HIGH SCHOOL MATHEMATICAL CONTEST IN MODELING OUTSTANDING PAPERS

HiMCM November 2003

The contest offers students the opportunity to compete in a team setting using applied mathematics in the solving of real-world problems.

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Editor's Comments

This is our sixth HiMCM Special Issue. Since space does not permit printing all of the nine national outstanding papers, this special section includes the summaries from six of the papers and edited versions of three. We emphasize that the selection of these two does not imply that they are superior to the other outstanding papers. They were chosen because they are representative and fairly short. They have received light editing, primarily for brevity. We also wish to emphasize that the papers were not written with publication in mind; the contest does not allow time to revise and polish. Given the 36-hour time limit, it is remarkable how well written many of the papers are.

We appreciate the outstanding work of students and advisors and the efforts of our contest director and judges. Their dedication and commitment have made HiMCM a big success. We also wish to note that this special section takes the place of our regular HiMCM Notes column, which will return in the next issue.

Contest Director's Article

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The High School Mathematical Contest in Modeling (HiMCM) completed its sixth year in excellent fashion. The growth of students, faculty advisors, and the contest judges is very evident in the professional submissions and work being done. The contest is still moving ahead, growing in a positive first derivative, and consistent with our positive experiences from previous HiMCM contests.

This year the contest consisted of 275 teams (a growth of about 30% from last year) from twenty-five states and from outside the USA. This year was the first year that schools were charged a registration fee of $45 for the first team and $25 for each additional team.

Thus our contest continues to attract an international audience. The teams accomplished the vision of our founders by providing *unique* and *creative mathematical* solutions to complex, open-ended, real-world problems. This year the students had a choice of two problems. Of the 275 teams, 156 submitted solutions to the B problem, and 119 submitted solutions to the A problem.

Problem A: What is it Worth?

In 1945, Noah Sentz died in a car accident and his estate was handled by the local courts. The state law stated that 1/3 of all assets and property go to the wife and 2/3 of all assets go to the children. There were four children. Over the next four years, three of the four children sold their shares of the assets back to the mother for a sum of $1300 each. The original total assets were mainly 75.43 acres of land. This week, the fourth child has sued the estate for his rightful inheritance from the original probate ruling. The judge has ruled in favor of the fourth son and has determined that he is rightfully due monetary compensation. The judge has picked your group as the jury to determine the amount of compensation.

Use the principles of mathematical modeling to build a model that enables you to determine the compensation. Additionally, prepare a short one-page summary letter to the court that explains your results. Assume the date is November 10, 2003.

Problem B: How Fair are Major League Baseball Parks to Players?

Consider the following major league baseball parks: Atlanta Braves, Colorado Rockies, New York Yankees, California Angles, Minnesota Twins, and Florida Marlins.

Each field is in a different location and has different dimensions. Are all these parks "fair"? Determine how fair or unfair is each park. Determine the optimal baseball "setting" for major league baseball.

	Outfield Dimensions					Wall Height			
Franchise	Left Field	Left Ctr	Ctr Field	Right Ctr	Right Field	Left Field	Ctr Field	Right Field	Area of Fair Ter
Angels	330	376	408	361	330	8	8	18	110,000
Braves	335	380	401	390	330	8	8	8	115,000
Rockies	347	390	415	375	350	8	8	14	117,000
Yankees	318	399	408	385	314	8	7	10	113,000
Twins	343	385	408	367	327	13	13	23	111,000
Marlins	330	385	404	385	345	8	8	8	115,000

COMMENDATION:

All students and their advisors are congratulated for their varied and creative mathematical efforts. The thirty-six continuous hours to work on the problem provided (in our opinion) a vast improvement in the quality of the papers. Teams are commended for the overall quality of work.

Again this year, many of the teams had female participation, which shows that this competition is for both male and female students. Teams again proved to the judges that they had "fun" with their chosen problems, demonstrating research initiative and creativity in their solutions.

JUDGING:

We ran three regional sites in December 2003. Each site judged papers for both problems A and B. The papers judged at each regional site were not from their respective region. Papers were judged as Outstanding, Meritorious, Honorable Mention, and Successful Participant. All regional finalist papers for the Regional Outstanding award were brought to the National Judging. For example, eight papers may be discussed at a Regional Final and only four selected as Regional Outstanding but all eight papers are brought and judged for the National Outstanding. The national judging chooses the "*best of the best*" as National Outstanding. The National Judges commend the regional judges for their efforts and found the results were very consistent. We feel that this regional structure provides a good prototype for the future of the contest's structure as it continues to grow.

JUDGING RESULTS:

National & Regional Combined Results

Problem	**National Outstanding***	Regional Outstanding	Meritorious	Honorable Mention	Successful Participant	Total
A	**4**	12	33	45	25	**119**
B	**5**	10	42	75	23	**155**
Total	**9**	14	51	104	33	**274**

GENERAL JUDGING COMMENTS:

The judges' commentaries provide comments on the solutions to each of the two problems. As contest director and head judge for the problems, I would like to speak generally about team solutions from a judge's point of view. Papers need to be very coherent, concise, and clear. Students need to restate the problem in their own words so that the judges can determine the focus of the paper. Papers that explain the development of the model, assumptions, and its solutions and then support the solution findings mathematically generally do quite well. Modeling assumptions need to be listed and justified but only those that come to bear on the team's solution (that can be part of simplifying the model). Laundry lists of assumptions that are never referred to in the context of the model development are not considered relevant and deter from the paper's quality. The model needs to be clearly developed, and all variables that are used need to be defined. Thinking outside of the "box" is also an ingredient considered important by judges. This varies from problem to problem but usually includes model extensions or sensitivity analysis of the solution to the teams' inputs. A clear conclusion and answers to specific scenario questions are all key components. The strengths and weakness section of the paper is where the team can reflect on the solution. Attention to detail and proofreading the paper prior to final submission are also important because the judges look for clarity and style.

CONTEST FACTS:

Facts from the 6th Annual Contest:

- A wide range of schools/teams competed including teams from Hong Kong.
- The 275 teams represented 60 institutions; 44 repeats and 16 new institutions.
- 44.36% of the teams had female participation. Forty-three of the 275 teams were all female.
- There were 953 student participants, 548 male (57.5%) and 395 female (42.5%).
- Schools from 25 states participated in this year's contest.

THE FUTURE:

This HiMCM contest that attempts to give the underrepresented an opportunity to compete and achieve success in mathematics endeavors appears well on its way in meeting this important goal.

We continue to strive to grow. Any school/team can enter the contest, as there will be no restrictions on the number of schools entering. A regional judging structure will be established based on the response of teams to compete in the contest.

Again, these are exciting times for our high school students. Mathematics continues to be more than learning skills and operations. Mathematics is a language that involves our daily lives. Applying the mathematical principles that one learns is key to future success. The ability to recognize problems, to formulate a mathematical model and solve it, to use technology, and to communicate and reflect on one's work is key to success. Students develop confidence by tackling ill-defined problems and working together to generate a solution. Applying mathematics is a team sport.

Advisors need only be a motivator and facilitator. They should permit students to be ***creative*** and ***imaginative***. It is not the technique that is fundamental, but the process that discovers how assumptions drive the techniques. Mathematical modeling is an art and a science. Through modeling, students learn to think critically; communicate efficiently; and be confident, competent problem solvers for the new century.

Contest Dates: Mark your calendars early: the next HiMCM will be held in November 2004. Registrations of teams are due in October 2004. Teams will have a consecutive 36-hour block within a window of about two weeks to complete their chosen problem. Teams can registrar via the worldwide web at www.comap.com.

HiMCM Judges Commentary

Problem A: What is it Worth?

Although some teams initially believed that this was a simple algebraic problem involving the time value on money (i.e., a Future Value = $P(1 + I)^R$ problem), it soon became apparent to the better teams that they needed to perform some modeling and critical analysis of the situation.

The judges were impressed with the creativity, quality of the analysis, and the writing by the teams modeling Problem A. Teams dealt with time value of money, and they dealt with the value of land over time. It was imperative for teams working with land value to assume or "pick" a location (county or state) in order to estimate through modeling the appreciation of the land's value. Doing this provided teams an opportunity to restate the problem in more manageable terms. This allowed successful teams to move from the general ill-defined problem to an attackable, more specific, defined problem.

Many teams attacked this problem only as a time value of money issue. A key aspect here was obtaining an interest rate or index growth rate over time. Research as to which values were the most appropriate was necessary. Many teams merely stated values. Judges expected the teams to develop a sub-model to determine this.

One of the items that distinguished the better papers was that those teams calculated the "worth" from various models (money, land, etc.) and then came to a final conclusion about compensation after considering all their possible outcomes.

Verification of models or model testing was also an important discriminator. Some teams tested their models to see if the results made common sense. Others compared their predictions with historical results that they were able to obtain via web sources. It is noted that some answers given by teams made little practical sense.

Problem B: How Fair are Major League Baseball Parks to Players?

