自由电子激光讲义

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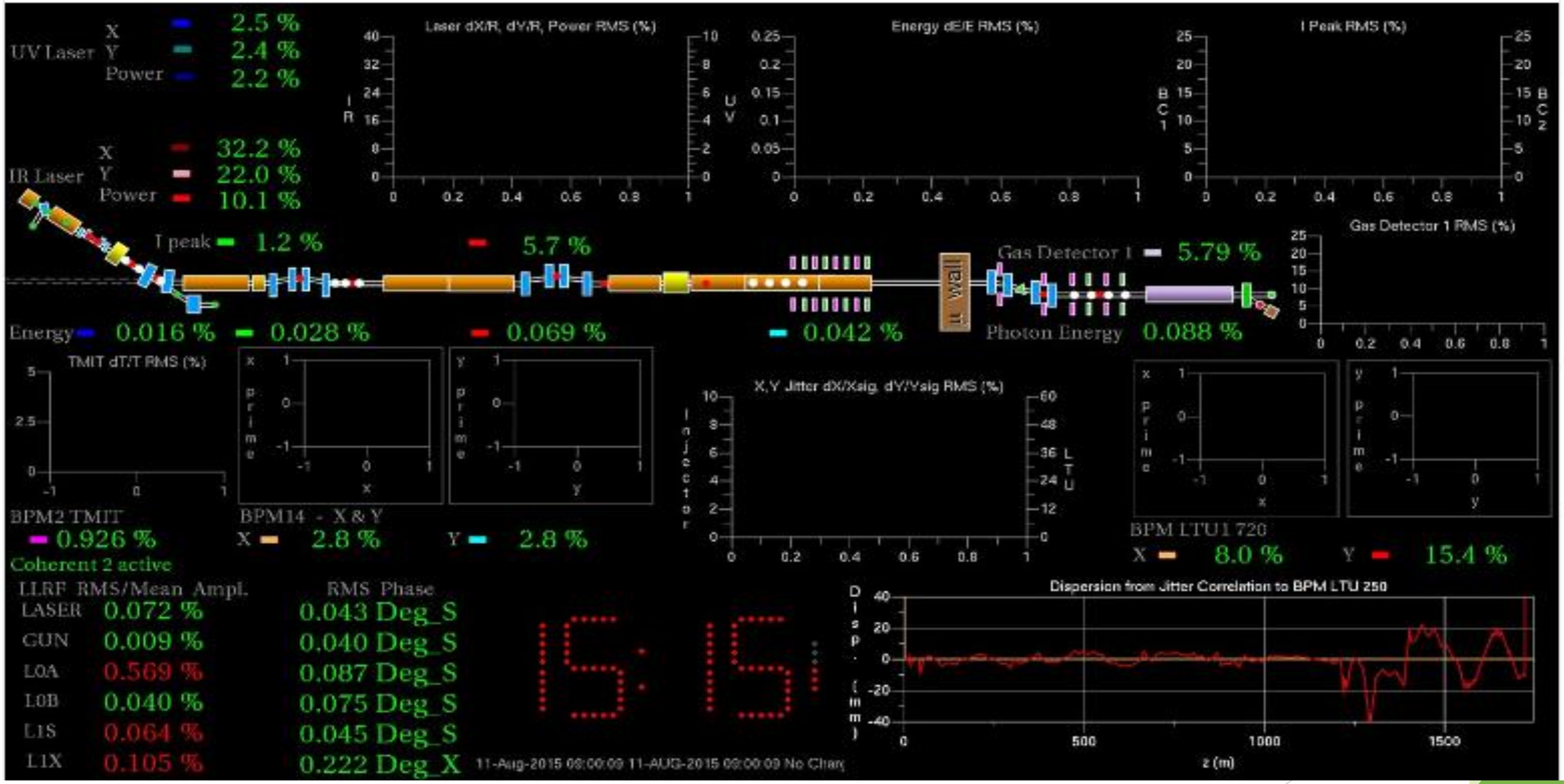
第三讲：高增益自由电子激光理论



3.1实现高功率输出的两种方法

1. 电子束回旋，多次放大，最后饱和输出

2. 电子束通过一个长波荡器，单次放大，最后饱和输出

3.2微聚束（Micro-bunching）

$$x(t)=\frac{K}{\gamma k_u}\sin(\omega_u t) \qquad z(t)=\bar{v}_z t-\frac{K^2}{8\gamma^2 k_u}\sin(2\omega_u t)$$

$$\gamma \rightarrow \begin{cases} \gamma \downarrow & x(t)\uparrow & z(t)\downarrow \\ \gamma \uparrow & x(t)\downarrow & z(t)\uparrow \end{cases}$$

电子在波荡器中运动，会导致电子集中在一个一个切片里，每个切片的长度远小于光波长 lamds，我们称其为微束团，微束团的位置又恰好处于电子束与光场交换能量最多的地方。

辐射

微束团 ⟷ 光场

加强群聚

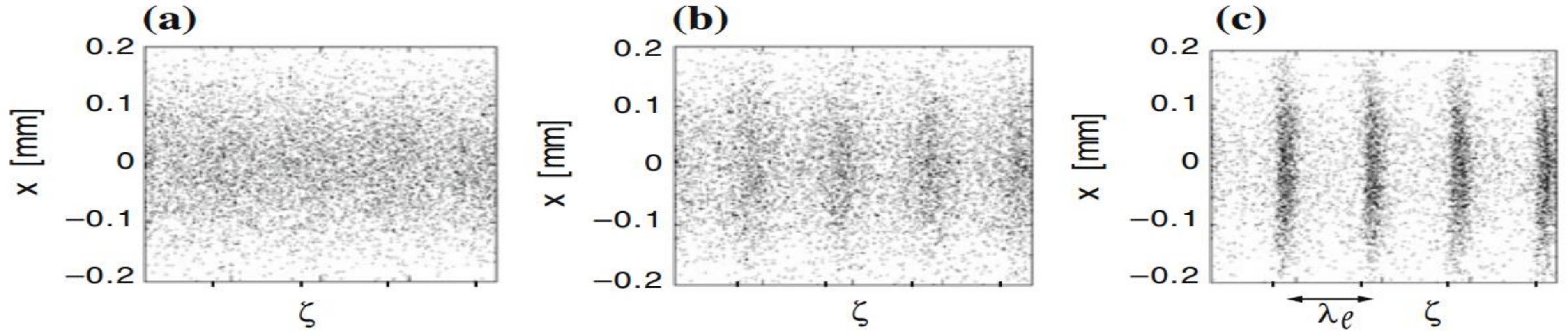


Fig. 4.1 Numerical simulation of microbunching. The particles are plotted as dots in a (x, ζ) plane, where x is the horizontal displacement from the undulator axis and ζ is the longitudinal internal bunch coordinate. **a** Initial uniform distribution, **b** beginning of microbunching, **c** fully developed microbunches with a periodicity of the light wavelength λ_ℓ. (Courtesy of S. Reiche).

