Report by Amplitude Analysis task subgroup

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on behalf of

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Spectroscopy WG meeting - March 7th 2016

Decay modes needing an amplitude analysis (in order of AA relative "easiness"):

Charged charmonium-like 4-quark $(c\bar{c}u\bar{d})$ spectroscopy

$B^0 \to J/\psi K^+\pi^-$

[Bari/TIFR]

[PRD 90 (2014) 112009] :

- **Observation (6.2σ) for** $Z_c(4200)^\pm \to J/\psi\pi^\pm$ **[$J^P=1^+$ preferred]**
- **Evidence for** $Z(4430)^\pm \to J/\psi\pi^\pm$
- **No evidence of** $Z_c(3900)^\pm \to J/\psi\pi^\pm$ (seen by BESIII/Belle/CLEO-c)! **Why?**

$B^0 \to \psi' K^+\pi^-$

[Bari/TIFR]

[PRD 88 (2013) 074026] :

- **Observation (5.2σ) for** $Z(4430)^\pm \to \psi'\pi^\pm$ **[$J^P=1^+$ preferred]**

[PRD 92 (2015) 112009 ; PRL 112 (2015) 222002]

- **Confirmation (13.9σ) for** $Z(4430)^\pm \to \psi'\pi^\pm$ **[$J^P=1^+$ preferred]**
- **Evidence for** $Z_c(4240)^\pm \to \psi'\pi^\pm$ **[$J^P=0^-$ preferred to $J^P=1^+$ by 1σ only but width becomes unrealiably large]**

(it cannot be excluded to be 's $Z_c(4200)^\pm$ **)**

It has ~same mass and width of $Z_2'(4250)^\pm$ ***reported by*** **[**PRD 78 (2008) 072004**]** ***but a 0^- state cannot decay strongly to*** $\chi_{c1}\pi^\pm$.

Y(4140) / Y(4274) charmonium-like 4-quark $(c\bar{c}s\bar{s})$

$B^+ \to J/\psi\,\phi K^+$

[Bogazici-MSGSU]

- Evidence (2009)/Observation (2011) from CDF for $Y(4140) \to J/\psi\,\phi$
- Evidence (2011) from CDF for $Y(4274) \to J/\psi\,\phi$
- $Y(4140)$ observation confirmed by CMS (>5σ); evidence by D0
- LHCb provided (2012) UL (at 90% CL, in tension with CDF by 2.4σ) (aiming an AA analysis)
- Naïve yield ratio estimate by CMS consistent with CDF

Additional modes are $B^0 \to J/\psi\,\phi K^0_S,\ J/\psi\,\phi\phi$ (some work started on the first one)

PentaQuark

$\Lambda_b \to J/\psi\, pK^-$

[CINVESTAV]

(just started; needs MC)

LHCb [PRL 115 (2015) 072001 ; Chin. Phys. C40 (2016) 011001]

- Observation (9.0σ , 12.0σ) for $P_c(4380)^+,\ P_c(4450)^+ \to J/\psi\, p$

[they have opposite parity; slightly favoured spin-parity $J^P=3/2^-, 5/2^+$]

(observed features can be explained by interference of two states with opposing parity)

Alternative approach : $B^0 \to J/\psi\, p\bar{p}$ (some work started @ Bari; needs MC)

OUTLINE

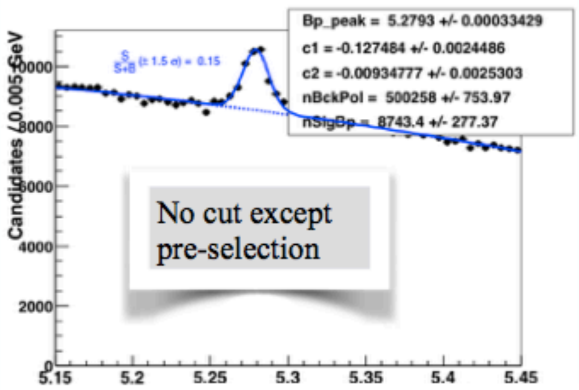
- **Signal extraction with high purity for the various decay modes (either cut-based & with BDT use)**

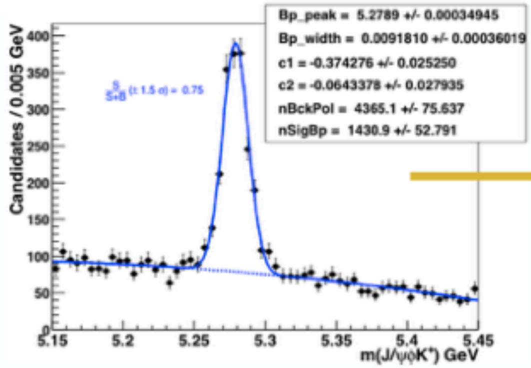
- **Reconstruction efficiency evaluation over the Dalitz Plots**

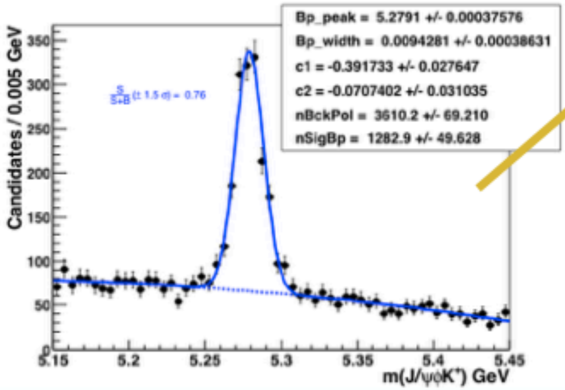
- **Full Amplitude Analysis fit : method & implementation status**

Signal extraction with high-purity

Let us consider first the $B^+ \to J/\psi \phi K^+$ decay mode. Cut-based selection results:





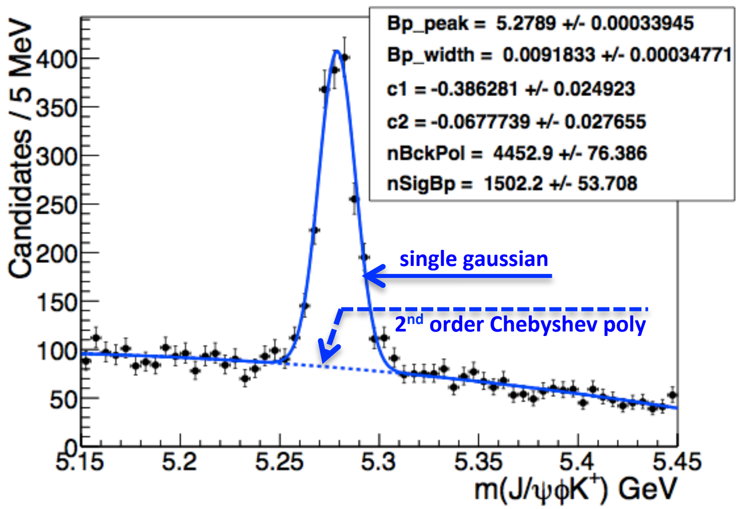


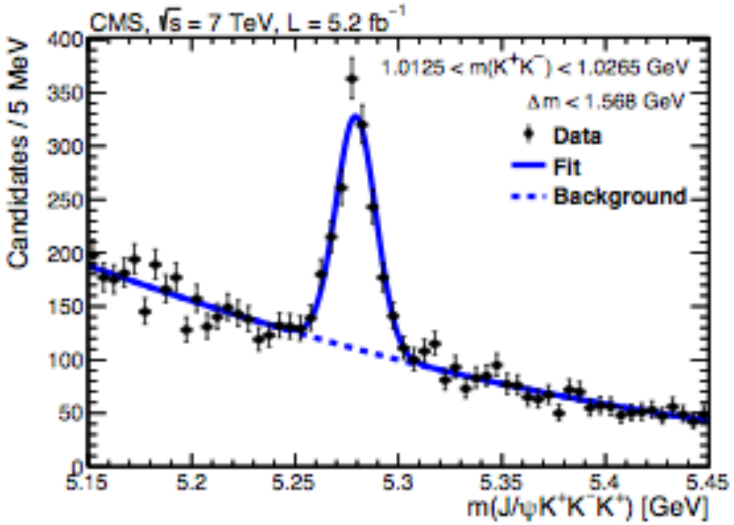
Selection of Cuts	Purity
• Kaon pt > 1.8 • B vtx prob > 0.2 • B sig > 8 • ϕ mass ± 6 MeV • ψ(2S) mass ± 80	FOM : (purity) **S/S+B=%69** **Signal : 1985** Bck : 8002
• Kaon pt > 1.8 • B vtx prob > 0.2 • B sig > 9 • ϕ mass ± 5 MeV • ψ(2S) mass ± 80	FOM : (purity) **S/S+B=%72** **Signal : 1804** Bck : 6362
• Kaon pt > 1.8 • B vtx prob > 0.3 • B sig > 9 • ϕ mass ± 4 MeV • ψ(2S) mass ± 80	FOM : (purity) **S/S+B=%75** **Signal : 1430** Bck : 4365
• Kaon pt > 1.9 • B vtx prob > 0.3 • B sig > 9 • ϕ mass ± 4 MeV • ψ(2S) mass ± 100	FOM : (purity) **S/S+B=%76** **Signal : 1282** Bck : 3610
• Kaon pt > 2.0 • B vtx prob > 0.3 • B sig > 9 • ϕ mass ± 4 MeV • ψ(2S) mass ± 100	FOM : (purity) **S/S+B=%76** **Signal : 1181** Bck : 3150

Trigger selection re-applied offline

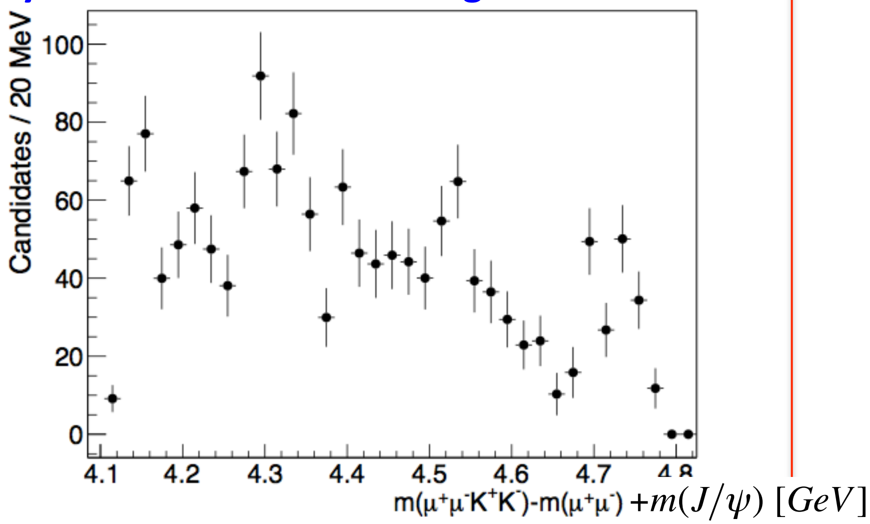
- **Attempts to use TMVA just started for this decay**

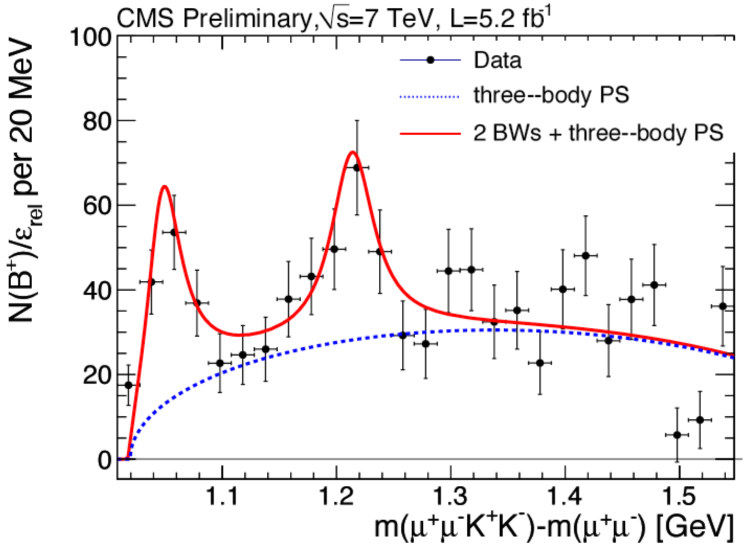
With this selection (delivering ~75% of purity) for $B^+ \to J/\psi \phi K^+$ we get for 2012 data [in comparison to a cleaner sample by tight cuts in 2011 data aiming to maximize stat. significance]:





By bin-wise method for bkg-subtraction :



