Artificial Neural Networks - #1

Classification using Single Layer Perceptron Model

- The following table shows sample data obtained from two different fruits.

	Weight (grams)	Length (cm)
Fruit 1 (Class C1)	121	16.8
	114	15.2
Fruit 2 (Class C2)	210	9.4
	195	8.1

- Train a single layer perceptron model using the above parameters to classify the two fruits.
- Using the model parameters you have obtained classify the fruit with weight 140gm and length 17.9cm.

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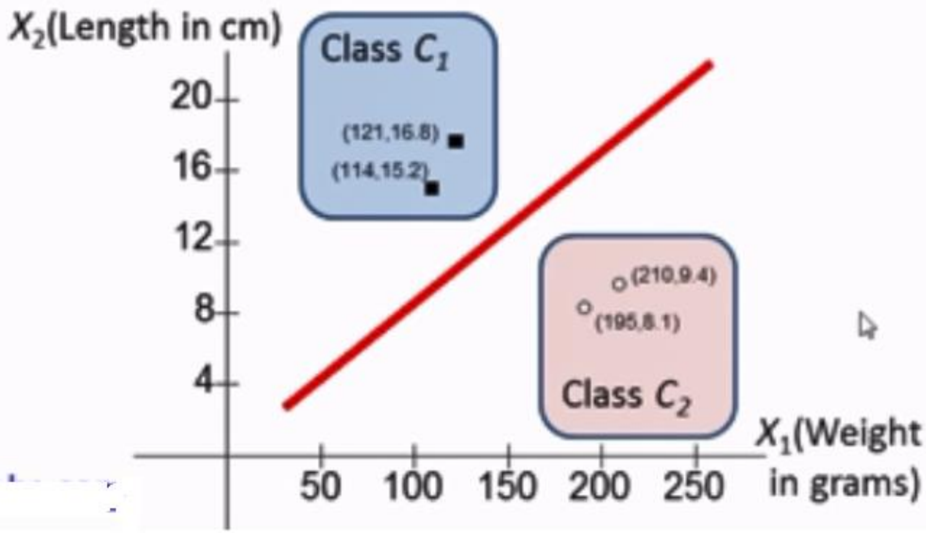
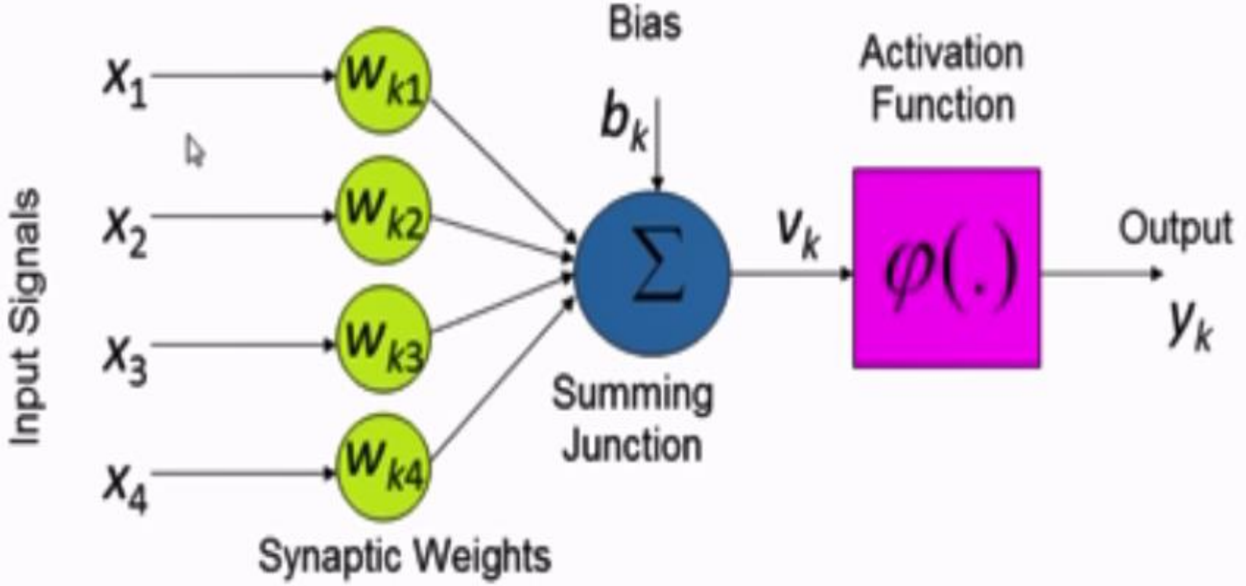
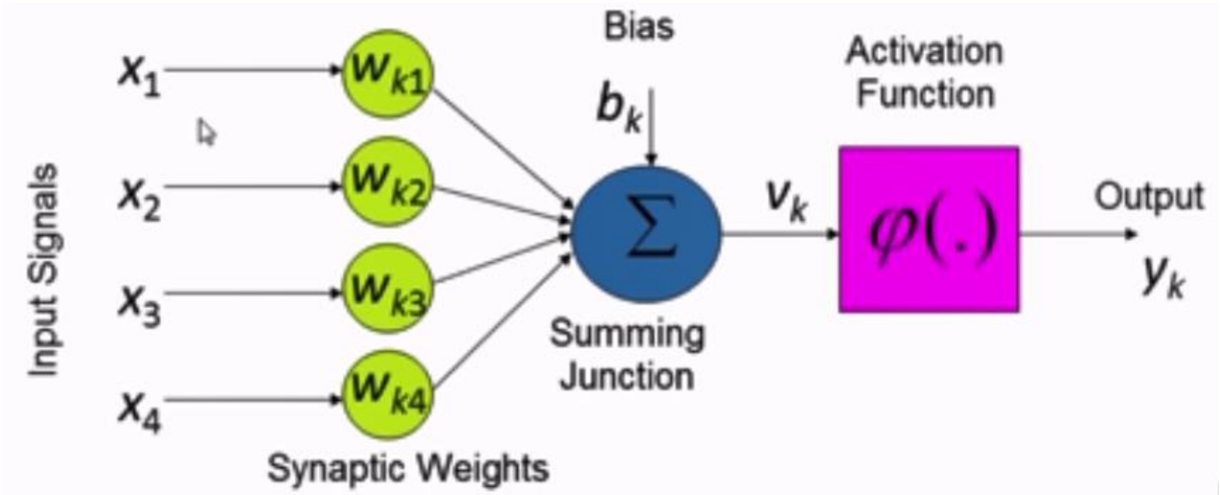


