Joint distribution $p(x,y) = p(x)\, p(y/x)$ | $P_X(x) = \sum_y p(x,y)$

$p(y,x) = p(y)\, p(x/y)$ | $P_Y(y) = \sum_x p(x,y)$

Entropy $\Rightarrow$ $\boxed{H(X) = -\sum_{x\in X} p(x) \log p(x)}$

Joint Entropy $\Rightarrow$

$$\boxed{H(X,Y) = -\sum_{x\in X}\sum_{y\in Y} p(x,y) \log p(x,y)}$$

x_i

$$\boxed{H(X,Y) = -E \log p(X,Y)} \quad \because \boxed{E(X) = \sum_i x_i\, p(x)}$$

Conditional Entropy $\Rightarrow$

$$\boxed{H(Y|X) = \sum_{x\in X} p(x)\, H(Y/x)}$$

$$\boxed{H(Y|X) = -\sum_{x\in X}\sum_{y\in Y} p(x,y) \log p(y/x)}$$

OR, $H(X|Y)$ = বিপরীত

$$H(Y|X) = -E \log p(Y/X)$$

Chain Rule (Entropy) $\Rightarrow$

$$H(X,Y) = H(X) + H(Y|X)$$

$$H(X,Y) = H(Y) + H(X|Y)$$

Relative Entropy $\Rightarrow$ $D(p\|q) = \sum_{x\in X} p(x) \log \frac{p(x)}{q(x)}$

$(-)$ নাই

$$= E \log \frac{p(x)}{q(x)}$$

Mutual Information $\Rightarrow$ $I(X;Y) = \sum_{x\in X}\sum_{y\in Y} p(x,y) \log \frac{p(x,y)}{p(x)\,p(y)}$

$I(X;Y) = H(X) - H(X|Y)$

$I(X;Y) = H(Y) - H(Y|X)$ $\quad = D(p(x,y)\,\|\, p(x)\,p(y))$

$$H(X,Y|Z) = H(X|Z) + H(Y|X,Z)$$

$$I(X;Y|Z) = H(X|Z) - H(X|Y,Z)$$

Conditional Relative Entropy $\Rightarrow$ $D(p(y|x)\,\|\,q(y|x)) = \sum_x \sum_y p(x,y) \log \frac{p(y|x)}{q(y|x)}$

