ECE5463: Introduction to Robotics

Lecture Note 5: Velocity of a Rigid Body

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Outline

- Introduction
- Rotational Velocity
- Change of Reference Frame for Twist (Adjoint Map)
- Rigid Body Velocity

Introduction

- For a moving particle with coordinate $p(t) \in \mathbb{R}^3$ at time t, its (linear) velocity is simply $\dot{p}(t)$

- A moving rigid body consists of infinitely many particles, all of which may have different velocities. What is the velocity of the rigid body?

- Let $T(t)$ represent the configuration of a moving rigid body at time t. A point p on the rigid body with (homogeneous) coordinate $\tilde{p}_b(t)$ and $\tilde{p}_s(t)$ in body and space frames:

$$\tilde{p}_b(t) \equiv \tilde{p}_b, \quad \tilde{p}_s(t) = T(t)\tilde{p}_b$$

Introduction

- Velocity of p is $\frac{d}{dt}\tilde{p}_s(t) = \dot{T}(t)p_b$

- $\dot{T}(t)$ is not a good representation of the velocity of rigid body
  - There can be 12 nonzero entries for $\dot{T}$.

$$\begin{bmatrix} 3\times 3 & 3\times 1 \\ 0 \;\; 0 \;\; 0 & 1 \end{bmatrix}$$

  - May change over time even when the body is under a constant velocity motion (constant rotation + constant linear motion)

- Our goal is to find effective ways to represent the rigid body velocity.

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Illustrating Example

- **Question:** Given the orientation $R(t)$ of a rotating frame as a function of time t, what is the the angular velocity?

- We start with an example for which we know the answer, then we generalize to obtain a formal answer

- **Example:** Suppose {b} starts with an initial orientation $R(0)$ and rotates about $\hat{\mathrm{x}}$ at unit constant speed (i.e. we know the angular velocity at time $t > 0$ is $\omega = (1, 0, 0)^T$), where

$$R(0) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad Rot(\hat{\mathrm{x}}_{s}; \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\theta & -s_\theta \\ 0 & s_\theta & c_\theta \end{bmatrix}$$

  Consider a point p rigidly attached to frame {b} with coordinates $p_s(t)$ and $p_b(t)$ in {s} and {b} frames.

Illustrating Example (Continued)

$R(t) = R_{sb}(t)$

- $p_s(t) = R(t)p_b \Rightarrow \dot{p}_s(t) = \dot{R}(t)p_b = \dot{R}(t)R^{-1}(t)p_s(t) = \boxed{\dot{R}(t)R^T(t)}\,p_s(t) \quad \cdots \text{①}$

From previous slide we know

$$R(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos t & -\sin t \\ 0 & \sin t & \cos t \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -\sin t & -\cos t & 0 \\ \cos t & -\sin t & 0 \end{bmatrix} \Rightarrow \dot{R}(t) = \begin{bmatrix} 0 & 0 & 0 \\ -\cos t & \sin t & 0 \\ -\sin t & -\cos t & 0 \end{bmatrix}$$

- Since we know the motion in this example, we must have $\dot{p}_s(t) = \omega_s \times p_s(t)$, where $\omega_s = (1, 0, 0)$ $= [\omega_s]\,p_s(t) \quad \cdots \text{②}$

By ① and ②, we know $\dot{R}(t)R^T(t) = [\omega_s] \quad \cdots \text{③}$

- Conclusion:

Please numerical verify ③ using the numbers

$$\Rightarrow \dot{R}(t)R^T(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Properties of Rotation Matrices

- **Property:** For any $\omega \in \mathbb{R}^3$ and $R \in SO(3)$, we have

$$R[\omega]R^T = [Rw] \; \cancel{\therefore}$$

see textbook page 66

- **Property:** Let $R(t) \in SO(3)$ be differentiable in t, then $\dot{R}(t)R^{-1}(t)$ and $R^{-1}(t)\dot{R}(t)$ are both skew symmetric, i.e. they are in $so(3)$.

