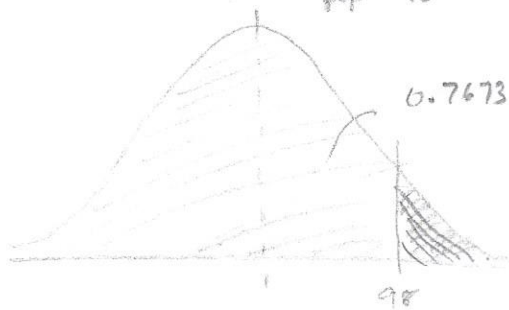
CENTRAL LIMIT THEOREM

$\mu_{pop} = 96$
$\sigma_{pop} = 16$



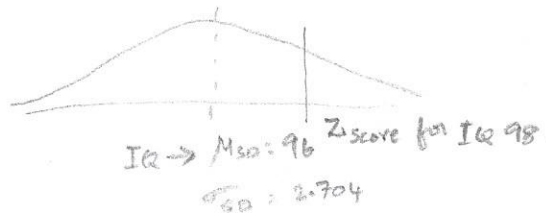
1. Average IQ required $\mu = 98$.

$\mu_{pop} = 96$

$\sigma_{pop} = 16$

Sample size $N = 35$

$\sigma_{SD} = \frac{\sigma_{pop}}{\sqrt{N}} = \frac{16}{\sqrt{35}} = \frac{16}{5.91} = 2.704$. $\mu_{pop} \cong \mu_{SD}$



Expectation 98. [consider one sample is represented as a value in pop distribution]

$Z\ score_{SD} = \frac{X_i - \mu_{SD}}{\sigma_{SD}}$

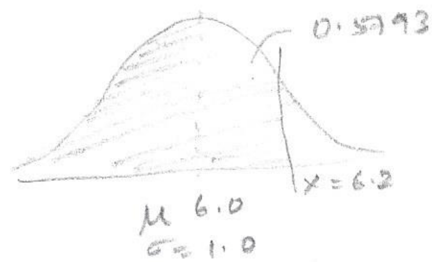
$= \frac{98 - 96}{2.704} = \frac{2}{2.704} = 0.739$. (Area with the Z score = (0.7673)

Probability of IQ ≥ 98 = $1 - 0.7673 = 0.2327$ or 23.27%.

2. Motorcycle Helmets → male head breaths

$\mu = 6.0$ in $\sigma = 1.0$ inch

a) One male is selected, what is probability that head breath is < 6.2 inch

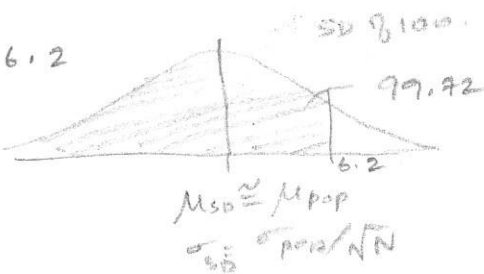


$Z_{6.2} = \frac{6.2 - 6}{1} = 0.2$

$Z_{6.2} \Rightarrow 0.2$ → Area under the curve is 0.5793

The probability is < 6.2 inches is 57.93 or 60%.

b) 100 randomly selected men. head breadth is < 6.2



$\sigma_{SD} = \sigma_{pop}/\sqrt{N} = 1.0/\sqrt{100} = 1/10 = 0.1$

$\mu_{SD} \cong \mu_{pop}$.

Find the Z score for $X = 6.2$

$Z_{SD\,6.2} = \frac{6.2 - 6}{0.1} = \frac{0.2}{0.1} = 2$ [Area under Z score 2.0 is 0.9972]

Probability that 100 randomly selected men have mean breadth < 6.2 = 99.72%.

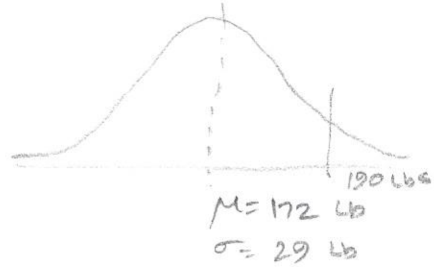
c. The problem with the decision is that though breadth less than 6.2 in fits 99.72%, the decision is based on a random sample of 100 men. [Sampling]

The decision should be made based on

- Multiple samples [Sampling distribution]
  - 100 is a very less sample size when it comes to all men who drive
  - Need samples based on stratified sampling (on height, weight, region, age etc
  - Multiple Sampling distribution based Mean will give a better estimate.