Model of Neuron

- The model consists of a set of synapses each of which is characterized by a weight or strength of its own.
- An adder, an activation function and a bias.





- In mathematical terms, a neuron k can be described by:

$$u_k = \sum_{j=1}^{m} w_{kj} x_j$$

and

$$y_k = \varphi(u_k + b_k)$$

where u_k is the linear combiner output due to input signals.

- Also $$v_k = u_k + b_k$$

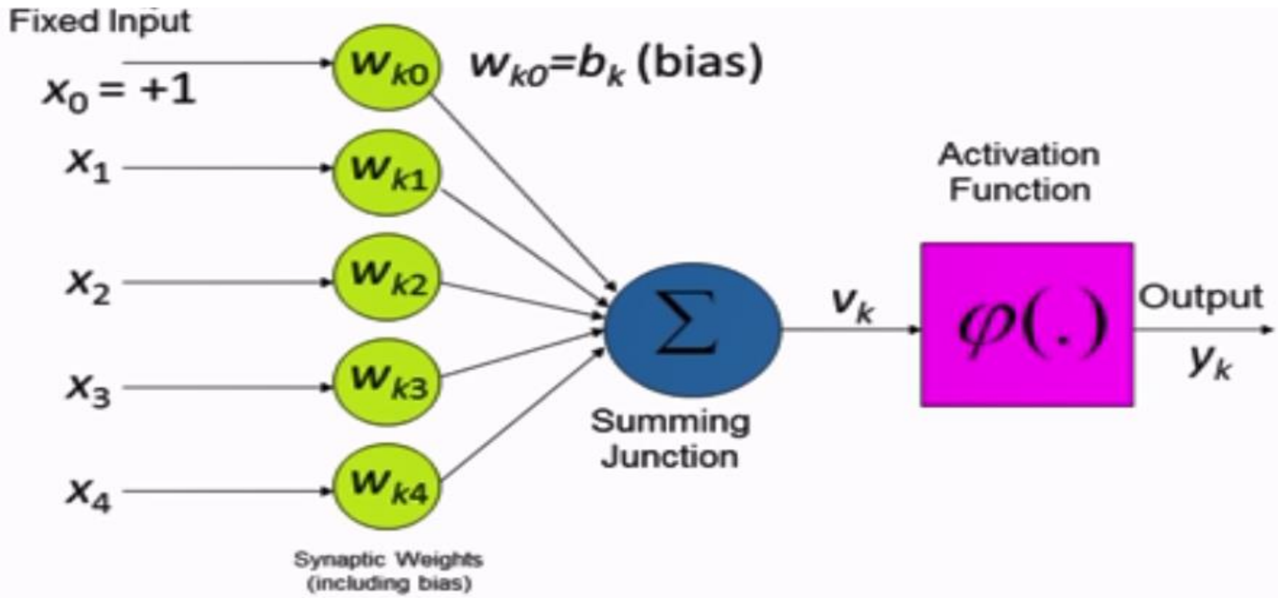
- The bias is an external parameter of artificial neuron and can be included into the equations as follows:

