

Math 19B: Practice Problems for the Midterm

1. Compute the following indefinite integrals:

$$\begin{array}{lll} \text{(a)} \int \frac{\arctan(2x)}{1+4x^2} dx & \text{(e)} \int x \tan^2(x) dx & \text{(i)} \int \frac{x}{\sqrt{1-x^2}} dx \\ \text{(b)} \int \sin^5(x) \cos^3(x) dx & \text{(f)} \int \sin^2(x) \cos^2(x) dx & \text{(j)} \int \frac{x^2}{\sqrt{1-x^2}} dx \\ \text{(c)} \int \frac{t^5}{\sqrt{t^2+4}} dt & \text{(g)} \int \frac{x^3+4}{x^2+4} dx & \text{(k)} \int \cos(x) \ln(\sin(x)) dx \\ \text{(d)} \int \sec^4(x) \tan^4(x) dx & \text{(h)} \int \sec^5(w) \tan^3(w) dw & \text{(l)} \int \frac{\arctan(x)}{x^2} dx \end{array}$$

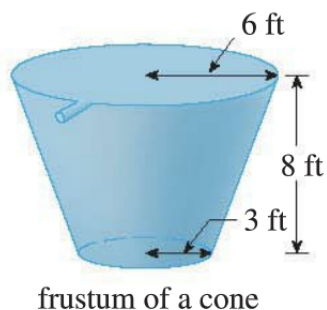
2. Compute the following definite integrals:

$$\begin{array}{lll} \text{(a)} \int_0^2 \frac{e^x}{1+e^{2x}} dx & \text{(c)} \int_{-1}^1 \frac{y}{e^{2y}} dy & \text{(e)} \int_0^1 \frac{x^2-5x+16}{(2x+1)(x-2)^2} dx \\ \text{(b)} \int_0^\pi \sin^5(x) \cos(x) dx & \text{(d)} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} t \cos^2(t) dt & \text{(f)} \int_{-1}^1 \frac{1}{\sqrt{1+x^2}} dx \end{array}$$

3. Let \mathcal{R} be the region in the first quadrant bound by the curves $y = x^3$ and $y = 2x - x^2$. Calculate the following quantities.

- The area of \mathcal{R} .
 - The volume obtained by rotating \mathcal{R} about the x -axis.
 - The volume obtained by rotating \mathcal{R} about the y -axis.
 - Let \mathcal{R} be the base of a solid. Cross-sections perpendicular to the x -axis are squares. Find the volume of this solid.
4. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bound by $y = \sqrt{x}$ and $y = x^2$ about $y = 2$.
5. A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm. How much work is done in stretching the spring from 12 cm to 20 cm?
6. A 1600-pound elevator is suspended by a 200-ft cable that weighs 10 pounds/foot. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 feet?
7. A circular swimming pool has a diameter of 24 feet, the sides are 5 feet high, and the depth of the water is 4 feet. How much work is required to pump all the water out over the side? (Use the fact that water weighs 62.5 lbs/cubic foot)

8. A tank (pictured below) is full of water. Find the work required to pump the water out of the spout. (Use the fact that water weights 62.5 lbs/cubic foot).



9. Find the derivative of the following functions:

(a) $F(x) = \int_0^x \frac{t^2}{1+t^3} dt$

(b) $G(x) = \int_x^1 \sqrt{t + \sin(t)} dt$

(c) $H(x) = \int_0^{x^4} \cos(t^2) dt$

10. If $\int_0^6 f(x) dx = 10$ and $\int_0^4 f(x) dx = 7$ find $\int_4^6 f(x) dx$

11. The velocity of a particle is given by $v(t) = t^2 - 4$ when $0 \leq t \leq 3$.

(a) Find the displacement of this particle from $t = 0$ to $t = 3$.

(b) Find the distance traveled by this particle from $t = 0$ to $t = 3$.

12. Does $\int_0^\infty e^{-x} dx$ converge? If so, what is its value?