We know $R(t)R^T(t) = I \Rightarrow \frac{d}{dt}\left(R(t)R^T(t)\right) = 0 \leftarrow$ zero matrix

$$\Rightarrow \dot{R}(t)R^T(t) + R(t)\dot{R}^T(t) = 0 \Rightarrow \dot{R}(t)R^T(t) = -R(t)\dot{R}^T(t)$$
$$= -\left(\dot{R}(t)R^T(t)\right)^T$$

$\Rightarrow \dot{R}(t)R^T(t) \in so(3)$

skew symmetric

Similarly, differentiating $R^T(t)R(t) = I \Rightarrow R^{-1}(t)\dot{R}(t)$ is skew symmetric.

Rotational Velocity Representation

- **Rotational Velocity in space frame:** Let $R_{sb}(t)$ be the orientation of a rotating frame {b} at time t. Then the (instantaneous) angular velocity vector w of frame {b} is given by

$$[\omega_s] = \dot{R}_{sb} R_{sb}^{-1}$$

where ω_s is the {s}-frame coordinate of w.

$$p_s(t) = R_{sb}(t) p_b \Rightarrow \dot{p}_s(t) = \dot{R}_{sb}(t) p_b = \underbrace{\dot{R}_{sb}(t) R_{sb}^{-1}(t)} p_s(t)$$

$$\Updownarrow$$

$$\dot{p}_s(t) = w_s \times p_s(t) = [w_s] p_s(t)$$

- Note the angular velocity w is a free vector, which can be represented in different frames.
- Its coordinates ω_c and ω_d in frames {c} and {d} satisfy

$$\omega_c = R_{cd} \omega_d$$

Rotational Velocity in Body Frame

- **Rotational velocity in body frame:** Consider the same set up as the previous slide where $R_{sb}(t)$ is the orientation of the rotating frame $\{b\}$.
  - ω_b denotes the body-frame representation of w, i.e. $\omega_b = R_{bs}(t)\omega_s = R_{sb}^{-1}(t)\omega_s$

$$\Rightarrow [\omega_b] = R_{sb}^{-1}\dot{R}_{sb}$$

Given $R_{sb}(t) \Rightarrow [w_s] = \dot{R}_{sb}R_{sb}^T$

$$w_s = R_{sb}w_b \Rightarrow [w_s] = [R_{sb}w_b] = \dot{R}_{sb}R_{sb}^T$$

$$\Rightarrow R_{sb}[w_b]R_{sb}^T = \dot{R}_{sb}R_{sb}^T$$

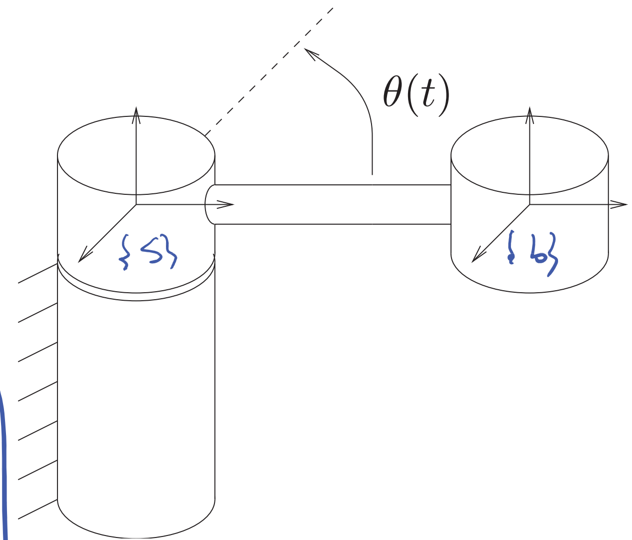
$$\Rightarrow [w_b] = R_{sb}^{-1}\dot{R}_{sb}$$

  - Note: ω_b is NOT the angular velocity relative to a moving frame. It is rather the velocity relative to the *stationary* frame that is instantaneously coincident with the rotating body frame.

