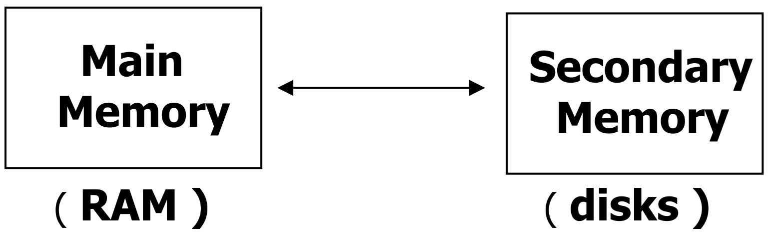
B-Trees

- Large degree B-trees used to represent very large dictionaries that reside on disk.

- Smaller degree B-trees used for internal-memory dictionaries to overcome cache-miss penalties.

B-Trees



$x \leftarrow$ a pointer to some object

DISK - READ(x)

operations that access and/or modify the fields of x

DISK - WRITE(x)

others operations that access but do not modify the fields of x

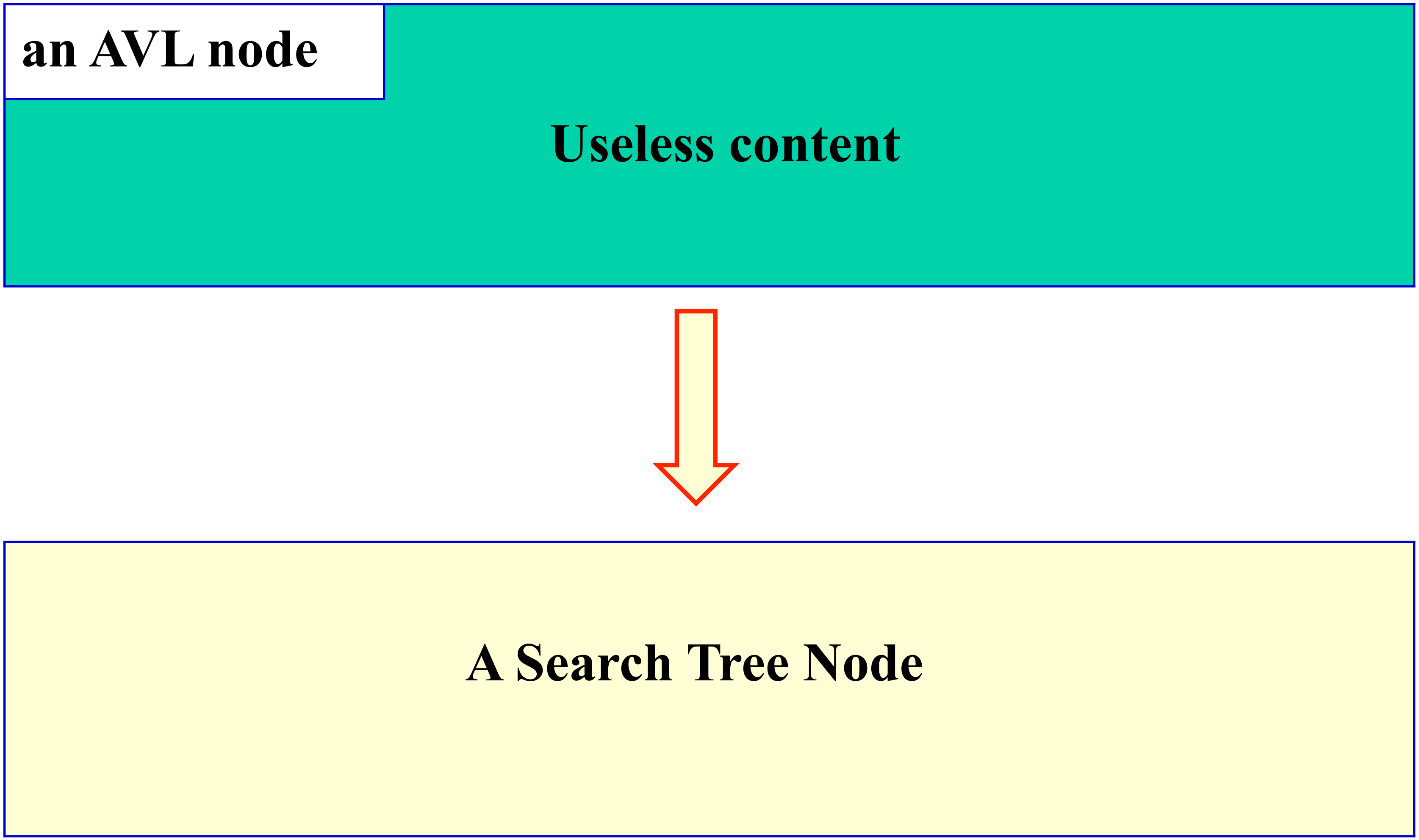
AVL Trees

- $n = 2^{30} = 10^9$ (approx).
- 30 <= height <= 43.
- When the AVL tree resides on a disk, up to 43 disk access are made for a search.
- This takes up to (approx) 4 seconds.
- Not acceptable.

Red-Black Trees

- $n = 2^{30} = 10^9$ (approx).
- $30 <= \text{height} <= 60$.
- When the red-black tree resides on a disk, up to 60 disk access are made for a search.
- This takes up to (approx) 6 seconds.
- Not acceptable.

