1 a) $u = \arctan(2x)$ $\qquad du = \dfrac{2}{1+4x^2}dx$

$$\frac{1}{2}\int \frac{2\arctan(2x)\,dx}{1+4x^2} = \frac{1}{2}\int u\,du = \frac{1}{4}u^2$$

$$= \boxed{\frac{1}{4}(\arctan(2x))^2 + C}$$

b) $$\int \sin^5(x)\cos^2(x)\cdot\cos(x)\,dx = \int \sin^5(x)\cdot(1-\sin^2 x)\cos(x)dx$$

$$= \int u^5(1-u^2)\,du = \int (u^5 - u^7)\,du$$

$[u = \sin(x) \quad du = \cos(x)dx]$

$$= \frac{1}{6}u^6 - \frac{1}{8}u^8 = \boxed{\frac{1}{6}\sin^6(x) - \frac{1}{8}\sin^8(x) + C}$$

Another method (or way to do this) is

$$\int \sin^5(x)\cos^3(x)dx = \int (\sin^2 x)^2\cos^3 x\cdot\sin x\,dx$$

$$= \int (1-\cos^2 x)^2\cos^3 x\cdot\sin x\,dx \qquad [u = \cos(x) \quad du = -\sin(x)dx]$$

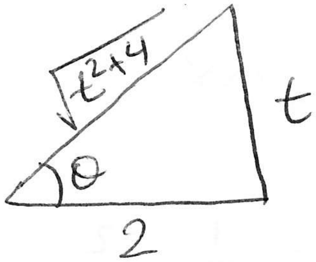
$$= -\int (1-u^2)^2\cdot u^3\,du = -\int [(1-2u^2+u^4)\cdot u^3]\,du$$

$$= \int (-u^3 + 2u^5 + u^7)\,du = -\frac{1}{4}u^4 + \frac{1}{3}u^6 + \frac{1}{8}u^8$$

$$= \boxed{-\frac{1}{4}\cos^4(x) + \frac{1}{3}\cos^6(x) + \frac{1}{8}\cos^8(x) + C}$$

(personally I prefer the first way)

c)



$2\tan\theta = t$, so $2\sec^2\theta\, d\theta = dt$

$2\sec\theta = \sqrt{t^2+4}$

$$\int \frac{t^5}{\sqrt{t^2+4}}\,dt = \int \frac{2^5 \cdot \tan^5\theta}{2\cdot\sec\theta} \cdot 2\sec^2\theta\, d\theta$$

$$= 32\int \tan^5\theta \cdot \sec\theta\, d\theta = 32\int (\tan^2\theta)^2 \cdot \tan\theta \cdot \sec\theta\, d\theta$$

$$= 32\int (\sec^2\theta - 1)^2 \tan\theta \sec\theta\, d\theta = 32\int (u^2-1)^2\, du$$

$[u = \sec\theta,\ du = \sec\theta\tan\theta\, d\theta]$

$$= 32\int (u^4 - 2u^2 + 1)\, du = 32\left[\frac{1}{5}u^5 - \frac{2}{3}u^3 + u\right]$$

$$= \frac{32}{5}\sec^5\theta - \frac{64}{3}\sec^3\theta + 32\sec\theta$$

$$= \boxed{\frac{1}{5}\left(\sqrt{t^2+4}\right)^5 - \frac{8}{3}\left(\sqrt{t^2+4}\right)^3 + 16\sqrt{t^2+4} + C}$$

d) $\int \sec^4(x)\tan^4(x)\,dx = \int \sec^2(x)\tan^4(x)\sec^2(x)\,dx$

$= \int (\tan^2(x) + 1)\tan^4(x)\sec^2(x)\,dx$ $[u = \tan(x),\ du = \sec^2 x\, dx]$

$= \int (u^2+1)u^4\, du = \int (u^6 + u^4)\, du = \frac{1}{7}u^7 + \frac{1}{5}u^5$

$$= \boxed{\frac{1}{7}\tan^7(x) + \frac{1}{5}\tan^5(x) + C}$$

e) $\int x\tan^2(x)\,dx = \int x(\sec^2 x - 1)\,dx = \int x\sec^2 x\,dx - \int x\,dx$

