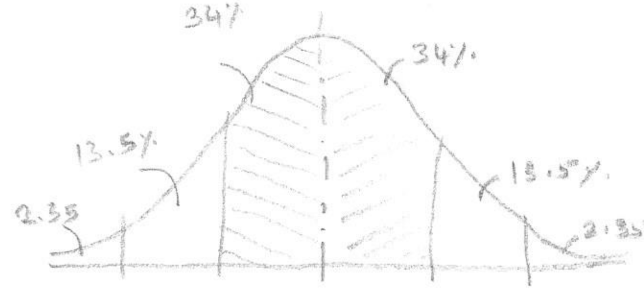
CONFIDENCE INTERVAL ASSIGNMENT

1. Find the average weight of the male.
Drawn random sample of 1000 men.
Population of men is 1,000,000
Average weight in sample is 180 lbs
Std deviation is 30 LBS.
95% CONFIDENCE INTERVAL.



$\sigma_{sample} = 30$ lbs

$\sigma_{SD} = \frac{\sigma_{sample}}{\sqrt{N}}$ | $\sigma_{SD} = \frac{\sigma_{pop}}{\sqrt{N}}$

$\sigma_{sample} \cong \sigma_{population}$

$M_{SD} = 180$ Lbs
$\sigma_{SD} = 30$ Lbs

$95\%\ CI = \mu \pm 2\sigma$

$= 180 + \frac{2.30}{\sqrt{1000}},\ 180 - \frac{2.30}{\sqrt{1000}}$

$= 180 \pm 1.897$

2. Assembly plant - Need to estimate the mean amount of time a worker takes to assemble a new component

$\sigma_{pop} = 3.6$ minutes

a) Sample of 120 workers $N = 120$; Average time to assembly is 16.2 minutes
Assumption 16.2 minuts is Sample Distribution Mean $M_{SD} = 16.2$ min and not $\bar{X}$

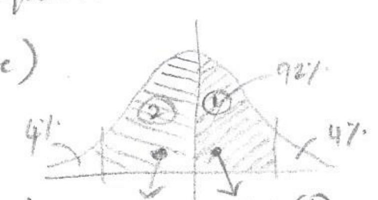
Construct 92% confidence interval

$M_{pop} \in [M_{SD} \pm Z \times SE]$

Standard Error SE = Sample distribution's Std deviation
$Z \times SE$ = Margin of error.

$SE = \sigma_{SD} = \frac{\sigma_{pop}}{\sqrt{N}} = \frac{3.6}{\sqrt{120}} = \frac{3.6}{\sqrt{120}} = 0.328$

Z value should be calculated from confidence interval required
(92% confidence interval is the 92% area under the curve)



so Zscore for 92% area is = 1.75
so 92% confidence interval of Sampling distribution mean is

$M_{pop} \in [M_{SD} \pm Z \times SE]$

$[16.2 \pm 1.75 \times 0.328]$

$\Rightarrow [16.2 \pm 0.574]$

b) How many workers needed to have assembly time estimated upto ±15 secs. with 92% conf

$M_{pop} \in (M \pm Z \times SE)$

$Z \times SE = 15$ | Zscore for 92% is 1.75

$1.41 \times SE = 15$

$1.41 \times \sigma_{SD} = 15$

$1.75 \times \frac{3.6}{\sqrt{N}} = 0.25$

$\frac{1.75 \times 3.6}{0.25} \quad \sqrt{N}$

0.25

$N = 635.04$
$N = 635$ workers

CONFIDENCE INTERVAL

3) Advocacy group would like to conduct a survey to find proportion 'p' of customers who bought a latest MP3 and were happy with it.

a) How large sample n should be to estimate p with 2% Margin of error and 90% CL.

