PSQF 4143: Section 3

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Statistics vs Parameters

- *Statistics* - A number that describes the sample; hopefully a representative number for the population
  - Notation: An English letter (i.e. X, $\bar{X}$)
- *Parameter* - A number to describe the population (i.e. the true value)
  - Notation: A Greek letter (i.e. μ, σ)

Notation

- An observed data point is represented with a capital letter, commonly X. This would represent the value for a single individual.
  - Example: X = 25
- Little x is used for a deviation score.
- The number of subjects in the population is denoted with a capital N. The number in the sample is denoted with a small n.
- If there are multiple observations or groups a subcript is used.
  - Example: A sample of 3 observations (n = 3): $X_1 = 12$, $X_2 = 23$, $X_3 = 8$.

Notation 2

- The greek captial letter, Σ, is used to indicate summation.
  - More formally: $\sum_{i=1}^{n} X_i = X_1 + X_2 + ... + X_n$
  - This means sum the value of X from 1 to n.
- Example: $X_1 = 12$, $X_2 = 23$, $X_3 = 8$.

$$\sum_{i=1}^{n} X_i = 12 + 23 + 8 = 43$$

Single number Descriptive Statistics

- *Central Tendency* - a set of descriptive statistics that describes a data set in a single number; also known as measures of central tendency as they describe where the center of the distribution lies
  - Mode
  - Median
  - Mean (Average)

Measure of Central Tendency

- Mode:
  - Indicates the most frequent value/category in the distribution.
- Calculating the mode:
  - Discrete Data: Value with the greatest frequency.
  - Categorized Continuous Data (i.e. Frequency Table): Midpoint of the interval with the greatest frequency.
  - Continuous Data: The highest point of the frequency distribution.

Mode Calculations

Central Tendency

Set 1	***Set 2***	***Set 3***	***Set 4***		***Set 5***	
2	10	26	Interval	Percent	Interval	Freq.
3	11	27	$0 - $10,000	12%	0 - 9	0
4	11	28	$10-$20,000	32%	10-14	20
4	11	28	$20-$30,000	27%	15-19	40
5	11	32	$30-$40,000	15%	20-24	15
7	12		$40-$50,000	8%	25-29	10
	13		$50,000 and up	6%	30-34	5
	19					

Figure 1: Practice Central Tendency

Measure of Central Tendency 2

- Median:
  - Is the value where half of the data fall above and half fall below.
- Calculating the median:
  - Nominal Data: Cannot be done meaningfully
  - Discrete Data:

1. Order values from highest to lowest, find the middle value.
2. Alternatively: find the value:

$$L = \frac{n+1}{2}$$

   * If L is a whole number, the median is the Lth value
   * If L is a decimal, the median is the average between the two values around L (i.e. the value rounded up and down from L)

Measure of Central Tendency 2

- Calculating the median:
  - Continuous Data:
    * determine where in the interval the median falls as a fraction, then go that fraction from the interval's lower limit to its upper limit
    *

$$Median = L + \frac{\frac{n}{2} - f_{below}}{f_{at}} * w$$

Where L is the lower limit of the of the interval containing the median n is total frequency f_{below} is the frequency below the interval f_{at} is the frequency at the interval w is the bin width

Median Calculations

2	10	26
3	11	27
4	11	28
4	11	28
5	11	32
7	12	
	13	
	19	

Interval	Percent
\$0-\$10000	12%
\$10000-\$20000	32%
\$20000-\$30000	27%
\$30000-\$40000	15%
\$40000-\$50000	8%
\$50000 and up	6%

Interval	Percent
0 - 9	0
10 - 14	20
15 - 19	40
20 - 24	15
25 - 29	10
30 - 34	5

Measures of Central Tendency 3

- Mean:
  - Is the average of the data, also known as the "balance point" or the place where the distribution would balance on a fulcrum.
  - The mean is used for interval or ratio data and the value is interpretted in the same metric as the raw data.
- Notation:
  - Sample: $\bar{X}$ - called "X-bar"
  - Population: μ - Greek letter "mu"

