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Probability Assignment

Question 1: Two dies are rolled at once. Find the probability for sum of numbers being even and one of the die shows 6

Answer: P(A) = Sum of numbers being even.

P(B) = One of the die shows 6.

P(A AND B) => ?

A and B are dependent events

D1 [6] \ D2	1	2	3	4	5	6
[SUM]	7	(8)	9	(10)	11	(12)

Total combinations of results when 2 dies are rolled.

= 6 × 6 = 36 [sample space]

3 instances when the sum is even.

So probability (D1 => 6 and D1+D2 (even)) = 3/36 = 1/12

P(A and B) = 8.3%

Question 2: Two dies are rolled at once. Find the probability for sum of numbers being less than 7.

Total matrix of possible combinations of results when two dies are rolled at once

Die 1 \ DIE 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(\text{Sum} < 7) = \frac{15}{36} = \frac{5}{12} = 41.6\%$

Question 3: Toss a fair coin three times. Given you have observed at least 1 head what is the probability of observing 2 heads

Answer: Probability of at least 2 heads

P (atleast 2 H) ⇒ ?

For each flip we have 2 possible occurances

So total no of possible combinations / occurances = $2 \times 2 \times 2 \Rightarrow 8$ (sample space)

Given we have already witnessed a Head. we have 2 more flips to predict

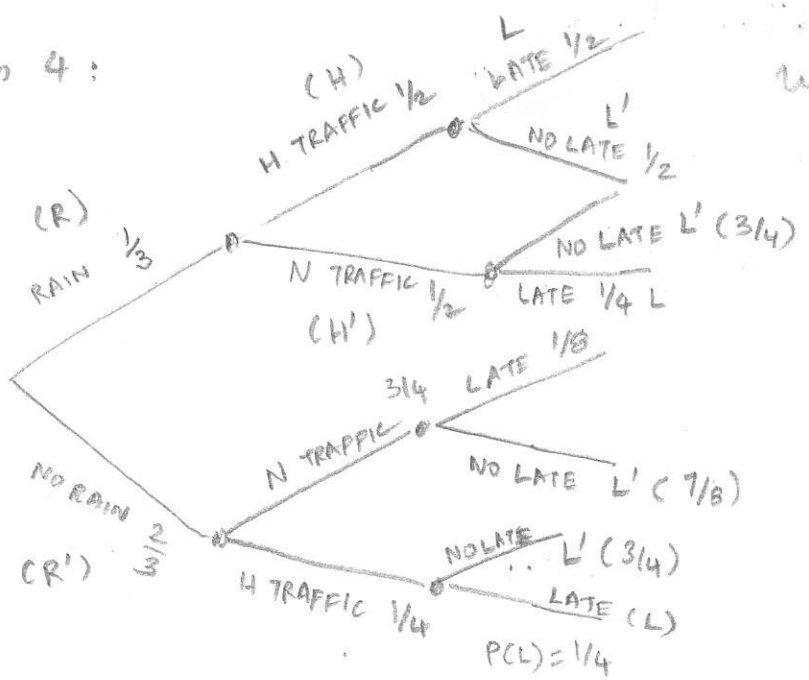
$P(X H_A H_B)$ =

X ⇒ Head already witnessed

1) H (H H)
2) H (H T)
3) H (T H)
4) H (T T)

So probability of at least 2 H = 3/4 = 75%

Question 4:



What is the probability

NOT RAINING, HEAVY TRAFFIC AND NOT LATE

$P(R' \cap H \cap L')$

$= P(R') \times P(H/R') \times P(L'/R'H)$

$= \frac{2}{3} \times \frac{1}{4} \times \frac{3}{4} \Rightarrow \frac{1}{8}$

(b) What is the probability that I am late?

$P(L) = P(\text{All the four probabilities of late scenarios})$

$= P(R'HL) + P(R'H'L) + P(RHL) + P(RH'L)$

$= \frac{2}{3}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{2}{3}\cdot\frac{3}{4}\cdot\frac{1}{8} + \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{4}$

$= \frac{1}{24} + \frac{1}{16} + \frac{1}{12} + \frac{1}{24}$

$= \frac{1}{6} + \frac{1}{16}$

$= \frac{11}{48}$

(c) $P(R/L) = \frac{P(R \text{ AND } L)}{P(L)}$

$= \frac{P(RHL) + P(RH'L)}{11/48}$

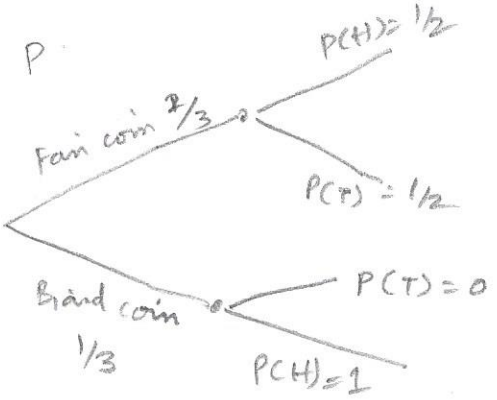
$= \frac{\frac{1}{12} + \frac{1}{24}}{11/48}$