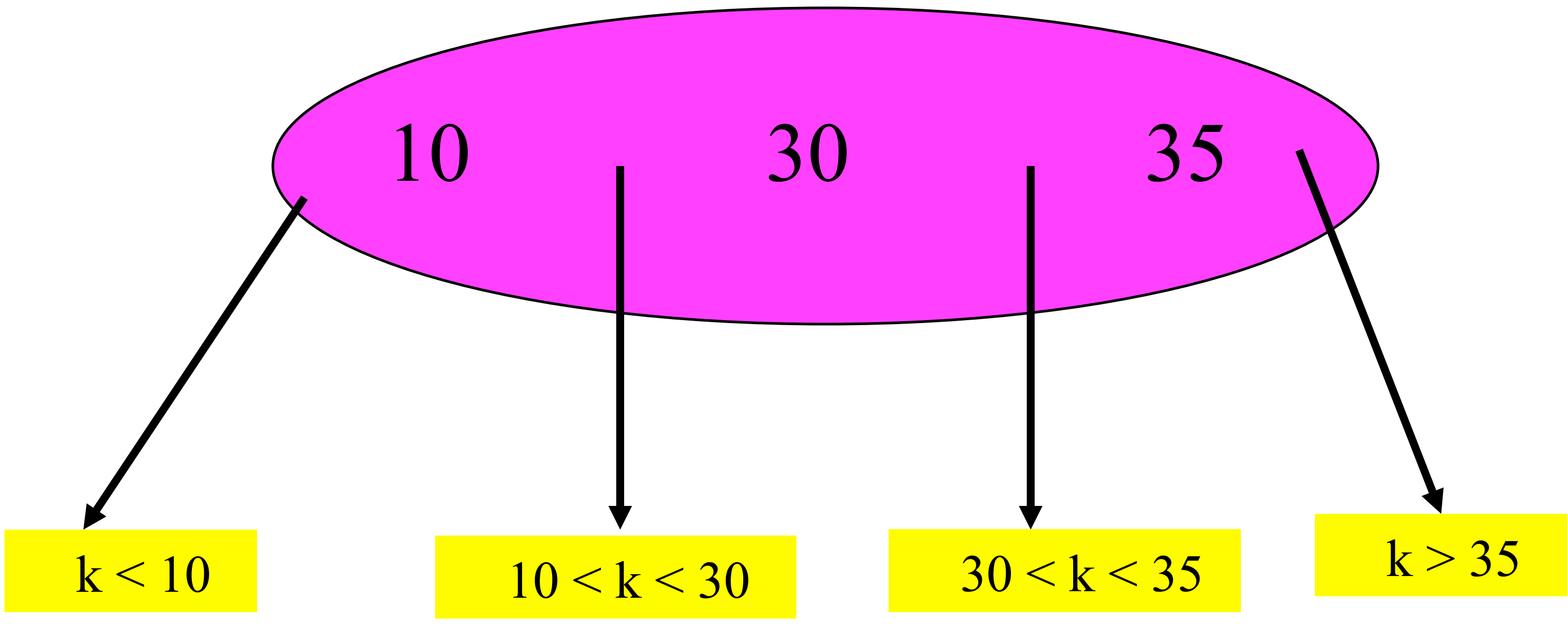
A Disk Page



m-way Search Trees

- Each node has up to $m - 1$ pairs and m children.
- $m = 2$ => binary search tree.

4-Way Search Tree



Maximum # Of Pairs

- Happens when all internal nodes are m-nodes.
- Full degree m tree.
- # of nodes $= 1 + m + m^2 + m^3 + \ldots + m^{h-1}$

  $= (m^h - 1)/(m - 1)$.
- Each node has $m - 1$ pairs.
- So, # of pairs $= m^h - 1$.

Capacity Of m-Way Search Tree

	m = 2	m = 200
h = 3	7	$8 * 10^{6} - 1$
h = 5	31	$3.2 * 10^{11} - 1$
h = 7	127	$1.28 * 10^{16} - 1$

Definition Of B-Tree

- Definition assumes external nodes (extended m-way search tree).
- B-tree of order m.
  - m-way search tree.
  - Not empty => root has at least 2 children.
  - Remaining internal nodes (if any) have at least ceil(m/2) children.
  - External (or failure) nodes on same level.

2-3 And 2-3-4 Trees

- B-tree of order m.
  - m-way search tree.
  - Not empty => root has at least 2 children.
  - Remaining internal nodes (if any) have at least ceil(m/2) children.
  - External (or failure) nodes on same level.

- 2-3 tree is B-tree of order 3.
- 2-3-4 tree is B-tree of order 4.

B-Trees Of Order 5 And 2

- B-tree of order m.
  - m-way search tree.
  - Not empty => root has at least 2 children.
  - Remaining internal nodes (if any) have at least ceil(m/2) children.
  - External (or failure) nodes on same level.

- B-tree of order 5 is 3-4-5 tree (root may be 2-node though).
- B-tree of order 2 is full binary tree.

Minimum # Of Pairs

- n = # of pairs.
- # of external nodes = n + 1.
- Height = h => external nodes on level h + 1.

level	# of nodes
1	1
2	>= 2
3	>= 2*ceil(m/2)
h + 1	>= $2*\text{ceil}(m/2)^{h-1}$

$n + 1 >= 2*\text{ceil}(m/2)^{h-1}, h >= 1$

Minimum # Of Pairs

$$n + 1 >= 2*\mathrm{ceil}(m/2)^{h-1},\ h >= 1$$

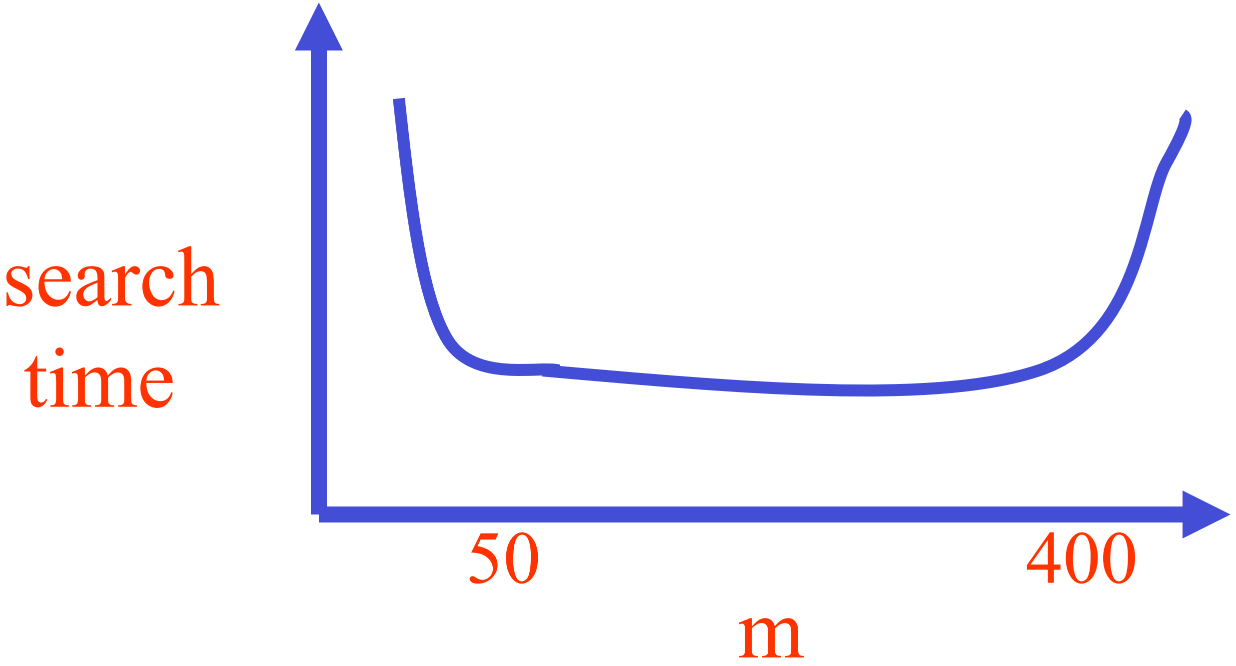
- m = 200.

height	# of pairs
2	>= 199
3	>= 19,999
4	$>= 2 * 10^6 - 1$
5	$>= 2 * 10^8 - 1$

$$h <= \log_{\mathrm{ceil}(m/2)} [(n+1)/2] + 1$$

Choice Of m

- Worst-case search time.
  - (time to fetch a node + time to search node) * height



- convention：
  - Root of the B-tree is always in main memory.
  - Any nodes that are passed as parameters must already have had a DISK_READ operation performed on them.
- Operations：
  - Searching a B-Tree.
  - Creating an empty B-tree.
  - Splitting a node in a B-tree.
  - Inserting a key into a B-tree.
  - Deleting a key from a B-tree.

Node Structure

$n\ c_0\ k_1\ c_1\ k_2\ c_2\ \ldots\ k_n\ c_n$

- c_i is a pointer to a subtree.
- k_i is a dictionary pair(KEY).

