at 11 am, running at the speed of 60 kmph. Train '*B*' leaves Mumbai Central for Lucknow by the same route at 2 pm on the same day, running at the speed of 72 kmph. At what time will the two trains meet each other?
   (a) 2 am on the next day
   (b) 5 am on the next day
   (c) 5 pm on the next day
   (d) 2 pm on the next day
   (e) None of these
4. A motor starts with the speed of 70 kmph with its speed increasing every two hours by 10 kmph. In how many hours will it cover 345 kms?
   (a) $2\frac{1}{4}$ hours (b) $4\frac{1}{2}$ hours
   (c) 4 hours 5 minutes (d) Cannot be determined
   (e) None of these
5. A boat takes 3 hours to travel from place *M* to *N* downstream and back from *N* to *M* upstream. If the speed of the boat in still water is 4 km/hr, what is the distance between the two places?
   (a) 8 km (b) 12 km
   (c) 6 km (d) Data inadequate
   (e) None of these
6. A boat has to travel upstream 20 km distance from point *X* of a river to point *Y*. The total time taken by boat in travelling from point *X* to *Y* and *Y* to *X* is 41 minutes 40 seconds. What is the speed of the boat?
   (a) 66 km/hr (b) 72 km/hr
   (c) 48 km/hr (d) Data inadequate
   (e) None of these
7. The speed of a car increases by 2 km after every hour. If the distance travelled in the first hour was 35 km, what was the total distance travelled in 12 hours?
   (a) 522 km (b) 456 km
   (c) 556 km (d) 482 km
   (e) None of these
8. A boat covers a distance of 30 km downstream in 2 hours while it take 6 hours to cover the same distance upstream. If the speed of the current is half of the speed of the boat then what is the speed of the boat in km per hour?
   (a) 15 kmph (b) 5 kmph
   (c) 10 kmph (d) Data inadequate
   (e) None of these
9. A man starts walking. He walked 2 km in the first hour. Then he walked two-thirds of the distance of the previous hour in each next hour. If he walked continuously then how long could he walk maximum?
   (a) 60 km (b) 6 km
   (c) 12 km (d) 8 km
   (e) None of these
10. Starting with the initial speed of 30 km/hr, the speed is increased by 4 km/hour every two hours. How many hours will it take to cover a distance of 288 km?
    (a) 4 (b) 6
    (c) 12 (d) 8
    (e) None of these
11. With a uniform speed a car covers a distance in 8 hours. Were the speed increased by 4 km/hr the same distance could be covered in $7\frac{1}{2}$ hours. What is the distance covered?
    (a) 640 km (b) 480 km
    (c) 420 km (d) Cannot be determined
    (e) None of these
12. A 300-metre-long train crosses a platform in 39 seconds while it crosses a signal pole in 18 seconds. What is the length of the platform?
    (a) 320 metres (b) 650 metres
    (c) 350 metres (d) Data inadequate
    (e) None of these
13. A 260-metre-long train crosses a 120-metre-long wall in 19 seconds. What is the speed of the train?
    (a) 27 km/hr (b) 49 km/hr
    (c) 72 km/hr (d) 70 km/hr
    (e) None of these
14. A 270-metre-long train running at the speed of 120 kmph crosses another train running in opposite direction at the speed of 80 kmph in 9 secs. What is the length of the other train?
    (a) 240 metres (b) 320 metres
    (c) 260 metres (d) 230 metres
    (e) None of these
15. A monkey ascends a greased pole 12 metres high. He ascends 2 metres in first minute and slips down 1 metre in the alternate minute. In which minute, he reaches the top ?
    (a) 21st (b) 22nd
    (c) 23rd (d) 24th
    (e) None of these

16. A man walks a certain distance and rides back in $6\frac{1}{4}$ h. He can walk both ways in $7\frac{3}{4}$ h. How long it would take to ride both ways ?
(a) 5 hours (b) $4\frac{1}{2}$ hours
(c) $4\frac{3}{4}$ hours (d) 6 hours
(e) None of these

17. There are 20 poles with a constant distance between each pole. A car takes 24 seconds to reach the 12th pole . How much time will it take to reach the last pole?
(a) 25.25 s (b) 17.45 s
(c) 35.75 s (d) 41.45 s
(e) None of these

18. A man is walking at a speed of 10 km per hour. After every kilometre, he takes rest for 5 minutes. How much time will be take to cover a distance of 5 kilometres?
(a) 48 min. (b) 50 min.
(c) 45 min. (d) 55 min.
(e) None of these

19. On a journey across Bombay, a tourist bus averages 10 km/h for 20% of the distance, 30 km/h for 60% of it and 20 km/h for the remainder. The average speed for the whole journey was
(a) 10 km/h (b) 30 km/h
(c) 5 km/h (d) 20 km/h
(e) None of these

20. In a 800 m race around a stadium having the circumference of 200 m, the top runner meets the last runner on the 5th minute of the race. If the top runner runs at twice the speed of the last runner, what is the time taken by the top runner to finish the race ?
(a) 20 min (b) 15 min
(c) 10 min (d) 5 min
(e) None of these

21. A man walks half of the journey at 4 km/h by cycle does one third of journey at 12 km/h and rides the remainder journey in a horse cart at 9 km/h, thus completing the whole journey in 6 hours and 12 minutes. The length of the journey is
(a) 36 km (b) $\frac{1332}{67}$ km
(c) 40 km (d) 28 km
(e) None of these

22. R and S start walking each other at 10 AM at the speeds of 3 km/h and 4 km/h respectively. They were initially 17.5 km apart. At what time do they meet?
(a) 2 : 30 PM (b) 11 : 30 AM
(c) 1 : 30 PM (d) 12 : 30 PM
(e) None of these

23. A train does a journey without stoppage in 8 hours, if it had travelled 5 km/h faster, it would have done the journey in 6 hours 40 minutes. Find its original speed.
(a) 25 km/h (b) 40 km/h
(c) 45 km/h (d) 36.5 km/h
(e) None of these

24. A train leaves station X at 5 a.m. and reaches station Y at 9 a.m. Another train leaves station Y at 7 a.m. and reaches station X at 10: 30 a.m. At what time do the two trains cross each other ?
(a) 7 : 36 am (b) 7 : 56 am
(c) 8 : 36 am (d) 8 : 56 am
(e) None of these

25. Cars C_1 and C_2 travel to a place at a speed of 30 and 45 km/h respectively. If car C_2 takes $2\frac{1}{2}$ hours less time than C_1 for the journey, the distance of the place is
(a) 300 km (b) 400 km
(c) 350 km (d) 225 km
(e) None of these

26. If I walk at 4 km/h, I miss the bus by 10 minutes. If I walk at 5 km/h, I reach 5 minutes before the arrival of the bus. How far I walk to reach the bus stand ?
(a) 5 km (b) 4.5 km
(c) $5\frac{1}{4}$ km / h (d) Cannot be determined
(e) None of these

27. A goods train leaves a station at a certain time and at a fixed speed. After 6 hours, an express train leaves the same station and moves in the same direction at a uniform speed of 90 kmph. This train catches up the goods train in 4 hours. Find the speed of the goods train.
(a) 36 kmph (b) 40 kmph
(c) 30 kmph (d) 42 kmph
(e) None of these

28. Without stoppages, a train travels certain distance with an average speed of 80 km/h, and with stoppages, it covers the same distance with an average speed of 60 km/h. How many minutes per hour the train stops ?
(a) 15 (b) 18
(c) 10 (d) 16
(e) None of these

29. If a man walks to his office at 3/4 of his usual rate, he reaches office 1/3 of an hour later than usual. What is his usual time to reach office.
(a) $\frac{1}{2}$ hr (b) 1 hr
(c) $\frac{3}{4}$ hr (d) 2 hrs
(e) None of these

30. If a man walks to his office at 5/4 of his usual rate, he reaches office 30 minutes early than usual. What is his usual time to reach office.
(a) 2 hrs (b) $2\frac{1}{2}$ hr s
(c) 1 hr 50 min (d) 2 hrs 15 min
(e) None of these

31. A train running between two stations A and B arrives at its destination 10 minutes late when its speed is 50 km/h and 50 minutes late when its speed is 30km/h. What is the distance between the stations A and B ?
(a) 40 km (b) 50 km
(c) 60 km (d) 70 km
(e) None of these

32. A car travels 25 km an hour faster than a bus for a journey of 500 km. If the bus takes 10 hours more than the car, then the speeds of the bus and the car are
(a) 25 km/h and 40 km/h respectively
(b) 25 km/h and 60 km/h respectively
(c) 25 km/h and 50 km/h respectively
(d) 25 km/h and 70 km/h respectively
(e) None of these

33. A train consists of 12 boggies, each boggy 15 metres long. The train crosses a telegraph post in 18 seconds. Due to some problem, two boggies were detached. The train now crosses a telegraph post in
(a) 18 sec (b) 12 sec
(c) 15 sec (d) 20 sec
(e) None of these

34. A man started running at a distance of 225 metre from the train. If the speed of the man is 6 km/h, then how much time should the train wait so that the man will be just able to catch it ?
(a) $2\frac{1}{4}$ min (b) 3 min
(c) $4\frac{1}{4}$ min (d) $4\frac{1}{2}$ min
(e) None of these

35. A man sitting in a train which is travelling at 50 kmph observes that a goods train, travelling in opposite direction, takes 9 seconds to pass him. If the goods train is 280 m long, find its speed.
(a) 62 kmph (b) 58 kmph
(c) 52 kmph (d) 50 kmph
(e) None of these

36. Two trains, one from Howrah to Patna and the other from Patna to Howrah, start simultaneously. After they meet, the trains reach their destinations after 9 hours and 16 hours respectively. The ratio of their speeds is:
(a) 2 : 3 (b) 4 : 3
(c) 6 : 7 (d) 9 : 16
(e) None of these

37. A train 75 metres long overtook a man who was walking at the rate of 6 km/h and passed him in 18 seconds. Again, the train overtook a second person in 15 seconds. At what rate was the second person travelling ?
(a) 3 km/h (b) 2.5 km/h
(c) 4 km/h (d) 1.5 km/h
(e) None of these

38. A jogger running at 9 kmph alongside a railway track is 240 metres ahead of the engine of a 120 metre long train running at 45 kmph in the same direction. In how much time will the train pass the jogger?
(a) 3.6 sec (b) 18 sec
(c) 36 sec (d) 72 sec
(e) None of these

39. Two trains are running at 40 km/h and 20 km/h respectively in the same direction. Fast train completely passes a man sitting in the slower train in 5 seconds. What is the length of the fast train?
(a) 23 m (b) $23\frac{2}{9}$ m
(c) 27 m (d) $27\frac{7}{9}$ m
(e) None of these

40. Two trains, 130 and 110 metres long, while going in the same direction, the faster train takes one minute to pass the other completely. If they are moving in opposite direction, they pass each other completely in 3 seconds. Find the speed of trains.
(a) 30 m/s, 40 m/s (b) 32 m/s, 48 m/s
(c) 40 m/s, 44 m/s (d) 38 m/s, 42 m/s
(e) None of these

41. A train overtakes two person who are walking in the same direction in which the train is going , at the rate of 2 kmph and 4 kmph and passes them completely in 9 and 10 seconds respectively. The length of the train is:
(a) 45 m (b) 50 m
(c) 54 m (d) 72 m
(e) None of these

42. Local trains leave from a station at an interval of 15 minutes at a speed of 16 km/h. A man moving from opposite side meets the trains at an interval of 12 minutes. Find the speed of the man.
(a) 4 km/h (b) 3.5 km/h
(c) 4.5 km/h (d) 3 km/h
(e) None of these

43. Local trains leave from a station at an interval of 14 minutes at a speed of 36 km/h. A man moving in the same direction along the road meets the trains at an interval of 18 minutes. Find the speed of the man.
(a) 8 km/h (b) 7 km/h
(c) 6 km/h (d) 5.8 km/h
(e) None of these

44. A train overtakes two persons walking along a railway track. The first one walks at 4.5 km/h. The other one walks at 5.4 km/h. The train needs 8.4 and 8.5 seconds respectively to overtake them. What is the speed of the train if both the persons are walking in the same direction as the train?
(a) 66 km/h (b) 72 km/h
(c) 78 km/h (d) 81 km/h
(e) None of these

45. Two trains running in opposite directions cross a man standing on the platform in 27 seconds and 17 seconds respectively and they cross each other in 23 seconds. The ratio of their speeds is:
(a) 1 : 3 (b) 3 : 2
(c) 3 : 4 (d) 2 : 1
(e) None of these

46. Two trains each of 120 m in length, run in opposite directions with a velocity of 40 m/s and 20 m/s respectively. How long will it take for the tail ends of the two trains to meet each other during the course of their journey :

(a) 20 s (b) 3 s
(c) 4 s (d) 5 s
(e) None of these

Directions (Qs. 47-48): Answer the following questions on the basis of the information given below:

(i) Trains A and B are travelling on the same route heading towards the same destination. Train B has already covered a distance of 220 km before train A started.

(ii) The two trains meet each other 11 hours after the start of train A.

(iii) Had the trains been travelling towards each other (from a distance of 220 km), they would have met after one hour.

47. What is the speed of train B in kmph?

(a) 100 (b) 180
(c) 116 (d) Data inadequate
(e) None of these

48. What is the speed of train A in kmph?

(a) 102 (b) 80.5
(c) 118 (d) Data inadequate
(e) None of these

CHAPTER 10

Mensuration

MENSURATION

Mensuration is the science of measurement of the lenghts of lines, areas of surfaces and volumes of solids.

Perimeter

Perimeter is sum of all the sides. It is measured in cm, m, etc.

Area

The area of any figure is the amount of surface enclosed within its boundary lines. This is measured in square unit like cm^2, m^2, etc.

Volume

If an object is solid, then the space occupied by such an object is called its volume. This is measured in cubic unit like cm^3, m^3, etc.

Basic Conversions :

I. $1\,m = 10\,dm$
$1\,dm = 10\,cm$
$1\,cm = 10\,mm$
$1\,m = 100\,cm = 1000\,mm$
$1km = 1000\,m$

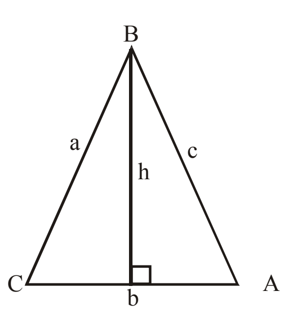
II. $1\text{ km} = \frac{5}{8}$ miles
$1\text{ mile} = 1.6\text{ km}$
$1\text{ inch} = 2.54\text{ cm}$

III. $100\text{ kg} = 1$ quintal
10 quintal $= 1$ tonne
$1\text{ kg} = 2.2$ pounds (approx.)

IV. $1\text{ litre} = 1000\text{ cc}$
$1\text{ acre} = 100\text{ m}^2$
$1\text{ hectare} = 10000\text{ m}^2$ (100 acre)

PART I : PLANE FIGURES

TRIANGLE



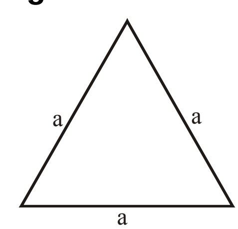
Perimeter $(P) = a + b + c$

$$\text{Area (A)} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{a+b+c}{2}$ and a, b and c are three sides of the triangle.

Also, $A = \frac{1}{2} \times bh$; where $b \rightarrow$ base
$h \rightarrow$ altitude

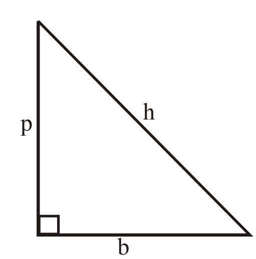
Equilateral triangle



Perimeter $= 3a$

$A = \frac{\sqrt{3}}{4} a^2$; where $a \rightarrow$ side

Right triangle



$A = \frac{1}{2} pb$ and $h^2 = p^2 + b^2$ (Pythagoras triplet)

where $p \rightarrow$ perpendicular
$b \rightarrow$ base
$h \rightarrow$ hypotenuse

EXAMPLE 1. Find the area of a triangle whose sides are 50 m, 78m, 112m respectively and also find the perpendicular from the opposite angle on the side 112 m.

Sol. Here $a = 50\text{ m}, b = 78\text{ m}, c = 112\text{m}$

$$s = \frac{1}{2}(50 + 78 + 112) = 120\text{m}$$

$s - a = 120 - 50 = 70\text{ m}$
$s - b = 120 - 78 = 42\text{ m}$
$s - c = 120 - 112 = 8\text{ m}$

$\therefore$ Area $= \sqrt{120 \times 70 \times 42 \times 8} = 1680$ sq.m.

$\because$ Area $= \frac{1}{2}$ base $\times$ perpendicular

$\therefore$ Perpendicular $= \frac{2\text{Area}}{\text{Base}} = \frac{1680 \times 2}{112} = 30\text{m}.$

EXAMPLE 2. The base of a triangular field is 880 m and its height 550 m. Find the area of the field. Also calculate the charges for supplying water to the field at the rate of ₹24.25 per sq. hectometre.

Sol. Area of the field $= \frac{\text{Base} \times \text{Height}}{2}$

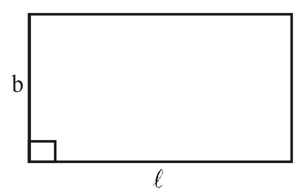
$$= \frac{880 \times 550}{2} = 242000 \text{ sq.m.} = 24.20 \text{ sq.hm}$$

Cost of supplying water to 1 sq. hm = ₹ 24.25

$\therefore$ Cost of supplying water to the whole field

= 24.20 × 24.25 = ₹ 586.85

RECTANGLE



Perimeter $= 2(\ell + b)$

Area $= \ell \times b$; where $\ell \rightarrow$ length

$b \rightarrow$ breadth

Shortcut Approach

If the length and breadth of a rectangle are increased by a% and b%, respectively, then are will be increased by $\left(a + b + \frac{ab}{100}\right)\%$.

EXAMPLE 3. If the length and breadth of a rectangle are increased by 10% and 8%, respectively, then by what per cent will the area of that rectangle be increased?

Sol. Given that, $a = 10, b = 8$

According to the formula,

Percentage increase in area $= \left(10 + 8 + \frac{10 \times 8}{100}\right)\%$

$$= \left(18 + \frac{80}{100}\right)\% = \left(18 + \frac{4}{5}\right)\% = 18\frac{4}{5}\%$$

EXAMPLE 4. If the length of a rectangle is increased by 5% and the breadth of the rectangle is decreased by 6%, then find the percentage change in area.