Example of Rotational Velocity

$$\dot{R}_{sb} = \begin{bmatrix} -\sin\theta(t) & -\cos\theta(t) & 0 \\ \cos\theta(t) & -\sin\theta(t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}$$

$$R = R_{sb}$$



$$R(t) = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ \sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{R}R^T = \dot{\theta} \begin{bmatrix} -\sin\theta(t) & -\cos\theta(t) & 0 \\ \cos\theta(t) & -\sin\theta(t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta(t) & \sin\theta(t) & 0 \\ -\sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \dot{\theta} \cdot \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = [w_s] \quad \text{where} \quad w_s = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix}$$

$$w_b = R_{sb}^{-1} w_s = \begin{bmatrix} C_\theta & S_\theta & 0 \\ -S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix}$$

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Change of Reference Frame for Twist

- Given two frames {c} and {d} with $T = (R, p)$ representing the configuration of {d} relative to {c}. The same rigid body motion can be represented in {c} or in {d} using the twist $\mathcal{V}_c = (\omega_c, v_c)$ or $\mathcal{V}_d = (\omega_d, v_d)$, respectively. How do these two twists relate to each other?

Let q be a point on the rigid body.

$\mathcal{V}_c = \begin{bmatrix} \omega_c \\ v_c \end{bmatrix}$ in {c} means $\Leftrightarrow$ $\dot{q}_c(t) = \omega_c \times q_c(t) + v_c$

$\mathcal{V}_d = \begin{bmatrix} \omega_d \\ v_d \end{bmatrix}$ in {d} $\Leftrightarrow$ $\dot{q}_d(t) = \omega_d \times q_d(t) + v_d$

$= [\omega_d] q_d(t) + v_d$

$\dot{\tilde{q}}_c(t) = [\mathcal{V}_c] \tilde{q}_c(t)$

$\dot{\tilde{q}}_d(t) = [\mathcal{V}_d] \tilde{q}_d(t)$

where $[\mathcal{V}_d] = \begin{bmatrix} [\omega_d] & v_d \\ 0 & 0 \end{bmatrix}$

$\Rightarrow T_{dc} \dot{\tilde{q}}_c(t) = [\mathcal{V}_d] T_{dc} \tilde{q}_c(t)$

$\Rightarrow \dot{\tilde{q}}_c(t) = T_{cd} [\mathcal{V}_d] T_{cd}^{-1} \tilde{q}_c(t)$

$$\Rightarrow \boxed{[\mathcal{V}_c] = T_{cd} [\mathcal{V}_d] T_{cd}^{-1}} \quad \cdots \quad ①$$

Change of Reference Frame for Twist (Continued)

We know $T_{cd} = T \doteq \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$

$$- [RW_d]\, p = -RW_d \times p = p \times RW_d = [p] RW_d$$

$$① \Rightarrow \begin{bmatrix} [w_c] & v_c \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} [w_d] & v_d \\ 0 & 0 \end{bmatrix} \begin{bmatrix} R^T & -R^T p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \underbrace{R[w_d]R^T}_{[RW_d]} & -R[w_d]R^T p + R v_d \\ 0 & 0 \end{bmatrix}$$

$$\Longleftrightarrow \quad = \begin{bmatrix} [RW_d] & [p] RW_d + R v_d \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow W_c = RW_d$

and $V_c = [p] R W_d + R V_d$

$\Downarrow$

- $\Rightarrow \begin{bmatrix} \omega_c \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \begin{bmatrix} \omega_d \\ v_d \end{bmatrix} \quad \cdots ②$

Adjoint Map

- Given $T = (R, p) \in SE(3)$, its **adjoint representation (adjoint map)** $[\mathrm{Ad}_T]$ is

$$[\mathrm{Ad}_T] = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \in \mathbb{R}^{6\times 6}$$

- Adjoint map changes reference frames for twist vector. If T is configuration of {d} relative to {c}, then the twists $\mathcal{V}_c$ and V_d in two frames are related by

