Math 19B Quiz: August 3, 2012 Solutions

Show all work and make sure to give exact answers. Good luck, and enjoy your weekend!

1. *(6 points)* Find the value of $H(-1), H'(0)$, and $H(1)$, where $H(x) = \displaystyle\int_{-1}^{x} \frac{1}{1+u^2}\,du$.

$$H(-1) = \int_{-1}^{-1} \frac{1}{1+u^2}\,du = \boxed{0} \text{ (by properties of the integral)}$$

The Fundamental Theorem of Calculus part 1 tells us that $H'(x) = \frac{1}{1+x^2}$, so

$$H'(0) = \frac{1}{1+0^2} = \frac{1}{1+0} = \boxed{1}$$

For the last part we will need to evaluate a definite integral. To do this we will use the Fundamental Theorem of Calculus part 2. Recall that the anti-derivative of $\frac{1}{1+u^2}$ is $\arctan(u)$ (since $\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$) which means that

$$H(1) = \int_{-1}^{1} \frac{1}{1+u^2}\,du = \arctan(x)\Big|_{-1}^{1} = \arctan(1) - \arctan(-1) = \frac{\pi}{4} - (-\frac{\pi}{4}) = \boxed{\frac{\pi}{2}}$$

2. *(7 points)* Find the value of $\displaystyle\int_0^1 (x^4 + 2x + e^x)\,dx$.

To evaluate a definite integral we will use the Fundamental Theorem of Calculus part 2. We first need to find an anti-derivative, then plug in the limits of integral.

$$\int_0^1 (x^4+2x+e^x)\,dx = \frac{1}{5}x^5+x^2+e^x\Big|_0^1 = \left(\frac{1}{5}+1+e^1\right)-(0+0+e^0) = \frac{1}{5}+1+e-1 = \boxed{\frac{1}{5}+e}$$

3. *(7 points)* Compute $\displaystyle\int \sin^2(x)\cos(x)\,dx$.

Recall that doing an indefinite integral simply means find the anti-derivative. To do this we will make a substation. Let $u = \sin(x)$. Then $du = \cos(x)\,dx$.

$$\int \sin^2(x)\cos(x)\,dx = \int u^2\,du = \frac{u^3}{3} = \boxed{\frac{\sin^3(x)}{3} + C}$$