Sol. Given that $a = 5, b = -6$

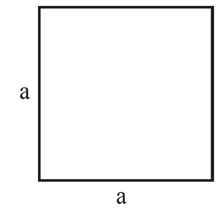
According to the formula,

Percentage change in area $= \left(5 - 6 - \frac{5 \times 6}{100}\right)\%$

$$= -1 - \frac{30}{100} = -1 - 0.30 = -1.3\% \text{(decrease)}$$

Negative value shown that there is a decrease in area.

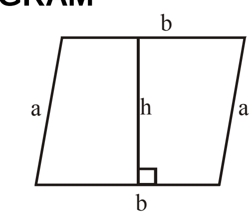
SQUARE



Perimeter = 4 × side = 4a

Area = $(\text{side})^2 = a^2$; where $a \rightarrow$ side

PARALLELOGRAM



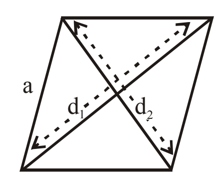
Perimeter = 2 (a + b)

Area = b × h; where $a \rightarrow$ breadth

$b \rightarrow$ base (or length)

$h \rightarrow$ altitude

RHOMBUS



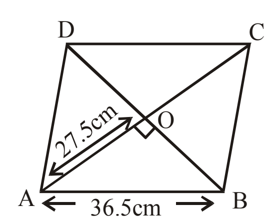
Perimeter = 4 a

Area $= \frac{1}{2} d_1 \times d_2$ where $a \rightarrow$ side and

d_1 and d_2 are diagonals.

EXAMPLE 5. The perimeter of a rhombus is 146 cm and one of its diagonals is 55 cm. Find the other diagonal and the area of the rhombus.

Sol. Let ABCD be the rhombus in which AC = 55 cm.



and AB $= \frac{146}{4} = 36.5$ cm

Also, AO $= \frac{55}{2} = 27.5$m

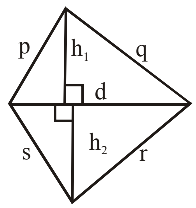
$\therefore$ BO $= \sqrt{(36.5)^2 - (27.5)^2} = 24$ cm

Hence, the other diagonal BD = 48 cm

Now, Area of the rhomhus $= \frac{1}{2} AC \times BD$

$$= \frac{1}{2} \times 55 \times 48 = 1320 \text{ sq.cm.}$$

IRREGULAR QUADRILATERAL



Perimeter $= p + q + r + s$

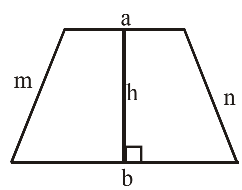
Area $= \frac{1}{2} \times d \times (h_1 + h_2)$

EXAMPLE 6. Find the area of a quadrilateral piece of ground, one of whose diagonals is 60 m long and the perpendicular from the other two vertices are 38 and 22m respectively.

Sol. Area $= \frac{1}{2} \times d \times (h_1 + h_2)$

$= \frac{1}{2} \times 60(38 + 22) = 1800$ sq.m.

TRAPEZIUM



Perimeter $= a + b + m + n$

Area $= \frac{1}{2}(a + b)h$; where (a) and (b) are two parallel sides;

(m) and (n) are two non-parallel sides;

h $\rightarrow$ perpendicular distance between two parallel sides.

EXAMPLE 7. A 5100 sq.cm trapezium has the perpendicular distance between the two parallel sides 60 m. If one of the parallel sides be 40m then find the length of the other parallel side.

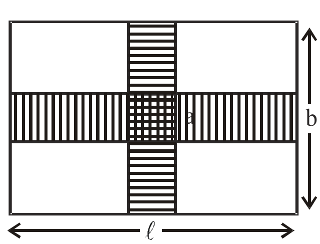
Sol. Since, $A = \frac{1}{2}(a + b)h$

$\Rightarrow \quad 5100 = \frac{1}{2}(40 + x) \times 60$

$\Rightarrow \quad 170 = 40 + x$

$\therefore$ other parallel side $= 170 - 40 = 130$ m

AREA OF PATHWAYS RUNNING ACROSS THE MIDDLE OF A RECTANGLE

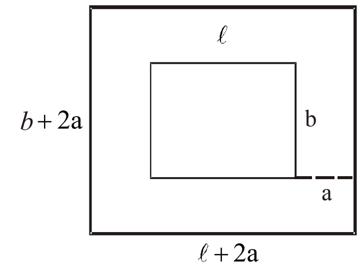


$A = a(\ell + b) - a^2$; where $\ell \rightarrow$ length

b $\rightarrow$ breadth,

a $\rightarrow$ width of the pathway.

Pathways outside

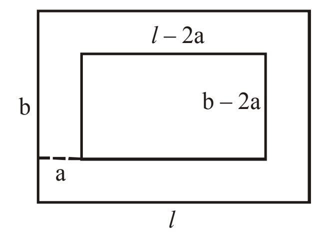


$A = (l + 2a)(b + 2a) - lb$; where $l \rightarrow$ length

b $\rightarrow$ breadth

a $\rightarrow$ width of the pathway

Pathways inside



$A = lb - (l - 2a)(b - 2a)$; where l $\rightarrow$ length

b $\rightarrow$ breadth

a $\rightarrow$ width of the pathway

☞ Shortcut Approach

If a pathway of width x is made inside or outside a rectangular plot of length l and breadth b, then are of pathway is

(i) $2x(l + b + 2x)$, if path is made outside the plot.

(ii) $2x(l + b - 2x)$, if path is made inside the plot.

EXAMPLE 8. There is garden of 140 m × 120 m and a gravel path is to be made of an equal width all around it, so as to take up just one-fourth of the garden. What must be the breadth of the path?

Sol. Since, path covers $\frac{1}{4}$th area of the garden, that means path is inside the garden.

Given, $l = 140$ m, $b = 120$ m, $x = ?$

According to the question,

$2x(l + b - 2x) = \frac{1}{4} \times l\ b \Rightarrow 2x(140 + 120 - 2x) = \frac{1}{4} \times 140 \times 120$

$\Rightarrow x(260 - 2x) = 2100 \Rightarrow x^2 - 130x + 1050 = 0 \Rightarrow x = 8.65$ or 121.3

Leaving 121.3, since width of the park cannot be greater than length.

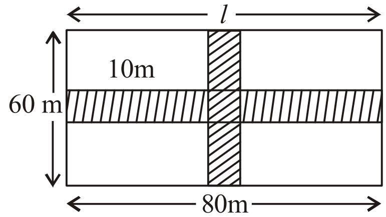
$\therefore$ Width of the park = 8.65 m

☞ Shortcut Approach

If two paths, each of width x are made parallel to length (l) and breadth (b) of the rectangular plot in the middle of the plot, then area of the paths is $x(l + b - x)$

EXAMPLE 9. A rectangular grass plot 80 m × 60 m has two roads, each 10 m wide, running in the middle of it, one parallel to length and the other parallel to breadth. Find the area of the roads.

Sol. Then, according to the formula,



Required area = x (*l* + b – x) = 10 (80 + 60 – 10) = 10 × 130 = 1300 sq m

EXAMPLE 10. A rectangular grassy plot is 112m by 78 m. It has a gravel path 2.5 m wide all round it on the inside. Find the area of the path and the cost of constructing it at ₹ 2 per square metre?

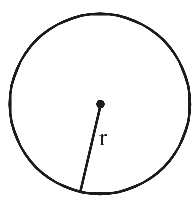
Sol. A = lb – (1 – 2a) (b – 2a)

= 112 × 78 – (112 – 5) (78 – 5)

= 112 × 78 – 107 × 73 = 8736 – 7811 = 925 sq.m

∴ Cost of construction = rate × area = 2 × 925 = Rs. 1850

CIRCLE



Perimeter (Circumference) = $2\pi r = \pi d$

Area = πr^2; where r → radius

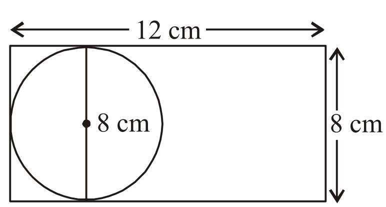
d → diameter

and $\pi = \frac{22}{7}$ or 3.14

> **Shortcut Approach**
>
> The length and breadth of a rectangle are *l* and b, then are of circle of maximum radius inscribed in that rectangle is $\frac{\pi b^2}{4}$.

EXAMPLE 11. Find the area of circle with maximum radius that can be inscribed in the rectangle of length 12 cm and breadth 8 cm.

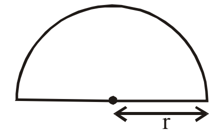
Sol. Here, l = 12 and b = 8



∴ Area of circle wiht maximum radius = $\frac{\pi b^2}{4}$

$$= \frac{\pi(8)^2}{4} = \frac{4\pi}{4} = 16\pi\, cm^2$$

SEMICIRCLE



Perimeter = $\pi r + 2r$

Area $= \frac{1}{2} \times \pi r^2$

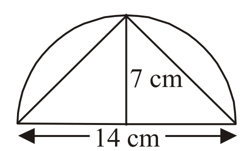
> **Shortcut Approach**
>
> The are a of the largest triangle incribed in a semi-circle of radius r is equal to r^2.

EXAMPLE 12. The largest triangle is incribed in a semi-circle of radius 7 cm. Find the area inside the semi-circle which is not occupied by triangle.

Sol. Given that, radius = 7 cm, diameter = 14 cm

According to the formula,

Area of the triangle = 7^2 = 49 sq cm

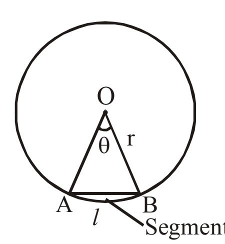


Area of semi-circle = $\frac{\pi r^2}{2} = \frac{\frac{22}{7} \times 7^2}{2}$ = 11 × 7 = 77 sq cm

Required answer = Area of semi-circle – Area of the largest triangle

= (77 – 49) sq cm = 28 sq cm

SECTOR OF A CIRCLE



Area of sector OAB = $\frac{\theta}{360} \times \pi r^2$

Length of an arc (*l*) = $\frac{\theta}{360} \times 2\pi r$

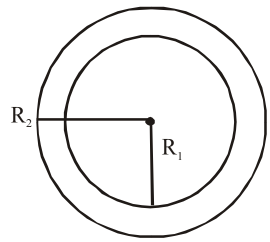
Area of segment = Area of sector – Area of triangle OAB

$$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

Perimeter of segment = length of the arc + length of segment

$$AB = \frac{\pi r\theta}{180} + 2r \sin\frac{\theta}{2}$$

RING



Area of ring $=\pi\left(R_2^2-R_1^2\right)$

EXAMPLE 13. A wire is looped in the form of a circle of radius 28 cm. It is re-bent into a square form. Determine the length of a side of the square.

(a) 44 cm (b) 45 cm
(c) 46 cm (d) 48 cm

Sol. (a) Length of the wire = Perimeter of the circle

$= 2\pi \times 28$

$= 176 \text{ cm}^2$

Side of the square $=\frac{176}{4}=44$ cm

EXAMPLE 14. The radius of a circular wheel is $1\frac{3}{4}$ m. How many revolutions will it make in travelling 11 km ?

Sol. Distance to be travelled = 11km = 11000 m

Radius of the wheel $=1\frac{3}{4}$ m $=\frac{7}{4}$ m

$\therefore$ Circumference of the wheel $=2\times\frac{22}{7}\times\frac{7}{4}=11$ m

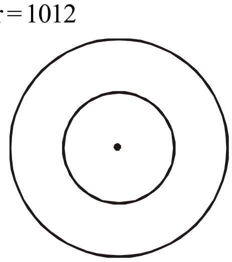
$\therefore$ In travelling 11 m, wheel makes 1 revolution.

$\therefore$ In travelling 11000 m the wheel makes $\frac{1}{11}\times 11000$ revolutions, i.e., 1000 revolutions.

EXAMPLE 15. The circumference of a circular garden is 1012m. Find the area of outsider road of 3.5 m width runs around it. Calculate the area of this road and find the cost of gravelling the road at ₹ 32 per 100 sqm.

Sol:

$A=\pi r^2, C=2\pi r=1012$



$\Rightarrow r=1012\times\frac{1}{2}\times\frac{7}{22}=161\text{m}$

$\therefore$ Area of garden $=\frac{22}{7}\times 161\times 161=81466$ sqm

Area of the road = area of bigger circle – area of the garden

Now, radius of bigger circle $=161+3.5=\frac{329}{2}$ m

$\therefore$ Area of bigger circle $=\frac{22}{7}\times\frac{329}{2}\times\frac{329}{2}=85046\frac{1}{2}$ sq.m

Thus, area of the road $=85046\frac{1}{2}-81466=3580\frac{1}{2}$ sqm.

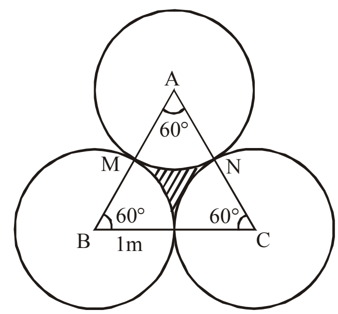
Hence, cost $=₹\frac{7161}{2}\times\frac{32}{100}=₹\,1145.76$

EXAMPLE 16. There is an equilateral triangle of which each side is 2m. With all the three corners as centres, circles each of radius1 m are described.

(i) Calculate the area common to all the circles and the triangle.

(ii) Find the area of the remaining portion of the triangle.

Sol.



Area of each sector $=\frac{\theta}{360}\times\pi r^2=\frac{60}{360}\times\frac{22}{7}\times 1\times 1$

$=\frac{1}{6}\times\frac{22}{7}=\frac{11}{21}\text{m}^2$

Area of equilateral triangle $=\frac{\sqrt{3}}{4}a^2$

$=\frac{\sqrt{3}}{4}\times 2\times 2=\sqrt{3}\text{m}^2$

(i) Common area = 3 × Area of each sector

$=3\times\frac{11}{21}=\frac{11}{7}=1.57\text{m}^2$

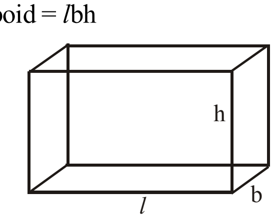
(ii) Area of the remaining portion of the triangle = Ar. of equilateral triangle – 3(Ar. of each sector)

$\sqrt{3}-1.57=1.73-1.57=0.16\text{m}^2$

PART-II SOLID FIGURE

CUBOID

A cuboid is a three dimensional box.
Total surface area of a cuboid = 2 (lb + bh + lh)
Volume of the cuboid = $l$bh



Area of four walls = 2(l + b) × h

☞ Shortcut Approach

If length, breadth and height of a cuboid are changed by x%, y% and z% respectively, then its volume is increased by

$$= \left[x+y+z+\frac{xy+yz+zx}{100}+\frac{xyz}{(100)^2}\right]\%$$

Note: *Increment in the value is taken as positive and decrement in value is taken as negative. Positive result shows total increment and negative result shows total decrement.*

EXAMPLE 17. If all the dimensions of a cuboid are increased by 100%, by what per cent does the volume of cuboid increase?

Sol. Here, x = y = z = 100

According to the formula,

Percentage increase in volume

$$= \left[x+y+z+\frac{xy+yz+zx}{100}+\frac{xyz}{100}\right]\%$$

$$= \left[100+100+100+\frac{100\times100+100\times100+100\times100}{100}+\frac{100\times100\times100}{(100)^2}\right]\%$$

$$= \left[300+\frac{10000+10000+10000}{100}+\frac{1000000}{10000}\right]\%$$

$$= \left[300+\frac{30000}{100}+100\right]\%$$

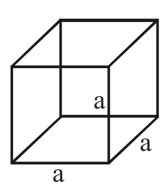
$$= (300+300+100)\% = 700\%$$

CUBE

A cube is a cuboid which has all its edges equal.

Total surface area of a cube = $6a^2$

Volume of the cube = a^3



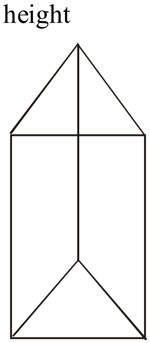
RIGHT PRISM

A prism is a solid which can have any polygon at both its ends.

Lateral or curved surface area = Perimeter of base × height

Total surface area = Lateral surface area + 2 (area of the end)

Volume = Area of base × height



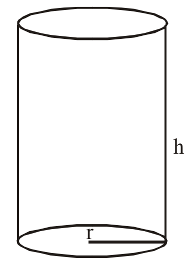
RIGHT CIRCULAR CYLINDER

It is a solid which has both its ends in the form of a circle.

Lateral surface area = $2\pi rh$

Total surface area = $2\pi r\,(r+h)$

Volume = $\pi r^2 h$; where r is radius of the base and h is the height



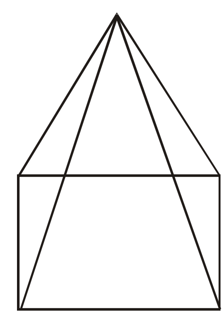
PYRAMID

A pyramid is a solid which can have any polygon at its base and its edges converge to single apex.

Lateral or curved surface area

$$= \frac{1}{2}\text{(perimeter of base)} \times \text{slant height}$$

Total surface area = lateral surface area + area of the base



Volume $= \frac{1}{3}$ (area of the base) × height

RIGHT CIRCULAR CONE

It is a solid which has a circle as its base and a slanting lateral surface that converges at the apex.

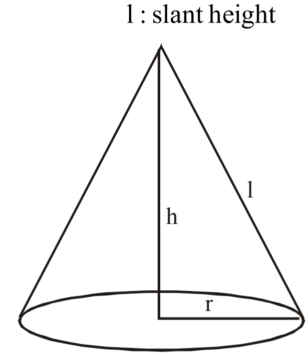
Lateral surface area = πrl

Total surface area = $\pi r\,(l+r)$

Volume $= \frac{1}{3}\pi r^2 h$; where r : radius of the base

h : height

l : slant height

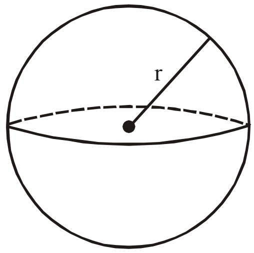


SPHERE

It is a solid in the form of a ball with radius r.

Lateral surface area = Total surface area = $4\pi r^2$

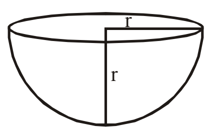
Volume $= \frac{4}{3}\pi r^3$; where r is radius.



