Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
A		2415921

Resource Availability and Sex Ratios

Summary

Lampreys are a primitive jawless fish. They are a special species in the vertebrate kingdom, with the characteristics of adaptive sex ratio variation. Its parasitic lifestyle and unique life cycle give it an important place in biological research and ecosystems. In this paper, we established a model to study the effects of resource availability and sex ratio on lampreys' populations and their impacts on the ecosystem.

First, in studying what happens when a population of lampreys can change its sex ratio and what that does to the larger ecosystem. We used a Logistic growth model to simulate individual population changes in lampreys. Because lampreys have a special reproductive mode, we changed the growth rate of the Logistic growth model to a parameter related to the sex ratio, in which the sex ratio model was used to obtain the sex ratio relationship. Then, the effects of lampreys on the ecosystem were analyzed by establishing a food web model based on the different predation amounts of different sexes.

Secondly, when analyzing the advantages and disadvantages of lampreys' population, we improved the Logistic growth model and made a comparative analysis with the common population.

In view of the impact of lampreys' population on ecosystem stability, the energy changes of other species derived from food web models can reflect the impact of lampreys' population on ecosystem stability to a certain extent.

In the end, lampreys' population is special, so the particularity of the ecosystem including lampreys' population cannot be ruled out. When studying whether the change of the sex ratio of lampreys can provide advantages for the ecosystem, the position of lampreys in the ecosystem can be replaced by common species in the food web model, and the difference between the two results can be compared at last. The analysis reveals the advantages that the change in lamprey's ratio provides to the ecosystem.

Key words: Logistic growth model, Food web model, Ecosystem model

1 Model establishment and solution

1.1 Problem 1: Model establishment and solution

1.1.1 Problem 1: Model establishment

To study the effects of lampreys on ecosystems, we should first know something about lampreys. There are three stages in the life cycle of lampreys: a longer-lasting juvenile stage, a special metamorphosis stage, and a relatively short adult stage. The life cycle of lampreys begins in a long (seven years or more) juvenile stage, with a very different lifestyle from that of adults. The larvae are called ammocoete, and as the name suggests, they hide in the sand and rarely leave the sand layer to enter the water body. The larvae feed on microscopic plankton or algae (the young's horseshoe mouth later morphs into the adult's fearsome pharynx). The larvae stay in the river bottom for about four years before morphing into parasitic fish and swimming into open water. Adult lampreys live for about a month and do not eat. And the important message is that lampreys die when they reproduce, so a male or a female can only reproduce once in a lifetime, and what this tells us is that it's not that the greater the proportion of females, the higher the population birth rate, but that the closer the ratio of males to females, the higher the population birth rate, so you can see that the population growth rate is related to the sex ratio parameter of the population. In the context of considering the sex changes of lampreys, we used a sex ratio model, in which we improved the male and female birth rates. During the development of lampreys, if they ate less, they would develop into males, and if they ate more, they would develop into females. We discussed the two cases of low and high availability of food resources respectively. In the case of low availability of food resources, a high male ratio can alleviate food tension, and after the low availability of food, the male ratio will decrease and the female ratio will increase. We assume that at one moment the ratio of male to female in a population replaces the ratio of male to female births in a population, and at the next moment this ratio will change.

Model assumption:

I. Let M (t) be the male ratio of lampreys' populations at time t, and in the case of low food availability, the male ratio at the next time is 0.01 more than that at the previous time, and vice versa.

II. Let N (t) be the proportion of females in the lampreys' population at time t, and in the case of low food availability, there are 0.01 less females at the next time than at the previous time, and 0.01 more females at the other time.

III. M_0 and N_0 were set as the ratio of males and females in Lampreys' population at the initial moment.

IV. Let Y (t) and Z (t) be the number of males and females in the lampreys' population at time t.

V. Let y_0 and z_0 be the number of males and females of lampreys at the beginning of the population.

Model building and solving:

The number of males is:

$$\begin{cases} Y(t_1) = (1+M_0-0.5)y_0\ , \\ Y(t_2) = (1+M(t_1)-0.5)Y(t_1), \\ Y(t_3) = (1+M(t_2)-0.5)Y(t_2), \\ Y(t_4) = (1+M(t_3)-0.5)Y(t_3), \\ \quad \cdots\cdots \end{cases} \tag{1.1}$$

The number of females is:

$$\begin{cases} Z(t_1) = (1+N_0-0.5)z_0\ , \\ Z(t_2) = (1+N(t_1)-0.5)Z(t_1), \\ Z(t_3) = (1+N(t_2)-0.5)Z(t_2), \\ Z(t_4) = (1+N(t_3)-0.5)Z(t_3), \\ \quad \cdots\cdots \end{cases} \tag{1.2}$$

The answer is:

$$\begin{cases} Y(t) = y_0 e^{(M(t)-0.5)t} \\ Z(t) = z_0 e^{(N(t)-0.5)t} \end{cases} \tag{1.3}$$

In the context of low food availability, males accounted for 78% and females 22%, while in the context of high food availability, males accounted for 56% and females accounted for 44%. The results obtained by bringing these two conditions into the model are shown in Figures 1.1 and 1.2.

