

## Introduction

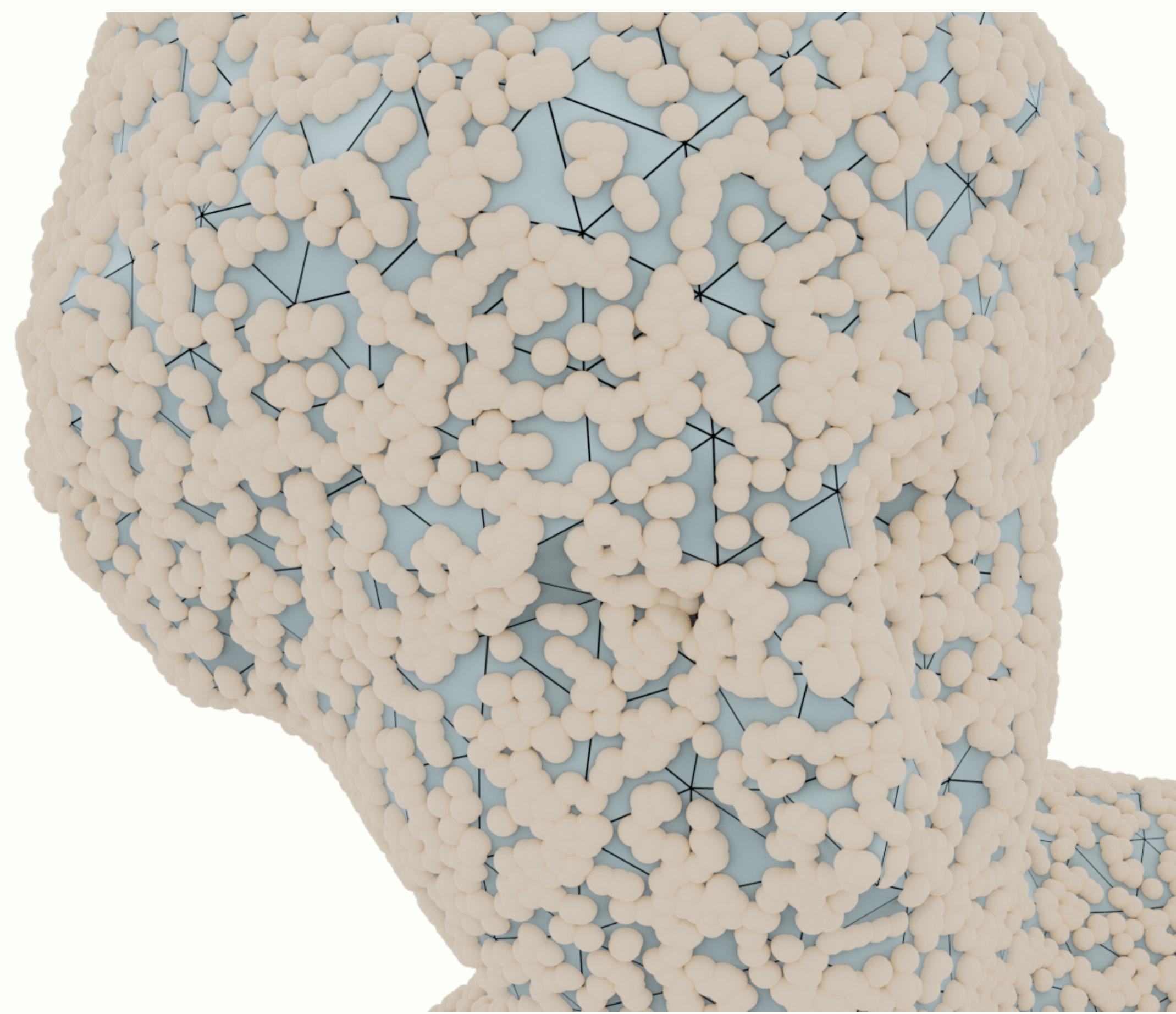
Sampling is a ubiquitous part of geometrical deep learning that operates with meshes. It allows the computation of geometric loss by converting a set of triangles into point-cloud.