4. Weights of adult males are normally distributed

a) One randomly selected male will weigh more than 190 pounds.

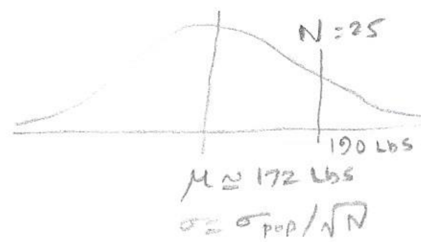


$$Z_{190\,Lbs} = \frac{190-172}{29} = \frac{18}{29} = 0.62$$

Area under this Z score is 0.7324 or 73.24%.

$P(X > 190) = 1 - 0.7324 = 0.2676$ or 26.76%

b) Sampling distribution N=25, what is the probability it will weigh > 190 Lbs



$\sigma_{SD} = 29/\sqrt{25}$

$= 29/5$

$\sigma_{SD} = 5.8$

Consider weight of 190 Lbs we need to find where this falls under the sampling distrubition (curve)

$$Z_{SD\,190} = \frac{190-172}{5.8} = \frac{18}{5.8} = 3.103$$

How much area does a Z score of 3.103 fall under is

$Z(3.103) = .9990$ or 99.9%

$P(X > 190) = 1 - 0.9990$

$= 0.0010$ or 0.1%

c) Men's fitness center has a elevator with maximum weight capacity is 4750 lbs

max = 4750 lbs

If 25 men cram into it the elevator

[Sampling distribution of N = 25]

$\mu_{SD} \cong \mu_{pop} = 172$ lbs

$\sigma_{SD} = \frac{\sigma_{pop}}{\sqrt{N}} = \frac{29}{\sqrt{25}} = \frac{29}{5} = 5.8$

Max weight = 4750 lbs

weight per person: $\frac{4750}{25} = 190$

What is the probability that average weight of a person in this distribution [person in the lift] is > 190.

$Z_{score\ 190lbs} = \frac{190 - 172}{5.8} = \frac{18}{5.8} = 3.103$

Area under the sample with Z score 3.103

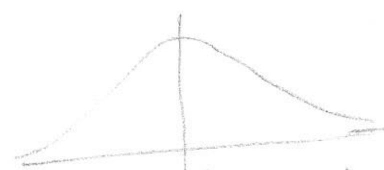
$= 0.9990$

$P(X > 190) = 1 - 0.9990 = 0.0010$ or 0.1%

5. Impurity in batch of chemical product is random variable

$\mu = 4.0g$

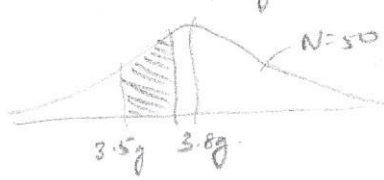
$SD = 1.5g$.



Sample (Sampling distribution)

$N = 50$

$\sigma_{SD} = \frac{\sigma_p}{\sqrt{50}} = \frac{1.5}{\sqrt{50}} = \frac{1.5}{\sqrt{2}\,5} = \frac{0.3}{\sqrt{2}} = 1.414$



$Z_{SD\ 3.8} = \frac{3.8 - 4}{1.414} = \frac{0.2}{1.414} = 0.141 \Rightarrow$ Area under $\Rightarrow 0.4443$

$Z_{SD\ 3.5} = \frac{3.5 - 4}{1.414} = \frac{0.5}{1.414} = -0.353 \Rightarrow$ Area under $\Rightarrow 0.3632$

$(-) = 0.0811$

$\Rightarrow 8.11\%$

$P(3.5 < X < 3.8) = 8.11\%$

6. Draw a random sample of N = 64 from a population with mean $\mu_{pop} = 50$. $\sigma_{pop} = 16$.

Sampling distribution Mean $\cong$ Population mean.

a1) Sample mean $\bar{x} = \frac{\sum_{i=1}^{64} X_i}{64}$.

a2) The Exceptation of $\bar{x}$ $E(\bar{x}) = \mu, \Rightarrow 50$.