Search

BT_Search(x, k)

$i \leftarrow 0$

while $i < n$ and $k > k_{i+1}[x]$

do $i \leftarrow i + 1$

if $i < n$ and $k = k_{i+1}[x]$

then return$(x, i + 1)$

if $leaf[x]$ then return NULL

else DISK-READ$(C_i[x])$

return B-Tree-Search$(C_i[x], k)$

- B-Tree-Created(T)：
  - Algorithm：

    B-Tree-Create(T)

    {

    $x \leftarrow \text{Allocate} - \text{Node}()$

    $\text{Leaf}[x] \leftarrow \text{TRUE}$

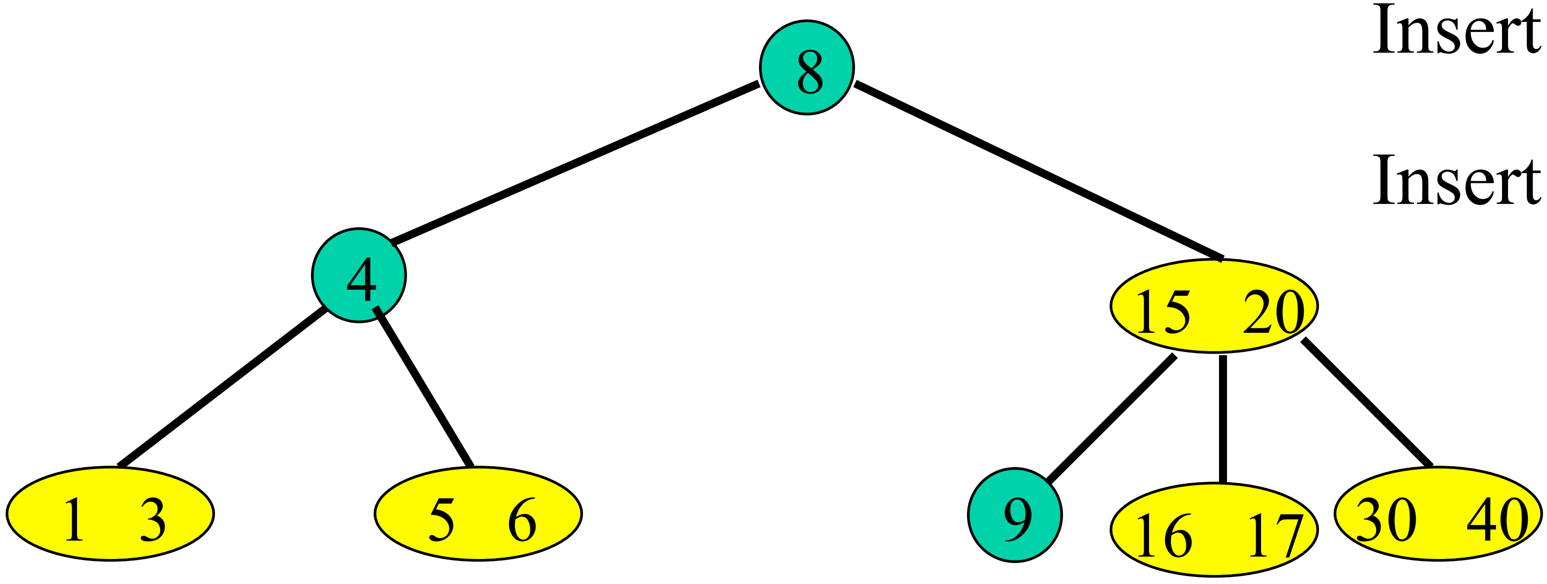
    $\text{n[x]} \leftarrow 0$

    DISK - WRITE(x)

    $\text{root[T]} \leftarrow \text{x}$

    }
  - time： $O(\mathbf{1})$

Insert



Insert 10?

Insert 18?

Insertion into a full leaf triggers bottom-up node ***splitting*** pass.

Split An Overfull Node

$m\ c_0\ k_1\ c_1\ k_2\ c_2\ \ldots\ k_m\ c_m$

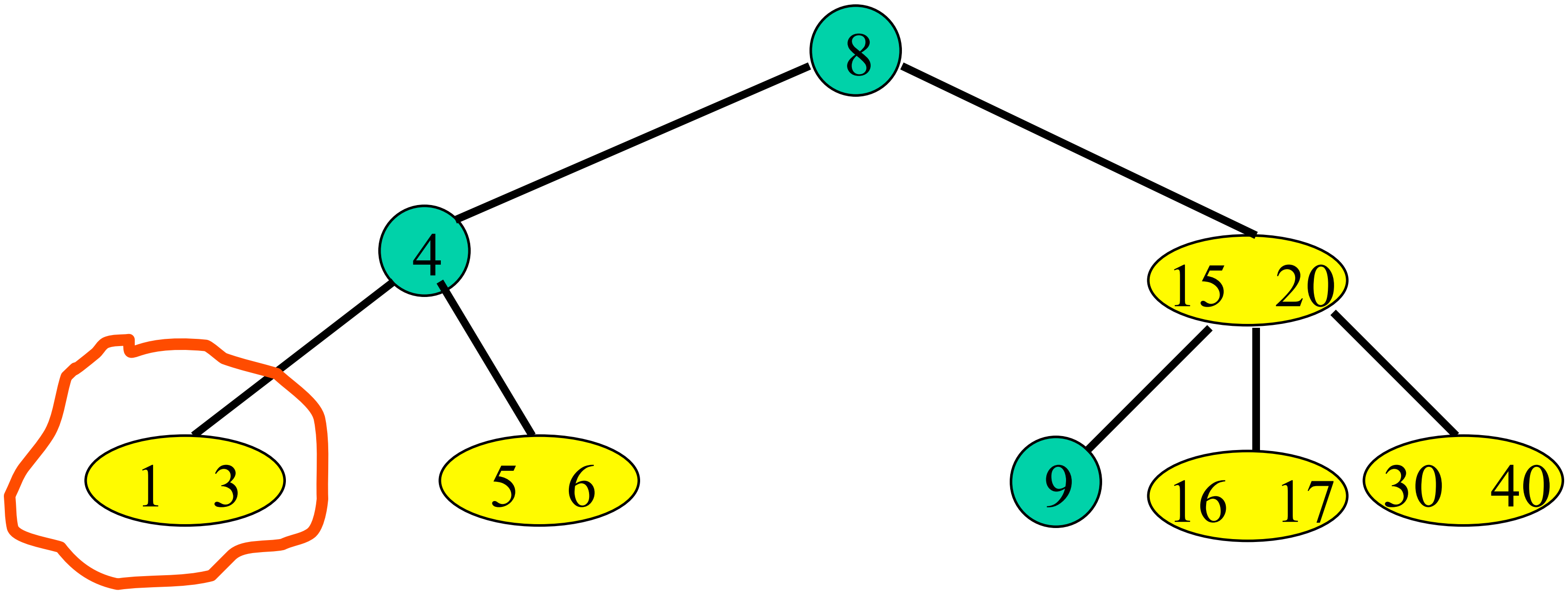
- c_i is a pointer to a subtree.
- k_i is a dictionary pair(KEY).