光场指数增益示意图

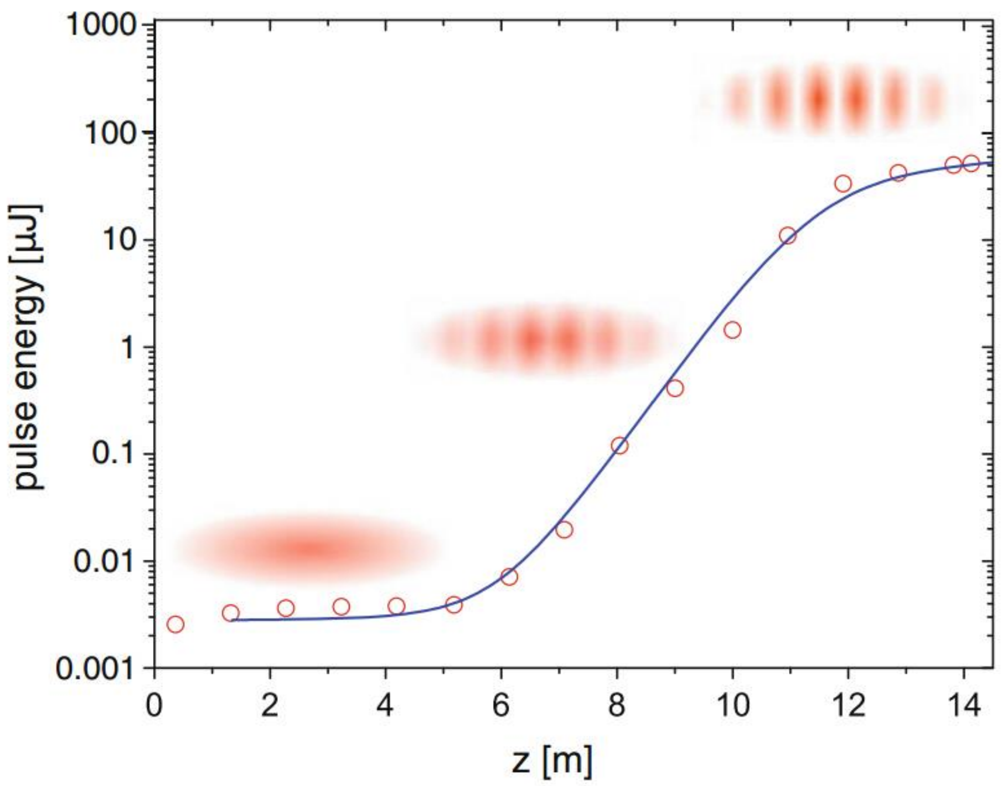


Fig. 4.2 The exponential growth of the FEL pulse energy as a function of the length z traveled in the undulator. The data (*open circles*) were obtained at the SASE FEL of the TESLA Test Facility [3], the electron energy was 245 MeV. The progressing microbunching is indicated schematically. Laser saturation sets in for $z \geq 12$ m. Here the microbunches are fully developed and no further increase in laser power can be expected.

3.3一维自由电子激光理论的基本假设

$$\tilde{E}_x(z,t)=\tilde{E}_x(z)\exp[i(k_l z-\omega_l t)] \quad \Rightarrow \quad E_x(z,t)=real\{\tilde{E}_x(z,t)\}$$

- 高增益自由电子激光的基本描述：

1.耦合的摆方程描述电子在(theta，p)相空间中的运动。
2.Maxwell波动方程描述光场中电场强度的变化
3.考虑纵向空间电荷力对微聚束的影响

- 假设初始时刻电子在相空间中均匀分布，当电子束与周期光场相互作用后，电子束会逐渐形成以光波长为周期密度的调制：

$$\tilde{\rho}(\psi,z)=\rho_0+\tilde{\rho}_1(z)e^{i\psi} \quad \Rightarrow \quad \rho(\psi,z)=\rho_0+real(\tilde{\rho}_1(z)e^{i\psi})$$

$\tilde{\rho}_1=\tilde{\rho}_1(z)$ 当电子束在波荡器运动时，电荷的一阶分量逐渐增大，直到饱和。

$$j_z = v_z \rho \quad \Rightarrow \quad \tilde{j}_z(\psi, z) = j_0 + \tilde{j}_1(z)\exp[i(k_l + k_u)z - i\omega_l t]$$

在高增益自由电子激光中，我们需要同时考虑大量电子，我们规定以光波的相位作为参考相位，并假设 psi0=0

$$\psi = (k_l + k_u)z - \omega_l t \quad pondermotive \quad phase$$

Note:本章在讨论一维FEL理论时，忽略电子的纵向振荡。

$$z(t) = \bar{v}_z t = \bar{\beta} c t, \qquad \bar{\beta} = 1 - \frac{1}{2\gamma^2}(1 + \frac{K^2}{2})$$

$$k_l + k_u \approx k_l / \bar{\beta}$$

电流密度可以写成下面的形式：

$$\tilde{j}_z(\psi, z) = j_0 + \tilde{j}_1(z)\exp[ik'(z - v_z t)] \quad k' = k_l / \bar{\beta}$$

3.4 波荡器中电场的变化

电子束辐射的电场满足的波动方程：

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\vec{E} = \mu_0 \frac{\partial \vec{j}}{\partial t} + \frac{1}{\varepsilon_0}\nabla\rho$$

在一维近似中，假设电场只有X方向，电荷密度对X 方向的倒数忽略，即不出现在场方程中，这样简化后的场方程变为：

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]E_x = \mu_0 \frac{\partial j_x}{\partial t}$$

其中在低增益FEL中x方向的场变化写成平面波的形式：

$$E_x(z,t) = E_0 \cos(k_l z - \omega_l t + \psi_0)$$

在1D高增益FEL中x方向的场变化写成平面波的形式：

$$E_x(z,t) = \tilde{E}_x(z)\exp[i(k_l z - \omega_l t)]$$

电场的群速度与相速度不同的影响将在后面讨论

将上述复振幅的电场表达式代入简化后的场方程：

$$[2ik_l\tilde{E}_x'(z)+\tilde{E}_x''(z)]\exp[i(k_lz-\omega_lt)]=\mu_0\frac{\partial\tilde{j}_x}{\partial t}$$

