

Proof:

Sufficient

For any natural number n , there are three prime numbers n , $n+2$, $n+4$.

Then by the Division Theorem, it can be

$$n = 3q + r (q \in \mathcal{N}, 0 \leq r \leq 2)$$

if $r = 0$, $n = 3q$, then n is divisible by 3. Since n is prime, n must be 3. So the three numbers are 3, 5, 7

if $r = 1$, $n + 2 = 3q + 1 + 2 = 3(q + 1)$, then $n+2$ is divisible by 3.

Since $n+2$ is prime, $n+2$ must be 3. So the three numbers are 1, 3, 5. It's impossible because n is prime.

if $r = 2$, $n + 4 = 3q + 2 + 4 = 3(q + 2)$, then $n+4$ is divisible by 3.

Since $n+4$ is prime, $n+4$ must be 3. So the three numbers are -1, 1, 3. It's impossible because n is prime.

there exist three number 3, 5, 7 whose sum is divisible by 3.

Necessary

$3 + 5 + 7 = 15$. So the sum is divisible by 3.

the argument have been proved.