

Perceptual Loss, GANs (part I)

Jun-Yan Zhu

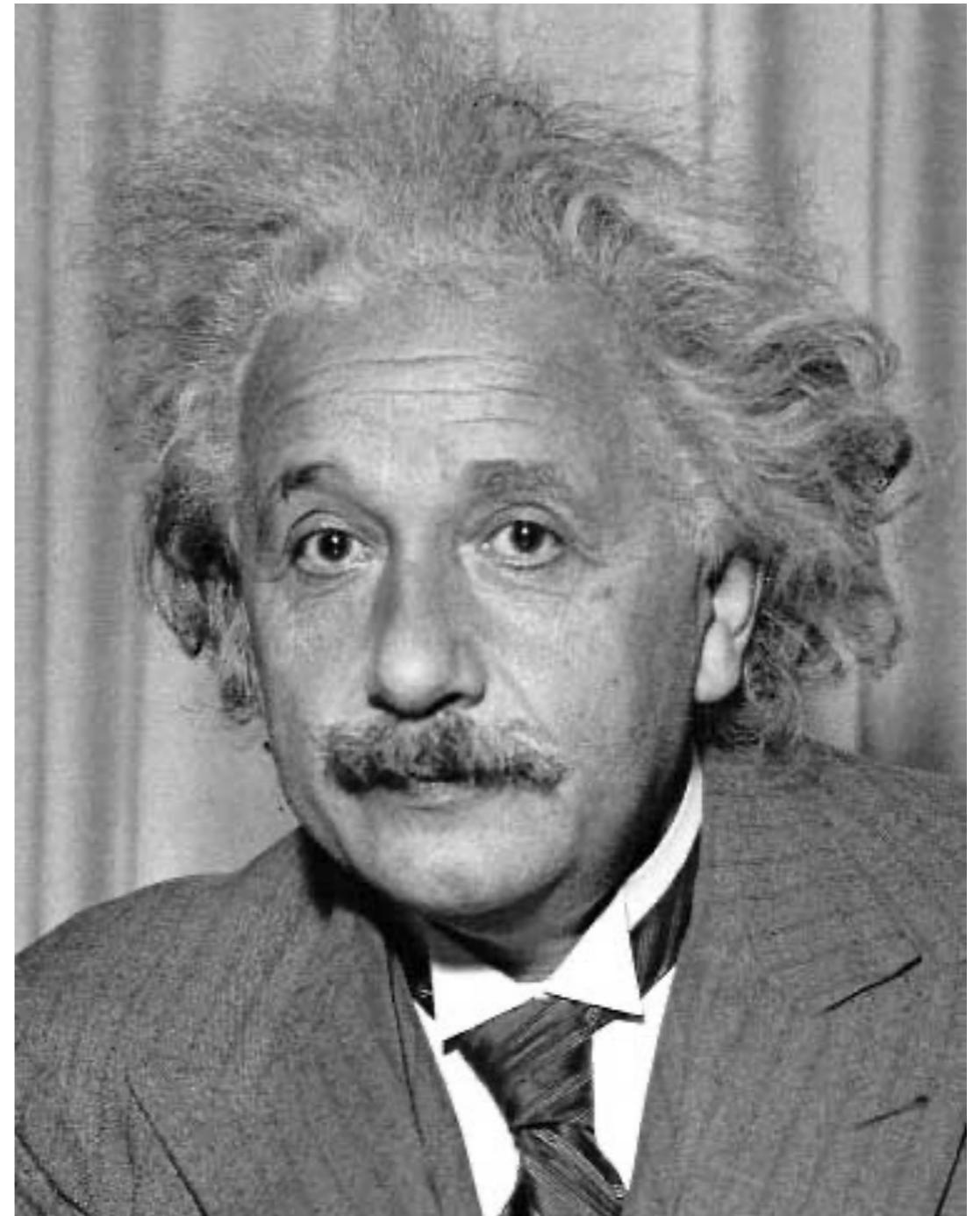
16-726 Learning-based Image Synthesis, Spring 2023

many slides from Alyosha Efros, Phillip Isola, Richard Zhang, James Hays, and Andrea Vedaldi, Jitendra Malik.

HW1 (hints)

Template matching

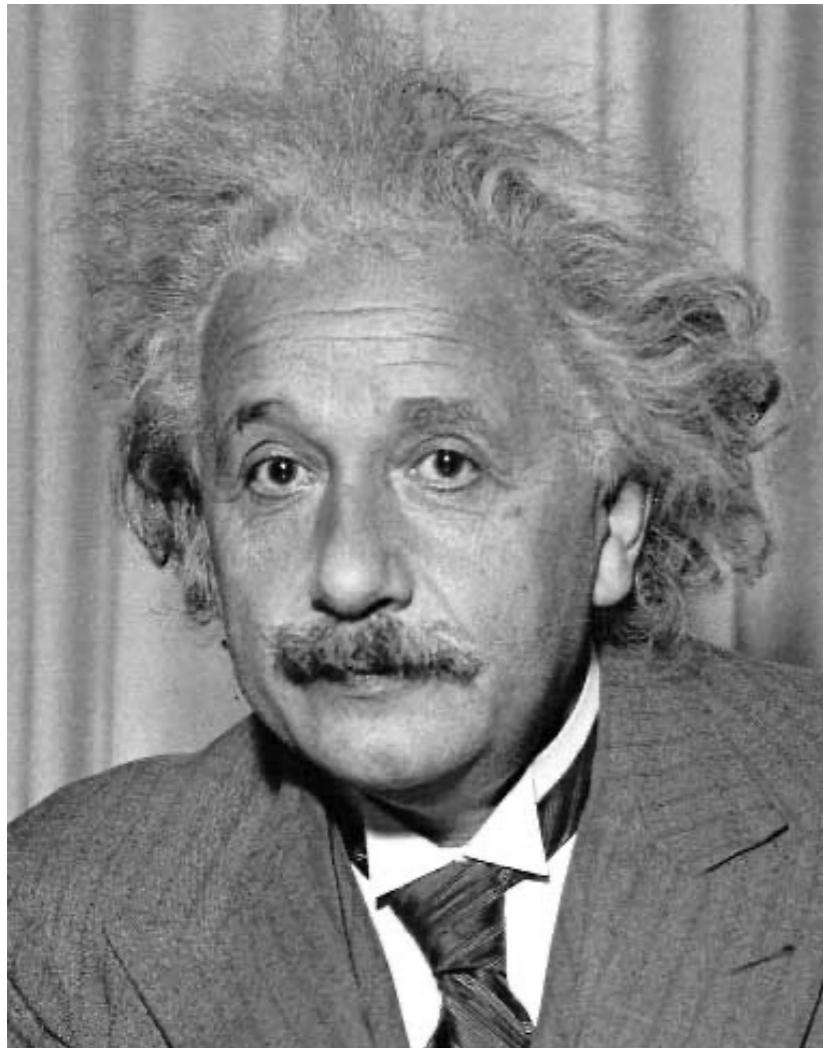
- Goal: find  in image
- Main challenge: What is a good similarity or distance measure between two patches?
 - Correlation
 - Zero-mean correlation
 - Sum Square Difference
 - Normalized Cross Correlation



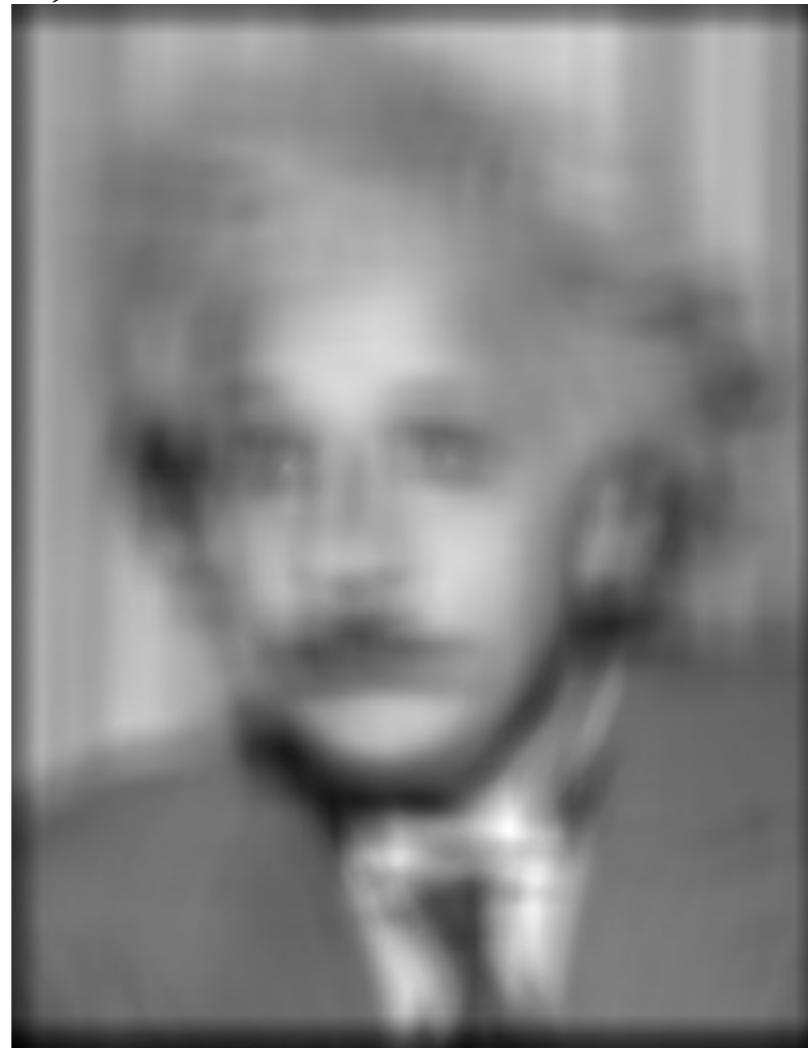
Matching with filters

- Goal: find  in image
- Method 0: filter the image with eye patch

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$



Input



Filtered Image

f = image
g = filter

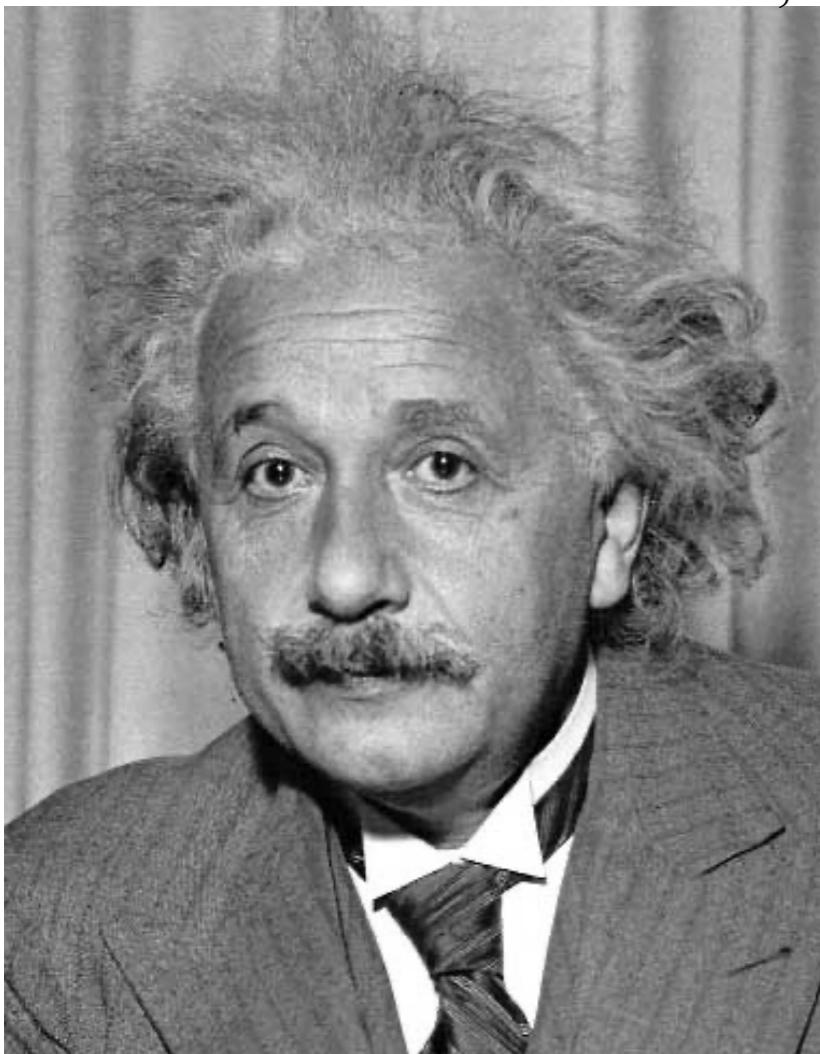
What went wrong?

Matching with filters

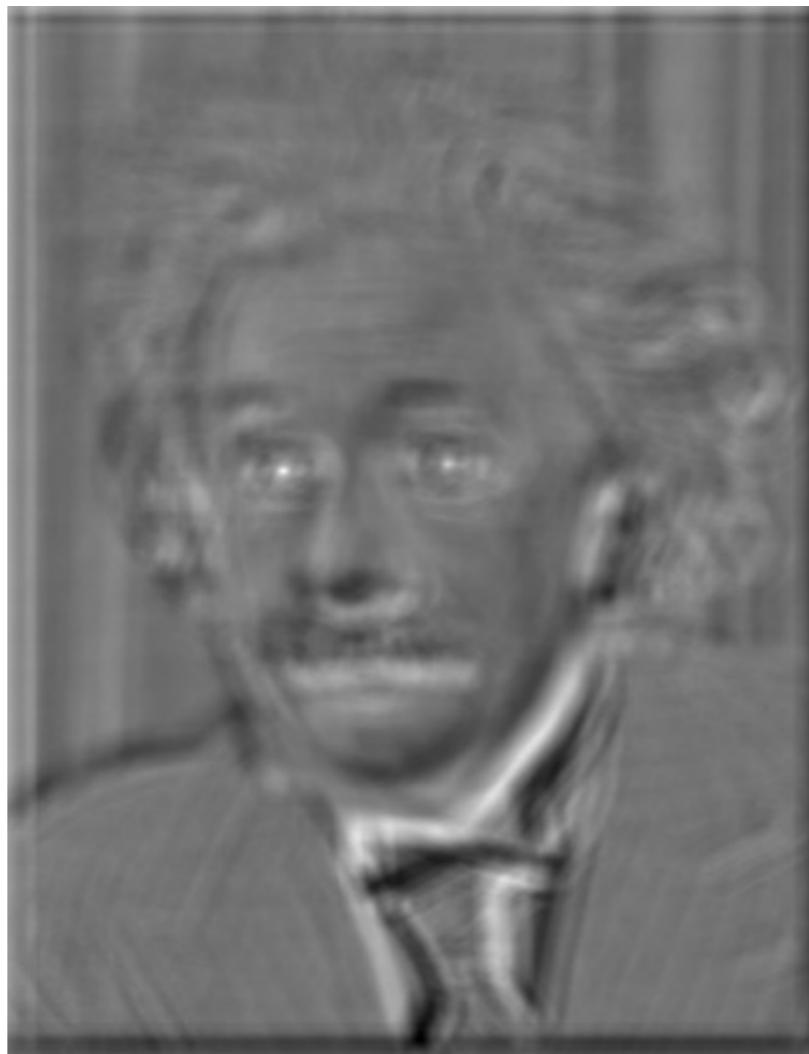
- Goal: find  in image
- Method 1: filter the image with zero-mean eye

$$h[m, n] = \sum_{k, l} (f[k, l] - \bar{f}) \underbrace{(g[m + k, n + l])}_{\text{mean of } f}$$

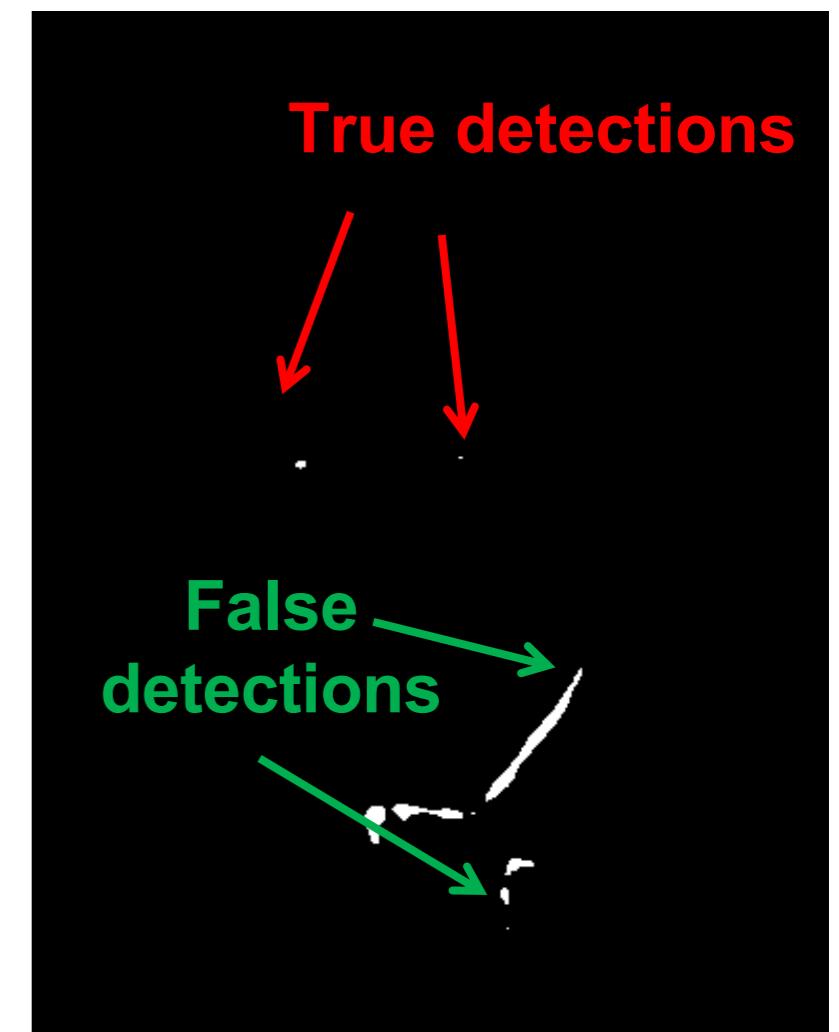
f = image
 g = filter



Input



Filtered Image (scaled)



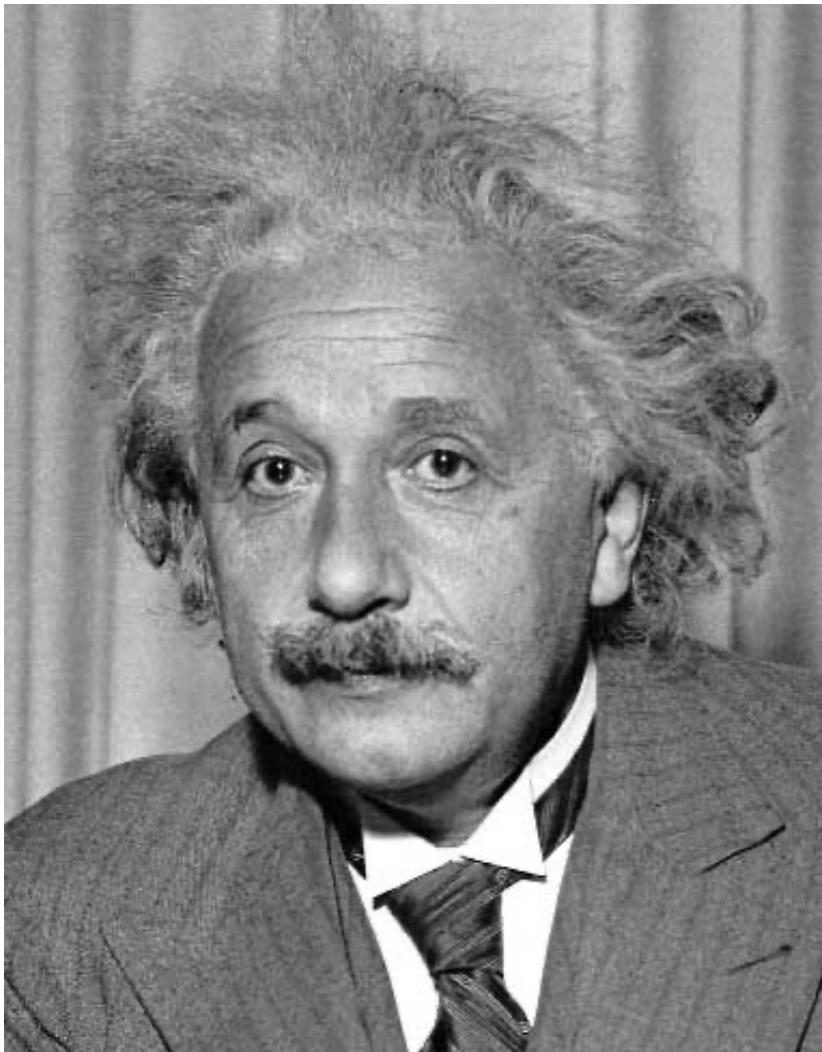
Thresholded Image

Matching with filters

- Goal: find  in image
- Method 2: SSD (Sum Square Difference)

$$h[m, n] = \sum_{k, l} (g[k, l] - f[m + k, n + l])^2$$

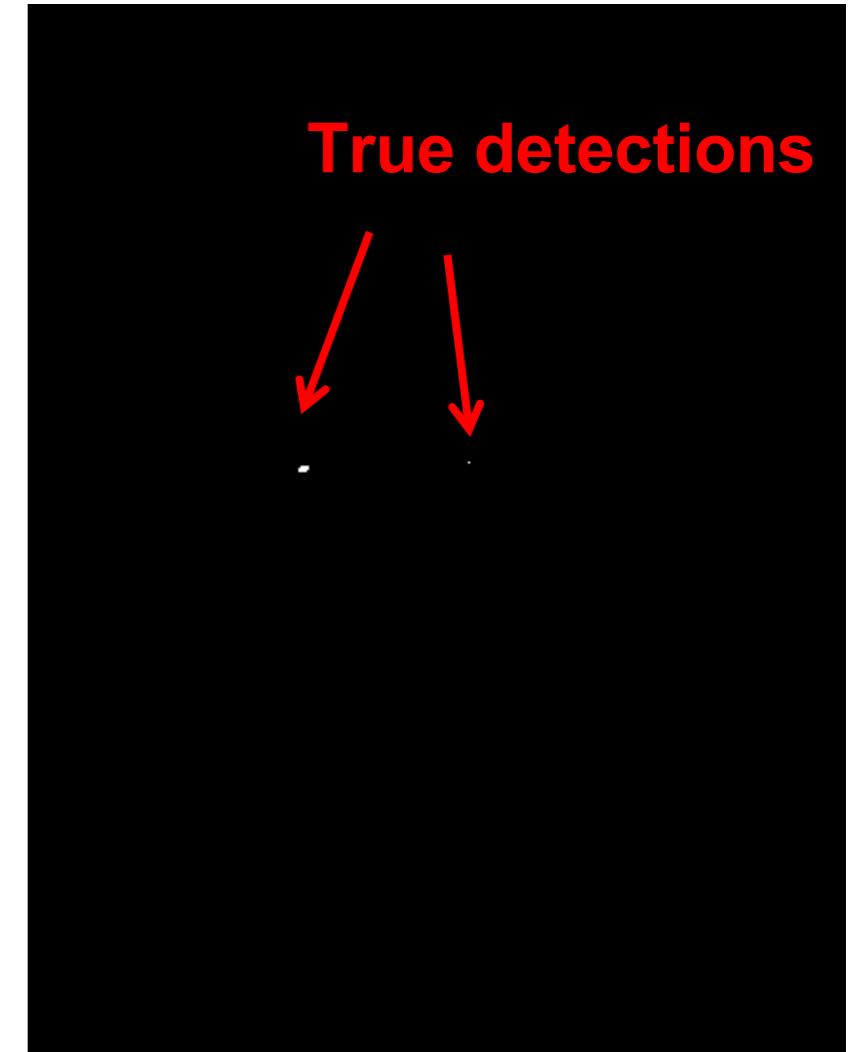
f = image
 g = filter



Input



1 - $\sqrt{\text{SSD}}$



True detections
Thresholded Image

Matching with filters

$$h[m, n] = \sum_{k, l} (g[k, l] - f[m + k, n + l])^2 \quad \begin{matrix} f = \text{image} \\ g = \text{filter} \end{matrix}$$

- Can SSD be implemented with linear filters?