This was the purest modeling problem of the two. This problem was ill defined because students needed to determine what *they* meant by "fair." Teams needed to step back and insure that they had a well-defined problem, defined "fair," and determined which aspects of "fair" they were trying to model.

Some teams did not define "fair" until after they completed their models, which was deemed too late by many judges.

The judges commented that the statement of assumptions with justification, style of presentation, and depth of analysis was very good. The better papers offered a good diversity of solutions. Problem B, in comparison to Problem A, appeared to lend itself better to consideration of a variety of assumptions and justifications.

It was critical for teams to define "fair" in order to compare the baseball parks. The judges were surprised that no team recommended changing the dimensions of ballparks in order to make them more "fair" based on their mathematical findings. The better papers considered such variables as altitude, wind, humidity, temperature, and other environmental factors in their model or their discussions of the model.

Many papers used computer code to determine the issue of fairness of the baseball parks. Computer codes used to implement mathematical expressions can be a good modeling tool. However, judges expect to see an algorithm or flow chart in the paper from which the code was developed. Successful teams provided some explanation or guide to their algorithm(s)—a step-by-step procedure for the judges to follow. The code that teams attached to their paper may only be read for those papers that reach the final rounds of judging. The results of any simulation or computer code need to be explained, and sensitivity analysis should be performed.

For example, consider an algorithm for the flip of a fair coin:

INPUT: Random number, number of trials

OUTPUT: Heads or tails

Step 1: Initialize all counters.

Step 2: Generate a random number between 0 and 1.

Step 3: Choose an interval for heads, like [0.0.5]. If the random number falls in this interval, the flip is a heads. Otherwise the flip is a tails.

Step 4: Record the result as a heads or a tails.

Step 5: Count the number of trials and increment: Count = Count + 1.

An algorithm such as this would be expected in the body of the paper with the code as an appendix.

The judges commend the teams for a truly outstanding job on a difficult, open-ended problem that provided some interesting reading.

GENERAL COMMENTS FROM JUDGES:

Executive Summaries:

These are still, for the most part, one of weakest parts of team submissions. These should be complete in ideas not details. They should include the "bottom-line" and the key ideas used. They should include the particular questions addressed and their answers. Teams should consider a three paragraph approach: restate the problems in their own words, give a short description of their method to model and solve the problem (without giving specific mathematical expressions), and state the conclusions, including the numerical answers in context.

Restatement of the Problem:

Problem restatements are important for teams to move from the general case to the specific case. They allow teams to refine many aspects of their thinking to give their model uniqueness and a creative touch.

Assumptions/Justifications:

Teams should list only those assumptions that are vital to the building and simplifying of their model. Assumptions should not be a reiteration of facts given in the problem description. Assumptions are variables (issues) acting or not acting on the problem. Every assumption should have a justification with it. Variables chosen need to be listed with notation and be well defined.

Model:

Teams need to show a clear link between the assumptions they listed and the building of their model or models.

Model Testing:

Model testing is not the same as testing arithmetic. Teams need to compare results or attempt to verify (even with common sense) their results.

Teams that use simulation must provide a clear step-by-step algorithm for the proposed simulation. Lots of runs and related analysis are required when using a simulation to model a problem. Sensitivity analysis is also expected to determine how sensitive the simulation is to the model's key parameters.

Conclusions:

This section deals with more than just results. Conclusions might also include speculations, extensions to the model, and generalizations of the model. Teams should ask themselves what other questions would be interesting if they had more time and then tell the judges about their ideas.

Strengths and Weaknesses:

Teams should be open and honest here. They should answer the question, "What could we have done better?"

References:

Teams may use references to assist in modeling the problem. However, they must also identify the sources. It is still required of the team to show how the model was built and why it was the model chosen for this problem. Teams are reminded that only inanimate resources may be used. Teams cannot call upon real estate agents, bankers, or any other real person to obtain any information related to the problem.

Adherence to Rules:

Teams are reminded that detailed rules and regulations are posted on the high school contests area of the COMAP Website. Teams are reminded that the 36-hour time limit is a consecutive 36 hours.

Problem A Summary: Clarkstown South High School

Advisor: Mary Gavioli

Team Members: Simi Bhat, Daniel Gendler, Mitchell Livingston, Terry Van Hise

When the rightful inheritance of a beneficiary is not given shortly after the probate court's original ruling, finding an appropriate amount of compensation for that inheritance is complicated by several factors. Noah Sentz's fourth child should have been awarded 1/4 of 2/3 of his estate according to state law at the time of Mr. Sentz's passing. However, the fourth child is now suing for compensation because he did not receive his portion of the estate. There were several factors we considered when calculating Mr. Sentz's compensation, the most important of which being inflation. Inflation is the progression of changing value of a dollar over time. This means that the items a dollar could purchase when Mr. Sentz passed away probably would not cover the price of that item at the present time. We found the inflation rate from the time that the three children sold their portions of the estate to the present to be 802.9%. This means that if the land was worth $1200 and other personal assets were valued at $100 then, then the land would be worth $9624 and the other personal assets would have a value of $802.90 now. We also took into account the revenue the fourth child would have been deprived of because he did not own the land since 1945. The total amount of revenue generated from the land would have been $53,568. However, the fourth son would have had to pay additional income tax on this revenue, which would amount to $13,894.80. He would have also had to pay property tax on this land for the past fifty-eight years, a total of $316.04. To find the final monetary compensation the son should be given we summed the values of all assets and income and subtracted all taxes giving Nick a final compensation of $49,784.06. We made a generalized model, allowing for the input of several variables such as state and federal income tax rate, original value of land, original value of personal assets, and such variables. To increase the ease of use of our model, we created a computer program in Java so that court officials could simply input the variables into the program and receive an output of the calculated monetary compensation the disputing beneficiary should receive. The math model we created uses data taken from government references and economic journals. We feel it is a feasible and accurate model.

Problem A Summary: Mills E. Godwin High School

Advisor: Ann Sebrell

Team Members: Derek Austin, Srini Sathyanarayanan, Matthew Walker, Zhiyuan Xu

We the jury have determined that the plaintiff, the fourth child of the late Noah Sentz, deserves monetary compensation for assets not awarded in the amount of $18,772.35. Through investigating the problem, we researched various aspects of inheritance laws as well as the economic factors influencing the asset price. Our amount of compensation is the modem value of his portion of the assets minus property taxes accrued since 1945. We found this amount of compensation through a mathematical model that we created.

We initially created a simple model in which the plaintiff would receive value only from land assets and built upon this. We established the property as rural farmland from Pennsylvania. We realized that rural farmland is highly influenced by inflation and the appreciation of land value. By tracking basic land value patterns, we were able to roughly estimate the value of land in 2003. Through researching Consumer Price Index we approximated the influence of the cost of living on the price of consumer goods, which we then included in the model as part of his compensation. We decided that the four children initially split the consumer goods while the mother received her portion of the estate only in farmland; in other words, we accounted for an uneven split in the type of assets distributed, even though the amount of assets for each group was even. Finally, we deducted a 9.5% property tax rate from the estimated value of the land portion of his share of assets to create a final amount of compensation.

Through thorough examination of various factors influencing the value of the estate, we can confidently conclude that the compensation for the fourth child's inheritance in this situation should be $18,772.35.

Problem B Summary: Maggie L. Walker Governor's School

Advisor: John Barnes

Team Members: Guilherme Cavalcanti, Thomas Fortuna, Mrinal Menon, Derek Miller

Our first step in evaluating whether or not the fields were fair was to hypothesize that teams with a greater penchant for hitting home runs would build smaller stadiums to hit more home runs, while teams that could not hit as many home runs would build larger stadiums in order to deprive other teams of home runs. We then gathered available team data for three to five years before each team built their current stadium, trying to determine if their home run performance in relation to their league affected the creation of their fields. No relationship was found between team performance and the pure dimensions of the field as far as home runs were concerned.

We then decided to test if the initial velocity of a baseball hit at each field was significantly different in a perfect, airless world. It was determined that there was no significant difference in initial velocity for each stadium. Research, however, revealed that there was a significant difference in the amount of home runs hit at each of the six assigned stadiums, specifically Coors Field. The only construction difference between Coors Field and all other stadiums was its extreme altitude. We then proceeded to modify our baseball projectile motion model to include air resistance, to account for changes in altitude and temperature. Two forms of Euler's method were used to model the trajectory of a baseball through a dense atmosphere. This updated model revealed that altitude and temperature were major factors in determining a field's fairness; hitting home runs at sea level fields required a much greater initial velocity.

We then proceeded to define fairness in our fields. Our first belief was that fields that are symmetrical about their centerlines (right and left field distance and wall heights are equal) are fair to both right-handed and left-handed hitters. Of the six stadiums, Pro Player Stadium of the Florida Marlins was the only one that could be considered fair. The remaining five teams in increasing fairness are the Denver Rockies, New York Yankees, Atlanta Braves, and Anaheim Angels. The Twins' stadium, being temperature controlled, did not apply to our model. We also believe that we can create a fair field distance by taking average distances of existing fields that are getting league average home runs, accounting for temperature and altitude variations. Using these two beliefs and our air resistance model, we can create a "fair" field knowing only the altitude, temperature, and uniform wall height.