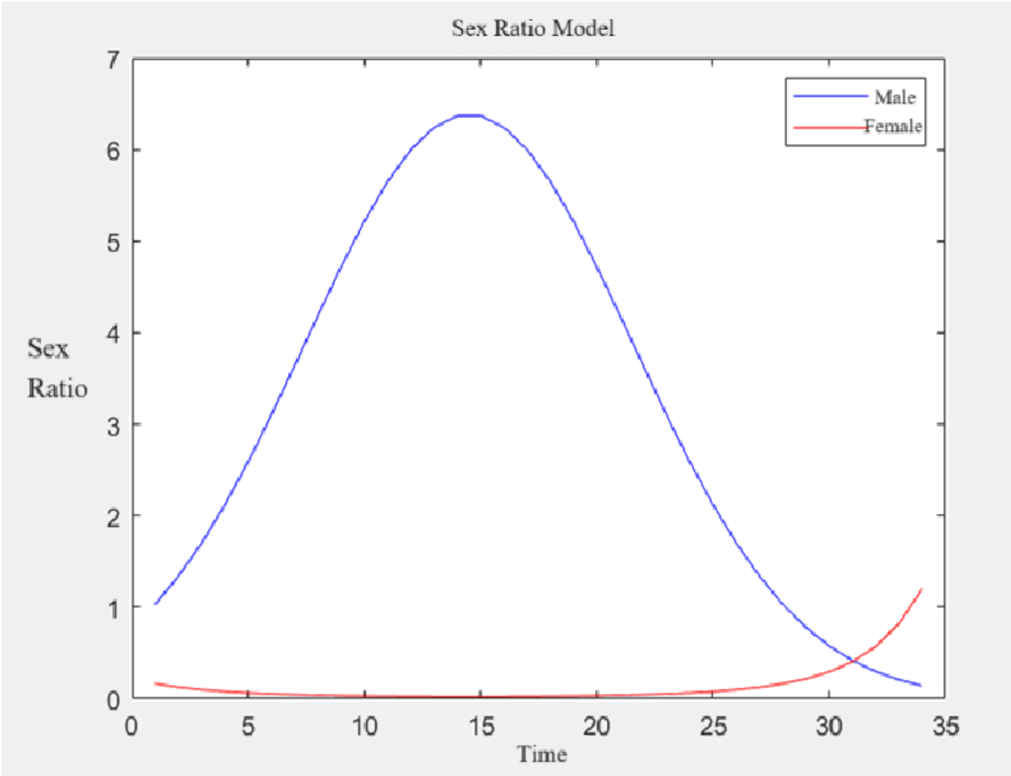


Figure 1.1

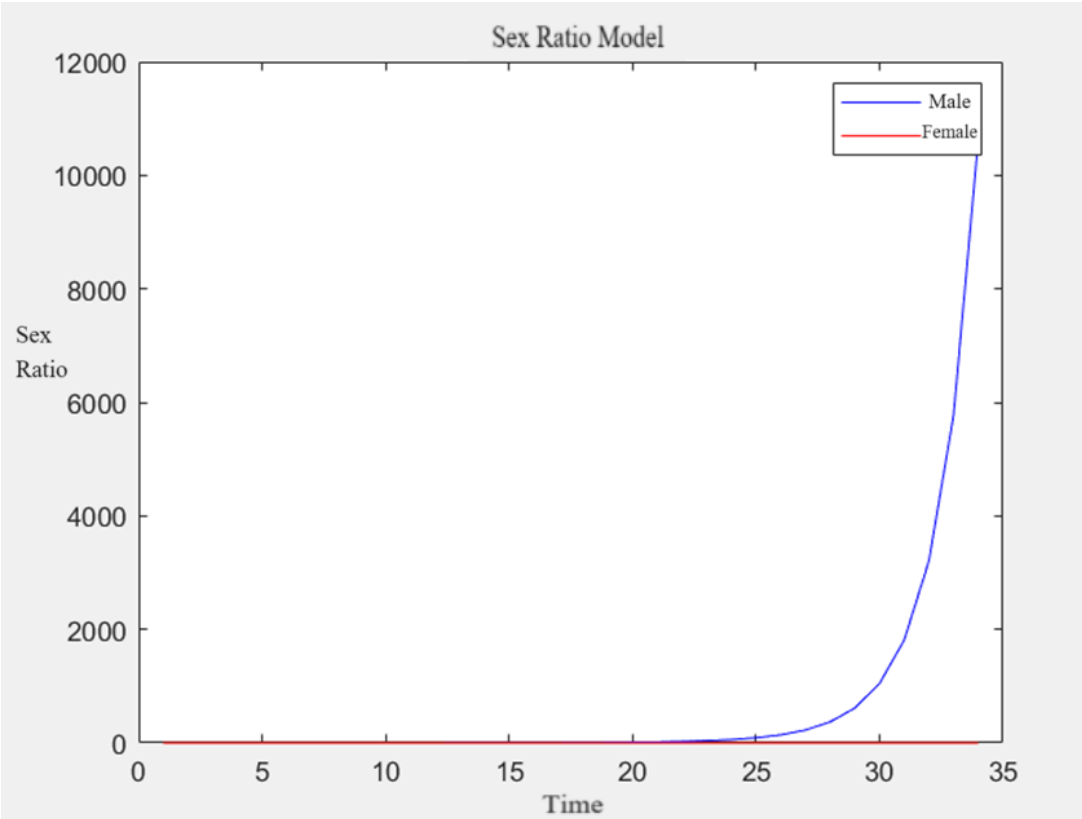


Figure 1.2

We know that the resources of lampreys are limited, and the limited resources will limit the infinite growth of the population, so the population growth rate will not remain constant during the simulation of population growth. We can assume that when the population reaches a certain value it stops growing, at which point the growth rate is 0. However, the reproductive mode and life cycle of lampreys are special. When using the Logistic growth model, the population growth rate of lampreys is not only affected by the limited environmental resources, but also by the sex ratio of lampreys. Since both female and male lampreys reproduce only once in their lives, we assume that the birth rate of lampreys is a function of the sex ratio, and the improved Logistic growth model is as follows.

Model assumption:

I. Population growth rate Q and E represent the growth rates of males and females of Lampreys' population respectively (growth rate = birth rate - death rate), Q (Y) and E(Z) are functions of Y (t) and Z(t), Q (Y)= Q-sY,E(Z)= E-Sz

II. The maximum number of population that can be accommodated by natural resources and environmental conditions is x_m, that is, when $x=x_m$, the growth rate r(x,b)=0.

Model building and solving:

From hypothesis I and II,

$$Q（Y）=Q(1-\frac{Y}{xm}), E（N）=E(1-\frac{Z}{xm}),$$

Then,

$$\begin{cases} \frac{dY}{dt}= Q(1-\frac{Y}{xm})Y, \\ \frac{dZ}{dt}= E(1-\frac{Z}{xm})Z, \\ \\ Y(t_0)=Y_0, \\ Z(t_0)=z_0 \end{cases} \quad (1.4)$$

Equation (1.4) is a separable equation whose solution is:

$$\begin{cases} Y（t）=\frac{xm}{\left(1+\frac{xm-Y_0}{Y_0}\right)e(-M(t)t)}, \\ Z（t）=\frac{xm}{\left(1+\frac{xm-z_0}{z_0}\right)e(-N(t)t)} \end{cases} \quad (1.5)$$

In this data, we still take M (t) and N(t) as the population birth rate, thus we can see that the growth rate of lampreys' population is a binary function of population number and birth rate, and we still bring the two cases into the model to get the following results in figure 1.3 and 1.4.