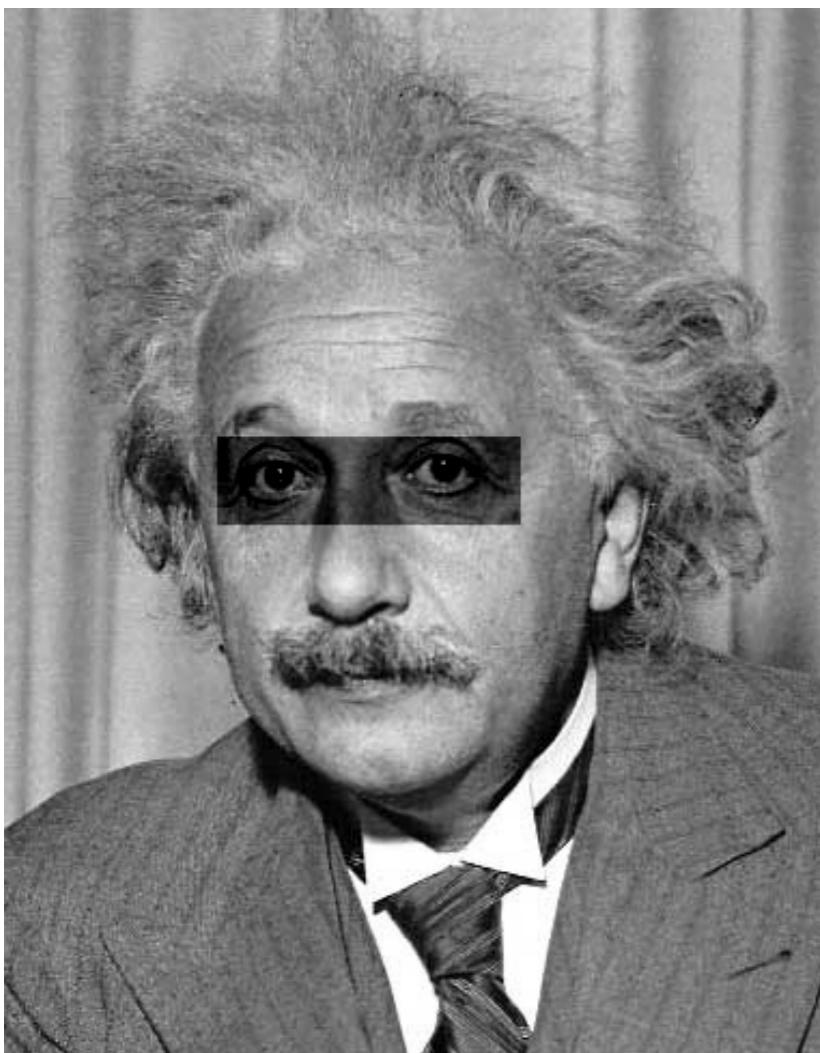
Matching with filters

- Goal: find  in image
- Method 2: SSD (Sum Square Difference)

What's the potential downside of SSD?

$$h[m, n] = \sum_{k, l} (g[k, l] - f[m + k, n + l])^2$$

f = image
g = filter



Input



1 - sqrt(SSD)

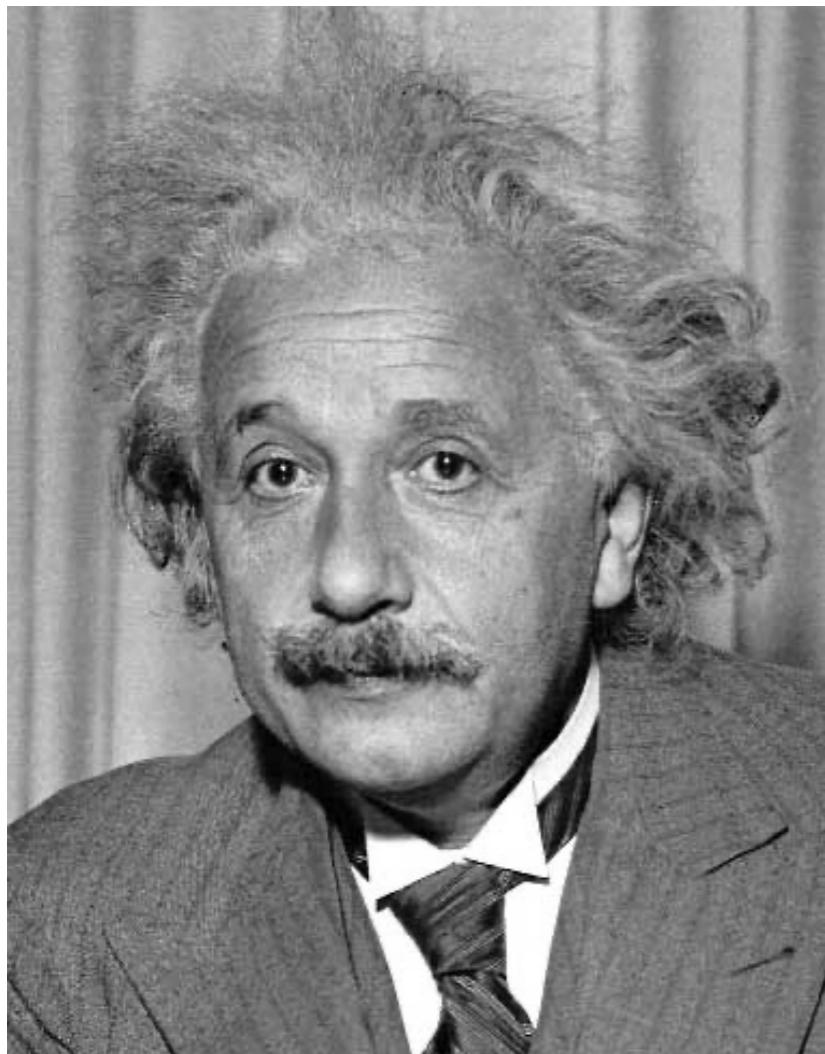
Matching with filters

- Goal: find  in image $f = \text{image}$
 - Method 2: Normalized Cross-Correlation $g = \text{filter}$

$$h[m,n] = \frac{\sum_{k,l} (g[k,l] - \bar{g})(f[m+k, n+l] - \bar{f}_{m,n})}{\left(\sum_{k,l} (g[k,l] - \bar{g})^2 \sum_{k,l} (f[m+k, n+l] - \bar{f}_{m,n})^2 \right)^{0.5}}$$

Matching with filters

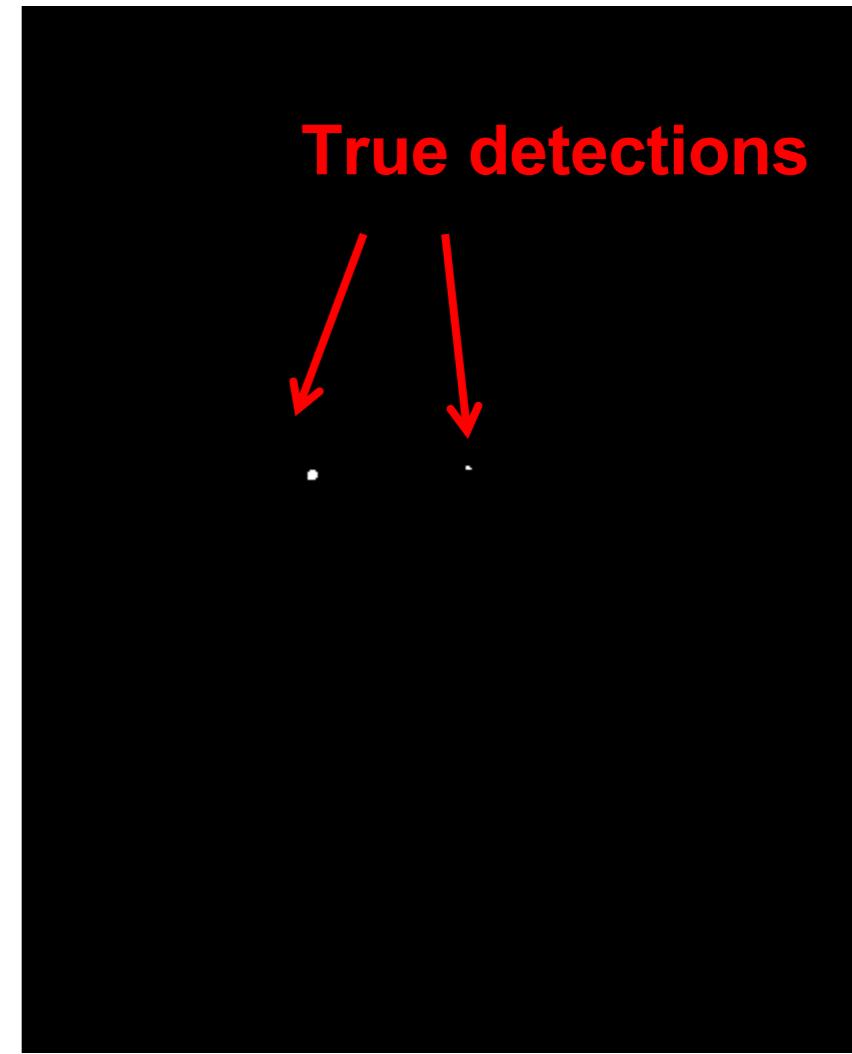
- Goal: find  in image
- Method 2: Normalized Cross-Correlation



Input



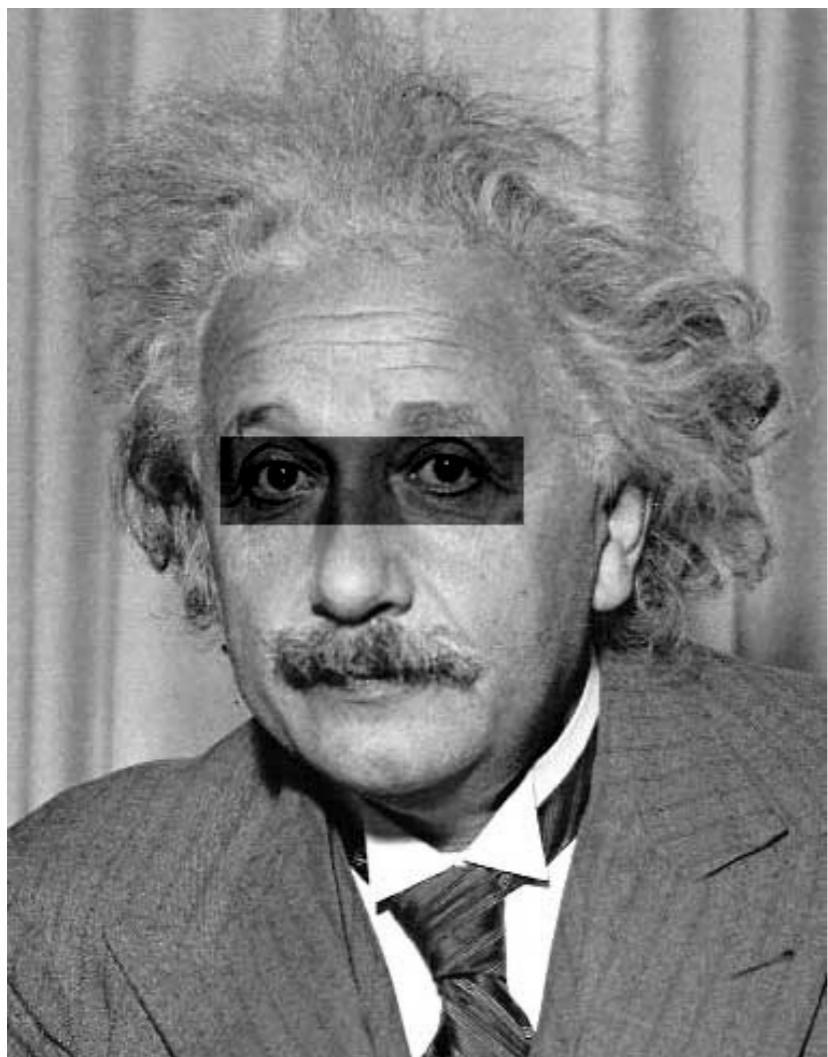
Normalized₀X-Correlation



True detections
Thresholded Image

Matching with filters

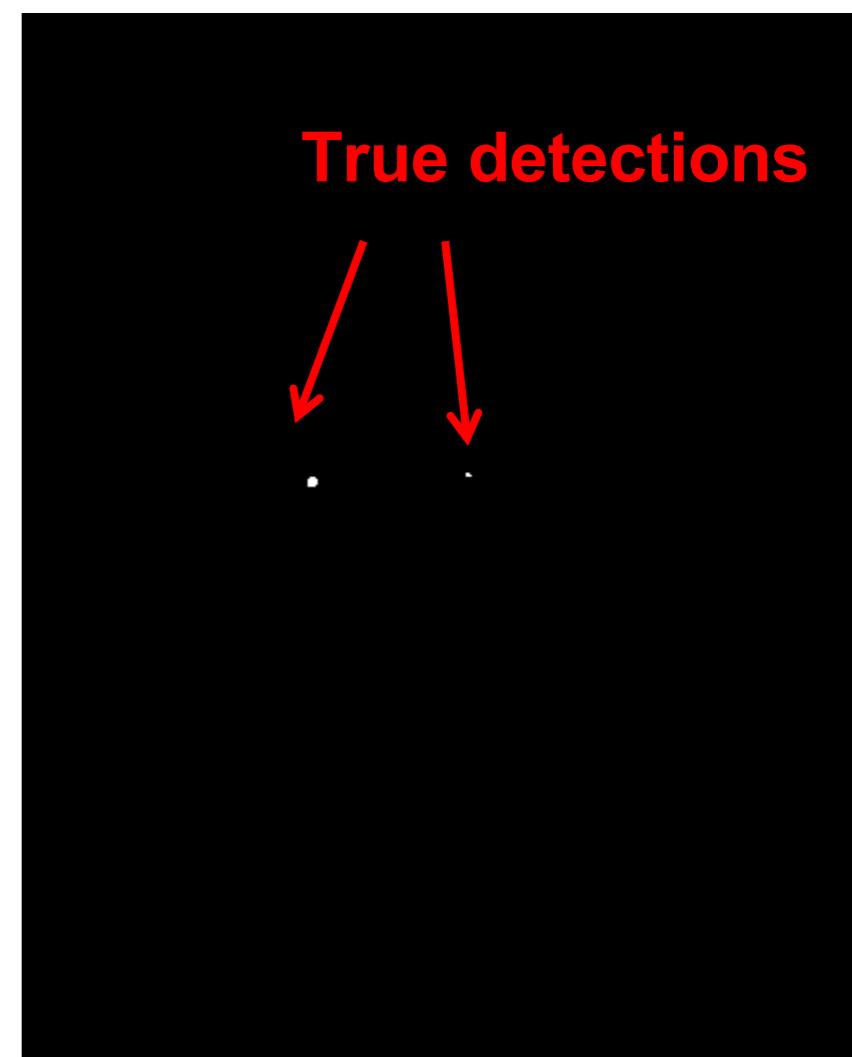
- Goal: find  in image
- Method 2: Normalized Cross-Correlation



Input



Normalized X-Correlation



Thresholded Image

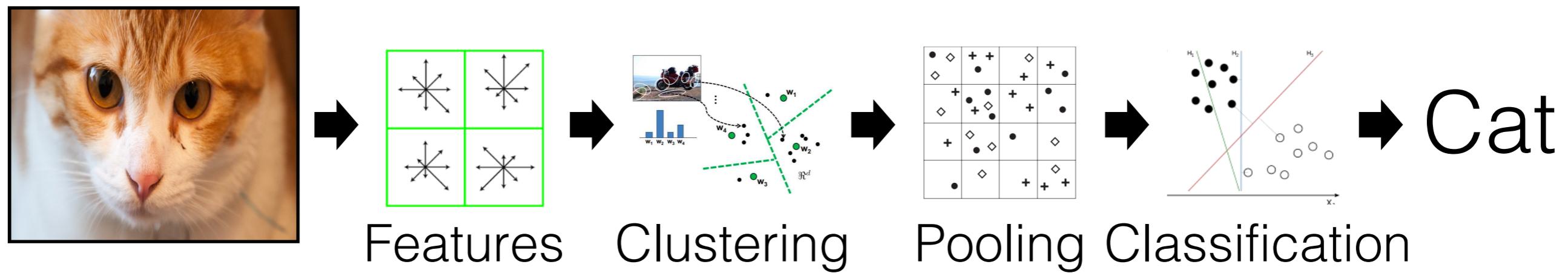
True detections

Q: What is the best method to use?

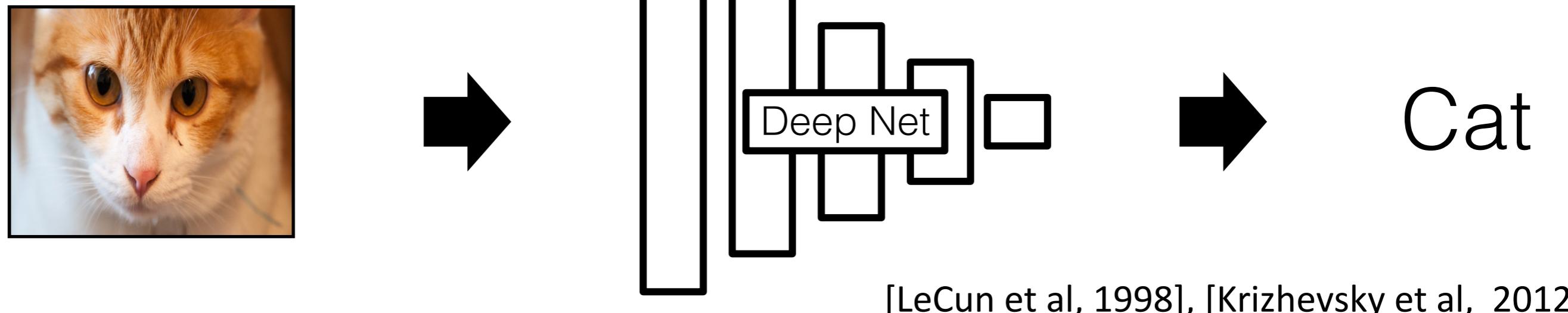
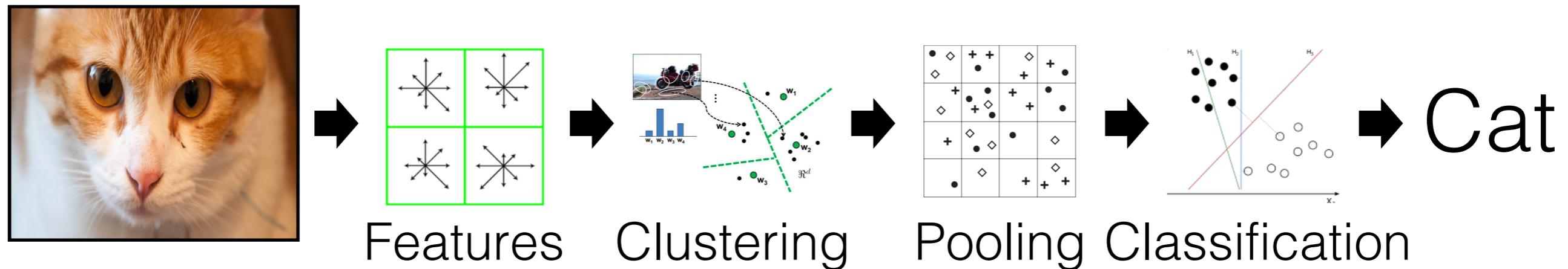
- Answer: Depends
- Zero-mean filter: fastest but not a great matcher
- SSD: next fastest, sensitive to overall intensity
- Normalized cross-correlation: slowest, invariant to local average intensity and contrast

Review (CNN for Image Synthesis)