Problem B Summary: Illinois Mathematics and Science Academy

Advisor: Steven Condie

Team Members: Jeffrey Chang, Alex Garivaltis, David Xu

To begin tackling the problem, we decided to use a national batting average of 0.25 as an indicator of fairness. Our vision of the "fairest" field was one that provided its players with a level of offensive and defensive opportunities consistent with the national average. Therefore, a ballpark that produced a batting average closest to the national average was considered to be the optimal setting. The model consisted of a computer program that would automatically calculate the players' positions on a Cartesian plane given the dimensions of a field. To determine the batting average for any of the given ballparks, we created an algorithm that would simulate 10,000 at-bats, taking into account appropriate ratios for strikeouts and foul-outs. The variables Θ (initial angle with respect to ground), v_0 (initial velocity of the ball), and Φ (angle of ball's direction of travel on Cartesian plane) were all randomized within reasonable ranges for each simulation.

The program then analyzed the trajectory of the ball, taking into account the wind speed and any effects from low air pressure. By calculating the position of the ball's landing spot, our model determined whether the ball would be caught (resulting in an out), would not be caught (resulting in a safe base advance), or would fly over the fence as a home run (counting as four base advances). Our simulation was able to compute the batting average by keeping track of the number of base advances over 10,000 simulations.

To assess the fairness of the stadiums played in by the Angels, Braves, Rockies, Yankees, Twins, and Marlins; we found the batting averages resulting from each ballpark's specific dimensions and environmental conditions. The resulting order of fairness for the ballparks, from fairest to least fair, was:

Yankees (0.287), Braves (0.294), Twins (0.308), Marlins (0.316), Angels (0.322), Rockies (0.341)

To determine the optimal setting in general we decided to employ an evolution-based simulation method. The program generated a random set of field dimensions within reasonable limits and calculated the expected batting average using 10,000 at-bat simulations. The computer then continued generating random ballparks, calculating the batting average every time. Whenever a ballpark resulted in a batting average that was closer to the national average than any previously checked ballpark, it became the new optimal setting. After repeating this process for a long time, the best setting naturally surfaced.

	Outfield Dimensions					Wall Height		
	Left Field	Left Ctr	Ctr Field	Right Ctr	Right Field	Left Field	Ctr Field	Right Field
Ideal Ballpark Dimensions (ft)	387	393	434	397	335	9.8	13.1	6.7

Problem B Summary: Dubuque Hempstead High School

Advisor: Karen Weires

Team Members: Tom Duggan, Josh Lichti, Cory McDermott, Brad Willenbring

To quantitatively compare the fairness of the given stadiums, we defined numerous variables and methods of measurement. A computer program was created to simulate the behavior of hits in each stadium, taking into account its dimensions and prevailing environmental conditions. The program also simulated the distance the outfielders need to move in order to field all non-homerun hits. This data was coupled with numerous other factors including backstop length and field orientation to determine each stadium's Defensive Bias Rating (DBR), a measure of how offensively or defensively biased the given ballpark is.

The DBR value was then factored into the calculation of another value, the Oppositional Equality Rating (OER), a measure of how well matched a given team is to their stadium. A team that hits well overall, stationed in a stadium that caters to a hitting team, is going to have a distinct advantage because they play 50% of their games on their home field.

The DBR and OER both proved to be very accurate models of real-life conditions among the stadiums in question. The Colorado stadium was determined to be the most unfair stadium based on its OER rating. Environmental conditions make hits in Colorado tend to fly further, coupled with a team that statistically (already) seems to prefer offense to defense. In other cases, such as Atlanta, the team's ability wasn't paired with its stadium, so the two canceled each other out in the OER rating.

Problem B Summary: Mills E. Godwin High School

Advisor: Ann Sebrell

Team Members: Deepa Iyer, Brandon Murrill, Omari Stephens, David Williamson

During our investigation of this situation, our team aimed to derive an optimal model for fairness in a baseball park. To achieve this goal, we created a list of ten factors that we felt could affect the fairness equilibrium of a field. We researched the specifications of these ten factors and found an abundance of data from various sources.

By analyzing the magnitude of the effect of each factor at all of the six parks, we created a score for each park that led us to our conclusions. We found that each park was biased toward the offense or the defense; none was completely fair.

We tested our models by comparing the classifications we generated to the historical conception of each park: whether the venue was considered more favorable for a hitter or a pitcher.

We then proceeded to develop an optimal, completely fair baseball environment. We strove to ensure that our park favored neither the offensive or defensive team. Finally, we generated two strategies to develop a fair stadium. The first method required different factors to favor either the offense or the defense; the total effect of all the factors remained at equilibrium. The second method called for nearly neutral values for every single factor, again yielding a neutral venue overall. We combined the two strategies to develop a realistic, optimal design for a truly fair field.

Problem A Paper: The Spence School

Advisor: Eric Zahler

Team Members: Jillian Bunting, Madeleine Douglas, Yi Zhou

PROBLEM RESTATEMENT

Noah Sentz died in a car accident in 1945. His wife received one-third of his estate, and the children received two-thirds. Over the next four years, three of the four children sold their shares back to their mother for $1300 each. Noah's assets were mostly comprised of 75.43 acres of land. In early November 2003, the fourth child sued for his rightful share. The judge ruled that he is due cash compensation.

LETTER TO THE COURT

We created a mathematical model for determining the plaintiff's compensation. We made several assumptions to produce an effective model. These included the dates of the death of Noah Sentz and the subsequent sale of his assets by his three sons, that the first son received only land, that the value per acre was uniform, that the non-land assets appreciated at the rate of inflation, and that the values of each child's share was equal in 1945.

Besides these, our model made no assumptions about the specific values of variables. Instead, all variables are available to be manipulated. This is a very effective way of determining compensation because it can be implemented in a variety of situations. This is especially important because certain factors (i.e., value per acre, appreciation of land) are specific to geographic regions.

The benefits of our model are that it gives a fairly good range of values for the compensation and that it leaves only one factor to be determined by the court (the value of land per acre in 1945). Even without determining the value per acre, our model tells us that the compensation due the fourth son is between $55,400.39 and $90,162.43. Our model is also strong because it made no arbitrary assumptions. The inflation rates between years were based on historical data. The land appreciation rate was inferred from national data.

We feel confident that our inflation and land appreciation rates are fairly accurate. Therefore, we respectfully recommend that the model be used to determine compensation after the price per acre is determined.

ASSUMPTIONS AND JUSTIFICATIONS

In 1945, the worth of each child's share was equitable. We assume this because none of the children sued within four years. The suit did not arise until 58 years later, implying that there was no problem for a long time.

The fourth son received non-land assets that did not appreciate as fast as the land. He is suing for the difference between what his share is worth now and one-sixth of what the estate would be worth today.

Mr. Sentz died in January of 1945. We assumed this in order to have a base date to calculate inflation.

The first, second, and third children sold their shares back to their mother in January 1947, January 1948, and January 1949, respectively. This allowed us to find inflation values. We chose dates spread evenly over the four-year period to reflect the changing inflations over those four years.

The value of the land per acre is uniform. This allowed us to relate our equations to one another, and simplifies the problem by eliminating a variable.

All other assets appreciate at the rate of inflation. This simplifies the problem.

The rate of land appreciation is constant and reflects the best-fit curve that we found (see **Figure 1** and **Appendix B).** This allowed us to create equations; with land appreciation as a variable, there is too much unknown. We considered it reasonable to assume that the land appreciation was roughly equal to the overall trend in the United States.

The first son only received land. We assumed this in order to simplify the problem and allow us to solve for the value of the land in 1945.

OUR APPROACH

Land appreciates faster than the rate of inflation (see **Appendix A** for a proof). Since other assets appreciate at the rate of inflation, their growth over a long period of time is less than that of land. As a result of a discrepancy in distributing land and other assets (the fourth child only received non-land assets), the fourth child's share has not appreciated at the same rate as the land. Thus, if we calculated how much a sixth of the entire estate would be worth now and how much the fourth child actually has, the difference would be how much the fourth child is entitled to.

VARIABLES

L, with subscripts corresponding to children and mother, is the amount of land in acres each received. A subscript of M refers to the mother, a subscript of 1 refers to the first child, a subscript of 2 refers to the second child, and so forth. V is the value per acre in 1945. A is the rate of appreciation of land per year since 1945. S, with subscripts corresponding to the subscripts of L, is the value of the non-land assets the children received in 1945. T is the total value of the estate in 1945. I is the inflation rate between 1945 and the given year. P is the total value of non-land assets in 1945. Finally, t is the number of years since 1945. From these variables, we generated these equations:

$P = S_2 + S_3 + S_4 + S_M$. The total value of the non-land assets is equal to the sum of what was distributed.

$T = 75.43V + P$. The total value of the estate is the sum of the value of the land and the value of the other assets.

