Syllabus for SUTD 40.616 (Fall 2025)
"Special Topics in Games, Learning, and Optimization"

1 General Information

Course Description: This is a graduate-level theory course providing a modern treatment of *online learning* and *learning in games*. The course is centered on the online learning framework as a paradigm for sequential decision making within strategic and non-stationary environments. Particular attention will be devoted to showing how online learning dynamics lead to equilibria in multi-agent, game-theoretic settings.

The course is designed for students interested in algorithmic game theory, multi-agent learning and optimization. Lectures will introduce the online learning paradigm, fundamental algorithms, key results, and analysis techniques, with the aim of preparing students to pursue research in this area.

Instructors: Anas Barakat, John Lazarsfeld, and Joseph Sakos (*Postdoctoral Research Fellows in ESD Pillar*)
Time: Tuesdays and Thursdays, 10am-12pm SGT (*starting on 2025.09.16*)
Location: SUTD Think Tank 11
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Prerequisites: Prior courses in calculus, linear algebra, and basic notions of probability and analysis. A prior course in optimization is helpful but not absolutely necessary. Prior knowledge of game theory is not required.

2 Overview of Topics

The course will consist of twice-weekly lectures organized in four parts – Part I: Online Learning, Part II: Learning in Normal-Form and Stochastic Games, Part III: Learning in Extensive-Form and Continuous Games, Part IV: Special Topics. A tentative schedule of lecture topics is as follows:

Part I: Online Learning – *Lectures given by John*

- (L01) – Introduction to Online Learning
  Prediction with expert advice, online convex optimization, regret, Multiplicative Weights Update and Online Gradient Descent.
- (L02) – Follow-the-Regularized-Leader: No-regret via Regularization
  Family of leader-based algorithms, analysis of Follow-the-Regularized-Leader (FTRL) via coupling with Be-the-Leader/Follow-the-Leader, Multiplicative Weights Update as FTRL, lower bounds for online learning.
- (L03) – Follow-the-Perturbed-Leader and Online Mirror Descent: No-regret via Perturbation and Penalty
  Follow-the-Perturbed-Leader (FTPL) analysis, equivalence between FTPL and FTRL, Online Mirror Descent analysis.
- (L04) – Online Learning with Bandit Feedback
  Bandit feedback model, expected regret and pseudo-regret, EXP3 algorithm for adversarial bandits, Explore-then-Commit and UCB algorithms for stochastic bandits.
- (L05) – Φ-Regret Minimization
  Beyond external regret: swap-regret, internal-regret, and Φ-regret framework. Blum-Mansour and Stoltz-Lugosi algorithms.
- (L06) – Blackwell Approachability and Regret Matching
  Blackwell's Approachability theorem, Regret Matching (RM) and Regret Matching+ (RM+) algorithms.

Part II: Learning in Normal-Form and Stochastic Games – *Lectures given by Anas*

- (L07) – Introduction to Normal-Form Games and Nash Equilibria
  Normal-form games, Nash equilibria (NE), game classes (potential, zero-sum, decomposition).
- (L08) – Online Learning in Potential Games
  Identical interest games, potential games, existence of pure NE, online learning in games paradigm, sublinear regret in potential games, learning approximate Nash equilibria via online learning in potential games.
- (L09) – Online Learning in Zero-Sum Games
  Zero-sum games, minmax theorem, online learning proof, learning approximate Nash equilibria via online learning.
- (L10) – Learning (Coarse)-Correlated Equilibria in General-Sum Games
  (Coarse)-correlated equilibria, time-average convergence via no-ϕ-regret learning, average vs. last-iterate convergence.
- (L11) – Optimistic Learning and Social Welfare of No-Regret Dynamics
  Optimistic FTRL algorithms, RVU bounds, individual vs. sum of regrets, fast convergence of social welfare.
- (L12) – Introduction to Stochastic Games and Multi-Agent Reinforcement Learning
  Introduction to Markov Decision Processes and Reinforcement Learning, definition of stochastic games, Shapley's minimax theorem, existence of Nash equilibria, independent and decentralized learning, zero-sum Markov games and Markov potential games, policy

gradient methods.

Part III: Learning in Extensive-Form and Continuous Games – *Lectures given by Joseph*

- (L13) – Introduction to Extensive-Form Games
  Game trees, imperfect information, perfect recall, strategy representations, Kuhn's theorem.
- (L14) – Learning Equilibria in Extensive-Form Games
  Counterfactual Regret Minimization (CFR) algorithm and speedups.
- (L15) – Introduction to Continuous Games
  Concave games, Rosen's theorem, variational inequalities, monotone games, zero-sum games and Gradient Descent Ascent (GDA), divergence of GDA in bilinear case.
- (L16) – Learning Equilibria in Continuous Games
  Proximal point method, Optimistic GDA and Extragradient algorithms for zero-sum games, learning equilibria in potential games, general concave games.
- (L17) – Price of Anarchy and Equilibrium Selection
  Braess's paradox, Pigou's network, smooth games, introduction to Price of Anarchy (PoA) bounds.

Part IV: Special Topics

The final six lectures will cover more advanced topics based on results in the field over the past five years:

- (L18) – Online Learning in Time-Varying Games (Anas)
- (L19) – (Multi-Agent) Online Nonstochastic Control (Anas)
- (L20) – Bridging Continuous-time and Discrete-time Learning in Games (John)
- (L21) – Unregularized Learning in Zero-Sum Games (John)
- (L22) – Sum-of-Squares Optimization in Games (Joseph)
- (L23) – Hidden Games (Joseph)

3 Assignments and Final Project

The course will involve two problem sets and a final project.

Homeworks: Each problem set will consist of several exercises based on topics covered in the lectures.

- *Problem Set A*: released Friday, 2025.09.18; due Friday, 2025.10.10.
- *Problem Set B*: released Friday, 2025.10.17; due Friday, 2025.11.14.

Final Project: Students will perform a final project based on reading and synthesizing several related papers about an active research area in the field. The instructors will propose a list of projects with a list of papers and will match students with topics based on interest. Students will give a short midterm presentation on their topics, prepare a final report, and give a final presentation in the last week of the course.

- *Project topic preference rankings*: due Friday, 2025.10.03.
- *Midterm presentations and feedback*: Tuesday, 2025.11.04. (5-10 minutes per student).
- *Final presentations*: Tuesday, 2025.12.16 and Thursday, 2025.12.18. (15-20 minutes per student).
- *Final report*: due Friday, 2025.12.19.

Grading: Will be based on the following percentages:

- **Homework**: 40% (20% per assignment)
- **Final Project**: 60% (midterm presentation: 10%, final presentation: 20%, final report: 30%).

More details on problem sets and final project will be covered in the first two lectures of the course.

4 References and Resources

Lecture notes and/or slides will be posted before each class. The content of the lectures draws from a variety of (both classical and modern) references, including the following texts:

- Cesa-Bianchi and Lugosi, 2006. *Prediction, Learning, and Games*.
- Hazan, 2016. *Introduction to Online Convex Optimization*.
- Nisan, Roughgarden, Tardos, and Vazirani, 2007. *Algorithmic Game Theory*.
- Orabona, 2019. *A Modern Introduction to Online Learning*.

Additional references and pointers will be given throughout the course.