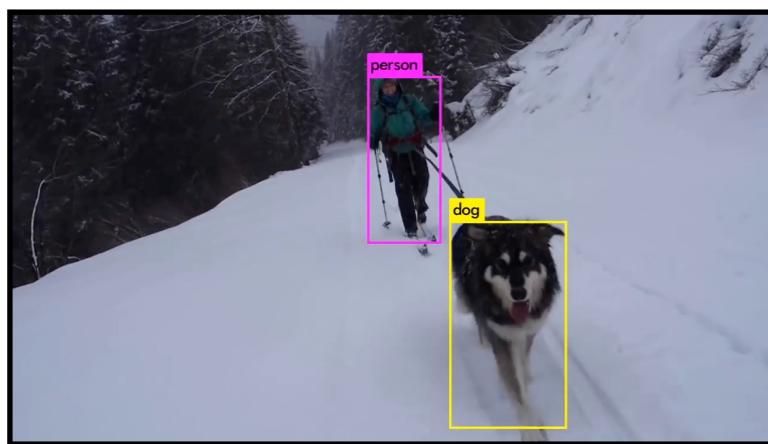
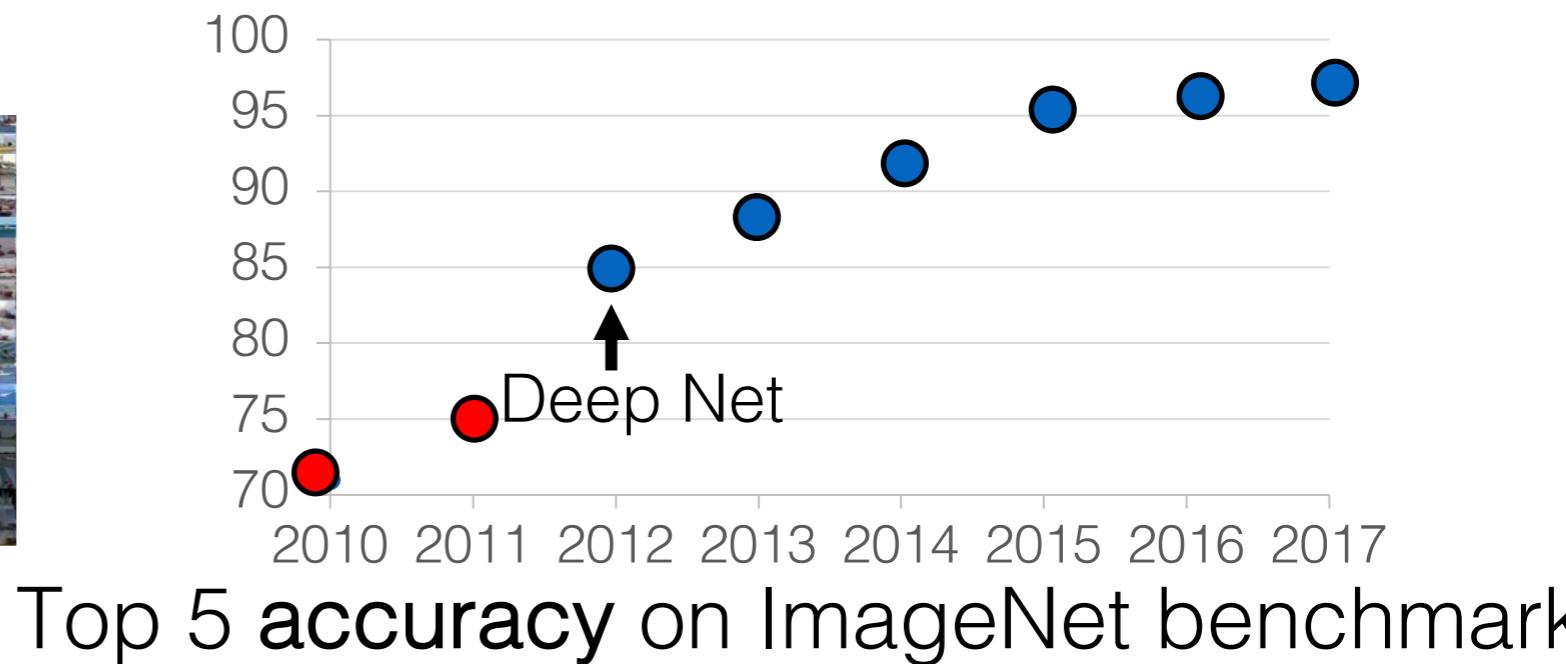
Computer Vision before 2012



Computer Vision Now

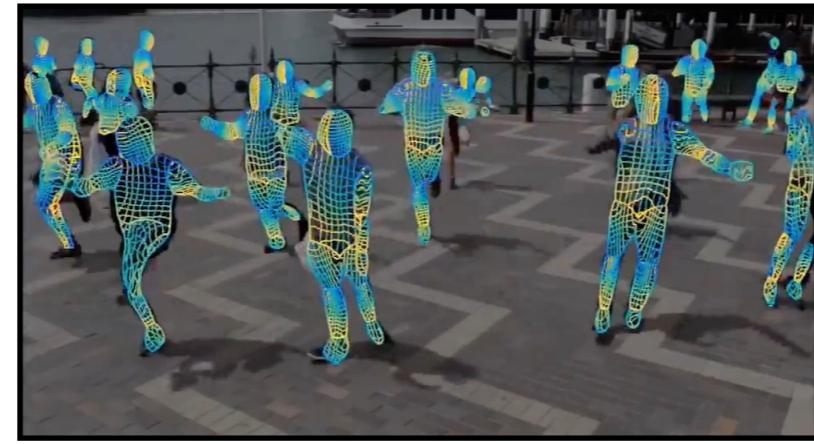


Deep Learning for Computer Vision



[Redmon et al., 2018]

Object detection



[Güler et al., 2018]

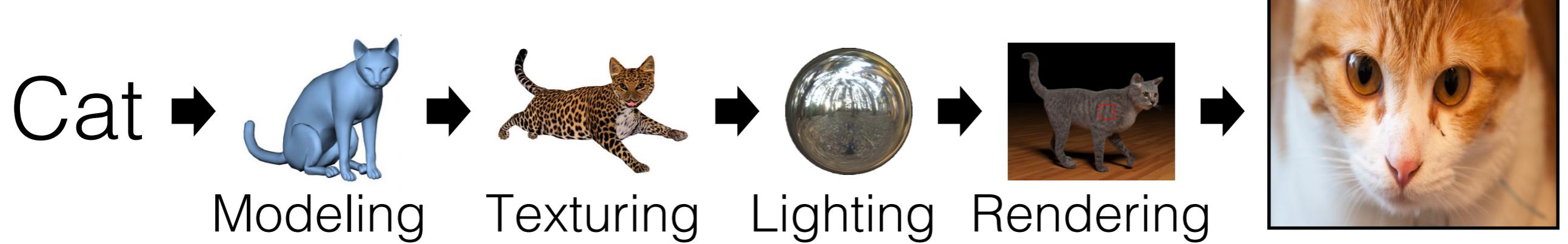
Human understanding



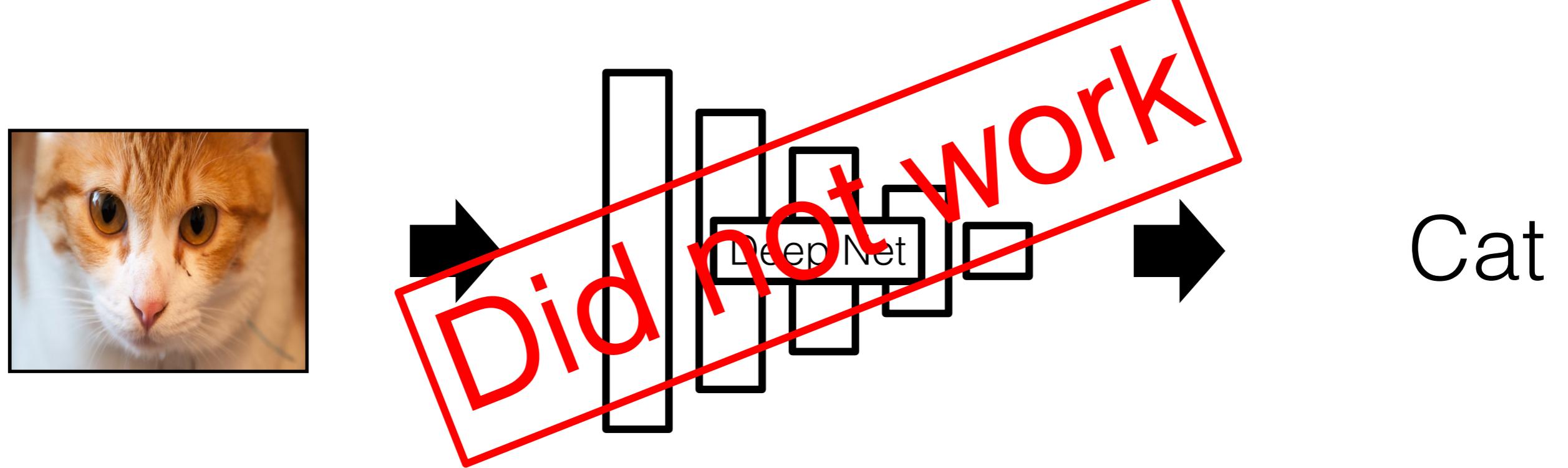
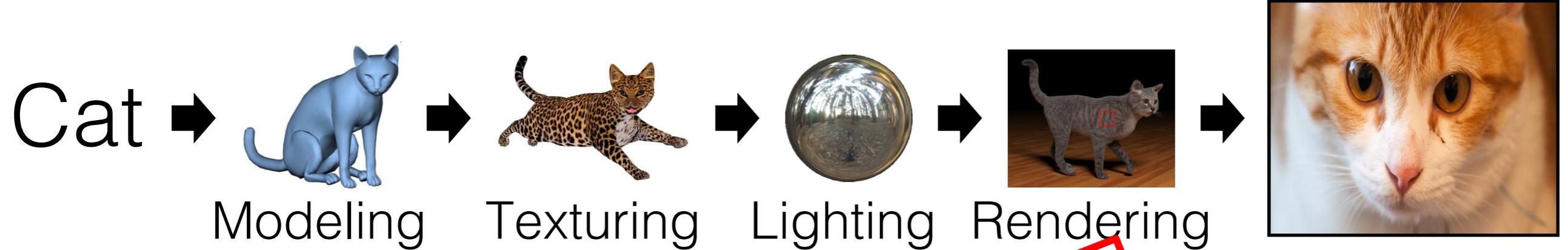
[Zhao et al., 2017]

Autonomous driving

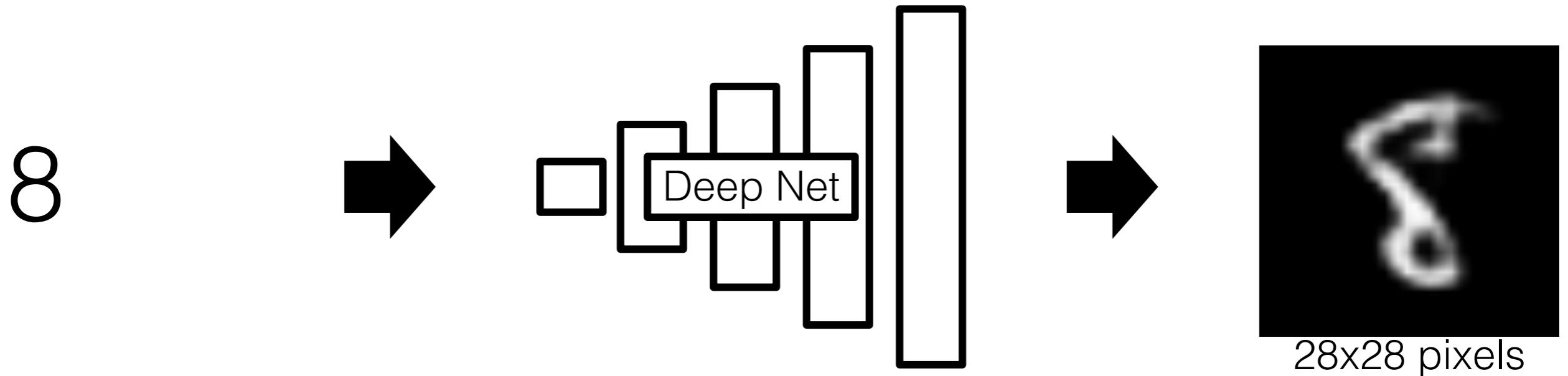
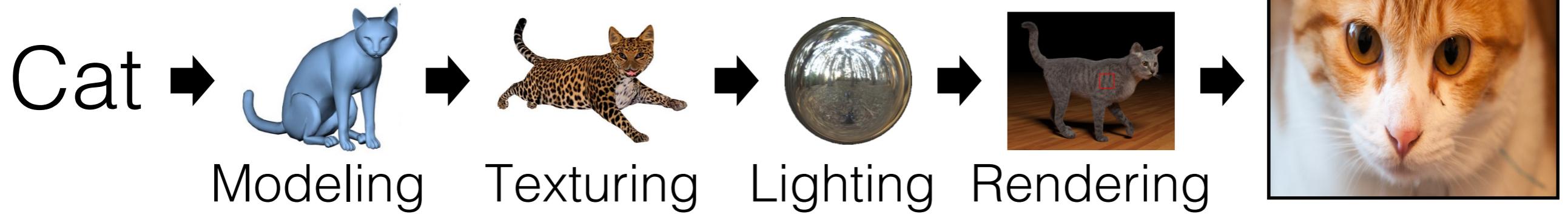
Can Deep Learning Help Graphics?



Can Deep Learning Help Graphics?

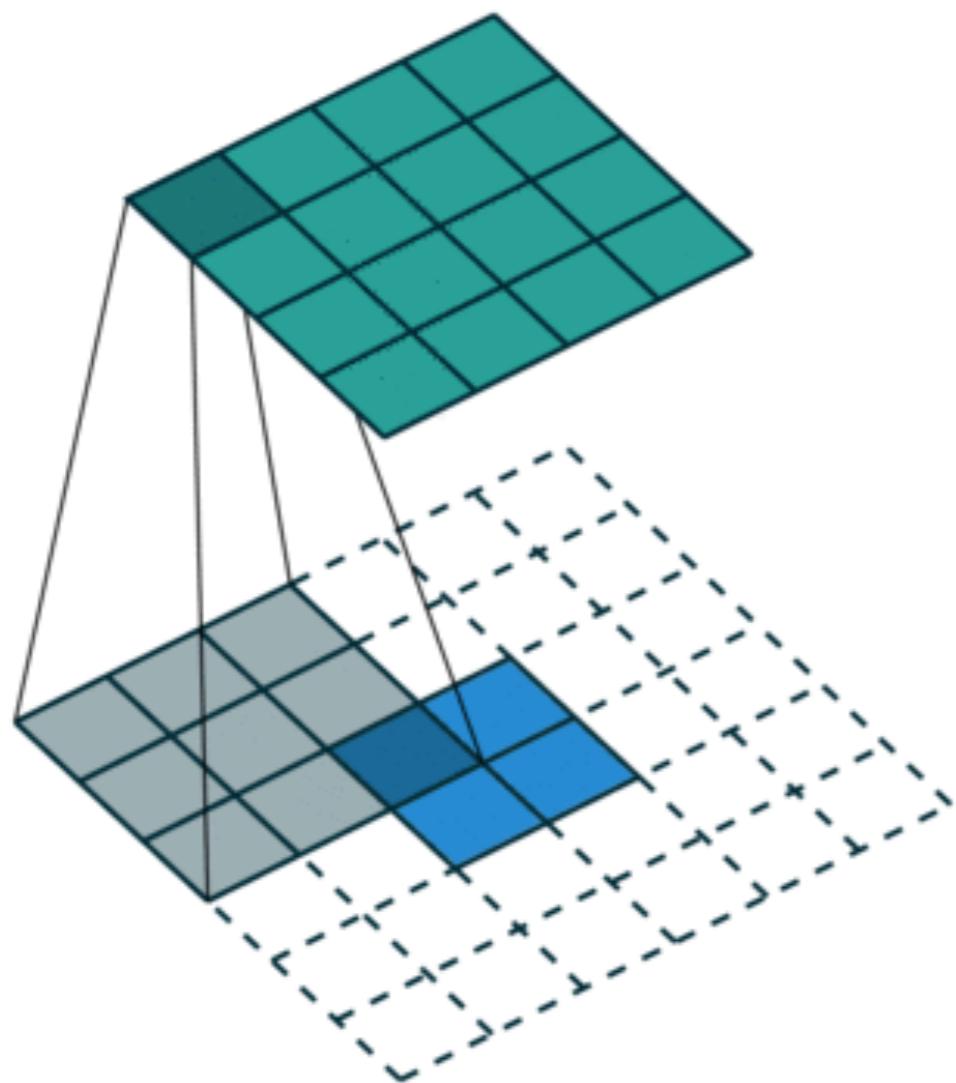


Generating images is hard!

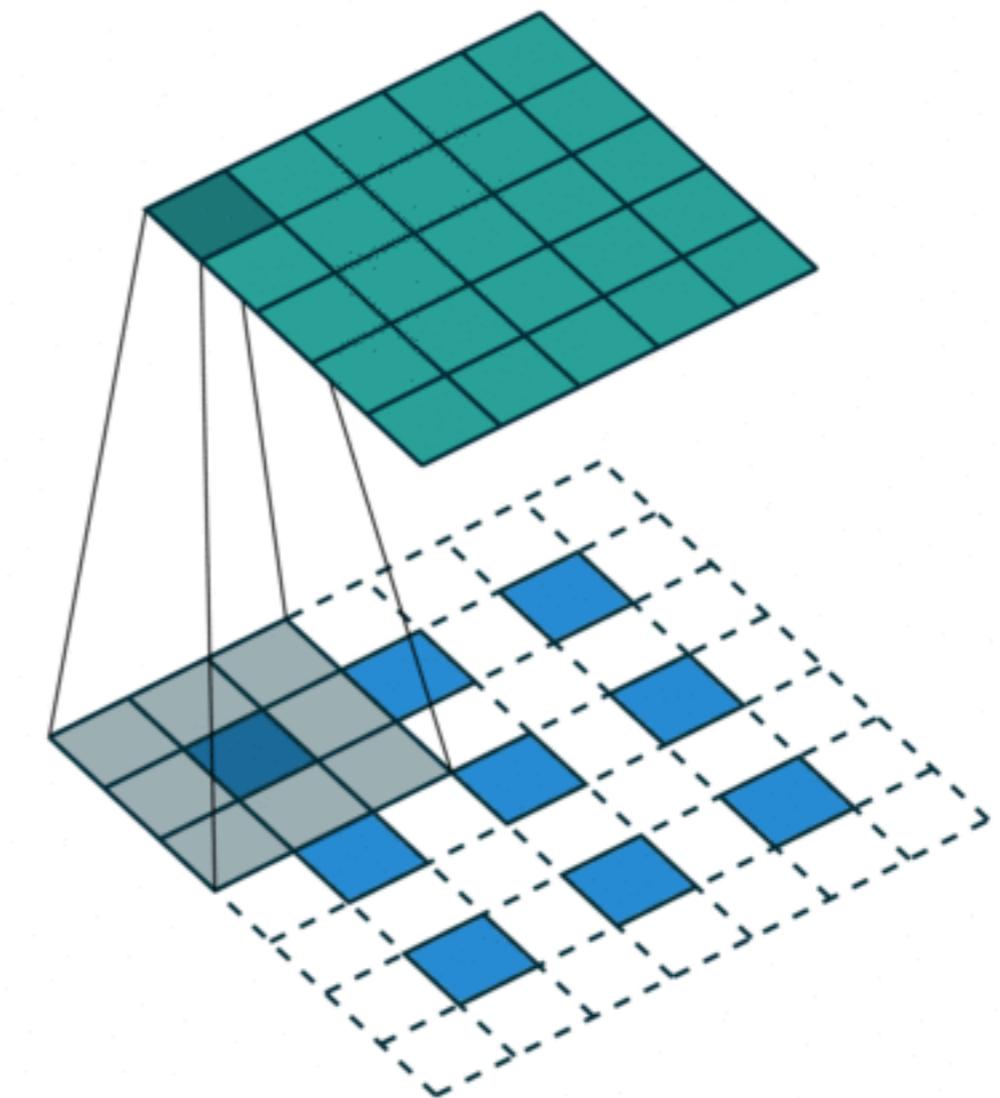


Better Architectures

Fractionally-strided Convolution

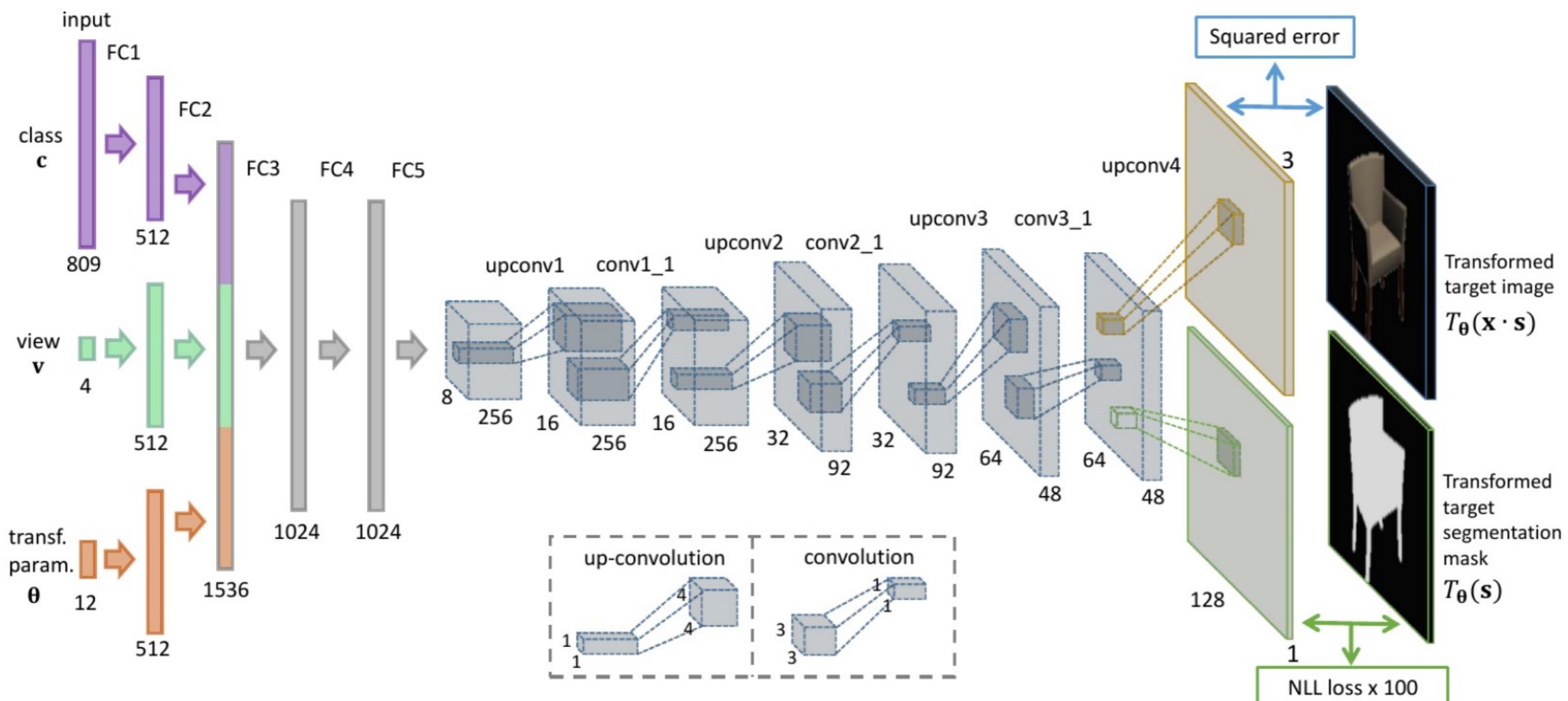


Regular conv



Fractionally-strided conv

Generating chairs conditional on chair ID, viewpoint, and transformation parameters



Dosovitskiy et al. Learning to Generate Chairs, Tables and Cars with Convolutional Networks
PAMI 2017 (CVPR 2015)

Interpolation between Two Chairs



Better Loss Functions

Simple L2 regression doesn't work ☹

Input



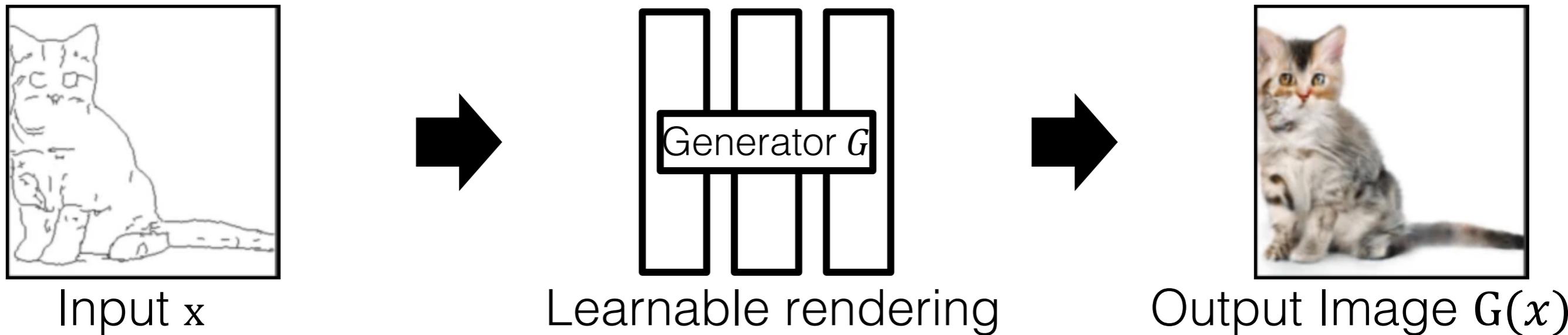
Output



Ground truth



Loss functions for Image Synthesis



What is a good objective \mathcal{L} ?

