Math 19B - Final Exam

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Notes: Show your work. In other words, just writing the answer, even if correct, may not be sufficient for full credit. Scientific calculators are allowed, but no programmable and/or graphing calculators. Make sure to give exact answers and simplify your answers when possible. And please put away your cell phones and other electronic devices, turned off or in airplane mode.

Your Name: _____

Problem 1:	out of 40
Problem 2:	out of 30
Problem 3:	out of 20
Problem 4:	out of 20
Problem 5:	out of 10
Problem 6:	out of 20
Problem 7:	out of 20
Problem 8:	out of 20
Problem 9:	out of 10
Problem 10:	out of 10
Total:	**out of 200**

Good luck and enjoy the rest of your summer!

1. *(40 points - 10 each)* Evaluate the following integrals:

(a) $\displaystyle\int_0^{\frac{\sqrt{\pi}}{2}} x\cos(x^2)\,dx$

(b) $\displaystyle\int x\sin x\,dx$

(c) $\int \left[t^3 - e^t + \cos(2t) + \frac{1}{t} - \frac{1}{1+t^2} \right] \, dt$

(d) $\int_0^{\frac{\pi}{3}} \sin^2 \theta \; d\theta$

2. *(30 points - 15 each)* Evaluate the following integrals:

(a) $\displaystyle\int \frac{5x^2 + x + 3}{x^3 + x}\, dx$

(b) $\displaystyle\int \frac{x^2}{\left(\sqrt{4 - x^2}\right)^3}\, dx$

3. *(20 points)* Using integration (either the method of washers or the method of shells), verify that the volume of a sphere with radius r is

$$V = \frac{4}{3}\pi r^3$$

Hint: The equation of a circle with radius r, centered at the origin is

$$x^2 + y^2 = r^2$$

4. (a) *(10 points)* Show that $\displaystyle\int_1^\infty \frac{x}{e^x}\,dx$ converges by finding its value.

(b) *(10 points)* Show that $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$ converges.

(c) *(Extra Credit, 5 points)* Find the sum of the infinite series in (b).

5. *(10 points)* If the **work** required to stretch a spring 1 foot beyond its natural length is 12 foot-pounds, how much work is needed to stretch it 9 inches beyond its natural length?

6. *(20 points - 10 each)* Test for absolute convergence, conditional convergence or divergence for the following series. Be sure to indicate which test you are using, explain why you can use it, and state your conclusion.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+\sqrt{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n^2)}{1+n^2}$

7. *(20 points - 10 each)* Find a power series representation for the following functions using any method you like. Write your answer using sigma notation. You do **not** need to find the radius and interval of convergence.

   (a) $f(x) = \ln(1 + x)$

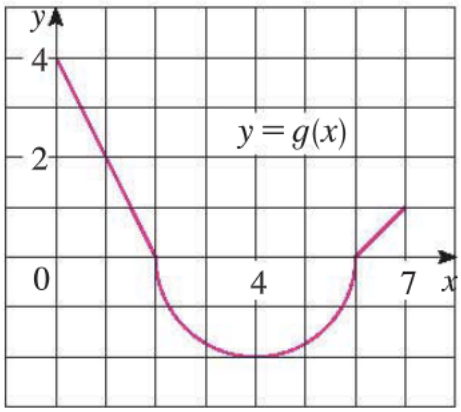
   (b) $g(x) = x \cdot \sin(2x)$

8. *(20 points - 10 each)* Find the radius and interval of convergence of the following power series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+4)^n}{n \cdot 7^n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)!(3x)^n}{(2n)!}$

9. *(10 points - 2 each)* The graph of g consists of two straight lines and a semi-circle, shown below. Let $f(x) = \int_2^x g(t)\,dt$.



Compute:

(a) $f(6)$

(b) $f(2)$

(c) $f(0)$

(d) $f'(1)$

(e) $f'(4)$

10. *(10 points - 5 each)* Let $\mathcal{C}$ be the curve $y = 1 + e^x$ from $x = 0$ to $x = 1$.

(a) Set up, but do **not** evaluate, an integral to find the arc length of $\mathcal{C}$.

(b) Setup, but do **not** evaluate, an integral to find the surface area of the object obtained by revolving $\mathcal{C}$ around the x-axis.

(c) *(Extra Credit, 5 points)* Compute the arc length of $\mathcal{C}$. The integral from part (a) must be set up correctly.