Standard deviation of $\bar{X}$ $\sigma_{\bar{x}} = \frac{\sigma_{pop}}{\sqrt{N}} = \frac{16}{\sqrt{64}} = \frac{16}{8} = 2$ $\sigma_{\bar{x}} = \sigma_{SD} = 2$

6 b) Approximately what is the probability that sample mean is above 54

Find the Z score for $M = 54$

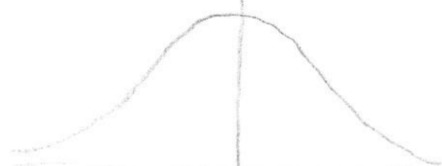
$$Z_{54} = \frac{54 - 50}{2} = \frac{4}{2} = 2 \quad \text{(Area under curve for } 2 \Rightarrow 0.9772$$

What is the $P(X > 54) = 1 - 0.9772$

$P(X > 54) = 0.0228$ (or) 2.28%

6(c) Assumption for (b) to be true is $M_{pop} \cong M_{SD}$
Sample is actually a sample distribution

7. Age of student graduates in Salem is normally distributed



$$\sigma_{SD} = \frac{3.1}{\sqrt{6}} = \frac{3.1}{2.449} = 1.265$$

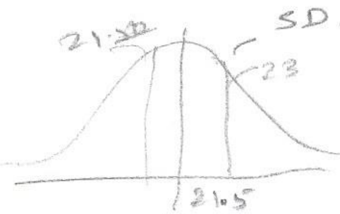
If we assume one of the sample (Mean) is 27
what is the probability that the M_{SD} is 27.

$$Z_{27} = \frac{27 - 23.1}{1.265} = \frac{3.9}{1.265} = 3.083$$

Area under the curve (is probability) = 0.9990

$P(X > 27) = 1 - 0.9990$
$= 0.0010$ (or) 0.1%

8. Average amount spent on food per table was \$21.50 SD of 2.22. What is the probability that 8 checks between \$20 and \$23.



N = 8

mean of sample of 8 checks is \$20. [SD1]
mean of sample of 8 checks is \$23 [SD2]
∴ mean of sample of 8 checks is between \$20 and \$23.

What is the probability that mean of SD is between these amounts.
Find the areas within (enclosed) within these amounts.

$$Z_{20} = \frac{20 - 21.50}{2.22/\sqrt{8}} = \frac{-1.50 \times \sqrt{8}}{2.22} = -1.911 \text{ (Area)} \Rightarrow 0.0281$$

$$Z_{23} = \frac{23 - 21.50}{2.22/\sqrt{8}} = 1.911 \text{ (Area)} \Rightarrow 0.9719$$

$$\begin{array}{r} 0.9719 \\ -\ 0.0281 \\ \hline 0.9438 \end{array}$$

$P(21.5 < X < 23) = 94.38\%$

9. Maths class grades $\mu = 75$ $SD = 5$

a) Randomly selected student was at least 83 $P(X \geq 83)$

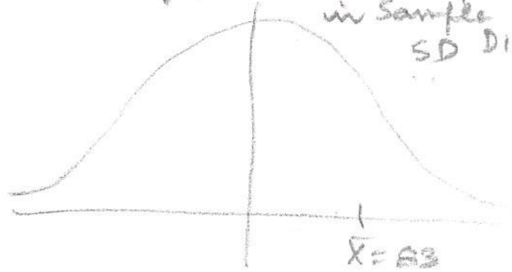
$$Z_{83} = \frac{83-75}{5} = \frac{8}{5} = 1.6$$ (Area under 1.6 is 0.4452)

$P(X \geq 83) = 1 - 0.4452$

$= 0.5548$ or 55.48%

μ_{SD} is mean of sample means.

Every point is a sample mean in sample SD Distribution



b) Sample size 5 students $N = 5$

Probability that sample mean is at least 83?

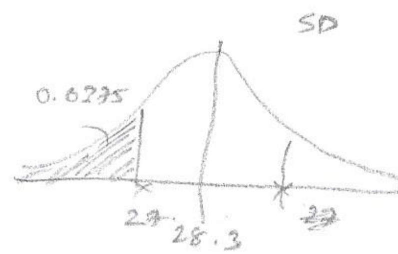
$$Z_{SD\,83} = \frac{83-75}{5/\sqrt{5}} = \frac{8\sqrt{5}}{5} = 1.6\sqrt{5} = 3.577$$

Area under 3.577 is not applicable, so there is no possibility or not probable.

10. Average age of Major League baseball players is 28.3 yrs and SD = 2.3 yrs

Ages are normally distributed. Randomly selected 10 players what is the probability that age < 27.

$\dot{X}$ what is the probability that one of the sample mean is > 27 ($P(\bar{x}) > 27$) among the sampling distribution.



$$Z_{SD\,27} = \frac{27-28.3}{2.3/\sqrt{10}} = \frac{-1.3\sqrt{10}}{2.3} = -1.787$$

Area under $Z(-1.787) = 0.0375$ or 3.75% (Probability) $P(x < 27)$.