Use parts for $\int x\sec^2 x\,dx$, where $u = x \quad dv = \sec^2 x\,dx$
$du = dx \quad V = \tan x\,dx$

$\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx = x\tan(x) - \ln|\sec x|$, so

$\int x\tan^2(x)\,dx = \boxed{x\cdot\tan(x) - \ln|\sec x| - \frac{1}{2}x^2 + C}$

f) $\int \sin^2(x)\cos^2(x)\,dx = \int \left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right)dx$

$= \frac{1}{4}\int [1-\cos^2(2x)]\,dx = \frac{1}{4}\int \sin^2(2x)\,dx = \frac{1}{4}\int \frac{1-\cos(4x)}{2}\,dx$

$= \frac{1}{8}\int [1-\cos(4x)]\,dx = \boxed{\frac{x}{8} - \frac{1}{32}\sin(4x) + C}$

g)
$$\begin{array}{r|l} & x \\ x^2+4 & x^3 + 0x^2 + 0x + 4 \\ & x^3 + 0x^2 + 4x \\ \hline & -4x + 4 \end{array}$$

$\frac{x^3+4}{x^2+4} = x + \frac{-4x+4}{x^2+4}$

$= x - \frac{4x}{x^2+4} + \frac{4}{x^2+4}$

$\int \frac{x^3+4}{x^2+4}\,dx = \int \left[x - \frac{4x}{x^2+4} + \frac{4}{x^2+4}\right]dx$

$= \frac{1}{2}x^2 - 2\int \frac{2x}{x^2+4}\,dx + \int \frac{\cancel{4}}{[(\frac{x}{2})^2+1]\cancel{4}}\,dx$

$= \frac{1}{2}x^2 - 2\ln|x^2+4| + \arctan\left(\frac{x}{2}\right)$

$(u = x^2+4,\ du = 2x\,dx)$

$= \boxed{\frac{1}{2}x^2 - 2\ln|x^2+4| + \arctan\left(\frac{x}{2}\right) + C}$

(h) $\int \sec^5(w)\tan^3(w)\,dw = \int \sec^4(w)\cdot\tan^2(w)\cdot\sec(w)\tan(w)\,dw$

$= \int \sec^4(w)\left(\sec^2(w)-1\right)\cdot\left[\sec(w)\tan(w)\right]dw$

$u = \sec(w) \qquad du = \sec(w)\tan(w)\,dw$

$= \int u^4(u^2-1)\,du = \int (u^6-u^4)\,du = \frac{1}{7}u^7 - \frac{1}{5}u^5$

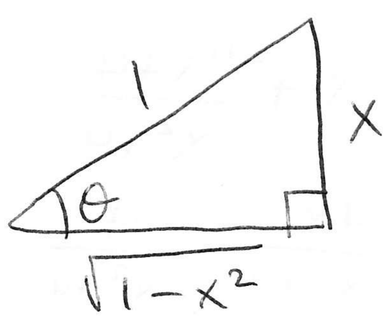
$= \boxed{\frac{1}{7}\sec^7(w) - \frac{1}{5}\sec^5(w) + C}$

(i) $-\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\cdot 2u^{1/2}$

$[u = 1-x^2,\ du = -2x\,dx]$

$= \boxed{-\sqrt{1-x^2} + C}$

(j)



$x = \sin\theta \qquad dx = \cos\theta\,d\theta$

$\sqrt{1-x^2} = \cos\theta$

$\int \frac{x^2}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^2\theta}{\cancel{\cos\theta}}\cancel{\cos\theta}\,d\theta = \int \sin^2\theta\,d\theta$

$= \int \frac{1-\cos(2\theta)}{2}\,d\theta = \frac{1}{2}\theta - \frac{\sin(2\theta)}{4}$