HEMISPHERE

It is a solid half of the sphere.
Lateral surface area $= 2\pi r^2$
Total surface area $= 3\pi r^2$

Volume $= \frac{2}{3}\pi r^3$; where r is radius



> ☞ ***Shortcut Approach***
>
> If side of a cube or radius (or diameter) of sphere is increased by x%, then its volume increases by $\left[\left(1+\frac{x}{100}\right)^3 - 1\right] \times 100\%$

EXAMPLE 18. If side of a cube is increased by 10%, by how much per cent does its volume increase ?

Sol. Here, x = 10
According to the formula,
Percentage increase in volume

$$= \left[\left(1+\frac{x}{100}\right)^3 - 1\right] \times 100\% = \left[\left(1+\frac{10}{100}\right)^3 - 1\right] \times 100\%$$

$$= \left[\left(1+\frac{10}{100}\right)^3 - 1\right] \times 100\% = \left[\left(\frac{11}{10}\right)^3 - 1\right] \times 100\%$$

$$= \left[\frac{1331}{1000} - 1\right] \times 100\% = (1.331 - 1) \times 100\%$$

$$= (0.331 \times 100)\% = 33.1\%$$

> ☞ ***Shortcut Approach***
>
> If in a cylinder or cone, height and radius both change by x%, then volume changes by $\left[\left(1+\frac{x}{100}\right)^3 - 1\right] \times 100\%$

EXAMPLE 19. If in a cylinder, both height and radius increase by 100%, by what per cent does its volume increase?

Sol. Here, x = 10
According to the formula,
Percentage increase in volume

$$= \left[\left(1+\frac{x}{100}\right)^3 - 1\right] \times 100\% = \left[\left(1+\frac{100}{100}\right)^3 - 1\right] \times 100\%$$

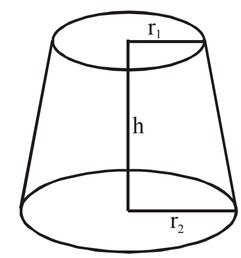
$$= [(2)^3 - 1] \times 100\% = (8 - 1) \times 100\% = 700\%$$

FRUSTUM OF A CONE

When a cone cut the left over part is called the frustum of the cone.
Curved surface area $= \pi l\,(r_1 + r_2)$

Total surface area $= \pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$



Volume $= \frac{1}{3}\pi h\left(r_1^2 + r_1 r_2 + r_2^2\right)$

EXAMPLE 20. The sum of length, breadth and height of a room is 19 m. The length of the diagonal is 11 m. The cost of painting the total surface area of the room at the rate of ₹10 per m² is :

(a) ₹ **240** **(b)** ₹**2400**
(c) ₹ **420** **(d)** ₹ **4200**

Sol. (b) Let length, breadth and height of the room be ℓ, b and h, respectively. Then,

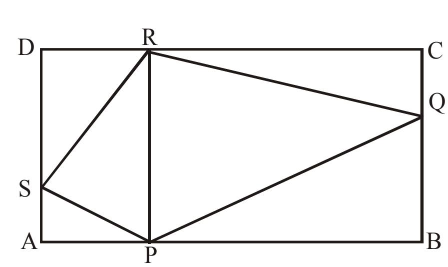
$\ell + b + h = 19$ ...(i)

and $\sqrt{\ell^2 + b^2 + h^2} = 11$

$\Rightarrow \ell^2 + b^2 + h^2 = 121$ ...(ii)

Area of the surface to be painted
$= 2(lb + bh + hl)$
$(\ell + b + h)^2 = \ell^2 + b^2 + h^2 + 2(\ell b + \ell h + h\ell)$
$\Rightarrow 2(\ell b + bh + h\ell) = (19)^2 - 121 = 361 - 121 = 240$
Surface area of the room $= 240\ m^2$.
Cost of painting the required area $= 10 \times 240 =$ ₹ 2400

EXAMPLE 21. ABCD is a parallelogram. P, Q, R and S are points on sides AB, BC, CD and DA, respectively such that AP = DR. If the area of the rectangle ABCD is 16 cm², then the area of the quadrilateral PQRS is :



(a) **6 cm²** **(b)** **6.4 cm²**
(c) **4 cm²** **(d)** **8 cm²**

Sol. (d) Area of the quadrilateral PQRS
$= \text{Area of } \Delta SPR + \text{Area of } \Delta PQR$

$$= \frac{1}{2} \times PR \times AP + \frac{1}{2} \times PR \times PB$$

$$= \frac{1}{2} \times PR(AP + PB) = \frac{1}{2} \times AD \times AB$$

(PR = AD and AP + PB = AB)

$$= \frac{1}{2} \times \text{Area of rectangle ABCD} = \frac{1}{2} \times 16 = 8\,\text{cm}^2$$

EXAMPLE 22. A road roller of diameter 1.75 m and length 1 m has to press a ground of area 1100 sqm. How many revolutions does it make ?

Sol. Area covered in one revolution = curved surface area

$\therefore$ Number of revolutions $= \dfrac{\text{Total area to be pressed}}{\text{Curved surface area}}$

$$= \frac{1100}{2\pi rh} = \frac{1100}{2 \times \frac{22}{7} \times \frac{1.75}{2} \times 1}$$

$= 200$

EXAMPLE 23. The annual rainfall at a place is 43 cm. Find the weight in metric tonnes of the annual rain falling there on a hectare of land, taking the weight of water to be 1 metric tonne to the cubic metre.

Sol. Area of land = 10000 sqm

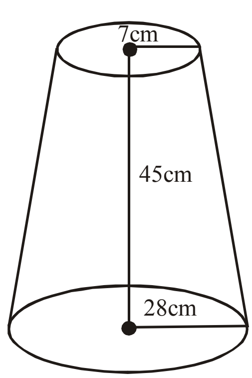
Volume of rainfall $= \dfrac{10000 \times 43}{100} = 4300\,\text{m}^3$

Weight of water = 4300 × 1 m tonnes = 4300 m tonnes

EXAMPLE 24. The height of a bucket is 45 cm. The radii of the two circular ends are 28 cm and 7 cm, respectively. The volume of the bucket is :

(a) $38610\ \text{cm}^3$ **(b)** $48600\ \text{cm}^3$
(c) $48510\ \text{cm}^3$ **(d)** None of these

Sol. (c) Here $r_1 = 7$ cm, $r_2 = 28$ cm and $h = 45$ cm



Volume of the frustum of a cone

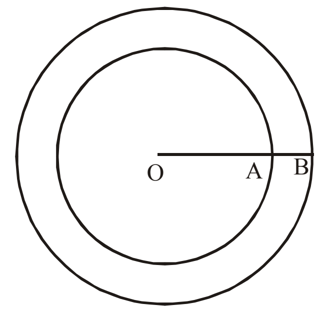
Volume of the bucket $= \dfrac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

Hence, the required volume

$$= \frac{1}{3} \times \frac{22}{7} \times 45(28^2 + 7^2 + 28 \times 7) = 48510\,\text{cm}^3$$

EXAMPLE 25. A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the external diameter be 50 cm and the length of the tube be 140 cm, find the number of cubic cm of iron in it.

Sol. Height = 140 cm
External diameter = 50 cm
$\therefore$ External radius = 25 cm



Also, internal radius OA = OB – AB= 25 – 2 = 23 cm

$\therefore$ Volume of iron $= V_{\text{external}} - V_{\text{internal}}$

$$= \frac{22}{7} \times 140\left(25^2 - 23^2\right) = 42240 \text{ cu. cm.}$$

EXAMPLE 26. A cylindrical bath tub of radius 12 contains water to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. What is the radius of the ball?

(a) $80\ \text{cm}^2$ **(b)** $84\ \text{cm}^2$
(c) $104\ \text{cm}^2$ **(d)** $76\ \text{cm}^2$

Sol. (b) Volume of the spherical ball = volume of the water displaced.

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi(12)^2 \times 6.75$$

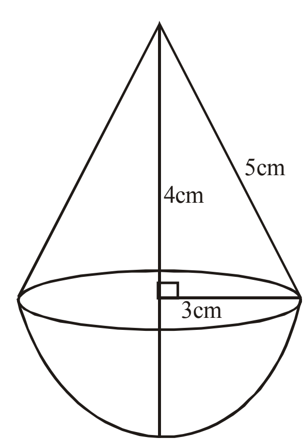
$$\Rightarrow r^3 = \frac{144 \times 6.75 \times 3}{4} = 729$$

or $r = 9$ cm,

EXAMPLE 27. A toy is in the form of a cone mounted on a hemisphere with the same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. Determine the surface area of the toy. (Use $\pi = 3.14$).

Sol. The radius of the hemisphere $= \dfrac{1}{2} \times 6 = 3$ cm

Now, slant height of cone $= \sqrt{3^2 + 4^2} = 5\text{cm}$



The surface area of the toy

= Curved surface of the conical portion + curved surface of the hemisphere

$= (\pi \times 3 \times 5 + 2\pi \times 3^2)\ cm^2$

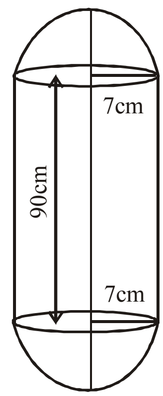
$= 3.14 \times 3\,(5 + 6)\ cm^2 = 103.62\ cm^2$.

EXAMPLE 28. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of Re. 1 per dm^2.

Sol. Let the height of the cylinder be h cm.

Then $h + 7 + 7 = 104$

$\Rightarrow h = 90$



Surface area of the solid

= 2 × curved surface area of hemisphere + curved surface area of the cylinder

$$= \left(2 \times 2 \times \frac{22}{7} \times 7 \times 7 + 2 \times \frac{22}{7} \times 7 \times 90\right) cm^2$$

$= 616 + 3960\ cm^2 = 4576\ cm^2$

Cost of polishing the surface of the solid

$$= ₹.\ \frac{4576 \times 1}{100} = ₹45.76$$

EXAMPLE 29. A regular hexagonal prism has perimeter of its base as 600 cm and height equal to 200 cm. How many litres of petrol can it hold ? Find the weight of petrol if density is 0.8 gm/cc.

Sol. Side of hexagon $= \dfrac{\text{Perimeter}}{\text{Number of sides}} = \dfrac{600}{6} = 100$ cm

Area of regular hexagon $= \dfrac{3\sqrt{3}}{2} \times 100 \times 100 = 25950$ sq.cm.

Volume = Base area × height

= 25950 × 200 = 5190000 cu.cm. = 5.19 cu.m.

Weight of petrol = Volume × Density

= 5190000 × 0.8 gm/cc

= 4152000 gm = 4152 kg.

EXAMPLE 30. A right pyramid, 12 cm high, has a square base each side of which is 10 cm. Find the volume of the pyramid.

Sol. Area of the base = 10 × 10 = 100 sq.cm.

Height = 12 cm

$\therefore$ Volume of the pyramid $= \dfrac{1}{3} \times 100 \times 12 = 400$ cu.cm.

EXAMPLE 31. Semi-circular lawns are attached to both the edges of a rectangular field measuring 42 m × 35m. The area of the total field is :

(a) 3818.5 m^2 **(b) 8318 m^2**
(c) 5813 m^2 **(d) 1358 m^2**

Sol. (a) Area of the field

$$= 42 \times 35 + 2 \times \frac{1}{2} \times \frac{22}{7} \times (21)^2 + 2 \times \frac{1}{2} \times \frac{22}{7} \times (17.5)^2$$

$= 1470 + 1386 + 962.5 = 3818.5\ m^2$

EXAMPLE 32. A frustum of a right circular cone has a diameter of base 10 cm, of top 6 cm, and a height of 5 cm; find the area of its whole surface and volume.

Sol. Here $r_1 = 5$ cm, $r_2 = 3$ cm and $h = 5$ cm.

$$\therefore \quad l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{5^2 + (5-3)^2} = \sqrt{29}\ \text{cm} = 5.385\ \text{cm}$$

$\therefore$ Whole surface of the frustum

$$= \pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$$

$$= \frac{22}{7} \times 5.385(5+3) + \frac{22}{7} \times 5^2 + \frac{22}{7} \times 3^2 = 242.25\ \text{sq.cm.}$$

$$\text{Volume} = \frac{\pi h}{3}(r_1^2 + r_1 r_2 + r_2^2)$$

$$= \frac{22}{7} \times \frac{5}{3}\left[5^2 + 5 \times 3 + 3^2\right] = 256.67\ \text{cu. cm.}$$

EXAMPLE 33. A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of one end, and the other end as its base. The volume of the cylinder, hemisphere and the cone are, respectively in the ratio :

(a) 2 : 3 : 2 **(b) 3 : 2 : 1**
(c) 3 : 1 : 2 **(d) 1 : 2 : 3**

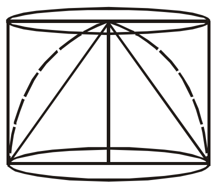
Sol. (b) We have,

radius of the hemisphere = raidus of the cone

= height of the cone

= height of the cylinder = r (say)

Then, ratio of the volumes of cylinder, hemisphere and cone



$$= \pi r^3 : \frac{2}{3}\pi r^3 : \frac{1}{3}\pi r^3 = 1 : \frac{2}{3} : \frac{1}{3} = 3 : 2 : 1$$

EXERCISE

1. The area of rectangular field is 460 square metres. If the length is 15 per cent more than the breadth ,what is the breadth of the rectangular field?
(a) 15 metres (b) 26 metres
(c) 34.5 metres (d) Cannot be determined
(e) None of these

2. What will be the cost of gardening 1-metre – broad boundary around a rectangular plot having perimeter of 340 metres at the rate of ₹ 10 per square metre?
(a) ₹ 3400 (b) ₹ 1700
(c) ₹ 3440 (d) Cannot be determined
(e) None of these

3. The cost of paint is ₹ 60 per kilogram. A kilogram paint covers 20 square feet. How much will it cost to paint the outside of a cube having each side 10 feet?
(a) ₹ 3000 (b) ₹ 900
(c) ₹ 1800 (d) ₹ 360
(e) None of these

4. 20 buckets of water fill a tank when the capacity of each bucket is 13.5 litres. How many buckets will be required to fill the same tank if the capacity of each bucket is 9 litres?
(a) 30 (b) 32
(c) 60 (d) Data inadequate
(e) None of these

5. The breadth of a rectangular hall is two-thirds of its length. If the area of the hall is 2400 sq metres, what is the length in metres?
(a) 120 (b) 80
(c) 60 (d) 40
(e) None of these

6. If a pair of opposite sides of a square is increased by 5 cm each, then the ratio of the sides of the new figure is 3 : 2. What is the original area of the square?
(a) 125 cm^2 (b) 225 cm^2
(c) 81 cm^2 (d) 100 cm^2
(e) None of these

7. An equilateral triangle, a square and a circle have equal perimeters. If T denotes the area of the triangle, S, the area of the square and C, the area of the circle, then :
(a) $S > T > C$ (b) $T > C > S$
(c) $T > S > C$ (d) $C > S > T$
(e) None of these

8. The capacity of a cylindrical tank is 246.4 litres. If the height is 4 metres, what is the diameter of the base?
(a) 1.4 metres (b) 2.8 metres
(c) 28 metres (d) 14 metres
(e) None of these

9. The internal measurements of a box with lid are $115 \times 75 \times 35$ cm^3 and the wood of which it is made is 2.5 cm thick. Find the volume of wood.
(a) 82,125 cm^3 (b) 70,054 cm^3
(c) 78,514 cm^3 (d) None of these
(e) None of these

10. The length and the breadth of a rectangle are in the ratio of 3 : 2 respectively. If the sides of the rectangle are extended on each side by 1 metre, the ratio of length to breadth becomes 10 : 7. Find the area of the original rectangle in square metres.
(a) 56 (b) 50
(c) 80 (d) Data inadequate
(e) None of these

11. A right circular cone is exactly fitted inside a cube in such a way that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is on the opposite face of the cube. If the volume of the cube is 343 cc, what **approximately** is the volume of the cone?
(a) 80 cc (b) 90 cc
(c) 110 cc (d) 105 cc
(e) 100 cc

12. If the length of a rectangle is increased by 20% and the breadth is reduced by 20%, what will be the effect on its area?
(a) 4% increase (b) 6% increase
(c) 5% decrease (d) 4% decrease
(e) None of these

13. The ratio between the length and the breadth of a rectangular plot is 7 : 5. If the perimeter of the plot is 144 metres, what is its area?
(a) 1320 sq. metres (b) 1260 sq. metres
(c) 1280 sq. metres (d) 1380 sq. metres
(e) None of these

14. The perimeter of a rectangle is equal to the perimeter of a right-angled triangle of height 12 cm. If the base of the triangle is equal to the breadth of the rectangle, what is the length of the rectangle"
(a) 18 cm (b) 24
(c) 22 cm (d) Data inadequate
(e) None of these

15. The squared value of the diagonal of a rectangle is $(64 + B^2)$ sq cm, where B is less than 8 cm. What is the breadth of that rectangle?
(a) 6 cm (b) 10 cm
(c) 8 cm (d) Data inadequate
(e) None of these

16. If the height of a triangle is decreased by 40%, land its base is increased by 40%, what will be the effect on its area?
(a) No change (b) 16% increase
(c) 8% decrease (d) 16% decrease
(e) None of these

17. A circular ground whose diameter is 35 metres, has a 1.4 metre-broad garden around it. What is the area of the garden in square metres?
(a) 160.16 (b) 6.16
(c) 122.66 (d) Data inadeuate
(e) None of these

18. The length of a rectangular plot is 20 metres more than its breadth. If the cost of fencing the plot at the rate of ₹ 26.50 per metre is ₹ 5,300, what is the length of the plot (in metres)?
(a) 40 (b) 120
(c) 50 (d) Data inadequate
(e) None of these

19. A rectangular plate is of 6 in breadth and 12 in length. Two apertures of 2 in diameter each and one aperture of 1 in diameter have been made with the help of a gas cutter. What is the area of the remaining portion of the plate?
(a) 62.5 sq. in (b) 68.5 sq. in
(c) 64.5 sq. in (d) 66.5 sq. in
(e) None of these

20. What would be the length of the diagonal of a square plot whose area is equal to the area of a rectangular plot of 45 m length and 40 m width?
(a) 42.5 m (b) 60 m
(c) 4800 m (d) Data inadequate
(e) None of these

21. What will be the ratio between the area of a rectangle and the area of a triangle with one of the sides of rectangle as base and a vertex on the opposite side of rectangle.
(a) 1 : 2 (b) 2 : 1
(c) 3 : 1 (d) Data inadequate
(e) None of these

