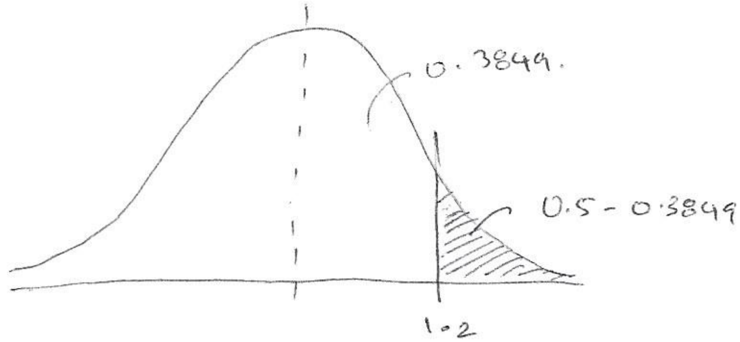
DISTRIBUTION ASSIGNMENT

1. DISTRIBUTION OF AGES FROM TITANIC DATASET - USING MATPLOTLIB

2. The average monthly sales of 2000 firms are normally distributed with mean 38,000 and std deviation Rs 10000, find

- $N \Rightarrow 2000$
- $\mu \Rightarrow 38,000$
- $\sigma \Rightarrow 10,000$



- The number of firms with sales more than 50,000

$X \Rightarrow 50,000$

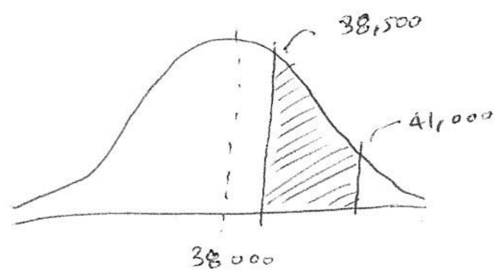
$$Z = \frac{X - \mu}{\sigma} = \frac{50000 - 38000}{10000} = \frac{12000}{10000} = 1.2$$

From Z score table. for 1.2 ⇒ 0.3849.

⇒ 0.5 - 0.3849 = 0.1151 or 11.5% companies. 11.5% of 2000 = 230 companies.

Percentage of firms with sales between Rs 38,500 and Rs 41,000

Find the Z scores



$$Z = \frac{38500 - 38000}{10000} = \frac{500}{10,000} \quad \frac{1}{20} = 0.05$$

$$Z = \frac{41000 - 38000}{10000} = \frac{3000}{10000} = \frac{3}{10} = 0.3$$

$Z_{0.05} = 0.0190$

$Z_{0.3} = 0.1179$

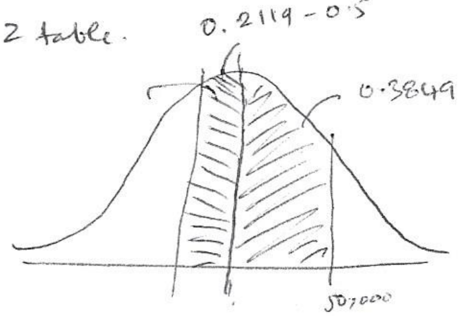
Area under the curve is 0.1179 - 0.0190

⇒ 0.0989 or 9.89% ⇒ 10% 200 companies (approx).

Number of firms between sales of 30,000 and Rs 50000

$$Z = \frac{30000 - 38000}{10000} = \frac{-8000}{10000} = -0.8 \Rightarrow 0.2119$$

From Z table.



Z = ~~50000~~ from above : 1.2 ⇒ 0.3849

= 0.5 - 0.2119 + 0.3849

= 0.2881 + 0.3849

= 0.6230 or 62.3% companies [1246 companies]

3. An unbiased die is tossed 700 times. Use normal approximation to binomial to find the probability obtaining

→ more than 124 aces

→ number of aces between 81 and 100

→ exactly 145 aces.

4. Random variable X follows Poisson distribution with parameter 25. Use normal distribution to poisson distribution to find probability that $X \geq 30$.

$\lambda = 25$.

5. A test is conducted with 25 MCQ's with 4 options. Determine the probability of the person answering exactly 5 wrong answers.

ASSUMPTION: ONLY ONE CHOICE GIVES THE CORRECT ANSWER.