$= \frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta$

$= \boxed{\frac{1}{2}\arcsin\theta - \frac{1}{2}x\sqrt{1-x^2} + C}$

k) $\int \cos(x)\cdot \ln(\sin(x))\,dx$ $\quad u = \sin(x)$, $du = \cos(x)dx$

$= \int \ln(u)\,du$ parts $w = \ln(u)$, $dw = \frac{1}{u}du$, $dv = du$, $v = u$

$= u\ln(u) - \int u \cdot \frac{1}{u}\,du = u\ln|u| - u$

$= \boxed{\sin(x)\cdot \ln|\sin(x)| - \sin(x) + C}$

l) Parts $\quad u = \arctan(x)$, $du = \frac{1}{1+x^2}dx$ $\quad dv = x^{-2}dx$, $V = -x^{-1}$

$\int \frac{\arctan(x)}{x^2}dx = -\frac{\arctan(x)}{x} + \int \frac{1}{x(1+x^2)}dx$ $\quad$ Partial Fractions

$\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+C}{1+x^2} \Rightarrow 1 = A(1+x^2) + (Bx+C)x$

$x=0)\ 1 = A$ $\quad 1 = 1 + x^2 + Bx^2 + Cx = 1 + x^2(1+B) + Cx$

$-1 = B,\ C=0 \Rightarrow 1 + B = 0$ and $C = 0$

$\int \frac{1}{x(1+x^2)}dx = \int \frac{1}{x}dx - \frac{1}{2}\int \frac{2x}{x^2+1}dx$ $\quad \left[\begin{matrix} u = x^2+1 \\ du = 2x\,dx \end{matrix}\right]$

$= \ln|x| - \frac{1}{2}\ln|x^2+1|$

$= \boxed{\ln|x| - \frac{1}{2}\ln|x^2+1| + C}$

(2) a) $\int_0^2 \frac{e^x}{1+e^{2x}}dx$ $\quad u = e^x$, $du = e^x dx$

$= \int_1^{e^2} \frac{du}{1+u^2} = \arctan(u)\Big|_1^{e^2}$

$= \arctan(e^2) - \arctan(1)$

$= \boxed{\arctan(e^2) - \frac{\pi}{4}}$

b) $\int_0^{\pi} \sin^5(x)\cos(x)dx$ $\quad u = \sin(x)$, $du = \cos(x)dx$

$x = 0 \Rightarrow u = 0$

$x = \pi \Rightarrow u = 0$

$= \int_0^0 u^5 du = \boxed{0}$

c) $\int_{-1}^{1} y e^{-2y} dy$ $\quad u = y$, $du = dy$ $\quad dv = e^{-2y}dy$, $V = -\frac{1}{2}e^{-2y}$

$= -\frac{1}{2}ye^{-2y}\Big|_{-1}^{1} + \frac{1}{2}\int_{-1}^{1} e^{-2y}dy$

$= -\frac{1}{2}e^{-2} - \frac{1}{2}e^{2} + \frac{1}{2}\left[-\frac{1}{2}e^{-2y}\right]\Big|_{-1}^{1}$

$= -\frac{1}{2}e^{-2} - \frac{1}{2}e^{2} - \frac{1}{4}e^{-2} + \frac{1}{4}e^{2}$

$= \boxed{-\frac{3}{4}e^{-2} - \frac{1}{4}e^{2}}$

(6)

d) $\displaystyle\int_{-\frac{\pi}{2}}^{\pi/2} t\cos^2(t)\,dt = 0$ since $t\cos^2(t)$ is odd

e) $$\frac{x^2-5x+16}{(2x+1)(x-2)^2} = \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2-5x+16 = A(x-2)^2 + B(2x+1)(x-2) + C(2x+1)$$

$x=2)$ $4-10+16 = 10 = 5c \Rightarrow \boxed{c=2}$

$x=-\frac{1}{2})$ $\frac{1}{4}+\frac{5}{2}+16 = A\left(-\frac{5}{2}\right)^2 = \frac{25A}{4}$