22. Two roads XY and YZ of 15 metres and 20 metres length respectively are perpendicular to each other. What is the distance between X & Z by the shortest route?
(a) 35 metres (b) 30 metres
(c) 24 metres (d) 25 metres
(e) None of these

23. What will be the area of a semi-circle of 14 metres diameter?
(a) 154 sq metres (b) 77 sq metres
(c) 308 sq metres (d) 22 sq metres
(e) None of these

24. The area of a right-angled triangle is two-thirds of the area of a rectangle. The base of the triangle is 80 percent of the breadth of the rectangle. If the perimeter of the rectangle is 200 cm, what is the height of the triangle?
(a) 20 cm (b) 30 cm
(c) 15 cm (d) Data inadequate
(e) None of these

25. The area of a rectangular plot is 15 times its breadth. If the difference between the length and the breadth is 10 metres, what is its breadth?
(a) 10 metres (b) 5 metres
(c) 7.5 metres (d) Data inadequate
(e) None of these

26. A rectangular garden has a 5-metre-wide road outside around all the four sides. The area of the road is 600 square metres. What is the ratio between the length and the breadth of that plot?
(a) 3 : 2 (b) 4 : 3
(c) 5 : 4 (d) Data inadequate
(e) None of these

27. Four sheets of 50 cm × 5 cm are to be arranged in such a manner that a square could be formed. What will be the area of inner part of the square so formed?
(a) 2000 cm^2 (b) 1600 cm^2
(c) 1800 cm^2 (d) 2500 cm^2
(e) None of these

28. In order to fence a square Manish fixed 48 poles. If the distance between two poles, is 5 metres then what will be the area of the square so formed?
(a) Cannot be determined (b) 2600 cm^2
(c) 2500 cm^2 (d) 3025 cm^2
(e) None of these

29. The area of a side of a box is 120 sq cm. The area of the other side of the box is 72 sq cm. If the area of the upper surface of the box is 60 sq cm then find the volume of the box.
(a) 259200 cm^3 (b) 86400 cm^3
(c) 720 cm^3 (d) Cannot be determined
(e) None of these

30. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. What is the area of the circle ?
(a) 88 cm^2 (b) 154 cm^2
(c) 1250 cm^2 (d) 616 cm^2
(e) None of these

31. The cost of carpeting a room 18m long with a carpet 75 cm wide at ₹ 4.50 per metre is ₹810. The breadth of the room is:
(a) 7 m (b) 7.5 m
(c) 8 m (d) 8.5 m
(e) None of these

32. If the perimeter and diagonal of a rectangle are 14 and 5 cms respectively, find its area.
(a) 12 cm^2 (b) 16 cm^2
(c) 20 cm^2 (d) 24 cm^2
(e) None of these

33. In an isoscele right angled triangle, the perimeter is 20 metre. Find its area.
(a) 9,320 m^2 (b) 8,750 m^2
(c) 7,980 m^2 (d) 6,890 m^2
(e) None of these

34. When the circumference and area of a circle are numerically equal, then the diameter is numerically equal to
(a) area (b) circumference
(c) 4 (d) 2 π
(e) None of these

35. In a parallelogram, the length of one diagonal and the perpendicular dropped on that diagonal are 30 and 20 metres respectively. Find its area.
(a) 600 m^2 (b) 540 m^2
(c) 680 m^2 (d) 574 m^2
(e) None of these

36. The diameter of a garden roller is 1.4 m and it is 2 m long. How much area will it cover in 5 revolutions ? $\left(\text{use } \pi = \frac{22}{7}\right)$
(a) 40 m^2 (b) 44 m^2
(c) 48 m^2 (d) 36 m^2
(e) None of these

37. The area of a triangle is 615 m^2. If one of its sides is 123 metre, find the length of the perpendicular dropped on that side from opposite vertex.
(a) 15 metres (b) 12 metres
(c) 10 metres (d) 9 metres
(e) None of these

38. A horse is tethered to one corner of a rectangular grassy field 40 m by 24 m with a rope 14 m long. Over how much area of the field can it graze?
(a) 154cm^2 (b) 308 m^2
(c) 150 m^2 (d) 407 m^2
(e) None of these

39. How many plants will be there in a circular bed whose outer edge measure 30 cms, allowing 4 cm^2 for each plant ?
(a) 18 (b) 750
(c) 24 (d) 120
(e) None of these

40. From a square piece of a paper having each side equal to 10 cm, the largest possible circle is being cut out. The ratio of the area of the circle to the area of the original square is nearly :
(a) $\frac{4}{5}$ (b) $\frac{3}{5}$
(c) $\frac{5}{6}$ (d) $\frac{6}{7}$
(e) None of these

41. A square carpet with an area 169 m^2 must have 2 metres cut-off one of its edges in order to be a perfect fit for a rectangualar room. What is the area of rectangular room?
(a) 180 m^2 (b) 164 m^2
(c) 152 m^2 (d) 143 m^2
(e) None of these

42. A picture 30" × 20" has a frame 2½" wide. The area of the picture is approximately how many times the area of the frame?
(a) 4 (b) $2\frac{1}{2}$
(c) 2 (d) 5
(e) None of these

43. A rectangular plot 15 m ×10 m, has a path of grass outside it. If the area of grassy pathway is 54 m^2, find the width of the path.
(a) 4m (b) 3m
(c) 2m (d) 1m
(e) None of these

44. If the area of a circle decreases by 36%, then the radius of a circle decreases by
(a) 20% (b) 18%
(c) 36% (d) 64%
(e) None of these

45. The floor of a rectangular room is 15 m long and 12 m wide. The room is surrounded by a verandah of width 2 m on all its sides. The area of the verandah is :
(a) 124 m^2 (b) 120 m^2
(c) 108 m^2 (d) 58 m^2
(e) None of these

46. A typist uses a paper 12" by 5" length wise and leaves a margin of 1" at the top and the bottom and a margin of ½"on either side. What fractional part of the paper is available to him for typing ?
(a) $\frac{2}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{5}{7}$
(e) None of these

47. A rectangular lawn 70 m × 30 m has two roads each 5 metres wide, running in the middle of it, one parallel to the length and the other parallel to the breadth. Find the cost of gravelling the road at the rate of ₹ 4 per square metre.
(a) ₹ 2,000 (b) ₹ 1,800
(c) ₹ 1,900 (d) ₹ 1,700
(e) None of these

48. A circular grass lawn of 35 metres in radius has a path 7 metres wide running around it on the outside. Find the area of path.
(a) 1694 m^2 (b) 1700 m^2
(c) 1598 m^2 (d) 1500 m^2
(e) None of these

49. A cylindrical bucket of height 36 cm and radius 21 cm is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed, the height of the heap being 12 cm. The radius of the heap at the base is :
(a) 63 cm (b) 53 cm
(c) 56 cm (d) 66 cm
(e) None of these

50. The radius of the wheel of a bus is 70 cms and the speed of the bus is 66 km/h, then the r.p.m. (revolutions per minutes) of the wheel is
(a) 200 (b) 250
(c) 300 (d) 330
(e) None of these

51. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. The area of the triangle is
(a) 72 cm^2 (b) 60 cm^2
(c) 66 cm^2 (d) 65 cm^2
(e) None of these

52. The cross section of a canal is a trapezium in shape. If the canal is 7 metres wide at the top and 9 metres at the bottom and the area of cross-section is 1280 square metres, find the length of the canal.
(a) 160 metres (b) 172 metres
(c) 154 metres (d) 165 metres
(e) None of these

53. It is required to fix a pipe such that water flowing through it at a speed of 7 metres per minute fills a tank of capacity 440 cubic metres in 10 minutes. The inner radius of the pipe should be :
(a) $\sqrt{2}$ m (b) 2m
(c) $\frac{1}{2}$ m (d) $\frac{1}{\sqrt{2}}$ m
(e) None of these

54. The area of a square field is 576 km^2. How long will it take for a horse to run around at the speed of 12 km/h ?
(a) 12 h (b) 10 h
(c) 8 h (d) 6 h
(e) None of these

55. The area of a rectangular field is 144 m^2. If the length had been 6 metres more, the area would have been 54 m^2 more. The original length of the field is
(a) 22 metres (b) 18 metres
(c) 16 metres (d) 24 metres
(e) None of these

56. A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet?
(a) 46 (b) 81
(c) 126 (d) 252
(e) None of these

57. A farmer wishes to start a 100 square metres rectangular vegetable garden. Since he has only 30 m barbed wire, he fences three sides of the garden letting his house compound wall act as the fourth side fencing. The dimension of the garden is:
(a) 15 m × 6.67 m (b) 20 m × 5 m
(c) 30 m × 3.33 m (d) 40 m × 2.5 m
(e) None of these

58. A rectangular tank measuring 5m × 4.5 m × 2.1 m is dug in the centre of the field measuring 13.5 m × 2.5. The earth dug out is spread evenly over the remaining portion of a field. How much is the level of the field raised ?
(a) 4.0 m (b) 4.1 m
(c) 4.2 m (d) 4.3 m
(e) None of these

59. A rectangular paper, when folded into two congruent parts had a perimeter of 34 cm for each part folded along one set of sides and the same is 38 cm when folded along the other set of sides. What is the area of the paper?
(a) 140 cm^2 (b) 240 cm^2
(c) 560 cm^2 (d) 160 cm^2
(e) None of these

60. The length and breadth of the floor of the room are 20 feet and 10 feet respectively. Square tiles of 2 feet length of different colours are to be laid on the floor. Black tiles are laid in the first row on all sides. If white tiles are laid in the one-third of the remaining and blue tiles in the rest, how many blue tiles will be there?
(a) 16 (b) 24
(c) 32 (d) 48
(e) None of these

61. Four equal circles are described about the four corners of a square so that each touches two of the others. If a side of the square is 14 cm, then the area enclosed between the circumferences of the circles is :
(a) 24 cm^2 (b) 42 cm^2
(c) 154 cm^2 (d) 196 cm^2
(e) None of these

CHAPTER 11 Clock and Calendar

CLOCK

Introduction

- A clock has two hands : Hour hand and Minute hand.
- The minute hand (M.H.) is also called the long hand and the hour hand (H.H.) is also called the short hand.
- The clock has 12 hours numbered from 1 to 12.

Also, the clock is divided into 60 equal minute divisions. Therefore, each hour number is separated by five minute divisions. Therefore,

Shortcut Approach

- One minute division $= \frac{360}{60} = 6°$ apart. ie. In one minute, the minute hand moves 6°.
- One hour division $= 6° \times 5 = 30°$ apart. ie. In one hour, the hour hand moves 30° apart.

  Also, in one minute, the hour hand moves $= \frac{30°}{60°} = \frac{1}{2}°$ apart.
- Since, in one minute, minute hand moves 6° and hour hand moves $\frac{1}{2}°$, therefore, in one minute, the minute hand gains $5\frac{1}{2}°$ more than hour hand.
- In one hour, the minute hand gains $5\frac{1}{2}° \times 60 = 330°$ over the hour hand. i.e. the minute hand gains 55 minutes divisions over the hour hand.

Relative position of the hands

The position of the M.H. relative to the H.H. is said to be the same, whenever the M.H. is separated from the H.H. by the same number of minute divisions and is on same side (clockwise or anticlockwise) of the H.H.

Any relative position of the hands of a clock is repeated 11 times in every 12 hours.

(a) When both hands are 15 minute spaces apart, they are at right angle.

(b) When they are 30 minute spaces apart, they point in opposite directions.

(c) The hands are in the same straight line when they are coincident or opposite to each other.

- In every hour, both the hand coincide once.
- In a day, the hands are coinciding 22 times.
- In every 12 hours, the hands of clock coincide 11 times.
- In every 12 hours, the hands of clock are in opposite direction 11 times.
- In every 12 hours, the hands of clock are at right angles 22 times.
- In every hour, the two hands are at right angles 2 times.
- In every hour, the two hands are in opposite direction once.
- In a day, the two hands are at right angles 44 times.
- If both the hands coincide, then they will again coincide after $65\frac{5}{11}$ minutes. i.e. in correct clock, both hand coincide at an interval of $65\frac{5}{11}$ minutes.
- If the two hands coincide in time less than $65\frac{5}{11}$ minutes, then clock is too fast and if the two hands coincides in time more than $65\frac{5}{11}$ minutes, then the clock is too slow.

Shortcut Approach

- **Shortcut Approach for finding degrees minutes and hours is**

  $\theta = \left(\frac{11}{2}M - 30H\right)$

  Where, M = minutes
  and, H = Hours
- When value of θ becomes more than 360, subtract 360 from the value of θ and complete the calculation.

EXAMPLE 1. The angle between the minute hand and the other hour hand of a clock when the time is 8:30 is

(a) 80 degrees **(b) 75 degrees**
(c) 60 degrees **(d) 105 degrees**

Sol. Degree required $(\theta) = \left[\frac{11}{2}M - 30H\right]$

$= \frac{11}{2} \times 30 - 30 \times 8 = 165 - 240 = 75$ degree

EXAMPLE 2. At what time between 4 and 5 will the hands of a watch

(i) coincide, and

(ii) point in opposite directions.

Sol. (i) At 4 O' clock, the hands are 20 minutes apart. Clearly the minute hand must gain 20 minutes before two hands can be coincident.

But the minute-hand gains 55 minutes in 60 minutes.

Let minute hand will gain x minute in 20 minutes.

So, $\frac{55}{20} = \frac{60}{x}$

$\Rightarrow \quad x = \frac{20 \times 60}{55} = \frac{240}{11} = 21\frac{9}{11}$ min.

$\therefore$ The hands will be together at $21\frac{9}{11}$ min past 4.

(ii) Hands will be opposite to each other when there is a space of 30 minutes between them. This will happen when the minute hand gains $(20 + 30) = 50$ minutes.

Now, the minute hand gains 50 min in $\frac{50 \times 60}{55}$ or $54\frac{6}{11}$ min.

$\therefore$ The hands are opposite to each other at $54\frac{6}{11}$ min past 4.

EXAMPLE 3. What is the angle between the hour hand and minute hand when it was 5 : 05 pm.

Sol. 5.05 pm means hour hand was on 5 and minute hand was on 1, i.e. there will be 20 minutes gap.

$\therefore$ Angle $= 20 \times 6° = 120°$ $\quad [\because$ 1 minute $= 6°]$

INCORRECT CLOCK

If a clock indicates 6 : 10, when the correct time is 6 : 00, it is said to be 10 minute too fast and if it indicates 5 : 50 when the correct time is 6 : 00, it is said to be 10 minute too slow.

- Also, if both hands coincide at an interval x minutes and $x < 65\frac{5}{11}$, then total time gained $= \left(\frac{65\frac{5}{11} - x}{x}\right)$ minutes and clock is said to be 'fast'.
- If both hands coincide at an interval x minutes and $x > 65\frac{5}{11}$, then total time lost $= \left(\frac{x - 65\frac{5}{11}}{x}\right)$ minutes and clock is said to be 'slow'.

EXAMPLE 4. My watch, which gains uniformly, is 2 min slow at noon on Sunday, and is 4 minutes 48 seconds fast at 2 pm on the following Sunday. When was it correct.

Sol. From Sunday noon to the following Sunday at 2 pm = 7 days 2 hours = 170 hours.

The watch gains $\left(2 + 4\frac{48}{60}\right) = 6\frac{4}{5}$ minutes in 170 hours.

$\therefore$ The watch gains 2 minutes in $\frac{2}{6\frac{4}{5}} \times 170 = 50$ hours

Now, 50 hours = 2 days 2 hours

2 days 2 hours from Sunday noon = 2 pm on Tuesday.

EXAMPLE 5. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Sol. In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand.

55 min. are gained in $\left(\frac{60}{55} \times 60\right)$ min. $= 65\frac{5}{11}$ min.

But, they are together after 65 min.

$\therefore$ Gain in 65 min. $= \left(65\frac{5}{11} - 65\right) = \frac{5}{11}$ min.

Gain in 24 hours $= \left(\frac{5}{11} \times \frac{60 \times 24}{65}\right)$ min. $= 10\frac{10}{143}$ min.

$\therefore$ The clock gains $10\frac{10}{143}$ minutes in 24 hours.

EXAMPLE 6. A man who went out between 5 or 6 and returned between 6 and 7 found that the hands of the watch had exactly changed place. When did he go out?

Sol. Between 5 and 6 to 6 and 7, hands will change place after crossing each other one time. ie. they together will make 1 + 1 = 2 complete revolutions.

H.H. will move through $2 \times \frac{60}{13}$ or $\frac{120}{13}$ minute divisions.

Between 5 and 6 $\rightarrow \frac{120}{13}$ minute divisions.

At 5, minute hand is 25 minute divisions behind the hour-hand.

Hence it will have to gain $25 + \frac{120}{13}$ minute divisions on the hour-hand $= \frac{445}{13}$ minute divisions on the hour hand.

The minute hand gains $\frac{445}{13}$ minute divisions in $\frac{445}{13} \times \frac{12}{11}$ minutes $= \frac{5340}{143} = 37\frac{49}{143}$ minutes

$\therefore$ The required time of departure is $37\frac{49}{143}$ min past 5.

CALENDAR

INTRODUCTION

An ordinary year has 365 days. Every year which is divisible by 4, is a leap year and has 366 days, But century year has 365 days except for year divisible by 400 which has 366 days.

An ordinary year contains 365 days i.e., 52 weeks + 1 day i.e. 1 odd day.

A leap year contains 366 days i.e. 52 weeks + 2 days i.e. 2 odd days.

A century (100 years) contains = 24 leap years + 76 ordinary years = 24 × 2 + 76 = 124 odd days = 17 weeks + 5 odd days

Similarly,

200 years contains 2 × 5 – 7 = 3 odd days

300 years contains 3 × 5 – 14 = 1 odd day

400 years contains 4 × 5 + 1 – 21 = 0 odd days

First January, 1 A.D. was Monday.

A solar year contains 365 days 5 hours 48 minutes 48 seconds.

The first day of a century must either be Monday, Tuesday, Thursday or Saturday.

Months	Odd days
January	3
February	0/1 (ordinary/leap)
March	3
April	2
May	3
June	2
July	3
August	3
September	2
October	3
November	2
December	3

To find a particular day witihout given date and day

Following steps are taken into consideration to solve such questions

Step I Firstly, you have to find the number of odd upto the date for which the day is to be determined.

