1. The following recursive function implements a math function. What is this math function? (10%)

```
int fun(int x, int y)
    { if (x == 0)   return y;
    else   return fun(x - 1,   x + y);}
```

2. Rewrite insertion sort using recursion. (10%)

3. An N by N magic square is a two-dimensional array in which integers 1, 2, 3, ... N^2 are placed into the array such that sums of all the rows, all the columns, and the two diagonals are the same. Write a program to generate all 3 by 3 magic squares. (20%)

4. You are given a set of N positive integers, and you'd like to partition the set into M subsets in such a way that the sum of the integers of every subset is the same. Write a program to ask the user for N, the integers, and M, M <= N, and then determine if the partition can be accomplished. The output of your program is either "Yes" or "No". (20%)

5. You are given a pile of discs which alternate in color and are arranged from the largest disc at the bottom to the smallest at the top, as shown in Figure 1 below. You are required to recursively separate the input discs into two piles, each having its own color, with smaller discs stacked on top of larger ones, as shown in Figure 2. This is a variation of the Tower of Hanoi problem, and like that problem, you cannot place a larger disc above a smaller disc during the solution process. (20%)

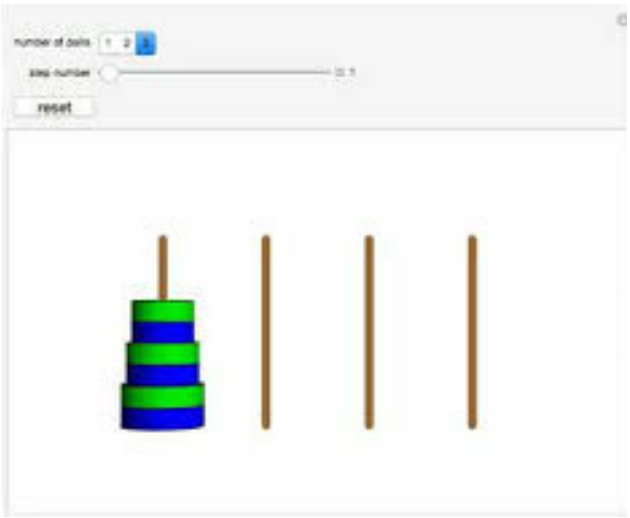

Figure 1

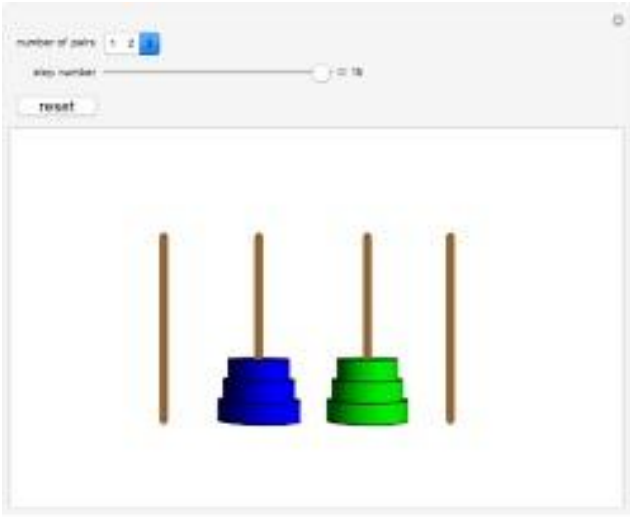

Figure 2

6. Tic-Tac-Toe is a game for two in which each player takes turn to place 'X' or 'O' into an empty square in a 3 by 3 game board. The first player starts the game and always places 'X' during the course of a game, whereas the second player always places 'O'. Given a game board represented by a 3 by 3 two-dimensional array with some X's, O's, and empty cells, you are to determine recursively if the given game position is reachable from the start of a game. If it is reachable, output "Yes"; otherwise, output "No". (20%)