$1+10+64 = 25A \Rightarrow 75 = 25A \Rightarrow \boxed{A=3}$

$x=1)$ $1-5+16 = A + -3B + 3C$

$12 = 3-3B+6 \Rightarrow 3 = -3B \Rightarrow \boxed{B=-1}$

$$\int_0^1 \frac{x^2-5x+16}{(2x+1)(x-2)^2}dx = 3\int_0^1 \frac{dx}{2x+1} - \int_0^1 \frac{dx}{x-2} + 2\int_0^1 \frac{dx}{(x-2)^2}$$

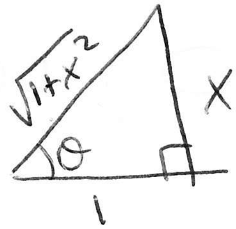
$$= \frac{3}{2}\ln|2x+1|\Big|_0^1 - \ln|x-2|\Big|_0^1 - \frac{2}{x-2}\Big|_0^1$$

$$= \frac{3}{2}\ln(3) - [\ln(1)-\ln(2)] - \left[\frac{2}{1-2} - \frac{2}{-2}\right]$$

$$= \frac{3}{2}\ln(3) + \ln(2) - [-2+1]$$

$$= \boxed{\frac{3}{2}\ln(3) + \ln(2) + 1}$$

(f) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{1+x^2}}\,dx$



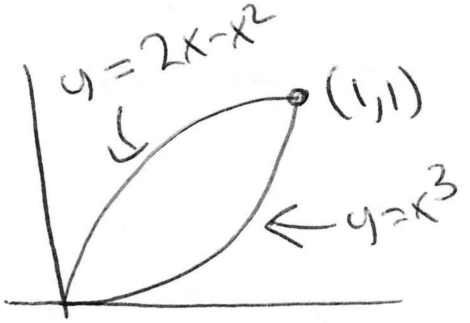
$x = \tan\theta$
$dx = \sec^2\theta\,d\theta$
$\sec\theta = \sqrt{1+x^2}$

If $x=1$, then $\theta = \frac{\pi}{4}$
If $x=-1$, then $\theta = -\pi/4$

$$\int_{-\frac{\pi}{4}}^{\pi/4} \frac{\sec^2\theta}{\sec\theta}\,d\theta = \int_{-\frac{\pi}{4}}^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big|_{-\frac{\pi}{4}}^{\pi/4}$$

$$= \boxed{\ln|\sqrt{2}+1| - \ln|\sqrt{2}-1|}$$

(3)



$x^3 = 2x - x^2$
$x^3 + x^2 - 2x = 0$
$x(x+2)(x-1) = 0$
$x=0$ or $x=-2$ or $x=1$

a) $$\int_0^1 [(2x-x^2) - x^3]\,dx = \int_0^1 [2x - x^2 - x^3]\,dx$$

$$= x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4\Big|_0^1 = 1 - \frac{1}{3} - \frac{1}{4} = \frac{12}{12} - \frac{4}{12} - \frac{3}{12}$$

$$= \boxed{\frac{5}{12}}$$

b) $$V = \pi\int_0^1 (R_{out}^2 - R_{in}^2)\,dx$$

$R_{out} = (2x - x^2) - 0$
$R_{in} = x^3 - 0$

$$V = \pi \int_0^1 \left[(2x - x^2)^2 - (x^3)^2\right] dx$$

$$= \pi \int_0^1 \left[4x^2 - 4x^3 + x^4 - x^6\right] dx$$

$$= \pi \left[\tfrac{4}{3}x^3 - x^4 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7\right]\Big|_0^1$$

$$= \pi\left[\tfrac{4}{3} - 1 + \tfrac{1}{5} - \tfrac{1}{7}\right] = \pi\left[\tfrac{140}{105} - \tfrac{105}{105} + \tfrac{21}{105} - \tfrac{15}{105}\right]$$

$$= \pi\left[\tfrac{41}{105}\right] = \frac{41\pi}{105}$$

c) $$V = 2\pi \int_0^1 r \cdot h \, dx \qquad r = x \qquad h = (2x - x^2) - x^3$$

$$= 2\pi \int_0^1 (2x^2 - x^3 - x^4) dx$$

$$= 2\pi \left[\tfrac{2}{3}x^3 - \tfrac{1}{4}x^4 - \tfrac{1}{5}x^5\right]\Big|_0^1$$

