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Continuous Approximation for Skip-Stop Operation in Rail Transit

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Abstract

Operating speed of a transit corridor is a key characteristic and has many consequences on its performance. It is generally accepted that an increased operating speed for a given fleet leads to reduced operating costs (per kilometer), travel and waiting times (three changes that can be computed precisely), an improved comfort and level of service, which can attract new passengers who are diverted from automobile (items harder to estimate precisely). That is why several operation schemes which aim to increase the operating speed are studied in the literature, such as deadheading, express services, stop skipping, etc.

A novel category of solutions to this problem for one-way single-track rail transit is to perform accelerated transit operations with fixed stopping schedules. The concept is quite simple: as the time required for stopping at each station is an important part of travel time, reducing it would be a great achievement. Particular operations that take advantage of this idea already exist. This paper focuses on one of them: the skip-stop operation for rail transit lines using a single one-way track. It consists in defining three types of stations: AB stations where all the trains stop, and A and B stations where only half of the trains stop (stations type A and B are allocated interchangeably). This mode of operation is already described in the literature [1, 2, 3] and has been successfully implemented in the Metro system of Santiago, Chile.

This work tackles the problem with a continuous approximation approach. The problem is described with a set of geographically dependent continuous parameters like the density of stations for a given line. Cost functions are built for a traditional (all-stop) operation and for skip-stop operation as described above. A simple example is presented to support this discussion. Finally, a discussion about the type of scenarios in which skip-stop operations are more beneficial is presented.

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Keywords: Transit operations; Skip-stop operation; continuous approximation; network design

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1. Introduction

Many subway systems face very high demands during peak periods reaching levels close to the capacity being offered. Once these high occupancy ratios exceed what is considered acceptable the operator is expected to increment the capacity of the system. Increasing the capacity via investments in new trains or subway lines is often very expensive. The social benefits of investing in extra trains are often reduced since the new trains are only strictly needed during peak periods. Alternatively, the operator could increment the capacity by increasing the operating speed. Increasing the operating speed not only increases the capacity of the system, but also improves the level of service offered to the users by reducing waiting and travel times and reducing the passenger density inside the trains. These short-term effects will often attract extra passengers from other transit modes.

A few metro systems around the world use some type of non-conventional operation mode to speed-up trains in which they do not stop in every station. These operational schemes can be classified as either express or skip-stop. A system operating with express services needs more than one track or takeover facilities, and allows very different services to coexist. Some of them may skip several stations acting as a fast trunk line while others may stop in every station acting as feeder. Express operations are quite common in sub-urban railways lines, such as the Paris RER, the Madrid RENFE, the Sydney and Melbourne suburban lines although it is also used in urban contexts such as in the New York Metro system. Although increasing operational speeds on a rail corridor is always challenging, it is especially so when a single one-way track is available for operations. In these cases, in which no vehicle overtaking is allowed, a skip-stop operation can be implemented.

Under a skip-stop operation, each train visits only a fixed subset of the stations. By skipping some stations the cycle time is reduced, increasing the operational speed. However, operating on a single-track imposes a severe constraint since trains must respect a certain distance between trains and all stations must be visited. The A/B Skip-Stop Express Services consist in operating trains alternately as type A and type B. Very busy stations are labeled as AB where all trains stop. Other stations located between AB stations are labeled alternately as type A, where only A trains stop, and type B, where only B trains stop [1]. Figures 1 and 2 (both taken from [3]) display this type of operation schematically. Fig. 1 highlights the stations visited by each of the trains A and B, while Fig. 2 presents time-space diagrams for standard (S) and skip-stop (S-S) operations.

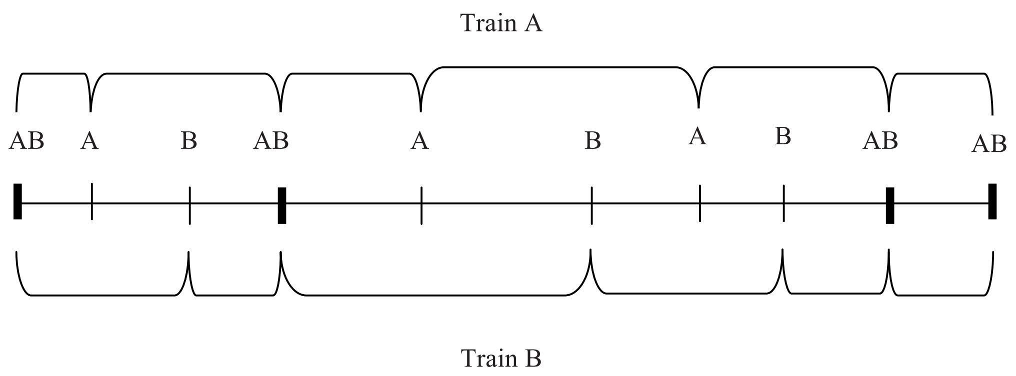

Fig. 1. Skip-stop operation

As far as we know, skip-stop operation was first developed for the Chicago Metro system in 1947 (Chicago-L, 2010), and later implemented in Philadelphia and New York. Currently none of these Metro systems offer a skip-stop service. However, since 2007, skip-stop operation has been implemented in Santiago, Chile [5, 6].

The Santiago Metro first implemented skip-stop operation on a line where it was facing severe capacity problems; crowding inside the trains reached an average of more than 7 pax/m^2 across all trains during a 15-min interval (on the heaviest inter-station segment of the line). The skip-stop operation was initially implemented only in the less loaded direction and in the morning peak period. Users adopted them quickly. It was shown that each skipped station lead to a reduction of 47 seconds in total travel time (leading to a 4.3 minute saving for a trip joining the extreme stations of the line), and that frequency grew from 34.5 trains/hr to 38 trains/hr with the same fleet, while operational and maintenance costs per km-train dropped. These results lead the Metro authorities to implement skip-stop operation in both directions and both peak periods of line 4 and extend the scheme into other lines of the system. Although the system remains well evaluated by users and Metro authorities, skip-stop operation has not been tried during off-peak periods. Also, Metro of Santiago is reluctant to implement this operational scheme in the most important line which captures the most demand, has the shortest average inter-station distance and operates with the highest frequency. Thus, although the operational scheme can provide significant benefits, the firm thinks that it is not beneficial under any circumstances. Also, as Block [4] states: "*Speed is increased, as fewer stops are made, but the question is, does this advantage outweigh the inconvenience of a person's having to switch from the A line to the B line through the intermediation of an AB stop – or having to go backwards of the line is outlaid as follows:* A_1, B_1, AB_1, A_2, B_2, AB_2*..., and one wants to travel from* A_1 *to* B_1*? (In this case one would have to proceed from* A_1 *to* AB_1*, and then back to* B_1*.)*". Notice that if a third type of train stopping at each station is operated to provide a direct trip to every passenger, the cycle time of the line is affected. Indeed, since we consider a single track with no overpassing facilities, this slower train will govern the cycle time. So one of the main goals of skip-stop operation would be lost.