Such a sampler must be

- Fast
- Differentiable
- Robust and producing low-variance estimate

## Limitation of Random Uniform Sampler (RUS)

- Points are drawn independently.
- Sampling patterns are subject to clustering.
- Distance estimate with a low number of points are subject to a large variance.



## Optimization on a measure space

The triangle mesh  $T$  can be written as a measure  $\mu_c^T$  carried by a union of simplexes

$$\mu_c^T(B) = \frac{1}{|T|} \sum_{t_i \in T} \int_{B \cap t_i} d\mathcal{H}^2(x).$$

## Supervised learning problem

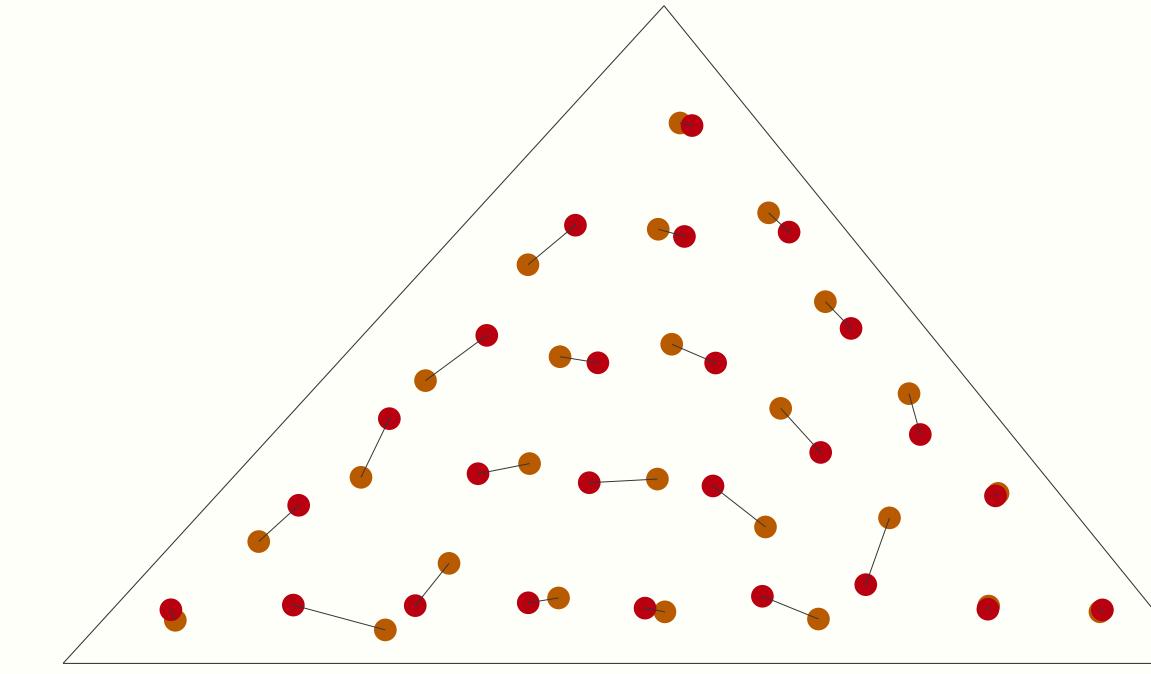
Given a sampling  $S$  and  $\ell$  sampled points, MongeNet  $f_\theta(t, \ell, p)$  minimizes:

$$\mathcal{L}(t, \ell, p, S) = \underbrace{W_2^\varepsilon(f_\theta(t, \ell, p), S)}_{\text{fidelity}} - \alpha \underbrace{W_2^\varepsilon(f_\theta(t, \ell, p), f_\theta(t, \ell, p'))}_{\text{diversity}}$$