Measures of Central Tendency 3

- Calculating the Mean
  - Sample:
$$\bar{X} = \frac{\sum X}{n} = \frac{X_1 + X_2 + ...}{n}$$
  - Population:
$$\mu = \frac{\sum X}{N} = \frac{X_1 + X_2 + ...}{N}$$

Mean Calculations 1

2
3
4
4
5
7

Mean Comic

Figure 2: Comic

Measures of Central Tendency 3

- Calculating the Mean
  - Categorized Data (Frequency Table):

$$\bar{X} = \frac{\sum f * X_{midpoint}}{\sum f}$$

where f and $X_{midpoint}$ are the frequency and midpoint of each bin/class.

Mean Calculations 2

Interval	Percent
0 - 9	0
10 - 14	20
15 - 19	40
20 - 24	15
25 - 29	10
30 - 34	5

Interval	Percent
\$0-\$10000	12%
\$10000-\$20000	32%
\$20000-\$30000	27%
\$30000-\$40000	15%
\$40000-\$50000	8%
\$50000 and up	6%

Mean is Mathematically Tractable

- This is fancy math lingo meaning that the mean is able to be manipulated algebraicly.
- Proof:

$$\frac{\sum X}{n} = \bar{X}$$
$$n\frac{\sum X}{n} = n\bar{X}$$
$$\sum X = n\bar{X}$$

Weighted Mean

- Used when groups are different sizes.

$$\bar{X} = \frac{n_a\bar{X}_a + n_b\bar{X}_b + n_c\bar{X}_c}{n_a + n_b + n_c}$$

- More generally:

$$\bar{X} = \frac{\sum n_j\bar{X}_j}{\sum n_j}$$

Weighted Mean Example

Boys: 5, 8, 3, 7, 7
Girls: 10, 12, 16, 8, 4, 8

Weighted Mean Example 2

District 1: n = 3347; Mean = 78.4
District 2: n = 200; Mean = 52.8
District 3: n = 334; Mean = 55.2

