PSQF 4143: Section 12

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Example 1

- Does speed reading help or hurt reading comprehension?
- Random sample of UI freshman students, $n = 100$
- Divide randomly in 2 groups of 50
  - **Independent** groups
  - No matching
  - No equating
- Experimental group ($n_e = 50$): take speed reading course
- Control group ($n_c = 50$)

Example 1 (cont).

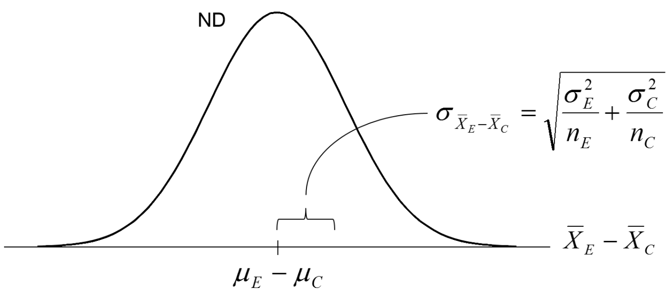
- State the statistical hypotheses.
  - H_0: The treatment has no effect
  - H_0: The two groups are the same (after the speed reading course)
  - H_0: $\mu_E = \mu_C$
  - H_0: $\mu_E - \mu_C = 0$
- Conduct Experiment:

	Experimental	Control
n	50	50
$\bar{X}$	35	31
σ	8	6

- Is $\bar{X}_E - \bar{X}_C = 4$ an unlikely result?
- To answer this, we need a probability distribution for $\bar{X}_E - \bar{X}_C$

Sampling Distribution $\bar{X}_E - \bar{X}_C$

- The sampling distribution of the difference between two independent means (σ_E and σ_C known).



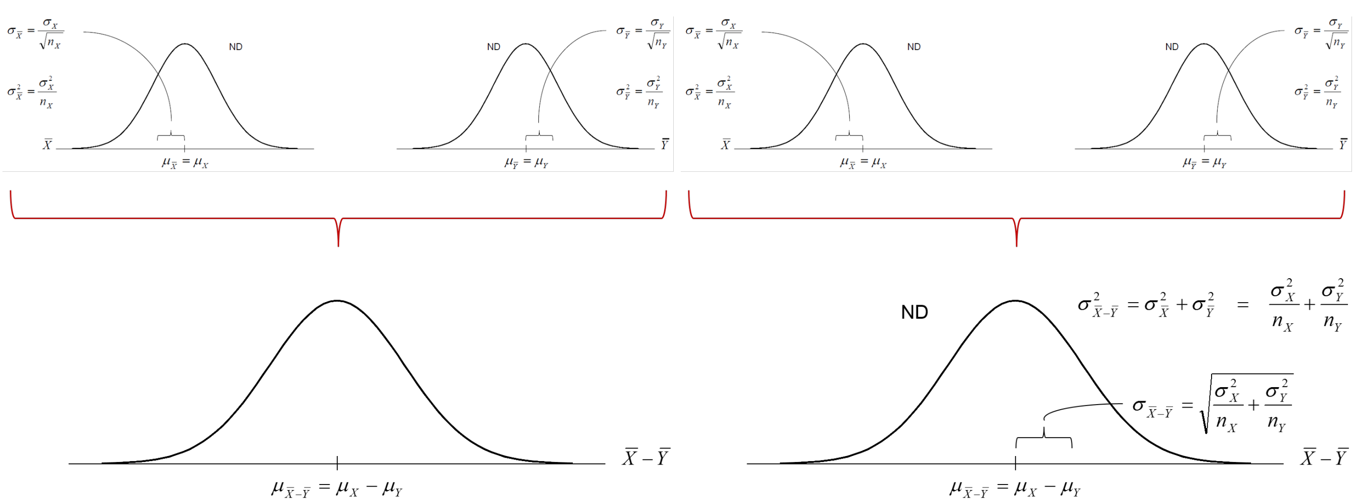
Example 1 (cont.)