$V(L_1 + L_2 + L_3 + L_M) = 75.43V$. The sum of all land distributed is equal to the total value of the land.

$L_1V = L_2V + S_2 = L_3V + S_3 = S_4 = (L_MV + S_M)/2$. This comes from the assumption that at the time of distribution, each child received a sixth of the estate, and the mother received a third.

At t years from January 1945, the value of person X's inheritance can be expressed: $(L_XV)(1 + A)^t + (S_X)(1 + I_{1945M})$.

The trend in a graph of national trends in land appreciation (Figure 1) appeared exponential. We ignored the dip at the end of the graph because the trend of exponential growth continues, as evidenced by **Figure 2.** We estimated coordinates on the land-value graph in Figure 1. From these we found an exponential model (see Appendix B). The general form of the exponential model was $y = ab^x$, where a is the starting value and b is the increase per year. We determined b was 1.0857. Thus land appreciates at a rate of 8.57% per year. Our calculated a was 100. Using this equation, we calculated the value of each child's share based on the value of the first brother's share, which was entirely land.

$1 + A = b = 1.0857$

$(L_1V)(l + A)^2 = 1300$

$(L_1V)(l.0857)^2 = 1300$

$(L_1V) = \$1102.87$, which is the value of each child's share in 1945.

To determine the value of the estate in 1945, we multiplied 1102.87 by 6 and obtained \$6617.21 = T. We looked up the inflation rate between January 1945 and November 2003: 940.45% (see Bibliography). So the fourth brother's share is now worth:

November 2003 Value = $1102.87(1 + I_{2003})$

$\$11474.81 = 1102.87(10.4045)$.

We were told that the estate was comprised mainly of land, which we took to mean that the land's value was more than 50% of the total estate value, but less than or equal to five-sixths of it, since the fourth brother received only non-land assets comprising one-sixth of the total. Thus, we created a range of compensation. If the land is half the total value, then its 1945 value is \$6617.21/2 = \$3308.605. Then the current value of the entire estate is:

$(1.0857)^{58}\ (3308.605) + 10.4045(3308.605) = \$424200.76$.

His portion would be a sixth of that, or \$70700.13. Then he should receive the difference between that figure and \$11474.81: \$59225.32. The value per acre would be: 75.43V = 3308.605, or \$43.86.

On the other hand, if land constituted everything but his portion, 5/6 of the total estate, the value of the whole is:

$(1.0857)^{58}\ (5514.34) + 10.4045(1102.87) = \$661101.91$.

Then his portion should be \$110183.65, which means he should receive \$98708.84. The value per acre is: 75.43V = 5514.34, or \$71.11 (see **Figure 3**). The range of compensation is then between \$59225.32 and \$98708.84. It should be closer to the higher value, as "mainly" seems to imply closer to five-sixths than to half.

The function that determines the compensation is (see **Appendix C**):

General: $$\frac{(1+A)^t(75.43V)+(1+I_{2003})(T-75.43V)}{6} - S_4(1 + I_{2003})$$

Specific: f(V) =
$$\frac{1.0857^{58}(75.43V)+10.4045(6617.21-75.43V)}{6} - 1102.87(10.4045).$$

STRENGTHS OF MODEL

1. The general model needs few assumptions because most of its components are variables. It is flexible in that if only a few values of variables are known, by manipulating the equations a solution can be determined. In this case, only land appreciation and rate of inflation, both of which were found online, were necessary to find a compensation range. Finding the best value in that range requires only the value of the land per acre.

2. The general model accounts for assets and property, as well as the percentage of the total each comprises.

3. The model is easy to use and is not caught up in accounting for thousands of possibilities that arise from ambiguity of the variables. To account for every possible range of values of each variable would spawn a convoluted model that may not yield an accurate answer.

4. The data needed to use the model are readily available.

5. The model is fairly comprehensive because it accounts for various factors, including land appreciation and inflation rate.

6. The model does not require a jury to use high-level math. Plugging in values and adding to get a total requires only sixth-grade skills.

WEAKNESSES OF THE MODEL

1. The assumption that the value of land was uniform was necessary for the general method. A difference in the value would make the problem more complicated, as it is nearly impossible to account for disparities without knowing other factors such as location.

2. The assumption that the assets appreciate at the inflation rate is unrealistic.

3. The assumption that the land appreciation trend is perfectly exponential ignores other factors that affect it, such as natural disasters and location. Also, the land appreciation was derived from one graph.

4. The assumptions of the date of Noah's death and the dates when the brothers sold their shares affect the rate of inflation and land appreciation. This would alter the total calculated value of the estate in 1945.

5. The assumptions that the first brother only inherited land and that he was the first to sell his shares are crucial. While the latter assumption is logical (the brother with the most land had to have sold his shares first or else his land would have appreciated and not have been worth the same as the assets of the other brothers), the former assumption was for convenience and affects the estate value in 1945.

6. The assumption that the inequity arose because of unequal land distribution is the basis for the entire model. However, it is not explicitly stated that such is the case.

APPENDIX A:

PROOF THAT THE RATE OF LAND APPRECIATION IS GREATER THAN THAT OF INFLATION

$$\frac{1}{6}\mathrm{T}(1 + \mathrm{I}_{2003}) < \frac{75.43V(1+A_{2003})^{58} + P(1+I_{2003})}{6}$$

We know that $\mathrm{I}_{2003} = 940.54\%$ (or 9.4045).

$$\frac{1}{6}\mathrm{T}(1 + 9.4045) < \frac{75.43V(1+A_{2003})^{58} + P(1+9.4045)}{6}$$

$$10.4045\mathrm{T} < 75.43\mathrm{V}(1 + \mathrm{A}_{2003})^{58} + 10.4045\mathrm{P}$$

$$10.4045\mathrm{T} < 75.43\mathrm{V}(1 + \mathrm{A}_{2003})^{58} + 10.4045(\mathrm{T} - 75.43\mathrm{V})$$

$$10.4045\mathrm{T} < 75.43\mathrm{V}(1 + \mathrm{A}_{2003})^{58} + 10.4045\mathrm{T} - 784.81\mathrm{V}$$

$$784.81\mathrm{V} < 75.43\mathrm{V}(1 + \mathrm{A}_{2003})^{58}$$

$$10.4045 < (1 + \mathrm{A}_{2003})^{58}$$

Therefore, at t years from 1945, $(1 + \mathrm{A}_{1945+t})^t > 1 + \mathrm{I}_{1945+t}$. Thus land appreciation is always greater than the rate of inflation.

$$1 + \mathrm{A}_{2003} > 1.041$$

$$\mathrm{A}_{2003} > 0.041$$

APPENDIX B:

CALCULATION OF EXPONENTIAL FUNCTION OF LAND VALUES

Approximate points:

(0, 100) In 1945, the price of land was 100 billion dollars.

(32, 1000) In 1977, the price of land was 1000 billion dollars.

(35, 2000) In 1980, the price of land was 2000 billion dollars.

(38, 3000) In 1983, the price of land was 3000 billion dollars.

$f(0)/f(32) = (ab^0)/(ab^{32})$
$100/1000 = 1/b^{32}$
$100b^{32} = 1000$
$b^{32} = 10$
$b = 1.081$

$f(0)/f(35) = (ab^0)/(ab^{35})$
$100/2000 = 1/b^{35}$
$100b^{35} = 2000$
$b^{35} = 20$
$b = 1.089$

$f(0)/f(38) = (ab^0)/(ab^{38})$
$100/3000 = 1/b^{38}$
$100b^{38} = 3000$
$b^{38} = 30$
$b = 1.094$

We took the average of the three b-values to get $1.0857 = 1 + \mathrm{A}$.

APPENDIX C:

DERIVATION OF GENERAL FORMULA

$$\mathrm{T} = 75.43\mathrm{V} + \mathrm{P}$$

$$\mathrm{P} = \mathrm{S}_2 + \mathrm{S}_3 + \mathrm{S}_4 + \mathrm{S}_\mathrm{M} \text{ or } \mathrm{P} = \mathrm{T} - 75.43\mathrm{V}$$

$$\mathrm{T} = 75.43\mathrm{V} + \mathrm{S}_2 + \mathrm{S}_3 + \mathrm{S}_4 + \mathrm{S}_\mathrm{M}$$

Since land appreciates uniformly, and non-land assets appreciate at the rate of inflation, the value of the estate in November 2003 is:

$$75.43\mathrm{V}(1 + \mathrm{A})^{58} + (\mathrm{S}_2 + \mathrm{S}_3 + \mathrm{S}_4 + \mathrm{S}_\mathrm{M})(1 + \mathrm{I}_{2003})$$

$$75.43\mathrm{V}(1 + \mathrm{A})^{58} + (\mathrm{T} - 75.43\mathrm{V})(1 + \mathrm{I}_{2003})$$

One-sixth of the estate value today is:

$$\frac{75.43V(1+A)^{58} + (T - 75.43V)(1+I_{2003})}{6}$$

The value of the fourth brother's share today is $\mathrm{S}_4(1 + \mathrm{I}_{2003})$, which we know is less than $\frac{75.43V(1+A)^{58} + (T - 75.43V)(1+I_{2003})}{6}$.

