punto 3

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sea por definicion

$$x_m = \frac{a+b}{2}$$

$$h = b - a$$

$$\int_a^b f(x)dx \cong \int_a^b p_2(x)dx$$

$$p_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}f(b)$$

entonces se reinterpreta la integral como

$$\int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}f(b)dx$$

entonces las primera suma es

$$f(a)\int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}dx$$

$$f(a)\int_a^b \frac{(x-b)(x-\frac{a+b}{2})}{(a-b)(a-\frac{a+b}{2})}dx = f(a)\frac{2}{h^2}\int_a^b (x-b)(x-\frac{a+b}{2})dx = f(a)\frac{2}{h^2}\int_a^b x^2 - \frac{ax+bx}{2} - bx + \frac{ab+b^2}{2}dx$$

$$= f(a)\frac{2}{h^2}(\frac{x^3}{3} - \frac{ax^2+bx^2}{4} - \frac{bx^2}{2} + \frac{abx+b^2x}{2})\int_a^b = f(a)\frac{2}{h^2}(\frac{b^3-3ab^2+3a^2b-a^3}{12}) = f(a)\frac{2}{h^2}\frac{h^3}{12} = f(a)\frac{h}{6}$$

la segunda suma seria

$$f(x_m)\int_a^b \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}dx$$

$$f(x_m)\int_a^b \frac{(x-a)(x-b)}{(\frac{a+b}{2}-a)(\frac{a+b}{2}-b)}dx = -f(x_m)\frac{4}{h^2}\int_a^b (x-a)(x-b)dx = -f(x_m)\frac{4}{h^2}(\frac{x^3}{3} - \frac{bx^2}{2} - \frac{ax^2}{2} + abx)\int_a^b$$

$$= f(x_m)\frac{4}{h^2}\frac{h^3}{6} = f(x_m)\frac{4h}{6}$$

y la tercera seria

$$f(b)\int_a^b \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}dx$$

$$f(b)\int_a^b \frac{(x-a)(x-\frac{a+b}{2})}{(b-a)(b-\frac{a+b}{2})}dx = f(b)\frac{2}{h^2}\int_a^b (x-a)(x-\frac{a+b}{2})dx = f(b)\frac{2}{h^2}\int_a^b x^2 - \frac{ax+bx}{2} - ax + \frac{ab+a^2}{2}dx$$

$$= f(b)\frac{2}{h^2}(\frac{x^3}{3} - \frac{ax^2+bx^2}{4} - \frac{ax^2}{2} + \frac{abx+a^2x}{2})\int_a^b = f(a)\frac{2}{h^2}(\frac{b^3-3ab^2+3a^2b-a^3}{12}) = f(b)\frac{2}{h^2}\frac{h^3}{12} = f(b)\frac{h}{6}$$

dando asi la suma en total

$$\int_a^b f(x)dx \cong \int_a^b p_2(x)dx = f(a)\frac{h}{6} + f(x_m)\frac{4h}{6} + f(b)\frac{h}{6} = \frac{h}{6}(f(a)+4f(x_m)+f(b))$$

si redefinimos h como h=(b-a)/2 queda

$$\frac{h}{3}(f(a)+4f(x_m)+f(b))$$