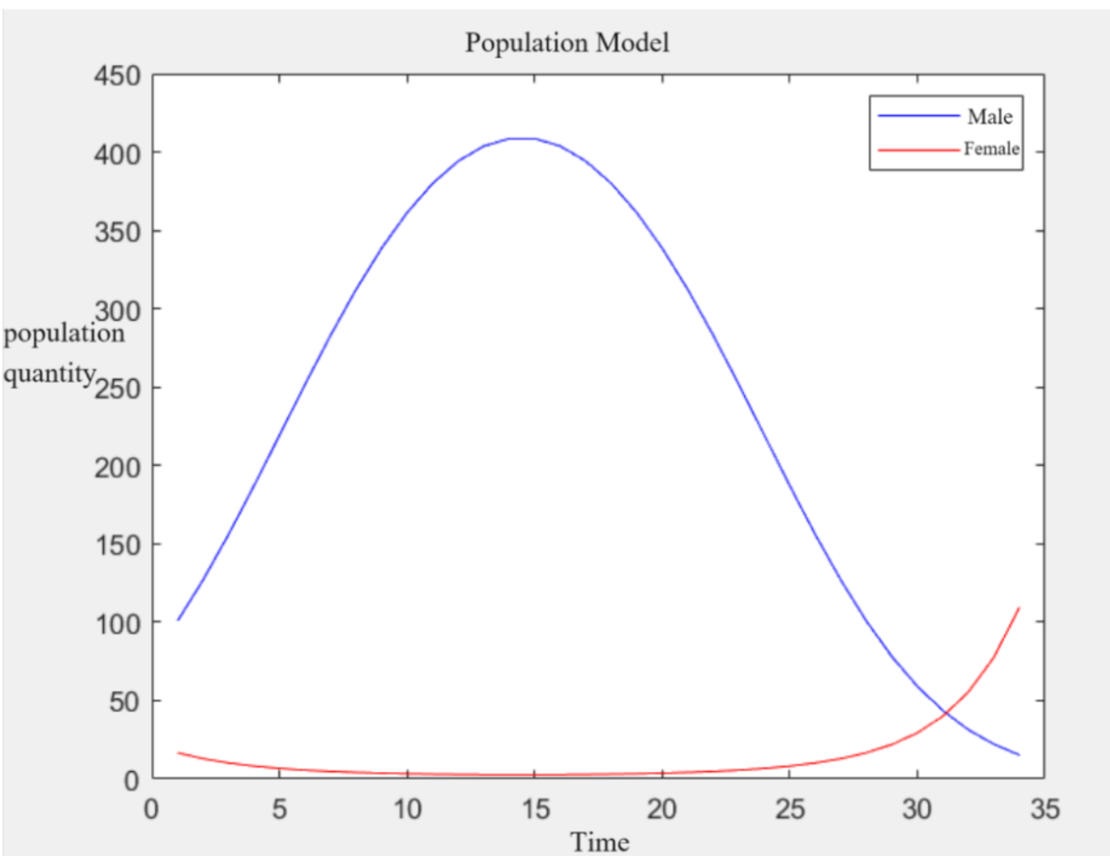


Figure 1.3

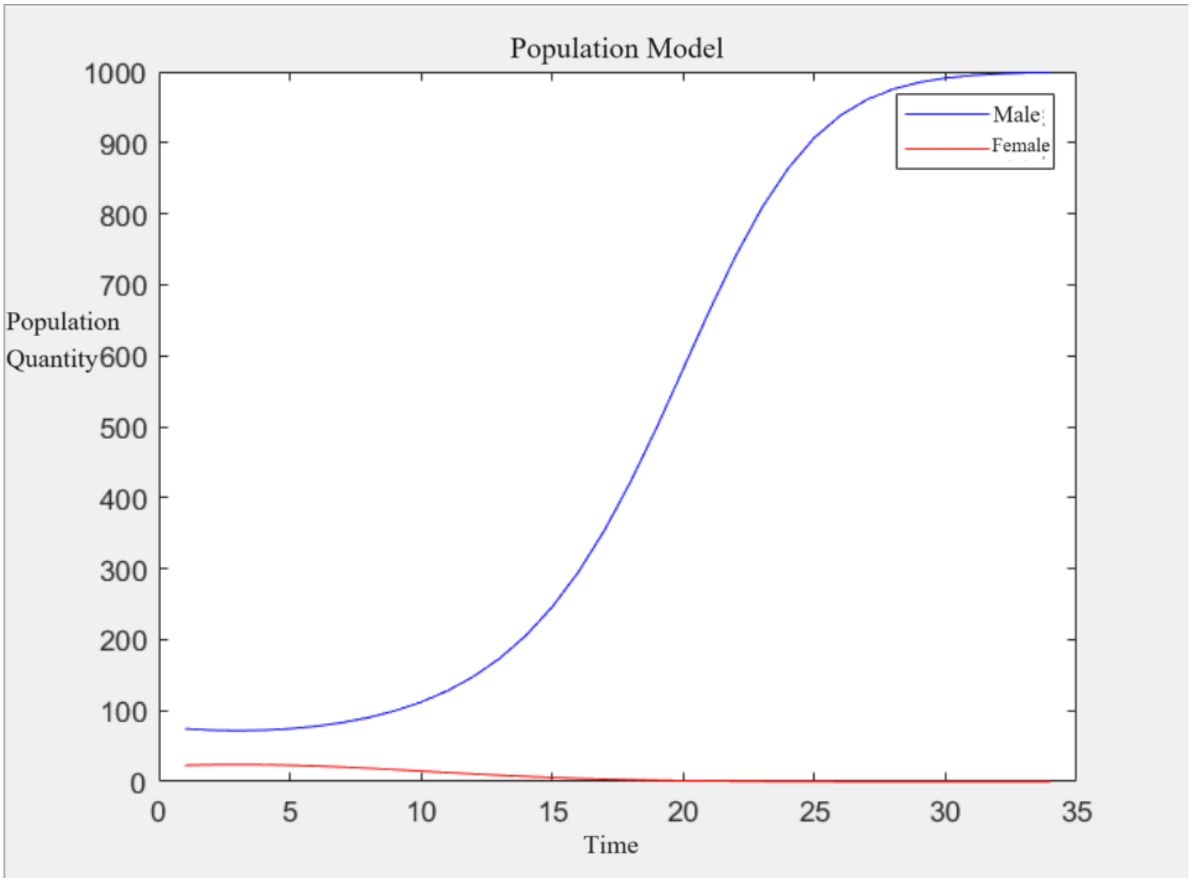


Figure 1.4

The above two figures show the numbers of males and females in the lampreys' population in two situations respectively. After obtaining the numbers of two sexes, a food chain model can be established according to the different food intake of females and males, and then the impact of lampreys' population on the ecosystem can be analysed. When establishing the food chain model, considering that lampreys have few natural predators and the fishing rate is greatly reduced due to the implementation of environmental protection policies, we stipulate that the energy inflow rate of lampreys' population into the upper level is much smaller than the energy intake rate of lampreys' population. Moreover, we stipulate that the population that can be preyed on by lampreys and the population that preys on lampreys are unified into one level respectively. The predation energy ratio of lampreys for different sexes was replaced by the average mass ratio. By searching the literature, we found that the mass W after birth of females (gonadal stage VI) was 2.8495-04L3.5377, and the mass W after birth of males (gonadal stage VI) was 1.3908-03L2.9867 [1].

Model assumption:

I. It is assumed that the upper-level energy quantity R of lampreys' population is only affected by predation and self-growth rate r of lampreys' population, where self-growth rate is constant.

II. The predation rates of male and female lampreys were set as m.n, and the ratio of the two was the ratio of individual mass.

III. Assuming that the upper-level energy source T of lampreys' population is only preying on lampreys and is not affected by its own mortality, the predation rate is w.

IV. The initial energy of the upper and lower stages is R_0,T_0.

Model building and solving:

Models are built by I and II,

$$\begin{cases} R=R_0-mY-nz+rR \\ Z=Z_0+wY+wZ \end{cases} \tag{1.6}$$

The results obtained after bringing in the data can reflect the impact of lampreys' population on the ecosystem under certain circumstances, and the results are as follows figure 1.5 and 1.6.

