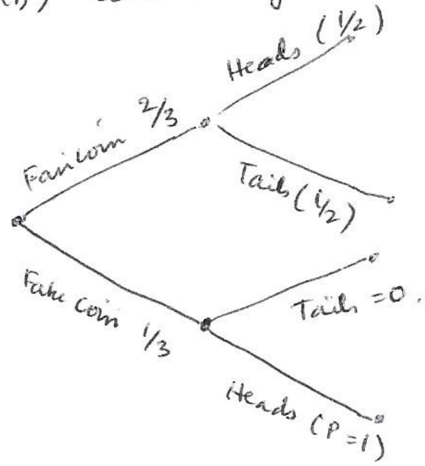
Probability Assignment

Q5 - Box contain 2 fair coin, 1 fake (two-sided coin) P(Heads) = 1. Pick a coin at random and toss it.

(a) What is the probability that it lands heads up.

(b) Coin toss gets heads. What is the probability it is a two-headed coin.



a) Probability it is head is

$P(\text{FAIR} \cap \text{HEADS}) + P(\text{FAKE} \cap \text{HEADS})$

$= 2/3 \times 1/2 + 1/3 \times 1$

$= 1/3 + 1/3$

$= 2/3$ or 66.7%

(b) $P(\text{Fake coin} / \text{Heads}) = \dfrac{P(\text{Fake coin AND HEADS})}{P(\text{HEADS})}$

$= \dfrac{1/3 \times 1}{P(\text{FAIR} \cap \text{HEADS}) + P(\text{FAKE AND HEADS})}$ [a]

$= \dfrac{1/3 \times 1}{2/3}$

$= 1/2 = 0.5 \Rightarrow 50\%$

Q7 - A population has mean of 50 and SD of 6.

(a) Mean and SD of a sampling distribution of the mean of N = 16.

$\mu_{pop} = 50$

$SD_{pop} = 6$

$\mu_{SD} \cong \mu_{pop} = \boxed{50}$

[Sample Distribution]

$\sigma_{SD} = \dfrac{\sigma_{pop}}{\sqrt{N}}$

$\sigma_{SD} = \dfrac{6}{\sqrt{16}} = \dfrac{6}{4} = \dfrac{3}{2} = \boxed{1.5}$

(b) Find μ_{SD} and σ_{SD} if N = 20.

$\mu_{SD} = \mu_{pop} = \boxed{50}$

$\sigma_{SD} = \dfrac{\sigma_{pop}}{\sqrt{20}} = \dfrac{6}{4.47} = \boxed{1.34}$

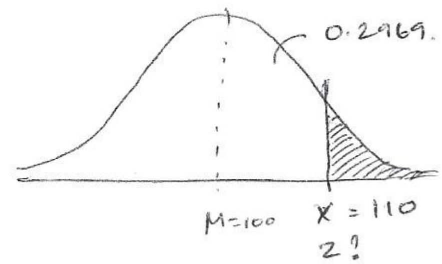
Q8: Given test is normally distributed with mean = 100 SD = 12.

(a) Probability that a single score drawn at random will be greater than 110.

$X_i = 110$

$$Z = \frac{X_i - \mu}{\sigma} = \frac{110 - 100}{12} = \frac{10}{12} = \frac{5}{6} = 0.833$$

~~or 83.3%~~

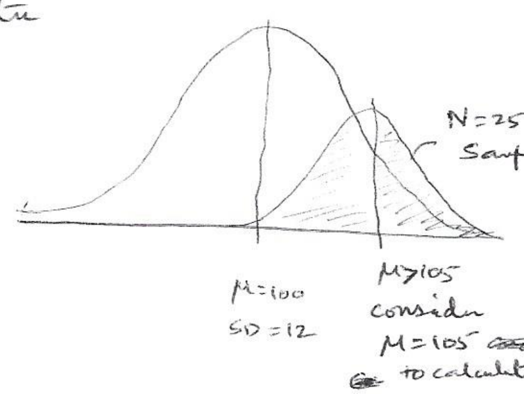


Area for Z score = 0.5 − 0.2969

Probability = 0.2131 or 21.3%

(b) Probability that a sample of 25 scores will have a mean greater than 105.

Consider the μ of the Sample is 105 and find the area (probability) more than 105



$$Z = \frac{X_i - \mu}{\sigma} \qquad \sigma_{SD} = \frac{\sigma}{\sqrt{N}}$$

$$= \frac{105 - 100}{2.4} \qquad \sigma_{SD} = \frac{12}{\sqrt{25}} = \frac{12}{5}$$

$$= \frac{5}{2.4} = 2.083 \qquad \boxed{\sigma_{SD} = 2.4}$$

Area for Z score = 0.5 − 0.4812

= 0.0188 (0.019)

Probability = 1.9%

(c) The probability that a sample of 64 scores will have mean greater than 10

The question should actually state SAMPLING DISTRIBUTION

$$\sigma_{SD} = \frac{12}{\sqrt{64}} = \frac{12}{8} = \frac{3}{2} = 1.5$$

$$Z = \frac{105 - 100}{1.5} = \frac{5}{1.5} = 3.33$$

Area for the Z score = 0.4996 ~ 0.5

Probability = 0.0004 or 0.04%

(d) The probability that mean of a sample of 16 scores will be either less than 95 or greater than 105.

$\sigma_{SD} = \frac{12}{\sqrt{16}} = \frac{12}{8} = 1.5$

$Z = \frac{95-100}{1.5} = \frac{-5}{1.5} = -3.33$ (Area under Z score) = 0.0004

$Z = \frac{105-100}{1.5} = \frac{5}{1.5} = 3.33$ (Area under Z score) = 0.9996

$= 0.9996 - 0.0004$

Probability. $= 0.9992$ (or) 99.92% (between mean 95 to 105).