假设电场在一个波荡器周期内变化很小，及慢变包络近似（SVP），目的是为了抛掉电场变化的二阶项：

$$|\tilde{E}_x'(z)|\lambda_l<<\tilde{E}_x(z)\quad\Rightarrow\quad\tilde{E}_x'(z)<<k_l|\tilde{E}_x(z)|$$

$$|\tilde{E}_x''(z)|\lambda_l<<|\tilde{E}_x'(z)|\quad\Rightarrow\quad|\tilde{E}_x''(z)|<<k_l|\tilde{E}_x'(z)|$$

由此得到波荡器中电场变化的一阶方程：

$$j_z=v_z\rho\quad\Rightarrow\quad\tilde{j}_z(\psi,z)=j_0+\tilde{j}_1(z)\exp[i(k_l+k_u)z-i\omega_lt]$$

$$\tilde{j}_x\approx\tilde{j}_z\frac{v_x}{c}=[j_0+\tilde{j}_1(z)\exp[i(k_l+k_u)z-i\omega_lt]]\frac{K}{\gamma}(\frac{e^{ik_uz}+e^{-ik_uz}}{2})$$

$$\frac{d\tilde{E}_x}{dz}=-\frac{\mu_0cK}{4\gamma}\tilde{j}_1(z)$$

空间电荷引起纵向场的变化：

$$\frac{\partial \vec{E}_z(z,t)}{\partial z} = \frac{\tilde{\rho}_1(z)}{\varepsilon_0}\exp[i(k_l + k_u)z - i\omega_l t]$$

$$\Rightarrow \tilde{E}(z) \approx -\frac{i\mu_0 c^2}{\omega_l}\cdot \tilde{j}_1(z) = i\frac{4\gamma_r c}{\omega_l K}\cdot\frac{dE_x}{dz}$$

一阶耦合方程：

$$\frac{d\psi}{dz} = 2k_u\eta$$

$$\frac{d\eta}{dz} = -\frac{e}{m_e c^2\gamma_r}R\left\{(\frac{\hat{K}\hat{E}_x}{2\gamma_r} + \hat{E}_z)e^{i\psi}\right\} = -\frac{e}{m_e c^2\gamma_r}R\left\{(\frac{\hat{K}\hat{E}_x}{2\gamma_r} - \frac{i\mu_0 c^2}{\omega_l}\cdot\tilde{j}_1(z))e^{i\psi}\right\}$$

$$\frac{d\tilde{E}_x}{dz} = -\frac{\mu_0 cK}{4\gamma}\tilde{j}_1(z)$$

假设初始的电子分布为 detla 函数

$$S(\psi)=\sum_{n=1}^{N}\delta(\psi-\psi_n) \qquad \Rightarrow S(\psi)=\frac{c_0}{2}+R\left\{\sum_{k=1}^{\infty}c_k\exp(ik\psi)\right\}$$

$$c_k=\frac{1}{\pi}\int_0^{2\pi}S(\psi)\exp(-ik\psi)d\psi=\frac{1}{\pi}\int_0^{2\pi}\sum_{n=1}^{N}\delta(\psi-\psi_n)\exp(-ik\psi)d\psi=\frac{1}{\pi}\sum_{n=1}^{N}\exp(-ik\psi_n)$$

常系数对应电子束直流部分，一阶系数对应电流一阶分量：

$$j_0=-ec\frac{2\pi}{A_b\lambda_l}\frac{c_0}{2} \qquad -ec\frac{2\pi}{A_b\lambda_l}=j_0/\frac{c_0}{2}=2\pi j_0/N \quad (\frac{c_0}{2}=\frac{N}{2\pi})$$

$$\tilde{j}_1=-ec\frac{2\pi}{A_b\lambda_l}\cdot c_1=-ec\frac{2\pi}{A_b\lambda_l}\cdot\frac{1}{\pi}\sum_{n=1}^{N}\exp(-ik\psi_n)=2j_0(\frac{1}{N}\sum_{n=1}^{N}\exp(-i\psi_n))$$

耦合的一阶微分方程：

$$\begin{cases} \dfrac{d\psi_n}{dz} = 2k_u\eta_n \\ \dfrac{d\eta_n}{dz} = -\dfrac{e}{m_e c^2\gamma_r} R\left\{ \left(\dfrac{\hat{K}\hat{E}_x}{2\gamma_r} - \dfrac{i\mu_0 c^2}{\omega_l} \cdot \tilde{j}_1(z) \right) \exp(i\psi_n) \right\} \\ \dfrac{d\tilde{E}_x}{dz} = -\dfrac{\mu_0 c K}{4\gamma} \tilde{j}_1(z) \\ \tilde{j}_1(z) = 2 j_0 \dfrac{1}{N} \sum_{n=1}^{N} \exp(-i\psi_n) \qquad n = 1, 2, 3 \ldots N \end{cases}$$

N是一个数值很大的数，这是一个多体问题，没有解析解，需要通过计算机进行数值模拟，得到满足一定精度的结果。

粒子分布函数：

在1D的FEL理论中粒子分布函数只写在二维的纵向相空间中，则单位相空间粒子数可以写成如下形式：

$$dn_e = n_e F(\psi, \eta, z) d\psi d\eta$$

我们做如下假设：电子束被激光周期调制，所以分布函数包括周期变化项：

$$F(\psi,\eta,z) = R\left\{\tilde{F}(\psi,\eta,z)\right\} = F_0(\eta) + R\left\{\tilde{F}_1(\eta,z)\cdot e^{i\psi}\right\}$$

则电荷密度的一阶分量，可写成如下形式：

$$\tilde{\rho}_1(z) = \rho_0 \int_{-\delta}^{\delta} \tilde{F}_1(\eta,z) d\eta \qquad \tilde{j}_1(z) = j_0 \int_{-\delta}^{\delta} \tilde{F}_1(\eta,z) d\eta$$

由刘维尔定理得到分布函数的 Vlasov Equation:

$$\frac{dF}{dz} = \frac{\partial F}{\partial z} + \frac{\partial F}{\partial \psi}\cdot\left(\frac{d\psi}{dz}\right) + \frac{\partial F}{\partial \eta}\cdot\left(\frac{d\eta}{dz}\right) = 0$$

重新写出 Maxwell Equation，给出电场强度的变化方程：

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\vec{E} = \mu_0 \frac{\partial \vec{j}}{\partial t} + \frac{1}{\varepsilon_0}\nabla\rho$$

