Digital Images Project

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1 Features

1.1 Image moments

The first kind of features we use is called image moments. It is a purely mathematical way to describe the distribution of pixels in an image. The p, q moment of a 2D set of pixels $\mathcal{S}$ is defined by :

$$m_{p,q} = \sum_{(x,y)\in\mathcal{S}} x^p y^q$$

Low moments have intuitive explanations : $m_{0,0}$ is the number of pixels, $\hat{x} = \frac{m_{1,0}}{m_{0,0}}$ is the mean value of abscisse, $\hat{y} = \frac{m_{0,1}}{m_{0,0}}$ is the mean value of ordinate.

To obtain translation invariance, we take a new origin $(\hat{x}, \hat{y})$ and define central moments :

$$\mu_{p,q} = \sum_{(x,y)\in\mathcal{S}} (x-\hat{x})^p (y-\hat{y})^q$$

Now we would like to have features with invariance by rotation. Hu's paper gives seven of them (page 7). Let us prove that the first one is indeed invariant :

First, we suppose the image is centered, which means $\hat{x} = \hat{y} = 0$.

$$\phi_1 = \mu_{2,0} + \mu_{0,2} = \sum_{(x,y)\in\mathcal{S}} (x-\hat{x})^2(y-\hat{y})^0 + (x-\hat{x})^0(y-\hat{y})^2 = \sum_{(x,y)\in\mathcal{S}} x^2 + y^2$$

Now define $(x' = x\cos\theta + y\sin\theta, y' = -x\sin\theta + y\cos\theta)$ obtained from (x, y) by rotation of angle θ.

$$\begin{aligned}
\phi_1' &= \sum_{(x,y)\in\mathcal{S}} x'^2 + y'^2 = \sum_{(x,y)\in\mathcal{S}} (x\cos\theta + y\sin\theta)^2 + (-x\sin\theta + y\cos\theta)^2 \\
&= \sum_{(x,y)\in\mathcal{S}} (x^2\cos^2\theta + y^2\sin^2\theta + 2xy\cos\theta\sin\theta) + (x^2\sin^2\theta + y^2\cos^2\theta - 2xy\cos\theta\sin\theta) \\
&= \sum_{(x,y)\in\mathcal{S}} (x^2+y^2)\underbrace{(\sin^2\theta + \cos^2\theta)}_{=1} \\
&= \phi_1
\end{aligned}$$

The last step is scale invariance. We define the standardardized moments by $\eta_{p,q} = \frac{\mu_{p,q}}{\mu_{0,0}^{\left(1+\frac{i+j}{2}\right)}}$.

Suppose we have $(x' = \alpha x, y' = \alpha y)$ obtained frome (x, y) by $\alpha > 0$ scaling.

$$\Phi_1' = \eta_{2,0}' + \eta_{0,2}' = \frac{\mu_{2,0}' + \mu_{0,2}'}{\mu_{0,0}'^2} = \frac{\alpha^2(\mu_{2,0} + \mu_{0,2})}{(\alpha\mu_{0,0})^2} = \frac{\mu_{2,0} + \mu_{0,2}}{\mu_{0,0}^2} = \Phi_1$$

Similarly, we get 7 features $\Phi_1, \ldots, \Phi_7$ as given in Hu, that are invariant under translation, rotation and scale.

1.2 tes trucs Etienne

2 Classification

2.1 Method

kNN ? neural nets ?

2.2 Distance

3 Tests

3.1 Classification

3.2 Distanciation