Step II Your required day will be according to the following conditions

(a) If the number of odd days = 0, then required day is Sunday.
(b) If the number of odd days = 1, then required day is Monday.
(c) If the number of odd days = 2, then required day is Tuesday.
(d) If the number of odd days = 3, then required day is Wednesday.
(e) If the number of odd days = 4, then required day is Thursday.
(f) If the number of odd days = 5, then required day is Friday.
(g) If the number of odd days = 6, then required day is Saturday.

NOTE : *February in an ordinary year gives no odd days, but in a leap year gives one odd day.*

EXAMPLE 7. What day of the week was 15th August 1949?

Sol. 15th August 1949 means

1948 complete years + first 7 months of the year 1949 + 15 days of August.

1600 years give no odd days.

300 years give 1 odd day.

48 years give {48 + 12} = 60 = 4 odd days.

[∵ For ordinary years → 48 odd days and for leap year 1 more day (48 ÷ 4) = 12 odd days; 60 = 7 × 8 + 4]

From 1st January to 15th August 1949

Odd days :

January – 3
February – 0
March – 3
April – 2
May – 3
June – 2
July – 3
August – 1

17 ⇒ 3 odd days.

∴ 15th August 1949 → 1 + 4 + 3 = 8 = 1 odd day.

This means that 15th Aug. fell on 1st day. Therefore, the required day was Monday.

EXAMPLE 8. How many times does the 29th day of the month occur in 400 consecutive years?

Sol. In 400 consecutive years, there are 97 leap years. Hence, in 400 consecutive years, February has the 29th day 97 times and the remaining eleven months have the 29th day 400 × 1100 = 4400 times

∴ The 29th day of the month occurs (4400 + 97) or 4497 times.

EXAMPLE 9. Today is 5th February. The day of the week is Tuesday. This is a leap year. What will be the day of the week on this date after 5 years?

Sol. This is a leap year. So, next 3 years will give one odd day each. Then leap year gives 2 odd days and then again next year give 1 odd day.

Therefore (3 + 2 + 1) = 6 odd days will be there.

Hence the day of the week will be 6 odd days beyond Tuesday, i.e., it will be Monday.

EXAMPLE 10. What day of the week was 20^{th} June 1837 ?

Sol. 20^{th} June 1837 means 1836 complete years + first 5 months of the year 1837 + 20 days of June.

1600 years give no odd days.

200 years give 3 odd days.

36 years give (36 + 9) or 3 odd days.

1836 years give 6 odd days.

From 1^{st} January to 20^{th} June there are 3 odd days.

Odd days :

January : 3
February : 0
March : 3
April : 2
May : 3
June : 6

17

Therefore, the total number of odd days = (6 + 3) or 2 odd days.

This means that the 20^{th} of June fell on the 2^{nd} day commencing from Monday. Therefore, the required day was Tuesday.

EXERCISE

1. If the two hands in a clock are 3 minutes divisions apart, then the angle between them is
(a) 3° (b) 18°
(c) 24° (d) 60°
(e) None of these

2. At what approximate time between 4 and 5 am will the hands of a clock be at right angle?
(a) 4 : 40 am (b) 4 : 38 am
(c) 4 : 35 am (d) 4 : 39 am
(e) None of these

3. What will be the acute angle between hands of a clock at 2 : 30?
(a) 105° (b) 115°
(c) 95° (d) 135°
(e) None of these

4. In 16 minutes, the minute hand gains over the hour hand by
(a) 16° (b) 80°
(c) 88° (d) 96°
(e) None of these

5. A clock is set right at 1 p.m. If it gains one minute in an hour, then what is the true time when the clock indicates 6 p.m. in the same day?
(a) $55\frac{5}{61}$ minutes past 5 (b) 5 minutes past 6
(c) 5 minutes to 6 (d) $59\frac{1}{64}$ minutes past 5
(e) None of these

6. At what time between 9'o clock and 10'o clock will the hands of a clock point in the opposite directions?
(a) $16\frac{4}{11}$ minutes past 9 (b) $16\frac{4}{11}$ minutes past 8
(c) $55\frac{5}{61}$ minutes past 7 (d) $55\frac{5}{61}$ minutes to 8
(e) None of these

7. A clock gains 15 minutes per day. It is set right at 12 noon. What time will it show at 4.00 am, the next day?
(a) 4 : 10 am (b) 4 : 45 am
(c) 4 : 20 am (d) 5 : 00 am
(e) None of these

8. What is the angle between the 2 hands of the clock at 8:24 pm?
(a) 100° (b) 107°
(c) 106° (d) 108°
(e) None of these

9. In a watch, the minute hand crosses the hour hand for the third time exactly after every 3 hrs., 18 min., 15 seconds of watch time. What is the time gained or lost by this watch in one day?
(a) 14 min. 10 seconds lost
(b) 13 min. 50 seconds lost
(c) 13min. 20 seconds gained
(d) 14 min. 40 seconds gained
(e) None of these

10. At what time between 3 and 4 o'clock, the hands of a clock coincide?
(a) $16\frac{4}{11}$ minutes past 3 (b) $15\frac{5}{61}$ minutes past 3
(c) $15\frac{5}{60}$ minutes to 2 (d) $16\frac{4}{11}$ minutes to 4
(e) None of these

11. A watch which gains uniformly is 2 minutes low at noon on Monday and is 4 min. 48 sec. fast at 2 p.m. on the following Monday. When was it correct?
(a) 2 p.m. on Tuesday (b) 2 p.m. on Wednesday
(c) 3 p.m. on Thursday (d) 1 p.m. on Friday
(e) None of these

12. If a clock strikes 12 in 33 seconds, it will strike 6 in how many seconds?
(a) $\frac{33}{2}$ (b) 15
(c) 12 (d) 22
(e) None of these

13. At what time between 7 and 8 o'clock will the hands of a clock be in the same straight line but, not together?
(a) 5 min. past 7 (b) $5\frac{2}{11}$ min. past 7
(c) $5\frac{3}{11}$ min. past 7 (d) $5\frac{5}{11}$ min. past 7
(e) None of these

14. At what time between 8 and 9 o'clock will the hands of a watch be in straight line but not together?
(a) $10\frac{11}{10}$ min. past 8 (b) $10\frac{10}{11}$ min. past 8
(c) $11\frac{10}{11}$ min. past 8 (d) $12\frac{10}{11}$ min. past 8
(e) None of these

15. At what time between 5.30 and 6 will the hands of a clock be at right angles?
(a) $43\frac{5}{11}$ min. past 5 (b) $43\frac{7}{11}$ min. past 5
(c) 40 min. past 5 (d) 45 min. past 5
(e) None of these

16. Find the angle between the hour hand and the minute hand of a clock when the time is 3.25.
(a) 45° (b) $37\frac{1}{2}^\circ$
(c) $47\frac{1}{2}^\circ$ (d) 46°
(e) None of these

17. How much does a watch lose per day, if its hands coincide every 64 minutes?
(a) $32\frac{8}{11}$ min. (b) $36\frac{5}{11}$ min.
(c) 90 min. (d) 96 min.
(e) None of these

18. An accurate clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?
(a) 144° (b) 150°
(c) 168° (d) 180°
(e) None of these

19. The first Republic Day of India was celebrated on 26th January, 1950. It was :
(a) Tuesday (b) Wednesday
(c) Thursday (d) Friday
(e) None of these

20. What will be the day of the week on 1st January, 2010 ?
(a) Friday (b) Saturday
(c) Sunday (d) Monday
(e) None of these

21. The calendar for the year 2005 is the same as for the year :
(a) 2010 (b) 2011
(c) 2012 (d) 2013
(e) None of these

22. If 09/12/2001 happens to be Sunday, then 09/12/1971 would have been at
(a) Wednesday (b) Tuesday
(c) Saturday (d) Thursday
(e) None of these

23. What was the day of the week on 15th August, 1947 ?
(a) Wednesday (b) Tuesday
(c) Friday (d) Thursday
(e) None of these

24. The last day of a century cannot be :
(a) Monday (b) Wednesday
(c) Friday (d) Tuesday
(e) None of these

25. The reflex angle between the hands of a clock at 10:25 is?
(a) 180° (b) $192\frac{1}{2}^\circ$
(c) 195° (d) $197\frac{1}{2}^\circ$
(e) None of these

26. A clock gains 5 minutes. in 24 hours. It was set right at 10 a.m. on Monday. What will be the true time when the clock indicates 10:30 a.m. on the next Sunday ?
(a) 10 a.m. (b) 11 a.m.
(c) 25 minutes past 10 a.m. (d) 5 minutes to 11 a.m.
(e) None of these

27. At what angle the hands of a clock are inclined at 15 minutes past 5 ?
(a) $72\frac{1}{2}^\circ$ (b) 64°
(c) $58\frac{1}{2}^\circ$ (d) $67\frac{1}{2}^\circ$
(e) None of these

28. Find the day of the week on 16th July, 1776.
(a) Tuesday (b) Wednesday
(c) Monday (d) Thursday
(e) None of these

29. On January 12, 1980, it was Saturday. The day of the week on January 12, 1979 was –
(a) Saturday (b) Friday
(c) Sunday (d) Thursday
(e) None of these

30. The year next to 1991 having the same calendar as that of 1990 is –
(a) 1998 (b) 2001
(c) 2002 (d) 2003
(e) None of these

31. A clock is set right at 5 a.m. The clock loses 16 min. in 24 hours. What will be the true time when the clock indicates 10 p.m. on the 4th day ?
(a) 11 p.m. (b) 10 p.m.
(c) 9 p.m. (d) 8 p.m.
(e) None of these

32. Find the exact time between 7 am and 8 am when the two hands of a watch meet ?
(a) 7 hrs 35 min (b) 7 hrs 36.99 min
(c) 7 hrs 38.18 min (d) 7 hrs 42.6 min
(e) None of these

33. A watch which gains 5 seconds in 3 minutes was set right at 7 a.m. In the afternoon of the same day, when the watch indicated quarter past 4 O'clock, the true time is –
(a) 4 p.m. (b) $59\frac{7}{12}$ minutes past 3
(c) $58\frac{7}{11}$ minutes past 3 (d) $2\frac{3}{11}$ minutes past 4
(e) None of these

CHAPTER 12

Permutation and Combination

INTRODUCTION

FACTORIAL

The important mathematical term "Factorial" has extensively used in this chapter.

The product of first n consecutive **natural numbers** is defined as **factorial of n**. It is denoted by n! or $\lfloor n$. Therefore,

$$\boxed{n! = 1 \times 2 \times 3 \times .... \times (n-1) \times n}$$

For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Note that :

$\frac{n!}{r!} \neq \left(\frac{n}{r}\right)!$

$0! = 1$

The factorials of fractions and negative integers are not defined.

EXAMPLE 1. Prove that n! + 1 is not divisible by any natural number between 2 and 'n'.

Sol. Since $n! = 1 . 2 . 3 . 4 ..... (n-1) . n$

Therefore n! is divisible by any number from 2 to 'n'.

Consequently n! + 1, when divided by any number between 2 and 'n' leaves 1 as remainder.

Hence, n! + 1 is not divisible by any number between 2 and 'n'.

Fundamental Principles of Counting

1. **Principle of Addition :** If an event can occur in 'm' ways and another event can occur in 'n' ways independent of the first event, then either of the two events can occur in (m + n) ways.
2. **Principle of Multiplication :** If an operation can be performed in 'm' ways and after it has been performed in any one of these ways, a second operation can be performed in 'n' ways, then the two operations in succession can be performed in (m × n) ways.

EXAMPLE 2. In a class there are 10 boys and 8 girls. The class teacher wants to select a student for monitor of the class. In how many ways the class teacher can make this selection ?

Sol. The teacher can select a student for monitor in two exclusive ways

(i) Select a boy among 10 boys, which can be done in 10 ways OR

(ii) Select a girl among 8 girls, which can be done in 8 ways.

Hence, by the fundamental principle of addition, either a boy or a girl can be selected in 10 + 8 = 18 ways.

EXAMPLE 3. In a class there are 10 boys and 8 girls. The teacher wants to select a boy and a girl to represent the class in a function. In how many ways can the teacher make this selection?

Sol. The teacher has to perform two jobs :

(i) To select a boy among 10 boys, which can be done in 10 ways.

(ii) To select a girl, among 8 girls, which can be done in 8 ways.

Hence, the required number of ways = 10 × 8 = 80.

EXAMPLE 4. There are 6 multiple choice questions in an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next three have 5 choices each?

Sol. Each of the first three questions can be answered in 4 ways and each of the next three questions can be answered in 5 different ways.

Hence, the required number of different sequences of answers = 4 × 4 × 4 × 5 × 5 × 5 = 8000.

EXAMPLE 5. Five persons entered a lift cabin on the ground floor of an 8-floor house. Suppose that each of them can leave the cabin independently at any floor beginning with the first. What is the total number of ways in which each of the five persons can leave the cabin at any of the 7 floors?

Sol. Any one of the 5 persons can leave the cabin in 7 ways independent of other.

Hence the required number of ways = $7 \times 7 \times 7 \times 7 \times 7 = 7^5$.

Method of Sampling :

Sampling process can be divided into following forms :

1. The order is IMPORTANT and the repetition is ALLOWED, each sample is then a SEQUENCE.
2. The order is IMPORTANT and the repetition is NOT ALLOWED, each sample is then a PERMUTATION.
3. The order is NOT IMPORTANT and repetition is ALLOWED, each sample is then a MULTISET.
4. The order is NOT IMPORTANT and repetition is NOT ALLOWED, each sample is then a COMBINATION.

PERMUTATION

Each of the arrangements, which can be made by taking, some or all of a number of things is called a PERMUTATION.

For Example : Formation of numbers, word formation, sitting arrangement in a row.

The number of permutations of 'n' things taken 'r' at a time is denoted by ${}^nP_r$. It is defind as, ${}^nP_r = \frac{n!}{(n-r)!}$.

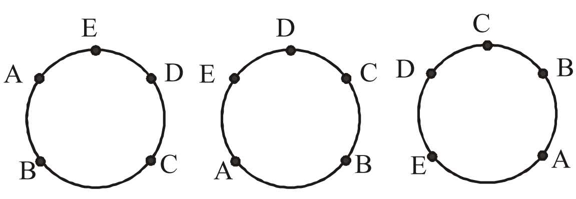
Note that:

${}^nP_n = n!$

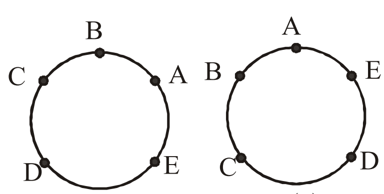
Circular permutations:

(i) Arrangements round a circular table :

Consider five persons A, B, C, D and E to be seated on the circumference of a circular table in order (which has no head). Now, shifting A, B, C, D and E one position in anticlockwise direction we will get arrangements as follows:



(i) (ii) (iii)



(iv) (v)

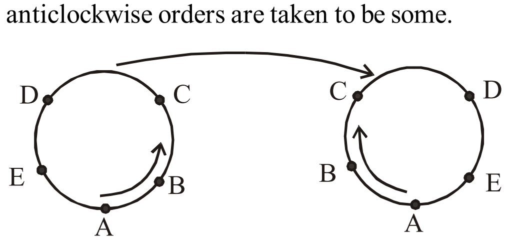
we see that arrangements in all figures are same.

$\therefore$ The number of circular permutations of n different things taken all at a time is $\frac{{}^nP_n}{n} = (n-1)!$, if clockwise and anticlockwise orders are taken as different.

(ii) Arrangements of beads or flowers (all different) around a circular necklace or garland:

Consider five beads A, B, C, D and E in a necklace or five flowers A, B, C and D, E in a garland etc. If the necklace or garland on the left is turned over we obtain the arrangement on the right, i.e., anticlockwise and clockwise order of arrangements are not different.

Thus the number of circular permutations of 'n' different things taken. all at a time is $\frac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken to be some.



EXAMPLE 6. Prove that ${}^nP_r = {}^{n-1}P_r + r.\,{}^{n-1}P_{r-1}$

Sol. ${}^{n-1}P_r + r.{}^{n-1}P_{r-1} = \frac{(n-1)!}{(n-1-r)!} + r\frac{(n-1)!}{(n-1-r+1)!}$

$$= (n-1)!\left\{\frac{1}{(n-1-r)!} + \frac{r}{(n-r)!}\right\}$$

$$= (n-1)!\left\{\frac{n-r+r}{(n-r)!}\right\} = \frac{n!}{(n-r)!} = {}^nP_r$$

EXAMPLE 7. Prove that ${}^nP_r = (n-r+1)\ {}^nP_{r-1}$

Sol. We have

$$(n-r+1)\ {}^nP_{r-1} = (n-r+1)\frac{n!}{(n-r+1)!}$$

$$= (n-r+1)\frac{n!}{(n-r+1)(n-r)!}$$

$$= \frac{n!}{(n-r)!} = {}^nP_r$$

EXAMPLE 8. The number of four digit numbers with distinct digits is :

(a) $9 \times {}^9C_3$ **(b)** $9 \times {}^9P_3$

(c) ${}^{10}C_3$ **(d)** ${}^{10}P_3$

Sol. (b) The thousandth place can be filled up in 9 ways with any one of the digits 1, 2, 3, ..., 9. After that the other three places can be filled up in ${}^9P_3$ ways, with any one of the remaining 9 digits including zero. Hence, the number of four digit numbers with distinct digits = $9 \times {}^9P_3$.

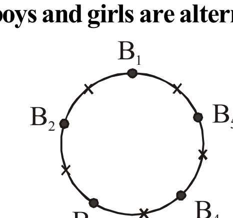
EXAMPLE 9. The number of ways in which 10 persons can sit round a circular table so that none of them has the same neighbours in any two arrangements.

Sol. 10 persons can sit round a circular table in 9! ways. But here clockwise and anticlockwise orders will give the same neighbours. Hence the required number of ways

$= \frac{1}{2}(10–1)! = \frac{1}{2}9!$

EXAMPLE 10. In how many different ways can five boys and five girls form a circle such that the boys and girls are alternate?

Sol. After fixing up one boy on the table the remaining can be arranged in 4! ways.



There will be 5 places, one place each between two boys which can be filled by 5 girls in 5! ways.

Hence by the principle of multiplication, the required number of ways = 4! × 5! = 2880.

EXAMPLE 11. In how many ways can 5 boys and 5 girls be seated at a round table no two girls may be together ?

Sol. Leaving one seat vacant between two boys may be seated in 4! ways. Then at remaining 5 seats, 5 girls any sit in 5! ways. Hence the required number = 4! × 5!

Conditional Permutations

1. **Number of permutations of n things taking r at a time, in which a particular thing always occurs =** $r.\,^{n-1}P_{r-1}$.