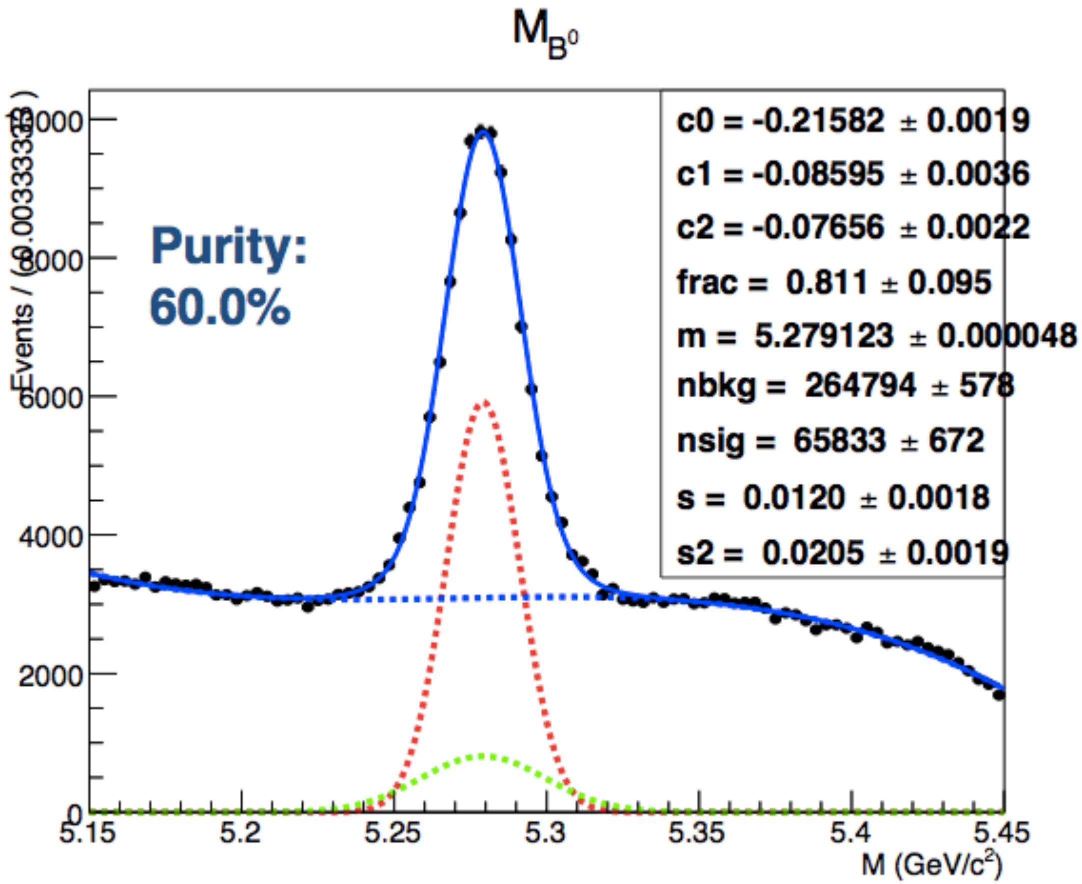


No more Δm cut. Study on B_s reflection is ongoing [$B_s \to \psi(2S) K^+K^- \to J/\psi \pi^\pm \pi^\mp K^+K^-$].

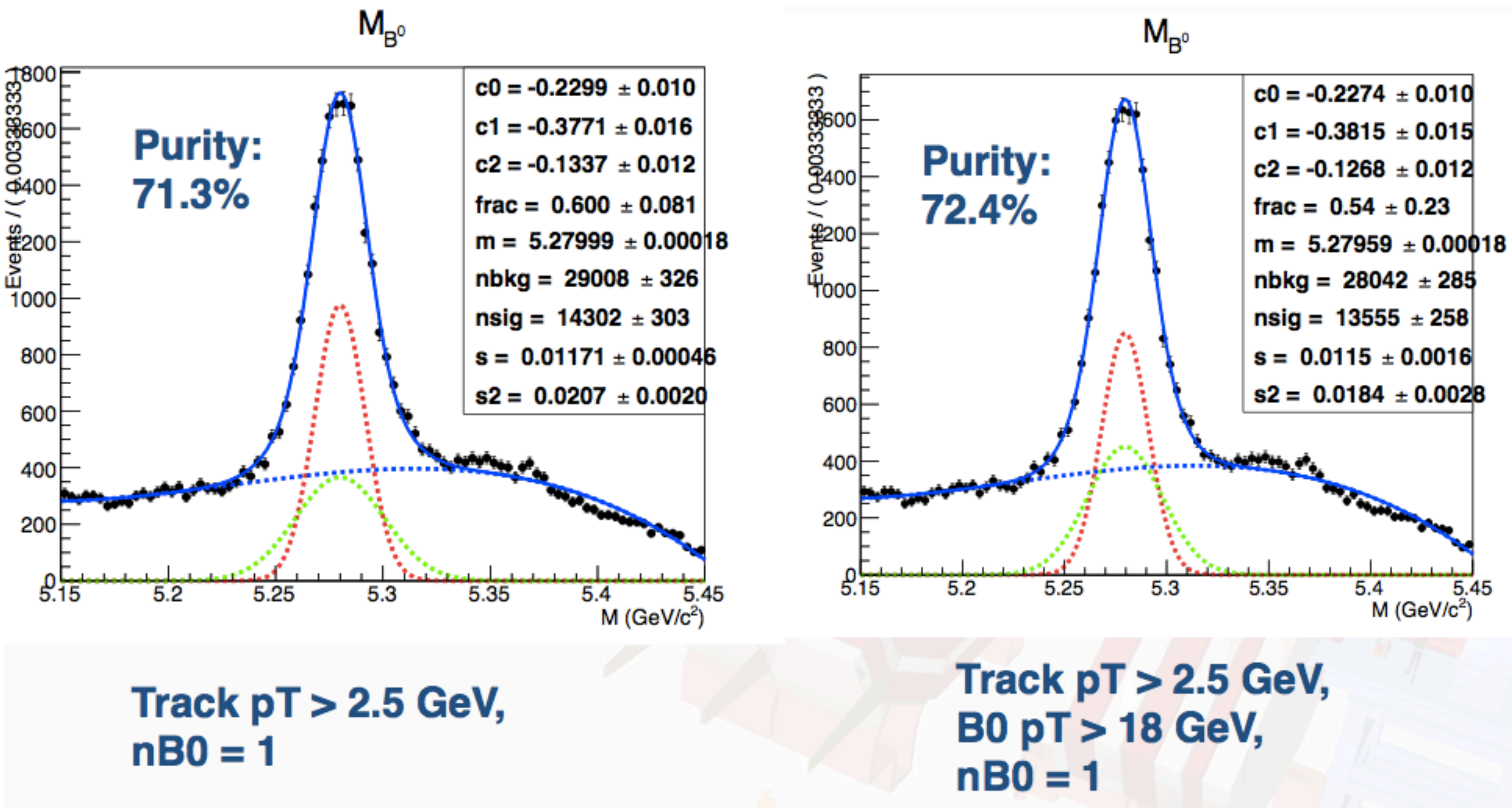
Cut-based selection targeting high-purity for $B^0 \to J/\psi K^+\pi^-$

- MuChi2/NDF <3
- Mu Strip hits > 5
- Mu Pixel hits >0
- Mu Dz <20
- Mu Dxy <0.3

- B0Vtx_CL > 0.09
- B0CosAlphaPV > 0.9985
- B0CTauPV/B0CTauPVE > 9.0
- MuMuVtx_CL > 0.02
- jpsip4.DeltaR(track) < 1.0
- trackChi2/NDF < 7.0
- track Strip hits > 10
- track Pixel hits >0
- B0 Pt > 8
- track pt > 0.45
- (*MuMuMass)[jpsi_index] - jpsi_mass < 0.12

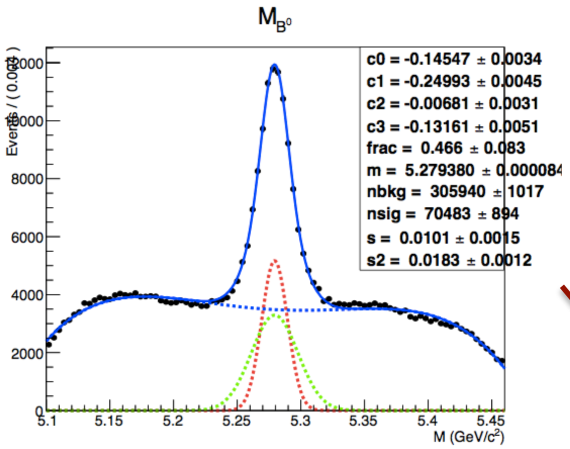


After some selection optimization (fine tuning) for $B^0 \to J/\psi K^+\pi^-$:

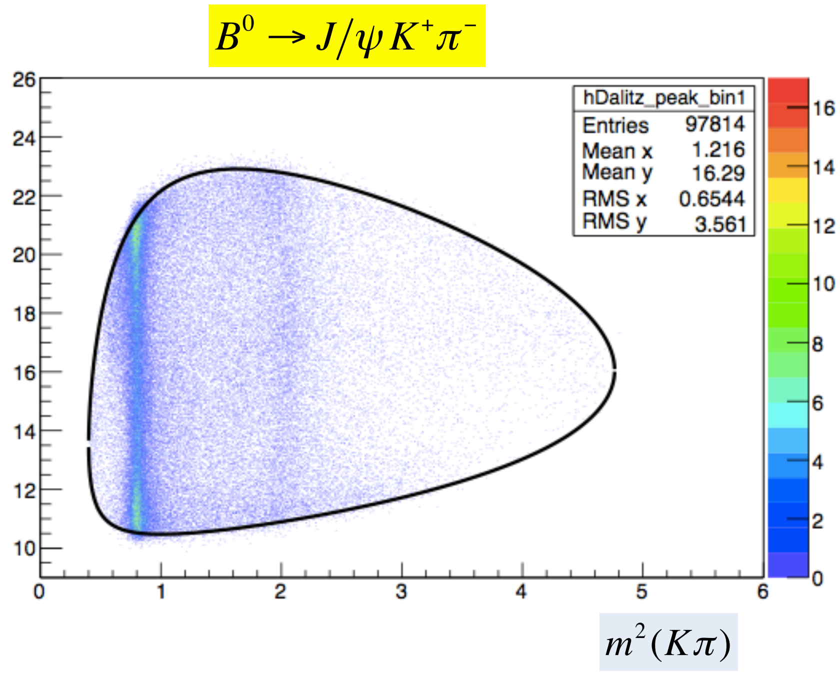


- **It can benefit from the application of TMVA (BDT algorithm) [see next decay mode]**

From some softer selection, the Dalitz Plot already shows clear K*(892) & K*(1430) signals:

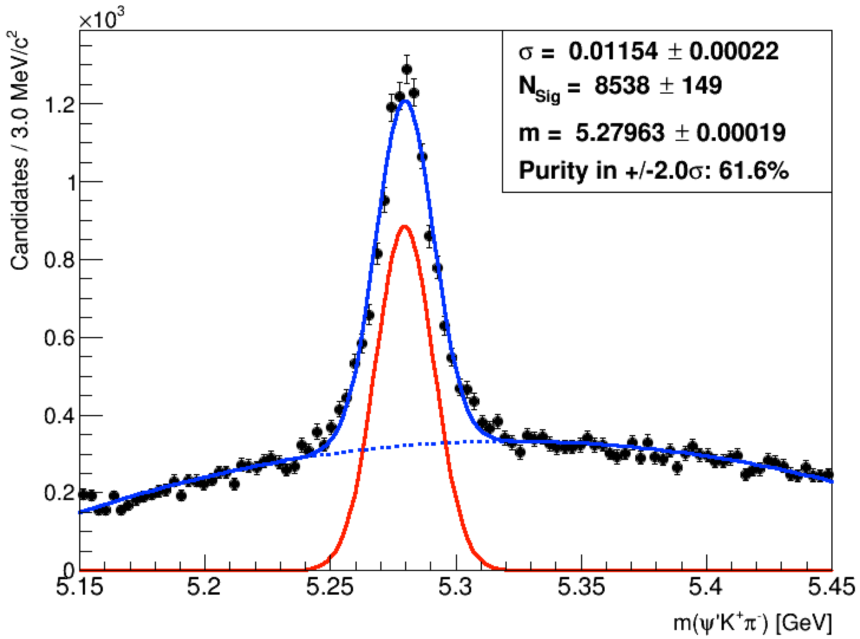


within $\pm 1\sigma$ of the core gaussian :