Margin of error = Z × SE

= 95% × SE (See explanation below)

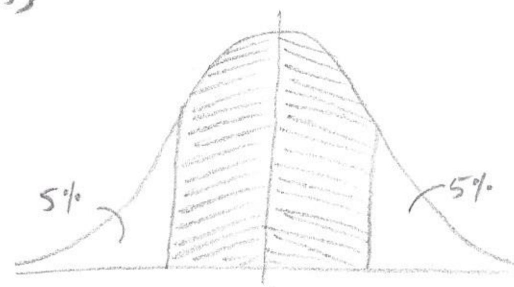
For a proportion $SE = \sqrt{\frac{pq}{n}}$

[P = 0.5]

$0.02 = 1.64 \times \sqrt{\frac{pq}{n}}$

$\sqrt{\frac{pq}{n}} = \frac{0.02}{1.64}$

$\sqrt{\frac{pq}{n}} = 0.01219$



$\frac{\sqrt{(0.5)(1-0.5)}}{\sqrt{n}} = 0.01219$

$\frac{\sqrt{0.25}}{\sqrt{n}} = 0.01219$

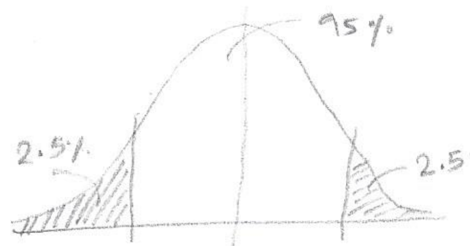
[SINCE IT IS CENTRAL LIMIT THEOREM PROBLEM
If CL = 90%, the area under the curve is represented as above, and the Z score is calculated as area under 95%

$\sqrt{n} = \frac{0.5}{0.012} = 41.66 \qquad n = 1736$

(b) Random sample = 1000 consumers.

$P = \frac{400}{1000}$ happy customers

$P = 0.4 \quad q = 0.6$



Confidence level of 95% Find the Confidence Interval.

$CI = P_s \pm Z\sqrt{\frac{P_s q_s}{n}}$

$= 0.4 \pm 1.96 \sqrt{\frac{0.4 \times 0.6}{1000}}$

$= 0.4 \pm 1.96 \frac{\sqrt{0.24}}{31.622}$

$= 0.4 \pm 0.03$

$= [0.43 - 0.37]$

4. Standard weight of 1 gm. weighed 4 times [0.95, 1.02, 1.01, 0.98]

$\bar{X} = \Sigma \bar{x} / n = 3.96/4 = 0.99$ (Sample Mean]

$\mu_{pop} = 1$ gm [ASSUMPTION?]

$$\sigma^2 = \frac{\Sigma(x - \bar{x})^2}{N}$$

$$\sigma^2 = \frac{(-0.04)^2 + (0.03)^2 + (0.02)^2 + (-0.01)^2}{4} = \frac{0.0016 + 0.0009 + 0.0004 + 0.0001}{4}$$

$$= \frac{0.0030}{4} = 0.00075$$

$\sigma^2 = 0.00075$

$\sigma_{sample} = 0.027$ in

$\sigma_{sample} \simeq \sigma_{population}$

95% confidence interval

Confidence Interval ($\mu_{sample} \pm 1.96\sigma_{sample}$) with 95% confidence level.

95% CL = 1.96 (Zscore)

$[\,0.99 \pm 1.96(0.027)\,]$

$(0.99 \pm 0.05292] \Rightarrow [1.04 - 0.94]$

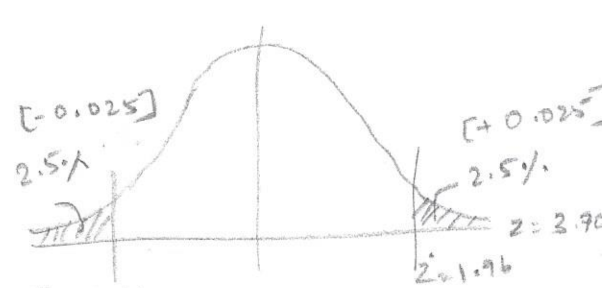
(b) 5% SIGNIFICANCE LEVEL

NULL HYPOTHESIS → THE WEIGH SCALE IS NOT ACCURATE

1. CONFIDENCE LEVEL = 95% (100-5)
2. CRITICAL REGION = 5% (Already given SIGNIFANCE LEVEL = 5%.)
3. Since CL = 95% it is 2σ region
4. Since the measure is looking for 5% critical region and NOT if the weight of 1 gram (std) is > or < a certain weight, it is two-tailed region

Z_{score} for 97.5 - 2.5

1.96 - (-1.96)



$Z_{score} = \underset{\text{Sample Mean}}{0.99} - \underset{\text{(pop mean)}}{1} / 0.027 = 0.1/0.027$
$= 3.703$

The Z score is in the Rejection region. so the NULL Hypothesis is rejected and ALTERNATE is accepted ⇒ The weigh scale is accurate.