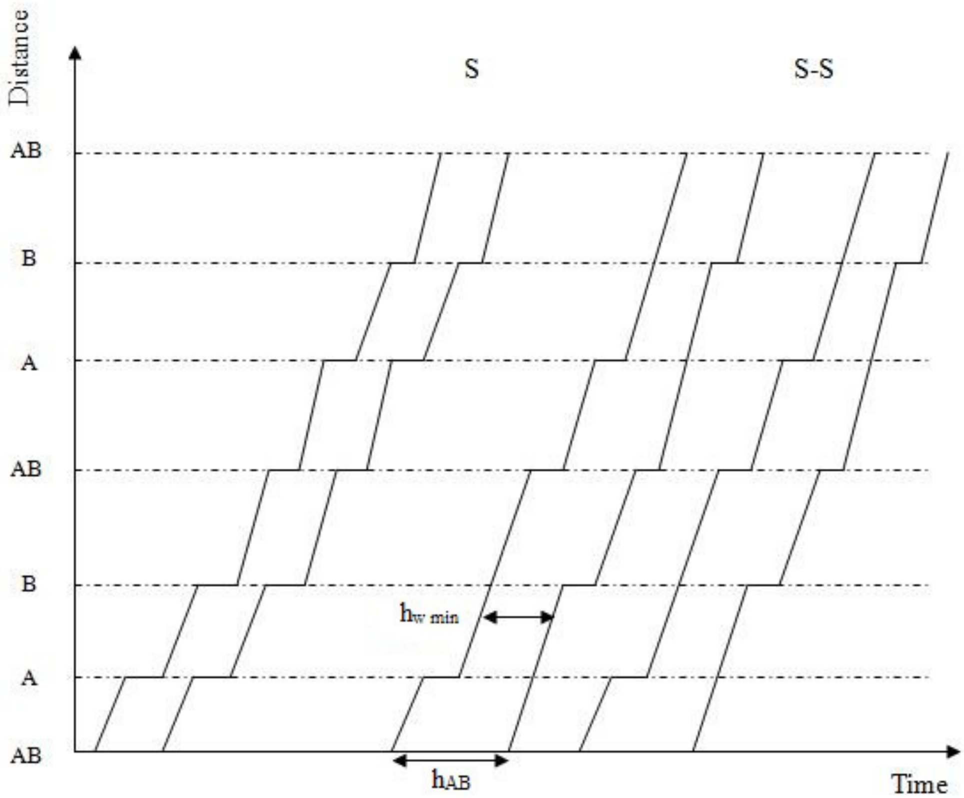

Fig. 2. Time-space diagram for standard and skip-stop operation

Thus, to consider implementing skip-stop operations, Metro operators need a clear methodology to determine if, for a given fleet, skip-stop operation is more beneficial than standard operation where each train visits all stops. In this comparison skip-stop operation must be evaluated under its optimal operation; i.e. the best distribution of A, B and AB stations. These observations lead to the question of which characteristics of a Metro line make skip-stop operation more attractive, how to obtain the optimal density of AB stations and how the characteristics of the line affect this optimal density. This paper is devoted to answer these questions.

Vuchic [2] indicates that stations A and B should be those with the smallest number of passengers, that the total number of passengers in all A stations should be similar to the total for all B stations to maintain even loadings of trains, that the number of A and B stations should be the same to maintain uniform headways at AB stations, and that there should be as few consecutive A-B station pairs as possible to minimize the number of links between A and B stations that cannot be travelled without reversing. The author also suggests that given the more complicated operation, a more rigorous control of train movements is needed. Also, it mentions that skip-stop should only be introduced on lines – or during periods of the day- for which headways are rather short (below five to six minutes) to avoid long waiting at A and B stations.

Metro operators should appreciate a tool to quickly determine using little information if skip-stop operation might be attractive. Also, if it would be considered, operators should be interested in knowing approximately how distant AB stations should be allocated in the line and in having some sensibility on which parameters are key on these decisions. Such a tool would allow an operator to have a preliminary idea on where and when skip stop operation could be beneficial, and to obtain an estimation of its impacts. If skip-stop is considered convenient, a deeper study with more complete data to get more precise results should be requested. On the other hand using an approximated tool could avoid an expensive data collection and analysis if skip-stop operation will clearly not provide significant benefits. Vuchic [3] provides some guidelines on how to compute the effects of skip-stop operation versus standard operation, and therefore estimate its convenience by describing each one of the effects for different trip types. However, determining the optimal distribution of AB stations would require an exhaustive analysis of all the alternatives (which may prove impracticable for long enough lines) or solving a mixed integer non linear problem quite intense in detailed origin destination demand.

This work provides an alternative attempt to quantify the operational benefits of skip-stop operation. For this purpose the system will be modeled using a Continuous Approximation Approach (for a description of the methodology and logistics applications, see [7]). The system is modeled with a set of geographically dependent continuous parameters like the density of stations for a given line. Cost functions are built for a traditional (all-stop) operation and for skip-stop operation. The second one has to be optimized in terms of the density of AB stations before comparing it with the first one. A graphical sensitivity analysis is conducted to see the type of scenarios in which skip-stop operation are more beneficial.

The remainder of this paper is divided into three sections. Section 2 gives the description of the proposed model, including the main assumptions, the notation that will be used for the different variables and parameters, and the complete formulation of the objective function for the case of all-stations and skip-stop operations. Section 3 presents a numerical analysis of the proposed model with a sensitivity analysis of the main parameters. Finally, Section 4 presents our conclusions, including a summary of the study's main contributions and some final comments on topics for future research.

2. Problem Formulation

Consider a fleet of trains operating on a bidirectional single-track circular line of length 2L, i.e. the length on each direction is L, in which half of the fleet operates each direction, as shown in Figure 3. The first goal is to determine which would be the optimal AB station density at each point of the circle based on the local conditions (regular station density and affluence). The second goal is to compare the total cost involved with skip-stop operation and compare it with a standard service. We will reach these goals based on a continuous approximation model.

The relevant costs involved in this comparison can be divided into those faced by users which include in-vehicle travel time, waiting time and transferring costs, and those faced by the operator, i.e. operational costs. We will neglect maintenance costs, which have shown to be lower in the case of skip-stop operation due to significantly less door activity.

We will assume that trains never stop, have infinite capacity and that dwell times are identical in all stations, independently of the number of boarding and alighting passengers. We will assume that under standard operation trains heading in each direction are evenly spaced along the track, so headways between consecutive trains are identical. In the case of skip-stop operation the headways between consecutive trains of the same type are also identical. In the case of skip-stop operation we will always have the same number of A and B stations between each pair of AB stations, and we will assume that the number of AB stations is even. Finally, to avoid considering an OD matrix for the system, we will assume that the destination of all passengers entering the system through any station is evenly distributed across all the remaining stations, i.e. all the remaining stations are equally likely to be the destination of a single trip originated in the origin station. This assumption of evenly distributed demand tend to overestimate the number of trips requiring a transfer, since an optimal design of common stations should take advantage of trip patterns concentrated at some stations, which in practice tend to be chosen to operate as common station. This assumption also means that as the system operates in both directions, the maximum trip length is L and the average trip length is a slightly over L/2.