- Capture realism
- Calculate image distance
- Adapt to new tasks/data.

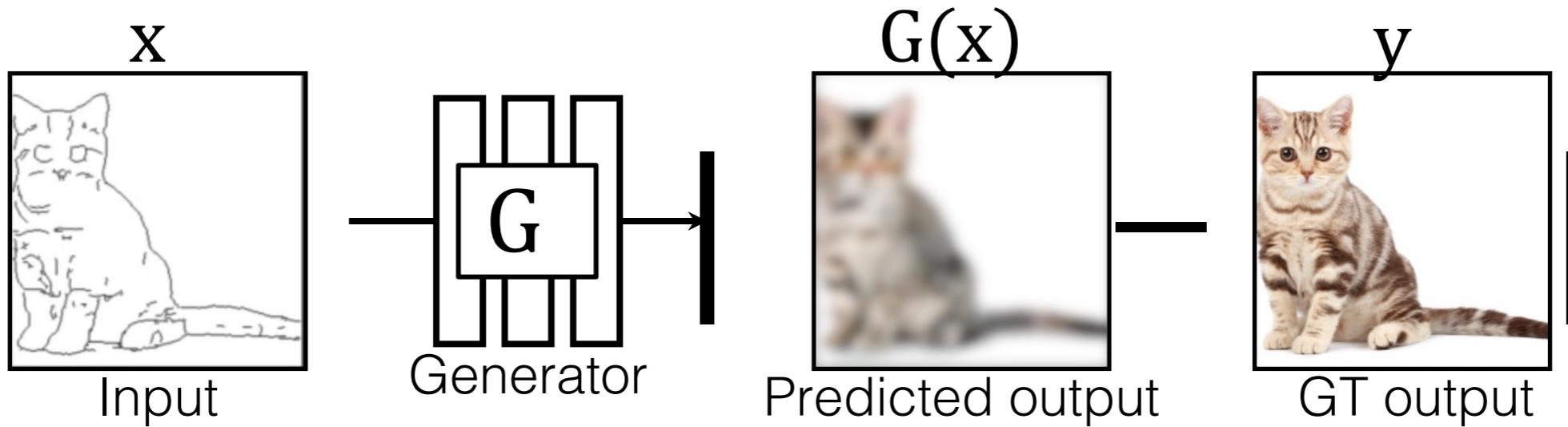
Problem Statement

$$\arg \min_G \mathcal{L}(G(x), y)$$

Generator Input Output image

Loss function

Designing Loss Functions

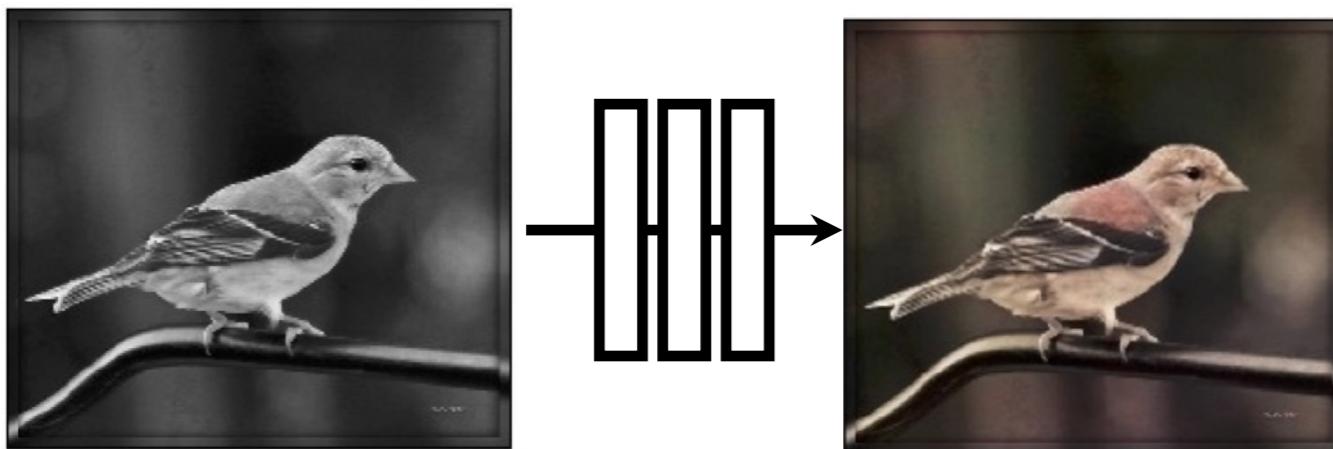


L2 regression

$$\arg \min_G \mathbb{E}_{(x,y)} [\|G(x) - y\|]$$

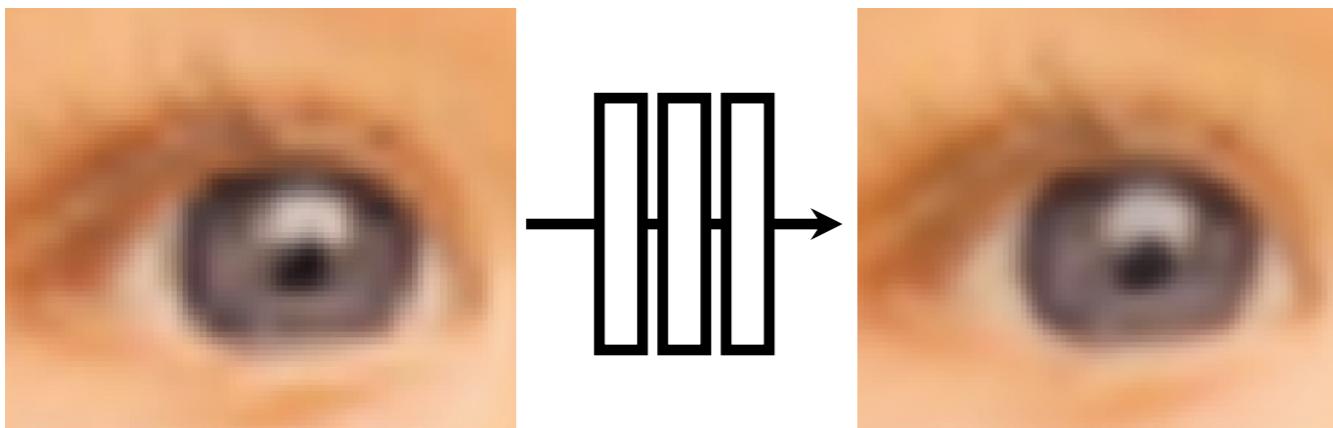
Designing Loss Functions

Image colorization



L2 regression

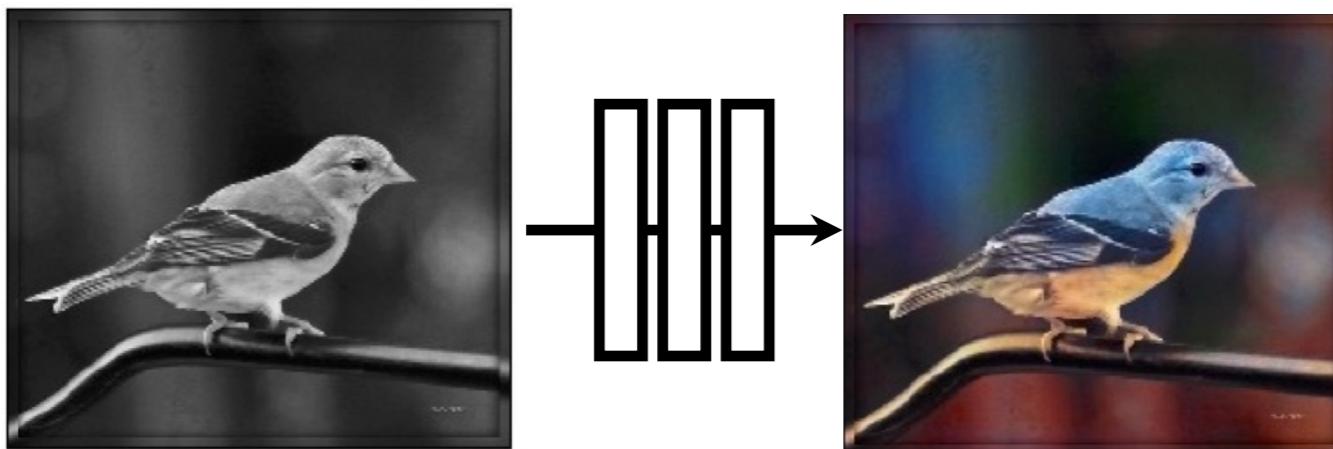
Super-resolution



L2 regression

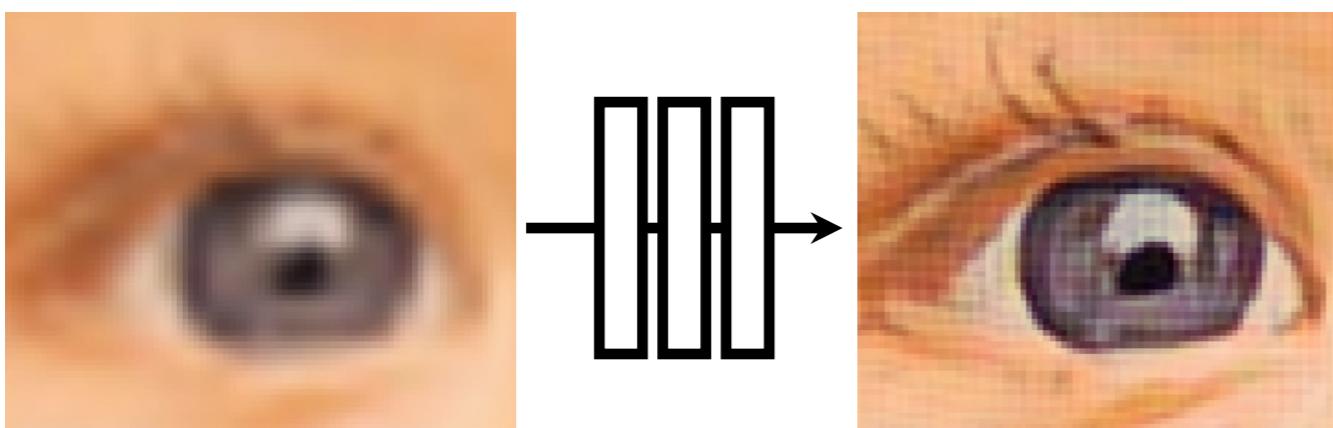
Designing Loss Functions

Image colorization



[Zhang et al. 2016]

Super-resolution



[Gatys et al., 2016], [Johnson et al. 2016]
[Dosovitskiy and Brox. 2016]

Classification Loss:
Cross entropy objective,
with colorfulness term

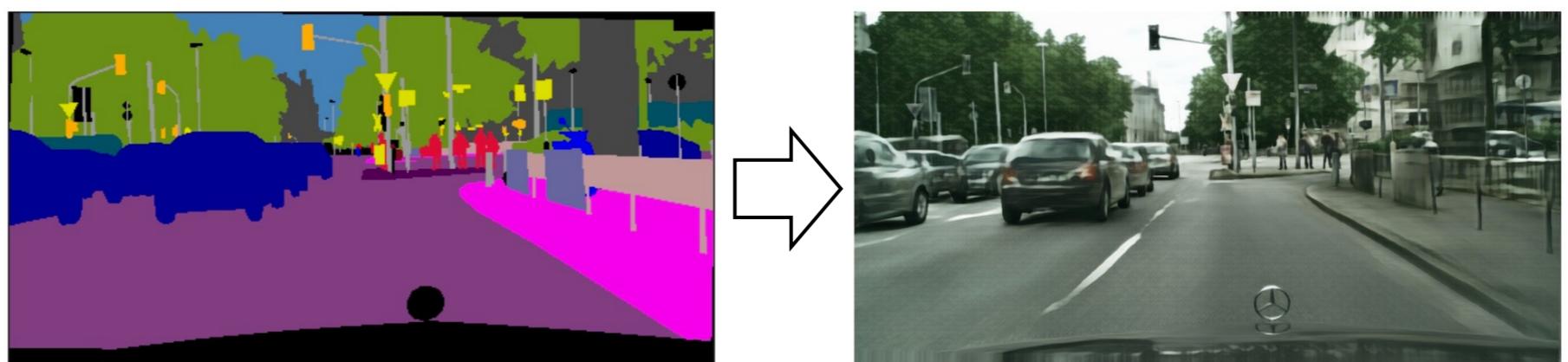
Feature/Perceptual loss
Deep feature matching
objective

“Perceptual Loss”

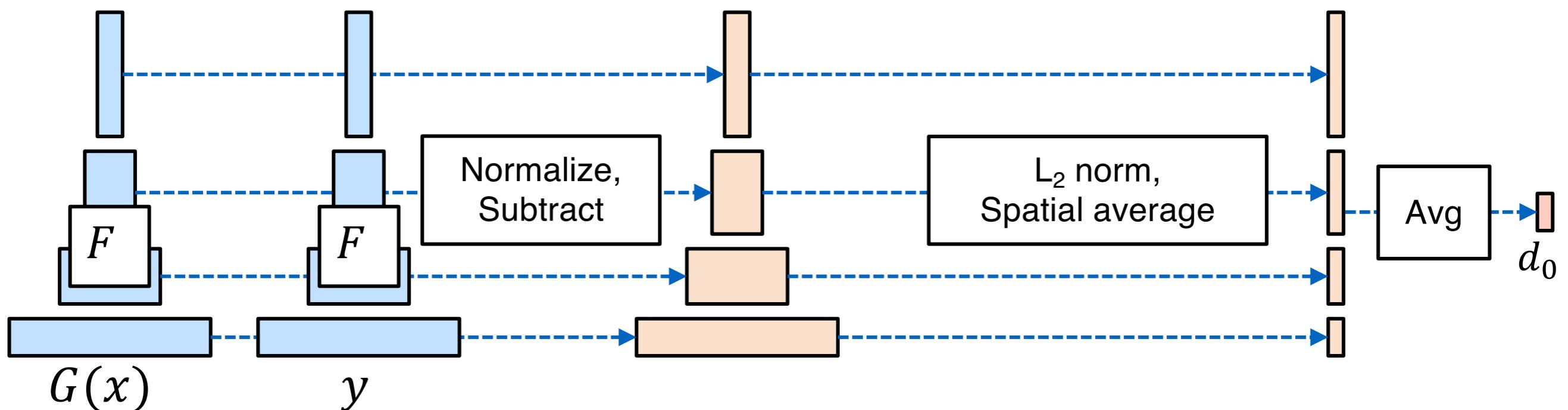
Gatys et al. In CVPR, 2016.
Johnson et al. In ECCV, 2016.
Dosovitskiy and Brox. In NIPS, 2016.



Chen and Koltun. In ICCV, 2017.



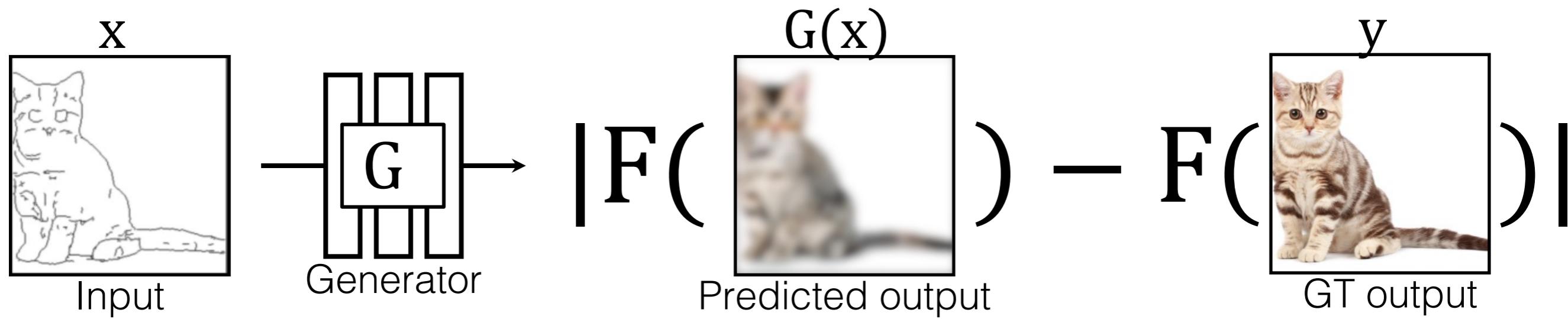
CNNs as a Perceptual Metric



(1) How well do “perceptual losses” describe perception?

c.f. Gatys et al. CVPR 2016. Johnson et al. ECCV 2016. Dosovitskiy and Brox. NIPS 2016.

CNNs as a Perceptual Metric



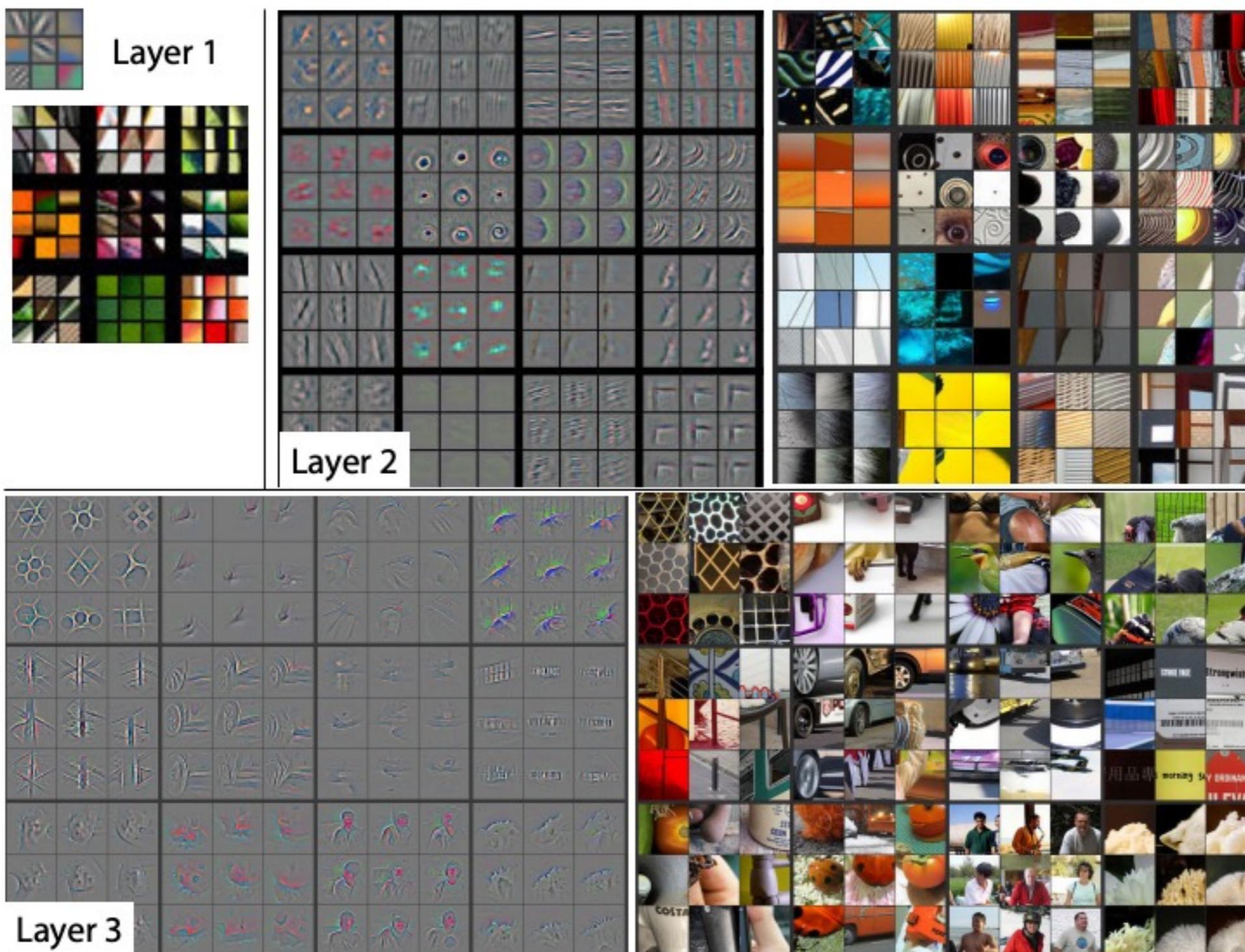
F is a deep network (e.g., ImageNet classifier)

Perceptual Loss

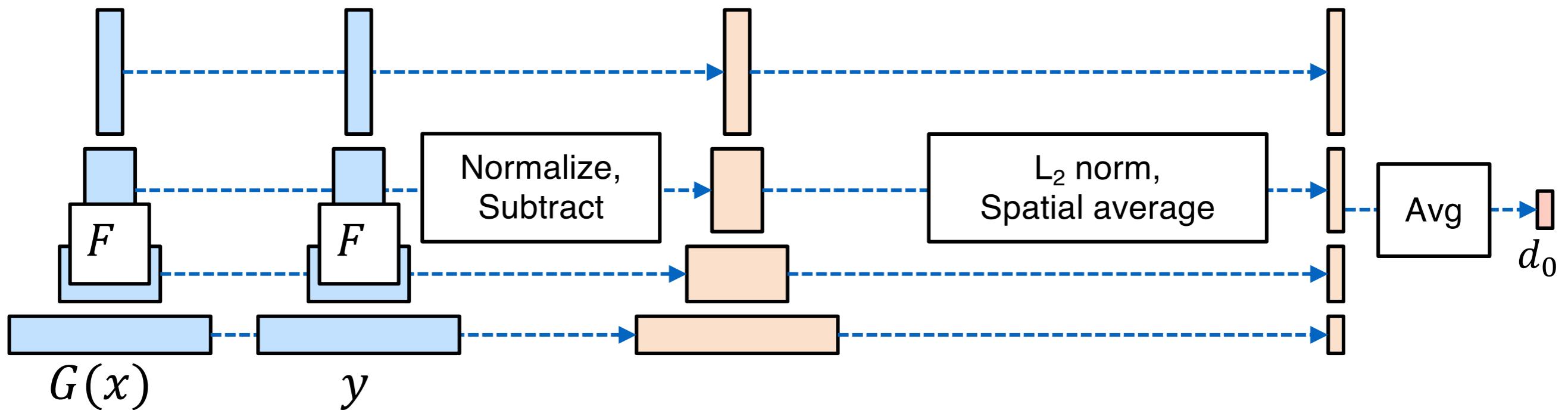
$$\arg \min_G \mathbb{E}_{(x,y)} \sum_{i=1}^N \lambda_i \frac{1}{M_i} \| F^{(i)}(G(x)) - F^{(i)}(y) \|_2^2$$

weight
The number of elements in the (i)-th layer

What has a CNN Learned?