Distinguishable Permutations

Suppose a set of n objects has n_1 of one kind of object, n_2 of a second kind, n_3 of a third kind, and so on, with $n = n_1 + n_2 + n_3 + \ldots + n_k$, Then the number of distinguishable permutations of the n objects is $\dfrac{n!}{n_1!\,n_2!\,n_3!\ldots..n_k!}$

EXAMPLE 12. In how many distinguishable ways can the letters in BANANA be written?

Sol. This word has six letters, of which three are A's, two are N's, and one is a B. Thus, the number of distinguishable ways the letters can be written is

$$\frac{6!}{3!\,2!\,1!} = \frac{6\times5\times4\times3!}{3!\,2!} = 60$$

EXAMPLE 13. How many 4 digits number (repetition is not allowed) can be made by using digits 1-7 if 4 will always be there in the number?

Sol. Total digits (n) = 7

Total ways of making the number if 4 is always there $= r \times {}^{n-1}P_{r-1} = 4 \times {}^6P_3 = 480$.

2. **Number of permutations of n things taking r at a time, in which a particular thing never occurs =** ${}^{n-1}P_r$.

EXAMPLE 14. How many different 3 letter words can be made by 5 vowels, if vowel 'A' will never be included?

Sol. Total letters (n) = 5

So total number of ways $= {}^{n-1}P_r = {}^{5-1}P_3 = {}^4P_3 = 24$.

3. **Number of permutations of n different things taking all at a time, in which m specified things always come together** $= m!(n-m+1)!$.

4. **Number of permutations of n different things taking all at a time, in which m specified things never come together** $= n! - m!(n-m+1)!$

EXAMPLE 15. In how many ways can we arrange the five vowels, a, e, i, o & u if :

(i) two of the vowels e and i are always together.

(ii) two of the vowels e and i are never together.

Sol. (i) Using the formula $m!(n-m+1)!$

Here n = 5, m = 2(e & i)

$\Rightarrow$ Required no. of ways $= 2!(5-2+1)! = 2 \times 4! = 48$

Alternative :

As the two vowels e & i are always together we can consider them as one, which can be arranged among themselves in 2! ways.

Further the 4 vowels (after considering e & i as one) can be arranged in 4! ways.

Total no. of ways = 2! × 4! = 48

(ii) No. of ways when e & i are never together

= total no. of ways of arranging the 5 vowels

– no. of ways when e & i are together = 5! – 48 = 72

Or use $n! - m!(n-m+1)! = 5! - 48 = 72$

5. **The number of permutations of 'n' things taken all at a time, when 'p' are alike of one kind, 'q' are alike of second, 'r' alike of third, and so on** $= \dfrac{n!}{p!\,q!\,r!}$.

EXAMPLE 16. How many different words can be formed with the letters of the world MISSISSIPPI.

Sol. In the word MISSISSIPPI, there are 4 I's, 4S's and 2P's.

Thus required number of words $= \dfrac{(11)!}{4!\,2!\,4!} = 34650$

6. **The number of permutations of 'n' different things, taking 'r' at a time, when each thing can be repeated 'r' times =** n^r

EXAMPLE 17. In how many ways can 5 prizes be given away to 4 boys, when each boy is eligible for all the prizes?

Sol. Any one of the prizes can be given in 4 ways; then any one of the remaining 4 prizes can be given again in 4 ways, since it may even be obtained by the boy who has already received a prize. Hence 5 prizes can be given $4 \times 4 \times 4 \times 4 \times 4 = 4^5$ ways.

EXAMPLE 18. How many numbers of 3 digits can be formed with the digits 1, 2, 3, 4, 5 when digits may be repeated?

Sol. The unit place can be filled in 5 ways and since the repetitions of digits are allowed, therefore, tenth place can be filled in 5 ways.

Furthermore, the hundredth place can be filled in 5 ways also.

Therefore, required number of three digit numbers is 5 × 5 × 5 = 125.

EXAMPLE 19. In how many ways 8 persons can be arranged in a circle?

Sol. The eight persons can be arranged in a circle in (8 – 1)! = 7! = 5040.

EXAMPLE 20. Find the number of ways in which 18 different beads can be arranged to form a necklace.

Sol. 18 different beads can be arranged among themselves in a circular order in (18 – 1)! = 17! ways. Now in the case of necklace there is no distinct between clockwise and anticlockwise arrangements. So, the required number of arrangements $= \dfrac{1}{2}(17!) = \dfrac{17!}{2}$

☞ **Shortcut Approach**

Number of ways to declare the result where 'n' match are played = 2^n

EXAMPLE 21. In a cricket tournament 5 matches were played, then in how many ways result can be declared?

Sol. Total ways to declare the result = $2^n = 2^5 = 32$ ways

COMBINATION

Each of the different selections that can be made with a given number of objects taken some or all of them at a time is called a COMBINATION.

The number of combinations of 'n' dissimilar things taken 'r' at a time is denoted by ${}^nC_r$ or C(n, r). It is defined as,

$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

EXAMPLE 22. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, find r.

Sol. ${}^{15}C_{3r} = {}^{15}C_{r+3}$

$\Rightarrow$ Either $3r = r + 3$ or $3r + r + 3 = 15$

$\Rightarrow$ Either $r = \frac{3}{2}$ or $r = 3$

Since, r cannot be a fraction, so r = 3.

EXAMPLE 23. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the values of n and r are

(a) 4, 3 **(b)** 3, 2

(c) 4, 2 **(d)** None of these

Sol. (b) We have, ${}^nP_r = {}^nP_{r+1}$

$$ \Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!} \Rightarrow \frac{1}{(n-r)} = 1 $$

or $n - r = 1$ ...(1)

Also, ${}^nC_r = {}^nC_{r-1} \Rightarrow r + r - 1 = n \Rightarrow 2r - n = 1$ ...(2)

Solving (1) and (2), we get r = 2 and n = 3

EXAMPLE 24. Prove that

$$ {}^nC_{r-2} + 3.\,{}^nC_{r-1} + 3.\,{}^nC_r + {}^nC_{r+1} = {}^{n+2}C_{r+2} $$

Sol. ${}^nC_{r-2} + 3\,{}^nC_{r-1} + 3\,{}^nC_r + {}^nC_{r+1}$

$= {}^nC_{r-2} + {}^nC_{r-1} + 2({}^nC_{r-1} + {}^nC_r) + ({}^nC_r + {}^nC_{r+1})$

$= {}^{n+1}C_{r-1} + 2.\,{}^{n+1}C_r + {}^{n+1}C_{r+1}$

$= ({}^{n+1}C_{r-1} + {}^{n+1}C_r) + ({}^{n+1}C_r + {}^{n+1}C_{r+1})$

$= {}^{n+2}C_r + {}^{n+2}C_{r+1} = {}^{n+3}C_{r+1}$

EXAMPLE 25. If ${}^nP_r = 720\,{}^nC_r$, then r is equal to

(a) 3 **(b)** 7

(c) 6 **(d)** 4

Sol. (c) ${}^nP_r = 720{}^nC_r$

or $\frac{n!}{(n-r)!} = \frac{720(n!)}{(n-r)!r!}$

$\Rightarrow r! = 720 = 1 \times 2 \times 3 \times 4 \times 5 \times 6!$

or $r = 6$

EXAMPLE 26. In how many ways a hockey team of eleven can be elected from 16 players?

Sol. Total number of ways = ${}^{16}C_{11} = \frac{16!}{11! \times 5!} = 4368.$

$$ = \frac{16 \times 15 \times 14 \times 13 \times 12}{5 \times 4 \times 3 \times 2 \times 1} = 4368. $$

REMEMBER

★ ${}^nC_0 = 1,\ {}^nC_n = 1;\ {}^nP_r = r!\,{}^nC_r$

★ ${}^nC_r = {}^nC_{n-r}$

★ ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

★ ${}^nC_x = {}^nC_y \Rightarrow x + y = n$

★ ${}^nC_r = {}^nC_{r+1} = {}^{n+1}C_r$

★ ${}^nC_r = \frac{n}{r}\,.\,{}^{n-1}C_{r-1}$

★ ${}^nC_r = \frac{1}{r}(n-r+1)\,{}^nC_{r-1}$

★ ${}^nC_1 = {}^nC_{n-1} = n$

Conditional Combinations

1. **Number of combinations of n distinct things taking r** $(\le n)$ **at a time, when k** $(0 \le k \le r)$ **particular objects always occur** = ${}^{n-k}C_{r-k}$.

2. **Number of combinations of n distinct objects taking** $r(\le n)$ **at a time, when k** $(0 \le k \le r)$ **particular objects never occur** = ${}^{n-k}C_r$.

3. **Number of selections of r things from n things when p particular things are not together in any selection** = ${}^nC_r - {}^{n-p}C_{r-p}$

4. **Number of selection of r consecutive things out of n things in a row** = $n - r + 1$

5. **Number of selection of r consecutive things out of n things along a circle**

$$ = \begin{cases} n, \text{ when } & r < n \\ 1, \text{ when } & r = n \end{cases} $$

6. **The number of Combinations of 'n' different things taking some or all at a time**

$= {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots\ldots\ldots + {}^nC_n = 2^n - 1$

EXAMPLE 27. In a class of 25 students, find the total number of ways to select two representative,

(i) if a particular person will never be selected.

(ii) if a particular person is always there.

Sol. (i) Total students (n) = 25

A particular students will not be selected (p) = 1, So total number of ways = ${}^{25-1}C_2 = {}^{24}C_2 = 276$.

(ii) Using ${}^{n-p}C_{r-p}$ no. of ways = ${}^{25-1}C_{2-1} = {}^{24}C_1 = 24$.

NOTE : *If a person is always there then we have to select only 1 from the remaining 25 – 1 = 24*

Shortcut Approach

Let there are n persons in a hall. If every person shakes his hand with every other person only once, then total number of handshakes

$$= {}^nC_2 = \frac{n(n-1)}{2}$$

Note: *If in place of handshakes each person gives a gift to another person, then formula changes to = n (n – 1)*

EXAMPLE 28. In a party, every person shakes his hand with every other person only once. If total number of handshakes is 210, then find the number of persons.

Sol. Let number of persons be n. Then, according to the question,

${}^nC_2 = 210$

$$\Rightarrow \frac{n(n-1)}{2} = 210$$

$$\Rightarrow n(n-1) = 420 = 21 \times 20$$

$$\Rightarrow n = 21$$

EXAMPLE 29. There are 10 lamps in a hall. Each of them can be switched on independently. The number of ways in which the hall can be illuminated is

(a) 10^2 **(b)** 1023

(c) 2^{10} **(d)** 10 !

Sol. Since each bulb has two choices, either switched on or off, therefore required number = $2^{10} - 1 = 1023$.

7. **The number of ways of dividing 'm + n' things into two groups containing 'm' and 'n' things respectively**

$$= {}^{m+n}C_m \; {}^nC_n = \frac{(m+n)!}{m!n!}$$

8. **The number of ways of dividing 'm + n + p' things into three groups containing 'm', 'n' and 'p' things respectively**

$$= {}^{m+n+p}C_m \cdot {}^{n+p}C_p = \frac{(m+n+p)!}{m!\,n!\,p!}$$

(i) **If m = n = p i.e. '3m' things are divided into three equal groups then the number of combinations is**

$$\frac{(3m)!}{m!\,m!\,m!\,3!} = \frac{(3m)!}{(m!)^3\,3!}$$

(ii) Buf if '3m' things are to be divided among three persons, then the number of divisions is $\frac{(3m)!}{(m!)^3}$

9. **If mn distinct objects are to be divided into m groups. Then, the number of combination is**

$\frac{(mn)!}{m!\,(n!)^m}$, when the order of groups is not important and

$\frac{(mn)!}{(n!)^m}$, when the order of groups is important

EXAMPLE 30. The number of ways in which 52 cards can be divided into 4 sets, three of them having 17 cards each and the fourth one having just one card

(a) $\frac{52!}{(17!)^3}$ **(b)** $\frac{52!}{(17!)^3\,3!}$

(c) $\frac{51!}{(17!)^3}$ **(d)** $\frac{51!}{(17!)^3\,3!}$

Sol. Here we have to divide 52 cards into 4 sets, three of them having 17 cards each and the fourth one having just one card. First we divide 52 cards into two groups of 1 card and 51 cards. this can be done in $\frac{52!}{1!\,51!}$ ways.

Now every group of 51 cards can be divided into 3 groups of 17 each in $\frac{51!}{(17!)^3\,3!}$.

Hence the required number of ways

$$= \frac{52!}{1!\,51!} \cdot \frac{51!}{(17!)^3\,3!} = \frac{52!}{(17!)^3\,3!}$$

NUMBER OF RECTANGLES AND SQUARES

(a) Number of rectangles of any size in a square of size n × n is $\sum_{r=1}^{n} r^3$ and number of squares of any size is $\sum_{r=1}^{n} r^2$.

(b) Number of rectangles of any size in a rectangle size n × p (n < p) is $\frac{np}{4}(n+1)(p+1)$ and number of squares of any size is $\sum_{r=1}^{n}(n+1-r)(p+1-r)$.

EXAMPLE 31. The number of squares that can be formed on a chessboard is

(a) 64 **(b)** 160

(c) 224 **(d)** 204

Sol. (d) A chessboard is made up of 9 equispaced horizontal and vertical line. To make a 1 × 1 square, we must choose two consecutive horizontal and vertical lines from among these. This can be done in $8 \times 8 = 8^2$ ways. A 2×2 square needs three consecutive horizontal and vertical lines, and we can do this in $7 \times 7 = 7^2$ ways. Continuing in this manner, the total number of square is

$$8^2 + 7^2 + 6^2 + ... + 2^2 + 1^2 = \frac{8(8+1)[(2\times 8)+1]}{6} = 204.$$

Shortcut Approach

If there are n non-collinear points in a plane, then

(i) Number of straight lines formed = ${}^nC_2$

(ii) Number of triangles formed = ${}^nC_3$

(iii) Number of quadrilaterals formed = ${}^nC_4$

EXAMPLE 32. In a plane, there are 16 non-collinear points. Find the number of straight lines formed.

Sol. Here, n = 16

$\therefore$ Required number of straight lines formed = ${}^nC_2$

$$= {}^{16}C_2 = \frac{16!}{2!(16-2)!} = \frac{16\times 15\times 14!}{2\times 14!}$$

$= 8 \times 15 = 120$

Shortcut Approach

If there are n points in a plane out of which m are collinear, then

(i) Number of straight lines formed = ${}^nC_2 - {}^mC_2 + 1$

(ii) Number of triangles formed = ${}^nC_3 - {}^mC_3$

EXAMPLE 33. In a plane, there are 11 points, out of which 5 are collinear. Find the number of triangles made by these points.

Sol. Here, n = 11, m = 5

Then, required number of triangles = ${}^nC_3 - {}^mC_3 = {}^{11}C_3 - {}^5C_3$

$$= \frac{11\times 10\times 9}{3\times 2\times 1} - \frac{5\times 4\times 3}{3\times 2\times 1}$$

$= 165 - 10 = 155$

Shortcut Approach

Number of diagonals in a polygen of n sides = ${}^nC_2 - n$

EXAMPLE 34. How many diagonals will be there in an 5 - sided regular polygon?

Sol. The number of diagonals = ${}^5C_2 - 5 = \frac{5\times 4}{2\times 1} - 5 = 5$

EXERCISE

1. In how many different ways can be letters of the word SOFTWARE be arranged in such a way that the vowels always come together?
   (a) 13440 (b) 1440
   (c) 360 (d) 120
   (e) None of these
2. In how many different ways can a group of 4 men and 4 women be formed out of 7 men and 8 women?
   (a) 2450 (b) 105
   (c) 1170 (d) Cannot be determined
   (e) None of these
3. A bag contains 2 red, 3 green and 2 blue balls. 2 balls are to be drawn randomly. What is the probability that the balls drawn contain no blue ball?
   (a) $\frac{5}{7}$ (b) $\frac{10}{21}$
   (c) $\frac{2}{7}$ (d) $\frac{11}{21}$
   (e) None of these
4. In how many different ways can the letters of the word BOOKLET be arranged such that B and T always come together?
   (a) 360 (b) 720
   (c) 480 (d) 5040
   (e) None of these
5. In a box there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?
   (a) $\frac{7}{19}$ (b) $\frac{2}{3}$
   (c) $\frac{3}{4}$ (d) $\frac{9}{21}$
   (e) None of these
6. In how many different ways can the letters of the word RUMOUR be arranged?
   (a) 180 (b) 720
   (c) 30 (d) 90
   (e) None of these
7. 765 chairs are to be arranged in a column in such a way that the number of chairs in each column should be equal to the columns. How many chairs will be excluded to make this arrangement possible?
   (a) 6 (b) 36
   (c) 19 (d) 27
   (e) None of these
8. In how many different ways can the letters of the word JUDGE be arranged so that the vowels always come together?
   (a) 48 (b) 24
   (c) 120 (d) 60
   (e) None of these
9. How many words can be formed from the letters of the word SIGNATURE so that the vowels always come together?
   (a) 720 (b) 1440
   (c) 3600 (d) 2880
   (e) None of these
10. In how many ways a committee consisting of 5 men and 6 women can be formed from 8 men and 10 women?
    (a) 266 (b) 86400
    (c) 11760 (d) 5040
    (e) None of these
11. Out of 15 students studying in a class, 7 are from Maharashtra, 5 are from Karnataka and 3 are from Goa. Four students are to be selected at random. What are the chances that at least one is from Karnataka?
    (a) $\frac{12}{13}$ (b) $\frac{11}{13}$
    (c) $\frac{10}{15}$ (d) $\frac{1}{15}$
    (e) None of these
12. 4 boys and 2 girls are to be seated in a row in such a way that the two girls are always together. In how many different ways can they be seated?
    (a) 120 (b) 720
    (c) 148 (d) 240
    (e) None of these
13. In how many different ways can the letters of the word DETAIL be arranged in such a way that the vowels occupy only the odd positions?
    (a) 120 (b) 60
    (c) 48 (d) 32
    (e) None of these
14. In a box carrying one dozen of oranges, one-third have become bad. If 3 oranges are taken out from the box at random, what is the probability that at least one orange out of the three oranges picked up is good?
    (a) $\frac{1}{55}$ (b) $\frac{54}{55}$
    (c) $\frac{45}{55}$ (d) $\frac{3}{55}$
    (e) None of these
15. Letters of the word DIRECTOR are arranged in such a way that all the vowels come together. Find out the total number of ways for making such arrangement.
    (a) 4320 (b) 2720
    (c) 2160 (d) 1120
    (e) None of these