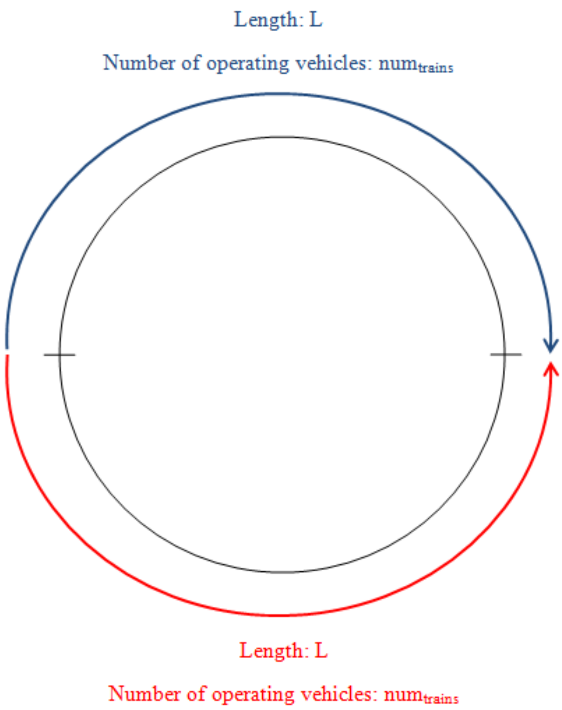

Fig. 3. Schematic of a line

We will initially assume that the conditions along the line are identical. We will call:
λ: station density [Stations/m]
N: number of passengers per unit of time starting a trip in each station [pax/s]
Also, we will need the following parameters:
n_T: number of trains operating in each direction
a: acceleration and deceleration rate of the trains [m/s²]
v_l: cruising speed of the subway [m/s]
t_s: dwell time in every station [s]
F: traction force of a train [N]
VTV: travel time value [$/s]
VTE: waiting time value [$/s]
CE: energy cost [$/W.s]
CT: transfer cost [$]

2.1. Standard Operation

In the case of standard operation, the average in-vehicle travel time per passenger can be approximated as:

$$\frac{L+\frac{1}{\lambda}}{2v_l}+L\frac{\lambda}{2}t_s$$

Where the first term represents the average in-vehicle travel time while the train is moving, and the second term represents the time waiting while other passengers board and alight the train.

In this operation scheme, the waiting time can be formulated as half of the ratio between the cycle time and the fleet size operating in each direction:

$$\frac{T_c}{2n_T} = \frac{\frac{2L}{v_l} + 2L\lambda t_s}{2n_T}$$

All passengers are offered a direct trip, so nobody needs transferring.

The operation costs considered in this analysis will only take into account electrical consumption. In a more complete model some costs associated to opening and closing doors and their maintenance could be added. We will distinguish two operational stages: high-consumption traction strain and low-consumption phase for maintaining cruise speed. We will consider that energy lost when braking is not recuperated elsewhere in the system as in modern systems.

We will consider that each train has a maximum traction force *F*, and a maximum acceleration rate *a*. To start moving, the train has to exert this force until reaching the cruise speed v_l. Thus, the required power *P* for a departing train is:

$$P = F\frac{v_l}{2}$$

And the energy consumed *E* corresponds to the product of the power and the duration that this power is needed, t_a:

$$E = Pt_a = F\frac{v_l^2}{2a} \qquad \text{since } t_a = \frac{v_l}{a}$$

Thus, the total traction cost ($C_{op,trac}$) per cycle can be computed as the product of E and the number of stops in a cycle:

$$C_{op,trac}^{n}{}' = (2L\lambda)\left(F\frac{v_l^2}{2a}\right)CE$$

The final step is to express the energy cost of all trains (in both ways) during the studied period (here one hour, 3600 seconds):

$$C_{op,trac}^{n} = 2C_{op,trac}{}'num_{trains}\frac{3600}{t_c^n}$$

Finally dividing by the number of stations to adapt this cost function to the cost of all trains during the studied period, expressed in CL \$/hour.station:

$$C_{op,trac}^{n} = 2C_{op,trac}{}'num_{trains}\frac{3600}{t_c^n}\frac{1}{(2L\lambda)}$$

During cruise speed, it is assumed that the power required to keep a train moving is a fraction *b* of the power needed to speed it up to the cruise speed. Then:

$$P = F\frac{v_l}{2}b$$

In this case the consumed energy for maintaining cruising speed between two stations is:

$$E = \left(F\frac{v_l}{2}\right)b\left(\frac{\frac{1}{\lambda} - \frac{v_l^2}{a}}{v_l}\right) = F\frac{b}{2}\left(\frac{1}{\lambda} - \frac{v_l^2}{a}\right)$$

And then the total energy cost per cycle ($C_{op,ma}$) can be computed as:

$$C_{op,ma}^{n}{}' = (2L\lambda)\left(F\frac{b}{2}\left(\frac{1}{\lambda} - \frac{v_l^2}{a}\right)\right)CE$$

The final step is to express the energy cost of all trains during the studied period, expressed in CL $/hour-station:

$$C_{op,ma}^{n} = 2C_{op,ma}{}'num_{trains}\frac{3600}{t_c^n}\frac{1}{(2L\lambda)}$$

2.2. Skip-Stop Operation

In the case of skip-stop operation we need to distinguish five different types of trips depending on their origin and destination. To describe these trip types we will use the concept of a bay as the distance between two consecutive AB stations; we will consider that both AB stations will not be part of the bay. The trip types are described as follows and illustrated in Figure 4:

- Type 1: From an AB to an AB station. All passengers take the first arriving train.
- Type 2: From an AB to an A (or B) station or vice versa. Some passengers might have to board the second train going through their origin station.
- Type 3: From an A to another A station, or from a B to a B station. Again in these trips some passengers might have to board the second train going through their origin station.
- Type 4: From an A station to a B station such that both stations do not belong to the same bay. These passengers will also have to transfer to the subsequent train at an AB station.
- Type 5: From an A to a B station belonging to the same bay. These passengers will have to travel on the opposite direction of their destination on some leg.

The parameters describing the operation are the same as those used for the Standard Operation, but we need to add a decision variable: the AB stations density that will be denoted as δ [stations/m], equals to the inverse of the length of a bay. This density cannot be lower than $\frac{1}{L}$ or greater than λ.

Of course, the number of passengers making each type of trip will be different and needs to be known *a priori* to compute the costs functions. For simplicity purposes, in this research we assume that all stations get the same number of passengers coming from any other specific station. In this model there are $2\lambda L$ stations in total, of which $2\delta L$ correspond to AB stations, and $(\lambda - \delta)L$ to A and B stations. Thus, there are $(2\lambda L)(2\lambda L) = 4(\lambda L)^2$ different possible trips from which $(2\delta L)(2\delta L) = 4(\delta L)^2$ are of type 1. Regarding trips of type 2, we have $4(2\delta L)(\lambda - \delta)L = 8(\lambda - \delta)\delta L^2$ of them, since we must consider trips joining AB stations with a and B stations, and where AB stations are at the origin or the destination of the trip. Following this argument, the proportion of trips of type 1, 2 and 3 are estimated as:

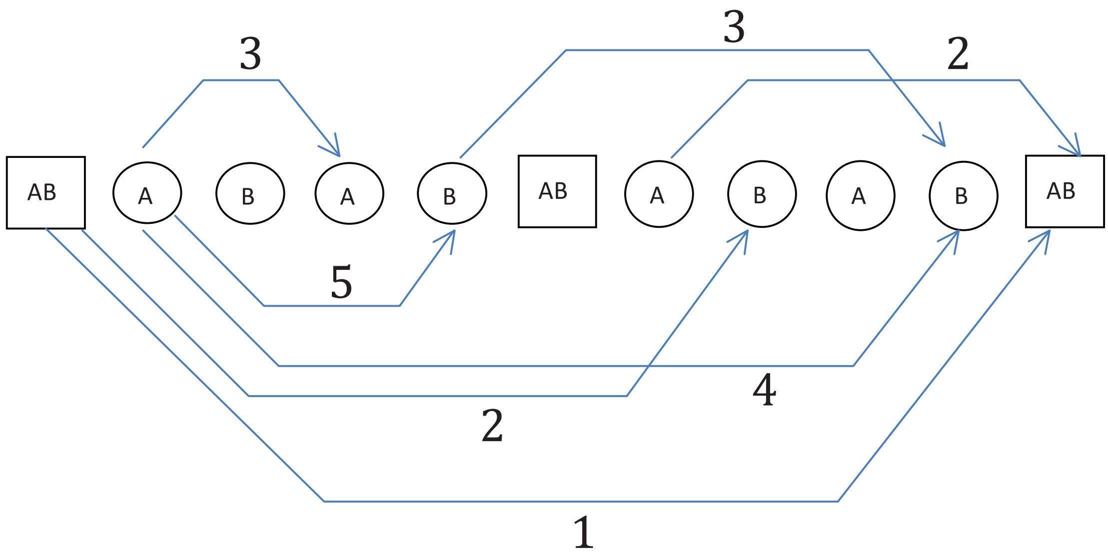

Fig. 4. Examples for the 5 types of trips

$$P_1 = \frac{\delta^2}{\lambda^2}$$

$$P_2 = \frac{2(\lambda - \delta)\delta}{\lambda^2}$$

$$P_3 = \frac{2\left(\frac{\lambda - \delta}{2}\right)^2}{\lambda^2} = \frac{(\lambda - \delta)^2}{2\lambda^2}$$

Where P_i is the proportion of trips type i.

In trips of type 4 and 5 the trip does not originate at an AB station. Once the origin of the trip is selected, the probability that the destination lays beyond its bay (in each direction) corresponds to of $\frac{2L\delta - 1}{(2L\delta)}$. Thus, we estimate:

$$P_4 = \frac{2\left(\frac{\lambda - \delta}{2}\right)^2}{\lambda^2} \frac{2L\delta - 1}{(2L\delta)}$$

$$P_5 = \frac{2\left(\frac{\lambda - \delta}{2}\right)^2}{\lambda^2} \frac{1}{(2L\delta)}$$

It can be easily verified that these five proportions add up to one. From now on sub-index i will be used to make reference of each trip type.

In-vehicle travel times depend on the number of stops made by the train. In the case of skip-stop operation, the number of stops varies depending on the type of trip taken by the user. So before computing in-vehicle travel times, the number of stops in each trip type needs to be determined (excluding the origin and destination stops from the trip).

Type 1: There are ($\frac{\lambda}{\delta}-1$) stations in a bay ($\frac{1}{\delta}$ is the distance between two AB stations). Then, going from an AB station to the next one, a skip-stop train makes $\frac{1}{2}\frac{\lambda-\delta}{\delta}+1=\frac{\lambda+\delta}{2\delta}$ stops if the stop at the AB station is included.

The number of different type 1 destinations is $2L\delta-1$. Thus, since we have an even number of AB stations the average number of stops must be:

$$\frac{1}{2L\delta-1}\left[\sum_{k=1}^{L\delta-1}2\left\{k\frac{\lambda+\delta}{2\delta}\right\}+\frac{\lambda+\delta}{2\delta}L\delta\right]=\frac{L^2\delta(\lambda+\delta)}{2(2L\delta-1)}=\frac{L(\lambda+\delta)}{2(2-\frac{1}{L\delta})}$$

Type 2: Let the origin station be an AB. We will first determine the number of stops from the last AB station visited until the destination. If we denote the number of stations between two consecutive AB station as x (then $x=\frac{\lambda}{\delta}-1$), then it can be shown that the average number of stops is $\frac{x}{4}+\frac{1}{2}=\frac{\frac{\lambda}{\delta}-1}{4}+\frac{1}{2}=\frac{\lambda+\delta}{4\delta}$.

Now, to reach that last AB station the passenger must stop $\frac{\lambda+\delta}{2\delta}$ times for each AB stations visited. Thus, the average number of stops in a trip Type 2 can be estimated us:

$$\frac{1}{L\delta}\sum_{k=0}^{L\delta-1}\left\{\frac{\lambda+\delta}{2\delta}\right\}k+\frac{\lambda+\delta}{4\delta}=\frac{(\lambda+\delta)L}{4}$$

Type 3: In this case the procedure is simpler since we can say that in average, approximately half of AB stations and half of A (or B) stations are visited. Thus, the average number of stops could be estimated as:

$$L\frac{\delta}{2}+L\frac{\lambda-\delta}{4}=L\frac{\lambda+\delta}{4}$$

A more precise calculation of this result is presented in the Annex of this paper where the validity of this approximation is shown.

Type 4: In this case the number of stops is similar to the type 3 trip, but slightly different. In the annex we show that the number of stops corresponds to:

$$\frac{L^2\delta(\lambda+\delta)}{2(2L\delta-1)}$$

Which can be effectively approximated to $\frac{L(\lambda+\delta)}{4}$ if $L\delta$ is assumed to be sufficiently large.

Type 5: As will be shown later in the document, the number of stations between AB stations rarely exceeds 8 stations since then type 5 trips become too common and too inconvenient. In that range, the average number of stops of this type of trips can be approximated as $\frac{x}{4}+\frac{3}{2}$ where x is the number of stations between two AB stations (2, 4, 6, 8…). Thus, the average number of stops in this case is estimated as

$$\frac{\lambda-\delta}{4\delta}+\frac{3}{2}=\frac{\lambda+5\delta}{4\delta}$$

Table 1 provides a comparison of these derived approximations and the exact average number of stops for a line with 20 AB stations. Four different scenarios are considered: bays holding 2, 4, 6 and 8 stations (as will be seen later it is hardly beneficial to have more stations in each bay than 8). As can be seen, the approximations are quite accurate.

The average trip time under a skip-stop operation should be:

$$T_{v,i}^{e}{}' = \frac{L+\frac{1}{\lambda}}{2v_l}+\frac{L\delta}{2}\frac{(\lambda+\delta)}{2\delta}t_s+P_5(extra\ travel\ time+extra\ stops) \qquad for\ i=1\ldots3$$

Table 1: Approximated and exact average number of stops for all destinations of each type.

		Stations per bay			
		2	4	6	8
Type 1	Approx	10,53	15,79	21,05	26,32
	Exact	10,53	15,79	21,05	26,32
Type 2	Approx	10,00	15,00	20,00	25,00
	Exact	10,00	15,00	20,00	25,00
Type 3	Approx	10,53	15,38	20,34	25,32
	Exact	10,53	15,38	20,34	25,32
Type 4	Approx	10,53	15,79	21,05	26,32
	Exact	10,53	15,76	21,01	26,25
Type 5	Approx	2,00	2,50	3,00	3,50
	Exact	2,00	2,50	3,11	3,75

To be able to compute the waiting time of a passenger, the cycle time of a train is needed:

$$t_c^e=\frac{2L}{v_l}+2L\frac{\lambda+\delta}{2}t_s$$

And now waiting time is calculated for each type of trip:

Type 1:

$$T_{e,1}^{e}{}' = \frac{t_c^e}{2num_{trenes}}$$

$$T_{e,1}^{e}{}' = \frac{\frac{2L}{v_l} + 2L\frac{\lambda+\delta}{2}t_s}{2num_{trenes}}$$

Type 2: There are two cases:

- The first train suits and the waiting time is $T_{e,1}^{e}{}'$
- The first train does not suit, passengers have to wait the same time plus the headway between two trains, the waiting time is $3T_{e,1}^{e}{}'$

