ບົດແກ້

1\. $x, y \in I$ $\quad x + xy + y = 9$

$$x(1+y) + (1+y) = 10$$

$$(1+y)(1+x) = 10$$

x	0	1	9	4	-2	-3	-11	-6
y	9	4	0	1	-11	-6	-2	-3

$$S = \{(0,9); (1,4); (9,0); (4,1); (-2,-11); (-3,-6); (-11,-2); (-6,-3)\}$$

2\. $m = ?$ $\quad x^3 - mx^2 - 6x - 8 = 0$; $x_1 \neq x_2 \neq x_3$

ໂດຍວ່າ x_1; x_2; x_3 ປະກອບເປັນອັນດັບທະວີຄູນ

ເຊິ່ງເຮົາໄດ້ວ່າ $x_2 = x_1 r$ ແລະ $x_3 = x_1 r^2$

ຕາມຫຼັກການວີແອັດ ~~ເຊິ່ງ~~ ເຮົາມີລາຍ

$$\begin{cases} x_1 x_2 x_3 = 8 & (1) \\ x_1 + x_2 + x_3 = -6 & (2) \\ x_1 x_2 + x_1 x_3 + x_2 x_3 = m & (3) \end{cases}$$

ພິຈາລະນາ (1); $x_1 x_2 x_3 = 8$ ໄດ້ວ່າ $x_1 (x_1 r)(x_1 r^2) = 8$

$$(x_1 r)^3 = 2^3$$

$$x_1 r = 2 = x_2$$

ພິຈາລະນາ (3); $x_1 x_2 + x_1 x_3 + x_2 x_3 = m$

ໄດ້ $\quad 2x_1 + \cancel{x} x_1 x_3 + 2x_3 = m$

$$2(x_1 + x_3) + x_1 x_3 = m$$

ເມື່ອ $x_2 = 2$ ຈາກ (1) ເຮົາໄດ້ $x_1 x_3 = 4$

ເມື່ອ $x_2 = 2$ ຈາກ (2) ເຮົາໄດ້ $x_1 + x_3 = -8$

ເມື່ອນັ້ນ ເຮົາໄດ້ $m = -12$

3. ອັນດັບ $\{u_n\}$ ກຳນົດດ້ວຍ $\begin{cases} u_1 = 11 \\ u_{n+1} = 10u_n + 1 - 9n \end{cases}$ $; \forall n \in \mathbb{N}$

- $u_{n+1} = 10u_n + 1 - 9n$

$u_{n+2} = 10u_{n+1} - 8 - 9n$

ໄດ້ $u_{n+2} - u_{n+1} = 10(u_{n+1} - u_n) - 9$

ໃຫ້ $V_n = u_{n+1} - u_n$; $V_{n+1} = 10V_n - 9$

$\because$ $V_{n+1} = pV_n + q$; $\alpha = \frac{q}{1-p}$ ເຊິ່ງເຮົາໄດ້ $V_n = (V_1 - \alpha)p^{n-1} + \alpha$

ເຊິ່ງເຮົາໄດ້ $V_n = 9 \cdot 10^n + 1$

ໂດຍວ່າ $u_{n+1} - u_n = V_n$ ເຮົາໄດ້ $u_n = u_1 + \sum_{k=1}^{n-1} V_k$

$u_n = 11 + \sum_{k=1}^{n-1}(9 \cdot 10^k + 1) = 11 + 10^n - 10 + n - 1$

$u_n = 10^n + n$

4. ແກ້ສົມຜົນ:

$(1 + \tan x)\cos^3 x + (1 + \cot x)\sin^3 x = \sqrt{2\sin 2x}$

$\cos^3 x + \sin x \cos^2 x + \sin^3 x + \cos x \sin^2 x = \sqrt{2\sin 2x}$

$\cos^2 x(\cos x + \sin x) + \sin^2 x(\sin x + \cos x) = \sqrt{2\sin 2x}$

$(\sin x + \cos x)(\sin^2 x + \cos^2 x) = \sqrt{2\sin 2x}$

$\sin x + \cos x = \sqrt{2\sin 2x}$ ຍ2

$1 + \sin 2x = 2\sin 2x$

$\sin 2x = 1$

ຈະໄດ້ $2x = \frac{\pi}{2} + 2\pi k$, $k \in I$

$\therefore S = \left\{\frac{\pi}{4} + \pi k\right\}$; $k \in I$

5. $\{x_k\}$ ສຳລັບ : $x_k = \frac{1}{2!} + \frac{2}{3!} + \dots + \frac{k}{(k+1)!}$