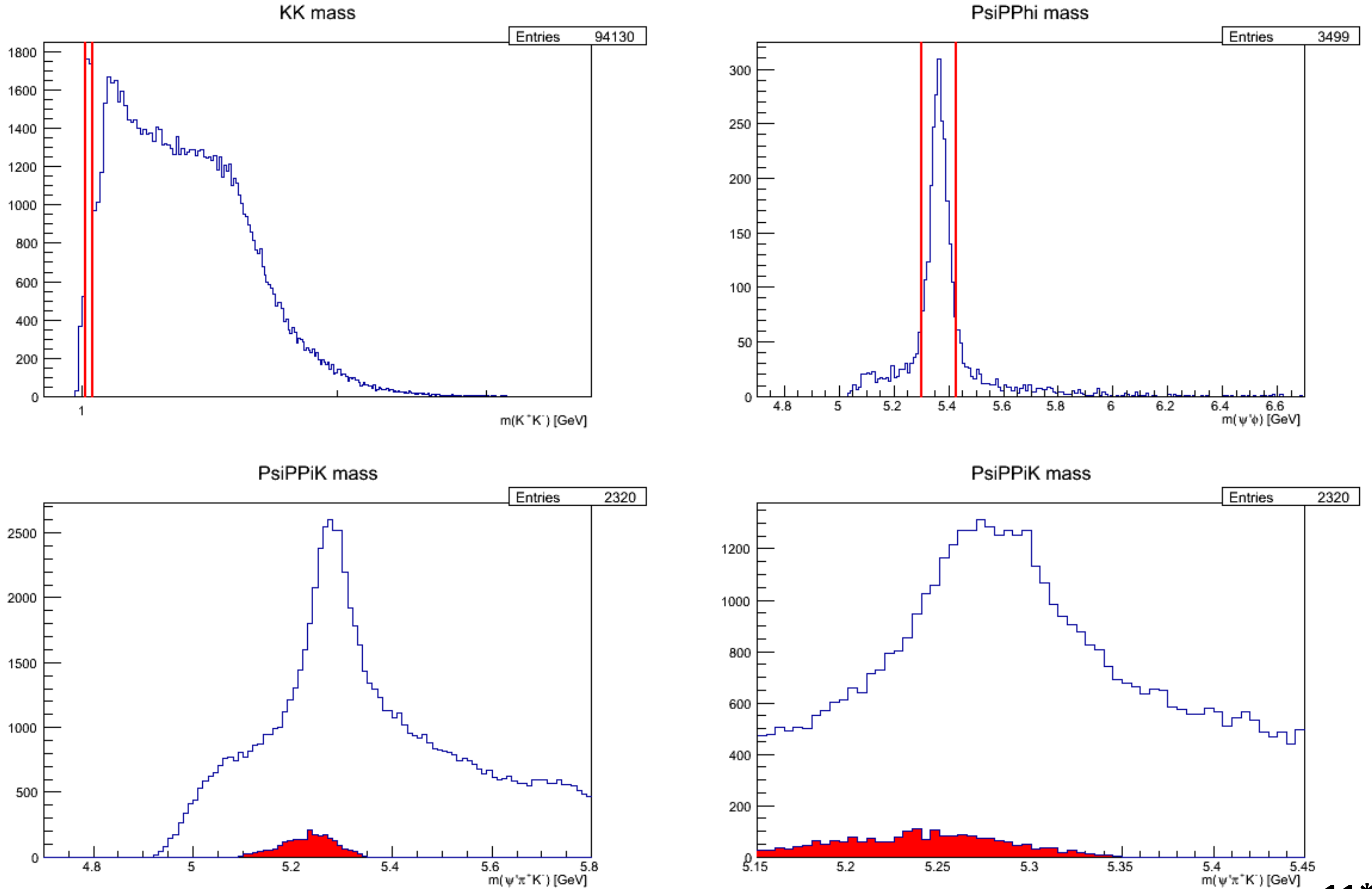
Cut-based selection targeting high-purity for $B^0 \to \psi' K^+ \pi^-$:

when applying *blindly* the previous selection criteria for
the J/ψ channel to this one … a purity of ~62% is achieved (instead of ~70%) :



- This is partially due to the fact that J/ψ is cleaner than ψ'
- Reached purity is low: try TMVA (se next slides).

Some study of reflections; for instance B_s :



Alternative TMVA (BDT) selection for $B^0 \to \psi' K^+ \pi^-$:

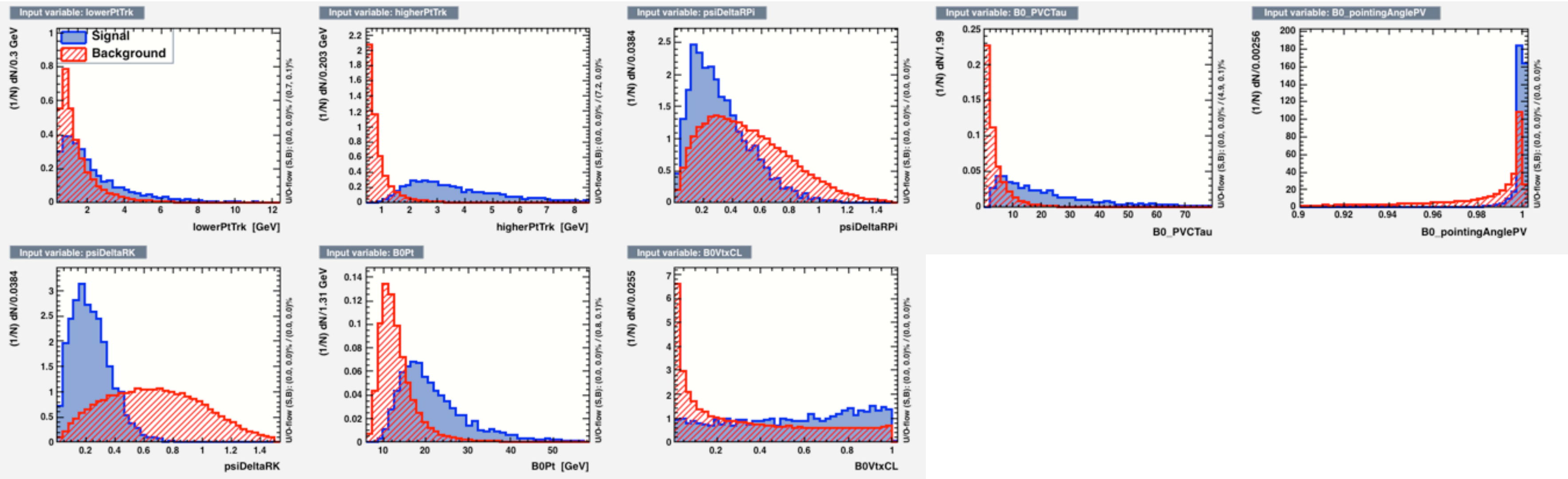
- **Apply a slight preselection and leave BDT working**
  (better than make BDT starting from some selection)

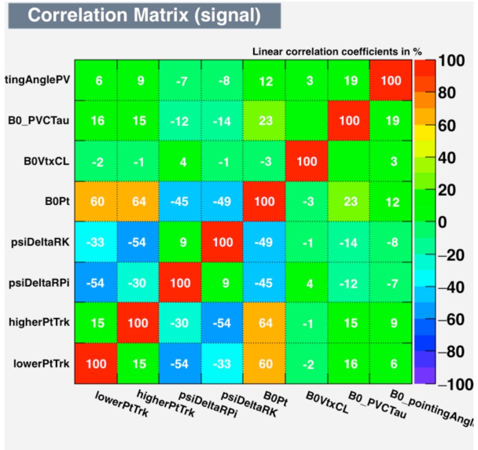
- **BDT is the best performing algorithm (from preliminary study)**

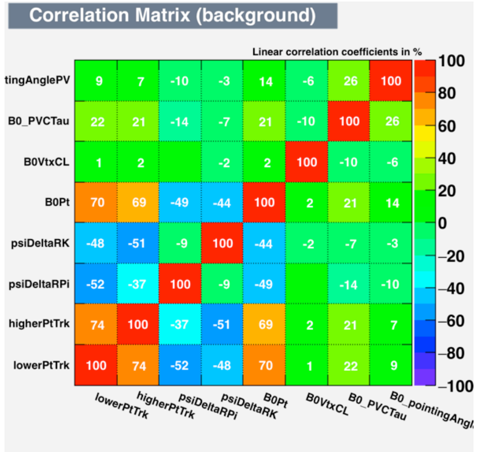
- **<u>Signal</u> : truth-matched signal candidates from signal PHSP MC**
  (1 candidate per event selected to further clean from twins) [older studies not reported here]

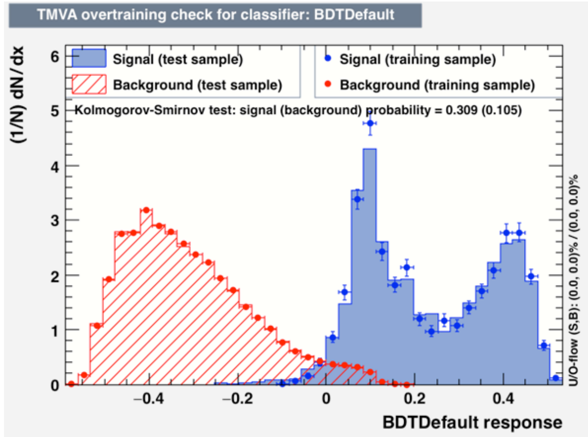
- **<u>Bkg</u> : from data sidebands**

- **Used variables: distributions for signal & bkg and correlation matrices (next slide)**

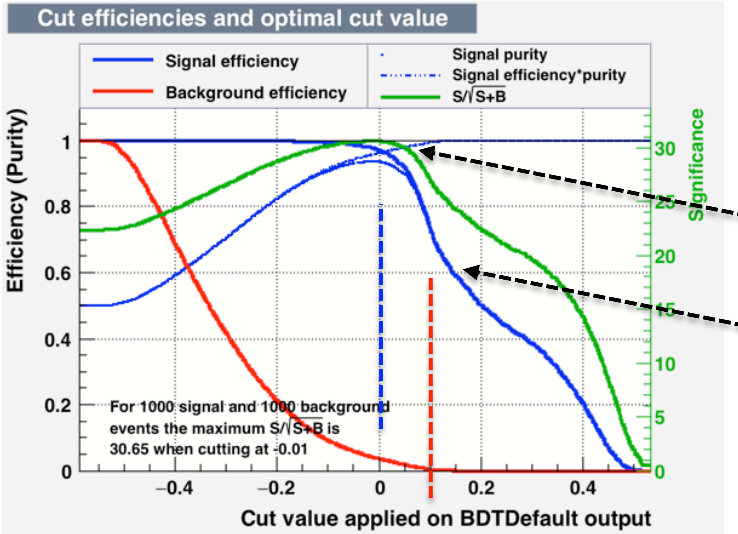








BDT output: choice of working point :

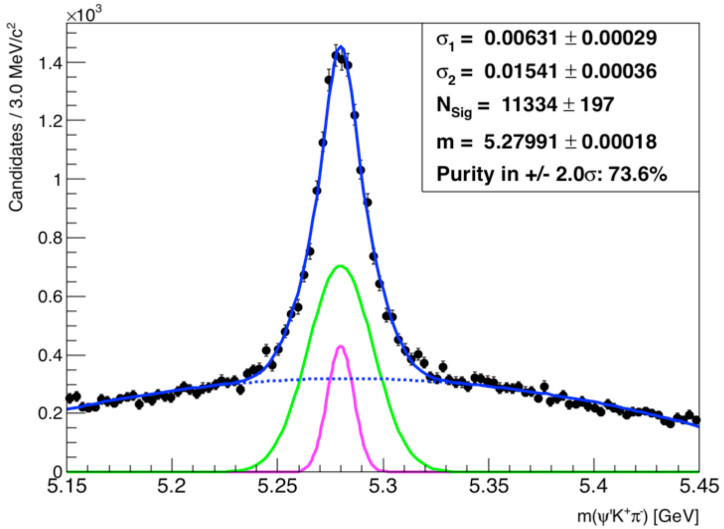


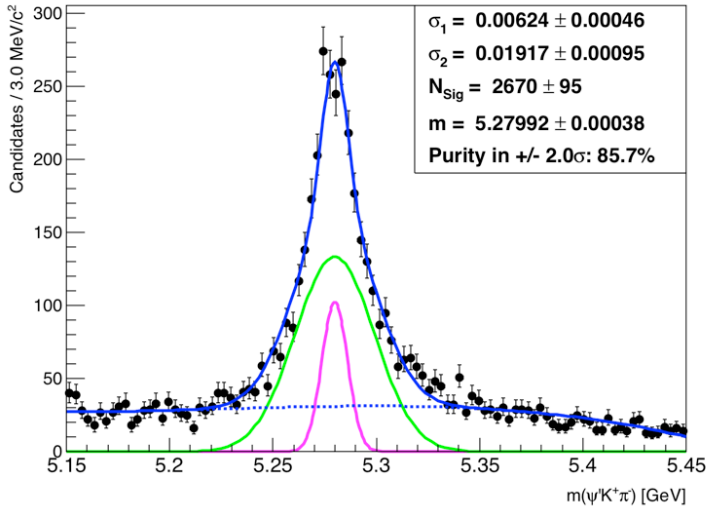
signal purity

signal efficiency

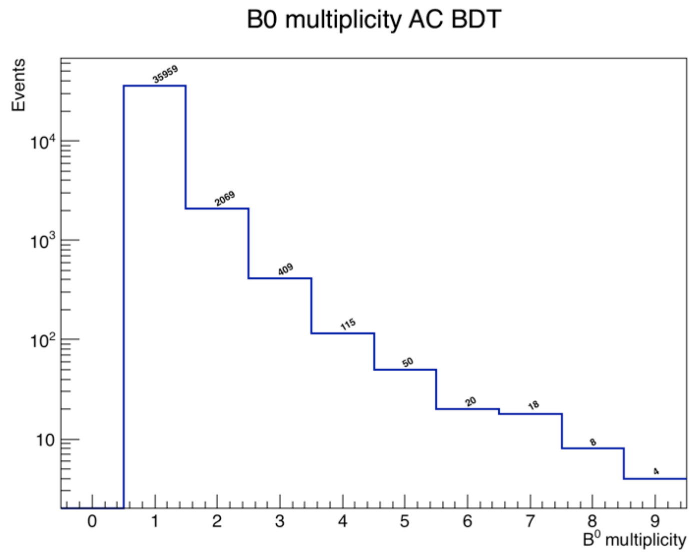
try cuts @ 0.0 & 0.1 :