Thus, the average waiting time is: $T_{e,2}^{e}{}' = 2T_{e,1}^{e}{}'$

Type 3: It is the same as for type 2, then $T_{e,3}^{e}{}' = 2T_{e,1}^{e}{}'$

Type 4: There are two cases:

- The first train suits, passengers have to wait $T_{e,1}^{e}{}'$ in the initial station and then have to wait the headway between two trains in the transfer station, the waiting time is $3T_{e,1}^{e}{}'$
- The first train does not suit, passengers have to wait $3T_{e,1}^{e}{}'$ in the initial station and then have to wait the headway between two trains in the transfer station, the waiting time is $5T_{e,1}^{e}{}'$

Thus, the average waiting time is: $T_{e,3}^{e}{}' = 4T_{e,1}^{e}{}'$

Type 5: There are two cases:

- The first train suits, passengers have to wait $T_{e,1}^{e}{}'$ in the initial station and then $T_{e,1}^{e}{}'$or $3T_{e,1}^{e}{}'$ in the transfer station. The waiting time is $3T_{e,1}^{e}{}'$
- The first train does not suit, passengers have to wait $3T_{e,1}^{e}{}'$ in the initial station and then $T_{e,1}^{e}{}'$or $3T_{e,1}^{e}{}'$ in the transfer station. The waiting time is $5T_{e,1}^{e}{}'$

Thus, the average waiting time is: $T_{e,3}^{e}{}' = 4T_{e,1}^{e}{}'$

These results can be seen in Fig. 5. With all that, total waiting cost for all passengers who travel during the studied period can be computed as follows:

$$C_v^e = \sum_{i=1}^{5} NP_i T_{e,i}^{e}{}' VSE$$

Only types 4 and 5 trips are involved with transferring in intermediate stations. For type 4, it is considered that time required to achieve the transfer is null because there is no need to walk to make a connection, passengers only have to go out from a train and wait on the same platform until the next train arrives. The additional waiting time is not included in this cost since it has been already taken into account in the waiting time part. Therefore transfer cost only should consider the inconvenience of having to change from one train to another. This parameter ($C_{T,4}$) is not well known (studies exist about transfer cost but generally deal with transfer from one transport mode to another, that includes walking time, that is not the case here) and then various values should be tested in application on numerical examples. For type 5, the transfer cost includes inconvenience, as for type 4, but also time required to change platform. In this case inconvenience is greater because passengers have to make a longer trip to reach their destinations, concept that must be included in the transfer cost ($C_{T,5}$).

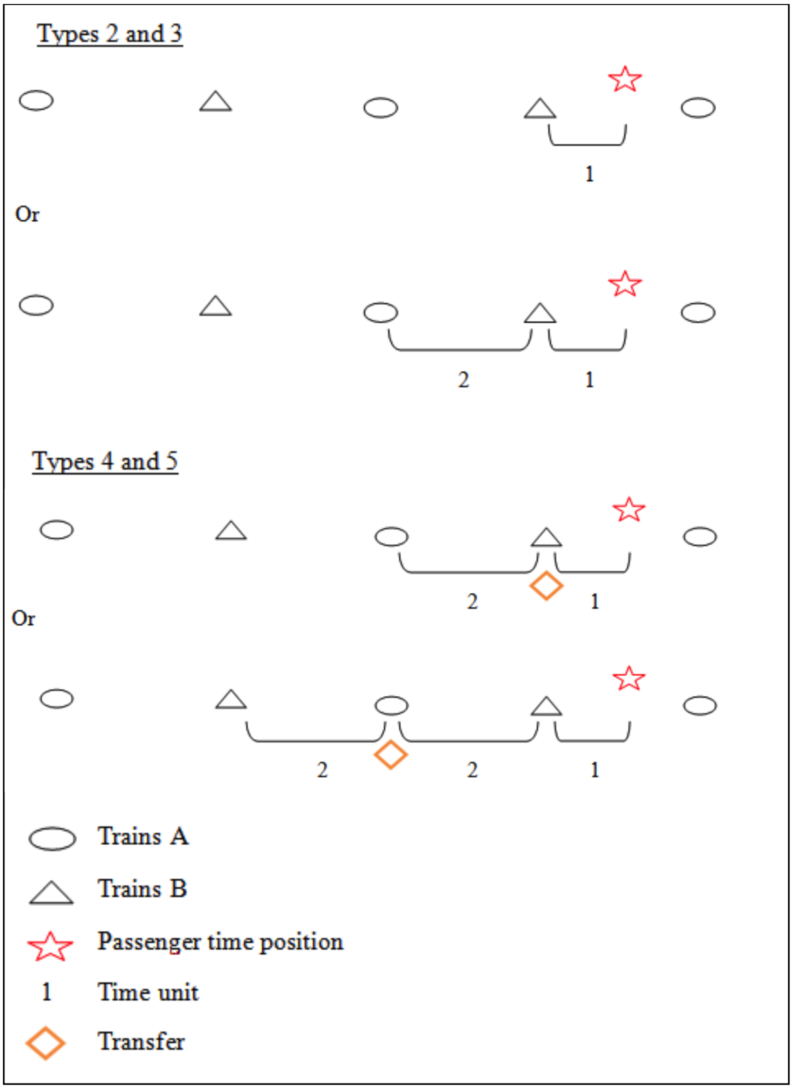

Fig. 5. Waiting time for all types of trip

Total transfer cost (C_{transb}) can be computed as follows:

$$C_{transb} = N(P_4 C_{T,4} + P_5 C_{T,5})$$

Now that numbers of stops are known for all types of trips, travel times for one passenger can be computed as follows for types 1,2 and 3:

$$T_{v,i}^{e}{}' = \frac{L + \frac{1}{\lambda}}{2v_l} + n_{s,i}t_s \qquad \text{for } i = 1,\ldots,3$$

But the in-vehicle travel times are different for trips type 4 and 5, since these types do not allow trips on all the line. For type 4 trips, the trip length can take values between $\frac{1}{2\delta}$ (in order to reach the first AB station) and $L + \frac{1}{\lambda}$, thus:

$$T_{v,4}^{e}{}' = \frac{L + \frac{1}{\lambda} + \frac{1}{2\delta}}{2v_l} + n_{s,4}t_s$$

As for type 5 trips, it was found that the average motion time is $\frac{2}{3\delta}$ †, thus:

$$T_{v,5}^{e}{}' = \frac{\frac{2}{3\delta}}{v_l} + n_{s,5}t_s$$

And then the total time for all passengers is transformed in monetary travel cost:

$$C_v^e = \sum_{i=1}^{5} NP_i T_{v,i}^{e}{}' VSV$$

2.3. Objective Function

In order to assess the benefits of skip-stop operations for a complete line, total costs functions are computed for both types of operation. These functions are built, according to what was presented in the last subsection, studying each station separately. Thus the objective functions (*FO*) for a specific station are:

- Standard operation: $FO^n = C_v^n + C_e^n + C_{op,trac}^n + C_{op,ma}^n$
- Skip-stop operation: $FO^e(\delta) = C_v^e(\delta) + C_e^e(\delta) + C_{transb}(\delta) + C_{op,trac}^e(\delta) + C_{op,ma}^e(\delta)$

In the first case, the value does not depend on the AB stations density δ, hence this is not an optimization problem. In the second case, the objective function, FO^e, that has to be minimized, depends on the variable δ. FO^n is computed and FO^e is minimized for all stations of the line, and then to get a valid result for an entire line, the costs of all stations have to be summed up. If the sum of FO^n for all stations is greater than the sum of FO^e, in terms of total costs it is worth considering the implementation of skip-stop operation.