CNNs as a Perceptual Metric



Perceptual Loss

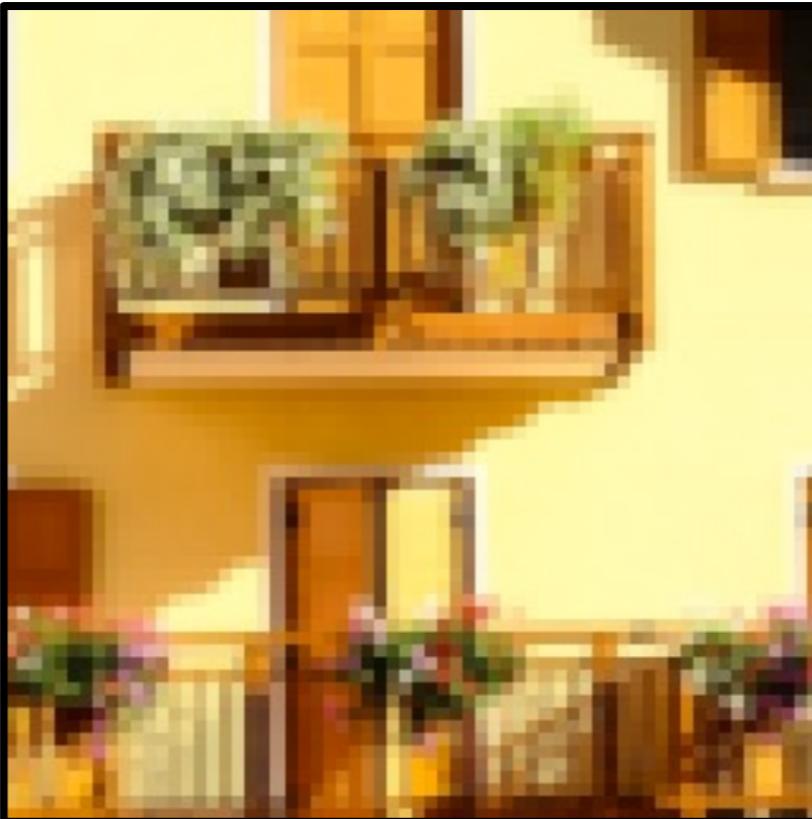
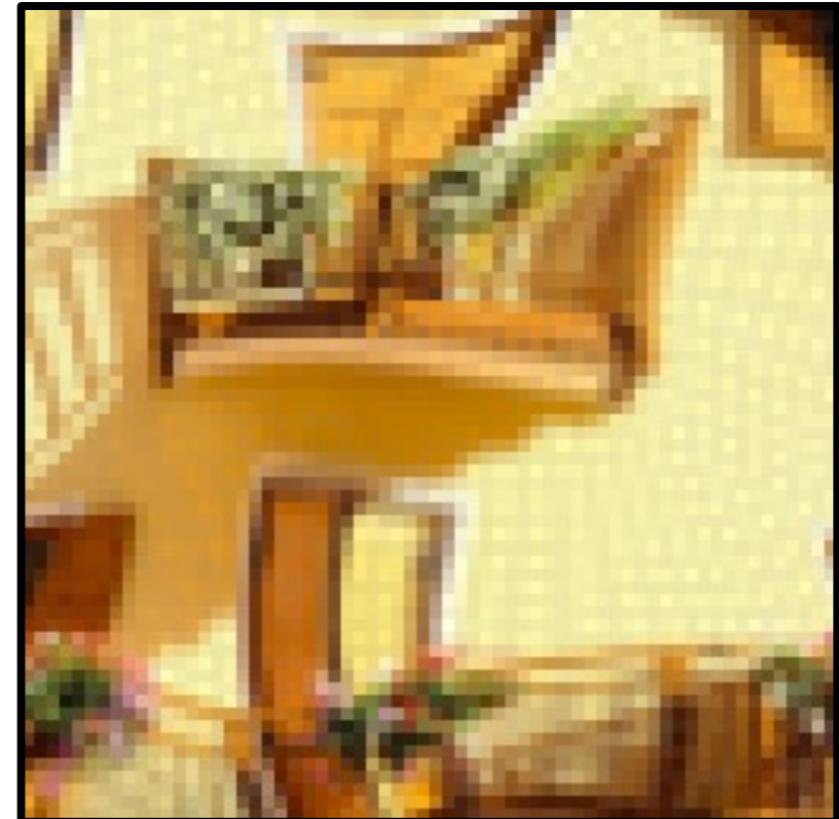
$$\arg \min_G \mathbb{E}_{(x,y)} \sum_{i=1}^N \lambda_i \frac{1}{M_i} \left\| F^{(i)}(G(x)) - F^{(i)}(y) \right\|_2^2$$

The number of elements in the (i)-th layer

weight
↓
 λ_i
↓
 M_i
↑
 $F^{(i)}$
↑
 $G(x)$ y

(i)-th layer

How Different are these Patches?

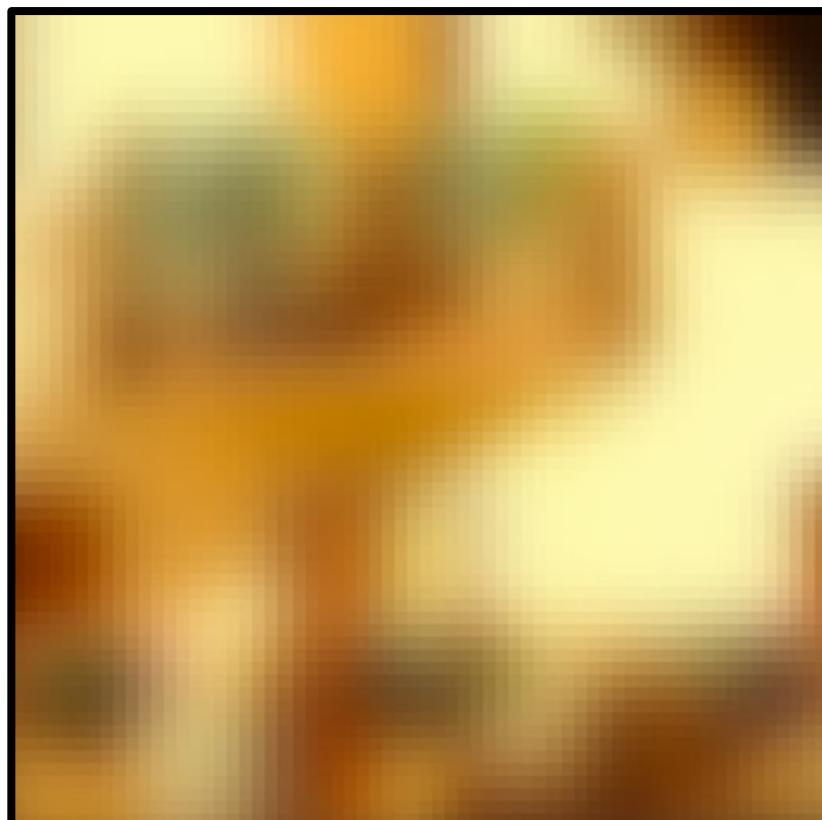
$D($  , )

Zhang, Isola, Efros, Shechtman, Wang.

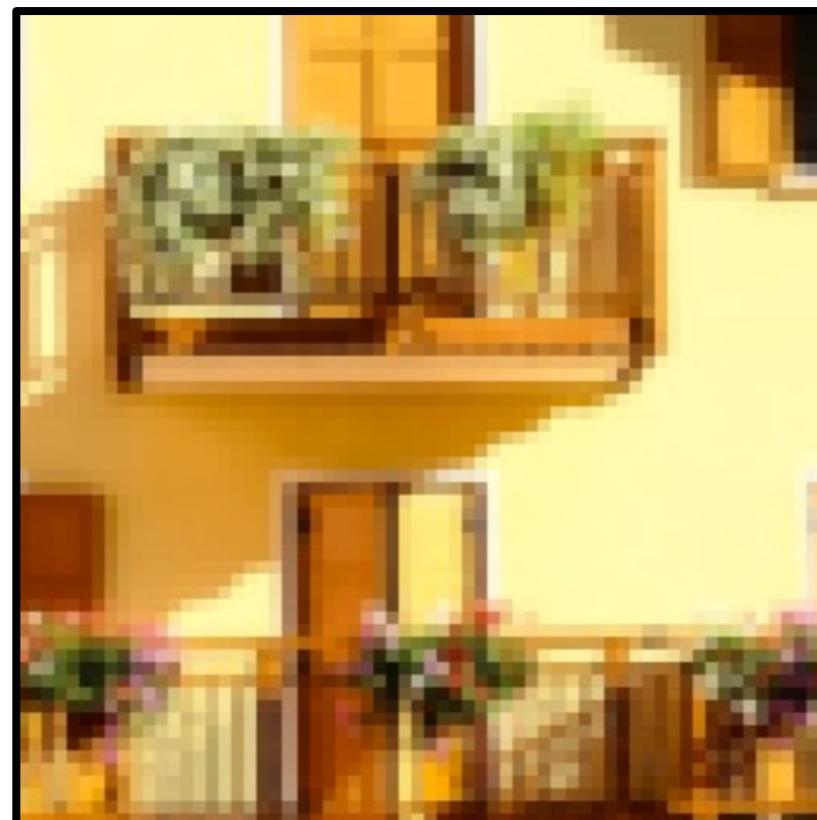
The Unreasonable Effectiveness of Deep Features as a Perceptual Metric. In *CVPR*, 2018.

Slide credit: Richard Zhang

Which patch is more similar to the middle?



< Type 1 >



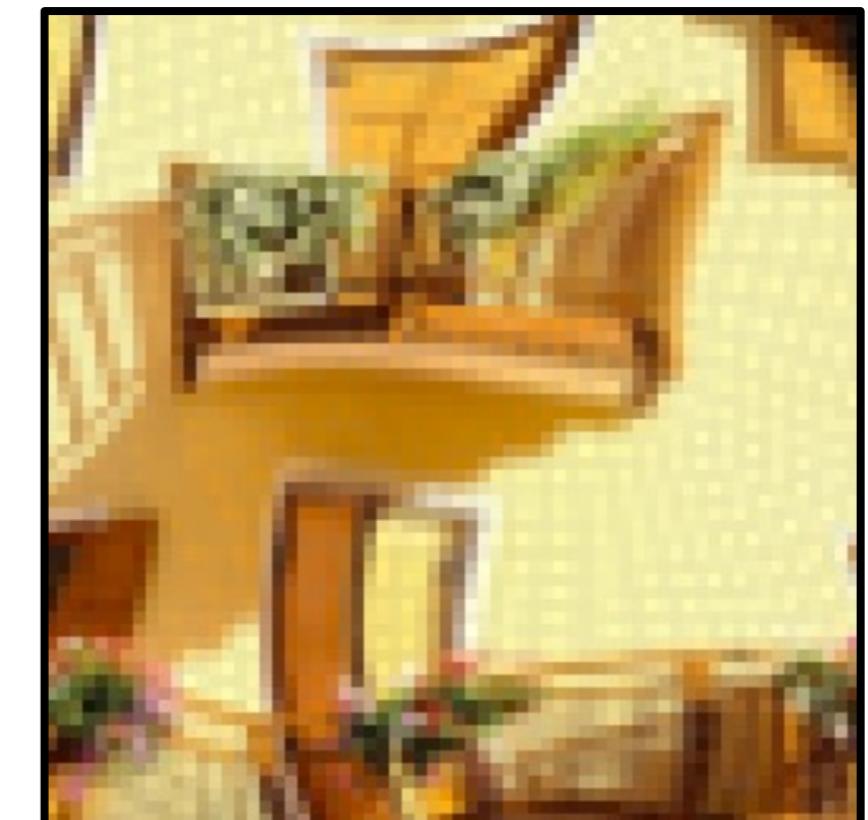
< Type 2 >

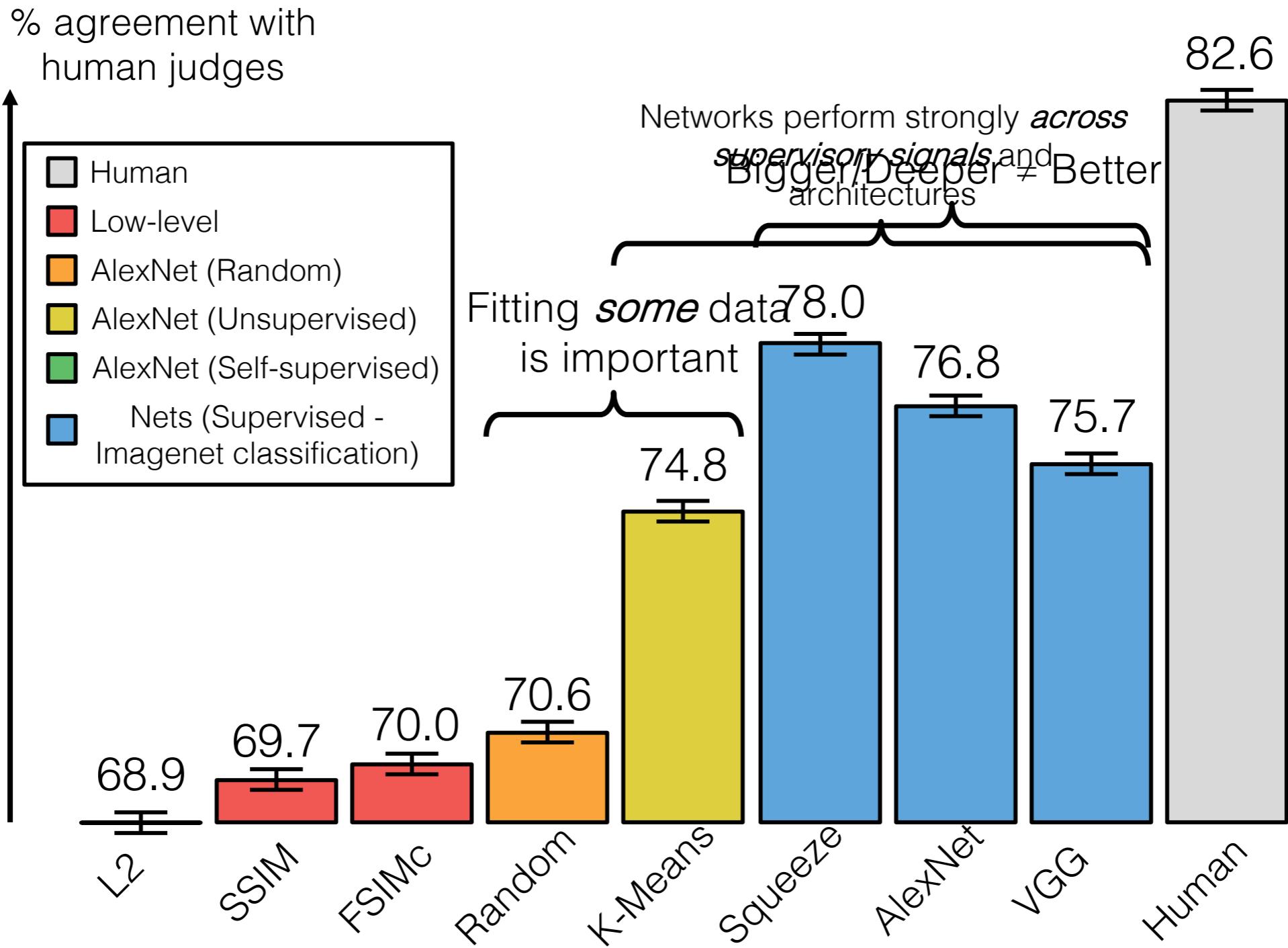
Humans

L2/PSNR

SSIM/FSIMc

Deep Networks?





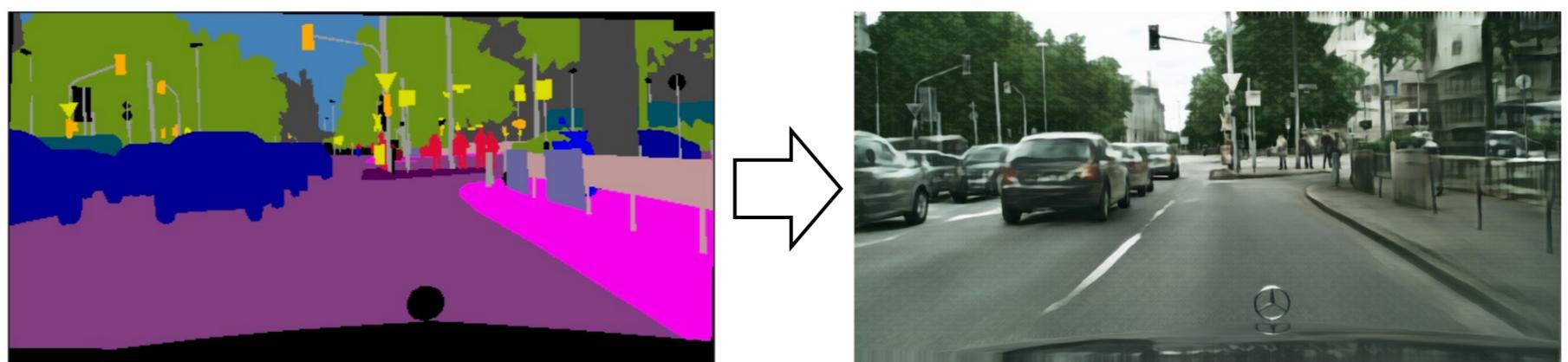
VGG ("perceptual loss") correlates well

“Perceptual Loss”

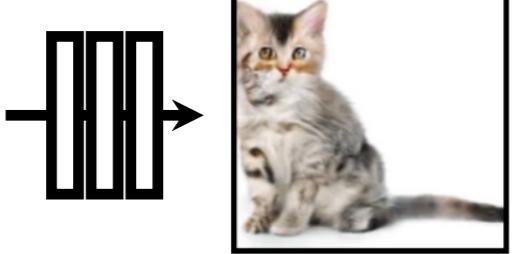
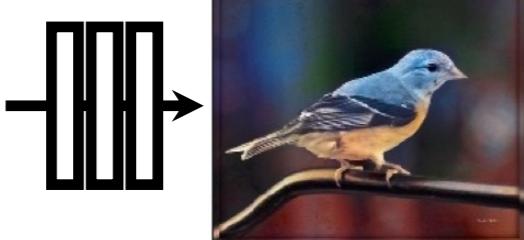
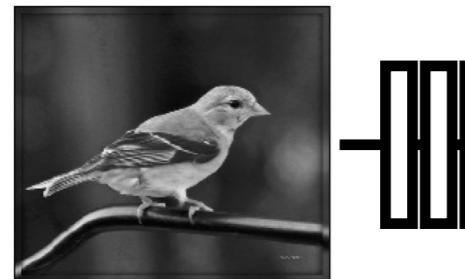
Gatys et al. In CVPR, 2016.
Johnson et al. In ECCV, 2016.
Dosovitskiy and Brox. In NIPS, 2016.



Chen and Koltun. In ICCV, 2017.



