Report on Dropping Test Masses in Mine Shafts to Measure Height

By Talya Lebson

I Introduction

In a mine shaft, there are many forces at play that need to be understood. These forces include gravitational force, which is dependent on the distance from the center of the Earth, drag force, which is dependent on the drag constant, and Coriolis force, which is affected by the Earth's rotation. The gravitational force is caused by the gravitational pull of the Earth and acts in the downward direction (towards the center of the Earth). The drag force is caused by the air that is being pushed out of the way while the object is falling and acts in the upward direction (away from the center of the Earth). The Coriolis force is caused by the Earth rotating and acts both in the transverse direction and in the downwards direction (towards the center of the Earth).

In this report for our mining company, I did many calculations to understand the effects of all of these forces and to see how these forces will affect measuring the vertical shaft by dropping a test mass. I explored if the test mass will reach the bottom before hitting the side of the shaft and what the expected fall time is for the test mass under different circumstances.

II Calculation of fall time (including drag and variable g).

To calculate the fall time, I used the solve_ivp function in Python. The equation used initially to calculate the fall time of the test mass with no drag or variable gravity was $\frac{d^2y}{dt^2} = -g + \alpha\left|\frac{dy}{dt}\right|^{\gamma}$ with α set to zero and γ set to 2. Since this is a second derivative and solve_ivp can only handle first derivatives, I needed to split this equation into two first derivatives. I set $\frac{dy}{dt} = v$ and then set $\frac{dv}{dt} =- g + \alpha|v|^{\gamma}$. The fall time found for a 4 km mine shaft with no drag and constant gravity was 28.3 seconds. To confirm that this was an accurate fall time, I compared it with a theoretical calculation which used the equation $t = \sqrt{2 * d/a}$.. For the acceleration in that equation, I plugged in g0, the gravitational acceleration of the Earth, which is 9.81 m/s^2. The theoretical time that I calculated was 28.6 seconds. The theoretical fall time and the fall time calculated with the integral were only 0.3 seconds different, which is insignificant, so the theoretical fall time was confirmed. The fall time calculated with gravity dependent on the distance from the radius was 28.5 seconds. This was calculated with solve_ivp as used before, but -g was replaced with $g(r) = g_o\left(\frac{r}{R_\oplus}\right)$, with R being the radius of the Earth and r being the distance from the center of the Earth. Lastly, the fall time was calculated with a drag force. The same equation that was used for the second calculation was used for this calculation, but alpha

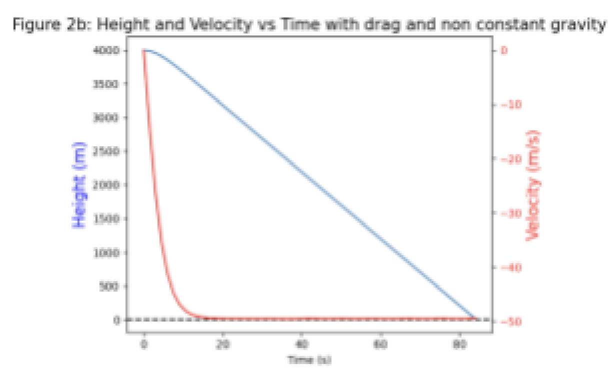

Figure 1: This is a graph of height and velocity as a function of time. The blue line represents height and the red line is velocity. It is clear that the test object will hit a terminal velocity at -50 m/s and the object hits the bottom at around 84 seconds.

was calibrated to be 0.004. Alpha was calibrated with the knowledge that the terminal velocity of the test object should be -50 m/s. I manually adjusted the value of alpha until the integral had a final velocity of -50 m/s. The final fall time calculated with drag and a variable gravity was 84.1 seconds. This shows that varying gravitational acceleration did not have a very significant effect on fall time with a shaft that is 4 km deep (0.2 seconds longer than no variable gravity), but once drag was added it had a very significant effect on fall time (55.6 seconds longer than no drag or variable gravity).

III Feasibility of depth measurement approach (including Coriolis forces)

The next calculation I conducted is the fall time with no drag, but with the Coriolis force. This is a force caused by the rotation of the Earth that acts in the x and y directions. The equation I used to calculate this force is ax = 2*omega*vy, and ay = -g(r) - 2*omega*vx. Ax is the acceleration in the x direction, omega is the earth's rotation rate, vy is the velocity in the y direction, ay is the acceleration in the y direction, g(r) is the same variable gravity acceleration as used above, and vx is the velocity in the x direction assuming it begins falling from the middle of the shaft. I used these two equations in solve_ivp to calculate the position in the x and y direction. When looking at the graph of the traverse position, it becomes clear that the test mass will hit the side before it is able to hit the bottom. It will hit the side at time=29.6 seconds when it is at a depth of 2.7 km, which is not all the way to the bottom of the shaft. When drag was added using the same equation as above, the test mass still appears to hit the side before it hits the ground, with it hitting the side at 29.6 seconds when it is at a height of 2.7 km.

