Math 19B - Midterm

August 17, 2012, Chris LeBailly

Notes: Show your work. In other words, just writing the answer, even if correct, may not be sufficient for full credit. Scientific calculators are allowed, but no programmable and/or graphing calculators. Make sure to give exact answers and simplify your answers when possible. And please put away your cell phones and other electronic devices, turned off or in airplane mode.

Your Name: _____

Problem 1:   out of 40

Problem 2:   out of 10

Problem 3:   out of 20

Problem 4:   out of 10

Problem 5:   out of 5

Problem 6:   out of 10

Problem 7:   out of 5

Total: **out of 100**

Good luck and have a great weekend!

1. *(40 points - 8 each)* Evaluate these definite and indefinite integrals.

(a) $\int xe^{3x}\,dx$

(b) $\int_1^{\sqrt{2}} x(x^2-1)^4\,dx$

(c) $\int_0^{\pi/6} \sec^4(2t)\tan^3(2t)\,dt$

(d) $\displaystyle\int \frac{5x - 2}{(x + 2)(x - 2)}\, dx$

(e) $\displaystyle\int \sqrt{4 - x^2}\, dx$

2. *(10 points - 5 each)* The velocity function $v(t) = 2\cos\left(\dfrac{\pi t}{6}\right)$ is given for a particle moving (back and forth) on a straight line. Assume $v(t)$ is measured in meters per second. Find the following:

(a) The displacement of the particle between time $t = 0$ and time $t = 4$.

(b) The total distance traveled by the particle between time $t = 0$ and time $t = 4$.

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3. *(20 points)* Let $\mathcal{R}$ be the region bound by $y = \sin x$, $y = 1$ and $x = 0$.

   (a) *(5 points)* Sketch the region $\mathcal{R}$ and compute its area.

   (b) *(15 points)* Using any method that you want, set up integrals (but do **NOT** evaluate) to find:

       i. the volume of the solid obtained by revolving $\mathcal{R}$ about the x-axis.

       ii. the volume of the solid obtained by revolving $\mathcal{R}$ about the y-axis.

   iii. the volume of the solid obtained by revolving $\mathcal{R}$ about $y = 1$.

4. *(10 points - 5 each)* **Differentiate** the following functions.

(a) $F(x) = \int_{-1}^{x} \ln(5t - 1)\ dt$

(b) $G(x) = \int_{0}^{x^2} \dfrac{3t}{2 + \sin(t^2)} dt$

5. *(5 points)* Does $\int_{1}^{\infty} \dfrac{1}{x^2}\ dx$ converge? If so, compute its value.

6. *(10 points)* A large aquarium is in the shape of a rectangular box as shown below. The aquarium is full of water but is filthy and needs to be cleaned. Find the amount of work needed to empty the aquarium (use the fact that water weights 62.5 lbs/cubic foot and that $62.5 = 125/2$).

10 ft.

4 ft.

10 ft.
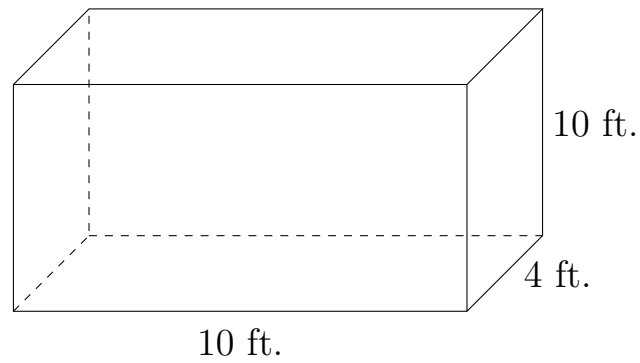
7. *(5 points)* Suppose $\int_0^5 f(x)\,dx = 5$, $\int_0^8 g(x)\,dx = 10$ and $\int_5^8 g(x) = 3$.

Compute $\int_0^5 \left(2f(x) + 3g(x)\right) dx$.

8. **Extra Credit** *(5 points)* Compute the volume of **one** of the solids from problem 3b. You can pick any one of the three, but indicate which one you have chosen. (The integral must be set up correctly for credit)