16. A box contains 5 green, 4 yellow and 3 white marbles, 3 marbles are drawn at random. What is the probability that they are not of the same colour?
(a) $\frac{13}{44}$ (b) $\frac{41}{44}$
(c) $\frac{13}{55}$ (d) $\frac{52}{55}$
(e) None of these

17. How many different letter arrangements can be made from the letters of the word RECOVER?
(a) 1210 (b) 5040
(c) 1260 (d) 1200
(e) None of these

18. How many three digit numbers can having only two consecutive digits identical is
(a) 153 (b) 162
(c) 168 (d) 163
(e) None of these

19. How many total numbers of seven-digit numbers can be formed having sum of whose digits is even is
(a) 9000000 (b) 4500000
(c) 8100000 (d) 4400000
(e) None of these

20. How many total numbers of not more than 20 digits that can be formed by using the digits 0, 1, 2, 3, and 4 is
(a) 5^{20} (b) $5^{20}-1$
(c) $5^{20}+1$ (d) 6^{20}
(e) None of these

21. The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 so that digits do not repeat and the terminal digits are even is
(a) 144 (b) 72
(c) 288 (d) 720
(e) None of these

22. Three dice are rolled. The number of possible outcomes in which at least one dice shows 5 is
(a) 215 (b) 36
(c) 125 (d) 91
(e) None of these

23. The number of ways in which ten candidates $A_1, A_2, ..., A_{10}$ can be ranked so that A_1 is always above A_2 is
(a) $\frac{10!}{2}$ (b) $10!$
(c) $9!$ (d) $\frac{8!}{2}$
(e) None of these

24. How many total number of ways in which n distinct objects can be put into two different boxes is
(a) n^2 (b) 2^n
(c) $2n$ (d) 3^n
(e) None of these

25. In how many ways can the letters of the word 'PRAISE' be arranged. So that vowels do not come together?
(a) 720 (b) 576
(c) 440 (d) 144
(e) None of these

26. There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2; task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many ways can the assignment be done?
(a) 144 (b) 180
(c) 192 (d) 360
(e) None of these

27. The number of ways in which one or more balls can be selected out of 10 white, 9 green and 7 blue balls is
(a) 892 (b) 881
(c) 891 (d) 879
(e) None of these

28. If 12 persons are seated in a row, the number of ways of selecting 3 persons from them, so that no two of them are seated next to each other is
(a) 85 (b) 100
(c) 120 (d) 240
(e) None of these

29. The number of all possible selections of one or more questions from 10 given questions, each question having one alternative is
(a) 3^{10} (b) $2^{10}-1$
(c) $3^{10}-1$ (d) 2^{10}
(e) None of these

30. A lady gives a dinner party to 5 guests to be selected from nine friends. The number of ways of forming the party of 5, given that two of the friends will not attend the party together is
(a) 56 (b) 126
(c) 91 (d) 94
(e) None of these

31. All possible two factors products are formed from the numbers 1, 2, 3, 4, ..., 200. The number of factors out of total obtained which are multiples of 5 is
(a) 5040 (b) 7180
(c) 8150 (d) 7280
(e) None of these

Directions (Qs. 32-33): Answer these questions on the basis of the information given below:

From a group of 6 men and 4 women a committee of 4 persons is to be formed.

32. In how many different ways can it be done so that the committee has at least one woman?
(a) 210 (b) 225
(c) 195 (d) 185
(e) None of these

33. In how many different ways can it be done so that the committee has at least 2 men?
(a) 210 (b) 225
(c) 195 (d) 185
(e) None of these

34. In how many different ways can the letters of the word ORGANISE be arranged in such a way that all the vowels always come together and all the consonants always come together?
(a) 576 (b) 1152
(c) 2880 (d) 1440
(e) None of these

CHAPTER

13 Probability

INTRODUCTION

Random Experiment :

It is an experiment which if conducted repeatedly under homogeneous condition does not give the same result.

The total number of possible outcomes of an experiment in any trial is known as the **exhaustive number** of events.

For example

(i) In throwing a die, the exhaustive number of cases is 6 since any one of the six faces marked with 1, 2, 3, 4, 5, 6 may come uppermost.

(ii) In tossing a coin, the exhaustive number of cases is 2, since either head or tail may turn over.

(iii) If a pair of dice is thrown, then the exhaustive number of cases is $6 \times 6 = 36$

(iv) In drawing four cards from a well-shuffled pack of cards, the exhaustive number of cases is ${}^{52}C_4$.

Events are said to be **mutually exclusive** if no two or more of them can occur simultaneously in the same trial.

For example,

(i) In tossing of a coin the events head (H) and tail (T) are mutually exclusive.

(ii) In throwing of a die all the six faces are mutually exclusive.

(iii) In throwing of two dice, the events of the face marked 5 appearing on one die and face 5 (or other) appearing on the other are not mutually exclusive.

Outcomes of a trial are **equally likely** if there is no reason for an event to occur in preference to any other event or if the chances of their happening are equal.

For example,

(i) In throwing of an unbiased die, all the six faces are equally likely to occur.

(ii) In drawing a card from a well-shuffled pack of 52 cards, there are 52 equally likely possible outcomes.

The **favourable cases** to an event are the outcomes, which entail the happening of an event.

For example,

(i) In the tossing of a die, the number of cases which are favourable to the " appearance of a multiple of 3" is 2, viz, 3 and 6.

(ii) In drawing two cards from a pack, the number of cases favourable to "drawing 2 aces" is ${}^{4}C_2$.

(iii) In throwing of two dice, the number of cases favourable to "getting 8 as the sum" is 5, : (2, 6), (6, 2), (4, 4), (3, 5) (5, 3).

Events are said to be **independent if the happening** (or non-happening) of one event is not affected by the happening or non-happening of others.

CLASSICAL DEFINITION OF PROBABILITY

If there are n-mutually exclusive, exhaustive and equally likely outcomes to a random experiment and 'm' of them are favourable to an event A, then the probability of happening of A is denoted by P (A) and is defined by $P(A) = \frac{m}{n}$.

$$P(A) = \frac{\text{No. of elementary events favourable to A}}{\text{Total no. of equally likely elementary events}}$$

Obviously, $0 \le m \le n$, therefore $0 \le \frac{m}{n} \le 1$ so that $0 \le P(A) \le 1$.

P(A) can never be negative.

Since, the number of cases in which the event A will not happen is 'n – m', then the probability P ($\overline{A}$) of not happening of A is given by

$$P(\overline{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$$\Rightarrow \boxed{P(A) + P(\overline{A}) = 1}$$

The **ODDS IN FAVOUR** of occurrence of A are given by $m : (n-m)$ or $P(A) : P(\overline{A})$

The **ODDS AGAINST** the occurrence of A are given by $(n-m) : m$ or $P(\overline{A}) : P(A)$.

EXAMPLE 1. Two dice are thrown simultaneously. The probability of obtaining a total score of seven is

(a) $\frac{1}{6}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{2}{7}$ **(d)** $\frac{5}{6}$

(e) None of these

Sol.

(a) When two are thrown then there are 6×6 exhaustive cases $\therefore n = 36$. Let A denote the event "total score of 7" when 2 dice are thrown then A = [(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)].

Thus there are 6 favourable cases.

$\therefore m = 6$ By definition $P(A) = \frac{m}{n}$

$\therefore P(A) = \frac{6}{36} = \frac{1}{6}$.

EXAMPLE 2. A bag contains 5 green and 7 red balls. Two balls are drawn. The probability that one is green and the other is red is

(a) $\frac{5}{132}$ **(b)** $\frac{7}{132}$ **(c)** $\frac{35}{66}$ **(d)** $\frac{31}{66}$

(e) None of these

Sol.

(c) There are 5 + 7 = 12 balls in the bag and out of these two balls can be drawn in ${}^{12}C_2$ ways. There are 5 green balls, therefore, one green ball can be drawn in ${}^5C_1$ ways; similarly, one red ball can be drawn in ${}^7C_1$ ways so that the number of ways in which we can draw one green ball and the other red is ${}^5C_1 \times {}^7C_1$.
Hence, P (one green and the other red)

$$= \frac{{}^5C_1 \times {}^7C_1}{{}^{12}C_2} = \frac{5}{1} \times \frac{7}{1} \times \frac{1.2}{12.11} = \frac{35}{66}.$$

EXAMPLE 3. A bag contains 5 white and 7 black balls and a man draws 4 balls at random. The odds against these being all black is :

(a) 7 : 92 (b) 92 : 7 (c) 92 : 99 (d) 99 : 92

(e) None of these

Sol.

(b) There are 7 + 5 = 12 balls in the bag and the number of ways in which 4 balls can be drawn is ${}^{12}C_4$ and the number of ways of drawing 4 black balls (out of seven) is ${}^7C_4$.
Hence, P (4 black balls)

$$= \frac{{}^7C_4}{{}^{12}C_4} = \frac{7.6.5.4}{1.2.3.4} \times \frac{1.2.3.4}{12.11.10.9} = \frac{7}{99}$$

Thus the odds against the event 'all black balls' are

$$(1 - \frac{7}{99}) : \frac{7}{99} : \text{i.e., } \frac{92}{99} : \frac{7}{99} \text{ or } 92 : 7.$$

EXAMPLE 4. The letters of the word SOCIETY are placed at random in a row. The probability that the three vowels come together is

(a) $\frac{1}{6}$ **(b)** $\frac{1}{7}$ **(c)** $\frac{2}{7}$ **(d)** $\frac{5}{6}$

(e) None of these

Sol.

(b) The word 'SOCIETY' contains seven distinct letters and they can be arranged at random in a row in ${}^7P_7$ ways, i.e. in 7! = 5040 ways.
Let us now consider those arrangements in which all the three vowels come together. So in this case we have to arrange four letters. S,C,T,Y and a pack of three vowels in a row which can be done in ${}^5P_5$ i.e. 5! = 120 ways.
Also, the three vowels in their pack can be arranged in ${}^3P_3$ i.e. 3! = 6 ways.
Hence, the number of arrangements in which the three vowels come together is 120 × 6 = 720
∴ The probability that the vowels come together

$$= \frac{720}{5040} = \frac{1}{7}$$

EXAMPLE 5. There are three events E_1, E_2 and E_3. one of which must, and only one can happen. The odds are 7 to 4 against E_1 and 5 to 3 against E_2. The odds against E_3 is

(a) 4 : 11 (b) 3 : 8 (c) 23 : 88 (d) 65 : 23

(e) None of these

Sol.

(d) Since, one and only one of the three events E_1, E_2 and E_3 can happen, therefore $P(E_1) + P(E_2) + P(E_3) = 1$ .....(1)
∵ Odds against E_1 are 7 : 4

$$\Rightarrow P(E_1) = \frac{4}{4+7} = \frac{4}{11} \quad .....(2)$$

∵ Odds against E_2 are 5 : 3

$$\Rightarrow P(E_2) = \frac{3}{3+5} = \frac{3}{8} \quad ......(3)$$

From (1), (2) and (3), we have, $\frac{4}{11} + \frac{3}{8} + P(E_3) = 1.$

i.e. $P(E_3) = 1 - \frac{4}{11} - \frac{3}{8} = \frac{88-32-33}{88} = \frac{23}{88} = \frac{23}{23+65}$

Hence odds against E_3 is 65 : 23.

ALGEBRA OF EVENTS

Let A and B be two events related to a random experiment. We define

(i) The event "A or B" denoted by "$A \cup B$", which occurs when A or B or both occur. Thus,
$P(A \cup B) =$ Probability that at least one of the events occur

(ii) The event "A and B", denoted by "$A \cap B$", which occurs when A and B both occur. Thus,
$P(A \cap B) =$ Probability of simultaneous occurrence of A and B.

(iii) The event " Not - A" denoted by $\overline{A}$, which occurs when and only when A does not occur. Thus
$P(\overline{A})$ = Probability of non-occurrence of the event A.

(iv) $\overline{A} \cap \overline{B}$ denotes the " non-occurrence of both A and B".

(v) "$A \subset B$" denotes the " occurrence of A implies the occurrence of B".

For example :
Consider a single throw of die and following two events
A = the number is even = {2, 4, 6}
B = the number is a multiple of 3 = {3, 6}

Then $P(A \cup B) = \frac{4}{6} = \frac{2}{3}$, $P(A \cap B) = \frac{1}{6}$

$$P(\overline{A}) = \frac{1}{2},\ P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - \frac{2}{3} = \frac{1}{3}.$$

ADDITION THEOREM ON PROBABILITY

1. ADDITION THEOREM : If A and B are two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. ADDITION THEOREM FOR THREE EVENTS : If A, B, C are three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$$
$$- P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

3. If A and B are **two mutually exclusive events** and the probability of their occurence are P(A) and P(B) respectively, then probability of either A or B occuring is given by
$P(A \text{ or } B) = P(A) + P(B)$
$\Rightarrow P(A + B) = P(A) + P(B)$

EXAMPLE 6. A and B are two events odds against A are 2 to 1. odds in favour of $A \cup B$ are 3 to 1. If $x \le P(B) \le y$. then the ordered pair (x, y) is :

(a) $\left(\frac{5}{12}, \frac{3}{4}\right)$ **(b)** $\left(\frac{2}{3}, \frac{3}{4}\right)$ **(c)** $\left(\frac{1}{3}, \frac{3}{4}\right)$ **(d)** $\left(\frac{1}{2}, \frac{3}{7}\right)$

(e) None of these

Sol.

(a) $P(A) = \frac{1}{3}; \ P(A \cup B) = \frac{3}{4}$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{3}{4}$$

$$\Rightarrow \frac{1}{3} + P(B) - P(A \cap B) = \frac{3}{4}$$

$$\Rightarrow P(B) = \frac{5}{12} + P(A \cap B) \ge \frac{5}{12}$$

$$\text{Also, } P(B) = \frac{5}{12} + P(A \cap B) \le \frac{5}{12} + \frac{1}{3} = \frac{3}{4}$$

$$\left[\because P(A \cap B) \le P(A) = \frac{1}{3}\right]$$

Hence, (x, y) is $\left(\frac{5}{12}, \frac{3}{4}\right)$.

EXAMPLE 7. Two cards are drawn from a pack of 52 cards. The probability that either both are red or both are kings is

(a) $\frac{1}{2}$ **(b)** $\frac{1}{321}$ **(c)** $\frac{325}{1326}$ **(d)** $\frac{1}{327}$

(e) None of these

Sol.

(d) 2 cards can be drawn from the pack in ${}^{52}C_2$ ways. Let A be the event " Two cards are red" and B be the event "Two cards drawn are kings".
The required probability is $P(A \cup B)$.
From addition theorem, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B). \quad ...(1)$$

Now, P (A) = Probability of drawing two red cards

$$= \frac{{}^{26}C_2}{{}^{52}C_2} \quad [\because \text{ There are total 26 red cards}]$$

P (B) = Probability of drawing two king cards

$$= \frac{{}^{4}C_2}{{}^{52}C_2} \quad [\because \text{ There are 4 king cards}]$$

$P(A \cap B)$ = Probability of drawing 2 red king cards

$$= \frac{{}^{2}C_2}{{}^{52}C_2} \quad [\because \text{ There are just 2 red kings}]$$

Substituting the values in (1), we get

$$P(A \cup B) = \frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^{4}C_2}{{}^{52}C_2} - \frac{{}^{2}C_2}{{}^{52}C_2} = \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}$$

$$= \frac{55}{221}.$$

EXAMPLE 8. If A and B are two events, the probability that at most one of these events occurs is :

(a) $P(A') + P(B') - P(A' \cap B')$

(b) $P(A') + P(B') + P(A \cup B) - 1$

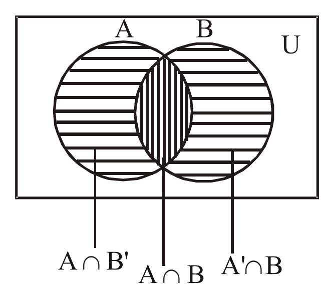
(c) $P(A \cap B') + P(A' \cap B) + P(A' \cap B')$

(d) All above are correct.

(e) None of these

Sol.

(d)



At most one of two events occurs if the event $A' \cup B'$ occurs.

$$P(A' \cup B') = 1 - P(A \cap B)$$

Now, $\quad P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$

$$= P(A') + P(B') - [1 - P(A \cup B)]$$

$$= P(A') + P(B') + P(A \cup B) - 1.$$

Finally, since

$$P(A' \cup B') = P[(A')' \cap B'] + P[A' \cap (B')'] + P(A' \cap B')$$

$$= P(A \cap B') + P(A' \cap B) + P(A' \cap B')$$

$$[\because P(A \cup B) = P(A' \cap B) + P(A \cap B') + P(A \cap B)]$$

[See the Venn diagram].

CONDITIONAL PROBABILITY

Let A and B be two events associated with a random experiment.

Then $P\left(\frac{A}{B}\right)$, represents the conditional probability of occurrence of A relative to B.

Also, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ and $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

For example :

Suppose a bag contains 5 white and 4 red balls. Two balls are drawn one after the other without replacement. If A denotes the event "drawing a white ball in the first draw" and B denotes the event "drawing a red ball in the second draw".

P (B/A) = Probability of drawing a red ball in second draw when it is known that a white ball has already been drawn in the first draw $= \frac{4}{8} = \frac{1}{2}$

Obviously, P (A/B) is meaning less in this problem.

MULTIPLICATION THEOREM

If A and B are two events, then

$$P(A \cap B) = P(A)\,P(B/A), \text{ if } P(A) > 0$$
$$= P(B)\,P(A/B) \text{ if } P(B) > 0$$

From this theorem we get

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \text{ and } P(A/B) = \frac{P(A \cap B)}{P(B)}$$

For example :

Consider an experiment of throwing a pair of dice. Let A denotes the event " the sum of the point is 8" and B event " there is an even number on first die"

Then A = {(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)},
B = {(2, 1), (2, 2), ..., (2, 6), (4, 1), (4, 2),..... (4, 6), (6, 1), (6, 2),....(6, 6)}

$$P(A) = \frac{5}{36},\ P(B) = \frac{18}{36} = \frac{1}{2},\ P(A \cap B) = \frac{3}{36} = \frac{1}{12}$$

Now, P(A/B) = Prob. of occurrence of A when B has already occurred = prob. of getting 8 as the sum, when there is an even number on the first die

$$= \frac{3}{18} = \frac{1}{6} \text{ and similarly } P(B/A) = \frac{3}{5}.$$

INDEPENDENCE

An event B is said to be independent of an event A if the probability that B occurs is not influenced by whether A has or has not occurred. For two independent events A and B.

$$P(A \cap B) = P(A)\,P(B)$$

Event $A_1, A_2, \ldots\ldots A_n$ are independent if

(i) $P(A_i \cap A_j) = P(A_i)P(A_j)$ for all i, j, $i \neq j$, That is, the events are pairwise independent.

