ECE5463: Introduction to Robotics

Lecture Note 2: Configuration Space

Prof. Wei Zhang

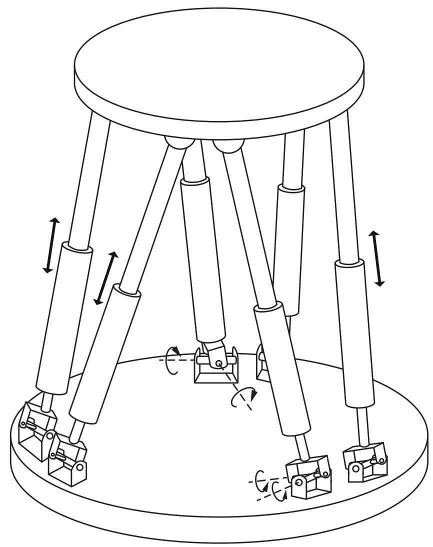
Department of Electrical and Computer Engineering
Ohio State University
Columbus, Ohio, USA

Spring 2018

Outline

- Mechanical Structure of a Robot
- Configuration Space
- Representation of Configuration Space

Typical Mechanical Structure



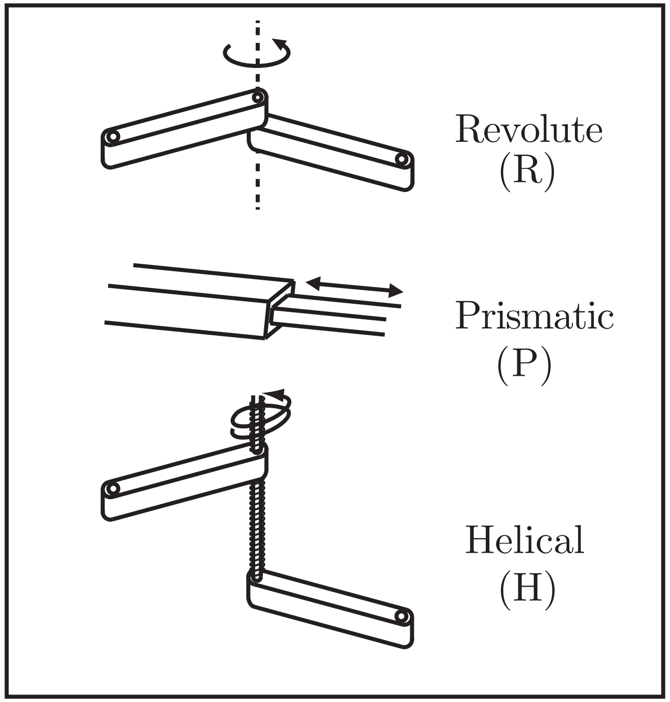
- A robot is mechanically constructed by connecting a set of bodies, called **links**, to each other using various types of **joints**.

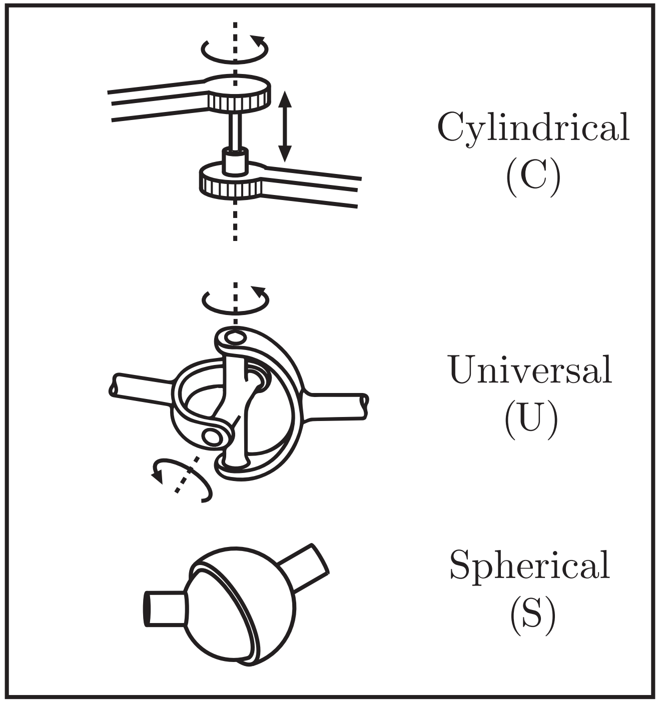
- Links are usually modeled as rigid bodies