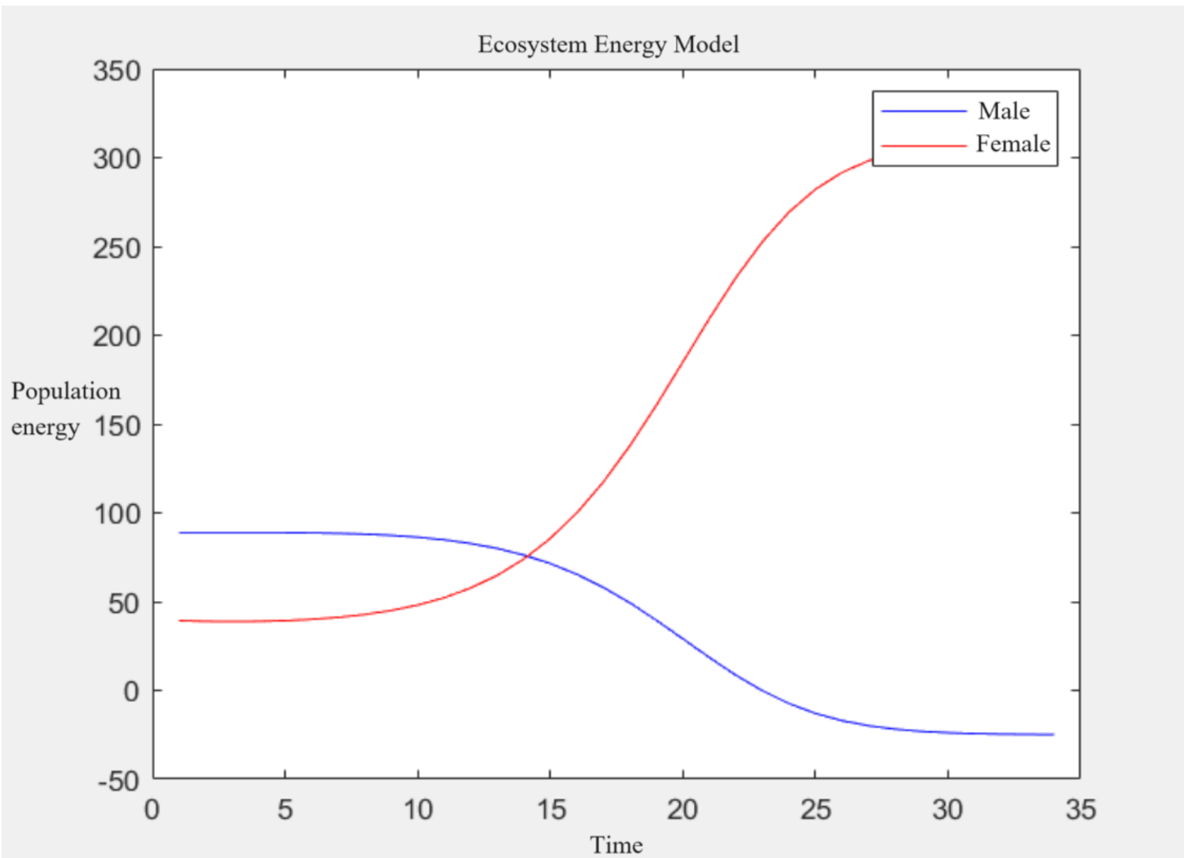


Figure 1.5

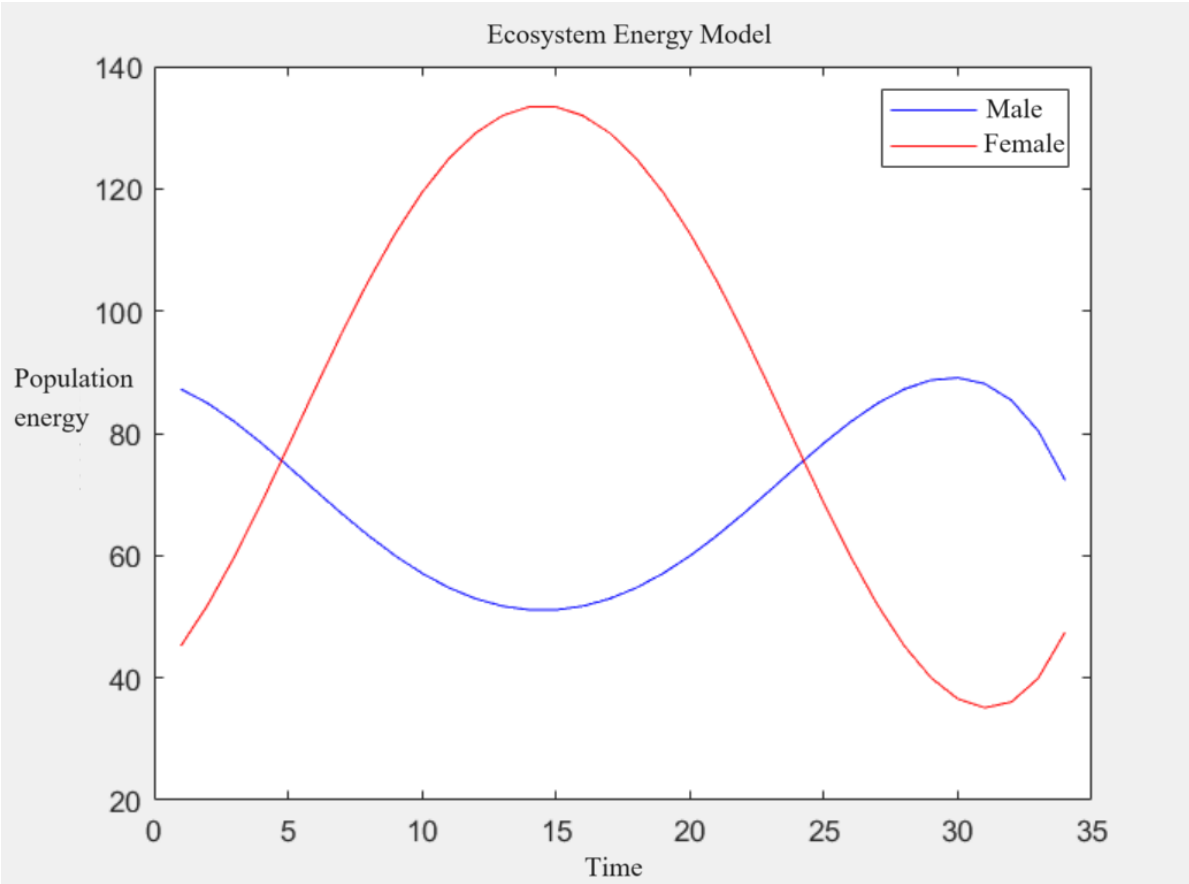


Figure 1.6

1.1.2 Problem 1: Result analysis

In order to solve this problem, the indirect effects of changes in the upper and lower levels of lampreys' population on the response ecosystem were analyzed. Firstly, the ecological impact of low food availability in lampreys' population was analyzed. At the beginning, lampreys did not prey on the next level in large quantities due to the shortage of food resources, nor did the upper-level prey on lampreys in large quantities. Moreover, the number of males of lampreys was significantly more than that of females, while males preyed less than females. Lampreys use a small amount of energy to keep their population as stable as possible. Then, the impact of lampreys on the ecosystem in an environment with high food availability of lampreys' population was analyzed. According to figure 1.6, in an environment with sufficient food, lampreys' population can not only ensure the stability of the next level, but also ensure that the upper level can obtain enough energy and maintain stability. Therefore, lampreys' population can maintain ecosystem stability to a certain extent.

1.2 Problem 2: Model establishment and solution

1.2.1 Problem 2: Model establishment

In order to study the advantages and disadvantages of lampreys' population, the focus is on the particularity of lampreys' population, and the particularity is reflected in the sex ratio and parasitic mode. And the sex ratio directly affects the individual reproductive efficiency, that is, the population birth rate. Before establishing the model, it is necessary to understand that lampreys do not eat after breeding, that is, lampreys reproduce once in a lifetime [2]. When the sex ratio is not 1:1, the sex with the largest proportion will have an individual without a mate, so we can assume a parameter, which is a function of the sex ratio and can affect the population birth rate. The improvement of the Logistic growth model is as follows.

Model assumption:

I. Let X be the parameter related to the sex ratio, where X is the proportion of the minority divided by the proportion of the majority sex.

II. Let Y (t) and Z (t) be the number of males and females in the lampreys' population at time t.

III. Let y_0 and z_0 be the number of males and females at the initial moment of lampreys' population.

IV. Population growth rate Q, E represent male and female growth rates of Lampreys' population respectively (growth rate = birth rate (C) -death rate (S)), Q (X, Y), E(X,Z) are functions of X and Y,Z, Q (X, Y)= CX-sY,E(X,Z)=XC-sZ.

V.The maximum number of population that can be accommodated by natural resources and environmental conditions is x_m, that is, when $x=x_m$, the growth rate r(x,b)=0.

Model building and solving:

From the assumption,

$$Q(X,Y)=QX(1-\frac{Y}{xm}),\ E(X,N)=EX(1-\frac{Z}{xm}),$$

Then,

$$\begin{cases} \frac{dY}{dt}= QX(1-\frac{Y}{xm})Y, \\ \frac{dZ}{dt}= EX(1-\frac{Z}{xm})Z, \\ Y(t_0)=Y_0, \\ Z(t_0)=z_0 \end{cases} \quad (2.1)$$

Equation (2.1) is a separable equation whose solution is:

$$\begin{cases} Y(t)=\frac{xm}{\left(1+\frac{xm-Y0}{Y0}\right)e(-M(t)Q(X)t)}, \\ Z(t)=\frac{xm}{\left(1+\frac{xm-z0}{z0}\right)e(-N(t)E(X)t)} \end{cases} \quad (2.2)$$

After the model is obtained, the two conditions of food availability of lampreys' population are introduced into the model respectively, and the results are shown in figure 2.1 and 2.2. In order to highlight the advantages and disadvantages of lampreys, the number of common species needs to be obtained, that is, the species growth rate is not affected by the sex ratio but only by the limited environmental resources, and figure 2.3 is obtained.