$$v_k = \sum_{j=0}^{m} w_{kj} x_j$$

- and

$$y_k = \varphi(v_k)$$

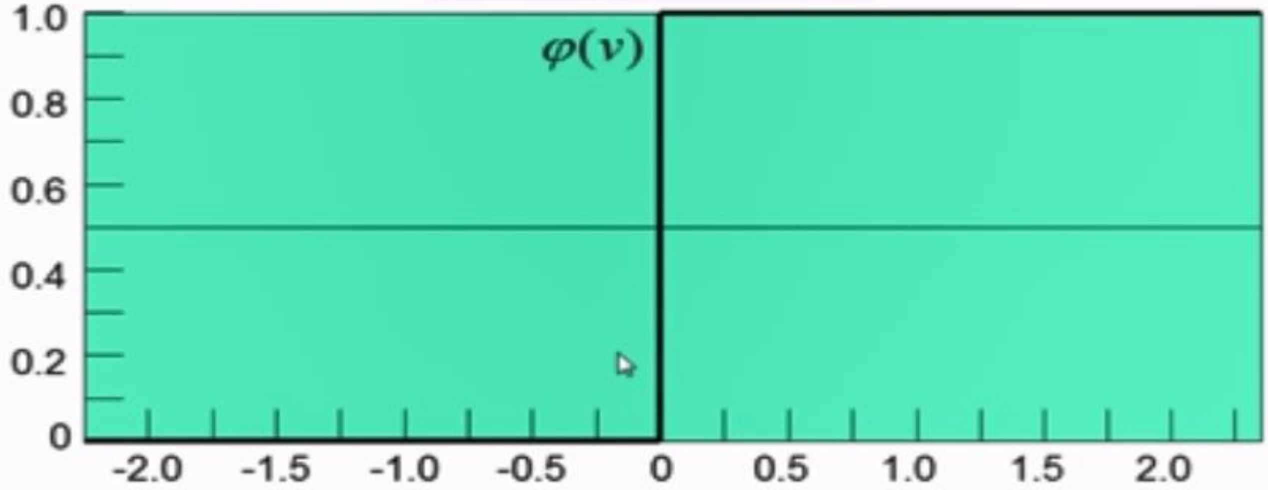
- Note the change of limits of j from 1 to 0.



Types of Activation Functions

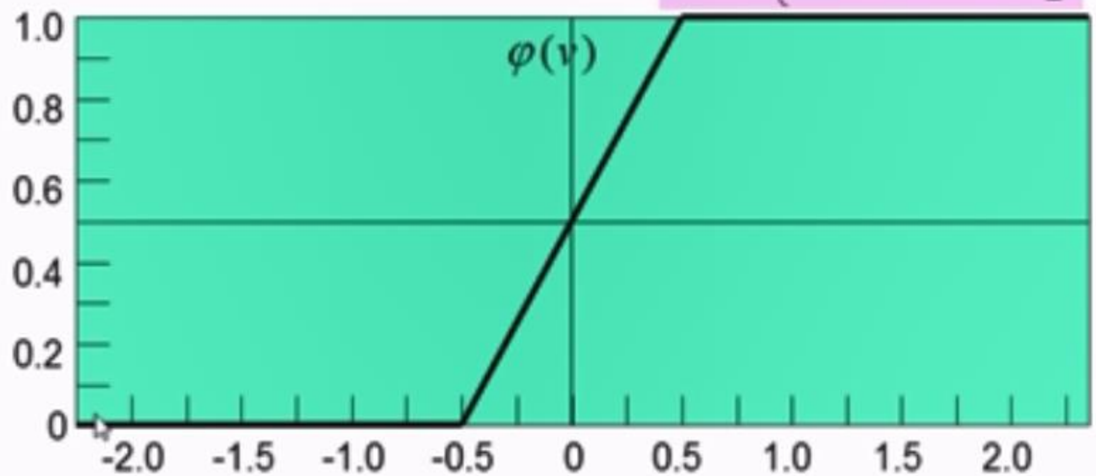
- Threshold Function or Heaviside Function:

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{if } v < 0 \end{cases}$$



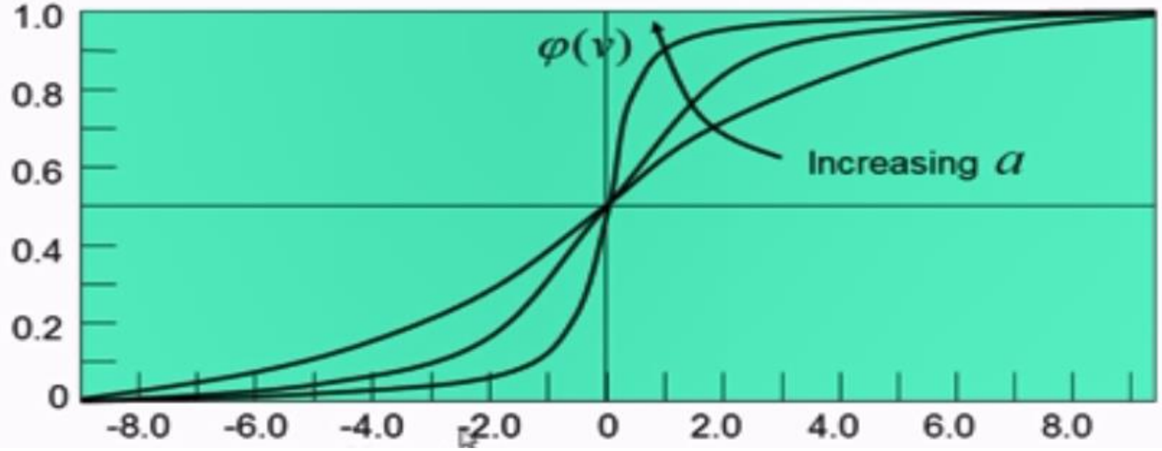
- Piecewise-Linear Function:

$$\varphi(v) = \begin{cases} 1, & v \geq +\frac{1}{2} \\ v, & +\frac{1}{2} > v > -\frac{1}{2} \\ 0, & v \leq -\frac{1}{2} \end{cases}$$



- Sigmoid Function: $\varphi(v) = \dfrac{1}{1+\exp(-av)}$

  where a is the slope parameter of the sigmoid function.



Single Layer Perceptron

- The neuronal model we have just discussed is also known as a perceptron.
- The perceptron is the simplest form of a neural network used for the classification of patterns said to be linearly separable.
- Basically, it consists of a single neuron with adjustable synaptic weights and bias.
- Now we will look at a method of achieving **learning** in our model we have formulated.

Perceptron Convergence (Learning) Algorithm

- Variables and Parameters

$$\mathbf{x}(n) = (m+1)\times 1 \quad \text{input vector}$$
$$= \left[+1, x_1(n), x_2(n), \ldots, x_m(n)\right]^T$$
$$\mathbf{w}(n) = (m+1)\times 1 \quad \text{weight vector}$$
$$= \left[b(n), w_1(n), w_2(n), \ldots, w_m(n)\right]^T$$
$$b(n) = \text{bias}$$
$$y(n) = \text{actual response}$$
$$d(n) = \text{desired response}$$
$$\eta = \text{learning-rate parameter, a postive constant less than unity}$$

1. ***Initialization***. Set $\mathbf{w}(0) = \mathbf{0}$. Then perform the following computations for time step $n = 1,2, \ldots$
2. ***Activation***. At time step n, activate the perceptron by applying input vector $\mathbf{x}(n)$and desired response $d(n)$.
3. ***Computation of Actual Response***. Compute the actual response of the perceptron:

$$y(n) = \mathrm{sgn}[\mathbf{w}^T(n)\mathbf{x}(n)]$$

where $\mathrm{sgn}(.)$ is the signum function.