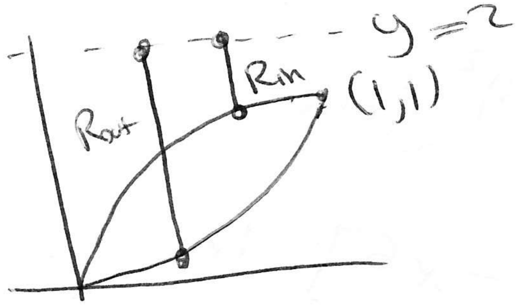
$$= 2\pi\left[\tfrac{2}{3} - \tfrac{1}{4} - \tfrac{1}{5}\right] = 2\pi\left[\tfrac{40}{60} - \tfrac{15}{60} - \tfrac{12}{60}\right]$$

$$= \boxed{\frac{13\pi}{30}}$$

d) $$V = \int_0^1 A(x) dx \qquad A(x) = (2x - x^2 - x^3)^2$$

$$= \int_0^1 (2x - x^2 - x^3)^2 dx = \boxed{\frac{22}{105}}$$

(4)



$R_{out} = 2 - x^2$

$R_{in} = 2 - \sqrt{x}$

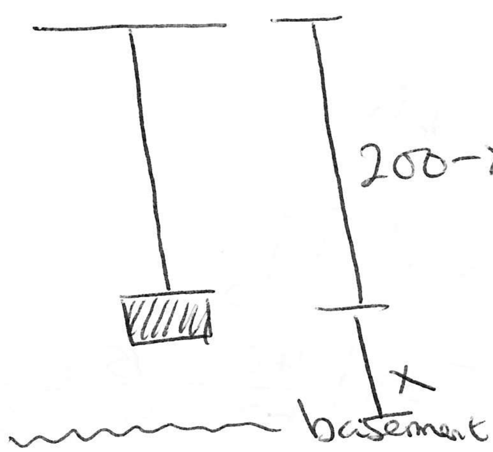
$$V = \pi \int_0^1 (R_{out}^2 - R_{in}^2)\,dx = \pi \int_0^1 \left[(2-x^2)^2 - (2-\sqrt{x})^2\right]dx$$

(5) $F = Kx$, $30 = K(0.03)$ Stretch in meters.

$\Rightarrow K = 1000$

$$W = \int_0^{0.08} 1000x\,dx = 500x^2 \Big|_0^{0.08} = 500(.08)^2 = \boxed{3.2\text{ J}}$$

(6)



$F = 10(200-x) + 1600$

$$W = \int_0^{30} (10(200-x)+1600)\,dx$$

$$= \int_0^{30} (2000 - 10x + 1600)\,dx$$

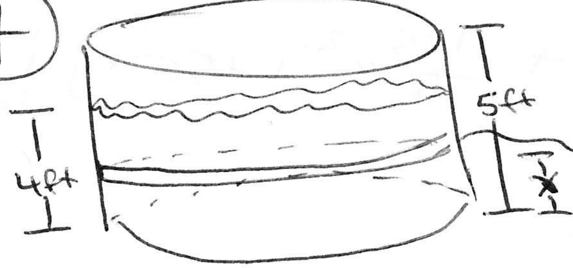
$$= \int_0^{30} (3600 - 10x)\,dx$$

$$= 3600x - 5x^2 \Big|_0^{30} = 108{,}000 - 4{,}500$$

$$= \boxed{103{,}500 \text{ ft-lbs}}$$

10

(7)



Slice looks like

$V_{slice} = \pi r^2 \Delta x$
$= 144\pi \Delta x$ ft^3

$F_{slice} = V_{slice} \cdot (62.5) = V_{slice}\left(\frac{125}{2}\right) = 9{,}000\pi\Delta x$ lbs