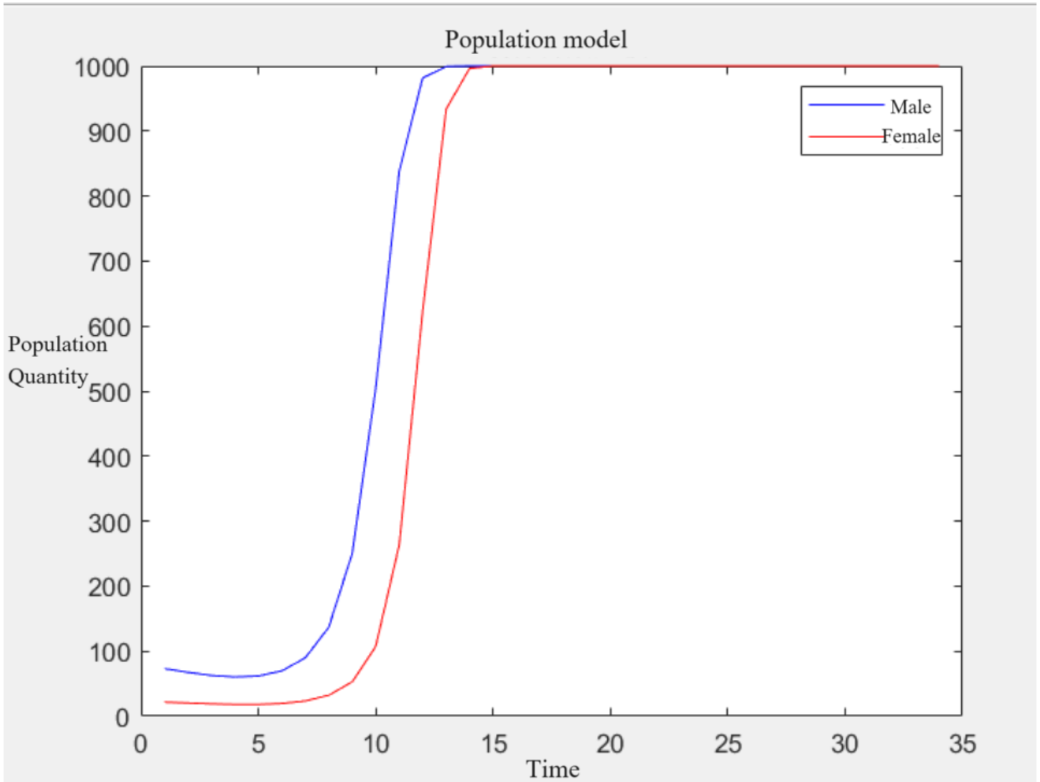


Figure 2.1

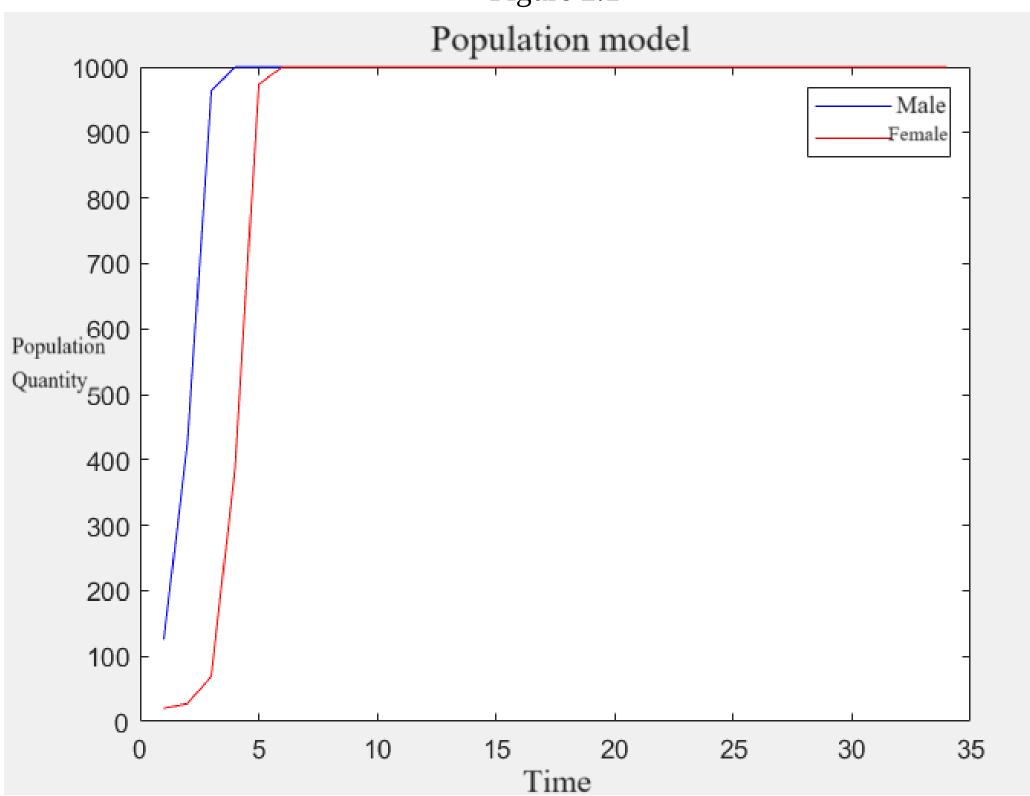


Figure 2.2

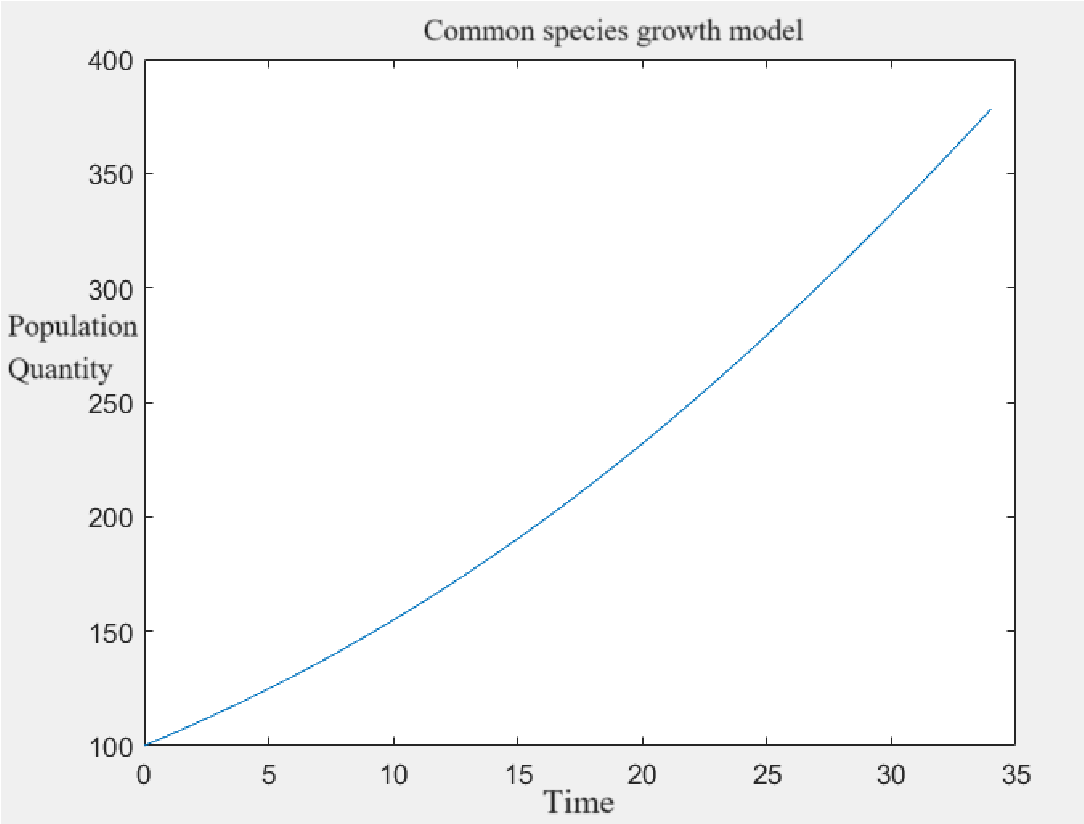


Figure 2.3

The comparison of population changes of common species shows that lampreys have a strong ability to adapt to both abundant and scarce environmental resources. It can be seen from the results that in the case of shortage of environmental resources, it takes 10 units of time for the population of Lampreys to reach the maximum number of individuals, while in the case of adequate environmental resources, it takes less time (5 units of time) for the population of Lampreys to reach the maximum number of individuals, and 180 units of time for the general population. It can be seen that when the population is greatly reduced in a short period of time due to natural disasters or other accidents, the lamprey's population can recover more quickly to the maximum population size. The disadvantage of lampreys is also obvious, when a few lampreys are transferred to other areas, the number of lampreys in this area will explode rapidly, and if not all of them can be treated, the resilience of the population will be very strong, which will cause a huge blow to the local ecological environment.