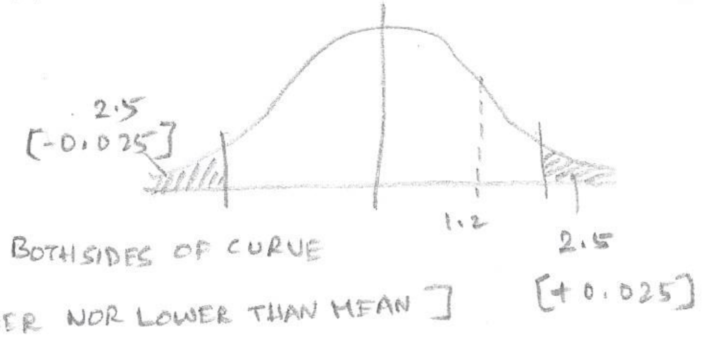
5. Time needed to complete a maze follows normal distribution with mean of 45 seconds. $\mu_{pop} = 45$ sec

Group of nine college vigorously for 30 minute and then complete the maze.

Sample mean and Std dev. $\mu_s = 49.2$ sec $\sigma = 3.5$ secs.

Find the hypothesis at 5% level of significance.

NULL HYPOTHESIS → WITH VIGOROUS EXCERCISE, THE MEAN TIME TO COMPLETE MAZE CHANGES



1. Confidence level is 95%.
2. CRITICAL REGION: (100 − 95) ⇒ 5%.
3. TAIL AREA WILLBE 2σ
4. AREA ACROSS THE NORMAL CURVE = BOTHSIDES OF CURVE

[IF THE MEAN CHANGES NEITHER HIGHER NOR LOWER THAN MEAN]

$$Z_{SCORE} = \frac{49.2 - 45}{3.5} = \frac{4.2}{3.5} = \frac{6}{5} = 1.2$$

[Area of 47.5 and -47.5 is represented in Zscore as +1.96 and −1.96.

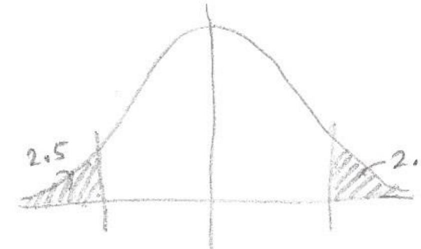
1.2 is within $\in [+1.96, -1.96]$ ⇒ There is no significant change due to vigorous exercise

Therefore, NULL HYPOTHESIS IS REJECTED, AND ALTERNATE HYPOTHESIS IS ACCEPTED

6. Installation of hardware takes random time with SD = 5 minutes

Technician does it on 64 different computers, average time of 42 minute.

Compute a 95% confidence interval for the mean installation time.



$\sigma_{pop} = 5$ minutes

Sample population $N = 64$

$M_{sample} = 42$ minute.

$\sigma_{sample} = \sigma_{pop}/\sqrt{N} \Rightarrow 5/\sqrt{64} = 5/8 \Rightarrow 0.625$

[Zscore for 97.5 area ⇒ 1.96]

95% Confidence interval ⇒ Confidence level (95%)

$$\begin{aligned} CI &= 42 \pm Z \times SE \\ &= 42 \pm 1.96 \times 0.625 \\ &= 42 \pm 1.225 \\ &= [43.23 - 40.775] \end{aligned}$$

7. TOPIC OF INTEREST IN OPTHAIMOLOGY, DOES SPHERICAL REFRACTION DIFFER IN LEFT & RIGHT EYE ON AVERAGE.

NULL HYPOTHESIS $\Rightarrow$ DOES THE SPHERICAL REFRACTION DIFFER IN LEFT AND RIGHT EYE ON AN AVERAGE

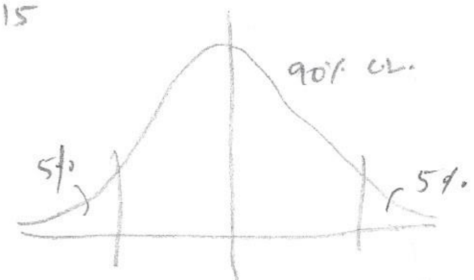
SAMPLE OF 17 PATIENTS WERE STUDIED (LEFT & RIGHT EYE) $N=17$

$\Sigma d_i = -3.50$ d_i = diff in left and right eye $\mu_{sample} = -3.50/17 = -0.205$ $(\bar{X})$