$$= E_{p(x,y)}\left[\log \frac{p(Y|X)}{q(Y|X)}\right]$$

$H(1/2, 1/4, 1/8, 1/8) = 7/4$
$H(1/4, 1/4, 1/4, 1/4) = 2$
$H(0, 1/2, 1/4, 1/4) = 3/2$

M	E	E	T	M	E	A	F	T	E	R	T	H	E	T	O	G	A	P	A	R	T	Y
f																						

Lab 2

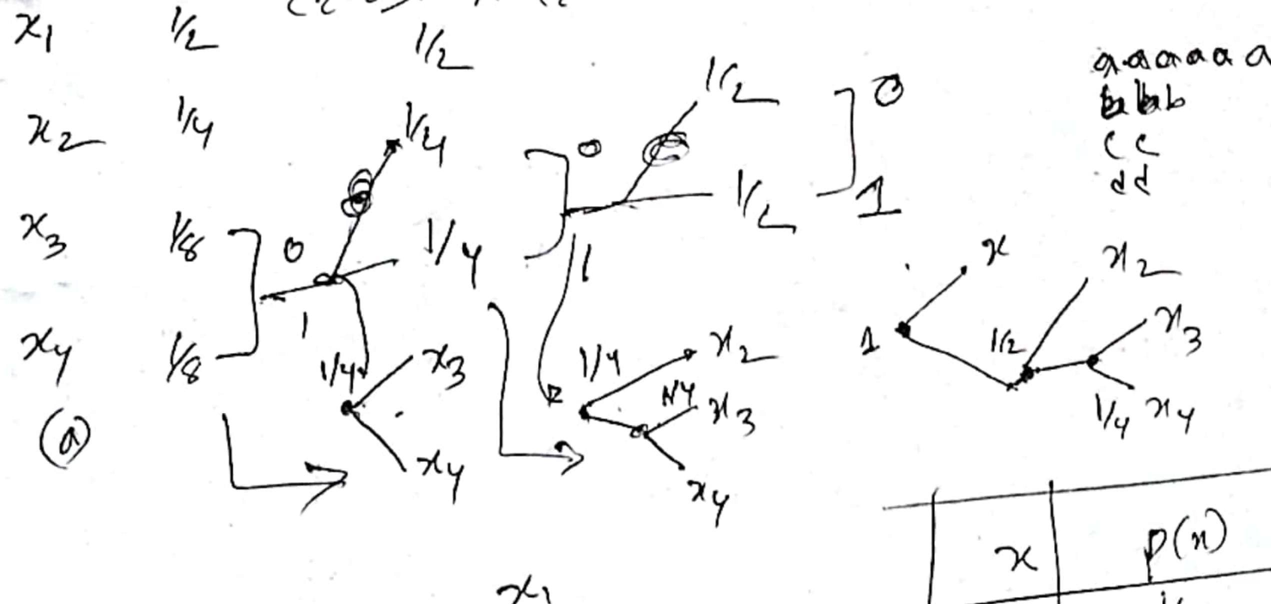
Haffman Coding

- ↳ important class of prefix code. → Haffman code
- ↳ Compact Code
- ↳ minimize excepted code word length.
- ↳ use for lossless data compression

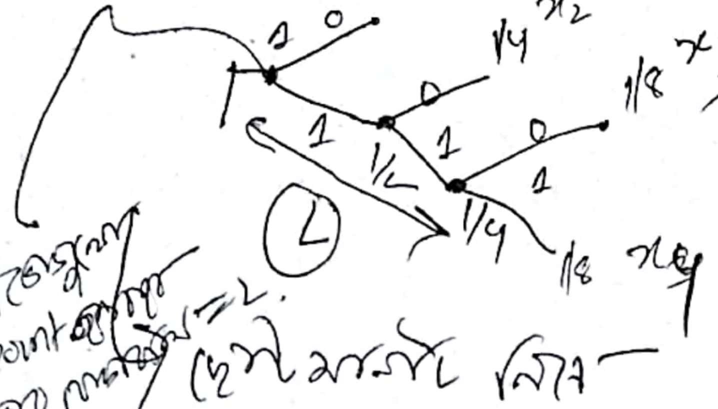
For example,

x	x_1	x_2	x_3	x_4
$p(x)$	$1/2$	$1/4$	$1/8$	$1/8$

→ বড় থেকে ছোট সাজাতে হবে



aaaaaa
bbb
cc
dd



x	$p(x)$	Haffman code
x_1	$1/2$	0
x_2	$1/4$	10
x_3	$1/8$	110
x_4	$1/8$	111

যেখান থেকে point থেকে ... = 2. যেখানে আসবে নিতে —

উপর থেকে নিচে যেতে হবে

heap = [[2, ['E', '']], [3, ['A', '']], [6, ['C', '']]

lo = heapq.heappop(heap)

lo = [2, ['E', '']]

hi = [3, ['A', '']]

pair[1] = '0' + pair[1] ⇒ প্রথমটিতে 0 বসানো হল

↪ [['E', '0']

২য় হল E ['A', '1'] ⇒ 1 বসানো হল

Lab-8 joint entropy

Conditional

mutual information

matrix = [

[1/8, 1/16, 1/32, 1/32],

[1/16, 1/8, 1/32, 1/32],

[1/16, 1/16, 1/16, 1/16]

[1/4, 0, 0, 0]

]

The marginal distribution of X is (1/2, 1/4, 1/8, 1/8)

of Y is (1/4, 1/4, 1/4, 1/4)

Entropy of X,

$$H(X) = -\sum_{x \in X} p(x) \log_2 p(x)$$

$$= (-\tfrac{1}{2}\log_2 \tfrac{1}{2} - \tfrac{1}{4}\log_2 \tfrac{1}{4} - \tfrac{1}{8}\log_2 \tfrac{1}{8} - \tfrac{1}{8}\log_2 \tfrac{1}{8})$$

$$= 7/4$$

$$H(Y) = -\sum_{y \in Y} p(y) \log_2 p(y)$$

Conditional entropy, $H(X|Y)$

$$H(X|Y) = \sum_{i=1}^{4} p(Y=i)\, H(X|Y=i)$$

$$= \tfrac{1}{4} H(\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{8}) + \tfrac{1}{4} H(\tfrac{1}{4}, \tfrac{1}{2}, \tfrac{1}{8}, \tfrac{1}{8}) + \tfrac{1}{4} H(\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}) + \tfrac{1}{4} H(1, 0, \ldots$$

all row multiplied by 4

$$H(Y|X) = \sum_{i=1}^{4} p(x=i)\, H(Y|X=i)$$ 2 [illegible]