To find his compensation, find the difference between these:

$$\frac{75.43V(1+A)^{58} + (T - 75.43V)(1+I_{2003})}{6} - \mathrm{S}_4(1 + \mathrm{I}_{2003}).$$

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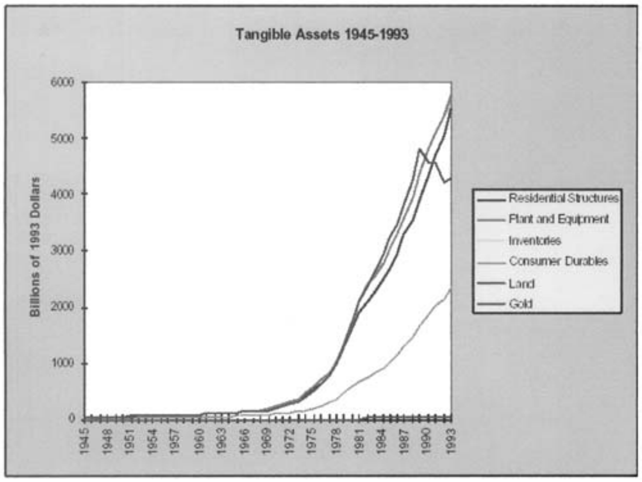

Figure 1. Land Appreciation Trend 1945–1993

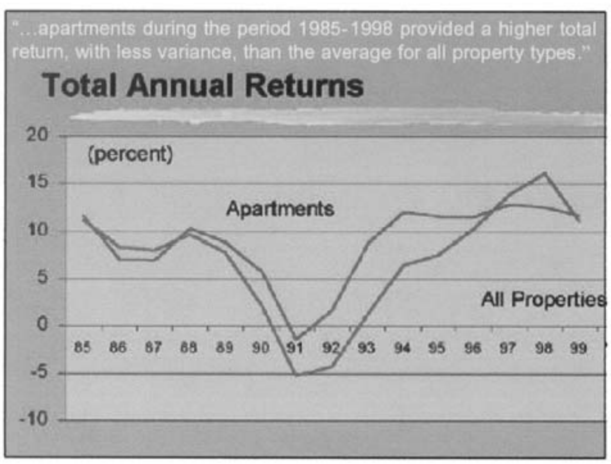

Figure 2. Total Annual Returns on Land 1985–1999

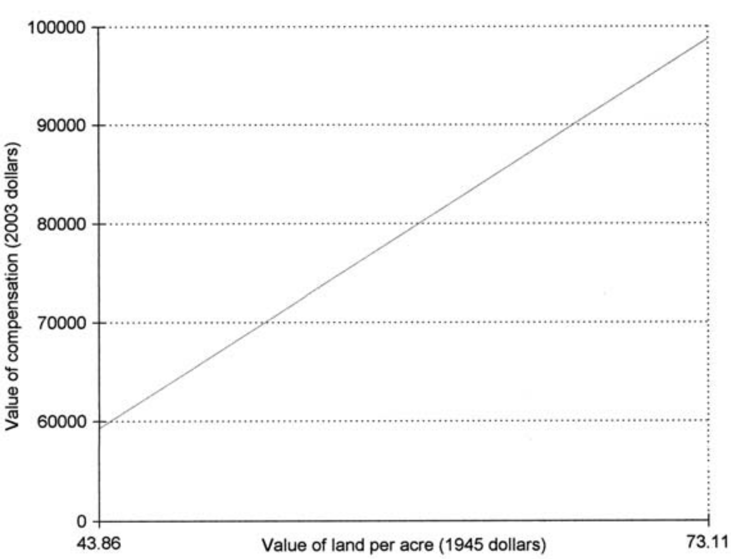

Figure 3. Compensation Range

Problem B Paper: Evanston Township High School

Advisor: Peter DeCraene

Team Members: Erica Cherry, Chris LeBailly, Eli Morris-Heft, Jean Rudnicki

OUR PARAMETERS FOR FAIRNESS

1. Left-handed batters should gain no advantage over right-handed batters. To ensure this, our field is symmetric about a line through home plate and second base.
2. Given a ball speed (V_{BALL}) and angle of elevation (φ), a ball should be a home run no matter the angle (θ) with respect to the foul lines at which it is hit.
3. The field should comply with major league rules and traditions, one of which is that the distance from home plate to the centerfield fence should be more than the corresponding distances down the foul lines. To adhere to this and comply with parameter 2, the fence is lowered in the middle such that a ball that would barely clear the right-field (or left-field) fence would, counting for distance, barely clear the centerfield fence. The fence is smoothly lowered from right field to center and then smoothly raised from center to left accordingly.
4. The current average number of home runs is about two per game. We aim to keep this figure static.
5. We recognize that, even as we strive to achieve parameter 1, there is no way to resolve the fact that a left-handed batter gets a one-step advance towards first base. We believe that the time it takes to step across home plate is negligible, so this discrepancy is not corrected.
6. We want the amount of fair territory to be consistent with the information given in the problem.

PROCEDURE

We constructed a computer simulation that picked a batter of random handedness, threw a random pitch, hit the ball at a random velocity and angle, and determined whether it was a home run. With each simulation of a game, we found the number of home runs. Through research, we found that the number of hits per game is about 20, and we worked with the dimensions of our park until, in accordance with fairness parameter 4, about two home runs were scored per game. We also ranked in fairness the parks given in the problem.

Our simulation variables are summarized in **Table 1** and **Figures 1 and 2**.

The parameter bounds were chosen based on our research. 10% of the general population is left-handed; we were unable to find the statistic for baseball players. The fastball is the pitch from which the most home runs are hit and is also the straightest pitch and the one that flies farthest when hit. The range of θ is derived naturally from the park's shape.

PROCEDURE

We researched equations to describe the flight of a hit baseball based on the speed of the pitch, the speed of the bat, and the initial angle of elevation, taking air friction into account. Our first equation shows how much kinetic energy is transferred to the ball:

$$KE_{BALL} = \frac{1}{3}(KE_{PITCH} + KE_{BAT}) \quad (1)$$

Only a third of the energy is transferred—the rest goes to vibration of the bat and to friction. Replacing KE with $\frac{1}{2}MV^2$, we have:

$$\frac{1}{2}M_{BALL}V_{BALL}^2 + \frac{1}{3}\left(\frac{1}{2}M_{BALL}V_{PITCH}^2 + \frac{1}{2}M_{BAT}V_{BAT}^2\right) \quad (2)$$

$$V_{BALL}^2 = \frac{1}{3}\left(V_{PITCH}^2 + \frac{M_{BAT}}{M_{BALL}}V_{BAT}^2\right) \quad (3)$$

$$V_{BALL} = \sqrt{\frac{1}{3}\left(V_{PITCH}^2 + \frac{M_{BAT}}{M_{BALL}}V_{BAT}^2\right)} \quad (4)$$

Thus we have an equation for the velocity of a hit ball.

Since we are designing the park for the real world, we have to consider air friction. Taking $\vec{v}$ to be the velocity vector of the ball (note that V_{BALL} is a scalar) and $\vec{f}$ as the drag vector, we have:

$$\vec{f} = -D\left|\vec{v}\right|\left(\vec{v}\right) \quad (5)$$

We have a negative on the right side because $\vec{f}$ is contrary to the motion of the ball. Breaking $\vec{f}$ into component vectors, we get:

$$\vec{f} = \left\langle \vec{f_x}, \vec{f_y} \right\rangle = \left\langle -D\left|\vec{v}\right|\left(\vec{v_x}\right), -D\left|\vec{v}\right|\left(\vec{v_y}\right) \right\rangle \quad (6)$$

Totaling the sum of the forces acting on the ball:

$$\sum F_x = -D\left|\vec{v}\right|\left(\vec{v_x}\right) = m\vec{a_x} \text{ and } \sum F_y = -mg - D\left|\vec{v}\right|\left(\vec{v_y}\right) = m\vec{a_y} \quad (7)$$

$$\vec{a_x} = \frac{D}{m}\left|\vec{v}\right|\left(\vec{v_x}\right) \text{ and } \vec{a_y} = -g - \frac{D}{m}\left|\vec{v}\right|\left(\vec{v_y}\right) \quad (8)$$

D is a constant dependent on air density (ρ), the ball's silhouette surface area (A), and the drag coefficient (C):

$$D = \frac{\rho CA}{2} \quad (9)$$

We researched ρ for each city in which we were given and for Evanston, where we located our ideal park. The silhouette surface area is easily determined because major league regulations state that the ball "shall…measure not less than nine nor more than $9\frac{1}{4}$ inches in circumference." Taking the circumference as $9\frac{1}{8}$ inches, A is $\frac{5329}{256\pi} \approx 6.626$ in^2. We modeled C on a graph in *The Physics of Baseball*. We tried many regression curves and found that the best fit was the cubic:

$$C(V_{BALL}) = (-7.773173 \times 10^{-5})V_{BALL}^3 + 0.013016 \times V_{BALL}^2 - 0.726969 \times V_{BALL} + 0.999441 \quad (10)$$

In order to derive a function for the position of the ball at time t, we split the forces acting on the ball into x- and y-components. To

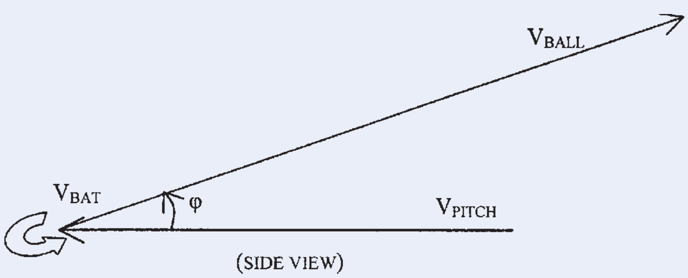

Figure 1.