$$\sigma_{\bar{X}_E - \bar{X}_C} = \sqrt{\frac{\sigma_E^2}{n_E} + \frac{\sigma_C^2}{n_C}}$$

- What is the $Pr(\bar{X}_E - \bar{X}_C \geq 4)$?

$$z = \frac{(\bar{X}_E - \bar{X}_C) - (\mu_E - \mu_C)_{HYP}}{\sigma_{\bar{X}_E - \bar{X}_C}}$$

Sampling Distribution Explanation



When population SDs are unknown

- Recall: $\sigma_{\bar{X}-\bar{Y}} = \sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}$
- Assumption: $\sigma_X^2 = \sigma_X^2 = \sigma^2$

- Then: $\sigma_{\bar{X}-\bar{Y}} = \sqrt{\frac{\sigma^2}{n_X} + \frac{\sigma^2}{n_Y}}$
- Now: $\hat{\sigma}_{\bar{X}-\bar{Y}} = \sqrt{\frac{\hat{\sigma}^2}{n_X} + \frac{\hat{\sigma}^2}{n_Y}} = \sqrt{\hat{\sigma}^2 \left(\frac{1}{n_X} + \frac{1}{n_Y}\right)}$

When population SDs are unknown 2

- $\hat{\sigma}_{\bar{X}-\bar{Y}} = \sqrt{\frac{\hat{\sigma}^2}{n_X} + \frac{\hat{\sigma}^2}{n_Y}} = \sqrt{\hat{\sigma}^2 \left(\frac{1}{n_X} + \frac{1}{n_Y}\right)}$
- $\hat{\sigma}^2$ is a weighted average of the unbiased estimates of σ_X^2 and σ_Y^2
- Since σ_X^2 and σ_Y^2 are both not known to us: $\hat{\sigma}_{\bar{X}-\bar{Y}} = \sqrt{\frac{n_X S_X^2 + n_Y S_Y^2}{(n_X - 1) + (n_Y - 1)} \left(\frac{1}{n_X} + \frac{1}{n_Y}\right)}$

Putting all together: Pop SDs unknown

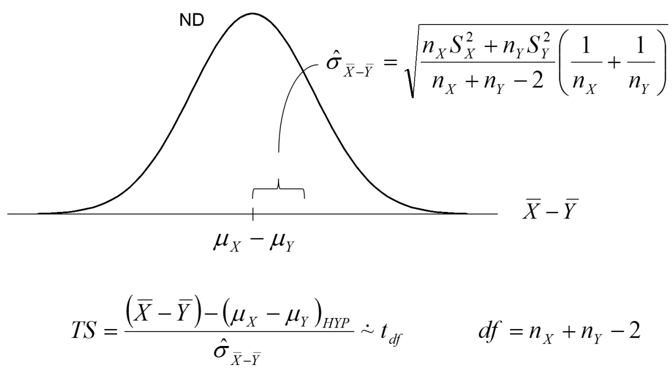


Figure 1:

Example 2

- Does speed reading help or hurt reading comprehension?
- Random sample of UI freshman students, $n = 100$
- Divide randomly in 2 groups of 50

	Experimental	Control
n	50	50
$\bar{X}$	35	31
S	8	6

- Conduct the hypothesis test at $\alpha = 0.01$ significance level.

Summary

- If Population SDs are known:

$$\sigma_{\bar{X}-\bar{Y}} = \sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}$$

$$TS = \frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)_{HYP}}{\sigma_{\bar{X}-\bar{Y}}} \sim Z$$

- If Population Sds are **NOT** known:

$$\hat{\sigma}_{\bar{X}-\bar{Y}} = \sqrt{\frac{n_X S_X^2 + n_Y S_Y^2}{(n_X - 1) + (n_Y = 1)} \left(\frac{1}{n_X} + \frac{1}{n_Y}\right)}$$

$$TS = \frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)_{HYP}}{\hat{\sigma}_{\bar{X}-\bar{Y}}} \sim t_{df}$$

$$df = n_X + n_Y - 2$$

Example 3

- 2 methods (A & B) of teaching addition of fractions
- Is method A more effective than Method B?
- Randomly sample 2 classes
- Results:

	A	B
n	10	24
$\bar{X}$	115	105
S	14	17

- Conduct the hypothesis test at the $\alpha = .05$ significance level.