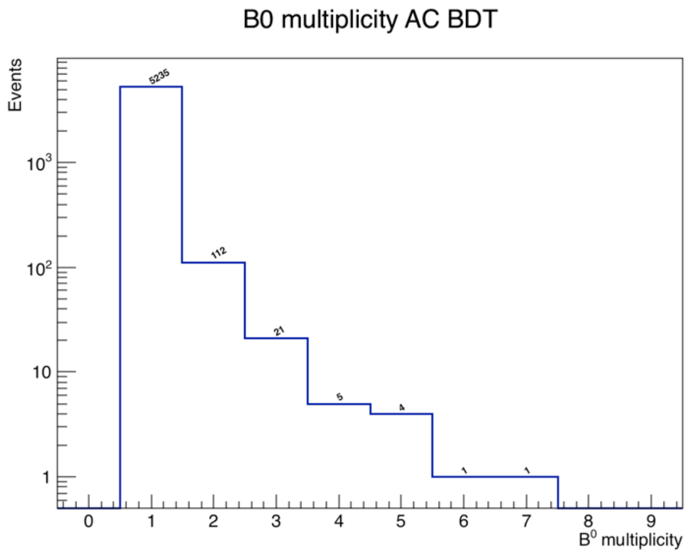
(results on next slide)





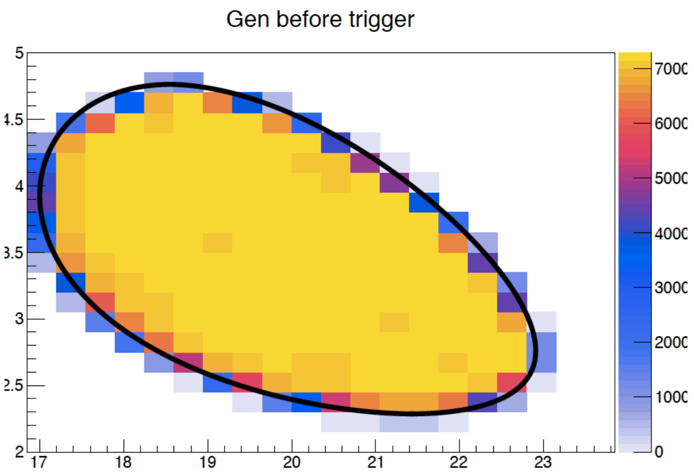
Residual multiplicity:

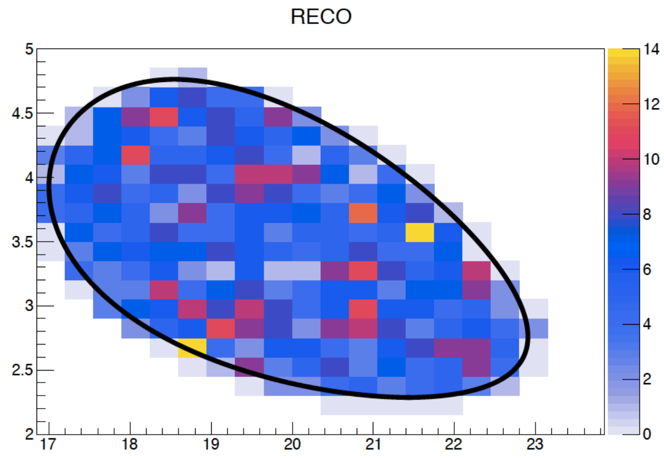


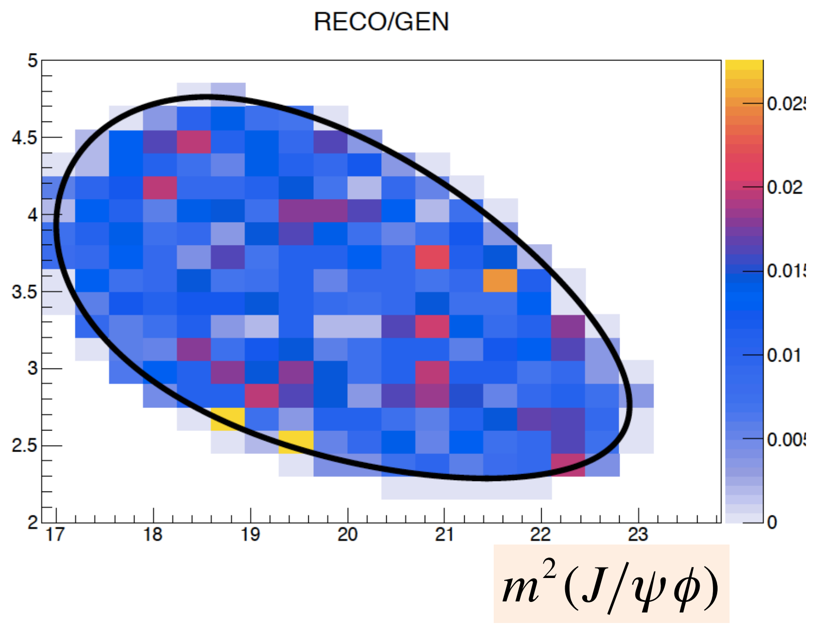


Relative/absolute reconstruction efficiency over the DP

Absolute efficiency for $B^+ \to J/\psi\,\phi K^+$ decay mode. Official PHSP MC results:

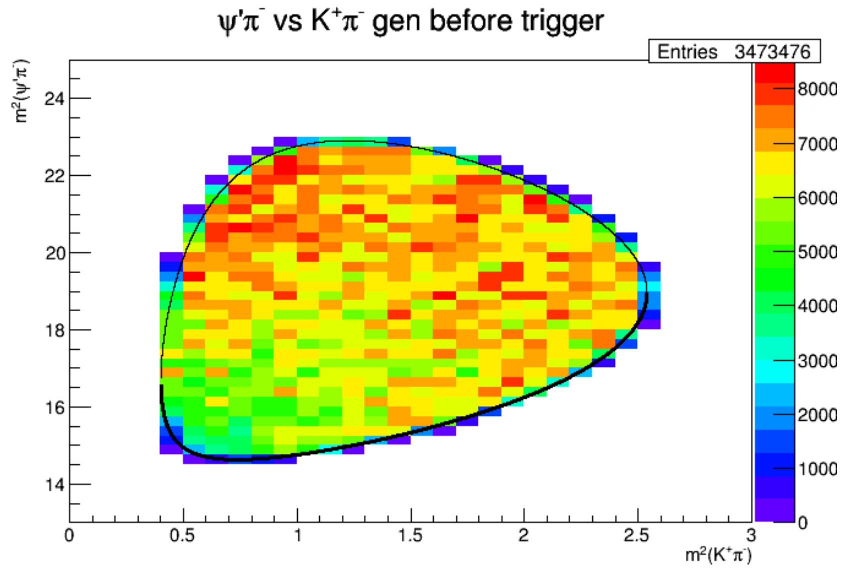


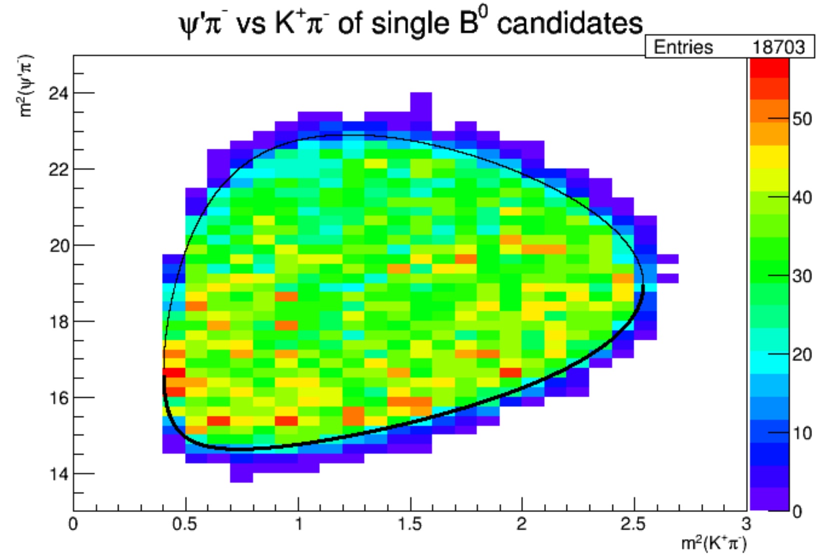


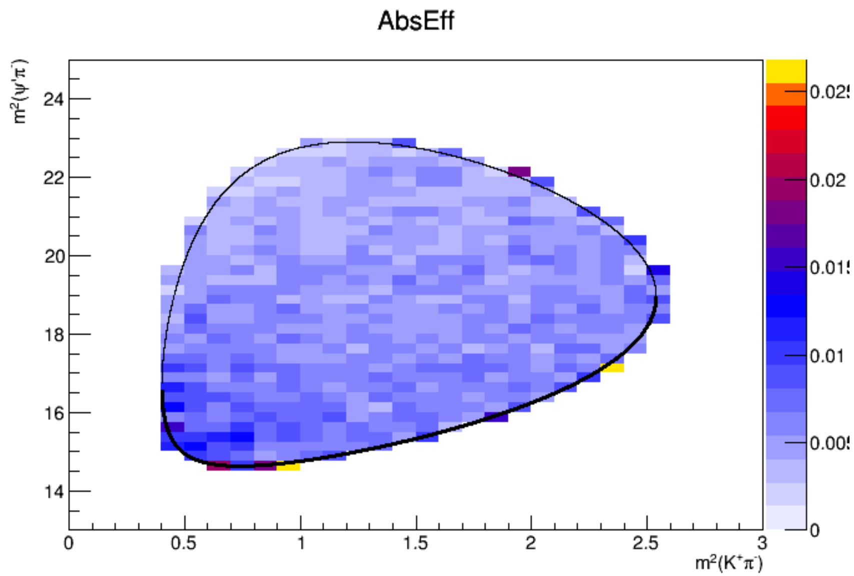


Absolute efficiency over the Dalitz Plot for $B^0 \to \psi' K^+ \pi^-$

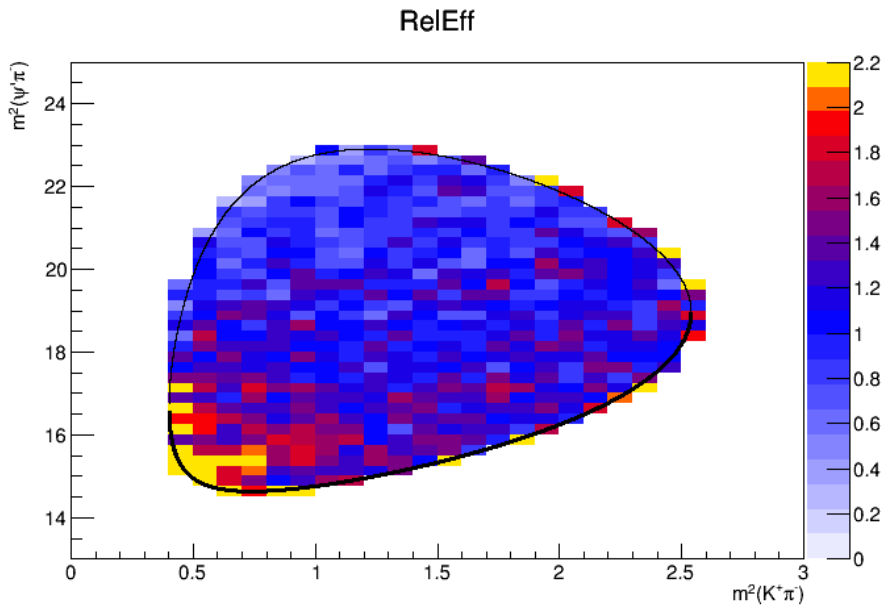
Official PHSP MC results (biased by a p_T filtering; waiting for new production)

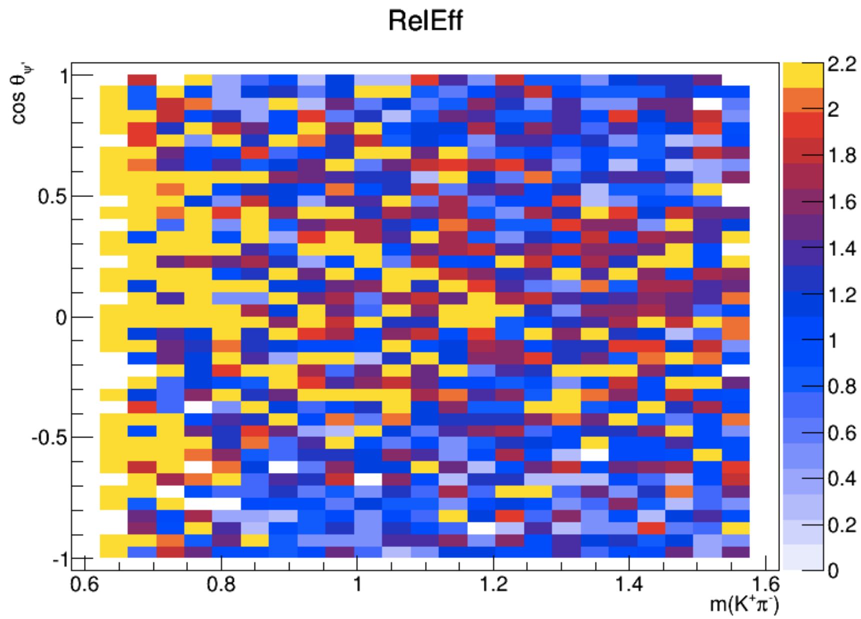






Relative efficiencies over the Dalitz Plots (traditional & rectangular) efficiency
Official PHSP MC results (biased by a p_T filtering)





What does mean doing a full Amplitude Analysis ?