$ceil(m/2)\text{-}1\ c_0\ k_1\ c_1\ k_2\ c_2\ \ldots\ k_{ceil(m/2)-1}\ c_{ceil(m/2)-1}$

$m\text{-}ceil(m/2)\ c_{ceil(m/2)}\ k_{ceil(m/2)+1}\ c_{ceil(m/2)+1}\ \ldots\ k_m\ c_m$

- $k_{ceil(m/2)}$ plus pointer to new node is inserted in parent.

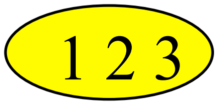
Insert



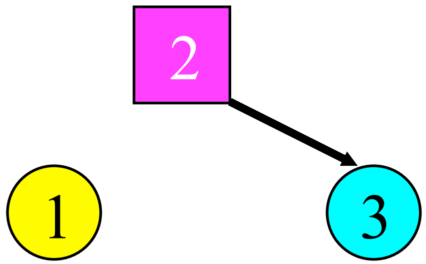
- Insert a pair with key = 2.
- New pair goes into a 3-node.

Insert Into A Leaf 3-node

- Insert new pair so that the 3 keys are in ascending order.

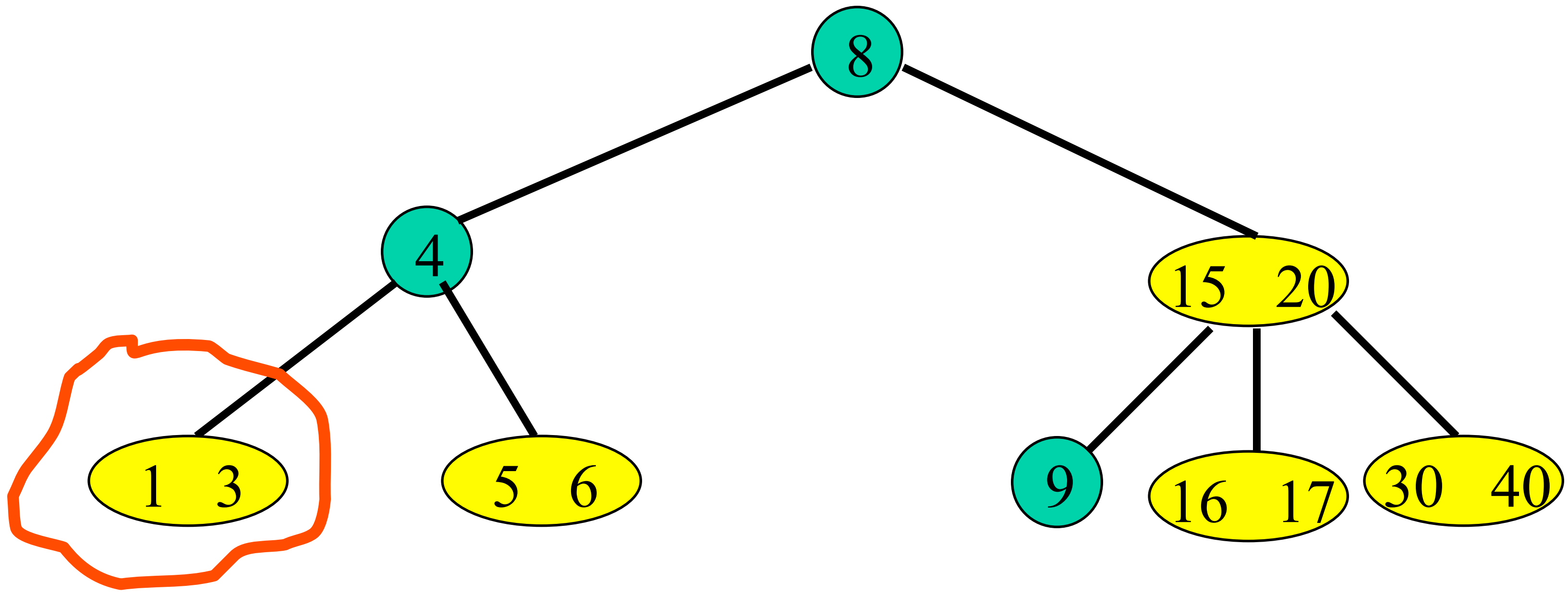


- Split overflowed node around middle key.



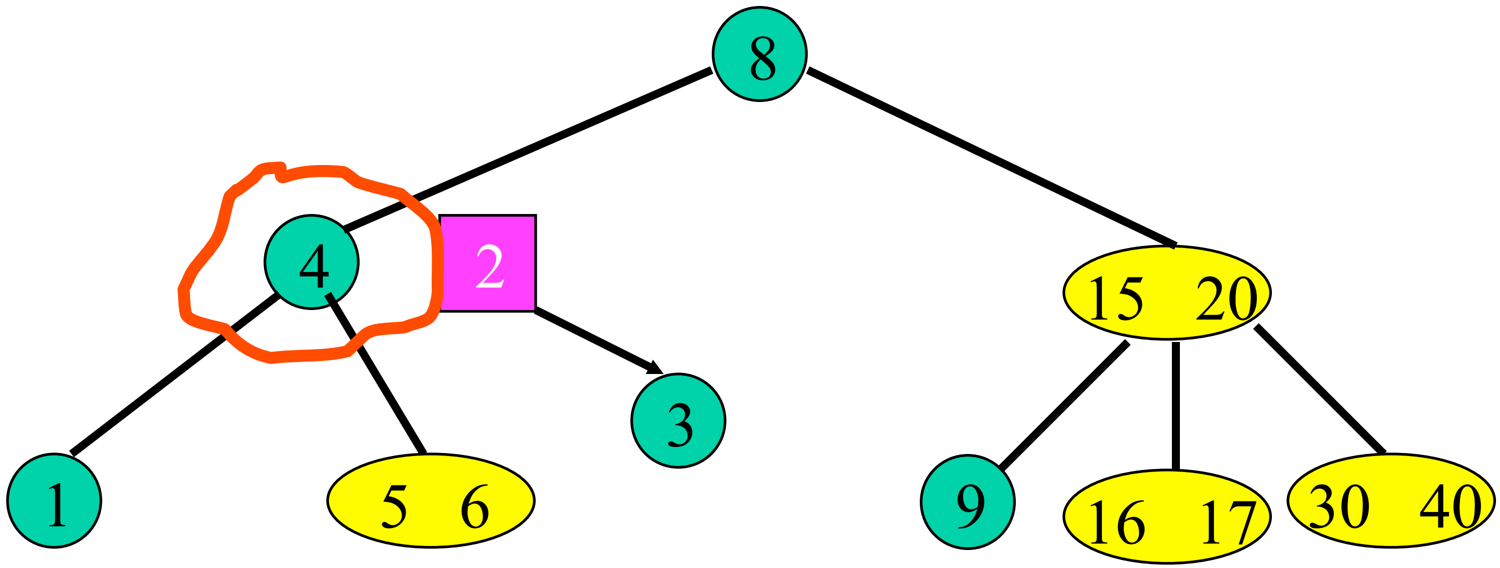
- Insert middle key and pointer to new node into parent.