通过上述微分方程，做适当近似与推导后，得到电场强度变化（只和Z相关）的三阶微分方程：

$$\frac{\tilde{E}_x'''}{\Gamma^3}+2i\frac{\eta}{\rho_{FEL}}\frac{\tilde{E}_x''}{\Gamma^2}+\left(\frac{k_p^{\ 2}}{\Gamma^2}-\left(\frac{\eta}{\rho_{FEL}}\right)^2\right)\frac{\tilde{E}_x'}{\Gamma}-i\tilde{E}_x=0$$

此方程包含了高增益FEL中重要的皮尔斯参数：

$$\rho_{\mathrm{FEL}}=\frac{\Gamma}{2k_u}=\frac{\lambda_u}{4\pi\sqrt{3}L_{G0}}\approx\frac{1}{4}\left[\frac{1}{2\pi^2}\frac{I_{pk}}{I_A}\frac{\lambda_u^{\ 2}}{\beta\varepsilon_N}\left(\frac{K}{\gamma}\right)^2\right]^{\frac{1}{3}}$$

我们假设三阶微分的解为指数形式 $\tilde{E}_x(z)=Ae^{\alpha z}$，做单能近似，并且忽略空间电荷效应，我们可以得到代数方程：$\alpha^3=i\Gamma^3$

$$\tilde{E}_x(z)=\begin{cases}A\exp(\alpha_1 z)=A\exp\left(\frac{i+\sqrt{3}}{2}\Gamma z\right)\\ A\exp(\alpha_2 z)=A\exp\left(\frac{i-\sqrt{3}}{2}\Gamma z\right)\\ A\exp(\alpha_3 z)=A\exp(-i\Gamma z)\end{cases}\Rightarrow P(z)\sim\tilde{E}_x\tilde{E}_x^{\ *}\sim\exp[2R(\alpha_1)]=\exp(\sqrt{3}\Gamma z)\equiv\exp\left(\frac{z}{L_{G0}}\right)$$

由此我们得到设计FEL的重要参数：1D Gain Length:

$$L_{G0}=\frac{1}{\sqrt{3}\Gamma}=\frac{1}{\sqrt{3}}\left[\frac{4\gamma_r^{\ 3}m_e}{\mu_0\hat{K}^2e^2k_un_e}\right]^{\frac{1}{3}}$$

1.Energy detuning: $W \neq W_r, \eta \neq 0, \sigma_\eta = 0$

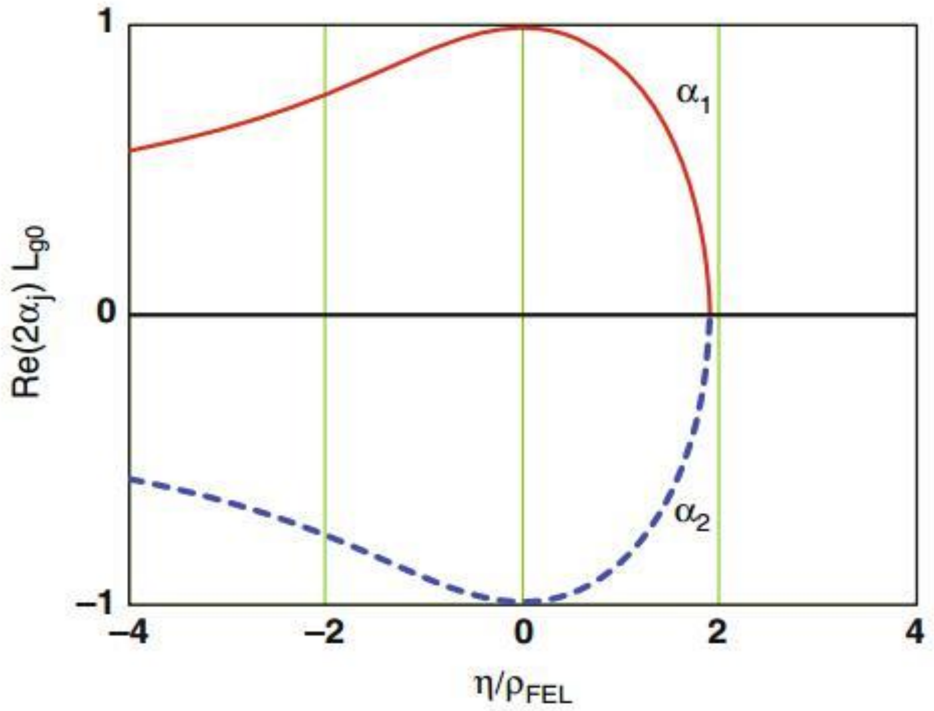


Fig. E.1. The real part of the first and second eigenvalue, multiplied with $2L_{g0}$, as a function of η/ρ_{FEL}, the relative energy deviation divided by the FEL parameter. Note that $\Re(\alpha_1)$ (*continuous red curve*) is positive, corresponding to exponential growth of the eigensolution $V_1(z) = \exp(\alpha_1 z)$. However, the real part vanishes above $\eta; \approx 1.88\rho_{\mathrm{FEL}}$ which means that the exponential growth stops if the electron energy W exceeds the resonant energy W_r by more than $\Delta W = 1.88\rho_{\mathrm{FEL}} W_r$. The real part of α_2 (*dashed blue curve*) is always negative, hence the eigenfunction $V_2(z)$ drops exponentially. Finally, $\Re(\alpha_3) \equiv 0$, so $V_3(z)$ oscillates along the undulator axis