[with reference to the decay $B^0 \to \psi K^+ \pi^-$ **]**

$\left(\psi \equiv J/\psi, \psi' \to \mu\mu\right)$

Assuming the only intermediate 2-body states are the K*s let us consider the following …

DECAY CONVENTION : $B^0 \to \psi(\to \mu\mu)K^*(\to K^+\pi^-)$ $\Longleftrightarrow$ $a \to 1(\to 34)2(\to 56)$

The *decay amplitude* is calculated in a 4-dimensional parameter space:

$$\Phi = (m^2_{K\pi}, m^2_{\psi\pi}, \vartheta_\psi, \varphi_{\psi K^*}) \quad \Longleftrightarrow \quad \Phi = (m^2_{56}, m^2_{16}, \vartheta_1, \varphi_{12})$$

… where :

ϑ_ψ : ψ helicity angle (angle between the K* and μ- mometum in ψ rest frame)

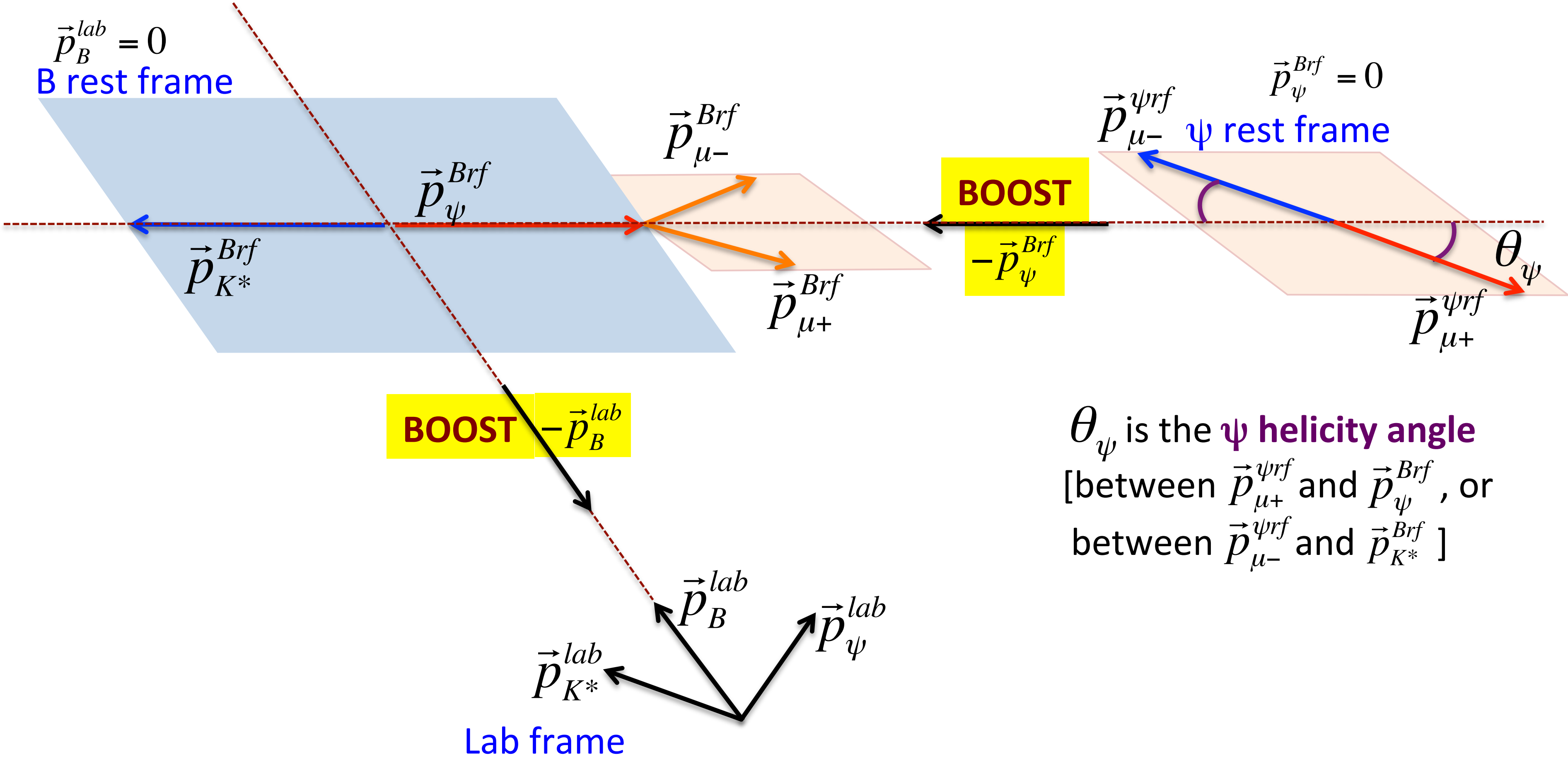
$\varphi_{\psi K^*}$: angle between ψ and K* decay planes

To understand better the definition of these two angles is better to check next 2 slides.

Traditional Dalitz Plot analyses deal with 3-body decays without any vector state as a daughter (many analyses of B,D meson decays @ the B-factories dealt with pions and kaons, either charged and/or neutral). In that case the decay amplitudes are calculated in a 2-dimensional parameter space, namely the Dalitz Plot space itself.

To calculate θ_ψ we need $\vec{p}_{\mu+}^{\,\psi rf}$ and $\vec{p}_{\psi}^{\,Brf}$. However we measure $\vec{p}_{\mu+}^{\,lab}$ and $\vec{p}_{\psi}^{\,lab}$!
We need to apply boosts as follows:

1) $\vec{p}_{\psi}^{\,lab} \xrightarrow{-\vec{p}_B^{\,lab}} \vec{p}_{\psi}^{\,Brf}$ & 2) $\vec{p}_{\mu+}^{\,lab} \xrightarrow{-\vec{p}_B^{\,lab}} \xrightarrow{-\vec{p}_\psi^{\,Brf}} \vec{p}_{\mu+}^{\,\psi rf}$



θ_ψ is the ψ **helicity angle**
[between $\vec{p}_{\mu+}^{\,\psi rf}$ and $\vec{p}_{\psi}^{\,Brf}$, or
between $\vec{p}_{\mu-}^{\,\psi rf}$ and $\vec{p}_{K*}^{\,Brf}$]

$\vec{p}_B^{\,lab}$ is the pseudo-momentum we measure; it is $\neq \vec{p}_B^{\,Bmother\ rf}$ that is of course unknown since the mother is not reconstructed

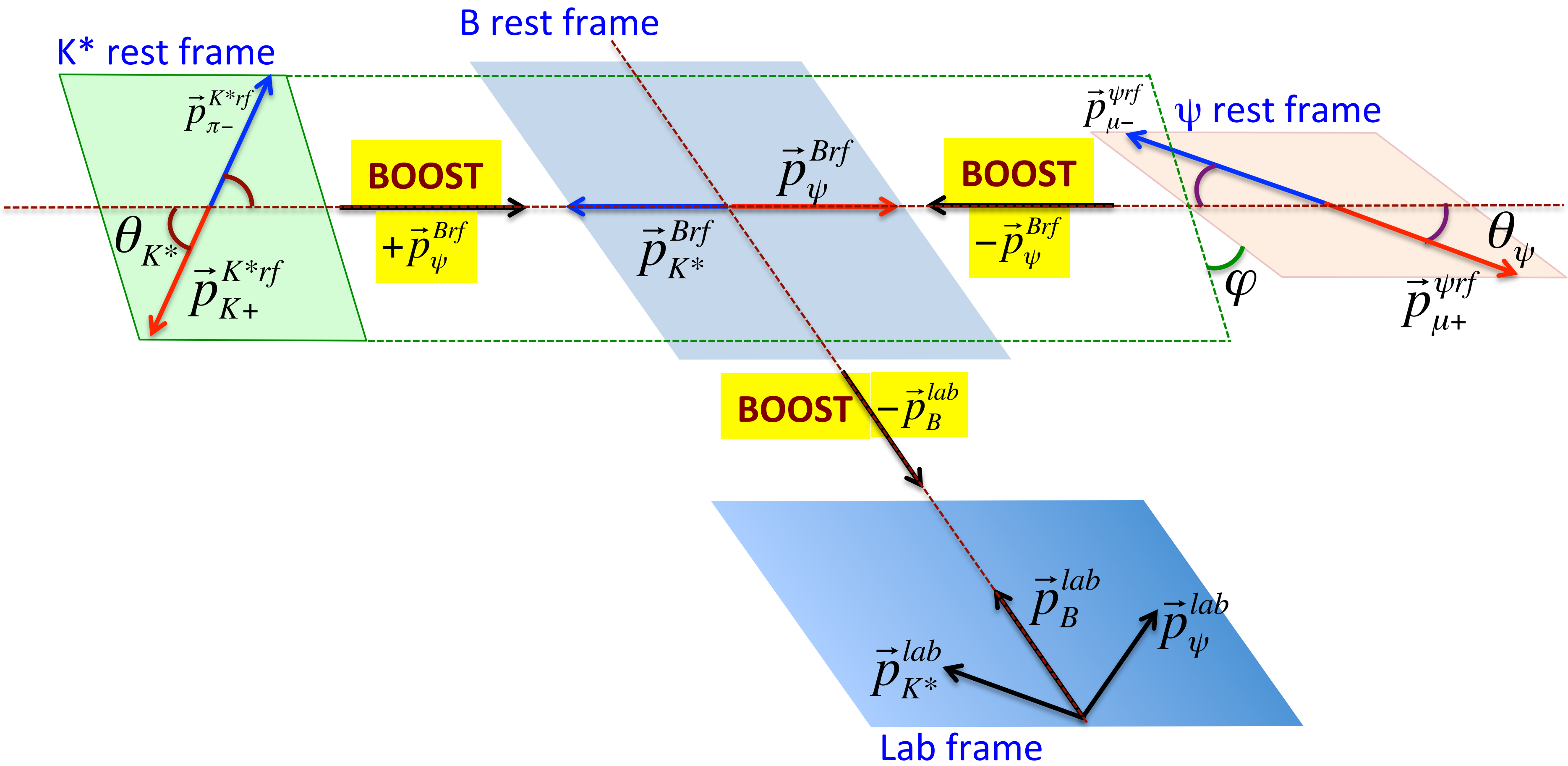
The actually code used to calculate this helicity angle:

```
GetCosThetaJPsi(TLorentzVector BVec, TLorentzVector JPsiVec, TLorentzVector MuPlusVec, float
BeamEnergy, float JPsiPDG , float muonPDG) {

  TVector3 JPsiInBFrame, MuInBFrame, MuInJPsiFrame, MuInJPsiFromBFrame;
  TLorentzVector JPsiInBFrameTLVec, MuInBFrameTLVec;

  // B0 -> J/psi K pi
  // get the momentum of the J/psi in the in the B rest-frame : JPsiInBFrame
  GetMomentumInMotherFrame( BVec , JPsiVec , BeamEnergy, JPsiInBFrame);
  JPsiInBFrameTLVec.SetPtEtaPhiM(JPsiInBFrame.Perp() , JPsiInBFrame.Eta(),  JPsiInBFrame.Phi() ,
JPsiPDG);

  // get momentum of the mu+ in J/psi rest-frame
  GetMomentumInMotherFrame( BVec , MuPlusVec, BeamEnergy, MuInBFrame); // B boost
  MuInBFrameTLVec.SetPtEtaPhiM(MuInBFrame.Perp() , MuInBFrame.Eta(),  MuInBFrame.Phi() ,muonPDG);

  GetMomentumInMotherFrame( JPsiInBFrameTLVec, MuInBFrameTLVec, BeamEnergy, MuInJPsiFromBFrame);
  float thetaJPsi = MuInJPsiFromBFrame.Angle(JPsiInBFrame);

  return TMath::Cos(thetaJPsi);
}


GetMomentumInMotherFrame( TLorentzVector Mother, TLorentzVector Particle, double BeamEnergy ,
TVector3 &Particle_rotated) {
  //Mother momentum in lab frame
  TVector3 bMother = Mother.BoostVector();
  Particle.Boost(-bMother);     // Particle momentum in Mother rest frame
  Particle_rotated = Particle.Vect(); // me: particle coordinates in the rotated frame
}
```

He have just seen the definiton of ϑ_{ψ} .

