MTP 290
Problem Set 5

1. Write a MATLAB script for implementing the LU decomposition (Doolittle's factorization) for a 3×3 matrix.

2. Let
$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & -2 \\ -2 & 1 & 1 \end{pmatrix}.$$
Find Doolittle's factorization of the above matrix. Further, for $b = [1, 1, 1]^T$, solve the system $Ax = b$.

3. Solve the following linear system by LU decomposition (Cholesky) Method
$$\begin{aligned} 16x_1 + 4x_2 + 4x_3 - 4x_4 &= 32 \\ 4x_1 + 10x_2 + 4x_3 + 2x_4 &= 26 \\ 4x_1 + 4x_2 + 6x_3 - 2x_4 &= 20 \\ -4x_1 + 2x_2 - 2x_3 + 4x_4 &= -6. \end{aligned}$$

4. Write a MATLAB script for implementing the Gauss-Jacobi method to solve the system $Ax = b$, where A is a non-singular matrix.

5. Solve the following linear system by Gauss-Jacobi method, with tolerance $= 10^{-4}$ in the l_∞ norm. Choose initial guess as $x_1 = 1/2 = x_2$
$$\begin{aligned} 10x_1 + x_2 &= 11 \\ x_1 + 10x_2 &= 11. \end{aligned}$$

6. Solve the following linear system by Gauss Jacobi method and choose initial guess as $x_1 = x_2 = x_3 = 0$
$$\begin{aligned} 4x_1 + x_2 - x_3 &= 3 \\ 2x_1 + 7x_2 + x_3 &= 19 \\ x_1 - 3x_2 + 12x_3 &= 31. \end{aligned}$$

7. Use Gauss-Jacobi Method to solve (Generate any initial guess in your programme)
$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 + 6x_5 &= -1 \\ -3x_1 + 9x_2 + x_3 - 2x_4 + 7.4x_5 &= 2 \\ 2x_1 - 1x_2 - 7x_3 + x_4 + 6.7x_5 &= 3 \\ 4x_1 + 3x_2 - 5x_3 + 7x_4 + 9x_5 &= 0.5 \\ 2x_1 + 3.5x_2 + 6.1x_3 - 4x_4 - 8.1x_5 &= 3.1. \end{aligned}$$

8. Use Gauss-Jacobi Iterations to attempt solving the linear system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 5 \\ 2x_1 - x_2 + 2x_3 &= 1 \\ 3x_1 + x_2 - 2x_3 &= -1. \end{aligned}$$

   (Whether the method converges?)

9. Write a MATLAB script for implementing the Gauss-Seidel method to solve the system $Ax = b$, where A is a non-singular matrix.

10. Redo the problem 5 and 6 using Gauss-Seidel method, with tolerance $= 10^{-4}$ in the l_∞ norm.

11. Use Gauss-Seidel Iterations to attempt solving the linear system

$$\begin{aligned} 2x_1 + 8x_2 + 3x_3 + x_4 &= -2 \\ 2x_2 - x_3 + 4x_4 &= 4 \\ 7x_1 - 2x_2 + x_3 + 2x_4 &= 3 \\ -x_1 + 5x_3 + 2x_2 &= 5. \end{aligned}$$

   (Whether the method converges?)