Handedness of Batter	90% right; 10 % left
Velocity of Pitch (V_{PITCH})	90 ± 5 mph
Velocity of Bat (V_{BAT})	71 ± 2 mph
Initial Angle of Elevation of Hit Ball (φ)	35° ± 5°
Angle of Hit Ball with respect to line A (θ)	–45° to 45°

Table 1. Simulation variables

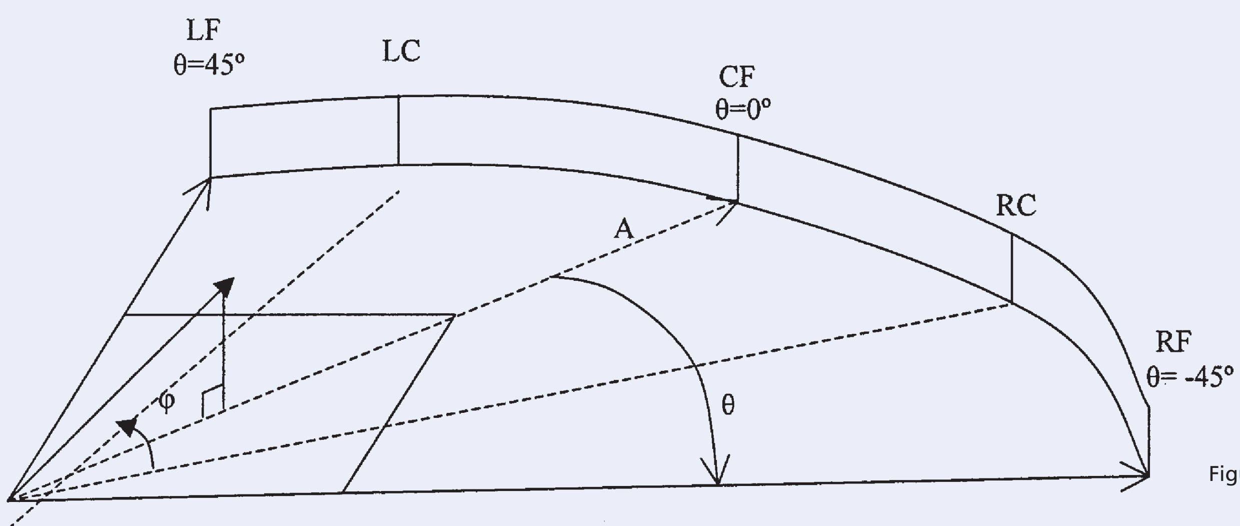

Figure 2.

derive the x-portion of the ball's position, we started with the equation:

$$F_x = ma_x \tag{11}$$

We also know that:

$$F_x = -kv_x^2 = ma_x = m\frac{dv_x}{dt} \tag{12}$$

because the only force acting on the ball in the x-direction is air friction. We are using k to represent the drag coefficient (about 0.0007). Using symbol manipulation, we find:

$$\frac{1}{v_x^2}dv_x = \frac{k}{m}dt \tag{13}$$

Integrating both sides, we get:

$$\int_{v_{0x}}^{v_x(t)} \frac{1}{v_x^2}dv_x = \int_0^t -\frac{k}{m}dt \tag{14}$$

$$\frac{1}{v_x(t)} - \frac{1}{v_{0x}} = \frac{k}{m}t \tag{15}$$

A little more manipulation gives us:

$$v_x(t) = \left(\frac{k}{m}t + \frac{1}{v_{0x}}\right)^{-1} \tag{16}$$

This gives us the velocity of the ball in the x-direction at time t. We also want the position at time t, which we derive by integrating:

$$p_x(t) = \int v_x(t)dt = \int\left(\frac{k}{m}t + \frac{1}{v_{0x}}\right)^{-1}dt = \frac{m}{k}\ln\left|kv_{0x}t + m\right| \tag{17}$$

In the y-direction, equations must take into account both air friction and gravity. We can start the same way as with the x-direction:

$$F_y = g - kv_y^2 = ma_y = m\frac{dv_y}{dt} \tag{18}$$

We can make this into a first-order differential equation by dividing m from the second and fourth terms in equation (18):

$$\frac{dv_y}{dt} = \frac{g - kv_y^2}{m} \tag{19}$$

Using equation (19), we formed a slope field and used the initial condition $v_y = 52.45 \sin(35°)$ to plot a solution to the differential equation. We took points from this curve and did a polynomial regression:

$$v_y(t) = -5.471729x^4 + 0.032844x^3 - 0.432209x^2 - 7.650204x + 52.326 \tag{20}$$

Thus, position function for the y-direction is:

$$v_y(t) = \int v_y dt \approx -1.094346 \times 10^{-4}t^5 + 0.008211t^4 - 0.14407t^3 - 3.825102t^2 + 52.326t + 1 \tag{21}$$

If we combine equations (17) and (21), we can find equations for the position and velocity of the ball at time t:

$$\vec{p}\left\langle\overrightarrow{p_x(t), p_y(t)}\right\rangle = \left\langle\overrightarrow{\frac{m}{k}\ln\left|kv_{0x}(t) + m\right|, -1.094346\times10^{-4}t^5 + 0.008211t^4 - 0.14407t^3 - 3.825102t^2 + 52.326t + 1}\right\rangle \tag{22}$$

$$\vec{v}\left\langle\overrightarrow{v_x(t), v_y(t)}\right\rangle = \left\langle\overrightarrow{\left(\frac{k}{m}t + \frac{1}{v_{0x}}\right)^{-1}, -5.471729x^4 + 0.032844x^3 - 0.432209x^2 - 7.650204x + 52.326}\right\rangle \tag{23}$$

We used a computer simulation to calculate the position of the ball. The simulation uses a fourth-order Runge-Kutta method. Using the changing acceleration and velocity vectors, it calculates each position of the ball based on the previous. As this method only works with first-order differential equations, the acceleration vector presents a problem. We resolved the problem by substituting in such a way that the acceleration vector's second-order equation became a system of first-order equations.

By using equation (8) to find the acceleration in both the x- and y-directions, it was possible to use Runge-Kutta to solve the system. Each time the Runge-Kutta algorithm was called, it moved the baseball forward one time step. Since there was a change in velocity in this step, it was necessary to recalculate the drag coefficient according to equation (10). In this simulation, the time step was 0.05 seconds. The Runge-Kutta algorithm was iterated until y-displacement was negative, meaning the ball had hit the ground.

When the Runge-Kutta method is called, the position and velocity vectors are stored in a matrix. We wanted to find the ball's height at a given horizontal displacement. To do this, the program searched the matrix and interpolated between the point with x-displacement just greater than what we wanted and the point with x-displacement just less than what we wanted.

This algorithm was very useful for figuring out if a hit was a home run. If we knew how far the ball had to travel to reach the wall, we used the previous algorithm to find the horizontal displacement. If the vertical distance was greater than the height of the wall, then the hit was a home run.

The next task was to find the distance from home plate to the wall. We knew the distance from home plate to left field, left centerfield, centerfield, right centerfield, and right field. Since we did not know what function modeled the shape of the wall, we assumed that it was linear. We broke it up into even sections, each with a range of 18° **(Figure 3)**.

Determining the value of θ depended on the batter's handedness. Right-handers tend to hit the ball towards left center, and left-handers tend to hit it towards right center. A Gaussian distribution was created to randomly pick the direction the ball was hit. For right-handers, the mean was $\theta = 22.5°$ (left centerfield) and for left-handers, the mean was $\theta = -22.5°$ (right centerfield). For both of these distributions, the standard deviation was set to 5°. A random number generator picked values according to the normal curve. These values were then altered to fit the appropriate distribution.