$$\mathrm{sgn}(x) = \begin{cases} +1, & \text{if} \quad x \geq 0 \\ -1, & \text{if} \quad x < 0 \end{cases}$$

4. ***Adaptation of Weight Vector***. Update the weight vector of the perceptron:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \eta\left[d(n) - y(n)\right]\mathbf{x}(n)$$

where

$$d(n) = \begin{cases} +1 & \text{if } \mathbf{x}(n) \text{ belongs to class } C_1 \\ -1 & \text{if } \mathbf{x}(n) \text{ belongs to class } C_2 \end{cases}$$

5. ***Continuation***. Increment time step n by one and go back to step 2.

Decision Boundary

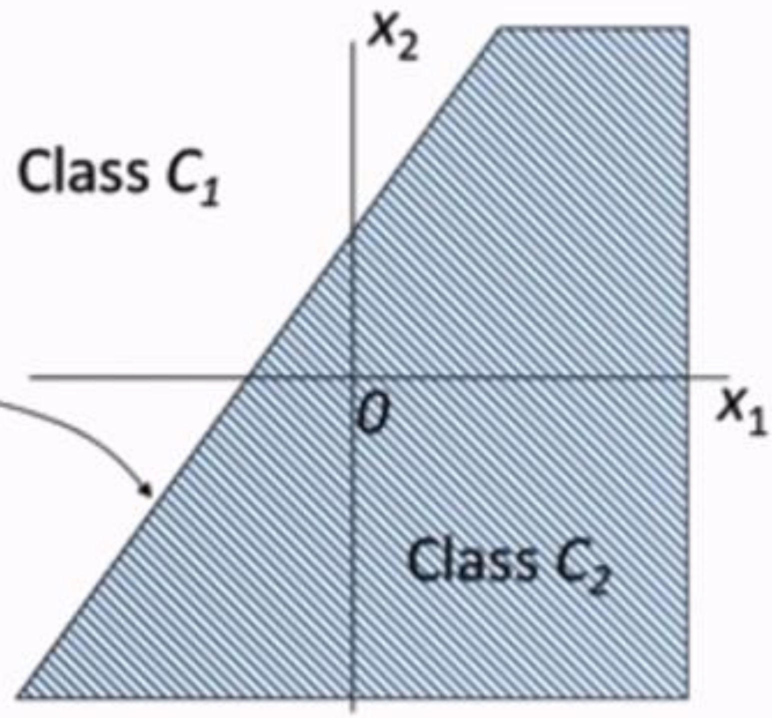
- The hyper-plane

$$\sum_{i=1}^{m} w_i x_i + b = 0$$

or

$$w_1 x_1 + w_2 x_2 + b = 0$$

is the decision boundary for a two class classification problem.



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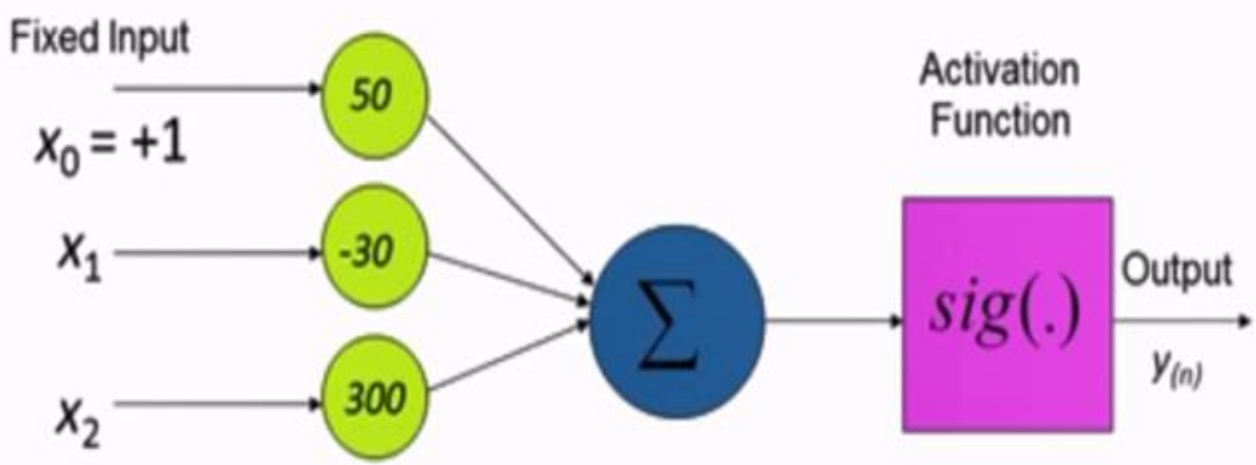
Solution to the Example
(with correct initial weights and bias)