- **Actuators**, such as electric motors, deliver forces and torques to the joints, thereby causing motion of the robot

- **End-effector**, such as gripper or hand, is attached to a specific link

Typical Joints





- Revolute Joint (R): hinge joint
- Prismatic Joint (P): linear joint
- Helical Joint (H): screw joint
- Cylindrical Joint (C): independent translation and rotation
- Universal Joint (U):
- Spherical Joint (S): ball and socket joint

Outline

- Mechanical Structure of a Robot
- Configuration Space
- Representation of Configuration Space

Configuration Space Definitions

- **Configuration**: a complete specification of the position of every point of the robot. geometry / structure information of the robot is know a priori

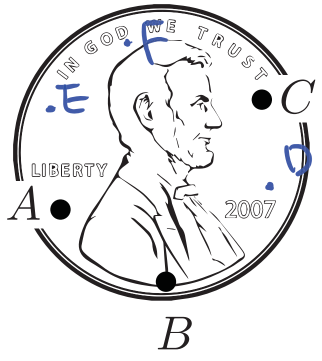
- **Degree of Freedom (dof)**: The minimum number of real-valued coordinates needed to represent the configuration

- **Configuration Space (C-space):** The space (set) that contains all possible configurations of the robot.

- Effective representation of the C-space is essential for many aspects of robotics

How to find the dof?

- Example: coin on a table



consider a few points on the coin. (coin's geometry is known e.g. we know dist(A,B) is known)

the coordinates of all the points can not be specified independently

- The first pt A (x_A, y_A) can be chosen arbitrarily $\Rightarrow$ dof ≥ 2

- Then the coordinate of , (x_B, y_B) must satisfy $\underbrace{d_{AB} = \sqrt{(x_A-x_B)^2+(y_A-y_B)^2}}_{1 \text{ constraint}}$

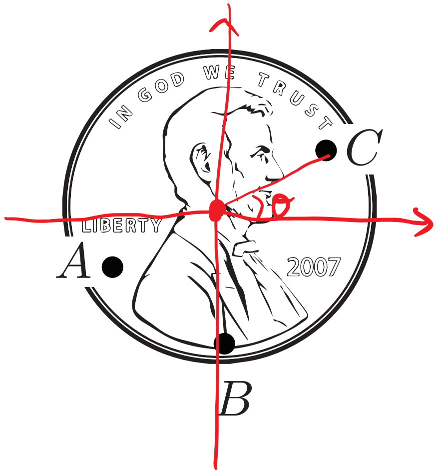
  $\Rightarrow (x_B, y_B)$ ⊕ constraint $\Rightarrow$ we have 1-more dof

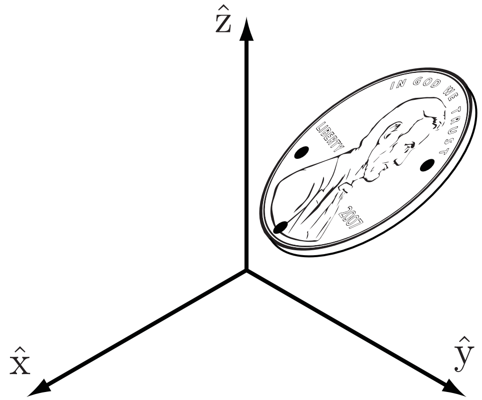
- After specifying (x_A, y_A) and (x_B, y_B), all other points' coordinates are fully determined (i.e. can be determined based (x_A, y_A) & (x_B, y_B))