$\Sigma\Sigma d_i \wedge 2 = 19.13$

$$\sigma^2 \text{ (variance)} = \frac{\Sigma (X-\bar{X})^2}{N-1} \text{ or } \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N-1} = \frac{19.13 - \frac{(-3.5)^2}{17}}{17-1}$$

$$= \frac{19.13 - \frac{12.25}{17}}{16} = \frac{19.13 - 0.72}{16} = 1.15$$



Zscore of 45% cum area = 1.65

$$\sigma_s = \sqrt{1.15} \Rightarrow 1.07$$

$$90\% \text{ CI} = \mu_{sample} \pm Z\,SE$$

$$= -0.205 \pm Z \times \sigma_s$$

$$= -0.205 \pm 1.65 \times 1.07$$

$$= -0.205 \pm 1.7655$$

$$= [-1.970 \text{ to } +1.560]$$

8. Smallest sample size required to provide a 95% confidence Interval for a mean. Interval should not be more than 1 cm.

Population variance is $9\ cm^2$

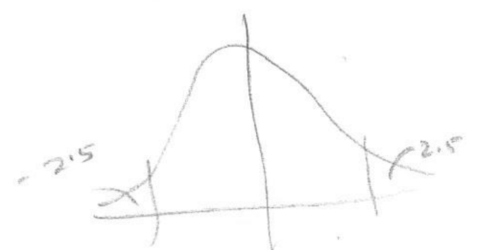
$\sigma_{pop} = 9$

C.I should not be > 1.

Zscore of 95% CL =

Zscore for area of 47.5

= 1.96



$$CI = \mu_{sample} \pm \frac{Z\sigma_{pop}}{\sqrt{N}}$$

$$1.0 \geq \left(\mu_{sample} + \frac{Z\sigma_{pop}}{\sqrt{N}}\right) - \left(\mu_{sample} - \frac{Z\sigma_{pop}}{\sqrt{N}}\right)$$

Minimum value of N will be obtained if we assume diff is 1

$$\Rightarrow \frac{1.96 \times 9}{\sqrt{N}} + \frac{1.96 \times 9}{\sqrt{N}} \leq 1$$

$$\frac{35.28}{\sqrt{N}} \leq 1$$

So $\sqrt{N} = \frac{35.28}{1}$ $N = 35.28^2$

$= 1244$

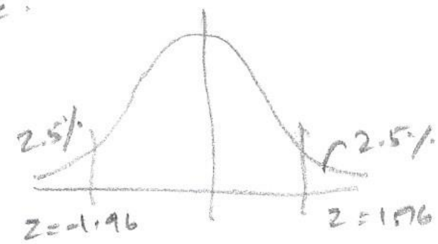
min sample size.

9) Recommended retail price is \$150. (Mean price) $M_{pop} = 150$.

Sample of 16 Retailers price of jeans is \$141. $\sigma_{SD} = 4$

If prices are randomly picked and are normally distributed

Construct a 95% confidence Interval for average sale price.



$CI = M_S \pm Z\,SE$ | 95% CL. Z score $= 1.96$

$= 141. \pm 1.96 \times 4$

$= 141 \pm 7.84$

$CI = 148.84 - 133.16$

10. Alcohol abuse is the important cause of deaths in young adults.

Survey of 17,096 students in U.S. 4 year college.

Frequent binge-drinking $\Rightarrow$ 5 or more drinks in a row, 3 or 4 times in past 2 weeks

3,314 students were classified as frequent binge-drinkers.

Construct a 90% confidence interval and true proportion of binge drinkers.

$N = 17096$.

No of students classified as frequent binge drinkers $= 3314$ students

$$p = \frac{3314}{17096} = 0.193$$

Construct a 90% confidence Interval around True proportion of binge drinkers

$P_s = 0.193$

$q_s = 1 - 0.193 \Rightarrow 0.807$

$N = 17096$

N is large.

Z score for 90% confidence interval

Z score for 45% area $= 1.65$

Confidence Interval for binomial distribution with proportion

$$= p_s \pm Z_s \sqrt{\frac{p_s q_s}{n}}$$

$$= 0.193 \pm 1.65 \sqrt{\frac{0.193 \times 0.807}{17096}}$$

$$= 0.193 \pm 0.005$$

$= [0.198 - 0.188]$ True proportion of binge drinkers.