Supplementary information for:
Noise correlations for faster and more robust learning
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Signal-to-noise preserving noise correlations through signal amplification.

Another approach to manipulating noise correlations while balancing signal-to-noise ratio (total signal/σ^2_{pool}) is to hold variance of individual neurons constant, but change the signal magnitude according to the level of noise correlations. This process might be akin to the population receiving an additional input that amplifies both signal and noise in the task-relevant dimension. Under these assumptions the σ^2_{pool} can be expressed in terms of a the variance of individual neurons and the level of noise correlations:

$$\sigma^2_{pool} = \sigma^2_{unit}\,(n + n(\mathrm{n}-1)\,\phi)$$

Where n reflects the number of neurons in the population, ϕ specifies the level of noise correlations, and σ^2_{unit} reflects the variance on individual units. We can write the total population signal as the sum of the signal contributed by similarly tuned neurons (S_{neuron}) :

$$total\ signal = n\ S_{neuron}$$

and derive the signal for individual neurons that would maintain a signal-to-noise ratio of one:

$$1 = \frac{n\ S_{neuron}}{[\sigma^2_{unit}\,(n + n(\mathrm{n}-1)\phi)]^{0.5}}$$

$$[\sigma^2_{unit}\,(n + n(\mathrm{n}-1)\phi)]^{0.5} = n\ S_{neuron}$$

$$\sigma^2_{unit}\,(n + n(\mathrm{n}-1)\phi) = n^2 S^2_{neuron}$$

$$\frac{\sigma^2_{unit}\,(1 + (\mathrm{n}-1)\phi)}{n} = S^2_{neuron}$$

$$S_{neuron} = \sqrt{\frac{\sigma^2_{unit}\,(1 + (\mathrm{n}-1)\phi)}{n}}$$

So – as $\phi \rightarrow 0$, signal should be proportional to the single unit standard deviation divided by the square root of the number of units in the population:

$$S_{neuron} = \sqrt{\frac{\sigma_{unit}^2}{n}} = \frac{\sigma_{unit}}{\sqrt{n}}$$

In contrast, as $\phi \rightarrow 1$, the units all represent the exact same information, and thus the signal necessary to produce an SNR=1 simply reduces to the standard deviation of a single unit:

$$S_{neuron} = \sqrt{\frac{(\sigma_{unit}^2 + n\sigma_{unit}^2 - \sigma_{unit}^2)}{n}} = \sqrt{\frac{(n\sigma_{unit}^2)}{n}} = \sqrt{\frac{(\sigma_{unit}^2)}{1}} = \sigma_{unit}$$