Since FO^e is a polynomial function of fifth grade in δ, we could not get an analytical expression for the optimum density of AB stations that would minimize FO^e. That is why a numerical analysis was

† See Appendix A.

performed with parameters similar to the Santiago case. These experiments are presented in the next section.

3. Case Study Results and Sensitivity Analysis

A numerical analysis was performed using Maple, and assuming typical Chilean values for the system parameters. There are four key parameters considered: (i) initial stations density λ, which for the Chilean case is approximately 1 station every 500 meters; (ii) number of passengers who start their trips in a specific station during the studied period, N=5,000 pax/hr; (iii) length of the line 2L, L=10Km; and (iv) available fleet, num_{trains} = 25. Notice that λ and N are local parameters, they change for each station, whereas L and num_{trains} are global parameters. The λ value is obviously the maximum value that can take the unknown δ, because if δ is greater than λ, it would mean that new stations have to be built.

There is another parameter but it was absorbed in the hypothesis that has been made: the distribution function of trip length f_D. In this work it was supposed that all stations get the same passengers load.

Other sets of relevant parameters in our model are economic parameters. There are five: VTV, VTE, CE, CT_4 and CT_5. The last two of them refer to transfer cost and are not known precisely, that is why various values will be tried. Costs associated with travel and waiting time are related to value of time, which we assume that depends directly on income. According to the International Monetary Fund [8], Chile has a gross domestic product per capita of about US$ 10,000, whereas India gets about US$ 1,000 and the United States about US$ 45,000. The ratios between these numbers are used to calculate VTV and VTE for India and the United States. As for electricity prices, a review was conducted and there are no real differences for industrial uses in these countries [9]. That is why only one value is used.

Finally, there are other parameters that have minor influence on the results of the model. For all of them average values of existing subway lines in Santiago are used. These are five: a=1 m/s, b=0.1, v_l = 60Km/h, t_p = 20 s, and F = 300,000 N, which is normal value for traction strain at start of an electrical locomotive.

3.1. Effect of Different Parameters on the Objective Function

A preliminary study can be made about the trade-off that is at stake when skip-stop operation is considered. For a given line with λ known (1/500 m^{-1} in our case), Fig. 6 shows how variations of the unknown δ affect each type of cost. As it can be seen, travel cost and waiting cost are widely dominant here. As expected, waiting times and transfer costs decrease when δ increases, but operational costs increase when δ increases.

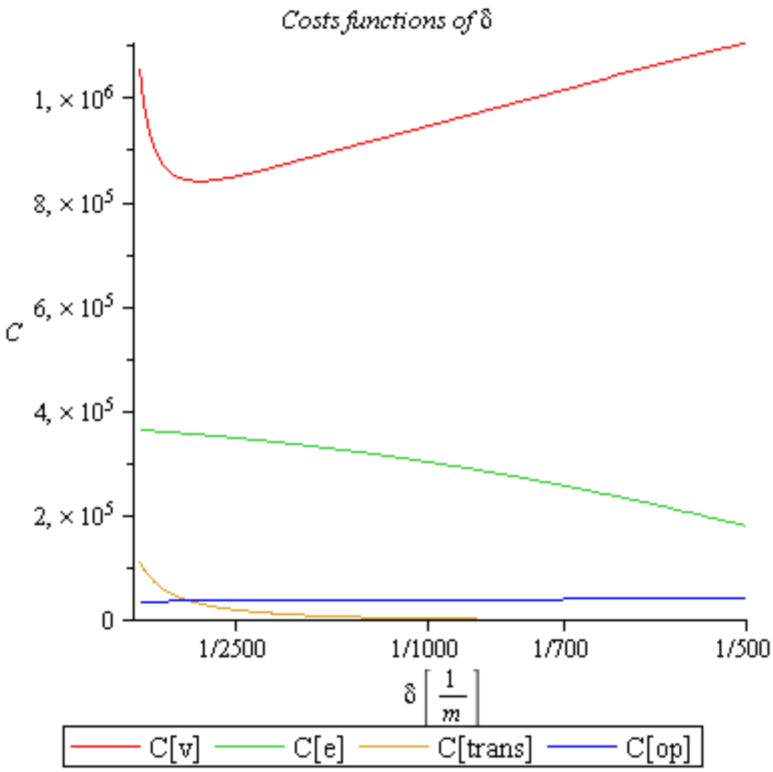

Fig. 6. Different Costs (in CL $/hour-station) as a function of δ

3.2. Sensitivity Analysis on Fleet Size (Frequency)

To study the impact of fleet size (or frequency) for a given line a comparison of the previous case changing the fleet size from 25 to 18 trains is performed. This new fleet size corresponds to headway of 185 seconds in standard operation. Fig.7 presents the result:

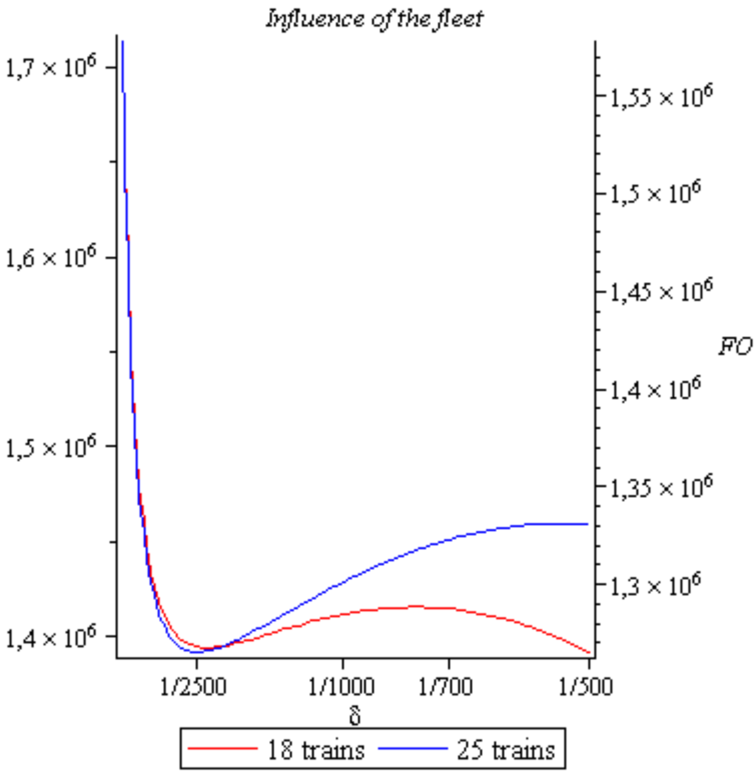

Fig. 7. Objective function function of δ with lower frequency

In the case of lower frequency there is again a local minimum but the objective function is greater in $\delta=\lambda$ than in this minimum. That means that in this case skip-stop operation should not be recommended.

In general, in low demand periods in which frequency is low standard operation should be maintained, because in these periods frequency is too low to make skip-stop operation attractive. For the same reason, rail lines with low frequency such as regional trains or interurban services are not good candidates for skip-stop operation implementation.

