Math 19B: Practice Problems for the Final Exam

1. Compute the following definite and indefinite integrals:

   (a) $\displaystyle\int_0^1 \arcsin(x)\,dx$

   (b) $\displaystyle\int \frac{dx}{\sqrt{x^2+4}}$

   (c) $\displaystyle\int_3^4 \frac{x}{x^2-4}\,dx$

   (d) $\displaystyle\int_2^3 \frac{dx}{(x-1)^{4/3}}$

   (e) $\displaystyle\int \tan^5\theta \sec^3\theta\,d\theta$

   (f) $\displaystyle\int e^{\sqrt{t}}\,dt$

2. Do the following series converge absolutely, converge conditionally, or diverge? Make sure you justify your work by indicating which test you are using and make sure the hypothesis are satisfied before conducting the test.

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n}{n+2}$

   (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+2}$

   (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^3+2}$

   (d) $\displaystyle\sum_{n=1}^{\infty}n^2e^{-n^3}$

   (e) $\displaystyle\sum_{n=1}^{\infty}\frac{3^n\cdot n^2}{n!}$

   (f) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^{n-1}}{5^n}$

   (g) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2n^2+1}{3n^2+7n+2}\right)^n$

   (h) $\displaystyle\sum_{n=1}^{\infty}3\left(\frac{5}{4}\right)^n$

   (i) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin(1/n)}{n^2}$

3. Set up integrals to find the areas and volumes below, but do not evaluate. Let $\mathcal{R}$ be the region bound by the curves $y = x^2 - 3x + 2$ and $y = 3x - 3$. Compute:

   (a) The area of $\mathcal{R}$.

   (b) The volume of the solid generated when $\mathcal{R}$ is rotated around the line $y = -2$.

   (c) The volume of the solid generated when $\mathcal{R}$ is rotated around the y-axis.

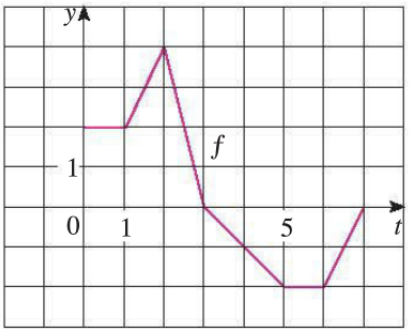
4. Let $\mathcal{C}$ be the curve $f(x) = \sin(x)$ where $0 \le x \le \pi$. Set up (but do **not** evaluate) integrals that finds

   (a) the length of $\mathcal{C}$

   (b) the surface area of the surface obtained by rotating $\mathcal{C}$ around the x-axis.

5. A force of 20 N is required to stretch a spring from its rest length of 10 cm to 20 cm. How much work is needed to stretch the spring from its rest length of 10 cm to 30 cm?

6. Let $g(x) = \int_0^x f(t)\,dt$ where f is the function whose graph is shown.



(a) Evaluate $g(0), g(1), g(2), g(3),$ and $g(6)$.

(b) On what interval is g increasing?

(c) Where does g have a maximum value?

7. Does $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ converge? If so, what is its value?

8. Find a power series representation for the following functions (using any method).

(a) $f(x) = \dfrac{x}{1+x^2}$

(b) $h(x) = \arctan(x)$

(c) $g(x) = e^{x^2}$

(d) $k(x) = \sin(x^4)$

9. Determine the interval and radius of convergence for the following power series.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2 \cdot 5^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n \cdot 4^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{(n+2)!}$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$