2.Space Charge and Energy Spread

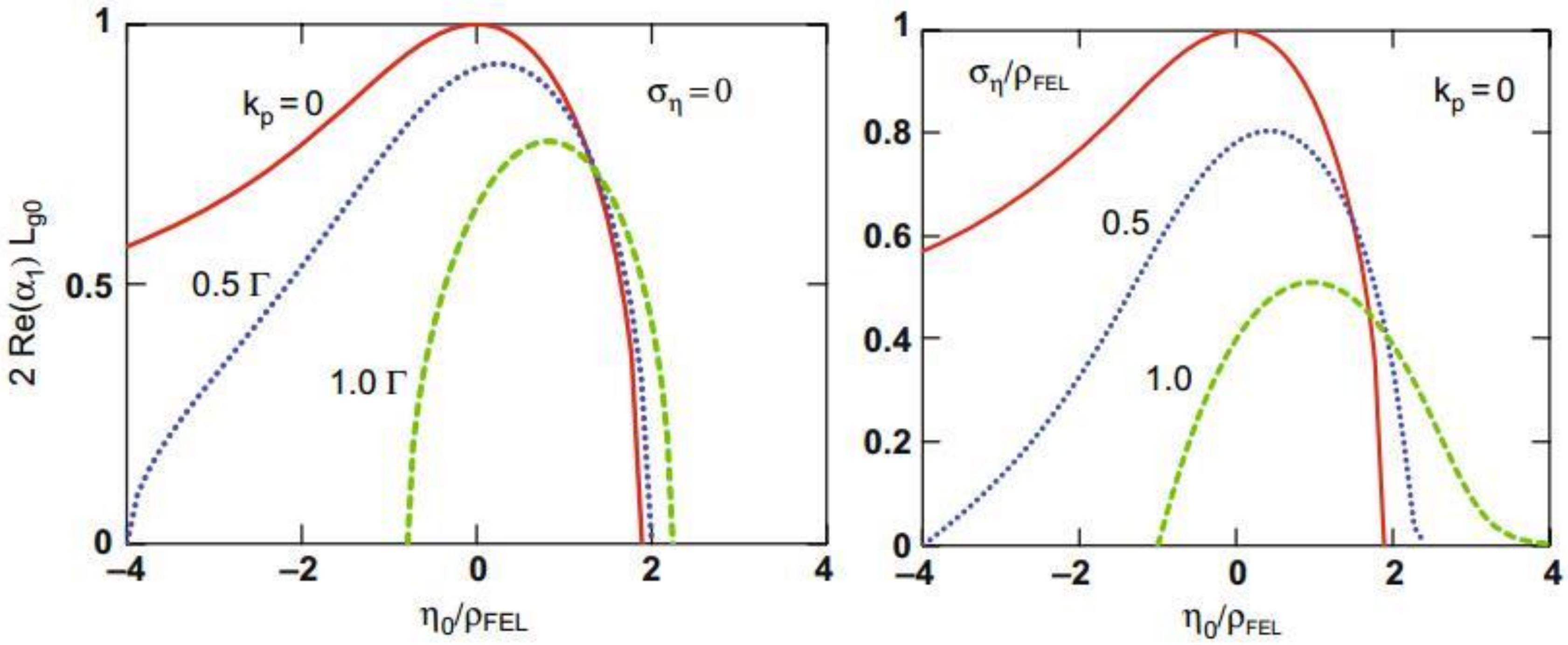


Fig. E.2. *Left*: the growth rate function $f_{gr}(\eta_0) = 2\Re(\alpha_1(\eta_0))L_{g0}$ plotted versus $\eta_0/\rho_{\mathrm{FEL}}$ for different values of the space charge parameter k_p. *Continuous red curve*: $k_p = 0$, *dotted blue curve*: $k_p = 0.5\,\Gamma$, *dashed green curve*: $k_p = 1.0\,\Gamma$. The energy spread is put to zero. *Right*: the growth rate function $f_{gr}(\eta_0) = 2\Re(\alpha_1(\eta_0))L_{g0}$ is plotted versus $\eta_0/\rho_{\mathrm{FEL}}$ for different values of the relative beam energy spread $\sigma_\eta = \sigma_W/W_r$. *Continuous red curve*: $\sigma_\eta = 0$, *dotted blue curve*: $\sigma_\eta = 0.5\,\rho_{\mathrm{FEL}}$, *dashed green curve*: $\sigma_\eta = 1.0\,\rho_{\mathrm{FEL}}$. Here the space charge parameter is set to zero

3.Transverse dimensions of the electron bunch :

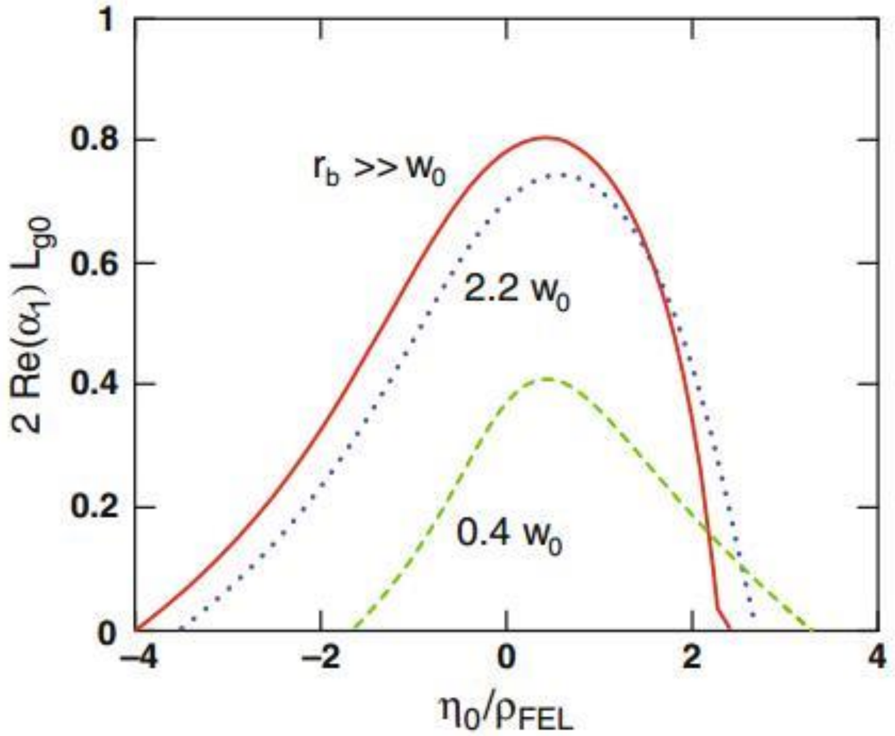


Fig. 6.3. The growth rate function $f_{\mathrm{gr}}(\eta_0) = 2\,\Re(\alpha_1(\eta_0))L_{\mathrm{g0}}$ plotted vs. $\eta_0/\rho_{\mathrm{FEL}}$ for a cylindrical beam with energy spread ($\sigma_\eta = 0.5\,\rho_{\mathrm{FEL}}$). *Continuous red curve*: wide beam with a radius $r_b \gg w_0 = \sqrt{L_{\mathrm{g0}}\lambda_\ell}$, *dotted blue curve*: narrow beam $r_b = 2.2\,w_0$, *dashed green curve*: very narrow beam, $r_b = 0.4\,w_0$