Technical minimum headway is a very important element in this problem. Indeed a line whose fleet is sufficient to reach technical maximum frequency will have no benefits implementing skip-stop operation because frequency will not be improved. But in the case technical minimum headway is improved, skip-stop operation should be considered.

3.3. Sensitivity Analysis on Line (Trip) Length

It is also interesting to see how the line length can affect the optimum AB stations density found solving the problem, and that with all other parameters given. Except from the fleet that has to be adjusted to get the same frequency. For example if L=10,000, number of trains will be 25 and if L=20 000, number of trains will be 50.

If the line is long, average trip length will be high. In this context and with the already described hypothesizes, by intuition, two effects are expected: Travel cost starts being much higher than other costs;

and there are less passengers in case 5 (have to make longer trips) because this type mainly concerns short trips.

Fig. 8 shows the results for L=15,000m and L=7,000m. For L=15,000m, the optimum AB stations density is lower than for a shorter line and social gain is greater. As for L=7,000m, skip-stop operation should not be recommended. Therefore, short lines and or lines with shorter trips are not good candidates to implement skip-stop operation.

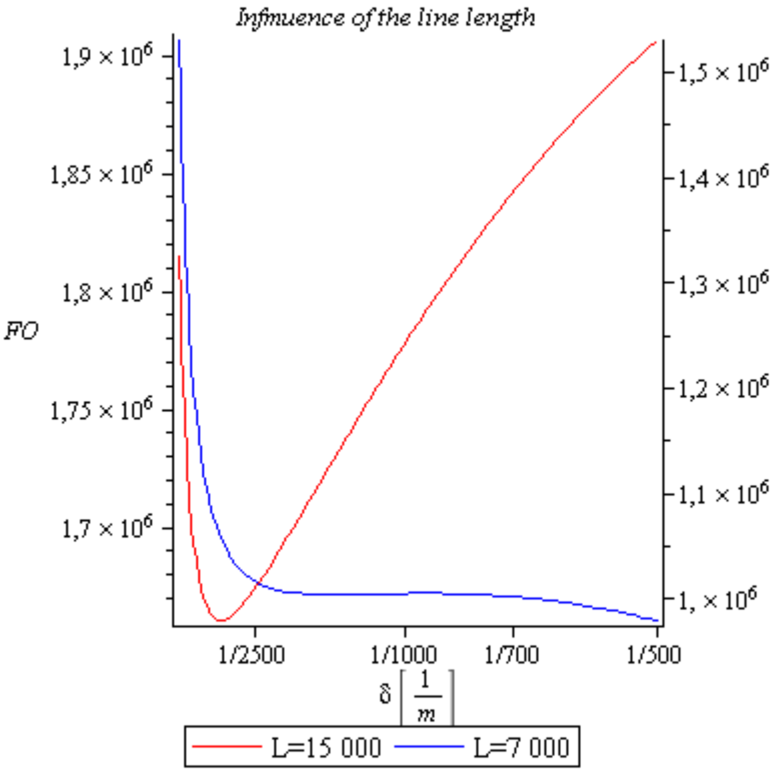

Fig. 8. Influence of the line length

3.4. Sensitivity on Value of Time

Now the influence of value of time is studied, comparing the Chilean case with the Indian one and then with the American one. Here, waiting time value is always twice higher than travel time value.

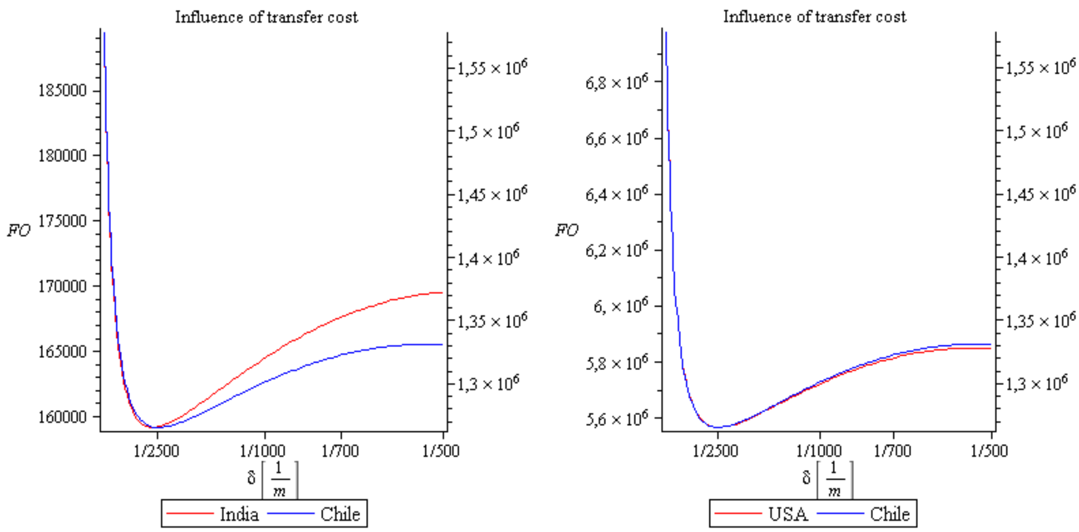

Fig. 9. Comparison between Chile and India; and Chile and USA.

The value of time does not have a great influence if all other parameters stay constant. The curves not only have the same form but also present the same optimum.

3.5. Sensitivity on Initial Station Density

Let see how the optimum AB stations density evolve when initial stations density of the line change. Fig. 10 presents a comparison of total costs for initial station density of one every 300 meters and 1 per kilometre.

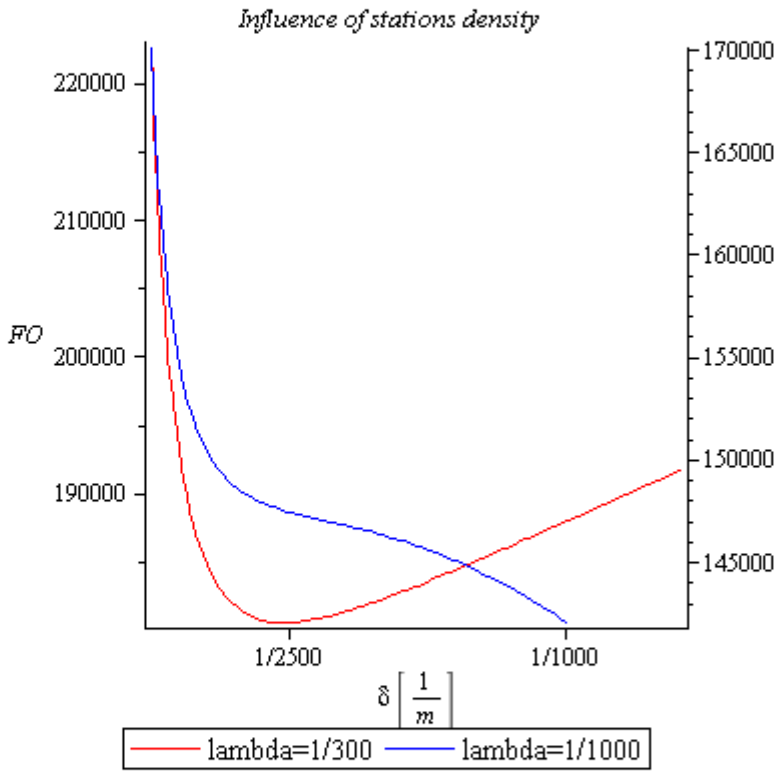

Fig. 10. Influence of stations density

Two conclusions can be drawn from this figure. First, when initial stations density is too low (blue curve), skip-stop operation should not be considered. The whole logic of this mode of operation is to save time by skipping stations, it is then logical that if there are few stations, the operation is pointless. Second, for $\lambda = \frac{1}{500}$ and $\lambda = \frac{1}{300}$ the optimum AB stations density is the same: $\delta = \frac{1}{2500}$. Therefore the optimal station density appears to be robust to changes in station density. Nevertheless, the proportion of AB stations changes from 0.2 for $\lambda = \frac{1}{500}$ to 0.12 for $\lambda = \frac{1}{300}$.