1.3 Problem 3: Model establishment and solution

1.3.1 Problem 3: Model establishment

To study the effects of lampreys' populations on ecosystems, we can simulate responses through a model that reflects changes in populations or factors other than lampreys in the ecosystem. Ecosystems are complex and diverse, and we can assume that a system (Figure 3.1) consists of a population of lampreys, a population that preys on lampreys (hereinafter referred to as the upper level), a population that is preyed on by lampreys (hereinafter referred to as the lower level), a population that parasitize the above three populations (hereinafter referred to as the parasitic population), and a population of lampreys producing economic value. We assume that the higher rate of increase is a function of the number of individuals who feed on lampreys, their own birth rate, their own death rate, and the number of individuals who feed on lampreys, independent of other factors. The next level is the same, the next level of growth is their own birth rate, their own death rate and the number of individuals that are preyed upon by the lamprey's population. The parasitic population is related to the individual numbers of the above three populations. In this study, we only consider the economic value generated by the lamprey population, so we stipulate that the economic value is only related to the number of individuals in the lamprey population. The upper level obtains energy through predation, so the amount of predation in the upper level directly affects the birth rate in the upper level. The next level of mortality is also directly affected by the amount of predation by lampreys. The lifestyle of the parasitic population is parasitic, so we stipulate that the birth rate and death rate of the parasitic population are affected by the number of individuals of the above three populations.

Model assumption:

I. Assume that the initial number of individuals in the upper and lower levels is A_0 and B_0.

II. Assume that A(t) and B(t) are the number of individual changes at the upper and lower levels at time t.

III. Suppose that the higher birth rate is c_1 and is related to the number of preys on lampreys. (Birth rate = reproductive rate X number of prey lampreys)

IV. Assume that the death rate of the upper level is s_1 and constant.

V. Assume that the next level of birth rate is c_2 and constant.

VI. Suppose that the next level of mortality is s_2 and is related to the number of predations by lampreys. (Mortality = aging rate X predation by lampreys)

VII. Suppose that the economic value of U is proportional to the number of lampreys' population individuals.

VIII. Suppose that the change in the number of parasitic populations is V and is A function of a, B, M, and N, V=Aa+Bb+Mm+nN(A,b,m, and n are the number of parasitic populations in the upper and lower level individuals, male and female lampreys, respectively).

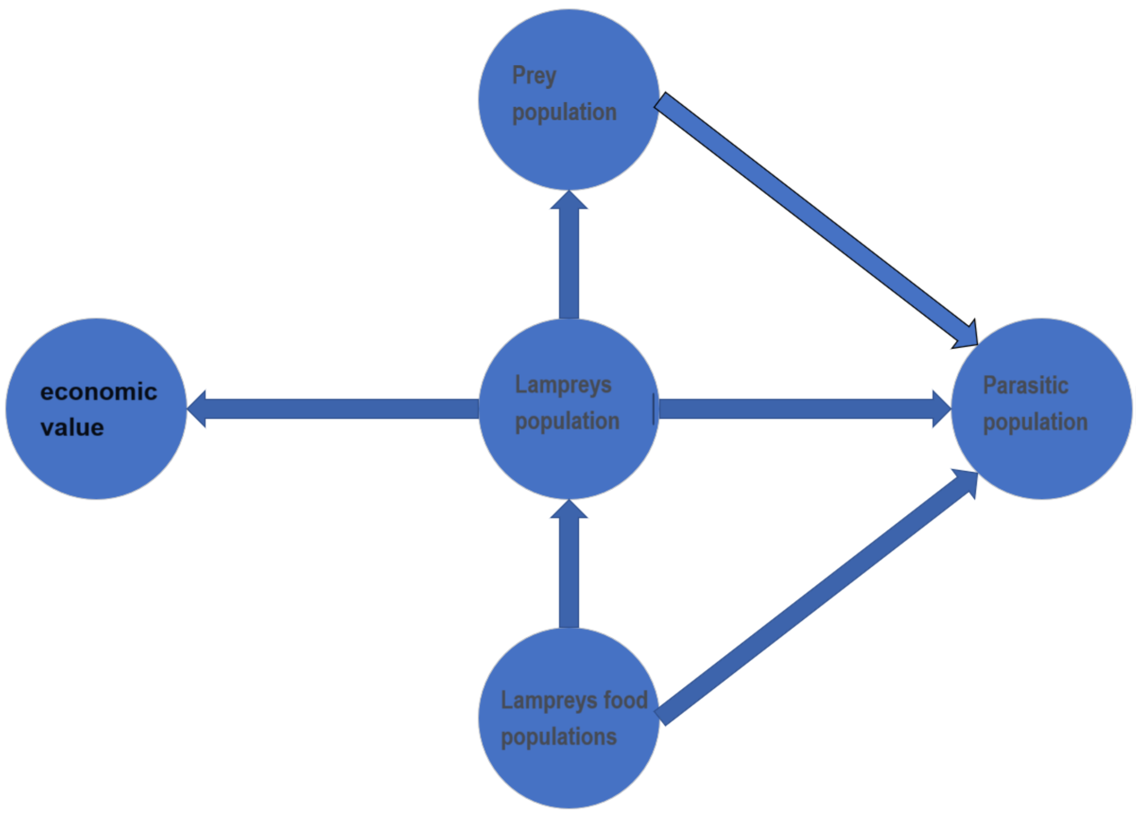
Model building and solving:

From the assumption,

$$\begin{cases} A（t）= c_1XA_0-s_1A_0 \\ B（t）= c_2XB_0-s_2B_0 \\ V=Aa+Bb+Mm+Nn \\ U=1000X(M+N) \end{cases} \quad (3.1)$$

Bringing the data into the model yields Figures 3.2 and 3.3, The birth rate and death rate of the upper-level population and the lower-level population are not only affected by predation, but also by natural disasters, virus transmission, human fishing and other influences. Therefore, the model used in this study is an ideal model, which can reflect certain results within a certain range.