To find the initial speed of a hit ball, we had to know the speed at which it was pitched and how fast the batter swung. From these, we used equation (4) to determine the speed of the ball off the bat. We knew that the average fastball is thrown at about 90 miles per hour. From the Gaussian distribution (with a mean of 90 and a standard deviation of 5), we randomly selected the speed of the

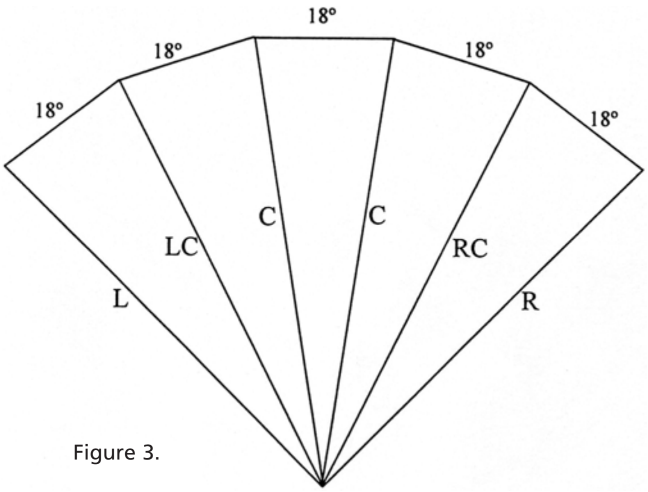

Figure 3.

pitch that fit the described distribution. Using the same method (with a mean of 71 and a standard deviation of 2), we found the bat speed. To calculate the angle of elevation (φ) we used the same method (with a mean of 35 and a standard deviation of 5). It was then possible to model the trajectory of the ball.

Given the distance from the fence to home plate, the simulation calculated the height of the ball and decided whether it was a home run, which only happens if the height of the ball is greater than the fence. However, the park specifications only include distances for left field, left centerfield, centerfield, right centerfield, and right field. We needed a way to transition smoothly from one given distance to the next. Though our park has a curved back wall, we assumed it was linear between points for simplicity.

In **Figure 4,** a and b are two of the six lengths given for each park. Let the third side of the triangle be c. The angle marked by the arrow is 22.5° because the fair territory is 90° and is divided into 5 sections (Figure 3). Let the distance to the fence be y (Figure 4). Let α be a varying angle, thus determining y. Note also angle β opposite side b. By the law of cosines:

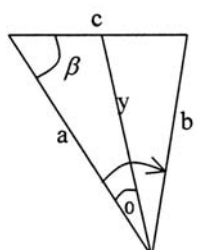

Figure 4.

$$c = \sqrt{a^2 + b^2 - 2ab\cos(22.5°)} \tag{24}$$

By the law of sines:

$$\beta = \arcsin\left(\frac{b\sin(22.5°)}{c}\right) \tag{25}$$

By the law of sines:

$$\frac{y}{\sin(\beta)} = \frac{a}{\sin(180° - \beta - \alpha)} \tag{26}$$

Solving for y gives us:

$$y = \frac{a\sin(\beta)}{\sin(180° - \beta - \alpha)} \tag{27}$$

We were able to use equations (24), (25), and (27) to find intermediate distances. These three equations were entered into the simulation to define where the wall was so the computer could designate whether a hit was a home run.

We then had to determine the height of the wall so that the probability of hitting a home run was independent of θ. After looking at various major league fields, we decided that the distance to the wall along each foul line should be 350 feet and that the distance to the centerfield wall should be 400 feet. We then needed an equation for the distance to the wall as a function of θ. We decided that the shape of the outfield wall should be an ellipse and used the polar equation of an ellipse:

$$r = \frac{2800}{\sqrt{30\sin^2\theta + 49}} \tag{28}$$

We needed to keep a few things constant. We wanted the wall height to be at least 8.5 feet at all angles to prevent outfielders from reaching over the wall. We set the ball speed and angle of elevation so that the ball just cleared the fence at all values of θ. To do this, we determined that the speed of the ball just after being hit should be 53.45 m/s and the angle of elevation should be 40°. After setting these constants, we ran the simulation.

According to our fairness parameters, a hit that just cleared the wall at one angle should just clear the wall at every other angle. Therefore, we used the simulation to determine the largest wall height that still allows a home run to clear the wall. We ran the simulation at 1° increments of θ between –45° and 45°. We then used the data and regression to find an equation for the wall height in meters in terms of θ:

$$Height(\theta) = 10.412518\sin(3.304047\theta - 1.570796) + 13.082055 \tag{29}$$

In order to get the area of fair territory, we took an integral of the ellipse:

$$\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{4}} Height(\theta)^2 d\theta = 113873.328773 \text{ square feet} \tag{30}$$

The results in **Tables 2 and 3** show that the park we designed was the fairest. The difference between the percentages of right- and left-handed home runs was the smallest. It is also a symmetric field about the line from home plate to centerfield. In accordance with our second parameter, the probability of hitting a home run is indeed independent of θ because of the varying wall height. Any ball that is a home run at one angle would be a home run at any other angle. Originally, we wanted our average home runs per game to be about 2. In our simulation, it was closer to 1. However, our simulation assumes that all pitches are fastballs, so our estimate for the number of hits per game is probably low. Thus, we feel that we have achieved that parameter. Our last parameter was that our park should have an area of fair territory that is comparable to other parks. Our park is in the median range of the parks given in the problem.

The Braves' park had the second-smallest discrepancy between the right- and left-handers. However, this park had the third highest number of home runs per game, so it is an easy park whether the batter is left-handed or right-handed.

The Marlins' park had a higher discrepancy between left- and right-handers, but at least their number of home runs per game

Team	% of hits that are home runs	Number of home runs per game	% right-handed hits that are home runs	% left-handed hits that are home runs
Angels	39.97	7.994	35.63	78.88
Braves	27.88	5.576	26.69	39.59
Rockies	63.65	12.73	60.17	95.09
Yankees	21.05	4.21	19.34	37.42
Twins	23.99	4.798	18.82	70.64
Marlins	20.82	4.164	19.05	35.69
Evanston	5.53	1.106	5.51	5.69

Table 2. Data for First Simulation Run

Team	% of hits that are home runs	Number of home runs per game	% right-handed hits that are home runs	% left-handed hits that are home runs
Angels	39.7	7.94	35.29	79.66
Braves	27.39	5.478	25.86	40.74
Rockies	62.80	12.56	59.13	96.91
Yankees	21.24	4.248	19.54	37.21
Twins	24.23	4.846	19.47	68.06
Marlins	21.43	4.286	19.70	36.76
Evanston	5.41	1.082	5.37	5.75

Table 3. Data for Second Simulation Run

was closer to the league average of 2. The Yankees' park's fairness was similar to the Marlins'.

The Rockies' park had the third highest advantage for left-handers. However, the percentage of left-handed hits that were home runs was 95.09% and 96.91%; it should not be that easy for batters to hit home runs. In addition, the number of home runs per game was significantly above the league average of 2. Therefore, this park was one of the least fair.

The Twins' park and then the Angels' had the largest advantage for left-handers, making both very unfair. The Angels' park had more home runs per game, so it was slightly less fair than the Twins', but both were significantly less fair than Evanston.

Figure 5 is a diagram of our ideal park.

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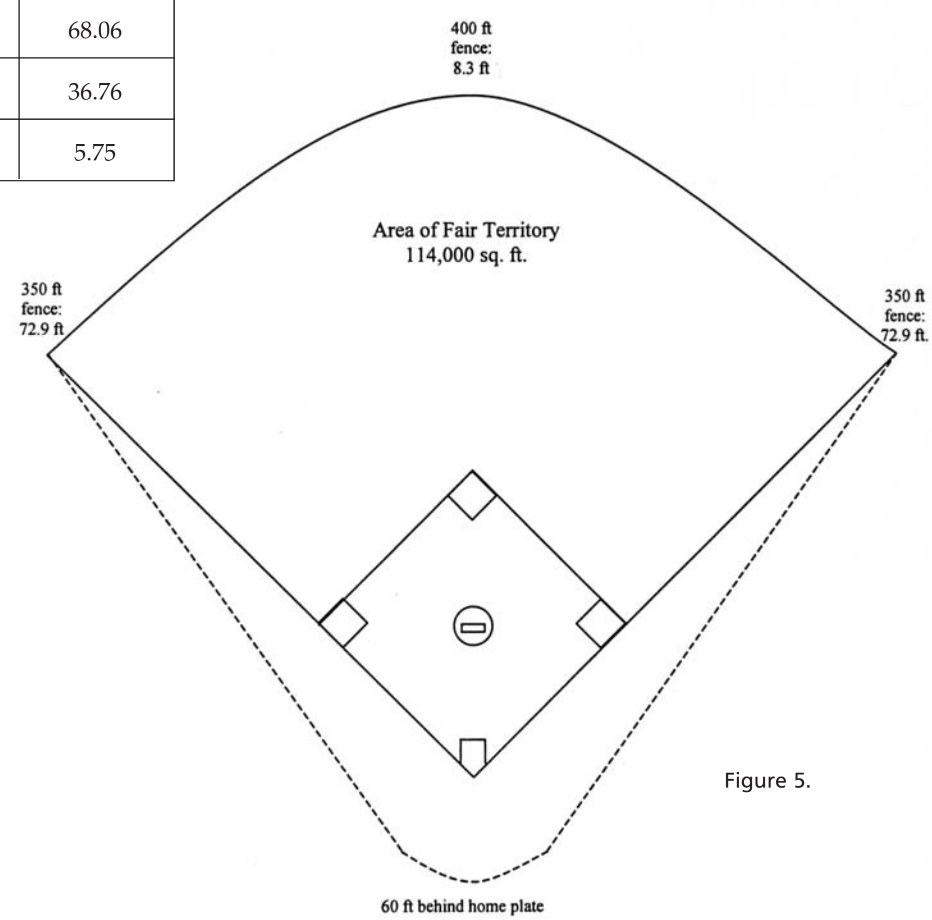

Figure 5.