The problems follows BINOMIAL DISTRIBUTION:

Probability of Success ⇒ $P(CA) = \left(\frac{1}{4}\right)$ | Probability of Incorrect answer $P(IA) = \frac{3}{4}$

CORRECT ANSWER

Out of 25 questions what is the probability of exactly answering 5 question wrong.

$$P(X=5) = 25C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{20}$$

$$= \frac{25!}{5!(25-5)!} \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{20}$$

$$= \frac{25 \times 24 \times 23 \times 22 \times 21}{5!} \left(\frac{3^{20}}{4^{25}}\right)$$

$$= \frac{25 \times 24 \times 23 \times 22 \times 21}{120} (0.25)^{5} (0.75)^{20} = 0.16 \text{ or } 16\% \text{ probability.}$$

6. In an astronomy experiment, average rate of photons reaching telescope is 4 photons per second. (Poisson random variable with mean of 4). Find the probability that no photon reaches the telescope.

→ The solution should follow the Poisson distribution.

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}, \quad \lambda = 4.$$

For given one second.

(probability) of $r=0$.

$$P(X=0) = \frac{e^{-4}4^0}{0!} = e^{-4} = 0.0183$$

⇒ 1.83%

7. No of calls coming to a cust support is Poisson random variable with mean 3.

$\lambda = 3$.

→ Probably that no calls come in a given one minute period

$r = 0$

$$P(X=0) = \frac{e^{-3} 3^{0}}{0!} = e^{-3} = 0.0497 = 4.97\%$$

→ ~~Probability that~~ in 2 differnt minutes are independent. Find probably that atleast Calls arriving
2 calls will arrive in a given two minute period. Which is a sum of 2 (1 sec) time periods.

Poisson Distribution – Cumulative Density Function.

$P(X \geq 2) \Rightarrow$ atleast 2 calls.

$P(X<2) + P(X \geq 2) = 1$

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - e^{-\lambda} \sum_{i=0}^{r} \frac{\lambda^{-i}}{i!}$

$= 1 - e^{-3}\left(\frac{\lambda^{-0}}{0!} + \frac{\lambda^{-1}}{1!}\right) + \cancel{\frac{\lambda^{-2}}{2!}}$

$= 1 - e^{-3}(1 + \lambda^{-1})$

$= 1 - e^{-3}(3^{-0} + 3^{-1})$

$= 1 - e^{-3}(1 + 3^{-1})$

~~$= 1 - e^{-3}(-2)$~~

~~$= 1 + 2e^{-3}$~~

$= 1 - e^{-3}(1 + \frac{1}{3})$

$= 1 - e^{-3}(1.33)$

$= 1 - 0.0497(1.33)$

$= 1 - 0.06621$

$P(X \geq 2) = 0.9337$ or 93.37%.

$\boxed{\text{Not sure } \lambda = 3 \text{ or } \lambda = 6}$

Prudhvi confirmed $\lambda = 6$.

$= 1 - e^{-6} \sum_{i=0}^{1} \frac{6^{-i}}{i!}$

$= 1 - e^{-6}\left(\frac{6^{-0}}{0!} + \frac{6^{-1}}{1!}\right)$

$= 1 - e^{-6}\left(1 + \frac{1}{6}\right)$

$= 1 - e^{-6}(1.66)$

$= 1 - 0.0024(1.66)$

$= 1 - 0.0041$

$= 0.9958$ or 99.58%.

8. Probability of first defective part after 3 good parts. 20% defective rate

~~P = 80%~~

Probability of a defective part = 20% = 0.2   Probability of good part = 80% = 0.8

$P(X=4) = 0.8 \times 0.8 \times 0.8 \times 0.2 = 0.512 \times 0.2 = 0.1024 \Rightarrow 10.24\%$

9. Probability student is admitted to a prestigious college = 0.3. If 5 students apply what is the probability that atmost 2 are accepted.