Figure 3.1



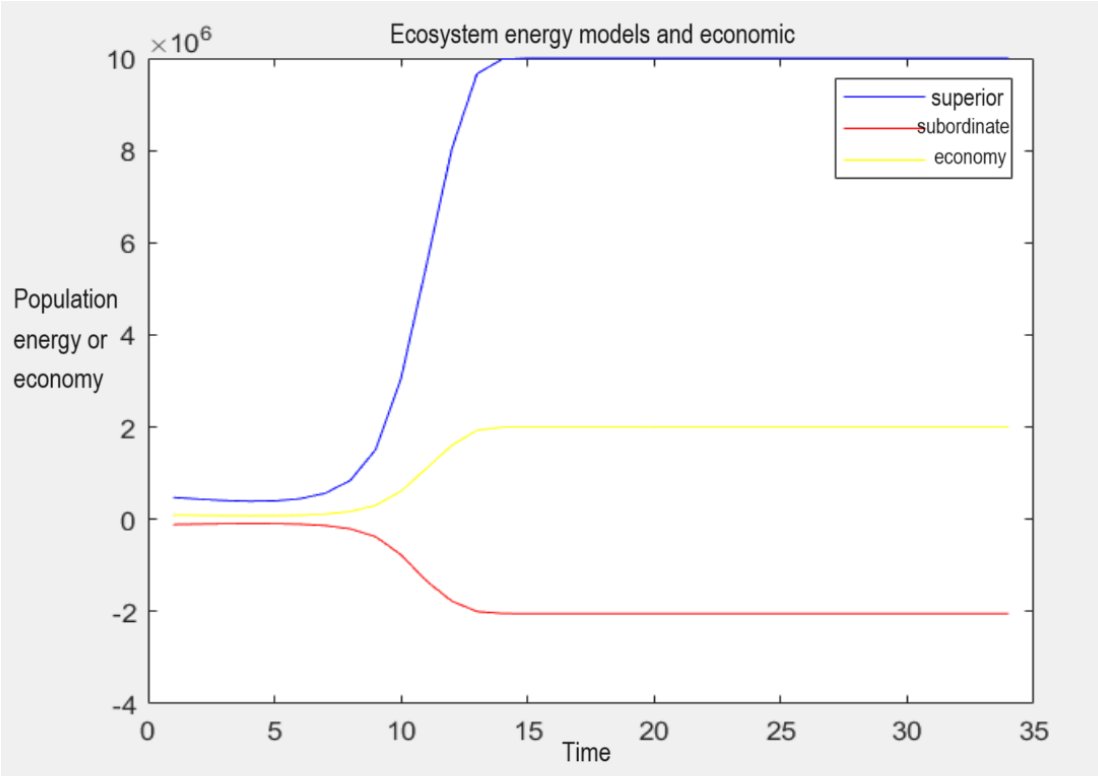


Figure 3.2

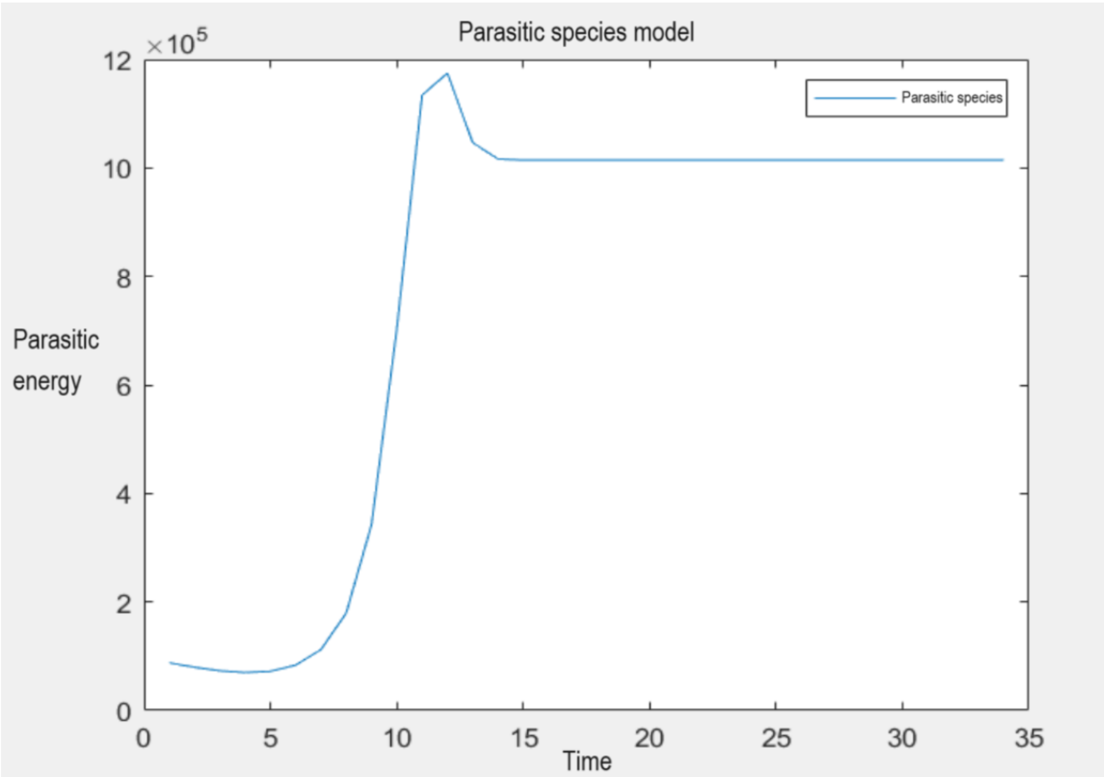


Figure 3.3

1.3.2 Problem 3: Result analysis

According to the results of the above graph, we found that changes in the sex ratio of lampreys would have multiple impacts on the ecosystem. Changes in the sex ratio of lampreys can alter the number of individuals in the lamprey's population and thus affect the number of individuals in other species.

When the female proportion of lampreys increased, the reproductive ability of the species increased, and the number of individuals in the population increased. Due to the approximately constant environmental resources, when lampreys were predators, the number of preys decreased. When lampreys are prey, the number of predators increases. The number of individuals competing with lampreys would also decrease.

When the female proportion of lampreys decreased, the reproduction ability of the species was weakened, and the number of individuals in the population decreased. Due to the approximately constant environmental resources, the number of preys increased when lampreys were predators. When lampreys act as prey, the number of predators decreases. The number of individuals competing with lampreys will also increase.

Lampreys play a specific role in the food chain, and fluctuations in the number of individuals caused by changes in their sex ratio will also cause fluctuations in the number of individuals up and down the food chain, which will create a chain reaction in their ecosystem. When the number of lampreys fluctuates too violently, it will have a great impact on the stability of the ecosystem. We know from consulting relevant information that the main predator of lampreys is humans, so this species is prone to overbreeding. After sea lampreys invaded the Great Lakes, they caused great damage to the fishery resources and ecosystem of the waters. Sea lampreys cling to the fish through their mouth suckers and sharp teeth, sucking the body fluids of the fish, and eventually causing wounds on the fish, resulting in death.

A single sea lamprey can kill about 40 pounds of fish during its parasitic life in a sea lamprey camp [3]. In this regard, we recommend increasing the population control of lampreys (through drug control, barriers, traps, etc.) to maintain the stability of the ecosystem in which they live.

The number of parasitic populations is closely related to the number of individuals in other populations, and the death of population individuals will affect the number of parasitic species. We can find from the image that the number of parasitic population individuals increases sharply and maintains approximately the same number after a certain number.

1.4 Problem 4: Model establishment and solution

1.4.1 Problem 4: Model establishment

Through comparative analysis, we can better understand the advantages and disadvantages of various options and make more informed decisions. In order to study whether the change of sex ratio of lampreys’ population can cause ecosystem advantage, a common ecosystem model can be established again. In order to reflect the environmental advantage of the ecosystem more comprehensively, plant population and oxygen are added to this model. We assume that part of the next level population is herbivores, and there are no herbivores in the upper-level population. The plants in this ecosystem are not affected by environmental seasons, and the plant population has a fast growth rate. In addition, we assume that this ecosystem is a closed ecosystem with no other species introduced except the above species. All the oxygen in this ecosystem comes from the plant species in this ecosystem, and all the oxygen consumed by all the species in this ecosystem comes from this ecosystem. The growth pattern of the upper-level species and the lower-level species remains unchanged.

Model assumption:

I. Assume the initial number of individual plant species H_0.

II. Suppose that the number of individual plant species H (t) at time t, H (t) is related to its own growth rate r and the number of individual herbivorous species in the next level.

III. Suppose that the oxygen content O (t) at time t is related to the number of individuals in the plant population, the number of individuals in the upper level, the number of individuals in the lower level, the number of individuals in the parasitic species, and the number of individuals in the hypothetical species population (or the number of individuals in the lamprey population).

Model building and solving:

From the assumption,

$$\begin{cases} H(t) = H_0 - (0.3X0.5)B + rH_0 \\ O(t) = 5H(t) - 3(A+B+M+N) \end{cases} \quad (4.1)$$

The above are models of plant species individual number change and oxygen change. The hypothetical common species and lamprey species are introduced into this model and the model used in question 3 to obtain Figures 4.1 and 4.2.

