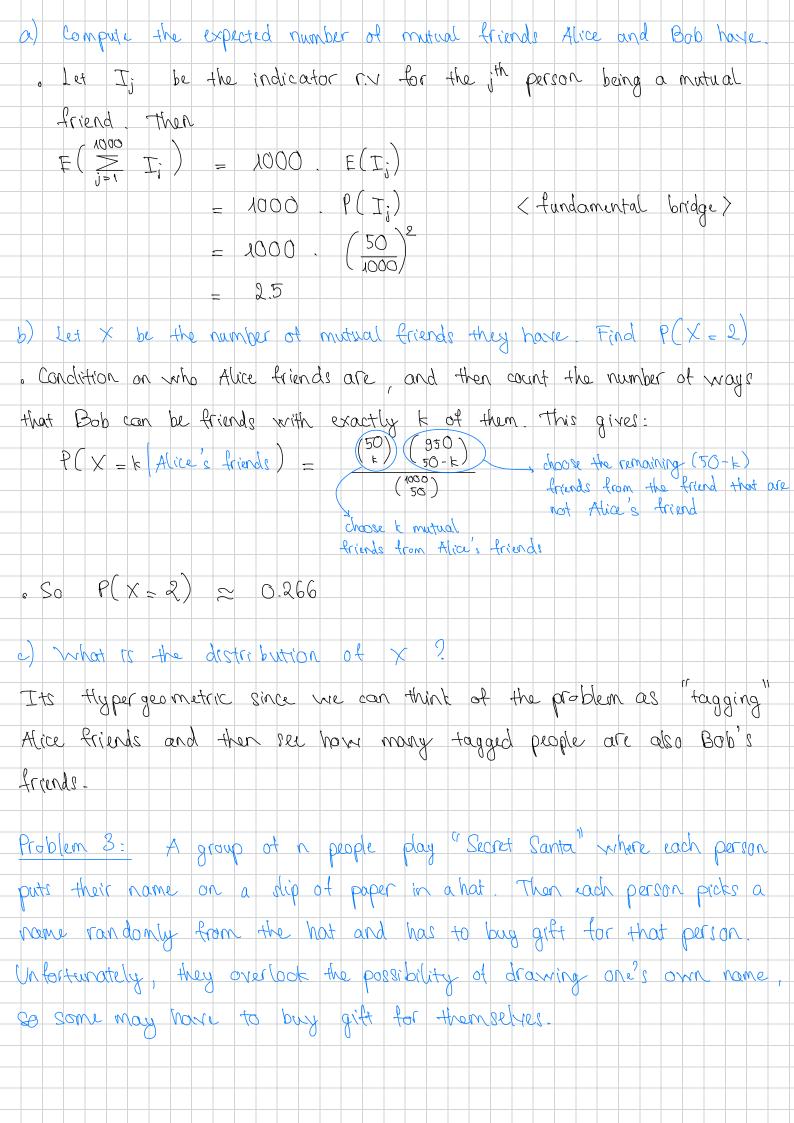
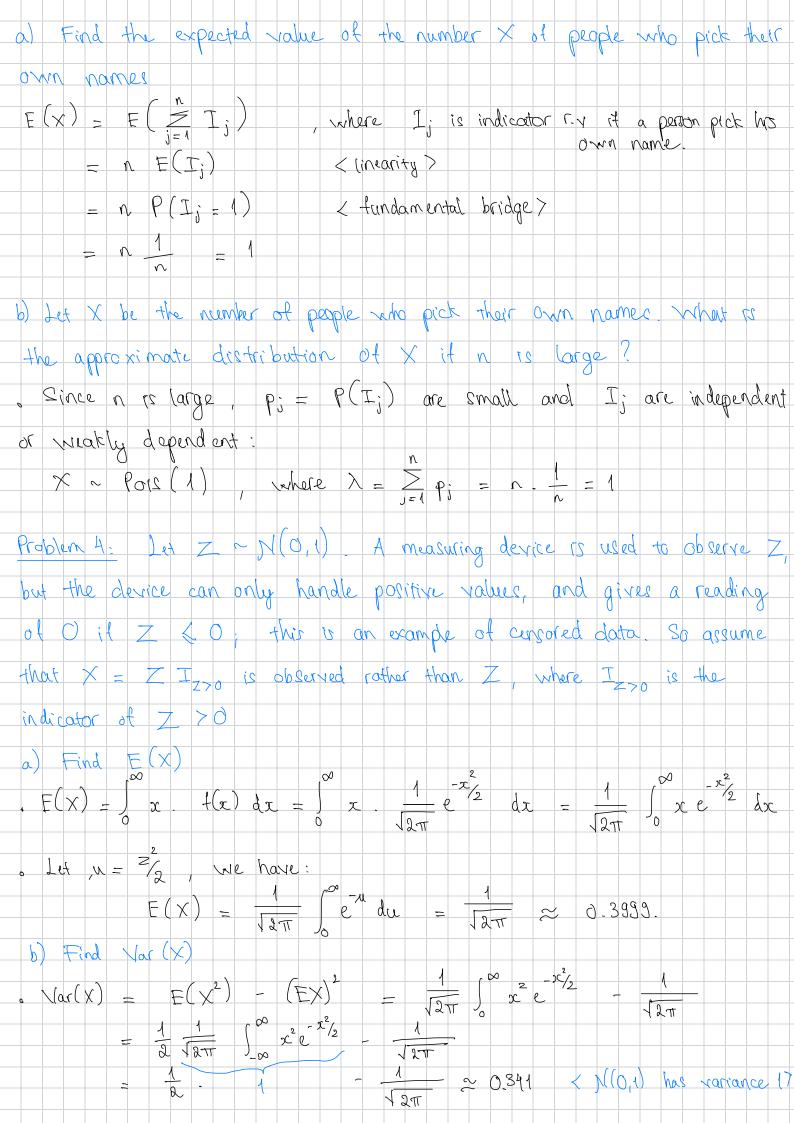


5.3 Unit 5 Practice Problems: Problem 1. Randonly k distinguishable balls are placed into a distinguisable boxes, with all possibilities equally likely. Find the expected number of empty boxes as a function of k and n . Let I; be indicator r.v that ith box is empty, so the expected number of empty boxes:  $E(X) = E(I_1 + I_2 + \dots + I_n)$  $= E(\underline{T}_1) + E(\underline{T}_2) + \dots + E(\underline{T}_n)$  $=\sum_{j=1}^{p}E(I_{j})$ . To find E(I;), we need to calculate the probability that a given box j is empty after placing all k balls. A box is empty if none of the k balls are placed in it. Since each ball is placed in box i with probability the probability that a ball is not placed in ith box is (1 - 1). Since the balls are placed independently probability that all k balls not placed in box j is  $P(box j empty) = (1 - \frac{1}{n})^k$ By the fundamental bridge:  $P(box j entg) = F(Tj) = (1-\frac{1}{n})^k$ So expected number of empty boxes is:  $E(X) = \sum_{i=1}^{n} E(T_i) = n \left(1 + \frac{1}{n}\right)^k$ Problem 2: Alice and Bob each has 50 friends out of 1000 people in town. They think that they are unlikely to have friends in common since each of them are friends with only 5% of the people in town, so their 5% are unlikely to overlop"