4 [illegible]

$$= \tfrac{1}{2} H(\tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{8}, \tfrac{1}{2}) + \tfrac{1}{4} H(\tfrac{1}{4}, \tfrac{1}{2}, \tfrac{1}{4}, \ldots) + \tfrac{1}{8} H(\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}, \ldots)$$ 8 [illegible]

$$+ \tfrac{1}{8} H(\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}, 0)$$

Joint entropy, $H(X,Y) = H(X) + H(Y|X)$

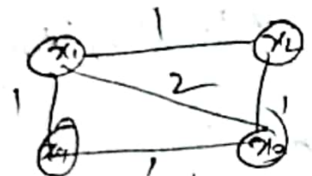
or $= H(Y) + H(X|Y)$

mutual information, $I(X;Y) = H(X) - H(X|Y)$

OR $= H(Y) - H(Y|X)$

Lab 7

chapter-3+4+5 ⇒ slide 47



The conditional probability are $P_{ij} = \dfrac{w_{ij}}{\sum_j w_{ij}}$

The state transition matrix.

$$P = \begin{pmatrix} 0 & 1/4 & 2/4 & 1/4 \\ 1/2 & 0 & 1/2 & 0 \\ 2/4 & 1/4 & 0 & 1/4 \\ 1/2 & 0 & 1/2 & 0 \end{pmatrix}$$

with $W = 6$

$w_1 = 4,\ w_2 = 2,\ w_3 = 4,\ w_4 = 2$

We get stationary distribution, $\mu_i = \left(\dfrac{w_i}{2W}\right)$

$= \left(\dfrac{4}{12}, \dfrac{2}{12}, \dfrac{4}{12}, \dfrac{2}{12}\right)$

And the entropy rate

$$H_\Delta(X) = H\Big(\underbrace{\tfrac{1}{12}, \tfrac{2}{12}, \tfrac{1}{12}}_{x_1}, \underbrace{\tfrac{1}{12}, \tfrac{1}{12}}_{x_2}, \underbrace{\tfrac{1}{12}, \tfrac{2}{12}, \tfrac{1}{12}}_{x_3}, \underbrace{\tfrac{1}{12}, \tfrac{1}{12}}_{x_4}\Big) - \mathcal{M}$$

$$= 1.33$$

$$\to H(X) = H\left(\frac{W_{ij}}{2W}\right) - H\left(\frac{W_i}{2W}\right)$$

Lab-6 optimality

$$L = \sum_i p(x_i)\, l_i$$

l_i → Binary codeword length

$$H\left(\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{8}\right)$$

$$= 1 + \tfrac{1}{2} + \tfrac{1}{4} \qquad = -\tfrac{1}{2}\log$$

$$= 1.75 \qquad = 1.75$$

$$L(P) \geq H(P) \rightarrow \text{optimal}$$

Avg-length ⇒

len(pair[1]) * 6/13

→ Codeword length

A = 6

Total = 13

freq[pair[0]] ⇒ 6

pair[0] = A

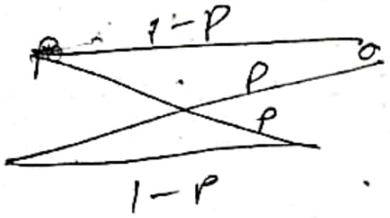
Unit 3 BSC

↳ Communication Channel model — When an error occurs, a 0 is received as a 1 and vice versa

↳ Transmitter send a bit (0/or 1) and receiver will receive a bit.

↳ Introduce bit flip errors

↳



$Pr[Y=0 \mid X=0] = 1-P$

$Pr[Y=1 \mid X=1] = 1-P$

$Pr[Y=0 \mid X=1] = P$

$Pr[Y=1 \mid X=0] = P$

↳ used in information theory (frequently)