With correct initial weights and bias

$$\left.\begin{aligned} &w_1(0) = -30,\ w_2(0) = 300, \\ &b(0) = 50,\ \eta = 0.01 \end{aligned}\right\} given$$



$$\mathrm{sgn}(x) = \begin{cases} +1, & \text{if} \quad x \geq 0 \\ -1, & \text{if} \quad x < 0 \end{cases}$$

$$\left.\begin{aligned} w_1(0) = -30,\ w_2(0) = 300, \\ b(0) = 50,\ \eta = 0.01 \end{aligned}\right\} given$$

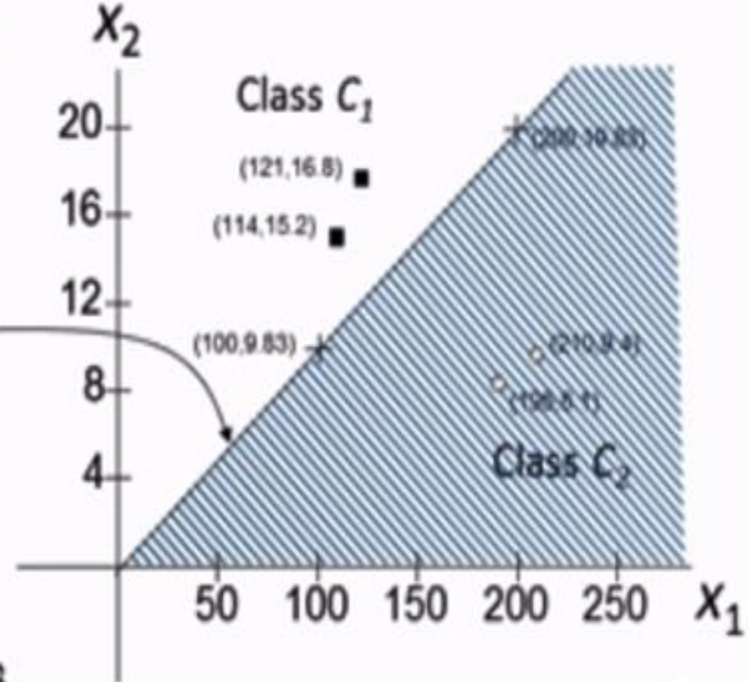
Therefore the Initial Decision Boundary for this example is:

$$w_1 x_1 + w_2 x_2 + b = 0$$

$$-30x_1 + 300x_2 + 50 = 0$$

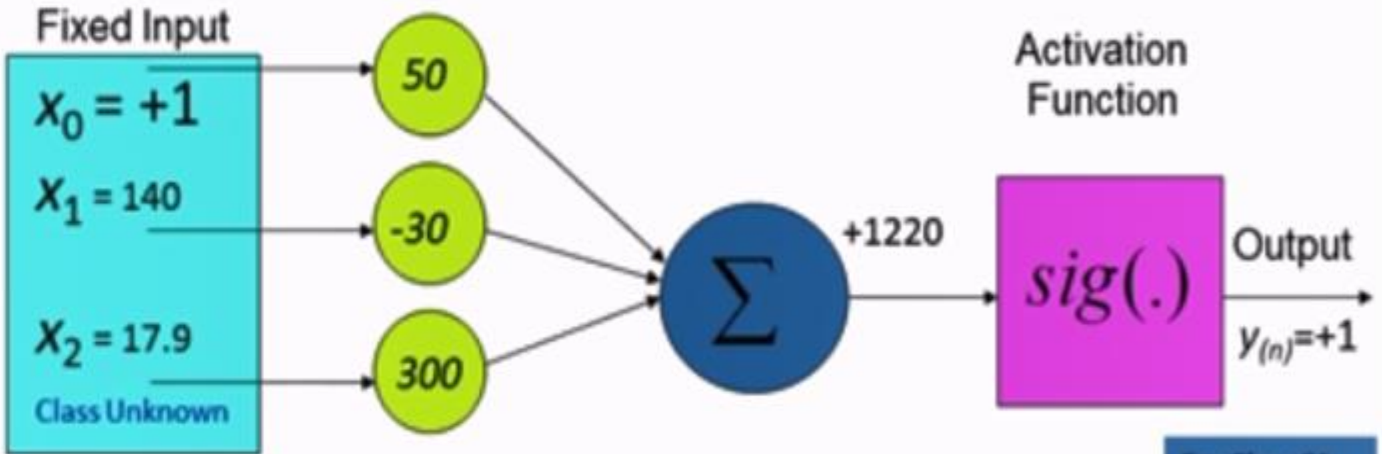
$$x_1 = 100,\ x_2 = \frac{30\times 100 - 50}{300} = 9.83$$