Insert



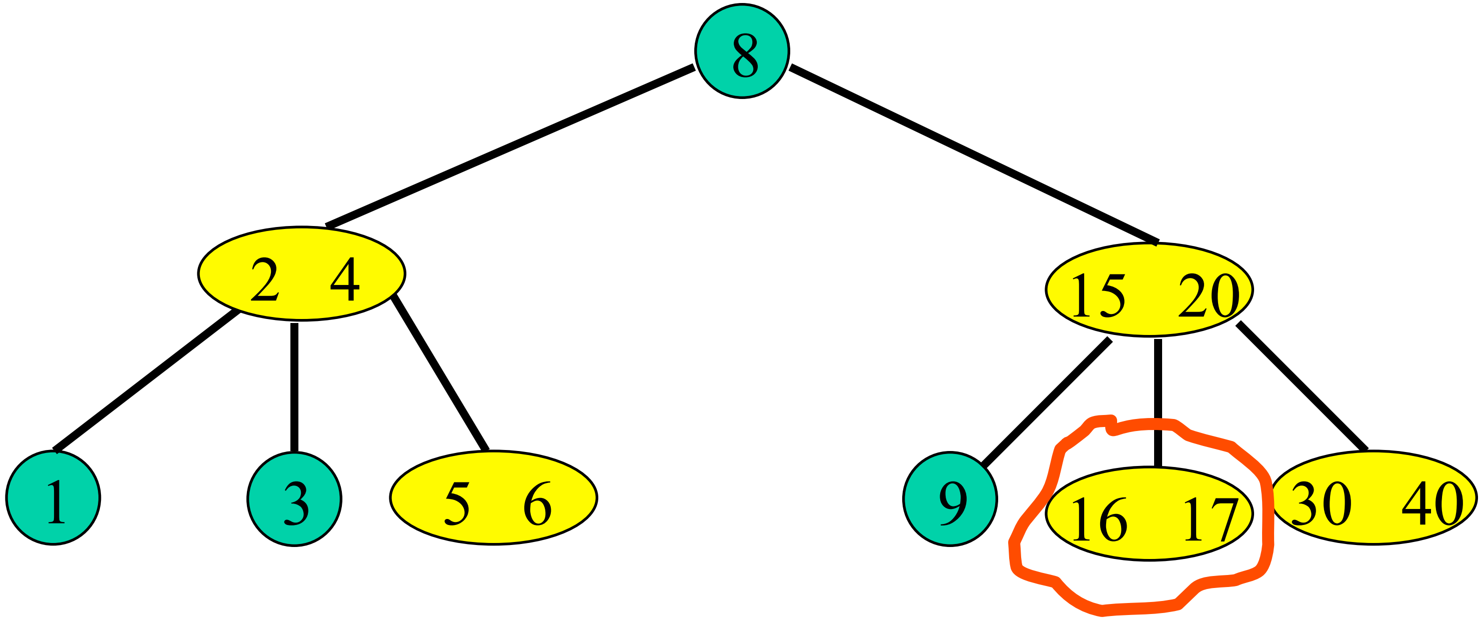
- Insert a pair with key = 2.

Insert



- Insert a pair with key = 2 plus a pointer into parent.

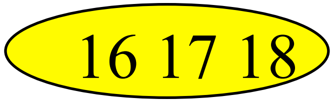
Insert



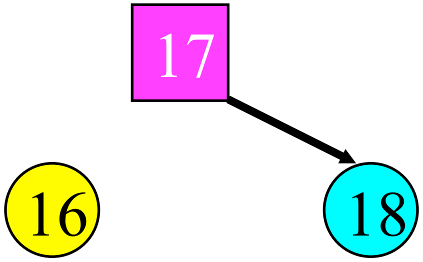
- Now, insert a pair with key = 18.

Insert Into A Leaf 3-node

- Insert new pair so that the 3 keys are in ascending order.

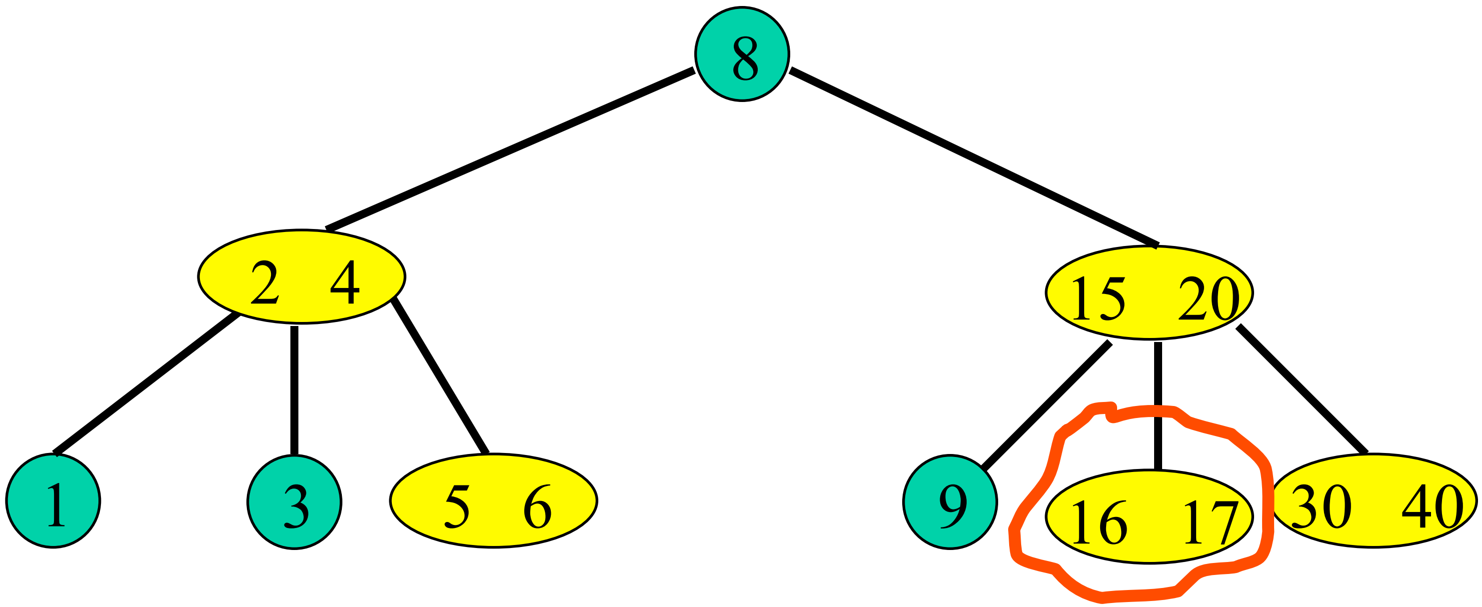


- Split the overflowed node.