Assumptions

1. Each sample is drawn at random from its respective population
2. The two samples must be independently selected.
3. Sampling with replacement, or \$ n < 5%\$ of N.
4. The two populations (of raw scores) are normally distributed.
5. The unknown population variances are equal (homogeneity of variances assumption).

Assumptions 1 - 3

- In Example 2, we analyzed the data as if we had drawn each of the two samples
  - independent (assumption 2)
  - from its respective population randomly (assumption 1)
  - with replacement (assumption 3)
- However, that is not how the experiment was done.
  - First, we randomly sampled without replacement from a single population
  - Then, we randomly assigned each participant to one of two groups (experimental or control)
  - This is called a randomized experiment

Assumptions 1 - 3 (cont)

- In other words, a random sampling model (Independent Means t-Test) was used to analyze data that came from two groups formed by random assignment
- Technically, a random assignment model (Permutation Test) should have been used to analyze the data
- Two mistakes when analyzing the data as we did
  1. Draws were made without replacement (inflates the SE)
  2. There was a slight degree of dependency between the two groups (deflates the SE)

Assumptions 1 - 3 (cont)

- It is a lucky break that in randomized experiments, the first mistake outweighs the second
  - Thus, when the random sampling model is applied to randomized experiments, the statistical test is conservative (the SE tends to be overestimated)
- For most randomized experiments, using the random sampling model yields the same statistical conclusions as the random assignment model
- However, with only random assignment of available participants, conclusions do not generalize beyond
  - the participants studied, and
  - the conditions of the study

Random Sampling vs Random Assignment

- Random Sampling
  - a method for obtaining a sample from an experimentally accessible population

- a basic assumption underlying statistical inference
- this is what makes it possible for us to make statistical conclusions based on theoretical probability models, and generalize those conclusions to a population

- Random Assignment
  - a method for dividing an available group of participants into two or more subgroups
  - its purpose is to make the comparison groups randomly equivalent to each other prior to any treatment(s)
  - it controls for extraneous variables (known and unknown)

The normality assumption

- The independent means t-test is robust with respect to violations of the normality assumption
  - If the two sample sizes are equal, the t-test gives fairly accurate p-values for a broad range of population distributions, provided the populations
    * have similar shapes
    * are unimodal
    * have no outliers
  - This holds for sample sizes as small as $n_X = n_Y = 5$
- When n_X and n_Y are both greater than 30, the normality assumption is unimportant, thanks to the central limit theorem

The homogeneity of variance assumption

- The independent means t-test is robust with respect to violations of the assumption of equal population variances, provided that $n_X = n_Y$
- There is a statistical test of the equality of population variances
  - F-test
- If the population variances are unequal, and if $n_X \neq n_Y$, the sample variances should not be pooled
  - A modified t-statistic with modified df can be used, often times called the Welch t-test

Confidence Intervals

- Recall the speed reading data:

	Experimental	Control
n	50	50
$\bar{X}$	35	31
S	8	6

- Find a 99% CI for $\mu_E - \mu_C$
- Confidence form looks as follows:

$$(\bar{X} - \bar{X}) \pm t_{crit}\hat{\sigma}_{\bar{X}-\bar{Y}}$$

Confidence Interval Interpretations

- Interpretations for 99% CI [0.24, 7.76]:

  1. Does μ fall in this interval?
  2. Is there a 99% chance that μ falls in this interval?
  3. If 100 intervals were constructed, about how many intervals would contain μ?
  4. If we constructed an infinte number of intervals, how many would contain μ?

Using confidence intervals to conduct a two-tailed hypothesis

- You can use a confidence interval to conduct a two-tailed hypothesis of any null hypothesis.

  - If the hypothesized value falls within the CI, fail to reject H_0
  - If the hypothesized value falls outside the CI, reject H_0

- Using the previous example [0.24, 7.76], would we fail to reject or reject H_0?