Generated images



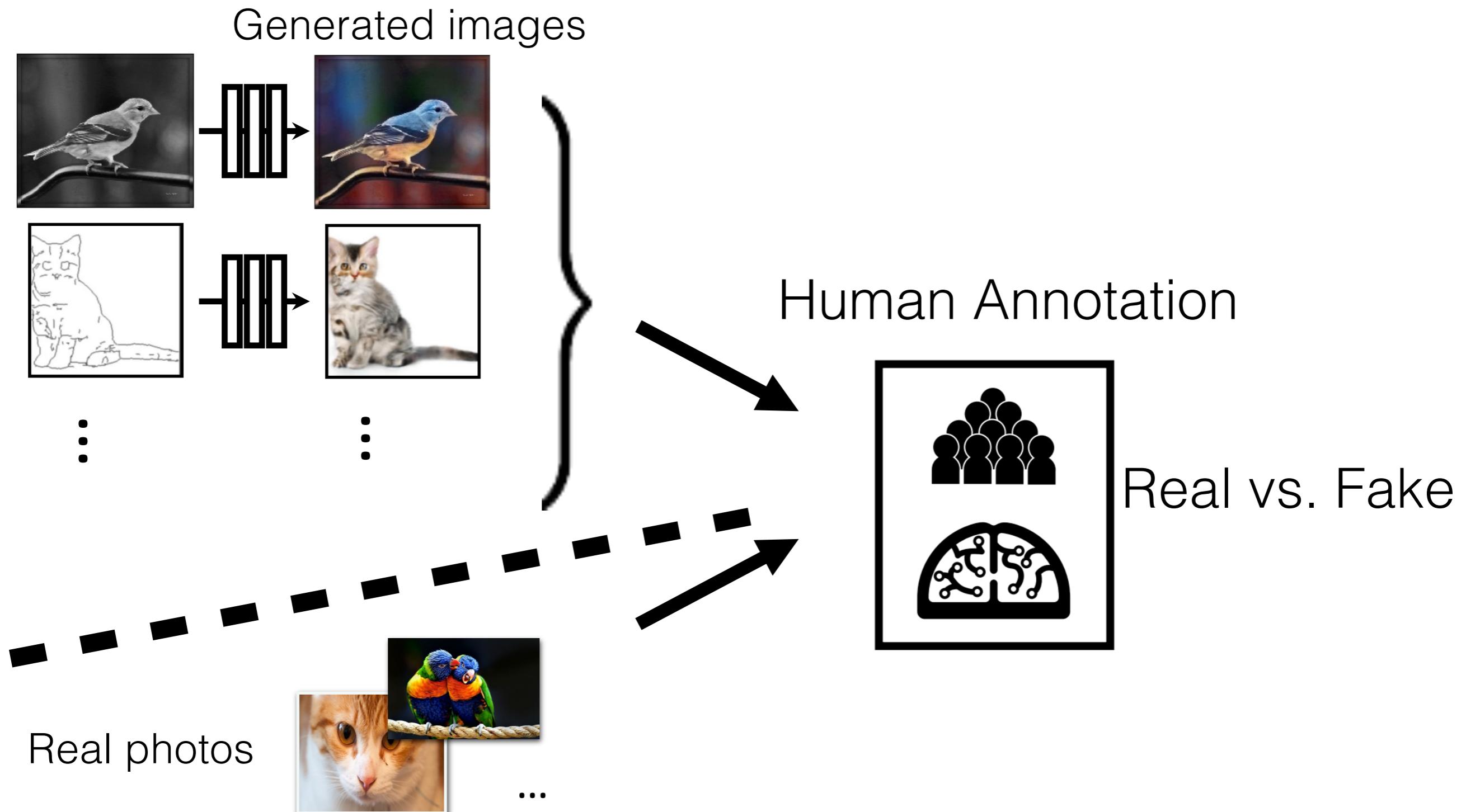
⋮

⋮

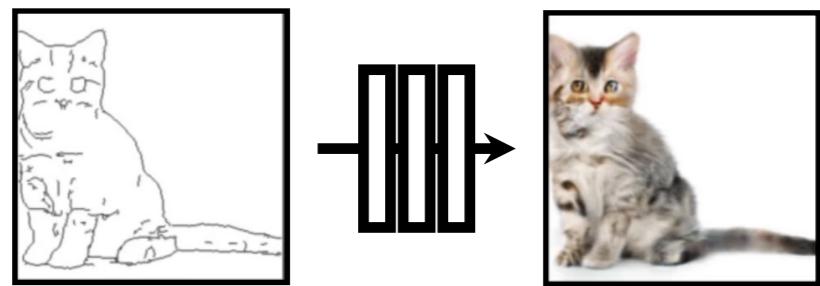
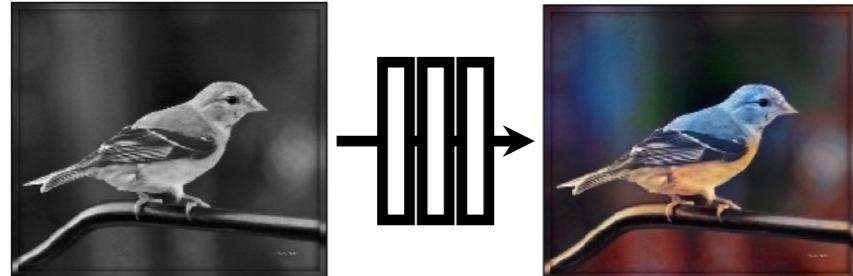


Universal loss?

Learning with Human Perception



Generated images



:

:

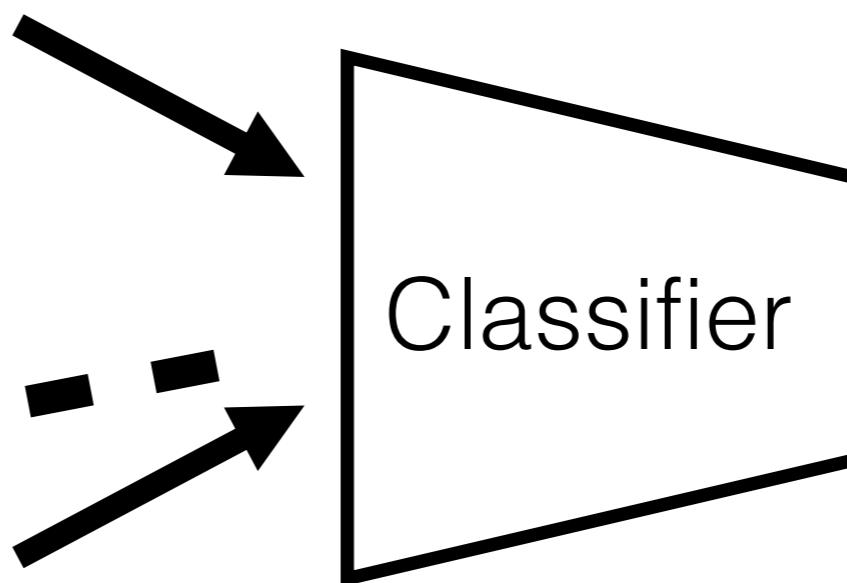


Generative Adversarial Network (GANs)

Real photos



...

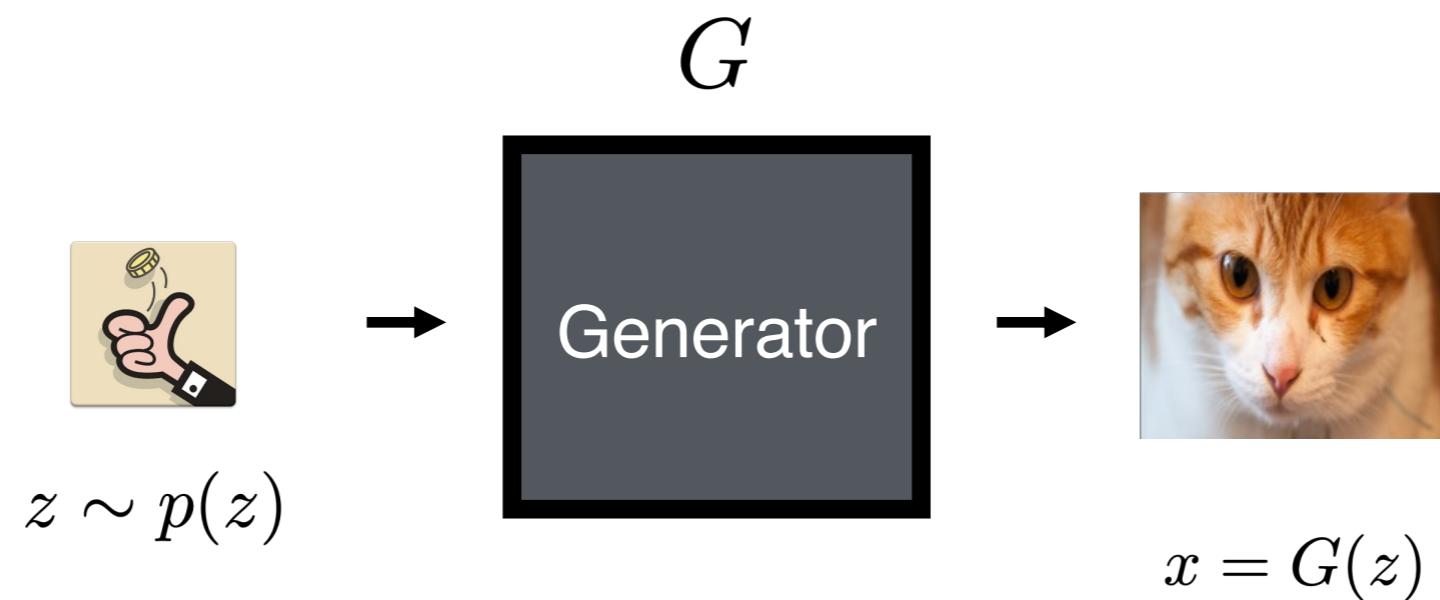


Real vs. Fake



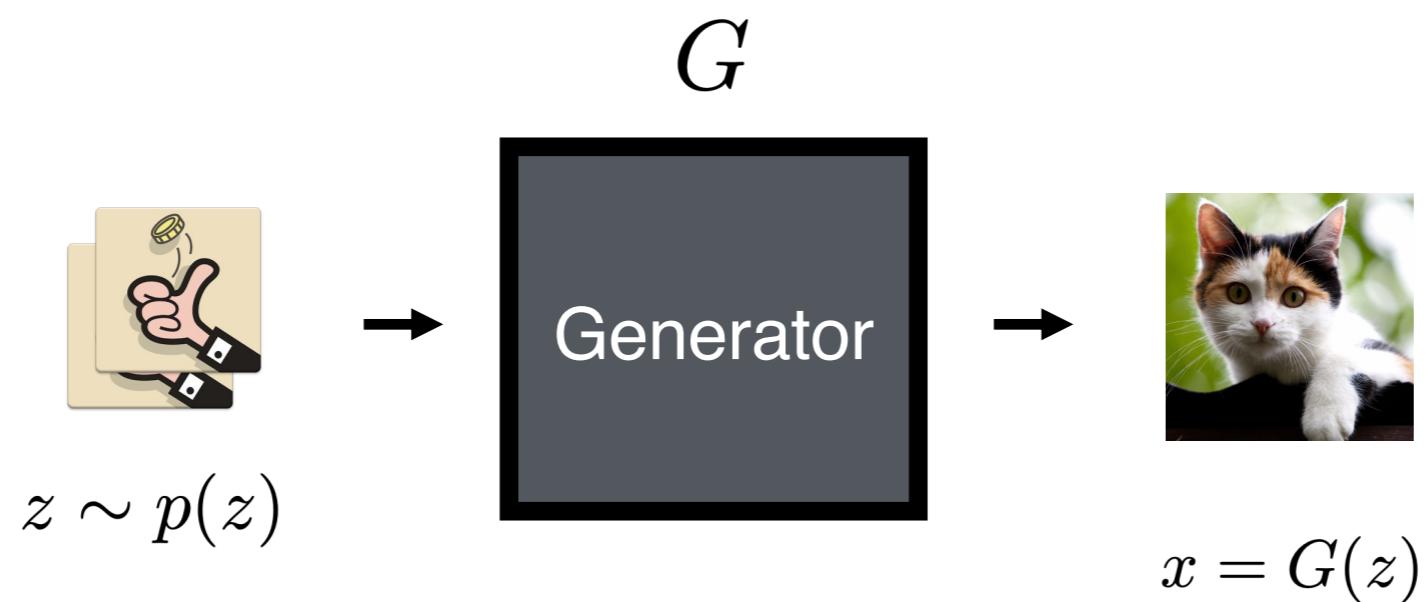
[Goodfellow, Pouget-Abadie, Mirza, Xu,
Warde-Farley, Ozair, Courville, Bengio 2014]

Image synthesis from “noise”



Sampler
 $G : \mathcal{Z} \rightarrow \mathcal{X}$
 $z \sim p(z)$
 $x = G(z)$

Image synthesis from “noise”



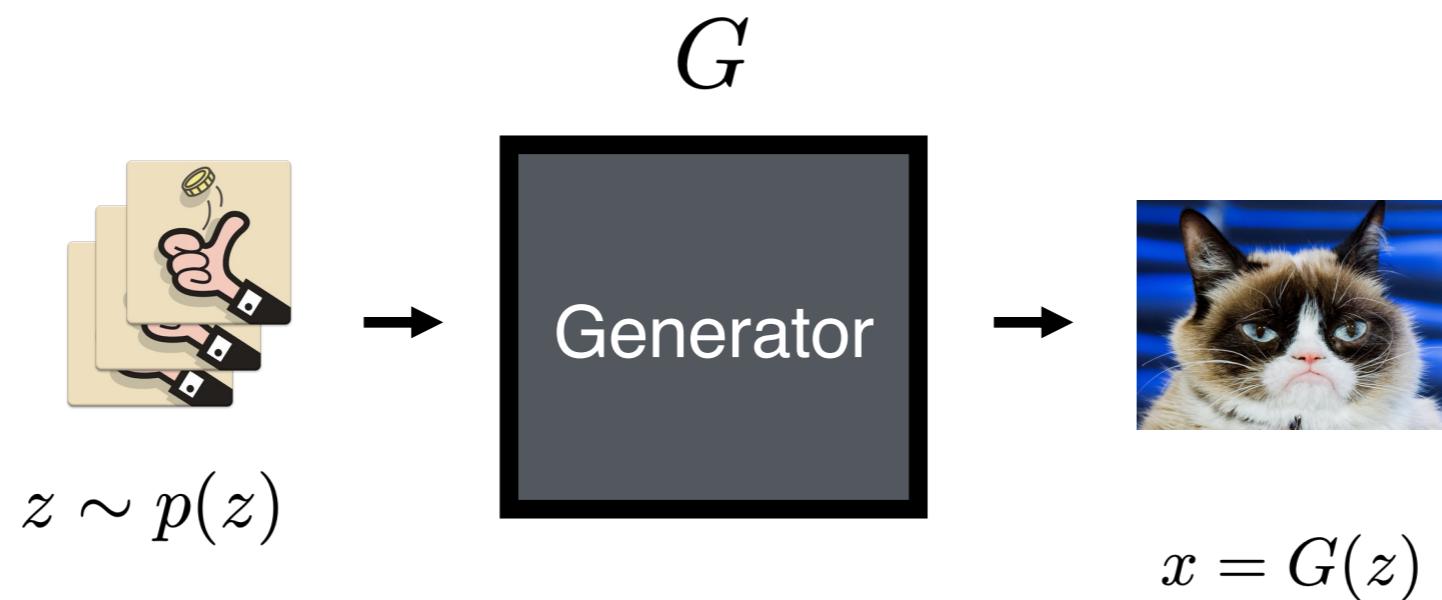
Sampler

$$G : \mathcal{Z} \rightarrow \mathcal{X}$$

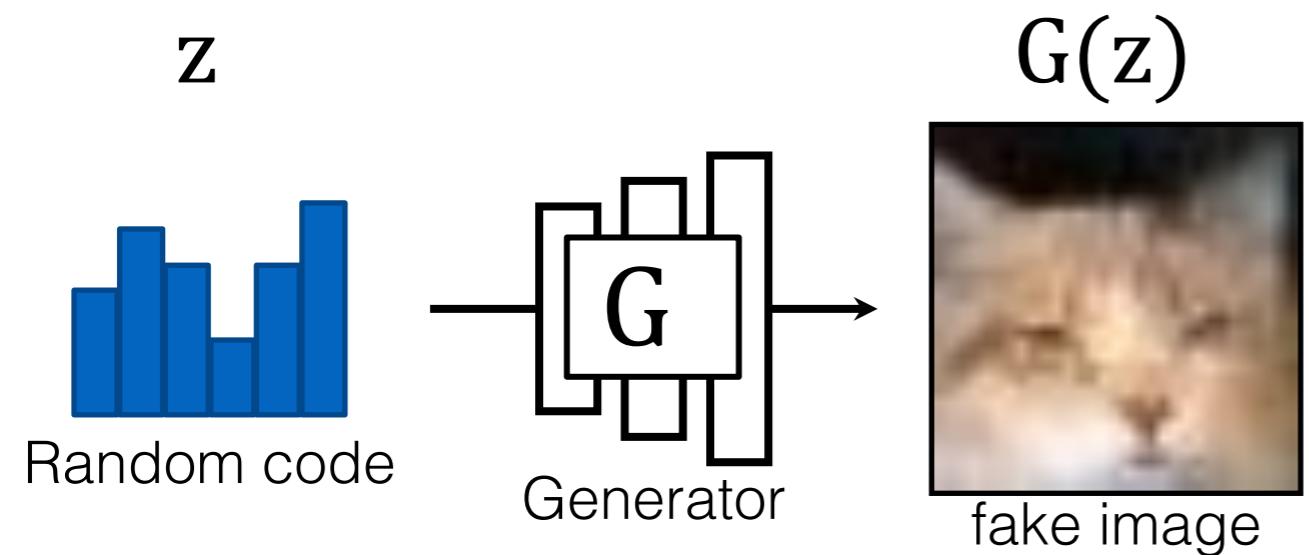
$$z \sim p(z)$$

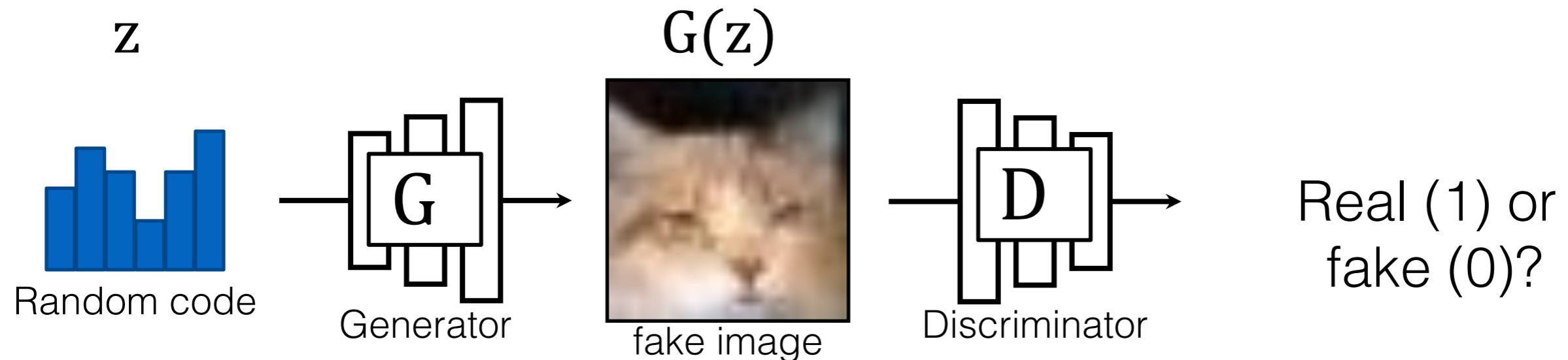
$$x = G(z)$$

Image synthesis from “noise”



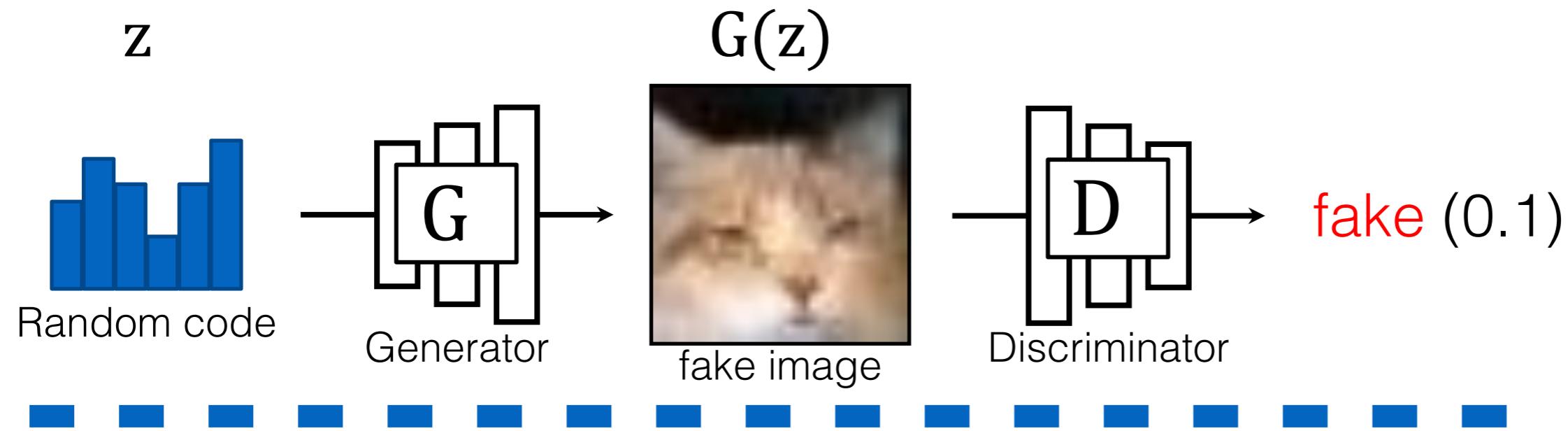
Sampler
 $G : \mathcal{Z} \rightarrow \mathcal{X}$
 $z \sim p(z)$
 $x = G(z)$





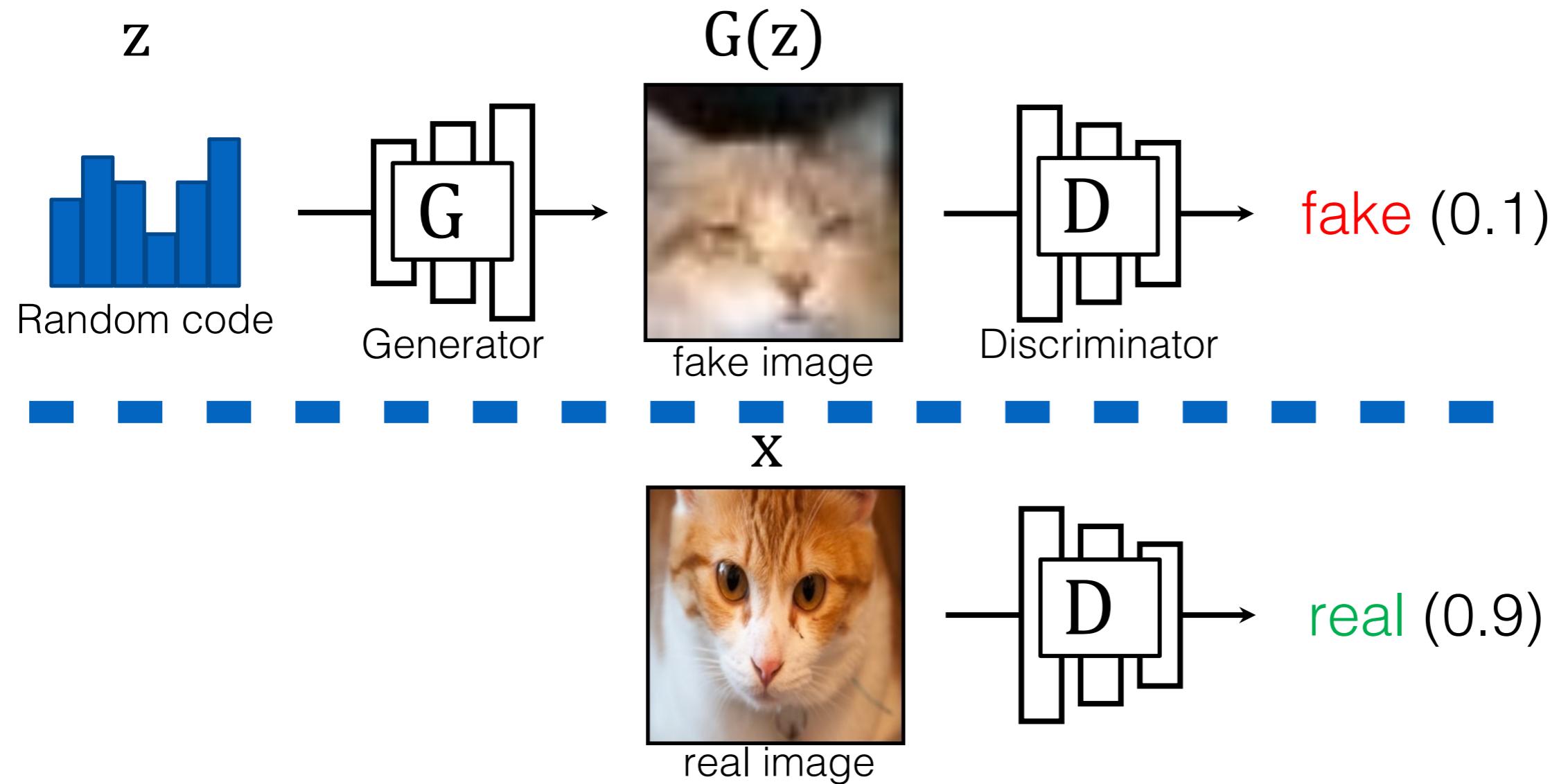
A two-player game:

- G tries to generate fake images that can fool D .
- D tries to detect fake images.