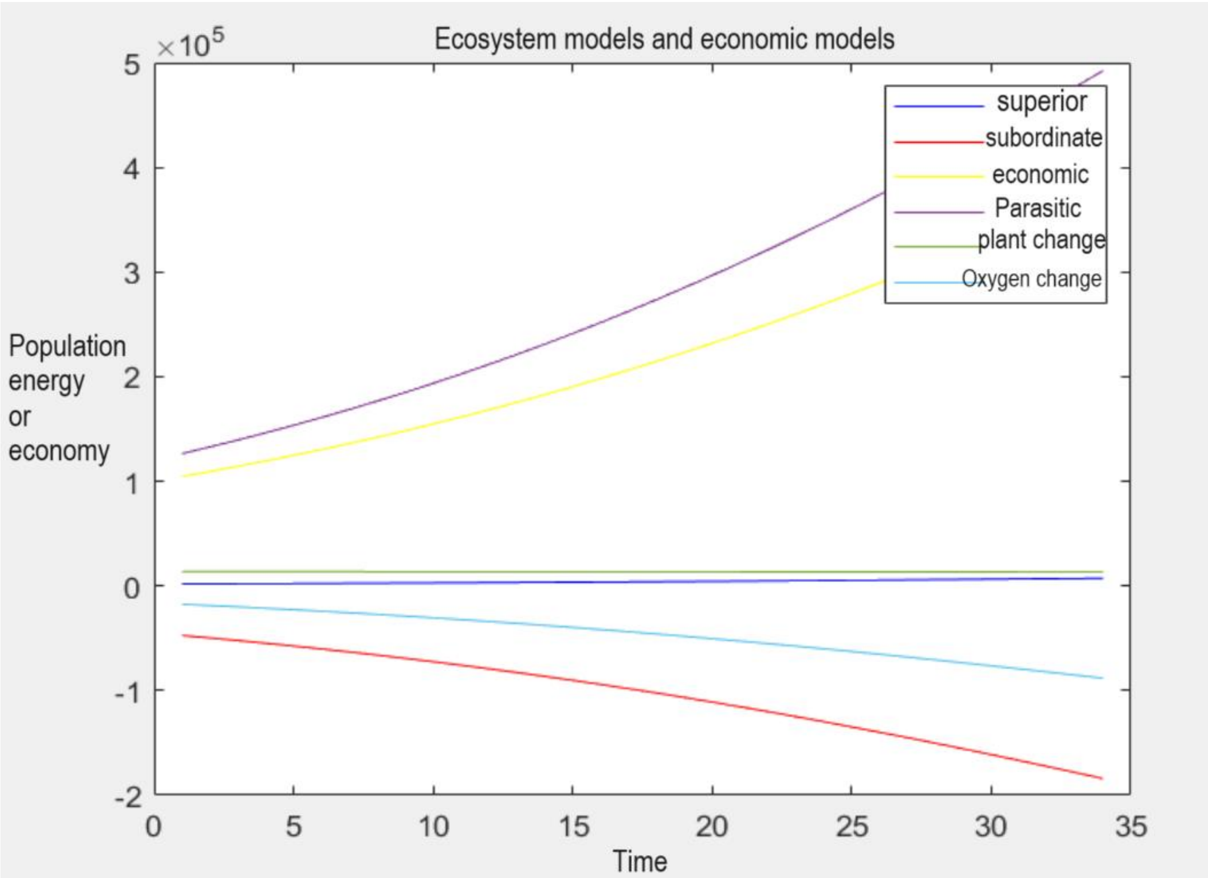
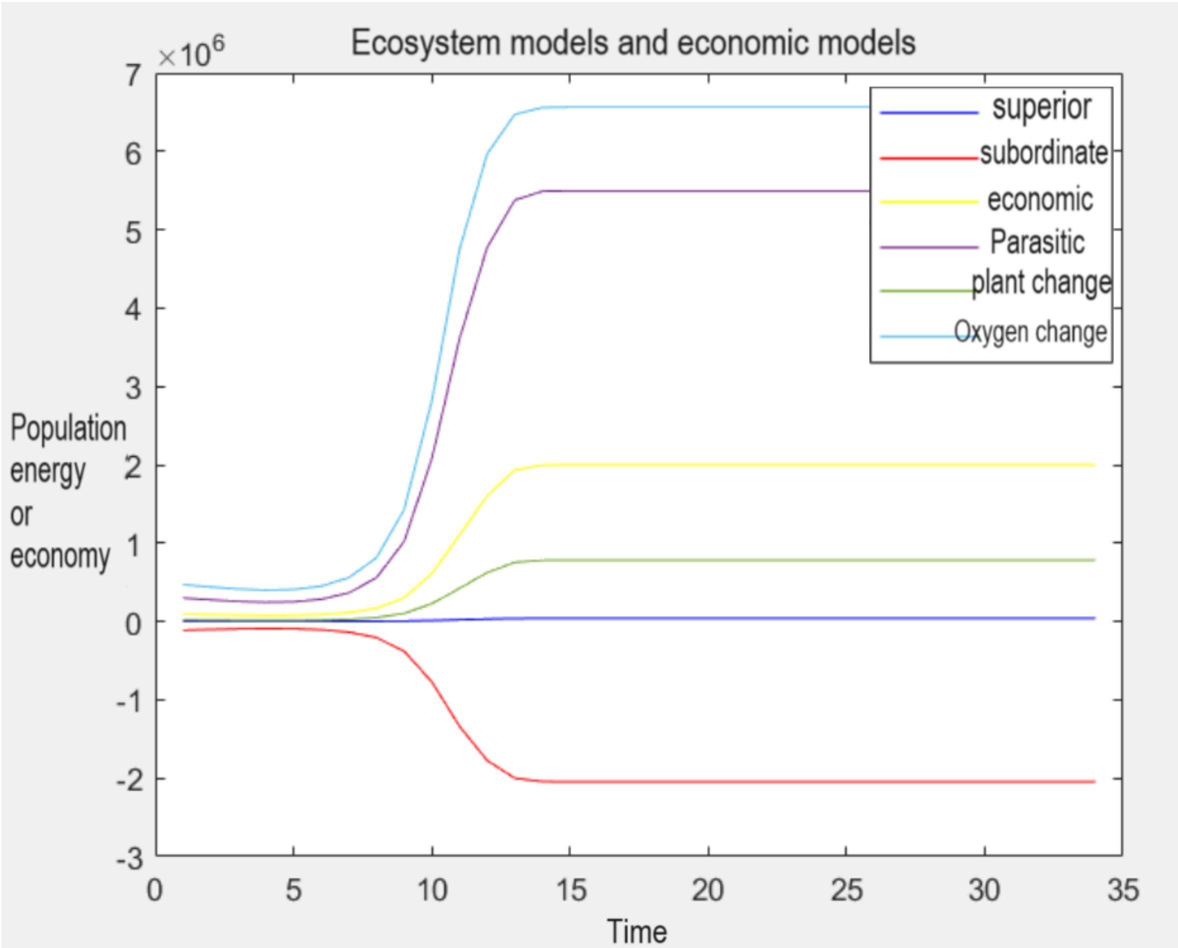


Figure 4.1


Ecosystem models and economic models
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Time
Population
energy
or
economy
superior
subordinate
economic
Parasitic
plant change
Oxygen change

Figure 4.2

1.4.2 Problem 4: Result analysis

According to the images, our analysis found that the change in the sex ratio of lampreys would change the amount of predation, the range of predation and the amount of captured feed in the ecosystem where the lampreys were located, which would have a direct impact on the upper and lower nutrient levels of the food web where the lampreys were located, and we found that this effect was beneficial. It keeps the number of species in the food web, including lampreys, close to a certain number, thereby maintaining the stability of the ecosystem.

When the ratio of male to female of lampreys' changes, its ecological niche will also change. When the proportion of females in lampreys increases, the population increases, and this change provides more food for their predators. When the proportion of lampreys is lower, the population decreases, and this change will provide more opportunities for the species that lampreys prey on and those that compete with lampreys. In addition, changes in the sex ratio of the lampreys' population can also affect their interactions with other species, because female lampreys are more susceptible to parasites and when the proportion of female lampreys increases, it also affects the number and distribution of parasites.

When in a closed ecosystem, changes in the sex ratio of lampreys will also affect changes in resources such as oxygen. When the proportion of female lampreys increases, the number of individuals in the next trophic level population decreases, which will increase the number of plants, and since the oxygen consumption of consumers is basically the same, the number of plants increases, and the amount of oxygen produced will also increase. This also provides more opportunities for other species to survive.

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