- overall, dof $= 3$

DoF of Planar and Spatial Rigid Body

Coin in $\mathbb{R}^3$





- planar rigid body has dof = 3

similarly, we can find out that

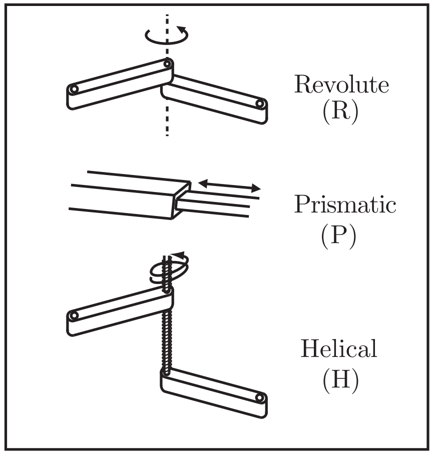
Spatial rigid body in R^3

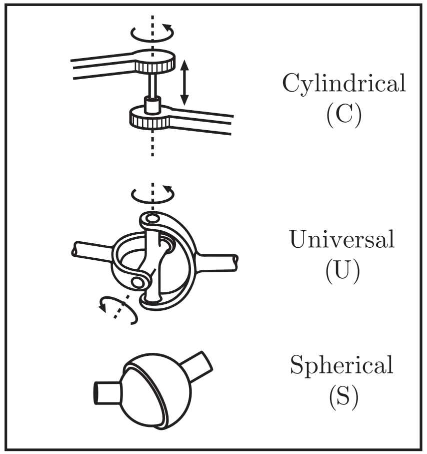
dof = 6

DoF of Joints

- Joint can be viewed as providing freedoms to allow one rigid body to move relative to another.
- Dof of a joint: minimum # of variables needed to represent the configuration of a joint
- Joint can also be viewed as providing constraints on the possible motions of the two rigid bodies it connects

these numbers are only true for joints that connect 2 bodies





Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

DoF of Mechanisms (Linkages)

$$\boxed{\text{dof} = (\text{sum of freedoms of the bodies}) - \text{number of } \textbf{independent} \text{ constraints}} \tag{1}$$

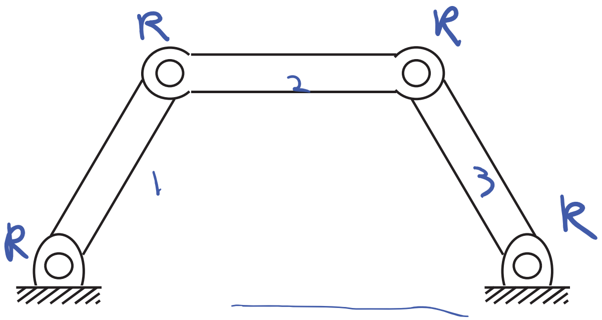
only true if all constraints are indep

- **Grübler's Formula:** dof $= m(N-1-J) + \sum_{i=1}^{J} f_i$

- N: # of links (including ground/frame as a link)
- m: # of dof of a body (m=3 for planar, m=6 for spatial)
- J: # of Joints
- f_i: # of dof of ith joint
- C_i: # of constraint provided by ith joint
  - Note: $f_i + C_i = m$

$$\text{dof} = m\cdot(N-1) - \sum_{i=1}^{J} C_i$$
$$= m\cdot(N-1) - \sum_{i=1}^{J}(m - f_i)$$
$$= m(N-1-J) + \sum_{i=1}^{J} f_i$$

