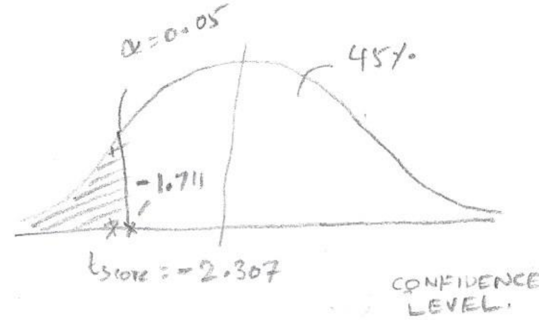
T-Test Assignment Questions

1. Average population heart rate $\mu = 72/min$

Sample size
$N = 25$
$\bar{X} = 69/min$
$S = 6.5$

$$t_{score} = \frac{69-72}{6.5/\sqrt{25}}$$

$$= \frac{-3}{6.5/5} \Rightarrow \frac{-15}{6.5} = -2.307$$



Assume we are evaluating the heart rate reduction for 5% significance level

$\alpha = 5\% = 0.05$

NULL HYPOTHESIS: AVG HEART BEAT IS 72/min (WHAT IS ALREADY ESTABLISHED)

ALTERNATE HYPOTHESIS: AFTER TRAINING AVG HEART BEAT < HEART BEAT BEFORE TRAINING

AVG Heart Beat < 72 b/min

Find the area under the critical region (5% significance level) $\Rightarrow A = 0.05$

Degrees of freedom = 25 − 1 = 24.

From the t table = $t_a = 1.711$

−2.307 is in Rejection region, so NULL HYPOTHESIS IS REJECTED.

ALTERNATE HYPOTHESIS IS ACCEPTED, Avg Heart rate after exercise is < 72 b/min.

2. Manufacturer's shoe model avg life is 15 months. $\mu = 15$

R&D has come up with a new product with avg life $\bar{X} = 17$ $n = 30$ $S = 5.5$ months
[Sample parameters]

NULL HYPOTHESIS: THE AVG LIFE IS = 15 months

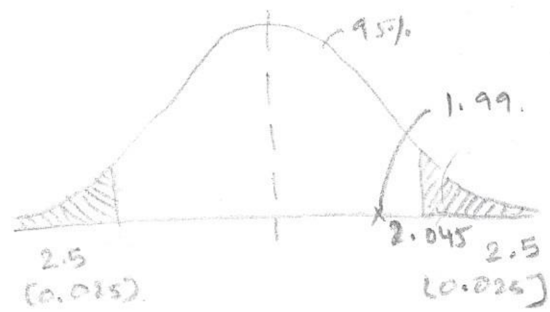
ALTERNATE HYPOTHESIS: THE AVG LIFE IS = 17 months

[So, it has to be two-tailed test]

Significance level $p < .05$

$$t_{SCORE} = \frac{17-15}{5.5/\sqrt{30}} \cdot \left(\frac{\bar{X}-\mu}{s/\sqrt{n}}\right)$$

$$= \frac{2\sqrt{30}}{5.5} \Rightarrow 1.99$$



Per tscore table df = 29, α = 0.025
[Two-tailed table]
$t_\alpha = 2.045$

Degree of freedom df = 30 − 1 = 29

Significance level $p < 0.05$

tscore 1.99 < p(0.05) So the NULL HYPOTHESIS ACCEPTED and AVG LIFE IS STILL 15 months

5. Big boss's department is recieving mean of 16 complaints a month $\mu = 16$.

Sample

$\bar{X} = 18$

$S = 2.05$

$n = 10$ months.

NULL HYPOTHESIS : COMPLAINTS = 16

ALT HYPOTHESIS : COMPLAINTS <> 16

[Since ALT HYPOTHESIS is checking if <>16, it will be a TWO TAILED TEST]

$t_{score} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{18-16}{2.05/\sqrt{10}} = \frac{2\sqrt{10}}{2.05} = 3.08$

$\alpha = 0.05$ [ASSUMPTION] $\quad df = 10 - 1 = 9$

$t_{std} = 2.262$.

$t_{score} > t_{std}$. so the t_{score} is in the critical region.

Hence NULL HYPOTHESIS IS REJECTED, ALT HYPOTHESIS IS ACCEPTED

BIG BOSS WILL FIRE THE MANAGER.