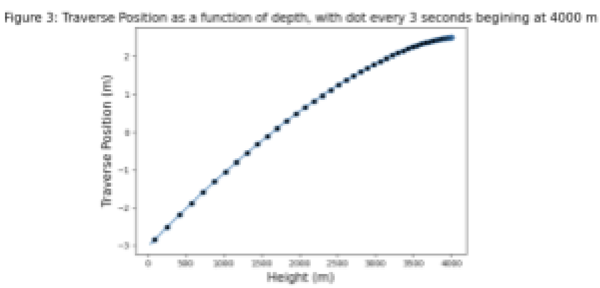

Figure 2: In this figure Traverse Position is plotted as a function of height of the mine. It is clear from this graph that the test mass will hit the wall (at y=0) before it gets to the bottom of the shaft (x=4000). The black dots on the graph are every 3 seconds.

IV Calculation of crossing times for homogeneous and non-homogeneous earth.

The last calculation done in the report was calculating the position and velocity of the test mass, assuming the Earth is not of uniform density. The Earth becomes denser closer to the center of the Earth, which can affect the force of gravity on the test object. The equation used to calculate the M with respect to the density of the Earth is M $= 4\pi \int_0^{R_\oplus} \rho(r) r^2 \, dr$ with $\rho(r) = \rho_n \left(1 - \frac{r^2}{R_\oplus^2}\right)^n$. P(n) was calibrated for each value of n using quad to ensure that the mass of the Earth stayed the same throughout all values of n. This was integrated using quad, which adds up the area under the graph to calculate the integral. The M(r) found using this equation was plugged into the equation $F = G * M(r)/r^2$. This was then plugged into solve_ivp to find the fall times at different density concentrations with G being the gravitational constant. The fall time to the center of the earth with n=0, which is the lowest density concentration, was 1.267e3 seconds with a final velocity of -7.9e3 m/s, and the fall time with n=9, the highest density concentration, was 9.44e2 seconds with the final velocity of -1.8e4 m/s. These calculations show that when the density concentration is higher, the object will take less time to fall because the object falls at a faster velocity.

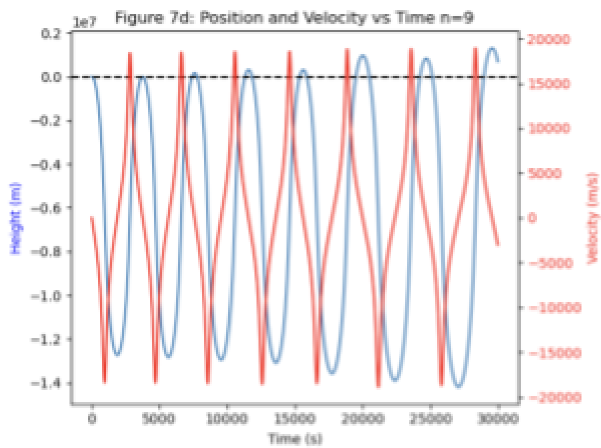

Figure 3: In this figure it can clearly be seen that the position and velocity oscillate as the test mass goes through the gravitational tunnel.

The last calculation made for this report was the fall time to the center of the moon. The fall time for the moon is calculated using the formula $a = -G * Mm * r/R^3$, where G is the gravitational constant, Mm is the mass of the moon, r is the distance from the center of the moon, and R is the radius of the moon. This was plugged into solve_ivp, and the fall time to the center of the moon was 1.6e3 seconds. The fall time to the center of the Earth calculated above was 1.3e3 seconds. It takes 300 more seconds to travel to the center of the moon than to the center of the Earth. The ratio difference in density between the Earth and the Moon is .6. When .6 is raised to -.5, then the result is the ratio between the fall time to the center Earth and the fall time to the center of the Moon (1.3). This shows that the fall time ratio is proportional to density ratio ^ -.5.

V Discussion and Future Work

Based on the findings above, I do not recommend using the test mass technique to measure the height of the shaft. For starters, drag is very complicated to take into account and involves a lot of guesswork, but it has a very big impact on the fall time of the test mass. The difference in fall time before and after drag was added was 55.6 seconds, which was more than double the original fall time with no drag or variable gravity. Additionally, I calibrated the drag constant based on previous knowledge, but that is not necessarily an accurate measurement, which would affect future calculations of the height of the shaft. On top of that, based on the Coriolis Force calculations, the test mass would likely hit the side of the shaft before it hits the bottom, which would mess up all the future calculations of the height of the shaft.

There were many assumptions made in these calculations, such as γ must equal 2, and alpha being calibrated by the researcher. Additionally, I assumed that the test mass would fall right in the center of the shaft, but that would likely not occur correctly either. On top of that, I assumed that the Earth is perfectly round, which it is not. Future research should calibrate the drag constant and γ , through experimentation, not guesswork to make it more accurate. Additionally, more calculations should be done assuming the test mass is falling from various points in the traverse direction to see if that has any impact on whether the test mass will hit the wall first. Lastly, the calculations should include a more accurate shape of the Earth.