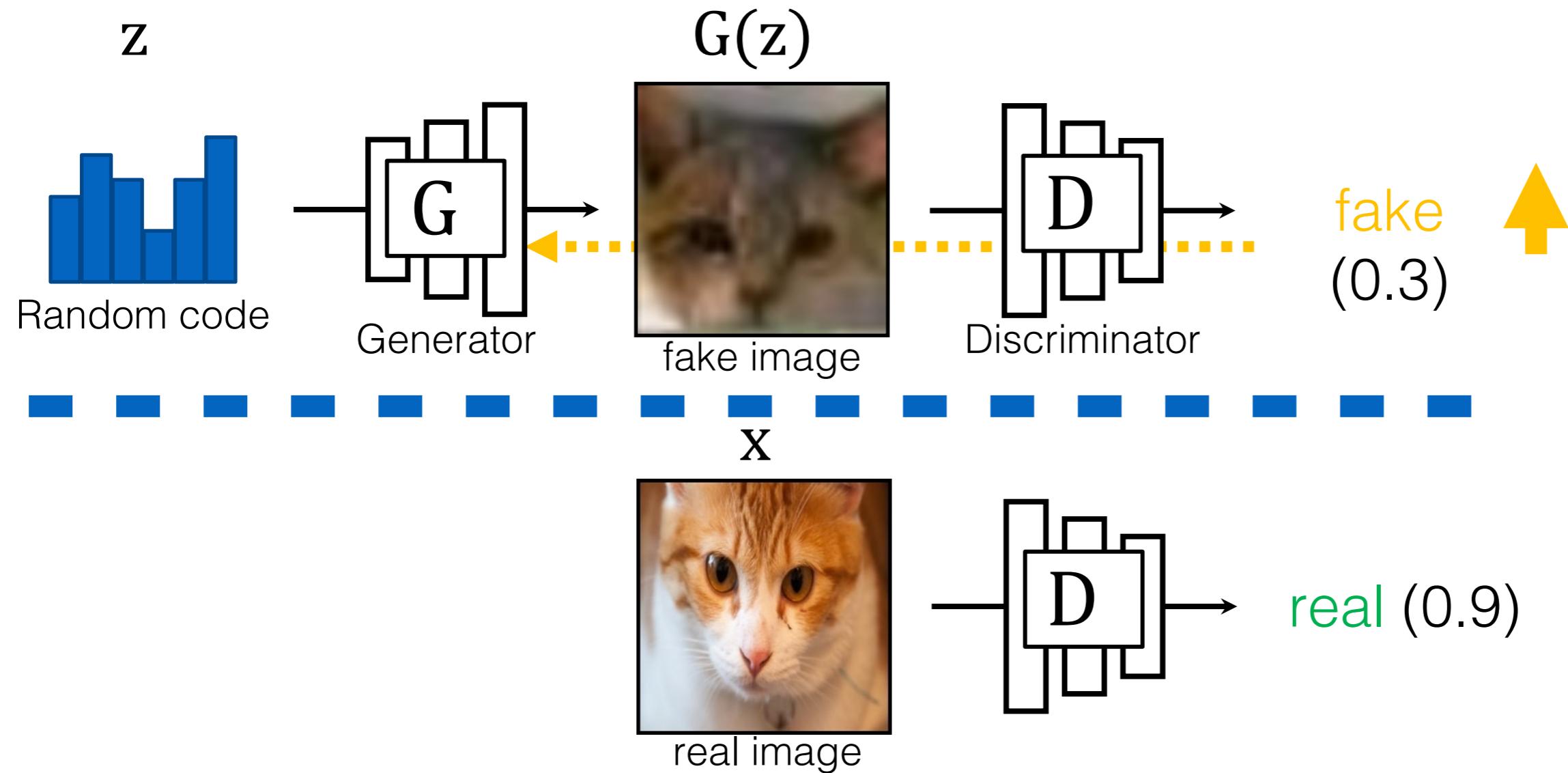
Learning objective (GANs)

$$\min_G \max_D \mathbb{E}_z [\log(1 - D(G(z)))]$$



Learning objective (GANs)

$$\min_G \max_D \mathbb{E}_z[\log(1 - D(G(z)))] + \mathbb{E}_x[\log D(x)]$$



Learning objective (GANs)

$$\min_G \max_D \mathbb{E}_z[\log(1 - D(G(z)))] + \mathbb{E}_x[\log D(x)]$$

GANs Training Breakdown

- From the discriminator D's perspective:
 - binary classification: real vs. fake.
 - Nothing special: similar to 1 vs. 7 or cat vs. dog

$$\max_D \mathbb{E}[\log(1 - D(\text{blurry image}))] + \mathbb{E}[\log D(\text{real cat image})]$$

GANs Training Breakdown

- From the discriminator D's perspective:
 - binary classification: real vs. fake.
 - Nothing special: similar to 1 vs. 7 or cat vs. dog

$$\max_D \mathbb{E}[\log(1 - D(\text{dog}))] + \mathbb{E}[\log D(\text{cat})]$$

- From the generator G's perspective:
 - Optimizing a loss that depends on a classifier D
 - We have done it before (Perceptual Loss)

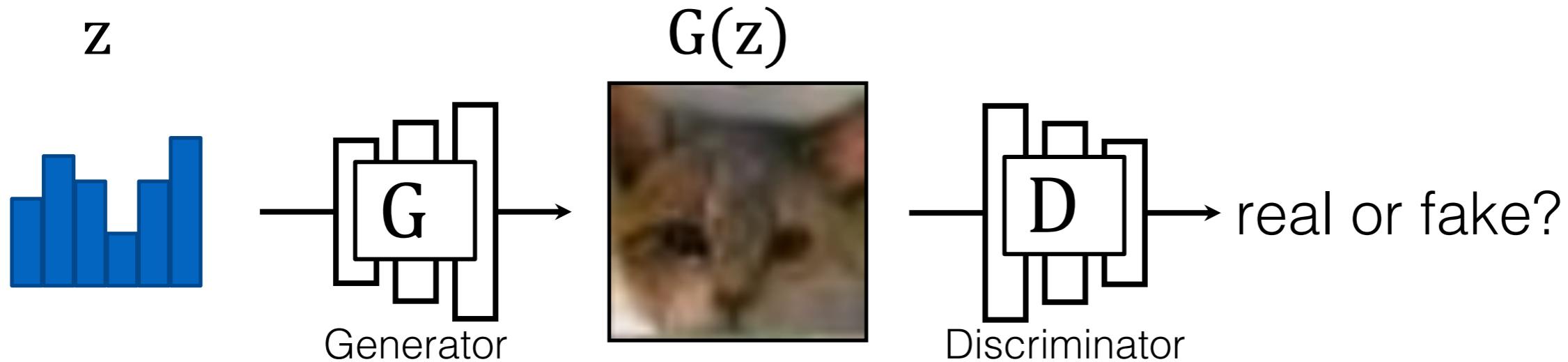
$$\min_G \mathbb{E}_z [\mathcal{L}_D(G(z))]$$

GAN loss for G

$$\min_G \mathbb{E}_{(x,y)} \|F(G(x)) - F(y)\|$$

Perceptual Loss for G

GANs Training Breakdown



G tries to synthesize fake images that fool **D**

D tries to identify the fakes

- Training: iterate between training D and G with backprop.
- Global optimum when G reproduces data distribution.

$p_g = p_{data}$ is the unique global minimizer of the GAN objective.

Proof

Optimal discriminator given fixed G

$$\begin{aligned} C(G) &= \mathbb{E}_{\mathbf{x} \sim p_{data}} [\log D_G^*(\mathbf{x})] + \mathbb{E}_{\mathbf{x} \sim p_g} [\log(1 - D_G^*(\mathbf{x}))] \\ &= \mathbb{E}_{\mathbf{x} \sim p_{data}} \left[\log \frac{p_{data}(\mathbf{x})}{P_{data}(\mathbf{x}) + p_g(\mathbf{x})} \right] + \mathbb{E}_{\mathbf{x} \sim p_g} \left[\log \frac{p_g(\mathbf{x})}{p_{data}(\mathbf{x}) + p_g(\mathbf{x})} \right] \end{aligned}$$

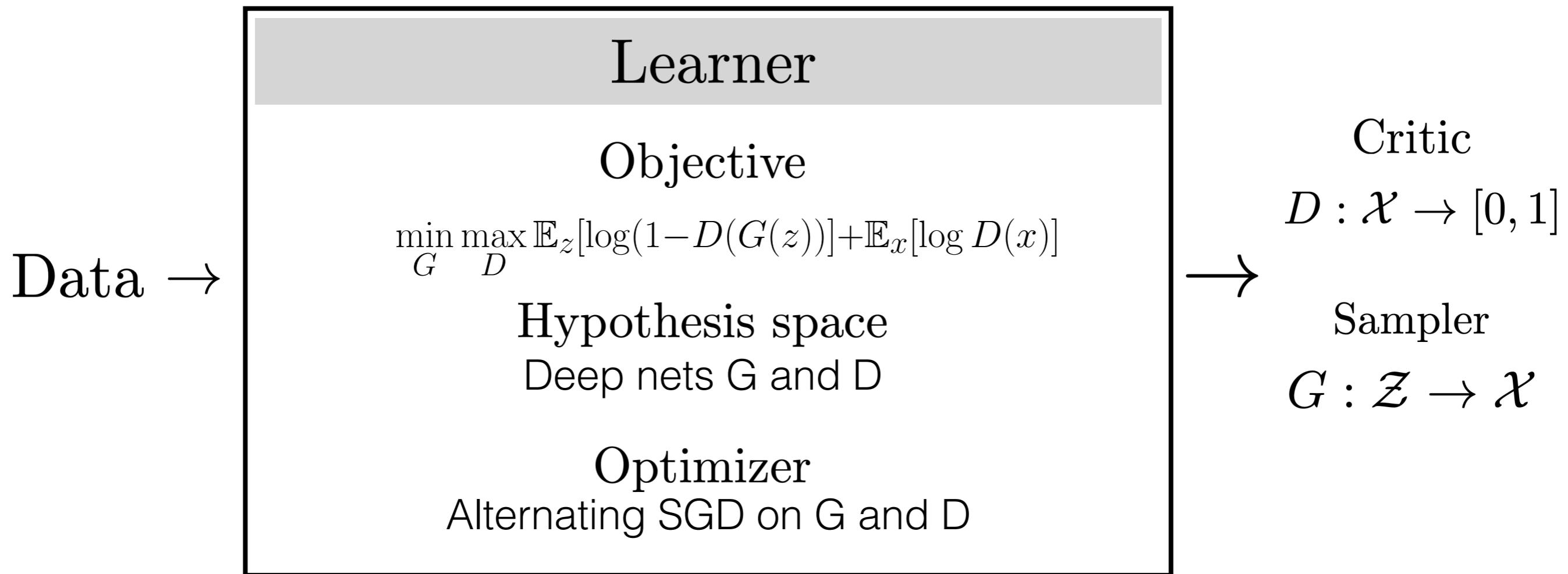
$$C(G) = -\log(4) + KL \left(p_{data} \middle\| \frac{p_{data} + p_g}{2} \right) + KL \left(p_g \middle\| \frac{p_{data} + p_g}{2} \right)$$

$$\begin{aligned} C(G) &= -\log(4) + 2 \cdot JSD(p_{data} \| p_g) \\ &\underbrace{\geq 0, \quad 0}_{\geq 0, \quad 0 \iff p_g = p_{data}} \quad \square \end{aligned}$$

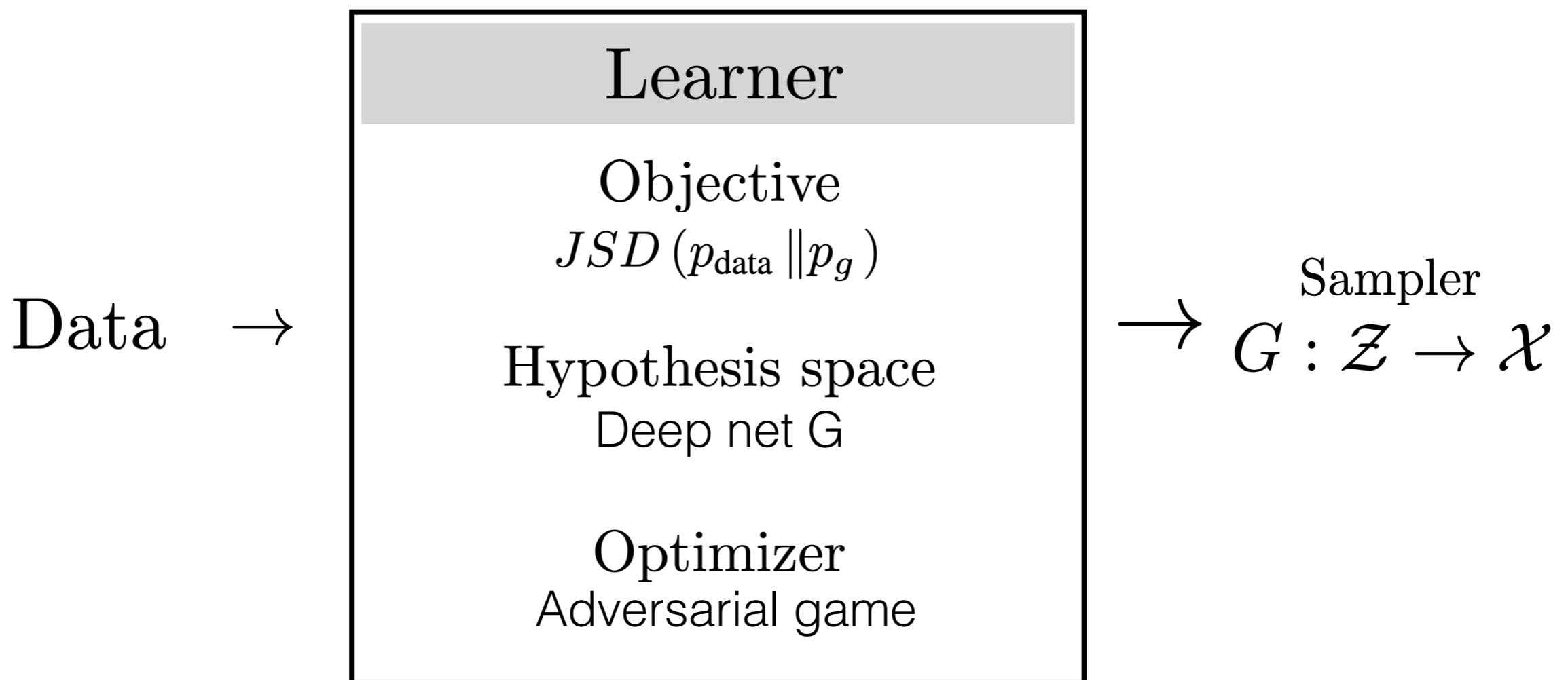
KLD (Kullback–Leibler divergence): $\mathcal{KL}(p||q) = \int p(x) \log \frac{p(x)}{q(x)} dx$

JSD (Jensen–Shannon divergence): $\mathcal{JSD}(p \| q) = \frac{1}{2}\mathcal{KL}(p \| \frac{p+q}{2}) + \frac{1}{2}\mathcal{KL}(q \| \frac{p+q}{2})$

Generative Adversarial Network



Generative Adversarial Network



What has driven GAN progress?

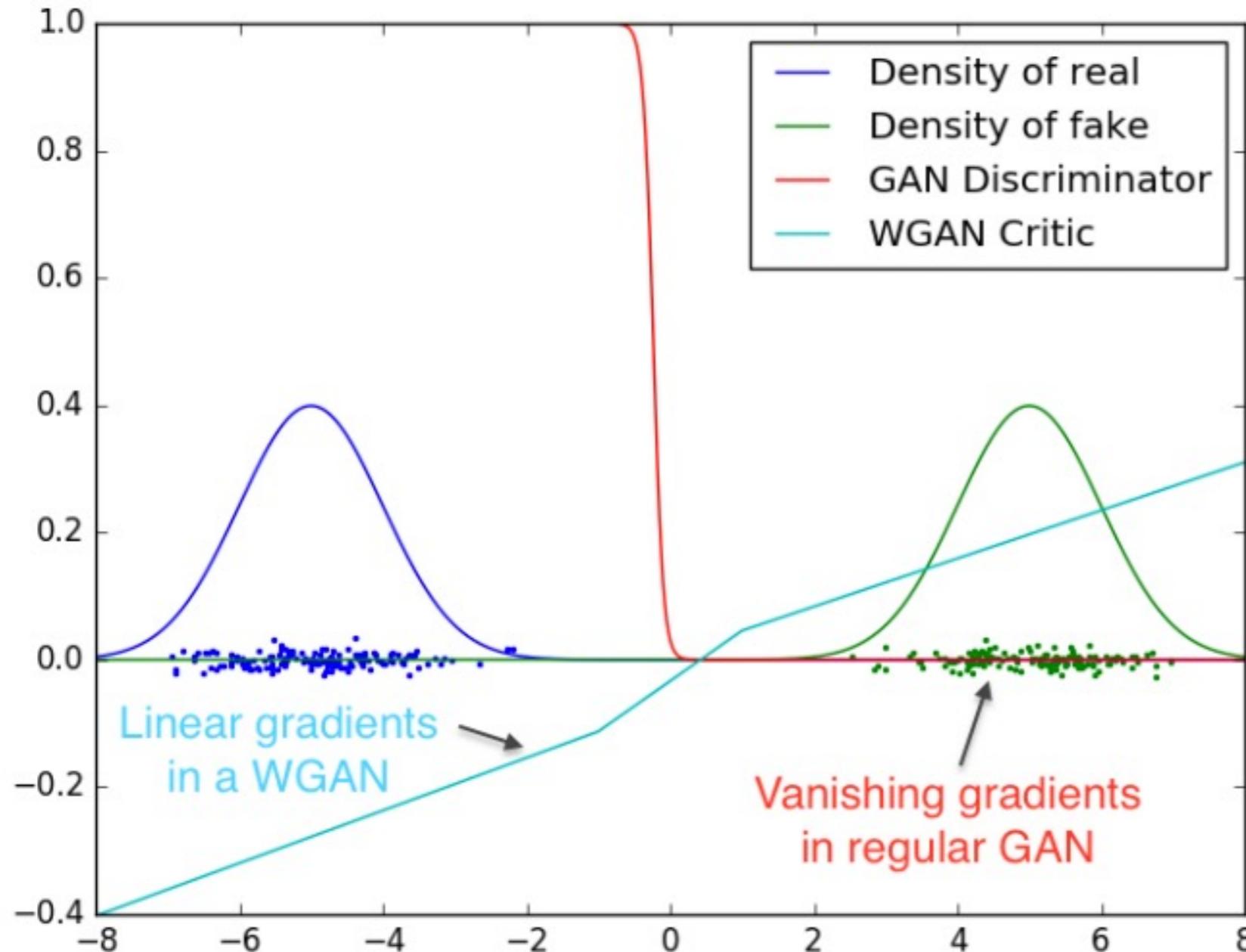


Ian Goodfellow @goodfellow_ian · Jan 14

▼

4.5 years of **GAN progress** on face generation. arxiv.org/abs/1406.2661
arxiv.org/abs/1511.06434 arxiv.org/abs/1606.07536 arxiv.org/abs/1710.10196
arxiv.org/abs/1812.04948

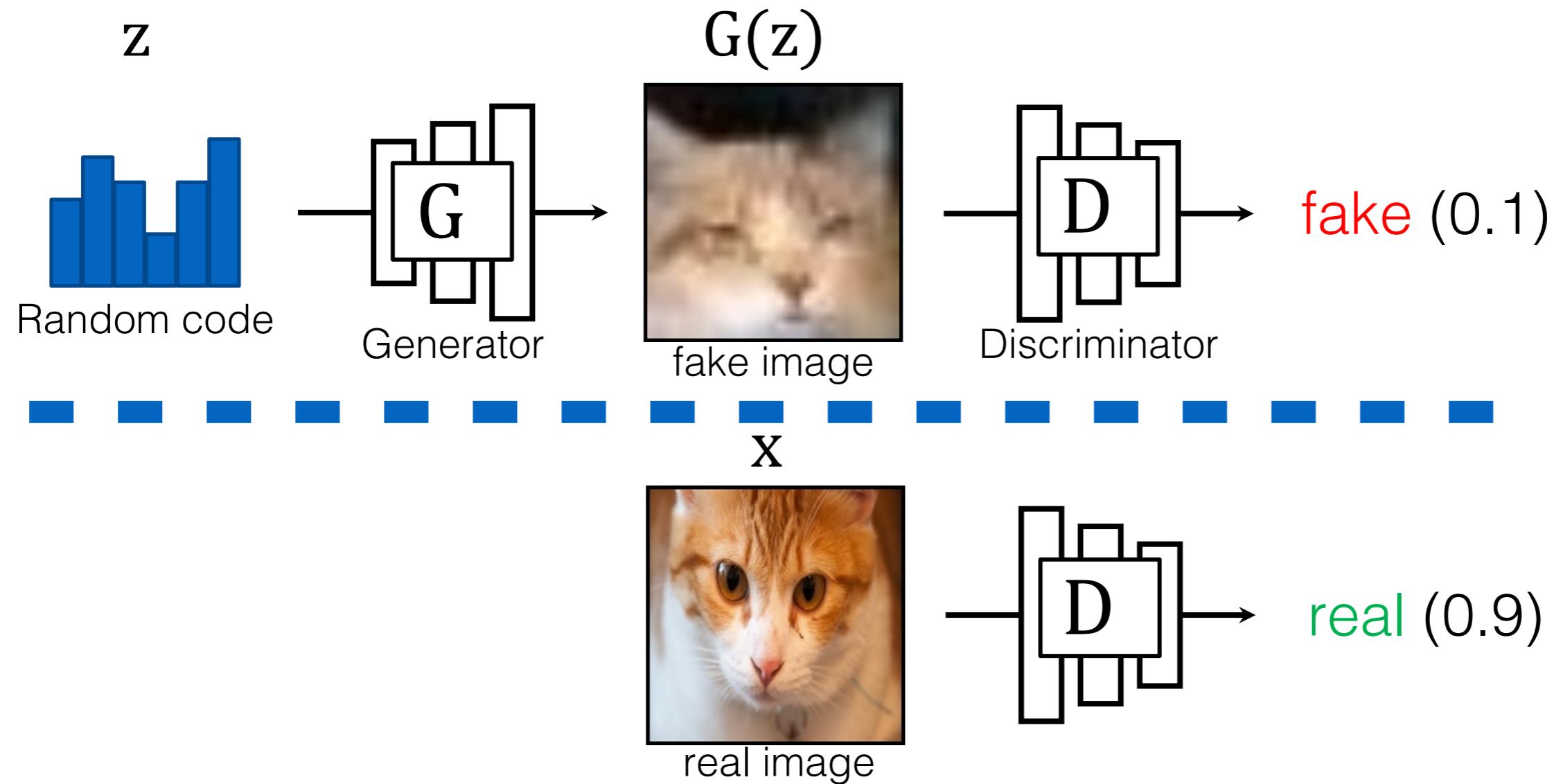




$$D_G^*(\mathbf{x}) = \frac{p_{data}(\mathbf{x})}{p_{data}(\mathbf{x}) + p_g(\mathbf{x})}$$

