# MASTER TEACHER'S GUIDE

## Unit Title: Formalism of Entanglement & Bell's Theorem (Week 3)

This module expands the computational framework from single-qubit matrices to multi-qubit tensor products. It formally defines entanglement not as "magic," but as the mathematical inability to factorize a state vector, culminating in the experimental proof of non-locality (CHSH Inequality).

| Field | Detail |
| --- | --- |
| **Target Audience** | **Tier 3 - Undergraduate / Developer Level** |
| **Design Principle** | **Multi-Qubit Formalism.** Concepts require students to construct $4 \times 4$ matrices using the Kronecker product ($\otimes$) and logically prove non-separability by solving systems of linear equations. |
| **Learning Progression** | **Tensor Products $\otimes$ → State Separability vs. Entanglement → Bell State Construction → CHSH Inequality/Non-Locality.** |
| **Duration** | **1 Week** (approx. 4×60-90 minute sessions) |
| **Teacher Guidance** | Proficiency in calculating tensor products of vectors and matrices is essential. Emphasize that "Entanglement" is strictly defined as a state that cannot be factored. |

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## 2. Pedagogical Framework: The Entanglement Engine

This unit uses **Linear Algebra** to rigorous define "multi-particle systems" and **Probability Theory** to prove Bell's Theorem. The goal is to move students from "linked coins" to "tensor product spaces."

| Focus Area | Objective (The student will be able to...) | Bloom's Level |
| --- | --- | --- |
| **Science/Literacy** | Explain the "Local Hidden Variable" (LHV) theory and how Bell's Theorem (CHSH) experimentally disproves it using statistical correlation limits. | **Understanding, Analyzing** |
| **Mathematics** | Calculate the **Tensor Product** of two state vectors and two matrices (e.g., $H \otimes I$). Prove a state is **Entangled** by showing it has no separable solution. | **Applying, Evaluating** |
| **Computational Logic** | Implement multi-qubit circuits in Qiskit to create Bell States. Simulate the CHSH game to | **Applying, Creating** |

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| | experimentally violate the classical win-rate limit (75%). | |

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### 3. Computational Logic Refinements (Week 3)

#### A. The Tensor Product (⊗)

| Concept | Explanation | Mathematical Description |
|---|---|---|
| **State Expansion** | Combining independent systems increases dimensionality ($2^N$). | $\binom{0}{1} \otimes \binom{1}{0} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ |
| **Matrix Expansion** | Gates acting on multi-qubit systems are larger matrices constructed from single-qubit gates. | $H \otimes I = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$ |

#### B. Entanglement vs. Separability

| Concept | Explanation | Mathematical Description |
|---|---|---|
| **Separable State** | A multi-qubit state that *can* be factored into individual qubit states. | $\|\Psi\rangle = \left(\frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle\right) \otimes \|0\rangle$ |
| **Entangled State** | A state where no such factorization exists. | $\|\Phi^+\rangle = \frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$ |
| **Bell Basis** | The four maximally entangled states that form a basis for the 2-qubit space. | $\|\Phi^+\rangle$ ; $\|\Phi^-\rangle$ ; $\|\Psi^+\rangle$ ; $\|\Psi^-\rangle$ |

#### C. Non-Locality & CHSH

| Concept | Explanation | Key Mathematical Action |
|---|---|---|
| **LHV Limit** | The maximum win rate for any classical (local hidden variable) strategy in the CHSH game. | $P(win)_{classical} \leq 0.75$ |
| **Quantum Violation** | The maximum win rate using entangled qubits and specific measurement angles. | $P(win)_{quantum} = \frac{2+\sqrt{2}}{4} \approx 0.8535$ |

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### 4. Exemplary Lesson Plan: Breaking Reality

**Module: Proving Non-Locality** This lesson focuses on the CHSH game, moving from the theoretical proof of the classical limit to the computational simulation that violates it.

**Coding Lab: Entanglement & CHSH**

| | |
|---|---|
| **Objective** | **Students will use Qiskit to construct Bell states via circuits and run a Monte Carlo simulation of the CHSH game to statistically prove quantum advantage.** |
| **Required Resources** | Python Environment (Jupyter), Tier3_W3_Entanglement_coding.ipynb, Tier3_Week3_worksheet.docx |

**Step-by-Step Instructions**

**Part 1: The Math (Pen & Paper - Worksheet)**

1. **Tensor Practice (Problem 1):** Calculate $|\psi\rangle = |+\rangle \otimes |-\rangle$. Result: $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$.
2. **Separability Check (Problem 2):** Determine if $\frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle$ is entangled. (Answer: No, it factors to $|1\rangle \otimes |+\rangle$).
3. **Proof (Problem 3):** Attempt to factor a state like $\frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{3}}|10\rangle$ and find the contradiction in the system of equations.

**Part 2: The Code (Qiskit Implementation)**

1. **Task 1 (Tensor):** Use *Statevector.tensor()* to programmatically verify the manual calculations from the worksheet.
2. **Task 2 (Bell Circuit):** Build the circuit $H \otimes I$ followed by $CNOT$ to generate $|\Psi^+\rangle$. Verify the output state vector.
3. **Task 3 (CHSH Game):**
   - Implement the **Classical Strategy**: Return fixed bits (e.g., $a = 0, b = 0$). Run 1000 times. Max win rate $\approx 75\%$.
   - Implement the **Quantum Strategy**: Create an entangled pair. Rotate measurement bases based on input questions $(x, y)$. Run 1000 times. Win rate $\approx 85\%$.

**Part 3: Assessment**

- **Quiz Question 2:** Identify the definition of a non-separable state.
- **Quiz Question 7:** Determine the result of applying $CNOT$ to the state $|10\rangle$.
- **Quiz Question 10:** Interpret the experimental violation of Bell's Inequality (rejecting Local Realism).

### 5. Resources for Curriculum Implementation (Week 3)

| Resource Name | Type | Purpose in Curriculum |
|---|---|---|
| **W3T3_Draft** | **Lecture Notes (DOCX)** | Detailed derivation of the tensor product, the matrix form of the Bell State circuit, and the logic of the CHSH game. |
| **Tier3_W3_Entanglement_coding** | **Lab Notebook (IPYNB)** | Students implement separability checks and run the full CHSH experiment simulation. |
| **Tier3_Week3_worksheet** | **Assessment (DOCX)** | Rigorous math problems to verify manual calculation of tensor products and non-separability proofs. |
| **Tier3W3_Entanglement_Quiz** | **Quiz (IPYNB)** | **Knowledge Check:** 10 multiple-choice questions covering tensor algebra, circuit construction, and Bell's theorem. |

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### 6. Conclusion and Next Steps

This **Tier 3, Week 3** module creates the distinct separation between classical and quantum logic. By proving **Non-Separability** mathematically and **Non-Locality** experimentally (via simulation), students accept Entanglement as a usable computational resource.

**Key Takeaway:** Multi-qubit spaces are formed by **Tensor Products**. Entanglement is the absence of a tensor product factorization. This resource allows correlations stronger than any classical system (Bell Violation).

**Next Steps:** Week 4 will utilize this entangled resource to perform **Quantum Teleportation**, moving a state vector from one qubit to another using Bell measurement and classical communication.