4. Conclusions

It was found that there are characteristics that are less favourable for skip-stop operations: short lines with few stations, since in these cases possible social gains are too low; low initial stations density, for the same reason; and low/medium frequency lines, because in this case skip-stop operation leads to great waiting time that is not compensated by time saved travelling. Indeed, we found that headways higher than three minutes between trains are prohibitive for this operation mode. This last point is interesting because it suggests that not only systems with low frequency such as regional trains should go on with standard operation, but also subway lines during no-peak hours when frequency decreases. Another interesting finding to note is that for systems that are already operating at technical maximum frequency, no benefits can be expectable from skip-stop operation since frequency cannot be increased. But in the case technical minimum headway shall be reduced, those lines could be candidates for the skip-stop operation mode.

If the studied line does not include one or more of the specifications above mentioned, skip-stop operation should be considered. But before being able to compare it with standard (all-stop) operation, skip-stop operation has to be optimized. This optimization consists in finding the AB stations density which minimizes an objective function composed of passengers' costs and operating costs. That is why objective function was drawn function of this variable for different instances of the problem, and a typical curve was found. This curve can be separated in three parts that correspond to the trade-offs made between travel time, waiting time, transfer cost and operating costs, and includes a local minimum that is the point of interest.

It was tested if this local optimum AB stations density evolved drastically when parameters of the problem changed and it was found that very few parameters have a great influence on it. One of them is the line length: it was found that the longer the line and or the trips, the more attractive is skip-stop operation whenever frequency is high. As it was supposed that average trip length was proportional to line length, this lead to the indirect conclusion that longer trips are favourable for the skip-stop operation. At contrary results were that global parameters (that do not change along the line) such as value of time or electricity prices as well as local parameters (that change along the line) such as initial stations density and amount of passengers starting their trips in the station did not really impact the value of the optimal AB station density. This latter fact is important because it means that the optimum solution is the same all along the line and is then a global optimum. An example was studied and it was determined that for a 20 kilometres long line and with a fleet of 50 trains, optimum AB stations density was one station every 2.5 kilometres (with an initial stations density of one station every 500 meters).

The model developed in the present work, based on few and easily accessible information, can serve as a guide for a rail corridor operator considering implementing a skip-stop operation. The model can also be used to simultaneously decide station density under skip-stop operations. In such cases the station density, λ, will become a variable instead of a parameter of the model. We have chosen to model the simplest network that captures the most important elements affecting the suitability of skip-stop operation. Its strongest assumption that demands are evenly distributed across stations tends to underestimate the benefits of skip-stop operation since in practice the demand is usually concentrated in some stations that will be chosen as common stations, thus reducing the number of trips that are forced to transfer (types 4 and 5) by the operational scheme. Additionally, the demand is responsive to the operation. In the case of Santiago the fraction of users observed transferring was smaller than predicted assuming that passengers would maintain their previous O-D stations, since passengers tend to accommodate the origin or destination station of their trip to avoid this inconvenience.

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Appendix A.

A.1. Type 3

We want to obtain an expression for the average number of stops of a trip type 3. There are $\frac{(\lambda-\delta)}{2\delta}2L\delta-1$ possible destinations, all equally likely. We will divide all possible destinations into two groups: i) those located in the same bay as the origin and those located in the exactly opposite bay along the line, and ii) those located in the other $2L\delta-2$ bays in between.

In the first case the sum of the number of stops to all the type 3 destinations located in the opposite bay must visit at least all stops in between both bays, that is:

$$\frac{(\lambda+\delta)}{2\delta}(L\delta-1)+1$$

for each one (there are $\frac{(\lambda-\delta)}{2\delta}$ type 3 destinations in that bay.

We need to add all the local stops within the local and the opposite bay. It can be easily shown that adding for all these destinations the number of such stops equals (recall that the trips take the shortest trip among both directions):

$$2*\left(1+2+3+\ldots+\frac{(\lambda-\delta)}{2\delta}-1\right)+\frac{(\lambda-\delta)}{2\delta}=\frac{2\left\{\frac{(\lambda-\delta)}{2\delta}-1\right\}\frac{(\lambda-\delta)}{2\delta}}{2}+\frac{(\lambda-\delta)}{2\delta}$$

After some basic algebra, the total number of stops for all type 3 destinations in the local and opposite bays can be computed as:

$$\frac{L(\lambda^2-\delta^2)}{4\delta}$$

The second case consists in analyzing the trips to all type 3 destinations in the rest of the bays. Notice that there are:

$$2\frac{(\lambda-\delta)}{2\delta}(L\delta-1)$$

of such destinations

The total number of stops must include those needed to reach the first AB station. The average number of stops to reach the first AB station can be estimated as:

$$\frac{(\lambda+\delta)}{4\delta}$$

The total number of stops visited after the first AB station is reached, can be decomposed in stations of the same type as the origin and AB stations. The sum equals:

$$2\left\{\sum_{k=1}^{\frac{(\lambda-\delta)}{2\delta}(L\delta-1)} k+\sum_{k=0}^{(L\delta-2)}\frac{(\lambda-\delta)}{2\delta}k\right\}$$

By adding the total number of stops for both cases and then divide by the number of destinations (i.e. $\frac{(\lambda-\delta)}{2\delta}2L\delta-1)$ we get the following estimation for the average number of stops per type 3 trip:

$$\frac{L(\lambda-\delta)(\lambda+\delta)}{4\left(\lambda-\delta-\frac{1}{L}\right)}$$

Which can be approximated to $\frac{L(\lambda+\delta)}{4}$ if L is assumed to be sufficiently large.

A.2. Type 4

As before, we want to obtain an expression for the average number of stops of a trip type 4. There are $\frac{(\lambda-\delta)}{2\delta}(2L\delta-1)$ possible destinations, all equally likely.

All trips type 4 have to reach an AB station. We can estimate the number of stops involved as:

$$\frac{(\lambda+\delta)}{4\delta}$$

The total number of stops visited after the first AB station and until the last AB station is reached, can be computed as:

$$\sum_{k=1}^{(L\delta-1)} \frac{(\lambda-\delta)}{2\delta}\frac{(\lambda+\delta)}{2\delta}(2k-1) = \frac{(\lambda-\delta)}{2\delta}\frac{(\lambda+\delta)}{2\delta}[L^2\delta^2 - 2L\delta + 1]$$

Finally, we need to add the stops involved with the destinations located in the opposite bay. The number of stops in this bay for each of these destinations can be estimated as:

$$\frac{(\lambda+\delta)}{4\delta}$$

Thus, by adding the total number of stops and then dividing by the number of destinations we get the following estimation for the average number of stops per type 4 trip:

$$\frac{L^2\delta(\lambda+\delta)}{2(2L\delta-1)}$$

Which can be approximated to $\frac{L(\lambda+\delta)}{4}$ if $L\delta$ is assumed to be sufficiently large.