$= \frac{3/24}{11/48}$

$= \frac{1/8}{11/48}$

$= \frac{1}{8} \times \frac{48}{11} \Rightarrow \frac{8}{11} \Rightarrow 72.7\%$

5) Question: Box has 3 coins, 2 fair and 1 biased ($P(H) \Rightarrow 1$)
Pick at random and toss it. What is the probability of Heads



$$P(H) = P(H/F) + P(H/B)$$
$$= \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1$$
$$= \frac{1}{3} + \frac{1}{3}$$
$$P(H) = \frac{2}{3}$$

(b) If a coin at random, is tossed up and it gets heads, what is the probability it is a two-headed (Biased) coin

$$P(B/H) = \frac{P(B \text{ and } H)}{P(H)}$$
$$= \frac{1/3 \times 1}{2/3}$$
$$= \frac{1/3}{2/3}$$
$$P\left(\frac{\text{Biased coin}}{\text{Head}}\right) = \frac{1}{2} = 50\%$$

6) Question: Customers of a coffee shop

$P(coc) = 70\% = 0.7$
$P(poc) = 40\% = 0.4$
$P(coc \cap poc) = 20\% = 0.2$

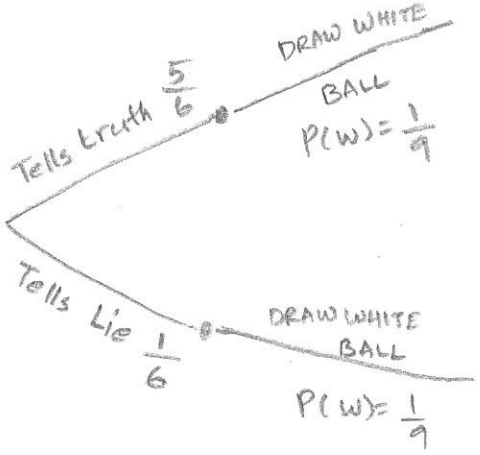
$$P(coc/poc) = \frac{P(coc \text{ AND } poc)}{P(poc)}$$

$$P\left(\frac{\text{coffee}}{\text{cake}}\right) = \frac{0.2}{0.4} = \frac{1}{2} = 50\%$$

Question 11: P(A tells Truth) = $\frac{5}{6}$

White ball is drawn from a bag (8 black + 1 white ball)

Find the probability that white ball was drawn.



$$P(W) = P(Th) \cdot P(W/Th) + P(Lie) \cdot P(W/Lie)$$

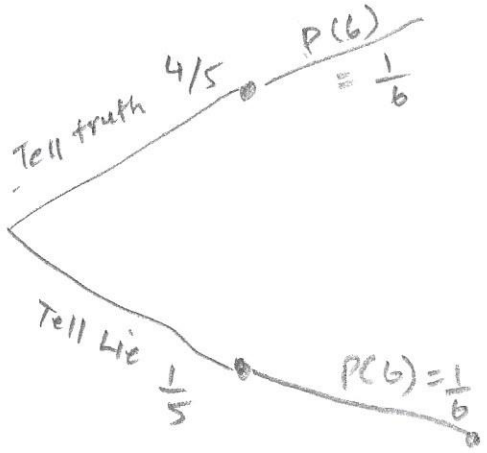
$$= \frac{5}{6} \times \frac{1}{9} + \frac{1}{6} \times \frac{1}{9}$$

$$= \frac{5}{54} + \frac{1}{54}$$

$$P(W) = \frac{6}{54} = \frac{1}{9} = 0.11 \text{ (or) } 11\%$$

Question 12: A speaks truth 4 out of 5 times. A die is tossed, and it is 6.

What are the chances it was actually 6.



$$P(6) = \frac{1}{6} \times \frac{4}{5} + \frac{1}{6} \times \frac{1}{5}$$

$$= \frac{2}{15} + \frac{1}{30}$$

$$= \frac{4+1}{30}$$

$$P(6) = \frac{5}{30} \Rightarrow \frac{1}{6} \text{ (or) } 16.7\%$$