4.Increase the Gain Length by the 3D effects(Xie Ming formula)

$$L_g = L_{g0}(1+\Lambda)$$

$$X_\gamma = \frac{L_{g0} 4\pi \sigma_\eta}{\lambda_u} \quad \text{energy spread parameter}$$

$$X_d = \frac{L_{g0}\lambda_\ell}{4\pi\sigma_r^2} \quad \text{diffraction parameter}$$

$$X_\varepsilon = \frac{L_{g0} 4\pi \varepsilon}{\beta_{av}\lambda_\ell} \quad \text{angular spread parameter .}$$

$$\Lambda = a_1 X_d^{a_2} + a_3 X_\varepsilon^{a_4} + a_5 X_\gamma^{a_6} + a_7 X_\varepsilon^{a_8} X_\gamma^{a_9} + a_{10} X_d^{a_{11}} X_\gamma^{a_{12}} + a_{13} X_d^{a_{14}} X_\varepsilon^{a_{15}} + a_{16} X_d^{a_{17}} X_\varepsilon^{a_{18}} X_\gamma^{a_{19}}$$

5.Simulation of Microbunching

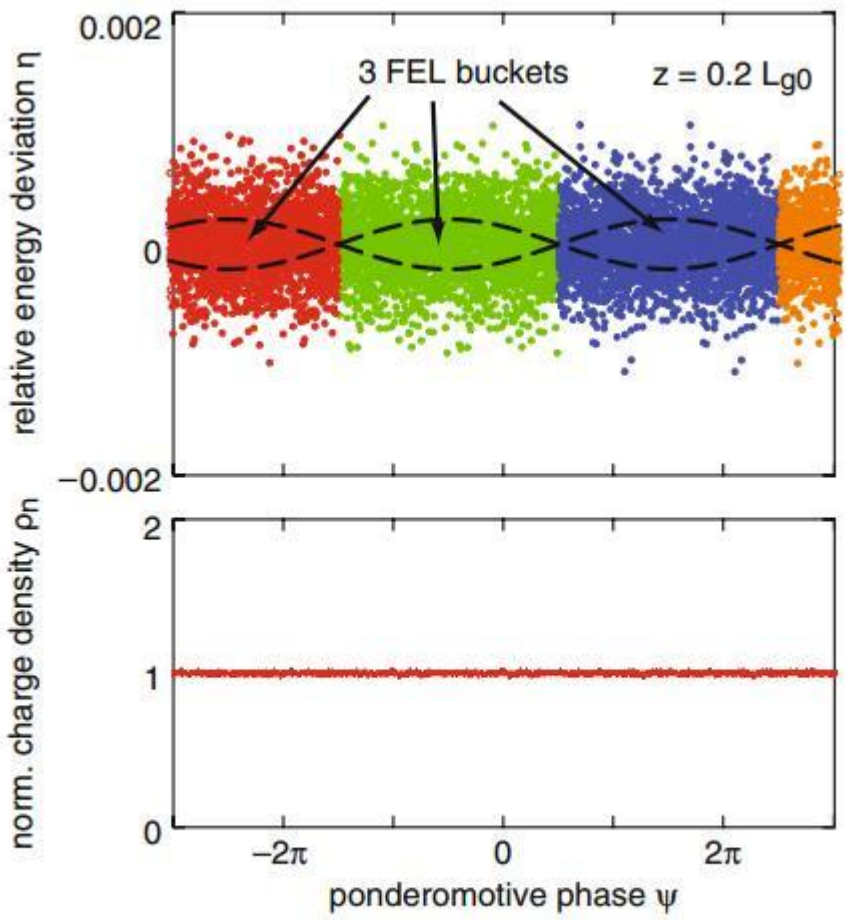


Fig. 5.8. Top: distribution of particles in the (ψ, η) phase space at $z = 0.2\,L_{g0}$. The separatrix is indicated by the dashed curves. Shown are several adjacent FEL buckets. Bottom: normalized charge density $\rho_n(\psi) = |\tilde{\rho}(\psi)|\,/\rho_0$, plotted as a function of ψ. Remember that the phase ψ corresponds to the internal bunch coordinate according to $\zeta = \lambda_\ell \cdot (\psi + \pi/2)/(2\pi)$