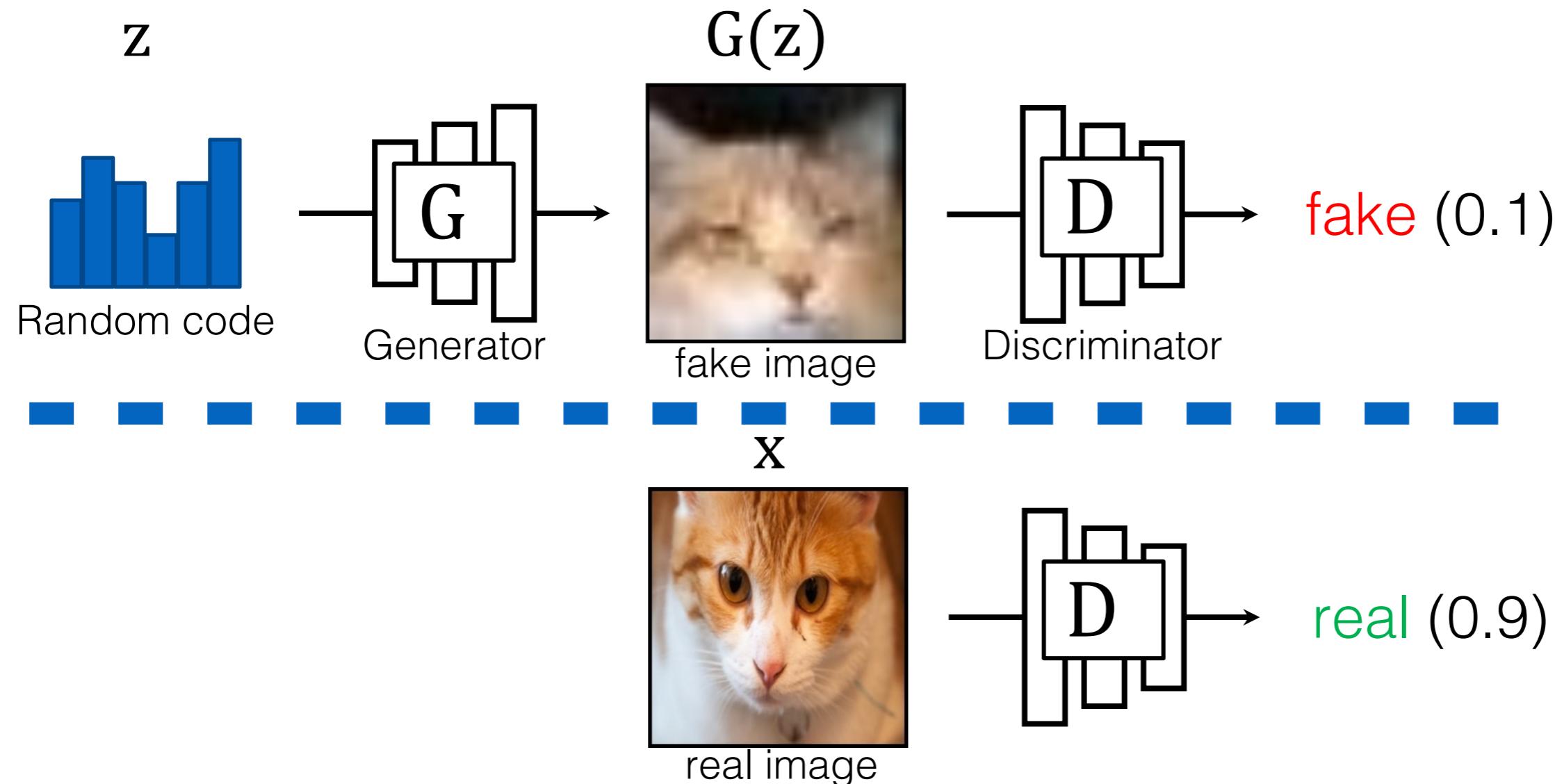
$\log D(G(\mathbf{z})) \rightarrow -\infty$

from [Arjovsky, Chintala, Bottou, 2017]



Learning objective (GANs)

$$\min_G \max_D \mathbb{E}_z[\log(1 - D(G(z)))] + \mathbb{E}_x[\log D(x)]$$

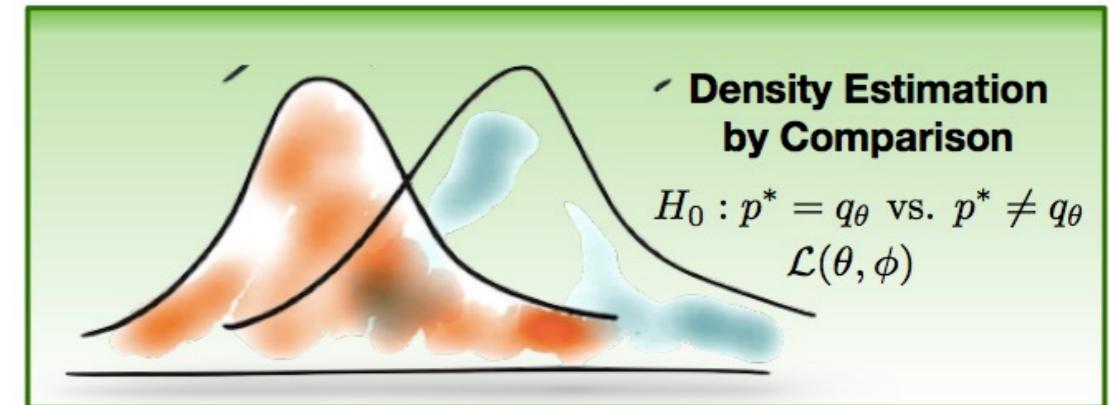


Learning objective (GANs variants)

$$\min_G \max_{f_1, f_2} \mathbb{E}_z [f_1(G(z))] + \mathbb{E}_x [f_2(x)]$$

EBGAN, WGAN, LSGAN, etc

Other divergences?



from [Mohamed & Lakshminarayanan 2017]

$$\min_G \max_{f_1, f_2} \boxed{\mathbb{E}_z[f_1(G(z))]} + \boxed{\mathbb{E}_x[f_2(x)]}$$

Convenient choice

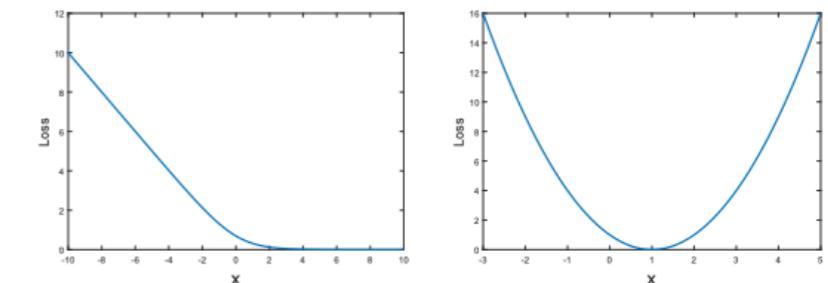
$$\begin{aligned} f_1 &= -f \\ f_2 &= f \end{aligned}$$

Different choices of f_1 and f_2 correspond to different divergence measures:

- Original GAN —> JSD
- Least-squares GAN —> Pearson chi-squared divergence

$$\min_D V_{\text{LSGAN}}(D) = \frac{1}{2} \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [(D(\mathbf{x}) - 1)^2] + \frac{1}{2} \mathbb{E}_{\mathbf{z} \sim p_z} [(D(G(\mathbf{z})) - 1)^2]$$

$$\min_G V_{\text{LSGAN}}(G) = \frac{1}{2} \mathbb{E}_{\mathbf{z} \sim p_z(\mathbf{z})} [(D(G(\mathbf{z})) - 1)^2].$$



Other divergences?

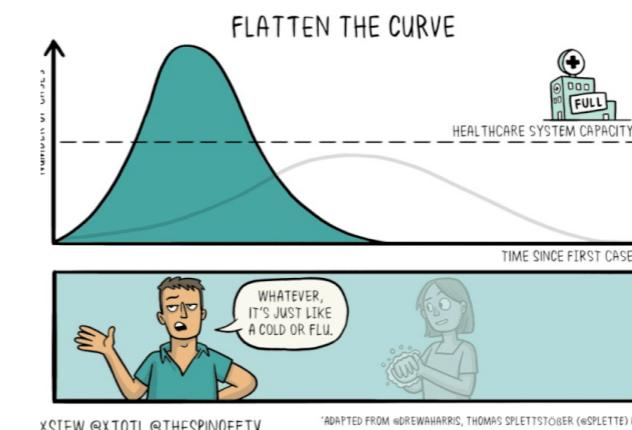
$$KL(p_{\text{data}} || p_{\theta}) \leftarrow \mathbb{E}_{x \sim p_{\text{data}}} [\log p_{\theta}(x)]$$

$$KL(p_{\theta} || p_{\text{data}}) \leftarrow \text{Reverse KL — mode seeking, intractable}$$

$$JS(p_{\text{data}}, p_{\theta}) \leftarrow \text{Jensen-Shannon, original GAN}$$

$$W(p_{\text{data}}, p_{\theta}) = \inf_{\gamma \in \Pi(p_{\text{data}}, p_{\theta})} \mathbb{E}_{(x,y) \sim \gamma} [\|x - y\|] \leftarrow \text{Wasserstein}$$

Earth-Mover (EM) distance
/ Wasserstein distance



Maximum log likelihood, KL, and JSD

KLD (Kullback–Leibler divergence): $\mathcal{KL}(p||q) = \int p(x) \log \frac{p(x)}{q(x)} dx$

JSD (Jensen–Shannon divergence): $\mathcal{JSD}(p \parallel q) = \frac{1}{2}\mathcal{KL}(p \parallel \frac{p+q}{2}) + \frac{1}{2}\mathcal{KL}(q \parallel \frac{p+q}{2})$

$$\mathbb{E}_{x \sim p_{\text{data}}(x)} [\log p_{\theta}(x)] = \int_x p_{\text{data}}(x) \log p_{\theta}(x) dx$$

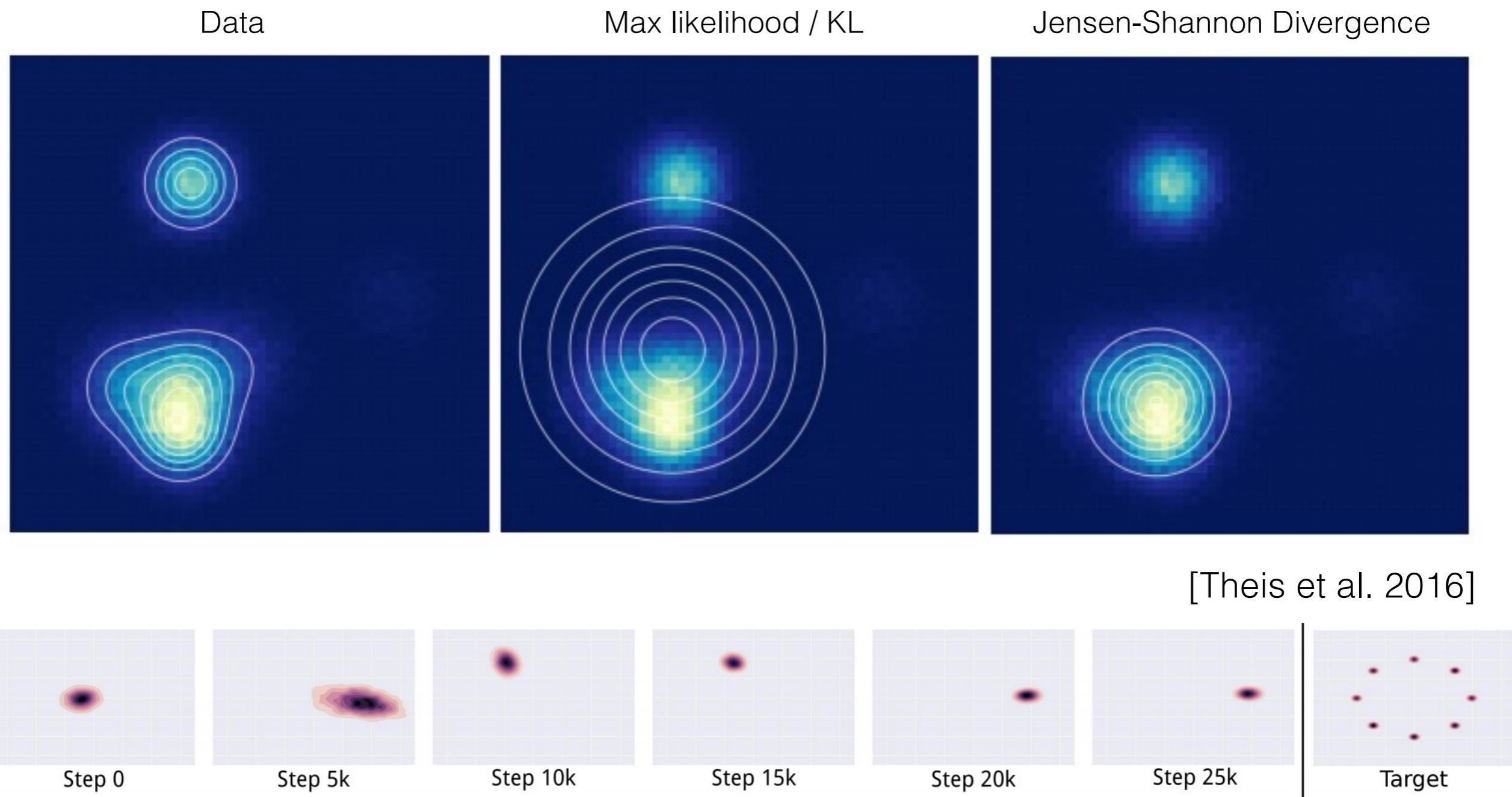
$$\mathcal{KL}(p_{\text{data}}(x) || p_{\theta}(x)) = \int_x p_{\text{data}}(x) \log \frac{p_{\text{data}}(x)}{p_{\theta}(x)} dx$$

$$= \int_x p_{\text{data}}(x) \log p_{\text{data}}(x) dx - \int_x p_{\text{data}}(x) \log p_{\theta}(x) dx$$

Constant
(independent of θ)

Maximize log likelihood= minimize KLD

Maximum log likelihood/KL vs. JSD



Other divergences?

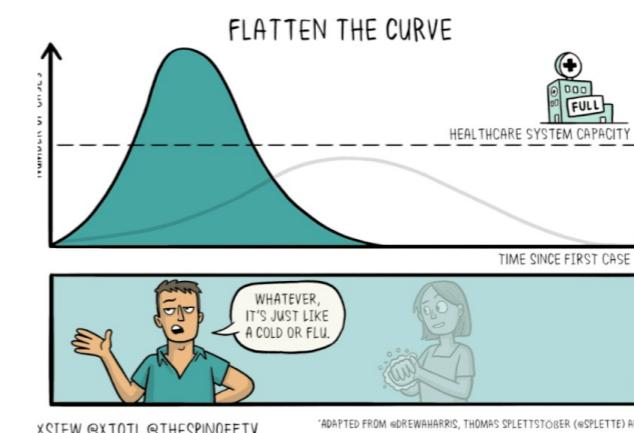
$$KL(p_{\text{data}} || p_{\theta}) \leftarrow \mathbb{E}_{x \sim p_{\text{data}}} [\log p_{\theta}(x)]$$

$$KL(p_{\theta} || p_{\text{data}}) \leftarrow \text{Reverse KL — mode seeking, intractable}$$

$$JS(p_{\text{data}}, p_{\theta}) \leftarrow \text{Jensen-Shannon, original GAN}$$

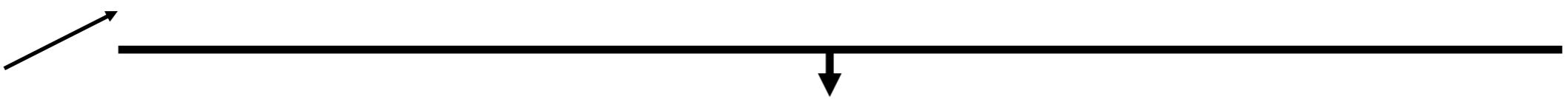
$$W(p_{\text{data}}, p_{\theta}) = \inf_{\gamma \in \Pi(p_{\text{data}}, p_{\theta})} \mathbb{E}_{(x, y) \sim \gamma} [\|x - y\|] \leftarrow \text{Wasserstein}$$

Earth-Mover (EM) distance
/ Wasserstein distance



Wasserstein GAN

[Arjovsky, Chintala, Bottou 2017]

$$\arg \min_G \max_{\|f\|_L \leq 1} \mathbb{E}_{\mathbf{z}, \mathbf{x}} [\boxed{-f(G(\mathbf{z}))} + \boxed{f(\mathbf{x})}]$$


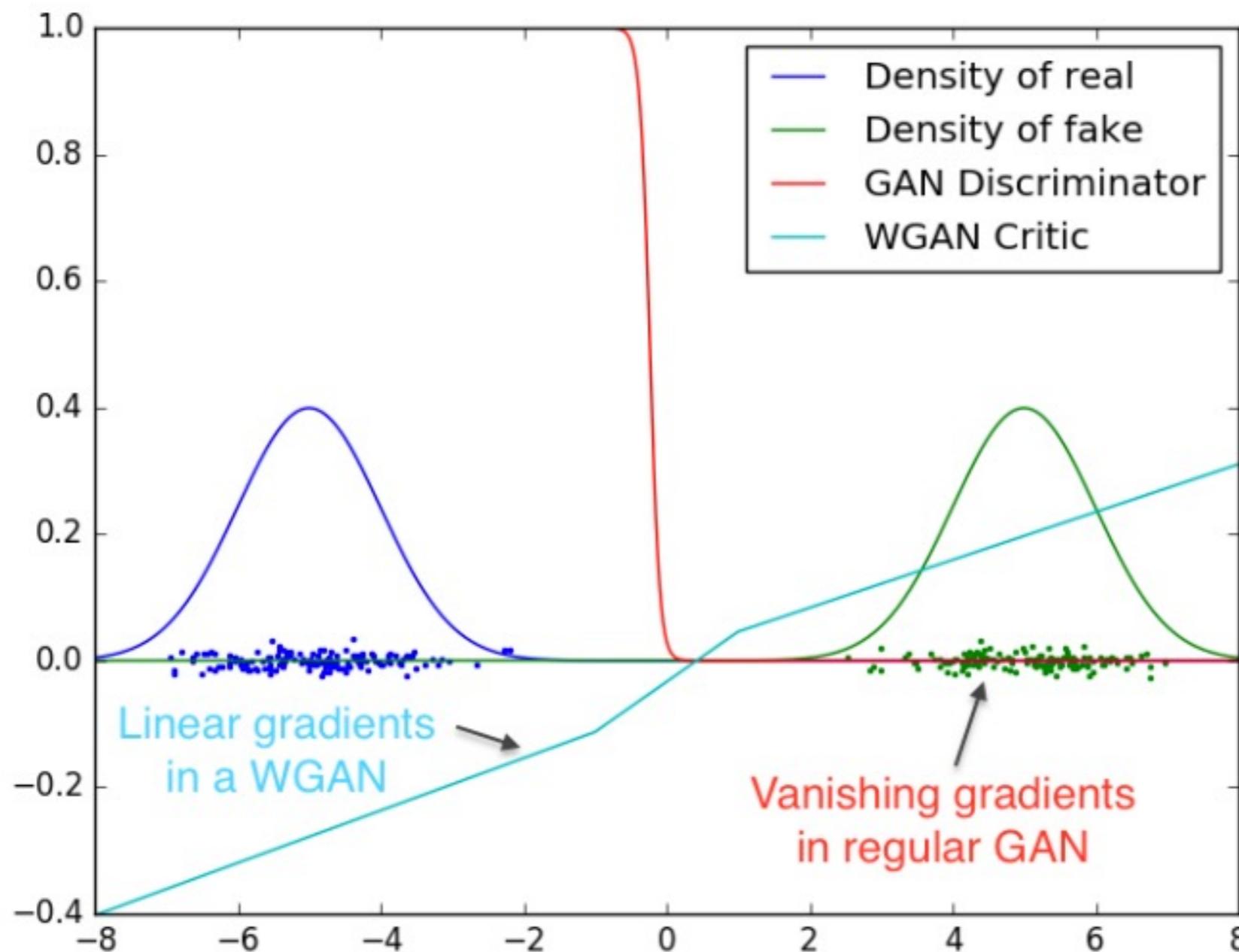
Lipschitz continuity
 $|f(x) - f(y)| \leq |x - y|$

$$W(p_{\text{data}}, p_{\theta}) = \inf_{\gamma \in \Pi(p_{\text{data}}, p_{\theta})} \mathbb{E}_{(x, y) \sim \gamma} [\|x - y\|]$$

wGAN GP [Gulrajani et al., 2018]:

$$\arg \min_G \max_f \mathbb{E}_{\mathbf{z}, \mathbf{x}} [\boxed{-f(G(\mathbf{z}))} + \boxed{f(\mathbf{x})}] + \underline{\lambda \mathbb{E}_{\hat{\mathbf{x}} \sim \mathbb{P}_{\hat{\mathbf{x}}}} [(\|\nabla_{\hat{\mathbf{x}}} f(\hat{\mathbf{x}})\|_2 - 1)^2]}$$

Gradient penalty (GP)



from [Arjovsky, Chintala, Bottou, 2017]

To be continued...

Thank You!



16-726, Spring 2023

<https://learning-image-synthesis.github.io/>