Digital Images Project

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1 Pretreatment

The goal here is to reduce noise as much as possible. It can be an issue to estimate perimeter for example. We chose here to use what we have seen in class, e.g. erosion/dilation, which can be combined to form closing/opening. A good noise-filter is to combine those once again. With a 3x3 block we retrieve a connex shape (most of the time) but the shape itself is not that great (not as smooth as we would want it to be). Yet it erase noise which can be useful for some features.

2 Features

2.1 Handmade features

2.1.1 Thickness

This feature is $\frac{perimeter}{area}$. Its goal is to roughly estimate the thickness of the object depicted. Its sole purpose is to be able to differentiate an apple (low coefficient) from a bone (high coefficient).
It is obviously invariant under translation, rotation, rescaling. Yet it is subject to noise. As said before, pretreatment results in a connex but not smooth image, which increase the perimeter and thus this feature.

2.1.2 Trick 1 : size

This one is a little "trick". It doesn't give any information on the object itself but on the image. It is $\frac{area}{size}$. Bells for example cover most of the image but bones only a small part. It helps distinguishing them.
This trick is obviously invariant to everything. Noise doens't affect it much.

2.2 Image moments

The first kind of features we use is called image moments. It is a purely mathematical way to describe the distribution of pixels in an image. The p, q moment of a 2D set of pixels $\mathcal{S}$ is defined by :

$$m_{p,q} = \sum_{(x,y)\in\mathcal{S}} x^p y^q$$

Low moments have intuitive explanations : $m_{0,0}$ is the number of pixels, $\hat{x} = \frac{m_{1,0}}{m_{0,0}}$ is the mean value of abscisse, $\hat{y} = \frac{m_{0,1}}{m_{0,0}}$ is the mean value of ordinate.

To obtain translation invariance, we take a new origin $(\hat{x}, \hat{y})$ and define central moments :

$$\mu_{p,q} = \sum_{(x,y)\in\mathcal{S}} (x-\hat{x})^p (y-\hat{y})^q$$

Now we would like to have features with invariance by rotation. Hu's paper gives seven of them (page 7). Let us prove that the first one is indeed invariant :

First, we suppose the image is centered, which means $\hat{x} = \hat{y} = 0$.

$$\phi_1 = \mu_{2,0} + \mu_{0,2} = \sum_{(x,y)\in\mathcal{S}} (x-\hat{x})^2(y-\hat{y})^0 + (x-\hat{x})^0(y-\hat{y})^2 = \sum_{(x,y)\in\mathcal{S}} x^2 + y^2$$

Now define $(x' = x\cos\theta + y\sin\theta, y' = -x\sin\theta + y\cos\theta)$ obtained from (x, y) by rotation of angle θ.

$$\begin{aligned}
\phi_1' &= \sum_{(x,y)\in\mathcal{S}} x'^2 + y'^2 = \sum_{(x,y)\in\mathcal{S}} (x\cos\theta + y\sin\theta)^2 + (-x\sin\theta + y\cos\theta)^2 \\
&= \sum_{(x,y)\in\mathcal{S}} (x^2\cos^2\theta + y^2\sin^2\theta + 2xy\cos\theta\sin\theta) + (x^2\sin^2\theta + y^2\cos^2\theta - 2xy\cos\theta\sin\theta) \\
&= \sum_{(x,y)\in\mathcal{S}} (x^2 + y^2)\underbrace{(\sin^2\theta + \cos^2\theta)}_{=1} \\
&= \phi_1
\end{aligned}$$

The last step is scale invariance. We define the standardardized moments by $\eta_{p,q} = \frac{\mu_{p,q}}{\mu_{0,0}^{\left(1+\frac{i+j}{2}\right)}}$.

Suppose we have $(x' = \alpha x, y' = \alpha y)$ obtained frome (x, y) by $\alpha > 0$ scaling.

$$\Phi_1' = \eta_{2,0}' + \eta_{0,2}' = \frac{\mu_{2,0}' + \mu_{0,2}'}{\mu_{0,0}'^2} = \frac{\alpha^2(\mu_{2,0} + \mu_{0,2})}{(\alpha\mu_{0,0})^2} = \frac{\mu_{2,0} + \mu_{0,2}}{\mu_{0,0}^2} = \Phi_1$$

Similarly, we get 7 features $\Phi_1, \ldots, \Phi_7$ as given in Hu, that are invariant under translation, rotation and scale.

3 Classification

3.1 Method

kNN ? neural nets ?

3.2 Distance

4 Tests

4.1 Classification

4.2 Distanciation