Q9: Try in Python.

Q10: Population is normally distributed with SD = 2.8

(a) Sample: [8, 9, 10, 13, 14, 16, 17, 20, 21]

Mean = 14.2

95% confidence interval of the mean = $\mu \pm 2\sigma$

$= 14.2 \pm 2.8 \times 2$

= [~~[illegible]~~ 19.8 - 8.6

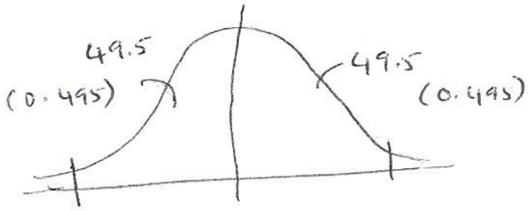
(b) 99% confidence interval for same data

$\cancel{\mu_{popu} \pm Z \times SE}$

$\mu_{sample} \pm Z \times SE$ (Sample Standard Deviation is called Standard Error]

~~[illegible]~~ 2σ = 95% confidence interval

99% ~~or~~ (0.99) of the ~~nom~~ area under the normal curve.



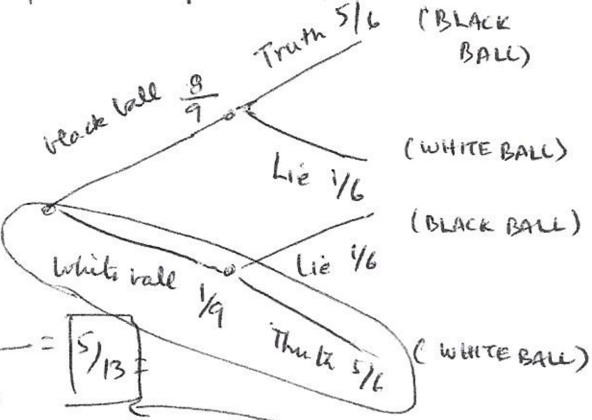
Z = 2.57.

$\Rightarrow 14.2 \pm 2.57 \times 2.8$

14.2 ± 7.196 (or) 14.2 ± 7.2

$\Rightarrow$ [21.4 - 7]

Q11: A is known to tell truth 5 out of 6 cases. (Bag contain 8 black and 1 white ball).
Find the probability that a white ball was drawn.



$$\frac{1/9 \times 5/6}{13/9\times 6} = \boxed{5/13}$$

$P(W / \text{Reported white}) = \cancel{1/9 \times 5/6} \;\; \frac{P(W \text{ AND REPORTED WHITE})}{P(W)}$

Probability of WHITE BALL) ~~called out as WHITE~~

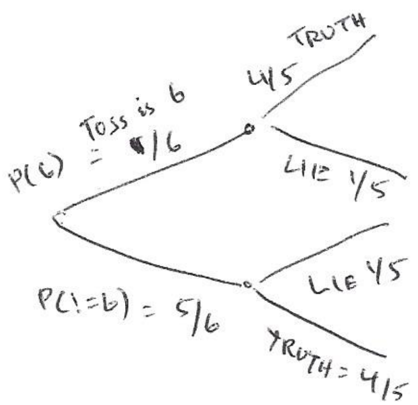
$= P(B \cap L) + P(W \cap T)$

$= \frac{8}{9} \times \frac{1}{6} + 1/9 \times \frac{5}{6}$

$= \frac{8+5}{9\times 6}$

$= \frac{13}{9 \times 6} = 0.2407$ (or) 24%

Q12: A speaks the truth 4 out of 5 times. A die is tossed. A reports it as 6.
What are the chances that there actually was a 6.



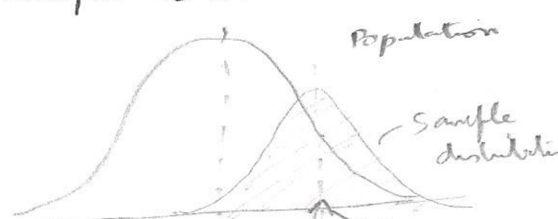
$P(\cancel{\text{Actually was}}\ 6 / \text{Reported } 6)$

$P(6 / \text{Reported } 6) = \frac{P(6 \text{ AND REPORTED } 6)}{P(6)\text{ Reported}}$

$= \frac{1/6 \times 4/5}{1/6 \times 4/5 + 5/6 \times 1/5}$

$= \frac{4}{4+5} = \frac{4}{9} = 0.44$ (or) 44.4%

Q9: Population has mean SAT score of 1000. Which would most likely to get a sample distribution mean of 1200. [Randomly sampled 10 students or Randomly sampled 30 students.



$\mu_{pop} = 1000$

$\mu_{SD} = 1200$ [Consider]

$N1 = 10$

$N2 = 30.$

$\sigma_{SD} = \frac{\sigma_{pop}}{\sqrt{N}}$

$\sigma_{SD=10} = \frac{\sigma_{pop}}{\sqrt{10}}$

$\sigma_{SD=30} = \frac{\sigma_{pop}}{\sqrt{30}}$

So $\sigma_{[10]}$ will be greater than $\sigma_{[30]}$ for a given σ_{pop}.

If we consider μ_{SD} as X_i in population distribution

$Z_i = \frac{1200-1000}{\sigma_{pop}} \Rightarrow Z_i = \frac{200}{\sigma_{pop}}$

Covers more area of the curve so higher probability $Z_i = \frac{200}{\sqrt{10}\,\sigma_{SD}}$ (or) $\frac{200}{\sqrt{30}\,\sigma_{SD}}$

So N=10 has more likely to get a sample distribution mean of 1200 rather than N=30.