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Problem A Paper: Arkansas School for Mathematics and Sciences

Advisor: Bruce Turkal

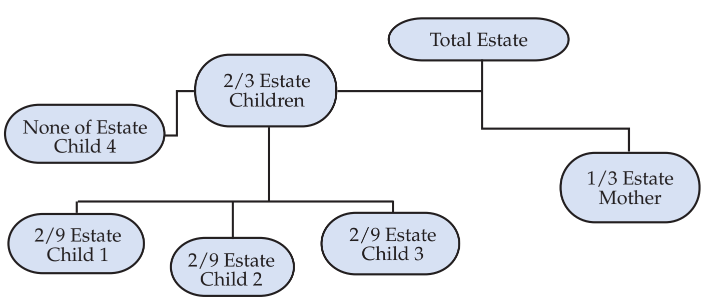
Team Members: Katherine Herring, Audrey Morris, Alex Wong, Johnson Wong

RESTATEMENT OF THE PROBLEM:

In 1945, the death of Noah Sentz resulted in the division of his estate among his wife and four children. According to state law, one-third of his property and assets went to his spouse and two-thirds went to his children. His estate mostly consisted of 75.43 acres of land. From 1946 to 1949, three of the children sold their shares back to their mother for $1,300 each. In the process of distributing the assets, his fourth child was left out under some unknown circumstances. This week the fourth child filed a lawsuit against the estate for his original inheritance from the probate case. The judge has ruled that the son shall receive his inheritance in the form of monetary payment. Our objective is to decide how much the fourth child will rightfully receive.

ASSUMPTIONS AND JUSTIFICATIONS:

- The problem occurs in the continental United States because the amount for which the siblings sold their assets is given in dollars.
- No claims have been filed against the estate.
- The fourth child was ignored in the distribution of the assets, and the three children who sold their land received two-ninths of the land, which was worth $1,300 each. If the land had been distributed appropriately, then each child would have received one-sixth of the total assets. This would make each lot 12.5717 acres and worth $975.



- The jury does not decide who pays the compensation.
- Assets other than the 75.43 acres are negligible.
- The federal estate tax was paid before the assets were distributed.
- Inheritance tax is not taken into account because:

  a. The $1,300 that each of the other three children sold back to the mother was assumed to be the actual value of two-ninths of the land. The calculation of current value of the assets was directly based on this value.

  b. It is taken after compensation is awarded. Therefore, the jury does not need to take it into consideration.

Solutions 1 and 2:

- Farmland value increase was a representation of the increase in value of the inherited land since 1946 to 1949.
- The land was unaltered from its state in 1945.

Solution 3:

- The fourth child received one-sixth of the assets, 12.57166667 acres.
- The land would have been sold for $975 between 1946–1949.
- The $975 was held in a bank account accumulating interest.
- The interest on the account compounds yearly.
- The interest rate on the account changes every year, following the average values for interest.
- Interest for 2003 has already been accrued.

Solution 4:

- The estimated 2003 value was assumed to be correct due to past trends.

MODEL:

We decided that there are four possible ways to calculate the amount of compensation for the fourth son. Solution 1 calculates the current value of the land based on a ratio of the land values in the 1940s to that in 2003. Solution 2 calculates the present value of the acreage that the fourth child would have received. Solution 3 awards him the worth of one-sixth of the land in 1949 plus the interest accrued. Solution 4 gives him the worth of the land in 1949 plus inflation.

Solution 1:

First, the average value of an acre of land for the entire United States from 1946 to 1949 according to the Economic Research Service was calculated. Next, the average value of an acre of land for 2003 according to the Economic Research Service was divided by the average price per acre to obtain a conversion factor for the increased value of land. Then the value of the fourth child’s assets was multiplied by the conversion factor to acquire a current value of the son's land, resulting in an equation:

$$V_{Total} = \left[\frac{P(y)}{\frac{P(y_1) + P(y_2) + P(y_3) + P(y_4)}{4}} \right] * F$$

where $P(y)$ is an average value of an acre of land for each year, y is the year, F is the fourth child's asset value, and V_{Total} is the total current value of the sons' land.

$$V_{Total} = \left[\frac{P(03)}{\frac{P(46) + P(47) + P(48) + P(49)}{4}} \right] * F$$

$$= \left[\frac{1270}{\frac{53 + 60 + 64 + 66}{4}} \right] * 975$$

$$= \left[\frac{1270}{60.75} \right] * 975$$

$$= 20382.72$$

Solution 2:

The average value per acre of land in 2003 according to the Economic Research Service was multiplied by the fourth son's share of the land, which was one-sixth of the total assets. The result is a current value of his land, which could be expressed as an equation: $V_{Total} = \mathrm{P}(y) * L$, where V_{Total} is the total current value of the son's land, $P(y)$ is an average value of an acre of land for the United States in 2003, and L is the fourth son's share of land in acres.

$$V_{Total} = \mathrm{P}(03) * L$$

$$= 1270 * 12.57166667$$

$$= 15966.02$$

Solution 3:

This solution is a calculation of the compensation according to the 1946 to 1949 value of land, which is \$975, with the interest that would have accrued had the son received and sold his property at the same time as his three siblings. Since the interest was assumed added yearly, a slightly modified version of the equation below was used:

$$F = P(1 + i)^n$$

where F is the amount in the account after interest, P is the amount in the account at the first of the interest period, i is the interest rate, and n is the number of periods. The following is the modified version of the equation:

$$F_n = F_{n-1}(1 + i)$$

where F_n is the amount after interest and i is the interest rate. F_0, the initial amount in the account, is \$975. A program was written to take the interest rates from 1946 to 2003 to determine the final amounts in an account that could have started in 1946, in 1947, in 1948, or in 1949. The program then averaged these four values, returning the value \$13,972.10.

Solution 4:

This solution used the 1946 to 1949 consumer price index conversion factors to estimate his inheritance from the 1940s in 2003 dollars. The conversion values we found used 2003 as the base for the other conversion factors, meaning the conversion factor for 2003 is 1.000 and the other factors were based on this value. The 1946 to 1949 values were averaged to give 0.12175. We divided what his assets would have been in 1946 to 1949, \$975, by this number to yield \$8008.21 as his compensation.

DISCUSSION:

For the first two solutions, the use of farmland values was inaccurate due to the unknown location and type of the estate's land. Although Solution 1 uses a ratio of increase in land values, making it better for estimation than a straight calculation of present value as in Solution 2, it is unlikely that all types of land change at the same rate. Even in the information found for only farmland, regional changes from 1945 to 2003 varied widely. Solution 2 assumes that the fourth son would have kept the land and sold it for present value. Because the other three children sold their properties back to the mother, it is doubtful that the fourth son would have kept his share only to sell it after fifty-eight years. Once again, the land value varies regionally. If the son had been able to sell his land along with his other siblings, it is very likely that he would have put the money in a bank, as modeled by Solution 3. The interest rates used were average short-term yearly rates, but he may have put the money in an account that either compounds more or less often. The fourth scenario is improbable; the fourth child would have at least invested the money, instead of letting it depreciate.

Through this analysis, it was concluded that Solution 3 is the fairest compensation. Solution 3 emulates what the son probably would have done if he had received his inheritance at the proper time. \$13,972.10 should be awarded to the son as his rightful inheritance.

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November 10, 2003

Dear Judge Robinson:

The jury has made a decision after much deliberation. This letter is intended to explain the jury’s method for determining the appropriate amount bestowed upon the son of the deceased Noah Sentz. The sum that will be given to Mr. Sentz should be $13,972.10.

We came up with four different methods to determine the proper compensation. The first method calculates the value of the land using a designed ratio as a conversion factor of increase to assess the current value for the property, based on the prices given for the land in acres. The second method calculates the present value of the land that he would have received using current average value per acre of land in the United States. The third method awards the beneficiary his rightful share of the initial value of the land plus interest that would have been accrued. The fourth method gives the beneficiary his legal portion of the initial value of the estate taking inflation into account. Through the analysis of these processes, we concluded that the third method gives the beneficiary the fairest compensation. The third method emulates what the most probable actions the beneficiary would have taken had he received his inheritance at the proper time. A program was created to simulate the accumulation of interest with a changing rate over a 58-year period. The solution produced by the program showed the appropriate compensation that should be granted to the beneficiary.

Respectfully,

The Jury