ຈົ່ງຫາ $\lim_{n\to\infty} \sqrt[n]{x_1^n + x_2^n + \dots + x_{2012}^n}$

ພິຈາລະນາ $x_k = \frac{1}{2!} + \frac{2}{3!} + \dots + \frac{k}{(k+1)!}$

$$x_k = \left(\frac{1}{1!} - \frac{1}{2!}\right) + \left(\frac{1}{2!} - \frac{1}{3!}\right) + \dots + \left(\frac{1}{k!} - \frac{1}{(k+1)!}\right)$$

ໄດ້ $x_k = 1 - \frac{1}{(k+1)!}$

ໃຫ້ $L = \lim_{n\to\infty} \sqrt[n]{x_1^n + x_2^n + \dots + x_{2012}^n}$

$$\ln(L) = \lim_{n\to\infty} \frac{\ln(x_1^n + x_2^n + \dots + x_{2012}^n)}{n}$$

ນຳໃຊ້ ໂລປີຕານ

$$\ln(L) = \lim_{n\to\infty} \frac{x_1^n \ln(x_1) + x_2^n \ln(x_2) + \dots + x_{2012}^n \ln(x_{2012})}{x_1^n + x_2^n + \dots + x_{2012}^n}$$

$$\ln(L) = \lim_{n\to\infty} \frac{\ln(x_1)}{1 + \left(\frac{x_2}{x_1}\right)^n + \dots + \left(\frac{x_{2012}}{x_1}\right)^n} + \dots + \frac{\ln(x_{2012})}{\left(\frac{x_1}{x_{2012}}\right)^n + \left(\frac{x_2}{x_{2012}}\right)^n + \dots + 1}$$

ເນື່ອງຈາກ $x_{k+1} > x_k$ ສຳລັບ $\forall_k \in \mathbb{N}$

ເພື່ອສະນັ້ນໄດ້ $\ln(L) = \ln(x_{2012})$

$$L = x_{2012} = 1 - \frac{1}{2013!}$$

ດັ່ງນັ້ນໄດ້ $\lim_{n\to\infty} \sqrt[n]{x_1^n + x_2^n + \dots + x_{2012}^n} = 1 - \frac{1}{2013!}$

6. $f(x) = 2x^2 + x - 2$, ຈົ່ງພິສູດວ່າ $f(f(x)) - x$ ຫານລົງໃຫ້ $2x^2 + 2x - 1$.

ພິຈາລະນາ $f(f(x)) - x = f(2x^2 + x - 2) - x$

$$= 2(2x^2 + x - 2)^2 + (2x^2 + x - 2) - 2 - x$$

$$f(f(x)) - x = 2(2x^2 + x - 2)^2 + 2x^2 - 4$$

ຈາກ ສຳລັບ $\forall_x$ ທີ່ເຮັດໃຫ້ $2x^2 + 2x - 1 \in \mathbb{Z}$

ສຳລັບ $\forall_x$ ທີ່ເຮັດໃຫ້ $2(2x^2 + x - 2)^2 + 2x^2 - 4 \in \mathbb{Z}$

ຕາມທິດສະດີບົດ ແລະ ຄຸນສົມບັດເສດເຫຼືອໄດ້

$$2(2x^2 + x - 2)^2 + 2x^2 - 4 \equiv r(x) \mod (2x^2 + 2x - 1)$$

$$2(2x^2 + x - 2 - (2x^2 + 2x - 1))^2 + 2x^2 - 4 - (2x^2 + 2x - 1) \equiv r(x) \mod (2x^2 + 2x - 1)$$

$$2(x + 1)^2 - 2x - 3 \equiv r(x) \mod (2x^2 + 2x - 1)$$

$$2(x^2 + 1 + 2x) - 2x - 3 \equiv r(x) \mod (2x^2 + 2x - 1)$$

$$2x^2 + 4x + 2 - 2x - 3 \equiv r(x) \mod (2x^2 + 2x - 1)$$

$$2x^2 + 2x - 1 \equiv r(x) \mod (2x^2 + 2x - 1)$$

$$r(x) \equiv 0 \mod (2x^2 + 2x - 1)$$

ດັ່ງນັ້ນພິສູດໄດ້ວ່າ ຫານລົງແທ້