Information capacity of BSC, $C = 1 - H(P)$ bits

Conditional probability $H(P)$ or $H(Y/X)$, as

$$H(P) = (1-P)\log\frac{1}{1-P} + P\log\frac{1}{P}$$

↳ Discrete memoryless channel

DMC ⇒ $P(Y/X)$

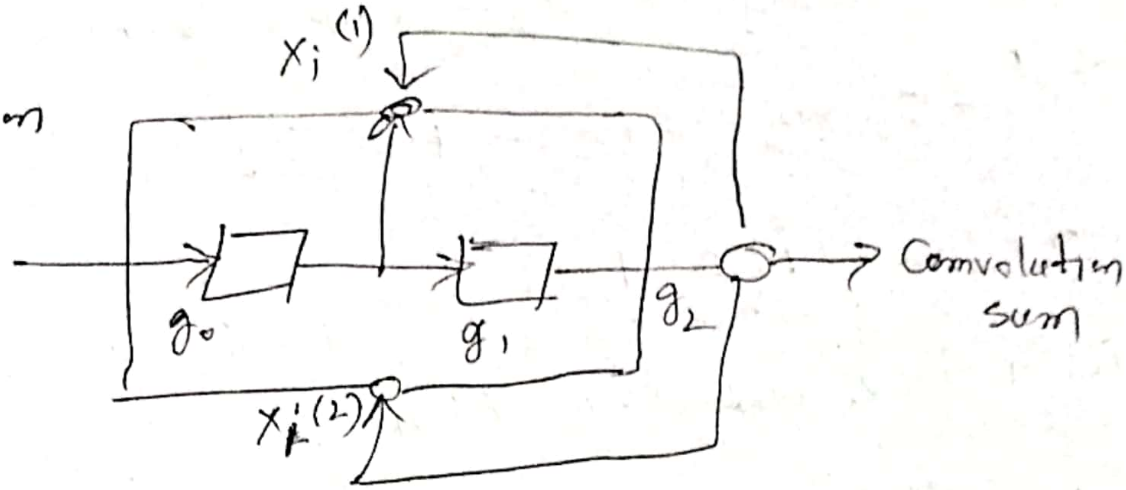
$P(Y/X)$

X \ Y	0	1
0	1-P	P
1	P	1-P

↳ $C = 1$ there are no error

$C = 0$ Completely unreliable

Lab2 Convolution



Input

Top output path : $(g_0^1, g_1^1, g_2^1) = (1,1,1)$

Bottom " " : $(g_0^2, g_1^2, g_2^2) = (1,0,1)$

message bit sequence $= (m_0, m_1, m_2, m_3, m_4) = (1,0,0,1,1)$

Sol^n We know that,

$$\cancel{x_i^j = \sum_{i=0}^{m} g_l^j m_{i-l}}$$

When $j=1$ and $i=0$ then

$$x_0' = g_0' m_0$$
$$= 1 \times 1$$
$$= 1 \% 2 = 1$$

$x_1' = g_0' m_1 + g_1' m_0 = 1 \times 0 + 1 \times 1 = 0+1 = 1 \% 2 = 1$

$x_2' = g_0' m_2 + g_1' m_1 + g_2' m_0 = 1 \times 0 + 1 \times 0 + 1 \times 1 = 1 = 1 \% 2$

$x_3' = g_0' m_3 + g_1' m_2 + g_2' m_1 = 1 \times 1 + 1 \times 0 + 1 \times 0 = 1 = 1 \% 2 =$

$x_4{}' = g_0{}' m_4 + g_1{}' m_3 + g_2{}' m_2 = 1\times1 + 1\times1 + 1\times0 = 2 \% 2 = 0$

$x_5{}' = g_1{}' m_4 + g_2{}' m_3 = 1\times1 + 1\times1 = 1+1 = 2 \% 2$

$x_6{}' = g_2{}' m_3 = 1\times1 = 1 = 1 \% 2 = 1$

$x_i{}' = 1111.001$

When $j = 2$ and $i = 0$ then

$x_0{}^2 = g_0{}^2 m_0 = 1\times1 = 1 \% 2 = 1$

Then for successive i, we

$x_1{}^2 = g_0{}^2 m_1 + g_1{}^2 m_0 = 1\times0 + 0\times1 = 0+0 = 0$

$x_1{}^2 = g_0{}^2 m_2 + g_1{}^2 m_1 + g_2{}^2 m_0 = 1\times0 + 0\times0 + 1\times1 = 1 = 1 \% 2 =$

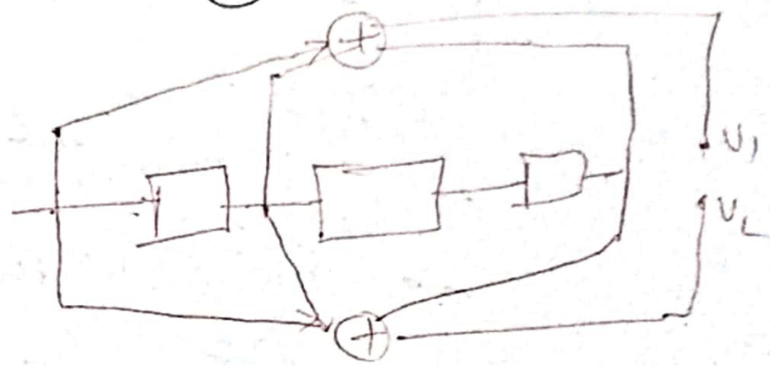
$x_i{}' = 1111001$

$x_i{}^2 = 1011111$

So. $x_i = 1110\,1111010111$

$\cancel{V_1}(x) = g_1(x)\, m(x)$ —— ①

$V_2(x) = g_2(x)\, m(x)$ —— ⑪



$m = \overset{4}{1}\overset{3}{0}\overset{2}{1}\overset{1}{0}\overset{0}{1} = ⑤$

$m(x) = 1\times x^4 + 1\times x^2 + 1\times x^0$

$= x^4 + x^2 + 1$

$\cancel{g_1(x)} = g_1 = (\overset{3}{1}\overset{2}{1}\overset{1}{1}\overset{0}{1})$