$T = T_{cd}$

$$\mathcal{V}_c = [\mathrm{Ad}_T]\mathcal{V}_d \quad \text{or equilvalently} \quad \boxed{[\mathcal{V}_c] = T[V_d]T^{-1}}$$

$\mathcal{V}_c = [\mathrm{Ad}_{T_{cd}}]\mathcal{V}_d$

- **Properties of Adjoint:**
  - Given $T_1, T_2 \in SE(3)$ and $\mathcal{V} = (\omega, v)$, we have

$$[\mathrm{Ad}_{T_1}][\mathrm{Ad}_{T_2}]\mathcal{V} = [\mathrm{Ad}_{T_1 T_2}]\mathcal{V}$$

e.g. $T_1 = T_{ab}$, $T_2 = T_{bc}$ $\quad T_1 T_2 = T_{ac} \Rightarrow [\mathrm{Ad}_{T_1T_2}]\mathcal{V}_c = \mathcal{V}_a$

  - For any $T \in SE(3)$,

$$[\mathrm{Ad}_T]^{-1} = [\mathrm{Ad}_{T^{-1}}]$$

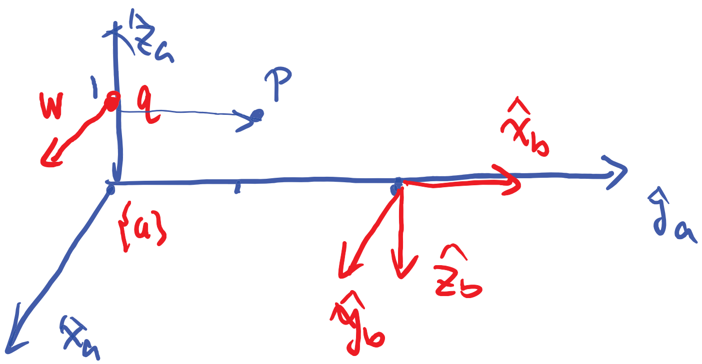
$[\mathrm{Ad}_{T_2}]\mathcal{V}_c = \mathcal{V}_b$

$[\mathrm{Ad}_{T_1}]\mathcal{V}_b = \mathcal{V}_a$

Example: Change reference frame for twist

Two frames {a} and {b} and configuration of {b} relative to {a} is $T = (R, p_0)$ with

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad p_0 = (0, 2, 0)$$



- rotating about w at unit speed
- the corresponding twist in {a}

consider ODE:

$$\dot p_a(t) = w_a \times (p_a(t) - q_a) = \begin{bmatrix}1\\0\\0\end{bmatrix} \times p_a(t) - \begin{bmatrix}1\\0\\0\end{bmatrix} \times \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$= \begin{bmatrix}1\\0\\0\end{bmatrix} \times p_a(t) + \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$\Rightarrow \mathcal{V}_a = \begin{bmatrix}1\\0\\0\\0\\1\\0\end{bmatrix} = \begin{bmatrix}w_a\\v_a\end{bmatrix}$$

What is $\mathcal{V}_b$? In Frame {b}:

- The same motion can be described by

$$\dot p_b(t) = w_b \times (p_b(t) - q_b) = \begin{bmatrix}0\\1\\0\end{bmatrix} \times p_b(t) - \begin{bmatrix}0\\1\\0\end{bmatrix} \times \begin{bmatrix}-2\\0\\-1\end{bmatrix}$$

Example: Change reference frame for Twist (Continued)

$$= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times p_b(t) - \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{w_b} \times p_b(t) + \underbrace{\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}}_{v_b}$$

$$\Rightarrow \mathcal{V}_b = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ -2 \end{bmatrix}$$

We know that $\mathcal{V}_a = [Ad_{T_{ab}}] \mathcal{V}_b \iff \mathcal{V}_b = [Ad_{T_{ba}}] \mathcal{V}_a$

V.F.Y. with the ~~nn~~ numbers that the above are true

$$\boxed{Ad_{T_{ab}} = \begin{bmatrix} R & 0 \\ [p_0]R & R \end{bmatrix} \qquad p_0 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \qquad [p_0] = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}}$$

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Derivation of Spatial Velocity of a Rigid Body

- **Question:** Given configuration $T_{sb}(t) = \underline{(R_{sb}(t), p_{sb}(t))}$ of a moving rigid body, how to represent/find the velocity of the rigid body?