Puzzler - Simpson's Paradox

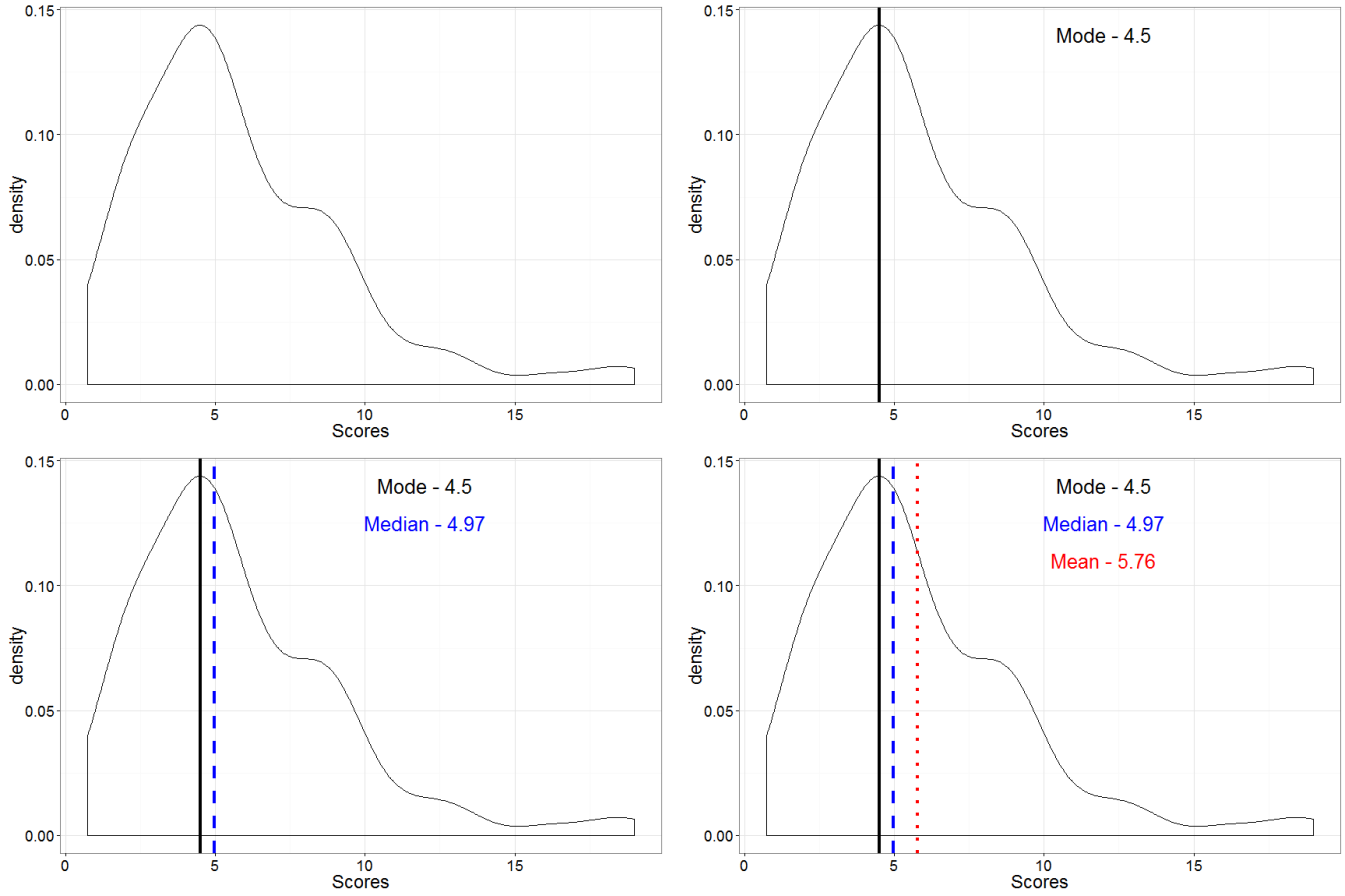
- The boys at Elm have a higher mean than the boys at Oak.
- The girls at Elm have a higher mean than the girls at Oak.
- The combined mean at Elm is lower than the combined mean at Oak.

Choosing a Statistic

- What statistic would you choose if the distribution was normally distributed? Why?
- What if the distribution was skewed?

What to use for skewed distributions?

Suppose we have the following distribution:



Strengths/Weaknesses of the Mode

- Strengths:
  - The mode is simple, easy to compute, and can be used with qualitative variables.
- Weaknesses:
  - However, it can change easily or be undefined for multimodal data.
    - This is often called sampling stability.
  - Not mathematically tractable.

Strengths/Weaknesses of the Median

- Strengths:
  - The median is the most typical number in the distribution.
    - The median minimizes $\sum |X - Mdn| \leq \sum |X - c|$ where c is any other value.
    - As such, the median is commonly used with income or prices.
  - The median is not affected by outliers
    - This is due to the median is not based on the value of the scores, but rather the ranking of scores.
    - The median would only be affected if the ranking of scores would change.
  - Easy to compute and computed for open-ended groups.
  - Is the 50th percentile
- Weaknesses:
  - Poor sampling stability compared to the mean (better than the mode).
  - Not as mathematically tractable compared to the mean.
  - Does not use all data in the calculation.
  - Can not be calculated for qualitative data

Strengths/Weaknesses of the Mean

- Strengths:
  - Mathematically tractable
    - Sum of deviation scores equals 0 ($\sum(X - \bar{X}) = 0$)
    - Minimizes the sum of squared deviations scores ($\sum(X - \bar{X})^2$)
  - The mathematic properties make the mean the balance point of the distribution.
  - Takes into account value of all numbers in the distribution.
  - Used in many advanced statistical techniques.
- Weaknesses:
  - Sensitive to outliers
  - Unable to calculate for open-ended groups.
  - Can not be calculated for qualitative data