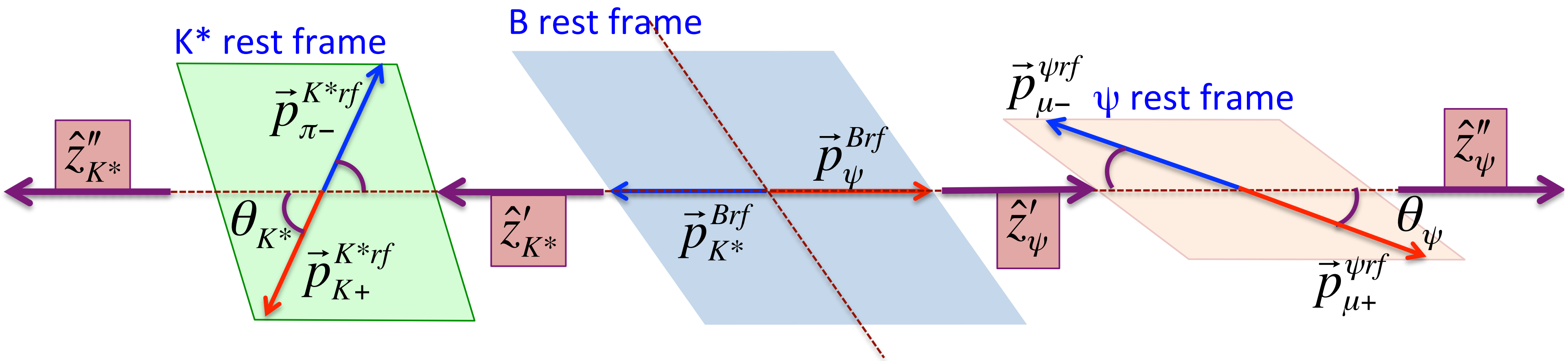
Similarly one can define the K* helicity angle ϑ_{K^*} (the angle between the ψ and the π momenta in the K* rest frame).

Of course, having the ψ and K* planes, the $\varphi \equiv \varphi_{\psi K^*}$ is defined as the angle in between:



More precisely we need to consider the coordinate systems in the next page :

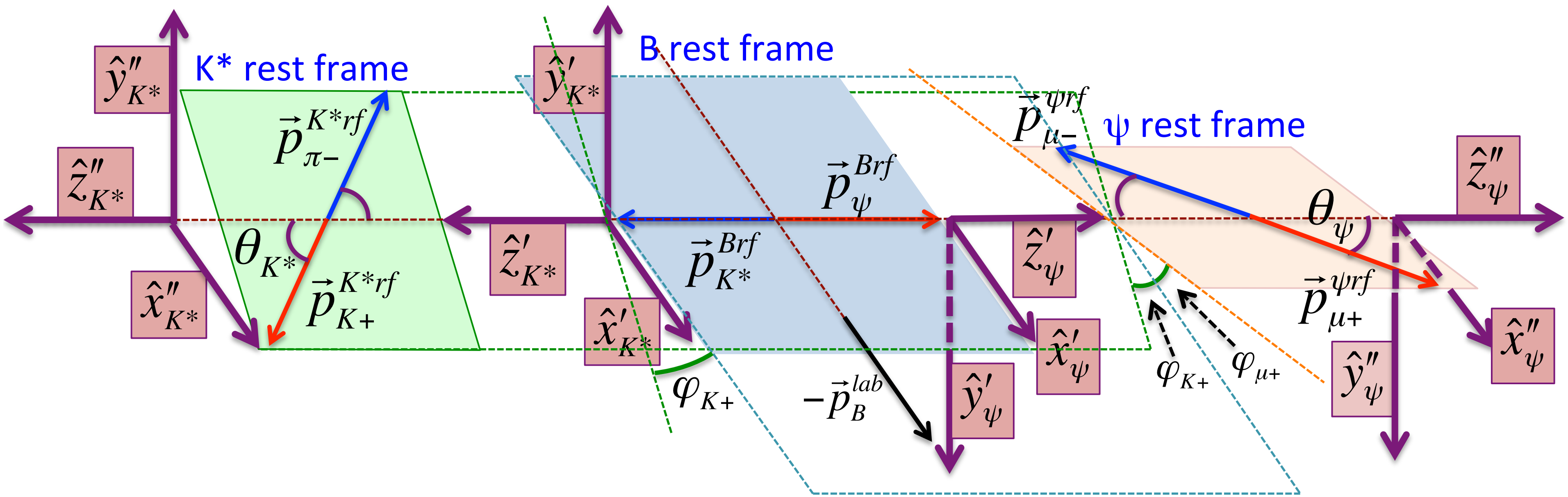
The ***z***-axis in the rest frame of the particle ***a*** (with spin ***J***) that decays is the arbitrarily defined spin-quantization axis to calculate M (here ***a=B meson*** and ***J=0***).
The ***z"***-axis in the rest frame of ψ is not arbitrary: it is the direction of $\vec{p}_{\psi}^{\,Brf}$ so that the spin projection along ***z"***-axis is $M_{\psi} = \lambda_{\psi}$.
Similarly ***z"***-axis is parallel to $\vec{p}_{K^*}^{\,Brf}$ (thus $M_{K^*} = \lambda_{K^*}$) on the other side.



The ***coordinate systems*** $(\hat{x}''_{K^*}, \hat{y}''_{K^*}, \hat{z}''_{K^*})$ and $(\hat{x}''_{\psi}, \hat{y}''_{\psi}, \hat{z}''_{\psi})$ are those defined in the ψ and *K** rest frames respectively. They can be considered as those **obtained by boosting** (to these rest frames) the systems $(\hat{x}'_{K^*}, \hat{y}'_{K^*}, \hat{z}'_{K^*})$ and $(\hat{x}'_{\psi}, \hat{y}'_{\psi}, \hat{z}'_{\psi})$ that can be defined in the B rest frame.

Since boosts are along the two ***z"***-axes the $\hat{z}'_{K^*}$ and $\hat{z}'_{\psi}$ unit vectors are as in figure.

In the B rest frame we have the freedom to choose the axes $\hat{x}'_{K*}$ and $\hat{x}'_{\psi}$ **to be the same** (i.e parallel; and in particular – for instance – parallel to the boost $-\vec{p}_B^{\,lab}$ applied to obtain this B rest frame). Since $\hat{z}'_{K*}$ and $\hat{z}'_{\psi}$ are anti-parallel also the 3rd axes must be anti-parallel and pointing in such a way to ensure both the coordinate systems are **right-handed**!

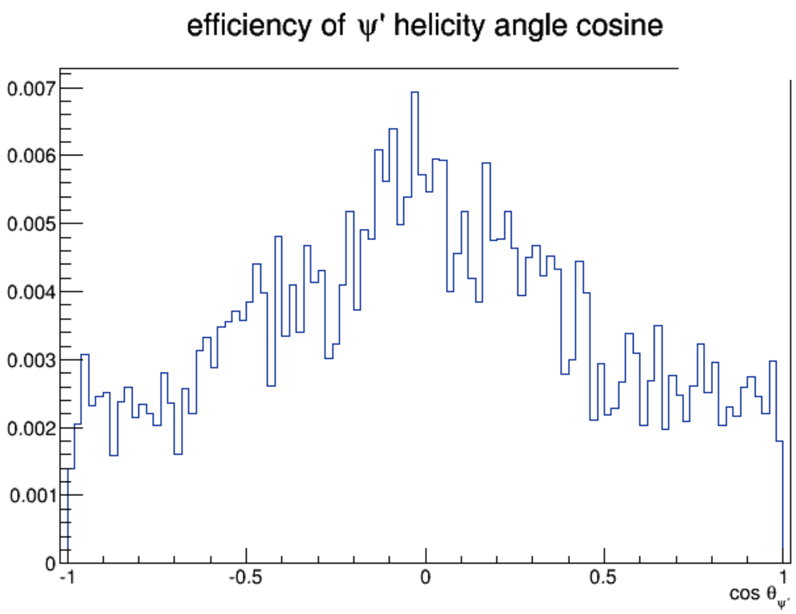


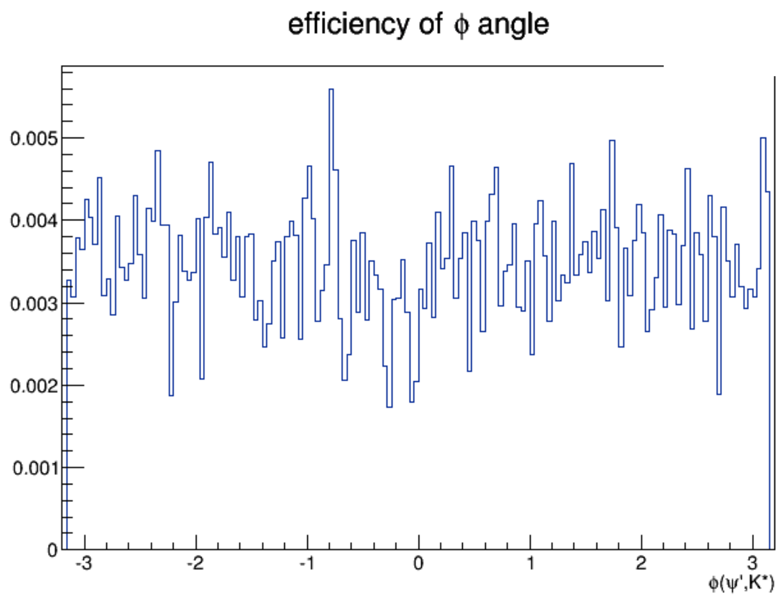
In the boost along the two ***z"***-axes, the other axes are obtained by rigid traslation and thus are represented as in the figure.

The angle φ is given by : $\varphi = \varphi_{K+} + \varphi_{\mu+}$ where the latter are the azimuthal angles in the primed coord. frames. Actually our code does something different (see next slide) :

```
GetPhi(TLorentzVector LoreVecB0, TLorentzVector LoreVecMum, TLorentzVector
LoreVecMup, TLorentzVector LoreVecK, TLorentzVector LoreVecPi) {

   // # B0 -> K+ pi- mu mu #

         TVector3 boostB0 = LoreVecB0.BoostVector();

         LoreVecMum.Boost(-boostB0);
         LoreVecMup.Boost(-boostB0);
         LoreVecK.Boost(-boostB0);
         LoreVecPi.Boost(-boostB0);

         TVector3 MuMuPlane = LoreVecMup.Vect().Cross(LoreVecMum.Vect());
         TVector3 KstPlane  = LoreVecK.Vect().Cross(LoreVecPi.Vect());

         if ( MuMuPlane.Cross(KstPlane).Dot(-LoreVecB0.Vect()) > 0.0 )
           phiKstMuMuPlane = MuMuPlane.Angle(KstPlane);
         else
           phiKstMuMuPlane = -MuMuPlane.Angle(KstPlane);

  return phiKstMuMuPlane;
}
```

Efficiency for the 2 angular variables :





- **Need to be checked with new official MC (no track p_T filtering)**

Performing a full amplitude analysis means performing an UML fit in the 4-dim space Φ .
The function to be minimized is:

LIKELIHOOD FUNCTION : $$F = -2\sum_{data} \ln\left((1-b)\frac{S(\Phi)}{\sum_{MC} S(\Phi)} + b\frac{B(\Phi)}{\sum_{MC} B(\Phi)} \right)$$

… where :

$S(\Phi)$ **: SIGNAL EVENT DENSITY FUNCTION**

$B(\Phi)$ **: BACKGROUND DENSITY FUNCTION (in the signal region)**

b **: fraction of the BACKGROUND EVENTS (in the signal region)**

… thus *Purity* is: $P = \frac{N_S}{N_S + N_B} = \frac{N_S}{N_{TOT}} = (1-b)$

the sum $\sum_{data}$ **runs over data events**

the sum $\sum_{MC}$ **runs over MC events, PHSP-generated and reconstructed using the same selection as in data.**

Note : the signal PDF is normalized by summing over simulated events so that it takes into account the nonuniformity of the reconstruction efficiency by implementing a 4D efficiency correction without the use of a parametrization. Similarly for the bkg, but it requires a parametrization of the bkg shape before fitting the data (from *B^0* mass sidebands in our case).