$$x_1 = 200,\ x_2 = \frac{30\times 200 - 50}{300} = 19.83$$



Initial hyper-plane does separate the two classes.

Classification of the Unknown Fruit



For Class C1, Output = +1

Now use the above model to classify the unknown fruit.

$$\mathbf{x}(\text{unknown}) = [+1, 140, 17.9]^T$$

$$\mathbf{w}(3) = [50, -30, 300]^T$$

$$y(\text{unknown}) = \text{sgn}\left(\mathbf{w}^T(3)\mathbf{x}(\text{unknown})\right) = \text{sgn}(50 \times 1 - 30 \times 140 + 300 \times 17.9)$$

$$= \text{sgn}(1220) = +1$$

$\therefore$ this unknown fruit belongs to the class C_1.

Solution to the Example
(with unknown initial weights and bias)

With unknown initial weights and bias

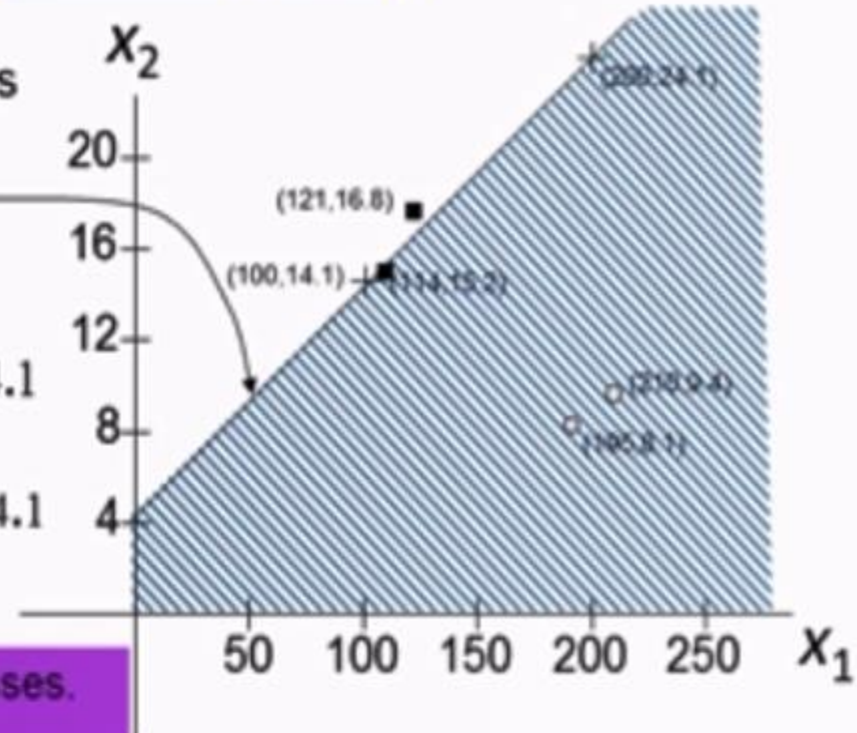
$$\left.\begin{aligned} w_1(0) = -30,\ w_2(0) = 300, \\ b(0) = -1230,\ \eta = 0.01 \end{aligned}\right\} given$$

Therefore the Decision Boundary for this case:

$$-30x_1 + 300x_2 - 1230 = 0$$

$$x_1 = 100, x_2 = \frac{30 \times 100 + 1230}{300} = 14.1$$

$$x_1 = 200, x_2 = \frac{30 \times 200 + 1230}{300} = 24.1$$



Initial hyperplane does not separate the two classes. Therefore we need to **Train** the Neural Network