5,4 Unit 5 Homework Problems Problem 1: Babo, the amoeba, currently liver in a pond. After one minute, Baloo will either die split into 2 amorbas, or stay the same, with equal probability. Find the expectation and variance of the number of amorbas in the pond after one minute a Let X be the number of amorbas in the pond after one minute:  $E(x) = P(x=0) \cdot x_0 + P(x=1) \cdot x_1 + P(x=2) \cdot x_2$  $=\frac{1}{3} - 0 + \frac{1}{3} - 1 + \frac{1}{3} - 2$  $Var(X) = E(X^2) - (EX)^2$  $= \left( \frac{1}{3} \cdot 0^2 + \frac{1}{3} \cdot 1^2 + \frac{1}{3} \cdot 2^2 \right) - 1$ Problem 2: Two researchers independently solect simple random samples from a population size M, with sample sizes on and n. Find the expected size of the overlap of the 2 samples as a function of 14, n and m. For m= 20, n= 30, H= 100, find the expected size of the overlap of the two samples. . Let I; be indicator r.v that the jth sample in population N that belongs to both m and n samples. Then:  $E\left(\sum_{i=1}^{N} T_{i}\right) = N \cdot E\left(T_{i}\right)$  < linearity > = M. P(I; =1) < fundamental bridge>  $= \mathcal{N} - \left(\frac{n}{H}\right)\left(\frac{n}{H}\right) \left(\frac{n}{H}\right) \left(\frac{n}{H}\right) = \mathbb{P}\left(\frac{n}{2} + \frac{n}{2}\right) = \mathbb{P}\left(\frac{n}{2} + \frac{n}{2}\right)$  $=\frac{m.n}{N}=\frac{20.30}{100}=6$ 

