

Proof: By Case

Since n is an integer, it must be an odd number, an even number or 0.

if n is odd, then n^2 and n are also odd. So the sum of three numbers $n^2 + n + 1$ is odd.

if n is even, then n^2 and n are even. So the sum of two even numbers plus one $n^2 + n + 1$ is odd.

if n is 0, then $n^2 + n + 1 = 1$ is odd.

the following conclusion is true.