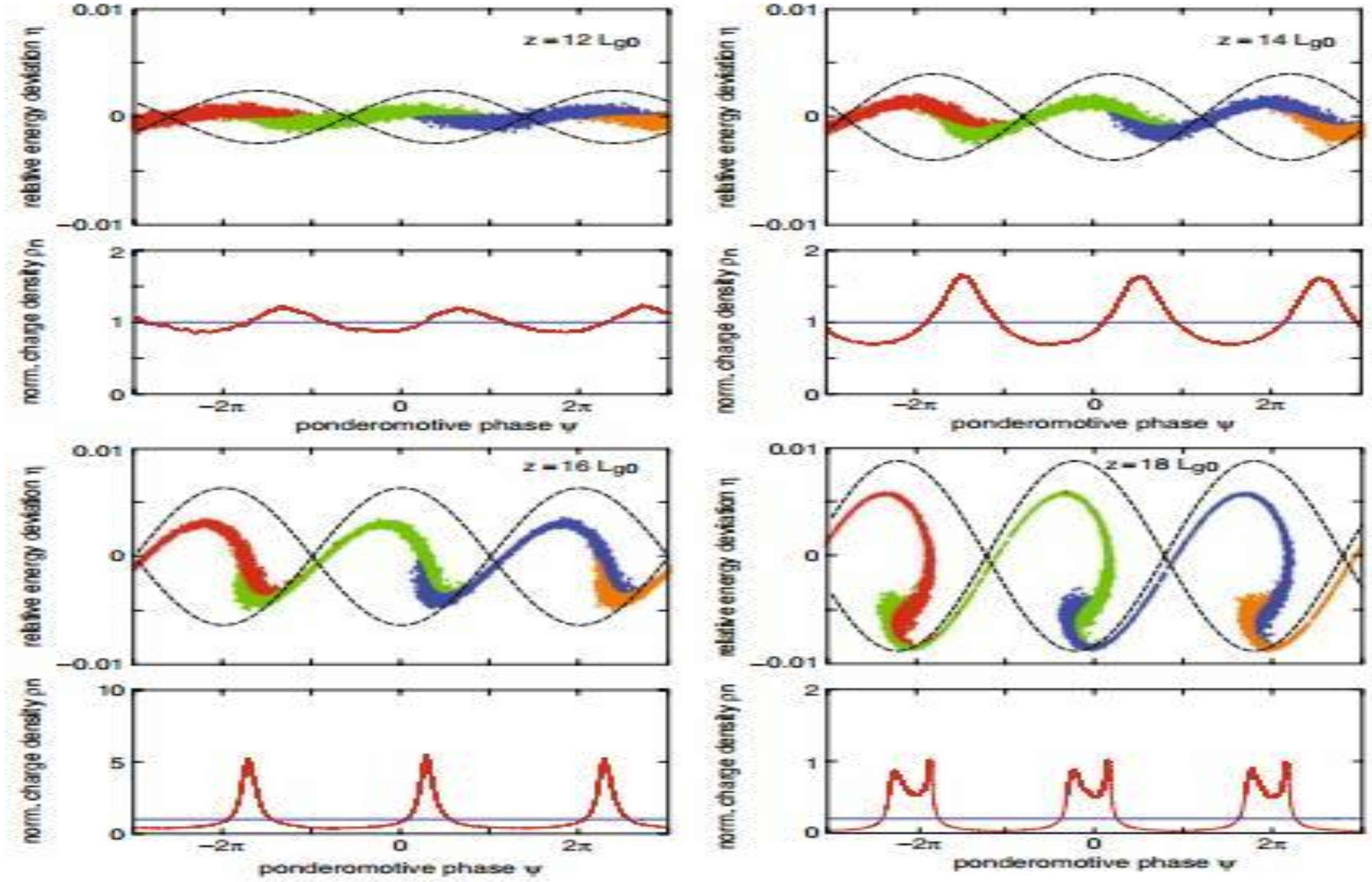


Fig. 5.9. Evolution of the microbunch structure. *Top left*: $z = 12L_{g0}$, *top right*: $z = 14L_{g0}$. *Bottom left*: $z = 16L_{g0}$, *bottom right*: $z = 18L_{g0}$. The upper subplots show the distribution of the particles in the (ψ, η) phase space. The separatrices are indicated by the *dashed curves*. The lower subplots show the normalized particle density as a function of ψ

6.Phases of Electric Field and Current Density

$$\tilde{E}_x(z) = \frac{E_{in}}{3}\sum_{j=1}^{3}\exp(\alpha_j z) \equiv \left|\tilde{E}_x(z)\right|\exp(i\varphi_E(z))$$

$$\tilde{j}_1(z) = -\frac{4\gamma_r}{\mu_0 cK}\tilde{E}'_x(z) = -\frac{4\gamma_r}{\mu_0 cK}\sum_{j=1}^{3}\alpha_j\exp(\alpha_j z) \equiv |\tilde{j}_1(z)|\exp(i\varphi_{j1}(z))$$

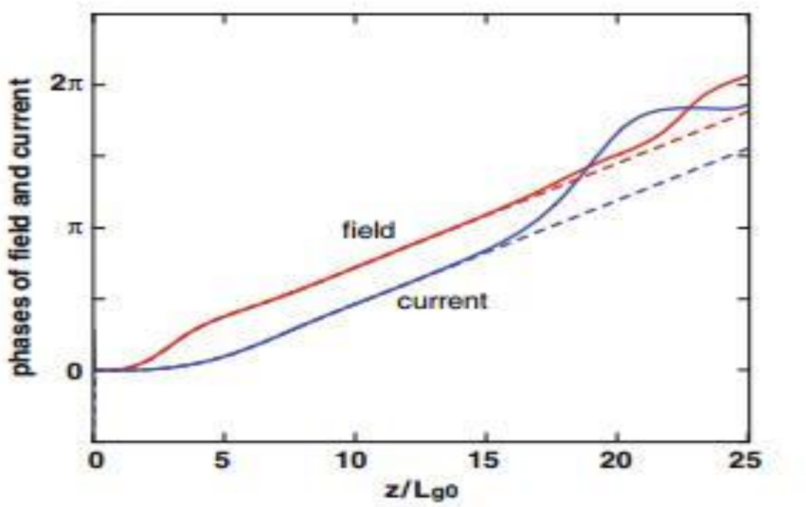


Fig. 5.11. The phase $\varphi_E(z) - \varphi_E(0)$ of the complex field $\tilde{E}_x(z)$ (*solid red curve*) and the phase $\varphi_{j1}(z) - \varphi_{j1}(0)$ of the modulated current density $\tilde{j}_1(z)$ (*solid blue curve*) as a function of z/L_{g0}. The phases have been computed using the coupled first-order equations. In the linear regime there is perfect agreement with the phases derived from (5.20), (5.21). The dashed lines show the extrapolation of the linear theory into the nonlinear regime $z > 17\,L_{g0}$

7.Phase of Center Buckets and Micro-bunch

$$\psi_b(z)-\psi_b(0) = -[\varphi_E(z)-\varphi_E(0)]$$
$$\psi_m(z)-\psi_m(0) = -[\varphi_{j1}(z)-\varphi_{j1}(0)]$$