3. INVESTIGATE the usefulness of relaxation training. Random sample of 30 people are studied

SAMPLE 1 : CONTROL (NO TRAINING)

$\bar{X} = 30$, $S = 6.63$, $n = 15$

SAMPLE 2 : RELAXATION (TRAINING)

$\bar{X} = 26$, $S = 6.20$, $n = 15$

Null Hypothesis → The training did not make a difference

$\bar{X}_1 = \bar{X}_2$

Alternate Hypothesis → The training did make the difference

$\bar{X}_1 \neq \bar{X}_2$

Firstly, we need to find the Test Statistic

$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_1^2/N_1 + S_2^2/N_2}}$ If $N_1 = N_2$ then $T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2 + S_2^2}{N}}} = \frac{30 - 26}{\sqrt{\frac{6.63^2 + 6.20^2}{15}}} = 1.706$

Since the Alternate hypothesis checks if the mean are not equal, this should be a Two-tailed test with significance level of $\alpha = 0.05$

Degrees of Freedom $df = \frac{S_1^2/N_1 + S_2^2/N_2}{(S_1^2/N_1)^2/(N_1-1) + (S_2^2/N_2)^2/(N_2-1)} = \frac{S_1^2 + S_2^2 / N}{\frac{(S_1^2/N)^2 + (S_2^2/N)^2}{(N-1)}}$

$= \frac{S_1^2 + S_2^2}{N} \times \frac{(N-1)}{(S_1^2/N)^2 + (S_2^2/N)^2} = \frac{14}{15} \times \frac{82.4}{\frac{1}{15^2}(S_1^4 + S_2^4)} = \frac{14 \times 15}{} \times \frac{82.4}{3410}$

$= 5.07$ (or) 5

t_α for $df = 5$ and $\alpha = 0.05$ and two tailed tscore curve, ⇒ 2.571.

$T < t_\alpha$ so NULL HYPOTHESIS IS ACCEPTED and so TRAINING DID NOT MAKE A DIFFERENCE

4. Continuation of #3 when the samples are paired based on dimension of sex and job type.

Pairs	Control	Relaxation	diff
1	38	35	3
:			
15	22	21	1

Evaluate the experiment using the criteria $p < 0.05$.
Assume two-tailed test.

Based on excel calculations

Ms of diff between paired sample $= 4 \; (\bar{X})$

S of the sample $= 2.56 \; (S)$

Standard error $\Rightarrow SE(\bar{X}) = S/\sqrt{n}$ (Samples std dev $= S/\sqrt{n}$) For population std dev $= \sigma$ or $[S]$

$= 2.56/\sqrt{15}$

$= 0.66$

NULL HYPOTHESIS : There is no difference in μ (means) of the two paired samples

$\bar{X}_C - \bar{X}_R = 0$

ALTERNATE HYPOTHESIS : There is difference in μ (mean) of the two paired samples

$\bar{X}_C - \bar{X}_R \neq 0$

Confidence Interval $= \bar{X} \pm t_{score} \times SE(\bar{X})$

$= 4 \pm t_{\alpha/2} \times 0.66$

$= 4 \pm 2.145 \times 0.66$

$= 4 \pm 1.417$

$= 5.417 - 2.58$

$df = 15 - 1 = 14$
significance level $= 0.05$
t-tailed test
tscore $= 2.145$ at 0.05 signific

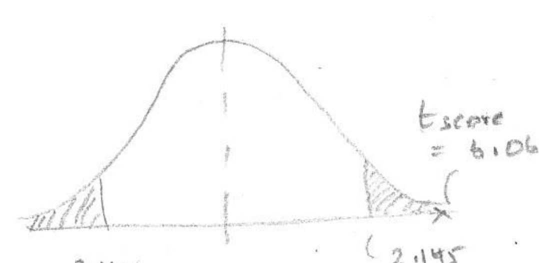
TEST THE NULL HYPOTHESIS : $t_{score} = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$

$= \dfrac{4 - 0}{0.66}$

$= 4/0.66$

$= 6.06$

($\mu = 0$ as this is the NULL HYPOTHESIS)
($\mu = 0$ mean the two samples are the same [diff between two samples is zero)



t score is in critical region so NULL HYPOTHESIS IS REJECTED

$\Rightarrow$ There is a difference between the stress levels before and after the course.