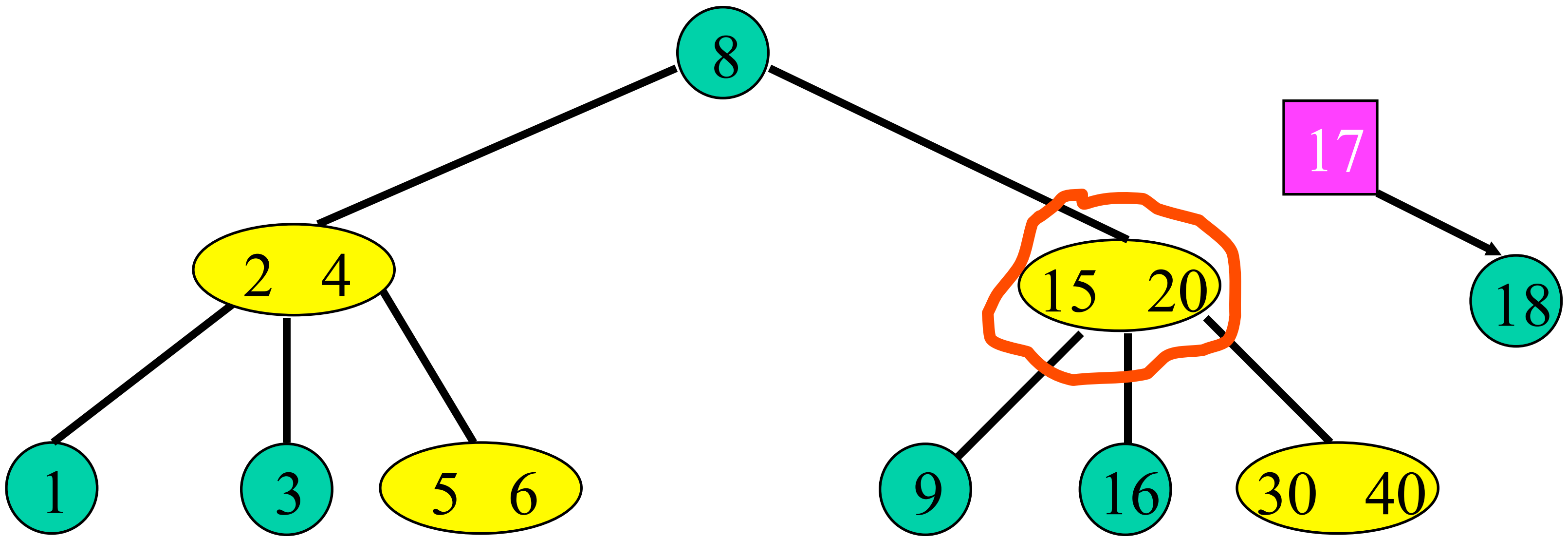
- Insert middle key and pointer to new node into parent.

Insert



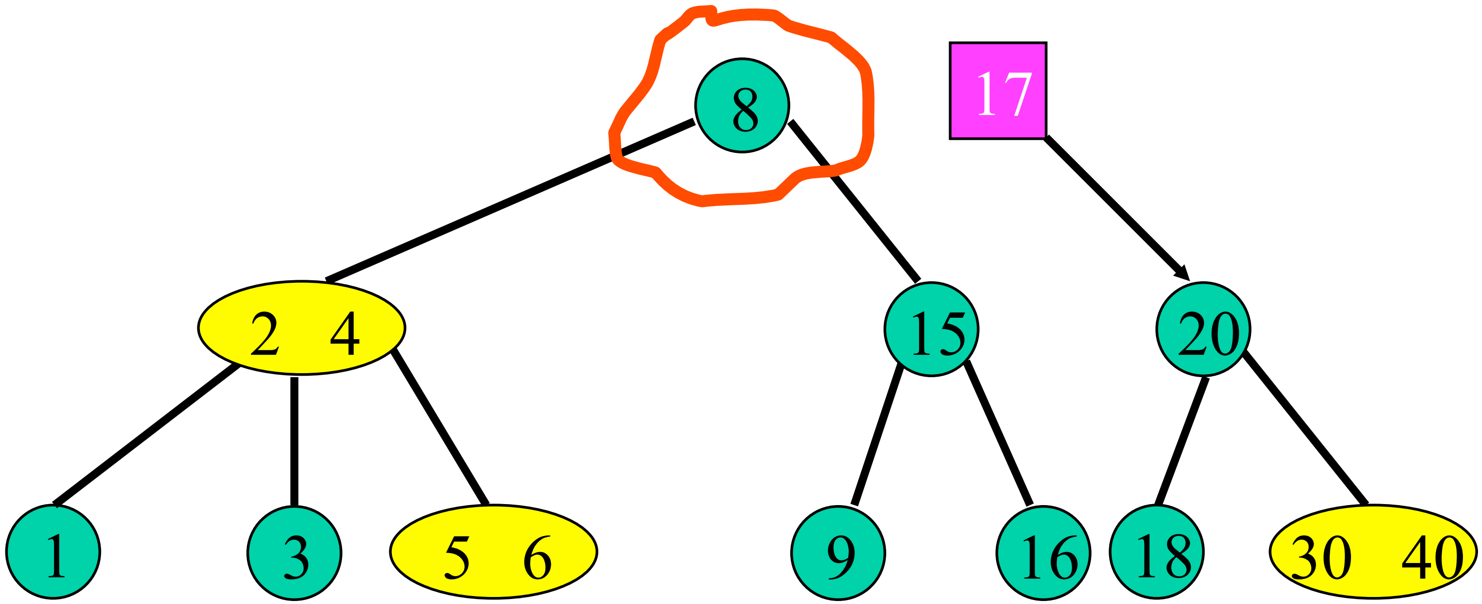
- Insert a pair with key = 18.

Insert



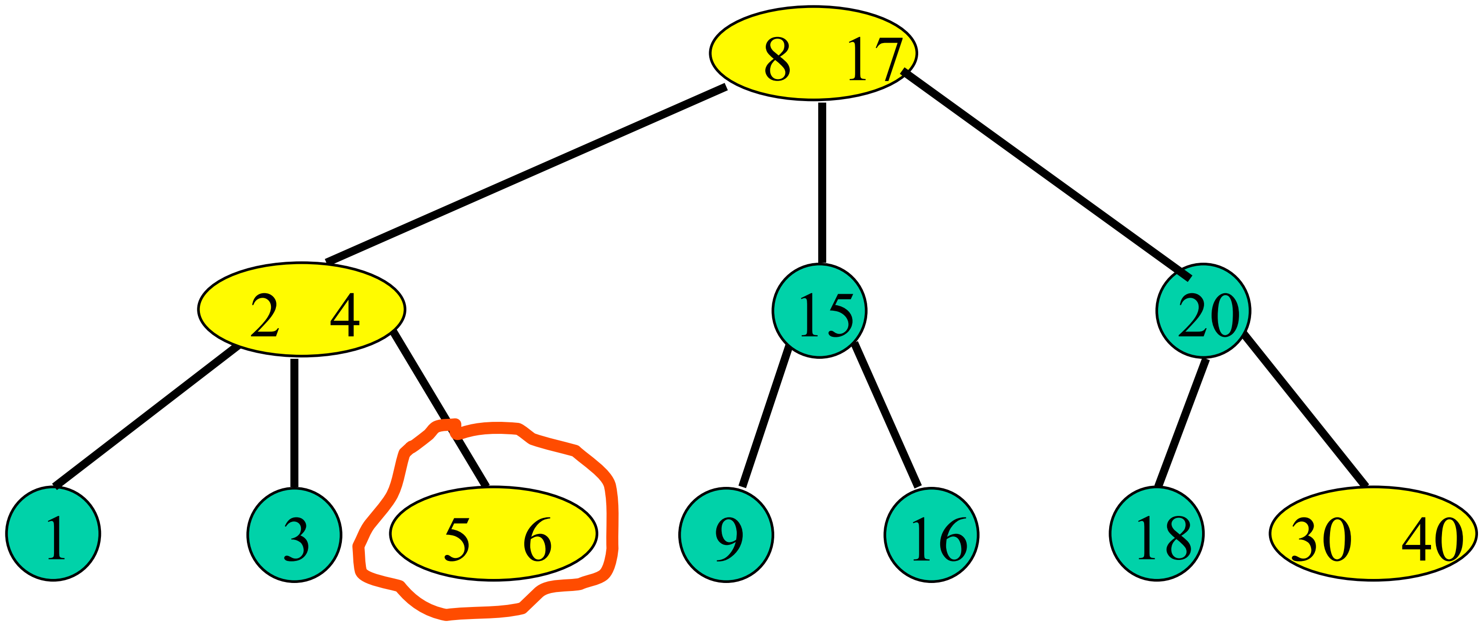
- Insert a pair with key = 17 plus a pointer into parent.