DoF Examples



$m=3$

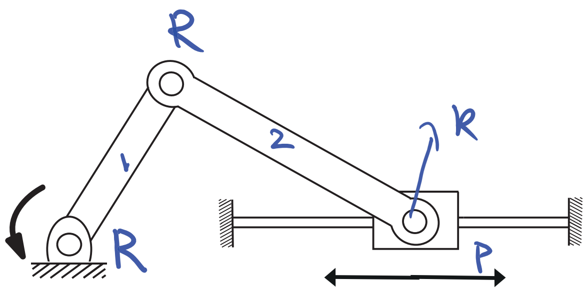
$N=4$

$J=4$

$f_i=1, \quad i=1,2,3,4$

$dof = 3(4-1-4)+1\cdot 4$

$= -3+4 = 1$



$N=4 \qquad m=3$

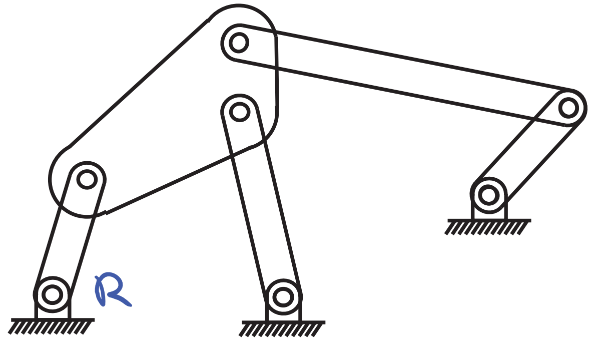
$J=4$

$f_1=f_2=f_3=1$

$f_4=1$

$dof = 3(4-1-4)+1\cdot 4$

$=1$

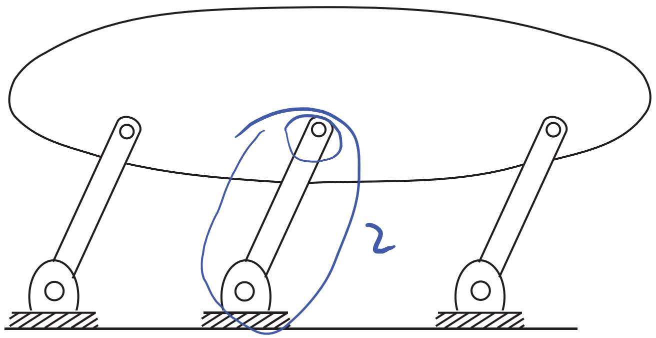


- $N=6$
- $J=7$
- $f_i=1$

$dof = 3(6-1-7)+7\cdot(1)$

$= 3\times(-2)+7 = 1$

DoF Examples



Blindly using Grübler's formula

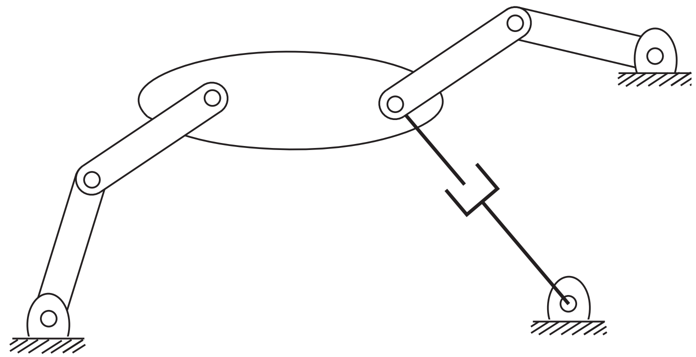
$N = 5$, $J = 6$, $f_i = 1$

dof $\cancel{=}$ $3(5-1-6) + 6\cdot(1)$

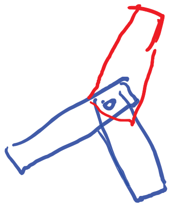
$= 3\cdot(-2) + 6 = 0$

Assuming all links are the same

In this case, link 2 is redundant, & the joint does not provide indep constraint, can be viewed as if there was no link 2



Consider a joint that connects three bodies



dof $= 3+1+1 = 5$, equivalently, this joint provides $9-5=4$ constraints

or equivalently, we can view 3-body joint as 2 2-body joint staying on top of each other

