Hello Everyone !

Welcome to the Intermediate classes !

FAQS

→ Notes will be uploaded after the class
→ Assignments are unlocked after the class ends
→ Consistency

If you want to go fast, go alone.
If you want to go far, go together.

Q1) Count of factors

factor of n ⇒ any number which divides n completely.

$n \% i == 0$

$n = 24 \Rightarrow \{1, 2, 3, 4, 6, 8, 12, 24\}$

$n = 10 \Rightarrow \{1, 2, 5, 10\}$

```
int countOfFactors(int n){
    int count = 0;
    for(int i=1; i<=n; i++){
        if(n % i == 0){
            count++;
        }
    }
    return count;
}
```

n=10

count = 0 1 2 3 4

i = 1 2 3 4 5 6 7 8 9 10

Execution time depends on the value of n.

[Assumption :- 10^8 iterations in 1 sec]

$[1, 10^8] \rightarrow 10^8$ nos. $[1, n] \rightarrow n$ nos.

n	no. of iterations	Execution Time
10^8	10^8	1 sec
10^9	10^9	$10^9/10^8$ sec = 10 sec.
9	9	$9 * 10^{-8}$ sec.
n_0	n_0	$(n_0/10^8)$ seconds
10^{18}	10^{18}	$(10^{18}/10^8)$ sec = 10^{10} secs.

you → child → grandchild → 4th → 5th ≈ 317 years

$10^8 \longrightarrow 1$ sec

$1 \longrightarrow \frac{1}{10^8}$ sec.

$n_0 \longrightarrow \left(\frac{1}{10^8} * n_0\right)$ seconds.

Optimise

$2 * 5 = 10 \Rightarrow 2, 5$.

$i * j = n$

$j = n/i \Rightarrow$ {i and n/i both are factors of n}

n = 24

i		n/i
1	≤	24
2	≤	12
3	≤	8
4	≤	6
6		4
8		3
12		2
24		1

n = 100

i		n/i
1	≤	100
2	≤	50
4	≤	25
5	≤	20
10	≤	10
20		5
25		4
50		2
100		1

$i \leq n/i$

$\Rightarrow i * i \leq n$

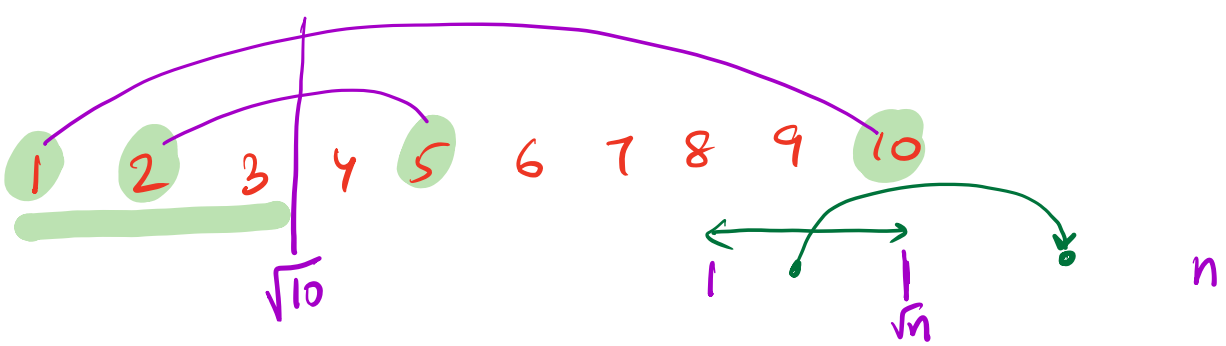
$\Rightarrow i^2 \leq n$

$\Rightarrow i \leq \sqrt{n}$

1 → 25

5 → 5

25 → 1



n = 60

i	→	n/i
1		60
2		30
3		20
4		15
5		12
6		10

7 → ✗

---

8 > 60/8

$i \leq \sqrt{n} = \sqrt{60}$

$= 7.74$

```
int countFactors(int n){
    int count=0;
    for(i=1; i*i<=n; i++){
        if(n%i==0){
            if(i==n/i)
                count++;
            else{
                // count both i and n/i
                count+=2;
            }
        }
    }
    return count;
}
```

n = 20

i = 1
i = 2
i = 3
i = 4 ~~✗~~

$n = 10^{18} \Rightarrow$ no. of iterations $= \sqrt{n} = 10^9$

$\Rightarrow$ execution time $= \frac{10^9}{10^8}$ secs $= 10$ secs.

