**Quantum Mechanics Math Practice Worksheet 📐**

This worksheet is designed to test your ability to perform core calculations in quantum mechanics, focusing on state vector manipulation and probability.

**Problem 1: Normalization Challenge (1 Qubit)**

Given the unnormalized qubit state vector:

**Task:**

1. Calculate the **norm squared**, .
2. Determine the **normalization constant,**
3. Write out the final **normalized state vector**,

**Problem 2: Born Rule Application**

The following is a normalized qubit state:

**Task:**

1. Calculate the **probability of measuring the state**
2. Calculate the **probability of measuring the state**
3. Verify that the probabilities sum to .

**Problem 3: Orthogonality Check (Inner Product)**

Determine whether the following two normalized states are **orthogonal** by calculating their inner product,

**Task:**

1. Write out the bra vector (the conjugate transpose of ).
2. Compute the inner product
3. State whether the vectors are orthogonal (Yes/No).

**Problem 4: Density Matrix for a Pure State**

A pure state is given by the vector:

**Task:**

1. Compute the **density matrix,**
2. What is the element (the probability of measuring )?

**Problem 5: Mixed State Density Matrix Construction**

A system is known to be in a **classical probabilistic mixture**:

* It is in the state with  **probability**
* It is in the state with  **probability**

**Task:**

1. Recall or derive the density matrices for and
2. Construct the **mixed state density matrix,**

**Problem 6: Tensor Product Composition**

Compute the composite state and express the final unnormalized vector in the basis

**Task:**

1. Perform the tensor product operation: .
2. Does the resulting 4-dimensional vector normalized? If not, normalized it.

**Solutions**

**Problem 1 Solution**

1. **Norm Squared:**
2. **Normalization Constant:**
3. **Normalized State Vector:**

**Problem 2 Solution**

1. **Probability**
2. **Probability**
3. **Verification:**

**Problem 3 Solution**

1. **Bra Vector** **:**
2. **Inner Product**
3. **Orthogonal?** Yes.

**Problem 4 Solution**

1. **Density Matrix**
2. **Element**

**Problem 5 Solution**

1. **Constituent Density Matrices:**
2. **Mixed State Density Matrix**

**Problem 6 Solution**

1. **Tensor Product Calculation:**

**Resulting Vector:**

Implementing matrix notation:

**Normalization Constant:**