**Tier 3: Week 2 Worksheet (Phase & Mastery)**

**Problem 1: Gate Application**

A qubit starts in the state. An gate is applied, followed by a gate.

**Task:**

1. Calculate the final state vector in Dirac Notation.
2. Identify the **single-qubit state** this final vector is physically equivalent to

*Hint: Remember that* *(ignoring a global phase) and*  *.*

**Problem 2: Relative Phase vs. Probability**

A qubit is prepared in the state:

**Task:**

1. Calculate the probability of measuring and

2. Calculate the probability of measuring the state

**Hint**: The amplitude for is The bra is

**Problem 3: Final State Vector Calculation**

A qubit starts in the state. The following sequence of gates is applied: followed by a .

**Task:**

1. Write the final state vector in the form .
2. Identify the **relative phase** of the component.

*Hint: The* *matrix is* *. The* *gate is*  *(ignoring global phase).*

**Problem 4: Global Phase Filtering**

Consider the two states below. Which one, if any, is physically equivalent to the state ?

**State B:**

**State C:**

*Hint: Find the constant* *such that* *or*  *If* *is a pure phase factor they are equivalent.*

**(OPTIONAL)Problem 5: Multi-Qubit Phase Application**

A two-qubit system is in the unnormalized state:

A gate (phase flip) is applied **only to the second qubit** The operation is represented by the tensor product

**Task:**

1. Calculate the resulting unnormalized state vector

*Hint: Apply the gate on first and second qubit accordingly like applying on 1 gate. can be read as act on and act on*

2. Explain the effect of the operation on the phase of the four basis components.

*Hint: The*  *operator flips the sign (applies a phase) only to those basis states where the second qubit is (i.e., and*

**Solution:**

**Problem 1:**

1. **Initial State:**
2. **Gate Matrices (Ignoring Global Phase):**
3. **Composite Operation:**

**Answer:** The final state is physically equivalent to the state. The two - rotations cancel each other out.

**Problem 2:**

1. Z-Basis Probabilities:

2. Basis Probability: Inner Product

**Probability**

**Conclusion:** The relative phase ( on ) makes the measurement result in the - basis distinct from a pure or state, proving its physical reality.

**Problem 3:**

1. **Initial State:**
2. **Apply** **:**
3. **Apply** **Gate**
4. The **relative phase** of the component is .

**Problem 4:**

**is physically equivalent to**

1. **Normalize** **:** The vector is , Norm squared is

**State B:** : The vector is , Norm squared is

**State C:** The vector is , Norm squared is . Thus, it is normalized.

**Conclusion:** States and have same relative phases with a difference in global phase. Therefore, they are **physically indistinguishable**. However, has different

**Problem 5:**

1. **Composite Operator Effect(mathematically):**  acts on the state

**Explanation of Effect:** The gate selectively applied a **relative phase of**  (a factor of ) to the complex amplitudes of the two components where the second qubit was in the state  *and* , effectively flipping the signs of the complex coefficients for those terms.