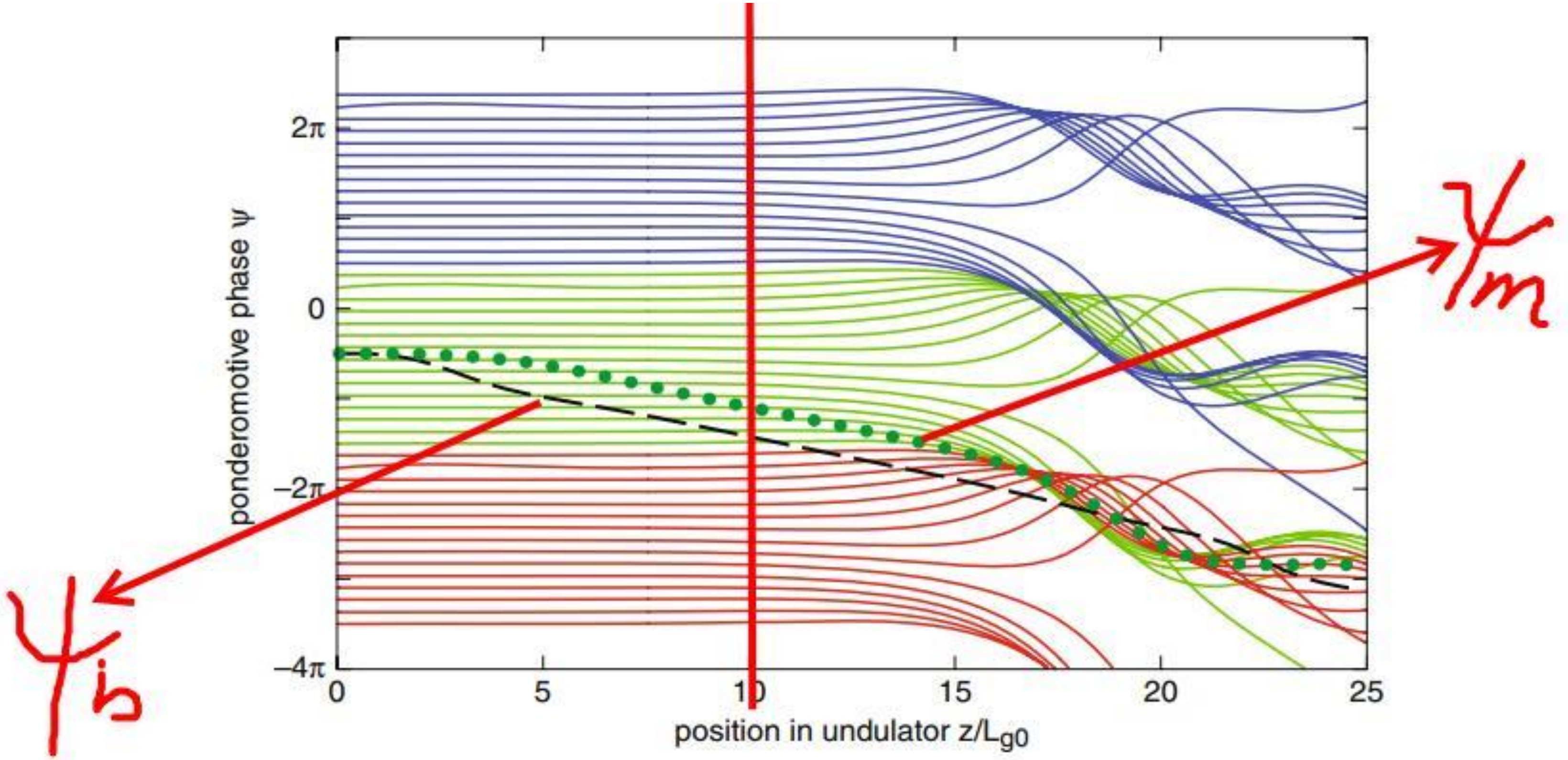
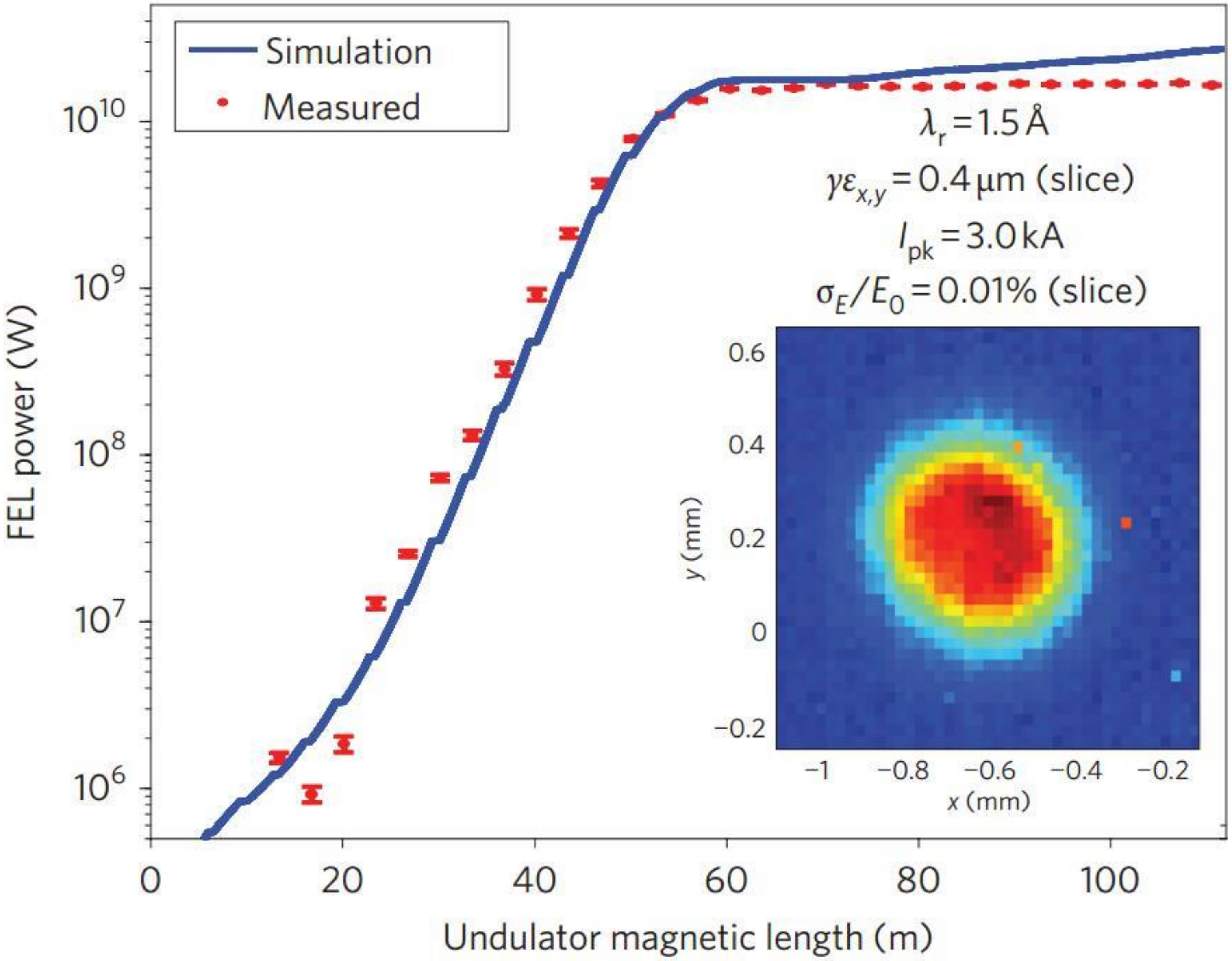


Fig. 5.12. Phase motion of selected particles in three adjacent FEL buckets during the exponential growth and saturation of the FEL power. The beam is monoenergetic and has an initial fractional energy deviation of $\eta = 0$. The dashed curve shows the variation of the FEL bucket center phase $\psi_b(z)$ according to (5.22). The dotted curve shows the motion of the microbunch phase $\psi_m(z)$ as given by (5.23)

P . Emma et. First lasing and operation of an angstrom-wavelength free-electron laser .nature photonics 4 (2010)



Thank you！

最后，衷心的祝愿各位前程似锦，一帆风顺，谢谢大家。

Peter schmüser / Martin Dohlus / Jörg Rossbach

"Ultraviolet and Soft X – Ray Free – Electron Lasers"