[Break till 10:41 PM]

Prime Number ⟶ 2 factors

n = 11 {1, 11} ✓

n = 10 {1, 2, 5, 10} ×

n = 23 {1, 23} ✓

n = 1 No
{1}

[1 is neither prime nor composite]

{10, 11, 23, 2, 25, 27, 31}

4 primes.

```
boolean isPrime(int n){
    if(countFactors(n)==2){
        return true;
    else
        return false;
}
```

return (countFactors(n) == 2);

Game [Gauss]

4th class

$$S = 1 + 2 + 3 + 4 + \cdots + 98 + 99 + 100$$
$$S = 100 + 99 + 98 + 97 + \cdots + 3 + 2 + 1$$

$$2S = 101 + 101 + 101 + 101 + \cdots + 101 + 101 + 101$$

$$= 101 * 100$$

$$\Rightarrow \boxed{S = \frac{100 * 101}{2}}$$

Sum of first n natural nos.

$$S = 1 + 2 + 3 + 4 + \cdots + n-1 + n$$
$$S = n + (n-1) + (n-2) + (n-3) + \cdots + 2 + 1$$

$$2S = (n+1) + (n+1) + \cdots + (n+1) + (n+1)$$

$$\Rightarrow 2S = n(n+1)$$

$$\Rightarrow S = \frac{n(n+1)}{2}.$$

Q) given n (a perfect square). Find sqrt(n).

$n = 25 \longrightarrow 5$

$n = 36 \longrightarrow 6$

$n = 1024 \longrightarrow 32$

$n = 30 \longrightarrow$ We will never get invalid input.

we need to find the i

s.t. $[i * i == n]$

$n = 16$

1 → ✗
2 → ✗
3 → ✗
4 → ✓
5
6
7
8
⋮
16

$i = \sqrt{n}$

```
int sqrt(int n){
    for(i=1; i<=n; i++){
        if(i*i==n)
            return i;
    }
    return -1; // Won't be reached
}
```

No. of iterations = $\sqrt{n}$.

Amazon MCQ

a) $\log_2 n$
b) n
c) $\sqrt{n}$ ✓
d) None of these.

8) Find $\lfloor sqrt(n) \rfloor$

$n = 40 \Rightarrow 6$

$n = 30 \Rightarrow 5$

$n = 200 \Rightarrow 14$

$n = 50$

i	i*i<=n
1	ans = 1
2	ans = 2
3	ans = 3
4	ans = 4
5	ans = 5
6	ans = 6
7	ans = 7
8	8*8 ≤ 50 ✗

```
int sqrt(int n){
    int i=1, ans=0;
    while(i*i<=n){
        ans=i;
        i++;
    }
    return ans;
}
```

no. of iterations = $\sqrt{n}$.

sqrt(n) —idea 1→ $\sqrt{n}$ iterations

sqrt(n) —idea 2→ $\log_2 n$ iterations [Advanced Module]

log Basics

$\log_b a$ ⇒ log of a to the base b.

[to what value we need to raise b to get the value a]

$\log_b a = c$ $\quad [b \in (0, \infty), b \neq 1]$

$\Rightarrow b^c = a$

$\log_2 64 = 6$ $\quad$ $\log_3 27 = 3$ $\quad$ $\log_4 1 = 0$

$2^{\boxed{6}} = 64$ $\quad$ $4^{\boxed{0}} = 1$

$\underbrace{2*2*2*2*2*2}_{6 \text{ times}} = 64$

$\lfloor \log_2 40 \rfloor = 5$

$2^5 \leq 40 < 2^6$

$n = 2^k \Rightarrow \log_2 n = k$.

$\log_a (a^n) = n$.

$a^{\boxed{n}} = a^n$

$\log_b a$ = How many times you need to multiply b to get a.

= How many times you need to divide a by b till it reaches 1.

$\log_2 32 = 5$

$32 \xrightarrow{/2} 16 \xrightarrow{/2} 8 \xrightarrow{/2} 4 \xrightarrow{/2} 2 \xrightarrow{/2} 1$

Q) Given +ve integer n, find how many times we need to divide n by 2 till it reaches 1.

[Homework]

n = 100

↓ /2

50

↓ /2

25

↓ /2

12

↓ /2

6

↓ /2

3

↓ /2

1

ans = 6

n = 324

ans = 8