MTP 290, Computational Laboratory, Tutorial Sheet 3

Please attempt all the question. Experimentation and exploring is encouraged. The tutorial is aimed to give you practice of the basics, but additional effort might be required to get comfortable with the tools used. Some questions might be open ended deliberately.

Question 3.1
Solve the equation $\cos(x) - x = 0$ for $x \in [0, \pi/2]$. Compare numerical results to determine which algorithm among the Newton's method and the bisection method performance better for this problem (For Newton's method, take the mid-point of the interval as the starting point).

Question 3.2
Use Newton-Raphson method to solve the equation $x^2 = 0$, using the initial estimate $x_0 = 0.1$. What is the observed order of convergence?

Question 3.3
Redo problem 3.2 using the modified Newton's method.

Question 3.4
Find the square root of 2, correct upto five decimal places using Newton's method as well as Secant method. Compute the order of convergence for both the methods.

Question 3.5
Consider the function

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Note that 0 is the only solution of $f(x) = 0$. Show that if $x_0 = 0.0001$, it takes more than one hundred million iterations of the Newton Method to get below 0.00005.

Question 3.6
Find, correct to 5 decimal places, the x-coordinate of the point on the curve $y = \log_e(x)$ which is closest to the origin. Use the Newton's Method and Secant Method.

Question 3.7
Let $f(x) = x^3 - 2x + 2$ and consider the initial approximation as $x_0 = 0$. Find the root of $f(x)$ using Newton's method.(Check whether this method converges or not?)