Insert



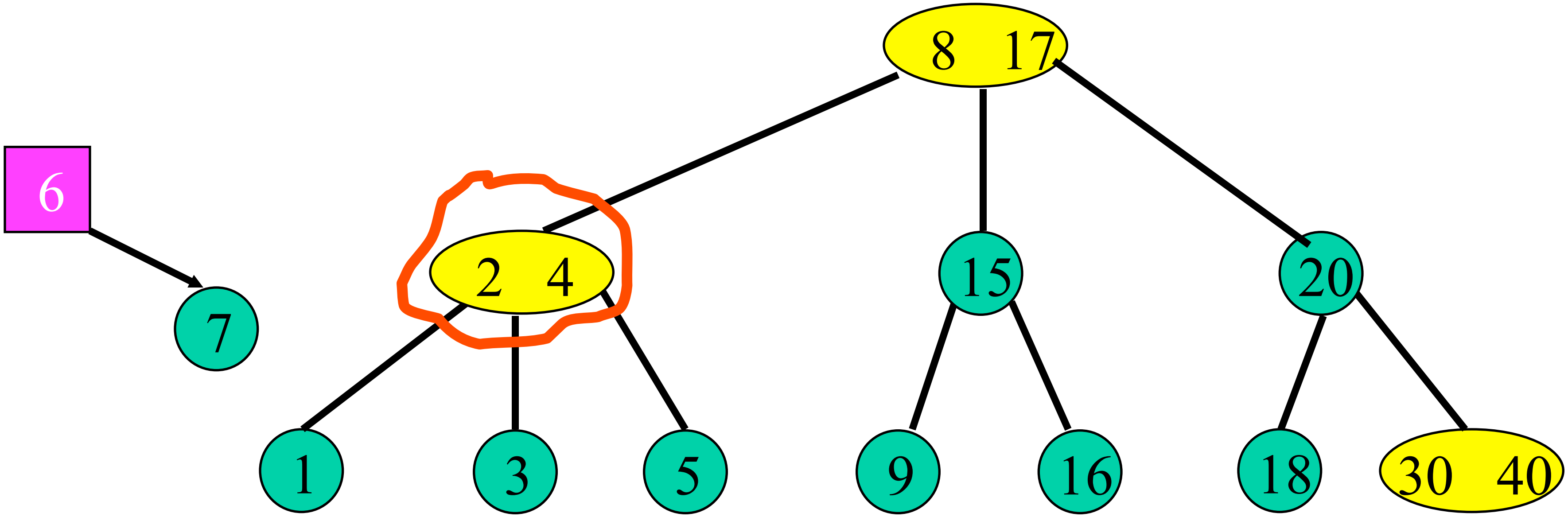
- Insert a pair with key = 17 plus a pointer into parent.

Insert



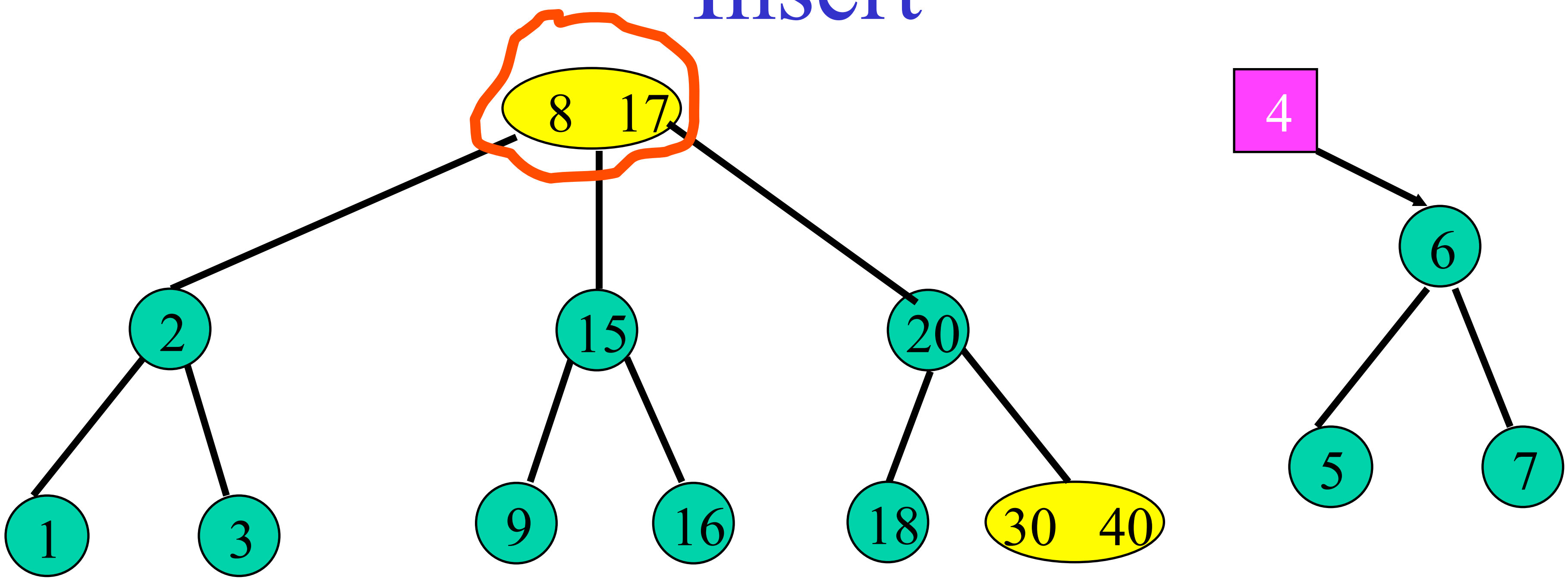
- Now, insert a pair with key = 7.