(ii) The probability of simultaneous occurrence of (any) finite number of them is equal to the product of their separate probabilities, that is, they are mutually independent.

For example :

Let a pair of fair coin be tossed, here S = {HH, HT, TH, TT}

A = heads on the first coin = {HH, HT}

B = heads on the second coin = {TH, HH}

C = heads on exactly one coin = {HT, TH}

Then P (A) = P(B) = P (C) = $\frac{2}{4} = \frac{1}{2}$ and

$$P(A \cap B) = P(\{HH\}) = \frac{1}{4} = P(A)P(B)$$

$$P(B \cap C) = P(\{TH\}) = \frac{1}{4} = P(B)P(C)$$

$$P(A \cap C) = P(\{HT\}) = \frac{1}{4} = P(A)P(C)$$

Hence the events are pairwise independent.

Also $P(A \cap B \cap C) = P(\phi) = 0 \neq P(A)P(B)P(C)$

Hence, the events A, B, C are not mutually independent.

EXAMPLE **9. The odds against P solving a problem are 8 : 6 and odds in favour of Q solving the same problem are 14 : 10 The probability of the problem being solved, if both of them try it, is**

(a) $\frac{5}{21}$ **(b)** $\frac{16}{21}$ **(c)** $\frac{5}{12}$ **(d)** $\frac{5}{7}$

(e) None of these

Sol.

(b) The odd against P solving a problem = 8 : 6.

$\therefore$ Probability of P not solving the problem $= \frac{8}{14} = \frac{4}{7}$

The odds in favour of Q solving problem = 14 : 10

$\therefore$ Probability of Q not solving the problem $= \frac{10}{24} = \frac{5}{12}$

Hence, the probability of P and Q not solving the problem

$$= \frac{4}{7} \times \frac{5}{12} = \frac{5}{21}$$

$\therefore$ Probability of the problem being solved

= 1 – probability of the problem not being solved

$$= 1 - \frac{5}{21} = \frac{16}{21}.$$

EXAMPLE **10. A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. The probability of occurrence of A is.**

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{5}{6}$ **(d)** $\frac{1}{6}$

(e) None of these

Sol.

(a) Let P (A) = a and P(B) = b Then $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A)P(B) = \frac{1}{6}$, because A and B are independent.

$\therefore ab = \frac{1}{6}$ .....(i)

Also $P(\overline{A} \cap \overline{B}) = [1 - P(A)][1 - P(B)];$.

$$\therefore [1-a][1-b] = \frac{1}{3} \Rightarrow 1 - a - b + ab = \frac{1}{3} \quad .....(ii)$$

From (i) and (ii) we have $a + b = \frac{5}{6}$ .....(iii)

Solving (i) and (iii) we get, $a = \frac{1}{2},\ b = \frac{1}{3},\ \therefore P(A) = \frac{1}{2}$.

EXAMPLE 11. In each of a set of games it is 2 to 1 in favour of the winner of the previous game. The chance that the player who wins the first game shall win three at least of the next four is

(a) $\frac{8}{27}$ **(b)** $\frac{4}{81}$ **(c)** $\frac{4}{9}$ **(d)** $\frac{2}{3}$

(e) None of these

Sol.

(c) Let W stand for the winning of a game and L for losing it. Then there are 4 mutually exclusive possibilities
(i) W, W, W (ii) W, W, L, W
(iii) W, L, W, W (iv) L, W, W, W.
[Note that case (i) includes both the cases whether he losses or wins the fourth game.]
By the given conditions of the question, the probabilities for (i), (ii), (iii) and (iv) respectively are

$$\frac{2}{3}.\frac{2}{3}.\frac{2}{3};\ \frac{2}{3}.\frac{2}{3}.\frac{1}{3}.\frac{1}{3};\ \frac{2}{3}.\frac{1}{3}.\frac{1}{3}.\frac{2}{3} \text{ and } \frac{1}{3}.\frac{1}{3}.\frac{2}{3}.\frac{2}{3}.$$

Hence the required probability

$$= \frac{8}{27} + \frac{4}{81} + \frac{4}{81} + \frac{4}{81} = \frac{36}{81} = \frac{4}{9}.$$

[$\because$ The probability of winning the game if previous game was also won is $\frac{2}{1+2} = \frac{2}{3}$ and the probability of winning the game if previous game was a loss is $\frac{1}{1+2} = \frac{1}{3}$].

EXAMPLE 12. Three numbers are selected at random without replacement from the set of numbers {1, 2, ..., N}. The conditional probability that the third number lies between the first two, if the first number is known to be smaller than the second, is

(a) 1/6 (b) 1/3 (c) 1/2 (d) 3/4

(e) None of these

Sol.

(b) The number of ways of choosing three numbers out of N is ${}^NC_3$. If these numbers are a_1, a_2 and a_3, they must satisfy exactly one of the following inequalities for a successful outcome.

$$a_1 < a_2 < a_3 \qquad a_1 < a_3 < a_2, \qquad a_2 < a_1 < a_3,$$

$$a_2 < a_3 < a_1, \qquad a_3 < a_1 < a_2, \qquad a_3 < a_2 < a_1.$$

Thus the number of ways of arranging the three numbers in a given order is $({}^NC_3)$ (6), and there are 3 ways in which the first number is less than the second. Now if A denotes the event : the first number is less than the second number, and B the event : the third number lies between the first and the second, we need to find P(B|A). Since

$$P(B \cap A) = \frac{{}^NC_3}{({}^NC_3)(6)} = \frac{1}{6} \text{ and } P(A) = \frac{({}^NC_3)(3)}{({}^NC_3)(6)} = \frac{1}{2},$$

We get $P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{1/6}{1/2} = \frac{1}{3}$.

EXAMPLE 13. Given two bags A and B as follows : Bag A contains 3 red and 2 white balls and bag B contains 2 red and 5 white balls. A bag is selected at random, a ball is drawn and put into the other bag, then a ball is drawn from the second bag. The probability that both balls drawn are of the same colour is

(a) $\frac{187}{1680}$ **(b)** $\frac{901}{1680}$ **(c)** $\frac{439}{1680}$ **(d)** $\frac{437}{1679}$

(e) None of these

Sol.

(b) The whole event consists of the following mutually exclusive ways.

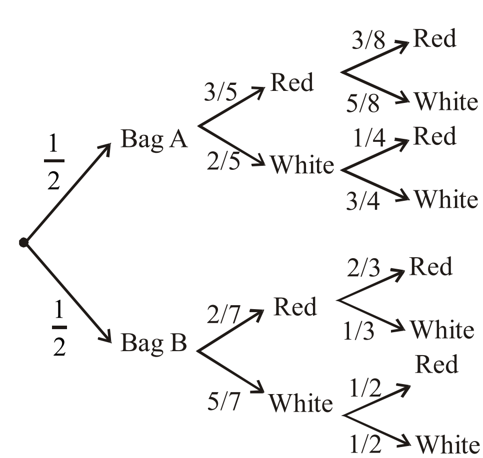
(1) Selecting the bag A, drawing a red ball from A and putting it into bag B and then drawing a red ball from B.

(2) Selecting the bag A, drawing a white ball from A and putting it into bag B and then drawing a white ball from B.

(3) Selecting the bag B, drawing a red ball from B and putting it into A and then drawing a red ball from A.

(4) Selecting the bag B, drawing a white ball from B and putting it into A and then drawing a white ball from A.

The tree diagram of the above processes are shown below, with respective probability of each step



The required probability is

$$= \frac{1}{2} \times \frac{3}{5} \times \frac{3}{8} + \frac{1}{2} \times \frac{2}{5} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{7} \times \frac{2}{3} + \frac{1}{2} \times \frac{5}{7} \times \frac{1}{2}$$

$$= \frac{9}{80} + \frac{3}{20} + \frac{2}{21} + \frac{5}{28} = \frac{901}{1680}$$

EXERCISE

1. In a given race the odds in favour of three horses A, B, C are 1 : 3; 1 : 4; 1 : 5 respectively. Assuming that dead head is impossible the probability that one of them wins is

(a) $\frac{7}{60}$ (b) $\frac{37}{60}$

(c) $\frac{1}{5}$ (d) $\frac{1}{8}$

(e) None of these

2. A man and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. The probability that only one of them will be selected is

(a) $\frac{6}{7}$ (b) $\frac{4}{35}$

(c) $\frac{6}{35}$ (d) $\frac{2}{7}$

(e) None of these

3. The probability that the 13^{th} day of a randomly chosen month is a Friday, is

(a) $\frac{1}{12}$ (b) $\frac{1}{7}$

(c) $\frac{1}{84}$ (d) $\frac{1}{13}$

(e) None of these

4. If a leap year selected at random, the chance that it will contain 53 Sunday is

(a) $\frac{3}{7}$ (b) $\frac{1}{7}$

(c) $\frac{2}{7}$ (d) $\frac{4}{7}$

(e) None of these

5. A Positive integer N is selected such that $100 < N < 200$. The probability that it is divisible by either 4 or 7 is :

(a) $\frac{38}{99}$ (b) $\frac{24}{99}$

(c) $\frac{34}{99}$ (d) $\frac{14}{99}$

(e) None of these

6. In a single throw with four dice, the probability of throwing seven is

(a) $\frac{4}{6^4}$ (b) $\frac{8}{6^4}$

(c) $\frac{16}{6^4}$ (d) $\frac{20}{6^4}$

(e) None of these

7. If A and B are two independent events with P(A) = 0.6, P(B) = 0.3, then $P(A' \cap B')$ is equal to :

(a) 0.18 (b) 0.28

(c) 0.82 (d) 0.72

(e) None of these

8. If three vertices of a regular hexagon are chosen at random, then the chance that they form an equilateral triangle is :

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$

(c) $\frac{1}{10}$ (d) $\frac{1}{2}$

(e) None of these

9. Six dice are thrown. The probability that different number will turn up is :

(a) $\frac{129}{1296}$ (b) $\frac{1}{54}$

(c) $\frac{5}{324}$ (d) $\frac{5}{54}$

(e) None of these

10. Four balls are drawn at random from a bag containing 5 white, 4 green and 3 black balls. The probability that exactly two of them are white is :

(a) $\frac{14}{33}$ (b) $\frac{7}{16}$

(c) $\frac{18}{33}$ (d) $\frac{9}{16}$

(e) None of these

11. The probability that a person will hit a target in shooting practice is 0.3. If he shoots 10 times, the probability that he hits the target is

(a) 1 (b) $1-(0.7)^{10}$

(c) $(0.7)^{10}$ (d) $(0.3)^{10}$

(e) None of these

12. The probability that at least one of the events A and B occurs is 0.7 and they occur simultaneously with probability 0.2. Then $P(\overline{A})+P(\overline{B}) =$

(a) 1.8 (b) 0.6

(c) 1.1 (d) 0.4

(e) None of these

13. The probability that A can solve a problem is $\frac{2}{3}$ and B can solve it is $\frac{3}{4}$. If both attempt the problem, what is the probability that the problem gets solved?

(a) $\frac{11}{12}$ (b) $\frac{7}{12}$

(c) $\frac{5}{12}$ (d) $\frac{9}{12}$

(e) None of these

14. Three integers are chosen at random from the first 20 integers. The probability that their product is even, is

(a) $\frac{2}{19}$ (b) $\frac{3}{29}$

(c) $\frac{17}{19}$ (d) $\frac{4}{29}$

(e) None of these

15. A die is loaded in such a way that each odd number is twice al likely to occur as each even number. If E is the event of a number greater than or equal to 4 on a single toss of the die, then P(E) is :

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

(e) None of these

16. The probability that two integers chosen at random and their product will have the same last digit is :

(a) $\frac{3}{10}$ (b) $\frac{1}{25}$

(c) $\frac{4}{15}$ (d) $\frac{7}{15}$

(e) None of these

17. Seven people seat themselves indiscriminately at round table. The probability that two distinguished persons will be next to each other is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

(e) None of these

18. Two dice are thrown. The probability that the sum of the numbers coming up on them is 9, if it is known that the number 5 always occurs on the first die, is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

(e) None of these

19. A speaks the truth in 70 percent cases and B in 80 percent. The probability that they will contradict eact. other when describing a single event is

(a) 0.36 (b) 0.38

(c) 0.4 (d) 0.42

(e) None of these

20. One ticket is selected at random from 100 tickets numbered 00, 01, 02,.....99. Suppose S and T are the sum and product of the digits of the number on the ticket, then $P(S = 9 / T = 0)$ is

(a) $\frac{19}{100}$ (b) $\frac{1}{4}$

(c) $\frac{2}{19}$ (d) $\frac{1}{50}$

(e) None of these

21. A die is loaded such that the probability of throwing the number i is proportional to its reciprocal. The probability that 3 appears in a single throw is :

(a) $\frac{3}{22}$ (b) $\frac{3}{11}$

(c) $\frac{9}{22}$ (d) $\frac{20}{147}$

(e) None of these

22. The probability of getting 10 in a single throw of three fair dice is :

(a) $\frac{1}{6}$ (b) $\frac{1}{8}$

(c) $\frac{1}{9}$ (d) $\frac{1}{5}$

(e) None of these

23. The probability that when 12 balls are distributed among three boxes, the first will contain three balls is,

(a) $\frac{2^9}{3^{12}}$ (b) $\frac{{}^{12}C_3.2^9}{3^{12}}$

(c) $\frac{{}^{12}C_3.2^{12}}{3^{12}}$ (d) $\frac{{}^{12}C_3.2^{11}}{3^{11}}$

(e) None of these

24. A and B toss a fair coin each simultaneously 50 times. The probability that both of them will not get tail at the same toss is

(a) $\left(\frac{3}{4}\right)^{50}$ (b) $\left(\frac{2}{7}\right)^{50}$

(c) $\left(\frac{1}{8}\right)^{50}$ (d) $\left(\frac{7}{8}\right)^{50}$

(e) None of these

25. If n integers taken at random are multiplied together, then the probability that the last digit of the product is 1, 3, 7 or 9 is

(a) $\frac{2^n}{5^n}$ (b) $\frac{4^n-2^n}{5^n}$

(c) $\frac{4^n}{5^n}$ (d) $\frac{4^n}{7^n}$

(e) None of these

26. A coin is tossed 5 times. What is the probability that head appears an odd number of times?

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{1}{2}$ (d) $\frac{4}{25}$

(e) None of these

27. Atul can hit a target 3 times in 6 shots, Bhola can hit the target 2 times in 6 shots and Chandra can hit the 4 times in 4 shots. What is the probability that at least 2 shots (out of 1 shot taken by each one of them) hit the target ?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{5}{6}$

(e) None of these

28. A bag contain 5 white, 7 red and 8 black balls. If 4 balls are drawn one by one with replacement, what is the probability that all are white ?

(a) $\frac{1}{256}$ (b) $\frac{1}{16}$

(c) $\frac{4}{20}$ (d) $\frac{4}{8}$

(e) None of these

29. A dice is thrown 6 times. If 'getting an odd number' is a 'success', the probability of 5 successes is :

(a) $\frac{1}{10}$ (b) $\frac{3}{32}$

(c) $\frac{5}{6}$ (d) $\frac{25}{26}$

(e) None of these

30. A bag has 4 red and 5 black balls. A second bag has 3 red and 7 black balls. One ball is drawn from the first bag and two from the second. The probability that there are two black balls and a red ball is :

(a) $\frac{14}{45}$ (b) $\frac{11}{45}$

(c) $\frac{7}{15}$ (d) $\frac{9}{54}$

(e) None of these

31. Two dice are tossed. The probability that the total score is a prime number is :

(a) $\frac{1}{6}$ (b) $\frac{5}{12}$

(c) $\frac{1}{2}$ (d) $\frac{7}{9}$

(e) None of these

32. A bag contains 3 white balls and 2 black balls. Another bag contains 2 white balls and 4 black balls. A bag is taken and a ball is picked at random from it. The probability that the ball will be white is:

(a) $\frac{7}{11}$ (b) $\frac{7}{30}$

(c) $\frac{5}{11}$ (d) $\frac{7}{15}$

(e) None of these

33. Suppose six coins are tossed simultaneously. Then the probability of getting at least one tail is :

(a) $\frac{71}{72}$ (b) $\frac{53}{54}$

(c) $\frac{63}{64}$ (d) $\frac{1}{12}$

(e) None of these

34. A bag contains 2 red, 3 green and 2 blue balls. 2 balls are to be drawn randomly. What is probability that the balls drawn contain no blue ball ?

(a) $\frac{5}{7}$ (b) $\frac{10}{21}$

(c) $\frac{2}{7}$ (d) $\frac{11}{21}$

(e) None of these

35. A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss equals

(a) $\frac{1}{2}$ (b) $\frac{1}{32}$

(c) $\frac{31}{32}$ (d) $\frac{1}{5}$

(e) None of these

36. The probability that the birth days of six different persons will fall in exactly two calendar months is

(a) $\frac{1}{6}$ (b) ${}^{12}C_2 \times \frac{2^6}{12^6}$

(c) ${}^{12}C_2 \times \frac{2^6-1}{12^6}$ (d) $\frac{341}{12^5}$

(e) None of these

37. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

(e) None of these

38. A and B are events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, $P(\overline{A}) = \frac{2}{3}$ then $P(\overline{A} \cap B)$ is

(a) $\frac{5}{12}$ (b) $\frac{3}{8}$

(c) $\frac{5}{8}$ (d) $\frac{1}{4}$

(e) None of these

39. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

(e) None of these

40. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$

(c) $\frac{7}{20}$ (d) $\frac{3}{20}$

(e) None of these

41. 2n boys are randomly divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is

(a) $\frac{n}{2n-1}$ (b) $\frac{n-1}{2n-1}$

(c) $\frac{2n-1}{4n^2}$ (d) $\frac{n+1}{2n+1}$

(e) None of these

42. Fifteen persons, among whom are A and B sit down at random at a round table. The probability that there are 4 persons between A and B is

(a) $\frac{9!}{14!}$ (b) $\frac{10!}{14!}$

(c) $\frac{9!}{15!}$ (d) $\frac{1}{7}$

(e) None of these

43. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probbility that they will not win a prize in a single trial is

(a) $\frac{1}{25}$ (b) $\frac{24}{25}$

(c) $\frac{2}{25}$ (d) $\frac{1}{27}$

(e) None of these