SIGNAL EVENT DENSITY FUNCTION : $S(\Phi) \overset{(*)}{=} \sum_{\xi=1,-1} \left| \sum_{K^*} \sum_{\lambda=-1,0,1} A_{\lambda\xi}^{K^*}(\Phi) + \cancel{\sum_{OTHER-R(\to\psi\pi)} \sum_{\lambda'} A_{\lambda'\xi}^{R}} \right|^2$

ONLY-K* ASSUMPTION

(for the time being)

where: $A_{\lambda\xi}^{K^*}(\Phi)$ **: decay amplitude for one K* resonance**

λ **: helicity of the ψ**

ξ **: helicity of the lepton pair that is produced in the EM decay via a virtual** γ

(this implies: $\xi = 1,-1$**)**

FURTHER ASSUMPTION (for the time being, for discussion purposes) :

let us assume the only intermediate 2-body state is a particular K* resonance ($\bar{K}^*$**) !**

$$S(\Phi) = \sum_{\xi=1,-1} \left| \cancel{\sum_{K^*}} \sum_{\lambda=-1,0,1} A_{\lambda\xi}^{\bar{K}^*}(\Phi) \right|^2 \Rightarrow S(\Phi) = \sum_{\xi=1,-1} \left| \sum_{\lambda=-1,0,1} A_{\lambda\xi}^{\bar{K}^*}(\Phi) \right|^2$$

… and we will discuss how the relative decay amplitude $A_{\lambda\xi}^{\bar{K}^*}(\Phi)$ **can be written.**

Previous assumptions mean :

$$\underset{0^-}{B^0} \longrightarrow \underset{1^-}{J/\psi(\psi')} \quad \underset{1^-}{K^{*0}(892)} \,/\, \underset{0^+}{K_0^*(1430)} \,/\, \underset{2^+}{K_2^*(1430)}$$

$$J/\psi(\psi') \longrightarrow \underset{1/2}{\mu^+} \;\; \underset{1/2}{\mu^-}$$

$$K^* \longrightarrow \underset{0^-}{K^+} \;\; \underset{0^-}{\pi^-}$$

Otherwise :

$$\underset{0^-}{B^0} \longrightarrow \underset{J_Z^{P_Z}}{Z^\mp} \quad \underset{0^-}{K^\pm}$$

$$Z^\mp \longrightarrow \underset{1^-}{J/\psi(\psi')} \;\; \underset{0^-}{\pi^\mp}$$

$$J/\psi(\psi') \longrightarrow \underset{1/2}{\mu^+} \;\; \underset{1/2}{\mu^-}$$

SIGNAL EVENT PDF :

$$S(\Phi)=\overset{(*)}{\sum_{\xi=1,-1}}\left|\sum_{K^*}\sum_{\lambda=-1,0,1}A^{K^*}_{\lambda\xi}(\Phi)+\cancel{\sum_{OTHER-R(\to\psi\pi)}\sum_{\lambda'}A^{R}_{\lambda'\xi}}\right|^2$$

ONLY-K* ASSUMPTION
(for the time being)

(*) Note : *isobar model* is implicitely assumed by writing this sum [total decay amplitude treated as a coherent sum of sub-2-body decays (where one daughter is a spectator)].

Here it is a sum of BW contributions for different intermediate 2-body states;
by default (only-K* assumption) it includes all the known $K\pi$ resonances!
For instance (with $\psi\equiv\psi'$):
- below threshold ($\cong 1593 MeV/c^2$) :

$m(B^0)-m(\psi')$

$K_0^*(800)$

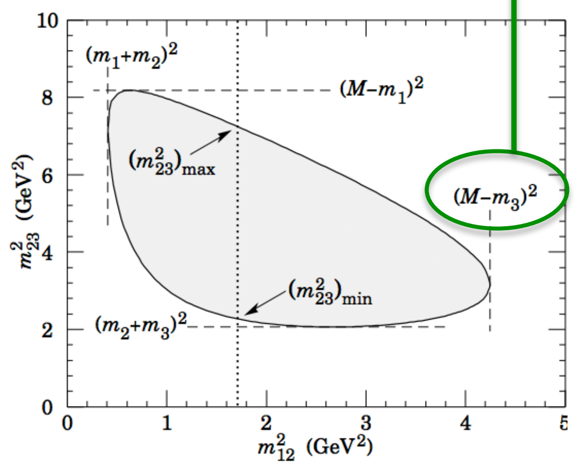
$K^*(892)$

$K^*(1410)$

$K_0^*(1430)$

$K_2^*(1430)$

Resonance	Mass (MeV/c^2)	Γ (MeV/c^2)	J^P
$K^*(892)^0$	895.81±0.19	47.4±0.6	1^-
$K^*(1410)^0$	1414±15	232±21	1^-
$K_0^*(1430)^0$	1425±50	270±80	0^+
$K_2^*(1430)^0$	1432.4±1.3	109±5	2^+
$K^*(1680)^0$	1717±27	322±110	1^-
$K_3^*(1780)^0$	1776±7	159±21	3^-



- and the first above threshold : $K^*(1680)$

BACKGROUND EVENT PDF : $B(\Phi) = P_n(m^2_{K\pi}, m^2_{\psi\pi}) \cdot P_m(\cos\vartheta_\psi) \cdot P_\ell(\varphi)$

… with $n / m / \ell$ - order polynomials

- **the bkg shape can be determined using (in our case) *B^0* mass sidebands (by means of an UML minimizing :** $F = -2\sum_{data} \ln\left(\frac{B(\Phi)}{\sum_{MC} B(\Phi)}\right)$ **)**

  In this way the resulting $B(\Phi)$ is efficiency-corrected; need some inclusive-B MC !

- **need to consider if contribution coming from hadronic mis-ID ($\phi \to KK,\ K^0_S \to \pi\pi$) (instead of vetoing them) and *twins* (instead of rejecting them asking for 1 cand per event as a last cut) can be parametrized so that we can write:**

  $B(\Phi) = B(m^2_{K\pi}, m^2_{\psi\pi}) \cdot P_m(\cos\vartheta_\psi) \cdot P_\ell(\varphi)$ **… where …** $B(m^2_{K\pi}, m^2_{\psi\pi}) = B_{SMOOTH} + B_{mis-ID} + B_{TWINS}$

- **the bkg might not depend on the angular variables**

The *amplitude of the decay* $B^0 \to \psi K^+ \pi^-$ in Φ-space for only one K* resonance is :

$$A_{\lambda\xi}^{K^*}(\Phi) = \boxed{A^{K^*}(m_{K\pi}^2)} \cdot \boxed{H_\lambda^{K^*} \cdot d_{\lambda 0}^{J(K^*)}(\vartheta_{K^*}) \cdot e^{i\lambda\varphi} \cdot d_{\lambda\xi}^1(\vartheta_\psi)}$$

angle-independent part

angle-dependent part

the angle dependent signal density function is obtained by using the helicity formalism :

… where :

$\Phi \leftarrow$

ϑ_ψ : ψ helicity angle (angle between the *K** and μ- mometum in ψ rest frame)

$\varphi_{\psi K^*} \equiv \varphi$: angle between ψ and *K** decay planes

ϑ_{K*} : *K** elicity angle

H_λ^{K*} : helicity amplitude for the decay via the intermediate resonance *K**

$J(K*)$: spin of the intermediate resonance *K**

λ : helicity of the ψ (the quantization axis being parallel to the *K** momentum in the ψ rest frame)

- for *K** resonances with spin 0, only $\lambda = 0$ is allowed

ξ : helicity of the *lepton pair*

$d_{\lambda 0}^{J(K*)}$, $d_{\lambda\xi}^1$: "small" Wigner functions (they are real !)

Angle-independent part of the amplitude $A^{K^*}(m^2_{K\pi})$

Let us consider the two-body intermediate resonance $R \equiv K^* \ (m_R \equiv m_{K\pi})$:

$$A^R(m_R^2) = \frac{F_B^{(L_B)} F_R^{(L_R)} \boxed{(p_B/M_B)^{L_B} (p_R/m_R)^{L_R}}}{M_R^2 - m_R^2 - iM_R\Gamma(m_R)}$$

with M_B, M_R the nominal (PDG) masses

orbital momentum part

… where: $F_B^{(L_B)}$, $F_R^{(L_R)}$ **decay form factors (Blatt-Weisskopf)**

(superscript denotes the orbital angular mom. of the decay)

- standard "hadron scale" : $r \equiv 1.6 GeV^{-1}$
- for *K** resonances with non-zero spin *J* …
  … L_B can take several values (*J-1*, *J*, *J+1*);
  tipically the default value is taken as the lowest (other @ syst.)

$p_{B(R)}$ **B^0 (*R*) meson daughter's momentum in the B^0(*R*) rest frame**

$\Gamma(m_R)$ **energy-dependent width of the resonance *R*, parametrized as :**

$$\Gamma(m_R) = \Gamma_0 \cdot \left(\frac{p_R}{p_{R0}}\right)^{2L_R+1} \cdot \frac{M_R}{m_R} \cdot F_R^2$$

with p_{R0} being *R* daughter's momentum calculated for the pole mass of *R*

For the non-resonant amplitude :

- ***angle-dependent part*** **:**

  same as the resonant angular amplitude
  but with relative angular momentum of the $K\pi$ system (instead of the spin of a K^*)

- ***angle-independent part*** **(**similar to the resonant amplitudes but without the denominator**) :**

$$A^{NR}(m_{K\pi}^2) = F_B^{(L_B)} F_{K\pi}^{(L_{K\pi})} (p_B / M_B)^{L_B} (p_{K\pi} / m_{K\pi})^{L_{K\pi}}$$

How can the angular part of the amplitude of the decay be derived ?

By applying the helicity formalism for a two-body decay applied recursively. Thus …

…let us consider first a decay process $a \to 1+2$

where a has **mass** $\boldsymbol{m_a}$, **spin** $\boldsymbol{J}$ and **spin projection** $\boldsymbol{M}$ along an arbitrarily-chosen *z*-axis

Let us choose the rest frame of a **in which its state vector is** $|J,M\rangle$ **.**

The final state vector $|f\rangle$ **of the** $1+2$ **system is given by …**

… the two-particle plane-wave helicity state $|\vec{p}_1,\lambda_1;\vec{p}_2,\lambda_2\rangle$**, which, in the CM frame, can be written as** $|p_f,\vartheta_f,\varphi_f,\lambda_1,\lambda_2\rangle$ **where the two angles concern the decay axis (direction along which the two particles travel back-to-back w.r.t the CM frame) with respect to the z-axis that is the spin-quantization axis of** a **.**

Namely ϑ_f,φ_f **are the polar angles of** $\vec{p}_f=\vec{p}_1=-\vec{p}_2$ **.**

Then …

… the amplitude of the decay is: $A(a \to f) \propto \sum_{J_f M_f} \sqrt{\frac{2J+1}{4\pi}} D^{J_f *}_{M_f \lambda(=\lambda_1-\lambda_2)}(\varphi_f, \vartheta_f, -\varphi_f) \delta_{M_f M} \delta_{J_f J} A_{\lambda_1 \lambda_2}$

$$\Leftrightarrow \boxed{A(a \to f) \propto \sqrt{\frac{2J+1}{4\pi}} D^{J*}_{M \lambda(=\lambda_1-\lambda_2)}(\varphi_f, \vartheta_f, -\varphi_f) A_{\lambda_1 \lambda_2}}$$

The angular distribution of the decay is: $\frac{d\sigma}{d\Omega_f}(\vartheta_f, \varphi_f) \equiv \left|A(a \to f)\right|^2$ $\boxed{d\Omega_f = d(\cos\vartheta_f) d\varphi_f}$

If experiment doesn't measure final state helicities λ_1, λ_2 **, the latter must be summed over :**

$$\frac{d\sigma}{d\Omega_f}(\vartheta_f, \varphi_f) \equiv \sum_{\lambda_1 \lambda_2} \left|A(a \to f)\right|^2 \propto \frac{2J+1}{4\pi} \cdot \sum_{\lambda_1 \lambda_2} \left| D^{J*}_{M \lambda(=\lambda_1-\lambda_2)}(\varphi_f, \vartheta_f, -\varphi_f) A_{\lambda_1 \lambda_2} \right|^2$$

The whole angular dependence is contained in the complex Wigner D-functions !
The helicity amplitude can be considered as complex parameters; once they are known, all the angular distributions would be known. They can be either measured or calculated.
Sometimes a complete calculation is impossible or measurement is not performed. To overcome this issue: relation between the helicity amplitudes formalism and the LS (or partial-wave) formalism.