- Similar to the rotational velocity, we consider a point q attached to the body and derive its differential equation in {s} frame.

$$q_s(t) = R_{sb}(t)q_b + p_{sb}(t) \quad \Rightarrow \quad \dot{q}_s(t) = \omega_s \times q_s(t) + v_s$$

$$\begin{bmatrix} q_s(t) \\ 1 \end{bmatrix} = T_{sb}(t) \begin{bmatrix} q_b \\ 1 \end{bmatrix}$$

we can see that $q_b = R_{sb}^{-1}(t)\left(q_s(t) - p_{sb}(t)\right)$

$$\dot{q}_s(t) = \underline{\dot{R}_{sb}(t)}\, q_b + \dot{p}_{sb}(t)$$

$$= \left(\dot{R}_{sb}(t) R_{sb}^{-1}(t)\right) q_s(t) - \left(\dot{R}_{sb}(t) R_{sb}^{-1}(t)\right) p_{sb}(t) + \dot{p}_{sb}(t)$$

$\Rightarrow [\omega_s] = \omega_s \times (\ \ )$

dropping "t" for simplicity

$$= [\omega_s] q_s + v_s$$

where

$[\omega_s] \triangleq \dot{R}_{sb} R_{sb}^{-1}$

$v_s \triangleq \dot{p}_{sb} + \omega_s \times (-p_{sb})$

$\Rightarrow \mathcal{V}_s = \begin{bmatrix} \omega_s \\ v_s \end{bmatrix}$ spatial twist

Spatial Twist and Body Twist

- Given $T_{sb}(t) = (R(t), p(t))$. Spatial velocity in space frame (called **spatial twist**) is given by

$$\mathcal{V}_s = (\omega_s, v_s), \text{ with } [\omega_s] = \dot{R}R^T, v_s = \dot{p} + \omega_s \times (-p)$$

- Change reference frame to body frame will lead to **body twist**:

$$\mathcal{V}_b = (\omega_b, v_b) = [\mathrm{Ad}_{T_{bs}}]\mathcal{V}_s, \text{ where } [\omega_b] = R^T\dot{R}, v_b = \underbrace{R^T\dot{p}}_{\text{velocity of origin of } \{b\} \text{ in body-frame}}$$

$\cancel{\mathcal{V}_b} =$

$$T_{bs} = \begin{bmatrix} R^T & -R^T p \\ 0 & 0 \end{bmatrix} \Rightarrow [\mathrm{Ad}_{T_{bs}}] = \begin{bmatrix} R^T & 0 \\ [-R^T p]R^T & R^T \end{bmatrix}$$

From last slide: $\tilde{q}_s(t) = T_{sb}(t)\tilde{q}_b \Rightarrow \dot{\tilde{q}}_s(t) = \dot{T}_{sb}(t)\tilde{q}_b = \boxed{\dot{T}_{sb}(t)\, T_{sb}^{-1}(t)}\,\tilde{q}_s(t) \Rightarrow \boxed{[\mathcal{V}_s] = \dot{T}_{sb} T_{sb}^{-1}}$

$$\begin{bmatrix} [\omega_s] & v_s \\ 0 & 0 \end{bmatrix}$$

$$[\mathcal{V}_b] = T_{bs}[\mathcal{V}_s]T_{sb} = T_{bs}\dot{T}_{sb}T_{sb}^{-1}T_{sb} = T_{sb}^{-1}\dot{T}_{sb}$$