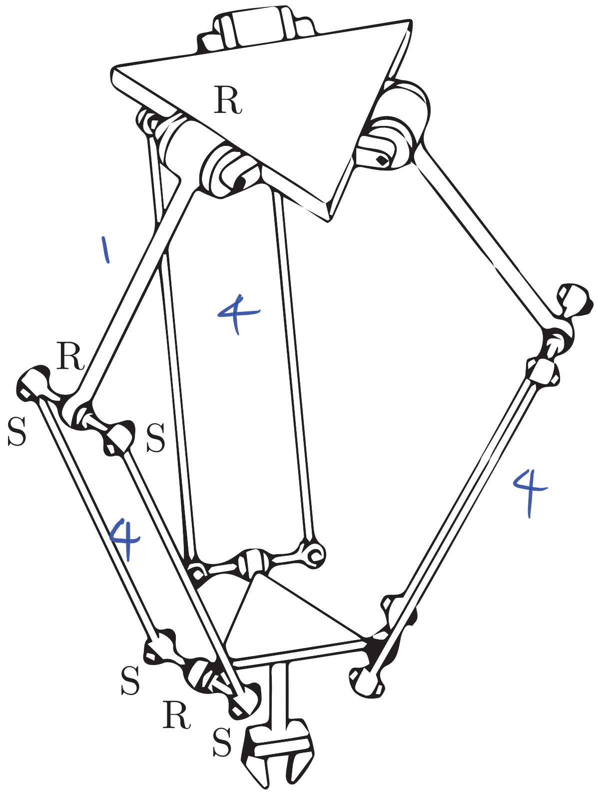
$N = 7+1 = 8$, $J = 8(R) + 1(P)$,

dof $= 3(8-1-9) + 9 = 3\times(-2) + 9 = 3$

$N = 4$, $J = 4$, $f_i = 1$

dof $= 3\cdot(4-1-4) + 4 = 1$

DoF Examples



Delta Robot :

$N = 5 \times 3 + 1 + 1 = 17$

$J = 3 \times 3\ (R) + 4 \times 3\ (S)$
$= 9(R) + 12(S)$

$f_R = 1\ ,\ f_S = 3$

$dof = 6 \times (17 - 1 - 21) + 9 \times (1) + 12 \times (3)$
$= 6 \times (-5) + 9 + 36 = 45 - 30 = 15$

Outline

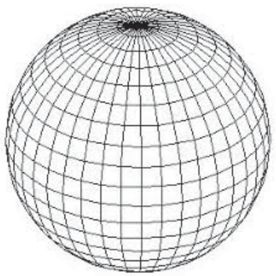
- Mechanical Structure of a Robot
- Configuration Space
- Representation of Configuration Space

Issues for Explicit Parameterization

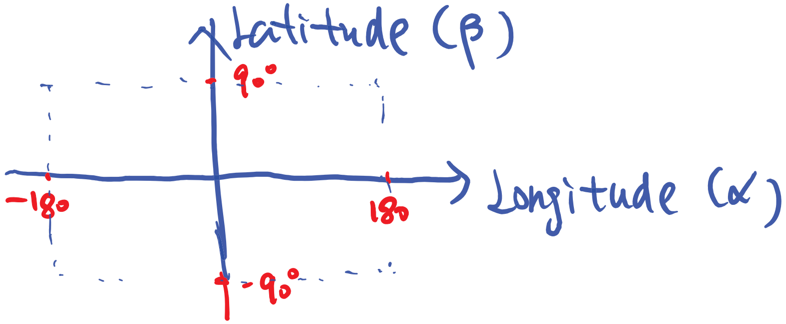
- Representation of Euclidean space: choose reference frame and represent point as a vector
- Representation of *curved space* is more tricky than it appears
- **Explicit parameterization** uses the same number of coordinates as the space dimension (suffer from singularity)

dof

Sphere $\mathcal{S}^2$



2-dim space, can be represented by two coordinates (Lat/Lon)



① only valid for a range, need bookkeeping steps to reset coordinate

② problematic in representing velocity as time rate of change of coordinate $(\dot{\alpha}, \dot{\beta})$

suppose one travels as a constant speed, but longitude change $\dot{\alpha}$ becomes unbounded as β approaches North/South pole