Simplest example: decay of a particle a (with **spin** J and **spin projection** M) into 2 spinless particles

($\lambda_1 = 0 = \lambda_2$): $\left.\frac{d\sigma}{d\Omega_f}(\vartheta_f, \varphi_f)\right|_{\lambda_1=\lambda_2=0} \propto \frac{2J+1}{4\pi} \left| D^{J*}_{M0}(\varphi_f, \vartheta_f, -\varphi_f) A_{00} \right|^2 = \left| Y^M_J(\vartheta_f, \varphi_f) \right|^2 \cdot \left| A_{00} \right|^2$

The helicity formalism provides in the general case of the sequential 2-body decays:

$$a \to 1(\to 34)2(\to 56)$$

… with the analogy being : $\bar{B}^0 \to \psi(\to \mu\mu)K^*(\to K^-\pi^+)$

$A(a \to 3456) =$

we need to sum over the helicities of the intermediate particles because they cannot be measured !

$$= \sum_{\lambda_1\lambda_2} D^{S_2 *}_{-\lambda_2(\lambda_5-\lambda_6)}(\varphi_2,\vartheta_2,-\varphi_2) D^{S_1 *}_{+\lambda_1(\lambda_3-\lambda_4)}(\varphi_1,\vartheta_1,-\varphi_1) D^{J*}_{M(\lambda_1-\lambda_2)}(\varphi,\vartheta,-\varphi) C_{\lambda_5\lambda_6} B_{\lambda_3\lambda_4} A_{\lambda_1\lambda_2}$$

… where : $(\varphi,\vartheta,-\varphi)$ **are calculated in the** a **rest frame with an arbitrary z axis**

$(\varphi_1,\vartheta_1,-\varphi_1)$ **are calculated in the** 1 **rest frame with** $z' \| \vec{p}_1^{\,a-rf}$

$(\varphi_2,\vartheta_2,-\varphi_2)$ **are calculated in the** 2 **rest frame with** $z'' \| \vec{p}_2^{\,a-rf}$

The 1 rest frame is obtained by boosting the system to it and the primed coord. system. The latter is chosen with $z' \| \vec{p}_1^{\,a-rf}$ **so that if particle 1 has helicity** λ_1 **in the** a **rest frame, then it will have a spin-component in its own rest frame of** $M_1 = \lambda_1$ **along the z' axis.**
(ϑ_1,φ_1) **are the angles defined by the direction of 3 in the primed coordinate system.**

Therefore, by rewriting for our specific decay modes :

$$A(B^0 \to J/\psi K^* \to \mu\mu K\pi) = \sum_{\lambda_{J/\psi},\, \lambda_{K^*}} A(K^* \to K\pi)\cdot A(J/\psi \to \mu\mu)\cdot A(B^0 \to J/\psi\ K^*)$$

$$\propto \sum_{\lambda_{J/\psi},\, \lambda_{K^*}} D^{S_{K^*}\,*}_{-\lambda_{K^*},\,(\lambda_K-\lambda_\pi)}(\varphi_{K^*},\vartheta_{K^*},-\varphi_{K^*})\cdot A_{\lambda_K\lambda_\pi}\cdot$$

$$D^{S_{J/\psi}\,*}_{\lambda_{J/\psi},\,(\lambda_{\mu^+}-\lambda_{\mu^-})}(\varphi_{J/\psi},\vartheta_{J/\psi},-\varphi_{J/\psi})\cdot A_{\lambda_{\mu^+}\lambda_{\mu^-}}\cdot$$

$$D^{J_{B^0}\,*}_{M_{B^0},\,(\lambda_{J/\psi}-\lambda_{K^*})}(\varphi_{B^0},\vartheta_{B^0},-\varphi_{B^0})\cdot A_{\lambda_{J/\psi}\lambda_{K^*}}$$

Further considerations can be done (*B^0* meson has spin-zero,…; *K^** can have spin 0,1,2).
At the end …

… the following expression can be obtained

for the angle dependent part of the amplitude $A_{\lambda\xi}^{K*}(\Phi)$:

$$H_{\lambda}^{K*}\cdot D_{\lambda 0}^{J(K*)*}(\varphi_{K^+},\vartheta_{K*},0)\cdot D_{\lambda\xi}^{1*}(\varphi_{\mu^+},\vartheta_{\psi},0)=$$

$$=H_{\lambda}^{K*}\cdot e^{i\lambda\varphi_{K+}}\cdot d_{\lambda 0}^{J(K*)}(\vartheta_{K*})\cdot e^{i\lambda\varphi_{\mu+}}\cdot d_{\lambda\xi}^{1}(\vartheta_{\psi})=$$

$$\boxed{\varphi=\varphi_{K^+}+\varphi_{\mu^+}}$$

$$=H_{\lambda}^{K*}\cdot d_{\lambda 0}^{J(K*)}(\vartheta_{K*})\cdot e^{i\lambda\varphi}\cdot d_{\lambda\xi}^{1}(\vartheta_{\psi})$$

[this is the expression discussed at the beginning]

Note: the helicity amplitudes for the intermediate resonances *K** are complex parameters in the AA fit.

Tipically for the strongest resonance, the *K*(892)* among the *K** in this case, is taken as a reference; this implies taking :

- the absolute value fixed to 1
- the phase fixed to 0

Which is the AA fit implementation status ? Pilot decay mode is $B^0 \to J/\psi K^+\pi^-$

Started from an incomplete & undocumented version of the Belle's AA fit (in Fortran!).
Many pieces have been put in place (some examples in the next slides).

Essentially:

- **The AA UML fit scheleton is in place (ported from Belle)**
- **All the machinery for the signal resonances is in place**

purity (from data)

$$F = -2\sum_{data} \ln\left((1-b)\frac{S(\Phi)}{\sum_{MC} S(\Phi)} + b\frac{B(\Phi)}{\sum_{MC} B(\Phi)} \right)$$

- **The background parametrized shape is foreseen in the code implementation.**
  Right now putting an actual bkg shape obtaned by Adriano Di Florio on B^0 sidebands candidates.
- **The PHSP MC is still missing in order to compute the normalization of signal.**
  For the time being (first tests of the fit functionality) we will assume a flat unitary relative efficiency. For the bkg thinking to use already existing inclusive BtoJPsi MC samples to handle realistic sidebands.

For each K* resonance, we need to calculate the Wigner d-matrices for different λ

$$d^j_{m'm}(\beta) = (-1)^\lambda \binom{2j-k}{k+a}^{1/2} \binom{k+b}{b}^{-1/2} \left(\sin\frac{\beta}{2}\right)^a \left(\cos\frac{\beta}{2}\right)^b P_k^{(a,b)}(\cos\beta),$$

where $a, b \geq 0$.

$$k = \min(j+m,\, j-m,\, j+m',\, j-m').$$

$$\text{If} \quad k = \begin{cases} j+m: & a = m'-m; \quad \lambda = m'-m \\ j-m: & a = m-m'; \quad \lambda = 0 \\ j+m': & a = m-m'; \quad \lambda = 0 \\ j-m': & a = m'-m; \quad \lambda = m'-m \end{cases}$$

$$b = 2j - 2k - a,$$

$P_k^{(a,b)}(\cos\beta)$ = Jacobi Polynomials

```
//================ Jacobi Polynomial ================
//Jacobi polynomial - order n
double jacobi_Pn (int n, double a, double b, double x)
{

    if (n==0){
        return 1.0;
    }
    else if (n==1){
        return  0.5 * (a - b + (a + b + 2.0)*x);
    }
    else {

        double p0, p1, a1, a2, a3, a4, p2=0.0;
        int i;
        p0 = 1.0;
        p1 = 0.5 * (a - b + (a + b + 2)*x);

        for(i=1; i<n; ++i){
            a1 = 2.0*(i+1.0)*(i+a+b+1.0)*(2.0*i+a+b);
            a2 = (2.0*i+a+b+1.0)*(a*a-b*b);
            a3 = (2.0*i+a+b)*(2.0*i+a+b+1.0)*(2.0*i+a+b+2.0);
            a4 = 2.0*(i+a)*(i+b)*(2.0*i+a+b+2.0);
            p2 = 1.0/a1*( (a2 + a3*x)*p1 - a4*p0);

            p0 = p1;
            p1 = p2;
        }

        return p2;
    }

}
//================ Jacobi Polynomial ================
```

```
//================ factorial ========================
int Factorial(int x)
{
    if (x==0) { return 1; }
    return (x == 1 ? x : x * Factorial(x - 1));
}
//================ factorial ========================
```

```
//================ combination ======================
int Combination(int n, int r)
{
  return (Factorial(n)) / ((Factorial(n-r)) * Factorial(r));
}
//================ combination ======================
```

```
//================ wigner d calculations =============

double wigner_d (int j, int m1, int m2, double theta )
{
    int array[] = {j+m1, j-m1, j+m2, j-m2};
    int k = *min_element(array,array+4) ;
    int a = fabs(m1-m2);
    int lambda;
    if (k == j+m1) { lambda = 0;}
    else if (k == j-m1) { lambda = m1-m2;}
    else if (k == j+m2) { lambda = m1-m2;}
    else if (k == j-m2) { lambda = 0;}

    int b = 2*j-2*k-a;

    double value = pow(-1,lambda) * pow(Combination(2*j-k,k+a),0.5) * pow(Combination(k+b,b),-0.5) *
pow(sin(0.5*theta),a) * pow(cos(0.5*theta),b) * jacobi_Pn(k,a,b,cos(theta));

    return value;

}
//================ wigner d calculations =============
```

```
//================ Blatt-Weisskopf Form Factors ======
// l = relative angular momentum
// q = momentum from "dec2mm"
// q0 = momentum from "dec2mm" with PDG mass
// r = meson radial parameter (hadron scale)
double bwff(int l, double q, double q0, double r)
{
    double z = r*r*q*q;
    double z0 = r*r*q0*q0;
    double f;
    //########### spin 0 ###############
    if (l == 0) {
        f = 0;
    }
    //########### spin 1 ###############
    if (l == 1) {
        f = sqrt((1+z0)/(1+z));
    }
    //########### spin 2 ###############
    if (l == 2) {
        f = sqrt((z0*z0+3.0*z0+9.0)/(z*z+3.0*z+9.0));
    }
    //########### spin 3 ###############
    if (l == 3) {
        f = sqrt((z0*z0*z0+6.0*z0*z0+45.0*z0+225.0)/(z*z*z+6.0*z*z+45.0*z+225.0));
    }

    return f;
}
//================ Blatt-Weisskopf Form Factors ======
```

$$A^R(M_R^2) = \frac{F_B^{(L_B)} F_R^{(L_R)} (\frac{p_B}{m_B})^{L_B} (\frac{p_R}{M_R})^{L_R}}{m_R^2 - M_R^2 - i m_R \Gamma(M_R)},$$

```
//================ Breit-Wigner Amplitude ============
// m0 = resonance mass (pdg)
// w0 = width (pdg)
// m = invariant mass of two daughters of the resonance
// m_d1, m_d2 = daughter masses
// l = relative angular momentum
// f = BW form factor
// q = momentum from "dec2mm"
// q0 = momentum from "dec2mm" with PDG mass

complex<double> bwamp(double m0,double w0,double m,double m_d1,double m_d2,int l,double f,double q0,double q)
{
    double width = w0*pow((q/q0),2*l+1)*(m0/m)*f*f;
    double deno = (m0*m0 - m*m)*(m0*m0 - m*m) + m0*m0*width*width;
    double rl = f*pow((q/m),l)*(m0*m0 - m*m)/deno;
    double imag = f*pow((q/m),l)*m0*width/deno;
    complex<double> val(rl,imag);

    return val;
}

//================ Breit-Wigner Amplitude ============
```

Strategy and infrastructure for testing the AA UML fit

- **Test strategy is to start with a naïve but realistic model feeding the generated-level simulated events. We have produced privately either $B^0 \to J/\psi K^*(892)$ & 3-body PHSP decay (NR contribution) and we will mix the events in a settled proportion.**
  The aim is to establish functionality.
  After that we will add either a scalar or/and a tensor K*(1430) in given proportion; again purpose is to obtain back by the fit the features fed into it (validation).

- **Test infrastrucure is already in place. The variables needed have been identified (*). They are put into a root-uple after applying the cuts and the AA fitter takes it as input. The analyzer providing variables for a specific mixture of simulated events (& applying the selection obtained offline) is ready.**

(*)

1. Measured B0 mass from event
2. Measured KPi mass from event
3. Measured J/Psi (MuMu) mass from event
4. Measured J/PsiPi mass from event
5. B0 3-momentum (modulus)
6. KPi 3-momentum
7. Jpsi 3-momentum
8. K 3-momentum
9. Pi 3-momentum
10. Theta K* (K* helicity angle)
11. Theta J/psi
12. Phi (angle between K* and J/ψ decay planes)

STATUS & OUTLOOK

- Signal extraction with purities interesting to allow performing an AA fit is on the way.
  Further enhancement can be reached by applying a BDT method.
  First attempts look promising.

- For a proper efficiency description the official PHSP MC productions need to be carried out (they are stuck since 3 months already!)

- Lot of work has been done to implement an AA UML fit.
  Functionality testing & validation are starting very soon.