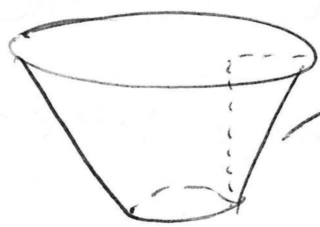
$D_{slice} = 5 - x$

$W_{slice} = 9{,}000(5-x)\Delta x$

$W = \int_0^4 9{,}000(5-x)\,dx = 45000x - 4500x^2\Big|_0^4$

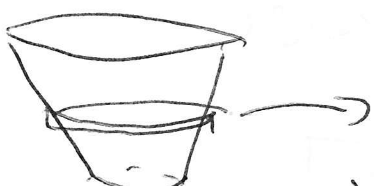
$= 180{,}000 - 72{,}000 = \boxed{108{,}000 \text{ ft-lbs}}$

(8)



$\frac{3}{8} = \frac{m}{x} \Rightarrow 3x = 8m$

$m = \frac{3}{8}x$



Slice looks like

$r = 3 + \frac{3}{8}x$

$V_{slice} = \pi r^2 \Delta x = \pi\left(3 + \frac{3}{8}x\right)^2 \Delta x$

$F_{slice} = \pi\, 62.5\left(3 + \frac{3}{8}x\right)^2 \Delta x$

$D_{slice} = 8 - x$

$W_{slice} = \pi\, 62.5(8-x)\left(3 + \frac{3}{8}x\right)^2 \Delta x$

$W = \pi \int_0^8 62.5(8-x)\left(3 + \frac{3}{8}x\right)^2 dx$

$$= \pi\left[-\frac{1125}{512}x^4 - \frac{375}{16}x^3 + \frac{1125}{4}x^2 + 4500x\right]\Big|_0^8$$

$$= \boxed{33{,}000\,\pi \text{ ft-lbs}}$$

(9) a) $\boxed{F'(x) = \dfrac{x^2}{1+x^3}}$

b) $G(x) = -\int_1^x \sqrt{t+\sin(t)}\,dt$

$$\boxed{G'(x) = -\sqrt{x+\sin(x)}}$$

c) Let $h(x) = \int_0^x \cos(t^2)\,dt$. Then $H(x) = h(x^4)$, so by the chain rule,

$H'(x) = h'(x^4)\cdot 4x^3 = \boxed{\cos(x^8)\cdot 4x^3}$

(10) $\int_4^6 f(x)\,dx = \int_0^6 f(x)\,dx - \int_0^4 f(x)\,dx$

$= 10 - 7 = \boxed{3}$

(11) a) $\int_0^3 (t^2-4)\,dt = \frac{1}{3}t^3 - 4t\Big|_0^3 = 9 - 12 = \boxed{-3}$

b) $\int_0^3 |t^2-4|\,dt = -\int_0^2 (t^2-4)\,dt + \int_2^3 (t^2-4)\,dt$

$= \left(4t - \frac{1}{3}t^3\right)\Big|_0^2 + \frac{1}{3}t^3 - 4t\Big|_2^3$

$$= \left(8 - \frac{8}{3}\right) + \left((9-12) - \left(\frac{8}{3} - 8\right)\right)$$

$$= 8 - \frac{8}{3} + -3 - \frac{8}{3} + 8 = 16 + \frac{16}{3} - 3$$

$$= \frac{48}{3} + \frac{16}{3} - \frac{9}{3} = \boxed{\frac{55}{3}}$$

(12) $\int_0^\infty e^{-x}\,dx = \lim_{t\to\infty} \int_0^t e^{-x}\,dx$

$$= \lim_{t\to\infty} -e^{-x}\Big|_0^t = \lim_{t\to\infty}\left[-e^{-t} + e^0\right]$$

$$= \lim_{t\to\infty}\left[1 - e^{-t}\right] = 1$$

$\int_0^\infty e^{-x}\,dx$ $\boxed{\text{Converges}}$ to $\boxed{1}$