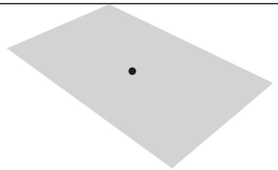
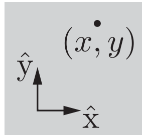
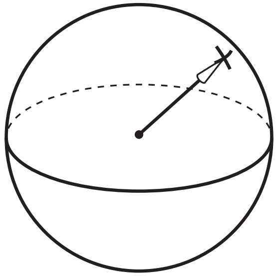
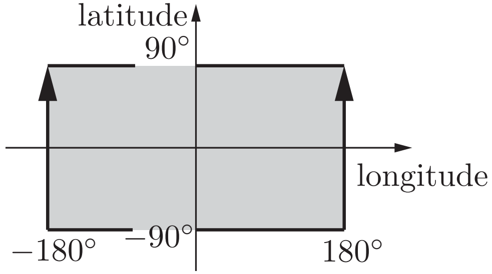
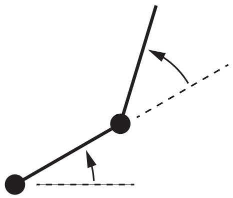
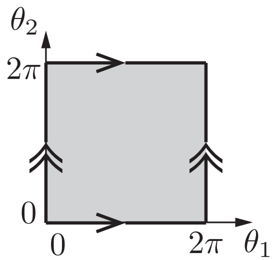
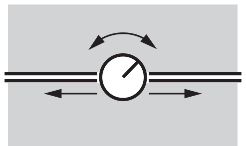
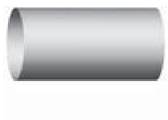
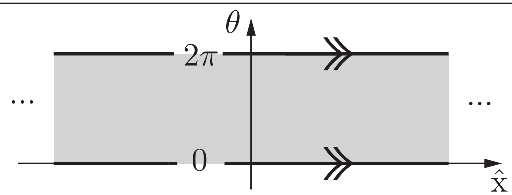
Topology

- Explicit parameterization of sphere (latitude/longitude) suffers from singularity because sphere and plane have different topologies.
- Roughly, two spaces are *topologically equivalent* if one can be continuously deformed into the other without cutting or gluing.

- Topologically **distinct** 1-d spaces:
  - **circle**: S (1-dim sphere)
  - **line**: $E = \mathbb{R}$ $\Longleftrightarrow$ a b $\longleftrightarrow$ $(a,b) \subset \mathbb{R}$
  - **closed interval**: $[a,b]$

Topology

- Examples of topologically different 2-d spaces

system	topology	sample representation
 point on a plane	 $\mathbb{E}^2$	 $\mathbb{R}^2$
 spherical pendulum	 S^2	 $[-180°, 180°) \times [-90°, 90°]$
 2R robot arm	 $T^2 = S^1 \times S^1$	 $[0, 2\pi) \times [0, 2\pi)$
 rotating sliding knob	 $\mathbb{E}^1 \times S^1$	 $\mathbb{R}^1 \times [0, 2\pi)$

S^n: n-dim sphere

surface of ball in R^{n+1}

n-dim

$T^n = \underbrace{S \times S \times \cdots \times S}_{n}$

$\neq S^n$

Implicit Representation of C-Space

- **Implicit Representation:** View n-dim space as embedded in a higher dimensional Euclidean space subject to constraints.

- Use more coordinates than the minimum, but can avoid singularities

- Example: Sphere $\mathcal{S}^2$

$$S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\} ,$$

View S^2 as a surface embedded in 3d space

It use more variables than dof of S^2, but there are no singularities.

A point moving smoothly around the sphere is represented by smoothly changing (x, y, z) even at North/South poles

- In this class, we will primarily use the implicit representation

Summary Questions

- What is the configuration space (C-space) of a robot?
- What is dof of C-space, and how to find dof?
- What is topological equivalence?
- Pros and cons for explicit and implicit representation of C-space
- Further reading: chapter 2 of Lynch and Park.