Insert



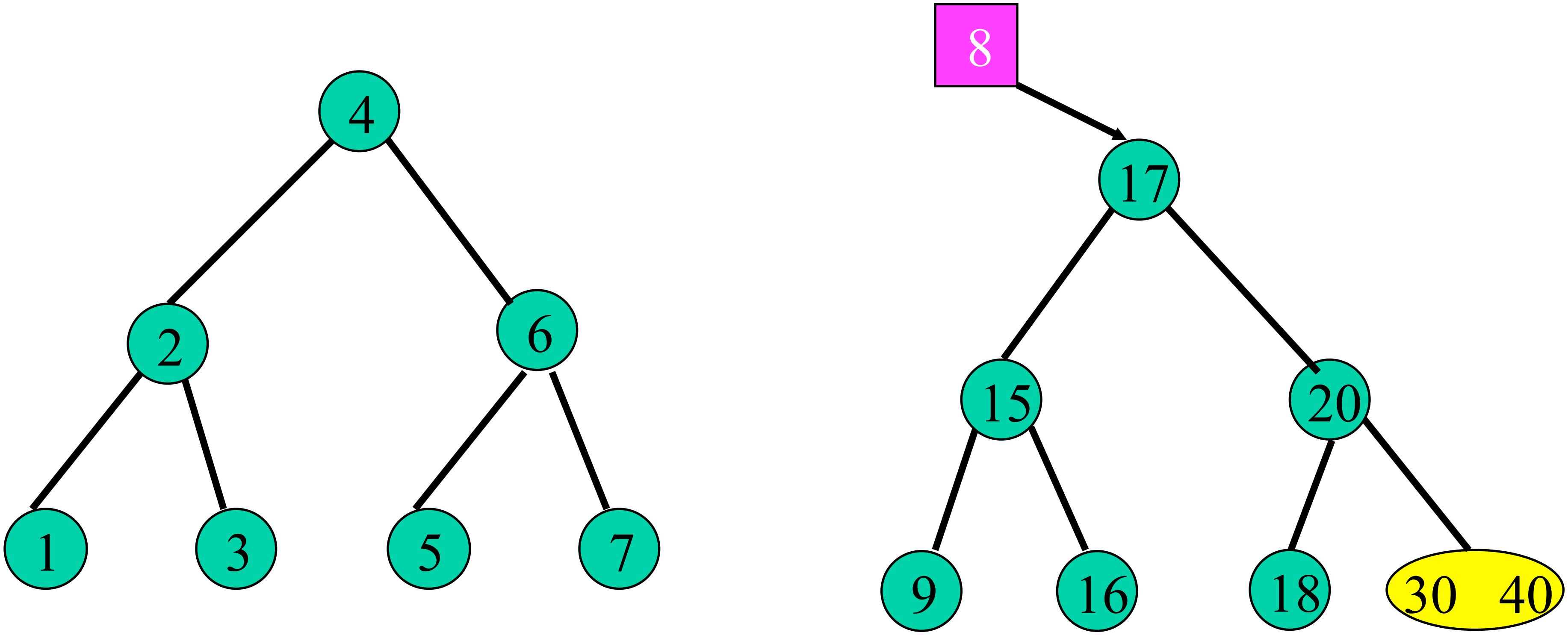
- Insert a pair with key = 6 plus a pointer into parent.

Insert



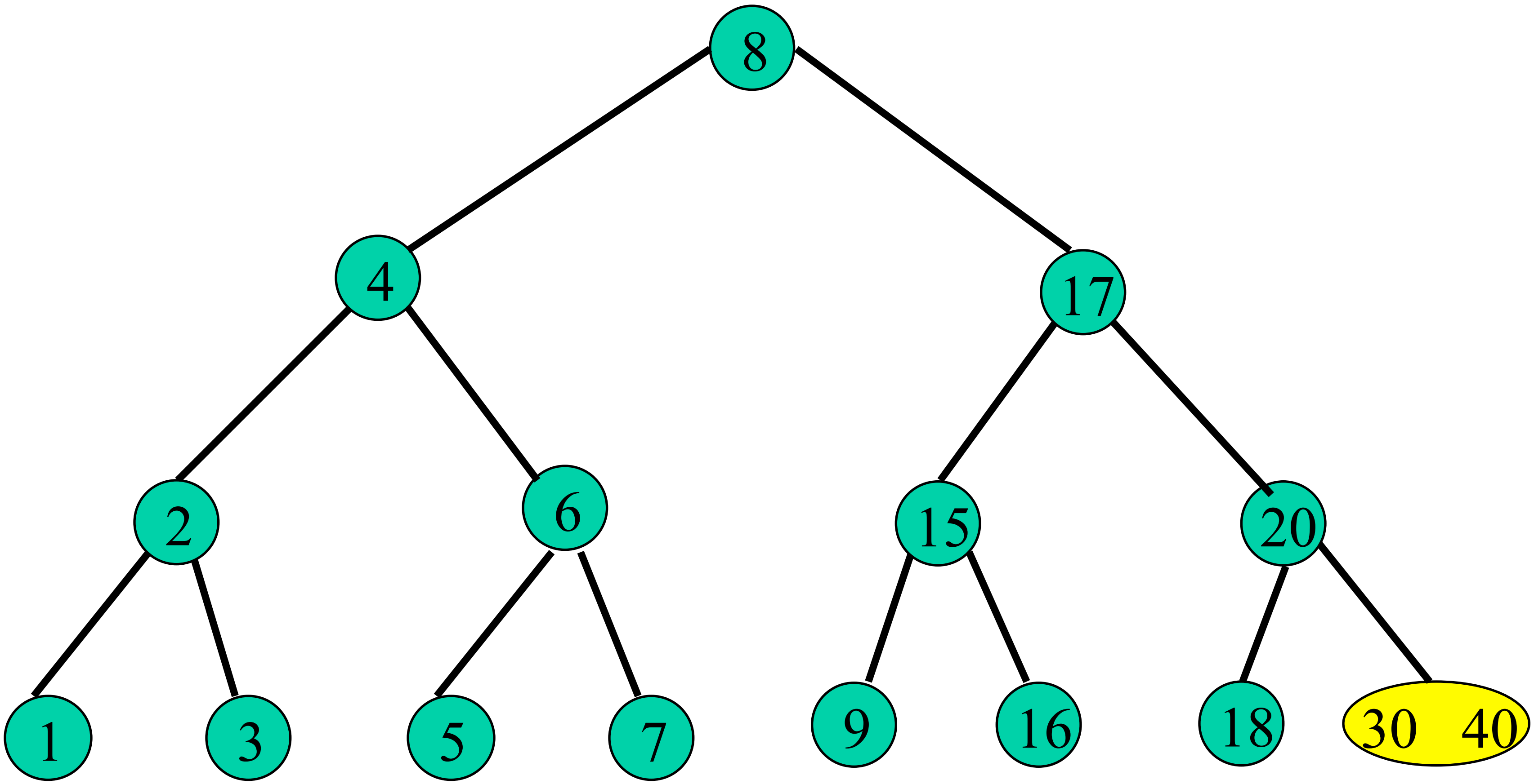
- Insert a pair with key = 4 plus a pointer into parent.

Insert



- Insert a pair with key = 8 plus a pointer into parent.
- There is no parent. So, create a new root.

Insert



- Height increases by 1.

- Btree::InsertNode(Key k, Element e)

```
{
    bool overflow = Insert(root, k, e);
    if (overflow)
        <Key, Node*> newpair= split(root);
        root = new Node(root, newpair);
    return;
}
```

- Bool Insert(node* x, Key k, Element e)

```
{
    if (leaf(x))
        insertLeaf(x, k, e);
        if (size(x) > m-1) return true;
        else return false;
    idx = keySearch(x, k);
    bool overflow = Insert(x->C[idx], k, e);
```

```
if (overflow)
    <Key, Node*> newpair = split(x->C[idx]);
    InsertPair(x, newpair);
    if(size(x) > m-1)
        return true;
    else return false;
}
```

- **Exercises: P609-3**