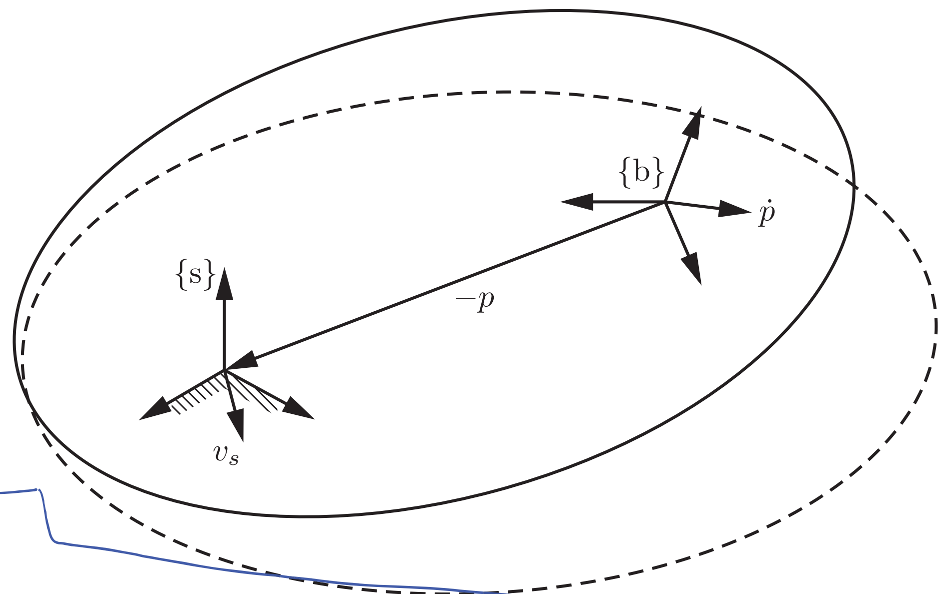
Spatial Twist and Body Twist: Interpretations

- ω_b and ω_s is the angular velocity expressed in {b} and {s}, respectively.
- v_b is the linear velocity of the origin of {b} expressed in {b}; v_s is the linear velocity of the origin of {s} expressed in {s}

$$T_{sb}(t) = \begin{bmatrix} R_{sb}(t) & p_{sb}(t) \\ 0 & 1 \end{bmatrix}$$

twist has $\left\{ \mathcal{V}_s = \begin{bmatrix} w_s \\ v_s \end{bmatrix}, \quad \mathcal{V}_b = \begin{bmatrix} w_b \\ v_b \end{bmatrix} \right.$

$$w_b = R^{-1}\dot{R} \qquad w_s = \dot{R}R^{-1}$$



$$v_b = R_{sb}^T \dot{p}_{sb} \qquad v_s = \dot{p}_{sb} + w_s \times (-p_{sb}) \quad \cdots \text{①}$$

Imagine the body infinitely large and {s} is also attached to the body

then v_s in ① is the velocity of the origin of {s} in {s}-frame

Example of Spatial/Body Twist I

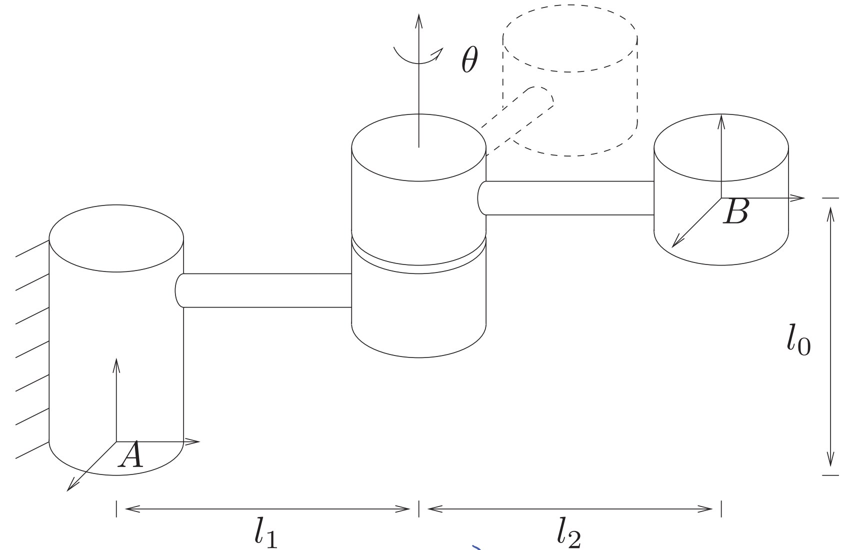
Homework 4 :