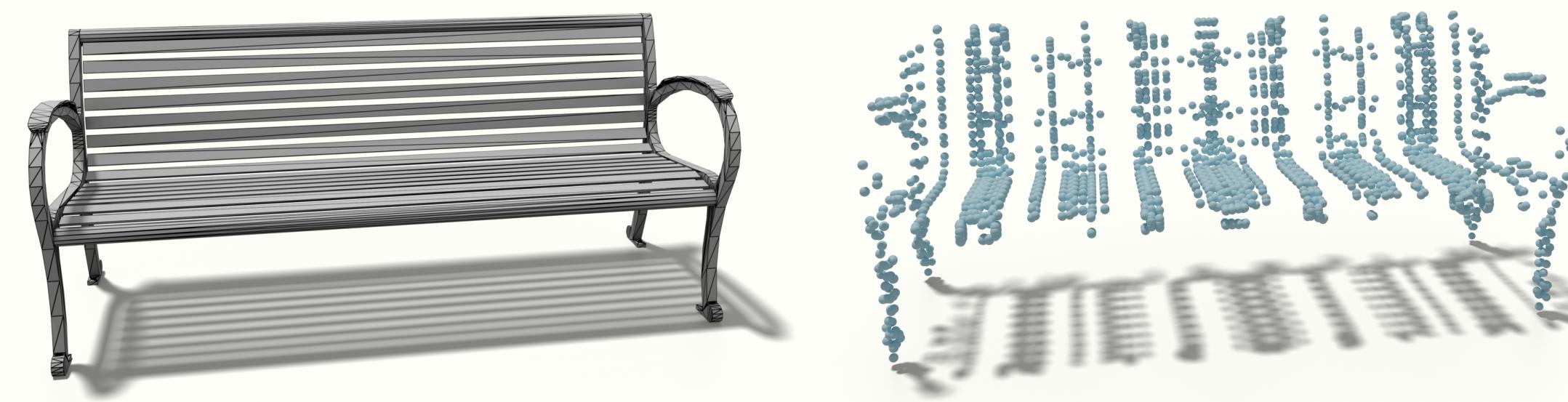
with  $W_2^\varepsilon$  the  $\varepsilon$ -regularized optimal transport [1, 2] and  $p, p' \sim \mathcal{N}(0, 1)$ , and  $f_\theta$  a MLP.

## Encouraging entropic samples

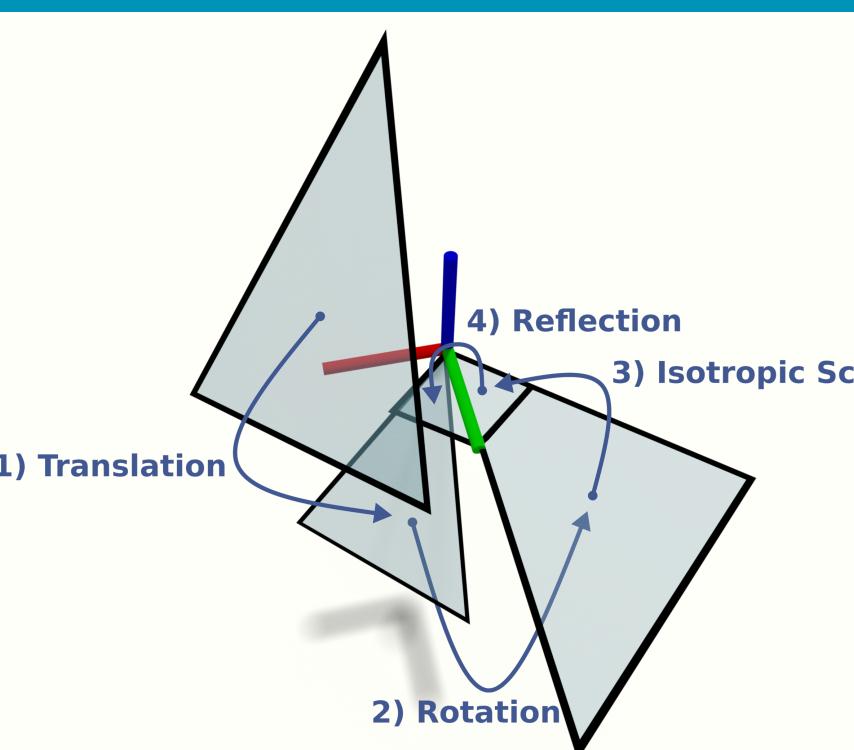
The point generation is conditioned by  $p$ . Two different  $p, p'$  result in a different sampling pattern.



