Lecture 7

Operations on functions 2

* Restriction: $f: A \to B$ and $C \subset A$, then

$f|_C$ is the function

$f|_C: C \to B;\ x \to f(x)$

* Composition: $f: A \to B$ and $g: B \to C$

$g \circ f: A \to C;\ x \to g(f(x))$

* Quotient of two functions

$f: D \to R$ and $g: D \to R \backslash \{0\}$

$\frac{f}{g}: D \to R;\ x \to \frac{f(x)}{g(x)}$

<u>Remark:</u>

$f \circ g \neq g \circ$ *in general*

<u>Examples</u>

$h: R\backslash\{0\} \to R\backslash\{0\};\ x \to \frac{1}{x}$

let $D = \{x \in R \mid x > 0\}$

$h|_D$ has a graph which is part of that of f

$f: R \to R;\ x \to sin(x)$ and $g: R \to R;\ x \to x^2 + 1$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{sin(x)}{x^2+1}$

We can also write $f.(h\circ)$ for $\frac{f}{g}$

$$(f.(h \circ g))(x) = f(x).(h \circ g)(x)$$

$$= f(x).h(g(x))$$

$$= f(x).\frac{1}{g(x)} = \frac{f(x)}{g(x)}$$

$$so\ \ f.(h \circ g) = \frac{f}{g} \text{ if } 0 \ is\ not\ in\ the\ range\ of\ g$$

Intervals

These are subsets $I \subset R$ such that:

if $a \in I, b \in I, c \in R$ between a and b

then $c \in I$.

Examples

* $\{x \in R \mid x > 0 \text{ and } x < 2\}$

* $\{x \in R \mid x \geq -1 \text{ and } x \leq 1\}$

* $\{x \in R \mid x > 0\}$

* $\{x \in x^2 > 1\}$ *not an interval*

Above can be expressed as the union of the 2 intervals

$x < -1$ and $x > 1$

For $a \in R$ and $b \in R$ and $a < b$:

$$(a, b) = \{x \in R \mid x > a \text{ and } x < b\} \ \ \textit{open interval}$$

$$[a, b] = \{x \in R \mid x \geq a \text{ and } x \leq b\} \ \ \textit{closed interval}$$

$$[a, b) = \{x \in R \mid x \geq a \text{ and } x < b\} \ \ \tfrac{1}{2} \ \textit{open}$$

$$(a, b] = \{x \in R \mid x > a \text{ and } x \leq b\} \ \ \tfrac{1}{2} \ \textit{open}$$

When $a = b$: $[a, a] = \{a\}$ *contains a single point*.

$$(-\infty, a) = \{x \in R \mid x < a\}$$

$$(-\infty, a] = \{x \in R \mid x \leq a\}$$

$$(a, \infty) = \{x \in R \mid x > a\}$$

$$[a, \infty) = \{x \in R \mid x \geq a\}$$

$$[-\infty, \infty) = R$$

Remark:

∞ and $-\infty$ are not real numbers so they are never included in any interval (or any other subset of R).