

# Feature Engineering

Linear Relationship among numerical  
variables/features

Day	ABC Returns (%)	XYZ Returns (%)
1	1.1	3
2	1.7	4.2
3	2.1	4.9
4	1.4	4.1
5	0.2	2.5

Daily returns for two stocks using the closing prices

Does Stocks ABC and XYZ are related or not?

How much strong these two stocks are related?

# Covariance

Covariance measures how two variables move together. It measures whether the two move in the same direction (a positive covariance) or in opposite directions (a negative covariance).

# Covariance

$$\text{Covariance} = \frac{\sum (\text{Return}_{ABC} - \text{Average}_{ABC}) * (\text{Return}_{XYZ} - \text{Average}_{XYZ})}{(\text{Sample Size})}$$

# Interpretation

In the example there is a positive covariance, so the two stocks tend to move together. When one has a high return, the other tends to have a high return as well.

If the result was negative, then the two stocks would tend to have opposite returns; when one had a positive return, the other would have a negative return.

# Correlation

Covariance can tell how the stocks move together, but to determine the strength of the relationship, we need correlation.

## Quantifying the relationship

- *Correlation* describes the strength of the *linear* association between two variables.
- It takes values between -1 (perfect negative) and +1 (perfect positive).
- A value of 0 indicates no linear association.



## Correlation

Since  $Cov(X, Y)$  depends on the magnitude of  $X$  and  $Y$  we would prefer to have a measure of association that is not affected by changes in the scales of the variables.

The most common measure of *linear* association is correlation which is defined as

$$\rho(X, Y) = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$$

$$-1 < \rho(X, Y) < 1$$

Where the magnitude of the correlation measures the strength of the *linear* association and the sign determines if it is a positive or negative relationship.

Linear Relationship among factor  
variables/features

# Correlation Analysis (Categorical Data)

- $\chi^2$  (chi-square) test

$$\chi^2 = \sum \frac{(\textit{Observed} - \textit{Expected})^2}{\textit{Expected}}$$

- The larger the  $\chi^2$  value, the more likely the variables are related
- The cells that contribute the most to the  $\chi^2$  value are those whose actual count is very different from the expected count

# Chi-Square Calculation: An Example

	Play chess	Not play chess	Sum (row)
Like science fiction	250(90)	200(360)	450
Not like science fiction	50(210)	1000(840)	1050
Sum(col.)	300	1200	1500

- $\chi^2$  (chi-square) calculation (numbers in parenthesis are expected counts calculated based on the data distribution in the two categories)

$$\chi^2 = \frac{(250 - 90)^2}{90} + \frac{(50 - 210)^2}{210} + \frac{(200 - 360)^2}{360} + \frac{(1000 - 840)^2}{840} = 507.93$$

- Compute p-value from  $\chi^2$  value.

If  $p\text{-value} < 0.05$  then the variables are not independent otherwise we conclude that the variables are dependent.

# Common Tasks we do in Feature Engineering

1. Remove the variables that contains NA's more than some threshold
2. Remove the near-zero variance features
3. Handle the feature which are collinear i.e., whose correlation is higher
  - a) Remove the features such that all pair-wise correlations are higher than some given threshold.
  - b) Build the new features with linear combination of existing features so that they are independent.