$\mathcal{V}_s$

$\mathcal{V}_b$

$[\mathcal{V}_s] = \dot{T} T^{-1}$

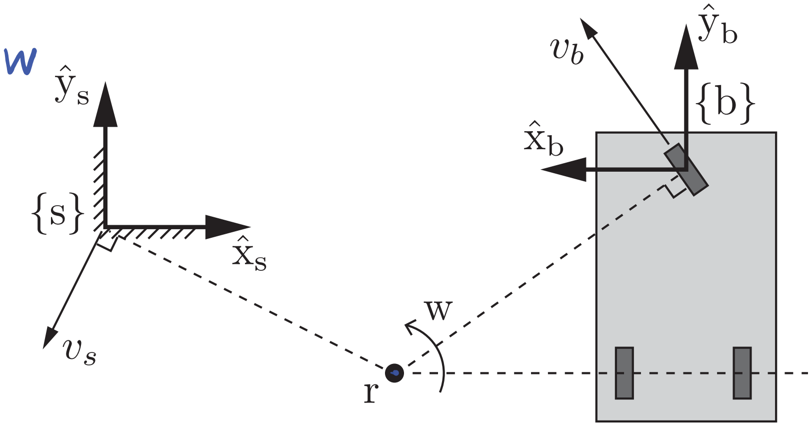
$[\mathcal{V}_b] = T^{-1} \dot{T}$



$$T(t) = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 & -l_2\sin\theta(t) \\ \sin\theta(t) & \cos\theta(t) & 0 & l_1 + l_2\cos\theta(t) \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example of Spatial/Body Twist II

suppose the car wheel ~~an~~ lead to pure rotation about w



what is $\mathcal{V}_s = (w_s, v_s)$ and $\mathcal{V}_b = (w_b, v_b)$

Note: $\hat{z}_s$ out of page $\quad$ $\hat{z}_b$ points into the page

$r_s = (2, -1, 0)$, $r_b = (2, -1.4, 0)$, w=2 rad/s

$$T_{sb} = \begin{bmatrix} -1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0.4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Screw axis: $\left\{ \hat{S}_s = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ h = 0,\ q_s = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \right.$

$$\Rightarrow w_s = \hat{S}\dot{\theta} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \quad v_s = -w_s \times q_s + \cancelto{0}{h\hat{S}\dot{\theta}} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \times \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ 0 \end{bmatrix}$$

$$\mathcal{V}_s = \begin{bmatrix} 0 \\ 0 \\ 2 \\ -2 \\ -4 \\ 0 \end{bmatrix}$$

In $\{b\}$-frame: screw-axis: $\hat{S}_b = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$, $h = 0$, $q_b = \begin{bmatrix} 2 \\ -1.4 \\ 0 \end{bmatrix}$

$$\Rightarrow w_b = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}, \quad v_b = -w_b \times q_b = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \times \begin{bmatrix} 2 \\ -1.4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1.4 \\ 0 \end{bmatrix} = \begin{pmatrix} 2.8 \\ 4 \\ 0 \end{pmatrix}$$

More Discussions

$$V_b = \begin{bmatrix} 0 \\ 0 \\ -2 \\ 2.8 \\ 4 \\ 0 \end{bmatrix}$$

Use the interpretations on slide 21:

$$\omega_s = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \quad v_s = \omega_s \times (-r_s) = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \times \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ 0 \end{bmatrix}$$

$$\omega_b = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}, \quad v_b = \omega_b \times (-r_b) = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \times \begin{bmatrix} 2 \\ -1.4 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 4 \\ 0 \end{bmatrix}$$