Deterministic sampling generates structured patterns



## Dimensionality reduction of the learning problem



We project the triangle of  $\mathbb{R}^3$  on a canonical space with angle preserving transformation to reduce the complexity of the learning task.

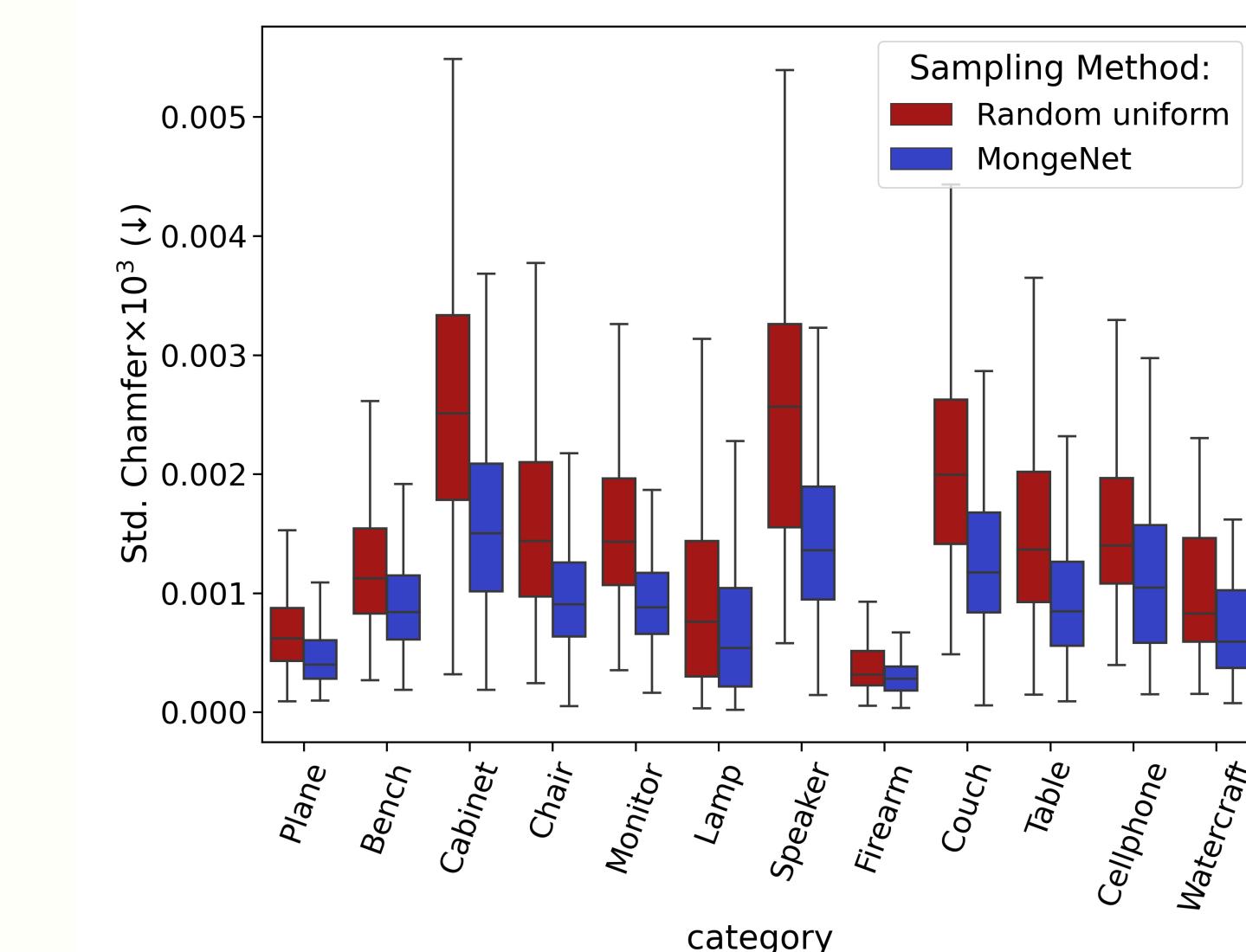
## Runtime vs. Pytorch3D

# Faces	10k	20k	30k	40k	60k	80k
RUS	1.14 ms	1.50ms	1.53ms	1.52ms	1.53ms	1.53ms
MongeNet	2.89 ms	5.41 ms	7.90 ms	10.5 ms	16.0 ms	21.7 ms

## Applications

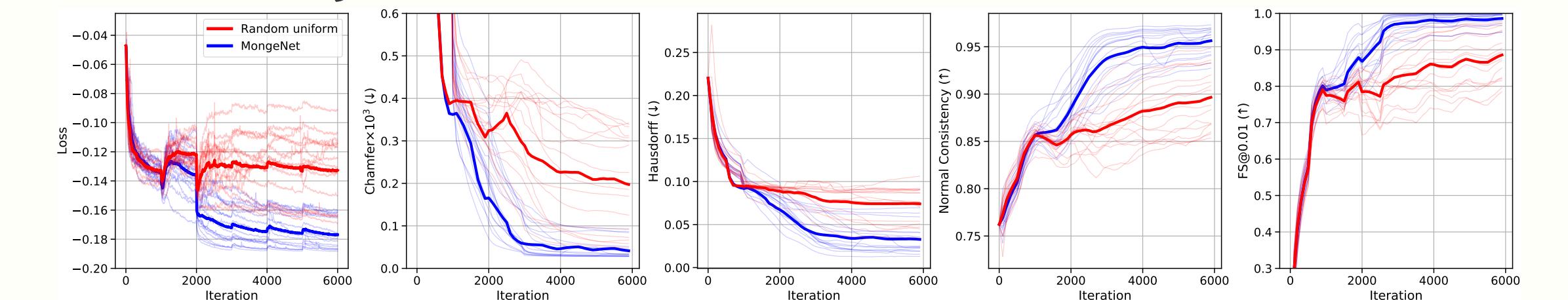
We establish the usefulness of the proposed method

### ▪ For metric evaluation



MongeNet reduces the evaluation variance.

### ▪ In a learning context with Point2mesh [3]



MongeNet allows training geometric deep learning models better and faster.

## Easy to use

```
from src.mesh_sampler import MeshSampler
mesh_sampler = MeshSampler(mongenet, num_sampled_points, compute_normals, bs).to('cuda')
points, face_ids, normals = mesh_sampler(vertices, faces, lenghts)
```

### Code repo

<https://github.com/lebrat/MongeNet>

### Contact us

fon022@csiro.au or leo.lebrat@gmail.com.

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## References

- [1] Marco Cuturi. Sinkhorn distances: Lightspeed computation of optimal transport. In *Advances in neural information processing systems*, pages 2292–2300, 2013.
- [2] Jean Feydy, Joan Glaunès, Benjamin Charlier, and Michael Bronstein. Fast geometric learning with symbolic matrices. *Advances in Neural Information Processing Systems*, 33, 2020.
- [3] Rana Hanocka, Gal Metzger, Raja Giryes, and Daniel Cohen-Or. Point2mesh: A self-prior for deformable meshes. *ACM Trans. Graph.*, 39(4), 2020.