$g_1(x) = 1\times x^3 + 1\times x^2 + 1\times x^1 + 1\times x^0$

$= x^3 + x^2 + x + 1$

$g_2 = (\overset{3}{1}\ \overset{2}{1}\ \overset{1}{0}\overset{0}{1})$

$K = 4$

$n = 2$

$g_2(x) = 1\times x^3 + x^2 + x^0$

$= x^3 + x^2 + 1$

$\oplus = +$

$\therefore V_1(x) = (x^3 + x^2 + x + 1)(x^4 + x^2 + 1)$

$= x^7 + \cancel{x^5} + \cancel{x^3} + x^6 + \cancel{x^4} + \cancel{x^2} + \cancel{x^5} + \cancel{x^3} + x + \cancel{x^4} + \cancel{x^2} + 1$

$= x^7 + x^6 + x + 1 \Rightarrow \boxed{11000011}$

$V_2(x) = (x^3 + x^2 + 1)(x^4 + x^2 + 1)$

$= x^7 + x^5 + x^3 + x^6 + \cancel{x^4} + \cancel{x^2} + \cancel{x^4} + \cancel{x^2} + 1$

$= x^7 + x^6 + x^5 + x^3 + 1 \Rightarrow \boxed{11101001}$

$\Rightarrow$ 11 11 01 00 00 01 00 10 11

λ max length

$= n(l + k - 1)$

$= 2(5 + 4 - 1) = 16$ bits

length = L

output bits $= n(m+L)$ or $= n(L+K-1)$

Constraints length $K = (m+1)$ and code rate, $r = \frac{L}{n(m+L)}$

$= \frac{1}{n}$

L = message length

m = No of shift register

n = no of modulo-2 adder

↳ Also known Trellis Code

↳ Improve error-Correcting capabilities.

↳ Encoder Rate is $R_t = \frac{1}{2}$ bits

$R_t = \frac{n_i}{n_c}$ bits → n_i = information bits

n_c = Codeword bits

lab 3 Lempel-Ziv

A → 0
B → 1

↳ Lossless data compression algorithm

↳ Graphics Interchange Format used in → (GIF)

↳ dictionary based encoding

A|A B|A B B|B|A B A|A B A B|B B|A B B A|B B

1 2 3 4 5 6 7 8 9

Position	1	2	3	4	5	6	7	8	9
Sequence	A	AB	ABB	B	ABA	ABAB	BB	ABBA	BB
Numerical Representation	ϕA	1B	2B	ϕB	2A	5B	4B	3A	~~4B~~ 7
Binary encoded Block	0000	0011	0101	0001	0101	1011	1001	0110	0111

↳ It is accomplised by dividing data into segments.

Lab-4 Hamming Code

message = 1010

1 — 001	2 — 010	3 — 011	4 — 100	5 — 101	6 — 110	7 — 111
P_1	P_2	1	P_3	0	1	0

1 0 1 1 0 1 0

0

$P_1 = 3\ 5\ 7$

$1 \oplus 0 \oplus 0 \rightarrow 1$

$P_2 = 3 \oplus 6 \oplus 7$

$1\ 1\ 0 \rightarrow 0$

$P_3 = 5\ \cancel{6}\ 7$

$0\ 1\ 0 \rightarrow 1$

$D_0 = 1\ 3\ 5\ 7$

$1\ 0\ 0\ 0 \rightarrow 1$

$D_1 = 2\ 3\ 6\ 7$

$0\ 0\ 1\ 0 \rightarrow 1$

$D_2 = 4\ 5\ 6\ 7$

$1\ 0\ 1\ 0 \rightarrow 0$

11

P = Parity

Even parity check

No of 1's even = 0

$\boxed{2^n = P}$

- Error Correction Code
- Can detect up to two bits errors
- Correct one bit errors
- minimum distance 3.
- Class of binary linear Code
- Rate of Hamming Code, $R = k/n = (1-r)/2^r-1$
- block length, $n = 2^r - 1$
- parity value 1 idicates there is an odd number
- " " 0 " " " even "