Binomial distribution solution

$N = 5$ and $N = 5$ $\quad P = (0.3) \quad q = (0.7)$.
$R = 2 \quad\quad R = 2$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= 5C_0 (0.3)^0 (0.7)^{5-0} + 5C_1 (0.3)^1 (0.7)^{5-4} + 5C_2 (0.3)^2 (0.7)^{5-3}$

$= 1 \times 1 \times 0.7^5 + 5 \times 0.3 \times 0.7^4 + 10 \times 0.3^2 \times 0.7^3$

$= 0.7^5 + 1.5 \times 0.7^4 + 0.9 \times 0.7^3$

$= 0.7^3 (0.7^2 + 1.5 \times 0.7 + 0.9 \times 1)$

$= 0.7^3 (0.49 + 1.05 + 0.9)$

$= 0.7^3 (0.49 + 1.95)$

$= 0.8369$ (or) 83.69%

10. Max weight elevator can adopt is 800 kg. Avg weight of adult is 70 kg variance of 200g. Probability the lift ~~reaches~~ reaches safely when 100 adults? what if 12 adults.

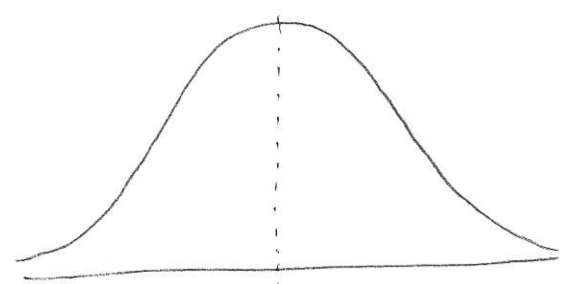
Adult $\mu = 70$kg

$\sigma^2 = 200g. \ (\sigma = \sqrt{200}) \Rightarrow 14.14$

For 12 adults $\rightarrow$ Total weight is $12 \times 70 = 840$ kg.

Total variance is $.200 \times 12 = 12/5 = 2.4$ kg.

Looks like a [CENTRAL LIMIT THEOREM]



$\mu = 840$ kg
$\sigma^2 = 2.4$ kg

11. MCQ type examination with 2 choices. Total 50 questions, get atleast 20 questions for a pass. What is the probability he will clear the exams.

$P(X \geq 20)$ [Solution falls under BINOMIAL DISTRIBUTION]

$P(X \geq 20) + P(X \leq 20) = 1$

$P(X \leq 20) = 1 - P(X \geq 20)$ $\quad \sum_{i=0}^{n} C_i^n p^i q^{n-i}$

$P(X \geq 20) = 1 - P(X < 20)$

$= 1 - \left[50C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{50-0} + 50C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{49} + \dots \right]$

$= 1 - \frac{1}{2^{50}} \left[50C_0 + 50C_1 + 50C_2 + 50C_3 + \dots \right]$

$= 1 - 0.0594$

$= 0.9405$ or 94.05%

If the choices were 4 for a correct answer. $p = 1/4$ $q = 3/4$

$P(X \geq 20) = 1 - \left[50C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{50-0} + 50C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{49} + 50C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^{48} + \dots \right.$

$= 1 - \frac{1}{(4^{50}} \left[3^{50}\, 50C_0 + 3^{49}\, 50C_1 + 3^{48}\, 50C_2 + \dots \right]$

$= 0.0139$ or 1.39%

12. A company manufactures LED bulbs with a faulty rate of 30%. what is the probability that $X = 2$ are faulty.

$P = 0.7$ $\quad Q = 0.3$ $\quad N = 6$ bulbs

$P(X=2) \Rightarrow 6C_2 (0.7)^2 (0.3)^{6-2}$

$= \frac{6 \times 5}{2} \cancel{(0.49)} \times 0.49 \times 0.0081$

$= 15 \times 0.49 \times 0.0081$

$= 0.059$

$= 5.9\%$

13. For a writer, effecicy is 6 errors per hour, entering 77 words per minute. What is the probability of 2 errors in 322 word report.

Efficiency is 6 errors per hour entering 77 words per minute

Time period = 1 hours.

This Solution follows a poisson distribution

~~λ = 6 errors.~~

322 word report = takes how many hours 322/77 = 4.18 ~~hours~~ minutes.

6 errors - 60 minute $= \dfrac{6 \times 4.18}{60} = 0.418$ errors.

? errors - 4.18 minutes

$\lambda = 0.418$ errors/minute

$r = 2$ error/minute

$$P(X=2) = \frac{e^{-0.418} \times 0.418^2}{2!}$$

$= 0.0575$ or 5.75%